

SOCIAL FOUNDATIONS OF THE
MATHEMATICS CURRICULUM:
A RATIONALE FOR CHANGE

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ABSTRACT

The nature of educational aims as criteria for worthwhile curriculum practice is explored and a cross-section of aims for mathematics education is discussed. An aim for mathematics education which emphasises the social aspect of the subject in its being, its conduct and its applications is identified and epistemological foundations for such a view of the nature of the subject are explored. It is argued that such an epistemological perspective of mathematics would be reflected in the social context of the mathematics classroom, arising from a methodology in which the subject would become more problematic and open to change, investigation and hypothesis.

The aims of two major mathematics curriculum development projects (the Nuffield Mathematics Project and the School Mathematics Project) are examined to determine the extent to which their aims may take the 'social' nature of mathematics into account. The probable social context of mathematics classrooms using their materials is postulated in an attempt to characterise the nature of the subject as it is reflected in these materials. A view of the nature of mathematics held by practising teachers and by pupils is then established by drawing upon, and extrapolating from, evidence relating to the social context of mathematics classrooms at primary and secondary level.

Conclusions follow, which suggest that fundamental change in mathematics education requires, as a first step, the adoption of a new epistemological perspective of the subject in order that the pursuit of the aim which emphasises the social nature of mathematics is achieved. It is suggested that this, in turn, ultimately could lead to the desired balance in the mathematics curriculum which hitherto has been lacking.

PREFACE

The search for the evidence in Chapters 4 and 5 of this study was undertaken, in the first instance, for the Cockcroft Committee of Inquiry into the Teaching of Mathematics. It will appear in part and in altered form in the final report of that Committee.

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INTRODUCTION

Change in mathematics education: A statement
of the problem

Developments in the teaching and learning of mathematics in the recent past have entailed a dramatic reappraisal of the mathematics taught in schools, to pupils of all ages. These changes have proved to be definitive, but, at the same time, controversial, and have resulted in concern about the quality of mathematics education at all levels in schools. This concern has culminated in the setting up by the government in 1978 of a Committee of Inquiry under the chairmanship of Dr W.H.Cockcroft, the brief of which is to consider the teaching of mathematics in schools and to make recommendations. The constituting of such a Committee provides clear evidence of the measure of concern about mathematics education in this country and the need to identify problem areas in which we are not succeeding and, if possible, to determine why we are not succeeding. This study will be concerned with some aspects of mathematics education which the author believes to be of fundamental importance to the solution of these wider problems.

A view of the problem

Curriculum development in mathematics has made heavy demands upon all those involved in mathematics education including not only teachers and pupils, but parents and employers as well. However, it is clear that past efforts have not achieved a "coherently conceived response to all the various pressures on the curriculum", as Howson (1979) points out. (p.136) The reasons why more positive success has not been achieved are doubtless many, but Otte (1979) identifies what is probably the most basic of these when he states:

"The hitherto insufficient elaboration and inaccurate identification of problems, the rampant catchwords, the hasty syntheses and judgements - these are all related to the general lack of clarity and stability of the goals of mathematics teaching and mathematics teacher education." (p.128)

Without clarity and general agreement about what it is we hold as aims for mathematics education, it seems likely that further development and future practice will continue to flounder. To reach such a consensus, however, it is also necessary to agree upon what mathematics *is*; that is, we must be clear about epistemological questions concerning the nature of the subject, how it comes into being and how it grows. Any attempt to define the aims of mathematics education without taking such matters into account is clearly to limit our view of what the potential of the subject is, how we can relate it to our environment, how we can use it to control that environment and how it can enrich our lives.

The contention of this study is that some of the fundamental re-thinking with respect to epistemological matters that has evolved over recent years and the relation of such thinking to mathematics as a discipline, need to be taken into account in the derivation of aims for mathematics education. Our particular concern is to show how different epistemologies are reflected in how mathematics is done in the classroom and, more specifically, in the effect they can exert on the social context in which the teaching and learning of mathematics takes place. It will be argued that the social nature of mathematics has been neglected and that this, in turn, has led to a perception of the subject that has imposed an inflexible, rigid pedagogy in the mathematics curriculum which has detracted from both teachers' and pupils' appreciation of the subject and possibly their success in it.

An outline of approach

The approach to the study will be through a consideration of goals in education and their role in the provision of criteria for worthwhile practice in the educational context. Aims for mathematics education from a variety of sources will then be critically examined with a view to producing a set of aims, the balance and acceptability of which will be argued. Included in these aims will be the notion of mathematics as a social activity. We shall then consider contrasting epistemological approaches to mathematics to determine the extent to which a view of mathematics seen in terms of a social activity may theoretically be substantiated. Following on this, in order to gain some idea of the aims of

mathematics education in practice, the aims of two curriculum development projects (the Nuffield Mathematics Project and the School Mathematics Project) will be examined and compared with the set of aims derived in the previous chapter. These influential projects have been selected for the purpose of illustrating the disparity that can exist with respect to the goals of mathematics education, how this reflects different views of the nature of mathematics and how it results in considerably different curricular outcomes in the classroom. The degree to which each project has been informed by theoretical considerations will also be examined.

In the latter part of the study, we shall be concerned with the presentation of such evidence as exists that provides some insight into the social context in which the teaching and learning of mathematics takes place at primary and at secondary level. This will be done in order to gain some notion of how mathematics would appear to be perceived in classrooms today and how these perceptions may affect the implementation of the mathematics curriculum. The issues involved will be of a socio-psychological nature and will entail an examination firstly, at primary level and then at secondary level, of institutional factors that may affect the mathematics curriculum, a consideration of teachers in the context of the mathematics classroom and a consideration of pupils in a similar context. The evidence from these areas of consideration will be drawn together to provide a composite picture of the social context in which primary and secondary pupils learn mathematics. We shall consider the view of mathematics that emerges and that is reflected in these results. Conclusions will follow.

Critical comment

In undertaking a study of this kind, it is particularly important to be alert to the nature of the exercise in order not to be drawn into the pitfalls of subjectivism. We are concerned here with what is essentially an evaluative exercise of a qualitative kind relating to mathematics education and approached through a consideration of relevant aims. Tawney (1973) writes that different approaches to evaluation can be compared by considering "the extent of what is being questioned". (p.5) There can be few broader bases from which to approach the evaluation of any educational or curricular problem than through a

consideration of aims, and maintaining an appropriate degree of objectivity can prove difficult in such circumstances. However, as Harlen (1973) points out:

"The uncommitted evaluator is faced with the problem of communication: if he tries to remain impartial and objective, he may end up also ignorant of many things which he should understand to do his job properly." (p.19)

Clearly, there is a balance to be sought between commitment and an appropriate degree of objectivity and in this study, Popper's (1972) interpretation of objectivity will be adopted and, hopefully, will guide us in attempting to achieve this balance. Objectivity, in his view, is conferred upon knowledge by its being made public and open to criticism. The process is one of building upon the critical thought of others and, in turn, presenting the results of the application of criticism in a form accessible to others for similar critical appraisal and examination. Commenting on Popper's approach, McNamara (1978) suggests that this view of objectivity "does not imply lack of interest or lack of passion, but it does imply a preparedness to be critical and to subject all tentative solutions to sceptical analysis." (pp.35-6) It is with this critical awareness and employing this analytic approach that we shall proceed to examine our problem.

CHAPTER 1

Aims in Mathematics EducationIntroduction

Our purpose in approaching the mathematics curriculum through the concept of aims is to explore the way in which aims may be considered to act as criteria for worthwhile practice in education, and ultimately to identify such criteria for mathematics education. In this chapter, therefore, we shall first of all examine the notion of aims in an educational context. We shall then consider a range of aims for mathematics education held by theorists and by practising teachers, and draw comparisons between the two. Finally, we shall examine lists of aims for mathematics education from two major sources to determine what consensus exists between the two and to gain some idea of the kind of balance that would be considered desirable for aims in mathematics education, particularly with respect to any 'social' criteria that may exist.

AIMS IN EDUCATION

One of the most important aspects of educational activity that characterises it as 'educational' is the fact that it is an intentional enterprise. (Oakeshott 1962, Hirst and Peters 1970) Whether these intentions are stated in terms of highly specific objectives (Whitfield 1972) or whether they are in the form of an hypothesis describing the likely outcomes of a learning situation devised according to a particular strategy (Stenhouse 1971), there is inherent in both approaches an intended outcome of a particular kind. Governing these outcomes are the educational aims to which we aspire in pursuing the more immediate tasks within the curriculum.

There has been much discussion about aims and objectives in the curriculum and the difference in nature between the two (e.g. Wheeler 1967, Pring 1971a). Hirst (1974) suggests that the difference lies in the degree of specificity demanded by the level at which matters are being discussed and "on the character of the issues at stake", a demarcation similar to Popper's (1972) notion of the difference between situations that demand clarity as opposed to precision. (p.16) Thus we may refer

to aims characterised in broadest terms as 'educational' when discussion is concerned with the intended outcomes of the educational process as a whole. An example of such an aim might be 'to develop autonomy in pupils'. The next level of discussion may be seen to be at the level of the curriculum where certain outcomes may be considered common to several subject areas, for example 'to develop observational skills'. At the level of a separate discipline such as mathematics, it may appear that the issues involved are specific enough to be referred to in terms of objectives rather than aims. However, at such a point it is the *character* of the issues being discussed and to which Hirst (1974) refers, that becomes the criterion in determining the use of the term aim or objective. While the subject of mathematics and the teaching of it may seem a very specific enterprise, there is much mathematical knowledge and practice from which to choose for inclusion in the curriculum, and there are reasons for choosing some aspects of that knowledge and practice and not others. It is at the point of making such choices that the aims of mathematics education demand attention. What is being considered is what it is thought mathematics education *should* set out to achieve and hence the issue involved becomes one of an ethical nature. Once having made judgements at this level with respect to intended outcomes of a long-term nature relating to the mathematics curriculum as a whole, then the more specific objectives leading to the attainment of our aims may follow. In considering educational aims, therefore, the ethical questions raised make it necessary and important to be clear about what we mean when we talk of aims in such a context.

The nature of aims in an educational context

Peters (1973) writes, "The very fact that education involves multiple criteria is perhaps one of the underlying reasons why statements of aim seem so necessary." (p.20) Because the intended outcomes of education lack a "determinate point" a statement of aims may help to focus attention "on some neglected priority". He further characterises an aim as conveying the notion of an objective that is not easily attainable and such that, whatever the activity required, no obvious structure is suggested in relation to its attainment. For example, to identify 'to develop basic numeracy in pupils' as an aim of mathematics education does not give any indication as to whether this means that pupils are simply to learn the four basic operations using arabic numerals or whether they are

also to develop skill and understanding in their application. The statement merely suggests the nature of an end in view in relation to some content but not a highly specific end, nor a means for its achievement. On the other hand, the lack of identification of such an aim would undoubtedly result in the neglect of a priority in mathematics education.

Peters (1973) also draws attention to the importance of being wary of accepting such notions as 'the self-realization of the individual' as educational aims. Statements of so-called aims couched in such terms do not take into account the educational context in which they are set. They do not "prescribe any specific direction or content" which provide the guiding principles that qualify a pursuit as 'educational'. Their vagueness, indeed, would seem to suggest an individual living in something of a vacuum with no shared social context, whereas Peters (1973) points out that such individual development can only take place "within the framework of some socially structured pursuit" into which a person can be initiated. (p.24) This is to echo Dewey (1916) who suggests that "When it is said that education is development, everything depends on *how* that development is conceived." (p.59 author's italics) Rather than being educational aims, Peters (1973) suggests that phrases such as 'the self-realization of the individual' refer to kinds of procedure that take place within an educational context and are not, in themselves, aims. He emphasises that although aims cannot be separated from procedures, it is not appropriate to view procedures in themselves as aims. This would be to accept means in themselves as an end in the curricular context whereas it is generally held that means and ends in such a context are contingent upon each other (e.g. Pring 1971b, Sockett 1974). It is impossible to separate the ends from the means because there are important principles implicit in both. There are principles of a moral nature, for example, involved in the grouping of pupils in a mathematics learning situation where certain resources may have to be shared within a group and individuals come to learn to respect the rights of others in this sharing. At the same time, there are principles of an intellectual nature implicit in the content of mathematical learning such as the ability to identify the features of a mathematical problem relevant to the structuring of a solution. These roles may be reversed so that the means or methodology may carry the intellectual message

while the content conveys something of an ethical nature. Thus method and content become inseparable when seen in terms of what we set out to achieve and in order for aims to perform their function of providing a focal point for educational activity, they should be stated in such a way as to provide direction for that activity and at the same time, take into account the social context in which it is set.

The ethical aspect of educational aims

The conceptual analysis of Hirst and Peters (1970) with respect to educational aims helps further to clarify their ethical nature. In the process of the identification and selection of such aims, we are judging what we consider to be worth while pursuing as educational goals. The view of education the authors put forward is expressed in terms of a commitment to "processes which assist the development of desirable states in a person involving knowledge and understanding". (p.40) However the question then arises, "how do we determine which states are desirable?". If one accepts this view of what education is about and that the school curriculum provides the institutionalised medium through which such developmental processes may occur, the issue is thus raised of the value judgements that are of necessity inherent in curriculum decision-making. Attention is drawn to the fact that in exercising choice, we are judging what we consider are desirable states to be developed and what are not, what knowledge they are to involve and how they are to be related to that knowledge. Referring again to the mathematical aim 'to develop numeracy in pupils', this aim may be selected because it is considered worthy of pursuit whereas the aim 'to develop numeracy in base five' may be rejected as an aim because it limits mathematical pursuit in what is thought to be a detrimental fashion. However, while conceptual analysis may make educators aware of the nature of the choices to be made, Hirst and Peters (1970) point out that "it cannot, of itself, provide answers to the ethical issues which it helps to make explicit". (p.40) In short, it cannot identify what aims are 'good' or 'bad', 'right' or 'wrong'. The onus for such judgements, then, is placed upon those who devise and implement the curriculum, be they curriculum developers or teachers. As Dearden (1968) suggests, teachers' views of what educational consequences are desirable constitute their views of what "the *aims* of education ought to be" and there is no system of education in which teachers can "escape responsibility for the

direction which things take." (p.13 author's italics) Aims in the educational context, therefore, reflect the values of the persons who determine them and implicit in those aims must lie the criteria for what is considered worthy of pursuit and achievement. This is stressed by Hirst and Peters (1970) when they state that "It is essential for a teacher to try to get a bit clearer about his aims; for unless he does this he will not have criteria by reference to which he can determine satisfactorily the content and methods of his teaching." (p.28)

AIMS IN MATHEMATICS EDUCATION

The foregoing discussion of the concept of aims in the curriculum is helpful in clarifying their relevance and importance in the context of mathematics education. By highlighting their function of providing guidelines for practice and establishing value criteria in so doing, the necessity for being clear about aims in mathematics education is emphasised. Without the identification of what is considered worthy of pursuit in the mathematics curriculum, there would be no criteria by which to judge the value of our specific intentions in what we choose to do in the day-to-day teaching of the subject. Aims thus formulated should provide answers to questions which ask why particular content is adopted and why it is taught in a particular way. If some aspect of mathematics curricular practice were not related to the attainment of at least one or other of the aims identified, then such a procedure could be said to be redundant or, indeed, obstructive insofar as it does not contribute to the satisfaction of any of the criteria of 'worthwhileness' manifested in those aims. If one were, for example, daily to pursue the practice of setting pupils exercises in computation in base five alone, clearly this would be obstructive to the attainment of the aim 'to develop numeracy' since it would be to present a very narrowly conceived view of number. This raises, however, the question of the degree to which certain practices may be undertaken and lead, to some extent at least, to the achievement of one or other aim. Conceivably, *some* practice in base five could result in the development in pupils of a broader understanding of number and mathematics as a whole which may be considered a desirable aim. This is essentially a question of balance in aims for mathematics education and in the attainment of them and one which will be considered later in this chapter. However, the ethical nature of educational aims as statements of intent that

embody what is considered worth while pursuing has been established. To this extent, the aims of mathematics education may be said to have implicit in them criteria for 'good' practice, in prescribing what mathematics education should set out to achieve. Some views on what these aims should be will now be considered.

A diversity of views of aims in mathematics education

The immediate problem that arises in a discussion of the aims of mathematics education is the wealth of sources available upon which to draw. Sometimes aims are not clearly defined but are imbedded within a general statement of a pedagogical or philosophical stance with respect to the teaching and learning of the subject. One such example is given by the Association of Teachers of Mathematics (1977) in the introduction to their publication "Notes on Mathematics for Children". In a more specific approach, eight aims listed by Perry in 1901 and quoted in a Ministry of Education Pamphlet (HMSO 1958) is examined and placed in a modern perspective, going so far as to refer to pupils and not to boys as in the original! (Howard, Farmer and Blackman, 1968). Arguments concerning the aims of mathematics education have also been expounded as a direct result of the introduction of 'new' mathematics into curricula. (Kline 1973) The plethora of sources of which these are but a few, necessitates selecting views of aims for discussion that appear to be representative of a cross-section of opinion. Therefore, interpretations of some educationists as well as those of practising teachers will be examined. Recent developments in mathematics curricula suggest that it could be of value to this exercise also to include a 'classical' view of the aims of mathematics education in order to discover what changes, if any, may have occurred with respect to those currently held as against those held in the past. For this purpose, the aims identified by Whitehead (1932) in his chapter dealing with the mathematics curriculum in "The Aims of Education" will be examined.

A classical view of mathematical aims

In his discussion of the mathematics curriculum, Whitehead (1932) uses the word 'recondite' to describe the discipline, a word which sums up what is perhaps the popular view of the subject as being obscure and abstruse. He elaborates on his use of 'recondite' when he states that

"By this word I do not mean difficulty, but that the ideas involved are of highly special application, and rarely influence thought." (p.117) He then proceeds to argue that "mathematics, if it is to be used in general education, must be subjected to a rigorous process of selection and adaptation." (p.119) The overall purpose of such a process would be to eliminate the reconditeness of the subject as taught in the curriculum and thus limit the general goals to be achieved in the light of the reality imposed by the intellectual capacities of the pupils for whom the curriculum is intended. The point is made that not all pupils intend to study, nor indeed are capable of studying, mathematics at a higher level after they have left school. With this in mind, the aims of mathematics education Whitehead (1932) identifies are: (1) to introduce pupils to abstract thought; (2) to illustrate the general use of mathematics by means of having pupils do practical examples and (3) to train pupils in logical method. He suggests that adaptation to the differences in the intellectual capacity of pupils be achieved by stressing the selection of content based upon considerations of those aspects of the subject that possess the greatest potential for generalisability. Of particular interest (for reasons that will be shown) is Whitehead's (1932) rejection of the learning and doing of mathematics for its own sake as a general aim of mathematics education for this, in his opinion, is where reconditeness enters the curricular scene. He suggests that the reasons which make the subject "a delight to its students" are, at the same time, the "reasons which obstruct its use as an educational instrument". (p.118) In other words, the more pleasurable aspects of the discipline are not of a nature to permit it to be enjoyed by all since they lie in the more highly abstract regions of the subject. As a result these very characteristics are, he states, only of value to those of "keenest intellects" and except for these few, "they are fatal in education". (p.119)

To appreciate the significance of Whitehead's (1932) rejection of the development of an intrinsic interest and of pleasure in the subject of mathematics as an aim for all pupils, it is necessary to view this rejection against a background of the aims of mathematics education held by others. While, as it will be seen, the three aims that are acceptable to Whitehead are still held to be important by many mathematics educators today, the acceptance or rejection of the aim of taking pleasure in the learning and doing of the subject is one which appears to be more

controversial.

Some contemporary views of aims of mathematics education

Freudenthal (1973) accepts that an aim of mathematics education should be the introduction of pupils to the world of abstract ideas. In his view, however, high importance must be placed upon what is taught and priorities carefully selected, emphasising, as does Whitehead (1932), that only a very few pupils are likely to become pure mathematicians. As a discipline of the mind and thus as an aid to the development of logical thought, he suggests that it is mathematical method rather than content which is important and that an aim should be to make mathematical method as explicit as possible in the teaching of the subject. Freudenthal (1973) examines applicability as an aim of mathematics education and states that while he does not "urge that the pupil learns applied mathematics, I do wish that he learns how to apply mathematics". (p.75) However, he insists that applicability should arise from "the lived-through reality of the learner" and that the connection should not be made in some artificial way for, after all, "Reality is the framework to which mathematics attaches itself." (p.77) He considers that the aim of studying mathematics for the pleasure of it is an hypocrisy "for even pleasure knows a scale of values" and he suggests that a more honest consideration is that pupils should study it because it is an important aspect "of their being as human beings". (p.68)

Kline (1973) agrees to some extent with Freudenthal (1973) on the latter point but in his more extreme view, Kline (1973) sees mathematics so linked with the real world that he believes it should be taught wholly in relation to other interests and not as a separate discipline. He considers that any intrinsic interest and enjoyment of the subject would be "by-products of the larger goal of showing what mathematics accomplishes". (p.147) He also argues that pupils are unlikely to be motivated by the intellectual challenge of the subject or by its "beauty" since neither quality is likely to be appreciated by "novitiates" in the course of their struggle in learning the subject.

Ormell (1972), like Kline (1973), sees the applicability of mathematics as the key to the aims of mathematics education but he differs

from him in his approach to what this entails. Ormell (1972) suggests how a new interpretation of applicability has specific implications for the general aims of mathematics education, the novelty of which lies in the main role of mathematics in society, as he sees it. This role is "to explore the predictable implications of 'possibilities'; i.e. suggestions, propositions, ...plans, ...theories" and the mathematical model is envisaged as the means for accomplishing this. (p.1) The model can act in a problem-solving or explanatory capacity and can lead to the discussion of the implications of other possibilities which may appear interesting. Because such activity occurs mainly in the realm of ideas, imagination is important and Ormell (1972) describes this as "the intuitive capacity to entertain mentally states of affairs which are not present in actuality." The development of such an "intuitive capacity" in relation to mental states could clearly be likened to the development of the ability to think in an abstract way, but also implied is a form or direction for that abstract thought. In Ormell's (1972) view, this shifts the emphasis from the abstractness of mathematics to "'integration' with other forms of knowledge' as the worthwhile goal". It is to play down the "logico-deductive method" of mathematics and to introduce in its place a new mathematical methodology in the form of mathematical model-making. Thus his view of the aims of mathematics education, dominated as it is by the applicability of the subject, also contains within it the notion of abstract thought embedded in imagination, and mathematical method in which the logico-deductive approach is subjugated to that of mathematical modelling. Ormell (1972) does not make specific statements with respect to the development of intrinsic pleasure in the subject as an aim of mathematics education but goes so far as to suggest that there is a demand for a new approach to "educate mathematically" the aim of which he sees as to "encourage the rising generation to think critically, skillfully and constructively in general terms (i.e. with mathematical models) about all aspects of the human condition." (p.10) This may be likened to the notion of developing an awareness of the relevance of mathematics through the reality of the pupils' world as put forward by Freudenthal (1973).

The perspective of practising teachers at primary level

The discussion of aims in relation to mathematics education thus far has been concerned with those of mathematics educators whose

interests lie largely in the realm of theory and outside of schools. It is necessary and desirable to include the views of practising teachers because of the importance of the effect a difference in perspective might have in characterising their views. Bearing in mind that what is being sought in particular here are aims that will help to identify criteria for good practice in mathematics education, the opinions of teachers as to what these might be are clearly highly relevant. It is important to note that the kind of evidence available relating to such a topic differs between the primary and secondary sectors. Teachers in the secondary sector are subject specialists and as such, have national professional bodies which represent and publish their views in reports dealing with individual subjects, while the primary sector does not. Thus evidence of the opinions held by primary teachers with respect to mathematical aims must be sought in research and further relevant evidence may be found in publications of Her Majesty's Inspectorate.

A major study investigating teachers' opinions of the aims of primary education was carried out by Ashton et al. (1975) involving 1513 primary teachers. While the study concerned the teachers' beliefs with respect to the whole curriculum, it is possible to extract evidence relating to their views of the aims of mathematics education from the data reported. In the course of the investigation, seventy-two aims for primary education were identified by the teachers themselves and then ranked according to their perceived importance. Of the 72 aims, only three were related specifically to mathematics but since each of these comprises a fairly lengthy and comprehensive statement, they are reported here in full and are as follows:

"18. The child should know how to compute in the four arithmetic rules using his knowledge of, for instance, number, multiplication tables and different units of measurement.

19. The child should know how to think and solve problems mathematically using the appropriate basic concepts of, for example, the number system and place value, shape, spatial relationships, sets, symmetry and the appropriate language.

20. The child should know how to use mathematical techniques in his everyday life; for instance, estimating distances, classifying objects, using money." (Ashton et al. 1975, p.240)

For purposes of brevity, the investigators referred to the first of the above aims as "arithmetic - the four rules", the second as "modern maths" and the third as "everyday maths". (p.58) Of the three, know-

ledge of everyday maths was ranked fifteenth in importance, the four rules of arithmetic came twentieth and modern maths was thirty-fourth. Of all the curricular aims considered, the one to achieve first place was that all pupils should be "happy, cheerful and well-balanced" and at fifth place came the aim that pupils should experience "enjoyment in school work".

The information provided by the data raises many interesting questions concerning the relative importance placed upon mathematics with respect to the curriculum as a whole but our concern here must be with mathematical aims and those that relate to them. For instance, there appears to be some lack of logic in giving precedence to the aim of achieving a knowledge of 'everyday maths' over that of learning arithmetic since little of the former can be gained without first being aware of the relevant skills and concepts to be applied that are encompassed in the latter. Another disturbing feature is that in classifying all 72 aims according to "knowledge", "skills" and "qualities", all of the mathematical aims are placed in the category of skills. (p.240) This is an anomalous situation which suggests a somewhat distorted view of the discipline of mathematics in not ascribing to it a cognitive element. Referring to the curriculum as a whole, it is clear that the primary teachers in this study placed great importance on the happiness of their pupils and the fact that they should enjoy their work.

An investigation by Bishop and McIntyre (1969) also helps to throw some light upon primary teachers' opinions with respect to the aims of mathematics education. In a questionnaire completed by 245 primary teachers, each was asked to place the following goals in mathematics in order of strength of emphasis, on a five-point scale: "'its application to everyday life'; 'as a foundation for more advanced mathematics'; 'as an enjoyable and satisfying activity'; 'as a foundation for scientific study' and 'for training children to think logically'." (p.34-5) Of the five goals, the application of mathematics to everyday life was rated as most important and the authors state that "Next in importance, for most teachers, was either the enjoyment and satisfaction to be gained from doing mathematics or its value in training children to think logically. Laying foundations for more advanced maths and for scientific study came a poor fourth and fifth respectively."

In both studies, comparisons were drawn between the priorities placed on the aims of mathematics education by the sample of primary teachers and a sample of secondary teachers. Ashton et al. (1975) found that of a sample of 459 secondary teachers, the priority was placed on the learning of the four rules of arithmetic at tenth place, followed by a knowledge of everyday mathematics ranked eighteenth and modern mathematics in thirty-fifth place. Thus they reverse the order of the first two priorities selected by primary teachers and, as a result, the emphasis on aims seems more logically placed. The happiness of pupils and enjoyment to be taken in their work came third and fourth respectively. This follows the pattern of the primary teachers very closely with somewhat less emphasis on the pupils' happiness and slightly more on taking pleasure in their work. In the study by Bishop and McIntyre (1969), it was found that a sample of 131 secondary teachers attached "a significantly greater importance to mathematics as an enjoyable activity, their most common first choice" with "negligible differences" between emphasis on the other goals apart from placing little importance on mathematics as a preparation for scientific study. (p.35)

A secondary level perspective of aims of mathematics education

The Assistant Masters' Association (1973) presents a fairly detailed discussion of the aims of mathematics education within the secondary sector in their report on the teaching of mathematics. To begin with, the fact that the practical use of the subject is most often considered the main aim for its study is noted and condemned. It is stated that this view "derives from a misunderstanding of what mathematics is" and that "the sooner educationists and the public at large realize that mathematics is not simply numerical work and algebraic manipulation, the better will be the understanding of the place of mathematics in the school curriculum". (p.2) It is further suggested that mathematics "stands alongside the study of our language as a basic intellectual skill". With this in mind, the A.M.A. (1973) report goes on to place high importance on "pleasure, enjoyment and intellectual stimulation" as benefits it is hoped would be derived from the study of mathematics, the supporting argument being that pupils will not get very far with the subject unless they enjoy it. Thus emphasis is given to the aim of studying the subject for its own sake and suggestions are made as to the kind of mathematical content it is considered could lead to the enjoyment

of the subject by pupils (for example, the study of pattern). The difference in intellectual capacity of pupils is acknowledged when it is suggested that for more able pupils, the recognition of pattern may supply them with the key to the solution of a problem, while for the less able it may simply act as an aid in ordering their environment. The report then alludes to the aim of studying mathematics for its usefulness and, finally, to the aim of the development of logical thought since "it remains true that mathematics provides the study *par excellence* in which the nature of inductive and deductive thinking can be examined and understood". (p.3, author's italics) Indeed, the claim is made that the "analytical approach" inherent in doing mathematics is especially necessary in the curriculum today because of the stress on value judgements, opinions and attitudes in other integrated "'soft' subjects". At the same time as rejecting a belief in the transfer of training from subject to subject, it is suggested that only by employing analysis such as is taught within mathematics can "value judgements be detected, attitudes examined for their value in terms of possible fruitful results and opinions critically assessed". Couched in these terms, it may at first appear that the A.M.A. (1973) may be proposing the study of mathematics as a panacea for problems related to all learning. However, a more reasonable interpretation may be that mathematics is seen as providing the key to an approach to problem-solving in other curriculum areas as suggested by Ormell (1972), although the approach advocated by the A.M.A. is the logico-deductive approach rejected by Ormell (1972).

A comparison of the aims of theorists and practitioners

It is clear from the preceding discussion that differences of opinion concerning the aims of mathematics education exist amongst theorists and between them and practising teachers. The evidence suggests that the differences lie not so much with the aims identified as with where the emphasis should be placed amongst them. For example, while Whitehead (1932) suggests that the usefulness of mathematics be illustrated through the repetition of practical examples by pupils, Freudenthal (1973) insists that its applicability should not be learned in an artificial manner but in such a way as to make it real for the learner. Kline (1973) takes this even further in suggesting that the subject should be taught only in conjunction with other sub-

jects where it is useful, and not as a separate discipline. Again, Whitehead's aim of developing logical thought through the teaching of mathematics stresses one aspect of methodology while Ormell (1972) prefers to stress the notion of the mathematical model and to play down the logical, deductive aspect of mathematical method. However, what does stand out particularly strongly amongst the theorists is the fact that they tend either to reject strongly or not to be overly concerned with the aim of studying mathematics for its intrinsic value and the pleasure it may give. In the case of Whitehead (1932) and Freudenthal (1973), both stress the necessity for selectivity with respect to what is taught in the belief that only a very limited number of pupils are capable of appreciating, and hence of taking pleasure in, the beauty of mathematics which tends, they suggest, to lie in the higher levels of abstraction within it. For Kline (1973), the pupils' pleasure in doing mathematics would seem a questionable matter while for Ormell (1972) it would appear to be incidental.

This is a point of view which appears to be very clearly rejected by practising teachers. The evidence presented indicates that the pupils' pleasure in the subject is of great concern to teachers at secondary level and the studies of primary teachers' aims suggest that they also place high priority on pupils' enjoyment of their work. The difference in attitude in this respect between theorists and practitioners is so marked as to suggest a manifestation of the ever-present dichotomy between theory and practice in education. One can only question why it should be so, and whether it may be peculiar to mathematics.

At the practical level, it may be possible that teachers are indicating a belief that only pupils who are enjoying what they are doing, will work and learn. This has become a basic tenet of the child-centred movement at primary level and an emphasis on 'discovery' learning arising from interpretations of Piaget's work (e.g. Inhelder and Piaget 1958). As stated in the Plowden Report, "Piaget's observations support the belief that children have a natural urge to explore and discover, that they find pleasure in satisfying it and that it is therefore self-perpetuating." (D.E.S. 1967, p.195) Such statements are easily interpreted to suggest that if children are not enjoying what they are doing, they may not be learning in a meaningful way. The fact that secondary

teachers appear to rate enjoyment in doing mathematics as the most important aim at primary level may better be understood in the light of Bell's (1979) statement that "For adequate understanding of mathematics and good feelings about it, excellent and rich instruction must begin in the years before high school, and probably in the primary school grades." (p.312) However, the task set in maintaining such an attitude towards the subject at secondary level becomes difficult because the reconditeness to which Whitehead (1932) refers is more likely to intrude. His advocacy of selectivity with respect to content and that of Freudenthal's (1973), is a process again referred to by Bell (1979) when he suggests that an aim of reforms in mathematics education has been the "rooting out (of) bad mathematics and obsolete topics from school-books and making appropriate mathematical structures the basis for teaching at all school levels." (p.311) With less 'reconditeness' and more 'appropriate mathematical structures' could come more enjoyment on the part of pupils. It would seem possible, however, that Freudenthal's (1973) notion of aiming to educate mathematically through the reality of the learner could offer a compromise in this respect and help to make the learning of mathematics a more satisfying, if not pleasurable, pursuit. If the subject is taught in such a way as to become part of the pupils' reality, it is likely therefore to become more relevant to them and they thus could be better motivated to learn. This counters to some extent, at least, the A.M.A.'s (1973) contention that unless pupils enjoy an activity, they will not be motivated to pursue it. If it is 'real' to them, it is more possible that they will be motivated. (The question of mathematics and the 'reality' of pupils will be discussed at greater length in a later chapter in this study.) Nevertheless, the point remains that there appears to be a difference in belief between some theorists and practitioners as to the desirability of pursuing the aim of teaching mathematics so that pupils come to value it for its intrinsic worth and to enjoy it.

A point of agreement between some theorists and teachers would appear to be the importance of the aim of developing logical thought in the teaching of mathematics. Freudenthal (1973) advocates that method rather than content be stressed in the process while both the A.M.A. (1973) and Ormell (1972) suggest that the development of logical thought implicit in mathematical method places the subject in a special position with respect to the curriculum as a whole, although each for different

reasons. The A.M.A. (1973) make the case that it is the analytical approach of mathematics that should be pervasive throughout other areas of study while in Ormell's (1972) view it is the applicability of the mathematical model as a problem-solving device that he believes should place the subject in this special position. Whether or not the subject is ascribed any special status in this respect is irrelevant to the aim of the development of logical thought except, very importantly, insofar as it involves entry into the pupils' reality. Logico-deductive method contained within mathematics alone or problem-solving techniques applied only to numerical problems would be very arid mathematics education, indeed, since the real world of the pupils is, superficially at least, non-mathematical until they have been initiated into it. Perhaps this is why Kline (1973) insists so strongly that mathematics be taught in conjunction with other subjects since only then possibly will pupils appreciate the essential mathematical nature of their world.

The search for criteria for good practice

The foregoing discussion of a limited selection of the literature concerned with the aims of mathematics education has indicated that the aims identified by Whitehead (1932) are still present in the considerations of mathematics educators of more recent years. However, it has also indicated firstly, that even though the aims held may be similar, differences do occur with respect to where priorities lie amongst them and, secondly, that there would appear to be a divergence of opinion between theorists and practising teachers with respect to the inclusion of the aims of the pupils' enjoyment of mathematics or of studying it for its intrinsic worth.

If the aims are to supply criteria for good practice and thereby act as guidelines for the selection of what mathematics is to be taught and how, clearly emphasis upon one aim rather than another will produce different sorts of mathematics curricula. The limitations that would arise from stressing pupils' enjoyment of the subject, for example, are all too clear. If content and method were to be selected according to this criterion above others, there could undoubtedly be much necessary and worthwhile mathematics excluded from the curriculum. There is also the superficially obvious but relevant argument that it is impossible

to make such a selection so that all pupils will enjoy learning mathematics, with the result that a desirable optimum may then become a minimum. The dangers of emphasising the aim of learning mathematics in order to be introduced to abstract thought are equally clear and would be highly likely to result in a mathematics curriculum more heavily recondite, to use Whitehead's (1932) description, than would be desirable. Thus while there may be a degree of agreement about what the aims should be, the lack of consensus concerning where the emphasis should lie, and hence of what an appropriate balance of aims should be, becomes the all-important factor.

A CONSIDERATION OF AIMS FOR MATHEMATICS EDUCATION AT A NATIONAL LEVEL

Because there is no central control over the curriculum in this country, teachers exercise the power to take decisions with respect to curricular content and methodology, particularly at primary level where there is no immediate influence exerted by examinations. Guidance in taking such decisions is clearly vitally important. Primary teachers would most likely seek such guidance from their LEA or relevant publications by HMI while, as we have noted, at secondary level specialist associations such as the Association of Teachers of Mathematics or the Mathematical Association exist to provide help of this kind. It seems reasonable to assume that sources of aims which are most likely to determine and guide practice in schools are those that function at a national level of concern or that are nationally representative. For this reason, lists of aims for mathematics education at primary level compiled by HMI (D.E.S.1979a) and for all levels (including secondary) by the Mathematical Association (1976) (hereafter referred to as the MA) will critically be examined and assessed for the criteria they offer for good practice in mathematics education.

There may be some hesitancy in accepting HMI and a professional association as arbiters of good practice because, as Nash (1978) states, "assessments of teaching made by practitioners (advisers and HMI's) have long been regarded as problematic by researchers using objective techniques." (p.72) However, the views of HMI have been used in the past for such a purpose by Nash (1978) himself, for example, in a study involving the identification of types of head teacher in small rural schools and also in the D.E.S. (1977) investigation "Ten Good Schools:

A secondary school enquiry" (D.E.S. 1977). In both cases, the observations of HMI led to choices being made concerning the exemplification of good practice. While the views of the MA (1976) may similarly be questioned on the grounds of objectivity, there are two reasons for choosing this body as a source. Firstly, because they are a nationally representative body of specialist mathematics teachers, the MA may be held to represent the considerations of a cross-section of practitioners who are manifestly concerned with the quality of mathematics education their pupils receive. Secondly, in view of the differences of opinion found in the earlier considerations of aims between theorists and practising teachers, it is of interest to compare those of HMI who (although practitioners of a sort) are not actually engaged in teaching, with those of specialists who are actually engaged in teaching the subject.

As a final consideration in selecting these sources of mathematical aims, it is important that they should broadly agree in their interpretation of the purpose of a statement of aims and both bodies make their positions clear in this respect. HMI (D.E.S. 1979) state that "*Aims* are essentially declarations of intent that give direction and shape to a scheme of work" (p.5 author's italics) while the MA stipulate that in using the term 'aims' they refer to "broad strategies" and to "general declarations of intent" related to the outcomes of mathematics education. (p.1) These two points of view would appear to be appropriately close to each other to permit the assumption that HMI and the MA ascribe the same meaning to the notion of aims with respect to mathematics education. Each set of aims will be presented separately below, then discussed jointly.

Mathematical aims at primary level

HMI (D.E.S. 1979) state that the list of aims they present for the study of mathematics in primary schools is not exhaustive and "the order is to some extent arbitrary". They are described as "general" aims and are expressed as abilities and qualities to be developed by pupils.

- "(i) a positive attitude to mathematics as an interesting and attractive subject;
- (ii) an appreciation of the creative aspects of the subject and an awareness of its aesthetic appeal;

- (iii) an ability to think clearly and logically in mathematics with confidence, independence of thought and flexibility of mind;
- (iv) an understanding of mathematics through a process of enquiry and experiment;
- (v) an appreciation of the nature of numbers and of space, leading to an awareness of the basic structure of mathematics;
- (vi) an appreciation of mathematical pattern and the ability to identify relationships;
- (vii) mathematical skills and knowledge accompanied by the quick recall of basic facts;
- (viii) an awareness of the uses of mathematics in the world beyond the classroom. Children should learn that mathematics will frequently help them to solve problems they meet in everyday life or understand better many of the things they see, and provide opportunities for them to satisfy their curiosity and to use their creative abilities;
- (ix) persistence through sustained work in mathematics which requires some perseverance over a period of time." (p.5)

A final over-riding aim is described as "to maintain and increase confidence in mathematics, shown by the ability to express ideas fluently, to talk about the subject with assurance and to use the language of mathematics." The aims are given after a discussion of the context of primary mathematics education and it is made explicit that they have been drawn up with primary pupils in mind.

Mathematical aims for all levels

In their publication "Why, What and How?", the MA (1976) make clear that their considerations are not made with secondary pupils alone in mind. Indeed, in their discussion of aims, they suggest that "the phrase 'at an appropriate level' must be added to almost any statement made" and that 'appropriate' "would refer to a combination of the child's (or adult's) age, his maturity and what could loosely be called his 'mathematical age'." (p.2) They also remind us that in their consideration of mathematical aims, they are concerned with them in the context of education as a whole and present their list bearing in mind more general educational aims and how mathematics may contribute to their achievement. Thus they refer to "educational aims" in "mathematical terms" and these are listed as follows (numbered by the author for purposes of comparison).

- "(i) the acquisition of certain basic skills and knowledge necessary for everyday life;
- (ii) the acquisition of further skills and knowledge pertinent to particular courses and careers;

- (iii) the development of the ability to think and reason logically and coherently, not neglecting the development of spatial thinking;
- (iv) an appreciation of the formulation of a problem in mathematical terms (i.e. the idea of a mathematical 'model') and hence an appreciation of the role that mathematics can play in a wide variety of disciplines;
- (v) mathematics as "queen and servant" - as a tool in man's control of his environment and as an intellectual activity and human achievement involving pattern and structure;
- (vi) mathematics as a social activity, in its conduct, its existence and its applications, with a concurrent emphasis on communication skills - verbal, graphical and written;
- (vii) mathematics as a language;
- (viii) an appreciation of the problem-solving powers of mathematics through personal experience of investigation and open-ended situations."

While there are obvious similarities between the two lists of aims presented, closer analysis brings to light some important differences.

A comparison of the aims identified

It is difficult and cumbersome to draw comparisons and generally to analyse two lists of aims of such length. However, it is possible to achieve some degree of clarity by treating the aims of mathematics education identified in earlier discussion (with varying degrees of acceptance and rejection) as general criteria according to which aims in the more extensive lists drawn up by HMI and the MA may be categorised. These were (1) the development of logical thought in pupils; (2) the introduction of pupils to abstract thought; (3) the development of an awareness of, and an introduction to, the applicability of mathematics and (4) the development of an interest in mathematics and taking pleasure in doing it for its own sake. It is necessary to add a fifth category in order to accommodate some of the aims in the MA list; thus we have category (5) the development of an awareness of the social aspect of mathematics. If we treat these aims as general categories, the two sets of aims identified may be examined to see whether they fall within any of the five categories. It is to be expected that some will not fall neatly into one or other category but into more than one, and where such overlaps occur they will be noted. The table below sets out the aims for mathematics education accordingly, referring to those identified by HMI as 'PA' with the relevant number (e.g. PA(i)), and those identified by the MA as 'MA' with the relevant number (e.g. MA(iii)).

A comparative analysis of identified aims

Development of logical thought	Introduction to abstract thought	Applicability of mathematics	Development of intrinsic interest and pleasure	Awareness of social aspect of mathematics
PA (iii) PA (iv) *PA (vi)	PA (v) *PA (vi) PA (vii)	PA (viii)	PA (i) PA (ii) PA (ix)	
MA (iii) *MA (viii) *MA (iv)	*MA (iv) *MA (v) MA (vii)	*MA (viii) *MA (iv) MA (ii) MA (i)		*MA (v) MA (vi)
* = occurs more than once				

In the following discussion of the above comparison of aims, those referred to as primary aims (PA) will be dealt with first followed by the aims identified for all levels of mathematical education by the Mathematical Association (MA).

(1) the development of logical thought

PA HMI (1979) isolate two aims of mathematics education which may be said to relate to the general development of logical thought, one of which refers specifically to clear and logical thinking in mathematics and a second which specifies that mathematical understanding is to be developed by means of enquiry and experiment. It would seem reasonable to link the latter with the development of logical thought since the processes involved in enquiry and experiment in a mathematical context, demand at the very least a certain logic in the identification and clarification of the question to be answered as well as of the means chosen to reach a solution.

MA The MA (1976) identify, in a similar way, the aim of developing logical thought in pupils but also link it with the development of spatial thinking. Again, there is an aim which is a close counterpart

to that in the PA list which refers to personal investigation by pupils in order to come to an appreciation of the potential of mathematics in problem-solving.

(2) an introduction to abstract thought

PA Reference to 'the nature of number and space' and to 'basic structure' emphasised the abstract nature of mathematics. For pupils to gain 'an appreciation' of these aspects of the subject involves a grasp, however tentative, of the notion of abstraction and the representational nature of the subject in relation to numbers and space. Similarly, mathematical pattern and the identification of relationships contribute to the development of abstract thought and in so doing, would use and build upon the ideas of number and space. An awareness of the idea that numbers 'grow', for example, is necessary to identify and understand the recurrence of the digit '2' in a pattern or sequence such as 2, 12, 22, 32. Clearly the identification of relationships would also contribute to the development of logical thought. However, it is arguable that the logical aspect of mathematical relationships is dependent in the first instance on an understanding of abstraction at however basic a level (e.g. representation may be pictorial and not numerical, initially). Since the mathematical knowledge and 'basic facts' in the seventh primary aim are not related to practical problem-solving or to anything of a particular concrete nature, this suggests a kind of activity that warrants being categorised as abstract.

MA The formulation of mathematical models involves the presentation of evidence in an abstract form but it also involves logic in the selection of pertinent features of that evidence to be presented in such a form. Thus the aim contributes both to the development of logical thought and of abstract thought as well, in that the former is necessary to the process that leads to the abstraction of relevant information in the form of a mathematical model. The ability to identify that a given amount of money (P) will increase by a certain percentage each year (i) and to be able to calculate the total amount at the end of one year is a necessary step towards achieving the generalisation $A=(p+i)^n$ (where n may be any number of years). This example also illustrates that this aim may be classified as one which is related to the applicability of mathematics. Stressing intellectual activity in relation to pattern and structure clearly

involves the development of abstract thought, as does emphasising the notion of mathematics as a language which would promote an appreciation of the discipline as a symbolic representation of meaning.

(3) the applicability of mathematics

PA The single aim suggested by HMI (1979) which may specifically be associated with the application of mathematics refers to the practical use of mathematics in the solution of problems of 'everyday life' together with a 'use' of an aesthetic nature. Here it is suggested that pupils should be made aware of the potential mathematics offers in the use of creative abilities. In order to be clearer about what may be envisaged in the development of pupils' creative ability in this context, it is helpful to be reminded that creativity may be viewed as the developing "of the attempt to launch out on one's own and impose one's own stamp on a product". (Hirst and Peters 1970, pp.53-4) Thus in the context of mathematics this could conceivably entail confidence and imagination in problem-solving on the part of pupils and in the satisfaction of their curiosity through devising number games or in making structures using different shapes. However, possibly what is really meant with respect to the 'creativity' of primary pupils learning mathematics is simply the ability to identify in a situation, mathematics which they may not previously have met.

MA The aim of developing an appreciation of the problem-solving potential of mathematics through pupil experience and investigation is advocated which clearly would broaden the awareness of the applicability of the subject. Two other aims that refer to applicability are those that refer to the mathematics necessary for 'everyday life' and to 'skills and knowledge pertinent to particular courses and careers', thus alluding to the usefulness of the subject at a basic level and then in connection with activities at a higher level. The relevance and applicability of mathematics is also reinforced in the aim which refers to the use of the mathematical model in 'a wide variety of disciplines'.

(4) the development of an intrinsic interest, and pleasure, in doing mathematics

PA Three aims are identified by HMI (1979) which could be classified under the heading of developing an interest and pleasure in doing mathe-

matics for its own sake. One suggests the desirability of pupils developing a positive attitude towards the discipline as both 'interesting and attractive'; a second stresses the appreciation of the 'creative' and 'aesthetic' aspects of the discipline, while a third stresses the necessity for pupils to persevere in undertaking mathematical tasks and sustaining their application to problems. Although the latter does not stipulate that pupils should take pleasure in what they are doing, the aim does suggest that what is desirable, is the development of an interest in the subject sufficiently deep as to result in such sustained application.

MA This analysis of the MA's aims shows that it is not held to be an aim for all pupils that they should like mathematics or be interested in it for its own sake. Two aims use the phrase 'an appreciation of' in connection with aspects of mathematics but this is perhaps correctly interpreted to mean 'an awareness of' rather than necessarily implying that what follows is to be highly and intrinsically valued.

(5) an awareness of the social nature of mathematics

PA None of the aims identified by HMI (1979) can be included in this category.

MA Two aims identified by the MA (1976) can be included here. One stresses the fact that mathematics is used as a tool by man to control his environment and refers to the discipline in terms of 'human achievement'. The second aim in this category is entirely given over to the development of the notion of mathematics in social terms. It is stressed as a 'social activity' in how it comes into being, in how it is 'done' and in its applications. Throughout all of this, the various communication skills are emphasised so that mathematics is seen in terms not only of representational skills but of the spoken word as well.

A consideration of the analysis of the two sets of aims

The analysis and the classification of the aims for mathematics education in these two lists under the four broad headings identified earlier and the additional 'social' heading is helpful in highlighting differences and similarities between the two. Hence, it also helps to

clarify where any emphasis might be placed. While it must be admitted that such an analysis is open to the usual weaknesses inherent in a subjective exercise of this kind, an attempt has been made to give reasons for the classification of aims under particular headings, particularly where the matter may have appeared open to question.

If we are to take the number of aims falling in each category as a rough guide we must bear in mind that nine aims from HMI (PA) and seven from the Mathematical Association (MA) are under comparison. Both appear to place considerable emphasis upon the development of logical thought and of abstract thought as aims, and this would appear to be where the similarity ends. Probably one of the least expected differences between the two approaches to aims appears in connection with the applicability of mathematics. Almost half the MA aims can be related to this category whereas only one out of nine of those of the HMI can be viewed in this light. Some of the emphasis by the MA arises from relating the acquisition of skills and knowledge to the context of the requirements for everyday life and for further practical or academic pursuits. There is also stress placed upon 'personal experience' in investigations which stipulates some direct experience of the applications of the subject in problem-solving situations. The second major difference occurs in relation to the category of aim concerned with the development of an intrinsic interest in mathematics and taking pleasure in doing it. Three of the aims selected by HMI fall quite clearly in this category while this is not the case with any of those identified by the MA. Reference is made by HMI (1979) to an "over-riding aim" for primary pupils "to express ideas fluently" and "to use the language of mathematics" without specific mention of the importance of communication. Finally, there is the striking difference of the complete lack of any aim identified by HMI which refers to the conception of mathematics as a social activity or to any 'social' element in mathematics education. It is somewhat surprising that this facet of the teaching and learning of mathematics receives no attention in the aims identified by HMI, particularly for pupils at primary level, since it could be said that it is these characteristics of mathematical learning that, potentially, make the subject relevant for pupils in appealing to the reality which surrounds them.

CONCLUSIONS

Clearly, it would be inappropriate to draw very firm conclusions from this type of analysis of statements of aims for mathematics education by HMI (D.E.S. 1979) and the MA (1976) since the difficulty in formulating them has been acknowledged. Indeed, we must remember as suggested earlier in this chapter, that it is the lack of precision and the generally directive nature of such statements that characterises them as aims. An aim broadly describes the nature of some desirable outcome in connection with some content. However, although we may not expect precision, we may seek clarity. Thus it could be argued that, in the mathematical context, the aim which states 'the acquisition of certain basic skills and knowledge necessary for everyday life' is perhaps a better formulation than that which refers to the development of 'mathematical skills and knowledge accompanied by the quick recall of basic facts'. The former attempts broadly to qualify the mathematical content by including a reference to what is needed mathematically in 'everyday life' and at the same time implies the usefulness of what is to be learned; the latter does not. This may appear superficially to be unimportant, but remembering the role that such statements may play in providing teachers with guidance for practice, phrases such as 'necessary for everyday life' take on particular significance.

A further point must be established in connection with the aims listed by the MA (1976). Although none refers to the development of a liking for, or an intrinsic interest in, doing mathematics, having identified aims, the MA go on to list mathematical goals which, in their opinion, lead to the achievement of their aims. These are listed under several headings the last of which is Mathematical Appreciation, and at the top of which is that pupils should "enjoy and be interested in mathematics and have a good attitude towards the subject". (p.5) The fact that they choose not to make this a general aim but treat it as an objective in the affective sphere linked with the day-to-day doing of mathematics, suggests a degree of realism of which Freudenthal (1973) and Whitehead (1932) would approve. However, we believe there is a case for including an aim that refers to 'attitude towards' as opposed to the 'liking of' or 'taking pleasure in' the subject. While it may be unrealistic and even harmful (in Freudenthal's sense) to hold the aim that all pupils should like mathematics, it would seem reasonable to pursue

the aim of the development of a positive attitude on the part of pupils towards the learning of the subject. Thus an acceptable aim on these grounds could be formulated as 'the development of a positive attitude towards work in mathematics'. If, therefore, we were concerned to draw up a list of aims for mathematics education to act as criteria for good practice and to guide us in the construction of a mathematics curriculum, an apparently reasonable and balanced list of aims would appear to be contained in the list offered by the MA (1976), prefaced by the phrase 'at an appropriate level of development' and with the addition of an aim which advocated the development of a positive attitude towards doing mathematics on the part of pupils.

Finally, however, our particular concern is with the identification by the MA (1976) of an aim, the purpose of which is to stress mathematics as a social activity in its existence, in its conduct and in its applications. This 'social' element of aims for mathematics education is evident to some extent in Freudenthal's (1973) references to the applicability of mathematics being taught in terms of the "lived-through reality" of pupils and to an emphasis on the importance of mathematics to them as individual human beings. (p.77) Ormell (1972) also touches upon it in suggesting an aim which should involve an awareness of the applicability of mathematics to all aspects of the human condition. There is a suggestion inherent in such views that the social nature of mathematics is important and that an obvious way of approaching the study of the discipline as a social phenomenon is through its applications. However, reference to mathematics not only as a social activity in its applications but in its existence and its conduct, as well, suggests that the characterisation of the subject in social terms is a highly complex phenomenon. In viewing these two aspects of mathematics, (a) its existence and (b) its conduct, in this light, issues of an epistemological nature are raised by the former, while the latter poses questions related to the mathematics curriculum as a whole. Thus there are questions both of an epistemological nature and a socio-psychological nature, to be considered. This suggests that these two aspects of mathematics viewed in terms of social activity have considerable potential for affecting the way the subject is taught and learnt.

The fact that an aim with such pervasive potential has been identified for mathematics education suggests that the concept of mathe-

matics as a social activity warrants analysis, particularly in order to clarify what it means in connection with the existence and the conduct of the subject. This is all the more important since there is some indication that the social nature of mathematics possibly is not inherent in aims held for the primary sector, at least. We shall therefore now go on to examine the notion of mathematics characterised in social terms to determine what implications there may be for the mathematics curriculum in such a conception of the discipline.

Mathematics as a Social ActivityIntroduction

A consideration of the aims of mathematics education in Chapter 1 concluded with the analysis of two lists of aims which, arguably, would be influential in providing guidelines for practitioners in the structuring of a mathematics curriculum. Of particular relevance to this study was the identification of aims concerned with social aspects of mathematics as a discipline. One of the lists (D.E.S. 1979) did not include any aims which fell into this category. In the other (MA 1976), two aims advocated a social view of mathematics, one of which sought to emphasise mathematics as a social activity in its being, its conduct and its applications, and stressed the importance of communication skills in such a view. It was suggested that the notion of the social nature of the discipline in its applications has to some extent been acknowledged and developed (e.g. Freudenthal 1973, Ormell 1972) whereas the notion of the social nature of the foundations of mathematics in conjunction with how it is 'done' appears not to have gained a similar degree of attention. Mathematics seen as a social activity in its foundations was identified as raising epistemological questions while to view it from such a perspective in connection with its conduct was seen to raise socio-psychological issues relevant to the whole curriculum process.

The concern of this chapter will be to examine the epistemological issues thus raised, in the course of which we shall draw comparisons between viewpoints which support such a position and one that does not. These will then be considered with respect to the concept of mathematics as a social activity in its conduct. For purposes of this study, we shall take the meaning of mathematics as a social activity in its conduct to be 'how it is done in the classroom'. Thus in relating epistemological considerations to this area of mathematical activity, we shall project the likely effects of two different philosophical viewpoints upon the mathematics curriculum in the classroom. This will enable us to gain some insight into the extent to which the approaches to the teaching and learning of mathematics that evolve may differ. It will also help us to identify the culmination of these differences in social terms.

MATHEMATICS AS A SOCIAL ACTIVITY IN ITS EXISTENCE

It is perhaps difficult to conceive of a discipline such as mathematics characterised in social terms. Although in recent years some attention has been focussed upon the social foundations of knowledge through considerations of the sociology of knowledge, there has been little of direct relevance to teachers (or, indeed, to curriculum developers) that might affect the day-to-day enterprise of educational activities. The discussion of the sociology of knowledge in relation to education has largely centred upon such matters as questioning the organisation of knowledge in the curriculum (Young 1971) and the structuring of the transmission of educational knowledge (Bernstein 1971). The content of the debate which has been at a highly abstruse level calling into question what should count as educational knowledge, has caused Pring (1972) to suggest that "There are limits to what meanings can be negotiated or realities reconstructed, and there seems little ground for turning the classrooms into either a market place or a building site". (p.28) Nevertheless, a basic tenet of a current view of the sociology of knowledge which cannot be ignored is that all knowledge is socially determined (Berger and Luckmann 1966), a factor which may be particularly relevant when viewed in relation to mathematics. Consideration of this point suggests that mathematics has become more removed from its social origins than any other subject in the curriculum, and has tended to be seen to have taken on an existence of its own quite separate from its foundation in human endeavour (e.g. Mannheim 1936). In the not-too-distant past it was stated to have a reality of its own outside human existence. (Hardy 1941) Rather than being viewed as a form of knowledge that changes or adapts, "Mathematics is presented as an ever-increasing set of eternal, immutable truths". (Lakatos 1976 p.142) While such an attitude is very likely attributable to the level of abstraction of the subject and its recondite nature to which Whitehead (1932) refers, it is possible that such a complete separation of the subject from its social roots may lead all too easily to the ignoring of issues of considerable importance in mathematics education. These issues may be particularly relevant in the highly technological society that exists in this country where there is a premium placed upon mathematics and its applications in supporting such a society. The discipline is not only highly relevant to applied technological studies such as engineering but, as Barnard (1972) points out when referring to mathe-

matics courses at university level, "We already have the 'mathematical' adjective applied to chemistry, biology, economics and politics, as well as to physics." (p.12) This serves to illustrate the pervasiveness of mathematics in a variety of forms of activity that take place within, and contribute to, different facets of an industrial social framework. It also identifies mathematics as a 'prestige' component of the curriculum as elaborated by Bernstein (1971) in his discussion of the varying social valuations placed upon different kinds of knowledge. Thus the anomaly exists wherein what may be judged one of the most fundamental disciplines of the curriculum in terms of its wide social applicability may, at the same time, be the furthest removed from its basis in human activity. An examination of epistemological considerations helps to clarify how this situation has arisen and what theoretical alternatives exist that may offer a perspective which brings mathematics closer to its foundation in social activity.

Mathematics and 'formalism'

The distancing of mathematics from its social origins and its removal from the sphere of human activity to the degree just described, has largely been due to the 'formalist' approach to the discipline as described by Lakatos (1976). Lakatos refers to the development of the concept of 'metamathematics' and suggests that it "tends to identify mathematics with its formal axiomatic abstraction" which in turn leads him to define the "'formalist' school" of mathematics. (p.1) Formal systems are seen to have replaced mathematical theories and to provide the "syntax of mathematical language". Hence,

"None of the 'creative' periods and hardly any of the 'critical' periods of mathematical theories would be admitted into the formalist heaven, where mathematical theories dwell like the seraphim, purged of all the impurities of earthly uncertainty."
(p.2)

That is to say, problems that do not fall within the range of the abstract realms of metamathematics including such matters as those that relate to informal mathematics and its growth "and all problems relating to the situational logic of mathematical problem-solving" would, in the light of formalist mathematics, be meaningless. (Lakatos 1976, p.1)

The epistemological basis for this view is seen to lie in logical positivism. Hamlyn (1970) describes as a "central thesis" of positivism,

the tenet that "all propositions other than those about mathematics or logic are verifiable by reference to experience". (p.37) This helps to clarify Lakatos' (1976) interpretation of formalist mathematics and the exclusion from it of practical, day-to-day matters that would, in the normal course of events, be characterised as mathematical. To take a mundane example, the solution of a problem relating to the total distance travelled by a car averaging 55 miles per hour over a period of three hours, would not be characterised as mathematical in the formalist sense. Similarly, an exposition in which the history of the development of the concept of function in response to particular mathematical needs over the centuries would not be interpreted as mathematics since, in the formalist sense, mathematics exists in formal systems and is not viewed as resulting from processes of trial-and-error or the development of competing theories. As an epistemological foundation for mathematics, positivism thus dogmatically removes the discipline from the arena of everyday life both with respect to its foundation arising from human thought and experience, and its potential for problem-solving and growth. Hamlyn (1970) states that "Positivism has now gone so far out of fashion that it is perhaps difficult to understand why anyone should have ever supposed that it should be acceptable." (p.60) With respect to mathematics, it has resulted in the representation of mathematical thought as a 'given', unchanging, formal system which grows only, as it were, by the accretion of new 'truths' and not through the developmental process of questioning what already exists, proposing new hypotheses and testing them against the old. It is not a theoretical approach to knowledge which in any way helps to clarify mathematics as a social endeavour in its existence nor, indeed, in its conduct.

The sociology of knowledge

Superficially, it might appear helpful to look to the sociology of knowledge for a solution to the problem of finding a rationale which brings the foundation of mathematical thought more closely to its social origins. The fact that the sociology of knowledge holds that all knowledge is socially determined has already been noted, and this might be taken to be an appropriate basis from which to gain a clearer perspective for epistemological considerations of mathematics. Peters (1977) however, provides firm guidance in this respect when, referring to the sociology of knowledge, he states:

"In developing a generalized and fashionable anti-establishment line it has often ignored the crucial distinction between the social basis of knowledge itself and of the ways of trans-

mitting knowledge, and has proceeded in sublime ignorance of centuries of work in epistemology on problems of truth and objectivity." (p.171)

The problem of objectivity in this connection is also raised by Popper (1966). His criticism of the sociology of knowledge helps to clarify its inadequacy in providing a satisfactory theoretical account of how objective knowledge, such as that represented in the discipline of mathematics, not only comes into existence but continues to grow.

A major part of Popper's (1966) criticism stems from the fact that the sociology of knowledge is primarily concerned with how knowledge is constructed as opposed to the product of such a process. Inherent in a concern with the way in which something comes about is a concern with causes, and questions of why it happens. Popper argues that being concerned with causes is to adopt an essentially subjective approach, since it implies "the tendency to unveil the hidden motives behind our actions". (p.215) Motives, in turn, lead to explanation and justification and by trying to explain what it is we have done or how something has come to pass, we tend to revert to belief. Popper (1972) discounts belief as a basis for knowledge since, inevitably, belief is informed by personal ideology. Thus different meanings can be attributed to the same concept or phenomenon since individuals may argue from such different perspectives that they "fail to distinguish between objective and subjective knowledge". (p.25) Contradiction then becomes acceptable and with it, irrationality; hence, rational critical discussion becomes impossible. Popper contends that the study of the products of man's activity is "vastly more important than the study of the production, even for an understanding of the production and its methods". (p.114) Rather than concerning ourselves with causes, he suggests that the importance of the social aspect of knowledge lies in the fact that the theories and ideas proposed by individuals are publicly accessible and open to criticisable form that knowledge becomes objective, and it is through the testing of hypotheses and ideas in public debate that objective knowledge changes and grows. Thus the study of the effects of man's activity leads to the establishment of objective knowledge, while a study of causes, in his view, does not allow it to develop beyond the subjective stage. If this is the case, then the sociology of knowledge is open to criticism on the grounds of relativism, i.e. what counts as knowledge is relative to a certain position adopted by individuals or groups or societies.

Popper's criticism of the sociology of knowledge thus identifies in it a major weakness as an epistemological interpretation of the social origins of knowledge. However, the fact that the sociology of knowledge is not acceptable for this purpose because of its subjective nature must not lead, in turn, to the assumption that objective knowledge, even in the Popperian sense, is completely value-free. There is, as Musgrave (1974) points out, a degree of subjectivism in the choice of the theories or ideas which constitute objective knowledge, that are to be criticised or tested. As he suggests, "we must separate the justification of a *choice* of theory from the justification of the *theory* itself". (Musgrave 1974, p.584 author's italics) In justifying our choice of problem, clearly there must be a degree of subjectivism and such justification necessarily involves "the psychological attitudes individuals adopt towards knowledge." (p.585) Other philosophers have attempted to take this element of subjectivism into account in proposing other theories of knowledge.

Other epistemological approaches and the social aspect of knowledge

We have seen how Popper's (1972) notion of competing theories contributes to the growth of objective knowledge. Popper offers what he describes as "an objective criterion" for the prevailing of one theory over another:

"It is that the new theory, although it has to explain what the old theory explained, *corrects* the old theory, so that it actually *contradicts* the old theory: it contains the old theory, *but only as an approximation.*" (p.16 author's italics)

Thus because of the elimination of error which results from contradiction, some part of the old theory is discarded, a process referred to by Popper as 'falsification'. This assumes some means of testing one theory against another, described as the "critical method", to determine which theory or part of a theory will be refuted. Again in Popper's words, "It is a method of trial and the elimination of errors, of proposing theories and submitting them to the severest tests we can design." (p.16)

Other philosophers who approach the theory of knowledge from the point of view of growth and change in a way similar to Popper's, find difficulty in accepting his proposal of the existence of an 'objective criterion' for judging one theory 'better' than another. Kuhn (1970a)

states that, "Few philosophers of science still seek absolute criteria for the verification of scientific theories. Noting that no theory can ever be exposed to all possible relevant tests, they ask not whether a theory has been verified but rather about its probability in the light of the evidence that actually exists." (p.145) He argues that Popper's (1972) notion of falsification might just as well be called 'verification' since what is involved is the seeking of the superiority of one theory over another in terms of potential for substantiating an hypothesis. Kuhn prefers the notion of "anomalous experiences" which he describes as "experiences that, by evoking crisis, prepare the way for a new theory". (p.146) He does not see this anomalous experience as being identifiable with the kind that result in falsification. "Indeed," he states, "I doubt that the latter exist." Kuhn (1970b) also criticises Popper's rejection of the subjective, and therefore of the psychological, aspects of how knowledge comes about for, he argues, they "may explain the outcomes of choices that could not have been dictated by logic and experiment alone". (p.22) Kuhn (1970c) considers it important to an understanding of how knowledge grows, to take into account socio-psychological factors that are inherent in the process of making judgements and choices. He writes that "in many concrete situations, different values, though all constitutive of good reasons, dictate different conclusions, different choices. In such cases of value-conflict, (e.g. one theory is simpler but the other is more accurate) the relative weight placed on different values by different individuals can play a decisive role in individual choice." (p.262)

This clearly brings into a consideration of the growth of knowledge the acknowledgement of the social aspect which not only accounts for the 'public debate' to which theories are submitted and to which Popper (1972) refers, but also the values of the individuals who propose the theories and who criticise them. Kuhn's (1970c) view, at the same time, however, attracts the charge of relativism which, as we have seen, may also be levelled against the sociology of knowledge. Kuhn (1970c) counters the charge by stating that he is in agreement with the notion of competing theories and that "each was believed to be true in its time but was later abandoned as false". (p.264) However, having gone that far, he cannot accept the next step which he considers many other philosophers take which is "to compare theories as representations of nature, as statements about 'what is really out there'". (p.265) He sees in

such a step the assumption of the existence of an absolute truth which he rejects. What he suggests is that the different circumstances in which individuals work, their different individual psychologies with the accompanying difference in values and perspectives which they hold, are a determining influence upon each individual's criticism of one theory and the proposal of a new one. He holds this influence to be important whatever theory is held to be true at a given time. Acknowledging this importance, in his view, does not constitute relativism; nor does the fact that he refutes the notion of absolute truth and with it the idea of more closely approximating such truth. For Kuhn (1970c), "'truth' may, like 'proof', be a term with only intra-theoretic applications". (p.266)

Toulmin (1972) is also concerned with the search for a theoretical explanation of how knowledge changes and grows which, at the same time, takes into account socio-psychological influences. For him, the central issue is related to questions of "rational function and intellectual adaptation" rather than with questions of logical form (p.vii) which leads him to consider problems of what he terms 'conceptual change'. Taking an historical and ecological perspective, he poses the problem in the following way:

"If all men's concepts, interpretations, and rational standards - in morals or in practical life, in natural science or even mathematics - are historical and cultural variables, so that our habitual modes of thought are as much reflections of our particular time and place as our habitual modes of social behaviour, then the same fundamental problem arises in each case. What solid claims can *any* concepts and modes of thought have on our intellectual allegiance?" (p.50 author's italics)

Thus he raises the question of relativism, and goes on to suggest that "The rational demand for an impartial standpoint is pressing and legitimate." (p.51) Toulmin's answer to this demand is arrived at after ecological and historical considerations related to concepts in a variety of disciplines. The basis for establishing such impartiality lies in taking into account the accumulation of experience of whatever kind (for example, mathematical or any other form of thought) "in *all* cultures and historical periods". (Toulmin 1972,p.500 author's italics) As a result, he suggests, we reach an objective point of view "in the sense of being neutral", but at the same time "its conclusions are always subject to reconsideration". Hence the on-going experience of the growth and change in knowledge is continuous, and "our ideas about rational

strategies and procedures for dealing with the problems in any field are always open to reconsideration, revision and refinement."

The preceding brief consideration of aspects of the epistemological approaches of Popper, Kuhn and Toulmin indicates that, while each may differ from the other, they offer viable alternatives to a theory of knowledge based upon logical positivism such as has dominated mathematics for many years. Each, to a greater or lesser degree, in interpreting knowledge in terms of growth and change also acknowledges the importance of the social aspect of such a process. Toulmin (1972) in particular, offers an interpretation of objectivity which is helpful in countering the charge of relativism when taking into account the socio-psychological and historical determinism of theories and ideas and the conferring of objective status upon them.

Lakatos (1976) has also participated in this debate and has related his thought in particular to mathematics and its study (Lakatos 1976). For this reason, his contribution is now considered separately.

Lakatos' interpretation of the development of mathematical thought

Lakatos' (1976) identification of the adverse effect upon mathematics of a positivist philosophical approach to the discipline has already been touched upon. It has, he suggests, resulted in 'formalism' which "denies the status of mathematics to most of what has been commonly understood to be mathematics, and can say nothing about its growth". (p.2) Lakatos is concerned to show how the "methodology of mathematics" can contribute towards its growth, and

"to elaborate the point that informal, quasi-empirical, mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations." (p.5)

He illustrates this by means of a dialogue between teacher and taught through which a "*rationaly reconstructed or 'distilled' history*" of mathematics is evolved. (Lakatos 1976, p.5 author's italics) (The actual history relating to the dialogue appears in footnotes.) The dialectic he thus presents illustrates how mathematical knowledge can change and grow by means of formulating theories, testing them and reformulating them in a way likened to Popper's (1972) interpretation

of how knowledge grows.

One of the 'givens' in a traditionally held view of mathematics, the notion of 'proof', is examined by Lakatos (1976) and provides an example of the effects of a formalist approach. He states, "Many working mathematicians are puzzled about what proofs are for if they do not prove". (p.29) The acceptance of the notion that a proof is infallible is, he suggests, a result of "dogmatist indoctrination" with the conviction that pure mathematicians hold them to be so. This, in fact, is not held to be the case and examples are cited. For example, Wilder (1944) is quoted as saying that "a proof is only a testing process that we apply to suggestions of our intuition" while Polya (1945) explains that proofs are a device for connecting mathematical facts and helping us to remember them. Proofs thus offer an example of Toulmin's (1972) theory involving conceptual change. What holds as a proof at one moment may well be refuted by others, in other circumstances, at some time in the future which may give rise to conceptual change. This fallibilist approach, in Lakatos' view, is necessary to an understanding of growth in mathematics. Without it, mathematics reverts to an authoritarian discipline, tinged with a bit of mysticism, as it has traditionally been characterised. This is totally to discount the individual's intuition which prompts conjecture, as well as to ignore the circumstances in which such theories are postulated. He states that,

"There is no theory which has not passed through such a period of growth; moreover, this period is the most exciting from the historical point of view and should be the most important from the teaching point of view." (Lakatos 1976, p.140)

Lakatos (1976) sets out to explore what he refers to as the 'methodology of mathematics' which he equates variously with a heuristic approach and with Popper's (1972) "logic of discovery" or "situational logic". (p.3) He gives numerous other examples of how mathematical concepts have, in the course of centuries, been discarded or altered to meet new demands made upon them. His use of the heuristic method in providing such examples emphasises how theories or concepts are open to criticism and refutation or 'proof' of a sort, and also illustrates how individuals might play their parts in a discussion leading to the reformulation of an idea. The following extract is an example, where Theta, Alpha and Kappa are pupils:

"Theta: So the logical point of view is 'crankish', is it?"

Alpha: *Your* logical point of view, yes. But I want to make another remark. Whether deduction increases content or not - mind you, of course it does - it certainly seems to guarantee the *continuous growth of knowledge*. We start with a vertex and let knowledge grow forcefully and harmoniously to explain the relation between the number of vertices, edges and faces of any polyhedron whatsoever: an undramatic growth without refutations!

Theta (to Kappa): Has Alpha lost all his judgement? One starts with a *problem*, not with a vertex!" (Lakatos 1976, p.82 author's italics)

Thus we have an example not only of the struggle of an individual to come to grips with an aspect of methodology in arriving at a mathematical concept, but also of the idiosyncratic points of view of individuals that can arise in the social give-and-take of such a situation.

Summary of theories of knowledge in relation to mathematics as a social activity in its existence

The purpose in examining the foregoing approaches to the theory of knowledge was firstly, to identify how it has come about that mathematics has become removed, to the degree that it has, from its origins in human thought and intercourse. Secondly, the intention was to identify epistemological interpretations that would offer a theoretical basis for the foundation of mathematical knowledge that takes into account the social aspect of how that knowledge came to be. In other words, we have sought theoretical clarification of mathematics as a *social activity in its existence*.

Lakatos' (1976) exposition of how logical positivism has affected mathematics in producing what he has called a 'formalist' approach, indicates how the view of mathematics as having an existence of its own came to be established. This view has influenced mathematics curricula for many years, and an unquestioning acceptance of content as unchanging and unchallengeable has tended to prevail. Mathematics remains, probably in the minds of most, as a given body of abstractions that is unassailable in its truth and assured in its methodology. The fact that something referred to as 'new' mathematics has been introduced into curricula may well be rationalised as knowledge that was 'there' all the time but

to which, only of late, the lay public has been deemed worthy of exposure. If there is seen to be an element of change at all it is probably viewed as mechanical as opposed to organic change; that is to say, the new elements are simply added to the long, logically determined series of mathematical truths already in existence rather than coming into being as a result of the refutation of some previously held concept or the birth of a new one as the result of individual intuition. The notion of criticising a mathematical tenet, of argument in connection with it and the possibility of it no longer 'fitting' the theory of which it is a part, is probably quite foreign to most teachers and the pupils they teach. In writing about 'new' mathematics for parents, teachers and pupils, Rosenthal (1965) epitomises such a view when she states that "most of us feel that there are certain eternal verities about which everyone must agree, and mathematics should be one of these." (p.9) The moral imperative in such a statement merely adds to the strength of such 'feelings'.

Having drawn our attention to the adverse effects of positivism on the general view of mathematics, Lakatos has also given us examples of how mathematical knowledge has changed through the centuries and how the 'eternal verities', in some cases at least, have proven to be ephemeral. In doing so, he has provided us with an example of the kind of epistemological approach offered by Popper (1972), Kuhn (1970a,b,c) and Toulmin (1972) applied to the discipline of mathematics. Through an historical perspective, Lakatos has illustrated how the demands of a particular era or the intuition of an individual has resulted in change in mathematical knowledge.

Clearly the adoption of either formalism or of an epistemological viewpoint which takes into account growth and change of knowledge must affect how the discipline is taught, a claim which Lakatos himself makes strongly in connection with the latter. We shall now go on to relate the possible effects of these different perspectives upon the mathematics curriculum. Firstly, we shall examine the effects of formalism and secondly, the effects of a growth and change epistemology on mathematics as a social activity *in its conduct*, i.e. how mathematics is done in the classroom.

THE EFFECTS OF DIFFERENT EPISTEMOLOGICAL VIEWPOINTS ON MATHEMATICS
AS A SOCIAL ACTIVITY IN ITS CONDUCT

It was noted earlier that the notion of mathematics as a social activity in its conduct would, for purposes of this study, be interpreted as 'how mathematics is done in the classroom'. Our concern is with mathematics at the curricular level and therefore our interest in how mathematics is 'conducted' or how it is done is related to considerations at a similar level. This is not to disregard the fact that mathematics as a social activity in its conduct takes place at other levels from the purest and most abstract state of 'higher' mathematics to its application in everyday, practical situations. However, the case that will be presented here is based upon the acceptance of the fact that mathematics as a social activity in the classroom is inherent in the implementation of the mathematics curriculum. That is to say, it involves the four components of the curriculum identified by Kerr (1968) as a statement of aims and objectives, the selection of content, methodology and evaluation, in relation to mathematics and in relation to those involved in implementing the curriculum, i.e. the teachers and the pupils. Clearly, the latter are vital to our considerations since it is they who confer the status of 'social activity' on whatever takes place within the classroom.

It will now be argued that mathematics as a social activity in its conduct in the context of the classroom, is affected to a considerable degree by the epistemological viewpoints of those involved in the enterprise. In doing so, we shall consider the possible effects of both epistemological points of view, positivism and a growth and change theory, on each of the four components of the curriculum referred to above.

Positivism and how it may affect the mathematics curriculum

If we were to approach the mathematics curriculum with positivism as our underlying epistemology, it is tempting to suggest out-of-hand that the prevalent concern would be with content. Before making such an assumption, however, it may be helpful to consider the implications of a positivist approach for the mathematics curriculum by attempting to place it in a more general educational context. There is some analogy

to be drawn between the kinds of consideration relating to such an approach with those that are to be found in a "formal-discipline" theory of education as identified by Wynne (1964). While our primary concern here is not with theories of education but with matters relating to the mathematics curriculum, there is an identifiable common concern between the formalist approach to mathematics as defined by Lakatos (1976) and the formal-discipline theory of education which Wynne (1964) describes.

Wynne (1964) traces the history of the philosophical foundations of the formal-discipline theory from Descartes onwards and suggests that the foundations and implications of the theory lie in "the distinction between mind and matter" and that, within the theory, it has been acceptable "to think of the different mental activities as the operation of metaphysical faculties or powers of the mind." (pp. 15-6) What is demanded of education in the light of such a theory, therefore, is the training of the mind and Wynne states that a tenet of the theory is that "any well organized subject could supply such training equally well" although "traditional subjects", mathematics being one of them, are held to be adequate for performing this role. Wynne states that emphasis in all subjects is on form rather than on content, and he suggests that "In mathematics, emphasis is on rules of operation and formal examples rather than on practical problems". (p.11) Thus, in a manner similar to that attributed to positivists, a formal-discipline tradition would approach mathematics as an entity unto itself. The perspective would be one of viewing a particular kind of knowledge not to be questioned nor applied, but to be practised for the sake of the values assumed to be inherent in the processes and the knowledge that form the discipline.

Lakatos (1976) states that a "formalist philosophy of mathematics has very deep roots" and suggests that "It is the latest link in the long chain of *dogmatist* philosophies of mathematics". (Lakatos 1976, p.4 author's italics) He goes on to suggest that "In the great debate, in which arguments are time and again brought up to date, mathematics has been the proud fortress of dogmatism" (p.5) and that "The dogmatists hold that - by the power of our human intellect and/or senses - we can attain truth and know that we have attained it". (p.4) It seems reasonable to assume that some of the epistemological roots of the

formal-discipline theory of education described by Wynne (1964) may be held historically to have contributed to those referred to as 'dogmatist' by Lakatos (1976). In accepting this, it becomes possible, throughout the following considerations, to draw some similarities between Lakatos' view of a positivist approach to mathematics and the place of the discipline within a formal-discipline theory of education. This is not to say, of course, that logical positivists would necessarily embrace a formal-discipline theory of education. It is, rather, simply a means of gaining further insight into how mathematics, viewed in itself as a formal discipline, might affect various components of the curriculum.

Positivism and objectives in the mathematics curriculum

As positivists, a combination of our unquestioning acceptance of the truths that make up the formal systems within mathematics together with the notion that these are subject to kinds of operations governed by particular rules, would clearly suggest limitations in defining our objectives in the teaching of mathematics. We would essentially be concerned with the teaching of operations and the relevant kinds of facts upon which such operations would be carried out. Such facts would relate to the formal system within mathematics with which we might be concerned - algebra provides such an example. Presumably we would intend that pupils should be introduced to the logical processes involved in the derivation of the mathematical truths from the axioms which are 'given' in the system. An example of such an objective might be: 'To learn the nature of an algebraic proof including (a) the meaning of a theorem, its converse and its negative; (b) induction and deduction; (c) necessary and sufficient conditions, if and only if.' Our positivist persuasion might well limit us to this kind of objective alone, although one would hope that as mathematics educators, we would also intend at the same time to develop a positive attitude towards our subject on the part of pupils. Thus our objectives would essentially be concerned with operations related to certain mathematical facts and carried out according to particular rules; we might also be concerned with the attitudes of our pupils towards the learning of the discipline.

Bearing in mind that aims for mathematics education may be used as criteria by which to judge the worthwhileness of curriculum objectives in mathematics, the set of aims arrived at in Chapter 1 will be used to

this purpose here. To enable us conveniently to carry out such an exercise, the list of aims which is adopted from the Mathematical Association (1976) (except for Aim 9) appears once again below.

A set of aims for mathematics education

- (1) the acquisition of certain basic skills and knowledge necessary for everyday life;
- (2) the acquisition of further skills and knowledge pertinent to particular courses and careers;
- (3) the development of the ability to think and reason logically and coherently, not neglecting the development of spatial thinking;
- (4) an appreciation of the formulation of a problem in mathematical terms (i.e. the idea of a mathematical 'model') and hence an appreciation of the role that mathematics can play in a wide variety of disciplines;
- (5) mathematics as "queen and servant" - as a tool in man's control of his environment and as an intellectual activity and human achievement involving pattern and structure;
- (6) mathematics as a social activity, in its conduct, its existence and its applications, with a concurrent emphasis on communication skills - verbal, graphical and written;
- (7) mathematics as a language;
- (8) an appreciation of the problem-solving powers of mathematics through personal experience of investigation and open-ended situations;
- (9) the development of a positive attitude towards doing mathematics on the part of pupils.

If we consider likely objectives for a formalist mathematics curriculum against this list, we find that only two aims or criteria (Aims 1 and 2) are of a factual nature and refer to "the acquisition of basic skills and knowledge". Another, Aim 3, refers to the development of the ability "to think and reason logically" (but also includes spatial ability) while Aim 9 is concerned with the development of a positive attitude towards the subject. It is arguable that with respect to mathematics as a language (Aim 7), a positivist approach would be highly likely to stress mathematics as a symbolic system and, to that extent only, as a language, i.e. to stress the representational aspect of language as opposed to the communicative aspect.

Aims 4, 5 and 8 would remain unaccounted for. That is to say, our mathematical objectives would not be concerned with developing the notion of the mathematical model and its multi-disciplinary role, the use and development of mathematics as a tool for the control of man's

environment nor with the problem-solving potential of mathematics. Particularly important to the present argument would be the exclusion of objectives that could lead to the satisfaction of Aim 6, concerning the development of the notion of mathematics as a social activity. Adopting a positivist philosophy would preclude any concern with teaching our pupils the social nature of the origins of mathematics in human activity, the wide applications it has and the value of the discipline in containing within it a variety of communication skills. Least of all, perhaps, would we be concerned through the medium of our teaching of the subject, with the exemplification of how mathematical knowledge can be developed and how potentially it may grow through our own theorising. Objectives would relate only to our concern with the formal reproduction of facts, the repetition of operations and their practice within a system in something of what might be described as an incestuous manner.

Positivism and the content of a formalist mathematics curriculum

In a selection of the content of what mathematics we might teach, our positivist inclinations provide us with a variety of formal systems from which to choose, for example geometry, algebra and calculus. The foregoing discussion of possible objectives has indicated that the criteria according to which we might select content from these various systems would largely be governed by considerations of what facts and skills would provide practice in particular kinds of operations. This clearly is not very helpful and becomes somewhat circular since the kind of goal we would appear to have in sight is concerned with knowing facts and manipulating them for the purpose of practice in such manipulation.

It has already been noted that the formal-discipline theory of education holds that in mathematics, emphasis would be placed upon rules and operations in doing formal examples. Wynne (1964) goes on to state that:

"The main consideration is that whatever can be observed and memorized should be observed and memorized. Such things are to be given special attention, not because of their social utility or personal significance, but because of the opportunities they provide for mental exercise." (p.11)

Lakatos (1976) makes an observation which is similar in that it highlights the barrenness of formalist mathematics.

"According to formalists, mathematics is identical with formalised mathematics. But what can one *discover* in a formalised theory?" (pp.3-4 author's italics)

The answer he offers is either nothing but the solution to problems by the endless repetition of formal procedures, or possibly the solution to problems within the system arrived at by haphazard guesswork (since there is no 'outside information' given, i.e. nothing external to the system). He considers neither of these acceptable.

Hirst (1974) establishes a point which is helpful in considering the content of a mathematics curriculum of the kind which we are attempting to project here. He refers to forms of knowledge which he has identified and states that, "The labels that I have used for distinct forms of knowledge are to be understood as being strictly labels for different classes of proposition." (p.87) He then goes on to suggest that the names he has ascribed to forms of knowledge are often used in a curricular context to refer to more than one type of proposition. For example:

"Even a term like mathematics, which may appropriately label a great deal of one form of knowledge because of the distinctive features of mathematical propositions, is frequently used in educational institutions to cover a concern not only for propositions of this kind, but also a concern for truths about the physical world and occasionally the history and philosophy of mathematics." (p.87)

In trying to envisage the content of our mathematics curriculum with positivist epistemological foundations, we might imagine it to consist of the kind of mathematical propositions to which Hirst refers, in as pure a state as possible, unadulterated by historical, philosophical or, indeed, physical considerations. Clearly, however, because a mathematics curriculum is necessarily embedded in an educational context, such purity would be impossible to achieve. As Hirst points out, disciplines within the curriculum such as mathematics contribute not only to specialist skills and knowledge but also to ways of understanding and ordering our experience as human beings. Apart from this consideration, the very act of teaching the subject with inherent intentional outcomes mediates against the learning of pure, unadulterated, propositional knowledge which formalist mathematicians might consider desirable. Thus the content of a formalist mathematics curriculum must inevitably be scarred by the very fact that it is contained within a curriculum, and it would seem likely that the choice of content would be made purely on the grounds of its relative importance to the formal system from which it is drawn.

Methodology in a formalist mathematics curriculum

Stenhouse (1971) makes the point that methodology in a curriculum is closely allied to content. While mainly concerned with discounting statements of objectives as a universally acceptable first step in planning a curriculum, he suggests that "one could start from a specification of content" and subsequently, "One would rely on the consonance between content and method to provide the teacher with a vehicle through which an area of experience or knowledge could be explored appropriately". (pp.76-7) The notion of a method of teaching which allows appropriate 'exploration' related to formalist mathematics is not one which seems easily applicable to the learning of dogmatically held truths. As we have seen, our projected content would probably consist largely of facts which make up formal systems, and the skills used to perform operations using those facts, according to the rules of the system. This kind of learning of facts and manipulative skills suggests that possibly the most appropriate way of teaching such mathematics to our pupils would be entirely in a didactic manner accompanied by considerable manipulative practice. If we are concerned that our pupils unquestioningly accept and learn selected mathematical 'truths', there seems little room for discussion, exploration or questioning in the course of such a process.

If we look again to the formal-discipline theory of education, we find that methodology in this context is approached with a view to the exercising of mental faculties as noted earlier and, as Wynne (1964) states, "Traditional procedures known as the book method, the writing method, the lecture method, and the question method, have all been used to serve the ends of formal mental exercise". (p.12) While, in pursuing a formalist mathematics curriculum, we may not necessarily accept that our pupils are learning mathematics in order to develop a mental faculty, these are the kinds of procedures we might well adopt since they could be held to allow least opportunity for the distortion of the content to be learned. They are procedures which permit minimal dialogue or discussion, if any. The teacher's role could be imagined largely as one of 'telling' and of 'showing how', while the explanatory aspect of the role would merely be one of emphasising the logic of the content rather than relating it to extraneous circumstances since this would be seen to detract from the abstract nature of the mathematics involved.

As in the case of our discussion of the likely content of a formalist mathematics curriculum, what we have projected in terms of method may also appear to be somewhat extreme. It might be possible that some variety could be introduced into the teaching of formalist mathematics through the use of programmed learning or audio-visual aids, for example. However, it seems that the most likely possibility for consonance of method and content to exist in a formalist curriculum is through the use of a methodology that is essentially didactic.

Griffiths and Howson (1974) would appear to acknowledge the difficulty generally of considering methodology in relation to the teaching of mathematics (almost, as it were, by default). In writing of the four curriculum components specifically in connection with mathematics they state that, "The problems of *method* belong, however, more to the province of education than of mathematics, and are of a pedagogical nature lying outside the scope of this book". (pp.156-7 authors' italics) Although acknowledging that they make some references to problems of method in other parts of their book, there would seem to be implicit in the above statement, the idea that while method or pedagogy is of a distinctly educational concern, objectives, content and evaluation are somehow different. It is a strange delineation to make and certainly an unhelpful one when considering curriculum components in relation to mathematics. However, the authors redeem themselves to some degree by quoting the following anecdote:

"A friend, on taking up his first teaching post, was told by the Head of Department, that he was to teach Form IV and that 'normally I start with Pythagoras and go on from there'.... Nothing whatsoever was said about 'method'." (p.156)

Perhaps this provides some insight into why the authors choose not specifically to discuss method as a component of the mathematics curriculum. More importantly, perhaps it is an indication of how content could be presumed to dictate methodology in a formalist mathematics.

Evaluation in a formalist mathematics curriculum

The kinds of evaluation procedures used in a curriculum are related to the objectives of that curriculum since the purpose of evaluation is to determine the extent to which objectives have been achieved. (Kerr 1968) Once again, the objectives of a formalist mathematics curriculum

of the kind deduced earlier, suggest that the recall of facts and the skills of manipulation would be our paramount interest in evaluating what our pupils may have achieved in pursuing such a curriculum. This, in turn, suggests that assessment would be a matter of asking questions of a factual nature and of posing problems demanding the use of relevant manipulative skills within a mathematical system. We would not be interested to any great degree in probing the mathematical understanding of pupils (since we would not have concerned ourselves with teaching for such understanding in our approach to the subject) except insofar as to determine their grasp of the logic within the formal systems to which they may have been introduced. However, once again that logic would be assumed to be demonstrable by the appropriate recall and manipulation of facts and skills.

Referring to curriculum evaluation, Taylor (1967b) writes:

"If the concern is with knowledge or subject matter, this is a question about what is, for example, science, mathematics, history and so on *for teaching*, with all this implies in the selection of content from a larger body of knowledge. Moreover this last question cannot be answered adequately unless it is placed in the context of the wider question. What is science, history, mathematics?" (p.19 author's italics)

In teaching formalist mathematics with its underlying positivist assumptions, the answer to this question is provided for us. Mathematics would be viewed as a body of abstract formal systems, the content of which would be unquestioned truths together with their unchallenged, internal logical methodology. This would be the way in which the subject would be presented 'for teaching'. It is to be remembered that this excludes any considerations of context of the derivation of such mathematics and of the applications of any of the mathematics involved. For the evaluation of what is learned within such an approach, it would appear that the testing of the recall by pupils of facts and testing their ability to perform formal manipulative skills would be the means most likely to be adopted.

Summary of a formalist approach to the mathematics curriculum

Drawing together the projected effects of positivism upon the four components of the mathematics curriculum and viewed as culminating in a formalist mathematics curriculum the conclusions may be summarised as follows:

- (1) Objectives for our curriculum would likely be stated in terms of facts to be learned and skills to be mastered, with some concern for the logic inherent in both; some attention might be paid to fostering a positive attitude towards the subject.
- (2) Content would probably consist of factual information and relevant manipulative skills with no concern for context or application.
- (3) Method would likely mirror the content in that it would be of a didactic nature and whatever the medium of teaching, it would ideally interpose itself between the pupils and the content as little as possible so as not to distort the content.
- (4) Evaluation would be based on the recall of facts and testing the ability of pupils to perform formal manipulative skills.

Growth and change epistemology and how it may affect the mathematics curriculum

A brief examination of Lakatos' (1976) considerations of a growth and change epistemology in relation to mathematics has illustrated how the teaching of the subject might be affected by the adoption of such a perspective. A basic assumption is that knowledge is founded in the proposals of theories or ideas which change and grow as a result of criticism and testing through time. While the social processes which contribute to knowledge and its growth are acknowledged, objectivity is not ignored. Although different points of view exist among the philosophers to whom reference has been made, as to how this objectivity is conferred upon knowledge, it is accepted by each as necessary in order that rational discussion may take place. There is not, however, the acceptance of an idea of absolute truth which exists 'out there' and which eventually may be attained. This is particularly relevant with respect to mathematics. It will be remembered that logical positivists view mathematics, together with logic, as the only two areas of knowledge not derived from empirical evidence. That is to say, mathematics is viewed as having an existence of its own and removed from any foundation in man's experience. This is the crucial difference between positivists and growth and change theorists. The latter, as exemplified by Lakatos (1976), consider mathematics as open to question through conjecture, critical argument and the testing of hypotheses amongst proponents of particular theories, as in other areas of knowledge. It

is a view which holds that there are discoveries yet to be made and problems still to be solved in mathematics as in any other discipline.

We shall now go on to examine this view for its likely implications for the four components of the curriculum in order to compare the results with those already reached with respect to the considered effects of logical positivism.

Objectives for a mathematics curriculum founded upon a growth and change epistemology

Our intentions in teaching mathematics according to principles based upon a growth and change theory of knowledge would be of a more complex nature than those that were argued from a positivist stance. This complexity arises from the fact that content would no longer be accepted as 'given' and we would no longer be primarily concerned with mathematics purely as a body of formal systems. Rather, we would also be concerned with particular kinds of processes and the part they play in giving rise to content. Clearly, there would be a considerable body of facts and skills that would be essential for pupils to learn, a knowledge of number and the kinds of operations performed on number providing the most obvious example. In view of our concern that pupils should come to appreciate mathematics as a growing, changing body of knowledge, we would also wish to instil in them an awareness of the kind of change in content and emphasis that can occur within mathematics and, to some degree, how and why these changes come about. An obvious example would be the notion of working in different number systems other than the decimal system, linking these perhaps with different systems of measurement and, in the case of the binary system, with the rise in prevalence of the use of computers.

Because much of mathematics has come into existence as a means by which man solves problems, we would be concerned to stress the problem-solving potential of the discipline. Where possible, we would also hope to illustrate problems within mathematics itself, even if only at the level of acknowledging that they exist. Lakatos (1976) refers to "problem situations in growing mathematical theories, where growing concepts are the vehicles of progress, where the most exciting developments come from exploring the boundary regions of concepts, from

stretching them, and from differentiating formal undifferentiated concepts." (p.140) While such developments may appear beyond the reach of school mathematics curricula, an awareness at however simple a level, that these kinds of problems have existed and do exist, would emphasise the fallibility of mathematics and the potential challenge within the discipline that such problems pose. The dilemma faced by the Greeks in discovering that the square root of two could be constructed as the diagonal of a square with sides measuring one unit but could not be represented by the ratio of two integers, is a case in point. Their concept of number was based on the ratio of two magnitudes and the square root of two presented them with a crisis which was resolved by treating it as a magnitude but not as a number. Ultimately, as a result of this problem, the concept of irrational numbers was recognised and became part of the number system. This kind of example would help to make mathematics appear more open to question in the minds of pupils, not to mention in the minds of teachers.

With the complexity arising from our epistemological approach, comes an even greater need than usual for clarity about what our intentions are in teaching mathematics. The kind of problem we are faced with in this situation is the kind to which Popper (1972) refers when he stresses the need for clarity as opposed to precision. By viewing mathematics as something other than a rigid body of facts and skills to be learned, we are introducing kinds of knowledge other than propositional as pointed out in Hirst's comments relating to the content of the mathematics curriculum referred to earlier. We move away from the precision of rigidly demarcated content to a concern not only with facts and skills but with foundations, attitudes, processes and applications. What becomes vitally important is that, with such a variety of kinds of objectives in mind we should be clear about the balance that we hope to achieve in our mathematics curriculum and to identify our objectives accordingly.

In devising a mathematics curriculum based upon a growth and change theory of knowledge, it would seem, then, that our objectives would be concerned with the following: (1) the learning of certain facts and skills; (2) an appreciation of the origin of some of the selected content, the notion of objectivity in relation to it and of any problematic aspect within it where relevant; (3) the importance of inquiry, questioning and

critical discussion in exploring mathematical ideas, stressing the inherent logic and the need for clarity of purpose and expression in so doing; (4) the application of the facts and skills learned, in problem-solving situations; (5) the formulation of hypotheses for the solution of problems and testing them. Thus our objectives would be likely to bring pupils into contact with mathematics as a kind of knowledge that can be open to question, and would involve them in thinking mathematically in such a way as to develop their understanding of the discipline. They would become more aware of its social foundations by the adoption of something of an historical perspective and by stressing the demonstrated problem-solving capacity of the subject as well as its potential for solving problems devised by the pupils themselves. They would also gain some notion of how the status of objectivity is conferred upon ideas by the criticism and testing of theories through time. The discussion and inquiry procedures that contribute to such activities would involve pupils as members of a group which would also likely lead to some awareness of the social aspect of how mathematics can develop and be done.

The aims or criteria listed earlier which thus would appear to be most directly satisfied by the probable kinds of objective projected for such a mathematics curriculum would be Aims 1, 2, 3, 4, 6 and 8. Certain basic skills and knowledge would be taught. We would clearly wish to develop our pupils' ability to think and reason more logically. Indeed, taking Lakatos' (1976) approach as an example, the logical aspect of mathematics would be one to be stressed as much in methodology as it would in content. This implies, therefore, that our methodology would have to be selected carefully to ensure that this criterion is satisfied and encouraging pupils to formulate hypotheses about how particular problems might be solved and to test their theories would be part of such a methodology. Hence they would gain an appreciation of the idea of presenting a problem in a mathematical format and experience of testing its application in carrying it through to a solution. This would give pupils experience in doing mathematics at a personal level and, at the same time, by presenting proposed solutions to problems, they would also experience mutual critical discussion about their ideas, and the potential for success of these ideas. Learning to formulate a problem in mathematical terms could go hand-in-hand with developing an appreciation of the wide applicability of mathematics in problem-solving generally. The fact that such procedures would involve rational

criticism, discussion and the sharing of ideas would also suggest that mathematics is an activity to be engaged in with other people and the communication of such ideas would clearly be a very important aspect of these procedures. In bringing some emphasis to bear on an historical perspective with respect to mathematics, the awareness of this social dimension could be developed further.

The aims which seem most at risk with the kind of objectives for a mathematics curriculum based on our adoption of a growth and change theoretical approach, are Aims 5 and 7. Mathematics as an intellectual activity in its own right would have to be stressed by not only presenting the discipline in its guise as a problem-solving activity, but also as a symbolic system which has been devised by man as a means of representing the pattern and structure in the world around him the study of which has helped to shape and control that world (Aim 5). Perhaps the development of spatial thinking might more readily be associated with this kind of activity rather than with the development of logical thinking. With respect to mathematics as a language (Aim 7), this aspect of the subject would be important to the kinds of objective related to the exploration of mathematical ideas and the formulation of hypotheses in mathematical terms. It would seem that it would purposefully have to be emphasised in method and content of the proposed mathematics curriculum or the important function of mathematics as a language, both in the representational and the communication sense, might too easily be taken for granted. Through all of this projected mathematics curriculum, the underlying notion of growth and change would be intended to provide a perspective of a living, growing discipline and, in involving our pupils in mathematics in this way, to contribute towards the development of a positive attitude on their part towards the discipline (Aim 9).

Our discussion thus far of the likely kinds of objectives for our mathematics curriculum has borne out the fears raised earlier of being presented with a situation that requires clarity and critical judgement to ensure an appropriate balance in the mathematics curriculum that, as growth and change theorists, we would consider both desirable and effective. The situation has already been clarified to the degree where the overlap of objectives with content and methodology has been noted. Pupils can only develop their ability to exercise logical thought and carry out rational critical discussion by being placed in situations of

a nature that allow this to happen. Similarly, the content and skills selected to be learned and with which such processes are carried out, must also be at hand. It could be argued that the adoption of the procedure of stating objectives at the outset of devising our curriculum might be an attempt on our part to be overly precise about what we hope to achieve. In other words, we may be placing ourselves in a situation such as Popper (1972) describes where there is an attempt to attain too great a degree of precision when it is really clarity that matters. This apparently restrictive aspect of stating objectives is a basic criticism raised by Stenhouse (1971) in past debates about rational curriculum planning. He suggests that,

"the use of objectives as a master concept will tend towards the selection of hypotheses in the light of one's hopes." (p.80)

This is clearly the case, and a desirable one, since we set out with particular goals in minds for our pupils, goals of a mathematical nature in this case, and it would be nonsensical to suggest that we do not. Our basic problem lies in the degree of precision to be exercised in selecting the content which, together with our methodology, will allow the kinds of objectives we have in mind to be achieved in mathematics. Thus we have a situation in which means-ends analysis becomes important in devising our curriculum. The situation arises where, as Pring (1971b) suggests, method and content cannot be divorced from each other nor from our objectives, since they are all logically necessary to each other. This leads us to a situation quite different from that in which we found ourselves when projecting a curriculum based upon a positivist philosophical approach because of the complexity of the choices open to us. It is a situation which would appear to justify a consideration of content and method together since they are mutually bound to such a degree.

Content and method in a mathematics curriculum based on a growth and change theory of knowledge

Taylor (1967a) describes a "learning experiences model for the curriculum" which he states draws attention "to the modes of thinking used by pupils in particular subject areas: to their reasoning in mathematics, science and history, and to their thinking strategies." (p.161) In viewing content and method as being so closely bound in order to achieve the kinds of objectives we have in mind, what we are in fact considering, are the mathematical learning experiences we plan

for our pupils. This is not to say that the distinction between content and method is blurred but that one would not be considered without the other. Bruner (1968) in approaching the process of devising a curriculum that accommodates the notion of continuous change within disciplines, considers content and method in this light. He is concerned not only with change in terms of what we know within any discipline, but also with the rate of change of society itself and this is seen by him as reason to accept the notion that education be redefined with each new generation. Ultimately, this leads him to consider an emphasis upon skills in relation to particular kinds of content as being most important and in the process, emphasis would be upon "studying the possible rather than the achieved - a necessary step if we are to adapt to change." (p.36) He approaches the problem through the development of a theory of instruction, an important part of which is the structuring of what is to be learned so that it may easily be grasped by pupils. His view of 'structure' with respect to disciplines may be helpful to us in selecting content for our mathematics curriculum.

Bruner (1968) suggests that each area of subject matter involves different "ways of thought" peculiar to it and in each there exists a "set of connected, varyingly implicit, generative propositions" which occur throughout the discipline at increasing levels of complexity from the simple and concrete to the highly abstract. (p.154) (An example in mathematics might be the commutative law, $a + b = b + a$.) He proposes the notion of a "spiral curriculum" in which these propositions are returned to repeatedly, in varying degrees of difficulty, and used in problem-solving activities in which previous knowledge is brought to bear upon immediate and new problems. Thus pupils would "be given the chance to solve problems, to conjecture, to quarrel, as these are done at the heart of the discipline." (p.155)

This would seem an approach to content and method that would lend itself to the achievement of the kinds of objectives we have projected for a mathematics curriculum with its growth and change theoretical basis. Content could be selected according to the generative power of the relevant concepts and skills and methodology would involve logical inquiry, critical appraisal, the formulation of hypotheses and other processes involved in problem-solving. Bruner (1968) states that a discipline is "given life and direction by the conjectures and dilemmas

that brought it into being and sustained its growth" and we would intend our mathematics curriculum to nurture a corresponding appreciation of conjecture and dilemma in our pupils. (p.159) Our mathematics curriculum, therefore, would be likely to consist of "exercises in conjecture, in ways of inquiry, in problem finding" in connection with the learning of selected mathematical concepts and skills. (p.160) These processes would very often involve group participation as opposed to individual pupils following a solitary path. As well, throughout all of these procedures the power of logic would be stressed, for, as Freudenthal (1973) says, "Rather than teaching logic, the mathematics teacher shall use logic and he shall make conscious to the learner that logic the learner is using." (p.661) The bringing together of content according to the criteria referred to earlier and this approach to methodology would have important repercussions for pupils, for teachers and for the classroom as a whole, a matter which will be considered towards the end of this chapter.

Evaluation procedures and the projected curriculum

The facts noted earlier in connection with the evaluation of a formalist mathematics curriculum are equally applicable to the evaluation of a mathematics curriculum based on a growth and change epistemology. These are that procedures used in evaluation reflect the kinds of objectives identified for a curriculum (Kerr 1968), and with the extent to which the curriculum is concerned with a particular kind of knowledge as well as what constitutes that knowledge (Taylor 1967b). As we have seen, our objectives would include the acquisition of skills and knowledge but would also be viewed, to a considerable extent, in terms of the processes underlying these skills and knowledge and the use to which they are put. The importance to the learning of mathematics of inquiry, discussion and postulating solutions to problems, for example have all been identified. Clearly, to evaluate the degree to which objectives of this nature have been achieved would logically require making demands upon pupils of a similar kind in a test situation in order to make appropriate judgements in relation to their respective abilities in their activities. Thus, as well as the possibility of testing the ability of pupils to recall particular knowledge and their use of skills in relation to that knowledge, we would wish to devise means to assess the degree to which they could use such knowledge in various stages of problem-solving including the identification of problems, the postulation of possible

solutions, presenting such solutions in a mathematical format and carrying through the necessary operations to find a solution. Another approach to evaluation might be for pupils to present an hypothesis in mathematical terms and to test it. Whether there would be a right or wrong conclusion in such an instance could be considered of secondary importance to the ability shown by pupils to argue their case logically and critically. Again, as with proposals for content and methodology, the kinds of objectives postulated would demand somewhat radical evaluation procedures that could differ markedly from those of a formalist mathematics curriculum. It now remains to compare the two kinds of mathematics curricula, based on different epistemologies, to see how they may differ in emphasis, generally.

A comparison of mathematics curricula based upon a positivist epistemological approach and a growth and theory epistemological approach

In projecting the possible kinds of mathematics curricula based upon the two epistemological approaches under consideration, the fact emerges that each differs markedly from the other with respect to the four components of the curriculum. The consideration of a positivist philosophical outlook in relation to objectives, content, methodology and evaluation presents a picture which is clear-cut and precise when compared with a mathematics curriculum founded on a growth and change theoretical basis. The view of mathematics in terms of formal systems, the content of which is held to be knowledge composed of unchallengeable truths and with an existence of its own, would seem to lead to a curriculum with objectives mainly concerned with the transmission to pupils of that content together with related skills. While there may be choice exercised with respect to content within different mathematical systems, the subject matter would probably be taught using an essentially didactic approach since there would be considered to be little that could possibly be open to question or discussion. The emphasis would be upon learning the content and practice in exercising the skills and the logic within each of the systems with the intention of developing proficiency in manipulative processes. Such exercises would be internal to the systems and not related to external problems. Evaluation accordingly, would test the degree of recall of facts and proficiency in manipulative skills.

To project beyond this curriculum to the classroom in which it might be implemented, the situation would likely be one in which a formal atmosphere would prevail. The formality would arise from the emphasis on didactic methods, possibly with the teacher explaining some part of the content of a section in a mathematics textbook, after which the pupils would work through a series of exercises related to that content. It would be unlikely that any discussion of the material itself might take place, nor any general debate about the content, its meaning or its relevance. Any discussion probably would be on a one-to-one basis with individual pupils asking the teacher for clarification. There would seem to be little likelihood of mathematical discussion amongst pupils themselves since there would be no encouragement to do so. At its most rigid, an adoption of the approach would not permit the teacher to go beyond the system to draw any analogy to practical, problem-solving circumstances to help the understanding of pupils. The classroom situation could perhaps almost be characterised as 'anti-social' insofar as the task at hand would be viewed in terms of individuals grappling with abstractions, the validity and form of which would not be open to question nor change, and there would unlikely be any activity other than 'book work' taking place.

A mathematics curriculum founded on a growth and change epistemological approach, as we have seen, presents a different picture. We have found, to begin with, that objectives would not be so easily definable since we would be more concerned with kinds of activity related to mathematics than with formal systems of mathematical knowledge. This suggested that it was appropriate to consider content and method together since they were not only logically linked with each other, but with the kinds of outcomes envisaged for the curriculum as well. The components of the curriculum examined in this light resulted in a view of a mathematics curriculum which was concerned with the foundations of mathematics in human thought and activity, and with the application of that thought and related skills over a variety of areas of problem-solving with which mathematics is concerned. This suggested that the criteria for the selection of content would differ from those which would guide such a selection in a formalist mathematics approach. The concern would be with knowledge and skills basic to the discipline of mathematics but selected possibly according to criteria such as their generative power or their problem-solving capacity. There would be a

degree of emphasis upon activity in connection with mathematics and the classroom atmosphere would not necessarily always be one of quiet application to book work. The teacher would encourage pupils to ask questions, to make counter-suggestions, to explain how they view problems and, generally to test their ideas in discussion. Pupils would be presented with alternatives that may have occurred in the development of particular concepts and shown how and why one theory proved more effective than another. Emphasis would be placed upon man's active involvement in the development of the abstract formal systems of mathematics and of the on-going potential for the bringing about of change within those systems. Pupils would be encouraged to apply mathematics to solving problems in experimental situations where possible and would be made aware of the potential of the subject for solving problems over a wide variety of areas. The classroom picture that emerges is one that includes varieties of activity that would sometimes involve pupils in discussion and critical argument with the teacher as well as each other, in problem-solving activities of a practical nature or in quiet book work. The prevailing atmosphere might best be described as one of active, directed inquiry.

Thus we reach the conclusion that a mathematics curriculum based on a positivist epistemology would likely result in something of an authoritative classroom situation in the sense that teachers would accept the mathematical content of their lessons as given and unchanging and not to be questioned, and would present it as such to their pupils. The pupils in turn would pursue their study of the discipline individually and somewhat passively, by accepting and learning facts and skills, and by performing the required operations. A mathematics curriculum based upon a growth and change epistemological foundation, on the other hand, would likely result in a classroom in which interchange between pupil and teacher and between pupil and pupil would be encouraged. In the course of this, pupils would, for some proportion of the time, be actively engaged in doing mathematics either at the level of critical argument related to problem-solving or solving problems in a more practical, applied sense.

While, as we have seen, differences between the two curricula exist at a variety of levels, the culmination of these differences might best be described in terms of the social context of the mathematics classroom

each would produce. On the one hand, the choice of objectives, content and methodology results in a social context that appears to stress the isolation of the pupils but not in such a way that their individuality is taken into account in terms of their ideas, thoughts or problems with regard to the learning of mathematics; nor is the atmosphere conducive to inter-personal exchange amongst pupils themselves or with the teacher. On the other hand, we have identified a social context which results in the sharing of ideas and problems amongst pupils and between pupils and teacher, so that the learning of mathematics becomes essentially a 'social activity' in an education context; inter-personal exchange is positively encouraged as part of the learning process through critical, rational discussion. Clearly these different social contexts reflect different roles for both teachers and pupils as well as divergent views of the nature of the discipline and ultimately must affect the quality of the mathematical learning that takes place. As such, therefore, the notion of the social context of the mathematics classroom is a phenomenon of some importance.

CONCLUSIONS

The argument presented thus far in this study has identified certain aims or criteria of worthwhileness for mathematics education. By examining two influential sources of aims, criteria of a social nature have been identified and in particular, a criterion which concerns the aspect of mathematics as a social activity in its existence and in its conduct. We have gone on to examine what the implications for a mathematics curriculum inherent in such a criterion might be, by considering (a) epistemological matters in connection with mathematics as a social activity in its existence, and (b) in the light of this, mathematics as a social activity in its conduct as evinced in the mathematics curriculum. With respect to the former, two epistemological viewpoints were considered. Firstly, the interpretation of mathematics of logical positivists was put forward, where mathematics is presented as a body of formal systems of immutable truths, with an existence of its own and wherein the social foundations of mathematics as a form of thought are not acknowledged. We then examined alternative epistemological foundations referred to as 'growth and change' theories of knowledge, which support the notion of mathematics as a social phenomenon in its existence. Following this, the likely effects of both points of view upon the mathematics curriculum

were explored in order to gain some insight into the relationship between each of the epistemological approaches and mathematics as a social activity in its conduct. This was done by projecting the kinds of mathematics curricula that could arise from each of the approaches. The projected curricula have been found to be markedly different, with the differences culminating in contrasting social contexts in which the teaching and learning of mathematics could take place.

Having identified the potential importance of the 'social' criterion for worthwhile practice in mathematics education, we shall consider in the next chapter two influential mathematics curriculum development projects that have been undertaken in the last two decades. They will be examined with the intention of determining the degree to which the aims satisfy the criteria identified in Chapter 1 as presenting an acceptable balance of the kinds of aims to be pursued in a mathematics curriculum. Underlying these considerations will be a search for an acknowledgement, in particular, of the criterion which stresses mathematics as a social activity in its existence and its conduct.

CHAPTER 3

A Consideration of the Aims of Two Mathematics
Curriculum Development Projects

Introduction

It was noted in Chapter 1 that the 'rational' approach to curriculum planning would be adopted for purposes of reference and discussion in relation to the curriculum process throughout this study. This approach involves the identification of aims and objectives, the selection of content and methodology in accordance with these and finally, evaluation procedures. (Kerr 1968) The value of such a paradigm is that it separates out steps of the curriculum process so that each may be considered in the light of any educational theory that may be relevant. While it has the attraction of apparent simplicity, however, not all curriculum innovation necessarily follows this pattern.

Schwab (1969) commenting upon early efforts in curriculum renewal in the U.S., points out that each of the early projects was founded in a different theoretical perspective, e.g. psychological, sociological or historical, and concerned a different subject in the curriculum. The complexity of the situation in which any curricular innovation takes place, he suggests, cannot afford to adopt a single theoretical basis and ignore others; rather it may be that all will be involved. He states that "a curriculum grounded only in a view of social need or social change must be equally doctrinaire and incomplete" (p.23) and goes on to predict that, "It is equally clear, however, that there is not, and will not be in the foreseeable future, one theory of this complex whole which is other than a collection of unusable generalities." (p.24) Schwab (1969) views education as essentially a practical enterprise and his approach to curriculum development follows on this in that he suggests it, too, should be approached in a practical manner and should be eclectic in its dependence upon theoretical considerations. For example, in constructing a mathematics curriculum for 9 to 11 year olds, the aim (guided by philosophical considerations) referring to the acquisition of knowledge and skills 'at an appropriate level' would lead us to consider the content to be adopted in the light of what psychological learning and development theory can tell us about how individuals learn

mathematical concepts at that stage of development. The rational paradigm for curriculum change allows this to happen at each stage of the process. The first stage of any proposal for curricular innovation, as Griffiths and Howson (1974) point out, "should be accompanied by some plan of action outlining the aims of the project", followed by other considerations. (p.146) It is with this prescription in mind that we shall approach an examination of the aims of two mathematics curriculum development projects.

In concerning ourselves with aims already identified as criteria for worthwhile practice in the mathematics curriculum (see Chapter 2, p.49), we are suggesting that they present an informed consensus with respect to the balance (in terms of kinds of consideration that need to be taken into account) that is desirable in devising such a curriculum. These aims are seen as acting as guidelines in the selection of what is to follow in implementing the curriculum. The purpose of using them as a reference point against which to examine the aims of the two projects in the following sections is not connected with measuring the effectiveness of either project in terms of outcomes they may have had. Rather, they are fulfilling an evaluative role of a kind that is concerned with the statement of the *intentions* of each project, insofar as they are held to be manifestations of an informed and balanced approach to the construction of a mathematics curriculum. We shall examine them in particular, for the inclusion of some consideration of the 'social' criterion, the importance of which has been identified for the mathematics curriculum and conclude by drawing implications for the social context of a mathematics classroom in which the approach of either project might be implemented.

It has already been argued in the selection of these aims as criteria that, provided the phrase 'at an appropriate level' prefaces each, the list is applicable to a mathematics curriculum for pupils of any age. (See Chapter 1) It is, therefore, considered acceptable to use these aims with reference to two curriculum development projects intended for different, but overlapping, pupil age groups. (Rather than repeating the aims here, the reader is referred to Chapter 2, p.49, where the list appears.

The aims of the Nuffield Mathematics Project (hereafter referred to as the N.M.P.) and the School Mathematics Project (hereafter referred to as the S.M.P.) will be treated separately. Each will first of all be examined against the aims or criteria identified earlier to determine the extent to which these are satisfied. The likely effect of the possible exclusion of any of these on the balance of the mathematics curriculum will be considered, followed by the examination of the relationship between the aims of each project and any theoretical considerations which may have informed their selection. Implications for the use of each set of aims or criteria as guidelines for teachers in implementing the mathematics curriculum will be discussed and finally, the social context of the mathematics classroom which each is likely to produce will be projected.

The N.M.P. and the S.M.P. have been deliberately selected for the contrasts they provide. The N.M.P. was a publicly funded project carried out under the aegis of the Department of Education and Science. It could be described as a 'total' project in that it set out to change not one, but several aspects of the mathematics curriculum for 9 to 13 year olds, including methodology and content. Materials were to be produced for teachers but not for pupils. Stress was placed upon helping teachers to understand how a new approach to the subject as a whole was to be implemented through a particular methodology. The project was developed in state schools and had a specific life span.

The S.M.P., on the other hand, is a privately funded enterprise which set out to formulate a new syllabus for 11 to 18 year olds and thus has been content-oriented. In this case, the production of materials for pupils has been the focal point of the project and schools initially involved in their development were mainly in the private sector of education. The project is an on-going, commercial concern and carries on at present revising old materials and producing new ones.

Although the N.M.P. has tended to be linked with primary schools, it set out to cater for 11 to 13 year olds as well. Hence there is an overlap in the age range of pupils for whom each project was intended, as noted above, which strengthens the validity of using a common set of criteria for examining the aims of the two projects.

THE NUFFIELD MATHEMATICS PROJECT.

There is something of an anomaly in the fact that the N.M.P. has not attracted the attention of researchers either in curriculum development or in mathematics education. For a project that was held to have had potential for fundamental change in mathematics education in the primary years and beyond, studies and investigations concerning the nature and extent of this effect might reasonably have been expected. Writing about the project and referring to the obsolescence both of ideas and materials in old syllabi, Bassett (1971) states that there was a "recognition that, although the problem was with secondary-school and university-level studies, the reform of the content and the spirit of the subject in its introductory phase gave the best promise of improvement." (p.474) In spite of the obvious importance attributed to what was being attempted by the N.M.P. in introducing change in the earliest years of schooling, it was not until 1975 that someone outside the project (but attached to the Nuffield Foundation) attempted a critique of the project as a whole. Hewton (1975), who undertook this exercise, suggested at the time: "A great deal has been written about 'Nuffield Mathematics' in the form of articles and comments in the press, journals, teachers' bulletins, etc. but despite the fact that 10 years have now elapsed since its inception, there has not appeared a reasonably detailed profile of the Project, its cost and its results." (p.407) The purpose of his paper was to rectify that omission, and as it contains the most detailed and comprehensive analysis of the project available (indeed, apparently the only one), reference to the paper will occur frequently in the present discussion.

A brief descriptive profile of the N.M.P. is to be found in the publication of the Schools Council Information Centre of July, 1977. The duration of the Project is given as 1964-71, the age range of the pupils for whom it was intended is stipulated as 5-13 year olds and the grant is listed as "Nuffield Grant only". (Schools Council 1977, MA 05 01) The information relating to the grant could be somewhat misleading since it may too readily be assumed that the Nuffield Foundation undertook full responsibility for the project, while this was not the case. As Hewton (1975) states, "it should be noted, with regard to the overall responsibility of the scheme, that although the Foundation assumed full financial responsibility, the venture was, in fact, jointly sponsored by

Nuffield and the Department of Education and Science." (p.410) He goes on to point out that, "In real terms, D.E.S. sponsorship meant liaison (eventually through the newly created Schools Council) with interested L.E.A.'s, and advice and assistance with in-service training for teachers participating in the scheme. They were not, however, involved, with any financial grants to the project."

Reference to the 'newly created Schools Council' which was formally constituted in 1964 helps to place the N.M.P. in perspective within the curriculum development movement in this country over the past two decades. The very first Field Report of the Schools Council, entitled "New Developments in Mathematics Teaching" is described on the cover as "a first progress report on the joint Schools Council-Nuffield Foundation Project". (Schools Council 1966) The project was one of the earliest to be launched on a national scale and by the end of its 'official' life in 1971, 'Nuffield maths' had tended to become linked with much of modern mathematics undertaken within primary schools. For example, Williams (1971), in a discussion relating to primary mathematics and "Curriculum for the 70's", reports a reference to the fact that "the 1960's might be called the Biggs/Nuffield era..." (Miss Edith Biggs, H.M.I., having been a main instigator of change in primary mathematics education that led to the launching of the N.M.P.). (p.5) The area of inquiry is described in the Schools Council Profile as being concerned with devising a "contemporary approach to mathematics for children from 5-13". (Schools Council 1977, MA 05 01) The first project team consisted of three primary school teachers, one junior school inspector, one secondary teacher and one College of Education lecturer. (Hewton 1975) The project materials were comprised, at the outset, entirely of guides for teachers. These are described in the Schools Council Profile (1977) as "having been written by teachers" whereas Hewton (1975) refers to teachers being involved in the development of the materials "in a participatory role". (Hewton 1975, p.412) This makes it somewhat unclear as to whether or not the official Profile is referring only to teachers who were members of the project team as being directly involved in the production of materials. By 1967, the project organiser recognised a considerable demand for some means of assessing pupils' progress in mathematical understanding. (Hewton 1975) Since the work of the team was heavily influenced by the work of Piaget and his colleagues at the Institut des Sciences de l'Education in Geneva, they were approached and

asked in the project organiser's words, to "prepare evaluating procedures which would be related to the needs of teachers following the Nuffield guides." (p.421)

All the guides eventually produced fell within four categories: (1) Introductory guides; (2) Teachers' guides; (3) Weaving guides; (4) Check-up guides. There were three Introductory guides provided, the first of which ("I Do - and I Understand") set out the aims and rationale of the project. The second related the project to middle school and secondary years while the third was intended to explain the project to parents. The purpose of the Teachers' guides was to elaborate upon three main topics, 'Computation and Structure' (five guides), 'Shape and Size' (four guides) and 'Pictorial Representation and Graphs leading to Algebra' (three guides). Concepts within these topics were to be introduced repeatedly at different stages to enable the mathematics to develop in a spiral fashion. The Schools Council Profile (1977) states that, "The same concept is met over and over again, but illustrated in a different way at each stage." Only two Weaving guides were produced to give teachers guidance on specific subjects; these were 'Desk Calculators' and 'How to Build a Pond'. Three Check-up guides were published, Checking-up I, II and III, and were intended as forms of individual assessment for pupils. Hewton (1975) quotes the project organiser as describing the aim of the check-ups as to "try to show that children acquire concepts gradually and point to the difficulties they are likely to encounter during their progress" allowing the teacher "to judge just where the child is in his normal development in order to put him in an appropriate play situation, or to give him the right practice." (p.421)

After the 'official' life of the project had ended in 1971, five modules consisting of sets of cards for use in middle schools were published in 1973 and four sets of problems together with teachers' books. Other books connected with the project and aimed at parents and teachers, are also listed in the Profile, as well as films and a chart. The Profile also includes information about training and dissemination in connection with the project which, at that time, was still carried out in the form of regional courses and conferences. Thus although the 'official' life of the project ended in 1971 clearly a considerable amount of activity was carried on after that period. A final

important point to be noted in connection with the project is that, as a result of its inception, teachers' centres first came into being. They were originally conceived of as providing a milieu and focal point for the in-service training of teachers involved in the project and, as Hewton (1975) puts it, "The centres were not meant simply for courses; they were considered essential for backing up the work already started in the initial training period. They facilitated discussion and collaboration in producing learning materials and using equipment." (Hewton 1975, p.413)

Aims of the N.M.P.

The aims of the N.M.P. appear in the first introductory guide published in 1975 entitled "I do - and I Understand". (Nuffield Foundation, 1965a) (Possibly of some significance is the fact that the word 'teaching' appeared in the title of the project in some of the early drafts of materials but was soon dropped.) The first section of the guide gives a brief description of the general philosophy underlying the project and stresses how pupils learn mathematics and not what to teach. They state that "Running through all the work is the central notion that the children must be set free to make their own discoveries and think for themselves, and so achieve understanding, instead of learning off mysterious drills." (Nuffield Foundation, 1965a, p.1)

The project aims are imbedded in the second section of the guide under the title of "The approach to mathematics in the primary school". It is difficult easily to extract specific aims from such description since the language used tends to cloud the clarity of what is intended. However, the following extracts from the introductory guide suggest a list of five areas of mathematical development held to be important by the project team. Firstly, they see the justification for the inclusion of mathematics in the curriculum inherent in the "notion of pattern and relationships, for this is how mathematics has enabled man to discover something of the shape and pattern of the universe, and so move towards the gradual mastery of his environment." (Nuffield Foundation 1965a, p.4) They then suggest that the approach to the mathematics curriculum should be "akin to that of a science" where pupils are involved in experimentation, formulating hypotheses, testing them and communicating the results. This would introduce pupils to inductive reasoning. Next,

they suggest the importance of deductive reasoning and state that "All thought processes are seed beds for the growth of logic, and all practical investigation leads to thinking. Fostering the development of children's thinking implies fostering the growth of logic." (p.5) Communication is also seen to be important and pupils should be introduced to its function in mathematical situations firstly in words and then in symbols. The importance lies in the fact that "This recording of an experience is an early stage in the use of mathematics as a language." Finally, it is suggested that in the approach to mathematics being adopted, "the most vital factor" is entailed in the answer to the question "does the child *enjoy* and *succeed* in his work?". (author's italics) A discussion then follows concerning the adverse attitudes held by pupils towards mathematics in the past. The suggested reason for such attitudes is that "These children simply do not understand either the work they were required to do or what, in fact, mathematics was all about." (pp.5-6) Thus, there is a clear emphasis on the promotion of positive attitudes in pupils towards mathematics.

The following five aims for mathematics education for 5-13 year olds emerge:

- (1) to develop an awareness in pupils of the importance of mathematics by illustrating that, through a study of pattern and relationships, man has been enabled to order his experience and to control his environment;
- (2) to give pupils the opportunity to experiment and in so doing, to adopt a scientific approach and develop inductive reasoning;
- (3) to develop the ability of pupils to think logically by carrying out practical investigations;
- (4) to develop in pupils an appreciation of mathematics as a language through a variety of means of communications (e.g. verbal, graphical, written);
- (5) to develop a positive attitude on the part of pupils towards mathematics through enjoyment of, and success in, their work, based upon the achievement of understanding.

A comparison with the aims for mathematics education which we have adopted indicates some similarity between the two with the above five aims satisfying to some degree, some of the criteria. These are concerned with the following areas: (a) the development of logical thought

(although there is no reference to spatial thinking in the N.M.P. aims); (b) the development of an appreciation of mathematics as the result of human endeavour enabling man to control his environment; (c) the appreciation of mathematics as a language system with an emphasis on communication skills; (d) an awareness of mathematics in the environment through practical investigations and (e) the development of a positive attitude towards the subject.

The four criteria not accounted for are (a) the acquisition of basic skills and knowledge for everyday life; (b) the acquisition of further skills and knowledge pertinent to particular courses and careers; (c) the notion of mathematical models and their pervasive potential for problem-solving, and (d) mathematics as a social activity (excluding the communication aspect of this criterion which is accounted for).

It would appear that five out of nine of our criteria are satisfied by the aims of the N.M.P. However, Aim 2 which refers to the acquisition of skills and knowledge 'pertinent to particular courses and careers' is not one which, in the normal course of events, would be held for all pupils of the 5-13 year age range, so that we could consider five out of a possible eight criteria to be satisfied. It must also be noted that in proposing the aim of developing a positive attitude towards mathematics, the project team suggest two ways in which this might be accomplished. They are as follows:

- "(1) At all times and at all levels children should have a real understanding both of the problem involved and the possible ways in which it might be approached.
- (2) Means be found to enable children to gain some insight into the nature of the subject - that it is forged for man's purpose and therefore variable and that it is an imaginative and creative subject and therefore fascinating." (Nuffield Foundation 1965a, p.6)

Thus although not proposed as an aim of the project in itself, there is the suggested awareness of mathematics as a social activity in its existence and as such, open to change, contained in the second of the above statements. However, this is not apparently held to be an aim in itself but is associated only with the aim of developing positive attitudes towards mathematics on the part of pupils.

A consideration of the aims of the N.M.P.

As stated at the outset of this chapter, our concern with the examination of the aims of the projects under consideration is not to be seen in terms of any attempt to measure the effectiveness of either project in terms of what they may have accomplished. Rather, the concern is to use the aims as a perspective from which to view the development of a mathematics curriculum and in this sense, it is a prior question we are posing and attempting to answer by such an examination. We have identified criteria that specify the kinds of mathematical considerations which appear to have some consensus in providing an acceptable basis for such a curriculum (eight of the nine having been proposed by the Mathematical Association, 1976). Our considerations of the aims of the N.M.P. must turn now to comparing them with our criteria to determine (1) the potential effect upon the balance of a mathematics curriculum of the inclusion or exclusion of certain aims or criteria, (2) the theoretical considerations guiding such choice, and (3) how these aims would fulfill their role in offering guidance to teachers in constructing or implementing a mathematics curriculum. In short, what we are concerned with is the adequacy of the aims offered with respect to these three factors. Finally, we shall conclude by projecting the kind of social context that would be likely to arise in the classroom in adopting such an approach to the mathematics curriculum.

Balance within a mathematics curriculum guided by such aims

While superficially, the aims of the N.M.P. might appear to suggest a practical approach with an emphasis upon learning by doing in the mathematics curriculum, this impression is weakened on further examination. To begin with, none of the aims has a cognitive element. There is no reference to the need for pupils to know certain facts or to have certain skills in order to carry out practical investigations or to formulate hypotheses in mathematical problem-solving activities. Indeed, mention of pattern and relationships is the closest the authors come to suggesting anything that might characterise content as mathematical. Without a framework of relevant knowledge and skills, both problem-solving activities and practical investigations could take place within many other contexts, for example environmental studies. Both of these aims stressing activity raise an issue dealt with by Hirst (1974) in a

discussion of curriculum planning. He suggests that in planning the content and methods of a curriculum in order that pupils may solve problems, both of a practical and a theoretical nature, we must first of all decide "what pupils need to know in terms of sheer fact". (p.6) Thus to talk of aims of a mathematics curriculum in terms of practical investigation and problem-solving without also including aims which refer to knowledge and skills is, to use Hirst's phrase, to engage in "logical confusion". The assumption made by the authors that because practical investigation involves thinking, the development of logical thought necessarily follows, seems to compound this confusion. Matters must be organised in such a way as to allow this to happen which assumes a conceptual schema on the part of pupils; this, in turn, is dependent upon knowing certain facts and having certain skills.

We must also note the lack of an aim that stresses the wide application of mathematics in problem-solving generally. Rather than the *applicability* of mathematics, what seems to be stressed is the *extraction* of mathematics from the environment by means of active investigation and experiment. This, once again, is difficult to do without the ability to identify what aspect of a situation characterises it as mathematical and subsequently, to develop that characteristic in more sophisticated terms. (Although the Schools Council Profile (1977) describes the repeated introduction of concepts so that a spiral effect occurs in the development of mathematical understanding as a characteristic of the project, this does not come through in the early literature of the project itself.) At its simplest, in a situation where a pupil is faced with finding the rod in a set of rods which is the longest, the mathematical element of that situation is the concept of size or dimension. However, in order to progress beyond this mathematically, the pupil must become aware of mathematical attributes at different levels of complexity, for example in quantifying size and in qualifying it in simple terms first of all and eventually in a variety of ways such as area, volume and surface area. Once such knowledge is extracted from the environment by investigation (and it may or may not be correctly extracted), it then becomes learned through repetition (i.e. practice) and application in a variety of problem-solving situations (i.e. it is generalised). (Hilgard and Bower 1966) Without such repetition and generalisation, it is unlikely that the mathematics extracted from a situation will properly be learned.

The lack of an aim which emphasises mathematics as a social activity in its existence and conduct (or, indeed, its application) seems something of an anomaly in the N.M.P. perhaps because the project tends to be characterised in terms of 'activity'. We can detect something of this aspect of mathematics, in a sense lurking in the background of statements made in connection with the promotion of a positive attitude towards the subject. Some hint of this is suggested by viewing mathematics as generated for man's purpose and therefore open to variation and linking this with the achievement of understanding. It appears, however, that this path to increased understanding on the part of pupils may have been sacrificed for that of the pupils' enjoyment and anticipated success in the mathematics they do. It is difficult to accept enjoyment as a necessary criteria for increased understanding of mathematical concepts. Theories of learning generally agree success will reinforce learning provided pupils are aware of that success (Hilgard and Bower 1966) but we have already noted that mathematical theorists regard the enjoyment of mathematics as too ambitious a goal to be held as an aim in itself (e.g. Freudenthal 1973). We must seriously question whether or not it is reasonable to depend upon such enjoyment to increase mathematical understanding and whether such understanding necessarily follows upon taking pleasure in an activity.

It could be said that the social nature of 'doing' mathematics is catered for insofar as pupils are engaged in extracting mathematics from their environment but there is little evidence that the intention is that what they do will be *shared* with others except insofar as it is recorded. As Davis (1967) points out, "the Nuffield Project seeks to identify clearly-defined developmental stages in the child's growth, and to hang its curricular plans on these pegs - individualizing for each single child separately, so that the children do *not* move together as a group." (Davis 1967, p.35 author's italics) If this is the case, then it would appear to preclude any sharing amongst pupils (through meaningful discussion or otherwise) of the mathematics extracted from an investigation or experiment.

On balance, our examination of the aims of the N.M.P. thus far tends to reinforce the statement of a general description of the project which says, "The stress is on how to learn not on what to teach." (Schools Council 1977, MA 05 01) This emphasis has prompted the suggestion by

Bassett (1971) that "the project gives pride of place to the way in which the subject is learnt" and therefore it is difficult to decide whether to classify it as curriculum innovation or as basically concerned with methodology. (p.475) This would appear to be a valid point since four out of the five aims identified for the project are activity-oriented and do not make mention of content; the fifth, as we have seen, refers to pattern and relationships in the environment. Thus the balance of the aims is heavily loaded on the methodology side. It could be said that, as a result, they present a picture of a mathematics curriculum in which pupils are engaged in activities from which they are to extract mathematics without having acquired any prior mathematical knowledge or skills as tools with which to carry out such work. Whether or not this is a realistic representation of what was intended will emerge from further discussion.

The theory underlying the N.M.P.

The statement in the guide 'I do - and I Understand' which stresses that whatever their developmental level, pupils "should have a real understanding" of the mathematics with which they are involved reflects the fact that the project team was heavily influenced by the work of Piaget. (Nuffield Foundation 1965a, p.6) Hewton (1975) states that "early on, for most of those involved, Piaget's work was accepted as having direct relevance for what they were about to do." (p.411) In order to gain some appreciation of this relevance, it is necessary to consider briefly some aspects of Piaget's developmental theory with respect to intellectual growth in children.

Piaget views the intellectual growth of children as involving stages of a pre-determined order beginning with the sensori-motor, passing to the pre-operational stage, followed by the concrete operational stage and finally, the stage of formal operations. (Inhelder and Piaget 1958) It is held that an individual cannot progress from one stage to another until an adequate level of development in the preceding stage has been reached. It is vital, in attempting to grasp what Piaget envisages in such developmental phases, to understand what he means by the term 'operational'. He writes in terms of 'operative knowledge' which is constructed by the individual through action and he suggests that the logico-mathematical operative knowledge is developed through action upon

physical objects rather than merely the perception of properties of physical objects. The 'fiveness' of five is grasped by young pupils through active experience in which they 'operate' on various sets of five objects, whether buttons, rods, marbles or whatever. Thus operative knowledge is gained initially by the active interaction of the individual with something in the environment. However, Piaget views action not only in terms of physical activity but he sees it essential that it be followed by reflection upon the physical activity. Hence the word 'operational' in the Piagetian sense denotes that an individual's behaviour is not simply goal-directed but that it has become 'internalised' and reversible. Bruner (1963) interprets this quality of internalisation of knowledge to mean "that the child does not have to go about his problem-solving any longer by overt trial and error, but can actually carry out trial and error in his head." (p.36) Reversibility involves the individual in perceiving, where relevant, that an operation can be compensated for by an inverse operation. Thus at the concrete operational level, the pupil will relate 'fiveness' to the immediate perception of five objects while at the formal operations stage, the notion of five can be conjured up without the presence of five objects. Operative knowledge at the concrete level will enable the pupil to have grasped that, upon the removal of one object from the set of five, the 'fiveness' can be restored by replacing it with another.

Operative knowledge thus is viewed within the context of cognitive development as the result of physical action of the type just described together with a mental kind of action which is referred to as 'reflection', which become internalised over a period of time. Tamburrini (1978) states that "Piaget's notion of action is not synonymous with physical activity. Cognitive development requires both physical manipulation and reflection upon action." (p.98) In discussing children's memory in connection with the learning of logico-mathematical operations Piaget and Inhelder (1973) refer to the "causal sphere" of children's operations and whether they are pupil-instigated or teacher-instigated. They state that "it is essential to put a series of discreet questions or to demand a symbolic reconstruction, for in these alone can the child manifest his actual remembrance of causality, i.e. his remembrance of his own interpretation of the sequences he has observed, and not merely his remembrance of the results." (p.211) Hence the understanding, rather than mere memory of results, comes with what follows an action when the pupil

is presented with a situation by the teacher and is not one of the pupil's own devising. To ensure an understanding of the activity, reflection on the part of the pupil of what transpired in the course of performing that action is necessary.

Davis (1967) writes that "The kindest thing that can be said of Piaget's writing is that, at least in most English translations, it is obscure and frequently misunderstood." (p.38) These misunderstandings often based on over-simplification, clearly are crucial when they are put forward as guides to educational practice, as for example, in the report of the Plowden Committee where it is stated, "Until a child is ready to take a step forward, it is a waste of time to try to teach him to take it." (D.E.S. 1967, p.25, para 75b) Isaacs (1960) was already drawing attention to the dangers of such over-simplification seven years before the Plowden Report appears. Referring to Piaget's theory of developmental stages in the intellectual growth of children, he writes:

"Many of those who have followed Piaget's work have tended to see this slow inward process in *antithesis* to the action from without of teaching and education. Thus it has seemed as if the latter's scope were being challenged and indeed radically limited by the boundaries now apparently set by the true reality within. What could be taught appeared to become something extraneous and superficial which was meaningful only if it followed in the wake of each stage of inward growth and merely exploited what each of these made possible." (Isaacs 1960, p.34 author's italics)

The suggested 'waste of time' referred to in the earlier extract from the report of the Plowden Committee provides an example of how radical limitations might come to be placed upon educational practice as a result of the application of a simplistic interpretation of Piaget's developmental theory.

In considering the aims of the N.M.P. in the light of such a theoretical rationale, we are drawn to the conclusion that they, too, may be based upon an over-simplification of Piaget's thought applied to the mathematics curriculum. The stressing of activity on the part of the pupil within the project is clearly related to the notion of the individual attaining operative knowledge by interacting with the environment. However, what seems to have been neglected in the statement of aims is the important second attribute of operative knowledge, namely the reflection upon an activity by the pupil once it has been carried

through. If the project were adequately to satisfy the theoretical rationale upon which it is based, some mention in their intentions (which are heavily activity-oriented as we have seen) should be made of the importance of such reflection in order not to be misleading about what is involved. Without this, the whole Piagetian notion of 'action' is open to misinterpretation. While this may have been considered to be inherent in the aim which stresses mathematics as a language and therefore, as a means of communication, clearly this aim could too easily be interpreted in connection with the *results* of activities alone which would not qualify as reflection upon the procedures taken to achieve those results.

It seems relevant to refer to the language in which the aims of the N.M.P. are couched and to be reminded that those we have identified have been extracted from such statements as "Fostering the development of children's thinking implies fostering the growth of logic" and references to matters such as pupils being "set free to make their own discoveries". (Nuffield Foundation 1965a, pp.5,1) Statements of this kind do not do justice to the notion of activity or action as interpreted in the theory upon which they are based and consequently such statements of intent become educationally meaningless. They do not suggest criteria that would make such practical investigation and 'discovery' purposeful in leading appropriately to the cognitive development of pupils. Davis (1967) writes of Piaget's work in connection with mathematics, "Many important aspects of mathematics remain untouched, and in the case of some others the analogies with Piaget's tasks may be misleading rather than illuminating." (p.38) It would seem that the aims for N.M.P. as they are set out, may be a case in point. To refer again to Peters' (1973) injunction that aims must help us to focus on neglected priorities, in this instance they have failed in this respect in neglecting to identify an important aspect of Piaget's notion of 'action'.

N.M.P. aims and guidance in planning a mathematics curriculum

We have argued that a major function of a statement of aims in mathematics education is to provide guidelines in constructing a mathematics curriculum. We shall now consider the N.M.P. aims in this light.

A major lack of balance in the aims in favour of methodology and

to the exclusion of content has already been noted. A teacher or curriculum developer faced with such a list of guidelines could well decide that mathematics would not be taught separately but would become integrated with a subject such as environmental studies, similar to the approach advocated by Kline (1973). There is nothing which suggests what notion of content there might be from this list drawn from the first Introductory Guide. However, in the second such guide ("A Look Ahead") the question is posed, "What's the philosophy behind all this?" and part of the answer that is given states, "Much 'old' mathematics is still relevant, but new ideas are worth pursuing if they generate interest and understanding." (Nuffield Foundation 1965b, p.9) This raises the point that some, at least, of the content will be new and this, together with the essentially new methodological aims, caused Hewton (1975) to suggest that "from the teachers' standpoint the implied reforms were extensive." (p.411) Although there is no mention in the aims of concepts or skills, Hewton (1975), again referring to the demands on teachers, goes on to state that "In terms of content it seemed that they would be required to master and then to present 'new ideas and unifying concepts' whilst at the same time showing the relevance of these to the child's everyday life." None of this is even hinted at in the identified aims. It seems that they were not intended entirely to relate to mathematics as most teachers probably knew it. Even if the aims were to relate to a traditional view of mathematical content, however, there is little offered in the way of guidance as to how that content should be selected.

It has been noted that the theoretical rationale upon which this project is based has been interpreted somewhat incompletely. This would be reflected accordingly in any mathematics curriculum using the aims as guidelines insofar as it would stress the 'doing' part of activities engaged in by pupils and neglect the identification of conceptual content and skills or of the relevance of the actions involved in their investigations. There is no indication of the importance for pupils of following their activities with carefully designed procedures to enable them to recall each stage of such activities. The lack of a cognitive element in the aims and a lack of reference to the applications of the discipline as a tool in problem-solving, combine to produce a view of mathematics which, at best, may be described as nebulous. The danger of setting out aims in such a fashion is that teachers might too easily be tempted

to assume that pupils already have the basic knowledge and skills to engage in mathematical activity of this sort or that they need not be taught separately.

Finally, we must consider the lack of guidance contained in the N.M.P. aims with respect to the social nature of mathematics. The lack of any reference in the aims which may be construed as interpreting mathematics as a social activity in its existence and conduct may be worthy of special notice in a curriculum with a methodological bias stressing activity since again, a misguided assumption may be made that 'activity' in the context of a classroom implies a social situation. In fact, as we have seen, the kind of activity intended is with the development of individuals in mind and is not a socially oriented mode of learning. (Davis 1967) Investigation and experiment may, at times, be 'social' in the sense that they are not necessarily carried out by solitary individuals and that pupils *may* discuss what they are doing. However, the question would be how much of what they are doing could be characterised as mathematical and how they would recognise it as such. Once again we are drawn to the lack of a cognitive element in these aims and the lack of reference to relevant concepts and skills which would lead to the recognition of what is mathematical and what is not. It is clearly impossible, if aims are not to include mathematical knowledge and skills, for pupils to become aware of how these may have been socially determined and open to change and development.

Social context of the mathematics classroom

An examination of the aims of this project and the theoretical rationale underlying it have illustrated how the view adopted of mathematics as a subject may affect what is to follow in the classroom in terms of the social context in which mathematics will be taught and learned. The stress on 'activity' would seem to imply considerable exchange taking place amongst pupils and between pupil and teacher and would require considerable resources in terms of concrete learning materials for purposes of experimentation and investigation. Although the suggested result may appear to be a classroom atmosphere characterised by activity and discussion, there is some cause for concern with respect to the context projected and the concern is three-fold.

Firstly, it is open to question, as we have seen, what proportion of the content of any discussion that may take place could be said properly to be mathematical, given what appears to be a lack of emphasis on cognitive aspects of the discipline contained in the aims. It seems possible that the pupils may not be given the kind of concepts and skills necessary to render any exchanges mathematically meaningful and such interchange ceases to be a desirable feature of the classroom if a fair proportion of it is not directed towards mathematical ends. Secondly, the strong emphasis on 'doing' suggests a lack of the kinds of activities that would satisfy the demand inherent in a Piagetian view of action for reflection and upon the relevant mathematical learning at various stages throughout the investigations and experimentation undertaken. While there is an emphasis upon communication in a variety of forms such as verbal, graphical and pictorial, this suggests that communication may tend to be concerned with results alone and be of a representative nature. There is no specific mention in the aims, for example, of recording by the pupils *throughout* the activities in which they are involved, and which would suggest some opportunity for the shared consideration of the mathematics that has been undertaken. Thirdly, it has been suggested that the teacher's role is to guide the pupils through their mathematical experiences and we have noted that for this to be appropriate to the pupils' logico-mathematical development, this should take place at an individual level. (Piaget and Inhelder 1973) In the classroom context envisaged, there would seem little opportunity for this to happen and it would be extremely difficult for the teacher to manipulate matters in such a way as to ensure that it could. At the same time, it is the individual pupil for whom the project is attempting to cater. (Davis 1967)

The view of mathematics as founded in individual action and presented by the N.M.P. suggests, therefore, a social context in the classroom which, ironically, could easily militate against the learning of the discipline by individual pupils. The context would appear to present an imbalance in favour of activities on the part of pupils engaged in 'doing' but without the appropriate balance of activities of a consolidating nature at the individual level which would involve sharing their experiences through discussion, the posing of questions or the description of what they have done. The demands upon teachers in such a context in terms both of mathematical knowledge and of organisational skills

would be great. It would seem open to question how, in such a context, they could ensure that an acceptable proportion of classroom activities and talk could be said to qualify as mathematical. More importantly, it would be difficult to ensure that pupils themselves could identify the 'mathematics' within the activities they may have undertaken.

Summing up

What we have been engaged upon in this section is an attempt to determine the adequacy of the aims of the N.M.P. in terms of (a) the balance they present with respect to the mathematics curriculum, (b) the theoretical rationale upon which they are based and (c) their efficacy in providing guidelines for the development of a mathematics curriculum. Finally, we have attempted to project the resulting social context in the mathematics classroom that such a view of mathematics might produce. The overall impression gained is one of a lack of an appropriate degree of clarity and critical thought in relation to how the aims are stated and a lack of the direction and content which Peters (1973) advocates in educational aims. There would appear to be a considerable lack of balance in what the project set out to achieve in terms of kinds of mathematical considerations to which pupils were to be introduced. The most obvious gap is the lack of reference to knowledge and skills and to the applications of mathematics in problem-solving situations. Some consideration of the inclusion of the social criterion is complicated by the stressing of activity methods, but on examination, it would appear not to have been met.

The N.M.P. appears to be an example of the pitfalls referred to by Schwab (1969) that occur as a result of basing curriculum development on a single theoretical foundation, in this case the developmental psychology of Piaget. Although Piaget (1972) refers to his theories as developmental epistemology there is reflected in the N.M.P. aims a distinct lack of epistemological considerations in the more conventional sense. The majority of aims are concerned with methodology and since there is no suggestion of content contingent upon this methodology, it appears to be an end in itself, a situation that is not generally held to be desirable as we have seen. (Pring 1971b, Peters 1973, Sockett 1974) Of particular importance is the fact that, having chosen to base the project on Piaget's developmental theory, this in itself would appear to have

been interpreted inadequately with reference to what constitutes action necessary for cognitive growth in the logico-mathematical sphere.

As guidelines for developing a mathematics curriculum, all of the foregoing criticisms apply once again to the aims identified. What is seriously in question in this respect is that with such a set of aims, the active engagement of pupils in mathematical tasks would be of paramount concern and there would be difficulty in identifying what the nature of these mathematical tasks should be or what it is that is desirable for pupils to learn. Perhaps equally important in using such aims would be the tremendous onus they would place upon the teacher with respect to mathematical knowledge and organisational skills.

The projected social context that could arise from the view adopted of mathematics arising from the actions of the individual, suggests the possibility of a classroom within which a lot may be happening in terms of discussion and 'doing'. The concern would be, however, the possible lack of balance in not providing the opportunity for individual pupils to consolidate their mathematical learning through sharing their experiences in a variety of ways. The mathematics itself may otherwise be lost in activity.

We are left with an impression of pupils 'doing' but there is the temptation to ask, 'Doing what?'.

THE SCHOOL MATHEMATICS PROJECT

The School Mathematics Project arose directly from a conference held at Southampton University in 1961, under the chairmanship of Professor Bryan Thwaites. The conference was attended by mathematicians from schools, universities and industry and took the first positive steps towards establishing a project by forming an organised structure of committees. (Thwaites 1961) The various committees concerned were (1) an industrial committee, (2) a committee for transition from school to university, (3) a committee to consider school mathematics syllabi, (4) a committee concerned with the suitability of university syllabi for the majority of potential students of university mathematics and (5) a committee to study computing using the Pegasus computer at the University of Southampton. Of the five, it is the committee which set out to con-

sider mathematics syllabi in schools, with which we are concerned.

The composition (deduced from membership lists) of this committee and of the sub-committee set up to consider the general school course in mathematics is worthy of note in showing where the initial impetus and interest of the project lay. (There was also a sub-committee of 28 members set up to consider double-subject mathematics in the sixth form.) Of the 9 members of the main committee, 3 were from universities, 3 from public schools and 3 from grammar schools. The sub-committee was composed of 23 members 11 of whom were from public schools, 10 from grammar schools and 2 from comprehensive schools. The membership of the main committee indicates the coming together of people from universities and schools out of concern for what was viewed as a common problem, while the membership of the sub-committee is indicative of the fact that this concern was with the mathematical education of pupils of high ability. Indeed, the crux of the problem faced by the Conference as a whole was concerned with the provision of teachers of mathematics and "it was decided right from the start to devote the Conference's energies mainly to the study of the supply of graduates." (Thwaites 1961, p.6) Thus, in the first instance the concern of the project was to devise a mathematics syllabus which catered for academically able 11 to 18 year olds. While the main aim ultimately lay with the provision of able teachers of mathematics, the "production of bread-and-butter technologists" was also anticipated together with a wider area of interest as the following statement indicates:

"This book is also concerned (though at one remove inasmuch as we have refrained from considering, for example, secondary modern schools) with the methods of bestowing a general mathematical understanding, or numeracy as it has been called, upon the whole population." (Thwaites 1961, p.98)

As the project has developed, the large proportion of the population left behind by that 'one remove' has also eventually come to gain from any benefits the project might have to offer.

From the time of the publication in connection with the conference in 1961, no further major publication appeared until 1972, when "The School Mathematics Project: the first ten years" (Thwaites 1972) comprising the first nine annual reports of the project, was published. While we are concerned here with the initial stages of the project and what they set out to do, some matters contained in this second publication are of some relevance to considerations here.

Thwaites (1972), writing of the conference held ten years earlier, says it was "aimed specifically at producing an 'ideal' school mathematics syllabus" (p.ix) and in the intervening years, the project was entirely concerned with the production of textbooks for 11 to 18 year old G.C.E., C.S.E. and sixth form pupils. (The C.S.E. version of the syllabus first appeared in 1966-67.) Corresponding guides for teachers were published which included material on how to approach the teaching of particular topics. Eight schools were initially involved in the use of the materials in pioneering the project, 6 public schools and 2 grammar schools. All the materials were written by teachers according to their particular strengths and interests. There was no apparent recourse to theoretical considerations and the only aim of the project was identified as the production of a new syllabus. As Thwaites (1972) puts it (after the event):

"Of over-riding importance for us, however, is that syllabuses and the associated teaching methods should be developed as a practical outcome of classroom experience, rather than as a result of theoretical discussions round committee tables."
(Thwaites 1972, p.6)

This view is reinforced later on in the book when he states that,

"The cardinal feature of the S.M.P. is that it is a free association of school teachers of mathematics who have a common interest in improving the teaching of mathematics by developing syllabuses, texts and other classroom materials."
(Thwaites 1972, p.195)

Thus the project could probably best be described as an entirely pragmatic undertaking, with no underlying theoretical rationale.

We shall not list the materials produced by the S.M.P. (as we did in the case of N.M.P.) since these are too numerous to mention. The list continues to grow in view of the on-going nature of the project.

The lack of aims for the S.M.P.

The founding of the S.M.P., as has been indicated, was a direct response to the need to attract more pupils to study mathematics at a higher level in order, in turn, hopefully to produce more and better qualified teachers of mathematics. Without any discussion or identification of aims other than the production of a new syllabus, the only statements upon which to draw that give any indication of prior considerations appear in the report of the various committees that met at the conference in 1961. Clearly, this situation presents something of

an impasse with respect to what is being undertaken in this study. It is impossible to discuss such matters as balance within selected aims and the theoretical rationale upon which they are founded when neither exist. While it might well be possible to identify implicit aims and something of a rationale as the project progressed (e.g. from "The Schools Mathematics Project: the first ten years" by Thwaites (1972)), neither appeared to exist from the outset of the project to guide those involved in writing the new syllabus which constitutes the project. It would appear that such considerations may have evolved as the project grew (Quadling, 1972) but then rather than being aims and statements of intent, such statements were made from the wisdom of hindsight.

The only prior considerations given to the nature of mathematics and how it might best be presented to pupils appears in the deliberations of the Conference committees, particularly those concerned with school mathematics syllabi (Thwaites 1961, Chapter IV) and in the introductory chapter. As Thwaites remarks in the Preface of the book, the contents do "indeed represent the consensus view of the members of the meeting." (p.xiv) Thus it might be assumed that if there were any background or perspective which informed the work of the project team, it would be contained largely in the report of this committee and the two sub-committees formed to deal with general school and sixth form syllabi. Of the 64 people who made up these committees, however, only 6 were actually to become involved in writing materials for the project. (By 1972, there were 61 contributors to the textbooks of the project.) This may be seen to weaken support for considering the thought of the committees as a source of the kind of information we seek. However, since there is no other, we must draw upon the report for whatever evidence it may offer. The identifying of aims is necessarily precluded but some idea of the kinds of considerations which entered into the thinking of these committees might help to illuminate how they viewed mathematics and mathematics education.

Having extracted such views, we shall relate them to our list of aims (see Chapter 2, p.49) remembering that they act not only as statements of intent but as criteria for what is worthwhile in mathematics education. In this way, we may at least discover what features of mathematics education the founders of the project held to be important for a mathematics syllabus. We shall then discuss the implications of

the lack of a theoretical rationale for the project and what it may have offered in terms of guidance in curriculum development as a result. Finally, we shall note how the view of mathematics that is projected in the discussion of conference members would be likely to manifest itself in the social context of the classroom in which it taught.

Considerations of mathematics as a discipline

Mathematics is interpreted in the report as providing a bridge between the humanities and the sciences. Not only has it this function, but,

"At the same time mathematics enjoys an independent existence as one of the finest products of the human mind, testifying along with the other arts, by its nature and content, to man's creative ability." (Thwaites 1961, p.1)

Recognition is then given to the fact that the discipline has evolved over the past fifty years with respect to scope, content, methodology and applicability to the problems of the world. Thus mathematics is acknowledged to be something of the nature of a work of art, having an existence of its own, but which is seen to grow as a result of further creativity of the human mind. The report then goes on to state:

"If mathematics is to be attractive to boys and girls at school...it must be presented as a living and expanding subject, exciting both in itself and in its relevance to the demands of modern society." (p.7)

Following on this, attention is directed towards the question of what considerations are relevant to the achievement of this outlook in a new mathematics syllabus.

In order to make the subject attractive, the committee recognised the importance of the development of appropriate attitudes in pupils. They acknowledge that, as teachers of mathematics, they might be out of touch with the attitudes of pupils "as they are conditioned in this present era of mass communication", and go on to suggest that they "might well consider whether we ought to modify our courses to conform somewhat more closely to their view-point." (Thwaites 1961, p.27) Part of the approach to developing appropriate attitudes is seen to lie in the early introduction of a carefully designed "blend between work of a deductive nature and the informal introduction of new topics in which use can be made of previous mathematical experience." In considering further how more positive attitudes might be developed, they suggest

there may be certain parts of mathematics to which pupils "take more readily" and pose several questions, the answers to which might throw light on such matters.

Of particular significance for our considerations is firstly, a concern with "certain concepts, the understanding and appreciation of which are essential to the proper development of a mathematical education from the earliest age." (p.28) In teaching such concepts to attain meaning and insight, they refer to their gradual and repeated introduction in new situations so that new learning may occur as the result of the development of already familiar knowledge. This would provide continuity throughout the course to be studied. Concern is also expressed for the kind of mathematical work that provides "opportunity for creative activity" by the pupils. It is suggested that the traditional syllabus be examined "with a view to inspiring in children something of the modern attitude towards the structure, pattern and beauty of mathematics." (p.30) Finally, the question is raised of the possible value of work "which, at the time of presentation, is clearly relevant to the living and thinking experience of the pupil". (p.28)

Several points emerge from these considerations which, if they were to inform the project to follow, could have a positive influence on the way in which it developed. The identification of key concepts stresses an awareness of the need for pupils to have essential mathematical tools with which to work, at the earliest possible stage. The notion of repeatedly introducing these concepts in different circumstances not only supplies the desirable continuity identified, but satisfies the demands of psychological learning theory that if generalisation of what is being learned is to take place, then the same concept must be presented to pupils in a variety of situations. (Hilgard and Bower 1966) It, too, bears traces of Bruner's (1968) idea of a spiral curriculum wherein the same concept is introduced at different levels of difficulty throughout a course. It is also interesting to note that what is stressed in this process of repetition is the development of meaning and insight. The meaning of mathematics is illustrated by the repetition of a concept in a variety of ways, while insight is gained by pupils in the course of this. Thus, eventually they could come to recognise, in an insightful way, a familiar concept imbedded in new mathematical material hence the key to understanding the new will be provided.

The introduction of work of a deductive nature in the earlier years is advocated which suggests keeping content of the more traditional syllabus and combining it gradually with the new. The 'new' seems to include what is referred to as the "modern attitude towards the subject" which, as we have noted, stresses the structure, pattern and beauty of the subject. (Thwaites 1961, p.30) In deliberating about the characteristics of new mathematics to be introduced, particular mention is made of attempting to exclude obsolete mathematics from the course. The concern expressed in relation to pupils' attitudes to mathematics gives rise to a variety of suggestions as to how attitudes might be improved which, as we have seen, include mathematics that caters for the creativity of pupils and the fact that it should be relevant to the pupils' interests and the world in which they live. Lastly, there is the view of mathematics, as a subject, to be presented to the pupils. In the first instance, the subject is referred to as having an existence of its own as a product of the human mind but in considering its presentation to pupils the suggestion is made that it should be presented as "a living and expanding subject". (Thwaites 1961, p.7)

Some of these considerations meet demands of the criteria for worthwhile mathematics education set out earlier (see Chapter 2, p.49). The most easily identifiable are the development of a positive attitude towards the subject in pupils, the essential learning of particular concepts, the recognition of pattern and structure in mathematics and an introduction to deductive thought. Problem-solving, the wide applicability of mathematics and the notion of mathematics as a 'tool' are not dwelt upon although some aspects of these would necessarily arise in stressing the relevance of mathematics to the pupils' world which is suggested. Part of the reason for not emphasising the problem-solving potential of mathematics is to some extent explained in the Preface to the book where reference is made to the diffusion of the distinction between mathematics as an intellectual activity and as applied knowledge. (Thwaites 1961) Hence it is possible that by deliberately avoiding emphasis on this aspect of mathematics it was hoped to achieve such a diffusion of this demarcation line. As a result, the impression is given that mathematics should be taught for its own sake and thus predominantly as an intellectual activity. Reference to pattern, structure and beauty of the discipline and its 'exciting'

aspect, both in itself and in its relevance, bear out the fact that the prime reason for founding the project was to attempt to lure more able pupils to become students of higher mathematics. Stressing such factors would be the path that the committee logically would follow.

Along with the wide applicability of mathematics and its use as a tool in man's problem-solving and control of the environment, other criteria which have not been met in the general points raised by the committee are the development of logical thought, mathematics as a language and means of communication, the social criterion and the involvement of pupils in investigation and experimentation. However, mathematical language and elements of logical thought appear in the list of fundamental concepts that is offered, as does the viewing of equations as mathematical models. (The sixth form sub-committee see applied mathematics "as consisting largely of the building of mathematical models" so their importance and wide applicability in problem-solving are recognised at this level. (Thwaites 1961, p.32)) The fact that mathematics as a social activity in its existence and conduct is ignored is not surprising in the light of the nature of the other criteria which are missing. Equally, this is not surprising in view of the interpretation of mathematics as having an existence of its own, separate from human activity of a social, as opposed to individual, nature. That it is referred to as living and growing does not appear to assume shared activity in the course of the process, which further emphasises the lack of a social element in relation to mathematics and the mathematics curriculum.

Our conclusion with respect to the balance of considerations of the committee must, therefore, be that the primary aim of producing able mathematicians has led to an emphasis upon suggesting the teaching of mathematics for its inherent, attractive value and without apparent concern for applicability and problem-solving (at least in the earlier years). Indeed, the committee concerned with university mathematics reports that "The schools must make mathematics an enjoyable subject for intelligent students." (Thwaites 1961, p.54) We are reminded of Whitehead's (1932) injunction that mathematics must not be taught for its intrinsic beauty and with an aim that pupils should enjoy it since such aims are attainable only by the intellectually able. However, what the S.M.P. set out to do was to cater for just such an intellectual élite

and their considerations in approaching the syllabus they set out to create show this to be the case.

Lack of a theoretical rationale

There seems a degree of naiveté in some of the questions asked by the committee in their concern for how mathematics might be made more attractive to pupils and how their attitudes might be improved. For example, they pose the question (to which reference has already been made):

"Is there also special value in work which, at the time of presentation, is clearly relevant to the living and thinking experience of the pupil?" (Thwaites 1961, p.28)

They go on to suggest that other such questions which they pose would be "fruitful fields of enquiry" for educational research. Learning theorists generally, have agreed for some time that to achieve the desired motivation in pupils towards learning a subject, ensuring the personal relevance of what it is to be learned to the pupils' experience is one way of achieving this. Motivation of this kind achieved at the outset may be of an extrinsic nature, but may gradually lead to the more desirable (in terms of quality of learning) intrinsic motivation where their interest may become inherent in the subject. (Hilgard and Bower 1966) Piaget's (Inhelder and Piaget 1958) work is also relevant in this respect since the logico-mathematical development of the individual depends upon the active involvement of pupils in mathematical tasks appropriate to their stages of development, so that they may achieve adequately at one level in order to reach the next. Thus the thinking and doing experience of the individual is believed to be highly relevant to the attainment of mathematical concepts. It is somewhat surprising, therefore, that members of a committee concerned with devising a mathematics syllabus for schools were seemingly unaware of fundamental psychological information of this nature.

A problem which may have been minimised had there been some attention paid to research information arose in connection with an appropriate reading level for the textbooks produced. Griffiths and Howson (1974) point out that while the G.C.E. books are "readable", it is unfortunate that where the rest of the series is concerned, "the standard of literacy which they demand from pupils is extremely high". (p.92)

Another question which the committee posed itself related to teachers of mathematics:

"Has work in which the teacher is completely secure in his knowledge, and for which he has an infectious enthusiasm a special value?" (Thwaites 1961, p.27)

If the enthusiasm were, indeed, infectious, it would appear that the question contains the answer. What is not taken into account in posing such a question, however, is how the textbook can intervene between pupil and teacher (even the highly enthusiastic) and what steps may be taken to overcome such a barrier. On the other hand, the question of any "special value" in this context may be intended in relation to the teacher as a writer of textbooks rather than as a teacher of mathematics.

These are a few of the more obvious points which denote a total lack of reference to theory. It would seem that no disciplinary considerations of a philosophical, psychological or sociological nature have been taken into account in approaching the construction of this syllabus.

Possible relevance to the social context of the classroom

In viewing mathematics as having an existence of its own, it seems clear that members of the conference are represented as holding a traditional epistemological view of the discipline, of a logical positivist nature. The effect is one of approaching the subject as a body of knowledge apparently existing outside human activity and, once again, it conjures up the picture presented by Lakatos (1976) of mathematics consisting of perpetually increasing, eternal and immutable truths. The epistemological view of the subject would suggest a particular classroom context in which it would be taught and the social factors that arise within it, as noted earlier in this study. The transfer effect of this particular point of view to the conduct of mathematics in the curriculum suggests an emphasis on the individual, personal confrontation of pupils with mathematics. A set of textbooks was called for as a priority for the project, "which present the subject from a modern point of view" and it was stated that such books "*must* be written as soon as possible." (Thwaites 1961, p.30 author's italics) Such an emphasis gives rise to the connotation of a classroom context of a traditional sort in which, however new the content and presentation in textbooks, the effect would

likely be to perpetuate an essentially didactic approach to teaching the subject. Tamburrini (1978) describes a "traditional didactic teaching approach" as often involving "a strong emphasis on memorising compared with other cognitive activities such as questioning, problem-solving, inventing, checking and verifying." (p.99)

Writing of the textbook and the approach to teaching that accompanies it, Griffiths and Howson (1974) state, "Even its appearance, particularly in hardback form, proclaims authority and permanence." (p.93) This would seem an apt description of the likely context in which the S.M.P. content might be taught, with both pupils and teachers being constrained by the 'authority' of the text and leading to an unquestioning acceptance of the permanence of what it contains. There would seem little opportunity to break the habit of dependence on textbooks in the traditional manner and to allow discussion, investigation or the challenging of ideas unless the teachers concerned were extremely confident with respect to what they were teaching. This, however, certainly would have been unlikely in the earlier years of the project since so much of the content was new and in-service courses were necessary to bring teachers up-to-date with the mathematics to be taught. (Thwaites 1972) Once more, the view of the subject would act as a powerful constraint on how it is taught in the classroom.

The project as an approach to curriculum development

It is difficult to imagine how considerations given to the founding of the S.M.P. could provide useful guidelines to others who might wish to attempt to draw up a mathematics curriculum. In the beginning, it was a project which set out with a particular target population in mind and with a single methodology which was to produce and use textbooks. Some indication of content was given at the outset with the listing of main concepts to be covered which incorporated the new with the old. Emphasis was to be placed upon leading pupils to gain an intrinsic interest and pleasure in doing mathematics. These are such specific considerations that they are not generalisable to, nor do they offer guidance in, planning a mathematics curriculum development project other than one for which identical purposes are held. The S.M.P. appears to be something of an extreme example in development work which does not conform to any of Schwab's (1969) observations about theoretical con-

siderations in this connection. Indeed, considerations of this nature were spurned as 'time-wasting', as noted earlier. This seems somewhat unfortunate since some of the questions posed by the committee suggest an apparent lack of knowledge of psychological factors of a fairly basic nature with respect to pupils, teachers and methodology. The project might best be described as one of trial-and-error, possibly highly informed in mathematical matters but somewhat ill-informed with respect to educational theory. Underlying all of these considerations is the important fact that the project is independently financed, members are free to pursue matters as they see fit and the enterprise is a successful financial concern. Perhaps this may be the most telling and important single factor about the project. Indeed, doubt has been expressed as to whether the word 'project' is applicable in this context, especially in view of the popularity of the materials produced. (The Mathematical Association 1976)

Summary

The S.M.P., from its inception, may well be unique in the singularity of its aim and in the apparent lack of recourse to theoretical considerations of any kind in founding the project. Consideration was given to the kind of mathematics to be taught and the pupils (insofar as they were described as intellectually able) for whom the project was intended at the outset. The procedure adopted was, quite simply, to set about writing a new mathematics syllabus for 11 to 18 year olds of this intellectual calibre. In particular, the dominating theme from the beginning of the immediate production of textbooks appears to have precluded any serious reconsiderations of how the foundations of the subject *per se* might be presented to pupils. The likely social context of mathematics classrooms in which the textbooks are adopted is probably one of a traditional, didactic nature.

It must be admitted that it is exceedingly difficult to discuss, in a theoretical vein, a project which began with such a single-minded approach and which, at the earliest stages, offered so few considerations in terms of intentions. We have approached it as an example of an influential mathematics curriculum development project with a view to examining the aims that were set for it. In searching out aims for mathematics education, we have been particularly concerned with determining

the extent to which projects have adopted an aim characterised as 'social' in that the view of mathematics to be projected is as a social activity in its existence and conduct, and which we believe produces a particular kind of social context in the mathematics classroom. The lack of any such consideration in the S.M.P. is very clear.

CONCLUSIONS

There is little doubt that both the N.M.P. and the S.M.P. have had a great influence on mathematics education in this country over the last two decades. With respect to the N.M.P., Hewton (1975) writes that "Sales figures suggest that over 50 per cent of primary teachers have at some time acquired Nuffield Mathematics materials and it seems likely that the notions implicit in the scheme have spread much further and become intermixed with other approaches." (p.428) The influence of the S.M.P. is, of course, even greater since it is an on-going project and continues to produce materials. Griffiths and Howson (1974) describe it as "the leading mathematics project in Britain" (p.141) and it is estimated that more than a million pupils in the U.K. were using their materials by 1976. (The Mathematical Association 1976) Thus the pervasiveness of the projects cannot be questioned.

We have examined both projects from our adopted perspective of aims, firstly of aims in general and then, of the particular aim which embodies the 'social' criterion for mathematics education. Using aims identified earlier as criteria for what is worthwhile in mathematics education, we have attempted to judge the adequacy of those of the projects with respect to (1) balance achieved, (2) the underlying theoretical rationale and (3) the provision of guidelines in devising and implementing mathematics curricula. The search for some acknowledgement of the social criterion has been carried out in the belief that such an aim has particular relevance to the kind of social context likely to exist in classrooms adopting the materials and approach of each project has been inferred.

This exercise carried out with respect to the N.M.P. and the S.M.P. has presented an interesting contrast at each level of consideration. In the course of our examinations, some of the pitfalls that face curr-

riculum development projects generally have been highlighted. The lack of clarity of the aims of the N.M.P. and the adoption of a single theoretical perspective which appeared to be inadequately interpreted has led to the conclusion that the project seemed to lack appropriate balance in terms of kinds of mathematical activities in which pupils were to be engaged. The strong emphasis on methodology appeared to leave much to be desired in terms of what the mathematical content of pupils' experiences should be. Materials produced consisted of guidelines for teachers. It appeared that the onus on teachers would therefore be considerable since the situation envisaged demanded confidence in the mathematical knowledge and a high degree of organisational ability.

In contrast, the aim of the S.M.P. to construct a new mathematics syllabus was remarkably clear in its singularity. There was no recourse to theory to inform project members and to guide them in carrying this out, a lack which was noticeable from the beginning in the considerations of the schools syllabus committee. As the primary purpose of the project was to produce textbooks, the emphasis was on content in which the old and the new were to be brought together. Teachers adopting the project, therefore, had the support of a syllabus in the form of textbooks for pupils together with corresponding guides for themselves to aid in the presentation of new mathematical concepts. Thus the onus upon them was eased considerably in comparison with teachers adopting the N.M.P.

It is perhaps the contrast between the two approaches in terms of a stress on methodology on the one hand, and content on the other, which provides the strongest demarcation between the two projects in curricular terms. Hewton (1975) refers to the disappointment of teachers that materials were not produced by the N.M.P. for pupils and he notes:

"A balance had to be struck between intrusion and guidance; but with the desire to encourage teachers to work things out for themselves the balance was tipped away from the provision of too much formal structure in the guides." (Hewton 1975, p.428)

It would seem that the respect for the myth of the desire for autonomy on the part of teachers (Maclure 1968) possibly led to not enough of either intrusion or guidance. The case of the S.M.P. is quite the opposite. The textbooks they offer have been taken up by schools with alacrity. Griffiths and Howson (1974) observe that,

"It is perhaps surprising that the textbook should have continued to be used in traditional form by reformers; for by its very nature - its cost and comprehensiveness - it militates against changes in the curriculum." (Griffiths and Howson 1974, p.93)

This would appear to be a danger of which the S.M.P. has become aware, for Thwaites (1972) writing ten years after the project came into being, states, "The last thing that the Founders of the S.M.P. wanted is that the S.M.P.'s offerings should degenerate into a new classroom dogma". (p.x) When the project members eventually came to offer "A statement of philosophy" in their annual report of 1968-9 as a result of criticism of the project from the press and from professionals (e.g. Lyness, 1969) they conclude by saying, "Finally, this philosophy of the S.M.P. includes the concept of continual experiment and research" and in view of this, "Our aim, therefore, is to go on developing and perfecting - a process which, we suspect has no end." (p.198) The question arises whether, in the course of this process, the research referred to will incorporate matters other than those mathematical and whether the project, as a result, will continue with a more informed theoretical perspective.

Our final conclusion in contrasting these projects lies in comparing the different social contexts of the mathematics classroom projected for each as a result of the way in which the nature of the subject itself is conceived. The situation produced by a view of mathematics founded in individual activity in the N.M.P. seems to be one characterised by busy activity in experiment and investigation. The amount of teacher-pupil and pupil-pupil interaction appeared likely to be considerable. What seemed questionable, however, was the degree to which the activity was mathematical and whether it was appropriately balanced by periods of consolidation of a more contemplative nature on the part of pupils. The traditional view of mathematics as a body of knowledge separate from human activity envisaged in the S.M.P., suggested a classroom context in which, since textbooks were to dominate, there would be little or no discussion or activity. The teacher would likely adopt a didactic approach so that the prevailing atmosphere would be one of application to book work. The earlier consideration of generating pleasure and interest in the subject might well be lost in the turning of pages.

It may be argued that two such contrasting social contexts for the teaching and learning of mathematics are inevitable because one is in a primary/middle school setting and the other is in a secondary situation. The question may be asked, however, need this be so? If, as we have accepted, the notion of mathematics as a social activity in its existence

and its conduct is valid and is an aim to be pursued in mathematical education at all levels, then clearly, ignoring this aim must have an undesirable effect in terms of the lack of balance in the curriculum its absence would create. What we have seen in examining these two projects is that both have ignored this aim in their respective approaches to the development of mathematics curricula. While the adoption of the criterion clearly, in itself could not answer all the problems faced by each project, it seems possible that it could have brought about a more desirable balance in approach in both the N.M.P. and the S.M.P. The way in which the discipline is interpreted in its being, i.e. its epistemological foundations, would seem to be a powerful determinant of how mathematics is done in the classroom and this, in turn, contributes to the manifestation of a particular kind of social context in that classroom. Hopefully, this has been illustrated in the course of the discussion of these two projects.

The social context of mathematics education has not received investigative attention in the past, but in view of its apparent role as a manifestation of the social criterion for what is worthwhile in mathematics education, it would appear to be a concept of some importance. By examining the factors that contribute to the social context of the mathematics classroom, we may gain some insight into the variables that enter into it, how they interact with each other and how they may contribute positively to the teaching and learning of mathematics. We may also thus gain some insight into what the generally held view of the nature of mathematics, as a subject, is in schools. The next two chapters of this study will, therefore, be concerned with such an examination.

CHAPTER 4

The Social Context of the Primary
Mathematics Classroom

Introduction

Two factors which remain relatively constant for any one class of primary pupils during an academic year and for most of the curriculum, are the teacher who teaches them and the classroom in which they are taught. It may be taken for granted, therefore, that the social context resulting from the interrelationships of pupils with their peers and of teachers with pupils, and in which all of the teaching and learning takes place, may also be relatively constant. However, it seems likely that some aspects of this context may vary depending upon the subject which is being taught at any given time. For example, the atmosphere generated by a class of pupils during an art lesson is quite different from that of a class engaged in learning French. Differences arise as a result of the nature of the task being undertaken which, in turn, impose different constraints. Our concern here is to try to determine the kind of social context engendered by the various elements that enter into the classroom situation during a mathematics lesson. In this way, we hope to gain some insight into the nature of mathematics as it is perceived by teachers and pupils at primary level, how these perceptions are formed and how they may affect the mathematics curriculum.

In approaching an examination of the social context of mathematics education through a study of the mathematics classroom, therefore, it is important to be clear about what we see as constituting the 'social context' which we seek to examine. Although the identification of the concept has arisen from considerations of the influence of certain aims of mathematics education on how the mathematics curriculum is implemented, it warrants further clarification in order to ensure that each of the major elements contributing to that context is included in our examination.

The first consideration of relevance is that the mathematics classroom is contained within an *educational institution* and will duly be influenced by characteristics of that institution. The school as a whole will have certain aims and will choose to emphasise certain aspects

of the educational process rather than others. For example, there may be more or less academic emphasis within the curriculum, a strongly hierarchical form of organisation or a weak one, a rigid system of reward and punishment or none at all. The individual mathematics classroom is bound to be affected by such constraints imposed by the wider setting in which it is placed. There may be certain of these factors which are, to a degree, under the teacher's control and others over which teachers may have no influence whatsoever. These factors will combine, however, to affect the context in which mathematics is taught.

Secondly, *teachers of mathematics* will bring to the classroom situation their past experiences of success or failure in learning the subject and their attitudes towards it, together with the important element of their authoritative position. Not all teachers of mathematics may have enjoyed great success in the subject nor may they necessarily like it, with the result that they may possess favourable or unfavourable attitudes towards it (just as pupils do). These will be reinforced by their past experience of teaching the subject and whether they view this as successful or not. This success, or lack of it, may relate to matters other than the subject, for example, their general ability as teachers in relation to such matters as discipline and classroom management. How they view their role as teachers of mathematics and whether or not this view coincides with that of those in higher authority may also affect their actions in the classroom. They may consider institutional factors to be supportive of their intentions, or to undermine them.

Thirdly, there are *the pupils* themselves. They also bring to the mathematics classroom their past experiences, their attitudes to mathematics as well as to school, their success or failure with regard to their study of the discipline. The cumulative effect of these kinds of consideration will add to, or detract from, the working atmosphere that the teacher strives to create. It will likely determine, to some degree, pupils' relationships with their teachers and with their fellow pupils as well.

These three factors will combine to produce a complexity of interpersonal relationships which are the manifestation of the social context in which the teaching and learning of mathematics takes place. It is suggested here that this social context is an important factor in the

mathematics curriculum and one which may potentially be strongly affected by the perspective of the subject held by teachers and pupils. It is hoped, therefore, that by examining the three major contributing factors to the social context of the mathematics classroom, we shall gain some insight into how mathematics is perceived by pupils and teachers, and how the subject exerts an effect on that context. In order to do this, a range of relevant literature and research (where it exists) will be examined. In most cases, it will be necessary to extrapolate considerations arising from these sources to the mathematics classroom situation since little literature or research of this nature, specifically related to mathematics, exists.

We shall accordingly, approach the social context of the primary mathematics classroom through a consideration of these three main factors. Firstly, we shall explore how characteristics of the school as an institution may affect the mathematics curriculum. Secondly, the contribution of teachers, their mathematical backgrounds and the mathematical knowledge they bring to the teaching and learning situation will be considered. Finally, we shall concern ourselves with pupils in the primary mathematics classroom and how they may come to perceive the subject as a result of the individual experiences they may undergo.

INSTITUTIONAL ASPECTS OF PRIMARY SCHOOLS AND THEIR EFFECTS UPON THE MATHEMATICS CURRICULUM

In his discussion of the primary school as a social institution, Blyth (1965) suggests that implicit in the social cohesion and control existing within the school are the means by which "discipline and good tone are maintained." (p.21) At a fundamental level, he sees this as being based upon a pattern of role-expectations which includes "assumptions on the part of teachers, pupils, parents, and society at large about what adults and children in primary school should do, and about how they should behave." Although some scope for variation exists, a required minimum of social cohesion and control is recognised, the establishment of which rests upon norms which are seen to be determined to some extent by pupils and teachers but to be influenced "especially by the head teacher". (Blyth 1965, p.21) The development of the ethos of a primary school, therefore, appears to rest upon an existing pattern of role expectations which are influenced from within and without the

school, together with these somewhat flexible norms. Because the latter are particularly influenced by head teachers, it would seem reasonable to argue that their flexibility would depend, to a large extent, upon the degree of authority head teachers wish to exert within their schools.

The influence of primary head teachers

Blyth (1965) notes that the norms selected by a head teacher "may be affected by the tradition which is uppermost in his own attitudes". (p.98) For example, a head teacher might be characterised as 'progressive' in choosing to adopt a vertical method of grouping pupils according to age within the school or as 'traditional' in choosing to adopt a system of streaming. Whatever their tradition or attitudes, however, head teachers would reasonably be expected to exert a strong influence upon the teachers in their individual schools and hence, upon the curriculum (including mathematics). In doing so, they would be exercising their role as leader within the school. Morrison and McIntyre (1969) observe that head teachers are, indeed, "sometimes referred to as leaders of the staff of their schools." (p.86) They argue, however, that two characteristics of a leader about which there is fairly general agreement are that (a) that person is a member of a group and, (b) they exert more influence upon the group than any other member. Accepting these characteristics as essential to the role of leadership, head teachers would be expected to have frequent contact with most members of their staff, otherwise they would effectively remove themselves from membership of the group formed by the staff and without such contact, they would be unlikely to exert much influence. As Morrison and McIntyre (1969) put it, a head teacher who chooses to have little daily contact with most of his staff "cannot be considered a member of the staff group or therefore its leader."

In a study carried out by Ashton et al. (1975), head teachers of 201 primary schools were asked about the format of most of the consultations between themselves and their respective staffs. Approximately 69% of schools in the sample had a full-time staff of five or more teachers and 30% of the head teachers concerned were either full-time or nearly full-time in charge of a class. Of the 184 head teachers who replied to the question, only six had regular formal staff meetings and two had occasional formal meetings; 73.4% reported frequent informal meetings and

21.7% replied that they had both frequent informal staff meetings and occasional formal staff meetings. The net result, in the authors' words, is that "even an occasional staff meeting was a feature of the organisation of only one-quarter of the sample schools." (p.29)

Clearly, formal staff meetings in themselves do not constitute the only kind of contact that qualifies head teachers as members of the group formed by their staffs but there is a strong case to be made to support the contention that such meetings are necessary. The evidence gathered in the study indicates that a majority of head teachers (73.4%) do not meet with their staff on a regular basis but meet frequently and informally. It might be argued that frequent informal meetings may be sufficient to qualify them for group membership, but it is open to question how often head teachers might meet most members of staff on such an informal basis. Equally, it might be argued that approximately one-third of head teachers in the sample might qualify for group membership on the basis of their own teaching activities; however, it is more likely that their teaching may preclude contact with staff members because of other demands made on their non-teaching time.

With respect to the second criterion of leadership, it is doubtful whether frequent, informal contact would be adequate for head teachers to exert the degree of influence appropriate for the satisfaction of this criterion. Informal meetings imply irregularity. Without regularity, any contact could well lack the consistency that is desirable, if not necessary, to the quality of the influence to be expected of a head teacher as a leader. Where head teachers actively engaged with a class of their own are concerned, without regular staff meetings they would be in a position to influence staff primarily by example since, as already noted, other duties would leave little time to be spent with staff to influence them in a manner which might best be described as of a professional development nature.

It is clear that the results of the study suggest that the leadership provided by head teachers of primary schools may vary to a considerable degree, especially when judged against the two criteria identified. Further evidence from the study indicates that the variation in the kind of leadership given can affect staffs in specific ways. It was found, for example, that where no formal meetings were held between the head teacher and the staff as a whole, "teachers were significantly more

likely to opt for a traditional role" (characterised as 'societal' insofar as it prepares the pupil for society). (Ashton et al. 1975, p.79) Where regular staff meetings were held, "teachers were more likely to choose more progressive roles" (characterised as 'individual' insofar as it was seen to foster the development of the pupil's individuality, interests and independence).

The results relating to head teacher and staff contact in this study become relevant to our considerations of the mathematics curriculum when viewed in conjunction with results of the D.E.S. (1978) survey of primary education. In this survey Her Majesty's Inspectorate used lists of content items related to individual subjects which were "likely to be found" in that area of study, in order to explore the kind of curricula offered in primary schools in their sample. (p.76) While these items were found to be considered important by a substantial proportion of teachers, HMI point out that, "They do not represent a full range of curriculum which is considered desirable or even necessarily a minimum curriculum." (p.77) That is to say, the lists did not fully reflect similar lists drawn up by HMI themselves and which they considered to be representative of a full range of curriculum. Rather, items in the lists used were selected on the basis of having appeared, individually, in at least 80% of the classrooms surveyed. Where mathematics was concerned, only two-thirds of all classes in the survey were found to undertake work related to all of these items. When mathematics was grouped with English, less than two-fifths of all classes were found to do all of the work identified in both subjects. These results caused HMI to make the following observation:

"This would seem to suggest that in individual schools either some difficulty is found in covering appropriately the range of work widely regarded by teachers as worthy of inclusion in the curriculum, or that individual schools or teachers are making markedly individual decisions about what is to be taught based on their own perceptions and choices or a combination of these." (D.E.S. 1978, p.80)

The nature of this observation leads us to consider the suggestion of such idiosyncratic decision-making with respect to the curriculum in the light of the leadership that may be provided by head teachers. This may be viewed in two ways. If, indeed, it is the heads of individual schools who are making decisions that lead to this kind of imbalance in the curriculum, the situation could be one in which they may be exerting influence with undesirable results. They may be making unilateral

decisions with little or no staff consultation and participation, and which staff may find it difficult to implement. This is the kind of situation that could account for Bennett's (1976) finding that some primary teachers feel that their traditional authority is being undermined by the introduction of more 'modern' methods. There is implicit in such a view a certain dissatisfaction with the status quo and it may be that, in many cases, teachers have no part to play in deciding the extent to which different types of method should be adopted. If, on the other hand, it is the case that not head teachers of schools, but individual teachers are making their own decisions with respect to curricular matters, the imbalance found by HMI suggests that these decisions are not made in a sufficiently informed manner and that teachers may lack appropriate guidance in this respect. This may, in part, be responsible for the "widespread feeling of uncertainty" amongst some primary teachers in relation to the teaching of mathematics of which Ward (1979) found evidence. (p.57)

In the first of these two possibilities relating to decisions made in primary schools with respect to curricular matters, what is suggested is the imposition of decisions taken by the head teacher in a manner which does not satisfy the criterion of leadership which calls for group membership. Arguably, such head teachers may be said to be exerting influence over their staff but what is suggested by Bennett's (1976) evidence is, rather, that authority is being imposed. In the second of the possibilities where individual teachers may be responsible for making judgements leading to an imbalanced curriculum, head teachers could be said to be failing in leadership insofar as they appear not to exert an appropriate influence over their staffs, since we have argued that the latter cannot take place without group membership arising from staff contact, once again, therefore, the possible lack of leadership through lack of group membership is suggested.

Implications for the mathematics curriculum

The lack of regular formal contact between primary head teachers and their staffs is especially worrying with respect to the teaching of mathematics in the light of the considerable evidence suggesting that primary teachers do experience difficulty in establishing an appropriate balance within the mathematics curriculum. In the D.E.S. (1978)

survey, for example, in a sample of 1,127 classes, primary pupils were found to show a lack of ability to apply mathematics to problem-solving situations (a finding since reinforced by the results of the Assessment Performance Unit survey of primary school mathematics (D.E.S. 1980)). In the earlier survey, HMI suggests that, "Learning to operate with numbers may need to be more closely linked with learning to use them in a variety of situations than is now common." (p.57) They also found that "Techniques learned in mathematics were infrequently used in other areas of the curriculum or related to everyday situations, and children were seldom required to quantify as part of their recording, except in mathematics lessons." (D.E.S. 1978, p.44) Ward (1979) identified reservations in the minds of some primary teachers about the adequacy of the teaching of basic processes and hence a tendency to swing back to emphasising these. Primary teachers in his sample of 40 schools also questioned the value of practical work in mathematics and whether pupils were making the necessary mathematical conceptual links in the course of such work. More recently, Galton et al. (1980) found from detailed observation of 464 pupils in their sample of 19 primary schools, that of the 28.5% of pupil recorded time spent on mathematics, 14.0% was in connection with number work, 4.3% with practical mathematics and 10.2% with abstract mathematics. They state that their results indicate that there is little to suggest "that teachers have in any sense moved seriously away from what has always been regarded as the main function of the junior (or elementary) school: the inculcation of basic skills, or the grasp of elementary concepts relating to numeracy and literacy." (p.78) These results which indicate that primary teachers are teaching essentially basic, traditional mathematics in a mainly didactic manner, together with our previous evidence that less head teacher/staff contact leads teachers to opt for a more traditional role (Ashton et al. 1975), could be viewed as a manifestation of the kind of leadership offered by many primary head teachers.

Studies of the role of primary head teachers

There would appear to have been few investigations carried out concerning the role of the primary head teacher. A small study by Nash (1978) involving only twenty-six small rural schools showed that the style adopted by the head teacher can directly affect the achievement of pupils in that school although we are warned that, because of the limit-

ations of the data, this result must be treated with caution.

A study carried out in America by Marcus et al. (1976) in conjunction with a national evaluation of schools project, investigated the relationship between the type of 'administrative' leadership and pupil achievement in mathematics and in learning to read. Analysis of data from twenty-four schools showed that in schools where principals emphasised the importance of the selection of basic teaching materials and became involved in decision-making with respect to the curriculum and teaching, there tended to be greater gains in pupil achievement in both mathematics and reading. These conclusions are reinforced by Lezotte and Passalacqua (1978) who report that among the common features of schools characterised as especially effective is the fact that the principal accepts responsibility for the instructional leadership of the school.

Posts of responsibility for mathematics

Recently there has been a move towards establishing posts of responsibility for mathematics in primary schools to provide guidance and support with respect to the mathematics curriculum for class teachers. However, the primary school survey (D.E.S. 1978) found that in smaller schools (less than three-form entry), posts "with special responsibility for games were more common than posts for mathematics". (p.37) In larger schools, the number of such posts ranked second only to music. However, where these posts do exist and leadership in the provision of a more balanced curriculum might be expected, there was found to be a noticeable effect on the quality of work throughout only a quarter of such schools. There was some evidence of these teachers with posts of responsibility "planning programmes of work in consultation with the head, advising other teachers and helping to encourage a consistent approach" in mathematical work. The general conclusion would appear to be, however, that thus far, where this liaison exists, it has had limited success. This may be due to the fact that status-differentiation in primary schools is limited (Blyth 1965) and that the role itself has yet adequately to be defined. Generally speaking, the whole notion of graded posts may be "ill-adapted to a class-teacher system" as Blyth suggests, because each teacher is considered to be capable of teaching all subjects. (p.164) This stresses the need for a person with a post

of responsibility in mathematics to have some qualification with respect to the subject which is in some way superior to that of other members of staff, whether it be in terms of more formal knowledge (e.g. mathematics as a specialist B.Ed. subject) or wider experience (e.g. developed through in-service training). It is equally important that staff should be aware of what this extra expertise is on the part of such a person.

The hidden curriculum

There are more subtle ways in which organisational features of a school may affect the curriculum through a phenomenon which has come to be known as the 'hidden curriculum'. The idea of a hidden curriculum is one which is open to wide interpretation and which at times, may be in danger of becoming confused because it is so diffuse and encompasses so much. At its simplest, Gordon (1978) sees the "distinction between the explicit and the 'hidden curriculum'" as being those factors which link what is taught with the organisation of the school. (p.248) He suggests that it relates to such matters as the basis upon which pupils are organised (e.g. streaming or mixed ability grouping) and "the structure and legitimation of hierarchies in schools".

Organisational features of the primary school, as we have seen, are largely determined by the head teacher. The D.E.S. (1978) survey indicated that approximately one in twenty of classes of 11-year-olds was streamed while the rest of the sample was organised according to mixed ability grouping. However, nearly three-quarters of the classes were grouped according to ability for mathematics. Thus while, to all intents and purposes, the organisational climate of the schools as a whole may be one that does not emphasise differences in ability, this would appear to mask a within-classroom situation which does make such a distinction, at least with respect to mathematics. Nash (1973) comments that in any classroom there exists a "community of knowledge" held by pupils and teachers about the relative ability of each member of the class. (p.90) Clearly, the ability grouping within mathematics lessons would help pupils to identify more specifically their own position within the hierarchy as well as to be identified by their peers in this respect. The message of the hidden curriculum in this instance is one that potentially may strongly affect the pupil's self-image sometimes in a detrimental way (Nash 1973) and the fact that it is linked with the subject of mathematics

must inevitably affect attitudes of pupils towards the subject, especially towards the lower end of such a hierarchy.

A further organisational feature arising from the D.E.S. (1978) survey provides another example of how the hidden curriculum may be manifested. It was found that over three-quarters of classes in the sample were taught some of the time by teachers other than the class teacher. Where mathematics was concerned, this applied to only 5% of seven-year-olds, 10% of nine-year-olds and 10% of eleven-year-olds. The corresponding figures for English relative to each age were 30%, 35% and 30%. This could be interpreted as a reflection of the number of primary teachers who feel adequately secure in their mathematical knowledge to take other classes. It may, also be taken by pupils as a reflection of the relative value placed on each subject, with English receiving more 'specialist' attention coming out on top.

Other factors contributing to the organisation of the primary school that form part of the hidden curriculum are somewhat more obvious. The system of rewards and punishment, whether rigid or flexible, will give pupils some idea of the concept of justice and how it may be administered. The compulsory wearing of a school uniform carries with it a message of allegiance to, and identification with, the body of the school as an institution. While these aspects of the hidden curriculum individually may not relate specifically to the mathematics curriculum, collectively they culminate in an effect which is felt in the classroom. A strongly hierarchical structure together with an emphasis upon academic excellence could, for example, result in a particularly rigid approach to ability grouping within the classroom rather than a more flexible moving 'up' or 'down' of pupils according to their changing needs and achievement. At its most extreme, the influence exerted in this respect could result in the pupils' grouping for mathematics acting as the determinant of their class position for all subjects in the curriculum.

Summary

Consideration of institutional features of the primary school indicate that the head teacher has major control over matters of organisation. (Blyth 1965) Evidence suggests that (a) primary head teachers have little formal contact with their staffs, and (b) where there is

little formal contact of this kind, teachers tend to adopt a traditional approach in the classroom. (Ashton et al. 1975) Indications are that primary teachers tend to teach mathematics which caters for basic numeracy with little emphasis on problem-solving, application or abstract mathematics. (D.E.S. 1978, Ward 1979, Galton et al. 1980) It is suggested that this may, in part, reflect a type of leadership on the part of head teachers. Posts of responsibility for mathematics in primary schools would appear not to have made much impact on the quality of mathematics teaching as yet.

We have seen how the hidden curriculum is imbedded in organisational features in schools and how the mathematics curriculum in the primary school may be affected accordingly, perhaps particularly by the grouping of pupils according to mathematical ability within classes.

TEACHERS OF MATHEMATICS IN PRIMARY CLASSROOMS

Wilson (1962), in his analysis of the teaching profession, points out that because the type of service given is so diffuse and the value judgements made are so open to question, the service offered by teachers is not given due public regard. While teachers may or may not be over-concerned with their professional status in this respect, if Nash (1973) is to be believed, "demoralized cynicism" is the "occupational disease" of the profession and he suggests that teachers' "carefully preserved professional rights are more or less worthless" since, in his view, "No teacher can afford to act differently from the rest of the staff." (pp. 129-130) If this is indeed the case, the 'myth of the autonomous teacher' referred to by Maclure (1968) would appear to be exploded.

While our concern here is with teachers in relation to the teaching of mathematics in particular, such a background of judgements may be helpful in gaining a perspective of what may be a general atmosphere within the profession and which inevitably would affect the teaching of the subject at least as much as any other subject. We shall examine the situation of primary teachers of mathematics from three points of view:

- (1) teacher education in mathematics and their perception of the subject;
- (2) teachers' curricular decision-making with respect to mathematics;
- (3) teacher perception of pupils and the interpersonal relationships that result.

The mathematical background of primary teachers and their perception of the subject

The D.E.S. (1978) primary survey shows that three-quarters of the 5,884 teachers in their sample were women. Graduate status was found to be more usual among recently qualified teachers, one-tenth of the total sample being graduates and two-fifths of these holding a Bachelor of Education degree. Since the James Report appeared in 1972, there has been a movement towards making teaching an all-graduate profession but as these statistics indicate, the movement is still in its early stages. Judge (1975), himself a member of the James Committee, comments upon the "poverty of thought" and the lack of critical discussion "on the nature of the teacher and on the objectives and methods of teacher education" that were pervasive in the fifties and sixties. (p.8) Referring to the report, he goes on to say that "The conviction that there should be a body of theoretical knowledge at once philosophically sound and applicable in good practice was stronger than the capacity to say what it was." (p.9) Because of the visibility of the teaching profession, he suggests that there is less agreement about what new entrants need to know and do than in any other profession.

This has been a matter for some investigation with respect to new entrants into the profession and the teaching of mathematics. In a paper written three years after the James Report, Shuard (1975) states, "The new B.Ed. was intended as a professional degree which could improve on the Certificate" and goes on to note that chances of this happening at that time were not good. (p.18) Where mathematics in particular was concerned, although prospects appeared to be poor it was found that mathematics student recruits were "not worse qualified than the average student in other main subjects." In a survey carried out with 1975 entrants to college of education courses with mathematics as a main subject, returns from approximately 425 first year students indicated that they intended to stay on for a fourth year to take the B.Ed. (Shuard 1977) By the academic year 1976-7, it was found in a similar survey that numbers of main mathematics students had decreased by 14% but their qualifications had improved, with approximately 70% having Advanced level mathematics. (Shuard 1978)

In an earlier study, Lumb (1974) investigated the initial mathematics

qualifications of student teachers on entry to a college of education, and found that in his sample of 110 men and 186 women, only 55.2% had Ordinary level mathematics passes. They were tested on computation as well as some modern mathematics items with the result that 86.4% failed to score at all on the latter type of question. As an example of more general number work, in a question which involved placing five simple fractions in order of size, 76% failed to do so. Not surprisingly, the conclusion was drawn that there should be a compulsory mathematics course for all college of education students.

Some interest has been shown in investigating the attitudes of college of education students towards mathematics. Lumb and Child (1976) explored changes of attitude towards the subject by testing student attitudes on entry to college and at the end of the first year. Although a somewhat limited study in design, initial results suggested no substantial initial difference amongst those intending to teach in first, middle or secondary schools. However, by the end of the year there was a substantial improvement in the attitudes of those intending to teach in first schools.

Ray (1975) investigated factors which appeared to affect recruitment to main mathematics courses in colleges of education with a sample of 848 first year entrants. It was noted that most students had a favourable attitude towards mathematics but an unfavourable attitude towards teaching methods used in conjunction with the subject. More girls had dropped mathematics at school because they had seemed to be encouraged less to keep it up, and phrases such as 'not a girl's subject' frequently appeared in replies to the questionnaire. Only 73% of all the sample had done Ordinary mathematics. The criterion for acceptance on the main mathematics course was either a good Ordinary level pass or an attempted Advanced level and only 19% of the sample had these qualifications. Some concern was expressed by the author of the study about the fact that many students apparently did not realise that they were qualified to take the main mathematics course, with less than one-quarter of those who met one or other of the criteria demanded actually choosing to do so. The main reason given for studying mathematics was an interest in the subject; success in it was seldom referred to and other subjects were preferred more. Criticisms of the way students themselves had been taught mathematics at school included references to 'humiliation', a

heavy reliance on textbooks, a lack of individual attention and a lack of relevance to people and life. Contrary to what Lumb and Child (1976) found, Ray's results showed that those intending to teach the youngest age group began with the least favourable attitude but results were similar insofar as this group were found to show the greatest improvement in attitude by the end of the first year.

The results of these studies are reflected in Ward's (1979) survey undertaken for the Schools Council. He reports that the chief handicap of primary teachers with respect to teaching mathematics appears to be their lack of mathematical knowledge and, therefore, of confidence in their teaching of the subject. Less than 60% of teachers in his sample of 40 schools had Ordinary level passes and less than 5% had Advanced level passes in mathematics. As noted earlier, there was evidence of considerable uncertainty on the part of teachers about mathematical teaching and even some desire for strong, centralised direction with respect to the mathematics curriculum. Straker (1978) also reports the fact that "many primary teachers do feel inadequate" with respect to the mathematical knowledge they have. (p.13)

While these studies in themselves provide limited evidence upon which to base conclusions with respect to primary teachers and their perceptions of, or attitudes towards, mathematics, taken in conjunction with the D.E.S. (1978) primary survey, they present a revealing picture. Firstly, in spite of indications that some college of education students had poor opinions of how they were taught mathematics, the fact that some retained sufficient interest in the subject to want to teach it themselves as a main subject suggests an initial positive outlook. It would be reasonable to assume, in the light of the evidence quoted above, that this outlook would not likely include the approach to the teaching of mathematics which they themselves had undergone. In view of Ray's (1975) results in particular, this could be said to be especially the case with respect to girls who had opted to study main mathematics. However, when considering the information provided by the D.E.S. (1978) we find a situation in which most primary teachers (the majority of whom are women) appear to approach the teaching of mathematics in a way which matches (perhaps all too well) the situation of which some, at least, profess to have had an adverse opinion. According to the survey, most primary teachers group their pupils according to mathematical ability within the

classroom, assign pupils work on an individualised work card system, teach mainly in a didactic way and rarely link the mathematics studied with other areas of work or with the world at large. (D.E.S. 1978) These aspects of teaching mathematics clearly mirror the criticisms made by student teachers quoted from Ray (1975) above. They also provide a possible insight into the professional cynicism referred to earlier. If student teachers' interest in the subject is sustained and they ultimately enter the classroom with the intention of bringing a fresh approach to the teaching of mathematics different from that which they themselves had experienced, something happens to inhibit the majority from putting such intentions into practice. It may be, as HMI (D.E.S. 1978) suggests, that ability grouping takes place in mathematics because "in this subject the sequence of learning is fairly clearly defined", thus attesting to the visibility of the subject; it is no doubt also due to the teacher's desire "to present children with tasks which were matched to their competence". (p.22) However, evidence from the survey suggests that whatever their intentions may be with respect to presenting the subject to pupils in a more meaningful and balanced manner, primary teachers tend to revert to a view of the subject as mechanical and unrelated to life, and openly to categorise their pupils as those who readily learn mathematics and those who do not, with all the attendant consequence of such identification for the pupil. One of the consequences has been an increase in the use of individual work cards, with 67%, 52% and 51% of seven, nine and eleven-year-olds respectively, frequently being given such materials to work from. (D.E.S. 1978)

While some of the reversion on the part of teachers to a bias in favour of more traditional mathematical content and method may be ascribed to their own lack of mathematical knowledge, there exists some opinion which suggests that this is not the answer to all mathematical teaching problems. Begle (1979) has noted how the mathematical knowledge of prospective teachers has tended to dominate a consideration of the candidates entering mathematics education in the U.S.A. He has suggested that "it seems to be taken for granted that it is important for a teacher to have a thorough understanding of the subject matter being taught." (p.28) He then quotes studies which suggest that "this belief needs drastic modification" and that "once a teacher reaches a certain level of understanding of the subject matter, then further understanding contributes nothing to student achievement." (p.51) Evidence

in this country relating to teachers in primary mathematics education, however, indicates that the suggested optimum of mathematical knowledge may have yet to be reached.

Primary teachers and curricular decision-making

Ashton et al. (1975) identified three areas which polarised the views of the 1513 primary teachers in their sample as traditional or progressive, and which clearly identify major areas of their decision-making. These were: (1) the principles they employed in selecting curricular content; (2) the way in which they involved pupils in learning; (3) the way in which they themselves promoted learning. Each of these focal points for teachers' decision-making is open-ended in nature; there is, for example, no single set of principles to guide them in the selection of what they teach nor is there one way to involve pupils in learning. While the value-laden nature of these three very important aspects of their responsibility may appear to be clear, it may be the case that some primary teachers are not always aware of the part values play in determining their judgements and hence, how values affect their decisions in these vital areas. Values are determined by beliefs and as Finlayson and Quirk (1979) point out, a commitment to "'a belief' in something" is often what characterises ideology at the level of the individual. (p.52) Ultimately, therefore, it is the beliefs of teachers which provide the values that determine their aims and guide them in their decisions as to what constitutes good educational practice. The importance of this ethical aspect of teachers' decisions identified by Hirst and Peters (1970) was raised in our discussion of educational aims earlier (see Chapter 1). While an awareness of this facet of the decisions they make is clearly very important, it would appear that the apparently idiosyncratic decisions sometimes made by teachers rest upon personal ideologies which they themselves may not clearly have articulated.

Freeman and Kuhs (1980) identify difficulties entailed in the judgement and decisions of mathematics teachers in the USA. They observe that the teacher receives different "content messages" from various sources such as parents or the head of the school and that "it is apparent that some of these messages must be ignored." (p.22) They go on to state:

"Given restrictions in the time available for mathematics instruction, it is simply not possible to provide adequate coverage of all of the topics she will be asked to teach But what topics should she ignore?" (Freeman and Kuhs 1980, p.22)

The dilemma posed teachers in primary schools in this country is similar in nature. We recall that HMI (D.E.S. 1978) suggest that imbalance in the curriculum may be due, in part, to difficulty "in covering appropriately the range of work" generally found to be "worthy of inclusion". (p.80) Difficulty of this nature requires value judgements to be made as to which aspects of the curriculum are to be included and which are not. While it may be the case that the teaching of mathematics in most primary schools in this country is supported by a scheme of work or content guidelines (D.E.S. 1978), teachers must still make such choices. If teachers do not identify the values that underlie the rationalisation of their choices the possibility of an imbalanced mathematics curriculum is strengthened. This is a danger which is further compounded in the case of primary teachers in view of the fact that they are called upon to make such decisions and choices across the whole curriculum. The fact that these decisions are made with respect to a single class over a relatively long period of time would appear to place them in a favourable position to make informed critical judgements and choices with respect to the needs of pupils in their charge. However, even with such an apparent advantage, the relationships between teacher and pupils add to the complexity of such a task, as we shall see.

Primary teachers' perception of pupils and classroom relationships

Nash (1973) states that "All genetic and sociological factors are mediated and realized through the interaction between the teacher and child in the classroom." (p.123) The mediation of these factors appears to depend in the first instance on the mutual perceptions of teacher and pupils. The combination of the resultant individual teacher-pupil relationships within a class together with the relationships of pupils with their peers, form the basis of the social context in which the teaching and learning of mathematics takes place.

In view of the evidence of ability grouping within primary classrooms, the teachers' perceptions of pupils takes on added importance and we are led to consider the notion of the 'self-fulfilling prophecy' with respect to their judgements of the ability of pupils. This was explored

by Rosenthal and Jacobson (1968) when they attempted to manipulate teacher expectations by attributing false I.Q. scores to pupils. The intention was to study the outcome in terms of pupils' achievement and to discover the extent to which the syndrome of the self-fulfilling prophecy in fact existed. It was found that where a falsely high rating of a pupil's ability was given to a teacher, pupils achieved more than would have been expected from their actual I.Q. scores thus indicating that pupils tend to fulfill teachers' expectations of them in terms of academic achievement. Good and Brophy (1978) comment critically upon this study and argue that the situation arose because of the "credibility of the source" of information about the potential of the pupils; i.e. who it was who identified the high and low achievers. (p.69) In what they call "naturalistic studies", Good and Brophy (1978) suggest that teacher expectations are related to "differentiated teacher behaviour" and are the result of a series of cause-and-effect relationships. The results of earlier studies reported by Brophy and Good (1974) indicate what some of these behaviours might be. They noted that attitudes of attachment, indifference, concern and rejection towards a pupil can lead to the beginning of self-fulfilling prophecies. Evidence indicated that pupils to whom teachers exhibited attachment but showed little overt favouritism, were high achievers and conformed to a pattern of desirable classroom behaviour. Pupils shown an indifferent attitude by the teacher were characterised by passivity and inconspicuousness. Even when perceived by teachers as shy, unhappy or nervous, pupils apparently still did not elicit the concern of the teacher and teachers appeared to be "truly indifferent" to them. (Brophy and Good 1974, p.160) Pupils to whom teachers showed an attitude of concern were given much of the teacher's time and effort in help. The pupils to whom an attitude of rejection was shown superficially appeared to be little different from the 'concern' pupils but gained the teacher's attention primarily in the course of the maintenance of classroom discipline.

In this country, Nash (1972) carried out an investigation into teacher attitudes with a sample of eight primary teachers and 236 pupils. He used the repertory grid technique in which bi-polar constructs were obtained from the teachers, the eight most highly ranked constructs were chosen and converted into a rating scale and this, finally, was used to obtain a rank order of subject ability for all pupils in each teacher's class. Pupils were observed as objectively as possible by researchers

and their behaviour then reinterpreted in the light of teachers' perceptions of them. The results indicated the difficulty in understanding why teachers' attitudes towards particular pupils are as they are and how idiosyncratic they may be. At the same time, however, the importance of their attitudes and perceptions upon the achievement of pupils was established.

Some reference has already been made to the intrusion of the sex of a pupil upon teachers' perceptions of them in connection with learning mathematics (see Ray 1975). We are reminded once again of the importance of this factor by the results of a study by Morrison, McIntyre and Sutherland (1965) carried out in Scotland and involving a sample of sixteen primary schools. It was found that teachers of both sexes employed a less analytic approach to rating girls academically than they did with boys and in particular, they associated girls' attainment in arithmetic more with good behaviour than was the case with boys. Thus what is suggested is that teachers' perceptions of the achievement of girls in connection with mathematical learning was not generally related to any ability they might have had in this area. Clearly, in such a case, the potential mathematical ability of some girls could remain unidentified and not be encouraged to develop.

With the 'visibility' of mathematics arising from its sequential nature, comes the public character of the criteria for success or failure in connection with it. We have noted that teachers' judgements about the mathematical ability of primary pupils produce within-class grouping thus applying a kind of mathematical label to individuals. It seems possible that these factors could intrude adversely upon an already complex classroom situation in which the perceptions by teachers of pupils are determined. If, indeed, pupils are so susceptible to the actions and attitudes of teachers, the overt classification that takes place in mathematics classes may act as a vehicle for exaggerating the effects of the visibility of the subject. There thus would appear to exist a greater potential for a negative classroom atmosphere to prevail during a mathematics lesson because of the labelling that takes place when, in fact, primary teachers may simply be satisfying what they perceive to be the demands of the subject in relation to their perceptions of each pupil's ability. In this way, a circularity may, indeed, arise in the primary mathematics lesson with respect not only to teachers' perceptions

and pupil achievement, but to the quality of the social context in which the teaching and learning of mathematics takes place, as well.

Summary

Indications are that college of education students taking mathematics as their main subject have an interest in the subject but do not necessarily prefer it to others nor see themselves as successful in it; some have been found to have an adverse attitude towards methods by which they were taught mathematics in schools and there is evidence that girls are sometimes discouraged from studying it at a higher level. (Ray 1975) Primary teachers appear to revert to an approach to teaching mathematics of which evidence suggests some may themselves have been critical as pupils in schools. (D.E.S. 1978, Ray 1975) There are some indications of a lack of confidence on the part of primary teachers in their mathematical knowledge and hence, in their ability to teach the subject. (Ward 1979) Although Begle (1979) suggests there is an optimum level of mathematical understanding necessary for effective mathematical teaching and that any further mathematical knowledge does not enhance results, it may be that this optimum has yet to be reached by primary teachers in this country.

Investigation has identified factors which polarise the views of primary teachers as 'traditional' and 'progressive' (Ashton et al. 1975) which are value-laden in nature and relate to important choices teachers must make. It is suggested that evidence from the D.E.S. (1978) survey indicates that primary teachers may not always be aware of the nature of these choices nor identify the values which guide them in making such decisions. Various agencies and factors affect teachers' decision-making in the mathematics curriculum (Freeman and Kuhs 1978) and although most primary schools have mathematical schemes of work (D.E.S. 1978) teachers still must exercise choice as to what mathematics is to be taught.

Teachers' perceptions of pupils are likely to be based upon interpersonal, cause-and-effect relationships rather than on received information. (Good and Brophy 1978) Studies show that teachers' perceptions can lead to self-fulfilling prophecies where individual pupils are concerned, with those to whom teachers show attachment achieving well and the remainder adopting a passive role in the classroom. (Brophy and Good

1974) Nash (1972) has also shown the importance of the perceptions of teachers on pupil performance and has highlighted the idiosyncratic nature of teachers' judgements of pupils. The difference between some teachers' perceptions of girls (as opposed to boys) learning mathematics is noted. (Ray 1975, Morrison, McIntyre and Sutherland 1965) It is suggested that the visibility of mathematics together with the teachers' perceptions of pupils as manifested in grouping them according to mathematical ability potentially may lead to a negative classroom atmosphere and a circularity between such classification and pupil achievement.

PUPILS AND MATHEMATICS IN THE PRIMARY CLASSROOM

The effects pupils have upon the teaching and learning situation in the classroom may broadly be defined in two ways. Firstly, there are the characteristics which they, as individuals, bring to the mathematical learning situation such as capacity for learning and their stage of intellectual development. Secondly, there is the influence they exert through the part they play in the social arena of the classroom which arises from their interaction with the teacher and with their peers. Underlying both of these, as we have suggested, is the effect of the nature of the subject being studied at any given time and the way in which this intrudes upon the classroom atmosphere. In this section, we shall be concerned with examining primary pupils engaged in the learning of mathematics in the classroom and any influence the subject *per se* may exert upon the situation. Our purpose is not to investigate how pupils learn mathematics nor what methods may be more effective than others. Rather, it is to attempt to gain some insight into the individual pupil's situation when learning mathematics within the context of the classroom group.

Several factors of a socio-psychological nature appear to be particularly relevant to primary school pupils and the learning of mathematics. For example, the way in which pupils view themselves as individuals and their ability with respect to learning the subject would seem to be of considerable importance as does their perception of the subject and of the teacher. Other factors that may contribute to their mathematical success are external to the classroom to some degree, and include matters such as parental involvement and the linguistic ability of the pupil. Clearly, while some of these factors are common to any learning

the pupil undertakes, our purpose here is to relate them to the pupils' study of mathematics to discern whether there may be any particular effect exerted by the subject in combination with such factors upon the context in which it is taught and learned.

The primary pupil as an individual

Nash (1973) has found that not only is the knowledge held by teachers and pupils of the relative ability of each class member as a general characteristic of primary classrooms, but that it is estimated by individual pupils with "considerable accuracy". (p.90) In an investigation in 152 primary schools, his finding endorsed the interactionist theory which predicts that pupils who are perceived unfavourably by teachers, develop poor self-concepts which tend to be reflected "in the low class positions the children believe themselves to have." (p.91) The converse was also found to be the case, where pupils viewed in a favourable light by the teachers believed themselves to be in a high position within the class hierarchy. Blyth (1965) refers to this classificatory role of the teacher together with peer judgements and their joint effect upon pupils, and suggests that "As this process gains momentum, some children may earn an adverse stereotype which survives many pathetic attempts to overcome it." (p.52) This factor is reinforced by Good and Brophy (1978) who point out that the self-concept of pupils results from "their early experience and the subtle but systematic opportunities and rewards" they have had. (p.82) They go on to state that "Children are not born with inadequate self-concepts. Self-worth is learned in interaction with others." (p.83)

In order to relate this notion of the self-concept of pupils to the situation of pupils learning mathematics, it is helpful to examine the individual's self-concept in the light of the classroom as a whole. The classroom, in which a large proportion of the pupil's social interaction occurs, is characterised by Jackson (1968) as conveying a threefold lesson which pupils have to learn in order to survive and develop. They must learn: (1) to live in a crowd; (2) to adapt to the fact that they are under conditions of constant evaluation both by the teacher and their peers; (3) to understand the conditions of power that exist within the classroom with the teacher in authority and wielding power. It seems possible that the mathematics classroom may present an extreme example

of these three factors to which primary pupils have to accommodate. To take them in reverse order, pupils quickly come to realise that it is the teacher who will pass judgement on their mathematical ability or lack of it. In the within-class grouping of primary pupils according to mathematical ability, it is very likely the teacher alone who allocates individuals to particular groups. We have also seen that pupils are aware of their position in such a hierarchy. (Nash 1973) Jackson's second point relating to the constant pressure of evaluation by teacher and peers emphasises the point that it is not merely a situation of once being allocated to a group, the matter is forgotten. This form of identification becomes a reference point with which a pupil will be connected consistently, thus forming the stereotype to which Blyth (1965) refers. The first point made by Jackson (1968), the fact that pupils must learn to live in a crowd, is met with a strategy of selecting from the class a group of 'significant others' with whom they identify. (Nash 1973) This can provide either a defence mechanism to cope with what they see as a lack of success or a means of reinforcing success. Nash (1973) observed in his study that the criteria underlying the formation of such friendship groups were the teacher's favourable or unfavourable perceptions of the pupils concerned.

It would seem, therefore, that what appears to be a vicious circle can be built up for some pupils and a self-fulfilling prophecy may well come into play. First of all, there is the overt identification of poor mathematical ability and the stereotyping of individuals accordingly; then follows the resultant diminishing of the pupil's self-concept and a choice of friendship group based upon the criterion of fellow pupils who have been perceived in a similar light by the teacher. If there were positive signs of flexibility in the grouping of pupils according to ability, this circularity might be broken. However, commenting on the flexibility of such grouping, Morrison and McIntyre (1969) suggest that while it is recognised as desirable, "it is often difficult to move a child up from one group to another". (p.112) As a result, "It is therefore normal for a child who starts in a low-ability group to remain in it, though a child in a higher group can easily be demoted." Pupils themselves are likely quickly to become aware of such inflexibility and the perceived lack of potential for improvement manifested in this manner is likely further to strengthen their poor self-concept.

For those who are mathematically able, the identification of their ability would presumably have a positive effect on the development of their self-concept. Apart from the lurking threat of demotion noted above, however, there would appear to be disadvantages of another sort for those allocated to the most able group. It was found by HMI (D.E.S. 1978) that while the mathematical work assigned to pupils in the average and below average groups was "more consistently matched to children's capabilities", this was not the case with respect to many of the more able. They state:

"However, for the children who showed most marked mathematical ability the work was often too easy and it is a matter for concern that these children's abilities were not fully extended in their work in this subject." (D.E.S. 1978, p.57)

It would appear that within-class grouping according to the teacher's perception of a pupils' mathematical ability may have disadvantages for all pupils concerned, whatever the organisational advantages may be. It seems a strong possibility that the primary pupil studying mathematics soon comes to learn that the subject serves as a means by which they become stratified according to ability and that the nature of the subject (as it is presented to them) together with the visibility of success or failure in learning it, provide the means by which consequently they are labelled.

Once again, the matter of the pupil's sex in the context of mathematical learning must be mentioned as a matter of some importance. While there appears to be little reason to expect that boys and girls should necessarily differ in their potential for achieving in mathematics, the fact that few girls choose to study it at higher levels has given rise to research to investigate the underlying reasons why such a situation should exist (e.g. Berrill and Wallis 1976, Preece 1979). At primary level evidence in the U.K. (D.E.S. 1978) and the U.S. (Fennema 1979) show little significant difference between boys' and girls' mathematical scores. It has been suggested in connection with visuo-spatial ability, which is seen as a factor in determining mathematical ability (MacFarlane-Smith 1964), that the kind of experience boys enjoy at pre-school and primary ages gives them an advantage over girls. Boys, for example, may have done a lot of model-making involving a degree of imagery whereas girls probably will have not. At the same time, what appears in textbooks or work cards might be interpreted as active discrimination, where boys buy planes and trains and girls are given the choice of dolls or

doll houses.(Berrill and Wallis 1976) These are just some of the indications which suggest that problems relating to the sex of pupils and the learning of mathematics begin in the very early years of schooling.

Pupils' perception of the subject of mathematics

One of the main findings of the recent report on mathematical development in primary schools by the Assessment Performance Unit (D.E.S. 1980) in relation to pupil attitudes towards mathematics was that "the four rules are regarded by pupils as the topics which are most representative of the subject." (p.130) An illuminative concept proposed by Skemp (1979) of the existence of two kinds of understanding is helpful in gaining an insight into how such perceptions may be formed. Skemp (1979) is concerned with the different goal structures pupils and teachers may hold and views these in terms of two kinds of understanding which he calls instrumental and relational. He states, "Instrumental understanding, in a mathematical situation, consists of recognising a task as one of a particular class for which one already knows a rule." (p.259) Relational understanding, on the other hand, is seen to consist mainly of relating a task to a suitable schema. In the former case, the goal is simply for the pupils to get the right answer while in the latter, the goal is more complex. In the development of relational understanding, the teacher seeks some indication that the pupil can fit what has to be learned into an appropriate schema or, alternatively, construct such a schema, thus indicating that they not only know what is right, but why it is right as well. It could be said that instrumental understanding is concerned with a more factual type of learning while relational understanding is concerned with using facts appropriately in a problem-solving situation. Such as delineation highlights the potential difficulty for pupils if their interpretation of the kind of response expected of them by the teacher is wrong. It also emphasises two aspects of mathematics with which pupils must become familiar in order to succeed in mathematical learning, i.e. the learning of basic facts and skills and the development of problem-solving ability. However, we have already noted evidence (D.E.S. 1978; Galton et al. 1980) which suggests that most mathematics taught in primary schools is concerned mainly with instrumental understanding and little of the relational sort. Thus it would appear that the more visible, instrumental aspect of mathematics which involves obtaining correct answers may be stressed to the

neglect of the pursuance of goals which include the development of relational understanding with which to solve problems.

Trown and Leith (1975), in a study concerned with the mathematical learning of a sample of 432 primary pupils, found evidence of a strong relationship between teaching strategy and pupil anxiety, where the teaching strategy was described as either pupil- or teacher-centred. (For example, we have seen that the goals and methodology of the Nuffield Mathematics Project were essentially of a pupil-centred kind. (See Chapter 3)) They determined that the anxiety level of pupils was the distinguishing factor between those who did and did not benefit from a pupil-centred approach. A teacher-centred, supportive strategy, on the other hand, appeared to be equally effective whatever the level of anxiety. Bennett (1976) also found that teaching style seemed to have a stronger effect on pupil achievement in mathematics than in any other subject, and that gains appeared to be greatest when a formal, teacher-centred strategy was used. While this evidence may not appear superficially to be related to pupils' perceptions of mathematics as a subject, there lies beneath the surface some indication of the perceived nature of mathematics as it is presented to them. There would appear almost to be an assumption implicit in the dichotomy of teacher-centred/pupil-centred approaches in teaching mathematics that a teacher-centred strategy is concerned with the teaching of facts and skills (leading to instrumental understanding), while a pupil-centred strategy is concerned with problem-solving done by pupils (leading to relational understanding). In other words it would seem that one kind of activity - learning facts and skills - is taught by the teacher, while another kind of activity - learning to solve problems mathematically - is one which pupils may have to learn themselves. It is probable that often the anxiety shown by some pupils when placed in a mathematical problem-solving situation arises not because they do not know the facts, but because the vital recognition of their interrelationship and application is lacking. We have referred to the fact that some teachers are concerned that their pupils do not make the appropriate link between 'practical' work and the mathematical ideas inherent in it. (Ward 1979) Perhaps this may be because they have not been taught how to make that link. Brandau and Easley (1980) in the U.S. have found that in mathematical problem-solving, "Strategies ... are not usually discussed with students; they are left on their own to discover them." (p.55) There seems a

strong likelihood, in the light of the evidence discussed, that the same situation exists in this country and that pupils are being taught mathematics in a way that will lead to the achievement of instrumental understanding but not of relational understanding. If this is indeed the case, mathematics is bound to be viewed by pupils as a linear, mechanical subject which they will likely come to accept as a series of facts which are 'right' and about which they as pupils cannot, with impunity, be wrong.

Pupils' perceptions of teachers in the primary school

The mutual perceptions of teacher and pupil are of paramount importance in the interactive situation in the classroom. Such teacher-pupil perception is particularly complex at primary level because of the fact that pupils progress rapidly through a variety of stages in social development during these years. Blyth (1965) gives the example of the difference between the seven-year-old's perception of the teacher as an authority figure and that of the nine-year-old, and the reaction of each to any sign of weakness on the teacher's part. With the younger pupils he suggests the reaction would be of "bewildered anarchy" while the older ones would present the teacher with a kind of "group hostility". (p.102) The expectations of the younger pupils are that order will be maintained while the older ones expect, amongst other things, efficiency and a "fitting object for their loyalty and identification". More general conclusions in this respect reported by Morrison and McIntyre (1969) suggest that pupils "tend to be more concerned with the qualities of teachers as teachers" rather than as personalities and this would appear particularly to be the case with younger pupils. (p.109) They suggest that problems arise when pupils meet with considerable variation amongst teachers who teach them, some of whom may have well-defined rules of behaviour, for example, and some of whom may not.

We have noted the potential effect upon pupils of the teacher as an authority in general and, in particular, as a judge of their mathematical ability leading to the grouping within class that very often takes place. Clearly, if pupils' perceptions of the teacher's ability to teach is not a favourable one, the situation for those who are placed in lower groups may be exacerbated. If they view the teacher as 'not being good at teaching', they are more likely to resent their relegation to a lower

ability group and possibly blame the teacher for their lack of mathematical success. This is a possibility of some importance since, as we have seen, many primary teachers do appear to lack confidence in their ability to teach mathematics. (Ward 1979, Straker 1978) Not only pupils in lower ability groups but pupils generally, whatever their ability, may perceive this and react in an adverse way. Ward (1979) points out that mathematics, more than any other subject, can suffer from poor teaching because of its inherent linearity. (Indeed, as noted earlier, it has been suggested that it is this linearity and the sequential aspect of the subject which leads teachers to adopt ability-grouping in the first instance. (D.E.S. 1978)) The strategy teachers may tend to pursue may be to follow through topics in a step-by-step approach that possibly lacks breadth and depth or even appropriate concrete experience because they are not adequately confident in what they are doing to deviate from the narrow factual path. Clearly, this could have undesirable consequences for all pupils concerned, in not providing an appropriate base from which the average and below average may progress, and in not extending the more able. The presentation of the subject may, therefore, affect the pupil's perception of the teacher if there is a frustration on the pupil's part in failing either to come to understand mathematics or to achieve a satisfactory level in it, a failure which, rightly or wrongly, they may perceive to be a result of the teacher's limitations.

Parental influence

Mathematics at primary level was a subject in which parents once felt confident to judge their children's progress. This confidence was somewhat undermined by the introduction of 'new' mathematics into the syllabus to the extent where books were being published to help parents understand the new mathematical mysteries being unfolded to their children (e.g. "The New Mathematics for Parents" by Heimer and Newman 1965). This is indicative of the degree of concern parents have with respect to the development of their children's mathematical ability and provides an interesting comparison with, for example, environmental studies, about which one would be hard put to find a similar book.

Parental interest in curricular matters generally, appears to have increased over recent years according to Ashton et al. (1975) when com-

paring their findings with those of Douglas (1964). While the degree of parental contact with schools may have been attributed to membership of a particular class in the past, more recently there has been a growing awareness of the lack of what could be called typical behaviour either for the working or the middle class (Musgrave 1979), although specific class variables such as linguistic code may still act as an advantage or disadvantage in the pupil's learning (Bernstein 1971). As we have seen to be the case with pupils, their parents also appear to consider the most important aspect of the teacher's role to be the ability to teach. (Musgrove and Taylor 1969) The individual primary teacher's ability is likely to be judged by parents largely in terms of their own child's success and reports of day-to-day classroom events. Since all parents will consider they have some mathematical knowledge, they may feel adequately initiated into mathematics to make judgements about matters such as the teaching of multiplication tables and basic numeracy.

Bernstein (1975) identifies two contexts of schooling which are relevant to considerations of this sort, one which is 'open' where subject matter and methodology of the curriculum are less defined and less structured, and the other which is 'closed' where content becomes more rigidly demarcated and teaching methods more formal. He sees the spectrum ranging from 'open' to 'closed' in terms of the pedagogy moving from the 'invisible' to the 'visible', with the invisible pedagogy occurring in the early years. Thus while their children are at primary school, parents may find it difficult to make judgements about their progress in some areas of the curriculum because of the high degree of subject integration (as in the case of environmental studies referred to earlier, for example) and a supposedly less structured approach in the teaching of it. However, even at this stage the situation with respect to mathematics is different and once more the visibility of the subject is attested to. Pupils' knowledge of facts leading to basic numeracy is easily identifiable and parents may even be aware of the ability group their children have been placed in for mathematics, presenting a 'closed' and 'visible' pedagogical situation. Thus with the apparent obviousness of success or failure, there may be pressure placed upon the pupil by parents to achieve even more or to improve. Such a situation clearly could contribute further to the effects of the visibility of the subject as a potential source for the increased frustration and possibly the wounding of the self-concept of pupils in their efforts

to learn mathematics.

A further potential cause for frustration on the part of both parents and pupils lies in the situation where the more mathematically able teacher adopts a broader, deeper view of mathematics as a discipline. Because a pupil is proficient in the knowledge of tables and number bonds, parents may assume their child to be very able in mathematics. However, if the teacher is equally concerned with such matters as problem-solving and more abstract concepts such as tessellation or symmetry, success with number bonds alone will not necessarily characterise a pupil as mathematically able. The situation may arise where not only does the parents' conception of the mathematical goals of the curriculum differ from that of the teacher's, but a conflict of views could also arise with respect to the mathematical ability of individual pupils.

Language, primary pupils and mathematics

Pupils' language and the extent to which the code they use is restricted or elaborated has been recognised as an important factor in classroom learning generally. (Bernstein 1961) It is difficult to judge the extent to which any restriction may be overcome by experience at school since it is not known the depth to which personality patterns have been determined by the time pupils enter school which could affect the potential for change. (Barnes 1971) The pupil's language has been recognised as of vital importance in mathematical learning (Shuard 1979) and this must be especially so at primary level where difficulties are compounded by the fact that pupils faced with written mathematics schemes are, at the same time, in the throes of learning to read. Shuard (1979) has drawn attention to the kinds of problem raised in this connection which include matters such as style of writing, visual materials used and the ease with which ambiguities arise. The D.E.S. (1978) survey points out that where special English language needs are concerned, a third of all schools in their sample had some children whose first language was not English. With respect to English as the weaker language in the mathematical learning situation, Dawe (1978) suggests that most studies in this field have tended to dwell on the effects of bilingualism on the pupil's performance in mechanical arithmetic while few have attempted to study its effect on thinking processes which underlie the learning of mathematics.

Austin and Howson (1979) suggest that research into pupils' language and the learning of mathematics in this country "has tended to be directed on language-behaviour during the teacher-child interaction." (p.175) They go on to state that, "At a practical level, investigations would suggest that in most classrooms the teacher does most of the talking and that few pupils respond." This would seem to support the D.E.S. (1978) finding that most mathematical teaching in primary classrooms is of a didactic nature. Austin and Howson (1979) note that "In the early years of schooling we are very much concerned with coordinating the child's developing of understanding of *both* the language of instruction and that of mathematics." (p.174 authors' italics) Again, the point is raised that a lack of active participation of pupils in discussion must necessarily inhibit the growth of understanding, a factor already noted in our examination of the Nuffield Mathematics Project (see Chapter 3).

Summary

The importance of the formation of the self-concept of pupils during the early years of schooling has been noted, together with the fact that much of the social interaction that gives rise to the self-concept takes place in the classroom. (Nash 1973, Good and Brophy 1978) Once stereotypes are established for pupils, they are difficult to overcome (Blyth 1965) and it is suggested that the grouping of pupils according to mathematical ability within classrooms may give rise to adverse stereotypes and poor self-concepts, which in turn may engender poor attitudes towards the subject. On becoming members of a class, pupils have to learn to be part of a group, to come to terms with the fact that they are under constant evaluation by teachers and peers, and that the teacher is in a position of power. (Jackson 1968) It is suggested that each of these lessons may be particularly exaggerated where the study of mathematics is concerned primarily because of the early identification of mathematical ability (arising from the 'visibility' of the subject) and the tendency to group pupils accordingly. (D.E.S. 1978) There appears to be subtle differentiation in approaches to teaching mathematics to girls and boys which would seem to favour boys. (Berrill and Wallis 1976, Preece 1979)

It seems possible that pupils' perceptions of mathematics as a sub-

ject may be affected by the fact that stress appears to be laid upon instrumental understanding and little on relational understanding as identified by Skemp (1979). This arises from the fact that the subject tends to be presented in a somewhat restricted linear fashion by means of an essentially teacher-centred didactic approach and with little emphasis on problem-solving.

Pupils appear to value the teacher's ability to teach more than any other teacher characteristic. (Blyth 1965) There is evidence that primary teachers are not confident in their mathematical ability and hence in their ability to teach the subject. (Ward 1979, Straker 1978) It is possible that where such confidence is lacking, it may result in limitations upon the mathematics that is taught and the teacher's lack of confidence may also be communicated to pupils. If this is the case, pupils relegated to average or below average ability groups may resent the judgement made of their ability while the above average may resent the fact that they are not being adequately stretched in mathematics.

Parents also appear to consider the teacher's ability to teach to be of chief importance (Musgrove and Taylor 1969) and it is arguable that they see their own knowledge of mathematics as enabling them to make judgements about their children's mathematical ability and about what mathematics should be taught. The visibility of their children's success or failure in the subject may add to the pressure of such judgements which may in turn cause some anxiety for pupils and affect their attitude towards learning the subject.

Pupils' language is recognised as of particular importance in the learning of mathematics at primary level and difficulties arising from such matters as linguistic code (Bernstein 1971), ambiguities in meaning (Shuard 1979), English as a second language (Dawe 1980) and the problems of learning to read at the same time as they are learning mathematics, have been noted. Austin and Howson (1979) report that most of the talking in mathematics lessons is done by teachers thus supporting the findings of the D.E.S. (1978) which suggest that most primary mathematics teaching is of a didactic nature.

CONCLUSIONS

The picture that emerges from these considerations of the social

context in which mathematics is taught and learned in primary classrooms is one that is dominated by a dilemma in which many primary teachers appear to find themselves. It would appear that the approach adopted by the majority of them to mathematical teaching is one of which many themselves may be critical, but (and herein lies the dilemma) which they are unable to alter due to their apparent lack of appropriate mathematical knowledge or related expertise. It would also seem that, in most schools, their dilemma is not eased by positive leadership within the school to help them interpret mathematics in a manner which has greater breadth and depth.

There is a clear tendency to adopt an approach to teaching mathematics which is determined almost entirely by the numerical aspect of the discipline, one that is linear and mechanistic, which involves learning facts and skills but seldom involves their application in problem-solving situations, relating them to other areas of study or investigating the existence of mathematics in their immediate environment. The atmosphere in which mathematics is likely to be learned in primary classrooms would appear, therefore, to be fairly limited to one in which pupils are engaged in pursuing the learning of number bonds and computational skills, often individually. There would seem to be little indication of activity and discussion taking place either between pupil and teacher or pupil and pupil. Certainly there appears little, if any, evidence of mathematics being presented (or, indeed, viewed) as a subject which is 'open' and problematic like any other.

The situation is the culmination of what begins initially with the imposition of the teacher's limited view of the nature of mathematics on the mathematics curriculum, so that it not only determines content and methodology but provides the means by which pupils become labelled and grouped. Mathematics becomes so 'closed' and structured, its visibility apparently so dominant, that teachers' perceptions of pupils in their earliest years of mathematical learning can lead to such judgements being made. Thus the situation reflected is one in which many primary pupils may not only develop a narrow conception of what mathematics is, but they are also likely to develop adverse attitudes towards the discipline which few may ever overcome because of the associated development of feelings of inadequacy.

We shall now go on to examine the context in which the teaching and learning of mathematics takes place at secondary level to determine the extent to which this picture may alter as pupils progress from one level to another.

CHAPTER 5

The Social Context of the Secondary
Mathematics ClassroomIntroduction

While it may be the case that "education has always been characterized by low autonomy as a social institution" (Scotford-Archer and Vaughan 1971, p.60), it would be wrong to assume that there is little potential for variation from one secondary school to another and to assume a fair degree of uniformity amongst them. Although certain political constraints exist, individual schools have a substantial degree of freedom and choice which lies within the discretionary power of the head teacher as already noted in connection with primary schools. Peters (1966) states that "The degree of autonomy accorded to head masters, at every level in the school system in England, over matters to do with curriculum, syllabus, discipline and school organization is astonishing." (p.254) From the choice of what shall be taught in their schools to the principles according to which the school will be organised, head teachers have the potential to define the particular kind of institution they aim to develop and the character of the institution which they lead will thus reflect their judgement and choices. It is arguable that this potential becomes more marked at secondary level because of the hierarchical form of organisation that prevails and of the mediation of the head teacher's wishes through departmental heads.

In this chapter we shall attempt to gain some impression of the dominant features of the social context of the secondary mathematics classroom in a way similar to that employed in the previous chapter in connection with mathematics at primary level. We shall concern ourselves first of all with the effect of the school as an institution on the mathematics curriculum. Following this, we shall consider teachers of secondary mathematics, including their mathematical background and their contribution to the social context of the mathematics classroom. Finally, we shall consider pupils studying mathematics in secondary schools from a like perspective.

INSTITUTIONAL ASPECTS OF SECONDARY SCHOOLS AND THEIR EFFECTS UPON THE MATHEMATICS CURRICULUM

A substantial amount of interest has been shown in organisational aspects of secondary schools since comprehensivisation began some years ago. Studies such as those carried out by Richardson (1975), Francis (1975), Newbold (1977) and Rutter et al. (1979) all contribute to the building up of a general picture of the complexity of factors that characterise these institutions. Since we are concerned here explicitly with the teaching and learning of mathematics, it is not intended to go, in depth, into how such schools may be organised. Rather, the intention is to extrapolate relevant features to indicate how they may influence the mathematics curriculum.

The school as an organisation

Rutter et al. (1979) were concerned with investigating what they refer to as 'school processes' in secondary schools. These are viewed as components of the social organisation of the school and hence as creating the context in which teaching and learning take place and "which seem likely to affect the nature of the school experience for both staff and pupils." (p.106) They saw these organisational features as falling within seven broad conceptual areas: (1) academic emphasis; (2) teacher action in lessons; (3) rewards and punishments; (4) pupil conditions; (5) pupils' responsibilities and participation in the school; (6) stability of teaching and friendship groups; (7) staff organisation. The cumulative effect of such factors was found to produce an ethos peculiar to each school. Outcomes in terms of pupil achievement and behaviour were, in turn, found to be linked with this ethos, whereas they were not found to be related to the physical or administrative characteristics of the school. Thus academic and behavioural outcomes of individual schools appeared to be influenced by the kind of social atmosphere which characterised each of them. Clearly, the social atmosphere of an institution would be expected to be largely dependent upon the interpersonal relationships existing within it. Of the kinds of considerations entering into school processes noted above, three of the seven (numbers (2), (5) and (6)) can be seen directly to involve personal relationships while the remaining four have the potential powerfully to affect these relationships. For example, a strong academic emphasis may stratify pupils

rigidly according to ability and establish, as a result, a hierarchy amongst pupils which may inhibit the attainment of a desirable degree of social cohesion within the school. Ultimately, therefore, it would appear that the social aspect or ethos of a school as an institution arises from the combined roles of the people in it, pupil with pupil, teachers with pupils and teachers with teachers.

Rutter et al. (1979) suggest that school processes are open to modification rather than being fixed, external constraints since they are controlled to a greater or lesser degree by various members of staff. While this may be so, the number of staff in a position to effect such change is small. Where teacher action in lessons is concerned, for example, the teacher can supposedly decide the specific material to be taught during a mathematics lesson. On the other hand, the head of the mathematics department will have decided the syllabus from which that material will be drawn while the head teacher may have decided that, in spite of the head of department's wishes, the class the teacher takes will be a mixed ability class. Each has been involved in decision-making at different levels but it is clear that it would be difficult for the class teacher alone to instigate change and that the individual teacher is, in fact, subordinate to the decisions of others. As Francis (1975) reminds us, the rules of a school are drawn up by the head teacher and what happens at departmental level is determined by the head of department. Thus while the constraints imposed ostensibly may be open to change, they become increasingly rigid at the level of the class teacher.

Heads of mathematics departments

Since the immediate arbiter of the institutional rules of the school for mathematics teachers is the head of department, the way in which this role is conceived and implemented is clearly important for the context in which the subject is taught. The importance of the quality of leadership supplied by heads of department is emphasised by Neill (1978) and Hall and Thomas (1977). Hall and Thomas (1977) in reporting the results of a study involving 39 mathematics departmental heads, describe their role as "complex and obscure" and because of the ambiguous requirements, suggest that feelings of anxiety and job dissatisfaction as well as futility and even mistrust of colleagues tend to build up. (p.30) Departmental heads in their sample tended to view their role not merely

in terms of academic demands but of managerial and representational demands as well. They were happy to accept that they represented the general ethos of the school as determined by rules laid down by the head. At the same time, they chose not to hold regular meetings of their department. This was interpreted as an indication that they expected departmental members to accept their rulings just as they, in turn, accepted those of the head. Since the managerial aspect of their role would seem to include not only the organisation of materials and personnel but also direction of the department's aims and supervisory control of the work and standards of the curriculum, it may be assumed that without regular departmental meetings, there could be little involvement of mathematics staff in curricular decision-making.

In such circumstances, we are reminded once again that leadership arises from being a member of a group and from wielding greater influence over members of that group than any other individual. (Morrison and McIntyre 1969) Hall and Thomas (1977) found some evidence that heads of department were concerned to help unqualified members of staff, but "they were neither enthusiastic about the value of formal departmental meetings for this purpose nor prepared to accept automatically responsibility for the discipline problems faced by probationer teachers." (p.35) This is borne out by a study carried out by Shuard (1973) where sixteen heads of mathematics departments completed questionnaires to determine what their expectations were with respect to the duties of new members of department. The results indicated that probationers were expected to carry out the total work load of teachers of mathematics except in connection with planning of a long-term nature. Although the results of both of these studies are based on work with small samples, they provide some indication of the complexities and problems entailed in the head of department's role.

A further study carried out by Hall and Thomas (1978) into the role specifications for heads of mathematics departments as sent to applicants for such posts by schools, suggests that the complexities of the role have yet to be understood sufficiently or identified properly by head teachers themselves. Neill (1978) would appear to agree with this and suggests that heads of department need training. He sees the major responsibility for in-service training of members of mathematics departments as being an integral part of their role and believes that rather than

being concerned primarily with organisational matters, departmental heads should be more concerned with mathematical matters where the need is seen to lie. The importance of the quality of leadership offered by heads of mathematics departments is clearly identified by HMI in a recent survey of secondary schools (D.E.S. 1979b) when they state that "good leadership by the head of department had raised the level of teaching in a school substantially" by effective coordination of the work of the department and that a lack of understanding of schemes of work on the part of some teachers "suggest that more use could be made of systematically organised departmental meetings." (pp.144-5)

Clearly, at secondary level the institutional factors contributing to the social context of the mathematics curriculum become more complex as a result of the kind of hierarchical organisation represented by heads of department, as well, of course, as by the constraints of examinations. The situation differs from that of the primary school in that, although the head teacher is still the main architect of the character of a school through taking decisions or stipulating rules that affect all facets of school life, responsibility for conveying that ethos to staff is largely delegated to heads of department. Thus the head of the mathematics department mediates the values that determine the kind of institution the head wishes to lead.

If the evidence of Hall and Thomas (1977) is accepted as representative of heads of mathematics departments generally, it suggests that they accept this representational aspect of their role and begin to exercise autonomy from that point onwards. It would seem that while (like primary head teachers), as heads of department they do not seek formal meetings with their fellow mathematics teachers, they do, however, expect them to accept their rulings with respect to curricular matters apparently most often without consultation. Thus the individual mathematics teacher may, in fact, exercise very little autonomy with respect to what is taught or to the organisational features that determine how it is taught. Clearly, part of the reason for this can be attributed to the constraints imposed by the examination system which dominates secondary schools. At the same time, if we think in terms of leadership once more, with the suggested lack of regular or frequent contact with department members, heads of department could be said not to be members of the group of which the department is composed and therefore not to

lead it. Any influence would once again appear largely in the form of imposed authority and the resultant picture is one in which the individual mathematics teacher may be in a position of implementing a curriculum with which they are not in sympathy. In a sense, much of the context in which they teach is imposed upon them from the outset leaving little, apparently, to be determined by them.

An indication of the potential influence of the head of department is given by Hargreaves (1967) in the identification of factors affecting teachers' attitudes. He found that the system of the allocation of teachers to particular classes tended to be taken by them as an indication of the judgement of the head of department of their respective basic competence. Thus teachers assigned to teach a lower set in mathematics might assume this to be a manifestation of the value placed upon their ability. This is one of the more obvious ways in which staff relationships may be adversely affected and, in turn, contribute to the atmosphere in which mathematics is taught. While this phenomenon (insofar as it exists) may not be peculiar to mathematics as a subject, it becomes more relevant when considering teachers of mathematics and their qualifications which we shall consider later in this chapter.

The hidden curriculum

Apple (1980) states, "We see schools as a mirror of society, especially in the school's hidden curriculum." (p.1) Perhaps this may particularly be the case in secondary schools where a more structured organisation prevails than in the primary sector. Hargreaves' (1967) study, which concerned a secondary modern school, illustrates how this can happen. Because the school was a selective school for boys, the hidden curriculum would be related to this fact and certain characteristics that followed from it. Classes were streamed, the school had a non-academic, custodial atmosphere, there were few extra-curricular activities for pupils and there was a degree of culture clash between staff and pupils. Clearly the structure and hierarchies implicit in factors of this nature would enter, to some extent, into the teaching and learning situation concerned with the explicit curriculum. The hidden curriculum may pervade the classroom through what may be accepted unquestioningly, as ordinary organisational procedures which are, in reality, procedures selected at a higher level to promote the ethos of

the school. Many of these relate to the school processes identified by Rutter et al. (1979) such as the degree of responsibility allocated to pupils and the system of rewards and punishments adopted within a school.

The phenomenon of the hidden curriculum is currently coming under deeper scrutiny and analysis. Apple (1980) questions whether schools are merely "reproductive mirrors" as is sometimes supposed and suggests that if, as in other work areas, there are in schools "elements of contradiction, of resistance, of relative autonomy", then they have "transformative potential". (p.22) It is possible to extrapolate such a view to the level of the mathematics department or even to the individual classroom and to imagine such elements and postulate their effects. At department level, for example, staff could conceivably resist a head of department's determination not to involve them in curricular discussion and policy making by becoming involved with each other as a group in order to engage in such discussion and professional self-help. At the level of the mathematics classroom, the teacher could resist the apparently 'given' aspect of the hidden curriculum by ignoring the departmental dependence on textbooks and by striking out in the direction of work of a more investigative, problem-solving nature.

A further aspect of the hidden curriculum that exerts some influence at secondary level and that may be particularly relevant where mathematics is concerned is the question of subject status. Richardson (1975), in an investigation into "Authority and Organisation in the Secondary School", identifies mathematics as an 'academic' subject. It perhaps may not be surprising that it should be so described but, as Musgrave (1979) reminds us, the curriculum as a whole displays values and the question arises as to whether the notion of being academic ascribes greater value to the subject than to others. Gordon (1978) touches on this when he suggests that bringing together the collection of subjects that constitute the curriculum "raises questions relating to the status of subjects: whether it is 'given' and if there is any logical distinction between high status (mathematics and science) and low status (social studies and home economics) subjects." (p.148) The recognition of mathematics as 'academic' in nature and being ascribed high status carries with it certain implications. For example, Morrison and McIntyre (1969) point out that a potential source of conflict among teachers within a school is the status perceived by one teacher or department to be ascribed to

another teacher or department. Thus if mathematics is given high status in an obvious way within a school (in resource allocation, for example), this may lead to friction between mathematics teachers and members of staff from other departments. This friction may further be reinforced by the fact that mathematics is, arguably, the only subject that has not been integrated with another in the curriculum. Thus mathematicians have been able to retain an individual identity as an 'authority' unlike some historians, for example, who may have lost theirs in the thicket called 'humanities'.

Summary

The institutional factors of the secondary school that affect the curriculum may be viewed from the perspective of school processes; these are of a social nature and combine to give the school an ethos which has been found to be related to pupil achievement and behaviour. (Rutter et al. 1979) While these processes are seen to be open to change, it would appear increasingly difficult for this to happen at the level of the individual class teacher since the rules of a school are drawn up by the head (Francis 1975) and these are mediated through the head of department (Hall and Thomas 1977). There are indications that heads of mathematics departments do not involve mathematics staff in matters of curricular decision-making and that they tend to view their role with some dissatisfaction and as being ill-defined. (Hall and Thomas 1977) The effectiveness of mathematics departments has been found to be directly related to the leadership given by heads of department and an increase in formal departmental meetings in schools is advocated. (D.E.S. 1979b) The power of the head of department is sometimes assumed by teachers to be manifested in their allocation to teach particular classes which they see to be a reflection of the judgement of their competence by the head of department. (Hargreaves 1967)

The more highly structured organisation of the secondary school gives rise to a more complex and potentially powerful hidden curriculum than is the case in primary schools. Rather than mirroring society through the hidden curriculum, Apple (1980) suggests that inherent in it are the means to bring about change. Where mathematics is concerned, the hidden curriculum may manifest itself through superior status ascribed to the subject which could cause resentment amongst staff members in other

departments. (Morrison and McIntyre 1969)

TEACHERS OF MATHEMATICS IN SECONDARY CLASSROOMS

In Chapter 4 we noted the identification of a degree of cynicism within the teaching profession by Nash (1973). In a study of discipline in the comprehensive school, Francis (1975) strikes an equally pessimistic note when he states, "Conditions of work may account for teachers' caution, but they do not explain the full force of their cynicism, which can be ferocious." (p.151) Part of this cynicism he sees as arising from the fact that many teachers resent the position in which they find themselves where they are expected to accept values and methods handed down by other people who, themselves, may not be seen to be directly involved in representing or implementing them in the classroom. As we shall see in the following considerations of mathematics teachers, they do not escape this particular attribute of the profession in the course of their duties.

The mathematical background of secondary teachers of mathematics and their perception of the subject

The fact that teaching is moving towards becoming an all-graduate profession has brought with it a confusion in variety of the degrees that student teachers may take. With respect to mathematics, it is possible to study for a Bachelor of Education degree with mathematics as a main subject or, alternatively, a degree in mathematics with a further year taking the Post-Graduate Certificate of Education to provide a professional qualification. There also exists the possibility of reading for a degree in Mathematical Education. Added to these three degrees, a fourth type of training to become a specialist teacher of mathematics entails following a certificate course with mathematics as a main subject. The variety provided in these four approaches means that initial training for secondary mathematics teaching results in courses in which different kinds of expertise are stressed and which may pose related problems once teachers enter the classroom. One view is that the demand for high academic standards in colleges of education courses opens the trap of "too narrowly conceived academic standards in mathematics" which may be beyond the student's capacity to understand. (Royal Society 1976, p.18) (This lends support to Begle's (1979) contention with

respect to an optimum in teachers' mathematical knowledge referred to in the previous chapter.) At the same time, the Royal Society suggests that the view that appropriately high standards can only be attained in the academic courses offered at universities raises the problem that this may detract from the professional aspect of initial training. Thus the four paths to becoming qualified as a mathematics teacher at secondary level while offering a diversity of experience, would appear to have inherent problems with respect to achieving an appropriate balance between mathematical knowledge and the professional component of the qualification.

The information from studies quoted in connection with the primary sector (see Chapter 4) concerning college of education mathematics student teachers also relates to students entering the secondary sector. The results obtained by Lumb and Child (1976) are of particular relevance in showing that prospective secondary teachers in their sample did not show a substantial improvement in attitude at the end of their first year. If this tendency were to prevail to the end of their course, taken together with the results of Ray's (1975) study, it would suggest that those who follow this path to becoming teachers of mathematics at this level might do so out of an interest in the subject but with little positive attitude in terms of their success in, or liking for, the subject.

Referring again to the Royal Society (1976) report, they note that there has been a marked effort to bring theory and practice closer together in the education of graduate student mathematics teachers by integrating the theoretical disciplines and injecting more material of 'mathematical education' into courses in an effort to overcome difficulties which students have been found to have in this area. This is an attempt to overcome what the Germans call 'practice shock' which is seen to arise from the lack of practical teaching skills from which graduates appear to suffer, a factor which apparently in the past "has hardly been taken into account in the reform of teacher training in the Federal Republic." (Mies et al. 1975, p.36) In this country, a Teacher Education Project has been established in an attempt to come to grips with such problems. (Kerry 1978) The five main themes chosen in this project for development in courses leading to the Post-Graduate Certificate of Education are class management and control, mixed ability teaching, excep-

tional pupils, language across the curriculum and teaching skills. Presumably these themes are indicative of problem areas post-graduate mathematics student teachers have found most difficult. A further study of a similar nature is being undertaken at the University of Leicester.

There has been limited investigative attention paid to the plight of graduate mathematicians who have entered the teaching profession. A pilot study carried out by Shuard (1973) has already been referred to in this chapter. In a study of 47 first year mathematics graduates, Cornelius (1973) found that in the identification of problems, 31 re-referred to discipline as the major one with the next most difficult in order of frequency being the teaching of pupils of low ability and teaching mixed ability groups. A comment such as "Discipline, especially with the less able who are uninterested in school, work and mathematics and generally disillusioned with life" is indicative of their problems. Further comment such as "Inability to change anything...policy is sent down and the people at the wrong end of the department have the dirty work to do" has a slightly cynical ring about it which lends some support to the contentions of Nash (1973) and Francis (1975) in this respect. In a study concerned with the professional socialisation of graduate students generally, Hoad (1974) identifies the existence of a "'subject culture' transmitted from experienced teachers to newcomers" and suggest, with respect to a newcomer, "If he wishes to be accepted, he must usually offer, in exchange, his compliance to schools norms and often, his help in school activities, even when these have no bearing on his own professional development." (p.177)

Some of this cynicism can also be detected where teacher attitudes are concerned, in the results of a study carried out to investigate the promotion and careers of secondary teachers. (Hilsum and Start 1974) A sample involving 6,772 teachers from 881 schools from almost all Local Education Authorities in the country were asked to rank twelve factors they saw as favouring promotion and twelve factors they thought ought to favour promotion. With respect to the former, the first five factors identified in order of importance were: (1) being a graduate; (2) being a specialist in a shortage subject; (3) social contacts; (4) conformity with advisers and (5) good relations with the head teacher. The factors they felt ought to favour promotion were: (1) flexibility in teaching methods; (2) familiarity with new ideas; (3) ability to control pupils;

(4) concern for pupils' welfare and (5) having teaching experience in a variety of schools. Clearly, there is considerable discrepancy between these two sets of factors and a marked personal emphasis in the first compared with a more professional one in the second. This evidence would appear to suggest that secondary teachers do not feel that they are judged by objective, professional criteria when being considered for promotion.

The apparent belief that the second most important factor affecting promotional chances is to be a specialist in a 'shortage subject' could augur well for mathematicians in view of the continued shortage of mathematics teachers. However, the survey showed that mathematics teachers in the sample ranked seventeenth according to subject taught, in gaining a Scale 2 post. By the time Scale 4 was reached, they ranked eighteenth; as Deputy Heads they ranked equal fifth with English and geography and as Heads, they ranked fourth. These statistics would not appear to support the view that teaching a shortage subject (in this case mathematics) affects the promotion of individuals. The fact that secondary teachers appear to perceive the opposite to be the case offers one indication, however slight, that their views concerning promotion prospects may be somewhat misguided.

There appears to be little evidence upon which to draw to gain an insight into how mathematics is taught in secondary schools and what this can tell us about how teachers perceive the subject, other than surveys carried out by Her Majesty's Inspectorate for the D.E.S. The most recent survey suggests that mathematics teachers adopt a mainly didactic approach. (D.E.S. 1979b) Hilsum and Strong (1978), in a study investigating a breakdown of the secondary teacher's day, found that mathematics teachers spent most of their time instructing and the proportion of time spent doing so was greater for the graduate teacher than for the non-graduate. However, it was also found to be the case that mathematics teachers more than any other subject teachers called pupils to them for individual marking and consultation.

HMI found in their secondary survey that most secondary schools adopt a mixed ability system of grouping pupils for the first two or three years. (D.E.S. 1979b) A Schools Council (1977) study of mixed ability teaching of mathematics in 26 schools comments on the fact that

the linearity of mathematics is usually used as an excuse for not adopting mixed ability grouping but that various individualised approaches have capitalised on this linearity in order to produce materials for the individual pupil in a mixed ability situation. Rather than exploiting any potential flexibility of these materials, however, it was found that teachers tend to use a single approach. The 'whole class' method was the most popular, involving the use of commercially produced topic or graded booklets interspersed with class lessons. It is suggested that teachers are unaware of the variation in method possible with a mixed ability group and that some neglected exploration and investigation almost entirely for the performance of exercises, and relied too heavily on one type of material. Morgan (1977) has found in a study carried out in Scotland, that one such individualised scheme tended to inhibit any dialogue taking place between teacher and pupil.

Although the evidence we have examined relating to the mathematical background of secondary teachers and their adoption of particular approaches to teaching the subject may not provide a great deal of information with respect to how they perceive mathematics as a discipline, what is suggested once more is some lack of confidence in teaching the subject. (D.E.S. 1979b) In the case of teachers with a P.G.C.E., while they may be confident in their mathematical knowledge, there are indications that they find some difficulty in putting their subject matter across in the classroom; the Teacher Education Project bears witness to this. Kerry (1978) In the case of teachers with a B.Ed. degree specialising in mathematics, while their mathematical background on entry to colleges of education may have improved (Shuard 1977) there is the suggestion that they are being stretched beyond their mathematical capacity and not achieving an appropriate understanding of the mathematics they are being taught to teach. (Royal Society 1976) For the non-graduate teacher, the arguments based on the studies of Ray (1975) and Lumb and Child (1976) are relevant once again. It will be remembered that there were indications of adverse attitudes towards methods by which students themselves had been taught mathematics at school, an interest in mathematics but not a great liking for it and some evidence of a lack of improvement in attitude towards mathematics during at least the first year of their course.

Thus the variety of mathematical background and training that secon-

dary teachers possess is reflected in the difference in the kinds of problem they may face in the classroom. It would appear that, at one extreme, highly specialist knowledge of their subject may obstruct their pedagogical expertise while at the other extreme, lack of adequate mathematical knowledge or a weak understanding of what they do have, may undermine their confidence in their ability to teach the subject. Looking for indications as to how this variation in background may affect classroom practice, we find that most secondary teachers appear to employ a didactic approach and that graduate teachers spend a greater proportion of their teaching time in this way than do non-graduate teachers. Where mixed ability teaching is adopted, although the potential exists for flexibility and a variety of approaches, teachers apparently tend to adopt one to the exclusion of others, with little emphasis upon application and exploration. The overall picture of secondary mathematics teaching that emerges, therefore, is one in which we see again a tendency for the perceived nature of the subject to be factual and highly structured. It is an interesting point that mathematics teachers should be found, more than any other subject teachers, to consult with pupils individually over the work they produce. On the one hand, this could be encouraging in that it presents the possibility of dialogue between teacher and pupil which is all to the good. On the other, it could also be a further indication of the visibility of the subject and of the perceived inherent demand for the achievement of a correct answer.

Secondary teachers and decisions affecting the mathematics curriculum

The difference between the situation in which secondary teachers find themselves compared with that of primary teachers, lies in the diminished professional autonomy they enjoy. In seeing themselves as purveying values and adopting methods determined by others (Francis 1975) coupled with the domination of an examination system, they may well consider that most important decisions are taken out of their hands. This is a situation which has been identified in connection with the teaching profession in other countries. Arfwedson (1976) discusses the dichotomy between the system of goals and the system of rules which is present in the organisation of schools in Sweden. The goals of a school are not seen to be accompanied by sanctions since they are related to the pedagogical methods and attitudes adopted, whereas rules do carry sanctions. A teacher cannot, for example, disregard keeping to a timetable or recording pupils' attendance with impunity. As Arfwedson (1976)

points out, the power of the teacher is somewhat superficial since there would appear to be an inevitable conflict between rules and goals, and although the teacher has apparent pedagogical freedom, the rules impinge strongly. He states:

"On the one hand the teacher is a part of the hierarchical power-structure of the school organisation, on the other hand it is his duty to realise goals that are mainly democratic and anti-authoritarian." (Arfwedson 1976, pp.141-2)

Again, we see the pervasiveness of the hidden curriculum. In America, this is a situation that has been exacerbated with respect to mathematics by widespread legislation for minimum-competency-based mathematics instruction which has resulted in teachers tending to treat the established minimum as their ultimate aim in the mathematical achievement of pupils. Also, with a strong move towards individualisation in mathematics teaching (Webb 1980) the teachers decision-making usually associated with planning (Clark and Yinger 1980) has been usurped and consequently teachers are seen to be in danger of becoming 'de-skilled'. A similar fear has been expressed by Morgan (1977) in a study connected with the adoption of an individualised scheme in Scotland referred to earlier. It may be that, as members of what Edelman (1974) refers to as one of the 'helping professions', the cynicism of teachers may arise, in part at least, from the implicit assumption that the priority of their role is to help and to guide, while in practice they are thwarted in exercising their professional judgement in doing so.

Some attempt has been made in America to study the relationship between teacher characteristics and attitudes with pupil achievement in mathematics. The study called the National Longitudinal Study of Mathematical Abilities is reported by Begle (1979) who states his scepticism of the results. He concurs with evidence gathered by Rosenshine (1971) that the concept of teacher effectiveness "may not be valid" since it is a quality that may vary over a period of time. (p.37) Begle's scepticism led him to undertake further analysis of the data gathered in the original study and the main conclusion drawn was that "significant relationships between teacher variables and effectiveness scored were not frequent, appearing in fewer than 30 per cent of the possible cases." (p.50) Affective variables were found to have a greater effect than background variables (e.g. teacher's sex or marital status) and the stronger affective variables differed depending upon the age of the pupils taught by the teacher. For example, at the sixteen-year-old level the satis-

faction of the teacher's need for approval correlated most strongly with pupil achievement. Begle (1979) concludes that "our attempt to improve mathematics education would not profit from further studies of teachers and their characteristics." (p.53) This also suggests, as might be expected, that factors which lead teachers to make particular decisions are not always of a pedagogical nature.

Finally with respect to teachers' decision-making, Woods (1979) has studied how teachers in a secondary modern school (about to become comprehensive) in effect, came to decide pupils' subject choices for them. He argues that they were forced by "commitment and structural constraint" to accommodate to the particular strategies of the school to achieve a pattern of examinations identified as desirable by the head. (p.178) While subject choice was presented to pupils in the guise of their choice being guided by teachers, "In fact, teachers made most of the decisions, albeit by rather tortuous routes, which led some to protest and yearn for 'cleaner' decisions." He goes on to state that "The overall effect is to get the pupils to articulate the teachers' decisions." This indicates clearly once again a basis for the cynicism of teachers and the dilemma in which they find themselves. Selkirk (1974) in his study of pupils doing Advanced level mathematics concluded that there were specific grounds for encouraging some pupils and discouraging others, with respect to choice of mathematics at this level. Possibly it would be to the benefit of both teachers and pupils if teachers were in a position to make decisions in this respect, according to more realistic guidelines.

Secondary teachers' perceptions of pupils in the context of the mathematics classroom

We have already noted the fact that during the primary years of schooling, pupils tend to be classified by both teachers and peers. By the time they move on to secondary school, such judgements will already have affected the pupil's self-concept. Judgements continue to be made, however, and where those of teachers are concerned, there is an important difference in the grounds upon which they are based. Because secondary schools are organised as they are, teachers spend proportionately little of their teaching time with any one pupil. As a result, their judgements of pupils are made on the basis of little teacher-pupil contact. This was noted by Hargreaves (1967) who studied the development of the atti-

tudes of both teachers and pupils in the course of their adaptation to the system of a streamed, secondary modern school. He argues that because of the minimal contact most secondary teachers have with their pupils, teachers' assessment of them tends to be more indirect and based upon their expectations of pupil conformity to a role as opposed to being based upon cause-and-effect experiences as is the case in the primary sector. From this, there follows a categorisation of pupils by teachers on minimal evidence and teacher-pupil interaction will be defined within the constraints of this categorisation. Thus begins the self-fulfilling prophecy where, as Hargreaves (1967) suggests, the pupil will adjust to the teacher's categorisation by exhibiting behaviour appropriate to it.

When Nash (1973) followed his sample of pupils from five primary schools through to a single comprehensive, he found that old friendship groups broke up and new groups were formed which once again reflected fairly accurately the teachers' perceptions of pupils. He suggests that two possible reasons for this were either that the teacher's perceptions of pupils are influenced by the company they are seen to keep or, conversely, pupils' friendships are influenced by the teacher's perceptions of them. Nash (1973) found that in evaluating pupils, teachers used personal rather than academic constructs, the three most common being Hardworking/Lazy, Mature/Immature, and Well-behaved/Poorly-behaved. The reason for this may well be the lack of a great amount of direct teacher-pupil contact identified by Hargreaves (1967) but if pupils do, indeed conform to the kind of categorisation that labels them 'poorly behaved', the self-fulfilling prophecy then becomes a vicious circle. Roberts (1971) describes the general effect of teachers' perceptions of pupils when she writes, "The teacher must be made aware of the potency of his expectations. Research shows that very, very simple acts on the part of teachers result in astonishing behavioural changes in students." (p.174)

We are led once again to consider how such matters may affect the social context of the mathematics classroom. A more complete picture will emerge as we proceed next to consider secondary pupils in this context. However, it is worth noting at this stage that if, as it would seem, teachers' perceptions have such potential for shaping the behaviour of pupils, both academic and otherwise, mathematics may again be a vehicle for the production of an exaggerated effect through what we have described

as its 'visibility'. Pupils will inevitably react to readily apparent success or failure with varying degrees of strength and in particular ways. Where there is not a great deal of success, the cumulative effect very likely leads to the hardening of adverse attitudes. The possibility also exists where, with more mixed ability teaching and a tendency to adopt more individualised materials and where the only differentiation among pupils is the speed at which they work (Schools Council, 1977), the definitiveness with which teachers may label pupils in more controlled learning situations of this kind will increase. If, indeed, mathematical teaching in secondary schools does tend to be a concentration of instruction from the teacher or the use of individualised materials by pupils, the lack of dialogue between teacher and pupils would account to some extent for the fact that teachers' judgements tend to be based on behavioural evidence rather than that of a more academic nature, in the mathematics classroom at least.

Summary

Secondary teachers' perceptions of pupils are largely based on minimal contact with them which can lead to their categorisation of pupils and the beginning of a self-fulfilling prophecy wherein pupils conform to teachers' expectations. (Hargreaves 1967) Nash (1973) has found that teachers at secondary level tend to use personal rather than academic constructs in judging pupils and Roberts (1971) emphasises the potentially powerful effect of teacher expectations. It is suggested that any such effects may be more exaggerated in the mathematics classroom because of the visibility of the subject and especially with the use of a mainly didactic approach or individualised materials.

SECONDARY PUPILS AND THE STUDY OF MATHEMATICS

The socio-psychological factors that demand consideration with respect to the study of mathematics by secondary pupils are similar in kind to those considered in relation to primary pupils. Clearly, however, differences arise both as a result of the age of secondary pupils and from the different form of organisation in secondary schools. For example, there is the more complicated evolution of the self-concept during adolescence and the fact that pupils are faced, for the first time, with the element of choice in their selection of the subjects they study. Most

of the matters that potentially affect the social context in which secondary pupils learn mathematics nevertheless fall with categories we have used in our examination of primary pupils. We shall, therefore, begin by examining some of the characteristics of secondary pupils as individuals followed by evidence relating to their perception of mathematics as a discipline. The final areas of consideration will be pupils' perceptions of teachers and an examination of parental influence at secondary level.

The secondary pupil as an individual

The experience pupils undergo in different primary schools can vary greatly and Newbold (1977) refers to problems that arise when these different experiences are brought from a number of primary schools to a single comprehensive school. Mathematics is mentioned in particular where marked variation in the performances of pupils at the end of the first year were related to the primary schools from which they had come. This reflects the results of the primary survey (D.E.S. 1978) where it was noted that only two-thirds of all classes in the survey were given work related to all of the items identified by teachers as forming a mathematics curriculum. As a result of the kind of mathematical experience they will have had at primary level, and more particularly their achievement or lack of it with respect to the subject, pupils' attitudes are likely to be entrenched by the time they enter secondary school. Evidence suggests that these attitudes are unlikely to be favourable and that at lower secondary level, few pupils appear to like mathematics although they recognise its usefulness and the necessity of having some knowledge of it. (Duckworth and Entwistle 1974)

The attitudes of girls towards mathematics becomes of increased concern by the time they reach secondary level. One source of evidence of the discouragement of girls from studying mathematics at a higher level has already been quoted (Ray 1975). The effects of such discouragement by teachers have been more widely studied in the USA than in this country. For example, Luchins (1979) observes that high school counsellors often discourage girls from pursuing mathematics and preparing for 'quantitative' careers because they do not believe that these activities provide opportunities for them. Armstrong (1980) states that, "It is the active encouragement of parent, teachers and counsellors which seem to affect participation (in high school mathematics courses)"

and goes on to suggest specific techniques to assist girls in overcoming a negative self-image with respect to mathematics. (p.30) Another study offers evidence that girls do not wish to be seen to succeed in mathematics because of the supposed masculine overtones of the subject. (Horner 1968) In yet another American study, Fennema (1979) reports that all the more obvious affective factors in connection with girls learning mathematics are related to mathematics being viewed as a male domain.

In an investigation carried out in this country, Preece (1979) tested the computational ability and attitudes towards mathematics of a sample of 1250 secondary pupils of both sexes. The overall attitude rating of girls and boys did not differ markedly whereas the profile did, for example, with the boys showing greater self-confidence and the girls showing a greater intrinsic interest in the subject. In the course of a year, the overall scores for the boys increased slightly while those of girls decreased and their attitudes became more negative. This finding was endorsed by careers personnel who found clear indications of girls changing what had been preferred choices of career simply because mathematics was required. It was concluded that "girls can rival the boys in many aspects of mathematical ability in the second year at secondary school, but the female attitude to the subject is a deteriorating factor". (p.29) It is interesting to note in conjunction with this that the recent survey carried out with fifteen year olds by the Assessment Performance Unit indicates that boys had higher scores in all sub-categories of written tests, the difference on average being about 3.5% and they also had higher scores in the practical tests. (D.E.S. 1980b) Thus the effects of the nature of the subject become more complex throughout the secondary years when related to the sex of the individual pupil.

Musgrave (1979) considers the choice which pupils have to exercise at secondary level to be a vital new element in their academic careers but sees it as being complicated by an increased development in their self awareness. He quotes Hudson (1968) who postulates that at this age, pupils are able to differentiate among four different selves and as a result, reflective choice becomes more difficult for them. The four selves are the "actual self" which is who they believe they really are, the "ideal self" which is who they would like to be, the "perceived self"

who is the person their teachers perceive them to be and the "future self" the person they expect to be a few year hence. (Musgrave 1979, p.229) All four are seen to be interdependent and to affect the ways in which pupils make their choices. This view emphasises the complexity of the situation with which secondary pupils are faced when choosing subjects and suggests the kinds of conflict that may arise. For example, the 'future self' of a pupil may include the self-picture of a career in electronics while the 'actual self' may involve a poor self-picture with respect to mathematical achievement and therefore would appear to render the choice of mathematics necessary in one sense but impossible in another. Such reasoning also helps to clarify how a conflict situation may arise for girls in choosing to take mathematics when their 'ideal self' is viewed in terms of femininity while they may consider that their 'perceived self' in relation to the subject will be seen to contradict this.

Selkirk (1974) deduced from his investigation that there were grounds for encouraging pupils to take mathematics at Advanced level in unorthodox combinations with other subjects (e.g. with Latin and History) and that the choice should not always be determined by the combinations of subjects pupils had previously followed. He noted that pupils in his sample who had done this had achieved well, and encouragement given to the study of the subject outside the usual combinations would be likely to benefit girls in particular. It seems possible that the element of choice offered secondary pupils with regard to the option of mathematics may be limited by a context of traditional combinations which could well prohibit some potentially able pupils from taking it at a higher level. Perhaps this is where some change in teachers' decision-making related to guiding pupils' choices noted earlier could have a desirable effect.

Secondary pupils and their perception of mathematics as a subject

On the whole, pupils' perceptions of mathematics as a discipline must be deduced from studies concerned with their attitudes towards it. However, one study in the USA by Erlwanger (1973) has attempted to explore the pupil's conceptions of mathematics through case studies of individual pupils. He is concerned to show how conceptions differ from one pupil to another but that for each, it is a complex but stable and

cohesive system which is manifested in their expressed views, ideas and beliefs about mathematics. For example, the apparent confidence of one pupil was found to be based on the belief that he was capable of determining his own system of rules, while the confidence of another was associated with a belief that all that was needed was to adhere to established rules and a dependence upon the booklets used in the class. Erlwanger's studies indicate how a pupil's conception of mathematics built upon basic misunderstandings may go undetected through the years and result in a complexity of mistaken habits and beliefs that may be difficult to change.

Turning to evidence of the attitudes of secondary pupils towards mathematics, Aiken (1976) in his survey of work carried out in this field refers to investigations which show significant but low correlation between attitude and achievement. Jackson (1968) reviews such work with respect to school subjects generally and reports that most investigations linking attitude with achievement have found no significant relationship between the two. In a study of the attitudes of secondary pupils to mathematics in this country (Gopal Rao 1967) significant differences between different schools did not appear to relate to syllabus content, viewed in terms of traditional and modern. There appeared to be a significant correlation between attitude to mathematics and attitude to school generally, between the attitude of individuals with those of their peer group, and a significant but low correlation between parents' and pupils' attitudes. Duckworth and Entwistle (1974), in a study already referred to, investigated the attitudes of second- and fifth-year pupils to nine subjects studied and found that mathematics was rated consistently low in 'interest' at sixth and seventh place respectively by girls and boys in second year and equal seventh in fifth year. With respect to 'social benefit' it fell from second to fourth place, again for girls and boys, while it was rated fifth and third respectively in 'difficulty' in second year and equal fourth in fifth year. On the whole, the picture is one of deteriorating attitudes.

Factors affecting choice of mathematics as a subject at Advanced level have received some attention and supply further indications of pupils attitudes. In a study involving three sixth form classes, two

of whom had followed a traditional Ordinary level syllabus and one a modern syllabus, Kempa and McGough (1977) measured attitudes of over 300 pupils in terms of 'ease', 'usefulness' and 'enjoyment'. Once again, no difference was found between the attitudes of those who had followed a traditional or modern course. It also appeared that the difficulty factor was of less importance than the enjoyment factor in determining their choice of mathematics while views related to usefulness were differentiated according to whether pupils were following an essentially scientific course as opposed to some other combination of subjects. Selkirk (1974) also found that type of course previously followed had no significant effect on sixth form pupil's attitudes to mathematics and noted a decline in favourable attitudes between fifth and sixth form as well as between lower and upper sixth form. His results indicated that only approximately 5% of his sample of over 1000 pupils viewed the teaching of mathematics as an attractive career prospect.

It is difficult to draw very specific inferences or conclusions with respect to pupils' perceptions of mathematics from studies of this kind. There is some indication provided by Erlwanger's (1973) case studies of how pupils develop a view that may best be described as individualistic. However, the cohesive system which they see mathematics to be is a reflection of how they have been taught the subject as well as the individual personal characteristics which affect their perception. There is some indication of a lessening of influence of the perceived applicability of the subject on pupils' views in Duckworth and Entwistle's (1974) finding that mathematics was rated lower in terms of 'social benefit' by the time pupils reached fifth form. By the time sixth form is reached and pupils have opted to study the subject at this level, 'usefulness' of mathematics would appear to be most often clearly linked with other subject choices.

What appears to be a consistent factor throughout these studies (where it arises) is that where modern or traditional syllabuses were followed, the modern course did not engender better attitudes towards mathematics. This has particular relevance for mathematics curriculum development but where the pupil's perception of the subject is concerned, it seems only to tell us that 'mathematics is mathematics' to pupils, whatever the content. This seems a strong indication that it is the

way the discipline is taught and not the type of mathematical content that may be the predominant factor where the formation of perceptions and attitudes are concerned. It is difficult to predict, for example, the effects of mixed ability methods upon the attitudes of pupils in the early years of secondary school. Given evidence of a relatively strong relationship between pupil and peer group attitudes and the deterioration of attitudes during these years (Gopal Rao, 1967), it could be argued that mixed ability grouping may allow peer group influence to dominate. However, as we have noted, mixed ability teaching in mathematics has led to an increase in the adoption of individualisation in teaching methods which involves pupils working with materials that are self-expository and lessen the amount of teacher/pupil contact. (Morgan 1977) We have also noted that problem-solving and investigatory work tend to be neglected. Again, therefore, it seems possible that the linearity of mathematics becomes stressed at the expense of other aspects of the subject and hence pupils' perceptions of the subject may become biased in this direction at secondary level if they have not already been so.

It is possible that this kind of effect may be aggravated further by the pupils' perceived status of the subject in the school as a whole. If the mathematics department does not have specialist accommodation, for example, and the subject is taught wherever there are desks, chairs and a blackboard, the impression given of the subject will be that of an academic one which consists entirely of book work. The recent D.E.S. (1979b) survey suggests that this is the situation with respect to mathematical teaching resources in many schools and that reallocation of accommodation should take place in 27% of all schools surveyed. It is recognised that mixed ability teaching of mathematics, to be successful, requires a good variety of resources (Lingard 1976) and clearly, if the subject is going to be regarded as anything other than mechanical and linear in nature by pupils, such resources become even more essential.

Pupils' perceptions of teachers at secondary level

It would appear that the first thing secondary pupils expect of teachers is an ability to keep order and if they are not capable of doing so, they are regarded by pupils "as having broken the rules". (Nash 1973, p.128) Thus the pupils labelled as 'badly behaved' will

likely act in an intransigent way and feel justified in behaving disruptively. Although Francis (1975) suggests that the claim that classroom control is strongly allied with the subject being taught is based on a somewhat "dubious foundation", he does acknowledge that the subject is an important element. (p.70) We have noted that where mathematics is concerned, there is the possibility that the visibility of success or failure in the subject may aggravate a situation in which a pupil labelled 'badly behaved' is likely to meet the expectations of such a categorisation by the teacher.

In a study of a class of twelve-year-olds during their first term in secondary school, Nash (1974) attempted to identify how pupils tend to discriminate between different teacher behaviours. He found that six pairs of constructs emerged strongly in the way in which pupils described how teachers behave. These were (1) Keeps order - Unable to keep order; (2) Teaches you - Doesn't teach you; (3) Explains - Doesn't explain; (4) Interesting - Boring; (5) Fair - Unfair; (6) Friendly - Unfriendly. He suggests that the identification of these constructs shows how clearly "the pupils' view of what is appropriate teacher behaviour and what is not is well developed." (p.50) Nash also found that the pupils' view of their own role is passive and one in which they do not see themselves as actively involved in finding things out on their own, nor in attempting to control their own behaviour.

This passive role of the pupil can be seen to emerge once again in a study by Hargreaves (1972) in which he examined interpersonal relationships in classrooms at secondary level. He noted that pupils tend to share a generalised attitude towards the teacher and drawing on the work of Flanders (1967) in studying relationships among teachers, pupils and their attitudes, he classified teachers as direct or indirect according to the preponderance of the kind of statements made by teachers to pupils. The direct teacher tends to be a purveyor of information while the indirect teacher is seen as pupil-centred, allowing the initiation of ideas to come from the pupils. It is suggested that the indirect teacher produces better attitudes to learning and higher attainment on the part of the pupils; the teacher who takes into account the ideas and feelings of pupils is rated as 'good'. It would seem that the role of the direct teacher subsumes a passive role on the part of the pupil and Nash's finding that pupils themselves interpret their role in this way in turn

suggests that they may tend to view teachers in general as 'direct'. Hargreaves (1972) argues that as a result of the power the teacher holds and the pupil's recognition of it, the pupil's view of the teacher, whatever it may be, is a product of, and response to, the teacher's behaviour. Roberts (1971) also comments on the fact that "Student anxiety evidently centres on their helplessness in relation to the teacher's power." (p.174)

The study carried out by Yates (1978) of four mathematical classrooms provides examples of what could be identified as 'indirect' and 'direct' teaching of mathematics and the reactions of pupils to the different approaches. One teacher quoted (who could be characterised as 'indirect') uses an open question and pupils' subsequent answers and further questions to develop the idea of elimination in linear programming. Yates notes that "He is not afraid to listen to the pupils' interpretation of questions." (p.115) On the other hand, the dialogue between another teacher and his pupils indicates that he expects the pupils to get onto his 'line' by posing questions that have a highly specific answer and "He dismisses a point as not important without stopping to consider the importance for the pupil concerned." (p.105) The pupils react according to the value they perceive the teacher to place on their contribution to discussion by sitting there and "tolerating him, endeavouring to find his answers at appropriate moments." (p.107)

Keddie (1971) has shown how pupils' perceptions of the teacher can be linked with overt categorisation into bands or streams. She found that A-stream pupils appear to accept unquestioningly the teacher's perspectives and definitions with respect to the curriculum whereas C-stream pupils do not. Rather, what she refers to as the pupils' "scepticism" causes them "to question the teachers' mode of organizing their material". (p.151) They also appeared to question the assumptions underlying the content they were being taught which was a 'new' subject called 'social science'.

These studies suggest that secondary pupils have quite well-defined perceptions of the teacher. These perceptions appear to follow largely from recognition of the teachers' position of power and the pupils' own apparent lack of freedom under it. Thus if teachers choose to develop

a more 'indirect' role, it could be difficult to overcome the inertia of pupils that may arise from their perception that their role is, almost by definition, a passive one under the 'rule' of the teacher. Bearing in mind the fact that secondary pupils have many different teachers, it is unlikely that they will meet a consistent pedagogical approach across the curriculum. This could add to their resistance to their own adoption of a more active classroom role. Where mathematics is concerned, pupils' apparent passiveness could well be more exaggerated because of the perception of the subject they may have gained. A view of mathematics as a 'given' unquestionable body of knowledge will likely become further entrenched if they are left to progress through mathematical material at an individual pace, with little discussion with the teacher. A further factor of this nature is that any inability of the below average pupil to accept the teachers' perspective of the organisation and content of the curriculum suggests that they may have difficulty in accepting some of the topics included in modern mathematics syllabi as being what they have come to know as 'mathematics'. On the other hand, this tendency could be turned to good advantage by the teacher taking up such questioning and turning it into a constructive dialogue.

Secondary mathematics pupils and class and parental influence

A follow-up study of 114 pupils from the Plowden primary survey (D.E.S. 1967) through to secondary school appears to reinforce the need for parents to become informed. (Ainsworth and Batten 1974) The most important parental characteristics linked with high pupil achievement were found to be "ambition, literacy, interest and awareness". (p.123) Surprisingly, the single variable most strongly related to pupils' success was the size of family from which the father came, pupils with fathers who were 'only children' having the highest likelihood of success. This suggests that fathers with such a background may directly influence their children's academic progress by becoming involved in it either because they have come to recognise a value in academic achievement as a result of such pressures placed upon them or for some other reason. Cox (1979) suggests that the main implication of a study carried out with a sample of disadvantaged pupils is that for intervention procedures to be of any value to the pupils, it is vital to gain the interest and co-operation of parents. In his study, Newbold (1977) found that only about 50% of parents of low ability pupils showed an interest in their progress at school.

In a study in which questionnaires were administered to 3,400 pupils in thirty-six secondary schools, Witkin (1974) concludes that "It does not appear that the social structure of schools and the experience of the children within them can be profitably described in terms of the class culture conflict model." (p.323) Witkin suggests, however, that the influence of the family is felt in the way family background may limit the extent to which a pupil uses the value systems presented by the school, to good advantage. This appears to support Keddie's (1971) finding that C-stream pupils, who tended to be from a working class background, were not "able to work within the framework which the teacher constructs". (p.150) As Witkin suggests, they may accept the values but not be socially articulate enough to benefit from them, while middle class pupils may choose to reject them altogether. Thus the influence of parents is subtly manifested and while it may be oversimplifying to reason in terms of class culture conflict, it would still seem to be the case that "there are many parents who want their children to do well at school, but who have no idea of how to play this role of the good parent" and who do not demonstrate the knowledge and attitudes appropriate to it. (Musgrave 1979, p.249) Mellin-Olsen (1976) has studied the effects of class in Norway upon mathematical learning in particular. Referring to the pupil, he stresses the importance of the need "to know how he and his family experience school, how they define it, and what role it plays for them." (p.16) He sees this as necessary in order to understand the conflicting message systems to which pupils may be exposed and presumably, if one gained such knowledge of family background, it might then become possible to understand how to help the parent to play the role of the 'good parent' in an educational context.

It would seem that for parents to have a positive effect upon their children's academic attainment they must have, together with the appropriate attitudes, an awareness of the educational system and how to manipulate it to the advantage of their children. As mentioned earlier, a new factor in the lives of secondary pupils is the opportunity for them to exercise choice and it would be reasonable to assume that many parents have considerable influence on whatever choices are made. We have noted that it is possible that parents and teachers may use different criteria in judging a pupil's ability or lack of it in mathematics. Parents may look for visible signs of achievement in the success of their children in computational work but not be aware of their mathematical

potential in other aspects of the subject, a perspective which can persevere through to pupils' experience at secondary level. It is likely that some parents need more informing in this respect in order that they may appropriately influence their children in making the choices they do. Without this, pupils who are potentially able mathematically may be discouraged from studying the subject at higher levels or from applying themselves to mathematical learning earlier on.

Summary

Variation in primary mathematical experience can be identified in the different levels of achievement in pupils' first year of secondary school. (Newbold, 1977) Pupils' attitudes to mathematics at lower secondary level are likely to be entrenched and unfavourable, although the usefulness of the subject may be recognised (Duckworth and Entwistle 1974). Girls may be discouraged from studying mathematics at a higher level (Ray 1975, Luchins 1979) and some may choose not to because of the supposed masculinity of the subject (Horner 1968, Fennema 1979). The fact that secondary pupils are faced with choice for the first time in their academic careers is complicated by the development of the self-concept at this stage. (Musgrave 1979) Selkirk (1974) found that pupils who chose to study mathematics at Advanced level in unorthodox subject combinations achieved well and that possibly girls could benefit from being encouraged to study mathematics in such combinations.

Erlwanger (1973) has shown how the conception of mathematics varies from pupil to pupil. Low significant correlation has been found between attitude and achievement in mathematics (Aiken 1976) while Jackson (1968) reports that no significant relationship with respect to attitude and achievement in school subjects generally, has been found. Unfavourable attitudes to mathematics have been found to level out by about the third year and significant correlation identified between individual attitudes and those of the peer group and between attitude to mathematics and to school generally; a significant but low correlation has been identified between parents' and pupil attitudes. (Gopal Rao 1967) Mathematics was rated low in interest amongst nine other subjects by both boys and girls and some decrease in its perceived social benefit identified between second and fifth year pupils. (Duckworth and Entwistle 1974) Kempa and McGough (1977) found that type of mathematical course

did not produce a difference in attitude towards the subject and that enjoyment of the subject was more important than difficulty, in determining pupils' choice of mathematics. Selkirk (1974) also detected no significant effect of type of course on pupils' attitudes to the subject. The lack of correlation between attitudes and type of syllabus followed is seen to indicate that factors other than kind of content are likely to be dominant in affecting the formation of pupil attitudes. This, together with previous evidence of teaching methods adopted suggests that a linear, mechanistic view of mathematics is projected and in turn, determines how it is perceived by pupils. It is argued that for the view of the subject to be otherwise in a mixed ability setting in particular, a variety of resources, as advocated by Lingard (1976), is essential.

Secondary pupils' main expectation of teachers is that they should be able to keep order (Nash 1973) and because the subject being taught is an important factor entering into classroom control (Francis 1975), it is suggested that the obviousness of the pupil's mathematical success or failure may be especially important in this respect. The secondary pupil's view of what is appropriate teacher behaviour is well developed and they see their own role as a passive one. (Nash 1974) The adoption by teachers of a 'direct' role which involves an essentially didactic approach (Hargreaves 1972) could result in pupils adopting a passive role. Teachers are identified by pupils as being in positions of power and react accordingly (Hargreaves 1972) and may feel anxious and helpless in the face of it (Roberts 1971). Yates (1978) has provided examples of the consequences of teacher behaviour that could be classified as 'direct' and 'indirect' in a study of the mathematical classroom, while Keddie (1971) gives evidence of pupils' reactions to the academic labels overtly ascribed to them. It is suggested that the passive role of the pupil in the mathematics classroom may be reinforced by their perceptions of the nature of the subject.

Parental involvement is seen to affect the academic achievement of pupils. (Ainsworth and Batten 1974, Cox 1979) Witkin (1974) suggests that family background is subtly manifested in the extent to which pupils are able to adapt to the values of the school while Musgrave (1979) identifies the fact that many parents are not sufficiently informed to make a positive contribution to their children's school life. The importance for mathematical learning of parental views of what the school

experience consists of is raised by Mellin-Olsen (1976). It may be that because of the diverse nature of the subject, parents may need further information with respect to mathematics in order to help their children to make appropriate choices at secondary level.

CONCLUSIONS

The dominant factor to emerge from the foregoing considerations of matters affecting the social context of the mathematics classroom at secondary level is the predicament of the pupils concerned. While there clearly is some effort to avoid the overt categorisation of individuals according to mathematical ability by the increased adoption of mixed ability teaching, it would appear that this may be doomed to failure from the outset in view of the ability grouping within classes at primary level that most pupils will have experienced. Thus they arrive at secondary school with unfavourable attitudes toward the subject and with a well established self-picture of their ability in relation to it. The mixed ability situation in which most find themselves must therefore come as something of a shock, especially in view of the 'whole class' approach which apparently is fairly generally adopted. The move towards the use of individualised materials is, at the same time, less likely to be particularly helpful for pupils in view of the fact that it involves little in the way of injection and explanation from the teacher. It is clear that not much occurs in the secondary mathematics classroom that might alter pupils' perception of mathematics as mechanistic and sequential which we have argued is likely to be the end-product of their experience at primary level.

Added to this concern is the evidence which points to the fact that secondary pupils view their role as essentially passive in nature, under the dominance and power of the teacher. This is of importance in relation to the curriculum as a whole but it is particularly relevant to the case we are putting forward here. We are concerned with the satisfaction of an aim for mathematics education that emphasises the social nature of the subject and have argued that the teacher's perception of the subject will be reflected in how it is done in the classroom. Evidence suggests that methods used in the secondary mathematics classroom are such that they would reinforce such a passive role on the part of the pupils at the same time as reinforcing a rigid view of the subject

and detracting from any notion of mathematics as an activity founded in the mutual sharing of ideas or exploring of hypotheses.

While this may appear to be an exceedingly difficult situation in which to bring about change, the clear picture presented of the strong influence of the teacher over pupils suggests where such change could begin. It would seem that pupils are passive because that is the way they perceive teachers to want them to be. If mathematics teachers were to evince different expectations of pupils and overtly to draw them into active participation in mathematics lessons, it is possible that pupils would come to react accordingly to such expectations and any self-fulfilling prophecy could become a more positive one. The reasons this may not happen in mathematics lessons in particular are, no doubt, many, and some may well be related to the fact that mathematics teachers do not appear to be decision-makers in their own right nor to participate in determining curriculum policy. However, it seems highly likely that underlying the situation as a whole are the teachers' own perceptions of what mathematics is and how they project this to their pupils in the pedagogy they adopt.

We shall conclude by addressing ourselves, in the final chapter, to a consideration of these and other matters raised in relation to mathematics education and to change in the mathematics curriculum.

CHAPTER 6A Rationale for Change in the
Mathematics CurriculumA SUMMARY OF THE CASE

What we have attempted to show in this study is the interrelationship of the epistemology of mathematics, the aims of mathematics education and the social context in which the mathematics curriculum is implemented. It has been argued that aims for mathematics education reflect the view that is held of the foundations of mathematical knowledge and that, whatever that view may be, it will be reflected in the social context in which the teaching and learning of mathematics takes place in the classroom.

Our consideration of aims for mathematics education held today suggest a view of mathematics founded in social activity and strong epistemological support for such a perspective has been identified. Our examination of two mathematics curriculum development projects has shown that this aspect of the discipline has largely been ignored in what they set out to do. One (the Nuffield Mathematics Project) has stressed the activity of the individual pupil in a methodology based upon essentially psychological considerations to the exclusion of other theoretical guidelines and produced an unbalanced approach to the mathematics curriculum as a result. This appeared to be particularly the case with respect to philosophical matters, with a lack of provision of criteria to guide teachers in the selection of mathematical content and a concentration upon the adoption of a particular methodology. The second project (the School Mathematics Project) showed a similar disregard not only for philosophical matters but for other theoretical considerations as well, the aim being quite simply to produce a new secondary mathematics syllabus. In this case, the emphasis was on content (much of it new to teachers as well as pupils) presented in textbooks which appeared to dominate the methodology employed and to enforce an essentially didactic approach in the adoption of the Project materials. Thus neither project promoted the notion of mathematics as a social activity and the social context of the mathematics classroom in which each result appeared to be of pupils working in fairly isolated learning circumstances, in

one case in active exploration with little consolidation of any mathematical learning and in the other, in bookwork. In neither situation was there indication of much mathematical discussion either of pupil with pupil or teacher with pupil.

In order to gain some idea of how mathematics is perceived by both teachers and pupils in the day-to-day learning of the subject, we then attempted to derive a picture of the social context of mathematics classrooms at primary and secondary levels. While this exercise was somewhat indirect in nature insofar as much of the evidence used was not specifically related to mathematics and hence had to be extrapolated to the mathematics classroom, a picture has emerged. At both primary and secondary levels, we have been drawn to conclude that the view of mathematics presented to pupils is largely that of a subject that is linear and mechanistic in nature, with little to suggest its conduct or foundations in social activity and discourse. There appears to be little evidence of the subject being perceived as open-ended, in terms of questioning, conjecture or with the possibility of redundancy of ideas or potential for change. This has been evinced in the mainly didactic methodology apparently adopted in schools which has tended to stress the inherent linearity of mathematics with little allowance for divergence into problem-solving or investigation in which the pervasive existence and applicability of the subject might be appreciated. This restricted approach probably arises in the primary sector, in part, due to a lack of confidence on the part of many teachers in their mathematical ability and hence their ability to teach the subject. In the secondary sector this applies to a lesser degree but it also seems possible that a tendency towards the adoption of an inflexible response to mixed ability teaching and individualised methods may reinforce this view of mathematics. However, it is argued that, in both cases, what the classroom context basically reflects is the view of the essential nature of the subject held by teachers which would appear to be narrow, restricted and certainly not 'social' in character.

Hence we are drawn to conclude that, as was the case in the mathematics curriculum development projects examined, the aim of mathematics education stressing the social aspect of the discipline is not achieved in classrooms simply because it remains largely ignored. Having argued that a mathematics curriculum is unbalanced without such a goal, it is

clear that change of a more fundamental nature than heretofore is necessary in the mathematics curriculum to redress this imbalance.

A BASIS FOR CHANGE

In earlier discussion (see Chapter 2) we have noted how the foundations of mathematics may be seen to be supported by epistemological theory that accommodates growth and change in mathematical knowledge. It was suggested that a classroom interpretation of the subject based on such a theoretical perspective would necessarily involve dialogue, discussion and investigation on the part of pupils and teachers, together with the equally necessary learning of facts and skills. (This is in contrast to the giving and receiving of instruction in an inert, unquestioned body of knowledge which emanates from a positivist approach to the subject.) While we have detected little evidence of a mathematics curriculum being affected by such a view in practice, there are some indications that it is beginning to gain a place in the thoughts of mathematics educationists who appear all too aware of the limited success of the classroom teaching and learning of their subject. However, as Bauersfeld (1980) admits, we know little about the nature of mathematics as a social activity in relation to the classroom when he states that "we still do not have much information about the social dimensions of generating mathematical knowledge and of developing individual mathematical power within the classroom." (p.24) He argues that although other disciplines have provided relevant research in this regard; research in mathematics education has not paid much attention to these dimensions of a social nature.

What appears to have been missing to date is a lack of clarity with regard to theoretical considerations that could provide some cohesion in such a pursuit. In this study we have argued that a first step, of necessity, must be to be clear about what we conceive mathematics to be, in order that we can then be clear about the aims we pursue in teaching the subject. Aims in the teaching of any subject logically should be based upon a clear interpretation of the nature of that subject, otherwise elements of the curricular process may well become mutually obstructive. We have seen this to be the case in mathematics education where an arguably restrictive view of content has largely been allowed to dictate methodology and thus to limit the pedagogy of the discipline.

While the aim to develop in pupils the notion of mathematics as a social activity in its foundations and its conduct (and ultimately its applications) has gained some acceptance in the mathematics education community, it requires for its achievement the acceptance of a view of mathematical knowledge founded in social endeavour and interchange. A first step towards achieving such an aim must, therefore, be the adoption of what we have described as a growth and change epistemology in order to accommodate such a view. The author^{has} argued the relevance of such an epistemological approach at the level of the curriculum generally. (Nickson 1975) - In doing so the relationship was examined between a growth and change theory of knowledge (as exemplified by Popperian thought) and the curricular process. At the same time the relationship of such a theory to Piagetian developmental concepts and Bruner's theory of instruction was considered together with its general relevance to curricular renewal through a constructive, dialectical approach. Again at the level of the curriculum generally, Confrey (1980) observes that some curriculum theorists are wrong in claiming that once content has been selected, the design of a curriculum is a simple, technical task. She states:

"Such a claim ignores the fundamental interaction between the content and its method of presentation. These two are related through the theory of knowledge one accepts." (p.23)

She advocates an eclectic approach wherein the theory of knowledge selected should be "subject-matter specific" and sees this as applicable to mathematics as to any other subject. Higginson (1980) recognises the relevance of Popperian and Piagetian thought (the latter in terms of genetic epistemology) to the mathematics curriculum in particular and suggests:

"The emotional-affective potential of this view (and surely this is the area where mathematics teachers have unconsciously done so much damage in the past) in the classroom would seem to be very great." (p.8)

With the adoption of such a theoretical perspective, teachers of mathematics would appear to be fallible beings (like their pupils), as would the subject they teach; they would come to view their role more openly and not solely in terms of the somewhat rigid imparting of skills and concepts that appears to predominate. Possibly most important of all, with a view of mathematics grounded in human endeavour, the applicability of the discipline at a variety of levels could well become readily discernible and appear less artificially induced to the pupils learning the subject.

CURRENT DIRECTIONS OF CHANGE

To argue for a new perspective for change in mathematics education is not to imply that change in the form of curriculum development work and educational research is not continually proceeding. In recent years, centrally controlled projects on a national scale have given way to smaller efforts in mathematics curriculum development in an attempt to cater for local needs as, for example, in the Kent Project and the Fife Mathematics Project. There are also projects with more specific intentions such as the Secondary Mathematics Individualized Learning Experiment. However, some projects at a national level such as the School Mathematics Project and the Mathematics for the Majority with its Continuing Mathematics Project still exist. The impact of past projects generally has been varied and difficult to gauge for, as Griffiths and Howson (1974) point out:

"Evaluation of mathematics projects has not engaged a great deal of interest in Britain, where it has mainly degenerated into a form of 'voting with one's feet'; it being interpreted that those projects with large followings have been successful, whereas those which attracted few disciples, and failed to hold on to those which they had, were failures." (Griffiths and Howson 1974, pp.150-1)

Having noted the fact that SMP materials have been more widely adopted in this country than any other project materials (see Chapter 3), we can deduce that development work has produced little in the way of rethinking of the mathematics curriculum at a fundamental level as opposed to largely dwelling on the introduction of new content.

The direction of change in educational research generally in this country, is one from studies of individual pupils and teachers to studies of the classroom as a whole; from the study of outcomes in a controlled situation to the study of processes in order to determine how the teaching and learning context contributes to or detracts from pupil achievement. Such holistic studies have come to be labelled 'ethnographic' or, more simply, observational, since what is attempted is to gain some insight into how social systems represented in the classroom by pupils and teacher, combine to produce a context for teaching and learning. (Shulman 1978) A major work of this nature in this country is the investigation by Rutter et al. (1979) which set out through observation and analysis, to relate pupil achievement in secondary schools to the differing corporate effect of school processes

within each school (see Chapter 5). More recently, an example of observational research is the study "Inside the Primary Classroom" by Galton et al. (1980) (see Chapter 4).

What is noteworthy about research of this kind is the indication of a greater concern for a naturalistic approach in studying the classroom *as it is* and, as a result, an increased concern with affective variables. It sets out to explore the effects of social processes in the classroom and the social interaction which constitutes such processes. Where research specifically related to the teaching and learning of mathematics is concerned, this new direction has yet to take hold in this country. A study undertaken by Yates (1978) has involved the use of an observational and interpretative paradigm in her investigation of four mathematical classrooms, as has a study by Morgan (1977) of the affective consequences of the use of individualised materials in mathematics lessons in Scottish schools. In the main, however, mathematical studies have tended to remain at the level of measuring pupil outcomes in terms of methodology such as teaching mathematics to primary pupils using a problem-solving approach (Burton 1980) or a concern with how pupils learn specific mathematical concepts such as understanding ratio (Hart 1978).

A PROPOSAL FOR CHANGE

Rather than taking the predicament of teachers or pupils as a starting point from which to examine the mathematics curriculum, we have begun with the nature of the subject of mathematics itself and have attempted to infer from a variety of evidence, the effect that different interpretations of the subject may have on the classroom situation. We have begun with the subject and not the participants who implement the mathematics curriculum in order to develop the thesis that mathematics and the way in which it is viewed as a discipline, imposes particular constraints on methodology and thus upon the social context of the classroom in which it is taught and learned. By examining the social context of the mathematics classroom, therefore, we have, in turn, deduced a particular view of mathematics which would appear to be held by most teachers and hence, passed on to pupils.

The link between an epistemological point of view with regard to a

subject and classroom practice in teaching it is a logical one. What has been exposed here is an illogicality that exists between aims identified for mathematics education and what would appear to be classroom practice in mathematical teaching and learning. What clearly is an important aim relating to the essence of mathematical processes and thought appears not to be receiving due regard at classroom level if, indeed, it is receiving any at all. It is arguable that of the nine aims identified in Chapter 1 of this study, the aim referring to the emphasis of the foundations of mathematics in social activity is the most fundamental since the processes that bring a discipline into being must necessarily inform what follows in educating individuals in that discipline. It is not surprising, therefore, that with the disjunction that exists between aims and practice, little of the much sought after improvement in mathematics education has taken place, in spite of the time and effort put into research and development work. In order for such improvement to be made possible in the first instance, there must be general recognition and acceptance of the aim stressing the social nature of the discipline. Without it, it seems unlikely that the achievement of the remaining aims would be possible at a level at which, ideally, we would like to see happen. So long as there remains a mystique about the nature of the subject and the myth is perpetuated that it is pre-ordained and not man-made, so the discipline will remain closed to the minds of most and the cause of improvement in mathematics education will suffer.

The enormity of the task and the problems entailed in proposing such a plan to expose mathematics education to a new perspective of this kind, is very clear. It would seem, however, that to begin with paying heed to aims is 'to begin at the beginning'. It is probably unlikely that many teachers are aware of this epistemological view of mathematics as a discipline and that to change entrenched points of view would be a very difficult task. However, it is incumbent upon mathematics educators who are responsible for the mathematical education of teachers at whatever level, to ensure that they do become aware of the social nature of mathematics and to present the open, problematical and diverse nature of the discipline. It seems very likely that we do teachers an injustice by *not* presenting them with such an alternative and by indoctrinating them with a single epistemological point of view based on logical positivism (whether or not it is identified as such) with all the

restrictions that follow. The impact upon the classroom learning of mathematics of this approach has been all too clear.

In practical terms, there is no doubt that to convey the social aspect of mathematics as an activity, it is necessary for teachers and pupils to *make* or *do* mathematics together. What has been lacking in the past is an adequate rationale that places such activity in balance with other aspects of mathematical learning. The pursuit of this aim in the adoption of a growth and change epistemological foundation for mathematics could provide the unifying element in the mathematics curriculum rather than the element that separates the psychology of method from the philosophy of content as would appear to have been the case in the past. Making and doing mathematics should become part of the mathematics curriculum because this is how mathematics is generated, just as the use of scientific method underlies the science curriculum and creative writing forms a part of the English curriculum. Mathematics 'workshops' for pupils should be just as much a part of the school timetable as are science laboratory periods.

The difficulty of implementing a proposal of this nature is one of breaking into a circular situation that has existed in mathematics education for a very long time in which mathematics has been approached as a 'given', inert and unquestionable body of knowledge. Spradbery (1976), writing of pupil resistance to past attempts to change the mathematics curriculum, comments:

"They have not challenged the established pattern of social relationships within the classroom: teachers and pupils continue to play the relatively passive role of reproducers, rather than producers, of knowledge." (Spradbery 1976, p.236)

He refers to the fact that for mathematics to be 'proper Maths', as a number of children described it, it must be presented detached from everyday knowledge. (p.237) Once again we are reminded that pupils form such views largely from their perceptions of teachers and the view of the subject which they project. Clearly an important area of research would be to explore how mathematical knowledge can be 'generated' in the classroom (as Bauersfeld (1980) puts it) and how the social constraints of teacher-pupil perceptions and expectations may be altered in such a situation. The move towards a more holistic approach in classroom research renders such an undertaking a much stronger possibility than may have been the case in the past.

Lortie (1975) brings us back to the plight of teachers when he writes that "Teachers have a built-in resistance to change because they believe that their work environment has never permitted them to show what they can really do." (p.235) We have argued elsewhere that this may especially be the case with respect to teachers of mathematics at primary and secondary level. Primary teachers appear to be insecure in their knowledge of the subject and there is evidence of a need for more positive leadership in the curricular decisions they make. Secondary teachers, on the other hand, appear to have much of the decision-making capacity withdrawn from them and if their cynicism may be taken as evidence, they would appear to prefer to be more involved at the curriculum-formulating level. Resistance to change at the primary level with respect to the mathematics curriculum may partially, at least, be due to lack of appropriate within-school support. In the case of secondary teachers, some of the resistance seems likely to be due to the fact that they probably would not have been consulted about the proposed change beforehand. In both sectors it would appear, therefore, to be a matter of balance between teacher autonomy and an appropriate degree of leadership, which suggests strongly that coursesⁱⁿ leadership would be a desirable area for the investment of in-service training time and finance.

To alter the internal constraint of (in the first instance) the teacher's perception of mathematics would be a change at the most fundamental theoretical level. Bauersfeld (1980) suggests that this is what is needed when he writes that "Mathematics education is deeply in need of *theoretical orientation*." (p.36) This, we believe to be the case, and we have attempted to show how a new theoretical perspective based on a growth and change epistemological interpretation of mathematics can contribute to fulfilling such a need. The adoption of such a perspective could contribute to the achievement of a desirable balance in the mathematics curriculum by satisfying an important aim for mathematics education which hitherto appears largely to have been ignored. In so doing, the mathematical experience of teachers and pupils alike could be greatly enriched by an expanded view of the nature of the subject to which they would be introduced.

Popper (1969) offers encouragement in taking such a step when he writes:

"the very refutation of a theory - that is, of any serious tentative solution to our problem - is always a step forward that takes us nearer the truth. And this is how we can learn from our mistakes." (p.vii)

The refutation of the old and the adoption of a new epistemological perspective in mathematics education could be the beginning of a tentative solution to some of the most fundamental problems we face in planning and implementing the mathematics curriculum. We *can* learn from our mistakes and mathematics education could, indeed, benefit from such a first step.

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