Grammatical structure and mathematical activity: comparing examination questions

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The project "The Evolution of the Discourse of School Mathematics through the Lens of GCSE examinations" is studying the ways in which the mathematical activity expected of students has changed over the last few decades by analysing the discourse of examination papers, using linguistic tools. In this paper we present one aspect of this analysis, comparing the grammatical complexity of sentences in questions from 1987 and 2011. We discuss the implications of differences in grammatical structure for the nature of the mathematical activity demanded of students.

Keywords: examinations, discourse analysis, readability, grammatical complexity

Introduction

Over the last decades there has been on-going public and academic concern about the nature and standards of school mathematics. This concern has driven frequent revisions of curriculum and examinations, yet controversy continues and there are contradictory opinions about the effects of reforms. The project "*The Evolution of the Discourse of School Mathematics through the Lens of GCSE examinations*"[3] aims to investigate changes in school mathematics in England, asking what has changed since the introduction of the GCSE (General Certificate of Secondary Education) examination in the mathematics that pupils are expected to learn and in the way they are expected to approach mathematics. We take national examinations at 16+ to be our 'window' onto the evolution of mathematics discourse in English schools. The existence of an intimate relationship between high stakes examinations and curriculum and pedagogy has been well established (e.g., Broadfoot 1996) and has been an explicit focus of debate about the design of assessment tasks for school mathematics (e.g., Bell, Burkhardt, and Swan 1992). Therefore, although the discourse of examinations has distinct characteristics, we see changes in examinations as a good index of changes in school mathematics. High-stakes examinations such as GCSE play an important role in the mathematics students experience, influencing the content of teaching, the ways tasks are defined and the kinds of student responses that are valued.

Rather than comparing syllabi or teaching methods, we seek to probe deeply into the nature of the mathematical activity construed by examination texts and expected of students by developing and applying a discourse analytic approach, drawing on Social Semiotics (Bezemer and Kress 2009; Halliday 1978; Hodge and Kress 1988; Morgan 2006) and Sfard's theory of mathematical thinking as communicating (Sfard 2008). Studying discourse in this way allows a subtle characterisation of the nature of mathematics and of student mathematical activity constructed through the forms of language used in examination papers. We argue that

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the analysis of change produced by this approach will provide insight into how changes in curriculum and assessment may affect students' mathematical learning. In this paper, we present one small part of our developing analytic framework, focussing on the issue of grammatical complexity. We will discuss some examples taken from examination papers from different years, considering how differences in grammatical complexity may affect the nature of students' mathematical activity.

Language, Mathematics and Assessment

Our theoretical perspective on the relationship between language and mathematics sees difference in the form of language to be associated with different construal of the nature of mathematics and mathematical activity (Morgan 2006; Sfard 2008; Schleppegrell 2007). However, within the literature on assessment, concern has generally been with the effects of language on the difficulty of tasks. For example, studies of examination questions have identified factors such as the structure of the question (Pollitt et al. 1998), use made of diagrams, technical notation and language, the number of steps required and the demand for recall of knowledge or strategies (Fisher-Hoch, Hughes, and Bramley 1997) to affect the difficulty of questions. Shorrocks-Taylor and Hargreaves (1999) summarise the findings of research into the syntactic aspects of mathematical text that may make reading more difficult. One of the issues arising from this review may be characterised as grammatical complexity, including large numbers of subordinate clauses and the common use in mathematical text of nominal clauses as the subject of sentences.

In recent decades, examination boards and test designers have sought to reduce difficulties seen to arise from the language of examination questions in an attempt to construct instruments that provide measures of mathematical knowledge and skills that are not invalidated by student difficulties in reading the questions. We too are interested in the validity of examinations, but rather than conceptualising this as involving some "pure" mathematical meaning that may be in danger of being obscured by complex language, we are concerned with how simplification of syntax or other changes to the language of a question may alter the nature of the mathematical activity demanded of students.

Measuring grammatical complexity

Our intention in this paper is to investigate how grammatical complexity may have changed over the period of the GCSE examination and to consider how any such variation might affect the nature of mathematics involved in the examinations. It is thus necessary to characterise grammatical complexity. We choose to do this in a way that allows us to take account of the potential reading difficulty caused by subordinate clauses identified by Shorrock-Taylor and Hargreaves (1999). In the present context, therefore, the focal property of language is its *recursivity*, that is, the fact that a unit of language such as a clause, phrase or word can be decomposed into similar elements which, in turn, can be decomposed according to similar principles, and so on. Guided by ideas drawn from systemic functional linguistics (Halliday 1985), we focus our analysis on the fundamental functional components of the sentence: *Participant(s)*, *Process*, *Circumstance* and *Value*. Interpreting these functional components, we consider *Participants* to be the objects, whether mathematical or 'everyday' that are significant in the mathematical activity presented in the examination paper. *Processes* are the actions, *Circumstances* contextualise the activity and *Values* are the qualities assigned to objects. In this paper, we have space only to present analysis of the complexity of the *Participants*. (It is, of course, also possible to identify components within subordinate clauses as well as at sentence level but we have not pursued the analysis to this level.)

We have adopted an operational definition that uses the recursive depth of a component as a measure of complexity. Here, recursive depth is the maximum number of decompositions that can be performed. Decomposition is possible when a unit of language (clause, phrase, word) contains a unit of the same or higher rank. Halliday identifies a strict hierarchical scale of "rank" of grammatical units: "A sentence consists of clauses, which consist of groups (or phrases), which consist of words, which consist of morphemes." (1985 p.25) For example, the phrase "in the chart drawn from this data" contains a higher ranked unit, the clause "drawn from this data". This decomposition is represented using brackets: [in the chart [drawn from this data]].

The example in Table 1 shows the parsing of a *Participant* in one sentence from an examination question posed in 1987: the nominal phrase *the values of x where y has a maximum or minimum value*. The complexity, or recursive depth (in this case 3), is given by the number of rows needed to complete the decomposition.

Table 1: Example of recursive decomposition

the values of x where y has a maximum or minimum value				
the values of x		where y has a maximum or minimum value		
The values	of x	(where y has)	maximum or minimum	value

Note that, while the clause *where y has a maximum or minimum value* is not recursive, the nominal group *maximum or minimum value* can be decomposed recursively in that it contains a phrase (i.e. a constituent of the same rank) *maximum or minimum*. In the rest of this paper, to save space, we represent the decomposition using square brackets rather than a table:

[the values [of x] [where y has a [maximum or minimum] value]]]

Sampling strategy

In order to develop and test our analytic tools at this initial stage of the project, we have worked with a small sample, including the higher tier papers from one examination board for two years: 1987 (the joint 'O' level/CSE syllabus in the final pre-GCSE year) and 2011. Although the formal composition and names of examination boards have changed several times over the period included in this study, there has been a high degree of continuity in the regional, institutional and personnel composition, allowing us to consider them to be "the same" over time. Within each paper, the questions were divided into sentences. Using the software myWordCount (http://www.mywritertools.com/Products_wordcount.asp), the number of words per sentence was counted. Assuming that longer sentences tend to be more complex, we aimed to analyse the longest 20% in each paper. All sentences of the length at the 20th percentile were included, so in practice rather more than 20% of the sentences in each paper were selected for analysis.

Results

In this paper we have space to present and discuss only the results for the *Participant* components. The recursive depths of these components are shown in Table 2 below. Compared to 1987, the 2011 distribution is strongly skewed towards simpler components.

Table 2: Recursive depth of *Participant* components

Recursive depth	1987		2011		All papers	
1	15	23%	30	43%	45	33%
2-3	40	61%	33	47%	73	54%
4-6	11	18%	7	10%	18	13%
Total	66	100%	70	100%	136	100%

It is interesting to note that, among the most complex *Participants* in 2011, the majority can be characterised as 'everyday' objects, e.g.:

[the amount [of each ingredient [needed [to make 15 Flapjacks]]]]

[information [about the points [scored [by some students [in a spelling competition]]]]]

while the most complex *Participants* in 1987 include more specialised mathematical objects, e.g.:

[the graph [of the curve [y=5+3x-x^2 [for -2≤x≤5]]]]

[the volume [of material [required [to construct a pipe [of length [one metre]] [having this cross-section]]]]]

Compared to these 1987 examples of depths 4 and 5, the most complex specialised mathematical objects *Participants* in 2011 are of depth 3, e.g.:

[the expression [which is a factor [of $4n^2 - 1$]]]

[points [on the circumference [of a circle]]]

The reduction of complexity has thus affected the mathematical objects while everyday components retain complexity.

In 1987, there are several cases of *Participants* involving material objects to be measured to a given degree of accuracy. The recursive structure of these can be both deep and broad (see example below). In contrast, mensuration tasks in 2011 either do not specify the unit to be used or indicate it next to the answer space. Where the degree of accuracy is specified, this is done in a separate sentence. A task that might be presented in 1987 as:

> Calculate [the area [in cm^2 [to 2 decimal places]] [of the shaded region]].

would be likely in 2011 to be presented as:

> Calculate [the area [of the shaded region]].
>
> Give [your answer [correct to 2 decimal places]].
>
> ………………… cm^2

In the first case, the unit and the degree of accuracy are construed as properties of the area itself, whereas in the second case they are properties of the answer. This difference in the construal of the object "area" affects the nature of the activity of calculation. In the first case, this activity includes considering the unit and approximating as well as carrying out the necessary operations; in the second, calculation and approximation are separate activities. Moreover, there seems to be no expectation that the student should consider the unit as this is present in the framing of the answer rather than as part of the task.

Discussion

The move from long, relatively complex sentences to short simple sentences clearly improves readability according to general readability measures and is consistent with the aim of the examination boards to reduce difficulties caused by language factors. However, our analysis raises some questions about the mathematical consequences.

The use of nominal groups packed with information is typical of scientific and mathematical texts (Halliday and Martin 1993). In non-specialised language, the same quantity of information might be given in several separate sentences. For example, the single instruction (given in 1987), containing a nominal group of depth 6:

> Calculate [the volume [of material [required [to construct a pipe [of length [one metre]] [having this cross-section]]]]].

might in 2011 be given as:

> This is [the cross-section [of a pipe]].
>
> The pipe is one metre long.
>
> What [volume [of material]] is required to make it?

Here, the most complex *Participant* is of depth 2. While complex nominal groups may make reading more challenging, they are not an arbitrary characteristic of specialised text but allow the formation of precisely defined objects which can act as *Participants* in further processes. For example:

> Compare {the volume of material required to construct a pipe of length *one* metre having *this* cross-section} with {the volume of material required to construct a pipe of length *two* metres having *a different* cross-section}.

The analysis of the complexity of the *Participant* components reveals that the changes between 1987 and 2011 may not only affect ease of reading but also the construal of the nature of mathematical objects and activity. The dense nominal groups in the 1987 papers incorporate the results of several mathematical processes as qualities of a single object. A consequence of this is that the (apparently simple) instruction to "calculate" in fact demands analysis of the structure of the object to be calculated and consideration of the form of the answer. In 2011 it seems that the processes of analysis, approximation and consideration of units are separated from calculation and that mathematical objects, being generally less complex, contain less potential for further mathematical activity.

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