A LONGITUDINAL STUDY OF THE DEVELOPMENT OF PUPILS' ALGEBRAIC THINKING IN A LOGO ENVIRONMENT

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ABSTRACT

This thesis is based on research to investigate the hypothesis that programming in Logo will provide pupils with a conceptual basis of algebraic ideas which will enhance their work with "paper and pencil" algebra. The aims of the research were to:

• trace the development of the use and understanding of algebra related concepts within a Logo programming context by reference to the work of four case study pairs of pupils during their first three years of secondary schooling (11-14 years)

• develop and test out materials designed to help pupils link the conception of variable derived within a Logo to a non-Logo context

• relate the pupils' understanding of variable in Logo programming to their understanding in "paper and pencil" algebra

The research consisted predominantly of a three year longitudinal case study of pupils programming in Logo during their "normal" secondary school mathematics lessons. The data collected for this longitudinal study included video recordings of the pupils' Logo work together with their spoken language (which was subsequently transcribed for analysis). Initially it was found that the case study pupils did not naturally choose to use variable in Logo as a problem solving tool but it was possible to develop teacher devised tasks which provoked its use. Previous research suggests that pupils often use informal methods which cannot easily be generalised and formalised in algebra. However in the Logo context pupils were able to negotiate a generalisation by interacting with the computer and discussing with their partner to the point where they could then write a Logo procedure to formalise this generalisation.

Categories of variable use were derived from the data in order to provide a framework for analysing the pupils' use and understanding of variable. At the end of the three year case study a structured interview was administered to the pupils to probe their understanding of variable in both the Logo and the algebra context. Evidence from the research suggests that the Logo experience does enhance pupils' understanding of variable in an algebra context, but the links which pupils make between variable in Logo and variable in algebra depend more upon the nature and extent of their Logo experience than on any other factor.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND TO THE RESEARCH

For many pupils algebra forms a barrier to engaging in and understanding secondary school mathematics. Research into children's understanding of algebra has highlighted the problems children have with interpreting the meaning of letters and with formalising and symbolising a generalisable method (Küchemann, 1981; Booth, 1984). Vergnaud has pointed out that "algebra is a detour: students must give up the temptation of calculating the unknown as quickly as possible, they must accept operating on symbols without paying attention to the meaning of these operations in the context referred to" (Vergnaud & Corte, 1986, p. 320). He quite rightly says that we must find problems which provoke the use of algebra. This is not an easy task in "traditional" school mathematics. However the computer programming context does provide situations in which variable is a meaningful problem solving tool. The increasing availability of computers in schools over the last few years has been remarkable and there is no reason to suppose that this increase will not continue. It seems appropriate therefore to consider the ways in which the computer can enhance the learning of mathematics, and in particular, as far as this study is concerned, the learning of algebra.

This thesis is based on research to investigate the hypothesis that certain programming experiences in Logo will provide pupils with a conceptual basis of algebraic ideas which will enhance their work with "paper and pencil" algebra.

The aims of the research were to:

- trace the development of the use and understanding of algebra related concepts within a Logo programming context by reference to the work of four case study pairs of pupils during their first three years of secondary schooling (11-14 years)

- develop and test out materials designed to help pupils link the conception of variable derived within a Logo context to a non-Logo context

- relate the pupils' understanding of variable in Logo programming to their understanding in "paper and pencil" algebra

Some of the research data for this thesis was collected as data for the Logo Maths Project (Sutherland and Hoyles, 1987). This project investigated the potential of Logo in a wider range of contexts than the "algebra" related context of this thesis: the issues of the role of the teacher, the role of the pupil collaboration and the problem solving strategies developed by the pupils within a Logo programming environment were all addressed. The categories and hypotheses derived within the Logo Maths Project have provided a framework for this thesis.

1.2 TEACHING AND LEARNING MATHEMATICS: A THEORETICAL PERSPECTIVE

"Our job as researchers is to understand better the processes by which students learn, construct or discover mathematics so as to help teachers, curriculum and test devisers and other actors in mathematics education, to make better decisions.... Theory is essential, and it is also our burden to organise our knowledge on mathematics education in coherent descriptive and powerful conceptual systems" (Vergnaud, 1987, p. 43).

In this theoretical review, links will be made between developments in cognitive psychology, devlopmental psychology, social psychology, artificial intelligence and mathematics education in order to provide a theoretical framework for the present research into pupils' use and understanding of algebra related ideas within Logo.

Mathematics education has been heavily influenced by the work of the psychologist Piaget. He was radical in that he rejected the commonly held position at the time of the child as a passive receiver of innate ideas and put forward the idea of the child as active constructor of his or her knowledge. He was mainly interested in the development of logical and mathematical concepts in the child and he describes four general factors which influence cognitive growth. "The first of these is organic growth and especially maturation of the nervous system " (Piaget & Inhelder, 1968, p. 154). The second factor is experience of the physical world. He includes in this both physical experience and indirect logico-mathematical experience. The third factor is experience from the social world "even in the case of transmission in which the subject appears most passive, such as school teaching, social action is ineffective without an active assimilation by the child which presupposes adequate operatory structures" (Piaget & Inhelder, 1968, p. 156). The fourth and coordinating factor of the previous three factors is equilibration. Equilibration is crucial to Piaget's theory and is the organisational element of cognitive development. In Piaget's theory a child approaches a new situation with existing cognitive structures (or schemas) and by processes of assimilation and accomodation equilibration is reached. Assimilation is the application of an existing schema to a novel situation. Accomodation takes place when the existing schema (or schemas) are adapted to the new situation. "In any given equilibration, there will be a greater or lesser degree of assimilation and accomodation, though both will always be present to some extent" (Dubinsky & Lewin, 1986, p. 60). Equilibration is the central mechanism which drives cognitive growth. Equilibration is also the process by which conflicting schemas can be integrated into new structures. "It is a series of active compensations on the part of the subject in response to external disturbances and an adjustment that is both retroactive (loop systems or feedback) and anticipatory, constituting a permanent system of compensations" (Piaget & Inhelder, 1968, p. 157).

As a consequence of this theory of equilibration Piaget put forward the idea that children develop through different stages, each stage being characterised by a cognitive structure which is qualitatively different from the cognitive structure of the preceeding stage. The essence of this theory is that as people grow older they do not just acquire more knowledge, they develop new cognitive structures. It is this stage theory which is the most controversial aspect of Piaget's work although he himself maintained that his stage theory was developed as a way of categorising and organising his data for analysis. Writing about his stages he said "I would compare them to zoological or botanical classification in biology which is an instrument that must precede analysis" (Piaget, 1977, p. 817). Higginson has pointed out that "for educational purposes, this emphasis on stages is unfortunate.......it is one of the parts of the overall theory which now appears most vulnerable......preoccupation with stages has blinded educators to rather more fundamental aspects of the theory. In other words if we reject Piaget's stage theory we do not also have to reject his theory of the child actively constructing her own reality" (Higginson, 1980, p. 232).

Piaget placed an emphasis on the child actively constructing his or her knowledge but this word "active" has been misinterpreted by teachers as implying that children should always be manipulating concrete objects, whereas for Piaget "Authentic activity may take place in the spheres of reflection, of the most advanced abstraction, and of verbal manipulation" (Piaget, 1968). Hermine Sinclair maintains that "action is all behaviour which will bring about a change in the world around us or by which we change our own situation in relation to the world...in other words it is behaviour which changes the knower-known relationship" (Sinclair, 1987, p. 28). Central to the theory of constructivism is the idea of a "normative fact." These are operational invariants which "the subject feels to be both evident and necessary, and often can no longer imagine that at the some earlier time they were not present in his mind" (Sinclair, 1987, p.32). An example of a "normative fact" is the commutativity of addition. These "normative facts" or operational invariants are called "theorems in action" by Vergnaud because he says that it is essential to analyse them in mathematical terms. The pupil uses these "theorems in action" implicitly; they are embedded within action and cannot be made explicit by the use of a representational system. "But one must never forget that concepts are rooted in the experience of students with different kinds of situations, and in schemas they use to deal with these situations. Before being objects, concepts are cognitive tools, and many theorems should be "theorems in action" before being explicit theorems, especially at the primary and early secondary level" (Vergnaud, 1987, p. 52). More recently the idea of a tool-object dialectic has been developed by Douady "We say that a mathematical concept is a tool when our interest is focussed on the use to which it is put in solving problems. By object we mean the cultural object, which has a place in the body of scientific knowledge, at a given time, and which is socially recognised" (Douady, 1985, p. 35). The idea of a concept being first of all a "cognitive tool" to solve certain problems is an attractive one. This tool will also need a name and one or more symbolic representations. Vergnaud has elaborated on the idea of a concept into one of a conceptual field" a set of situations, the mastering of which requires a variety of concepts, procedures and symbolic representations tightly connected with one another" (Vergnaud, 1982, p. 36). This conceptual field is more precisely defined as a triplet (S,I, §) in which

S is a set of situations that make the concept meaningful
I is a set of invariants that constitute the concept
§ is a set of symbolic representations used to represent the concept, its properties and the situations it refers to

Vergnaud stresses that "The epistemological analysis of the subject matter must take place within a problem setting. Epistemology is first of all concerned with a problem of functionality. By this I mean that mathematical concepts, mathematics, procedures and mathematical representations are answers to problems that we must identify and analyse to understand how students deal with them and eventually discover or understand these answers" (Vergnaud, 1984, p. 18).

Within Piagetian theory "the origin of conceptualisation lies in the formulation of schema from the internalisation of action upon objects. In Piaget's terms the production of the sign happens in terms of grafting of signifiers onto existing concepts" (Walkerdine, 1982, p. 130). Vergnaud however stresses the importance of the set of symbolic representations because "the analysis of the isomorphic properties of signifiers and signified is inescapable" (Vergnaud, 1984, p. 20).

Vergnaud is one of a group of French Didacticians who have written extensively about the need to define carefully certain aspects of learning which are specific to the learning of mathematics within a classroom situation. "The didactics of mathematics is the study of the pupils acquisition of mathematical knowledge. The objective is the study of the situations and the processes which have been provoked with the intentions of giving the pupils a sound knowledge of mathematics" (Brousseau, 1981). They stress that theories of learning cannot fully be realised until the whole classroom situation is taken into consideration. Some of the most important ideas within this theory relate to the pupils' behaviour which result precisely because he/she is a member of a classroom and not a learner in some other setting. They stress that what is taught in the classroom is knowledge transposed for the classroom."Didactical transposition refers to the adaptive treatment of the mathematical knowledge to transform it into a knowledge to be taught" (Chevellard, 1980). They put forward the idea of a "didactical contract" which is the implicit expectations about learning which exist between the teacher and the pupils and between the pupils themselves within the classroom setting. Brousseau says that "We know that the only way to "do" mathematics is to search for and solve certain specific problems, and while doing so raise new questions. Thus what the teacher has to manage is not the communication of knowledge, but the devolution of a good problem. If this transfer works, the pupil enters the game, and if he ends up winning, learning has occurred" (Brousseau, 1984, p. 111). Brousseau describes the paradoxes caused by the nature of the didactical contract. From the teacher's perspective "Everything he does to make the pupil produce the behaviours he expects tends to deprive the latter of the conditions necessary for understanding and learning the notion concerned: if the teacher says what he wants, he can no longer obtain it" (Brousseau, 1984, p. 113). From the pupil's perspective "if he accepts that the teacher, according to the contract, teaches him the results, he will not attain them himself and thus will not learn mathematics, i.e. he will not really make mathematics his own...to learn for him, implies to reject the contract, and to accept being himself engaged in the problem. In fact, learning will not be based on the correct functioning of the contract, but rather on breaching it" (Brousseau, 1984, p. 113).

For the French didacticians "errors are not understood as mere failures of pupils but rather as symptoms of the nature of the conceptions which underly their mathematical activity." (Balacheff, 1984, p. 36). An Obstacle is "a conception, possibly a knowledge which has first been efficient to solve some type of problems but fails when faced with other ones" (Balacheff, 1984, p. 36). More specifically a didactical obstacle is an obstacle which has resulted from a previous didactical situation, whereas an epistemological obstacle is an obstacle which is "intrinsically related to the concept itself" (Balacheff, 1984, p. 36). These distortions and misconceptions have been the focus of many mathematics educators who have investigated the nature of pupil errors (for example Hart, 1981a). "Errors appear to be subject matter specific but the fact of their appearance seems to be relatively independent of the curriculum and the teaching methods "(Byers & Erlwanger, 1985, p. 214). It would seem that the existence of pupil errors supports the claim that pupils actively construct their own learning. "Typically children's errors are based on systematic rules...children's faulty rules have sensible origins. Usually they are distortions or misinterpretations of sound procedures" (Ginsburg, 1977, p. 128). Research in Artificial Intelligence, with its aim of modelling human capabilities by a computer has also concerned itself with pupil errors. "Contrary to the assumptions of earlier structuralist theories modern organisation theory suggests that the learner may organise the mathematics she is learning in her own way so that she remembers some things that she has not been taught. The resulting structures may improve her understanding but they may also produce distortions and misconceptions" (Byers & Erlwanger, 1985, p. 271).

The information processing model of the mind has certain similarities with the theory of Piaget in that in both the learner actively constructs knowledge. Information processing models of the mind however describe not only the way knowledge is structured but also how that knowledge is accessed. Originally theorists put forward the model that there exists a central processing mechanism in the mind. More recently there has been a development away from the idea that there is one organising structure. As explained by Kilpatrick "Theorists are challenging the idea that the mind is composed of all-purpose mechanisms...instead they are turning to the idea that there is a society of mind...In such a view the mind is a collective of partially autonomous smaller minds, each specialised to its own purpose, that operates in parallel rather than in sequential fashion" (Kilpatrick, 1985, p. 11). Many information processing models offer potential for modelling the learning process (Lawler, 1985; DiSessa, 1987; Papert, 1980; Minsky, 1977).

In particular Minsky has developed a "Frame" theory for the acquisition of knowledge. "Here is the essence of the theory: when one encounters a new situation (or makes a substantial change in ones view of the present problem) one selects from memory a substantial structure called a frame. This is a remembered framework to be adapted to fit reality by changing details as necessary...a frame is a data-structure for representing a stereotyped situation, like being in a certain kind of living room or going to a child's birthday party. Attached to each frame are several kinds of information. Some of this information is about how to use the frame. Some is about what one can expect to happen next. Some is about what to do if these expectations are not confirmed" (Minsky , 1977, p. 212). As can be seen from this description a frame is used to represent situation specific knowledge. Central to the idea of the theory is that when presented with a new situation the learner initially attempts to cue the retrieval of a frame from memory. "The matching process which considers whether a proposed frame is suitable is controlled partly by one's current goal and partly by information attached to the frame; the frames carry terminal markers ...This frame can be imagined to have a certain number of "slots", the top level of which is filled with invariants which are always true about the situation and the bottom level can be thought of as variable "slots" (Minsky, 1977, p. 218). Each individual's frame for the same situation is considered to be different. In this theory a frame will make a default evaluation of a variable "slot" if no input can be found from the situation under examination. This input will be derived from previous situations and may be inappropriate. Davis relates this process to Piaget's concepts of assimilation and accomodation. "When the judgement is made that the instansiated frame is an acceptable match to the input data, we can say that "assimilation" occurs. If the judgement is made that the match is unacceptable, we have a precondition for "accomodation" to take place" (Davis, 1984, p. 65). Davis provides an example of the "Symmetric Subtraction" frame. "At first, subtraction problems are of the form "5 - 3", but are never of the form "3 - 5". Hence following the "Law of Minimum Necessary Discriminations", students synthesise a frame that inputs the two numbers "3" and "5" and outputs "2". The frame ignores order since a consideration of order has never been important. In later years of course the student will need to deal with both "7 - 3" and "3 - 7", and will need to discriminate between them. Such discrimination capability has not been built into the frame (which is why it is called symmetric). Consequently in later years certain specific errors are easily predicted and are, in fact, precisely what one observes" (Davis, 1983, p. 270).

Within the scope of this thesis it is not possible to relate the theories derived from artificial intelligence to the theories of developmental psychology influenced by Piaget. Boden however has suggested that "Piaget's commitment to cybernetics, his formalism, his structuralism, and his semiotic mentalism all predisposed him to sympathy (which he occasionally expressed) for a computational approach to theoretical psychology" (Boden, 1982, p. 169). She argues however against an overall organised structure of mind". Work in artificial intelligence has suggested that knowledge may be modular, with limited opportunities for coordination between the various modules, and that potential contradictions can exist within a knowledge system without prejudicing its functioning" (Boden, 1982, p. 170). She does stress however that computational models are still very restricted and that "despite its vagueness, and the unclarity of its research implications, Piaget's theory of equilibration merits attention" (Boden, 1982, p. 172).

The theories derived from artificial intelligence all appear to take into account the context specific nature of knowledge in a way which was never specifically addressed by Piaget. However neither the artificial Intelligence theories nor Piaget have adequately taken account of the role of language in the learning process. The artificial intelligence models' inability to take account of the role of language are probably a reflection of the over simplification which still seems to be still a necessary part of any computer-based model of human intelligence. Whereas it has been argued by Light that "Piaget's rejection of any fundamental role for language in the genesis of concrete-operational thought seems to have been premised on a view of language as consisting largely of a collection of conventional signifier-signified relationships" (Light, 1983, p. 77).

For Piaget language was "grafted-on" (Walkerdine, 1982) to the child's existing mental structures. Piaget considered social interaction to be important in providing the child with a source of conflict to enable the child to reconstruct his or her knowledge or to "decentre" his or her thinking. He put forward the idea that egocentricism is the main obstacle in a child's progress. Cognitive conflict was seen as the key factor in encouraging the child to "decentre". Conflict is envisaged as the key to progress, whether it arises from differences in subjects' approaches to the same task or from deliberately created differences in their perspectives on the task" (Light, 1983, p. 72). Whereas there is some support for this view (Doise, 1975; Mugny, 1978; Glachan & Light, 1982), Russell suggests that conflict can only be productive if the child already posseses an "objective propositional attitude" to the task in which she is engaged (Russell, 1982). Conflict is seen as provoking a move from the subjective to the objective, thus allowing the learner to use knowledge which he already possesses.

Walkerdine is even more radical in her ideas "It has become an increasing problem to attempt to move beyond mere assertions that context is important towards actual attempts to understand how to theorise this term and therefore to more clearly understand its effect. I want to challenge the assumption that context can be seen as an effect which can be "welded on" to a Piagetian edifice left almost entirely intact" (Walkerdine, 1982, p. 130). She criticises many existing theories in their attempt to place context outside of the child. "I intend to develop the theme that children are engaged in a process in which the crucial moment of understanding lies in a specific relation of signified to signifier" (Walkerdine, 1982, p. 131). She believes that reasoning in the child is embedded within the discourse and that pupils are not necessarily reasoning formally when they are engaged in a heavily metaphoric task. "In approaching formal reasoning they actually have to suppress their metaphoric axis" (Walkerdine, 1982, p. 141). She also suggests that formal reasoning operates on the internal relationships of the language within a statement and that "we do not have to seek explanations in terms of the structures of the child's mind.....meaning is created at the intersection of the material and the discursive, the fusing of signifier and signified to produce a sign" (Walkerdine, 1982). She stresses that formal reasoning has to be learned and that the teacher has a crucial role in helping the child to move along the metonymic axis. "Teachers manage in very subtle ways to move the children from utterance to text by a process in which the metonymic form of the statement remains the same while the relations on the metaphoric axis are successfully transformed, until the children are left with a written metonymic statement in which the same metaphors exist only by implication. It is this process which is crucial to the process of abstract thinking" (Walkerdine, 1982, p. 153).

This emphasis on the role of the adult or teacher is also reflected in the theory of Vygotsky. He stresses the need for the adult to structure the learning environment in such a way that the child can reach his or her zone of proximal development "the distance between the actual development as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Forman and Cazdan have compared the role of peer collaboration from both a Piagetian and a Vygotskian perspective "A Piagetian perspective on the role of social factors in development can be useful in understanding situations where overt cognitive conflicts are present. However if one wants to understand the cognitive consequences of other social interactional contexts, Vygotsky's ideas may be more helpful. In tasks where experimental evidence was being generated and where managerial skills were required, by assuming complementary problem solving roles, peers could perform tasks together before they could perform them alone" (Forman & Cazdan, 1985, p. 343).

1.3 PUPILS' DIFFICULTIES WITH TRADITIONAL ALGEBRA : A REVIEW OF THE LITERATURE

Algebra as a mathematical language has developed over the centuries from its first introduction as a tool to solve equations in which a letter or symbol represented a particular but unknown number (at the time of Diophantus circa 250 AD) to classical generalised arithmetic in which symbols were used to represent relationships between variables (at the time of Vieta in the early Seventeenth century) to what we now know as modern algebra. Modern algebra can be thought of as a language which enables the similarities in structure between different mathematical systems to be made explicit. Algebra has played a central role in school mathematics for many years and although more recently the teaching of algebra has been given less emphasis Byers and Erlwanger stress that "we can no more dispense with teaching algebraic symbolism than teaching place-value notation. Symbolic expressions are transformed more easily than their verbal conterparts so that they not only save time and labour but they also aid the understanding of content" (Byers and Erlwanger, 1984, p. 265). Vygotsky believed that "the new higher concepts in turn transform their meaning of the lower. The adolescent who has mastered algebraic concepts has gained a vantage from which he sees arithmetic concepts in a broader perspective" (Vygotsky, 1934, p. 115).

Before considering the computer programming context, it is important to take into account previous work on pupils' conceptions of variable in algebra in order to provide a background for interpreting and understanding pupils' conceptions in Logo. There is a considerable amount of algebra research related to the manipulation of algebraic objects within the context of solving equations (for example Herscovics & Kieran, 1980; Kieran, 1984) but most of this work is not relevant for the present study because it is not suggested that programming in Logo will help pupils to solve algebraic equations. The author believes that pupils difficulties in algebra arise from both their informal methods in arithmetic and their lack of acceptance and understanding of the algebraic object. These are important issues which programming in Logo could help to address. This section will present the background research in these areas.

1.3.1 The Gap Between Algebra and Arithmetic

Filloy and Rojano in Mexico maintain that it is only through re-encountering the history of the development of algebraic thought and relating this to the teaching of algebra in the classroom that we can begin to understand some of the conceptual obstacles which pupils have at the beginning stages of learning algebra. They say that in the history of algebra the most significant change in symbolism is the step from the mathematical concept of the unknown to the mathematical concept of the variable. "Theoretical and historical considerations seem to indicate that there is a didactical cut in the evolution line that goes from an arithmetical to an algebraic thought" (Filloy & Rojano, 1984, p. 51) Filloy and Rojano have developed teaching sequences which are related to the historical development of algebraic thought and the epistemological obstacles overcome during the historical development. Their work has mainly focussed on children's solutions of problems of the form:

$$Ax \pm B = Cx \pm D$$

as they progress from the equations which can be solved by "plugging in" a specific unknown to those in which it is necessary to operate on the unknown. Harper (1987) has also attempted to relate the development of algebraic thought in the child to the historical perspective. "The step between the Diophantine to the Vietan system took place over a period of more than 1300 years. In the classroom this step must often be taken over a period of less than five years; the present indications are that few pupils actually achieve it (Harper, 1987, p. 86). While this research highlights the problems involved in teaching algebra it does not in the author's opinion provide any clear help on how to restructure the teaching of algebra.

Other researchers, although not attempting to relate pupils' diificulties with beginning algebra to the historical development of algebra, have pointed out that a substantial part of the problem which pupils have with formalising generalisations in algebra is caused by their use of informal methods when solving arithmetic problems (Booth, 1981; Booth, 1984; Ginsburg, 1977; Pettito, 1979). This means for example that pupils might find it difficult to express the area of a rectangle in the form A = WxL (where A,L and W are the respective area, length and width of the rectangle) because their informal method for solving area of rectangle problems in arithmetic is counting the number of squares in the rectangle and not to multiply the length of the rectangle by the width of the rectangle. It appears that often teachers expect pupils to use a formal method in algebra which does not match the pupils normal method for solving the problem. It is suggested that in the Logo context there does not have to be a gap between pupils' informal method and the formal representation of this method.

Pettito (1979) investigated the relationship between pupils' use of formal and non-fornal (intuitive) methods for solving algebraic equations. She presented nine ninth grade students with algebraic equations which were similar in structure but which increased in structural difficulty. The following is an example of two algebraic equations which are identical in structure:

$$\frac{1}{3} = \frac{2}{(x + 1)} \qquad \frac{14}{23} = \frac{56}{(x + 2)}$$

Both equations are identical in form but not in the numerical relationships embedded within them. She maintains that success on the first type of problem is based on a more intuitive approach whereas success on the second type of problem required the pupil to use a more formal "taught" method. She also concluded that the successful "equation solver" was more likely to combine a strategy of formal and non-formal approaches than use either approach on its own.

Booth, as a result of a study which involved both a teaching experiment and individual interviews with pupils aged from 13 to 15 concluded that "many children do not seem to have a formal representation of the methods they use in solving mathematical problems, and indeed they may not use the formal 'taught' methods, but may instead use more informal procedures of their own" (Booth, 1984, p. 94). This she points out has serious implications for the teaching and learning of algebra and if generalised arithmetic is considered as the use of letters to represent general statements in arithmetic then the non-use of formal structures in arithmetic could have serious consequences on pupils' ability in algebra.

1.3.2 Pupils' Acceptance and Understanding of Algebraic Objects.

The meaning of Letters Substantial past research has shown that pupils have condsiderable difficulties in accepting and using the idea of a letter as representing a variable in algebra (Collis, 1974; Booth, 1984; Küchemann, 1981; Wagner, 1981). In addition, the idea that the same letter can represent different numbers and that different letters can represent the same number is not often understood by pupils. These findings are particularly relevant because in Logo pupils encounter variables as a means of solving certain problems and the aim of this research is to investigate their conceptions and misconceptions in this area. Of course in algebra a letter can be used to represent a specific unknown of which the following equation is an example:

$$x + 5 = 10$$

or to represent an indeterminate of which the following identity is an example:

$$6x + 2 = 3x + 1$$

School algebra has usually first introduced pupils to a letter as representing a specific unknown and Freudenthal (1973) presents a valuable discussion on the relative merits of the different approaches to the teaching of algebra. He suggests that it is the ambiguous nature of the use of letters in algebra which is problematic for pupils. "The didactically weak spot of the ambiguous algebraic names is that their meaning, that is the sort of things they name, must again and again be mentioned explicity" (Freudenthal, 1973, p. 296). He points out that in natural language names also have ambiguous meanings but that unlike algebra, the name itself helps to clear up the ambiguity.

Küchemann has carried out research in which he analysed the meaning which pupils attach to letters. His research, as part of the Concepts in Secondary Mathematics and Science project (C.S.M.S) tested five thousand pupils within the age range 11-16 from 50 secondary schools throughout England (see appendix 1). In addition 27 children aged from 13-15 were interviewed on an individual basis. This research will be reviewed in depth because it provides a framework for the present thesis.

By analysing the results of the individual interviews and the pupils' responses to the test items Küchemann was able to identify six different ways that pupils used and interpreted letters (Küchemann, 1981, p. 104). These are:

Letter evaluated. This category applies to a response where the letter is assigned a numerical value from the outset.

Letter not used. Here the child ignores the letter, or at best acknowledges its existence but without giving it a meaning.

Letter as object. The letter is regard as a shorthand for an object or as an object in its own right.

Letter as specific unknown. The child regards a letter as a specific but unknown number, and can operate upon it directly.

Letter as generalized number. The letter is seen as being able to take several values rather than just one.

Letter as variable. The letter is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values.

"Generally the first three categories indicate a low level of response, and it can be argued that for children to have any real understanding of even the beginning of algebra they need to be able to cope with items that require the use of a letter as a specific unknown at least when the structure of such items is simple" (Küchemann, 1981, p. 105).

He also reported that very few children in their survey reached the level of understanding which interpreted a letter as a variable. Küchemann found that many pupils were able to successfully answer some of the questions by interpreting a letter as an object (for example 2a + 5a = can be interpreted as 2 apples + 5 apples and answered correctly). However this technique breaks down as soon as it is essential in order to solve the problem to distinguish between the object itself and the number of the object. For example in response to the following question:

> Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils and altogether it costs me 90 pence. If b is the number of blue pencils bought and if r is the number of red pencils bought, what can you write down about b and r?

Many pupils replied "b + r = 90" to stand for "blue pencils and red pencils cost 90 pence" treating the letter as referring to the objects themselves. These results are supported by some work carried out by Rosnick with 1st year Engineering students at the University of Massachusetts. One hundred and fifty students were given the problem:

> Write an equation using the variables S and P to represent the following statement "At this university there are six times as many students as professors". Use S for the number of students and P for the number of professors.

Thirty seven percent of the group were unable to write the correct equation, $S = 6P$ and the most common error was the reversed equation $6S = P$ (Rosnick, 1981). A modified form of this question was given to a further 119 university students studying a calculus course for social sciences. The results of both experiements led Rosnick to suggest that the common error is caused by the students interpreting S as standing for student (the object) and not for the number of students and similarly P as standing for Professor (the object) and not for the number of professors.

Many of the C.S.M.S. items were answered correctly by the students treating the letter as a specific unknown. The question:

> Add 4 onto $n + 5$

is an example of this. Pupils can successfully answer this question by thinking of n as representing just one specific value.

Whenever a letter is thought of as a generalised number it is able to take on more than one value. An example of a C.S.M.S. question which requires the pupil to perceive a variable in this way is:

> What can you say about c if $c + d = 10$ and c is less than d?

Küchemann maintains however that it is not until a child views a "letter as variable" that the full potential of the use of letters in algebra is realised. An item of the C.S.M.S. which, he maintains, cannot be answered correctly unless pupils regard a letter as a variable is:

> Which is larger 2n or $n + 2$? Explain................

"The point of this question was to see whether children would recognise that the relative size of two expressions (2n and $n + 2$) was dependent on the value of n" (Küchemann, 1981, p. 111). Only 6% of 14 year olds answered this question correctly, and Küchemann maintains that they did this by establishing a second-order relationship between 2n and n + 2, that is by accepting the idea that the relationship between one set of values is dependent on the changes in another set of values.

He suggests that one reason pupils why may have problems with the idea of letter as representing a variable is that so many of the questions which they normally encounter in algebra can be answered by interpreting the letter at one of the lower levels of his categories. If this is the case it is not surprising that he found that "On the algebra test the majority of 13, 14 and 15 year olds were not able to cope consistently with items that can properly be called algebra at all" (Küchemann, 1981, p. 118). Küchemann used his six categories of letter use, together with the structural complexity of items to identify four levels of understanding with respect to pupils' interpretation of letters in algebra. These were linked to Piagetian sub-stages (below late concrete, late-concrete, early-formal, late-formal). Although Küchemann is very cautious about linking these levels to particluar ages in the child there is an implicit message in their work which is that 1) pupils need to develop through the stages and 2) that most 14 and 15 year olds are still at a stage of concrete operations, and cannot therefore work within formal algebraic systems. In the author's opinion this has had an unfortunate repercussion in the educational system as it is now widely accepted in the U.K., that it is inappropriate to teach pupils any formal alegbra in the early years of secondary school. Hardly any account appears to have been taken of the relationship between the ways in which pupils learned algebra and their performance on the C.S.M.S. tests.

Wagner (1981a & b) has also carried out extensive work on pupils' understanding of the variable name in algebra. She points out that "the role of a variable may be described as that of a name, a placeholder, an index, an unknown, a generalised number, an indeterminate, an independent or dependent variable, or a parameter. Adding to this complexity is the fact that, generally speaking, different literal symbols can be used to represent the same thing, and the same literal symbol can be used to represent different things. At the same time, certain letters have acquired fixed connotations relative to particular contexts. It is no wonder that students have so much difficulty working with literal variables" (Wagner, 1981a, p. 165). In a study which investigated whether or not students were able to conserve equation and function under alphabetical transformations of variable names she carried out clinical interviews with 30 pupils aged between 10 and 15. She presented students with the two following identical equations and probed to find out if they thought they were identical.

$$7 \times W + 22 = 109$$
$$7 \times N + 22 = 109$$

From this study she reported two common misconceptions about variables "a) that changing a variable symbol involves changing the referent and b) that the linear order of the alphabet corresponds to a linear ordering of the number system" (Wagner 1981b, p. 116).

She also points out that the context in which a variable appears again affects its meaning in the pupils' view. She explains that it is only in mathematics that the context and the referent determine "the mathematical role of the variable" (Wagner, 1981a, p. 166). Whereas a change in the symbol does not usually effect the meaning of the variable, a change in the context or the referent could affect the role of the variable. So for example if we compare two algebraic expressions:

$$x + 2 = 2 + 3x \qquad (1)$$
$$x + 2 = 2 + x \qquad (2)$$

In the first expression x is a specific unknown and in the second expression x is a generalised number.

Booth has carried out a more detailed analysis of some of the C.S.M.S. algebra errors as part of a project which investigated the reasons for the most persistant errors identified by Küchemann. Booth's findings were consistent with Küchemann's in that she also found that "Children have difficulty in grasping the notion of letters as generalised numbers" and their "natural tendency is to interpret letters as standing for specific numbers" (Booth, 1984, p. 85).

Matz (1980) has carried out a detailed analysis of algebra errors made by secondary school pupils whilst solving algebraic equations. She based her work on the theory that "errors are the result of reasonable, although unsuccessful attempts to adapt previously acquired knowledge to a new situation" (Matz, 1980, p. 95). The ultimate aim of this work is to build a computational learning model of algebra. However her identification of "malrules" used systematically by the learner when presented with an unfamiliar algebraic equation is an important contribution to our understanding of the individual pupil's possible responses. She maintains that most errors can be divided into "those that are generated by an incorrect choice of an extrapolation technique and those that correspond to unmade developmental changes" (Matz, 1980, p. 101). She points out that if students initially fail to realise that a letter represents a number, then operating on the letter will appear to be totally underconstrained. She says that the only linking feature between the multiple uses of letters in algebra is their abstractness. This she maintains is an over generalisation of the concept of variable which hides the distinction between letter as constant, letter as parameter, letter as specific unknown, and letter as "variable".

Acceptance of an "Unclosed" Algebraic Expression as an Algebraic Object Many pupils cannot accept that an unclosed algebraic expression is an algebraic object (Booth, 1984; Collis, 1974; Jensen & Wagner, 1982). So for example pupils are unable to accept that an expression of the form X + 3 could possibly be the solution of a problem. Again this finding needs to be investigated in the Logo context because pupils can more naturally encounter situations in Logo in which these "unclosed" expressions occur as objects during a problem solving process.

Collis (1974) linked pupils' ability to tackle algebra problems to the Piagetian idea of concrete and formal thinking (Inhelder & Piaget, 1958). Collis suggested that the ability to accept lack of closure (ALC) linked to the pupils' cognitive level (Collis 1974). The level of "closure" with which the child is able to work depends on his ability to regard the outcome of an operation (or series of operations) as unique and "real". He suggests that it is not until the pupil reaches the final stage of development (at about 15+) that he is able to consider "closure" in any formal sense because he is able to work on the operations themselves and does not need to relate either the elements or the operations to a physical reality. He now becomes capable of dealing with variables as such because he can hold back from drawing a final conclusion until he has considered various possibilities, an essential strategy for obtaining a relationship as distinct from obtaining a unique result" (Collis, 1974, p. 6). Collis maintains that when the pupil can accept the idea of an unclosed operation he can then work with complex systems (or Multiple Interacting Systems (MIS)) where "complex systems are those where more than one system of co-variation is involved and any meaningful solution of a set of problems depends on working with the interaction of the two (or more) systems" (Collis, 1974, p.7).

In Booth's work she found that errors in algebra may arise as a result over confusion with algebraic notation, in particular with conjoining in algebraic addition. This is linked to their inability to accept for example p + q as a legitimate answer which relates to Collis's findings on pupils' inability to accept lack of closure (ALC) in an algebraic expression. Booth devised a teaching experiment specifically designed to remediate algebraic errors. She based a series of worksheets around the use of an "imaginary maths machine" which can be instructed to perform operations and solve problems (see Booth 1984 for a detailed description of this machine). "The main gains of this teaching experiment were that the pupils began to accept the idea of an unclosed answer in algebra (e.g. a + b)" (Booth, 1984). The teaching experiment did not however show any clear improvement in pupils' understanding of letter as representing a variable.

Thompson and Dreyfus suggest that if algebra is regarded as generalised arithmetic then "instruction in arithmetic might be adapted so as to anticipate operations of thought that students can readily generalise in their initial experiences in algebra" (Thompson & Dreyfus, 1985, p. 1). They stress that one of the aspects of algebra which beginning students find difficult is the substitution of expressions for variables possibly because they cannot conceive of an expression as being a single unit. Certainly if students perceive a variable as representing an object then how could the variable y (misconceived as one object) represent an expression a + b (misconceived as two objects).

1.3.3 Summary

This review of past literature related to pupils' conceptions in algebra has highlighted the main issues which effect pupils' use and understanding of variable in algebra. These can be summarised as:

- lack of understanding that a letter can represent a generalised number
- lack of understanding that a systematic relationship exists between two variable dependent expressions
- inability to accept an "unclosed" expression in algebra (for example a + 6) which relates to the inability to operate on these expressions
- the gap between arithmetical and algebraic thinking which relates to the use of informal methods in algebra.

1.4 AN OVERVIEW OF THE RESEARCH

This section will present an overview of the chapters which form this thesis.

Chapter 2 discusses the background rationale for using the computer, and more specifically the computer programming language Logo, as a basis for providing pupils with a conceptual basis of algebraic ideas. This chapter also presents some results of the Logo Maths Project (Sutherland & Hoyles, 1987) which provided a framework for this present research.

In Chapter 3 an overview of Logo as a programming language is presented and a detailed analysis is made of the conceptual field of variable under study.

The research consisted of two strands, a major longitudinal study of four pairs of pupils (aged 11-14) programming in Logo as part of their "normal" secondary school mathematics class, and a follow up study of a group of pre-algebra primary school pupils (aged 10-11).

1.4.1 The Longitudinal Study

Chapter 4 provides a description of the research methododology used for the longitudinal study, including descriptions of the classroom setting and the role of the researcher. This chapter also describes the data collected throughout the longitudinal case study, and provides a timetable of this data collection.

The first aim of this study was to investigate the nature of pupils' understanding of algebra related ideas in Logo in order to establish whether or not pupils have similar difficulties when programming in Logo as they have been found to have in "paper and pencil" algebra. The analysis of the data collected as part of the longitudinal study is presented in Chapter 5. At the end of this chapter a summary is made of each individual case study pupil's developing use and understanding of variable in Logo.

The nature of the materials developed to help pupils make links between algebraic ideas developed within a Logo environment and those used in a "paper and pencil" algebra context are presented in Chapter 6. This chapter also analyses the effect of the pupils' engagement with the "Function Machine" materials.

Chapter 7 presents the results of the structured interview, administered individually to the case study pupils at the end of the three year longitudinal study. This interview was aimed at probing both the pupils' understanding of variable in Logo and their understanding of variable in "paper and pencil" algebra.

1.4.2 The Pre-algebra Study

At the end of the three year case study a subsidiary one year study of a group of primary school pupils (aged 10-11) was carried out. These pupils were chosen because they had not been introduced to any formal algebra during their "normal" school mathematics. The Logo environment for these pupils was structured in order to overcome some of the obstacles to the understanding of variable which had arisen as a result of the longitudinal case study. The rationale and results of this study are presented in Chapter 8.

1.4.3 Summary

Finally in Chapter 9 a synthesis is made between the strands of the longitudinal case study and the pre-algebra study. The results are discussed from the perspective of the theoretical framework developed in Chapters 1 to 4. The final conclusions also include a discussion of the implications for future research which arise as a result of this study.

CHAPTER 2

THE COMPUTER AND THE LEARNING OF MATHEMATICS

2.1 THE ROLE OF PROGRAMMING

There is no doubt that computers will be part of the mathematics classrooms of the future. The interactive nature of the programming activity itself stimulates exploration, investigation and discussion,all activities encouraged in the Cockcroft Report (Cockcroft et al., 1982) and it has been observed that "Pupils in surprisingly large numbers are finding a joy and zest in some aspects of mathematics which they did not find before" (Fletcher, 1983, p. 2). Very little research exists on this subject and much of the rationale for computer programming stems from experiences derived from classroom practice. However although the readily observable motivational and attitudinal effects of computer programming must not be underestimated it is also important to justify the programming activity from the point of view of learning.

The recent advent of the microcomputer has radically changed the nature of programming as a problem solving activity. Only twenty years ago the most usual way of presenting a program to a computer was on a set of punched cards, the user often having to wait many days before a program was executed. This slow "turnaround time" meant that debugging was an arduous and frustrating task. Nowadays, when working with a compiled programming language on a microcomputer, typing and syntax errors can be corrected almost instantly. A problem can be easily broken into parts and each part can be tested and debugged separately. This allows for more flexibility in individual programming style. In addition with a programming language like Logo it is possible to start a session without having a clearly defined idea of the problem to be solved; the problem itself emerging through interacting with the computer. Despite this new technology much of what is taught as computer programming in schools at present is taught from what could be called a "mainframe perspective". That is the teacher often tries to push the students into a rigid problem solving mould. Brian Harvey makes the point that "Planning is one of the most fundamental problem-solving skills. But there are many kinds of planning. The kind in which every part of your program's behaviour is written down before you begin programming isn't very realistic in many contexts. Even in the large scale business or government projects that structured programmers like to talk about , it is very common that the ultimate users of a program change their mind about how it should work, once they have had some experience of using it" (Harvey, 1985, p. 165).

This chapter will review literature related to the role of computer programming, and more specifically programming in Logo, to the learning of mathematics. Past research on the cognitive effect of programming can be roughly divided into the following areas:

- work which attempts to demonstrate that the algorithmic nature of computer programming is crucial to the learning of mathematics (Johnson 1986; Johnson & Anderson, 1985; Knuth, 1974)

- work which attempts to link programming to the development of general problem solving skills (Clements, 1986; Clements, 1987; Clements & Gullo, 1984; DeCorte & Verschaffel, 1985; Pea & Kurland, 1984b; Salomon & Perkins, 1987)

- work which aims to describe the cognitive demands of learning a programming language (Hoc, 1977; Hillel & Samurçay 1985a; Leron, 1983; Mendelsohn, 1986; Papert, Watt et al, 1979; Pea & Kurland, 1984a; Rouchier & Samurçay, 1985; Rogalski, 1985; Rouchier, 1986; Samurçay 1986)

- work which aims to show that programming activity can help with the learning of mathematical content (Feurzig et al, 1969; Finlayson, 1985; Hart, 1981b; Hartley, 1980; Hillel, 1984; Howe, O'Shea & Plane, 1980; Kieran, 1985; Leron & Zaskis, 1986; Milner, 1973; Noss, 1986; Thomas & Tall, 1986).

2.1.1 Programming as an Algorithmic Activity

There is a school of thought which suggests that the algorithmic nature of programming is its most crucial aspect from the point of view of learning mathematics. An algorithm has been defined by Knuth as " a precisely defined set of rules telling how to produce specific output information from given input information in a finite number of steps" (Knuth, 1974, p. 323). Knuth suggests that programming an algorithm will help in the understanding and learning of the algorithm. This is based on the assumption that in programming a person is teaching the computer and "a person does not really understand something until he can teach it to the computer, i.e. express it as an algorithm" (Knuth 1974, p. 327). Johnson maintains that the whole nature of a concept changes when "the concept can be viewed as a procedure, i.e an ordered sequence of steps for doing a particular task, and hence a dynamic entity rather than a static definition or statement" (Johnson, 1986). He gives the example of the concept of a prime number, saying that in writing an algorithm (computer program) to generate prime numbers the pupil will develop an enhanced understanding of the concept of a prime number. This author believes that in focussing too heavily on algorithmics many of the crucial aspects and benefits of learning programming may be overlooked.

2.1.2 Programming as a Problem Solving Activity

Much of the research on programming as a mathematical activity has focused on the relationship between computer programming and problem solving with the process of programming itself being considered crucial. Enough is not yet known about the individual nature of problem solving strategies used by pupils when programming (these are likely to be language dependent as well as problem dependent) and so research which expects some sort of "idealised" problem solving skill to transfer from programming to other contexts appears rather naive in its approach. Pea and Kurland (1984a) have suggested that transfer of programming skills to other non programming contexts might only result from an advanced level of programming competency which most school children do not reach. Clements (1986) carried out an experiment to assess the effect of learning programming and computer assisted learning on specific cognitive skills (for example reflectivity, divergent thinking). The study lasted for 22 weeks with seventy-two 6 - 8 year olds being randomly assigned to either a Logo programming or a Computer Aided Instruction or a control group. Clements concluded from his study that "Logo programming can increase performance in specific cognitive and metacognitive skills and on measures of creativity" (Clements, 1986, p. 317). These findings contrast with findings of Pea and Kurland (1984b) derived from an experiment to investigate whether or not Logo experienced pupils developed planning skills. The research was carried out with 32 children, half of whom received Logo instruction for half a school year and the other half were a non-treatment control group. They reported that the Logo children showed no more evidence of having acquired planning skills than the non-Logo group. It seems very likely that the discrepancy between the results of Clements and those of Pea and Kurland are entirely due to the nature of the Logo treatment itself. De Corte and Verschaffel (1985) in reviewing the evidence for the effects of computer experience on children's thinking skills also conclude that there is very little evidence supporting the claim that computer programming will have positive effects on childern's thinking and problem solving skills. However they suggest that "longitudinal investigations should explicitly be process oriented, i.e. oriented towards a better understanding of the psychological processes that arise during computer learning, of the individual differences in those processes, and of the difficulties that children encounter while learning" (De Corte & Verschaffel, 1985, p. 12). They also stress that "to maximise the probability that children will apply the acquired knowledge and skills beyond the computer-learning environment, it is absolutely necessary to teach for transfer " (De Corte & Verschaffel, 1985, p. 13).

Much of the past research can be criticised on the basis of attempting to measure transfer of general problem solving skills without either taking into account the possible content specific nature of problem solving or without having analysed the nature of the Logo activity, from a problem solving perspective, in which the pupils have engaged.

2.1.3 The Cognitive Demands of Learning Pogramming.

A review of the literature on the cognitive demands of learning to program highlights the fact that very little is yet known. Pea and Kurland stress that it is nonsense to treat programming as a "unified homogeneous activity" saying that "what one needs in order to program will depend in fundamental ways on one's programming goals" (Pea and Kurland, 1984a, p. 4). It is likely that many of the computer programming demands will be language specific and all the research points to novice programmers developing many incorrect representations concerning the computer's functioning. From the perspective of this thesis the most relevant research is that which concerns the cognitive demands of learning certain aspects of programming which have particular relevance for the learning of mathematics. Mathematics educators in France for example have recently carried out studies on the cognitive demands of learning Logo. They quite rightly maintain that until these demands have been identified we will not be able to harness the potential of Logo within the Mathematics classroom. "Many researchers (Mendelsohn, 1986; Pea & Kurland, 1984a; Samurçay, 1985a) show that even at a simple level programming is a complex task and its learning implies acquisition of some specific concepts like variable, iteration and recursion. Although these concepts can be considered in conception (in terms of conceptual field) with the mathematical concepts of variable, they present complex relationships with them, in terms of acquisition" (Samurçay, 1985b, p. 76). As this thesis is concerned with the learning of algebra related ideas within Logo two studies by Samurçay and Hillel and Samurçay related to the cognitive demands of learning about variable in programming will be reported in detail.

The Cognitive Demands of Using Variable in Programming Samurçay in working with 15-16 year old students programming in Pascal discovered that the algebraic models which the pupils brought from mathematics to the programming situation often constituted obstacles to programming. She maintains that the mathematical conception of variable is insufficient because of its static nature and that a programming variable is more dynamic. "We argue that the algebraic conceptions of variable, equality sign and equation constitute a necessary but an insufficient model on which can be built the programming concepts of variable, assignment and loop-construction" (Samurçay, 1985a, p. 42). This research suggests that the difference in meaning between the equality sign in Pascal and algebra creates an obstacle for pupils. From this respect the programming language Logo has a distinct advantage over both Pascal and Basic.

Samurçay in collaboration with Hillel have carried out some important research into pupils' understanding of variable in Logo which as they quite rightly point out is a precursor to any potential algebra understanding. "We see clearly that aside from difficulties in defining a general procedure, there is more basically, a lack of an immediate sense of the necessity to define such procedures" (Hillel & Samurçay, 1985a, p. 8). In a study in which they observed two pairs of nine year olds as they worked with the variable concept they found several levels of conceptual difficulties in "identifying what is actually varying, understanding what the variable-name signifies, operating on the variable within a procedure and dealing with an input-dependent "interface". They identified three different types of variable activity:

> 1) Trying out specific instances of an already written general procedure. This involves assigning an initial value to the variable input which they reported did not present any conceptual difficulty to the children. They report that pupils might use a general procedure without necessarily identifying what is varying, and that this is particularly likely to occur when pupils have not defined the procedure for themselves.
>
> 2) Using general procedures as building blocks in more complex problems. In this instance pupils do need to identify what is varying and assign appropriate values to the variable. They may also need to construct a variable dependent interface in order to create their superprocedure.
>
> 3) Defining new general procedures. In this activity they need to identify what is varying, name and declare their variable and operate on the variable within the procedure (when operating on the variable means passing the variable to primitives or procedures and the action of modifying the variable within the procedure).

Hillel and Samurçay report that naming of variable can become problematic "in situations where the 'internal' and the 'external' variation are in less obvious relation ...... Children are sometimes confused about what the variable-name actually signifies....this is in part, because they may attach undue importance to the name as determining the function of the input" (Hillel &Samurçay, 1985b, p. 12). They also maintain very strongly that pupils understanding of variable in Logo is inextricably linked to their understanding of procedure and modularity.

2.1.4 Programming and the Learning of Mathematics

Most of the reviewed research related to the learning of mathematical content has expected too much in terms of the learning of specific mathematical ideas without either clearly analysing these ideas or clearly analysing from which programming ideas these might derive. In the Logo Maths project for example it was quite naively expected that pupils would learn about angle from programming in Logo. Analysis of the data indicated that some pupils never worked on problems which required reflection on turtle turn and its synthesis with angle and so did not learn anything about these concepts throughout their three years of programming in Logo (Hoyles & Sutherland, 1986). This study is concerned with the learning of algebra related ideas within a programming environment and so the research related to this area will be reviewed in depth.

Programming and Learning Algebra Before Logo became widely available in schools Basic was the language most often used in the mathematics classroom. Several studies have attempted to use this programming experience to help pupils learn certain algebra related ideas. In the Nottingham Programming Project about three hundred 11-12 year old secondary school pupils learned to program in Basic before learning any algebra. The following is an example of a problem posed to the pupils after about four or five lessons. "Count how many times your heart beats in one minute, store this number in the computer and use this to make the computer calculate how many times it beats in one hour, one day, one week, one year and since you were born" (Hart, 1985). Hart reported the following typical solution:

```
10 LET X = 78
20 LET H = X*60
30 LET D = H*24
40 LET W = D*7
50 LET Y = D * 365
60 LET T = 11*Y + 15* W + 3*D
70 PRINT H,D,W,Y,T
```

The pupils were given pre- and post-tests using items from the C.S.M.S algebra test (see appendix 1). The results of these tests indicated that the Basic experienced pupils achieved better results than the "norm" as represented by the C.S.M.S results. In particular the pupils were more successful with the item "If John has J marbles and Peter has P marbles , what could you write for the number of marbles they have altogether?"

Thomas and Tall worked with 42 mixed ability 12 year olds with no previous experience of algebra. The aim of their research was to test whether or not a computer based teaching programme devised by them had any effect on the pupils' understanding of the use of letters in algebra. These pupils were divided into matched pairs using results of the C.S.M.S algebra test (appendix 1). The experimental group were given an introduction to simple basic programming . An example of the type of problem worked towards would be for pupils to compare the output from the following two programs for different values of a and b.

```
10 INPUT a
20 INPUT b
30 c= 2*(a+b)
40 PRINT c
50 GOTO 10
```

```
10 INPUT a
20 INPUT b
30 c=2*a+2*b
40 PRINT c
50 GOTO 10
```

The pupils were also introduced to a "Maths Machine " (software developed for the project) in which they were asked to find solutions to problems of the form

For what value of x is $2x + 1 > 5$?

"This was achieved by inputting the formula 2x+1 as a function and then choosing values of x to input. The 'machine' displayed the value of the function for this value of x and so values giving a result greater than 5 could be recorded" (Thomas & Tall, 1986, p. 316). The experimental pupils were given a test based on the C.S.M.S test as a post-test and a delayed post-test. The results of their research showed that the experimental group performed "significantly better than the control group on questions requiring an understanding of the use of letters as a specific unknown and as a generalised number or variable (Thomas & Tall, 1986, p. 317).

Noss (1985, 1986) also worked with younger children as part of the Chiltern Logo Project. The aim of the project was to investigate the nature and extent of the mathematical environment created through young children (aged 8-11) learning Logo. The 118 children who took part in the project were distributed among five top junior classrooms. During the last six months of the eighteen month project Noss carried out an algebra study with eight of these pupils. The aim of this study was to investigate "the extent to which the pupils could a) construct meaningful symbolisations for the concept of variable and b) contruct formalised (algebraic) rules" (Noss, 1986, p385). He presented the pupils individually with a series of paper and pencil tasks during a taped interview. These questions were all adapted from those used by Booth (Booth , 1984). and were chosen as being appropriate to allow children, who had not yet encountered any formal algebra, the possibility of contructing their own formalisation and notation during the process of solution. During the previous period of Logo research no attempt had been made to show these pupils the links between their Logo work and algebra. One of the items presented to the children is given below:

> Peter has some marbles
> Jane has some marbles.
>
> What could you write for the number of marbles
> Peter and Jane have altogether?

Fig. 2.1: Noss's "Marbles" Item

From this research Noss found that as far as the concept of variable was concerned some of the pupils were able to construct variable names for unknowns in order to solve the problems presented to them and that the two pupils who were not able to construct names had not used the idea of variable in Logo. He gives an example of Nicola who when presented with the "marbles" problem (Fig. 2.1) said:*"You could use the input again"* (although she had not previously referred to the word input). When Noss asked her how she wrote down:

:PETER + :JANE = all the marbles

saying *"Peter plus Jane equals all the marbles. You use those two as the inputs, with as many marbles as you want"*. Noss then asked her what the dots in front of PETER and JANE were and she said *" They're to represent that it's an input."* When prompted about the meaning of input she said:*" That you can type in however size you want it or how many you want it. How ever many they want. How many they want Peter to have and how many they want Jane to have"* (Noss 1985, p. 412-415).

Noss also found instances of the children constructing names for unknowns which stood for a range of numbers. He suggests that " the Logo work may have helped to form the children's conception of variables as generalised numbers, namely that the metaphor of typing in a value at the keyboard may have presented a way of conceptualising a range of numbers while only necessitating the consideration of specific values (one at a time). In the context of inputs, Logo variables are assigned a single value at the time the procedure is "run". Although the name of the input may, of course, stand for an infinitely large range of possible values only one value is assigned at a time"(Noss 1985,

p. 424).

Noss concludes from his study that "The interpretation of the data offered here (and it should be emphasised that it is one possible interpretation), is that children may - under the appropriate conditions - make use of the algebra they have used in a Logo environment, in order to construct algebraic meaning in a non-computational context" (Noss, 1986, p. 354).

2.2 THE LOGO COMPUTER ENVIRONMENT: RESULTS FROM THE LOGO MATHS PROJECT

The Logo Maths Project was concerned with a wider range of issues than this thesis (Hoyles, Sutherland & Evans, 1985; Hoyles & Sutherland, 1986; Sutherland & Hoyles, 1988). In particular it investigated: the problem solving strategies used by pupils within the Logo programming environment; the nature and consequences of the teacher interventions in the learning process and the nature and extent of the collaboration between pupil pairs. Many of the results derived from the Logo Maths Project have provided a theoretical framework from which to analyse the results of this thesis and consequently this section will summarise these results. The results will be presented in the form of extended citations from the recent report of the Logo Maths Project (Sutherland & Hoyles, 1987).

2.2.1 Problem Solving Strategies Used by Pupils.

> "At the beginning of the project pupils were given the freedom to choose their own goals and develop their own problem solving and programming strategies. Although our interventions were often focussed on process in the form of encouraging the pupils to reflect, we did not impose any "idealised" problem solving strategies on the pupils since the computer is a new problem solving tool and we wished to investigate the problem solving strategies developed by the pupils for themselves. We have identified from the transcript data categories of programming activity which provide a framework for analysis. These categories are:
>
> Working at a Syntactical Level This activity consists of the use of primitives, procedures (or sequences of these) with a focus on screen output without any apparent reflection of how or why the output was achieved. Examples of such activity are random typing of commands, passively "copying" from other pupils or from a handbook or randomly putting inputs into the REPEAT command. Our observations have led us to believe that pupils who work at a syntactical level are not

provoked to think about the processes involved in their work and this tends to be detrimental to their learning.

"Making Sense of" This is exploratory activity in which pupils are trying out a new idea or procedure and reflecting on what is happening. Sometimes such activity is completely non-goal directed, sometimes it takes place within goal directed activity and sometimes a goal emerges from the activity. If pupils are to develop an understanding of the processes involved in Logo programming we would suggest that it is important that pupils are encouraged to "take time out" from working towards predefined goals to explore how a new process works.

Goal Directed This is activity aimed at achieving an outcome. From the research data two separate dimensions along which turtle graphics goals can be classified have been identified:

a) Loosely defined ....Well defined

b) Real World...Abstract

It is hypothesised that the 'position' of the goal with respect to these two dimensions will affect pupil interaction and behaviour.

a) Loosely Defined ......Well Defined

This dimension is concerned with the extent to which the pupils have defined and planned the final outcome of their work. On the one hand, loosely defined goals are characterised by a lack of detailed preplanned structure: they evolve out of exploratory "making sense of" activity. It is important here to separate out global from local structure. Within loosely defined goals at a local level, individual modules can be well defined given the modular nature of Logo; in other words the global looseness of the goal does not imply that a local subgoal need not be tightly structured by the pupils.

Well defined goals on the other hand have a well worked out overall structure and global product. At the local level an individual module forming part of the overall structure may not be well defined. The way it is composed may emerge from local exploratory activity. For example, when a pair of children worked on the well defined goal of writing a procedure for the word LONG they did not have a prescribed plan for defining the shape of each letter.... these emerged in an exploratory manner from their activity at the keyboard.

b) Real World .... Abstract

This dimension is concerned with the extent to which pupils aim to come up with an actual representation of 'reality'. It must be stressed that this dimension concerns the pupil's perception of the 'realness' of the representation they are producing. There is not necessarily anything objectively more real about the image of a flower than an image of a square but we have found that pupils' programming style appears to be influenced by how they see the image they are drawing, that is whether they see it as a picture of something in their 'real world' or whether they see it as an abstract pattern. Figure 2.2 illustrates pupil goals classified according to the above dimensions.

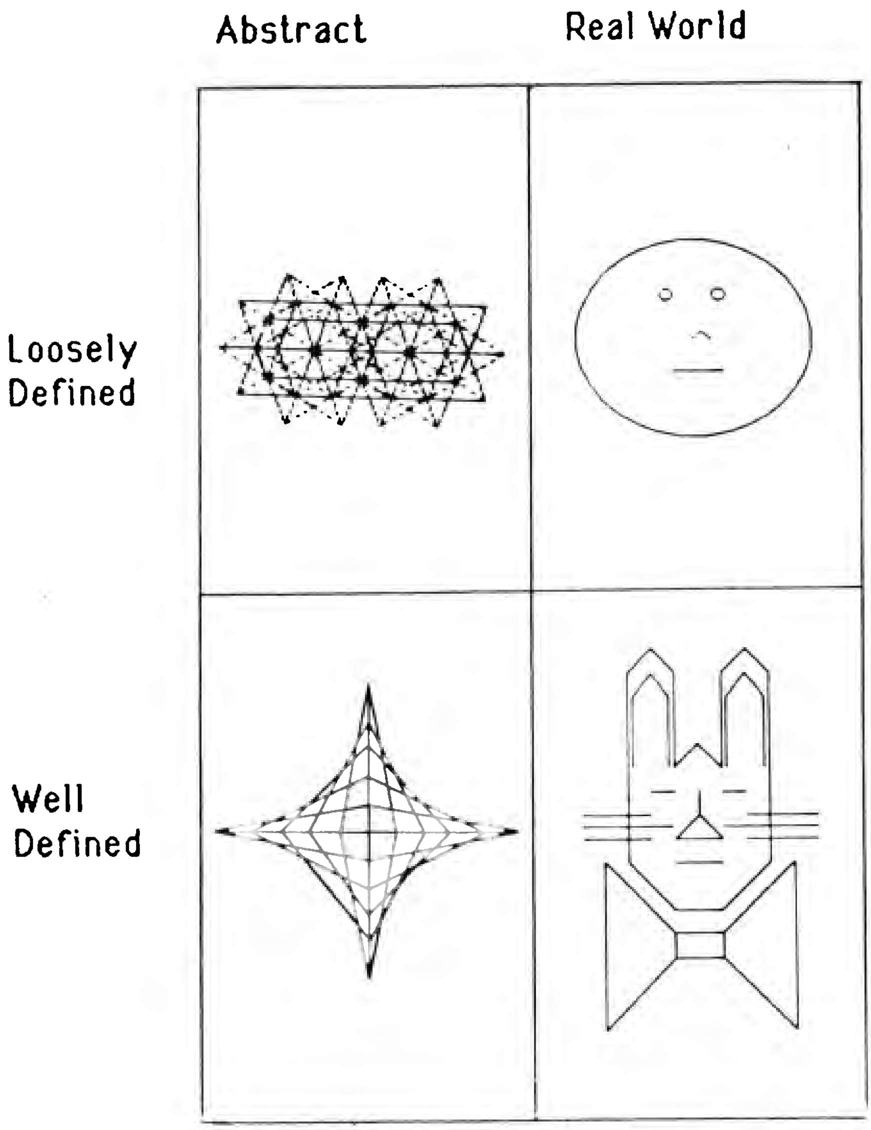


Fig. 2.2: Classification of pupil Goals in Logo

Within goal directed work, we have identified different subsets of activity: planning, implementing and debugging all of which can have either a local or global focus. Local activity focuses on the graphics or text output; while global activity focuses on a mental plan. These processes together with their interaction with the negotiation of a goal are represented in Fig. 2.3. The sequence of the activities depend on a pupil's individual programming style and the content and nature of the problem" (Sutherland & Hoyles, 1987, p. 45-48).

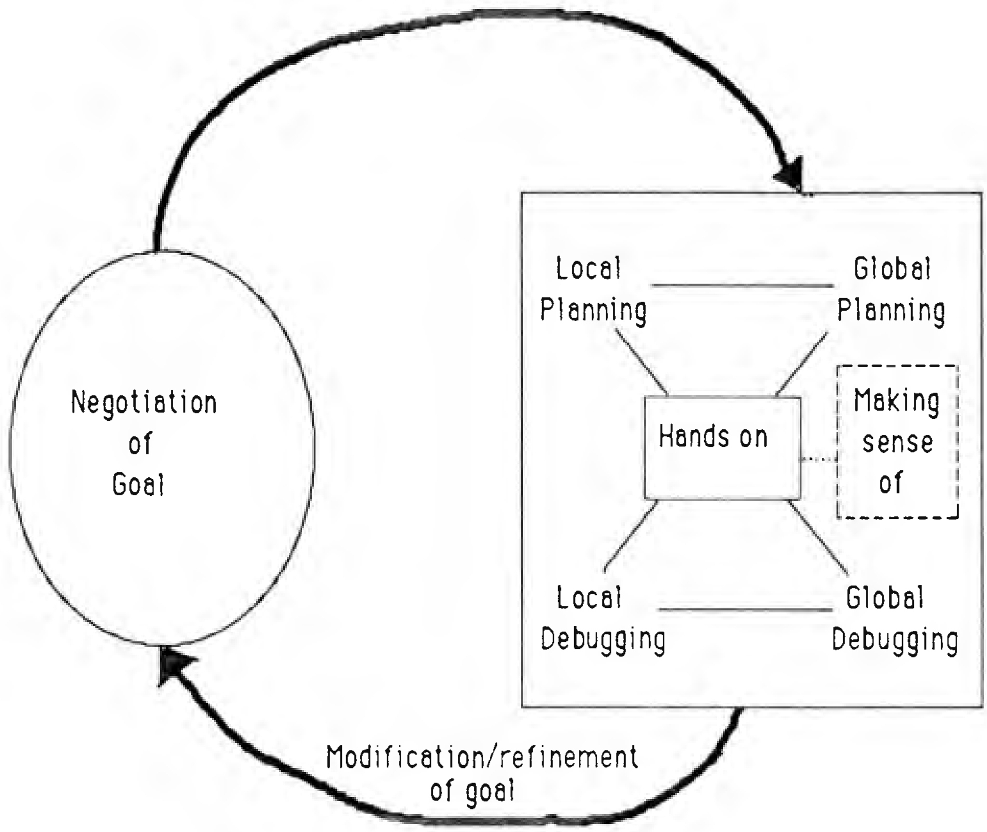


Fig. 2.3: Categories of Programming Activity

2.2.2 Pupils' Use of Structured Programming Ideas

"Pupils' use of structured programming design is influenced by both the nature and requirements of the pupil goals and the way these goals are perceived by the pupils. When pupils perceive their project to be one of working towards a real world representation the Logo commands are likely to become an extension of their drawing arm and subprocedures defined only as a way of storing commands in a shorter sequential manner. In such circumstances pupils do not perceive a need for their functional subprocedures to be reusable modules and consequently do not attempt to put interfacing commands into separate subprocedures. They also think out their commands in a step by step linear way and debug in a similar manner. When pupils work on well defined abstract goals they are likely to plan their work in such a way that more naturally suggests the idea of breaking a problem into parts and defining each part as a separate subprocedure. There is however considerable variability between pupils in their perception of modularity in any design. This may be associated with the pupil's level of field dependence/independence. In addition pupils are more likely to perceive modularity when a module is not embedded within a design. Projects consisting of 'disconnected' modules are therefore more suitable for introducing ideas of modularity to pupils. We now believe that a pupil's progress in being able to break down well defined goals into parts is a consequence of experience of building up subprocedures into loosely defined goals and defining superprocedures for the final image" (Sutherland & Hoyles, 1987, p. 195).

2.2.3 The Nature and Consequences of Teacher Intervention

"During the Logo Maths Project transcript data was continuously collected and analysed and the nature of our interventions changed on the basis of this ongoing analysis. We suggest that an important role for the teacher is to help pupils develop flexibility in their approach to programming: the pupil who naturally prefers to define procedures in the editor needs to know when it is appropriate to try out modules in direct drive; conversely the pupil who always works in direct drive needs to be shown the power of defining in the editor and be provoked to predict the output of procedures before they are run. We see a need for pupils to work on teacher devised tasks designed for specific learning outcomes and for teaching episodes in which the control of the interaction is more with the teacher than with the pupils. It is important however to maintain a balance between teacher initiated activities and pupil directed exploration. How to structure the learning situation while maintaining the pupils' sense of control and without inhibiting investigatory activity and extended project work are questions for which we are only beginning to find answers. We know on the one hand that we must sometimes carefully organise the pupils' learning environment yet we have observed pupils losing motivation because of 'over intervention'. Teachers must decide on the aims of the Logo work in their classrooms and then base their intervention strategies around these aims. Our research has also uncovered commonly occurring bugs in pupils' conceptions of how programming works in Logo. Teachers need to be aware of these potential pitfalls and help pupils understand the appropriate Logo structure and syntax which matches the pupil's problem solution.

Our overall strategy for intervention gradually changed over the period of research so that we were giving pupils more teacher directed tasks in order to encourage pupils to choose from a range of goals. We recognise the importance from a motivational point of view of the pupils choosing their own goals and we started our project with a strategy of encouraging this freedom" (Sutherland & Hoyles, 1987, 195).

2.2.4 The Role of Collaboration

"Turning to the question of discussion and collaboration, there is no doubt that Logo programming provides an engaging problem solving context. It was evident that not only were pupils provoked to talk but also that almost all the talk was task related. Despite marked variation between the patterns of interaction between pupil pairs, instances for each pair were recorded when collaborative exchanges:

- provided challenging ideas for projects and increased the range of projects chosen.
- kept a project going in the face of "obstacles".
- changed the level of representation of the work (conceptual to concrete and vice versa).
- provoked discussion and reflection on the computer feedback.
- facilitated the development of more flexible approaches to problem solving and programming.

Our research also identified specific individual conceptual development as a result of the three way interaction between pupil pair and computer. In such cases the computer environment provoked conflict through graphical feedback and also provided 'scaffolding' which allowed a pupil to move on from an earlier conception. The conflict was also found to be influential in provoking more elaborate and supported argument between pupils. We found however that collaborative work or discussion does not necessarily lead to individual learning gains in tightly specified circumstances. Pupil pairs tend to have implicitly negotiated individual dominance for particular aspects of any activity. This negotiation of dominance can impede individual acquisition of particular understandings. Thus the role of peer interaction in a computer environment involves issues which are extremely complex. It is difficult therefore for a teacher (or researcher) to predict with any precision what a pair jointly or individually will gain in any collaborative setting" (Sutherland &

Hoyles, 1987, p. 196)

2.3 OVERVIEW

Although much has still to be learned about the specific cognitive demands of programming it is clear that learning to program is a non-trivial task. Despite this pupils can and do learn to program in a way which would not have been predicted before the advent of the microcomputer. In addition within the domain of algebra there is some evidence that programming can provide pupils with a basis for an understanding of variable as representing a generalised number.

We must be careful however not to restrict our vision by previous research carried out at a time when the technology was in some way substantially different from that which is available today. Programming is a problem solving activity. The potential for interacting with the computer whilst engaged in the problem solving activity could radically change the nature of the activity. Very little of the reviewed research has, in the author's opinion, adequately dealt with this issue.

CHAPTER 3

LOGO AND VARIABLE

3.1 AN OVERVIEW OF THE PROGRAMMING LANGUAGE

Logo is a programming language derived from the Lisp family. It was developed by Papert and Feurzig in the Artificial Intelligence Laboratory at the Massachusetts Institute of Technology in the late 1960's and was designed so as to provide a mathematical environment accessible to children of all ages and abilities. More recently Logo has become available for a range of microcomputers used within the educational system.

As a programming language the most important features of Logo are:-

It is procedural A procedure is a group of commands which have been given a name (the procedure name). Procedures can communicate with each other via inputs and outputs and it is the procedural nature of Logo which encourages the user to break a problem into simpler components, working on and refining each component in a structured manner.

It is extensible An extensible language is one in which user-defined procedures look like primitive procedures. User defined procedures can therefore act as primitives of the language. This is very valuable for teaching purposes because a teacher can, for example, extend the language by adding new looping structures. "The right control structure for you is the one that best solves your immediate problem. But only an extensible language like Logo allows you the luxury of accepting this idea" (Harvey, 1985). A procedure can consist of Logo primitives or other procedures.

It is interactive Any Logo primitive or procedure is executed by typing it into the computer so that feedback is immediate and errors can be corrected as they occur. Before defining procedures in the editor pupils can test out their ideas in direct drive an important strategy when attempting a new challenging project. When they have defined a procedure the editing facilities of the language make it easy to correct mistakes.

The data structure of Logo are lists A list consists of an ordered sequence of elements which may be numbers, words or other lists. Lists provide a powerful means to create complex data structures (for example hierarchical tree structures). Lists can become bigger or smaller as the program executes and so do not have the problem of taking up a fixed amount of storage in the computer. In addition in Logo variables are not typed. This means that at the beginning of a procedure it is not necessary to specify whether the variables will be, for example, real numbers or character strings.

It is recursive A recursive procedure uses itself as a subprocedure. The facility to use recursive procedures enables simple and elegant programs encapsulating the essential structure of a problem to be used in complex structures. Although the ideas behind recursion are certainly not trivial using recursion in Logo could provide pupils with a basis for the use and understanding of recursive related ideas in mathematics (for a discussion of these ideas see Leron & Zazkis, 1986)

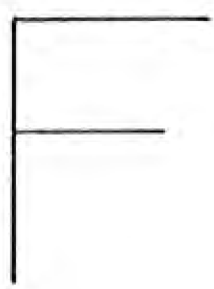
It is functional In a functional language such as Logo or Lisp the underlying model of an operation is a mathematical function. "The emphasis is not on what is going on inside the computer, but on how to link up function machines which the computer emulates to achieve a desired objective" (Klotz, 1986 ,p. 17).

For most pupils the entry point of Logo is through turtle graphics, which provides an important visual dimension at the beginning stages of learning a programming language. The programmer controls either a floor or a screen turtle to draw a graphical object. For example the following commands will draw the letter F (Fig. 3.1a)

a)

```
LT 90
FD 20
RT 90
FD 20
BK 20
LT 90
FD 15
RT 90
FD 25
```

b)



c)

```
TO F
LT 90
FD 20
RT 90
FD 20
BK 20
LT 90
FD 15
RT 90
FD 25
END
```

Fig. 3.1: The Letter "F"

These commands can be entered into the computer in direct mode, in which case the typing of each command will produce an effect on the screen. It is an important aspect of learning about the sequential nature of programming that the pupil sees that each command typed into the computer produces an effect and the visual outcome on the screen helps to reinforce this. If the pupil is satisfied with these commands he or she can define a procedure (Fig. 3.1c). In the version of Logo used throughout this project procedures were always defined in editor mode. Modifications to the procedure were also made in editor mode. When a procedure is being defined in the editor mode no graphical image is produced on the screen. In order to run the procedure the pupil returns to direct mode and types the procedure name into the computer (in this example the name is F). Two important points must be mentioned here. The first is that the geometry of turtle graphics can be pursued to a very high level from both a programming and a mathematical point of view (Ableson & Di Sessa, 1981) and secondly that Logo can be used in the way that other programming languages can be used to solve non-graphical problems. Logo has been chosen for the purposes of this research for the following reasons:

- the entry point via turtle graphics is accessible and motivating for a mixed ability range of pupils

- the functional aspect of the language models the properties of functions in mathematics

- the assignment statement does not use the "=" sign, a potentially confusing aspect of some programming languages from a mathematical viewpoint

- the structured nature of the language encourages the analysis and breaking down of problems into parts, an important mathematical activity

- Logo predominantly uses local variable within procedures and it is suggested that this facilitates the introduction of the variable concept

Appendix 3.1 gives a description of the Logo commands and structures which were most commonly used throughout this project.

3.2 THE USE OF VARIABLE IN LOGO

"It is difficult to talk about programming as if it is a unitary skill. The cognitive processes involved in a programming activity depend both on the programming environment used (language, machine, e.t.c.) and on the class of problems that are attempted to be solved. For example, the problems which can be solved in Logo do not belong to the same class of problems which are solved in Prolog (Hillel & Samurçay 1985b, p. 2)

In Logo variables are used as part of procedure definitions and although not the focus of this thesis the issues of subprocedure, modularity and sequencing are strongly related to the use of variable in Logo. Logo is both a functional and a modular programming language. In Logo variables can be defined either as global or local. A local variable is a parameter through which a value is passed to the procedure. The following is an example of the use of variable input to a procedure.

```
TO SQUARE "SIDE
REPEAT 4 [FD :SIDE RT 90]
END
```

Local and global variables The variable SIDE is named in the title line of the procedure and then used within the procedure. As a language Logo differentiates between the name and value of a variable. The use of quotes in the form "X denotes the name of a variable and the use of a colon in the form :X denotes the value of a variable. The variable SIDE is used as a means of passing a value to the procedure SQUARE. The value of SIDE is assigned when the procedure SQUARE is invoked. Typing SQUARE 30 will cause the computer to execute the procedure SQUARE by assigning the value 30 to the variable called SIDE. When a variable is used as an input to a subprocedure it only exists locally to that procedure and to any subprocedures called from within that procedure. It ceases to exist within the computer memory when the subprocedure has been processed. In contrast a global variable which is usually assigned by means of the MAKE statement exists within all procedures and subprocedures and only ceases to exist when the computer is turned off.

Local variables are inextricably linked to ideas of output and recursion in Logo. The author wanted to develop a consistent approach to the teaching and learning of Logo as a programming language and so decided to introduce pupils predominantly to local as opposed to global variables. In fact there was only one occasion when pupils used a global variable throughout the three years of the project. This meant that the pupils involved in this study did not (apart from this one occasion) use variable in the assignment statement MAKE. Apart from the aesthetic computer science perspective the author considers that local variables are more consistent with algebra usage.

Procedures which output Logo is a functional programming language, the underlying model of which is the mathematical idea of function, which takes a variable input, processes it and outputs a value. The following provides a simple example of a function which calculates the square of any number.

```
TO SQR "NUM
OUTPUT :NUM * :NUM
END
```

FORWARD SQR 50 will cause the value 50^2 to be calculated and output to be used as input to FORWARD. The idea of functions which output will be addressed in more detail in Chapter 6.

3.3 THE CONCEPTUAL FIELD OF VARIABLE IN LOGO

This research is concerned with pupils' use and understanding of algebra related ideas within a Logo environment. It is not concerned with the pupils' understanding of variable from a computer science perspective. In this respect the pupil is distancing herself or himself from the processes which are taking place within the machine itself. Obviously the pupil is interacting with a machine and the influence which this may have on the models developed by the pupil is one focus of this research.

In this research the concept of variable in Logo will be studied from the perspective of a "conceptual field" (Vergnaud, 1982). As explained in Chapter 1 a conceptual field is a set of problem situations "the mastery of which requires a variety of concepts, procedures and symbolic representations tightly connected with one another" (Vergnaud, 1982, p. 36). The idea of a conceptual field is used in order to put bounds on the concept under study and to allow for the inevitable overlap between concepts. In addition crucial to the idea of a conceptual field is the interrelationship between the set of problem situations which use the concept, the set of invariants which constitute the concept and the symbolic systems used to represent the concept.

This study is concerned with the use and understanding of algebra related ideas in Logo and this is the perspective from which the conceptual field of variable will be developed. At the beginning of the period of research it was not possible to carry out a precise "a priori" analysis of the conceptual field of variable relevant for this study because very little was known about either the types of problems and algebra related ideas which would be appropriate for use by pupils programming in Logo. The conceptual field of variable presented here developed throughout the first two years of the research. This analysis has been influenced by the work of Hillel and Samurçay who have analysed the different programming concepts underlying Logo (Fig. 3.2).Their analysis is valuable in setting out the relationship between the different uses of procedure in Logo. They define a simple procedure to be a procedure made up of Logo primitives only. If a procedure contains another subprocedure they refer to it as a composed procedure. In this study a composed procedure is called a superprocedure. They state that "from a cognitive psychology point of viewpoint the concept of variable represents an invariant. By that we mean that, in the case of a variable, changing the values of the inputs in a procedure still leaves both the inter- and intra-procedural relations invariant. This invariance is

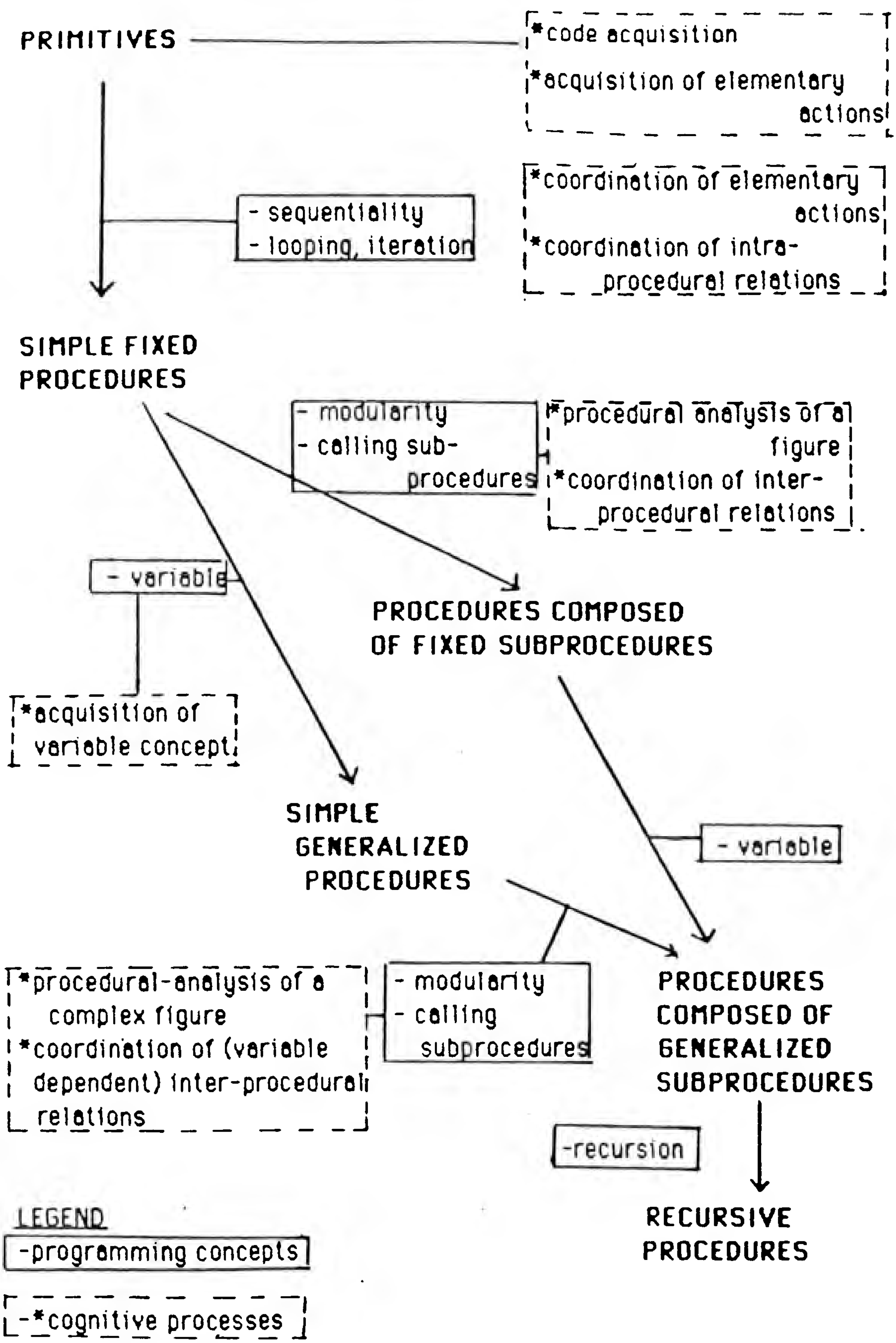


Fig. 3.2: Hillel and Samurçay's "Conceptual Field of Logo programming"

characterised by the attribution of a name to the variable and by the control of its value" (Hillel, Samurçay, 1985b, p. 6). Hillel and Samurçay also point out that in Logo programming the concept of variable is always encountered in conjunction with other concepts (for example procedure, recursion etc.).

3.3.1 Set of Problem Situations

The research project started with the aim of allowing pupils the freedom to choose their own goals. Analysis of the first eighteen months of transcript data indicated firstly that pupils rarely chose projects which 'needed' the concept of variable and, secondly that even when the researcher perceived a need for variable in a pupils' project or in a "teacher-given" task, and intervened appropriately, the pupils were resistant to using it. This was the case for both pupils with little and pupils with no experience of variable in "paper and pencil" algebra. Pupils could not conceive of a project to use the idea of variable until they had had some idea of its potential. It was decided therefore to introduce the concept of variable to all the pupils within a series of structured tasks. The first such task, the "Scaling Letter" task was aimed at provoking the pupils to use the concept as a tool to solve problems and then later to develop the idea of variable as an object for manipulation (Douady 1985). This task is presented,together with the aims of the task, in appendix 3.2. After the "Scaling Letters" task a range of teacher-devised tasks were developed to provoke pupils to use algebra related ideas within their Logo programming (these are presented in appendix 3.3). The four pairs of case study pupils did not all work on the same tasks throughout the project. They also worked on "Function Machine" tasks, which had been designed to help the pupils make links between variable in Logo and variable in "paper and pencil" algebra (see Chapter 6 for a fuller discussion). The set of problems in which variable was needed as a problem solving tool can be classified as:

- Simple graphical objects which was can be represented by a general procedure (Fig. 3.3).

- Composed graphical objects which can be represented by a fixed composed procedure which used a general subprocedure or a general composed procedure (Fig. 3.4).

- Graphical objects which can be represented by a linear tail recursive procedure (Fig. 3.5).

- Non graphical functions (Fig 3.6).

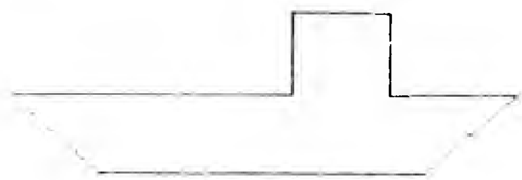


Fig. 3.3: Simple Graphical Object

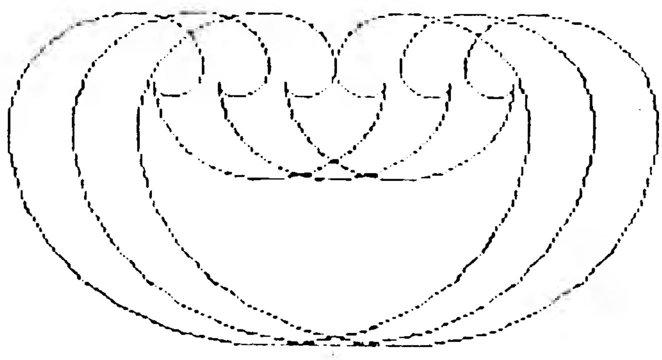


Fig. 3.4: Composed Graphical Object

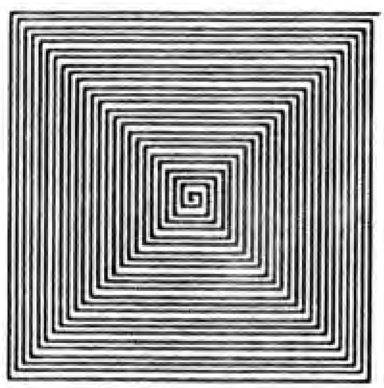


Fig. 3.5: Recursive Graphical Object

```
TO FUN "NUMBER
OP ADD 23 :NUMBER
END
```

Fig. 3.6 Logo Function

The problem situations were either teacher or pupil devised. Table 3.1 presents an overview of the case study pupils involvement in these problems.

Table 3.1: Overview of Case Study Pupils' Engagement in Variable Related Problems.

	Sally & Janet	George & Asim	Linda & Jude	Ravi & Shahidur
Simple Graphical Object	General Polygon Variable Letter General Flower Arrowhead Lollipop	Variable Letter Lollipop	General Polygon Variable Letter General Square Arrowhead Lollipop	Variable letter General Line Arrowhead Lollipop
Composed Graphical Object	Clown's Face Starbuster General Hexagon L O N G Variable Square Patterns Variable Rectangles Butterfly Row of Decreasing-Squares	Pythagorean Triangle Castle 3-D Word Variable Square Patterns Arrowhead Row of Decreasing-Squares	Nested Circles Row of Decreasing-Pines Composed Variable-Letters Row of Decreasing-Squares	Row of Decreasing-Pines Row of Decreasing-Squares
Recursive Graphical Object	Row of Pines Spiral	Circular Spiral Nested Circles Spiral	Nested Polygons	
Logo Function	Function Machines	Function Machines	Function Machines	Function Machines

This table provides a rough guide only. The classification depends on the pupils' interpretation of the task. This is why the tasks are classified differently for different pupil pairs. The detail of pupils' involvement in tasks is presented in Chapter 5. In addition one problem "type" mentioned in the table could have been the focus of many sessions work.

Solving these problems involved, not only the use of algebra related concepts, but also the use of the following mathematical and programming ideas. This list is not intended to be exhaustive but only to give an indication of the breadth of ideas which are involved when using general procedures in Logo.

Mathematical Ideas	Programming Ideas
Measure	Procedure
Decimal numbers	Superpocedure
Negative numbers	Modularity
Angle	State transparency
Ratio and proportion	Turtle state
Function	Tail Recursion
	Output

Within the Logo context general procedures are either:

A) Procedures which have an effect but do not output values.
B) Procedures which output values.

For type A procedures the domain of the variable input needs to be considered. For type B procedures both the domain of the variable input and the range of the variable output need to be taken into account. The following sets of numbers were used by pupils: Natural numbers; Integers; Real numbers. Logo words and lists were not part of domain.

3.3.2 Categories of Variable Use

It is important to analyse the contexts in which pupils use variable. By carrying out an ongoing analysis of the situations in which pupils use variable to define a general procedure, categories of variable use have been identified. They provide a framework for analysing the pupils' understanding of algebra related ideas in Logo.

(I) One variable input to a procedure.
(S) Variable as scale factor.
(N) More than one variable input to a procedure.
(O) Variable input operated on within a procedure.
(F) Variable input to define a mathematical function in Logo.
(G) General superprocedure.
(R) Recursive procedure.

This section will describe each of the above categories and also discuss the researcher's *a priori* analysis of the possible demands of a Logo task from the perspective of these categories. Within a turtle geometry domain a general procedure produces a "varying" effect on the screen and so it is suggested that defining general turtle geometry procedures is conceptually easier than defining non-turtle geometry procedures. At any level a general procedure can either arise out of a solution to a well-defined problem or it can arise out of loosely-defined activity in which the superprocedure is built up through interaction with the computer. It is likely that the dimension well-defined/loosely defined will effect the cognitive demands of the task (See Section 2.2.1 for a fuller discussion of this).

(I) One variable input to a procedure Only situations in which the variable has not been operated on within the procedure are included in this category (see for example Fig. 3.7). This variable could represent: (a) a positive integer in, for example, the number of 'REPEATs'; (b) a real number in, for example, a distance or angle command. When pupils use one variable input they are using variable as a place holder for a set of numbers. It was hypothesised at the beginning of this research that using variable in this way may aid the understanding of variable as a general unknown in algebra.

```
TO TRIANGLE "SIDE
REPEAT 3 [FD :SIDE RT 90]
END
```

Fig. 3.7: Procedure with One Variable Input.

(S) Variable input as scale factor In this situation the variable input is used to scale all the distance commands in a turtle graphics procedure. This type of variable input can be used by pupils as a way of generalising a fixed procedure (see for example Fig 3.8) without making explicit the geometrical relationships within the procedure. At the beginning of the research it was hypothesised that the idea of changing a fixed procedure to a general procedure by scaling distance commands would be conceptually easier for pupils to use than making a general relationship explicit by operating on a variable input within a procedure. When pupils use input as a scaling factor they can define a general procedure from a fixed procedure without reflecting on the invariants within their procedure.

(N) More than one variable input to a procedure This category is concerned with situations in which pupils use more than one variable input to their procedure often as a means of avoiding expressing a general relationship between variables within a procedure (see for example Fig. 3.9). Variable inputs can be added to a general procedure in order to avoid making a relationship explicit between several variables. It is suggested that using more than one input (N) in this way is conceptually easier than operating on a variable within a procedure (category O).

(O) Variable input operated on within a procedure In this category any general relationship between variables within a procedure is made explicit by operating on one or more variable inputs within the procedure (see Fig. 3.10). Pupils operate on a variable within a procedure when they need to make a general relationship explicit. In order to do this they need to identify what is variable and what is invariant within a procedure. Pupils often negotiate this relationship through their "hands on" interaction with the computer and within this project the researcher specifically intervened to provoke this "hands on" negotiation. When pupils use Logo to formalise a general relationship the computer feedback can inform them on whether or not they have correctly represented the generalisation.

```
TO TOM "SCALE
LT 90
PU
BK 90
PD
FD MUL :SCALE 60
LT 45
FD MUL :SCALE 20
RT 90
FD MUL :SCALE 20
RT 90
FD MUL :SCALE 20
RT 90
FD MUL :SCALE 20
END
```

Fig. 3.8: Variable as Scale Factor

```
TO KITE "YT "HT
RT 45
FD :YT
RT 90
FD :YT
RT 90
FD :YT
RT 90
FD :YT
BD :YT
RT 90
FD :YT
RT 45
FD :HT
END
```

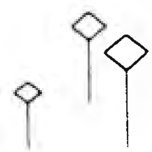


Fig. 3.9: More than One Input

```
TO SQUAN "NUM
LT 135
REPEAT 4 [FD :NUM RT 90]
LT 135
FD MUL :NUM 3
END
```

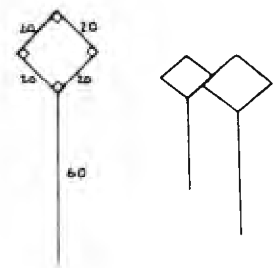


Fig. 3.10: Variable Operated On

(F) Variable input to define a mathematical function in Logo In this category variable is input to a procedure, which acts like a mathematical function, that is it is operated on within the procedure and the result is output from the procedure to be used by another Logo function or command (see for example Fig 3.6). At the beginning of the period of research nothing was known about the cognitive demands of using variable to define a mathematical function in Logo.

(G) General superprocedure This category refers to general superprocedures which use general subprocedures (see for example Fig 3.11). Logo is a structured programming language. This means that nested layers of superprocedures can be defined.

```
TO LONG "SCALE
STEP
L :SCALE
MOVE :SCALE
O :SCALE
MOVE :SCALE
N :SCALE
MOVE :SCALE
G :SCALE
END
```

Fig. 3.11: General Superprocedure

(R) Recursive procedure This category refers to general recursive superprocedures (see for example Fig 3.12). In the context of this research pupils only used tail recursive procedures.

```
TO CORRIDOR "DISTANCE
FD :DISTANCE
RT 90
CORRIDOR ADD :DISTANCE 1
END
```

Fig. 3.12: Recursive Procedure

3.3 SUMMARY

This chapter has presented an analysis of the class of variable related problems which were used by the pupils taking part in this study. This analysis will form the basis from which to trace the case study pupils' developing use and understanding of algebra related ideas within a Logo programming context.

CHAPTER 4

OVERVIEW OF THE LONGITUDINAL CASE STUDY RESEARCH

4.1 RESEARCH METHODOLOGY

4.1.1 A Theoretical Perspective

Research methodology is often characterised by the qualitative, quantitative dimension. Goetz and LeCompte (1984) maintain that this description over simplifies the methodological issues involved. They have suggested that one way of characterising research methodologies is by framing them along four dimensions. In order that the present research study can be more explicitly characterised these dimensions will first be described:

Deduction..Induction

"Purely deductive research begins with a theoretical system, develops operational definitions of the propositions and concepts of the theory, and matches them empirically to some body of data" (Goetz & LeCompte, 1984, p. 4) and "Purely inductive research begins with collection of data (empirical observations or measurements of some kind) and builds theoretical categories and propositions from relationships discovered among the data" (Goetz & LeCompte, 1984, p. 4).

Verification..Generation

Verification research aims to test out certain hypotheses developed outside of the ongoing research and attempts to find evidence that the hypotheses can be applied to more than one set of data. Generation research on the other hand attempts to generate propositions and constructs during the research and may start with no particular theoretical framework or be informed from the beginning by a particular theory. Generative research is often inductive and verification research is often deductive.

Enumeration..Construction

" Enumeration is a process by which previously defined units of analysis are subjected to systematic counting or enumerating; it is usually preceded by the aforementioned constructive process. A constructive strategy is aimed at discovering what analytic constructs or categories can be elicited from the stream of behaviour; it is a process of abstraction in which units of analysis become apparent in the course of observation and description" (Goetz and LeCompte, 1984, p. 5).

Subjectivity..Objectivity

Within a subjective approach to research the researcher constructs and reconstructs categories of analysis derived from the research data. An objective approach to research "applies conceptual categories and explanatory relationships brought by external observers to the analysis of unique populations (Goetz & LeCompte, 1984, p. 6).

When this research commenced very little was known about pupils' use and understanding of variable in Logo. It was decided that a methodology situated on the Inductive , Generative, Constructive, Subjective end of the above continua would enable the complex interrelationships between the pupils, the teacher and the computer feedback to be investigated. Throughout the research the aim was to generate theory by continuously refining category systems devised from the data. The learning situation was considered from a holistic point of view i.e. the interactions between the pupil pairs, the intervention from the teacher and the feedback from the computer were all the focus of analysis. This did not mean that all these aspects were considered simultaneously. The problem of analysis was complicated and the aim was that by continuously examining the transcript data through different frameworks it would be possible to discriminate within these frameworks, ultimately being able to use these multiple perspectives as "subconscious" tools to analyse the data from an holistic point of view. In other words the separate frameworks provide a way in, a first step in a model which aims eventually to analyse the whole situation. By using this method the research aimed to generate and continuously refine theories in a systematic and rigorous manner. At the beginning stages of ethnographic research the researcher "takes a stance of a radically naive observer" (Atkinson, 1979, p.53), trying to avoid "sharpening their problems into specific research hypotheses until considerable exploratory investigation has occurred (a process termed progressive focussing)" (Atkinson, 1979, p. 53). As hypotheses are developed attempts are made to "maximise the chances of discovering negative cases in order to highlight critical deficiencies in the ideas under exploration" (Atkinson, 1979). In a sense one aims to refute a hypothesis by the discovery of a counter example. "Ethnographers attempt to describe systematically the characteristics of variables and phenomena, to generate and refine conceptual categories, to discover and validate associations among phenomena, or to compare constructs and postulates generated from phenomena in one setting with phenomena in another setting " (Goetz & LeCompte, 1984, p. 8).

Because the ethnographic researcher is studying a natural setting it is not possible to prestructure and impose "a priori" restrictions on the research setting. In this project the Logo environment of the case study pupils was under the control of the researcher but the paper and pencil mathematics curriculum was not. For this reason it was important for the researcher to develop reflexivity i.e. "an attempt to render explicit the process by which the data and findings were produced" (Atkinson, 1979, p. 53). One way to develop reflexivity is by continuously re-analysing the data with the aim of developing alternative explanations of the phenomena observed and with particular attention to the interpretation of the effect of the researcher and the research setting on the data collected. The ethnographic research also attempts to triangulate the research findings by using other sources of data. In this project the triangulation has been obtained by collecting other sources of data in addition to the case study transcript data. Pupils were given structured interviews to probe their understanding of variable in Logo and they also visited the University laboratory in order to carry out specific structured tasks individually. In addition a further study was carried out with a group of primary pupils (Chapter 8) in order to confirm or refute some of the hypotheses developed from the main study.

4.1.2 The Author's Preconceptions

The author aimed to enter the field with an assumption of ignorance about the issues being investigated and with an attempt to supress all preconceived ideas related to these issues. Nevertheless the researcher did have some preconceived views and attitudes and this section will attempt to describe these as carefully as possible. She had spent several years teaching 16-18 year old pupils Advanced level mathematics and had found that many of these pupils had almost insurmountable misconceptions within the domain of algebra, and that these misconceptions provided serious obstacles to their learning of advanced level mathematics. She had also taught Basic and Logo (Sutherland, 1984, p. 23 - 32) programming to mathematically low attaining pupils and had found that many of these pupils were able to use algebra related ideas in a programming context. This led her to hypothesise that the computer programming context might provide a basis for learning certain algebra related ideas.

Based on her teaching experience the author also believed that pupil learning is more likely to occur when pupils are actively engaged in reflecting on the problem solving processes themselves. That the computer seemed to provide a context for provoking reflection was an observation made during classroom practice. She also believed that it was the teacher's role to foster pupil autonomy in the learning situation and believed that peer group work could help in this area.

These preconceptions gradually became informed by a theoretical background derived from Mathematics Education, Artificial Intelligence and Psychology. However at the beginning of the research the author attempted to suspend this knowledge in order to study and observe the classroom situation in as open a way as possible. As the research progressed the author aimed to link the theories developed from the research with existing theoretical frameworks.

4.1.3 Description of the Classroom Setting

The research class was chosen from a mixed ability mathematics class of an inner London comprehensive school. The pupils attending the school come from a wide range of ethnic and social backgrounds. The school is one of two lower schools which both feed pupils into the same upper school at the age of 14-15. Both the lower school sites and the upper school site are physically separated by several miles. The Mathematics Department covers all three sites, although some teachers teach predominantly on one site. The Mathematics Department is considered to be strong in terms of the unity and working relationships between the teachers and in terms of the approach to mathematical processes within the curriculum. The pupils in this school follow a scheme of work, SMILE (appendix 4.1) in which they work either in groups or individually with very little whole class teaching.

The class was chosen because of the experience and good practice of the mathematics teacher and because of her willingness to participate in the research. At the beginning of the period of research she had had very little experience of using the computer but her classroom and classroom practice provided an ideal context in which to introduce two computers. The two computers (RML 380z machines) were placed in the corner of the classroom.

One aim of the research was to discover how the cognitive and communicative functions of pair interaction might contribute to the learning process. The pairs were chosen by the mathematics teacher to achieve effective working groups taking into account friendship patterns, complementary learning styles and personality factors. The pupils took turns to work on the computer during their "normal" mathematics lesson. There were 26 pupils in the class and mathematics was timetabled for four 55 minute lessons a week. All the pupils in the class had approximately 45 hours of "hands on" Logo programming time throughout the period of research. During the second year of the project the original teacher left the school to work on the development of curriculum materials for Logo. The class had two other mathematics teachers during the three years of the project, both of these being very supportive of the Logo work.

4.1.4 Choice of Case Study Pairs.

"Ethnographers depend on conventions of pragmatically and theoretically informed selection rather than probabilistic sampling" (Goetz & LeCompte, 1984, p. 8). Four case study pairs of pupils were chosen from the working pairs to give a spread of mathematical attainment and an equal number of boys and girls (initially 2 girl pairs, 2 boy pairs and 2 girl/boy pairs). The teachers in this school derive a "SMILE level" for each individual pupil (see appendix 4.1). The pupils in the class were ranked according to this level (at the beginning of the period of research) and the ranked list was divided into quartiles. The four pairs were chosen so that two pupils represented each quartile although these two pupils were not necessarily working partners. The aim was for comparability and translatability of generated findings. Comparability means that the characteristics of the group under study should be clearly and precisely described so that other researchers can decide in which way this group can be used as a basis for comparison with other groups. Translatability means that the categories and tools of analysis are identified so explicitly that they can be used meaningfully in other related research settings.

Table 4.1: Overview of Case Study Pupils' Involvement in Project

	Year 1	Year 2	Year 3
Pupil 1 (Sally)	√	√	√
Pupil 2 (Asim)	√	√	√
Pupil 3 (George)	√	√	√
Pupil 4 (Janet)	√	√	√
Pupil 5 (Jude)	√	√	_
Pupil 6 (Ravi)	_	_	√
Pupil 7 (Linda)	√	√	√
Pupil 8 (Shahidur)	_	√	√

The working pairs were Sally and Janet; George and Asim; Linda and Jude; Ravi and Shahidur.

It was possible to collect data on one girl pair and one boy pair throughout the three years of the research. However after two years it was decided that the boy/girl pair were not working together productively and this pair was changed to an all girl pair. Both pupils belonging to the 'lowest' attainment pair left the school after the first year of the research and another pair was chosen. Then one of this 'new' pair left at the end of the second year of research (although not the focus of this research it is interesting to note the considerable difficulties encountered in trying to follow lower attainment pupils for three years). Table 4.1 presents an overview of the years in which each pupil was part of the project:

4.1.5 The Role of the Researcher as Participant Observer

Longitudinal case studies were made of the four pairs of pupils (aged 11-14) throughout the three years of the project. The researcher acted as a participant observer within the classroom. (There were three members of the Logo Maths Project (Hoyles, Sutherland & Evans, 1985) team, although only one acted as a participant observer at any one time. Within the rest of this thesis the use of the word "researcher" could refer to any one of these three researchers). A participant observer "directly observes, and to some extent takes part in the everyday life in a chosen setting" (Atkinson, 1979). The researcher was responsible for the pupils' learning of Logo and for making notes whilst in the classroom. The researcher was welcomed in the classroom and was free to interact with pupils during their "normal" mathematical activity. The class teacher adopted a role of working individually with the pupils by circulating around the classroom. The teacher either initiated the interaction with an individual pupil or was requested by the pupil raising his or her hand. There was a general rule within this classroom that the pupils did not ask the teacher for help until they had asked at least one other pupil. At any one time several pupils could have their hands raised requesting help and the researcher would often choose to offer help to these pupils. This was primarily so that she could become as familiar as possible with the mathematics scheme, but also so that she could obtain a general overview of the case study pupils "normal" mathematics work. In order that these pupils were not noticeably being singled out she would offer help and talk to all the members of the class. She wanted her presence to be accepted by the class. They seemed to assume that she had some kind of teaching/advisory role and did not appear to be surprised by this 'extra' person's presence in the classroom. This was possibly because the pupils were very used to visitors and student teachers in their classroom.

At the beginning of the research the overriding strategy for intervention was to leave the control for learning with the pupil in order to build up autonomy and reduce teacher-dependence. Strategies of intervention found to achieve this were :

- suggestions which were process rather than goal directed

- comments or follow-up questions which pushed responsibility back to the pupils

It was recognised however that the nature of the interventions might change as the research progressed.

4.2 OVERVIEW OF DATA COLLECTED

4.2.1 Classroom Transcript Data

Once the case study pupils had been chosen a video recording was made of each of their Logo sessions (by connecting the video recorder between the computer and the monitor). In addition both pupils wore a microphone connected to the video recorder. This meant that all the pupils' spoken language and the output from the computer was recorded. All the video recordings were transcribed and these formed the basis of the research data. They made it possible for the researcher to be able to move away from the computer and observe from a distance knowing that pupils' spoken language together with the computer commands were being recorded. The video recordings were supported by the following additional classroom data:

- hard copies of procedures written and graphical output
- pupils' written notes
- researcher's notes of each Logo session.

4.2.2 Pupil Profiles

The following data was collected in order to build up a pupil profile of each individual case study pupil:

- structured interview with case study pupils at the end of each academic year (appendix 4.2a)
- structured interview data from mathematics teacher (appendix 4.2b)
- written report from form tutor (appendix 4.2c)
- record of all school mathematics work carried out throughout the

three years of the project. (appendix 4.4)

In addition discussions with the mathematics teacher about the case study pupils were ongoing throughout the period of research.

4.2.3 Function Machine Data

Research evidence suggested that the case study pupils might not be able to relate the algebra ideas developed through their Logo programming to the 'paper and pencil' algebra context. Consequently materials were designed specifically to provoke this link. The pupils engaged in these materials towards the end of the research project. The results and analysis of the pupils' engagement in these materials is presented in chapter 6.

4.2.4 Individual Laboratory Tasks

At the end of the period of research all the case study pupils visited the University laboratory for one day in order to carry out individually a set of teacher devised Logo tasks. These tasks were both computer based programming tasks and "paper and pencil" tasks (appendix 5). They were designed to probe the individual pupils' understanding of algebra related ideas in Logo and the analysis of these tasks is presented in Section 5.7.

4.2.5 The "Arrowhead" Task

It was recognised that within the ongoing classroom transcript data it was not always possible to distinguish between the algebra related ideas which the pupils had used and understood themselves and those which were the focus of new teacher interventions. It was decided to administer a teacher devised task specifically designed to probe certain aspect of the pupils' understanding of algebra related ideas. This task (the "Arrowhead" task (appendix 3.4)) was the last Logo task within the period of research and was carried out by the pupils working in pairs in their mathematics classroom. Throughout the administration of this task teacher interventions were only made in order to keep the pupils on task. The results and analysis of this task are presented in Chapter 5.

4.2.6 Structured Interview Data

In order to probe the case study pupils' understanding of algebra related ideas in both Logo and 'paper and pencil' algebra the case study pupils were all given a structured interview (which included paper and pencil tasks) at the end of the period of research. The analysis of this interview gave new insights on the pupils' understanding of algebra related ideas which then provided another framework through which to reanalyse the transcript data. The structured interview is described in detail in Chapter 7.

4.2.7 The Comparison Group

A comparison group of pupils were chosen from a parallel class of pupils from the same school. The school used for research purposes has two lower schools for pupils aged 11-14 both feeding into the same upper school. The lower schools are on different sites and there is no contact between the pupils from the two lower schools, although they are both taught by teachers from the same Mathematics department. None of the pupils in the lower school , from which the comparison group was taken had used Logo and both the research class and the comparison class were mixed ability classes. The comparison pupils were given the "paper and pencil" algebra questions of the structured interview. They were not intended to be a control group but analysis of their results on the structured interview was used to provide an additional framework from which to analyse the responses of the case study pupils. The results from this data are presented in section 7.3.

4.3 TIMETABLE OF DATA COLLECTION

The following is a timetable of the three years of data collection:

Table 4.2: Timetable of Longitudinal Case Study Data Collection

October 1983 - March 1986:	Longitudinal classroom transcript data
June 1984, June 1985, June 1986:	Pupil profile interviews
February 1986 - April 1986:	Function Machine data
April 1986	Individual laboratory task data
May 1986:	The "Arrowhead" task data
June 1986:	Structured interview data

4.4 PHASES OF DATA COLLECTION

It is the nature of case study research that data collection and analysis are both ongoing processes. The following presents a brief summary of the phases of the research in order to show the relationship between analysis and data collection.

The Initial Phase (Oct 1983-Aug 1984) During this phase transcript data and classroom based researcher notes were collected in order to be able to reconstruct as much as possible about the research situation taking into account that "most ethnographers accept the more achievable goal of recording phenomena salient to major aspects of the topic they have defined" (Goetz & LeCompte, 1984). The researcher adopted a position of naive observer so that important aspects of classroom phenomena were not overlooked by the researcher's need to fit the classroom data to a pre-existing theory. During this phase the transcript data was continuously being analysed and the following category systems were derived from the data to provide frameworks for ongoing analysis of the data:

- categories of teacher intervention (appendix 4.2)
- categories of type of programming activity (Fig. 2.1)
- categories of pupil discourse (Sutherland & Hoyles, 1987)
- categories of variable use (section 3.3.2)

The Second Phase (Sept. 1984 - June 1986) During this phase the transcript data was examined systematically through each of the category systems developed from the data. In the light of analysis, these category systems were refined. Salient issues began to emerge and tentative hypotheses were developed. Data collection still continued at the same time as the analysis was being carried out. Analysis of the data effected the type of tasks which were presented to the pupils in the classroom and also the researcher's "way" of intervening in the learning. All the episodes of the transcript data which were related to the pupils use and understanding of algebra related ideas in Logo were taken out of the transcript data to form a sequential story. Preliminary analyses of these "stories" were carried out. Ongoing analysis influenced the tasks devised for the pupils (including the function machine tasks). In addition the pupils were given individual structured tasks when they visited the University laboratories. Additional research data was collected by carrying out structured interviews to probe the pupils understanding of algebra related ideas in both logo and "paper and pencil" algebra. Finally all the pupils in their pairs were given the "Arrows" task specifically designed to probe their understanding of algebra related ideas.

The Third Phase (July 1986- October 1987) All the case study data had been collected . Detailed analysis was carried out on: the fuction machine material; the individual laboratory day tasks; the "Arrowhead" task and the structured interview data. The preliminary hypotheses were refined and new hypotheses were devloped. The classroom transcript data was reanalysed using these hypotheses as a framework.

CHAPTER 5

THE CASE STUDY PUPILS' DEVELOPING USE AND UNDERSTANDING OF VARIABLE.

5.1 INTRODUCTION

This chapter presents the case study pupils' developing use and understanding of variable within a Logo context. It represents a final analysis of the case study transcript data. Conclusions which are drawn from this data are highlighted within the text. These conclusions are based on the analysis of the whole of the longitudinal data and not just the particular sequence in which the conclusion is made. The sequences are presented in detail and the analysis is given in a temporal form so that the pupils' development is kept in perspective. The sessions have been numbered so that the reader is aware of the intervening sessions, which were not included, in which the pupils did not use the idea of variable. Not all sessions are reported in equal detail. The detail is included only when it is critical from the point of view of the pupils' developing understanding. The reader may wish to refer to section 4.3 to locate the classroom data collection sessions within the overall perspective of data collected for the project. At the beginning of each episode the type of project is classified according to the dimensions:

Loosely defined...............Well defined
Real world......................Abstract

These categories are discussed in section 2.2.1. At the beginning of each episode the pupils' use of variable is classified according to the categories outlined in section 3.3.2.

Section 5.6 presents an overview of the "Arrowhead" task which was presented to all the case study pairs at the end of the period of research. The differing approaches of each pair will discussed from the perspective of the pupils' developing understanding of variable. Section 5.7 presents the results and discussion of the individual laboratory tasks which were given to each case study pupil at the end of the period of research. Finally in section 5.8 all the data is synthesised and an overview is presented of each individual pupils' development over the three year longitudinal study.

5.2 LONGITUDINAL CASE STUDY: SALLY AND JANET

Sally is an exceptionally shy girl. She is possibly very able, but her inability to articulate her ideas makes it difficult for her mathematics teacher to "get in touch" with her true potential. Nevertheless she reached a high level of attainment in mathematics throughout the project. Sally is certainly lacking in confidence and often during her Logo programming makes comments like *"it won't work"*. She enjoys mathematics but is not someone who shows her feelings and her mathematics teacher says of her *"she always works sensibly and quietly but without any apparent enthusiasm or self motivation - perhaps just a reflection of her very quiet personality."* Janet on the other hand is a very chatty and sociable girl. Her attainment in the class is average although her teacher says of her *" I would like to think that using Logo has helped her in the sense that she is quite a bubbly personality and it has given her a vent for her being able to be herself and have ideas in a mathematical context, which is not how she viewed doing mathematics would be"*. Sally and Janet worked together throughout the three years of the project. Initially there was not much spoken language from Sally during the sessions but it was discovered that she talked more if we moved away from the computer.

5.2.1 General Polygon

Year & Session No:	Year 1; Session 14 & 15
Type of goal:	Loosely defined abstract
Category of variable use:	(O) Variable operated on within a procedure

These two sessions are included in detail because they were the first sessions in which Sally and Janet were introduced to the idea of variable in Logo programming. They also illustrate a session in which the "teacher given" Logo formalism did not match the pupils' generalised method.

Sally and Janet through discussion between themselves and feedback from the computer negotiated the relationship between the number of sides and the turtle turn for a regular hexagon. After negotiating the relationship in direct mode they defined a procedure. They then continued with this process for a regular pentagon, octagon and a ten sided polygon. Their polygon procedures were non-state transparent and were all of the form of the HEX procedure given in Fig. 5.1a. Rotated shapes were produced for their regular polygon shapes by defining procedures of the form HEXHOUSE (Fig. 5.1b). The non-state transparent nature of the initial module (HEX) was crucial for the production of the rotated pattern (HEXHOUSE) and there was evidence that both Sally and Janet knew this. This was however the cause of the eventual mismatch between the pupils' solution and the "teacher given" Logo formalism. Sally and Janet produced these patterns by using a strategy of trying out the commands for the regular polygon module in direct drive before defining a procedure. **There is clear evidence that this negotatiation in direct drive mode together with their discussion was crucial in helping them develop an understanding of the relationship between the number of sides and the angle turned for a regular polygon.**

Throughout the session the researcher (denoted "Res." in the transcript text) nudged the pupils to reflect on the relationship as illustrated by the following extract:

Janet *"We divided 360 by 10."*

Res. *"And why do you divide 360 by 10?"*

Janet *"Cos a circle's 360."*

At the beginning of the next session the researcher intervened to nudge Sally and Janet into trying out other specific cases by asking them to draw an eleven sided regular polygon. The following discussion illustrates that they were still coming to terms with the relationship:

Janet *"That will be 36 won't it.....no..."*

Sally *"No this has got..."*

Janet *"Eleven into 36..."*

Sally *"It's 32.8...."*

Janet *"Why don't we do the 12...it won't have a point...12 into 36 goes three times.....so the angle is three...'*

They tried:

REPEAT 11 [FD 20 LT 3]

which was consistent with their strategy of defining regular polygons although it reflected Janet's incorrect calculation of a turtle turn of 3. The computer response indicated that this was not what they had predicted and they immediately dropped down a level and tried out the individual commands:

FD 20 LT 3 Fd 20 LT 3............

Again the computer response prompted Janet to say:

Janet *"No stop this is stupid....it can't be 3....."*

Sally *"It can be 3..."*

Janet *"12 into 360 ...it is 30..."*

They again tried out this idea in direct drive without using the REPEAT command and before the 12 sided shape was completed they typed in:

REPEAT 11 [FD 15 LT 30]

The computer response provoked Janet to say:

Janet *" It should be 12 times."*

Sally *"12 times will take it back to there again won't it....I know what I'm doing now..."*

They typed in an extra FD 15 and then confidently defined:

```
TO TWE
REPEAT 11 [FD 15 LT 30]
FD 15
END
```

They then defined the accompanying TWEHOUSE.

They had avoided defining an eleven sided regular polygon and so the researcher intervened to show them how to use the computer to calculate the turning angle of an eleven sided regular polygon by typing PRINT DIV 360 11. This gave 32.73 and in direct mode they tried out FD 20 LT 32.73 and then cleared the screen and tried out:

```
REPEAT 10 [FD 20  LT 32.73]
FD 20
```

They then defined the procedure ELE which had the same structure as all their other regular polygon procedures. They used a similar strategy to define a seven sided regular polygon procedure. In order to nudge Sally and Janet into reflecting on the general relationship within their polygon procedures the researcher then asked them how they would define a 9 sided regular polygon:

Sally *"Umm to get the angle you divide it by 360."*

Janet *"Miss if you want to repeat it you always do it one less....or else it will go back to there and if you want to do one of these patterns ...it will always be repeating itself ...it won't do that......"*

The researcher decided to intervene to tell Sally and Janet how to use variable to define a general polygon procedure. She had not however adequately observed the structure of the pupils' polygon procedures. She was preoccupied by her own solution to the problem as the following interchange illustrates. She first of all typed into the computer:

```
POLYGON "NUMBER
```

and then said:

Res. *"How would you make a five sided figure.....REPEAT 5...FORWARD whatever you want...and how would you get the LEFT bit...."*

The researcher was focussing on the angle turned and had not observed that Sally and Janet consistently used REPEAT N-1 for an N sided polygon. Janet was able to offer an explanation of how to get the turning angle:

Janet *"Divide it by 360 miss "*

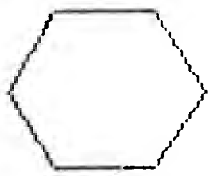
Res. *"So what we're going to do is we're going to write a program called POLYGON...which will do a shape for any number of sides."*

The researcher continued without observing the mismatch between her solution and the pupils' solution and typed:

```
TO POLYGON "NUMBER
REPEAT :NUMBER [FD 20 LT DIV 360 :NUMBER ]
END
```

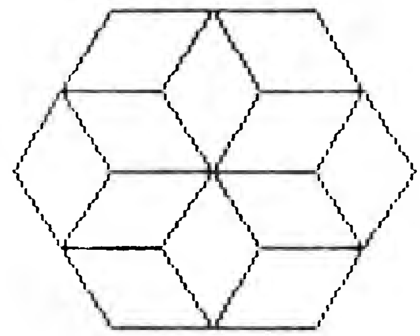
As the researcher worked through the program she continuously asked the pupils to reflect on the structure of a procedure for a 5 sided regular polygon Her constant empahsis was on the angle turned:

a)



```
TO HEX
REPEAT 5 [FD 40 LT 60]
FD 40
END
```

b)



```
TO HEXHOUSE
REPEAT 6 [HEX]
END
```

c)

```
TO POLYGON "NUMBER
REPEAT :NUMBER [FD 20 LT  DIV 360 :NUMBER]
END
```

Fig. 5.1: Sally and Janet - General Polygon

Res. *"If this was 5 ...this would be 360 divided by 5 wouldn't it....if that was 10....that would be what..."*

She then showed the pupils how POLYGON 9 would produce a 9 sided regular polygon. Finally she elaborated on this use of variable. This **intervention was crucial because it "signalled" for the pupils the possibility of using more than one variable input in a procedure.** Sally and Janet did in fact take up this idea in the subsequent session.

Res. *"So you see you can use the computer and it saves you a lot of work....another thing that you can do later...you can have more than one input......so if you wanted the possibility of changing that say (pointing to the side length)...you could call that something else....and as long as you had its name up here....and then as well as the number of sides...you'd have one number for the number of sides....and after that you'd put the length..."*

Also during this intervention the researcher explicitly told the pupils to use a meaningful variable name. **Later analysis of the data indicated that Sally, in particular, had attached too much significance to the meaningful variable name.** After defining the general polygon procedure the researcher told Sally and Janet to try out different inputs to their general polygon procedure. She also suggested they define a general polygonhouse procedure. Their subsequent discussion indicated that they were confused about the structure of the "teacher defined" general polygon procedure and felt that in some way it was not the same as their fixed polygon procedures. They were not however able to make this difference explicit, and were therefore not able to communicate their confusion to the researcher/teacher.

Janet *"I just want to see how she did this one.... besides I want to change it...oh no forget it...*

Janet did not have the confidence to attempt to modify the procedure. Instead she entered into the "didactical contract" of defining a general polygonhouse procedure.

```
TO POLYGONHOUSE "NUMBER
REPEAT :NUMBER
END
```

Janet *" I bet you it don't work..."*

Sally *" I dunno...I dunno if it will work..."*

They tried POLYGONHOUSE 3 which (because of the syntax error in the REPEAT command) produced an error message. They were confused and said:

Sally *"Let's change POLYGON ...."*

At this stage Janet's level of motivation was low and it is suggested that this is because of the mismatch between the pupils negotiated solution and the "teacher given" formalism.

Janet *"No we can't she's set it for us..I wan't to do a face"*

They looked at the POLYGON procedure and Janet said:

Janet *"I think we should change it somehow...I don't get it though you know....I still don't get it though you know...I hope they save our programs you know.....TWE...TWEHOUSE.....I liked that..."*

Despite this alienation they continued to accept the "didactical contract" of working with the general polygon procedure and typed in a number of inputs to POLYGON before the end of the session.

The computer feedback from their "hands on" work in direct mode and their discussion enabled Sally and Janet to negotatiate a general relationship within a regular polygon. They almost certainly did not fully understand this before the beginning of the session. In addition the researcher had nudged Sally and Janet into considering more specific cases in order to help them develop their understanding of the general relationship. The fact that the teacher-given Logo formalism did not match the pupils' general method caused the pupils to be alienated from the task and was detrimental to the pupils' learning about the "power" of Logo to represent a general relationship. **However within this session both Sally and Janet do appear to have understood that a variable name can be used as a place holder for "any number". However Janet's**

resistance to using decimals indicates that for her the idea of "any number' was restricted to positive whole numbers.

5.2.2 Clown's Face

Year & Session No:	Year 1; Session 16
Type of Goal:	Loosely defined abstract
Cateory of variable use:	(N) More than one variable input to procedure

Within this session Sally and Janet chose to use their general polygon procedure (Fig. 5.1c) as a tool in the construction of a clown's face. This session is important because it illustrates how Sally and Janet were able to take on some of the ideas introduced to them in the previous session.

Sally *"We want a round face...what about ...have we still got the POLYGON in there...."*

Janet *"Yeah"*

Sally *"How do we put it in?"*

Janet *"POLYGON 13"*

This interchange illustrates Sally's reliance on Janet for the details of the Logo syntax. This reliance persisted throughout the project. Using the command POLYGON 13 (see Fig. 5.1c for the POLYGON procedure) in direct mode they drew the outline of the face. They then moved the turtle into the correct position for the nose. At this point Sally again initiated the idea of using the POLYGON procedure to draw the nose, suggesting that they change the FD 20 in the procedure to a smaller amount.

Sally *"Miss.....do we always have to do 20....."*

Janet by elaborating on this idea indicated that she has also understood the idea:

Janet *"Miss you know the POLYGON.....the one we did....could we just change it so you leave a space....so whenever we want to we could put something in miss...?*

It seems that Sally and Janet had taken on the **idea of using a variable as a place holder for a general number. Janet's language also indicated a top down approach in her thinking (i.e. don't make the decision about the specific number until later).** She had taken on the idea but needed teacher support on the Logo syntax in order to define the following procedure:

```
TO POLYGON "NUMBER "LENGTH
REPEAT :NUMBER [FD :LENGTH LT DIV 360 :NUMBER]
END
```

Fig. 5.2: Sally and Janet - Clown's Face

They then used this procedure POLYGON 20 3 to draw the nose and continued to work in direct drive to complete the clown's face (Fig. 5.2). When they defined the procedure for the Clown's face they took account of their modified polygon procedure and typed in POLYGON 13 20 (as oppposed to the original POLYGON 13). This indicated an awareness of the structure and the associated syntax of their modified POLYGON procedure.

5.2.3 Starbuster

Year & Session No:	Year 2; Session 2
Type of goal:	Loosely defined abstract
Category of variable use:	(I) One variable input

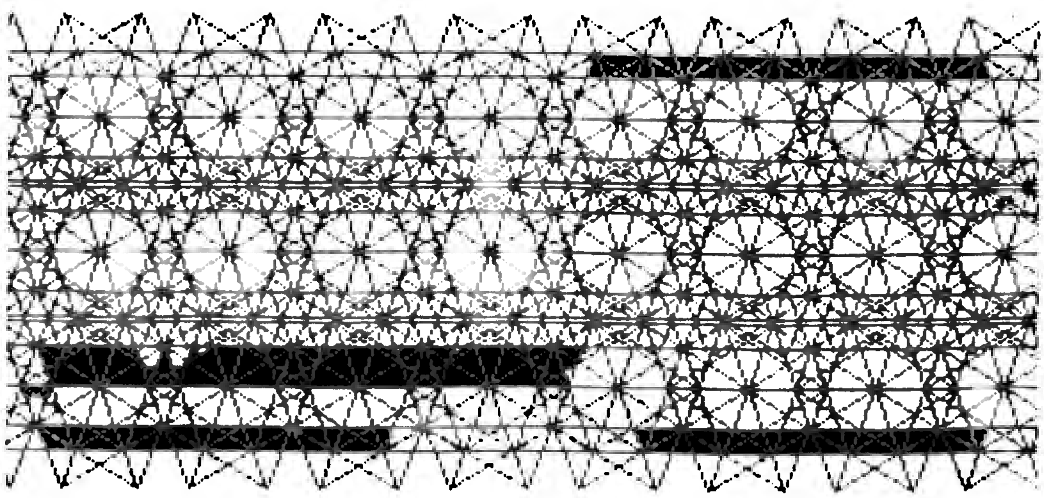
At the beginning of the second year of the project Sally and Janet were working on their own project.Within a loosely defined activity they were building up star patterns on the screen. They had built up a module SDS (Fig. 5.3) and they used this module to define:

```
TO SDDS              and          TO SDDDS
REPEAT 2 [SDS]                    REPEAT 3 [SDS]
END                               END
```

The researcher intervened to suggest that they used a variable input. At this stage neither Sally or Janet were able to articulate any of their previous use of variable. The researcher showed them how to define a general module SDNS (Fig 5.3) and Janet accepted this use of variable:

Janet *"So miss you put SDNS...say you want it 3 times...you put 3..."*

They then used this general procedure as part of a fixed superprocedure STARBUSTER (Fig 5.3).



```
TO STARBUSTER
MOVE
SDNS 9
MOVE2
SDNS 9
MOVE3
SDNS 9
END
```

```
TO SDNS "N
REPEAT :N [SDS]
END

TO SDS
SS
SFS
END
```

```
TO SFS
REPEAT 5 [ FS]

TO SS
REPEAT 5 [STAR]
END

TO STAR
REPEAT 4 [FD 40 LT 144]
END

TO FS
REPEAT 5 [40 RT 144]
END
```

Fig. 5.3: Sally and Janet - Starbuster

5.2.4 General Hexagon

Year & Session No: Year 2; Session 5

Type of goal: Well defined abstract

Category of variable use: (I) One variable input

This session illustrates the author's "hidden agenda" of variable leading to an inappropriate intervention. Sally and Janet were working on a pattern of tessellated hexagons and they were introduced to the following general hexagon module.

```
TO HEX "TWA
REPEAT 6 [FD :TWA  RT 60]
END
```

They were not however aiming to produce a general tessellated hexagon pattern and they only used the general hexagon with one input. Consequently at this stage a general hexagon procedure was an inappropriate tool. **The motivation level was low during the session, because of the researcher's inappropriate intervention, and Sally and Janet were not able to relate this module to their previously defined general polygon procedure** (Fig. 5.1c).

5.2.5 Variable Letters

Year & session No:	Year 2; Session 6 & 7
Type of goal:	Well defined abstract
Category of variable Use:	(S) Variable as scale factor
	(G) General superprocedure

By this stage in the research the author had decided that attempting to introduce variable to pupils within the context of their own goals often led to inappropriate suggestions to use variable to solve problems which from the pupil perspective did not need variable. She decided that it would be better if the "hidden agenda" was made explicit. The author therefore devised the "Scaling Letter" task (appendix 3.2) to be given to all of the case study pupils. At the beginning of the session Sally and Janet were given the "Scaling letter" handout (Fig. 5.4).

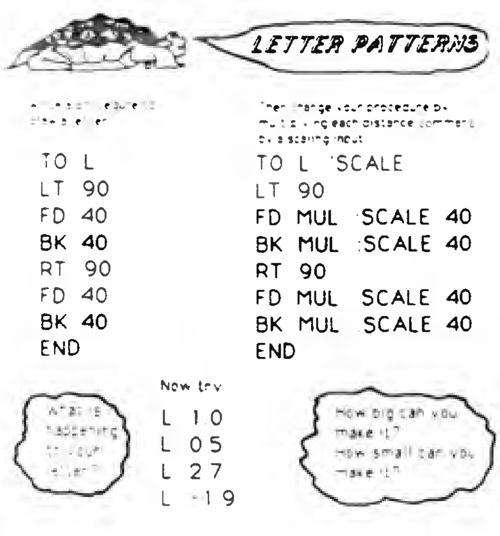


Fig. 5.4: The "Scaling Letter" task

They copied into the editor the procedure L and then modified the L procedure by scaling all the distance commands as instructed on the sheet. They tried out the variable L procedure with the inputs 1.3, 2, 0.5, 0.25, 0.1, 7.2 and 0.001. **This task had provoked them, for the first time, to use a range of decimal input.** (This was also the case for the other case study pupils).

Sally and Janet then decided that they would work towards the goal of producing the word L O N G. Their way of working on this project was typical of their way of interacting with the computer and will be described in detail. They moved the turtle to the left hand side of the screen, kept a record of their commands and then typed in L 1. The author suggests that Sally had a well worked out top down strategy for solving the problem although it was never made explicit. After the computer had drawn the L, Sally suggested:

Sally *"Make a move...and then we could use MOVE each time.."*

Joanne however was not so certain about the modularity and said

Joanne *"No 'cos it won't be the same distance......we can use separate moves between each procedures.....call them LO ON NG".*

Sally did not disagree with this idea and in direct drive they worked out the commands for the move between the L and the O, keeping a written record. Then in direct drive they drew a "square" O, again keeping a record of their commands. Sally again suggested that each move procedure could be the same if they were making each letter procedure the same width. **The "hands on" experience of interacting with the computer had now convinced Janet of the modularity of the move between each letter. Perceiving the "move commands" as a module turned out to be important at a later stage, because they eventually decided to make this into a general module.** At this stage they defined a MOVE procedure then defined a variable "scaled" O procedure from their written record. (They now had in the editor, an interfacing procedure (MOVE), a general L procedure (L), and a general O procedure (O)). They used the MOVE procedure to put the turtle in the correct position to draw the next letter and in direct drive drew the letter N. They used a systematic trial and error approach to obtain a "reasonable" length line for the diagonal of the N. In direct drive they had written down the following list of commands for the N:

```
LT 90
FD 40
LT 45
BK 57
RT 45
Fd 40
BK 40
PU
RT 90
BK 40
```

In their first introduction to the idea of scaling they had been given the L task (Fig 5.4) and in this L procedure all the distance commands which had been scaled were of length 40. It appeared that Sally and Janet took this as being of significance, interpreting the sheet as saying that it was only distances of length 40 which needed to be scaled. When they first defined a variable N procedure they only scaled the FD and BK 40 commands leaving untouched the BK 57 command. **This tendency to spuriously generalise from the given worksheet consistently recurred throughout the research.** The researcher intervened to correct their misunderstanding. It was now the end of the session. At the beginning of their next session they realised that they had not defined a procedure to place the turtle in the correct "start" position and immediately defined this (STEP, Fig. 5.5) from their written record. They then proceeded to enter:

```
STEP
L 1
MOVE
O 1
MOVE
N 1
MOVE
```

At this stage they decided to put all these commands into a superprocedure called LONG. In direct mode they then worked on the G and defined the general G procedure in the editor with reference to their written record. This time they scaled all the FD commands. When this fixed superprocedure LONG had been defined the researcher nudged them into making a general superprocedure:

Res *"Have you tried it with ...different sized letters...."*

Janet's reply indicated a good understanding of the processes which they had used in order to define their fixed superprocedure:

Janet *"Oh Miss....'cos what we did was....miss we put it on one....we didn't put it on scale..."*

Res. *"Now what you can do....you can make LONG with an input so that you can change the size..."*

The researcher showed them how to define:

```
TO LONG "SCALE
STEP
L :SCALE
MOVE
O :SCALE
MOVE
N :SCALE
MOVE
G :SCALE
END
```

They tried out LONG .5, which drew small letters with large gaps in between them, and the computer response provoked Sally to say:

Sally *"Miss however small it's going to be it's going to have the same distance apart isn't it...."*

Res. *"Is there anything you can do about it?"*

Sally *"Put it on SCALE".*

```
TO    LONG "SCALE
      STEP
      L :SCALE
      MOVE :SCALE
      O :SCALE
      MOVE :SCALE
      N :SCALE
      MOVE :SCALE
      END
```

```
TO N "SCALE
   LT 90
   FD MUL :SCALE 40
   LT 45
   BK MUL :SCALE 57
   RT 45
   FD MUL :SCALE 40
   BK MUL :SCALE 40
   LIFT
   RT 90
   BK MUL :SCALE 40
   PD
   END
```

```
TO STEP
   LIFT
   BK 150
   PD
   END
```

```
TO MOVE "SCALE
   LIFT
   FD MUL :SCALE 50
   PD
   END
```

```
TO L "SCALE
   LT 90
   FD MUL :SCALE 40
   BK MUL :SCALE 40
   RT 90
   FD MUL :SCALE 40
   BK MUL :SCALE 40
   END
```

```
TO O "SCALE
   FD MUL :SCALE 40
   RT 90
   BK MUL :SCALE 40
   RT 90
   FD MUL :SCALE 40
   RT 90
   BK MUL :SCALE 40
   RT 90
   END
```

```
TO G "SCALE
   RT 90
   BK MUL :SCALE 40
   RT 90
   BK MUL :SCALE 40
   FD MUL :SCALE 40
   RT 90
   FD MUL :SCALE 40
   RT 90
   BK MUL :SCALE 20
   RT 90
   FD MUL :SCALE 20
   LIFT
   FD MUL :SCALE 20
   RT 90
   BK MUL :SCALE 20
   RT 90
   PD
   END
```

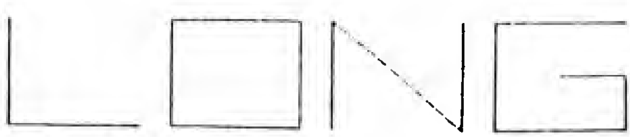


Fig. 5.5: Sally and Janet - L O N G

They immediately edited the MOVE procedure to scale all the distance commands confidently coping with the syntax. They did not however modify the calls of MOVE in the LONG procedure. When they typed in LONG .25 the following error message appeared:

> "LOGO CAN'T DO MOVE AT LEVEL 1 IN THIS LINE OF LONG BECAUSE THERE'S NO INPUT FOR MOVE."

Without any intervention from the researcher they negotiated the meaning of this error message:

Janet *"What does it mean...there's no input for MOVE..."*

Sally *"What does...has LONG got MOVE in...*

Janet *"Has LONG got MOVE in...yeah it has..."*

They looked at their LONG procedure in the editor and Janet edited all the MOVE calls to MOVE :SCALE (see Fig. 5.5). This seems to indicate at least an understanding of the necessary surface syntax. They tried out LONG 0.1 and discussed whether or not they should make their STEP procedure variable.

Janet *"Should we MUL that as well..no that wouldn't work would it...unless we got rid of STEP altogether...make another program called LONG2...and don't put STEP in it..."*

They did not in fact take up this idea. Within this session using a teacher devised task as a starting point Sally and Janet had extended the task to one of their own. This was associated with a high level of motivation. The researcher had nudged the pupils towards the idea of defining a general superprocedure. **However it was the computer feedback, unexpectedly producing small letters with large gaps, which provoked them into defining a general interface procedure.**

5.2.6 General Flower

Year & Session No:	Year 2; Session 8
Type of goal:	Loosely defined real world
Category of variable use:	(O) Variable operated on.

This session was important because Sally and Janet returned for the first time since their first session of variable use to the idea of making a relationship explicit by operating on a variable within a procedure. They had been asked to make a picture of different sized flowers. Sally immediately initiated the idea of defining a general procedure.

Sally *"If we can find some way of making it bigger or smaller...it'll save us doing all sorts of flowers wouldn't it..."*

Janet suggested starting in direct mode and drawing a specific sized **module. This continued to be their "normal" strategy** when defining **general procedures.**

Janet *"Yeah I know so first of all design a flower first"*

In direct drive they spent a considerable time negotiating the detail of their specific flower producing the following commands:

```
ARCR 10 360
ARCR 10 360
ARCR 10 90
ARCL 10 270
ARCR 10 360
ARCR 10 45
ARCL 5 360
CT
ARCR 10 90
FD 40
CT
ARCL 10 360
```

During these negotiations the researcher who until this point had not been observing their work said:

Res. *"Will you later think about how you can make that into a bigger flower...using a variable input..."*

Sally appears to be suggesting the use of variable as scale factor.

Sally *"Miss could you do sort of ARCR...MUL...ummm....."*

Res. *"Yeah or you might not want the MUL....which one makes it different sizes..."*

Sally *"The ten"*

The researcher suggested that they use one variable input with the idea that they would operate on this variable when appropriate.

Res. *"So you might want to do is just put in a name for the 10...you know you've done it lots of times now...you've done it with polygons haven't you....what does that 10 number in the ARCR stand for?*

Sally *"The radius"*

Janet *"The radius"*

Res. *"Well you could call it RADIUS then if you like....when you come to that and you need help ask me..."*

The researcher then left Sally and Janet to finish their flower. When she was out of earshot Sally said

Sally *"We gotta think about making a flower first...let alone making it bigger or smaller..."*

indicating that they needed to negotiate the details of their specific flower before they wanted to "take on" the idea of making it general. When they had finally finished the flower in direct mode they did accept the "didactical contract" of defining a general procedure. Sally started to define the fixed procedure:

```
TO FLO
ARCR 10 360
```

when Janet said

Janet *"Remember what miss said.....*

Janet appeared to be confident about using "half formed ideas". Sally on the other hand, although she often appeared to have a better understanding of a generalisation, was reluctant to use the Logo language to represent this generalisation. She said:

Sally *"We can't do...how do we do this the other way "*

Sally had used variable both as one input operated on within a procedure and also as a scale factor. It is suggested that she had developed two frames for these two ways of using variable. Her remark of *"how do we do this the other way"* seems to indicate that she has already identified two ways of defining general procedures in Logo. Janet said:

Janet *"Remember ARCR RADIUS"*

Sally *"Yeah but what is RADIUS.....they won't know what RADIUS means...just do it like this first it's easier..."*

It is difficult to interpret Sally's remark but it could be that she was confused about the use of the meaningful variable name RADIUS. This name had a meaning for her in mathematics, but she did not understand how it could have a meaning for the computer. **She did not, at this stage, understand that the name RADIUS was just a specific instance of a general set of names.** Sally again suggested that they define a fixed flower procedure first but Janet said:

Janet *"No 'cos then we'll only have to change it..."*

Janet often seemed to be motivated by the need for "economy of action". Sally agreed to define a general flower procedure and they then asked the researcher for help.

Janet *"Miss but it doesn't know what RADIUS is..."*

The researcher did not pick up on this confusion over the variable name. She asked them if all the radius inputs to ARCR were the same and they told her that the middle one was different. She then asked how the 5 was related to the 10 and they both replied *"It's half"* They replaced all the inputs of size 10 in the ARCR and ARCL commands by the word RADIUS. When they came to the command for the inner small circle their discussion indicated an understanding of the relationship of "dividing by two" but an uncertainty about how to use the Logo syntax to make the relationship explicit.

Janet *"No this is DIV."*

Sally *"How do we do that?"*

Janet *"It's RAD"*

Sally *"No"*

Janet *"DIV*

After asking for help with this command they finished defining the flower and tried out the general procedure. This produced a varying sized flower head with a fixed sized stem. They then decided that they wanted the length of the stem to be related to the size of the flower. **This is an example of the computer response provoking them into making a relationship explicit within their procedure.** Sally suggested:

Sally "Do that MUL business"

They changed the FD 40 (for the stem of the flower) in their procedure to FD MUL :SCALE 40. This use of "variable as scale factor" indicates a confusion between their two known ways of defining general procedures. They were asked if they wanted the length of 40 to be in any way related to the 10 in the radius of the circle. This provoked Sally to say:

Sally "Oh times it by four....

They were then helped to define the procedure FLO given in fig 5.6.

a)

```
TO FLO "RAD
ARCR :RAD 360
ARCR :RAD 360
ARCR :RAD 90
ARCL :RAD 360
ARCL :RAD 270
ARCR :RAD 360
ARCR :RAD 45
ARCL DIV :RAD 2 360
CT
ARCR :RAD 90
FD MUL :RAD 4
CT
ARCL :RAD 360
END
```

b)

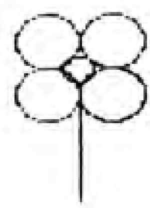


Fig. 5.6: Sally and Janet - A General Flower

This session highlights the conflict between both Sally and Janet's "(S) variable as scale factor" frame and "(O) variable operated on" frame. This conflict arises again in the next session in which it begins to be resolved.

5.2.7 Patterns of Squares (the length of this session was approximately three hours.)

Year & Session No:	Year 2; Session 11
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor
	(I) One variable input

This session is crucial in that after a series of nudges from the researcher Sally and Janet started to integrate their "variable as scale factor" and "one variable input" frames. All the case study pupils, in pairs, were carrying out some tasks at the University. One pair was given a handout (appendix 5.1) on which they were told that they were going to engage in a game. The object of the game was for one pair to define a procedure for a given shape and then use this procedure as a message to the other pair who then had to guess which picture it represented. They were told that they would be given the pictures at random but that they would have to write procedures for all the shapes (to push them into recognising the modular nature of the shapes). It is not clear however that they took on the game as "write a procedure so that the other pair can guess the shape but rather that they took it on as "write a procedure so that the other pair does not guess the shape". They started with Fig. 5.7a, negotiating the idea of defining a variable procedure for a square:

Janet *"Alright shall we start..I've got ito make a square...."*

Sally *"mmm"*

Janet *"And make one of them programs where you......"*

Sally *"MUL SCALE".*

Janet *"Yeah that's it...come on..."*

After this negotiation they moved the turtle into their desired starting position and after keeping a record of their commands defined a startup procedure (M1). At this point Janet said *"NO don't put M1..cos then they're going to realise it's move"* which seemed to indicate that she thought that the object of the game was that the other pair should NOT guess the shape from the procedure. At this point they negotiated the sizes of each square.

Janet *"Do it 40 30 20 10...*

Sally *"No wait a minute...20 15 10 5..."*

Sally *"Well shall we....yeah we should do this MUL business...'cos they won't understand will they.."*

Sally's question seems to refer to the idea of using variable as opposed to defining a fixed procedure and does not at this stage appear to be discriminating about which category of variable she plans to use. In direct mode they drew a square with sides of length 30 and then started to define:

```
TO SQU "SCALE
FD MUL :SCALE 30
RT 90
```

At this point it seems that Sally was still not convinced about using a general square module or perhaps she was more concerned with the state of the turtle.

Sally *"You do REPEAT 4 FORWARD 30, RIGHT 90...and that will bring it back to*
the *beginning..like that then we just have to move it up......do a square we'll have to move it forward about 5...and then do another*

square..then move it forward..and do another square..get me.."

Sally wanted to use REPEAT to make the square state transparent. Janet however suggested the idea of defining a general procedure

Janet *"But if we do this we can change it...we just put in a number and it will do that bit instead of drawing it all out..."*

Sally *"Just do REPEAT all the time".*

Janet *"Oh I get you but they'll understand...we're trying to make it so they don't understand..."*

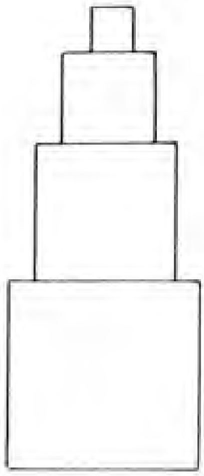
Sally *"But we can't do it the other way...dunno how."*

Janet *"Don't we only have to do one program..."*

Sally *"Alright..."*

The negotiation indicates a reluctance on Sally's part to use an idea with which she is not completely familiar.It seems that she possibly wanted to use the REPEAT command but was not sure how to do this in the context of using "variable as a scale factor". The issue was also confounded by them not wanting the other pair of pupils to "guess" their procedure. Despite Sally's lack of confidence they were able to define the general procedure BOX in Fig . 5.7b without any teacher intervention.

a)



b)

```
TO BOX "SCALE
FD MUL :SCALE 30
RT 90
FD MUL :SCALE 30
RT 90
FD MUL :SCALE 30
RT 90
FD MUL :SCALE 30
RT 90
END
```

c)

```
TO BOXS
MO1
BOX 1
MO2
BOX .75
MO3
BOX .5
MO4
BOX .25
END
```

```
TO MO1
PU
LT 90
BK 53
LT 90
PD
END
```

```
TO MO2
CT
PU
BK 25
PD
END
```

```
TO MO3
PU
RT 90
BK 15
RT 90
BK 5
RT 180
PD
END
```

```
TO MO4
PU
RT 90
BK 7
LT 90
FD 4.5
END
```

Fig. 5.7: Sally and Janet - Pattern of Squares (1)

They tried out BOX with an input of 30 (which drew a square of side length 90). This choice of input gives an insight into a possible conflict between "(S) variable as scale factor" and "(I) one variable input"

Janet *"It's got to be smaller...it's got to be something like .3"*

After trying BOX 3 and then BOX .3 Janet said

Janet *"It's not 3 and it's not .3 and its not 30....what is it...'*

Sally *"BOX 1...the normal way...."*

Janet *"Alright we'll do it the normal way then....*

Analysis of the transcripts indicates that their use of the term "the nomal way" refers to the original fixed shape drawn. They typed in BOX 1 and were satisfied with the effect. After much negotiation they ended up with the fixed superprocedure BOXS (Fig. 5.7c). They had used inputs of 1, 0.75, 0.5 and 0.25 to BOX. This gave them square side lengths of 30, 22.5, 15 and 7.5 instead of the 30, 20, 10, 5 which they had initially planned. Using variable as a scale factor had made the interface commands very difficult.

They were next asked to write a procedure to draw Fig. 5.8a. Because the "guessing" pair had taken so long to guess their first procedure BOXS Sally suggested:

Sally *"Just do 'em all joining sort of thing....I don't reckon we should do this MUL business..we'll just do it joining on to..*

Janet *"Right go on we'll let them have it easy..."*

They defined the procedure THISISGOOD (Fig. 5.7b) without using variable. The researcher intervened to ask them to do this figure again using their variable square module. Without any difficulty they built up the shape in direct drive and then defined a superprocedure EASY (Fig 5.8c).

The researcher asked them which solution they had found easier and Sally said:

Sally *"That one's quicker...but if you're working it out you can have the boxes...that's an easy pattern it just follows on anyway...".*

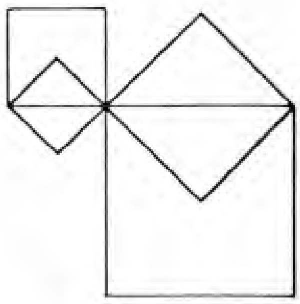
With the aim of helping Sally and Janet to integrate their two frames the researcher asked them to define a square procedure without scaling the distance commands. They defined:

```
TO SQUARE "SIDE
REPEAT 4 [ FD :SIDE RT 90]
END
```

When asked the difference between their BOX and their SQUARE procedure Janet said:

Janet *"Umm.....the SCALE is....you have to...it seems harder 'cos instead of putting in the actual number how long you want it to be...you have to put it to SCALE...so it's a bit harder working out what you actually want..."*

a)



b)

```
TO THISISGOOD
STAGE1
STAGE2
STAGE3
END
TO STAGE1
REPEAT 3 [BK 20 RT 90]
END

TO STAGE2
BK 60
RT 90
FD 40
RT 90
BK 40
RT 90
FD 40
END

TO STAGE3
LT 45
FD 10
BK 40
RT 90
BK 30
LT 90
FD 30
BK 40
RT 90
FD 10
RT 90
BK 10
CT
END
```

c)

```
TO EASY
BOX 2
LT 45
BOX 1.5
LT 135
BOX 1
LT 45
BOX .5
END
```

Fig. 5.8: Sally and Janet - Pattern of squares (2)

Sally *"If you put SCALE 30 and then you want BOX 1 it'll come out as 30...."*

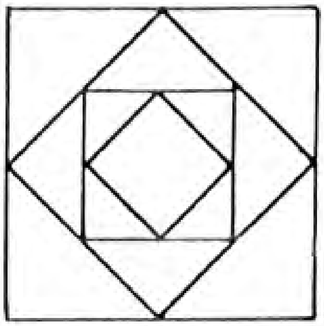
They were then asked to produce Fig. 5.9a using the SQUARE procedure. They worked in direct drive and defined DIASIDE (Fig. 5.9b). They were then asked to redo the pattern using their BOX module. Sally suggested

Sally *"If we change the SCALE to say 10...then it will be easier".*

Indicating that she was thinking through the process of the effect of the value 30 within their BOX (Variable as scale factor) procedure.

Janet *"Oh yeah.."*

a)



b)

```
TO DIASIDE
SQUARE 40
FD 20
RT 45
SQUARE 30
FD 15
RT 45
SQUARE 20
FD 10
RT 45
SQUARE 15
END
```

c)

```
TO DIAMULE
BOX 1.25
FD 20
RT 45
BOX 1
FD 15
RT 45
BOX .75
FD 10
RT 45
BOX .5
END
```

Fig. 5.9: Sally and Janet - Pattern of Squares (3)

DIASIDE and DIAMULE as far as Sally and Janet were concerned drew the same shape. They were not concerned with the discrepancy between square sizes which was reasonable because there was nothing in the way that the problem had been presented which suggested that "exact" lengths were important. The researcher asked them if they were exactly the same size and Janet said

Janet *"Round about"*

They were asked how they could be sure that they both drew exactly the same size image and they suggested comparing both images drawn on the screen. The researcher however asked them how they could tell from looking at their procedures.

Sally *"Well that's 30...and the one that's 25 is a quarter of 30...and you add those together and you get what it is..."*

Res. *"What's that?*

Sally *"Umm 7 and a half...'*

Res. *"So how long is it..."*

Sally *"37 and a half..."*

They again were asked which procedure they would prefer DIAMULE or DIASIDE

Janet *"Well I prefer that one '*(meaning DIASIDE) *'cos if you want it by 30....you just put 30 up there....you don't have to halve it...quarter it...whatever.."*

Sally *"You can see it anyway....you can see what you're doing...."*

Janet *"And you don't have to type as much as well."*

Sally *"Like err...say you get SQUARE 15....you know the sides are going to be 15......but if you get BOX 1...and you don't know what the SCALE is then you wouldn't know what it is..."*

These interventions were probably crucial in pushing them into discriminating between the use of variable in the category of "(O) one variable input" and "(S) variable as scale factor".

The Rectangle Finally at the end of this long session Sally and Janet were asked to define a procedure to draw a general rectangle. They were told that the width of the rectangle should be twice as long as the length of the rectangle. This intervention was crucial in influencing their choice of solution.

Sally *"The MUL thing..."*

Janet *"You have to umm...you have to do sort of like say one's 20 then.....you say umm yes you have to...say that's 10 right and that's 20...you still have to do square...no that's wrong....you have to do MUL blah blah blah blah...you have to do it on both of the sides won't you...but the only trouble is...how we're going to do it...how we going to do it.....'cos we have to do SCALE umm.....Use the SIDE one...it'll be easier...*

Sally *"No I was going to do the MUL..."*

Janet *"Alright do the MUL then....we should do the SIDE one...*

Sally *"Wait a minute look how long's that...20...MUL SCALE 20...and then do that one it'll be just FD..."*

Janet *"We'll do it with both...right and see what happens..."*

Sally *"Right we'll have to do one for the side and another one for the thing..."*

They decided to *"just do it normally first"* meaning try out a specific case. After drawing a fixed rectangle with side lengths of 20 and 10 they defined:

```
TO RECM "SCALE
FD MUL :SCALE 20
RT 90
FD MUL :SCALE 10
RT 90
FD MUL :SCALE 20
RT 90
FD MUL :SCALE 10
RT 90
END
```

They then tried out RECM 1. Their use of "(S) variable as scale factor" indicates that they had taken into account the need for a relationship between the length and the width of the rectangle.

It is suggested that at the beginning of the session Sally and Janet had not discriminated between the use of "(I) one variable input" and "(S) variable as scale factor" but that they began to do so during this session. The teacher intervention asking them to

compare DIAMULE and DIASIDE was probably critical in helping them to begin to discriminate.

5.2.8 Row of Pines

Year & Session No:	Year 2; Session 14
Type of goal:	Well defined abstract
Category of variable use:	(O) Variable operated on
	(R) General recursive superprocedure

During this session Sally and Janet were asked to reproduce a "Row of decreasing pine trees" (Fig. 5.11c). To solve this problem they needed to define a general procedure for a "pine tree". They could do this by using two unrelated variable inputs, variable as scale factor or variable operated, all categories of variable which had peviously been used by them. Janet initiated the discussion by suggesting

Janet *"Alright so start in the middle....a MUL or a SCALE..."*

She could be saying use variable operated on (MUL) or variable as scale factor (SCALE). They adopted their usual strategy of working on a fixed module in direct mode. Before defining the general module Janet said:

Janet *"So now from there we really have to make a SCALE or MUL command...and all we have to do is put the moves in between ...OK come on then which are we going to use MUL or SCALE."*

Two important points can be deduced from this statement. Firstly Janet has from a top down point of view solved the problem. She has analysed the pattern into a series of different sized pine trees (for which she plans to define a variable module) interfaced by a set of "moves." She also has a view that there are two possible ways of solving the "variable module" problem. Sally however said:

Sally *"SIDE"*

To which Janet replied:

Janet *"Oh Yeah use SIDE...it will be easier won't it."*

This reference to "SIDE" is almost certainly a reference to using "One variable" input and Janet's reply was a reference to their previous session (when solving the pattern of squares) in which they found that using "(S)variable as scale factor" to define a square had caused considerable "interfacing" difficulties. Janet then suggested that they start by drawing a specific sized shape.

Janet *"FD how much...let's make it...and then we can convert it can't we..."*

They worked out a pine tree in direct drive using the commands:

FD 130
LT 30
BK 30
FD 30
RT 60
BK 30

Fig. 5.10: Sally and Janet - Pine Tree

Their choice of the lengths 130 and 30 suggests that they had not thought through the need to make a relationship between these lengths explicit.

Janet *"Yeah now let's do a procedure..."*

Sally *"Wait a minute if we're going to use SIDE...how we're going to do it?"*

Sally appears to be referring to the problem of needing to take into account the "trunk" and the "branches".

Janet *"Call it PINE..call it SIDE...and then it's FD dot dot SCALE SIDE....can't remember..."*

Janet's language still indicates a confusion between a "(I) one variable input" frame and "(S) variable as scale factor" frame.

Res. *"Do you know what it means Janet when you put dots SIDE?"*

Janet *"It means instead of a number instead of 30 ..that's what it will be...but I was wondering you know...say that's 130 will these be just SCALE like that..."*

Res *"That was 130....and those were 30...so if that's SIDE what will those be....seewhat I mean..."*

Sally suggested the idea of using two unrelated variable inputs.

Sally *"You have to call them something different then..."*

At this point the researcher decided to nudge them into operating on a variable within their procedure. It is interesting to note that they had not initiated this for themselves although they had used the idea before.

Res. *"You can call it something different if you want...or you can call it by it relationship to SIDE....let's think of a more simple example...if it was 120 and 30...what would be the relationship between that bit there and that bit there..."*

Janet *"It's a quarter of it.."*

At this point the researcher engaged in a teaching episode with the pupils in which she explicitly explained how and why to define a general procedure in which the variable SIDE is operated on within the procedure. Sally and Janet with help defined PINE (Fig. 5.11a)

They were asked to explain the process.

Janet *"It goes up...then it turns that way...then it goes down a quarter of the SIDE...then it goes back up again..."*

Sally *"Then it turns up..."*

Janet *"It goes right 60 like that...then it goes down...then it goes up...then it goes down again..'*

They tried out PINE 120 their choice of input seems to indicate that they had understood that they were not in this instance scaling the distance commands. After this session they did not use "variable as scale factor" again. In direct mode they produced the row of pine trees with the following commands:

```
MOVE
PINE 120
MOVE2
PINE 110
MOVE2
PINE 100
MO
PINE 90
MOVE2
PINE 80
MOVE2
PINE 70
MOVE2
PINE 60
.............
............
```

They then wanted to define a superprocedure and Janet was convinced that there must be a simpler way of writing a procedure than by just entering all the commands again.

Janet *"Instead of typing all this out how are we going to make a big program..."*

Sally *"But we can't...we can't just type REPEAT MOVE PINE 120...'cos it's just going to keep on doing the same one all the time...*

Janet *"Yeah but...yeah I know but...is there any way we could do....no I don't think so..."*

Sally *"Miss is there any other way....you know how you do REPEAT it....so it won't do the same thing all the time..."*

This interchange was remarkable. They had seen that their series of commands had a structure which they did not think that the REPEAT command could deal with. The researcher decided to introduce them to a recursive structure and within a teaching episode they defined the procedure FOREST (Fig 5.11b).

a)
```
TO PINE "SIDE
FD :SIDE
LT 30
BK MUL :SIDE 0.25
FD MUL :SIDE 0.25
RT 60
BK MUL :SIDE 0.25
FD MUL :SIDE 0.25
LT 30
BK :SIDE
END
```

b)
```
TO FOREST "SIDE
PINE :SIDE
MOV2
FOREST SUB :SIDE 10
END
```

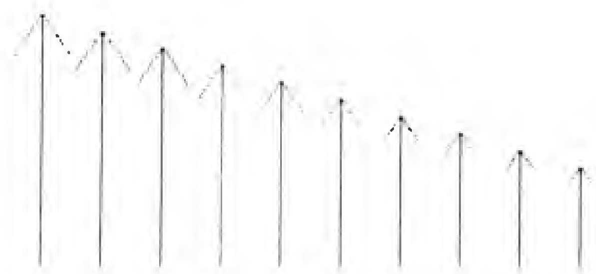
c) 

Fig. 5.11: Sally and Janet - Row of Pine Trees

During this session Sally and Janet had again been nudged by the researcher into operating on a variable within their procedure. Without this intervention they would almost certainly have used two unrelated inputs. When the general superprocedure FOREST was defined they seemed to understand the idea of operating on the variable in the recursive call as this represented the relationship which they had generated in direct mode. They used the variable name SIDE in both the subprocedure PINE and the general superprocedure FOREST.

5.2.9 Spirals

Year & Session No:	Year 3; Session 1
Type of goal:	Well defined abstract
Category of variable use:	(O) Variable operated on
	(R) Recursive procedure

This session illustrates how even within an apparently well defined task pupils can devise a "valid" solution which does not match the teacher's expected solution. During this session Sally again took on the role of negotiating the general relationship within the geometric object and Janet took on the role of negotiating the details within a specific case.

Sally and Janet were given a sheet containing several spiral patterns (Fig 5.12) and were told that they could choose any of the spirals to draw. They decided to draw the square spiral and perceived the task as one of drawing a similar square spiral shape and not one of representing exactly what was on the paper. Sally and Janet negotiated the structure of the spiral with Sally making a global analysis of the problem.

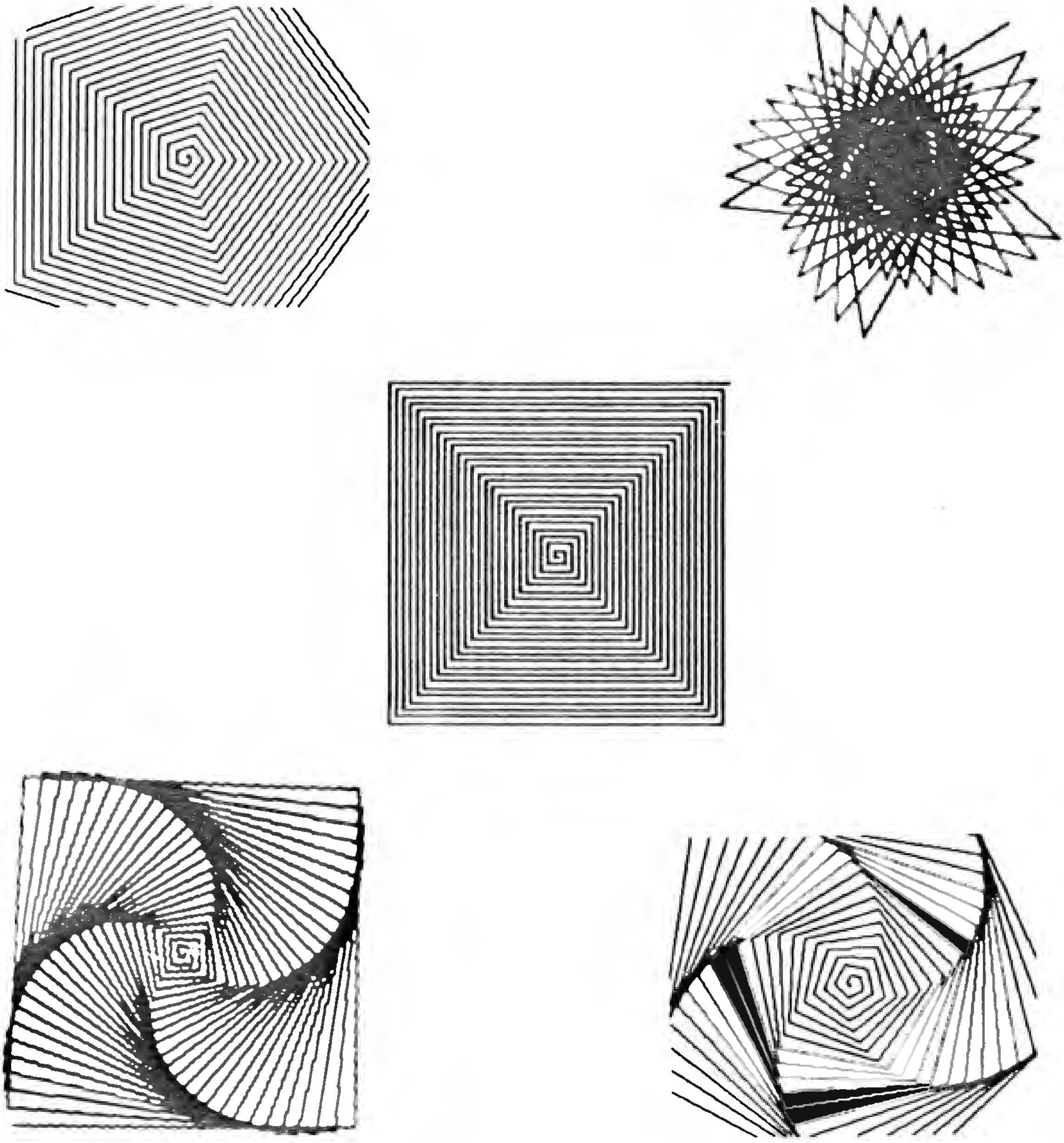


Fig. 5.12: The "Spiral" Task

Sally *"First we draw a line...then we turn 90 ...add 10 and then we continue doing that....turn 90...add 10.....add whatever number we want.*

Janet's next statement indicated a need to try out the plan at the "hands on" stage. She focused on the local details of the plan.

Janet *"Right let's just see if it works...so how long will the first line be....it can't be that long..."*

Janet's disagreement provoked Sally into explaining that her example was a generic example and not a specific case.

Sally *"No not 10 ....I know......I was just giving an example."*

Janet *"So how big will the first line ...be the actual line..*

Sally *'Three"*

Janet *"Three?"*

Sally *"Quite small"*

They type in: FD 3 RT 90 and then negotiated again.

Janet *"And what now?"*

Sally *"It would have to be...wait a minute...it's a square...'*

During Sally and Janet's collaborative work at the computer the "hands on" stage seemed to be very important in helping them to get started on a problem. After typing in several commands they entered into another planning stage with Janet more able to participate in the decision processes. They typed FD 4 and discussed the plan again:

Janet *"FD 4....no hang on this is what it would do look...."*

Sally *"You'd have to add two each time."*

Janet *"Look say you do that then that would be the same as that...but this one would have to be longer."*

Sally *"Yeah"*

Janet *"So that would be the shortest one...so say that would be 3...that would be 4 and that would be 4....and that would have to be one smaller...."*

Sally *"Work it out on here..look that's the first one....make it go down like that....that must be one longer....and that has to be one longer..."*

Janet *"And that has to be one longer..."*

Sally *"No those two can always be the same the same size....get me..."*

Janet *"Yeah"*

Sally *"Cos look....."*

Janet *"What you're really saying is those two are the same size...."*

Sally *"Just add two..."*

Whatever Sally's original plan she was willing to negotiate with Janet and between them they came to a shared understanding. Sally took the role of focusing the discussion on a global plan and Janet took on the role of attending to local details.

Janet *"So what you're really saying is those two are the same size..and those two are the same size....so how do we do that...."*

Sally *"Alright I'll make a quick REPEAT command".*

Janet realised that there was a problem with using the REPEAT command.. This was similar to the "Row of pines" problem.

Janet *"Yeah you'll have to....oh no you can't REPEAT...no you can't....'cos you have to....*

Janet's language was not very explicit but the comment was important as it registered the potential problem with using the REPEAT command. For the time being they returned to trying out their plan in direct drive. Again they were using "hands on" activity to give them space to negotiate the problem. After trying the sequence FD 3, RT 90, FD 4, RT 90, FD 4, RT 90, FD 4, they cleared the screen and started again with: FD 10, RT 90, FD 10, RT 90, FD 12, RT 90, FD 12, RT 90. At this point Janet put forward an alternative plan and suggested that they make the next two commands FD 12 (so there would be 4 consecutive FD 12 commands). Sally disagreed with this suggestion. Janet was still not clear and made her questioning more specific.

Janet *"No hang on...would it work if you said 10 10 12 12 12 12 14 14 14 14 14..."*

This question provoked Sally to elaborate her reply:

Sally *"No 'cos these go up by 4..* (meaning EF is 4 bigger than AB in Fig. 5.13) *'cos that is 2 and we want that to be 2 out as well....so it's 14...."*

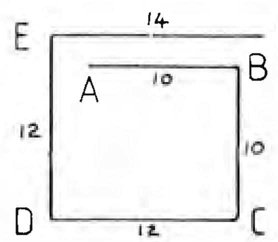


Fig. 5.13: Sally and Janet - Part of a Spiral

Janet was not sure about this but the disagreement was resolved pragmatically by Sally typing FD 14 into the computer. They continued to type in the commands according to Sally's plan and as the image emerged Janet suddenly gained insight into the structure of the solution.

Janet *"I'm enjoying myself now...it's all clear.."*

It is suggested that without this collaborative sequence and the computer feedback Janet would not have been able to reach the stage where formalising the generalisation by writing a Logo program would be meaningful.

Sally *"Right so we'd have to do......what do we have to do really..*

Janet demonstrated her understanding by expressing the generalisation in natural language:

Janet *"Emmm what do we have to do...you have to repeat it and add two"*

Sally took up the idea of repeating and said that they must decide how many times to repeat..they negotiated this, deciding on 100 repeats...after making this decision Janet said:

Janet *"Yeah...go on then..."*

At this point Sally again realised that there was a problem

Sally *"But I don't know what to do to make it go in two's..."*

Janet now decided that they should ask the researcher for help. The last ten minutes of the session consisted of a teaching episode. The researcher asked Sally and Janet to explain the way in which they have solved the problem.The researcher's solution would have been of the form:

```
TO SPI "X
FD :X
RT 90
SPI ADD 20 :X
END
```

The researcher however matched the Logo formalism to Sally and Janet's general method and by the end of the session the following procedure was defined:

```
TO TEN "TWO
FD :TWO
RT 90
FD :TWO
RT 90
TEN ADD :TWO 2
END
```

Fig. 5.14: Sally and Janet - Spiral Procedure

Previous analysis of the transcript data had indicated that the case study pupils were beginning to attach too much significance to the name of a variable and so in this session the researcher had decided to intervene to 'nudge' the pupils away from using variable names like SIDE and NUM. Their choice of procedure and variable names appears confusing but they understood the role of each named variable. **Within this session Sally and Janet had again operated on a variable in order to make a general relationship explicit.**

5.2.10 Spiral Extension

Year & Session No:	Year 3 ; session 2
Type of goal:	Well defined abstract
Category of variable use:	(N) More than one variable input

In this session the researcher nudged Sally and Janet into extending their spiral procedure to making the angle variable.

Res. *"Instead of always turning 90 ..you can turn a different angle."*

She had expected them to first change the value of 90 to another specific value but Janet immediately initiated the idea of making the angle variable.

Janet *"What you want is a program like this ....but instead of FD 10 you want to do the same thing for the angle...so you can just put in a number and it will do it..........so how do you do that....."*

Sally *"You'd have to do a name..."*

Janet *"Yeah so you'd have to have two names....so that could be TEN and that could be TWO...and that could be ONE..."*

They had taken on the idea that any variable name could be used.

Sally *"Umm shall we try it...."*

Janet *"How do we put another number.....miss what we're thinking is to do the same for the angles .....like we do for there....so we just have to type in ......we have to make a separate program..."*

Res. *"Oh you can put another one...just go up to the top line ...put dots...and put another one in....call it angle or whatever...."*

Janet *"What we going to call it..."*

They modified their program to add another variable but initially left out the second variable in the recursive call. This produced an error message and with help they were able to correct the bug to produce the procedure in Fig. 5.15.

```
TO TEN "TWO "ONE
FD :TWO
RT :ONE
FD :TWO
RT :ONE
TEN ADD :TWO 2 :ONE
END
```

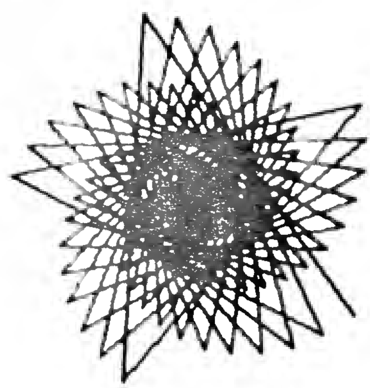


Fig. 5.15: Sally and Janet - Spiral Extended

They then used this procedure to investigate a wide range of spiral patterns using both positive and negative inputs.

5.2.11 General Butterfly

Year & Session No:	Year 3; session 4 & 5
Type of goal:	Well defined real/abstract
Category of variable use:	(N) More than one variable input
	(G) General Superprocedure

Sally and Janet were asked to produce any picture made up of variable sized triangles.

They decided first to draw a general triangle. They started to draw a triangle in direct mode. Without any intervention they defined a general triangle procedure:

```
TO ET "SIDE
FD :SIDE
RT 60
BK :SIDE
RT 60
FD :SIDE
RT 60
RT 180
END
```

They used ET in direct mode to produce a butterfly, keeping a record of their instructions.

Janet *"When we make that...if we want to make it bigger or smaller we'd make it like this....it'd have giant antlers....and if we make it bigger...it'd have these tiny little ones..."*

Janet saw the need for varying the size of antennae with respect to the size of the body. The exact relationship was not however important.

Janet *"So we'll just have to make a program for the things..."*

Sally *"So we'll change them as well."*

Janet *"Yeah....so let's make the program...what are you going to call it.....try and make it look more like a butterfly...instead of two triangles with two little things sticking out of it's head...."*

They continued to draw the butterfly in direct mode and then defined the butterfly procedure (Fig. 5.16).

Janet *"Put how big you want it...and how big you want the antlers...no not SIDE again...what about ONE and TWO...be easiest....ONE for that and TWO for that..."*

Sally *"Have to do another one after ONE*

Janet *"Yeah so ONE..then you'd have to put dots TWO."*

It seems that the naming of the variable in the title line of the procedure is crucial in pushing Sally and Janet to plan out what they want to make variable within their procedure. They are using the same variable names ONE and TWO as they had used in the previous project.

They now wanted to add another variable to draw a pattern (the line AB in Fig. 5.17b) on their butterfly.

Janet *"Alright this is the pattern... that would have to be BFLY ONE TWO THREE.."*

Sally added another input (THREE) to the title line. The procedure was now the one given in Fig. 5.17a.

```
TO BFLY "ONE "TWO
RT 90
ET :ONE
RT 180
ET :ONE
RT 30
FD :TWO
CT
FD :TWO
END
```

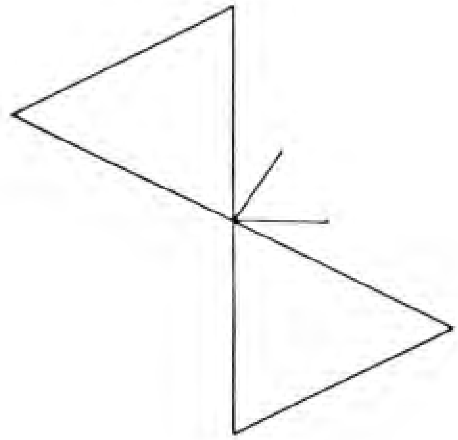


Fig. 5.16: Sally and Janet - Butterfly (1)

a)

```
TO BFLY "ONE "TWO "THREE
RT 90
ET :ONE
RT 180
ET :ONE
RT 30
FD :TWO
BK :TWO
RT 60
FD :TWO
BK :TWO
RT 45
FD :THREE
BK :THREE
RT 30
FD THREE
BK :THREE
BK :THREE
FD :THREE
LT 35
BK :THREE
FD :THREE
END
```

b)

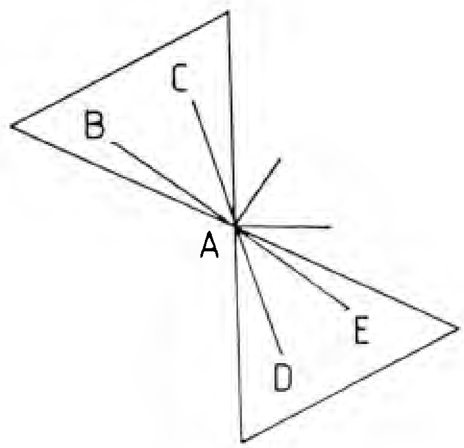


Fig. 5.17: Sally and Janet - Butterfly (2)

They had added a new variable name THREE although when using the procedure they always assigned TWO and THREE the same values. In this instance not specifying relationships between the variables did not effect the shape of the butterfly. They tried out BFLY 20 5 5 , BFLY 30 15 15, BFLY 20 10 10 and BFLY 40 20 20, always assigning the same value to the last two variables.

The aim for this session had been for the pupils to make a general relationship explicit by operating on a variable within a procedure. The pupils chose to solve the problem by using three unrelated variable inputs

because they did not see any necessity for making a relationship between them explicit.

5.2.12 Arrowhead

Year & Session No:	Year 3; Session 6
Type of goal:	Well defined abstract
Category of variable use:	(O) Variable operated on

All the case study pairs were given the "Arrowhead" task (Appendix 3.3) at the end of the period of research. The task was given after the pupils had been given the individual laboratory tasks (Chapter 7.0). The aim of the task was that the pupils would define a general procedure in order to draw a general "arrowhead" shape and the following intervention was made in order to make the goal explicit.

Res. *"I want this shape here....but I want it to be as big or as small as I want...I want them to be ...you know an enlargement...blown up or made smaller...so that it's similar..."*

Janet immediately suggested that they use three inputs. She analysed the shape into three varying parts and at this stage was not concerned with the interelationshp between the parts.

Janet *"One two three...different inputs...."*

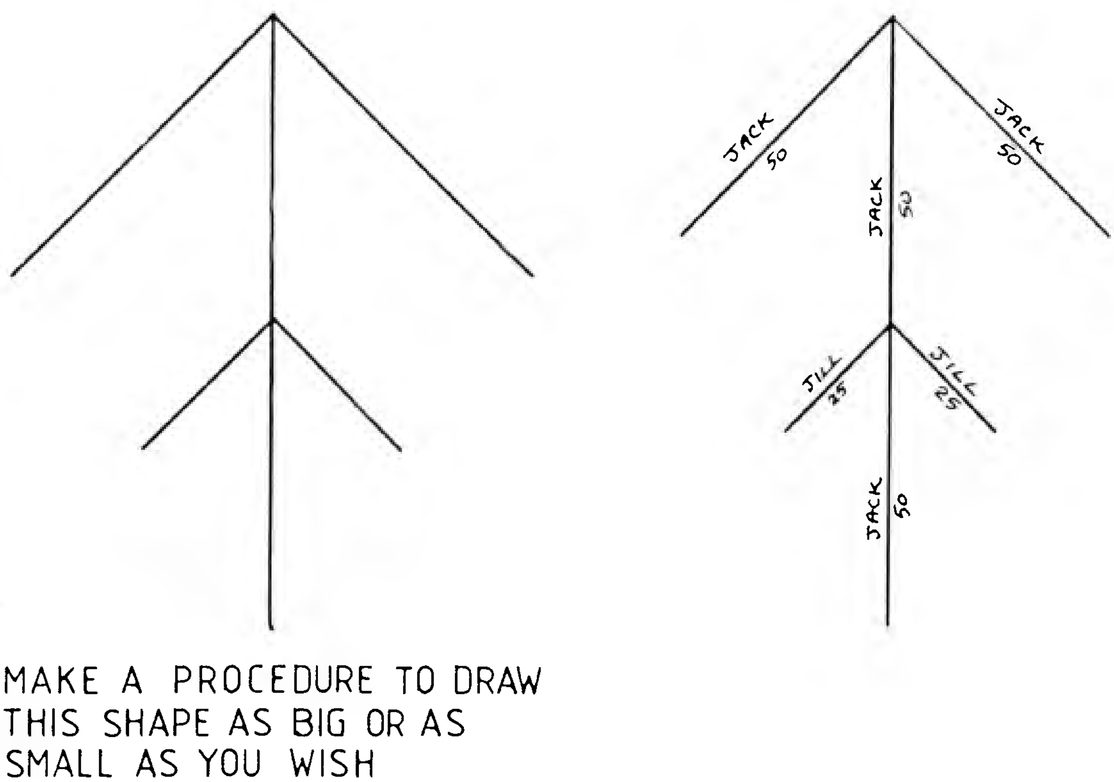
They used their usual strategy of first typing in a specific shape:

They had not taken as important the ratio between the component parts of the arrowhead, which were implicit on the handout (Fig. 5.18a). Janet was certain that they needed three inputs, although they had only used used two different lengths in their fixed arrow, when they drew it in direct mode. This suggests that she was influenced by their previous session in which they had defined a general procedure with three variable inputs.

Janet *"Alright now for this we need...we work it out 'cos that will have to be something called JACK ..that JOHN and that JILL...if you get what I mean..."*

Again this is evidence that the naming of the variables is provoking Janet to plan her use of variables. She is using a variable for each distinct part of the arrow head and has not analysed the shape for the relationship between the variables.

a) b)



c)

```
RT 90
BK 50
RT 45
FD 50
BK 50
LT 90
FD 50
BK 50
RT 45
FD  50
RT 45
FD 25
BK 25
LT 90
FD 25
BK 25
RT 45
FD 50
```

Fig.5.18: Sally and Janet - Arrowhead (1)

When they started to define the procedure however they typed the following title line using two inputs:

```
TO HILL "JACK  "JILL
RT 90
```

They then stopped to discuss their use of variables again.

Sally *"BK 50"*

Janet *"BK JACK"*

Sally *"Wait a minute..."*

Janet *"dot dot..."*

Sally *"Wait a minute ...you have to do it...BK MUL*

Janet *"That's multiplied..."*

Sally *"Yeah I know... "*

Janet *"BK...multiply 50 by what..."*

Sally *"By JACK.."*

It seems that Sally was suggesting using variable in the category of "(S)variable as scale factor". Janet however appears to be suggesting that they operate on a variable within their procedure (category(O)).

Janet *"No no no no...this is what you do....you say..umm...for this one you say...BK JACK...and for this one you multiply by 2 'cos that's half...."*

Sally *"No 'cos we want like..."*

Janet *"No.....'cos listen look....but anyway say that's 100...and we put in 100...then that would do that a 100...but you'd have to put in another number ...so instead of putting in two numbers...listen..."*

Sally *"But we're not going to put any old numbers in...'cos it won't be the same pattern.."*

Sally seems to understand that the relationship between the variables is important but at this stage it is not clear whether she wants to use "(S)Variable as scale factor" or make the relationship between the variables explicit by operating on them within the procedure.

Janet *"Yeah...but if you put in 75...then they're not going to be 75....they're going to be any old number..."*

Sally *"Yeah that's why we're going to multiply it..."*

Janet *"Yeah but you don't need to multiply it...that's what I'm saying...if you say..umm.....if that one say..that times by 2...it would be that wouldn't it...."*

Sally *"Divided by 2..."*

Janet *"Yeah...you know what I mean..."*

Sally *"Alright..."*

Janet *"But I don't know how we're going to do it...we can get rid of JILL".*

Sally *"Well think about it...umm that command think...."*

Janet *"Umm what about....right you know when we do...if we get down to* *there...it would be a FD multiply..."*

Sally *"No divide"*

Janet *"By 2.......then will it know..."*

Sally *"What we're talking about"*

It seems as if they know the relationship but are not convinced that they can teach it to the computer.

Janet *"Yeah...but what do we do for this here....do we just put BK JACK...or whatever..."*

Sally *"Yeah*

They did not at this stage remove the the second variable JILL and continued to define HILL in Fig. 5.19a until they came to the change the FD 25 command (see Fig. 5.18c), which was half the length of JACK.

a)

```
TO HILL "JACK "
RT 90
BK :JACK
RT 45
FD :JACK
BK :JACK
LT 90
FD :JACK
BK :JACK
RT 45
FD :JACK
RT 45
FD DIV :JACK 2
BK DIV :JACK 2
LT 90
FD DIV :JACK 2
BK DIV :JACK 2
RT 45
FD :JACK
END
```

b)

```
TO HILL "JACK
RT 90
BK SUB :JACK 10
RT 45
FD :JACK
BK :JACK
LT 90
FD :JACK
BK :JACK
RT 45
FD :JACK
RT 45
FD DIV :JACK 2
BK DIV :JACK 2
LT 90
FD DIV :JACK 2
BK DIV :JACK 2
RT 45
FD SUB :JACK 10
END
```

Fig. 5.19: Sally and Janet - Arrowhead (2 and 3)

Janet *"OK...so this goes...FD ..divided by 2...no it doesn't....you know when we did the input machine...how did we do that..."*

Janet knows that she can divide by a variable as this is what she did when defining a simple function in Logo (see section 6.2.2). Sally however does not seem to be able to recall this use.

Sally *"I don't know we never done it did we.....JACK divided by 2...is it divided first ..or what..."*

Janet *"FD divide JACK...divide by JILL"*

Sally *"No....right so we want it to go forward by half of JACK....so* *would that be JILL.."*

Sally was tentative with her suggestions although she appeared to understand the relationship.

Janet *"Yeah I know....but just forget about JILL for the moment....how do we do it..."*

Sandra looked up the syntax in a manual and then continued to define the command FD DIV :JACK 2 (Fig. 5.19a).

They deleted JILL from the title line of HILL and then tried out HILL 50.

Janet *"Hey it worked...I don't believe it..."*

Sally *"Do it again..."*

HILL 70

Without any intervention they had operated on a variable to make a relationship explicit within their procedure (HILL, Fig. 5.19a). The discussion appeared to be very important in helping them to reach the solution.

They had completed the task as they had understood it but the researcher decided to intervene to provoke them into thinking about whether this shape was exactly the same shape as the one on the sheet. They were asked to measure the figure and compare the measurements with their own figure. They discovered the lengths to be 5, 6 and 3 c.m.(see Fig. 5.18a). They were asked to produce exactly the same shape. In the following sequence they discuss the relationship between the different parts of the arrow:

Janet *"So what we have to do....we have to make say JILL now......listen do you see this bit here...we have to change this bit to JILL...and we have to make JILL be the long bits....you see at the moment it's hundred .....and it's getting 50..."*

In their initial conception of the problem JACK represented the length of AB ,BC,BE, and EG and JILL represented the length of DE and EF (see Figs 5.18a and b).

Sally *"Yeah"*

Janet *"But we have to change it so that it takes the 60....and it divides it to 30......"*

Sally's response indicates her need for precision and is a possible insight into her reluctance to commit herself to the Logo syntax unless she is absolutely certain that it is correct.

Sally *"Divides by 2..."*

Janet *"Yeah so this one becomes JILL....and then we have to find out which ones are them...no..."*

Sally *"Alright you do it..."*

Janet *"Look we went RT 90 and BK 50....and then went RT 90....so it's this one and this one we have to change...then we went back . It's this one and this one..."*

Sally *"Yeah"*

Janet *"So it's those two and those two we've got to change to JILL"*

Sally *"Can't we just change these bits.."*

janet *"What do you mean.."*

Sally *"Why don't we change these bits here...and that bit there...it would be exactly the same....."*

Janet *"Yeah I see what you mean...then we would have to put JILL first...then JACK..."*

Sally *"Alright"*

Janet typed in

HILL "JILL "JACK

Sally was however still concerned about the need to make the relationship explicit

Sally *"You can put any number in....but it won't be...."*

Janet *"No it won't..."*

Sally was searching for a relationship

Sally *"Have to find some way....subtract one isn't it...."*

Janet *"What do you mean".*

Sally *"To get that you have to subtract one..."*

Janet *"So to get that you have to go FD JACK".*

Sally *"SUB one".*

Janet *"No FD SUB JACK one".*

Sally *"Yeah ".*

Janet *"Or... ten...ten.."*

After measuring they had discovered that AB was 60 and BE was 50 so at this stage they have suggested making AB; JACK and BE; SUB :JACK 10. Sally however was concerned about this and started to think about the nature of the relationship when the shape becomes bigger.

Sally *"Wait if it gets bigger...would it still be 10?"*

Janet *"No...yes it would ...because ...let's just do it this way...and if it's wrong....it would be right...because there you've got your 100....ok you're subtracting 10...and you've got 90....you'll always be subtracting 10...'cos it's 10 less isn't it..."*

Sally *"Yeah....but if it's double as big.....then this must be double as small..."*

She probably meant that "the amount you subtract must be twice, i.e if 60 becomes 120 then 50 should become 100 i.e. 20 less than 120....instead she says twice as small. Janet changed the procedure top line again to

HILL "JACK

which indicated that she now thought all the lengths could be expressed as a relationship to JACK. This changing of the declaration of variables seems to be very important to Janet.

Sally *"Right if this was 12...and it was subtract 10...then we'd only need 11 there.....it wouldn't be the same..."*

(She meant that if AB were 120 and you take away 10 then BE would be 110 and then the figures would not be similar.)

Janet *"It would.."*

Sally *"It wouldn't...'cos we say subtract 1 from 6...would be 5.....but then this would be 10 next time...."*

Janet *"I dunno...let's just do it this way....because I dunno what you're talking about..."*

Sally *"Look"*

Janet *"Talk to me"*

Sally *"Look this is 5....and if we made it twice as big...that's 10...and this is 6...and if we made it twice as big...it would be 12...yeah....and if you said SUB JACK 10...that would give you 11...and that's too long for that..."*

Sally almost certainly understood the problems associated with similar figures but she was not able to explain her meaning very well to Janet and Janet was confused.

Janet *"Mmm...no 'cos we're going....that's 70....no that's 50...right..."*

Sally *"OK"*

They started to define:

```
TO HILL "JACK
RT 90
BK SUB :JACK 10
RT 45
FD :JACK
```

Sally *"Which ones are we changing?"*

Janet *"Those ones..."*

Sally *"It's the wrong one..."*

Janet *"It's the right one."*

Sally *"No we're changing this one here and that one here....and that one there......."*

Janet *"No we're not 'cos we're keeping that the same...and we're* *changing that to SUB.....can't we just say ADD 10".*

Janet seemed to have taken control changing the procedure to become the procedure given in Fig. 5.19b.

They tried out HILL 100 and HILL 50

Janet *"Is that right then..."*

Sally *"Probably"*

Janet *"Do you think this one would be 121"*

Sally *"No 11...the other one....HILL 100...the size should be 12....do HILL 100"*

Sally means that if BE is 100 then AB should be 120.

The researcher then asked them to try HILL 120 again and to measure the lengths on the screen and to compare these with the expected lengths.

Janet *"It's too long"* (meaning BE)."

Sally *"By about 5 isn't it".*

Janet *"So now I see what you mean"...the thing is if we were to have minus...so I think you have to add another 10...if you do something like...if it adds up to say 10....then subtract 10.. but if not subtract 20 do you get what I mean..."*

She was beginning to understand Sally's previous points. In order to confront Sally and Janet with their bug the researcher suggested that they try HILL 10 and the turtle only drew one arrow head.

Sally *"It's too small ,,,you can't see it...'cos we done minus 10 remember..."*

They knew there was a problem and continued to try to resolve it.

Sally *"It's like you know those cards...you put a number in and you get a number out....and you have to find the connnection..."*

Janet *"You could always do FD SUB JACK JILL....make JILL a number.."*

Janet understood the problem but did not know how to solve it

Janet *"But what you're basically saying is...that's 60 and that's 50...so that would be 120 and that would be 100..."*

It was the end of the session and the researcher explained how they could solve the problem. The problem had not been one of formalising in Logo but rather one of devising a correct general method to solve the problem.

This session indicates that Sally and Janet were able to solve their **"simplified" version of the problem** using a "halving and doubling" **strategy and in this context were able** to operate on a variable within a procedure. After an intervention they attempted to come to terms with the original problem and the main obstacle to their solution was their confusion associated with similar figures. By the end of the session they were beginning to understand that Janet's intuitive solution involving subtraction did not provide a general solution.

Sally and Janet's solution to the "Arrowhead" task is compared to the solutions of the other case study pupils in section 5.6. An overview of Sally and Janet's development throughout the three years of the research is presented in section 5.8.

5.3 LONGITUDINAL CASE STUDY: GEORGE AND ASIM

George is a very confident, articulate dominant boy who relates better to adults than he does to his peers. His favourite subject at school is Craft, Design and Technology, *"'cos I enjoy making things"*. His mathematics teacher considers that he is above average in the class and *"he works enthusiastically and perservered over all sorts of problems with a high level of concentration"*. She also says that *"he is an independent worker to the extent of being a loner and I still think that he doesn't discuss his work enough with others...even those sharing the same task... he is highly motivated but he doesn't take on board the ideas of others easily"*. He has enjoyed mathematics more at secondary school than at primary school but his perception of his own ability is *"I'm O.K.... but I'm not the best"*. When he was asked what he has enjoyed most about his Logo programming he said *"Getting away from maths while I'm doing it"*. He shows some anxiety about his enthusiasm for computer programming and when we asked him what he thought about not having a computer in the mathematics class he said *"Ummmm I wouldn't like it... I suppose people would get on with their maths and do more maths ... when you're using the computer everyone's walking around"*.

Asim is a reserved studious boy who worries about his mathematics work and English is not his first language. His mathematics teacher considers that he is also above average in the class *"I think Asim's attitude has broadened in the year and he now enjoys the more creative aspects of maths, though he has difficulty approaching investigative work."* She also said that *"he is a very organised and independent learner....highly motivated....preferring to think things put for himself."* Mathematics is one of Asim's favourite subjects. When asked what he likes doing most of all when he is not in school he said *"Usually I read or revise."* The mathematics teacher said *"I think Logo has helped him develop the less traditional aspects of lear/ning...allowing him scope for independence in setting his own problems and in relating his original narrower view of maths to a broader field."*

5.3.1 Pythagorean Triangle

Year & Session No:	Year 1; Session 15 & 16
Type of goal:	Well defined abstract
Category of variable use:	(N) More than one variable input (O) Variable operated on (G) General superprocedure

At the beginning stages of the research the aim was to introduce pupils to the idea of variable within their own projects. During Asim and George's first fourteen Logo sessions they did not choose to work on projects which needed the idea of variable. This tension led the researcher to make a tentative suggestion of a project in which she hoped variable would become a necessary problem solving tool.

Res. *"I want you to do some sort of pattern which is made up of l loads of different squares".*

George *"You mean a big square which is getting smaller and smaller and smaller".*

Res. *"Something like that...anything which is made of squares...we could call it a world of squares".*

Asim *"Square world".*

Res. *"Square world ...yeah anything you like that is made up in your imagination of squares of different sizes".*

This intervention was not explicit enough and Asim and George in fact chose to draw Fig. 5.20.

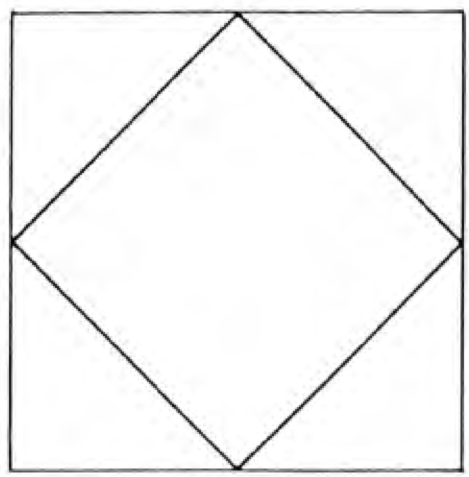


Fig: 5.20 Diamond Within a Square

Although it was not what the researcher had expected she still persisted with her "hidden agenda".

Res. *"If you like when you've done your procedure I'll show you how to change it so you can make it any size you want."*

Within this project George and Asim were estimating the length of the hypotenuse of a right angled triangle. At this point the researcher started to discuss with the them the possibility of using Pythagoras's rule to calculate the length of the hypotenuse of the triangle. She asked them what they knew about this rule and Asim said:

Asim *"This is the same area...if you put a square here....it's the same area as this one and this one....."*

The researcher then showed them how they could use Logo to calculate the length of the hypotenuse of a right angled triangle and then in a very teacher directed sequence showed them how to define the fixed procedure:

```
TO HYPOTENUSE
MAKE "AREA1  MUL 47.7 47.5
MAKE "AREA2  MUL 47.5 47.5
MAKE "AREA3  ADD :AREA1 :AREA1
PRINT SQT :AREA3
END
```

This was the first and only time in which any of the case study pupils used the Logo command MAKE. In retrospect the author now believes that the pupils should have been introduced to the idea of a procedure which output the desired result (See Chapter 3.0 for a discussion of this idea). The researcher then showed Asim and George how to modify this procedure to make it general.

```
TO HYPOTENUSE  "SIDE1  "SIDE2
MAKE "AREA1 MUL :SIDE1 :SIDE1
MAKE "AREA2 MUL :SIDE2 :SIDE2
MAKE "AREA3 ADD :AREA1 :AREA2
PRINT :AREA3
END
```

In the next session they were asked to reflect on the process within the procedure:

George "First of all you do..umm....SIDE....AREA SIDE1.....and then you multiply it by itself....SIDE1 times SIDE1...and then the same for SIDE2...and then you add them...each side areas to get AREA3...and then you print the square root of AREA3..."

George was beginning to understand the process. However during this session he was very much in control of the programming activity and Asim became a passive onlooker. The researcher asked them to draw an isosceles right angled triangle of side length 40 using the HYPOTENUSE procedure as a tool. (They were restricted to drawing isosceles right angled triangles because of the problem of calculating the angles of a non isosceles right angled triangle). They successfully did this for several isosceles right angled triangles and then the researcher suggested that they draw the squares on the sides of the triangle (Fig 5.21). They decided to write a superprocedure for this figure and when the researcher said:

Res. "You will have to make a different procedure for each square until I show you how to make squares of different sizes"

George immediately initiated the idea of using a variable.

George "It's easy....you write a program for a square...and instead of SIDE you put something like LENGTH..."

This indicated that George had already taken on the idea of using a variable to represent a range of numbers. They were shown how to define:

```
TO SQ "LEN
REPEAT 4 [:LEN RT 90]
END
```

Using this module in direct drive they built up Fig 5.21.

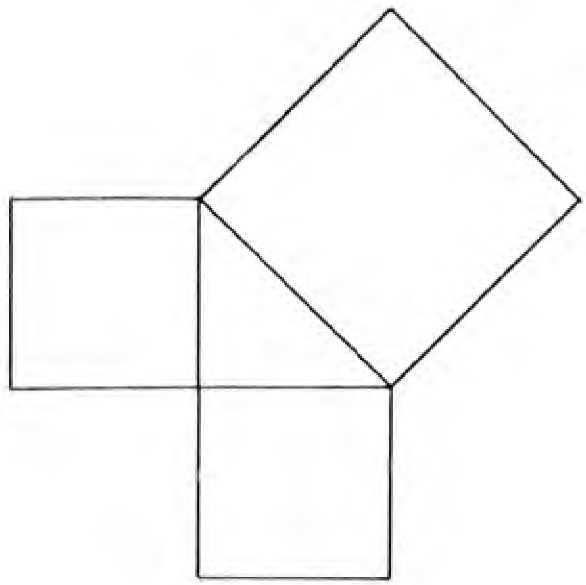


Fig. 5.21: George and Asim - Pythagorean Triangle

During this process Asim said:

Asim " I"m getting confused".

To which George replied:

George "I know why you're getting confused...it's because you don't understand what we just did...do you.".

George's reply indicated his need to be in control to the exclusion of involving Asim. During this session most of the discussion was between George and the researcher. After analysing this transcript the researcher became more aware of her effect on the collaboration between George and Asim and this awareness caused her to be more cautious about her interventions in future sessions. George needed a square module which would either draw a left square or a right square and he initiated the idea of making the turn variable

George "Miss can you do a different angle as well because....say change it from going LEFT 90...to RIGHT 90...I want to put on the top ANGLE...and instead of the RT 90..I"m going to put ANGLE 90."

This suggestion indicated a good understanding of the nature of variable as a place holder although it was not taken up by the reseracher. **George initiated the idea of defining a general triangle procedure** and he typed in:

```
TO TRIANGLE "LEN
LT 90
FD :LEN
RT 135
FD 42
RT 135
FD :LEN
END
```

at which point he said:

George *"Miss there may be a problem here...because how does it know which LEN it is...*

Res. *"Well they have to be different don't they...which two are both the same?"*

Asim at this point contibuted to the discussion:

Asim *"Hold it this one must be LEN2."*

This comment appears to indicate that Asim had an understanding of the mathematical invariance within this isosceles right angled triangle. However during this session he was not all involved with the Logo syntax and left this within George's control. The procedure was modified to that given in Fig. 5.22. They now had all the tools available to produce Fig. 5.21 in a range of different sizes.

Within these two sessions they had been introduced to the idea of operating on a variable to make a relationship explicit, to the idea of using more than one variable input, and to the idea of defining a general superprocedure. George had taken on the idea of using a variable to represent a range of numbers and seemed to have some understanding of using variables to make a general relationship explicit but Asim had not touched the keyboard or shown any evidence of being involved with the processes related to the writing of the Logo procedure. In retrospect it would have been better to introduce them to the idea of variable in a more carefully thought through task in which they were not introduced to so many new ideas at once. George's ability to take up the ideas, however, suggests that he could have been using variable at an earlier stage in the project.

```
TO TRIANGLE "LEN1 "LEN2
LT 90
FD :LEN1
RT 135
FD :LEN2
RT 135
FD :LEN1
END
```

Fig. 5.22: George and Asim - General Triangle Procedure

5.3.2 Battlements

Year & Session No:	Year 1; Session 17
Type of goal:	Well defined real
Category of variable use:	(I) One variable input.

George brought to this session a planned project (Fig. 5.23). He had initiated the idea of using a variable in the category of "(I) One variable input" within the flowing procedure BATTLEMENTS.

```
TO BATTLEMENTS "NUM
REPEAT :NUM  [BATTLE]
END
```

This again gives insight into George's developing understanding of variable as a place holder for a range of numbers. Asim had not been involved with the planning of this project.

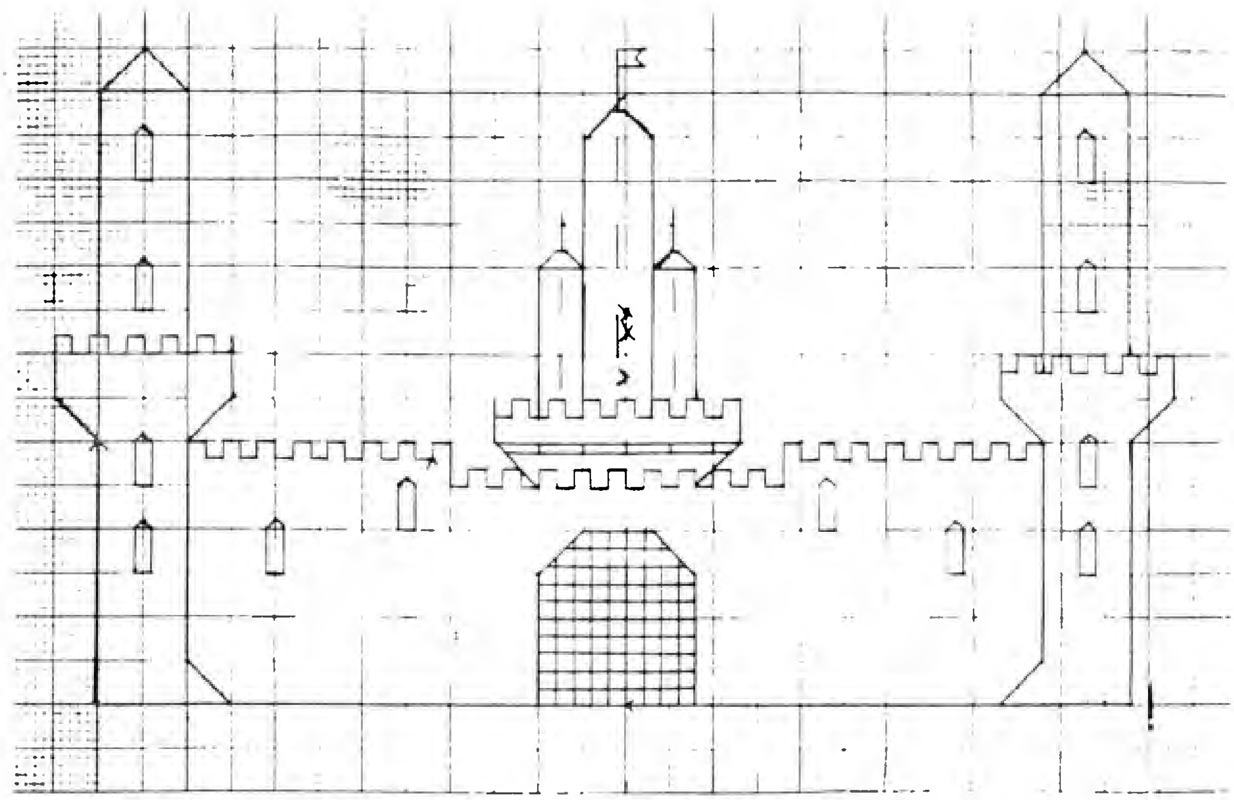


Fig. 5.23: George - Castle Project

5.3.3 Circular Spiral Task

Year & Session No:	Year 2; Session 5
Type of goal:	Well Defined/Loosely defined Abstract
Category of variable use:	(0) Variable operated on (R) Recursive procedure

At the beginning of this session George and Asim were asked to choose a task from a set of abstract images, with the aim of provoking them to use the idea of variable They chose to reproduce the spiral image (Fig. 5.24). The researcher said to them at the beginning of the task:

Res. *"Remember I want you to use input"*

George *"What do you mean?"*

Res. *"Remember....to make things different sizes..."*

Asim and George immediately decided to use the ARCR (see appendix 3.1) command to draw the image and spent a considerable amount of time experimenting in direct drive with different sequences of inputs. They typed in the startup commands and the researcher asked them:

Res. *"And what are you putting in for the input for ARCR...what sort of numbers are you putting in?"*

George was able to initiate the idea of using variable.

George *"10 90...what you could...so instead of the 10...have an input".*

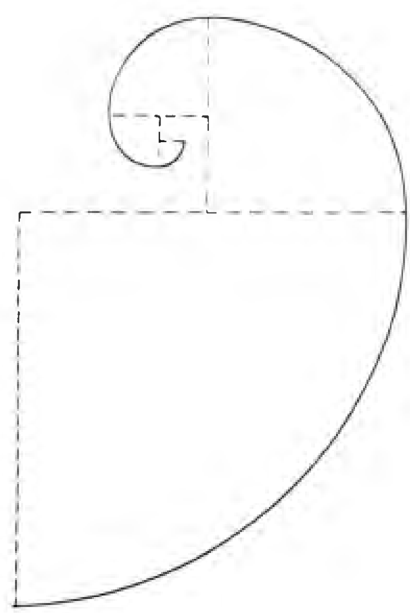


Fig. 5.24: Circular Spiral Task

The following extract illustrates the importance of the discussion in helping them to negotiate the relationship between the inputs to the ARCR command.

Asim *"That's twice as big as that...that's twice as big as that...no that's twice as small as that...which is twice as small as that......which is twice as small as that....."*

George *"Yeah...probably...how about making it...you know that 80....make it 60...it'd probably work as well".*

Asim *"80...60...it'd be different.."*

George *"I know...let's try..."*

Asim *"It would be different...look at that compared to that..wouldn't it...everything is twice as big as something...isn't it..."*

They finally decided on the following sequence:

ARCR 5 90
ARCR 10 90
ARCR 20 90
ARCR 40 90
ARCR 80 90

They then negototiated what to do next with George taking on the "didactical contract" of using variable.

George *"We've got to do this input command miss wanted us to do it..just do a circle...get bigger...bigger ..bigger...bigger..."*

Asim initiated an extension to the goal.

Asim *" Should we draw those square things as well?"*

George *"I've got another idea....do another one coming round there..."*

George knew how he was going to acheive this:

George *"So we just got to reverse all these."*

In direct drive they produced the following pattern with the commands:

ARCR 80 90
ARCR 40 90
ARCR 20 90
ARCR 10 90
ARCR 5 90
RT 180
ARCL 35 180

When George and Asim had finished drawing Fig. 5.25a in direct drive they wanted to write a procedure. The researcher first elicted their understanding of the relationship between the inputs of the ARCR commands in the right hand spiral and explained how to formalise this in Logo.

```
TO ANGLE "NUM
ARCR :NUM 90
ANGLE MUL :NUM 2
END
```

Because they had "experimented" with different sequences of numbers they had no problem in recognising the mathematical structure of their spiral and accepted the Logo formalisation. The researcher intentionally did not include a conditional STOP statement in the procedure at this stage. When they tried out the procedure the spiral carried on drawing, eventually hitting the edge of the screen. This provoked George and Asim to reflect further on the process without any intervention from the reserarcher.

George *"Oh it's multiplied by two...oh we've got..."*

Asim *"What did you do ...what happened....."*

George *"It's multiplied by two again...it didn't stop...."*

Asim *"You want how many times you've got to do it...."*

They had a general idea now and needed to focus on the particular values:

Res. *"On which ARCR command does it get too big?"*

George *"80...when it doubles 80.....".*

The researcher showed them the Logo syntax:

```
TO ANGLE "NUM
IF GRQ :NUM 80 [STOP]
ARCR :NUM 90
ANGLE MUL :NUM 2
END
```

They were not at all confident that this procedure would work. George appeared to think that the computer had some magical powers which he could control!

George *"Keep your fingers crossed".*

Asim *"I'm not superstitious".*

George *"Well keep you feet crossed then".*

Asim *"I'm still not superstitious".*

George *"I'm superstitious".*

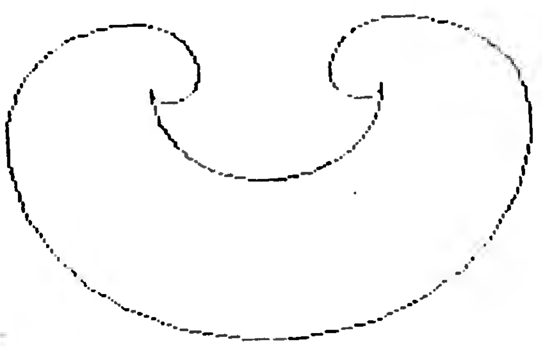
Nevertheless without any intervention from the researcher they started on the task of writing a procedure to draw the left hand spiral. They were aware that they could use the same input. George even realised that they could not use the same conditional statement.

George *"Miss we're going to have...we can't have...IF GREATER THAN 5...'cos it goes down to 5....so what should we have....below 5..."*

The researcher showed them the LESS THAN statement and they wrote the subprocedure finally putting all the subprocedures together in a superprocedure SPIRAL (Fig. 5.25b). George and Asim then built up a pattern on the screen with their SPIRAL procedure. They finally wrote a superprocedure SPIRAL 3 (Fig. 5.26). This was the first time that they had built up a pattern from an existing module in a loosely defined way.

George and Asim were very motivated throughout this session and Asim became more involved in the negotiation of decisions than he had done in the previous two sessions. **They had used variable to formalise a relationship which they had evolved and negotiated through discussion and interaction with the computer. George and Asim did not have any difficulty in accepting the idea of using variable in a conditional expression, in fact it had been essential as a tool in the solution of their problem.**

a)



b)

```
TO SPIRAL
START
ANGLE 5
ANGLE2 80
RT 180
ARCR 35 180
END
```

```
TO START
 RT 90
 PU
 BK 30
 PD
 END
```

```
TO ANGLE "NUM
IF GRQ :NUM 80 [STOP]
ARCR :NUM 90
ANGLE MUL 2 NUM
END
```

```
TO ANGLE "NUM
IF LSQ :NUM 5 [STOP]
ARCL :NUM 90
ANGLE2 DIV :NUM2
END
```

Fig. 5.25: George and Asim - Spiral Pattern

```
TO SPIRAL3
SPIRAL
PU
HOME
FD 20
PD
SPIRAL
PU
HOME
FD 40
PD
SPIRAL
END
```

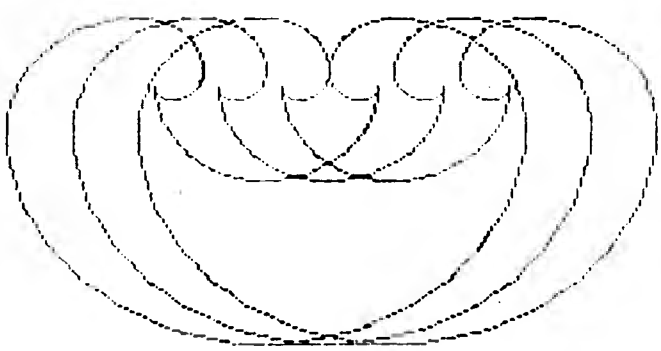


Fig. 5.26: George and Asim - Multiple Spiral Patterns

5.3.4 Nested Circles

Year & Session No:	Year 2; Session 6
Type of goal:	Well defined abstract
Category of variable use:	(O) variable operated on
	(R) Recursive Procedure

In this sesion George and Asim again choose from a set of projects, the image of nested circles (Fig 5.27a). As with the circular spiral they decided to use the ARCR command and then in direct drive negotiated the radius inputs to ARCR. They typed in:

```
ARCR 10 360
ARCR 15 360
ARCR 20 360
ARCR 24 360
```

The researcher intervened to nudge them into using variable.

Res. *"If you made it 25 360....you could then use input....can you see you're going up in equal stages...you could write one procedure which did the whole lot....can you remeber you did that last time...you could call it pattern...so you'd do the same thing over and over again...do you see what I mean..."*

They took up this idea of ARCR 25 360 and George also suggested.

George *"And then make it 30...'cos then we can do an input..."*

They typed in:

```
ARCR 25 360
ARCR 30 360
```

and then George asked for help with writing the Logo program.

George *"Now we've got to do this input thing...I can't remember how to do it...'miss how do you do input again.."*

Res. *"Well tell me what you want to do"*

George *"Emm...we want to add 5 each time.."*

With support they defined the following procedure:

```
TO CLAM "NUM
ARCR :NUM 360
CLAM ADD :NUM 5
END
```

The researcher tried to involve Asim in the process.

Res. *"Do you understand what we're doing Asim?"*

Asim *"Umm...you're going to change the number....to keep on adding 5..."*

George knew from his experiences in the previous session that they would need a conditional statement and he started to look in the handbook.

George *"It's this one GRQ...do we have to put anything else..*

Res. *"I want you to do it*

They typed in:

```
TO CLAM "NUM
ARCR :NUM 360
CLAM ADD :NUM 5
IF GRQ  :NUM 35 [STOP]    (the conditional in incorrect place)
END
```

and tried out CLAM 5 which did not stop because of the incorrect placing of the conditional statement.

Res. *"Why did it go on...can you work out why,..."*

George *"It goes ARCR....then a specific number 360..right round..then it does...then it adds 5..then it carries on".*

Asim was also involved in the process.

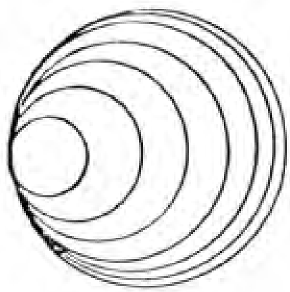
Asim *"Miss should we put the greater than 35 in front..."*

George *"Yes"*

Res. *"You see it never gets to that line....."*

They modified their procedure to:

a)



b)
```
TO CLAM "NUM
ARCR :NUM 50
IF GRQ :NUM 35 [STOP]
CLAM ADD :NUM 5
END
```

Fig. 5.27: George and Asim - Nested Circles

Res. *"How many circles should there be?"*

Asim *"Seven"*

George *"Seven"*

The procedure however drew eight circles.

Res. *"Now why has it done eight..."*

Asim *"'Cos it's one more greater...and after it's done that it should stop......than 30...it should stop then.."*

George *"No not greater than 30..because you want it doing it say 5 times..."*

Res. *"Why does it do a 40 as well?"*

Asim *"Cos umm it just umm...it's greater than 35....and after it's done 40 it stops..."*

George "Oh so you greater than 30...."
Asim "Why don't you ever believe me?"

This discussion indicates that Asim is clearly involved with the "mathematical" processes although George takes control of typing the commands into the computer. Using CLAM they built up a pattern on the screen. It was the end of the session and the researcher asked them to explain their CLAM procedure.

Res. "How does it work"

George "Well we've got ARCR NUM...and it's called CLAM NUM.....and the NUM stands for when you type in....you've got to put CLAM 5...then it will draw it with a diameter of 5...then it will add 5...and make it...and add another 5 each time...'

They then decided to write a procedure for their new pattern.

George "Write a quick program called CLAM2...then put a REPEAT 8 CLAM They wanted to define a fixed superprocedure. The researcher however told them how to write a general superprocedure.

```
TO CLAM2 "NUM
REPEAT 8 [CLAM :NUM RT 45]
END
```

They themselves did not need, and did not use, CLAM2 as a general **superprocedure but used it only with one fixed input.**

5.3.5 War Games

Year & Session No: Year 2; Session 7

George and Asim were working on a project of their own choice. This project involved simulating bombs falling on a tank. The researcher made an inappropriate intervention to suggest that they use variable, which was firmly rejected by George.

Res. "What you could do you know is you could use input to make bombs...you could make it look as if it's dropping bombs and you could do them in different sizes"

George "If it was directly above it you wouldn't get different sizes ...it would just be the same size all the way down".

Res. "Would it"

George "Yeah...anyway we're just going to have these marks where it's been and then a tank exploding...if we can find the program...which we haven't".

This discussion illustrates the pupils' capacity to reject the researcher's

inappropriate intervention.

5.3.6 Variable Letters

Year & Session No:	Year 2; Session 9,10,11
Type of Goal:	Well defined abstract
Category of variable use:	(S) variable as scale factor

As were all the case study pupils George and Asim were introduced to the idea of using "Variable as a scale factor" within the "Scaling Letter" task. (fig. 5.4).

Res. *"So what you start off with is to do a proceure for a letter...any letter....do a simple one to start with...and when you've done that I want you to change it...change that procedure so that it takes an input called SCALE...or whatever you want to call it...that you multiply every distance by....so that you make your letter different sizes...if you start off and do a procedure for a letter first...and then I'll come back and show you what to do."*

George and Asim did not want to draw the letter L and so started with the letter S trying out some commands in direct mode, and then defined SSSSSSS (Fig. 5.28a).

a)
```
TO SSSSSSS
    RT 180
    Fd 20
    ARCL 10 18
    FD 20
    ARCR 10 180
    FD 25
    END
```

b)
```
TO SSSSSS "SCALE
RT 180
FD MUL :SCALE 20
ARCLMUL10:SCALE10180
FD MUL :SCALE 20
ARCR MUL :SCALE 10 180
FD MUL :SCALE 25
END
```

Fig. 5.28: George and Asim - Variable Letter "S"

At this point George said:

George *"We should have done E".*

Asim *"Why".*

George *"Easy to scale down".*

They tried to follow the handout without any researcher/teacher support but finally had to ask for help to produce the general SSSSSS procedure in Fig. 5.28b. They then tried SSSSSS with inputs of 1.0, 0.5,1.7 and -1.0.

They then decided to draw the letter T which they first carried out in direct mode. Asim defined a general procedure TTTTTTT (Fig. 5.30) from the direct drive commands, clearly indicating confidence with the process.

Asim *"We're doing it in that MUL thing straight away. "*

Asim was typing and this physical involvement provoked him to reflect on the effect of making the angle variable.

Asim "What would happen if I put RT MUL?"

George "Instead of going 90 ..it would go 45 or 12".

Asim "So it would choose".

George "Yeah it would go any angle like that...woosh"

Asim " Must try that sometime."

Asim's use of the word "MUL" however does indicate a confusion between the prefix operator and the variable name. With reference to the initial value George said:

George "You've got to put that in otherwise it won't know it."

They typed in TTTTTTT 1.0 and George referred to this as the "normal" one.

It was now the end of the session and in the next session George brought plans for the letter A and the letter R (Fig 5.29). This was typical of his need for taking control and had the effect of keeping Asim out of the processes. They defined the "scaled" A from the written record and then the "scaled" R appearing to have no problems with the use of variable in this context. They then decided to build up a pattern using these letters and had considerable difficulty making all the letters the same height. They did not think of changing the value of the variable but attempted to make all the heights standard in their individual procedures. After finally building up a shape they wanted to define a superprocedure and asked about making the superprocedure general:

George "Miss when we do the program for the whole thing do we have scale in that or....

They were again told to define a general superprocedure although in fact they only needed a fixed superprocedure. They defined the procedure SSSSSSSTTTTTTTAAAAAAARRRRRRR with an input in the title line which was not used within the procedure (fig 5.28a). They did not initialise the input when they used the procedure. They then tried the procedure with an input of 1 (although this did not do anything) George realised this and said

George "Miss we didn't have to put scale in that program"

They removed the variable input rejecting the idea of defining a general superprocedure because they had not needed a general superprocedure. This illustrates how if pupils do not need an "idea" then intervention to provoke its use will have little effect.

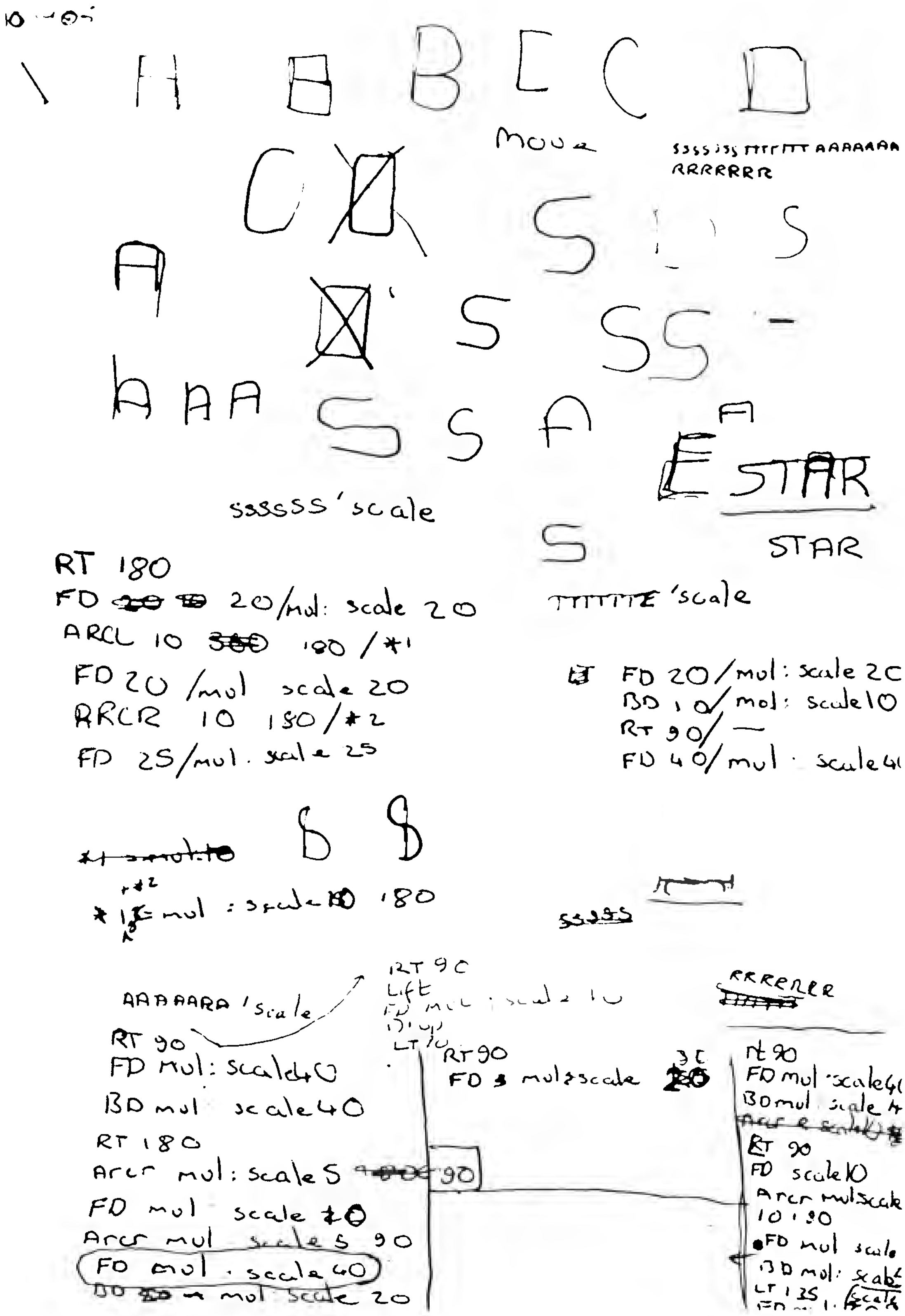


Fig. 5.29: George's - Planning for 3D STAR

```
TO SSSSSSSTTTTTTTAAAAAAARRRRRRR
    SSSSSSS 0.5
    HOME
    SSSSSSS 1.0
    HOME
    SSSSSSS 1.5
    HOME
    SSSSSSS 2.0
    HOME
    TTTTTTT 0.5
    HOME
    TTTTTTT 1.0
    HOME
    TTTTTTT 1.5
    HOME
    TTTTTTT 2.0
    SETX 200
    SETY 85
    SETH 0
    AAAAAAA 0.5
    SETX 200
    SETY 85
    SETH 0
    AAAAAAA 1.0
    SETX 200
    SETY 85
    SETH 0
    AAAAAAA 1.5
    SETX 200
    SETY 85
    SETH 0
    AAAAAAA 2.0
    SETX 245
    SETY 90
    SETH 0
    RRRRRRR 0.5
    SETX 245
    SETY 90
    SETH 0
    RRRRRRR 1.0
    SETX 245
    SETY 90
    SETH 0
    RRRRRRR 1.5
    SETX 245
    SETY 90
    SETH 0
    RRRRRRR 2.0
    END

TO SSSSSSS "SCALE
    RT 180
    FD MUL :SCALE 20
    ARCL MUL :SCALE 10 180
    FD MUL :SCALE 20
    ARCR MUL :SCALE 10 180
    FD MUL :SCALE 25
    END

TO TTTTTTT "SCALE
    FD MUL :SCALE 20
    BK MUL :SCALE 10
    RT 90
    FD MUL :SCALE 40
    END

TO AAAAAAA "SCALE
    RT 90
    FD MUL :SCALE 35
    BK MUL :SCALE 35
    RT 180
    ARCR MUL :SCALE 5 90
    FD MUL :SCALE 10
    ARCR MUL :SCALE 5 90
    FD MUL :SCALE 35
    BK MUL :SCALE 20
    RT 90
    FD MUL :SCALE 20
    END

TO RRRRRRR "SCALE
    RT 90
    FD MUL :SCALE 40
    BK MUL :SCALE 40
    LT 90
    FD MUL :SCALE 10
    ARCR MUL :SCALE 10 180
    FD MUL :SCALE 10
    BK MUL :SCALE 5
    LT 135
    FD MUL :SCALE 30
    END
```

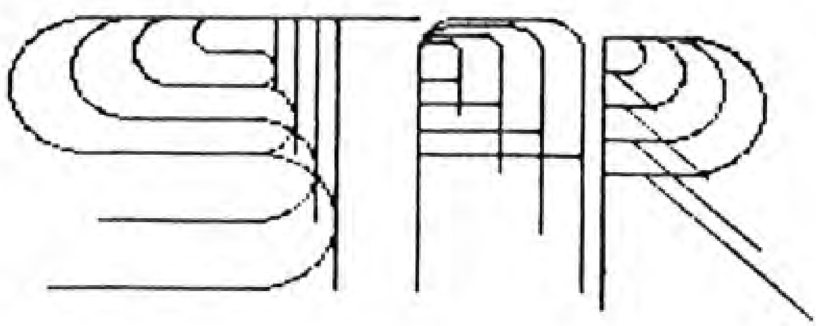


Fig. 5.30: George and Asim - 3D STAR

5.3.7 Patterns of Squares (A) (the length of this session was approximately 3 hours)

Year & Session No:	Year 2; Session 29
Type of goal:	Well defined abstract
Category of variable use:	(I) One variable Input
	(S) Variable as Scale Factor

At the end of the second year of the project the case study pupils visited the University laboratory to carry out a range of tasks (Appendix 5.1). George was not present on this day and Asim worked with Jude. Asim and Jude were asked to draw Fig 5.31 Their discussion indicated that **Asim was still very unclear about the appropriate Logo syntax to use in order to solve the problem.** It is suggested that this is because during the majority of previous sessions George had dominated the keyboard work. Asim and Jude first of all solved the task without using variable and then after an intervention from the researcher they attempted to define a general square.

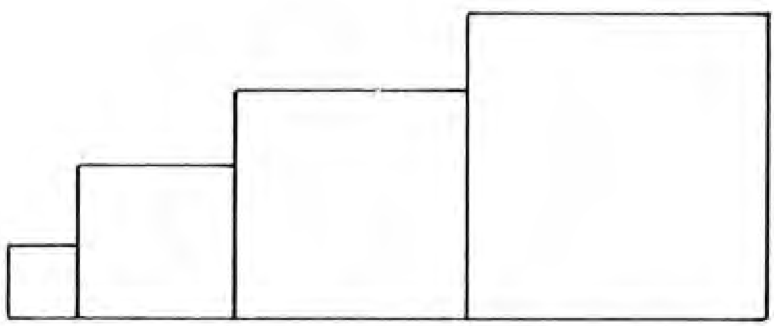


Fig. 5.31: Jude & Asim-pattern of Squares (A)

Jude *"That's the procedure...alright type SCALE at the top..and put err a comma there".*

It is almost certain that Jude was thinking in a "(S) variable as scale factor" frame.

TO LYNX "SCALE

Jude *"Now..where was...now put scale ...wait a minute you have to multiply".*

Jude was trying to reconstruct the syntax.

Asim *"You don't start off by putting scale".*

Jude *"We're doing a program".*

Asim *"I know you don't start there".*

Jude "*Oh yeah PU..come on PU...PU....now scale ...BK 100 that goes to scale*".

BK 100 was the startup command and not part of the square. Asim did not want to scale this command.

Asim "*No that should be on its own though*"

Jude "*Now BK SCALE...I think you put scale and those dots*".

They typed in BK SCALE :

Jude "*Do you put MUL 30*".

Asim "*The MUL before the scale*".

Res. "*You don't have to...you can just do...*"

Jude "*The number*".

Res. "*You can either do it MUL if you want to multiply every number by scale or you can just do the number so that for example if you wanted it to be 40 then you would say BK :SCALE...right and then you would put 40 for scale*".

The researcher was suggesting that they use Variable in the category of "(I) one variable input". Her use of the variable name SCALE was unintentionally confusing.

Jude "*And do you need these dots*".

Res. "*You would need dots before scale because you've called scale your input...every time you use it you have to have two dots before it*".

Asim "*Where do we put MUL*".

Res. "*You want to use MUL...you don't have to do it that way..I know that's what you did last time you did it but if you just want to change the number if you just want to make it different sizes then all you have to do is...can you remember doing that with George*".

Asim was now obviously confused between his "(I) one variable input" frame and his "(S) Variable as scale factor" frame.

Asim "*But we used MUL*".

The researcher now intervened considerably to help them sort out this conflict. Asim defined:

```
TO WHITE "SCALE
REPEAT 4 [FD :SCALE  LT :SCALE]
END
```

He had replaced both the input to FD and the input to LT by the variable SCALE. This seems to indicate that he was trying to "remember" the syntax without reflecting on the processes involved.

Jude *"You don't put scale for the things".*

Asim *"This is confusing".*

Without any intervention Asim changed WHITE to:

```
TO WHITE "SCALE
REPEAT 4 [FD :SCALE LT 90]
END
```

They then started to define the following interface procedure:

```
TO LYNX "SCALE
FD :SCALE
END
```

Asim *"We want to know if we do a program for each line?"*

Which seems to mean a procedure for each interface. Another researcher became involved at this stage.

Res. *"Now what do you want your program to do?"*

Jude *"Miss we want to increase its size by an input".*

Asim *"The teacher said to change the numbers".*

Res. *"Are you trying to write a program...I don't think you know quite what you are doing is that right?"*

Asim *"Yes I'm sure".*

Asim may have understood but the "new" researcher did not understand what they were trying to do and she helped them to change their LYNX procedure to a square procedure.

```
TO LYNX "SCALE
REPEAT 4 [FD :SCALE LT 90]
END
```

As Asim pointed out they had already defined a general square procedure(WHITE).

Asim *"Miss we've done that we've done that in a different one ...in WHITE."*

The researcher suggested that they try out WHITEand they typed in:

WHITE 2

The researcher then suggested that they tried LYNX:

LYNX 2

Res. *"So play around with that and see if you can build up your pattern from those."*

Jude and Asim knew that the intervention had been inappropriate.

Jude *"Miss we're supposed to do this procedure again with SCALE".*

Asim *"We just wanted to know if we can do it with one program".*

Res. *"Yes you can".*

They were not happy with these procedures and deleted them all from working memory. They then defined a JAGUAR program.

```
TO JAGUAR "SCALE
PU
BK :SCALE
PD
REPEAT 4 [FD :SCALE LT 90]
FD :SCALE
REPEAT 4 [FD :SCALE  LT 90]
FD :SCALE
REPEAT 4 [FD :SCALE LT 90]
FD :SCALE
REPEAT 4 [FD :SCALE  LT 90]
END
```

They had effectively replaced all the distance commands in their procedure by the variable SCALE without reflecting on the relationship between the different values of the inputs. When they tried JAGUAR 2 a row of small squares was drawn

Asim *"What's that...what is that".*

Jude *"It's supposed to be this".*

Asim *"Wait a minute..oh I know".*

Asim changed the interface commands to:

```
TO JAGUAR "SCALE
PU
BK :SCALE 100
PD
REPEAT 4 [FD :SCALE  LT 90]
FD :SCALE 10
REPEAT 4 [FD :SCALE  LT 90]
FD :SCALE 20
REPEAT 4 [FD :SCALE  LT 90]
FD :SCALE 30
REPEAT 4 [FD :SCALE  LT 90]
END
```

Again it seems that he was trying to 'remember" a previously learned rule for using "Variable as scale factor." This was being confounded by his previous experience of using "One variable Input." Asim then said

Asim *"No I think it's MUL....you have to put in MUL there".*

He then inserted a MUL before all the interface commands to produce:

```
TO JAGUAR "SCALE
PU
BK MUL  :SCALE 100
PD
REPEAT 4 [FD :SCALE  LT 90]
FD MUL  :SCALE 10
REPEAT 4 [FD :SCALE  LT 90]
FD  MUL :SCALE 20
REPEAT 4 [FD :SCALE  LT 90]
```

```
FD MUL :SCALE 30
REPEAT 4 [FD :SCALE  LT 90]
END
```

He tried JAGUAR 10 and nothing was drawn as the BK :SCALE MUL 100 command with an input of 7 took the turtle off the screen. They put another PD in their procedure and tried again...but still nothing was drawn. They changed a few PD commands and tried again.

Asim *"How come we can't see anything"* .

They decided to try using the trace command to trace through their procedure as it output to the screen. The researcher helped them to look at the process focusing on the size of each square within the procedure.

Res. *"Look all these are scale scale scale scale...did you want them to be all the same size".*

The researcher decided to define a variable square module for them.

```
TO S "SIDE
REPEAT 4 [FD :SIDE  RT 90]
END
```

and showed them how to use this to draw squares of different sizes. They modified their JAGUAR program to use the variable square S program although this only compounded the error.

```
TO JAGUAR "SCALE
PU
BK MUL :SCALE 100
PD
REPEAT 4 [S 10  LT 90]
FD MUL :SCALE 10
REPEAT 4 [S 20  LT 90]
FD MUL :SCALE 20
REPEAT 4 [ S 30  LT 90]
FD MUL :SCALE 30
REPEAT 4 [S 40  LT 90]
END
```

They tried JAGUAR 10 and again nothing was drawn so Asim modified the procedure by removing all the REPEAT 4 LT 90 so that the following was defined.

```
TO JAGUAR "SCALE
PU
BK MUL :SCALE 100
S 10
FD MUL :SCALE 10
S 20
FD MUL :SCALE 20
S 30
FD MUL :SCALE 30
S 40
END
```

He then tried JAGUAR 10 again but still nothing was drawn because of the first BK MUL :SCALE 100 command which took the turtle off the screen.

Res. *"Tell me why you've done BK MUL :SCALE 100 ".*

With this nudge they said:

Jude *"Miss it's going to go off the screen".*

Asim *"So take all the MULs off..."*

Their finally modified their procedure to become:

```
TO JAGUAR "S
PU
BK 100
S 10
FD 10
S 20
FD 20
S 30
FD 30
S 40
END
```

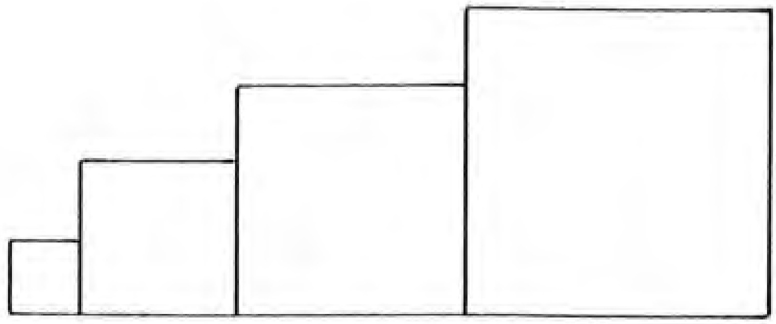


Fig. 5.32: Jude and Asim - Patterns of Squares (B) - Final Procedure

which finally worked.

This was the first time that Asim, without George, had had to make decisions about using variable. He seemed to have a clear modular idea of how he wanted to solve the problem. However there is evidence that he tried to "remember' previously learned rules for using variable and was initially using a variable to replace anything which varied without reflecting on the relationships within the procedure. This was probably due to his experience with the "Scaling Letters "task in which he would have been able to engage in the task in a rote manner. There is no evidence at this stage that he is mananging to coordinate his "(I) one variable input" frame and his "(S) variable as scale factor" frame.

5.3.8 Pattern of Squares B

Year & Session No:	Year 2; Session 15
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor

As George had been absent on the laboratory day it was decided to give George and

Asim one of the "Patterns of Squares" tasks (Fig 5.33a). They produced the following procedure using variable in the category of "(S) variable as scale factor". This may have been influenced by Asim's previous session with Jude.

```
TO SQ1 "SCALE
REPEAT 4 [FD 10 MUL :SCALE RT 90]
END
```

They had however made a syntax error and when they tried SQ1 10 an error message was produced. They corrected this and tried SQ1 1 George then typed in:

```
SQ1 1.5
RT 135
```

at which point Asim said:

Asim *"We haven't done all the squares yet".*

George *"I know we only have to do one square".*

Asim *"Yes we do".*

George *"No we don't".*

Asim *"There's four of them".*

George *"We don't".*

Asim *"We do".*

George *"Because we change the size of one square...the first squre so we just need one square...we just chaneg the size of it..."*

This discussion seems to indicate that Asim had not taken on the idea of using the general module in order to produce the whole pattern (possibly another indication that he was thinking from a "(S) variable as scale factor" perspective. They did however build up the pattern with the general square module. As the pattern emerged they negotiated the size of each module.

Asim *"You've got it wrong...the sides are too small.....1.5 is too small...*

George *"'Cos that is not twice the size of that and that is not twice the size of that...it's one and a half times*

When it was finished George said:

George *"I just wanted to ask miss something...when you do the program for all 4 squares do you have to do the scale again*

George was asking about whether or not to define a general superprocedure. This confusion seems to be directly linked to previous introductions of the idea when the researcher told them to define a general superprocedure when it was not a necessary tool within their problem solution.

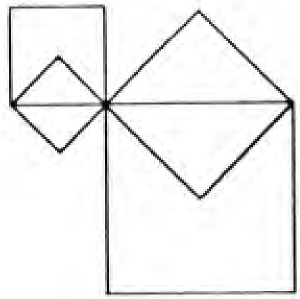
Res. *"Are you going to change it each time or are you going to tell it what it is"*

George *"No tell it what it is."*

Res. *"If you're going to tell it what it is then you don't actually need it OK".*

George seemed to understand this and defined a fixed superprocedure (Fig. 5.33b).

a)



b)

```
TO QUAD
RT 135
SQ1 1
RT 45
SQ1 2
RT 135
SQ1 3
RT 45
SQ1 4
END
```

Fig. 5.33: George and Asim - Pattern of Squares (A)

When this was finished the researcher asked them to reflect on the processes involved:

Res *"What size are they?"*

George *"1 2 3 and 4".*

Res. *"What that's the input".*

Asim *"We only use one square for the four".*

Res. *"So what does that make the size".*

Asim *"All the sides are 10 20 30 40".*

This interchange indicates that Asim was beginning to understand the processes involved in obtaining this solution.

They were then asked to produce another pattern of squares (Fig 5.34a) They started in direct mode uising their general square procedure. After drawing the outer square and moving the turtle to the position for the next inner square they discussed whether or not to make the interface procedure general.

George *"Oh that's good so we can do scale"* (meaning use Scale for the interface commands).

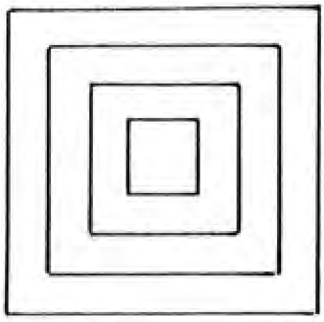
This seems to have confused Asim who was trying to discriminate between the use of variable in the category of "(I) one variable input" and variable in the category of "(S) variable as scale factor">

Asim *"We're not going to do that in scale are we".*

George *"Miss it's MUL then...oh yeah I know'..it's OK*

However when they came to the second set of interface commands they used the same set as before and so eventually defined a fixed interface procedure (MOVE, Fig. 5.34b)). They finally produced a fixed superprocedure, TUNNEL(Fig 5.34b) for this pattern.

a)



```
TO SQ1 "SCALE
REPEAT 4 [FD MUL :SCALE RT 90]
END
```

b)

```
TO TUNNEL
SQ1 1
MOVE
SQ1 2
MOVE
SQ1 3
MOVE
SQ1 4
MOVE
SQ1 5
MOVE
SQ1 6
MOVE
SQ1 7
MOVE
SQ1 8
MOVE
SQ1 9
MOVE
SQ1 10
END
```

```
TO MOVE
PU
BK 5
LT 90
FD 5
RT 90
PD
END
```

Fig. 5.34: George and Asim - Pattern of Squares (B)

5.3.9 Spirals

Year & Session No: Year 3; Session 3

Type of goal: Well defined abstract

Category of variable use:
(N) More than one variable input
(O) Variable operated on
(R) Recursive procedure

George and Asim were given the spiral task (appendix 3.5) which had also been given to Sally and Janet (see section 5.2.9). They immediately negotiated their solution to the pattern given in Fig. 5.35a.

George *"Shall we start with that. How shall we do it...FD 1 RT 90 FD 2 RT 90 ...FD 3."*

Asim *"Is it like that?"*

George *"Yes 'cos you go FORWARD 1 and then you have to go up 2 and go that way 3".*

Asim *"No hold it...can't you do REPEAT ...FD...something?"*

George *"And add 1"*

Asim *"REPEAT that"*

George *"I can't remember how to do that."*

They started to look through the Logo handbook . They, like Sally and Janet, had devised a solution but needed to find the Logo structure and syntax which matched their solution.

Res. *"What are you looking for?"*

George *"You know that MUL and all that..."*

Res. *"What is it you want to know?"*

George *"Adds...the add one..."*

Res. *"What are you going to do".*

George *"We're going to go Fd 1 RT 90...FD ADD 1 RT 90".*

Res. *"Are you going to write that in a program?"*

George *"That wouldn't work out...I dunno..Oh yeah...ummm...FD...if you put FD 1...RT 90....FD ADD 1 RT 90...then you know how do you get it to go FD 2 RT 90 FD 3 RT 90....."*

George and Asim had used recursion in two previous problem solutions and it seems that George was referring to this Logo structure but still needed support with the details of the syntax. The researcher suggested that they write a procedure to draw one part of the spiral.

Res. *"Write the program that does the FD 1".*

With help they defined:

```
TO CORRIDOR "DISTANCE
FD :DISTANCE
RT 90
END
```

Res. *"Right now I want you to try that out".*

They tried out:

```
CORRIDOR 1
CORRIDOR 2
CORRIDOR 3
```

Asim *"Can't we repeat this miss....?"*

George *"Repeat forward "distance " and add 1. Miss what about....if we do FD 1 and then you go FD 2...but then it will go back to FD 2 again won't it..."*

The researcher showed them how to modify the procedure to that given in Fig. 5.35a

George *"Miss what about...if we do FD 1...and then you go FD 2...but then it will go back to FD 2 again.."*

Res. *"No because everytime it does it adds one on each time...what you're doing is add one to distance each time.."*

George *"Yeah that means...miss that means that what is going to happen is it's going to go ...that means you're going to have to keep on typing CORRIDOR 1...and then CORRIDOR 2...and then CORRIDOR 3...and CORRIDOR 4...and all that.."*

They tried out:

CORRIDOR 1

George *"So now it won't stop".*

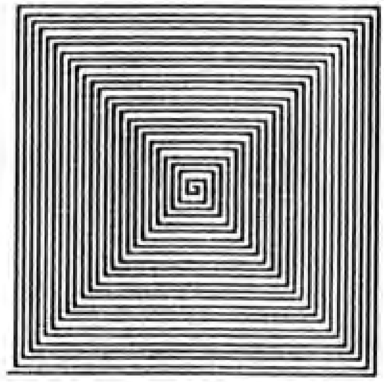
Asim *"When's it going to stop?"*

George *"It's not going to".*

a)

```
TO CORRIDOR "DISTANCE
FD :DISTANCE
RT 90
CORRIDOR ADD :DISTANCE 1
END
```

b)



c)

```
TO CORRIDOR "DISTANCE "ANGLE
FD :DISTANCE
RT :ANGLE
CORRIDOR ADD :DISTANCE 1
END
```

d)

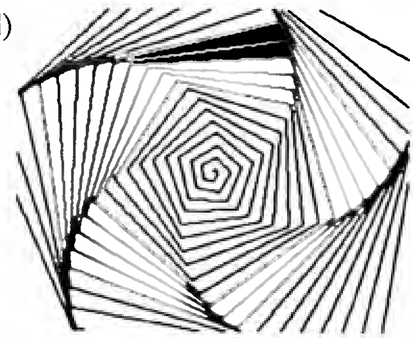


Fig. 5.35 George and Asim - Spiral and Spiral Extended

The researcher then nudged them into making one of the other spirals on the sheet (appendix 3.5).

Res. *"Is there anything you could do to your program to make this one*

George *"Yeah..turn it..instead of RT 90...turn it RT 95 or something...the problem is it looks here as if it goes FD and then RT 95 then FD ... then RT 85.*

They changed the RT 90 in their program to RT 95. George then initiated the idea of making the angle variable:

George *"Oh miss...is it possible to have two inputs...CORRIDOR 1 and then the angle..'*

They changed the angle in their program to RT 85.

George *"So miss up there we have to have another thing...distance and then angle...another quote..?"*

They changed their program to that given in Fig. 5.35c. They now investigated the effect of different angle inputs. **Within this session George and Asim defined and used a recur sive procedure. Although they had used this structure twice before they still needed support with the syntax. The "teacher given" Logo formalism matched their own solution, which they had negotiated without needing to interact with the computer.**

5.3.10 Arrowhead

Year & Session No:	Year 3; Session 8
Type of Goal:	Well defined abstract
Category of variable use:	(O) Variable operated on
	(G) General superprocedure

George and Asim were given the "Arrowhead" task (appendix 3.3) as their final task of the project. The task was given to them after the pupils had carried out the individual laboratory tasks (section 5.7). They first of all negotiated a plan.

Asim *"You just draw an arrow and MUL it"*

George *"How we going to half it by the way...."*

This interchange suggests that Asim wanted to use variable in the category of "(S) variable as scale factor" and George wanted to use variable in the category of "(O) variable operated on".

Asim *"What about....so we have to..."*

George *"It's more likely...it's actually MUL by 2......it's double...that is double that."*

Asim *"Double what?"*

George *"That is double that..."*

Asim *"I know ...that is obvious.."*

George *"Do the small one first".*

They then started to draw an arrow in direct drive:

```
LT 90
FD 20
RT 45
BK 10
FD 10
RT 90
FD 10
BK 10
LT 135
```

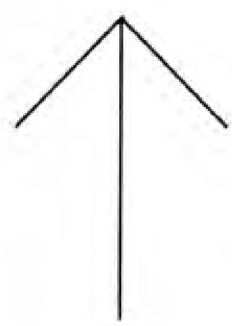


Fig. 5.36: George and Asim - Arrowhead in Direct Mode

At this point Asim thought that the task was complete because he saw the final form as being made up of one arrow module. George on the other hand did not perceive the problem in this way:

Asim *"But you only need one.."*

George *"How do you mean...that's the small one.."*

Asim *"I know...but you only need one arrow...then you can MUL it as you like....we're going to be making prison clothes... ARROW MUL...OK...."*

They started to define a procedure:

TO ARROW "NUM

Asim *"OK where do we start?"*

George started to analyse what was invariant and what was varying within the procedure.

George *"Actually we can't have that LT 90 there...because if we do that...every time we do arrow......so do FD 30...'cos that's always the same".*

Asim *"No it isn't".*

George *"It is 'cos that's the same distance as that...*

It seems that they were both using the word "same" in different ways. They typed into the procedure:

FD 20
RT 45

They then started to negotiate how they were going to take imake the "BK 10" command variable.

George *"Wait a sec'...it needs a MUL".*

Asim *"Miss when we draw these all over the place...will be drawing the whole thing ...or just one arrow..."*

Res. *"The whole shape".*

Asim *"BK two dots MUL"*

It seems that Asim was referring to his "(S) variable as scale factor" frame.

George *"No it's not..."*

Asim *"Yeah....BK 10 MUL....*

George *"No BK NUM".*

George appeared to be thinking from a "(O) variable operated on" frame.

Asim *"We forgot the 10..put the 10 in there...two dots.."*

George *"No it's alright...it's MUL.."*

Asim *"10....no you don't put MUL you only put NUM.....it's got nothing to do with MUL..."*

George "It has...."

Asim "It'll be NUM two dots 10..."

George "No it's backward 10...."

Asim "And then NUM..."

This interchange is difficult to interpret, but it seems that Asim is still confused between the various contexts of using variable.

George then expressed the relationship between the "shaft" and "head" of the arrow.

George "Multiplied by two each time......it's BK two dots NUM...."

Asim was still talking from a "(S) variable as scale factor" frame.

Asim "That's what I keep saying...why don't you listen to me....and then you put 10 afterwards...or 10 before it..."

They attempted to resolve their disagreement by asking the researcher/teacher.

George "Miss...when you have BK umm....do you have BK colon NUM

Res. "Yes...or whatever you've called your input..."

Asim "Well where do you put the number....or don't you put the number..."

George "Well if it's that it's doubled every time..."

Res. "So how are you going to do it...what's NUM standing for..."

George "The number...how far it is..."

Res. "So tell me how you're thinking of doing it..."

George "Umm...do one arrow..then multiply by 2 ....and do it again..."

Res. "Remember I want you to make the whole thing any size.....I'm going to let you work it out...OK..."

They continued to negotiate the relationship between the two lengths in their arrow module.

George "Listen....that is twice the size of that in distance right.....so we have FD NUM...and then backward NUM DIV 2..."

Asim "But then we're going to have to put DIV 2 in them".

George "What..."

Asim "Wait a minute......Do you put FD 20 NUM...or FD NUM 20..."

Asim was still referring to his "(S) variable as scale factor" frame. In the meantime George changed the program to become:

```
TO ARROW "NUM
FD :NUM
RT 45
```

Asim "We must have to put a MUL in".

Asim had not understood what George was trying to do. he kept suggesting "MUL" because his "(S) variable as scale factor" frame was predominant.

George *"No we don't...we have a DIV.....Asim 'cos look...what we can do is make it go back to the centre...and then do that again...'cos that is half that..."*

Asim *"What I want to understand OK... — is this arrow supposed to be one arrow like that...forget about the top.....or is it supposed to be like that...or is it supposed to be like that......or is it supposed to ba a whole thing..."*

George *"Yeah..that that that that..*

Asim *"They're supposed to be together...yeah but everything's going to be FD 20 ...oh yes....so that will be the NUM..."*

George *"mmm"*

Asim *"FD whatever it is...."*

George *"FD NUM t.....then it's DIV 2 NUM".*

Now for the first time Asim appeared to be talking from a "(O) variable operated on" frame.

Asim *"BK DIV 2 NUM..."*

They continued with:

```
BK DIV :NUM 2
FD DIV :NUM 2
RT 90
FD DIV :NUM 2
LT 135
END
```

They tried out ARROW 20 (see Fig. 5.37 for completed procedure) which worked. They then tried out:

```
LT 180
ARROW 20
CT
RT 90
ARROW 40
CT
LT 90
ARROW 40
CT
RT 180
ARROW 20
```

They started to define the superprocedure and George said:

George *"Why not change the arrow direction".*

He then defined:

```
TO ARROWPLUS "NUT
LT :NUT
ARROW 20
CT
LT :NUT
```

```
ARROW 40
END
```

They first tried out ARROWPLUS without an input and when this produced an error message they typed:

```
ARROWPLUS 90
```

Res. *"I want you to be able to make them different sizes"*

They immediately changed the program to take two inputs (Fig. 5.37)

```
TO ARROWPLUS "NUT "NUM
CT
LT :NUT
ARROW :NUM
CT
LT :NUT
ARROW MUL :NUM 2
END
```

```
TO ARROW "NUM
FD :NUM
RT 45
BK DIV :NUM 2
FD DIV :NUM 2
RT 90
FD DIV :NUM 2
LT 135
END
```

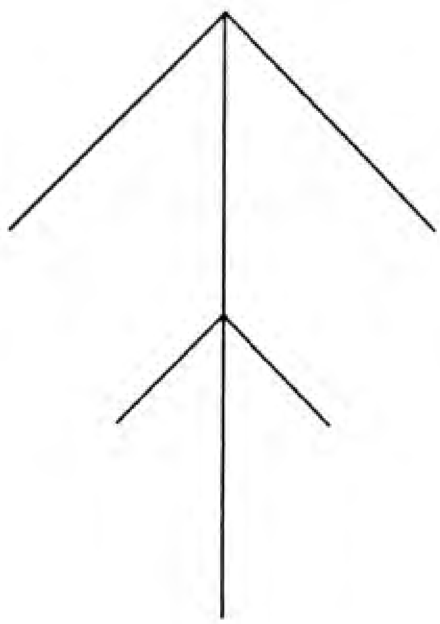


Fig. 5.37: George and Asim - Final Arrowhead Procedure

Thet tried out:

```
ARROWPLUS 90
```

and when this gave an error message typed in:

```
ARROWPLUS 90 20
```

They used this superprocedure to make a pattern of arrowheads all over the screen.

George and Asim had solved the original "Arrowhead" task by analysing the arrowhead into a simpler module. They operated on a variable within their procedure in order to produce this module. It is suggested that at the begining of the session Asim wanted to solve the problem by using variable in the category of "(S) variable as scale factor". George on the other hand wanted to use variable in the category of "(O) variable operated on". There is evidence that George dominated the session from the point of view of resolving this issue. However Asim persistently questioned George until he appeared to accept George's perspective. When George had decided that he wanted to change the orientation of the "Arrowhead" he had no difficulty in adding another variable NUT to represent a range of angle inputs. They did not however, until nudged by the researcher, take on the idea of making their ARROWPLUS procedure draw "any sized" arrowhead. This is possibly because of the original image (Fig. 5.37) which was presented to them. The issue of how to convince pupils that the problem is one of producing a general module when they are only presented with a specific shape needs to be tackled.

George and Asim's solution to the "Arrowhead" task is compared to the solutions of the other case study pupils in section 5.6. An overview of George and Asim's development throughout the three years of the research is presented in section 5.8.

5.4 LONGITUDINAL CASE STUDY: LINDA AND JUDE

Linda is a very friendly, talkative girl who is confident with adults. By the end of the first year of the project the mathematics teacher considered that Linda "*has a more positive and more confident attitude to maths now than when she started and I think this is reflected in her performance. I think she suffered from a lack of confidence in maths in primary school which accounts for her low entry grade*". At the beginning of the project she was not confident about her ability to do mathematics *"I wasn't good at maths at my primary school"* . Although during the first year of the project it was felt that Linda was gaining confidence in her ability to do mathematics this was not consistently maintained throughout the three years (the class did have three changes of mathematics teacher during the three years of the project). From her Logo work it appeared that she was very resistant to any form of number manipulation and by the end of the project she told us*"I'm not too keen on Maths 'cos I don't think I am any good at it"*. She acknowledged her success with her Logo work and positively enjoyed "working out sums" in Logo, but was not able to view her activities at the computer as related in any way to her potential in school mathematics. Linda's Logo work has been very important to her and she told us that when she talks to her friends in other classes about the computer *"they sort of get jealous because they don't do it and they really want to do it"*.

Jude gives the impression of being a quiet boy although he told us that he has been in quite a lot of trouble at school for *"mucking about"* . His mathematics teacher says that *"he is a bubbly personality, tending to mischievous naughtiness with very little ability to concentrate over a period of time"*. He is considered by his mathematics teacher to be *"below average in ability ....his level of motivation depends on the task he is doing, as he sometimes needs constant reminding to concentrate.. yet often gets engrossed in something... there appears to be no pattern to the topic or type of work involved"*. He is rather neutral about mathematics *"I like it alright miss..."* but is more enthusiastic about the Logo activities within his mathematics lessons *"Cos it was more exciting ... miss 'cos you're just doing the same thing everyday when you are writing cards"*. He gives another insight into why the computer is important to him *"It is better than paper to write 'cos it can't get lost as easily as paper"*. His mathematics teacher says *"I think that Logo has improved his ability to concentrate in mathematics"*. By the end of the second year of the longitudinal study it was decided that Linda and Jude were no longer collaborating productively. Linda was paired with another pupil, Elaine. For the purposes of this study Jude remained a case study pupil, although the transcript data was available for him for two years only.

Linda and Jude's first year of Logo programming was spent coming to terms with their idea of defining and editing a simple procedure. Analysis of the first year's transcript data indicated that within this context they had restricted themselves to using angle inputs of 45, 90 and 135. At the beginning of the second year of the project they were given tasks to provoke them to extend their range of angle input.

5.4.1 General Polygon

Year & Session No:	Year 2; session 7
Type of goal:	Well defined abstract.
Category of variable use:	(O) Variable operated on

In this session Linda and Jude were introduced to the idea of variable to draw a general polygon. When this session is compared with a similar session for Sally and Janet (section 5.2.1) it can be seen that Linda and Jude are much less able to articulate the general relationship. They had spent several previous sessions drawing regular polygons and the researcher first asked them to reflect on the relationship between the number of sides and the angle turned:

Linda *"Yeah...you've got to multiply it by 360".*

Jude *"Yeah like the 3 sided figure has to be timesed by 120 to get to 360".*

Res. *"What about 5 sides...how would you work out the angle for that..."*

Jude... *"mmm errrr"*

Res. *"What do the angles have to add up to?"*

Jude *"Err....360..."*

Res. *"So if it's got 5 sides....what's the angle..."*

Jude *"75"*

Linda *"72 and a bit...it was..."*

They were then asked to draw a nine sided regular polygon.

Res. *"What angle will you have to use"*

Jude *"Nine times...I don't know...nine divided by something...360 divided by nine..."*

With help they used the computer to do this division and typed in:

REPEAT 9 [RT 40 FD 40]

Their use of an input of 40 for both angle and distance indicates a possible lack of discrimination between these two inputs. The input of 40 turned out to be too large and so they modified this to 20.

Linda *"I suppose you want a ten sided shape now./..that's easy 'cos it will be 36".*

They tried: REPEAT 10 [RT 36 FD 20]

As the ten sided polygon appeared Linda said:

Linda *"Who'se got the brains today".*

They defined procedures for both the nine sided and the ten sided shape. They next worked on a 20 sided regular polygon.

Linda *"We're going to work out the angle for a 20 sided shape".*

Jude *"Angle's 18".*

They tried: REPEAT 20 [RT 18 FD 10]

The researcher again nudged them into articulating the general rule.

Res. *"If I said to you that I wanted to draw any sided shape no matter how many sides it has...what's the rule..could you tell me what the rule is for working it out?"*

Linda *"You divide how many sides you're doing by 360."*

Res. *"You mean the other way round".*

Linda *"360 by how many sides you're doing".*

Within a teaching episode Linda and Jude were told how to define the following regular polygon.

```
TO POLY "NUM
REPEAT :NUM [FD 10 RT DIV 360 :NUM]
END
```

Fig. 5.38: Linda and Jude: General Polygon Procedure

and showed how to use this procedure. They tried out POLY 12 and POLY 40.

Linda *"Do a little one...do POLY 4".*

This reference to "little" indicates that Linda saw the input as changing the global size of the polygon. She was not aware either of what was being effected in the procedure or of what aspects of the geometrical object were being effected.

Res. *"What will that be?"*

Jude *"A square".*

Linda *"Will it".*

As the square was drawn Linda appeared surprised. They tried out consecutively inputs of 5 to 42 to POLY (Fig. 5.38). The figure itself did not conflict with Linda's misunderstanding about the effect of the variable NUM. At this point the researcher intervened to show them how to define the following recursive procedure.

```
TO POLY "NUM
IF GRQ :NUM 50 [STOP]
REPEAT :NUM [FD 10 RT DIV 360 :NUM]
POLY ADD :NUM 1
END
```

In retrospect this was totally inappropriate and Linda said:

Linda *"I didn't understand all of it".*

There was no evidence from this session that Linda and Jude had understood the idea of using Logo to formalise a generalisation. **Analysis of this transcript indicates that Linda and Jude's understanding of the general relationship was very tentative and "telling" them the Logo formalism for this relationship did not help them develop their understanding. It is possible that if they had been allowed to spend more time "making sense" of the first general polygon procedure this might have helped their developing understanding. They do appear however to have taken on the idea that it is possible within Logo to use a variable to effect the size of geometrical objects.** Linda was not aware that the variable called NUM effected the number of sides of the regular polygon.

5.4.2 Nested Circles

Year & session No:	Year 2; Session 9
Type of goal:	Well defined abstract
Category of variable use:	(I) One variable input

In order to provoke Linda and Jude to use variable again they were asked to reproduce an image of nested circles. Jude immediately knew how to solve the problem.

Jude *"I know how to do it...you do ARCR one size...ARCR another size...and then keep on".*

They typed in:

```
ARCR 5 360
ARCR 10 360
ARCR 15 360
ARCR 20 360
ARCR 23 360
ARCR 25 360
ARCR 27 360
```

They were more interested in the visual effect on the screen than on the mathematical relationship between the "radius" inputs. After they had defined this as a fixed procedure the researcher nudged them into using the idea of variable:

Linda *"What you mean....so untold little ones...*(meaning whatever size you want) *like that".*

Res. *"Do you know how to write a procedure which can make a shape of any size...not only one size..."*

Linda *"No".*

Res. *"Didn't the other teacher show you how to use input?"*

Linda *"No".*

Res. *"I thought you used inputs to make things different sizes".*

It is difficult to understand why Linda was rejecting the idea of variable when two sessions previously she appeared to have accepted this idea. This however was her standard reply when asked by a teacher whether or not she remembered using a mathematical idea previously. The researcher then showed them the following procedure to draw a general square.

```
TO SQUARE "SIDE
FD :SIDE
RT 90
FD :SIDE
RT 90
FD :SIDE
RT 90
FD :SIDE
END
```

The researcher had first negotiated with Linda and Jude the number of FD and the number of turn commands in a square and matched the "teacher given" structure to their own method which was to draw a non state transparent square.

Res. *"See what happens if you put in a negative number"*

They tried SQUARE -88.

Jude *"It went back".*

Linda *"Yeah it went backwards".*

Linda's next comment provides evidence of her motivation

Linda *"I like experimenting like this..it's good".*

They tried SQUARE -8.3 and SQUARE -99.99 using both decimal and negative numbers and then produced a pattern of nested squares, writing a fixed superprocedure to draw these. The researcher then nudged them into defining a general triangle procedure.

Res. *"Do you think you could write a program for any size triangle?"*

Linda's reply indicated a developing understanding of the idea.

Linda *"Well won't it be the same....and just put TRIANGLE SIDE".*

Res. *"What were the commands for a triangle?"*

Jude *"urrrr 120....miss 'cos 360 miss....3 sides".*

They defined a general triangle procedure in the editor indicating a developing confidence with the syntax.

```
TO TRIANGLE "SIDE
FD :SIDE
RT 120
FD :SIDE
RT 120
FD :SIDE
RT 120
END
```

The fact that they made the triangle state transparent, whereas they had not made the square state transparent, is possibly influenced by the work in the previous session with a general polygon procedure. The triangle for them was a special case of a general polygon, whereas the square was not. They then used this procedure TRI to make a pattern of nested triangles. **Linda's final comment indicates that she was engaging in the task and that she was also developing confidence in using variable.**

Linda ***"My we are brainy today......haven't done this much work for ages...***

5.4.3 Variable Letters 1

Year & Session No:	Year 2; Session 11 & 12
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor
	(G) General superprocedure

As was the case with all the case study pupils Linda and Jude were given the "Scaling Letter" task (appendix 3.2). It was just a starting point in provoking them to engage in a range of extended tasks.

Res. *"Now what we want you to do is make your L so that it can be different sizes...instead of 40 you can multiply 40 by something that I've called SCALE...then when you run your program you put in different numbers for SCALE".*

Jude related this to his previous experience of using variable.

Jude *"Like we did before".*

However Linda again rejected the idea.

Linda *"I don't remember".*

The author suggests that her persistent rejection of an idea which it is known that she was previously motivated to use cannot be explained by any of the theories of learning which were presented in chapter 1. She may be rejecting the researcher as an authority figure or she may have learned to "play safe" when asked what she knows about a topic always giving a standard reply of "I don't remember". Linda and Jude modified the L procedure as instructed on the handout (appendix 3.2). Linda was clearly confused.

Linda *"I don't know how to do it.I don't really understand".*

Her lack of confidence at this stage will be contrasted with her confident approach to the use of variable in the category "(S) variable as scale factor" by the end of the project.

Res. *"So tell me what you think happens when you put in a number like 0.5".*

Jude *"Multiplies".*

Linda *"It multiplies on a scale of 40...no with 0.5 it takes away..it decreases on a scale of 40".*

Jude *"Yeah but taking away".*

Linda *"Something like that anyway".*

They clearly did not understand the effect of the scaling variable at this stage. The researcher went through the process of the procedure asking Linda and Jude to work out the lengths of the distance commands for an input of 2. They then decided to define a variable letter E. The researcher told them to draw a specific sized E first and she suggested:

Res. *"When you did this one ...the height of it was 40 before you changed it...so do an E whose height is 40".*

They did not take on this advice when drawing their fixed E, but when later they defined the general E procedure, they used the advice in a way that had not been intended by the researcher. They were developing a general rule for defining a general procedure.

Jude *"You know on the FD's and BK's miss...do we put the scale?"*

Linda *"Do we put the scale?"*

Res. *"Are you confident that this will work".*

Linda *"Well if it works on the L ....I don't see why it shouldn't work on an E".*

Linda's attitude and involvement with the task was clearly changing.

Jude *"Do you put the scale on the BK".*

Res. *"Yeah the FD and the BK".*

Linda had developed a working rule for which commands should be scaled.

Linda *"On the moving commands".*

They defined the general E procedure (Fig. 5.39b) from the direct drive commands in Fig. 5.39a. They had changed all the distance commands to length 40 in the variable procedure (Fig. 5.39b). In making this "spurious generalisation" from the handout (in which all the distance commands were of length 40) they indicated that they did not understand the process of scaling. The researcher intervened:

Res. *"Oh what have you done....you've changed all the 50's to 40's".*

Linda *"Yeah 'cos you said to do it on a 40 scale".*

The researcher had suggested that they should make the fixed E a standard height of 40. They chose to make it a height of 50 and then when scaling by a variable changed all the lengths to 40 (see Fig. 5.39a and 5.39b). They tried E 3 which drew Fig. 5.39c. They also tried E 1 and as in both cases a letter E was drawn the computer feedback did not provoke them into reanalysing their E procedure and finding the bug. They started to make a pattern composed of L's and E's. They were using the L and E module to extend the task for themselves.

a)

```
RT 90
FD 50
LT 90
LT 90
RT 90
FD 50
BK 50
LT 90
FD 25
RT 90
FD 50
BK 50
LT 90
FD 25
RT 90
FD 25
BK 25
```

b)

```
TO E "SCALE
RT 90
FD MUL :SCALE 40
LT 180

RT 90
FD MUL :SCALE 40
BK MUL :SCALE 40
LT 90
FD MUL :SCALE 40
RT 90
FD MUL :SCALE 40
BK MUL :SCALE 40
LT 90
FD MUL :SCALE 40
RT 90
FD MUL :SCALE 40
BK MUL :SCALE 40
END
```

c)

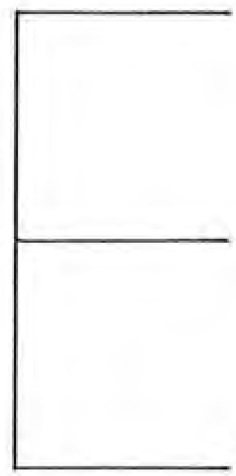


Fig. 5.39: Linda and Jude: Variable Letter E.

The transcript data was not available for the next session but it was known that they used their modules L and E to build up a pattern and then with help defined a general superprocedure LE:

```
TO LE "SCALE
REPEAT 200 [L :SCALE   E :SCALE   RT 12]
END
```

This had evolved out of a building up activity and falls into the category of a loosely defined goal, in that they did not plan this pattern at the beginning of the session. In

Session 13 they again tried LE 2.

Res. *"What does the 2 mean?.*

Linda *"Umm that's the size".*

Jude *"It multiplies the scale".*

Res. *"When you put LE 2 in what does the 2 do?"*

Linda *"It doubles the scale doesn't it..."*

Their responses indicated a developing understanding of variable as "scale factor. They next decided to draw the letter T and worked first in direct drive and then defined a fixed T procedure (Fig. 5.40a). When this was defined Jude said:

Jude *"We forgot to put the SCALE didn't we".*

They modified the T procedure to that given in Fig. 5.40b.

a)
```
TO T
LT 90
BK 25
FD 50
RT 90
FD 25
BK 50
CT
END
```

b)
```
TO T "SCALE
LT 90
BK MUL :SCALE 30
FD MUL :SCALE 30
RT 90
FD MUL :SCALE 30
BK MUL :SCALE 30
CT
END
```

c)

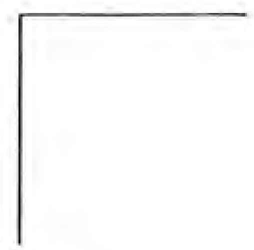


Fig. 5.40: Linda and Jude: Variable Letter T.

They have again deleted the original values in their fixed procedure and replaced all the distances by 30. They still seem to have the idea that all the distances in the scaled procedure must be the same (as they had been in the original given L procedure). They tried T 2 and Fig. 5.40c was produced. They did not understand this and used the computer to trace through their procedure.

Linda *"It's gone wrong there...but I don't see how".*

Jude's reply indicated that he was starting to think about the effect of scaling.

Jude *"We should have put 60 miss..."*

He changed the last BK command to BK MUL :SCALE 60. The inputs to the last two commands of the fixed T were in the ratio 2 to 1. Jude had made the inputs to the last two commands in the general procedure in the same ratio. This seems to indicate that he was beginning to reflect on the relationship between the component parts of the geometrical object. He tried: T 1 T 0.5 T 0.01. This time the procedure (although not a scaled version of the fixed T) drew a T and so the computer feedback had not provoked them to modify line 3 of the procedure to BK MUL :SCALE 60. They then built up another rotated pattern copying the structure of the LE procedure:

```
TO LET "SCALE
REPEAT 200 [L :SCALE  E :SCALE  T :SCALE  RT 10]
END
```

They had not tried the commands out in direct mode before defining in the editor. The pattern did not draw what they expected because of the CT (Centre the turtle) command in the procedure T.

Linda *"I can't see what's wrong with it...miss we've had a look at it and as far as we know there's nothing wrong with it..."*

They spent some time trying to find the bug.

Jude *"Do you know when we did the BK and the 60....I reckon it's something to do with that...."*

This indicates that Jude was not confident about his debugging of the letter T procedure.They were unable to find the bug on their own and finally needed help from the researcher.

Within these three sessionsLinda and Jude had extended the given task for themselves. **Although initially they did not understand the effect of the scaling variable the need to debug was provoking them to focus on the process within their procedure. Jude was beginning to think about the relationship between the distance commands in the original fixed module although there was no evidence that Linda was doing this yet.** In addition they had built up a pattern from their general modules and also wanted to make this "built up " pattern general. This provoked the need for a general superprocedure. Although the superprocedure originally arose from a "building up" activity when they defined the second superprocedure LET they knew what outcome they wanted and therefore were pushed into focussing on process in order to debug their errors. **At this stage in the context of using a variable to scale distance commands they always used the variable name SCALE, which is the name that had been used on the Scaling Letter handout.**

5.4.4 General Square (1)

Year & Session No:	Year 2; Session 16 & 17
Type of goal:	Well defined abstract
Category of variable use:	(I) One variable input

By this stage in the project it was recognised that Linda and Jude were not always collaborating in a constructive way. It was decided to pair Linda with Elaine during the next two sessions. They were working on their own goal of a pattern of squares with hearts in them and had defined a fixed procedure for a square. Elaine wanted to define another fixed procedure for a smaller square and Linda then initiated the idea of defining a general square procedure. This indicated that **Linda understood the idea of using a variable to "make bigger and smaller" and had developed a confidence in using the idea for herself.**

Linda *"Instead of doing that...do this..what you do is...have you done the MUL SCALE".*

Elaine *"This is quicker".*

Linda *"It isn't 'cos you can write this down and then change the size of your shape...miss..We decided to do it like that instead of with hearts so I was thinking as we haven't got much time to write it out as a MUL SCALE...as we did for that..."*

Res. *"Yeah you could do.... so you can make squares of different sizes..."*

Linda *"Yeah that's what we want to do...so we can get them smaller..."*

Res. *"You don't actually have to use MUL in it...you can just use."*

Linda *"Scale".*

Elaine *"Yeah or something...or side..."*

Res. *"Alright what you have to do is instead of saying FD 45 and RT 90...you want to be able to make it FD any number...don't you?"*

Linda *"Yeah so you write FD SCALE".*

Linda had suggested using variable in the category of "(S) variable as scale factor" but the researcher thought that in the context of defining a general square procedure " (I) one variable input" would be more appropriate. However the variable used was called SCALE which cannot have helped Linda to discriminate between her "(S) variable as scale factor" frame and her "(I) one variable input" frame.

```
TO LE "SCALE
REPEAT 4 [FD :SCALE RT 90]
END
```

Fig. 5.41: Linda and Elaine - General Square Procedure (1)

Her next comment indicated her confusion with the procedure LE.

Linda *"Don't we have to use that MUL?"*

It was the end of the session and they used their LE procedure with inputs of 3, 22.5 and 30. In the next session they again wanted to use LE and Linda said:

Linda *"Yeah remember we changed the LE to SCALE".*

They tried LE 4 LE 8 LE 16 LE 32 LE 64. Elaine was not happy with any of these images and suggested that they define a fixed square procedure.

Elaine *"No that's not right...scrap it...and we'll do 45 by 45. ....see we can't do LE 45...'cos that will make it like that and we want it like that..."*

Elaine wanted a "left turning " square and LE was a "right turning" square. Linda however wanted to use the general procedure.

Linda *"Get it down to there and then draw LE 30....whatever and draw the square".*

These two sessions were important because **for the first time Linda had initiated the idea of using variable. It is suggested that her engagement with the "Scaling letters" task had been a critical step in her accepting of the idea of a variable in Logo although at this stage she had not discriminated between "(S) variable as scale factor" and "(I) one variable input".**

5.4.5 Row of Pines

Year & Session No:	Year 2; Session 18
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor

The researcher aimed to give this task (appendix 3.4) to all the pairs at the end of the school year. Jude and Linda were paired together again in order to carry out this task. This session illustrates how Linda was still modifying the values used within her fixed procedure, in addition to scaling all the distance commands. Linda immediately initiated the idea of using variable as scale factor.

Linda *"We just have to do one don't we and then make a procedure and do Scale".*

Jude *"Yeah".*

In direct drive they had produced the commands for a pine tree given in Fig. 5.42a. and they then negotiated how to define a general procedure.

Linda *"Miss how do you do a scale thing..I've forgotten".*

Res. *"Well how do you want to change it?"*

Linda *"Well 'cos we have to do that pattern...I thought we might as well do Scale and then we can make it as big or as small as we want".*

Res. *"What you normally do is if you're using Scale like that...you say FD and multiply Scale by whatever length you want it to be..."*

Whilst working on the "Scaling Letters "task Linda and Jude had changed the lengths of their fixed module as well as scaling by a variable. They seemed to think that this was necessary and although they had already drawn a fixed "pine tree" in direct mode they now wanted to negotiate the lengths for their general procedure.

Linda *"What shall we multiply it by...4?"*

Jude *"8.....no 16..."*

Linda *"Multiplied by 16..it'll be massive ..."*

Linda appears to mean that if the length of the "tree trunk" were 16 and they then scaled this distance by a variable amount they could end up with a "massive" tree. She appears to be thinking of inputs as positive whole numbers only.

Fig. 5.42b gives their final general procedure. They had completely changed the lengths of their original fixed pine tree. They were obviously reflecting on the effect of using a scaling variable. They were not concerned with the exact internal ratio between their original 100 for the "trunk" and 30 for the "branches" but they did preserve a "good enough" ratio. The respective lengths in their general module were 8 and 4.

a)

```
LT 90
BK 100
FD 100
RT 45
BK 30
FD 30
LT 90
BK 30
```

b)

```
TO LINDA "SCALE
LT 90
BK MUL :SCALE 8
FD MUL :SCALE 8
RT 45
BK MUL :SCALE 4
FD MUL :SCALE 4
LT 90
BK MUL :SCALE 4
END
```

c)

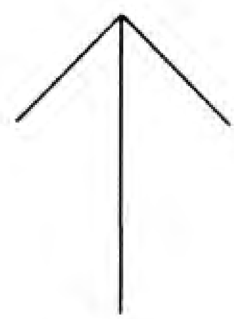


Fig. 5.42: Linda and Jude: Pine Tree

They tried LINDA 4 and LINDA 8.

Res. *"So what does the scale do...what does Linda 8 do?.*

Jude *"Err it tells the program...errr".*

Res. *"What does the 8 do?*

Jude *"8 multiplies the 8".*

Res. *"So how far is that distance there?"*

Jude *"That is 36".*

Linda *"Oh it's 8 8's is 64..."*

Res. *"So what about this one?"*

Linda *"4 ....8's...4 ....8's are 32".*

This interchange again provides evidence that they had reflected on the effect of the scale factor. The researcher again asked them:

Res. *"What does MUL SCALE 4 do?*

Linda *"It multiplies it by 4...whatever I type in.*

It is suggested that one reason why they were provoked into modifying the distances in their fixed module before scaling was Linda's reluctance to use a decimal input to make the pine tree smaller. **This session provides evidence that they are beginning to reflect on the effect of the variable on the constituent parts of the graphical object.** They still use the variable name SCALE. They successfully used their general module (LINDA) to draw the row of decreasing pine trees in direct mode.

5.4.6 <u>Variable Letters 2</u>

Year & Session No:	Year 3; Session 1,2 & 3
Type of goal:	Well defined/ loosely defined abstract
Category of variable use:	(S) Variable as scale factor
	(N) More than one variable input
	(G) General superprocedure

It was decided at the beginning of the third year of the project to pair Linda with Elaine. Elaine's previous experience of variable was restricted and so they were both again given the variable letter task. Linda was specifically asked to help Elaine. They worked on the L on the handout (appendix 3.2). They then decided to draw the letter "g" which finally ended up as a "q". In direct drive they produced the following commands (Fig. 5.43a):

a)

```
ARCR 7 360
PU
FD 7
RT 90
FD 3
PD
FD 30
LT 135
FD 10
BK 10
RT 135
BK 33
LT 90
BK 7
```

b)

```
TO QU "SCALE
ARCR MUL :SCALE 7 360
PU
FD MUL :SCALE 7
RT 90
FD MUL :SCALE 3
PD
FD MUL :SCALE 30
LT 135
FD MUL :SCALE 10
BK MUL :SCALE 10
RT 135
BK MUL :SCALE 33
LT 90
BK MUL :SCALE 7
END
```

Fig. 5.43: Linda and Elaine - Variable Letter q.

They then defined the general procedure. After help with the first ARCR command they produced the procedure given in Fig. 5.43b and tried out QU 2 which worked. They had not changed the original lengths of their fixed procedure as Linda had always done previously. This could be because Linda was developing an understanding of the idea of scaling the distance commands of a fixed shape. In the next session they typed in QU 2 again and the researcher asked them to reflect on the process within their procedure:

Res. *"Good now tell me what the 2 does?"*

Elaine *"I dunno...what do you put the 2 for Lee?"*

Linda *"It makes it bigger...it times it by 40 whatever it was at 40..it makes it 80 or whatever".*

Linda's reply indicated that she understood that each distance amount was multiplied by the value of the variable. However she referred to a distance of "40" which was the height of the fixed L in the original handout (Fig. 5.4).There was no length of 40 in the fixed q module.

Elaine *"Oh just multiply by 2".*

They built up a rotated pattern using L 1 and QU 2 and defined:

```
TO QE2
REPEAT 8 [QU 2  LT 45]
END
```

The researcher suggested that they make QE2 general.

Res. *"Do you think you could make your QE2 into a program which you could make bigger or smaller...by scaling".*

Linda *"Easy".*

They did not define a general superprocedure but modified the fixed QE2 to another fixed procedure and changed the name to QE3.

```
TO QE3
REPEAT 8 [QU 3  LT 45]
END
```

Res. *"What if you wanted to do it the same way as you made the original q bigger...could you make your QE2 bigger by using SCALE instead of putting 2 or 3 or 10 or 15...you put a word in".*

They were not sure how to do this so they were shown how to define:

```
TO QE2 "JIM
REPEAT 8 [ QU  :JIM  LT 45]
END
```

By now the researcher realised that they were attaching too much significance to the name SCALE and suggested another variable name. **Analysis of the transcript data indicates that many of the teacher interventions contributed to this overinterpretation of the name SCALE.** They were initially confused by this change of variable name and typed in:

QE2 JIM

using the word JIM for the input. After an intervention they tried QE2 0.2 and QE2 0.1, using decimal inputs because they wanted a small pattern. At the beginning of the next session they tried to use QE2 again without giving it an input. They had not understood the general nature of this superprocedure. However the computer response provoked Linda to reflect:

Linda *"Oh so what we've got to type is QE2 3 or whatever ".*

They tried inputs of 3 and 0.5.

Res. *"What does the 0.5 do?"*

Linda *"It makes it really small....what about that little one we did...it was really small wasn't it...0.2 wasn't it..."*

Linda appears to understand variable as making bigger or smaller. They then defined the following procedure:

```
TO LE2 "TIM
REPEAT 8 [L :TIM   LT 45]
END
```

Now that they knew that the variable name did not always have to be SCALE they had introduced another new name. They built up a pattern with QE2 and LE2 and the researcher showed them how to define:

```
TO EL2 "JIM "TIM
QE2 :JIM
LE2 :TIM
END
```

They had now defined two levels of nested general procedures. They had for the first time used two variable inputs in their general superprocedure. When they first used EL2 they only assigned a value to one of the variables. After an intervention they inputs of 2 2, 3 3, 1 1 ,0 0. Although they had used two separate inputs they always assigned them the same value. They were however very pleased with the result.

Linda *"You watch...it's fantastic.."*

Elaine *"We done it miss..."*

Linda *"We done it....that is with L's and Q's..."* (Fig. 5.45)

```
TO PARTYTIME
EL2 2 2
PU
RT 180
FD 120
RT 90
FD 70
RT 90
PD
EL2 0.5 0.5
PU
RT 90
FD 120
LT 90
PD
EL2 0.5 0.5
PU
FD 230
PD
EL2 0.5 0.5
PU
LT 90
FD 130
RT 90
PD
EL2 0.5 0.5
END
```

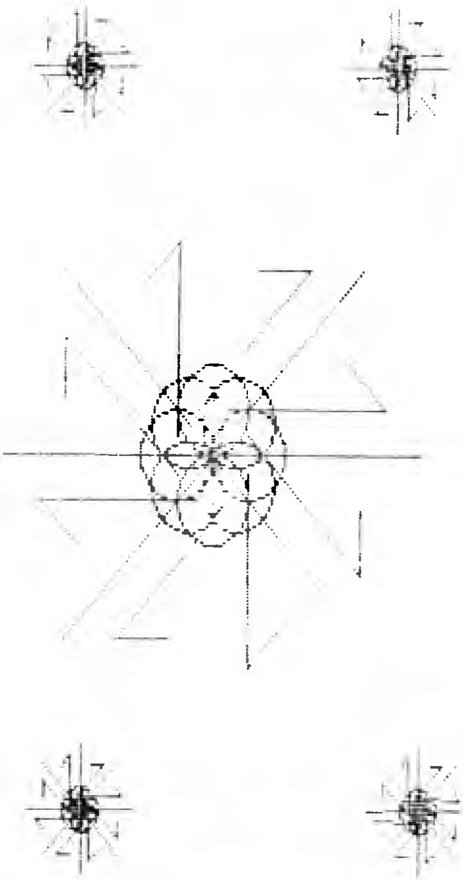


Fig. 5.44: Linda and Elaine - Procedure for Partytime **Fig. 5.45:** Partytime

The image on the screen had provoked them to extend the project and use the EL2 procedure with the inputs 0.5 0.5, another example of them using decimal inputs. In their next session they made a pattern with their EL2 procedure and define a fixed superprocedure PARTYTIME (Fig. 5.45). **Again the initial given task had provoked them into needing to use variable in the category of "(G) general superprocedure" and "(N) more than one variable input".** They had used a different variable name in the subprocedure QE2 from the name used in the subprocedure LE2 and probably thought that this was necessary. They had matched the name of the variable in the general superprocedure to the name of the variable in the general subprocedure. This was possibly why they wanted to use two variables in the general superprocedure.

5.4.7 Whynot

Year & Session No: Year 3; Session 4

Type of goal: Well/Loosely defined abstract

Category of variable use: (S) Variable as scale factor

At the beginning of the session the researcher suggested that they write a procedure to draw a variable letter "Y". Almost immediately they negotiated the name of their procedure. Naming seemed to be very important to them.

Linda *"And I know what we can call the program".*

Elaine *"WHYNOT".*

Linda *"WHY".*

Elaine *"I've got a brilliant one..WHYWHYNOT..because it will be one whole word..'cos there won't be a space".*

In direct mode they drew the letter Y using the commands given in Fig. 5.46a. When this was finished Elaine said:

Elaine *"So I want to scale it".*

Linda *"Yeah if you're going to put scale in it you've got to do a procedure".*

Res. *"You're going to call it SCALE...you could call it anything".*

Linda's reply indicates that she seems to have accepted the idea that any name can be used.

Linda *"Yeah I know...we called it TIM last time..didn't we".*

As they started to define the variable Y procedure there was again evidence that Linda was reflecting on the effect of the scale factor and thinking about the values that they would eventually assign to the variable input.

Linda *"Shall we put two in SCALE?"*

Elaine *"That will make it about that big".*

They defined:

a)

```
RT 90
FD 50
RT 180
FD 50
RT 45
PD
FD 35
BK 35
LT 90
FD 35
BK 35
RT 135
```

b)

```
TO WHY "SCALE
RT 90
FD MUL :SCALE 50
RT 180
FD MUL :SCALE 50
RT 45
PD
FD MUL :SCALE 35
BK MUL :SCALE 35
LT 90
FD MUL :SCALE 35
BK MUL :SCALE 35
RT 135
END
```

Fig. 5.46: Linda and Elaine - Why

When the procedure was defined they tried WHY 2 and WHY 1.3 again using decimal inputs. They used their strategy of building up a rotated pattern from the Y procedure using the command REPEAT 12 [WHY 0.5 RT 30]. In direct mode they produced a pattern of different sized rotated "y's" all over the screen. They then defined a fixed superprocedure WHYNOT to draw this pattern (Fig. 5.47).

```
TO WHYNOT
REPEAT 12 [ WHY 0.2 RT 30]
LT 180
PU
FD 200
PD
REPEAT 12 [WHY 0.2 RT 30]
RT 180
PU
FD 100
LT 45
FD 100
LT 45
FD 100
PD
REPEAT 12 [WHY 0.3 RT 30]
RT 180
PU
FD 100
RT 90
FD 100
PD
REPEAT 12 [ WHY 0.3 RT 30]
PU
BK 200
REPEAT 12 [WHY 0.3 RT 30]
PU
FD 100
LT 90
FD 100
LT 90
FD 100
PD
REPEAT 12 [WHY 0.3 RT 30]
END
```

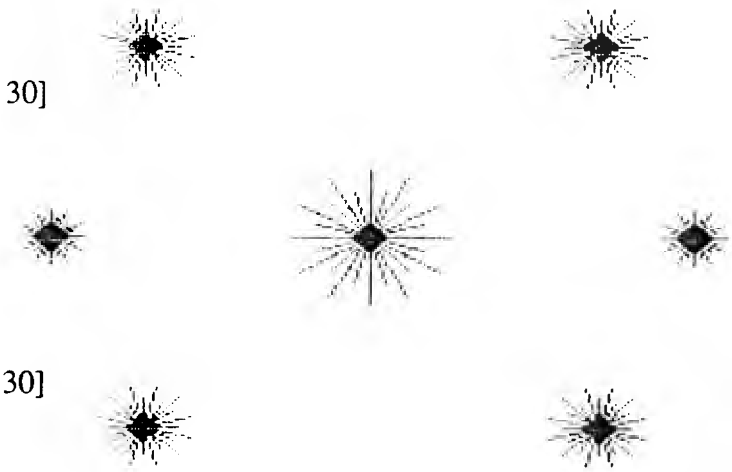


Fig. 5.47: Linda and Elaine - Whynot

5.4.8 General Square (2)

Year & Session No: Year 3; Session 5

Type of goal: Well defined abstract

Category of variable use: (S) Variable as scale factor

This session was aimed at consolidating Linda's understanding of 360 as a total turn. Linda and Elaine were asked to draw a square and use this square to make a complete rotated pattern investigating the link between the number of REPEATs and the turtle turn in the complete pattern. They used the following command to draw a square:

REPEAT 4 [FD 30 RT 90]

The researcher interevened to suggest that they make this square variable sized.

Elaine *"Alright you do this...cos we'll have to think and I can't remember the way".*

They typed in TO SQUARE

Linda *"Now we've got to make this thing any size".*

Elaine *"You haven't put anything up like....just do it exactly the same as if it was...as if we were doing it with the q or a y or something".*

They typed in TO SQUARE "SCALE.

Linda seemed to be trying to integrate her two variable frames.

Linda *"Can we do that with SCALE in as well?"*

Res. *"Yeah".*

Linda *"How do we manage that then..REPEAT 4... Umm scale...no forward....oh gosh.....I don't know if this is going to work".*

Without any intervention she started to define:

```
TO SQUARE "SCALE
REPEAT 4 [FD MUL :SCALE 2.......
```

and then Elaine said:

Elaine *"That will make it big" .*

So Linda changed the procedure to that given in Fig. 5.48a.

a)
```
TO SQUARE "SCALE
REPEAT 4 [FD MUL :SCALE 1  RT 90]
END
```

b)
```
TO SQ  "LEN
REPEAT 4 [FD :LEN RT 90]
END
```

Fig. 5.48: Linda and Elaine - General Square (2)

This reduction of the "amount to be scaled" to 1 could indicate that Linda had consciously reflected on the effect of the scale factor. Her procedure SQUARE was in fact equivalent to a procedure (see SQ detailed in Fig. 5.48b) in which one variable input is used. She then however used an input appropriate for a Scaling frame.

SQUARE 1

The computer response provoked her to change this to

SQUARE 3

They laughed as another "dot" was produce:

Elaine *"Put SQUARE 5 or something".*

Res. *"What does all that MUL business do then?"*

Linda *"It multiplies whatever number you put in".*

Elaine *"Well when you do the MUL it scales it down".*

Res. *"Do you have to put SCALE or could you use anything?"*

Linda *"You could use anything....Elaine ...Linda...Lulu".*

They finally used SQUARE 15 SQUARE 40 and SQUARE 25 SQUARE 50 in the context of drawing rotated patterns.

Within this session Linda defined a general square using " (S) variable as scale factor". The computer response confronted her two existing frames, that of "(I) one variable input" and that of "(S) variable as scale factor". In reducing her procedure so that the scale factor scaled a unit square she was defining a procedure which had the same effect as if she had used "(I) one variable input".

5.4.9 Arrowhead

Year & Session No:	Year 3; Session 7
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor
	(N) More than one variable input

This was Linda and Elaine's last session of the project. As were all the case study pairs they were given the "Arrowhead" Task (appendix 3.3). The task was given after the pupils had been given the individual laboratory tasks (described in detail in section 5.7). When presented with the "Arrowhead" Task their initial strategy was to draw a fixed arrow in direct drive. They produced the following commands (Fig. 5.49).

LT 90
PU
BK 50
PD
FD 100
LT 45
BK 60
FD 60
LT 90
FD 60
PU
LT 135
FD 42
RT 90
FD 20
LT 45
PD
FD 30
BK 30
RT 90
FD 30

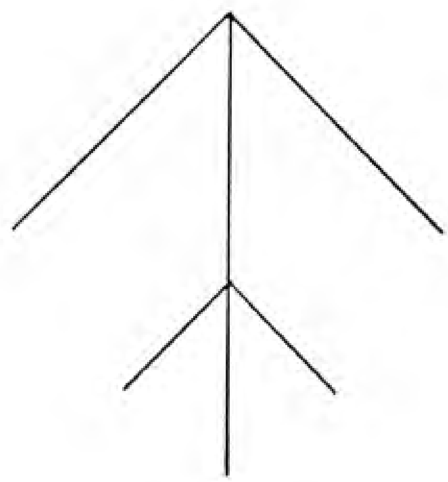


Fig. 5.49: Linda and Elaine - Arrowhead in Direct Mode

When this was finished they started to define a procedure by typing TO TREE at which point Elaine said:

Elaine *"We've got to make it on SCALE...how do we do that though....JIM...that's what that guy was called..."*

She typed TO TREE "JIM

Elaine *"Are you sure that's all we do?"*

Linda *"Yeah and then when the forwards come.....you change it to..."*

Elaine *"What a number ....or JIM...."*

Linda *"Yeah JIM......."*

Elaine recognised that all the forward distances were not the same length and said:

Elaine *"Yeah but the forwards aren't all the same...."*

Linda who seemed to be thinking from her "Variable as Scale Factor" frame said:

Linda *"Yeah it doesn't matter..."*

Elaine *"Are you sure ...miss...when we change the forwards to.....we changed them to JIM....but the forwards aren't all the same size so will that make a difference..."*

This task was designed for the researcher to elicit the pupils' understanding of variable and the researcher pushed Elaine back into making her own decision. As a result of the non-intervention Linda and Elaine had to resolve the conflict on their own.

Res. *"You have to work it out.....you decide...."*

Linda *"So what do we do?"*

Elaine *"Ummm I'm just trying to think what we did......shall we just try* *it...but I'm sure it's not going to work if we do it like that....go and ask Miss...'cos it will save us a lot of time....."*

Linda *"Miss come and help us...."*

Elaine *"Can you have two names at the top?"*

Elaine wanted to use two variable inputs.

Linda *"Yeah".*

Elaine *"So what are we going to have.....are we going to have FD MUL...or FD JIM...."*

Linda *"FD JIM".*

Elaine *"OK....so call the other one MARK......"*

TREE "JIM "MARK

The naming of the variables in the title line seems to be important in helping them to plan their use of variables in a general procedure.

Linda *"Oh we 've got to put something in the BK's as well....it's all the move commands...the drawing commands.."*

Linda was specifically referring to her "(S) variable as scale factor" frame.

Elaine *"But the BKs are different...'cos we've got BK there and Bk there..."*

Linda *"I know...but it's all the moving commands...the drawing ones..."*

Linda had developed a working rule of "scale all the moving commands".

Elaine *"So that means we've got to have three names up there".*

Linda *"No you don't have to have three names..I don't think..."*

Elaine *"I can't remember how to do it".*

They did not feel confident to resolve the conflict themselves and Linda tried to call George over saying "*George ...are you good on the computer...?*"At this point Elaine decided that they could work it out for themselves:

Elaine *"No...just do it like that Linda"*

Linda showed her lack of confidence

Linda *"I hope I'm doing this right...."*

Linda typed in:

BK JIM :SCALE

her use of syntax indicating a real confusion. She has used the variable name SCALE although they had decided to use JIM and MARK. Linda wanted to scale a specific amount.

Linda *"What we going to have for scale size too....."*

Elaine did not understand this:

Elaine *"You decide the scale when you do it..."*

Linda *"Yeah but you've got to put something in there..."*

Linda now changed the line to:

BK MUL :JIM

She was trying to reconstruct the syntax.

Linda *"There see BK MUL...and then you've got to have a number here..."*

Linda finished the line:

BK MUL :JIM 30

Elaine then noticed that they had used BK 50 in their written record and said:

Elaine *"Oh you've got BK 50...so maybe we should have 50....yeah just keep the same numbers as we've got in there..."*

The line now became:

BK MUL :JIM 50

Elaine was not sure:

Elaine *"This isn't going to work....I can tell you now...FD MUL colon JIM.....or is it MARK.....oh I'm willing for BK to be MARK and forward to be JIM...."*

Linda *"Why is JIM very forward..."*

Elaine *"No".*

Linda *"So is the forward MARK...."*

Elaine *"Yeah...oh no this isn't right...oh let's just do it anyway.....and we see what sort of weird shape we come out with..."*

```
PD
FD MUL :MARK 100
LT 45
```

Elaine *"Oh so BK's JIM...."*

They typed in:

```
BK MUL :JIM 60
FD MUL :MARK 100
LT 90
FD MUL :MARK 60
PU
LT 135
FD MUL :MARK 42
RT 90
FD MUL :MARK 20
LT 45
PD
FD MUL :MARK 30
BK MUL :JIM 30
RT 90
```

Linda *"Who's forwards Mark...."*

Elaine *"Yeah..."*

```
FD MUL :MARK 30
END
```

They had devised a strategy of using the variable name MARK for the forward commands and the variable name JIM for the backward commands. The completed procedure is given in Fig. 5.50.

They then typed in TREE without any inputs. They did not understand the error message and typed TREE again...which again produced an error message. At which Linda said

Linda *"I haven't done this for ages..."*

They tried TREE again and then Linda suddenly said:

Linda *"Oh you dozy trollop....you type in tree and a number..."*

This indicates their ability to work things out for themselves from the computer response. They tried TREE 15 and this still produced an error message. The researcher intervened.

Res. *"If you've got 2 inputs....you have to put two numbers in ...don't you..."*

The image was too big for the screen.

Linda *"Make it a bit smaller this time....so we can see it...make it a lot smaller....do it 5 5....."*

They tried TREE 0.1 0.1

Elaine *"Hey it worked...it worked..."*

Res. *"Why ....did you think it wouldn't..."*

Linda *"Cos Elaine thinks I'm stupid......I don't think I'm stupid.....but she thinks I'm stupid..."*

They tried:

```
TREE 1 1
TREE 0.5  0.5
TREE 1.5 1.5
```

They had completed the task.

The researcher wanted to provoke them into reflecting on whether or not the variables MARK and JIM had to be the same value. She suggested that they try TREE 1 0.5 which did not draw a tree. She then asked Linda and Emma if they thought that JIM and MARK had to be the same value.

Linda *"Yeah".*

Res. *"Why?"*

Elaine *"Because we went backwards and forwards like that..."*

Linda *"And it would make a difference if they weren't the same length...."*

Res. *"So do you think they should both be the same name or a different name..."*

Linda *"The same name.."*

Res. *"So use the same name.."*

Elaine *"So we have to change the names....do you want us to change the names on the top...or just on the...."*

Res. *"You decide...do you need one name ...or two names now..."*

Elaine *"Umm two ....I think we do...'cos of BK and FD..."*

Res. *"Yeah but the distance is the same for backwards and forwards.....you go the same distance..."*

Elaine *"Oh".*

Res. *"When you make those two different values JIM and MARK.....does it draw t the right sort of shape..."*

Elaine *"No see we got one that's 42....so that's totally different from the rest of them...."*

Elaine has difficulty in discriminating between what is varying and what is invariant. Linda seemed to be more able to take on the idea and started to change all the variable names to JIM...

Res. *"Explain why you are calling them all JIM..".*

Elaine *"Cos they're all the same.."*

Linda *"Cos it won't make any difference.."*

Elaine *"Yeah but what do we do...when we come to the forward 42.....and it's a MUL...."*

They then tried TREE 1 and found that it worked.

```
TO TREE "JIM "MARK
BK MUL :JIM 30
PD
FD MUL :MARK 100
LT 45
BK MUL :JIM 60
FD MUL :MARK 100
LT 90
FD MUL :MARK 60
PU
LT 135
FD MUL :MARK 42
RT 90
FD MUL :MARK 20
LT 45
PD
FD MUL :MARK 30
BK MUL :JIM 30
END
```

Fig. 5.50: Linda and Elaine - General Arrowhead Procedure

This was Linda and Elaine's last Logo session within the three years of the research. Linda appears to have taken on the idea of using variable as a scale factor and in using variable in this way she does not necessarily have to reflect on the relationship between the component parts of the geometrical object. At the beginning of this session Linda wanted to use variable in the category of "variable as scale factor" and Elaine wanted to use variable in the category of "(N) more than one variable input". They resolved their disagreement without any outside intervention and produced a working solution to the problem. Their solution however indicated a transitional stage in their thinking about variable. There is no evidence that Linda or Elaine yet integrated their " (S) variable as scale factor" and "(N) more than one variable input" frames.

Linda and Elaine's solution to the "Arrowhead" task is compared to the solutions of the other case study pupils in section 5.6. An overview of Linda and Jude's development throughout the three years of the research is presented in section 5.8.

5.5 LONGITUDINAL CASE STUDY: SHAHIDUR AND RAVI

Shahidur and Ravi were not case study pupils throughout the three years of the project. Shahidur joined the project in the second year when he was paired with Ann. Then at the end of the second year Ann left the school and Shahidur started to work with Ravi. It is only Shahidur and Ravi who have been case studied for this thesis. However it is important to follow Shahidur's progress with Ann before he started to work with Ravi and so these sesions will be presented in detail. Unfortunately only one year of longitudinal transcript data is available for Ravi.

When Shahidur started secondary school he was a very quiet boy who hardly spoke any English. He often missed mathematics lessons so that he could attend an "English as a second language" lesson. He is rather small for his age and certainly at the beginning of secondary school was not a pupil who would be easily noticed by a teacher. By the third year of the project his English and his confidence had improved remarkably. This did result in him becoming more disruptive in class. He was always very enthusiastic about using the computer and when asked what he like most about his mathematics lessons he said *"computing"* . His reply to what do you like least was *"homework"* . He became very keen to explore the computer system and at one point was banned from using the computer for several weeks because he had succeeded in erasing some programs from the class disk. At the beginning and throughout the three years of the project his mathematical attainment within the class was very low. In response to the question *"What do you think your mathematics teacher thinks about your maths"* he gave us the impression that he thought that the teacher gave him work which was too easy *"even though I could do. it...but I was still doing mistakes... "*. He loved drawing realistic images in turtle graphics *"'cos I'm quite good at drawing ..I draw the picture and I can do it.. a picture in Logo is easy"*. He also very much preferred to choose his projects himself *"so I can do what I want and what I like"*.

At the beginning of the project it was noticed that Ravi appeared to be very disruptive and did not find it easy to settle in class. His mathematical attainment was very low and although this improved throughout the project his attainment was still below average with respect to the rest of the class by the end of the project. Although he was not initially a case study pupil it was noticed that during the beginning stages of learning Logo he often became very frustrated by his work at the computer. He set himself very high standards and became angry by what he perceived as his failure to reach these standards. There was however a remarkable change in his computer work as he began to accept the debugging powers of Logo. His concentration level when working in Logo far exceeded that exhibited by him during his "normal" mathematics work. His favourite subject at school is graphical communication and when we asked him what that was he said *"It's all to do with architecture really.........that's what I want to be..."* . He told us that maths was his favourite subject although he talked about being *"only on level 4"* . He said that his teacher *"thought that I was a bit talking too much... but I got on with my work when I wanted.... and did a lot of homework"*. He prefers to choose his own projects and he also prefers to work with a partner.

5.5.1 Variable Letters 1

Year & Session No:	Year 2: Session 4,5 and 6.
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor.

This was Shahidur and Ann's first introduction to the idea of variable and they were introduced to the the variable letter task (appendix 3.2). Ann immediately asked how she could multiply by SCALE when the computer did not know what number it was. The researcher said that when they used the procedure they would tell the computer what number to use. Ann was amazed by this idea. They defined a fixed L and then when told to introduce a variable Ann again said:

Ann *"How can you do that if you don't know the number?*

Res. *"Well when you run the program you put a number in and SCALE becomes the number you tell it".*

Ann *"You can pick any number you want".*

```
TO  L "SCALE
LT 90
FD MUL :SCALE 40
BK MUL :SCALE 40
RT 90
FD MUL  :SCALE 40
BK MUL :SCALE 40
END
```

Fig. 5.51: Shahidur and Ann - Variable Letter L

Ann's remark illustrates her disbelief in the idea of using a variable to represent a range of numbers. The researcher showed them how to define a scaled L procedure (Fig 5.51). They tried L 5.

Res. *Try something a bit smaller to start with...try it with all sorts of numbers...and I want you to try it with some decimal numbers......like 0.5. see how big you can make it and how small you can make it".*

They tried L 1.0 then L 0.1 then L 0.01 and finally L 9.9

Ann *"Our littlest one is 0.1 and our biggest one is going to be 9.9."*

Ann wanted to try L 8.8 to which Shahidur said:

Shah. *"You can't have 8.8 'cos it will be the same size...this way should be smaller.."*

Shahidur had interpreted the decimal input 8.8 to be a code in which the first 8 effected the size of the vertical part of the L and the second 8 effected the size of the horizontal part of the L. They next tried L 8.7 L 7.5 L 4.2

Shah. *"Ah it's good....4.2 is that..."*

Ann *"Well we can make it bigger except it wouldn't look right...that's supposed to be shorter than that and it would look too long".*

The researcher asked them what the scale did and Ann said:

Ann *" It makes it go bigger or smaller".*

Res. *"And do you know how it does it?"*

Ann *"No".*

The process was explained to them.

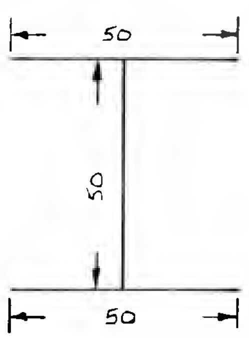
Res. *"It takes the 4.2...puts it in there....and then multiplies the 40 by 4.2. and then the next line it does the same..."*

They next drew a letter I in direct drive. When they wanted to define a procedure Ann said

Ann *"Miss you know on the "I"...will I put SCALE."*

However they first defined a fixed I procedure and then with the help of the researcher modified this to become a scaled I procedure (Fig 5.52c).

a)



b)

```
TO I
LT 90
FD 50
LT 90
FD 50
BK 100
FD 50
RT 90
BK 100
LT 90
FD 50
BK 100
END
```

c)

```
TO I "SCALE
LT 90
FD MUL :SCALE 50
LT 90
FD MUL :SCALE 50
BK MUL :SCALE 100
FD MUL :SCALE 50
RT 90
BK MUL :SCALE 100
LT 90
FD MUL :SCALE 50
BK MUL :SCALE 100
END
```

Fig. 5.52: Shahidur and Ann - Variable Letter I

They tried out their "I" procedure:

Shah. *"Now 0.01"*

Ann *"No this side has to be bigger than that one...1.0...no...0.10...where would you say..."*

Ann was still confused about the effect of the decimal "code". They tried I 0.01 and then I 4.3 which went off the screen. They could not understand this because L 4.2 had not gone off the screen. The researcher asked them to reflect on the height of their fixed I in comparison with their fixed L.

Res. *"Ah so it's 100 that way...so if you multiply 100 by 4..2what do you get".*

Shah. *"I dunno..I'm no good at multiplying".*

Ann *"Oh yeah so 1.1 will be the same length as we did...I would think".*

The fixed L had a height of 40 and then when they used an input of 4.2 to the scaled L procedure the height drawn was 168. The height of their fixed I was 100 and Ann appears to have estimated that an input of 1.1 to the scaled I would make the I about the same height as the L with an input of 4.2. It actually made the height of the I, 110, as opposed to 168 for the L. It is suggested that the order of the error indicates that Ann was reflecting on the process within the procedure.

At the beginning of the next session Ann and Shahidur used their scaled I procedure again. They were still confused about decimal numbers. They tried L 0.1 and Shahidur said

Shah. *"It's the other way round".*

Ann *"No it was 0.01 but that only made a line this way and it didn't make a line this way did it".*

They tried L 0.01 which produced a small dot. The researcher explained that there was a vertical and horizontal part of L being drawn but that the image was so small that the vertical component could not be seen.

Shah. *"Yeah miss but it can't go that way because you know we haven't done a number for this way...we just done a number for this way"* (meaning we've only done a number for the horizontal part).

The researcher again said that both parts were drawn but they remained unconvinced. They next tried I 4.2.

Shah. *"Miss how come it hit the edge and when we done the L it didn't?"*

Shahidur still did not understand about the process within the L and the I procedure. The researcher again explained that the vertical height of the fixed L was 40 and that the vertical height of the fixed I was 100. The researcher then suggested that they could modify their I procedure (Fig. 5.52c).

Res. *"So instead of making that 50 50 100 50 50 100 if you made it 20 40..".*

Ann *"So make all the 50's 20 and all the 100's 40".*

They made this modification and then tried I 4.2 and then L 4.2 and decided that they were about the same height. They next worked on an E in direct drive.

Res. *"Do you remember how your I was too big...would it be better to make your E the same sort of proportion as your I....because that was just 40 this way and 20 that way...wasn't it.".*

Shah. *"Yeah alright miss..."*

They typed in LT 90 (turtle now pointing vertically upwards) and then negotiated the first distance

Ann *"So what do we do...that's going to be 20 isn't it?"*
FD 20

Ann *"Let's draw it on a bit of paper".*

Res. *"And what do you want these distances to be?"* (meaning AE)

Ann *"40"...we want that one to be about 25...*(AB) *that one 25..*(EF)*.and that one 20"* (CD) (Fig. 5.53).

Shah. *"No because L was 20 that way and I was 20 that way".*

Ann *"I know so this is 25 because that's smaller isn't it.".*

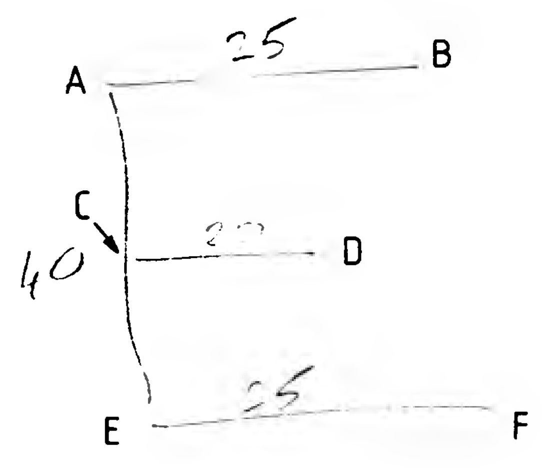


Fig. 5.53: Shahidur and Ann - Planning for General E Procedure.

After this negotiation they typed in the commands given in Fig. 5.54a in direct drive. They then defined the E procedure and Ann immediately said:

Ann *"No hold on...this needs SCALE dosn't it.."*

Without any intervention they defined a scaled E procedure (Fig. 5.54b).

They tried E 4.2 (their favoured input)) and the E procedure worked first time. They then used their general procedures to build up a pattern on the screen.

a)

```
LT 90
FD 20
BK 40
RT 90
FD 25
BK 25
LT 90
FD 20
RT 90
FD 20
BK 20
LT 90
FD 20
RT 90
FD 25
```

b)

```
TO E "SCALE
LT 90
FD MUL :SCALE 20
BK MUL :SCALE 40
RT 90
FD MUL :SCALE 25
BK MUL :SCALE 25
LT 90
FD MUL :SCALE 20
RT 90
FD MUL :SCALE 20
BK MUL :SCALE 20
LT 90
FD MUL :SCALE 20
RT 90
FD MUL :SCALE 25
END
```

Fig. 5.54: Shahidur and Ann - Variable Letter E.

At the beginning of the next session they decided to make a pattern with their general letter procedures. Ann wanted to try out an input of 1.5 to their L procedure and Shahidur thought that this would not draw an L. He still thought that the "1" of the 1.5 would effect the vertical part of the L and the "5" of the 1.5 would effect the horizontal part of the L. They tried L 1.5 and Shahidur started to "make sense of" the computer response.

Shah. *"Miss you know 1.5...is it 60 that way and that way"* .

He seemed to be beginning to understand the 1.5 multiplies the 40 in both the vertical and horizontal components of the l. The screen response to L 1.5 must have contributed to this understanding. Leaving the L on the screen they then started to redraw the letter I in direct mode without using their already defined general I procedure. Ann gave the following reasons.

Ann *"No if we start there we'll have to do the whole I....and it might come out the wrong proportions....so it's better just to do the whole thing ...isn't it..."*

This illustrates how readily pupils can avoid using their general modules, returning to work at a lower level, unless provoked to do so either by the task itself or by direct teacher intervention.

They produced LIE in direct mode and were asked to draw a small E. They typed in E 0.01

Res. *"What is 0.01 what does that mean..."*

Ann *"Well the original thing that we had...it multiplies it out of 100".*

Res. *"What does it multiply".*

Shah. *"Numbers miss...out of a 100".*

Res. *"It multiplies it by 0.01".*

Ann *"Yeah".*

They then tried I 4.2. The researcher then suggested that they draw a row of decreasing L's. They typed in L 4.2 ; the interfacing commands; L 3.1; the interfacing commands; L 2.1; the interfacing commands and finally L 1.1 producing Fig. 5.55.

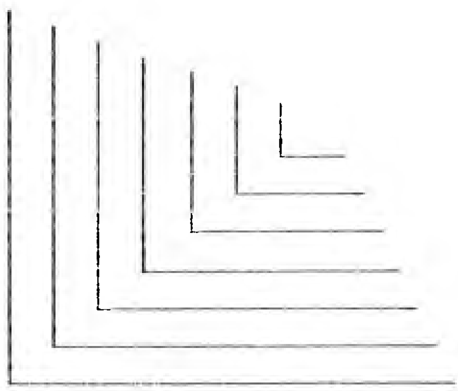


Fig. 5.55: Shahidur and Ann - Decreasing L's

Within these three session Shahidur and Ann, having started from a position of disbelief, have accepted the idea of using a variable to produce different sized images on the screen. They have started to reflect on the effect of the value of the variable input on the commands within their "scaled" procedure. Initially an obstacle to this understanding was their misconception about decimal numbers. They could not conceive of a decimal as a "whole" but thought about it as a made up of separate parts which acted on the separate parts of the geometrical object being constructed. The computer response was crucial in helping them to come to an understanding of a decimal number as a "whole" and as this understanding developed they were able to reflect on the effect of this "whole" on the distance commands within their "Scaled" procedure.

5.5.2 Line and Cross

Year & Session No:	Year 2; Session 10
Type of goal:	Well defined abstract
Category of variable use:	(I) One variable input.

During this session Shahidur and Ann used the following procedure LINE as a tool to draw various given shapes.

```
TO LINE "NUMBER
FD :NUMBER
BK :NUMBER
END
```

They accepted the use of variable in this context but were not provoked to reflect on the process within the procedure. **There is no evidence from later transcript analysis that Shahidur and Ann have learned anything from this procedure. This could be because it was "teacher given" and had not been constructed by them.**

5.5.3 Row of Pines

Year & Session No:	Year 2; Session 12
Type of Goal:	Well Defined Abstract
Category of Variable Use:	(S) Variable as Scale Factor

Shahidur and Ann were given the "Row of Pines" task (appendix 3.4) which had also been given to Sally and Janet and George and Asim. There is no transcript available for this session but from the researcher's notes it is known that Ann immediately said *"Oh I know...what was that we did to make it different sizes...MUL...."*.The researcher suggested that first of all they direct drive a fixed shape. When they had finished doing this they again asked "how to do the MUL thing". The researcher reminded them how to scale all the distance commands in their pine procedure and they were then able to do this for themselves (Fig 5.56).

```
TO ARROW'S "SCALE
BK MUL :SCALE 60
FD MUL :SCALE 100
LT 135
FD MUL :SCALE 40
BK MUL :SCALE 40
LT 90
FD MUL :SCALE 40
END
```

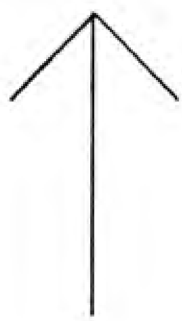


Fig. 5.56: Shahidur and Ann - Pine Tree

They used ARROW'S with an input of 1. Ann said that the next arrow had to be "three quarters" and the researcher told them to use 0.75. They then wanted a "half" arrow and Ann thought that this would be .55 but Shahidur said that it should be .5. They entered ARROW'S .5 and finally used ARROW'S with an input of .25.

In the next session the researcher asked them to write a superprocedure for the row of pines. They started to define STAIR'S (Fig. 5.57a)

a)
```
TO STAIR'S
MOVE
ARROW'S 1
MOVE1
ARROW'S  .75
MOVE1
ARROW'S  .50
MOVE1
ARROW'S  .25
MOVE1
ARROW'S  .15
END
```

b)


Fig. 5.57: Shahidur and Ann - Row of Pines

They then tried this out which produced Fig. 5.57b.

Res. *"So Shahidur what does ARROW'S .75 do?"*

Shah. *"Err miss it makes it smaller.. it multiplies the 60 miss...and miss if we do ARROW'S 1 it".*

Ann *"It multiplies it...."*

Shah. *"It does 60 yeah...."*

Res. *"And what does that SCALE thing do then?"*

Ann *"The SCALE is the number that you pick".*

Res. *"If I pick 2 what would it do.."*

Ann *"It would multiply it ...the SCALE by whatever number's here..."*

This discussion indicates that Shahidur and Amanda's understanding of variable has developed so that they now understand that a variable is used as a place holder for a range of numbers. They have also reflected on the effect of the variable input on the global size of the "scaled letter". They returned to the problem of trying to get the arrows all on one line.

Ann *"Now we can't change MOVE1 ...she said to change the whole lot of it...'cos it's not MOVE1 is it....it's the arrows..because MOVE1 just moves it here right..."*

Shah. *"Yeah but instead of changing the ARROW ...instead of taking it there...MOVE1 takes it there..."*

This discussion illustrates their understanding of process within their procedures with respect to turtle state and also their understanding of the relationship between the state of the turtle at the beginning and end of each procedure. This should be contrasted with their very low level of performance in their school mathematics. They spent the rest of the session modifying the MOVE1 interface procedure until they had solved the problem to their satisfaction.

Within this session Shahidur and Ann had initiated the idea of using variable as a scale factor in order to solve a given problem. There is evidence that they were not using the idea in a "rote" way They were beginning to understand the effect of an input on an individual scaled command. It is not suggested that they had reflected on the internal ratios within the procedure. In all the general procedures which they defined they used a variable name SCALE.

5.5.4 Variable Letters 2

Year & Session No:	Year 3; Session 4 & 5
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor

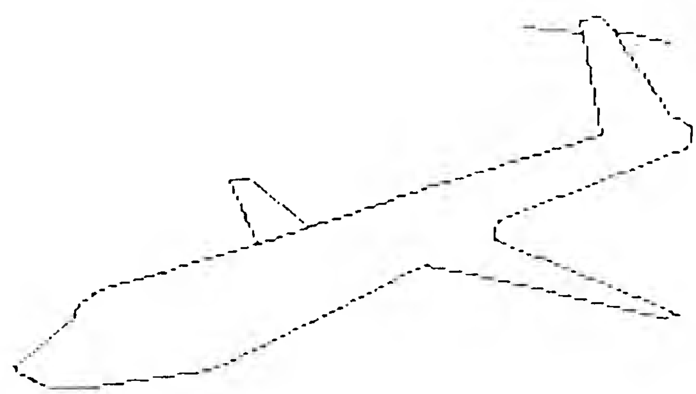


Fig. 5.58: Shahidur and Ravi - Aeroplane

At the beginning of the third year Ann left the school and so Shahidur was paired with Ravi. After three sessions of working together on a well defined real image of an aeroplane (Fig. 5.58), which had been their own choice, Ravi and Shahidur were given the "Scaling Letter" task (appendix 3.2). This was Ravi's first introduction to the idea of variable. They defined the given fixed L and the researcher helped them to scale all the distance commands. They then tried L 4.

Res. *"So that means that when it goes forwards it is multiplying the 40 by 4..."*

Shah. *"So it's 160".*

Res. *"So how far is it going to go forwards and backwards when you put 2 in?"*

Ravi *"80"*

They then tried L 0.5

Res. *"How big is the distance when you put 0.5 in?"*

Shah. *"20".*

Res. *"Why is it 20?"*

Shah. *"'Cos it's half."*

Shahidur understood the "local" effect of the scaling variable within the **procedure.**

They then decided to draw an R and drew this first in direct mode. They started to type in TO R and then negotiated how to make it general.

Shah. *"Oh yeah MUL..."*

Ravi *"What?"*

Shah. *Do the MUL".*

However they defined a fixed procedure without scaling the distance commands (Fig. 5.59a).

a)

```
TO R
LT 90
BK 40
FD 80
RT 90
ARCR 20 180
LT 125
FD 50
END
```

b)

```
TO R "SCALE
LT 90
BK MUL :SCALE 40
FD MUL :SCALE 80
RT 90
ARCR MUL :SCALE 20 180
LT 125
FD MUL :SCALE 50
END
```

Fig. 5.59: Shahidur and Ravi - Variable Letter R.

They started to change this to a general procedure by scaling all the distance commands and Ravi wanted to change FD 80 to FD MUL :SCALE 40 giving as his reason:

Ravi *"Rub the 80 out....you can't do it 80 ...'cos you've got to do it the same all the way round".*

Ravi, like other case study pupils, appears to have taken the idea from the given handout that all the distance commands within the scaled procedure should be of length 40. Shahidur however knows that this was not the case. He disagreed but needed to ask the teacher for confirmation.

Shah. *"Miss do you put the same all the way around...that was meant to be 80...but he said it has to be 40..."*

Res. *"Well you used 80 in your original program didn't you.."*

Ravi *"But it's going to be too big then...if it's going to double the size.."*

Ravi was thinking about the effect of scaling without being concerned with the internal ratios within the E shape. With help from the researcher they finally defined a general procedure (Fig. 5.59b). They discovered that this worked when they tried out an input of 2. They next decided to draw an S but drew the number eight instead using the commands in Fig. 5.60a

a)

```
ARCL 20 180
ARCR 20 180
ARCR 10 90
```

b)

```
TO EIGHT "SR
ARCL MUL :SR 20 180
ARCR MUL :SR 20 180
ARCR MUL :SR 10 180
END
```

c)

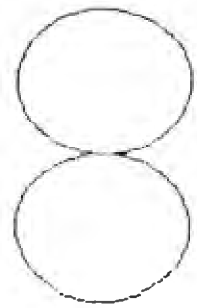


Fig. 5.60: Shahidur and Ravi - Variable Number 8.

This time they defined a scaled procedure without first defining a fixed procedure (Fig. 5.60b). **The researcher suggested that they use another variable name other than SCALE and Shahidur's response indicated that he thought that the name SCALE had some meaning.**

Shah. *"But if we want to scale it we have to put scale don't we".*

Res. *"You can call it anything you want..you can call it SHAHIDUR if you want..or RAVI."*

They tried out EIGHT 2. They then built up a pattern of S's using an input of 1.5. Their comments indicated their pleasure with the effect.

Shah *"Wicked ain't it.".*

Ravi *"It's dry"*

The value of this pleasure in terms of motivating the pupils to engage in the task must not be undervalued in terms of their eventual learning.

In the next session they used their L procedure with an input of 2 and the researcher asked them to reflect on the processes within the procedure.

Shah. *"After L we have to put you know the number we want".*

Res. *"And what does the 2 do?"*

Shah. "Well say we typed in 40** for that L ...well if we put 2 in it's **gonna do you** know...2 times 40...so it will be 2 **times 40".

Res. *Alright so what's all that scale business..what does that do . ...do you know Ravi."*

Ravi *"Doubles the sides".*

Res.	*"And what value does SCALE have then."*
Shah.	*"It adds the number that you put on...so you know we done the L and we put 40...so if we put 2 it's going to double the size..if we put 3 L 3..it'll be 120....so it multiplies the number that we put on".*
Res.	*"So if I did say 3 here...what would happen..."*
Ravi	*"It would be three times bigger....."*
Res.	*"What part of the circle would be three times bigger...would it be the whole circle...?"*
Shah.	*"It would be you know....no not the whole circle...say if it was about that wide ...well if we did it 3 then it's going to be about that wide innit...but it will still be a semi-circle".*

Shahidur is able to relate the effect of the variable input on each individual command within the procedure. Ravi appears to understand variable as effecting the overall size of the geometrical object. He has not yet shown evidence of being able to analyse the effect on the constituent parts of the geometrical object. They are also beginning to understand that the shape is invariant. i.e if it was a semicircle it will stay a semi circle. There is no evidence that they are aware of the relationship between the constituent parts of the geoemetrical object.

5.5.5 Arrowhead

Year & session No:	Year 3; Session 6
Type of goal:	Well defined abstract
Category of variable use:	(S) Variable as scale factor.

As were all the case study pupils, Shahidur and Ravi were both given the "Arrowhead" Task (appendix 3.3) at the end of the three years of the classroom research. (The session took place after the Laboratory tasks had been administered to the pupils individually). Unlike the other pairs they had only constructed procedures in which variable was used to scale distance commands before engaging in this session. **During this session the level of motivation was not high. It was as though they felt threatened by the teacher directed nature of this task. They nevertheless accepted the "didactical contract" of accomplishing the task.**

Res.	*"What we would like you to do is to write a program to draw exactly the same shape....but I want it to be a program which you could change...so that you could make that shape any size you like...and do lots of them all over the screen."*

They immediately started in direct mode to draw one arrow keeping a record of their commands. When they had finished this they started to define a fixed procedure until Shahidur said

Shah. *Oh yeah...we got to do the scale..."*

Ravi *"Yeah I know...afterwards though....we have to finish it all...and then scale..it's best..."*

Shahidur however added ARROW "SCALE to title line of the procedure.

Ravi *"MUL scale*

Shah. *"No just Scale.."*

Ravi *"You have to put MUL SCALE...innit miss....you have to put MUL SCALE".*

Shah. *"No SCALE".*

Ravi *"It's MUL SCALE".*

Shah. *"No on the top it ain't..."*

Ravi *"It does...you have to put it on all the things that go forward.."*

Shah. *"Yeah but...."*

Ravi *"It doesn't matter.....you have to do it on all of them....ask Miss...you have to do it on all of them..."*

Shah. *"No".*

Ravi *"You have to do it you do.....how much do you bet...."*

Shah. *"Ask miss.."*

The above interchange illustrated their unwillingness to collaborate on the task. They were both arguing about different matters of syntax but within the discussion they were not able to negotiate this. Instead they kept wanting to refer to the teacher as an authority figure. Shahidur typed in FD :MUL saying:

Shah. *"Miss don't you do that..."*

Res. *"Well the two dots...come before the SCALE......because MUL stands for multiply.....so you say MUL.....and then the two dots come before the scale..."*

Ravi *"That's what I was trying to tell him "*

This comment of Ravi's appears to be motivated by the need to "win" an argument.

They then defined the general ARROW procedure (Fig. 5.61). The procedure included the startup commands which had also been scaled by a variable. This indicates that Ravi and Shahidur had not identified exactly what was the invariant module within their procedure. They tried out ARROW 1 which worked. They then tried ARROW 2 which went off the screen provoking them to try ARROW .5. They had completed the arrows task and the rest of the session was taken up by them making a pattern of arrows on the screen.

This session illustrates that Shahidur and Ravi are able to initiate for themselves the idea of using variable in the category of "(S) variable as scale factor" in order to define a general procedure for a simple geometrical object. They were not able to negotiate issues of syntax between themselves and needed to refer to the authority of the reseracher/teacher.

```
TO ARROW "SCALE
PU
FD MUL :SCALE 50
PD
LT 180
FD 100
RT 135
FD MUL :SCALE 60
BK MUL :SCALE 60
RT 90
FD MUL :SCALE 60
BK MUL :SCALE 60
LT 45
FD MUL :SCALE 50
LT 45
FD MUL :SCALE 40
BK MUL :SCALE 40
END
```

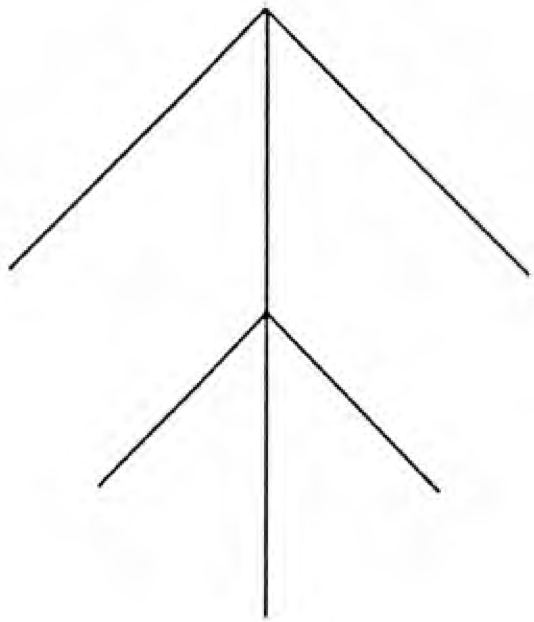


Fig. 5.61: Shahidur and Ravi - General Arrowhead Procedure

Shahidur and Ravi's solution to the "Arrowhead" task is compared to the solutions of the other case study pupils in section 5.6. An overview of Shahidur and Ravi's development throughout the three years of the research is presented in section 5.8.

5.6 OVERVIEW OF THE ARROWHEAD TASK

At the end of the period of research all the case study pairs carried out the "Arrowhead" task (appendix 3.3). The researcher's brief was not to intervene at all during the sessions, although occasional interventions were made to keep the pupils on task, and to provoke the pupils to extend their solution, once they had completed a solution from their own perspective. Each case study pairs' involvement in this task has been described in detail in sections 5.2.12, 5.3.10, 5.4.9 and 5.5.5.

The pairs Sally & Janet and George & Asim both produced programming solutions using variable in the category of "(O) variable operated on". The pairs Linda & Elaine and Ravi & Shahidur both produced solutions using variable in the category of "(S) variable as scale factor".

Sally and Janet's perception of the geometrical object was different from George and Asim's perception and consequently their programming solutions were different (see Fig. 5.19b and Fig. 5.37). They both chose to use "halving and doubling" ratios as opposed to the ones presented in the task. For Sally and Janet the arrowhead was one object, but for Asim and George (mainly due to Asim's influence) the arrowhead consisted of two parts, one smaller arrowhead placed on top of a bigger arrowhead. Analysis of the transcripts indicates that the task was not a trivial one for these pupils and that both pairs needed discussion and interaction with the computer in order to produce a working programming solution. Each partner within these two pairs brought a different perspective to the problem solution, but within the session they were able to negotiate in order to produce a common solution. Jointly both pairs were able to negotiate a programming solution in which a simple "halving and doubling" relationship was made explicit by operating on a variable. When Sally and Janet had completed their solution to the task they were nudged by the researcher into producing another solution in which the given ratios were made explicit. They attempted to come to terms with this but the mathematical ideas associated with similar figures bcame an obstacle to their solution of the task from this perspective.

Linda worked with Elaine whilst carrying out the "Arrowhead" task. There is clear evidence that Linda wanted to solve the task by using "(S) variable as scale factor" and Elaine wanted to solve the task by using "(N) more than one variable input". Eventually by interacting with the computer and by discussion they solved the task using "(S) variable as scale factor" (Fig. 5.50). Elaine initially devised a "working rule" of "all the backward commands are the same and all the forward commands are the same". This indicates that she was analysing the geometrical object for invariants although her solution was misconceived. Linda and Elaine's different approaches resulted in a solution in which they assigned one "scaling" variable for all the backward commands and one "scaling" variable for all the forward commands (Fig. 5.50). The researcher had to nudge them both into reflecting on their solution by asking them to try inputs of different values. They finally modified their solution to using only one variable in the category of "(S) variable as scale factor".

Ravi and Shahidur were not very motivated to carry out the "Arrowhead" task. They appeared to find the task threatening and instead of resolving conflict by discussion tended to ask the researcher/teacher to resolve their conflict. They were however able to produce a working solution to the task using "(S) variable as scale factor" (Fig. 5.61). Their final procedure for the arrowhead included the "navigating" command to place the turtle on the left hand side of the screen and they had also scaled this command. This suggests that they had not perceived the arrowhead module as separate from the navigation command. Results from the Logo Maths Project indicate that at the beginning stages of learning Logo many pupils are not able to perceive the "navigating" commands as separate from the commands which produce the geometrical object (Hoyles & Sutherland, 1984).

Analysis of this task indicates that all of the case study pupils could use variable to solve the "Arrowhead" task, but the nature of their solutions depended very much on their previous exerience of variable in Logo. One aim of the task was to investigate whether or not the pupils could use variable in the category of "(O) variable operated on". **Making a relationship explicit by operating on a variable was not a necessary problem solving tool for this particular problem. Pupils will always tend to devise solutions which require an "economy of action" and this has to be taken into account when devising problems designed to probe pupils' understanding.**

5.7 INDIVIDUAL LABORATORY TASKS

At the end of the period of research all the case study pupils spent a day at the University laboratory working individually on specific Logo tasks devised to probe their understanding of algebra related ideas within Logo. These results are important as they provide evidence of each individual pupil's understanding of variable. In almost all the other computer work pupils worked in pairs at the computer. This section will present the results of these tasks for each individual pupil.

5.7.1 Logo Programming Tasks

Three variable related Logo programming tasks were administered; the "Variable Square" task (Fig. 5.62); the "Row of Decreasing Squares" task (Fig. 5.63) and the "Lollipop" task (Fig. 5.64). The strategy of researcher intervention throughout the session was to give help only with Logo syntax and only when requested. Help on syntax was first provided by reference to a handout on variable (appendix 5.2) and if this was not sufficient by actual spoken communication with the pupils. Once the pupils had solved the task the researcher sometimes nudged them into thinking about an alternative solution. All the case study pupils' programming solutions to these tasks are presented in Table 5.1.

The pupils' performance on these tasks is part of the evidence which is being built up of their developing understanding of variable in Logo. All the pupils used variable in some form in order to solve both the "Variable Square" task and the "Lollipop" task. When first solving the Variable Square task Linda, Jude, Ravi and Shahidur all asked for help with Logo syntax. The only help which was given to Linda was the handout (appendix 5.2) and from that she devised all her solutions. Shahidur and Ravi used variable in the category of "(S) variable as scale factor" for the "Variable Square" task and needed help with Logo syntax. After this intervention Shahidur needed no more help to complete the other two tasks. However Ravi still needed help with syntax to complete the "Lollipop" task. Jude asked for *"help with the input"* when defining a variable square and was given the handout (appendix 5.2). This support was not sufficient and he still requested spoken support. He also asked for support with syntax when solving the "Lollipop" task.

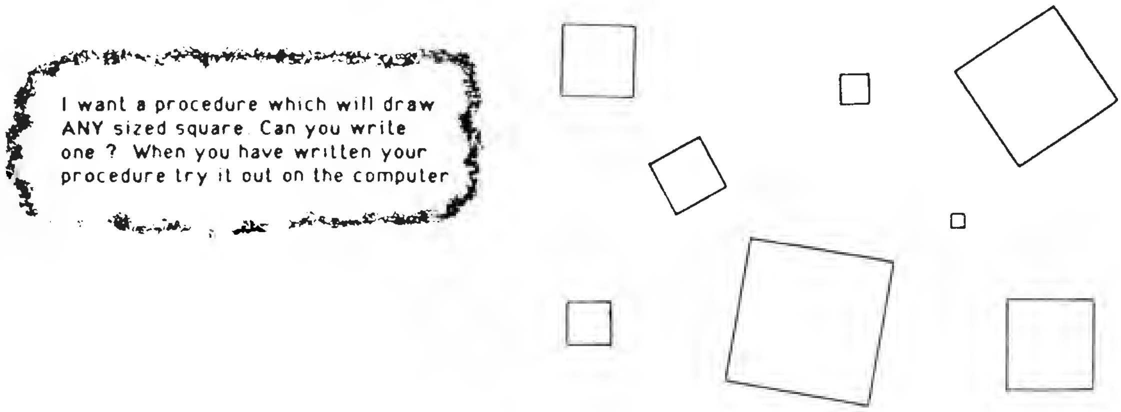
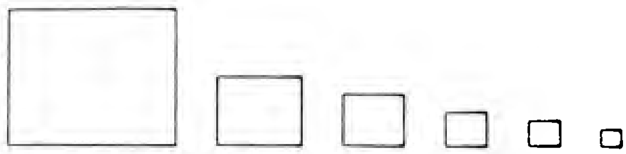


Fig. 5.62: The Variable Square Task

Write down a procedure to draw the following picture.



Now try out your ideas at the computer.

Fig. 5.63: The Row of Decreasing Squares Task

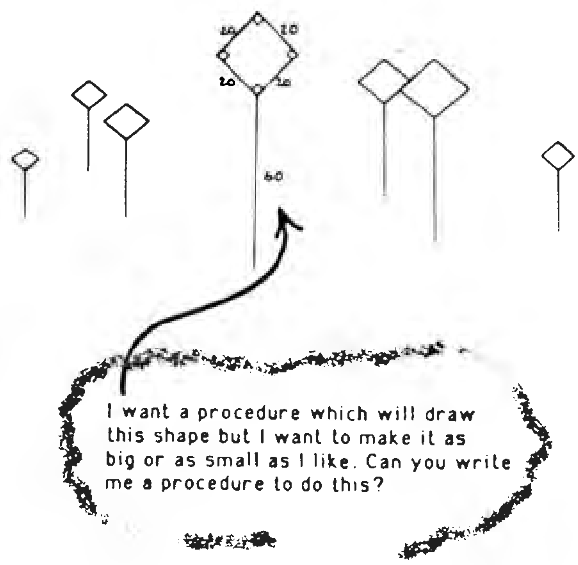


Fig. 5.64: The Lollipop Task

Table 5.1: Case Study Pupils' Solutions to "Hands on" Individual Programming Tasks.

	VARIABLE SQUARE TASK	ROW OF DECREASING SQUARES TASK	LOLLIPOP TASK
SALLY	TO WERT "SIDE REPEAT 4 [FD :SIDE RT 90] END TO BAG "SIDE REPEAT 4[FD :SIDE LT 90] PU FD ADD :SIDE 10 END	TO BIGBAG "SIDE TRAV BAG SUB :SIDE 10 BAG SUB :SIDE 20 BAG SUB :SIDE 30 BAG SUB :SIDE 40 BAG SUB :SIDE 50 BAG SUB :SIDE 10 END	TO STICK "SIDE1 "SIDE2 PU LT 90 BK 50 PD FD :SIDE1 RT 45 REPEAT 4 [FD :SIDE1 LT 90] END
ASIM	TO SQUARE "MUL REPEAT 4 [FD :MUL RT 90] END	TO STAIR PU BK 10 LT 90 PD SQJ 40 SAAB SQJ 20 SAAB SQJ 15 SAAB SQJ 10 SAAB SQJ 5 END TO SQJ "MUL REPEAT 7 [FD :MUL RT 90] END TO SAAB LT 180 PU FD 20 LT 90 PD END	TO KITE "RAF LT 90 FD :RAF LT 45 REPEAT 4[FD DIV :RAF 3 RT 90] END

	VARIABLE SQUARE TASK	ROW OF DECREASING SQUARES TASK	LOLLIPOP TASK
GEORGE	TO SQUAN "NUM REPEAT 4 [FD :NUM RT 90] END	TO SQ1 "NUM MOVE3 :NUM B1 :NUM B2 :NUM B3 :NUM B4 :NUM B5 :NUM B6 :NUM END	TO SQUARES "NUM LT 135 REPEAT 4 [FD :NUM RT 90] LT 135 END

```
TO B1 "NUM
SQ1 :NUM
RT 90
FD 5
BIT
END

TO B2 "NUM
SQ1 SUB :NUM 5
FD SUB :NUM 5
RT 90
FD 5
BIT
END

TO B3 "NUM
SQ1 SUB :NUM 10
FD :NUM
RT 90
FD 5
BIT
END

TO B4 "NUM
SQ1 SUB :NUM 15
FD SUB :NUM 15
RT 90
FD 5
BIT
END

TO B5 "NUM
SQ1 SUB :NUM 20
FD SUB :NUM 20
RT 90
FD 5
BIT
END

TO B6 "NUM
SQ1 SUB :NUM 25
CT
END

TO MOVE3 "NUM
PU
BK MUL :NUM 2
PD
END

TO BIT
LT 90
PU
FD 10
PD
END
```

	VARIABLE SQUARE TASK	ROW OF DECREASING SQUARES TASK	LOLLIPOP TASK
JANET	TO BOX "PIG REPEAT 4 [FD :PIG RT 90] END	TO CUBES M1 BOX 60 MO2 BOX 30 MO3 BOX 15 MO4 BOX 7 MO5 BOX 3 END TO M1 PU BK 120 PD END	TO KITES "YT "HT RT 45 FD :YT RT 90 FD :YT RT 90 FD :YT RT 90 FD :YT BK :YT RT 90 FD :YT RT 45 FD :HT END

TO MO2 FD 60 RT 90 FD 60 RT 90 PU BK 5 PD LT 180 RT 90 BK 30 LT 90 END	TO MO3 FD 30 RT 90 FD 30 RT 90 PU BK 5 PD LT 180 RT 90 BK 15 LT 90 END	MO4 moves Fd 15 MO5 moves FD 7 MO6 moves FD 3

JUDE	TO SQU "SIDE FD :SIDE RT 90 FD :SIDE RT 90 FD :SIDE RT 90 FD :SIDE RT 90 END	TO SQUARE PU BK 120 PD SQU 60 PU FD 70 PD SQU 50 PU FD 60 PD SQU 50 . ... FD 30 PD SQU 10 END	TO LOL "SIDE RT 45 FD :SIDE RT 90 FD :SIDE LT 45 FD MUL :SIDE 3 BK MUL :SIDE 3 RT 135 FD :SIDE RT 90 FD :SIDE END

	VARIABLE SQUARE TASK	ROW OF DECREASING SQUARES TASK	LOLLIPOP TASK
RAVI	TO BOX "SCALE RT 90 FD MUL :SCALE 40 RT 90 FD MUL :SCALE 40 RT 90 FD MUL :SCALE 40 RT 90 FD MUL :SCALE 40 END	NO variable used	TO TOM "SCALE LT 90 PU BK 60 PD FD MUL :SCALE 60 LT 45 FD MUL :SCALE 20 RT 90 FD :MUL :SCALE 20 RT 90 FD MUL :SCALE 20 RT 90 FD MUL :SCALE 20 END
LINDA	TO LEIGH "NUM FD :NUM LT 90 FD :NUM LT 90 FD :NUM LT 90 FD :NUM END	TO LEIGH2 PU BK 100 PD LEIGH 40 LT 90 PU FD 50 PD LEIGH 30 .... LEIGH 5 END	TO TRI "ANGLE LT 90 FD :ANGLE END TO ANGLE "TRI RT 45 FD :TRI LT 90 FD :TRI LT 90 FD :TRI LT 90 FD :TRI END
SHAH.	TO SIZE "SCALE FD MUL :SCALE 40 LT 90 FD MUL :SCALE 40 LT 90 FD MUL :SCALE 40 LT 90 FD MUL :SCALE 40 END	TO SIZEMOVE PU BK 60 PD SIZE 1.5 MOVE SIZE 1 MOVE SIZE .7 MOVE SIZE .4 MOVE SIZE .3 MOVE SIZE .2 END	TO KITE "SCALE LT 45 FD MUL :SCALE 20 LT 90 FD MUL :SCALE 20 LT 90 FD MUL :SCALE 20 LT 90 FD MUL :SCALE 20 RT 45 FD MUL :SCALE 60 END

To provide a framework from which to analyse the pupils' programming solutions they have all been categorised according to the categories of variable outlined in Section 3.8.2. This analysis is presented in the following table.

Table 5.2: Classification of Case Study Pupils' Solutions to Individual Laboratory Logo Programming Tasks

	Variable Square Task	Row of Decreasing Squares Task	Lollipop Task
Sally	(I) One variable Input	(G) General superprocedure (O) Variable operated on	(N) More than one input
Asim	(I) One variable Input	(I) One variable input	(O) Variable operated on
George	(I) One variable Input	(G) General superprocedure (O) Variable operated on	(O) Variable operated on
Janet	(I) One variable Input	(I) One variable input (for subprocedure)	(N) More than one input (unrelated)
Jude	(I) One variable Input	(I) One variable input (for subprocedure)	(O) Variable operated on
Ravi	(S) Variable as scale factor	No input used	(S) Variable as scale factor
Linda	(I) One variable Input	(I) One variable input (for subprocedure)	(N) More than one input (unrelated)
Shahidur	(S) Variable as scale factor	(S) Variable as scale factor	(S) Variable as scale factor

Sally, George and Asim have clearly demonstrated their facility to operate on a variable in a Logo program. However although Sally clearly demonstrated this ability in the "Row of Decreasing Squares" task she did not perceive a need to make a relationship explicit in the "Lollipop" task and used two unrelated inputs when solving this task. Janet did not operate on a variable when first solving the "Lollipop" and like Sally used two unrelated inputs. However when asked by the researcher to solve the problem with one variable only she immediately without any help removed the second variable HT from her procedure and replaced FD :HT by FD MUL :YT 3 in the final line of her procedure. This suggests that if we want pupils to operate on variables we need to devise tasks in which this need is made explicit. When Sally was asked to change her lollipop procedure STICK to use one variable only she chose to rewrite the procedure using variable as scale factor. This possibly indicates that, for her, the "Lollipop" task was similar to the letter tasks and thus her "(S) variable as scale factor" frame was invoked. Jude also operated on a variable in his solution to the "Lollipop" task. He solved the task by first drawing the lollipop in direct mode and then defining a procedure, adding one variable (for the side of the square) to the title line. When he came to the line of the procedure which would draw the "stick" of the "lollipop" he explicitly asked for help on how to multiply a variable by three. This provides evidence of his understanding of the existence of the idea of operating on a variable although without help he would not have been able to manage the syntax. Linda wrote two separate general modules (TRI for the "stick" and ANGLE for the "lollipop") in order to solve the "Lollipop" task and when asked to combine them into a general superprocedure demonstrated her confusion over doing this. She first tried:

```
TO KITE
TRI "ANGLE
ANGLE "TRI
END
```

and then tried:

```
TO KITE "LINE
TRI "ANGLE
ANGLE "TRI
END
```

but could not complete the task on her own.

Both Sally and Asim's planning work indicates that they had had clear "top down" plans of how they would solve the "Row of decreasing Squares" task (Fig. 5.65 and Fig 5.67). George and Janet's plans evolved in a more "bottom up" way and they both needed to negotiate with the computer as their plans emerged (Fig. 5.68 and Fig. 5.69). However Sally and Asim's "top down" approach does not preclude difficulties with local issues. Both Asim and Sally had difficulties with choosing the exact Logo syntax, which was not the case for George and Janet. In addition Asim had considerable difficulty with predicting the necessary orientation (i.e LEFT or RIGHT) of the turtle. In addition Janet and George's confidence in using the Logo syntax when carrying out these individual tasks was a reflection of their relative dominance over keyboard work throughout their three years of collaboration with their respective partners.

Janet, Sally, and Linda's use of variable names indicates that they understand that any name can be used. Within these tasks George only used the variable name NUM but as he completed these task quickly he was given an extension task in which he used a range of variable name. It is suggested that George also understands that any variable name can be used. Asim's use of variable names presents a more complicated picture. He initially chose the variable name MUL for his variable square module. This seems to indicate a persisting confusion between the prefix operator and the variable name and it probably stems from the "Scaling letters" task. He started to use the name MUL when solving the "Lollipop" task and was nudged by the researcher not to do so, resulting in his use of the name RAF. The other three pupils, Jude, Ravi and Shahidur restricted themselves to the variable name used when first introduced to the idea. So Jude used the variable name which stemmed from his "(I) one variable input" frame and Ravi and Shahidur used the variable name which stemmed from their "(S) variable as scale factor" frame.

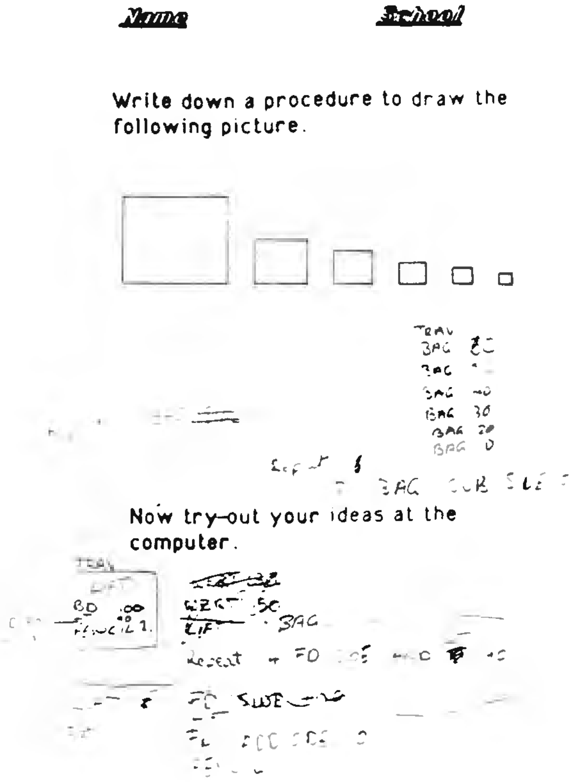


Fig. 5.65: Sally's Planning for Row of Decreasing Squares Task

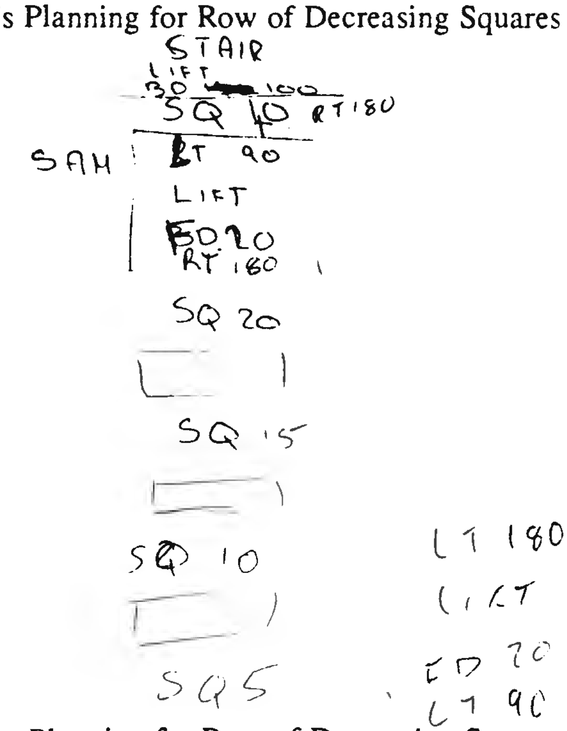


Fig. 5.66: Asim's Planning for Row of Decreasing Squares Task

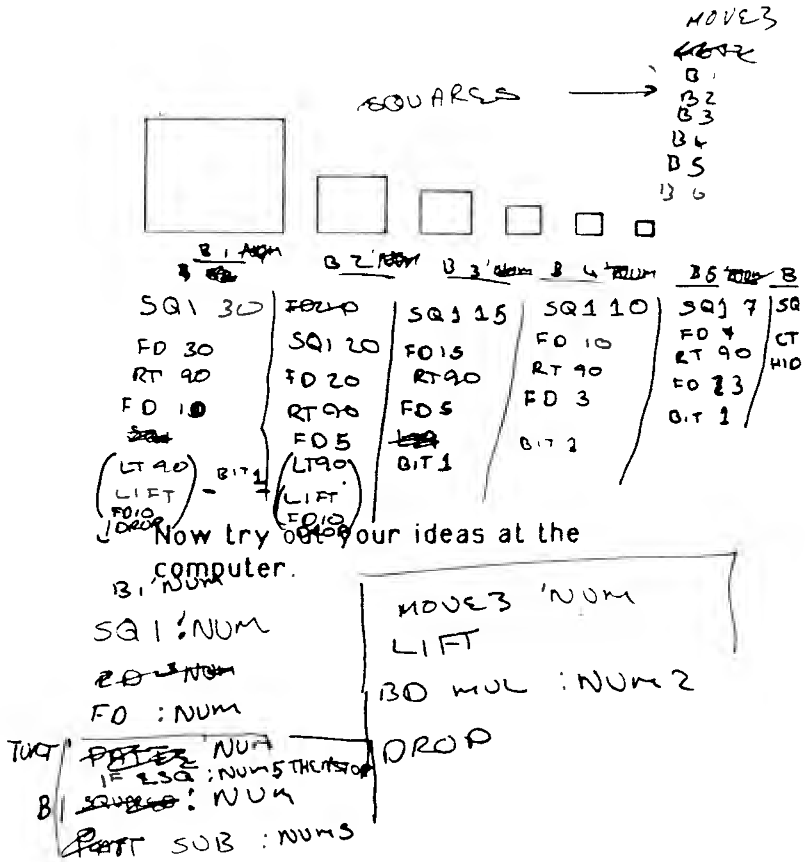


Fig. 5.67: George's Planning for Row of **Decreasing Squares Task**

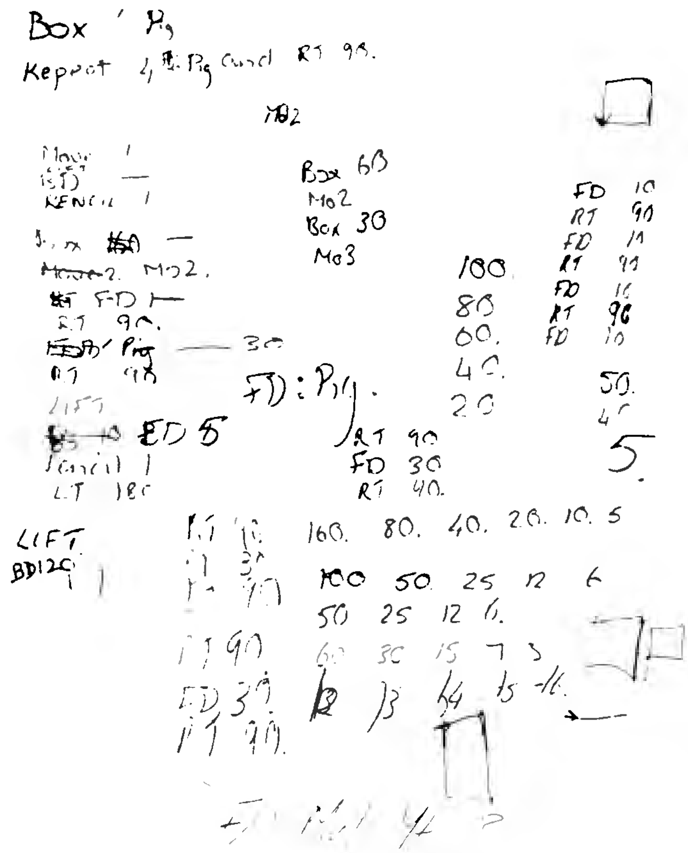


Fig. 5.68: Janet's Planning for Row of Decreasing Squares Task

5.7.2 "Paper and Pencil" Tasks

A series of "paper and pencil" tasks were adminstered to the pupils. Within each task the pupils had to trace out a procedure. The following is a list of the tasks:

RECTANGLE "FUN "SUN Procedure using variable in the category of "(I) More than one variable input" (appendix 5.3a)

RECTANGLE "C Procedure using variable in the category of "(O) Variable operated on" (appendix 5.3b)

PUZZLE "BIT Procedure using variable in the category of "(S) Variable as scale factor" (appendix 5.3c)

SURPRISE "SCALE Procedure using variable in the category of " (S) Variable as scale factor" (appendix 5.3d)

PUZZLE Fixed superprocedure using variable dependent subprocedure (appendix 5.3e)

PAT "NUM (G) General superprocedure using variable dependent subprocedure (appendix 5.3f)

MYSTERY "NUM (R) Recursive procedure (appendix 5.3g)

Table 5.3 presents an overview of the pupils' solutions to these tasks. The responses have been classified as correct from the point of view of interpretation and evaluation of the variable used within the procedure.

Sally and Asim's inability to answer the recursive procedure question (MYSTERY) correctly compared with George and Janet's correct solution appears to be a reflection of George and Janet's relative dominance over issues of Logo syntax when working with their partners. Asim's incorrect responses to the RECTANGLE questions indicates that he had decided on the global output of the procedures (he drew rectangles incorrectly oriented) without following through the procedure sequentially. Jude and Linda could interpret simple general procedures which used variable in the category of "(N) more than one variable input", " (S) variable as scale factor" or " (O) variable operated on" but were not able to interpret procedures which used variable in the category of "(G) general superprocedure".

but were not able to interpret procedures which used variable in the category of "(G) general superprocedure".

Table 5.3: Case Study Pupils' Solutions to Individual Laboratory "Paper and Pencil" Tasks

	Sally	Asim	George	Janet	Jude	Ravi	Linda	Shah.
RECTANGLE "FUN "SUN (N) 2 inputs	√	√#	√	√	√	√#	√	√#
RECTANGLE "C (O) Variable operated on	√	√ #	√	√	√	√#	√	√(N)
PUZZLE "BIT (S) Variable as scale factor	√	√	√	√	√	√	√	x
SURPRISE "SCALE (S)Variable as scale factor	√	√	√	√	√	√	√	√
PUZZLE Fixed superprocedure with general subprocedure	√	√	√	√	x	x	x	√
PAT "NUM (G) General superprocedure with general subprocedure	√	√	√	√	x	x	x	x
MYSTERY "NUM (R) Recursive procedure	x	x	√	√	x	x	x	x

√ represents correct Solution; x represents incorrect solution

√ (N) represents correct solution sfter nudge from researcher

√# represents solution to a rectangle task in which rectangle is drawn with incorrect orientation

Ravi was only able to interpret a general procedure which used variable in the category of "(S) variable as scale factor". Shahidur was able to correctly interpret variable in the category of "(S) variable as scale factor" when the variable name was SCALE but not when the variable was named BIT. When Shahidur responded to the RECTANGLE "C procedure he was initially puzzled by the single variable name C but after a nudge from the researcher was able to successfully interpret variable in the category of "(O) variable operated on". He was not able to interpret a procedure which used "(N) more than one variable input". He was able to correcty interpret a fixed superprocedure but was not able to interpret a general or recursive superprocedure.

5.8 OVERVIEW OF CASE STUDY PUPILS' DEVELOPMENT

5.8.1 Phases in Pupils' Developing Understanding of Variable in Logo

Analysis of the case study transcript data indicates that all the case study pupils have developed throughout the three years of the classroom based research in their ability to use and understand variable to produce turtle geometrical objects within Logo. It has been possible to identify certain phases in this development. These phases are very context specific, so that if a pupil is able to perform at one phase for a certain class of geometrical objects they will not necessarily be able to perform at this phase for another.

Phase 1 Pupils understand that using a variable effects the overall size of the geometrical object produced (i.e makes it bigger or smaller).

Phase 2 Pupils understand that a variable can be used to represent a range of numbers. They have not identified how (and are not necessarily aware that) changes in the geometrical object are related to changes in the value of the variable or variable input.

Phase 3 Pupils begin to relate the effect of assigning different values of a variable to a related change in the geometrical object produced.

Phase 4 Pupils are aware that a relationship exists between the component parts of a geometrical object and that the variables used can effect this relationship. They have not however identified the relationship and cannot therefore use variables to make the relationship explicit.

Phase 5 Pupils are able to identify the relationship between the component parts of a geometrical object and can make this relationship explicit within a Logo procedure. Whether they do or not depends on the task.

The interactive nature of Logo means that pupils can negotiate the nature of a general relationship whilst interacting with the computer. So for example if pupils are working on the "Spiral" task (appendix 3.5) they can interact with the computer to develop an understanding of the general relationship within the spiral (see Section 5.2.9 for a more detailed description of Sally and Janet's approach to this task).

5.8.2 Discussion of Each Individual Pupil's Development

Not surprisingly the pupils' performance on the individual laboratory tasks reflects their previous computer experience although there is a gap between what pupils have used in the classroom activity and what they can use without support (either from their partner or from the teacher) when working on their own. Pupils have been ranked (pupil 1 - pupil 8) according to their attainment on their school mathematics scheme (appendix 4.1). Table 5.4 presents an overview of each pupils' classroom use of variable during the three years of the research according to the categories outlined in Section 3.3.2. The table shows that there are considerable differences between the Logo experience of each individual case study pupil. Ravi, Jude and Shahidur's more limited use of variable was a consequence of them being both case study pupils for a shorter length of time than the other pupils and having a higher absence rate than the other pupils. This meant that the teacher was more reluctant for them to spend "hands on" computer time on Logo work during their "normal" mathematics lessons. In choosing to carry out research in a "normal" classroom over a period of three years it had to be accepted that for reasons beyond the researcher's control the pupils were not always available for a "planned" session.

Table 5.4 : Overview of General Procedures Written by Case Study Pupils

CATEGORY OF USE	Pupil 1 SALLY	Pupil 2 ASIM	Pupil 3 GEORGE	Pupil 4 JANET	Pupil 5 JUDE	Pupil 6 RAVI	Pupil 7 LINDA	Pupil 8 SHAH.
(I) One Variable Input	4	2	1	4	2	0	2	1
(S) Variable Input as Scale Factor	3	5	4	3	4	3	7	7
(N) More than One Variable Input	3	3	3	3	0	0	3	0
(O) Variable Operated on	6	6	6	6	1	0	1	0
(G) Input to General Superprocedure with Variable Subprocedure	2	3	3	2	3	0	4	0
(R) Recursive Procedure	2	3	3	2	0	0	0	0
(F) Input to Mathematical Function	2	2	2	4	1	2	3	2

Each individual pupils' development based on the analysis of the transcript data and the individual laboratory tasks will now be discussed.

Sally There is evidence throughout the three years of the classroom work that Sally tends to look for general relationships within a problem. However she does have some reluctance to commiting herself to formalising this generalisation in Logo. She has let Janet take control of the decisions related to the detail of the syntax. There is evidence from the transcript data that even when she understands a general relationship that she has both a difficulty in expressing it in natural language and in representing it with a Logo formalism. Without Janet's support Sally may never have been sufficiently motivated to engage in Logo programming tasks.

By the end of the period of research Sally could discriminate between using "(S) variable as scale factor", " (I) one variable input", "(N) more than one variable input", and "(O) variable operated on". and was able to use an appropriate category in order to solve a task. If she perceived a generalised relationship within a task she was able to make this relationship explicit by operating on the variables within her procedure. However she did not always perceive a need for a general relationship (for example the "Lollipop" task, Table 5.1) and would then use more than one variable to solve the task. There is some evidence that she also believes that using "(S) variable as scale factor" provides a simple way of defining a general procedure for a geometrical object. She initially suggested using this type of solution when working with Janet on the "Arrowhead" task. Finally however after considerable negotiation with Janet and with the computer they devised a solution to the "Arrowhead" task which used variable in the category of "(O) variable operated on". Sally initiated the idea of using a recursive structure to solve the row of decreasing squares task but was unsure of the associated formalism. There is strong evidence that she is able to perceive modularity within a task and this enables her to analyse a task into nested levels of general superprocedures. Her solution to the row of decreasing squares task indicates that she is able to use variable in the category of "(G) general superprocedure".

She understands that any variable name can be used but tends to be "conservative" in her choice of variable name. She understands that a variable represents a range of numbers and has confidently used decimal numbers and negative numbers.

Asim There is clear evidence that George dominated the Logo sessions at the beginning of the period of research and that Asim allowed George to dominate. In the first session in which they used the idea of variable (Chapter 5.3.1) Asim was not involved in any of the decisions related to the use of variable. Asim, like Sally, appears to be reluctant to use the Logo syntax for himself. At the end of the second year of the project when Asim worked with Jude (section 5.3.7) there is evidence that Asim had a clear idea of how he wanted to solve the task but was unable to match the Logo formalism to his own solution. Like Sally he often appears to have made a clear modular analysis of a task but is not so clear about how to link that analysis to a Logo solution. Within his collaboration with George he took on the role of providing the mathematical analysis of a problem (e.g the "Spiral" task, Section 5.3.3) but allowed George to take on the role of formalising in Logo.

Asim first started to become involved in decisions related to the use of the Logo syntax during the "Scaling Letters" task (Section 5.3.6). This is possibly because he was more comfortable with the directive approach of the handout (appendix 3.2) for this task. For almost the first time he took control of typing in the procedures during this session. There was very little risk associated with this because the method of solution had been clearly specified. After this session Asim usually invoked his "Scaling" frame when engaging in variable related task. He does not appear to have integrated this frame with his other variable frames. In particular when Asim and George were working on the "Arrowhead" task (Section 5.3.10) Asim was talking from his "Scaling" frame throughout most of the session. His own individual solutions to the individual laboratory tasks (Section 5.7) at the end of the third year of the case study research indicate that he was able to initiate the idea of operating on a variable to make a relationship explicit within a Logo program, although he still needed support with the syntax. There is no evidence that he can use variable to define a general superprocedure or a recursive procedure. His choice of variable names throughout the project indicates that he does not clearly underdstand that any variable name can be used. His choice of the variable name MUL to solve the tasks on the "Row of Decreasing Squares" task (Table 5.1) indicates a possible confusion between "(S) variable as scale factor" and "(O) variable operated on".

Asim's incorrect ordering of the inputs in his solution to the "paper and pencil" "RECTANGLE FUN SUN" task (Table 5.3) tends to suggest that he focuses more on global outcome and less on local details of a problem solution. Careful analysis of the data indicates that Asim's collaboration with George was detrimental to Asim's taking control of and subsequent understanding of variable in a Logo context.

George from the beginning of the project had shown a confidence and willingness to experiment with Logo syntax. He also needed to control the activity. This control often took the form of him planning a solution to a task before a session, so that during the session Asim could not become involved in the problem solving processes. George, like

Janet, needed the interaction with the computer in order to negotiate a general solution to a task. From the beginning George showed that he had taken on the idea of using a variable to represent a range of numbers and initiated the use of this idea in his own projects (see for example Chapter 5.3.2). There is evidence that George understands that any name can be used to represent a variable.

George did not always find it easy to analyse a problem in a "top down" way and his solution to the "Row of Decreasing Squares" task (Table 5.1) illustrates this. However after negotiating a solution with the computer he is able to take the risk of using a higher level Logo structure than the one he uses in direct mode (for example general superprocedure or recursion). George can operate on a variable to make a relationship explicit within a procedure. He has used the idea of tail recursion during several projects and used it when working on an individual "extension" to one of the laboratory tasks. He was able to correctly interpret the "paper and pencil" recursive procedure (MYSTERY, Table 5.3) administered on the same day. He is able to confidently define a general superprocedure and seems to particularly enjoy building up sets of nested subprocedures. It seems that George never took on board a "(S) variable as scale factor" frame. This could be because he had already understood the idea of operating on a variable within a procedure before engaging in the "Scaling letter" task.

Janet appears to need negotiation and computer feedback in order to come to terms with a general relationship. She is however not afraid of taking risks and trying out a solution even if this solution turns out to be incorrect. She appears to have integrated her "(S) variable as scale factor", her "(I) one variable input" and her "(O) variable operated on" frame and no longer used variable as scale factor by the end of the period of research. Janet was initially resistant to using decimal numbers as input to a variable but showed no such resistance by the end of the three year study. It is suggested that the "Scaling letters" task played an important role in this respect. She clearly understands that a variable can represent a range of numbers and she understands that a variable name can be any name although she does not often choose to use abstract variable names.

She did not choose to operate on a variable in any of her solutions to the Individual Laboratory tasks (see Table 5.1) but when nudged to do so by the researcher in the "Lollipop" task could do so without difficulty. Evidence from her solutions to the individual laboratory tasks (Table 5.2) and the "Arrowhead" task (Section 5.2.12) indicates that Janet is more likely to introduce a number of unrelated variables in order to solve a task than to use a variable to make a relationship explicit. Janet appears to use the naming of a variable in the title line of a procedure as a means of helping her to plan her use of variables within a procedure. This has also been reported by Hoyles and Noss (Hoyles & Noss 1988). It is suggested that her collaboration with Sally has been crucial in provoking her to come to an understanding of a general relationship within a problem.

Jude was only a case study pupil for the first two years of the project. Within that time he mainly used variable in the category of "(S) variable as Scale Factor" and "(I) one variable input". His solutions to the individual laboratory tasks (Table 3.2) indicates that he could use variable to solve the tasks although he did need "spoken" support from the researcher in order to reconstruct the Logo syntax. This was not surprising as he had only worked on Logo tasks three times during the third year of the longitudinal case study research (he was no longer worked with Linda as a case study pupil and the class teacher did not encourage him to use the computer). He solved the individual "Lollipop" task (Table 5.1) by operating on a variable. He was also able to solve the "paper and pencil" "Variable Operated on" interpretation task (Table 5.4). His understanding of variable in this category was unexpected and further analysis of the transcript data indicates that his first introduction to the idea of variable (when working with Linda) had been in the context of defining a general polygon (using "Variable operated on"). This session had been very "teacher directed" and he did not ever use variable again in this category during his Logo programming sessions. However when engaging in the "Scaling Letters" task (Chapter 5.4.3) he reflected on the relationship between the component parts of the letter being scaled and did not engage in the task in a rote manner.

His choice of variable names is still restricted to those used when he first encountered variable (e.g. SIDE, SCALE). The researcher specifically intervened to provoke the pupils to use any variable name in the third year of the study and because Jude was no longer a case study pupil he was not a recipient of this teacher direction. Evidence from the transcript data indicates that Jude has accepted the idea that a variable represents a range of numbers.

Ravi became a case study pupil at the beginning of the third year of the project and for this reason his use of variable within Logo was restricted to five sessions in which he used variable in the category of "(S) variable as scale factor" only. However he has accepted the idea that a variable in Logo represents a range of numbers and there is evidence that he is beginning to reflect on the effect of multiplying fixed numbers by a scale factor. There is no evidence that he treats a geometrical shape as a whole from the point of view of analysing the interrelationship between its component parts. His choice of variable name was almost entirely restricted to the name SCALE. When given the individual laboratory tasks (Table 5.1) he needed "spoken" help with syntax. His solutions to the "Variable Square" and the "Lollipop" task (Table 5.1) both used variable in the category of "(S) variable as scale factor". He did not use variable at all in the "Row of Decreasing Squares" task. When given the individual "paper and pencil" laboratory tasks (Table 5.4) he could only interpret the "(S) variable as scale factor" tasks.

Linda Evidence from the transcript data suggests that Linda predominantly uses a "(S) variable as scale factor" frame and she was clearly thinking from this perspective when working on the "Arrowhead" task (Section 5.4.9). However when solving the individual laboratory tasks she did not use variable in the category of "(S) variable as scale factor". This is possibly because when working on the first task she had asked for help with syntax and had been given a Logo handout on which variable was used in the category of "(I) one variable input" (appendix 5.2). This does tend to indicate that she can invoke either a "(I) one variable input" frame or a "(S) variable as scale factor" frame, or "(N) more than one variable input" frame but that she has not yet integrated these different frames. There is no evidence that she is able to analyse the relationship between the component parts of a geometrical object and consequently cannot use variable in the category of "(O) variable operated on".

Linda understands that any variable name can be used and particularly enjoys using nonsense names. She does not choose to use abstract variable names. She also understands that a variable represents a range of numbers and has extended her understanding of "range of numbers" throughout the three years of the longitudinal study. At the beginning of the research she was totally resistant to using decimal input but as a consequence of the "Scaling Letter" task started to use decimal numbers and negative numbers as inputs to variables. In the context of building up loosely defined goals Linda had defined general superprocedures but she had never used a recursive procedure. She was not able to interpret the "paper and pencil" general superprocedure or recursive procedure task (Table 5.4).

Shahidur Throughout his Logo work Shahidur has almost exclusively used variable in the category of "(S) variable as scale factor" and has almost always used the variable name SCALE. He was initially resistant to the variable called C in the RECTANGLE "C task (Table 5.3) and it is suggested that this is related to his inexperience of using single letter variable names within the classroom Logo context. He could not interpret the "paper and pencil" task which used a variable name BIT although he could interpret the similar task which used the variable name SCALE. Again this would seem to stem from his restricted choice of variable names throughout his Logo work. He was confidently able to solve the individual laboratory programming tasks (Table 5.3) and he used variable in the category of "(S) variable as scale factor" to solve all three tasks including the "Variable Square" task. He was not able to interpret the "paper and pencil" laboratory task which used two variable inputs (Table 5.3). He also could not interpret the general superprocedure or the recursive procedure task (Table 5.3). He was able to interpret the question which used variable in the category of "(O) variable operated on" (Table 5.3). It is suggested that within interpretation tasks there is no difference between "(S) variable as scale factor" and "(O) variable operated on". The differing demands of the two categories of variable use only become apparent when the pupil has to analyse a task in order to define a general procedure.

CASE STUDY PUPILS' USE AND UNDERSTANDING OF THE FUNCTION MACHINE MATERIAL

6.1 OVERVIEW

One aim of the research is to relate pupils' understanding of variable in Logo to their understanding of variable in 'paper and pencil' algebra. Research evidence suggests that most pupils do not on their own make links between similar concepts encountered in different contexts (DeCorte and Verschaffel, 1985; Lawler, 1985; Pea & Kurland, 1984) and findings from the Logo Maths Project also support the view that pupils' knowledge is very context specific (Sutherland and Hoyles, 1987). For example pupils were not able to relate their understanding of "360 degrees round a point" developed within a mathematical context to 360 degrees as a total turn within a Logo context. Similarly many pupils showed no evidence of being able to relate their knowledge of turtle turn in Logo to angle in "paper and pencil" mathematics (Hoyles and Sutherland, 1986). Thus there was no reason to suppose that the case study pupils would make links between variable in Logo and variable in algebra without specific teacher directed tasks designed to provoke these links.

Logo is a functional programming language, the underlying model of which is the mathematical idea of function. It is possible to define and build up functions, composite functions and inverse functions in Logo which model the behaviour of functions in mathematics. The following example based on an elementary mathematical function will serve to illustrate this point. The mathematical notation for function varies both historically and pedagogically and fig 6.1 represents a simple function by means of a) Logo notation b) mapping diagram and c) algebraic notation.

a) Logo Notation

```
ADDFOUR "X
OUTPUT ADD :X 4
END
```

b) Mapping Diagram

IN		OUT
3	->	7
-2	->	2
1.5	->	5.5

c) Algebra Notation

F(X) = X + 4

or X -> X + 4

Fig. 6.1: Function Representations

These can all be thought of as different representations of the same function. Associated with the function is a domain and this can be defined for both the mathematical and the

Logo function. Associated with each member of the domain is a unique image. In order to define a function in Logo it is necessary to use the idea of output. The composite function in Logo can also be represented in a way which matches the algebraic representation as the following example (Fig. 6.2) illustrates:

a) Algebra representation

FG(x) =F(G(x)) Where the functions F and G are defined by:

F(x)=x+4 G(x)=2x

b) Logo representation

```
TO FG "x
OP F G :x
END
```

where the functions F and G are defined by:

```
To F "x          TO G "x
OP :x +4         OP 2 * :x
END              END
```

(from now on the abbreviated version OP of OUTPUT will be used).

Fig. 6.2: Composite Functions

The inverse function can also be represented for example:

a) Algebra representation

$H(z) = z+4.5$
$H^{-1}(z) = z-4.5$

b) Logo representation

```
TO H "z          TO INVH "z
OP z+4.5         OP z-4.5
END              END
```

Fig. 6.3: Inverse Functions

In both the algebra and the Logo representation changing the name of the variable does not change the function itself. So for example $H(y) = y-7$ is the same function as $H(w)= w-7$ and in Logo:

```
TO H "y     is the same as     TO H "w
OP :y-7                        OP :w -7
END                            END
```

This thesis is concerned with pupils' understanding of variable and not function. It was decided however to base materials to help pupils make links between variable in Logo and variable in algebra on the idea of function because of the similarity in structure between the two. The role of variable in defining Logo functions is similar to the role of variable in defining algebraic functions. It was hypothesised that presenting pupils with these two similar contexts would help them make links between variable in Logo and variable in algebra. Although the primary aim of the function machine material was to use the similarity between function in Logo and function in algebra to help pupils make the link between variable in Logo and variable in algebra, subsidiary aims of the materials were:

- to extend pupils' use of variable to a non-graphics context in which a variable represents a number

- to move pupils from using words for variable names to single letters as this is what is normally encountered by them in the 'paper and pencil' algebra situation

- to extend pupils' experience of using 'unclosed' variable expressions in Logo

- to provoke pupils to use decimal and negative numbers and in doing so extend their understanding of a variable as representing a range of numbers

- to confront pupils with the idea that changing the symbol within a function does not imply changing that to which it refers

The pupils' function machine experience consisted of two types of activity.

a) a computer based activity and

b) a "paper and pencil" based activity

The following two sections describe these activities in detail. Before the materials were used with the case study pupils they were piloted with eight similar aged pupils in a separate secondary school. These pupils had been part of the Chiltern Logo project and had all had approximately 60 hours of "hands on" Logo time.As a consequence of the piloting the original handouts were modified and in addition one extra handout to make a guessing game explicit was devised. The function materials described in this chapter were influenced by "Number Mappings" which is one of a series of booklets prepared by the DIME Pre-Algebra Project (Giles 1984).

6.2 COMPUTER BASED ACTIVITY

6.2.1 Description of Materials

Working in pairs pupils were given a worksheet asking them to define a simple arithmetic function (appendix 6.1a). This was the first time that any of the case study pupils had used the Logo idea of output. The worksheet directed the pupils to try different inputs to the 'function' machine. The role of the researcher/teacher was to keep the pupils on task. They were asked to experiment with a range of inputs; the inputs were specifically chosen to include a decimal number and a negative number to extend the pupils' notion of "any number" and to "make sense" of this Logo procedure . The researcher/teacher asked the pupils to use a range of variable names including single letter names. The following are examples of some of the procedures which were defined:

```
LL "L                         HAZEL "NUT
OUTPUT MUL 1.5 :L             OUTPUT DIV :NUT 3
END                           END
(equivalent to x -> 1.5x)     (equivalent to y -> y/3)
```

When the researcher/teacher felt confident that the pupils had begun to develop an understanding of defining simple functions in Logo, they were given a worksheet (Appendix 6.1b) which:

1.Asked one of the pair to define a function machine without allowing their partner to see the function.

2. Asked the other one of the pair to put numbers into the function machine in order to work out the function. The "guesser" was asked to draw a mapping diagram as a problem solving tool. This "guessing game" was a critical element of the materials because it motivated both pupils to reflect on the process within the function machine.

3.Asked the "guesser" when she/he had worked out the function to define an identical function machine. In order to prevent their partner from guessing the function the pupils saw the necessity of choosing function names which were not linked to the effect of the machine.

4. Asked the pupils to convince themselves that both the functions were identical in structure although the names used might be different.

For example:

```
TO MAT "PIG              TO MULRED "RED
OP  MUL  14  :PIG        OP  MUL  14 :RED
END                      END
```

are both equivalent to z -> 14 z.

In order to provide a challenge for all the case study pupils two extensions sheets were prepared (Appendix 6.1c and 6.1d) which introduced the pupils to the idea of a composite function and an inverse function. Again the primary aim of this material was that the pupils should make links between variable in Logo and variable in algebra. The extra sheets were only introduced to pupils who seemed to need tasks of a more challenging nature than the initial task.

6.2.2. Analysis of Pupils' Use and Understanding of Function Materials

At a later stage in this thesis the effect of pupils' use of the function materials on their understanding of variable in algebra will be analysed. In order to provide a basis for this analysis this section will present a detailed description of each of the case study pupils' use of the materials. Only detail which is considered relevant to the pupils' understanding of variable will be included. Crucial aspects of the analysis are highlighted within the text. The use of these materials provided considerable insights into the pupils' understanding of decimal and negative numbers. This is part of the conceptual field of variable under study within this thesis (see chapter 3.3) and so this detail will be included. The case study pupils did not always work with their normal partner when working on these materials. The timetable in section 4.3 gives an overview of when these materials were administered in relation to the longitudinal collection of transcript data, the structured interviews and the final "Arrowhead" task.

Sally and Janet Session 1 & 2 Sally worked for two sessions both with Janet. When Sally was first given the sheet (Appendix 6.1a) she said *"like a calculator innit."* Both Sally and Janet seemed to quickly understand the idea of defining a function in Logo and defined an "add 8.25" and a "subtract 9" function. They used the variable name NUM for both functions (the name presented to them on the sheet) and **when asked if the name had to be NUM Janet said that it could be any name.** They then defined a "Multiply by ten" function using the variable name PIG .

```
TO MULTEN "PIG
OP MUL 10 :PIG
END
```

Janet specifically asked if it was possible to write a procedure to *"add any number to any* *number"*. **This requested intervention suggested that she had already taken on the idea of variable input as a place holder for a range of numbers.** They were shown how to modify their existing "Add Four" procedure to do this.

```
TO ADDY "NUM "PIG
OP ADD :NUM :PIG
END
```

They showed an understanding of the general nature of their procedure by changing the name ADDFOUR to ADDY. They then planned to define a procedure to "divide any number by six " but instead defined a procedure to divide 6 by any number.

```
TO DIVSIX "NUM
OP  DIV 6 :NUM
END
```

They tried out PRINT DIVSIX 5 and the result given was 1.2. The decimal number obviously confused them because Janet said *"we need to have a bigger number.."* She was expecting the input to be divided by 6 and the decimal result made her think that their chosen input was too small. The order of the inputs to DIV was not confronted by this example as neither of them reflected on the process within DIVSIX. **In fact none of the pupils reflected on the relationship between the input and the output and the process within the function procedure until they started to play the "guessing game".**

The researcher decided to introduce Sally and Janet to the "guessing game" (Appendix 6.1b). When Sally saw the mapping diagram on the sheet she said *"Like them DIME cards innit"* (Giles, 1984). **Sally and Asim were the only case study pupil who spontaneously related this work to her normal mathematics work.** Janet first defined a "multiply by 14 " function and initially chose the name "MULBOX" for her procedure. She seemed to think that the word MUL in the procedure name had some significance but the researcher pointed out that this name would help Sally guess the function and so Janet changed the name to MAT. This was an example of the pupils' programming action giving an insight into their misunderstanding and the ease with which it was possible for the teacher/researcher to present the pupil with a counter example. Sally guessed the function after trying three inputs and then defined her own "multiply by 14" function MULRED.

```
TO MAT "PIG            TO MULRED "RED
OP MUL 14 :PIG         OP MUL 14 :RED
END                    END
```

Sally had also used the word MUL in the name of her procedure. They tried out the functions and they were both satisfied that these procedures represented the same function. Sally then defined a multiply by 6 function:

```
TO SAND "WERT
OP MUL 6 :WERT
END
```

which Janet worked out after trying five inputs.

Two days later when Sally and Janet worked on the materials again Sally specifically asked if the function could do two things, meaning combine together more than one function. **This suggests that Sally had related the logo activity to her "normal" mathematics work in which she had already encountered the idea of a composite function.** They were both given the composite function sheet (Appendix 6.1c) and they spent some time making sense of the order in which the composite function was calculated at first making an incorrect prediction:

```
TO SUBSIX "Z          TO ADDTEN "A
OP SUB 6 :Z           OP ADD 10 :A
END                   END
```

Eventually they were able to predict correctly the order of execution of two simple functions composed together (ADDFOUR and SUBSIX). **In order to define these functions they had used the variable letter names Z and A another indication that they were beginning to relate this activity to their "paper and pencil' algebra work.**

Next they were given the inverse function sheet and were told to play the "guessing game" in the context of working out the inverse function. Janet defined a "multiply by 13" function (JOB).

```
TO JOB "J             TO UNDOJO "Q
OP MUL 13 :J          OP DIV :Q 13
END                   END
```

using the name JOB for the function and the name J for the variable. Sally after trying three inputs (1 2 3) decided she knew the function and defined the inverse UNDOJO. She tested her hypothesis that this was the inverse function with one input 2 by typing in PRINT UNDOJO JOB 2 and as this input gave the same output of 2 she decided that her inverse function was correct. (Although not the focus of the research this does suggest that materials need to be prepared in which the correct response from one example leads to an erroneous proof. Not only is this important for mathematics but it is also important in the area of testing and debugging programs). Sally then defined a subtract 5 function (LOT) and after trying four inputs Janet decided that she had worked it out and correctly defined the inverse UNLO.

```
TO LOT "W          TO UNLO "R
OP SUB :W 5        ADD :R 5
END                END
```

Again Sally and **Janet's use of function names and variable names** suggested that they **understood that "any name" could be used.**

Linda and Elaine Session 1 Linda and Elaine were handed the starting sheet (Appendix 6.1a) and they defined the given ADDFOUR function trying this out with the given inputs . Linda initiated the idea of defining a procedure to subtract and also said *"We don't have to use NUM do we?"* **indicating that she already understood the idea that any name can be used for a variable.** They defined SUB16 expecting the procedure to subtract 16 from any number.

```
TO SUB16 "SEAN
OP SUB 16 :SEAN
END
```

They tried out PRINT SUB16 27 and did not appear to be surprised by the -11 result. They then tried PRINT SUB16 59 and did not question the -43 result. When they tried PRINT SUB16 20 and when the result given was -4 they finally questioned the answer saying that it should have been +4. When they were asked to study the procedure Linda said *"16 minus 20..it can't do it...it would be minus..."*. **She was rejecting the idea of a negative number answer even though this was what the computer had produced.** It was as if negative numbers were not part of her understanding so she had not attended to the negative results produced by the computer; she had in fact denied them; they were meaningless to her so she had not bothered to make sense of them and there was nothing about the situation which provoked her to do so. She said *"You can't do it with numbers bigger than 16..It would be OK to enter 4........this would give 12"*. They then changed the order of the subtraction in their procedure with Linda saying *"I'm no good at subtraction"* . When they tried an input of 2 with the new function machine the output produced was -14. Linda said *"Oh because we took away a bigger number than 2."*. She said *"Do a big number"*. They tried inputs of 7056 and 243. In order to provoke them to reflect more on the processes within their procedures they were told to try out the guessing game and Elaine defined a "subtract 17" machine called POXIE with variable input called SEAN.

```
TO POXIE "SEAN
OP SUB :SEAN 17
END
```

Linda tried out an input of 8 and the machine returned -9 she then tried an input of 9 , by now confidently predicting that the result would be negative. She tried one more input and correctly worked out the function although she was not able to explain how she had done this. Within this session Linda and Elaine have both begun to reflect on the processes involved in defining simple function machines.

Linda and Janet Session 1 Linda with some help from the researcher started the session by defining a procedure to multiply by 10. Janet after trying four inputs guessed the function. Janet then defined a function JO, to subtract seven from any number. She initially typed OP SUB 7 :K but then changed this without any intervention to OP SUB :K 7. Linda tried the inputs of 1 and 2 and said confidently "It's minus 7". She defined the same function LEIGH, and checked that they were both identical using one input.

```
TO JO "K                    TO LEIGH "JO2
OP SUB :K 7                 OP SUB :JO2 7
END                         END
```

Linda then defined a function UGLY, to "Divide by 3" which after three inputs Janet guessed correctly.

```
TO UGLY "NIC
OP DIV :NIC 3
END
```

Janet then defined an "add 3.5" function (TWA) for Linda to guess.

```
TO TWA "M
OP ADD 3.5 :M
END
```

It had been observed from analysis of previous transcript data that Linda was very reluctant to use decimal numbers. In this context when presented with decimal output she was no longer able to use her previous knowledge of adding functions (developed during her previous function machine session). Linda produced the following mapping diagram:

IN	OUT
5	8.500
4	7.500
10	13.500

and reflecting on these numbers said *"ADD"* Janet prompted her by saying *"ADD what?"* Linda then gave the surprising answer *"ADD 1000"* . It appeared that Linda was completely confused by the zeros in the last two decimal places. In order to probe her understanding the researcher pointed to the screen and asked what the number said. Linda replied *"thirteen point five hundred"* . Linda then said, *"Oh it's add 1"* possibly now modifying the 1000 response on the basis of some previous learnt knowledge that the zeros after a decimal point have no effect on the number. She then tried:

IN	OUT
20	23.5
1	4.5

and said *"Yeah it's add"* . She said again *"It's add 1"*. Janet in order to help Linda said *"Why 1.....if you put 10 in and add 1 what would you get?..if you add one to two...do you get 5....you get 3..."* Linda continued to reflect and tried an input of 12 which gave 15.5. She was obviously having considerable difficulty and said *"Add.....what...I must be stupid..."*, to which Janet said *"You're not stupid....you're not trying...because you reckon you're stupid...just get on with it..."*. Linda was becoming desperate *"Give me a clue..."* . Janet then carried out a classical teaching episode "leading" Linda to the answer:

Janet *"What is 1 + 2"*

Linda *"3"*

Janet *"What is 15.5 - 12?"*

Linda *"13.5 ....no...3.5..."*

Janet *"So what is it?"*

Linda *"Add 12"*

Janet *"No"*

Linda *"ADD 3"*

Janet *"And if it'd be 3....it'd be.....?"*

Linda *" o.k.......add 3.5".*

Linda then defined an "Add 0.5" function for Janet called SLOAN. Her decision to use a decimal number indicated that the "didactical contract" of the game had pushed her into using decimal numbers as this is what Janet had used.

```
TO SLOAN "RANGER
OP ADD :RANGER 0.5
END
```

The researcher suggested to Linda that she made the function something other than "add 0.5"...and she clearly gave her reason for choosing 0.5. *"Well I wouldn't be able to work anything else out..."* . Janet tried three inputs and said *"Times by 1.5"* . The researcher suggested that she checked this by defining a "Times by 1.5" function and she defined EE.

```
TO EE "E
OP MUL 1.5 :E
END
```

She tried out one input 20 which produced 30 and said *"It's wrong"* She then after trying two more inputs said that the function must be add .5. She said that it had taken her a long time...because she thought that *"it would be timesed."*. Janet then defined:

```
TO TB "K
OP ADD 21.5 :K
END
```

Linda tried an input of 1 which gave 22.5 and then an input of 2 which gave 23.5. Linda said *"You've done it again..it's add 1.5 "* to which Janet replied *"No"*

Linda then tried an input of 3 which gave 24.5 and an input of 4 which gave 25.5.

Linda *"Right...it's add...'cos it keeps adding...as the number gets bigger...so is the answer...it's add 1 ".*

Janet *"No".*

Linda tried 10 which gave 31.5. The researcher asked them both how they could work out how much the function was adding by using the input and the output. Janet said *"take 31.5 and take away 10".* Linda did this and said *"It is add 21.5"* . **Throughout this session mapping diagrams had played an important role in helping both Janet and Linda reflect on the process of the function machine. Throughout the session Janet had consistently used single letter variable names and Linda had consistently used nonsense names.**

<u>Linda and Janet Session 2</u> Janet and Linda start the session by making sense of the function machines again. Janet defined:

```
TO SLO "PPY
OP ADD :PPY 13
END
```

and explained to Linda that *"You have to put it that way round otherwise you get minus numbers when you do subtract".* **Janet had devised a working rule of "Put the variable first" without understanding the process.**

Linda then defined:

```
TO NUT "D
OP SUB :D 56
END
```

Linda tried to reconstruct the "working rule" which she had devised in the previous session. *"for add machine you subtract the input from the answer; for subtract machine you subtract the answer from the input..".*
The researcher suggested that they try to construct a similar rule for multiplying and dividing. Linda said *"I'm hopeless on my times tables".*

```
TO WAL "NUT
OP MUL 9 :NUT
END
```

They tried an input of 4 which produced 36 and Janet said *"36 divided by 4 is 9"* . They both developed a new "working rule".

Janet *"For MUL it's divide"*

Linda *"And for divide do mul"*

They defined:

```
TO HAZEL "NUT
OP DIV :NUT 3
END
```

with Janet reiterating her "working rule" of *"It's the variable and then the number"* . They tried an input of 12. Janet then confidently defined a procedure with two variable inputs reconstructing what she had done in a previous session with Sally:

```
TO SHE "WO "MAN
OP ADD :WO :MAN
END
```

Linda said *"It'll add any number you put in to anything you want"* indicating that she did have a good understanding of the idea of using a variable in Logo to represent any number. They then defined:

```
TO H "M "AN
OP SUB :M :AN
END
```

The reseacher asked Linda *"What happens if you do the second one larger?"* and Linda's reply of *"You get a minus number"* indicated that she was reflecting on the process within the procedure and that she was also coming to an understanding of subtracting a larger number from a smaller number. They tried out PRINT HE 123456 123333 and the computer replied 123. They next defined:

```
TO BAT "TLE "CAT
OP MUL :TLE :CAT
END
```

and

```
TO SKEL "IT "ORE
OP DIV :IT :ORE
END
```

They tried to predict what the answer would be to PRINT SKEL 45 4. Janet said *"11"* and Linda said *"8"*. Neither of them calculated the decimal answer 11.25 correctly. They next tried PRINT SKEL 66 5 and Linda predicted an answer of 11 and Janet predicted an answer of 13. By the third try the previous computer responses appeared to have provoked them into taking into account the decimal fraction part of the answer and when they tried PRINT SKEL 93 15 Janet predicted *"6 and a half "* and Linda predicted *"6 and a bit"*. The computer replied 6.2 and Linda said *"I told you I'm no* *good at this."* to which Janet replied *"Cos you reckon you're no good...that's why....".*"They spent the rest of the session giving each other "sums" using their BAT and their PAT procedure. Not only did they both find this very motivating with Linda **explicitly saying that she was enjoying the session but there was clear evidence that the computer responses were helping them reflect on and "home in" to the correct solutions. Linda's expressed enjoyment of 'playing with" decimal numbers should not be underestimated, especially when put in the perspective of her resistance to using decimal input in the beginning stages of her turtle graphic work. It is suggested that one of the motivating factors throughout this session was the freedom which Linda and Janet had to choose any variable and any procedure name. In what other situation could pupils chose such extraordinary names for mathematical functions?**

Ravi and Shahidur Session 1 This was Ravi and Sawkat's first session with the function machine materials and they copied the ADDFOUR machine from the sheet (Appendix 6.1a) into the computer. They tried out the inputs 3 and 12.5 both specified on the sheet. They then define an ADDSIX function still using the variable name NUM. They tried out inputs of 5 and 29 and then defined an "ADD 13.5" function, choosing the decimal number themselves but being prompted to choose a variable name other than NUM by the researcher.

```
TO ADD13.5 "SUM
OP ADD 13.5 :SUM
END
```

It is almost certain that Ravi and Shahidur thought that the name "ADD13.5" was a significant part of the function definition. They tried out inputs of 4 and 18.5. It was then suggested that they try the guessing game. Jude with help from the researcher defined an "Add 23.6" function.

```
TO JUDE "PAPER
OP ADD 23.6 PAPER
END
```

Shahidur was reminded to use a mapping diagram to help the "guessing "and he wrote down:

IN	OUT
2	25.6
1	24.6

He then said *"I've got it now...it is Jude NUM then 23.6".* When asked what the function was doing to the 23.6 he replied *"adding to it".* His first comment seemed to

operator ADD. (It should be noted here that English is his second language and that when he first started secondary school he could hardly speak any English). Shahidur then defined:

```
TO MODRIS "SUM
OP :MODRIS
END
```

indicating considerable confusion about the procedure name, the variable name and the syntax of the Logo commands.With help from the researcher he finally defined:

```
TO MODRIS "SUM
OP ADD 15.9 :SUM
END
```

It is interesting to note that in contrast to Linda, Shahidur does not appear to be at all resistant to using decimals in this context. Ravi tried the inputs 3 and 1 and said *"All the time it's adding miss...so you have to..."* . After some support from the researcher he said that it was adding 15.9. Ravi then started to define:

```
TO KIYA "DIV
OP DIV
END
```

Again this use of syntax indicated a considerable level of confusion. The prefix operators (DIV MUL etc) appear to be adding to this confusion. With help from the researcher Ravi defined:

```
TO KIYA "EARS
OP DIV :EARS 6
END
```

Shahidur tried an input of 1 which produced 0.167 and an input of 2 which produced 0.333 and Ravi seemed surprised by these decimal numbers. Shahidur however said *"It can't be subtract...it's divide something"*. This reasoning indicates that Shahidur has a good confidence and facility with numbers .He eventually correctly said that is was *"divide by 6. "*

Ravi and Jude Session 1 This was Jude's first and Ravi's second session with the function machine material. They both started with the beginning sheet (Appendix 6.1a) and defined the ADDFOUR function trying this out with the inputs specified on the sheet. They then defined:

```
TO MULFIVE "NUM
OP MUL 5 :NUM
END
```

and tried an input of 5. They were then given the "guessing game" sheet (appendix 6.1b) and Ravi defined:

```
TO NOSE "NUM
OP MUL 6.5 :NUM
END
```

He had correctly used the syntax without any intervention but still using the variable name NUM which had been presented on the sheet. Jude tried inputs of 5 (giving 32.5) and 1 (giving 6.5). Jude was convinced that the operation was add and tried an input of 0 "*to see what number it has to add.*" He first said that the function is "*add 6.5*" and then after being asked to reflect said " *times by 6.5*". Jude then defined:

```
TO W "NUM
OP SUB :NUM 17
END
```

Ravi tried an input of 3 (producing -14.0) "*SUB ehh....it's a low number...it's minus innit....it's obvious innit*". He then tried 1 which produced -16 at which he gave an insight into a misconception on negative numbers by saying "*It's gone up hasn't it.*". Jude however indicated his understanding by saying "*No ...it's gone down*". Ravi then tried an input of 2 which produced -15.0. It was the end of the lesson and as Ravi could not work out the function Jude told him what it was. **During this session Jude had only used the variable name NUM .**

Shahidur & Fahid Session 1 This was Shahidur's second and Fahid's first session and they were given the beginning sheet (Appendix 6.1a). They defined the ADDFOUR function. Shahidur appeared to be confused. They needed support to get started. They tried PRINT ADDFOUR -14 and Shahidur predicted that the result would be -18. The researcher suggested that they study the procedure but Shahidur was still confused. Shahidur defined another function:

```
TO SUBSIX "NUM
OP SUB 6 :NUM
END
```

The researcher suggested that they predict what the result was going to be before pressing the return button. They predicted that an input of 10 would produce an output of 4. When this was not produced they discussed for some time whether the 6 was being taken away from the 10 or the 10 being taken away from the 6. They tried an input of 12 and correctly predicted an output of -6. Shahidur appeared to be beginning to understand the processes involved. The researcher asked them if the variable name had to be NUM and Shahidur said "*It could be anything*". He changed the name NUM in

SUBSIX to NO . They tried this out and the procedure still worked in the same way giving them concrete experience that the variable name is not significant. The researcher said *"Could it be just one letter like N?"* Shahidur replied *"maybe"* and tried this finding that the procedure still worked. Shahidur said *"That's because of the quotes and the dots"*. They were given the "guessing game" and Shahidur defined:

```
TO FATHEAD "SUM
OP :NUM DIV 3.7
END
```

indicating confusion between the naming of the variable and the syntax of DIV. The researcher explained his mistakes and also suggested that he used an operation other than DIV. He defined:

```
TO FATHEAD "NUM
OP SUB 5.5 :NUM
END
```

Although Fahid tried a range of inputs and made a number of guesses Shahidur finally told him the rule *"subtract the number from 5.5"*, indicating by his use of language an awareness of the order of inputs used. Fahid then defined:

```
TO MIEOW "N
OP SUB :N 3
END
```

Shahidur tried an input of 2 and when this gave a result of -1 he immediately said *"Is it 3....is it subtract 3..."*. He tried an input of 1 which produced an output of -2 and said *"It is subtract 3"*. **There is evidence from this session that Shahidur has a good understanding of the processes involved in defining simple functions. He does however still have difficulty with the formal Logo definitions and cautiously persists** with the variable name **NUM.**

George & Asim Session 1 & 2. This was George and Asim's first session and they defined an ADDFOUR function (taken from Appendix 6.1a) trying out the suggested inputs. They then defined:

```
TO MUL17 "NUM
OP MUL 17 :NUM
END
```

and

```
TO SUB5 "NUM
OP SUB 5 :NUM
END
```

They tried an input of 3 and were surprised by the output of 2. **As with all** the other pupils Asim and George **had incorrectly specified the order of** inputs to

the subtraction operation (SUB). This could be due to a confusion over the syntax or a misconception about the operation of subtraction. The researcher explained about the importance of the order of the inputs for the subtraction operation. They were then given the "guessing game" sheet and when asked if they thought that the variable name always had to be NUM both said no. Asim asked if *"it could do two things"* meaning could he put in a composite function and he was told that he would be shown how to do this at a later stage. **This request which was similar to one made by Sally indicates that he was relating the Logo work to his use of functions in his "normal' mathematics.** George defined the procedure RESEARCH:

```
TO RESEARCH "DIG          TO RESEARCH2 "NUM
OP DIV 99 :DIG            OP DIV 99 :NUM
END                       END
```

Asim tested this with:

IN	OUT
4	24.750
2	49.5000
1	99.000
3	33

Asim eventually came up with "Divide by 99" and the researcher told him to define a procedure and he defined the procedure RESEARCH2 which "divided 99 by any number". Within this session Asim had not adequately confronted the issue of the ordering of the inputs to the subtraction operator. Next Asim defined for George:

```
TO GEORGE "NUM
OP MUL 0 :NUM
END
```

The rest of the session highlighted George's misconceptions about multiplying and dividing by zero and the researcher carried out a teaching episode in which she used concrete examples to show George the effect of dividing by a number which becomes increasingly smaller (i.e tending to zero.)

In the next session George and Asim started with the composite function sheet (Appendix 6.12). They defined:

```
TO ADDFOUR "X and                 TO MULTEN "Y
OP ADD 4 :X                       OP MUL 10 :Y
END                               END
```

They tried ADDFOUR MULTEN 1 which gave 14 and ADDFOUR MULTEN -3 which gave -26. They predicted that ADDFOUR MULTEN -7 would give -66 which it did. George explained *"Multiply by 10 gives -70 and then you add 4 giving -66".* Asim thought that an input of -3 would give zero but George explained that it should give 10. They were given the undoing function sheet and they started to define:

```
TO UNDOADDFOUR "Y
OP SUB :Y 4
END
```

but after studying this George changed the OP line to OP SUB 4 :Y indicating a reflection on the order of the inputs to SUB. After making sense of the composition of a function and its inverse they were asked by the researcher to play the "guessing game' with the guesser defining the inverse of the "guessed" function and not the function itself. George started by defining the following function for which an inverse does not exist:

```
TO ASIM "P
OP MUL :P 0
END
```

Asim looked at the MULTEN function and said that the undoing function would be DIV. They defined:

```
TO UNDOMULTEN "B
OP DIV 10 :B
END
```

Asim tried out ASIM 4 which gave 0 and ASIM 2 which gave 0. Asim wanted to define the inverse procedure; he was confused about the naming; did it have to be called UNDOMULTEN? George told him to call it UNDOOMAR. **It seems that the name of the function may still have too much significance for Asim.** Asim predicted the "ASIM" procedure to be:

```
TO ASIM "Z
OP MUL :Z 0
END
```

He then defined:

```
TO UNDO-OMAR "X
OP SUB :X 0
END
```

The inverse of the given addition function had been subtraction and this is probably why Asim thought that he should use SUB here. He tried PRINT ASIM UNDO-OMAR 5 which returned 6 and was puzzled. George said that he should do UNDO-OMAR first. Asim said that it would not make any difference. He tried PRINT ASIM UNDO-OMAR

0 which returned 0. He thought that his inverse procedure was incorrect and changed it to:

```
TO UNDO-OMAR "X
OP DIV :X 0
END
```

When he tried out PRINT ASIM UNDO-OMAR 4 an error message was produced. He tried PRINT ASIM UNDO-OMAR 5 which still produced an error message. Asim said that if the function was MUL :X 0 then the inverse must be DIV :X 0. The researcher again spent time exploring the effect of dividing by 0. Asim did not appear to have taken on board any of the explanations from the previous session. George initially thought that 30 divided by 0 would give 30.

6.3 "PAPER AND PENCIL" BASED ACTIVITY

6.3.1 Description of Materials

Approximately one month after the case study pupils had worked on the "hands on" function material they worked away from the computer on a series of 'paper and pencil' tasks. The aim of these tasks was to make explicit to the pupils that a function could be represented by a formal Logo representation and by a formal algebra representation. All the pupils were handed the same worksheet (Appendix 6.2) and they then worked through them at their own pace. The tasks were of the following form:

1. Worksheets (a) to (e) directing pupils to write down Logo functions from a range of mapping diagrams (appendices 6.2a to e). These worksheets were designed to consolidate the link between mapping diagram and Logo representation.

2. A worksheet (f and g) giving the pupils the conversions between Logo and algebra notation and directing pupils to make some further conversions themselves (appendices 6.2f and g).

3. Worksheets (h and i) directing pupils to write down both the Logo and the algebra representation for some given mapping diagrams(appendices 6.2h &i).

When the pupils had completed these tasks they were asked to write down the algebra representation on sheets (a) to (e) (on which they had already previously written down the Logo representation). Pupils were told that they could discuss the task with other pupils as the aim of the task was to make links between representations and not to test if pupils knew the function.

6.3.2 Analysis of Pupils' Responses to 'Paper and Pencil' Tasks

Pupils freely discussed amongst themselves the nature of the function represented by the mapping diagram. Despite this free discussion pupils used different notations to represent the same functions. They were all able to complete the material from the point of view of writing down representations. Analysis of the pupils' responses to these questions indicates that from the point of view of understanding of variable the most relevant aspects of their responses were:

- their choice of variable names to define the Logo functions in sheets (a) to (e).

- their choice of variable names to define the algebra functions in sheets (a) to (e)

- the relationship between the algebra (written down at the end of the session) and the Logo representation (written down at the beginning of the session) for sheets (a) to (e). In particular an analysis was made between the consistency of the ordering of the inputs to the Logo and the algebra operations.

The following is an overview of each case study pupils' responses to the "paper and pencil" materials.

Sally used the same variable name for the algebra representations as she had done for the Logo representations. In addition her ordering of the algebra representation was consistent with the ordering of the Logo representation. So for example if she had written down:

```
TO SUBTWO "X
OP sub :X 2
END
```

she wrote down x -> x - 2 for the Logo representation. Her solutions to all of the questions were correct. There was evidence of her both changing the ordering of the Logo representation after she had written down the algebra representation and of her changing the ordering of the algebra representation after she had reflected on and compared it with the Logo representation. The evidence from her performance on this task is that she is able to convert between representations in this context and that neither

her "algebra" frame nor her "Logo" frame are dominant.

Asim used the variable name A for all the Logo functions and the variable name x for all the algebra functions. The ordering of his Logo and algebra functions was consistent when it was important (i.e for all subtraction and division functions). There was evidence that he changed the ordering of Logo functions after writing down the algebra representation but no evidence of him changing the algebra representation with respect to the Logo representation. It is suggested that for Omar his algebra frame was dominant the time when he engaged in this task.

George used a range of variable names for both the Logo and the algebra tasks and he did not use the same variable name when defining the same function. Like George his order was consistent when it was important but not otherwise. There was evidence that he changed the algebra representation after he had written it down and compared it with the Logo representation. So for example one of his correct Logo responses to a question was:

```
TO SUBTEN "A
OP SUB 10 :A
END
```

He initially wrote down A -> A - 10 as the algebra representation and then after comparing this with the Logo representation changed the algebra to A -> 10 - A (which was now consistent with the Logo representation. It is suggested that for George his Logo frame was dominant when he engaged in this task.

Janet used the same variable name for both the Logo and the algebra functions. The order of her algebra representations was always consistent with the order of her Logo representations even when they were both incorrect. The evidence was that she matched the algebra representation to the already completed Logo representation and that for her a Logo frame was dominant while she engaged in this task.

Linda started to use the variable name A for an ADD function, S for a subtract function, M for a multiply function and D for a division function when working on the Logo tasks. She used the same letter for the algebra representation as she had used for the Logo representation. Her ordering was not consistent for the items on sheets (a) to (c) but was entirely consistent for sheets (d) and (e). In particular on one item her Logo representation was:

```
TO MULFIVE "M
OP MUL :M 5
END
```

and her algebra representation was M -> M5. This indicates that whilst engaging in this task she was converting from an algebra to a Logo representation. Linda had never previously engaged in any algebra as part of her "normal" mathematics lessons.

Ravi used the same variable name A for his algebra representation as he had done for his Logo when answering the items on sheets (a) to (e). There was no observable pattern in his responses, sometimes the Logo response was correct and the algebra response was incorrect and vice versa. It seems as if he randomly chose the ordering of inputs to the operations when carrying out the tasks and did not attempt to match the two representations.

Jude used the letter A for all of both the Logo and the algebra representations. He consistently put the number first in both the Logo and the algebra representations. He seemed to be using a rote rule to generate the representations and there was no evidence that he was making links between representations within this context.

Shahidur generated the letters for his Logo representations in alphabetical order and then used the same letter for the algebra representation. His response on sheet (a) and the first one of sheet (b) were not consistent (with respect to the ordering of inputs) but then he started to modify the Logo representation after he had written down the algebra representation even when the alteration meant that the response was no longer correct. Shahidur had not carried out any algebra as part of his "normal" mathematics lessons. Although Shahidur wrote down algebra expressions of the form 4 + C and 4xG (with the number first and the variable second) when he came to write down the algebra representation for a "subtraction" or a "division" function he was not able to accept the idea of the variable at the end of the expression. So for example he had correctly written down:

```
TO DIVTWO "I
OP DIV 2 :I
END
```

as a Logo representation, but when he came to write down the algebra representation he incorrectly wrote down I -> I ÷ 2 and then changed the Logo expression to OP DIV :I 2. This provides evidence of him making links between representations.

6.4 DISCUSSION OF RESULTS

When defining a function in Logo the syntax of the representation is critical. Although some of the pupils initially found the specific nature of the Logo syntax difficult to remember, the practical nature of the Logo sessions meant that by the end of the sessions all the pupils could define simple functions whilst working at the computer. The "guessing" game was important in engaging both pupils in the "hands on " activity of defining functions. It also provided the motivation to provoke the pupils to reflect on the process within their defined functions. Before engaging in the "guessing game" the pupils tended not to reflect on the relationship of the input and output to the defined function. In addition there was evidence that the writing down on paper of the mapping diagram was important in provoking pupils to reflect on the processes involved in defining a simple function. The fact that pupils were free to choose any variable name and any function name also appears to have been important from the point of view of motivation. Pupils with their partners seemed to find their own level of working. So for example some pupils rapidly defined functions for all operations and others spent longer with addition functions.

It was remarkable to observe how involved Linda and Elaine became with the "guessing" game and that, although on the one hand Linda said that she was useless at subtraction in this context she started to say *"I like working it out."* She also initially related her own fear of negatives to the computer (It has been reported elsewhere that students often "attribute to the computer some of the semantic capacities of the human operator" (Rogalski, 1985). In the course of one session Linda showed by her predictions that she was coming to terms with the idea of a negative number as being a reasonable answer. It is suggested that pupils at this age are well aware of some of their weaknesses in mathematics and are also not at all happy about the situation. They therefore welcome a new approach which provokes them to think about problems which they normally find difficult. Although the pupils almost always used positive whole numbers as inputs to their function machines the resultant output provoked them to think about both decimal and negative numbers.

The pupils were first presented with the idea of a function machine by means of a written example (ADDFOUR) and they spuriously generated from this one example. They all thought that the name "ADDFOUR" of the function on the first handout was significant. It was only when the "guessing game" pushed them into changing the name that they realised that any name could be used. They also used the same variable name (i.e. NUM) as the one presented on the sheet until nudged by the researcher into choosing any name.

The pupils choice of variable names during the "hands on" programming sessions could give important insight into their confidence in and understanding of variable. The following is a summary of the variable names used by the pupils during the "hands on"

Logo Function Machine sessions:

Sally	NUM PIG RED X Q W
Asim	NUM X Y B Z X
George	NUM DIG X Y B L
Janet	NUM PIG RED X J II ORE CAT WO MAN NUT TLE D E K M
Jude	NUM
Ravi	SUM PAPER EARS NUM
Linda	SEAN JO JO2 RANGER NUT WO MAN CAT
Shah.	SUM PAPER

Janet and Linda seem to have been the most imaginative in their choice of variable names. Sally, George and Janet were able to use "nonsense" names, meaningful names and single letter names. Asim and Jude always used the variable name NUM. Was this associated with a restricted understanding of variable? Jude, Ravi, Linda and Shahidur either used the given variable names or made up their own "nonsense" names. It was only the pupils who had had some experience of algebra in their "normal" mathematics lessons (see Appendix 4.4) who chose to use single letter variable names in this context.

There was evidence that both Sally and Asim related the computer based function materials to previous algebra based function work which it is known that they had already carried out in their "normal" mathematics lessons. Janet and George however did not make any explicit reference to previously learned algebra based ideas when engaging in the function machine material at the computer.

Evidence from the "paper and pencil" tasks suggests that Sally, Asim, George and Janet have made links between the Logo and the algebra representations of a function. For George and Janet their Logo frame appears to be dominant. For Asim his algebra frame appears to be dominant and for Sally there is no evidence that either frame is dominant. Both Linda and Shahidur appear to be beginning to make links between the Logo and the algebra representation of function. There is no evidence that Ravi or Jude have made any links between the Logo and the algebra representation of a function. The results reported in this chapter will be further interpreted with respect to the rest of the data and synthesised in Chapter 9.

CHAPTER 7

THE STRUCTURED INTERVIEW: CASE STUDY ANALYSIS

This chapter presents an analysis of the results of the structured interview administered individually to all the case study pupils at the end of the period of research. The general aims of the structured interview were to investigate:

- whether or not the case study pupils could formalise a generalisable method in both the Logo and the algebra context
- the pupils' understanding of algebra related ideas in Logo
- the pupils' understanding of Logo derived ideas in "paper and pencil" algebra

These results provide yet another piece of evidence which is being built up of the case study pupils' understanding of algebra related ideas in both the Logo and the "paper and pencil" algebra context. They provide a means of triangulating the research findings from the transcript data.

The algebra questions of the structured interview were all taken from the Concepts in Secondary Mathematics and Science study (C.S.M.S. appendix 1). Within this study the percentage positive responses to the questions for just under 1000 secondary aged pupils aged 14+ were known. In addition the algebra questions were administered to a comparison group of eight pupils. The C.S.M.S and the comparison group results will be used in section 7.4 to provide a framework for further analysis of the case study pupils' results..

Four of the case study pupils had had no algebra experience of "paper and pencil" algebra in their "normal" mathematics lessons and four had had some experience. Appendix 4.4 presents an overview of this experience.

7.1 DESCRIPTION AND RESULTS OF STRUCTURED INTERVIEW

All the questions of the structured interview were first piloted with a group of eight 14-15 year old pupils who had had approximately 60 hours of "hands on" Logo experience and who had all used the idea of variable in their Logo programming. As a result of this pilot it was found that there was some ambiguity in the Logo questions and so several modifications were made. This section will describe these questions together with their respective aims and present the results derived from the administration of the structured interview.

7.1.1 The "General Polygon" (Logo Question 1)

The pupils were asked to:

a) Write down the Logo commands to draw a square
b) Write down the Logo commands to draw a hexagon
c) Write down a Logo procedure to draw a general polygon

The aim of this question was to probe the pupils' understanding of the relationship between turtle turn and angle within a regular polygon. The question is included here because it gives insight into the pupils' ability to represent a general relationship in Logo.

The most important result from the analysis of this question is that the four pupils (Sally, Asim, George and Janet) who were able to express the relationship between the number of sides of a regular polygon and the turtle turn (4 out of 8) in natural language were able to write down the Logo formalism for this generalisation.

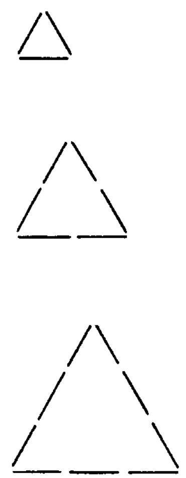
7.1.2 Formalising a Generalisation (Algebra question 2a and 2b)

It was decided to use two questions taken from the DIME (Giles, 1984) material in order to elicit the pupils' ability to articulate and formalise a generalisation in algebra. The questions used are given in Fig. 7.1a and Fig. 7.1b. If the pupils indicated that they were having considerable difficulty in completing question 2a they were not presented with question 2b.

Although all the pupils were able to express the general rule for question 2a in natural language only the "algebra experienced" pupils were able to formalise this rule in algebra. Table 7.1 provides an overview of the pupil responses.

(a) Equilateral triangles

1.



Length of side		Number of matches
1	⟶	3
2	⟶	6
3	⟶	
4	⟶	
5	⟶	
10	⟶	
50	⟶	

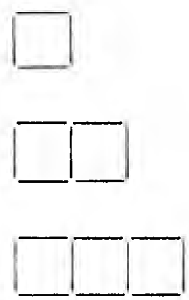
2 Explain in words how to get the second number from the first.

3 The whole mapping is given by

$n \longrightarrow$

(b) Row of squares

1.



Number of squares		Number of matches
1	⟶	4
2	⟶	7
3	⟶	
4	⟶	
5	⟶	
11	⟶	
60	⟶	

2 Explain in words how to get the second number from the first

3 The whole mapping is given by

$m \longrightarrow$

Fig. 7.1: Formalising a Generalisation-Algebra Test Questions for Case Study Structured Interview

Table 7.1: Categorisation of the Case Study Pupils' Responses to "Formalising a Generalisation" Questions

	Sally	Asim	George	Janet	Jude	Ravi	Linda	Shahidur
Question 2a								
Able to use natural language to generalise	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Able to represent generalisation in algebra	Yes	Yes	Yes	Yes	No	No	No	No
Algebra notation used	(n3)	3n	3n	nx3	-	-	-	-
Question 2b								
Able to use natural language to generalise	Yes	Yes	Yes	Yes but incorrect	Not Asked	Not Asked	Not Asked	Not Asked
Able to represent generalisation in algebra	Yes	Yes	Yes	Yes	Not Asked	Not Asked	Not Asked	Not Asked
Algebra notation used	m3+1	3m+1	3m+1	mx4-1*	-	-	-	-

**consistent* with natural language response

7.1.3 Interpretation of Variable (Algebra Questions 3A and logo Questions 3L)

In order to probe the pupils' understanding of variable in the algebra context it was decided to use questions from the Concepts in Secondary Mathematics and Science algebra test (C.S.M.S.; Appendix 1) which were directly related to pupils interpretation of letters in algebra. These questions had all been used with 1000 pupils in the age range 14-15 and it was intended to make comparisons between the case study pupils responses to these questions and the facility rate obtained from the C.S.M.S. test. For this reason it was decided not to modify any of the C.S.M.S. questions. In order to probe the pupils' understanding of variable in Logo questions were constructed which were similar in form to some of the algebra questions when appropriate. The algebra questions (3A) are presented below:

Write down the smallest and largest of these:

n +1, n + 4, n, n - 7.

Smallest........... Largest..................

Question 3A(i)

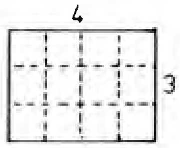
Which is the larger, 2n or n + 2?

....

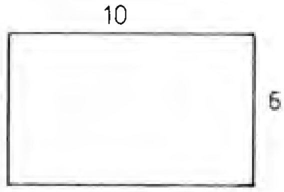
Explain..

Question3A(ii)

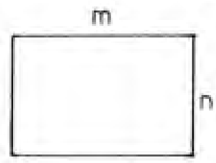
What are the areas of these shapes?



A = ...

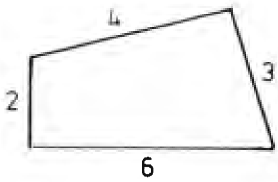


A = ...

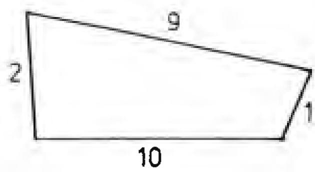


A = ...

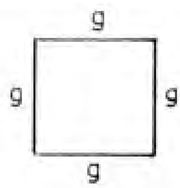
Question 3A(iii)



The perimeter of this shape is equal to 6+3+4+2 which equals 15

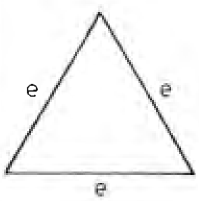


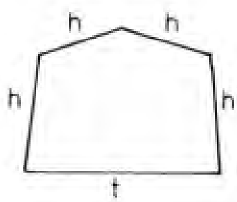
Work out the perimeter of this shape. p = ...

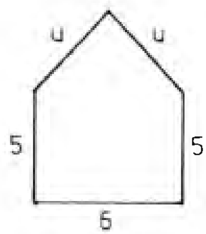


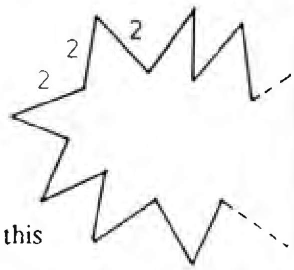
This square has sides of length g. So, for its perimeter we can write p=4g

What can we write for the perimeter of each of these shapes?









Part of this figure is not drawn

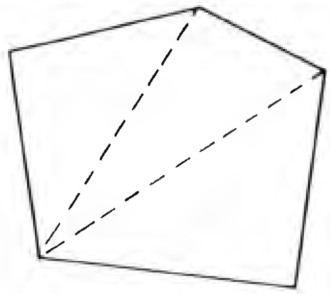
p = ... p = ... p = ... p=............................

Question 3A(iva) Question 3A(ivb) Question 3A(ivc) Question3A(ivd)

If John has J marbles and Peter had P marbles, what could you write for the number of marbles they have altogether?...

Question 3A(v)

In a shape like this you can work out the number of diagonals by taking away 3 from the number of sides.



So a shape with 5 sides has 2 diagonals.

a shape with 57 sides has .... diagonals

a shape with k sides has .... diagonals

Question 3A(vi)

What can you say about c if c + d = 10 and c is less than d ....

Question 3A(vii)

When are the following true - always, never, or sometimes?

Underline the correct answer:

A + B + C = C + A + B

Always. Never. Sometimes, when ....

Question 3A(viiia)

When are the following true - always, never, or sometimes?

Underline the correct answer:

L + M + N = L + P + N

Always. Never. Sometimes, when ....

Question 3A(viiib)

The Logo questions (3L) are presented below:

```
TO LINE1 "N
FD ADD :N 1
FD ADD :N 4
FD SUB :N 3
FD :N
FD SUB :N 7
END
```

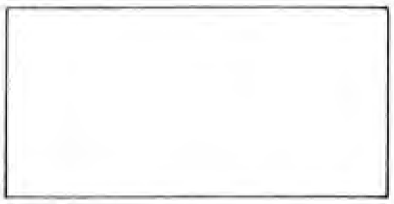
Which Logo command draws the shortest line?.......................
Which Logo command draws the longest line?.........................

Question 3L(i)

```
TO ROD "X
FD MUL 2 :X
FD ADD 2 :X
END
```

Which Logo command draws the longer line?........
Explain.........................

Question 3L(ii)



Complete the following procedure to calculate the area of ANY rectangle

```
TO RECTAREA
OP
END
```

Question 3L(iii)

```
TO MOVE "A "B "C
FD :A   FD :B   FD :C
RT 90
FD :C   FD :B   FD :A
END
```

When do these commands draw the same length line?
Always..................
Never..................
Sometimes, when...............

Question 3L(viiia)

```
TO ANN "L "M "N "P
FD :L   FD :M   FD :N
RT 90
FD :L   FD :P   FD :N
END
```

When do these Logo commands draw the same length line?
Always.......................
Never.......................
Sometimes, when.....................

Question3L(viiib)

Table 7.2 presents an overview of the pupils' responses to these questions. The related algebra and Logo questions have been grouped together in the table although the Logo questions were administered after the algebra questions had been completed by the pupils.

Table 7.2: Case Study Pupil Responses to C.S.M.S Algebra and Related Logo Questions

Question No.	Sally	Asim	George	Janet	Jude	Ravi	Linda	Shahidur
3A(i)	●	●	●	●	●	●	○	●
3L(i)	●	●	●	●	●	●	●	●
3A(ii)	R	R	R	R	○	○	○	○
3L(ii)	R	R	R	R	R	R	R	R
3A(iii)	●	●	●	●	○	○	○	●
3L(iii)	●	●	●	●	○	○	●*	○
3A(iva)	●	●	●	●	●	●	●	●
3A(ivb)	●	●	●	●	○	○	○	●
3A(ivc)	●	●	●	●	○	○	○	●
3A(ivd)	●	●	●	●*	○	○	○	●
3A(v)	●	●	●	●	○	●*	○	●*
3A(vi)	●	●	●	○	○	○	○	●
3A(vii)	●	●	●	●	○	○	●	○
3A(viiia)	●	●	●	●	○	○	○	●
3L(viiia)	●	●	●	●	●	●	●	●
3A(viiib)	●	○	○	○	○	○	○	○
3L(viiib)	●	○	●	●*	○	○	●*	○

● represents a correct response with no prompt from interviewer.
●* represents a correct response after a prompt from the interviewer.
○ represents an incorrect response.
R represents the response "2n" is bigger to question 3A(ii) and 3L(ii).

These results at first sight present a complicated picture. Within this section the most crucial aspects of the results from the point of view of each case study pupil will be discussed The interpretation of these results with respect to the pupils Logo experience is presented in Chapter 9.

Sally completed both the algebra and Logo questions quickly without any intervention from the interviewer. She answered all the algebra questions correctly apart from the question 3A(ii) to which she said that "2n is larger because it is doubled". She was the only case study pupil to answer the algebra question 3A(viiib) "When is L + M + N = L + P + N true" correctly. She gave consistent responses to both the algebra and the Logo equivalent questions.

Asim completed the algebra and Logo questions without any intervention from the interviewer. As with Sally his responses to the algebra and the equivalent Logo questions were consistent. His incorrect response to question 3A(viiib) "When is L+M+N = L + P +N true" is surprising.

George answered the CSMS algebra questions without any intervention from the interviewer. When answering the Logo question 3L(i) he asked if FD SUB :N 7 meant "subtract N from 7" and the interviewer told him that it meant "subtract 7 from N", after which he wrote down a correct solution. George was able to answer 3L(~~v~~ iiia) (the question about different variable names representing the same value) correctly without any intervention although he had answered the equivalent algebra question 3A(v) incorrectly. George's understanding of variable in Logo appears to be more elaborated than his understanding in algebra. This will be further analysed with respect to his Logo experience and "paper and pencil" algebra experience in Chapter 9.

Janet's responses were similar to those of George except that when she reached question 3A(ivd) she said that she did not understand the question. The researcher said "*So part of it is not drawn ...so you don't know what's left do you....so there are n sides altogether and each one is of length two*". After this prompt she immediately wrote down the correct response of nx2. She also needed a prompt on question 3L(viiib). She at first replied never and after the question "why" decided that the answer should be sometimes because "*you could put any number in and P could be any number .....so could M...*" As with George her understanding in Logo appears to be more elaborated than her understanding in algebra.

Jude was only able to respond positively to two algebra questions 3A(i) and 3A(iva).

Jude attempted to use his Logo understanding of variable in the algebra context when answering the C.S.M.S question 3A(iii) but his idea of "any number" soon became confused with
"anything" as the following example illustrates.

Jude *"Does M mean any number miss?"*

Res. *" M is any number and N is any number."*

Jude *"So I can just put anything I want."*

His response to the Logo questions was slightly better, although as with Ravi he performed substantially worse than the rest of the group. Jude has never carried out any "paper and pencil" algebra during his normal mathematics lessons.

Ravi, like Jude could answer very few algebra questions. He said *"J plus P"* as a response to question 3A(v) ("If John has J marbles and Peter had P marbles what can you say about the number of marbles they have altogether") but wrote down JP. Ravi has never carried out any "paper and pencil" algebra in his normal mathematics lessons.

Linda's correct response to the question 3A(vii) is surprising as she had never before engaged in any "paper and pencil" algebra in her "normal" school mathematics lesson. As with George and Janet she was able to answer considerably more of the Logo questions correctly.

Shahidur. For a non-algebra experienced pupil Shahidur's responses to the algebra questions are remarkable and in one instance he answered an algebra question (3A(iii)) correctly and the related Logo question (3L(iii)) incorrectly. The researcher was so surprised during the interview that she asked him *"Have you done this at home "* to which he replied *"No I've never done it before."* He needed a prompt to answer question 3A(v) (the marbles question). He first wrote down 9 as a solution saying "*'Cos John begins with J and there's four letters in John and Peter begins with P and there's five letters in Peter."* When the researcher suggested that the number of marbles should be changed to Q and R respectively he immediately wrote down Q + R as a solution. When presented with the perimeter question 3A(ivd) he wrote down 2 x n's as a solution. When asked to explain his solution he said *"Cos there's the size of them are 2....and there are n's of them ...so 2 times n will be the answer".*

7.1.4 Function Machine Question (Logo and algebra questions 4)

Pupils were presented with three simple functions (Fig. 7.2) in the form of mapping diagrams and asked to write down both the Logo and the algebra representation. The aim of this question was to investigate the pupils' ability to use variable to represent a simple function in both Logo and algebra. Table 7.3 presents an overview of the case study pupils' responses to Question 4.

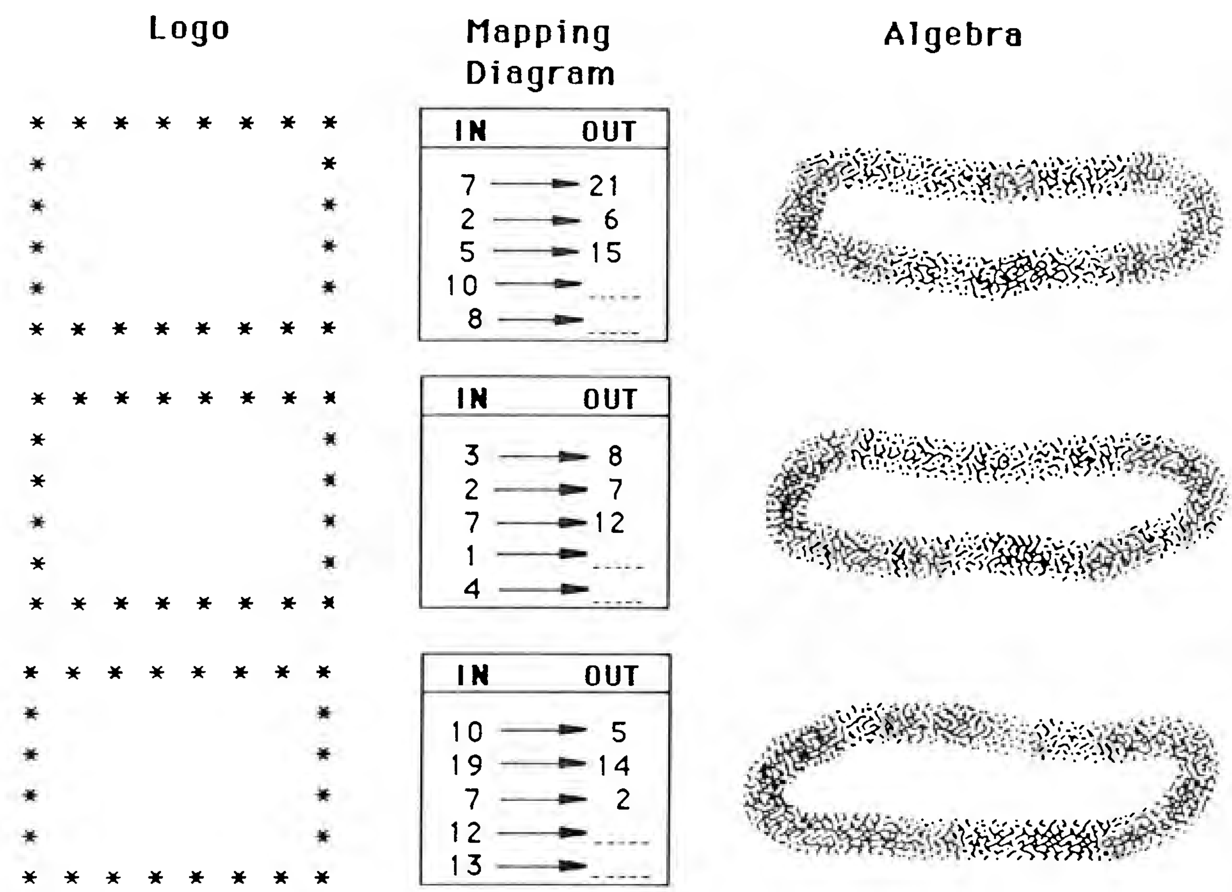


Fig. 7.2: Function Machine Questions

Table 7.3: Case Study Pupils Responses to Function Machine Questions

	Multiply by 3		Add 5		Subtract 5	
	Logo	Algebra	Logo	Algebra	Logo	Algebra
Sally	ADD 3 :A	A->A+3	ADD 5 :A	A->A +5	SUB 5 :A	A->A-5
Asim	MUL 3 :A	Y->3Y	ADD 5 :NUM	Y->Y +5	SUB 5 :NUM	Y->Y-5
George	MUL 3 :NUM	N->3N	ADD 5 :X	N->N+5	SUB 5 :Y	N->N-5
Janet	MUL 3 :N	n->3n	ADD 5 :N	n->5 +n	SUB :N 5	n->n-5
Jude	MUL 3 :NUM	3X	ADD 5 :R	R+3	SUB 5 :z	z - 5
Ravi	MUL NUM 3	-	ADD NUM 5	-	SUB NUM 5	-
Linda	MUL 3 :JIM	x->3x	ADD 5 :BOD	x->x+5	SUB :ANG 5	x->x-5
Shahidur	MUL 3 :NUM	x 3	ADD 5 :NUM	+5	SUB 5 :N	- 5

All the pupils (apart from Sally in the case of "multiply by 3 when she appears to have made a "careless" error) were able to write down a correct Logo representation for a "Multiply by 3" and an "Add 5 " function. All the pupils wrote down a representatiuon for the "subtract 4 " function but only Janet, Ravi and Linda gave the correct order for the inputs to SUB. What is more interesting is that Sally, Asim and George although writing down incorrect order of inputs to SUB wrote down the algebra representation correctly. This suggests that they were not converting from algebra to Logo representation (or vice versa) but were constructing the two representations separately.

In the context of this problem two of the pupils Jude and Linda who had not been able to write down any algebra representation for the question 2 questions were able to write down at least one algebra representation. For example Linda correctly wrote x -> 3x, x-> x + 5, x-> x - 5 and Jude wrote down 3x, R+3. Ravi and Shahidur were not able to write down any algebra representations.

Sally, Asim, George, Janet, Jude and Shahidur all used single letter variable names for at least one of the variables in the Logo representation. Ravi used the name NUM throughout (the name used when the function machine material was first introduced to him). Linda used three words JIM, BOD, and ANG. Ravi was the only pupil who omitted to put a colon (:) in front of the Logo variable name.

7.2 DISCUSSION OF CASE STUDY PUPILS' RESPONSES TO STRUCTURED INTERVIEW

In order to analyse the pupil responses to the structured interview questions the following categories have been devised:

Acceptance of the idea of variable This was deemed present if the pupils were willing to begin to attempt the interview questions. Some of the comparison pupils for example when presented with the algebra related questions said that they had never seen anything like it before and would not engage in the solution process). The questions used to provide evidence of this acceptance were: Algebra: All questions; Logo: All questions.

Understanding that a variable name represents a range of numbers The variable name in Logo or letter in algebra is seen as being able to represent several values rather than one. The questions used to provide evidence of this understanding were: Algebra: All questions; Logo: All questions.

Understanding that different names can represent the same value. The question used to provide evidence of the understanding that different names can represent the same value was: Algebra: 3A(viiib); Logo: 3L(viiib).

Acceptance of "lack of closure" in an expression. The questions used to provide evidence of the acceptance of "lack of closure" were:Algebra 3A(iv), 3A(v), 3A(vi), 4A; Logo 4L.

Ability to establish a second-order relationship between variables. The question used to provide evidence of this understanding was: Algebra3A(ii); Logo 3L(ii).

Ability to represent a generalised method (which involves operating on a variable). The questions used to provide evidence of this ability were: Algebra 2, 3A(iii), 3A(ivd); Logo: 1, 3L(iii).

Ability to use variable to represent a simple function. The questions used to provide evidence of this ability were: Algebra 4A; Logo 4L.

Table 7.4: Classification of Case Study Pupils' Responses to Structured Interview

	Acceptance of the idea of variable		Understanding that variable name represents a range of numbers		Understanding that different names can represent the same value		Acceptance of lack of closure		Ability to establish a second-order relationship between variables		Ability to represent a generalisation which has already been expressed in natural language		Ability to use variable to represent a "simple" function	
	Logo	Algebra	Logo	Algebra	Logo	Algebra	Logo	Algebra	Logo	Algebra	Logo	Algebra	Logo	Algebra
Pupil 1 Sally	●	●	●	●	●	●	●	●	○	○	●	●	●	●
Pupil 2 Asim	●	●	●	●	○	○	●	●	○	○	●	●	●	●
Pupil 3 George	●	●	●	●	●	○	●	●	○	○	●	●	●	●
Pupil 4 Janet	●	●	●	●	●	○	●	●	○	○	●	●	●	●
Pupil 5 Jude	●	●	●	○	○	○	●	○	○	○	●	○	●	●
Pupil 6 Ravi	●	●	●	○	○	○	●	○	○	○	●	○	●	○
Pupil 7 Linda	●	●	●	●	●	○	●	○	○	○	●	○	●	●
Pupil 8 Shaidur	●	●	●	●	○	○	●	●	○	○	●	○	●	○

● case study - positive response
○ case study - negative response

The case study pupils' responses to the individual test questions have been analysed according to the above categories. This analysis is presented in Table 7.4.

All the case study pupils accepted the idea of variable in both Logo and algebra, and that a variable name in Logo represents a range of numbers. All except for Ravi and Jude have carried this understanding to the algebra context. The understanding that variables with different names could represent the same value was beginning to be understood by half of the pupils in Logo but this understanding was only carried over to the algebra context for one of the case study pupils. All of the case study pupils accepted "lack of closure" in a Logo expression. All apart from Jude, Ravi and Linda accepted the idea in algebra. None of the case study pupils appeared to understand the nature of the interrelationship between two algebraic expressions in either Logo or algebra.

All the case study pupils could formalise a method generalised by them in Logo, whereas not all of them were able to formalise a method in algebra which they had already generalised in natural language. All could use variable in Logo to represent a "simple" function, and six of them could use variable in algebra to represent a "simple" function.

7.3 THE COMPARISON GROUP'S RESPONSES TO STRUCTURED INTERVIEW

A similar group of pupils to the case study pupils were interviewed in order to provide another perspective from which to analyse the case study pupils' responses to the algebra related questions of the structured interview. This group of pupils will be called the comparison group. The school used for the longitudinal case study research has two lower schools for pupils aged 11-14, both feeding into the same upper school. The lower schools are on different sites and there is no contact between pupils from the two lower schools. None of the pupils in the non case study "lower" school had used Logo. The mathematics classes in both "lower" schools are all mixed ability. The comparison group of pupils was chosen from the non case study "lower" school. All the case study pupils could be ranked according to their SMILE level (appendix 4.1).

The comparison group of pupils were chosen so as to cover a range of mathematical attainment and to be similar to the case study pupils with respect to the SMILE levels (Table 7.5). It should be noted that the highest ranked comparison pupil was one level higher than the highest ranked case study pupil.

Table 7.5: SMILE Levels for Case Study and Comparison Pupils at End of Three Year Longitudinal Study

Case Study Pupils		Comparison Pupils	
Sally	5.2	Simon	6.2
Asim	4.8	Bila	5.2
George	4.8	Bozi	5.0
Janet	4.1	Jamila	4.0
Jude	3.5	Susan	4.0
Ravi	3.5		
Linda	3.2	Phuong	3.2
Shahidur	2.8	Rahina	2.8

(It was only possible to interview seven comparison pupils due to absence)

Table 7.6 presents an overview of the comparison group's responses to all the algebra questions of the structured interview.

Table 7.6: Comparison Group's Responses to Algebra Structured Interview Questions

Question No.	Simon	Bila	Bozi	Jamila	Susan	Phuong	Rahina
2a	•	•	•	o	o	o	o
2b	•	•	•	o	o	o	o
3A(i)	•	•	•	o	o	o	o
3A(ii)	R	R	R	o	o	o	o
3A(iii)	•	•	•	o	o	o	o
3A(iv)a	•	•	•	•	•	o	o
3A(iv)b	•	•	•	o	o	o	o
3A(iv)c	•	•	•	o	o	o	o
3A(iv)d	•	o	o	o	o	o	o
3A(v)	•	•	o	o	o	o	o
3A(vi)	•	•	•	o	o	o	o
3A(vii)	•	•	•	o	o	o	o
3A(viiia)	•	•	•	o	o	o	o
3A(viiib)	o	•	o	o	o	o	o
4	•	•	•	o	o	o	o

• represents a positive response
o represents a negative response
R represents the response "2n" is bigger to question 3A(ii).

A third perspective on the case study pupils' responses to the structured interview questions was obtained by asking a practising SMILE teacher to say whether he considered it to be likely or unlikely that a pupil working at a specified SMILE level would be able to be able to give a correct response to each interview question (see appendix for a discussion of SMILE levels). These predictions are given in Table 7.7. In addition the percentage of correct responses to each of the C.S.M.S questions is given for the 1000 14- 15 year old pupils who took part in the C.S.M.S survey (appendix 1).

Table 7.7 : C.S.M.S % Correct Responses on Algebra Questions (14-15 year olds) and Predicted Likelihood of Pupils Being Able to Answer Questions Correctly According to SMILE Levels.

	SMILE levels					C.S.M.S % correct response for 14 - 15 olds
	6	5	4	3	2	
3A(i)	L	L	L	U	U	72 %
3A(ii)	L	U	U	U	U	6%
3A(iii)	L	L	U	U	U	68%
3A(iva)	L	L	U	U	U	94%
3A(ivb)	L	L	U	U	U	68%
3A(ivc)	L	L	U	U	U	64%
3A(ivd)	L	U	U	U	U	38%
3A(v)	L	L	U	U	U	63 %
3A(vi)	L	L	U	U	U	52 %
3A(vii)	L	L	U	U	U	30%
3A(viiia)	L	L	L	U	U	72 %
3A(viiib)	L	U	U	U	U	25%

L - teacher predicted that it is likely that pupils working at this level would be able to answer question correctly
U - teacher predicted that it is unlikely that pupils working at this level would be able to answer question correctly

The case study and comparison group of pupils have been divided into four quartiles according to their ranked order of SMILE levels and an analysis of their understanding of variable according to the categories presented in section 7.2 has been undertaken. These results are presented in Table 7.8.

Table 7.8: Proportion of Correct C.S.M.S Algebra Question Responses for Case Study and Comparison Pupils by Quartiles (According to Ranked Mathematical Attainment List)

	1st Quartile		2nd Quartile		3rd Quartile		4th Quartile	
	Case Study	Comp. Group	Case Study	Comp. Group	Case Study	Comp. Group	Case Study	Comp. Group
Acceptance of the idea of variable.	2/2	2/2	2/2	1/2	2/2	0/1	2/2	0/2
Understanding that variable name represents a range of nos.	2/2	2/2	2/2	1/2	2/2	0/1	2/2	0/2
Acceptance of "lack of closure"	2/2	2/2	2/2	0/2	0/2	0/1	1/2	0/2
Understanding that different variable names can represent the same value	1/2	1/2	0/2	0/2	0/2	0/1	0/2	0/2
Ability to establish 2nd order relationship	0/2	0/2	0/2	0/2	0/2	0/1	0/2	0/2
Ability to use variable to represent a "simple" function	2/2	2/2	2/2	1/2	1/2	0/2	1/2	0/2
Ability to use variable to represent a generalisation which has already been expressed in natural language	2/2	2/2	2/2	1/2	0/2	0/1	0/2	0/2

7.4 HYPOTHESES DERIVED FROM RESULTS OF STRUCTURED INTERVIEW

The hypotheses derived within this section will be used as a basis for the final synthesis of the case study data which will be presented in Chapter 9. This section will discuss the results of the structured interviews both from the point of view of the case study pupils' understanding in Logo and their understanding in "paper and pencil" algebra.

Acceptance of the idea of variable was deemed present if pupils were willing to attempt the structured interview questions. All the case study pupils accepted the idea of variable in Logo and all of the case study pupils accepted the idea of variable within the "paper and pencil" algebra questions. This contrasts with only three out of seven comparison pupils accepting the idea of variable. Four of the eight case study pupils had never carried out any algebra within their "normal' mathematics lessons and it is suggested that the Logo experience of variable has provided these pupils with a framework from which they can begin to develop an understanding. It is known that "paper and pencil" experience of mathematical ideas can often lead to pupils developing resistance to using the idea (an example from the Logo Maths Project is Linda's resistance to the use of decimals in the Logo context). It should not be underestimated that the use of variable in Logo has provided these pupils with a positive experience. It is suggested that it is only through the beginning acceptance of and use of an idea that understanding develops.

All the case study pupils accepted the idea that a variable name represents a range of numbers. All except for Ravi and Jude have carried this understanding to the algebra context. Evidence from the C.S.M.S (appendix 1) research and from the comparison group's responses suggests that the understanding that a variable name can represent a range of numbers would not have been expected from the case study pupils Janet, Linda and Shahidur. It is suggested that the understanding was derived from their Logo experience. The two pupils Jude and Ravi, who did not respond positively in the algebra context have both had a very limited experience of variable in Logo (appendix 4.2).

All of the case study pupils accepted lack of closure in a Logo expression. When compared with the comparison group more case study pupils than would have been expected accepted lack of closure in an algebra expression. This acceptance of lack of closure will be discussed with respect to the case study pupils' experience of Logo in chapter 9.

In order to test the pupils' understanding of whether or not a different variable name can represent the same value they were given the following Logo and algebra items:

When is the following true ?	When do these Logo commands draw the same length line?
L+M+N=L+P+N	TO LINES "L "M "N "P FD :L FD :M FD :N RT 90 FD :L FD :P FD :N END
Always. Never. Sometimes,when..................	Always. Never Sometimes,when......................

Only Sally responded positively to both questions but four out of the eight of the case study pupils responded positively to the Logo question. It is suggested that the pupils' Logo experience was sufficient for them to be able to use this idea within a Logo context but was not yet extensive enough for them to use this understanding in a "paper and pencil" algebra context.

None of the case study pupils could answer either the C.S.M.S algebra question "Which is the larger 2n or n+2? Explain........................" or the Logo related question correctly (Question 3L(ii)). Küchemann maintains that "An important feature of these relationships is that their elements are themselves relationships, so they can be called *'second order'* relationships" (Küchemann, 1981). He suggests that it is only when pupils have grasped this notion that they have fully understood the idea of variable. Analysis of the data indicates that none of the pupils had carried out any Logo tasks related to this idea.

All the case study pupils could formalise a method generalised by them in Logo. They were also able to express simple functions in Logo notation. Although there is no evidence that the case study pupils are more able to formalise a generalisation in algebra there is evidence that their experience of the function machine material has made them more able to formalise simple functions in algebra.

When the responses to the individual C.S.M.S questions are studied more closely, and taking into account the C.S.M.S % correct responses, the comparison group's responses and the teacher's predictions it would appear that the case study pupils' responses to the perimeter question (3A(ivd)) and the marbles question (3A(v)) are the most unexpected. These are both questions which require the pupil to think of "letter as a generalised number" (Küchemann,1981 and page 13 of this thesis).

The results presented in this chapter will be synthesised with the data obtained from the individual laboratory tasks (Chapter 6) and the longitudinal transcript data and presented in Chapter 9.

CHAPTER 8

PRE-ALGEBRA PUPIL STUDY

8.1 OVERVIEW AND AIMS OF THE STUDY

Throughout the case study research it was always recognised that four of the case study pupils were carrying out algebra tasks as part of their "normal' school mathematics lessons. For these pupils it was difficult to disentangle the effect of the "school algebra" work on their understanding of variable in Logo. It was decided therefore to carry out a study with pupils who had no experience of "paper and pencil" algebra to investigate whether or not these pupils could develop:

Acceptance of the idea of variable It was hypothesised that pupils would develop this acceptance by using variable within a range of Logo programming tasks.

Understanding that a variable name represents a range of numbers It was hypothesised that pupils would develop this understanding by engaging in variable related tasks which required a range of inputs (including decimal and negative numbers) for their solution

Understanding that the name of a variable itself is not significant. It was hypothesised (based on the longitudinal case study findings) that the pupils would develop this understanding by specific teacher intervention telling the pupils to replace their variable name by other variable names within their procedure, whilst keeping everything else fixed.

Understanding that different variable names can represent the same value A task was specifically devised to help pupils develop this understanding. Within the task the pupils were first asked to define a general rectangle procedure using two variable inputs and they were then asked to use this procedure to draw a square (appendix 8.1). By engaging in this task they would be using the idea that variables with different names could be assigned the same value.

Acceptance of "Unclosed" expressions It was hypothesised that engaging in the "Function Machine" task (appendix 6.1a and 6.1b) would help pupils develop this acceptance.

Understanding that a second-order relationship can exist between two simple Logo expressions A specific task was devised (appendix 8.2) to help pupils develop this understanding. Within this task pupils were asked to investigate the relationship between two simple Logo functions for different values of variable input.

8.1.1 Classroom Based Work: November 1986 -June 1987

A class of 27 top junior school pupils (aged 10-11) were chosen from a rural primary school. This school had been part of the Chiltern Logo Project (Noss, 1985). The class of pupils chosen had learned Logo in their first year of junior school (when aged 7-8) and then had not carried out any more Logo work until they started their fourth year of junior school. The class teacher had been involved in the Chiltern Logo project. The author visited the class twice before the research commenced and was impressed by the reflective way in which the pupils engaged in the Logo activity. One computer was permanently in the classroom and pupils took turns in pairs to program in Logo throughout the week. Before commencing the research most of the pupils in the class were able to write and debug simple procedures and many of them were able to combine their procedures into superprocedures. None of the pupils had used the idea of variable before the research commenced.

The aim was to introduce as many of the class as possible to the idea of variable throughout the academic year 1986/1987. It was intended to introduce the pupils to variable in the context of the "Scaling Letters" task (appendix 3.2). This task turned out to be inappropriate for these pupils. In order to carry out the task pupils needed to use the idea of multiplying by a decimal and this became an obstacle to their learning of variable. Instead, the idea of variable was introduced within the context of defining general letter procedures which used "More than one variable input" as illustrated by Fig. 8.1.

```
TO L  :LINE  :DAN
BK :LINE
RT 90
FD :DAN
END
```

Fig. 8.1: General L Procedure using Two Variable Inputs

The Logo used was LCSI Logo for the BBC Acorn computer. There are some differences between this Logo and that used by the longitudinal case study pupils and these are decribed in appendix 3.1.

The researcher visited the class three times in the Autumn term, twice in the Spring term and four times in the summer term. Each visit was for a whole morning (3 hours) and during the visit three of four pairs of pupils would work at the computer consecutively. During these visits, if appropriate, the researcher would introduce the idea of variable to the pupils working at the computer. The idea of variable was introduced either within preplanned tasks or within the pupils' own goals. The session would then be discussed with the class teacher at the end of the visit. Preplanned tasks which had been used during the session were given to the class teacher for her to use with other pupils, during the intervening weeks before the researcher's next visit, if she thought that it was appropriate. It was never the intention that all the pupils in the class should use the idea of variable, but only the pupils for which either the researcher or the class teacher decided that variable would be a meaningful problem solving tool. The pupils continued to engage in variable related tasks even when the researcher was not present.

During this period the data collected consisted of:

- researcher's observational notes of pupils interaction with the computer made during class visits

- pupils' written records of Logo programming made in their "Logo" exercise book.

- records of procedures written

The researcher's interventions and the preplanned tasks were informed by both the findings of the longitudinal case study research and the categories of variable outlined in Chapter 3.3.2.

8.1.2 Choice of Pupils for Pre-algebra Study

At the end of the third term of the pre-algebra study a structured Logo interview was carried out with eight pupils chosen from the class. These pupils were chosen from the pupils who had used the idea of variable in Logo during the year. They included four boys and four girls and, as far as possible, a spread of pupils from the point of view of mathematical attainment. This spread was obtained by using the teacher's ranked list , ranked according to a basic mathematics test administered to the pupils during the third term of the primary study. The following pupils were chosen:

First Quartile: Nicholas, Clare,

Second Quartile: Stuart, Richard, Joanne,

Third Quartile: Kelly, Craig, Helen

None of the pupils in the fourth quartile of the teacher's ranked list had used the idea of variable and so were consequently not chosen for the structured interview.

The Logo experience of these pupils has been categorised according to the categories of variable use outlined in Chapter 3.3.2 . An overview of this experience is presented in Table 8.1. Evidence for the pupils Logo experience was derived from both the researcher's classroom visits and the pupils' Logo records in their exercise books. All of the pupils have not had the same amount and extent of Logo experience.

Table 8.1: Overview of Pre-algebra Pupils' Logo Experience

	Nich.	Clare	Stuart	Richard	Joanne	Kelly	Craig	Helen
(I) One Variable Input	●	●	●	●	●	▨	●	▨
(S) Variable as Scale Factor	○	○	○	○	▨	○	○	○
(N) More than one Input	▨	▨	●	●	▨	●	●	▨
(O) Variable operated on	●	○	▨	▨	○	○	▨	○
(G) General Super-procedure	○	○	○	○	○	○	○	○
(R) Recursive Procedure	▨	○	○	▨	○	○	▨	○
(F) Logo Function	▨	▨	▨	▨	▨	▨	▨	▨

○ represents no use
▨ represents a small amount of use (Approx. 0 to 3 hours)
● represents extensive use (over 3 hours)

Table 8.2 presents an overview of the pre-algebra pupils' involvement in the three preplanned Logo tasks:

The "Rectangle" task (appendix 8.1) aimed at developing understanding that different variable names can represent the same value.

The "Function Machine" task (appendix 6.1a and b) aimed at developing acceptance of an "unclosed" algebraic object; extending pupils' understanding of a range of numbers; developing acceptance of single letter variable names.

The FUNNY/SUNNY task (appendix 8.2) aimed at developing an understanding of the second -order relationship between two algebraic objects.

Table 8.2: Pre-algebra Pupils Involvement in Pre-planned Logo tasks.

	Rectangle Task	Function Machine Task	Funny - Sunny Task
Nicholas	√	√	√
Clare	x	√	√
Stuart	x	√	√
Richard	√	√	√
Joanne	√	√	x
Kelly	√	√	x
Craig	√	√	√
Helen	x	√	x

√ represents task involvement
x represents no task involvement

8.2 DESCRIPTION AND RESULTS OF STRUCTURED LOGO INTERVIEW

A structured Logo interview was carried out with all the designated pre-algebra pupils. This interview was tape recorded and all the tape recording were transcribed. Many of the Logo questions administered to these pupils were identical to the Logo questions administered to the case study pupils (see chapter 7). This section will describe the interview questions together with their respective aims and present the results derived from the administration of these Logo questions.

8.2.1 The General Square: Procedure Writing Task (Question 1)

Working at the computer the pupils were presented with a handout which asked them to write a general square procedure (appendix 8.3a). The aim of this question was to investigate the pupils' ability to use "One variable Input". Table 8.3 presents the results of this question.

Table 8.3: Analysis of Pre-algebra Pupils' Responses to General Square Question

	Nicholas	Clare	Stuart	Rich	Joanne	Kelly	Craig	Helen
Correct solution using (I) One variable input	◆	◆	◆			◆		
Correct solution using (N) two variable inputs				◆	◆			
Variable Used but Incorrect solution							◆ TO WERD :HERD FD :WERD RT 45 END	
No variable Used								◆
Variable name/s used	S	LINE	LINE	R,H	LINE, SIDE	S	W/HERD	

◆ represents programming solution in this category

8.2.2 Interpretation of Procedure "Variable Operated on" (Question 2)

The following procedure was presented to the pupils

```
TO SHAPE :XX :YY
FD :XX
RT :YY
FD :XX + 30
END
```

They were then asked to draw out the computer response to SHAPE 30 60. The aims of the task was to investigate if the pupils could:

- interpret the value of a variable used to represent a length
- interpret the value of a variable used to represent an angle
- interpret the value of a variable which had been operated on within a procedure.

The results of this task are presented in Table 8.4.

Table 8.4: Analysis of Pre-Algebra Pupils' Response to Interpretation of "(O) Variable Operated On" Logo Task

	Nicholas	Clare	Stuart	Rich	Joanne	Kelly	Craig	Helen
FD :XX interpreted correctly	●	●	●	●	●	●		
RT :YY interpreted correctly	●	●	●	●	●	●		
FD :XX+30 correctly interpreted	●	●	(N)	●	●	●		
Says procedure draws a square							●	●

● represents category used (N) represents category used after nudge from researcher

8.2.3 Interpretation of Variable (Questions 3a to 3d)

All the following questions were presented to the longitudinal case study pupils. The aims of the questions were to investigate more precisely whether or not the pupils could:

- understand that a variable name represents a range of numbers
- understand that different variable names can represent the same value
- accept "lack of closure" in an expression
- establish a second-order relationship between simple Logo expressions

Question 3a This question had also been administered to the case study pupils (Question 3L(i) in Chapter 6.1.3). The pupils were presented with the following question the results of which are presented in Table 8.5.

```
TO LINE1 :N
FD :N + 1
FD :N + 4
FD :N - 3
FD :N
FD :N - 7
END
```

Which Logo command draws the shortest line?................

Which Logo command draws the longest line?.....................

Table 8.5: Pre-Algebra Pupils' Responses to Question 3a

	Nicholas	Clare	Stuart	Rich	Joanne	Kelly	Craig	Helen
Pupils' choice for longest line	:N +4	:N + 4	:N + 4	:N - 7	:N - 7	:N - 7	Not given	Not given
Pupils' choice for shortest line	:N - 7	:N - 7	:N - 7	:N	:N + 1	:N	Not given	Not given

This Logo question had originally been chosen because it matched the C.S.M.S algebra question 3A(i) (Section 7.1.3). In the Logo context the correct interpretation of negative inputs is not so clear if the absolute length of the lines are considered. So for example:

When $N \geqslant 6$ FD :N + 4 is longest
FD :N - 7 is shortest.

$N = 5$ FD :N + 4 is longest
FD :N - 7 and FD :N - 3 are both shortest.

$N = 4$ FD :N + 4 is longest and FD :N - 3 is shortest.

For $n \leq -7$ FD N - 7 is longest.

This means that the pre-algebra pupils' responses are difficult to interpret. Three of the pupils said that :N + 4 was the longest line and :N - 7 the shortest line. This can be considered correct from an algebra perspective. Richard's response varied according to the value of N and he talked about the "turtle going back on itself". This suggests that he may have been thinking about the issues discussed above although he was obviously not able to articulate these. Joanne said that FD :N - 7 would be longest because *"It doesn't matter about take or add because it would be going one way or another"*.

It is suggested that Nicholas, Clare, Stuart, Richard and Joanne had in this context accepted an unclosed expression in Logo. Kelly's response however indicated that she was not able to interpret an unclosed expression in this context. She said that FD :N - 7 would draw the longest line *"because it's the highest number"* and the The FD :N was the shortest because there *"was no number by it"*.

Question 3b This question was also administered to the longitudinal case study pupils (Question 3L(viiia) in section 7.1.3). The pupils were presented with the following question, the results of which are given in Table 8.6.

```
TO MOVE :A :B :C
FD :A  FD :B  FD :C
RT 90
FD :C FD :B FD :A
END
```

When do these Logo commands draw the same length line?
Always..............
never......................
Sometimes, when......................

Table 8.6: Pre-Algebra Pupils' Responses to Question 3b

	Nicholas	Clare	Stuart	Rich	Joanne	Kelly	Craig	Helen
Correct Response of "Always"	●	●	●	●	●	●	Not given	Not given
Never								
Sometimes								

● represents response in this category

All the pupils who were given this question were quite certain that both expressions would always draw the same length line. The following is a list of reasons given:

Nich. *"Because it doesn't matter which way you put it in... because in numbers say if you add say a million to two.... it wouldn't matter if you added two million... you'd still get the same answer".*

Clare *"Because you've got the same sizes... but they're just the other way round".*

Stuart *"Cos A, B and C are the same as C, B and A".*

Rich. *"It just adds on ... A... B and C... it's like adding 5, 6 and 7.... and then its added 7, 6 and 5"*

Joanne *"It doesn't really matter .... because you haven't added any number onto it...."*

Kelly *"Cos it's just the letters backwards"*

Question 3c This question was also administered to the case study pupils (Question 3L(viiib) in Chapter 7.1.3). The following question was presented to the pupils, the results of which are given in Table 8.7.

```
TO ANN :L :M :N :P
FD :L :FD :M FD :N
RT 90
FD :L :FD :P FD :N
END
When do these Logo commands draw the same length line?
Always..................
Sometimes .............
Never..................
```

Table 8.7: Pre-Algebra Pupils' Responses to Question 3c

	Nicholas	Clare	Stuart	Rich	Joanne	Kelly	Craig	Helen
Always								
Never		●				not given	not given	not given
Correct response of Sometimes	●		●	(N)	●			

● represents response in this category
(N) represents response in this category after nudge from researcher

The pupils who answered this question correctly gave the following reasons

Nich. *"Sometimes... say if M and P were the same they would draw the same length of line".*

Stuart *"Cos M and P might be the same".*

Rich. *"N and N are the same, L and L are the same... but you can put P the same as M .. but you could put P different to M".*

Joanne *"When they've both got the same amount of numbers".*

Clare's reply was interesting in that she did not think that M and P could be the same *"because it isn't really worth it... if they were the same ... I'd just put 3 letters in"*

Question 3d This question was also administered to the case study pupils (Question 3L(ii) in Chapter 7.1.3)The following question was presented to the pupils, the results of which are given in Table 8.8.

```
TO ROD :X
FD 2 * :X
FD 2 + :X
END
```

Which Logo command draws the longer line?.............................

Table 8.8: Pre-Algebra Pupils' Responses to Question 3d.

	Nicholas	Clare	Stuart	Rich	Joanne	Kelly	Craig	Helen
2 * :X is longer			●		●	●	Not given	Not given
2 + :X is longer								
Correct response of IT DEPENDS	●	●		●				

● represents response in this category

The pupils who replied "it depends" were quite clear about their reasons.

Nich. *"it depends ... if you had 2 they'd be equal... and when you had 3 then times would make it longer ..... I think just over 2... about 2...the plus one is bigger".*

Richard *"It depends how long x is ... say x is 1.. 2 times 1 is 2.. 2 plus 1 is 3.. but if its higher than that ... say it's 10... 2 times 10 is 20.. 2 plus 10 is 12.*

Clare *"It depends ....cos if x is 2 it would be the same....if it was 18 then 2 times x would be bigger...if x was 1 then 2 plus x would be bigger."*

8.2.4 Area of Rectangle: Procedure Writing Task (Question 4)

This question was also given to the longitudinal case study pupils (Question 3L(iii) in Section 7.3.1.) The aim of the question was to investigate whether or not the pupils could use variable to represent a generalised method. Table 8.9 presents the pre-algebra pupils' responses to this question.

Complete the following procedure to calculate the area of ANY rectangle.
TO RECTAREA
OP
END

Table 8.9: Pre-Algebra pupils' Responses to Area of Rectangle Question

	Nicholas	Clare	Stuart	Rich
Solution	:S * :Y	No solution	:CRAIG + :STU	:R * :H

Joanne, Kelly, Craig and Helen were not given this question.

8.2.5 General Letter H Procedure Writing Task (Question 6)

Working at the computer the pupils were presented with a handout asking them to write a general letter H procedure (appendix 8.3b). The aim of this task was to investigate whether they would use variable in the category of "(N) more than one variable" or "(O) variable operated on" in order to solve this task. Table 8.10 presents the results of the task.

Table 8.10: Pre-Algebra Pupils' Responses to General H Procedure Task

	Nicholas	Clare	Stuart	Richard	Joanne	Kelly	Craig	Helen
(I) One Variable Input						•U		
(N) More than One Variable Input		•S	•S	•S	•S		•U	
Variable Operated On	•S							
No Variable Used								•U
Variable Name	S	A,S,D F,G	STU, LINE	RE,RF, RF	LINE, SIDE	I	HID DIG	

•S represents category used successfully
•U represents category used unsuccessfully

Kelly's unsuccessful attempt involved first of all using one variable for all the angle inputs within her procedure. When this did not draw an H she changed the procedure to use one variable for all the FORWARD commands. When this did not work she changed her procedure to use one variable for all the BACKWARD commands. She did not manage to write a correct general H procedure. Craig's solution was:

```
TO HID:DIG
FD 200 BK 100 RT 90 FD 50 LT 90 FD 100 BK 200
END
```

This drew the letter H but gave insight into his superficial understanding of variable. Helen very confidently defined a fixed H procedure and when asked of she could make this variable she said *"Just do the same like that...but do it bigger."*

8.2.6 Interpretation of Variable Name

Whenever the pupils used a variable within the Logo questions they were asked:

(a) What does the variable do?

(b) What name does it have to be?

(c) What sort of numbers can you put in?"

The excerpts, from the taped interview, related to these questions will be presented in detail as they illustrate the pupils' understanding.

Nicholas

Res. *"What does the S do in your variable square procedure?"*

Nich. *"Well that's the length...you put the length of each side of the square".*

Res. *"So what sort of numbers could you put in there?"*

Nich. *"Any...if you did it too much...it might go off the screen".*

Res. *"And does it have to be called S?"*

Nich. *"No it could be called`anything."*

Clare

Res. *"You've used the word LINE there...so what does that do?"*

Clare *"Well if you put in the word LINE...then you can change the numbers".*

Res. *"Does it have to be the word LINE?"*

Clare *"No any word you want."*

Res. *"And what sort of numbers can you put in?"*

Clare *"Any number...but if you want the angle and the line...then you can put two numbers with a space inbetween".*

Stuart

Res. *"You've used LINE in your procedure..what does that do?*

Stuart *"So you can use any number..."*

Res. *"Does it have to be called LINE?"*

Stuart *"No it can be called anything."*

Res. *"And what sort of numbers can you put in?"*

Stuart *"Anything....up to about 500"*

Res. *"And down to what".*

Stuart *"Down to one".*

Richard

Res. *"So what do the variables in your procedure do?"*

Rich. *"They're both for making the size of the sides...and you can make a square with them...or you could make a rectangle".*

Res. *"Do they have to be called R and H?"*

Rich. *"It could be called` anything".*

Res. *"And what sort of numbers can you put in?"*

Rich. *"Up to whatever the computer will take".*

Joanne

Res. *"Explain to me what your LINE or SIDE do?"*

Joanne *"That means you can type in any number...and you can get the sides...how ever long you want...to save you typing out a procedure".*

Res. *"And does it have to be called LINE?"*

Joanne *"No...anything.."*

Res. *"What sort of numbers can you put in?"*

Joanne *"Any numbers you like really"*

Kelly

Res. *"What does your variable S do?"*

Kelly *"You could do anything........when you run the program and put the numbers in........"*

Res. *"Did you have to call it S?"*

Kelly *"No you could call it anything you like".*

Res. *"And what sort of numbers can you put in?"*

Kelly *"You can put any numbers in".*

Res. *"Like what?"*

Kelly *"Ummm....45.....67..."*

Craig

Craig's solutions to both the general square problem and the general H problem suggest that his understanding of variable was very restricted. However when carrying out Question 2 he gave the following responses:

Res. *"What do you think the XX and the YY does?"*

Craig *"That's the program and it makes it bigger or smaller".*

Res. *"In what way does it do that...tell me about it?"*

Craig *"Umm....say you put a number in...it'll make it bigger or smaller...depending on the number".*

Res. *"And what sort of number?"*

Craig *"Depends what sides they are...depends what it's going to be"*

Res. *"So give me an idea of the number"*

Craig *"Say to do a square....you'd do...equal sides..."*

His response to this question indicates that he assumes that the procedure will draw a square. The last time in which he had used variable had been in the context of defining a general rectangle procedure with two variable inputs and he had then used this rectangle procedure to draw a square.

Helen

Like Craig, Helen had not been successful in defining a general procedure for herself. The following interchange occured after she had carried out Question 2.

Res. *"What would this XX and this YY do?"*

Helen *"You could put any number in".*

Res. *"What sort of number"*

Helen *"Ummm.....say if you wantedto do a square...you could put ...."*

As with Craig Helen thought that SHAPE drew a square and she had also recently carried out the rectangle task (appendix 8.1).

8.3 DISCUSSION OF RESULTS

This section will discuss the pre-algebra pupils' responses to the structured interview from the perspective of the categories outlined in Section 8.1.1. Table 8.11 presents an overview of these responses.

Table 8.11: Overview of Pre-Algebra Pupils' Understanding of Variable According to Categories Outlined in 8.1.1

	Nicholas	Clare	Stuart	Rich	Joanne	Kelly	Craig	Helen
Acceptance of the idea of variable	●	●	●	●	●	●	●	●
Understanding that any variable name can be used	●	●	●	●	●	●	●	●
Undestanding that a variable name represents a range of numbers	●	●	●	●	●	●	o	o
Understanding that different variable names can represent the same value	●	o	●	●	●	o	o	o
Acceptance of lack of closure	●	●	●	●	●	o	o	o
Ability to establish a second-order relationship between variables	●	●	o	●	o	o	o	o
Ability to represent a generalisation already expressed in natural language	●	o	●	●	o	o	o	o

All the pre-algebra pupils accept the idea of variable. As hypothesised at the beginning of this chapter it appears that involvement in variable related tasks is sufficient to develop this acceptance. All the pupils also understand that any variable name can be used and again it apears that it is sufficient to demonstrate this idea to pupils within the context of a procedure which they have already defined.

All the pupils apart from Craig and Helen appear to understand that a variable represents a range of numbers. However their understanding of "range" varies between pupils. The six pupils who developed this understanding were able to interpret procedures which used abstract variable names. Clare, Stuart and Joanne however preferred to use meaningful variable names in their solutions to the "General Square" and the "General Letter H" task, whereas Nicholas, Richard and Kelly used abstract variable names.

Craig worked with Richard throughout the year and a superficial look at their joint procedures written gives the impresson that Craig had had extensive experience of variable (Table 8.1). Closer examination of their working relationship, from the detailed notes made by the researcher when she was present, indicates that Richard took control of the decisions related to the use of variable. By the end of the year Richard had an elaborated understanding of variable (Table 8.11) but Craig's understanding was very restricted and his solutions to the general square and the general letter H tasks indicate that he was using variable in a rote manner. He declared a variable at the top of his procedure and then did not use the variable in a meaningful way within his procedure. It is suggested that this is because he never had the opportunity to use variable for himself whilst working with Richard. Helen on the other hand did not attempt to use variable at all in her solution to the general square or the general H tasks. It is suggested that this is because her experience of variable throughout the year was limited in comparison with the other pupils (Table 8.1).

The four pupils (Nicholas, Stuart, Richard and Joanne) who developed an understanding that different variable names can represent the same value (as evidenced by question 3c) had all either engaged in the RECTANGLE task (appendix 8.1) specifically designed to provoke this understanding, or had, within the context of one of their own projects, used more than one variable and then assigned different variable names the same value when executing their procedure. Although it might be expected that Clare would have developed this understanding close analysis of her Logo work throughout the year indicates that she had never used variable in this category.

When engaging in the "Function Machine" task (appendix 6.1) all the pupils had defined simple functions of the form:

```
TO SEED :EED          TO HELEN :NUM
OP 67 * :EED          OP 8 + :NUM
END                   END
```

The evidence from the pupils responses to the structured interview questions indicates that five of the pre-algebra pupils developed an acceptance of the idea of an unclosed expression in Logo and it is suggested that the use of the function machine materials contributed to this acceptance.

Three of the pre-algebra pupils (Nicholas, Clare and Richard) developed an understanding of the second-order nature of a relationship between Logo expressions (as evidenced by question 3d). It is hypothesised that this was due to their engagement in the task FUNNY, SUNNY (appendix 8.2) which had been specifically designed to provoke this understanding. None of the pupils who had not engaged in this task developed this understanding.

Evidence from question 4 suggests that three of the pupils Nicholas, Stuart and Richard can use Logo to formalise a generalisation which involved operating on a variable. Although the response of Stuart to this question was incorrect it was consistent with his general method for calculating the area of a rectangle. Further analysis of these pupils' Logo work shows that they had all on at least one occasion operated on a variable in the context of defining a general procedure. There was no evidence that Clare, one of the pupils from the first quartile was able to use Logo to formalise a generalisation by operating on a variable. Analysis of the data indicates that she had never used variable in this way within her classroom Logo work.

It is suggested that the most critical contributing factor in the pre-algebra pupils' developing understanding of variable is the extent of their use of variable with respect to the categories of variable outlined in Section 3.3.2.. Closer analysis of this table (Table 8.1) indicates that the individual pupils' Logo experience was not uniform. As explained in section 8.1.1 the Logo experience of the pre-algebra pupils was only partially in the control of the researcher and the eight pre-algebra pupils chosen for the structured interview were chosen at the end of the period of research. The four boys were noticeably very enthusiastic about using the computer and had a tendency to dominate the programming activity. In addition they were the ones who tended to work throughout their lunch times. Under these circumstances, without specific teacher intervention, the girls allowed these boys to dominate, resulting in the heavy bias of "hands on" Logo experience in favour of the boys.

The most important results of the pre-algebra study are that:

- It is possible for 10 - 11 year old pupils who have had no experience of "paper and pencil" algebra to use variable to represent a general method in Logo. All the pupils who were able to do this, in the context of the structured interview, had previously used variable in the category of "(O) variable operated on" during their "hands on" Logo programming sessions.

- It is possible for 10 - 11 year old pupils who have had no experience of "paper and pencil" algebra to understand that a second-order relationship can exist between two simple Logo expressions provided they have engaged in tasks which use this idea in a Logo programming context.

CHAPTER 9

OVERALL CONCLUSIONS AND IMPLICATIONS OF RESEARCH

This chapter will synthesise research findings from the longitudinal case study with those from the pre-algebra study. The implications for classroom practice will be presented together with a discussion of the limitations of the study. In addition the author will discuss some directions for future research.

The categories outlined in Section 7.2 have provided a framework for the analysis of both the longitudinal case study and the pre-algebra study. The research has aimed to establish the extent to which pupils after experience with Logo activities can:

- Accept the idea of a variable
- Understand that a variable name represents a range of numbers
- Understand that different variable names can represent the same value
- Accept "lack of closure" in a variable dependent expression
- Understand the nature of the second order relationship between two variable dependent expressions
- Use variable to represent a generalised method
- Use variable to represent a simple function

The longitudinal case study pupils' understanding of variable within a Logo context was investigated first. The research then sought to find evidence as to whether Logo derived understanding of variable could act as a basis for the use and understanding of variable in a "paper and pencil" algebra context. The pre-algebra pupils' understanding of variable was investigated in a Logo context only.

Throughout the thesis the aim has been to analyse the pupils' understanding of variable with respect to their use of variable in Logo. In order to do this categories of variable use were derived (section 3.3.2). These are summarised below:

- (I) One variable input to a procedure
- (S) Variable as scale factor
- (N) More than one variable input to a procedure
- (O) Variable operated on within a procedure
- (F) Variable input to define a mathematical function in Logo
- (G) General superprocedure
- (R) Recursive procedure

9.1 THE LEARNING ENVIRONMENT

It was recognised at the beginning of the period of research that the interrelating roles of peer interaction, teacher intervention and computer response all contribute to learning. Analysis of the transcript data from the perspective of pupils' understanding of variable has highlighted the strength of the interrelationship between these three factors. Not only is it beyond the scope of this thesis to attempt to disentangle this interrelationship, it is probably an impossible task. Learning is more likely to occur when pupils are engaged in a task and pupils are more likely to be engaged in a task if they are motivated to carry out the task. This research has shown that the effect of peer interaction and computer feedback is to provoke task involvement. Evidence of this comes from the task related nature of the pupils' talk within the transcript data.

9.1.1 Factors Which Inhibit Task Involvement

One of the findings was that pupils do not naturally choose projects which need the idea of variable. The author attempted to introduce the idea of variable within the pupils' own projects because of her "hidden learning" agenda. These interventions were almost always inappropiate and were either rejected by the pupils or had the effect of inhibiting task involvement.

As the research progressed the author started to devise "teacher directed" tasks so that the pupils would need to use variable related ideas in order to solve them. Imposing tasks on pupils sometimes had the effect of decreasing motivation.

Although collaboration usually had the effect of increasing the motivation level and consequently provoking more task involvement this was not always the case. In particular at the beginning of the three year longitudinal study George's dominance and need to control had the effect of preventing Asim from becoming fully engaged in the programming activities. Even within what could be classified as "good collaborative work" detailed analysis of the data highlights the fact that pupils tend to divide their efforts within collaborative work. This was particularly noticeable with Sally & Janet and George & Asim. Both Sally and Asim lacked interest in the details of Logo syntax and allowed their partners to take control of these issues. They both found the choosing of variable and procedure names a difficult or perhaps tedious task. George and Janet on the other hand enjoyed experimenting with and taking risks with the Logo syntax.

Another factor which inhibited task involvement was the introduction by the author of a Logo formalism (in the form of Logo syntax) which did not match the pupil's own generalised method (for example the introduction of the state-transparent REPEAT 4 [FD 120 RT 90] to draw a square when the pupils had themselves generated in direct mode a non-state-transparent square). When this happened the pupils tended to reject the "teacher given" Logo formalism because they could not relate this to their prior activity.

9.1.2 The First Introduction of a New Idea

Whether pupils are working on their own goals or teacher devised goals the pupils' first introduction to a new idea plays a critical role in their subsequent developing understanding. Pupils tend to spuriously generalise from this first introduction and thus develop misconceptions. If the teacher is aware of these misconceptions the computer can be used to remediate them. An example of this phenomenom is when pupils think that the variable name itself has significance (e.g. SIDE, SCALE). If pupils are told to replace the variable name with another name throughout their procedure this is usually sufficient for them to develop an understanding that any variable name can be used. This is an example of the teacher/researcher intervening in a directive manner. In the computer context the pupil is able, by trying out the idea at the computer, to have immediate feedback as to whether or not the teacher's suggestion is correct. In a "paper and pencil" algebra context the pupil can usually only accept or reject the teacher's word. In this sense the Logo programming context effects the "didactical contract" which exists between the teacher and the pupils in the "normal" mathematics classroom.

9.1.3 The Crucial Role of the Teacher

The researcher/teacher played a crucial role in the learning of variable related ideas within the Logo programming context. The way the learning environment was structured enhanced the potential for learning and on the other hand inappropriate interventions destroyed this potential. The implications for teaching are that the challenge is for teachers to find a balance between allowing pupils to work on their own goals and asking them to work on teacher devised tasks.

9.2 NEGOTIATING A GENERALISED METHOD

The review of the literature presented in section 1.3.1 suggests that pupils often use informal methods which cannot easily be generalised and formalised. "If children do not have that structure available in the arithmetic case, they are unlikely to produce (or understand) it in the algebra case" (Booth, 1985, p.102). However in the Logo environment the longitudinal case study pupils were able to interact with the computer and negotiate with their peers so that their intuitive understanding of pattern and structure was developed to the point where they could make a generalisation and formalise this generalisation in Logo. There is evidence that in many cases they would not have been able to do this without the "hands on" interaction with the computer.

Sally and Asim did not appear to need the Logo syntax to negotiate their generalised method in the same way that the other pupils needed it and as discussed earlier they let their partners take decisions related to the local details of syntax. Analysis of Sally and Asim's use of spoken language at the beginning of a session indicates that they were often able to analyse what was invariant and what was variable within a problem solution. They were not always confident about this analysis and preferred to let their partners take the risk of trying out the ideas at the computer. For the other pupils the Logo syntax helped them to negotiate their understanding. It is possible that the reason why Sally and Asim did not "need" the Logo syntax in the same way that the other pupils did is that they had already developed their own abstract representation system to deal with the type of turtle geometry problems in which they engaged during the longitudinal study. The peer interaction provoked Asim and Sally to become involved in the production of a computer program, thus encouraging them to move from the global to the local details of their plan. This research cannot answer the question of whether working individually would have provoked Sally and Asim to use the Logo syntax for themselves or whether in this situation they would not have been motivated to learn to program at all.

There is evidence that declaring the variables in the title line of a procedure helps pupils come to terms with what is varying within a problem. Some pupils seem to use the entering of the title line (for example Janet's TO HILL "JACK "JILL in Section 5.2.12) as a way of structuring the problem environment for themselves. As they proceed through the procedure definition process they may decide to remove variables from the title line as they decide to make relationships between variables explicit within their procedure. This phenomena has also been observed by Hoyles and Noss (1988).

When teachers intervene to tell pupils about a new programming structure it is crucial that they match the "teacher given" Logo formalism to the pupils' own generalised method. Even within an apparently well defined task pupils can devise a solution which does not match the teacher's expected solution as Sally and Janet's solution to the "Spiral" task illustrates (section 5.2.9).

This research indicates that pupils' ability to use Logo to represent a general method is linked to their use of variable in the category of "(O) variable operated on". It is suggested that it is only when pupils are able to use variable in this category that they have made the break from arithmetical to algebraic thought (Filloy and Rojano, 1987). The author believes that if they have not made this break in the Logo domain they are unlikely to be able to do so in the algebra domain. Therefore more attention needs to be paid to devising tasks in which it is necessary to operate on a variable in order to solve the task.

There is evidence from the pre-algebra study that 10-11 year old pupils with no experience of "paper and pencil" algebra can use variable in the category of "(O) variable operated on". This evidence also suggests that pupils' ability to operate on a variable within a Logo procedure is not age related.

9.3 ACCEPTING THE IDEA OF VARIABLE AND UNDERSTANDING THAT A VARIABLE REPRESENTS A RANGE OF NUMBERS: THE ROLE OF THE "SCALING" LETTERS TASK

The Logo Domain This research study indicates that the "Scaling Letters" task (appendix 3.2) provided an important introductory context for the use of variable for all the longitudinal case study pupils apart from George. It is suggested that George had already taken on the idea of operating on a variable within a Logo context before engaging in the "Scaling Letters" task and so for him using variable in the category of "(S) variable as scale factor" was not a useful new tool.

When pupils first engage in the "Scaling Letters" task they can do so in a "rote" manner although even this "rote engagement" helps in the understanding that a variable effects the overall size of the geoemetrical object produced (Phase 1, section 5.8.1). All the longitudinal case study pupils who only used variable within the context of scaling letters developed both an acceptance of the idea of variable and an understanding that a variable name represents a range of numbers in the Logo programming context. From this respect the task was particularly valuable because it provoked the use of decimal numbers, thus extending pupils' understanding of "any number".

There is evidence that as the pupils continued to use variable in the category of "(S) variable as scale factor" they developed in their understanding of variable to the point where they began to become aware that a relationship exists between the component parts of a geometrical object (Phase 4, section 5.8.1). This is an important pre-cursor for being able to make this relationship explicit (phase 5, section 5.8.1). Using variable in this category does not however provoke them into needing to make the relationship explicit.

Although the task was "teacher devised" all the pupils extended the task in valuable ways from the perspective of variable use. Sally and Janet used variable in the category of "(G) general superprocedure" for the first time whilst working towards a well-defined goal (section 5.2.5), and Linda and Elaine extended the task in a more loosely defined way by building up patterns on the screen and then needing the idea of a general superprocedure to represent these patterns (section 5.4.6). In addition the "Scaling Letters" task provoked George, Asim, Sally, Janet, Linda and Shahidur to initiate the idea of using variable for themselves, which they had not done before they engaged in the task.

The "Scaling Letters" task was not appropriate for the pre-algebra pupils. This was because the idea of multiplying by a decimal turned out to confuse as opposed to help them use the idea of variable. It was possible to find other problem situations (predominantly using variable in the category of "(N) more than one variable input") for these pupils, so that by the end of the pre-algebra study all of them had accepted the idea of a variable in Logo and six of them understood that a variable represents a range of numbers.

The Algebra Domain All of the longitudinal case study pupils accepted the idea of a variable in the context of the "paper and pencil" algebra part of the structured inteview (Chapter 7). In addition six of the longitudinal case study pupils (two of whom had had no experience and one had had minimal experience of algebra as part of their "normal" school mathematics) understood within the context of the algebra structured interview that a variable represents a range of numbers. One of these pupils, Shahidur, had only used variable in the category of "(S) variable as scale factor" and Linda, had predominantly used variable in this category. Linda and Shahidur are both pupils who are very unlikely to be given any algebra work as part of the SMILE curriculum (appendix 4.4). In fact their teacher often expressed concern that they were meeting variable related ideas within the Logo context. It is suggested that using variable in the category of "(S) variable as scale factor" is sufficient to foster an acceptance of the idea of variable in the algebra domain. In addition using variable in this category in Logo makes it likely that pupils will understand that a variable represents a range of numbers in the algebra domain.

9.4 EXTENDING PUPILS' UNDERSTANDING OF VARIABLE

There is evidence from previous algebra research that pupils' initial understanding of a variable is both under and over constrained. The evidence from this research indicates that this is also true in the Logo domain. Pupils are not able to analyse what is invariant within a situation and this results in underconstraining. An example of this is when Linda and Elaine were solving the "Arrowhead" task (section 5.4.9). Their initial solution was underconstrained in that they used one variable to scale all the backward commands and another variable to scale all the forward commands, when both of these variables should have been identical (i.e. the necessary limits were not put on the variable in terms of the relationship between the constituent parts of the arrowhead). When pupils use variable in the category of "(O) variable operated on" they will necessarily have to analyse what is variable and what is invariant. If they use variable in the categories of "(I) one variable input", "(N) more than one variable input" and "(S) variable as scale factor" they do not necessarily have to address this issue.

9.4.1 Naming the variable

Before pupils can use and manipulate a variable they will have to name it. There is evidence from this research that when pupils are first introduced to variable they attach significance to the variable name. On the one hand choosing a meaningful variable name helps pupils accept the object, on the other hand the meaningful name encourages pupils to think that it has some meaning in itself. All of the longitudinal case study pupils apart from Shahidur were able to interpret a Logo procedure which used a single letter variable name. It is suggested that Shahidur's difficulty with this single letter name was due to his restricted use of variable names within his own "hands on" programming work. The author encouraged the pre-algebra pupils to use a range of variable names from the beginning of their use of variable. None of the six pre-algebra pupils who were able to use variable had any difficulty in interpreting a procedure which used single letter variable names. It is suggested that pupils need to be encouraged to use a range of variable names, including "nonsense" names (which they know have no meaning) and abstract and single letter names (which they will use in their algebra work).

9.4.2 Understandng that Different Variable Names Can Represent the Same Value

The Logo Context Pupils overinterpret the constraints on the variable name itself. Algebra research has shown that pupils do not understand that different variable names can represent the same value (Küchemann, 1981). Four of the longitudinal case study pupils understood this idea within the Logo environment. All of these had, within their Logo programming experience, defined a procedure with at least two variables and then in the context of using the procedure assigned both inputs the same value. For example after defining the procedure TREE with two variable inputs JIM and MARK Linda and

Elaine typed in commands of the form TREE 1 1; TREE 0.5 0.5; TREE 1.5 1.5. The longitudinal case study pupils who did not understand this idea had never used variable in this way. Further evidence for this was also provided by the pre-algebra study. It is suggested therefore that using variable in the category of "(N) more than one variable input" and assigning several of these variables the same value in the context of "hands on" Logo activity will helps pupils develop an understanding that different variable names can represent the same value in Logo. Again this is evidence that it is the using of an idea in a Logo programming context which is the crucial factor which influences understanding.

The Algebra Context Only the longitudinal case study pupil Sally showed any evidence of having developed an understanding that different variables can represent the same value in the algebra context. One cannot attribute this to her Logo experience as she had also carried out more "paper and pencil" mathematics work than the other case study pupils apart from Asim. Given Asim's involvement with "paper and pencil" algebra as part of his "normal" school mathemics (appendix 4.4) it is surprising that he could not answer correctly either the Logo or the algebra question related to this idea. There is evidence that Asim's understanding of variable in Logo was considerably more restricted that Sally's and he had never used variable in the way described above during his Logo work. It is suggested that if more explicit attention had been paid to the use of this idea in the Logo domain then pupils would have been more likely to be able to use this idea in the algebra domain.

9.5.3 Acceptance of "Lack of closure" in a Variable Dependent Expression

The Logo Context All the longitudinal case study pupils accepted "lack of closure in a variable dependent expression" and five out of eight of the pre-algebra pupils accepted "lack of closure" in Logo expressions. All of these pupils who accepted the idea had used "unclosed" expressions within the context of defining simple functions. It is suggested that the function machine materials provided a simple and valuable context within which pupils needed to construct "unclosed" variable dependent expressions.

The Algebra Context Five of the longitudinal case study pupils accepted lack of closure within the algebra context of the structured interview. This is more than would have been expected given their experience of "paper and pencil" algebra. It could be that once pupils are no longer resistant to the idea of a variable then accepting the idea of "unclosed" variable dependent expressions is not a difficult step. It is essential that within the algebra domain pupils accept that expressions like 3x+4 are objects. As discussed in section 1.3, previous algebra research (Booth 1984, Collis 1974) has shown that this is often difficult for pupils. Overcoming this barrier by providing pupils with relevant Logo experiences could be an important step in helping pupils to be able to manipulate "unclosed" expressions in the algebra domain.

9.5.4 Understanding the Nature of the Second Order Relationship Between two Variable Dependent Expressions

The Logo Context None of the longitudinal case study pupils appeared to understand the nature of the second order relationship between variable dependent expressions in Logo. Detailed analysis of the data indicated that none of these pupils had worked with these ideas throughout the three years of the study. A task was developed for the pre-algebra pupils (appendix 8.2) specifically designed to develop this understanding. Three of the five pre-algebra pupils who engaged in this task showed, within their structured interview, that they had developed an understanding of the idea. This demonstrates that it is possible for pupils, if they used this idea during their "hands on" Logo programming sessions, to develop an understanding of this idea in Logo. Again this provides evidence that a crucial factor in learning is first the use of an idea within a problem solving situation.

9.5 THE ROLE OF THE FUNCTION MACHINE MATERIALS IN HELPING PUPILS MAKE LINKS BETWEEN LOGO AND "PAPER AND PENCIL" ALGEBRA

There is evidence that at least six of the longitudinal case study pupils made some links between variable in Logo and variable in "paper and pencil" algebra and this thesis has highlighted the extent to which the pupil's understanding of variable in algebra is related to their use of variable in Logo.

There is no evidence that using the "function machine" materials provided obstacles for the pupils with respect to making links and it is suggested that one of the most important aspects of the function machine material in helping the pupils to make links was that it provoked the pupils to use a range of variable names, including single letter names. Evidence from the "paper and pencil" function machine tasks (section 6.3.2) suggests that George and Janet's Logo frame is dominant in that they made conversions from the Logo representation to the algebra representation. Asim's algebra frame appears to be dominant in that he made conversions from the algebra to the Logo representation only. For Sally there is no evidence that either frame is dominant. Both Linda and Shahidur showed evidence of converting from the Logo to the algebra representation as they carried out the "paper and pencil" function machine tasks. These were both non-algebra experienced pupils who exhibited an unexpected understanding of variable in the "paper and pencil" structured algebra interview.

Jude had the least "hands on" time with these materials and both Jude and Ravi's involvement in the "Function Machine" tasks was less extensive than the other pupils from the point of view of range of functions defined and range of variable names used. These two pupils were the only two who were not able to use their Logo understanding that a variable represents a range of numbers in the algebra context of the structured interview.

The author suggests that the evidence from Linda and Shahidur's engagement in the "Paper and Pencil Function Machine" materials and their unpredicted understanding of algebra ideas, and Ravi and Jude's lack of engagement and their corresponding lack of understanding of algebra ideas indicates that pupil engagement with these materials helped them make links between Logo and "paper and pencil" algebra.

The nature of the research was such that it is not possible to say whether the pupils who did link their understanding from a Logo to a "paper and pencil" context could have done so without the "Function Machine" materials. It is the author's belief, however, that this is not likely to be the case.

9.6 FACTOR'S CONTRIBUTING TO PUPILS' SYNTHESIS OF VARIABLE USE

This section will discuss how far the pupils have synthesised their knowledge of variable in Logo from the perspective of the following categories of variable use (all categories which are related to the production of a simple graphical object (section 3.3.1)).

(I) One Variable input
(S) Variable as scale factor
(M) More than one variable input
(O) Variable operated on

The idea of frame, derived from Minsky (1977) has been used throughout this thesis because it has been found to be useful in attempting to describe the context dependent nature of learning which was revealed by the transcript data. Pupils have been described as thinking from, for example a "(S) variable as scale factor" frame or a "(I) one variable input" frame. It is not suggested that pupils are conscious of these frames or that for example Asim's "(S) variable as scale factor" frame is the same as Janet's "(S) variable as scale factor" frame.

What is clear from the transcript data is that during the beginning stages of variable use pupils unconsciously bring to the problem situation their most recently used variable frame. Within the problem situation as a result of negotiation with their partner and negotiation with the computer they may begin to discriminate between different variable frames. Apart from one occasion there was no attempt throughout the research to make the pupils more conscious of the various ways in which they used variable. The one occasion was when Sally and Janet had defined two general square procedures, one using "(I) one variable input" and the other using "(S) variable as scale factor" and the researcher asked them to compare the processes within both procedures (section 5.2.7). Because in this instance teacher intervention was shown to be important it is suggested that teacher devised tasks could be used to help pupils become more aware of and thus more able to discriminate between their own frames with the ultimate aim of helping them to develop a synthesis.

It is beyond the scope of this thesis to use the transcript data to elaborate on Minsky's frame theory. However there is sufficent evidence that within the Logo context learning is very fragmented. Pupils do not naturally make links between their various variable frames. These unconscious and subjective frames appear to derive from the pupils' previous use of variable and are thus very related to the categories of variable use outlined in section 3.3.2.

9.6 IMPLICATIONS FOR FURTHER RESEARCH

Within this project pupils mainly engaged in problems within a turtle geometry domain. Further work needs to be carried out on problems taken from the non-graphical domain. The author suggests that the facility to negotiate a general method whilst interacting with the computer will still be a crucial aspect of the formalisation process. Specific Logo microworlds may need to be devloped to provide pupils with the facility of using the Logo syntax to develop an understanding of a general relationship within other domains. So for example the author suggests that it would not be sufficient for pupils to write a Logo procedure to calculate areas of regular shapes in order for them to understand about area. Rather they need new Logo primitives which they can manipulate in direct mode so that they can develop more of an intuitive understanding of area before they write an abstract Logo procedure to calculate this area. This suggestion would need to be investigated by future research.

There is a need for a further longitudinal study with pupils for whom algebra is not considered an appropriate part of their school mathematics curriculum to establish whether or not these pupils can, given a suitable Logo experience, learn algebra. It was a limitation of the present study that some of the longitudinal case study pupils use of Logo was restricted. This was due to factors outside the control of the researcher. It is often the case, however, that pupils who are not attaining in their school mathematics work are given less time on the computer than pupils who are attaining well. This is undoubtedly due to the pressure of the curriculum. It will always be difficult to control this factor in any classroom based research but future research projects should try to ensure that all pupils being studied have similar Logo "hands on" time. It should be added that there was no evidence throughout this research that pupils' understanding of variable was related to Piagetian developmental stages (Piaget, 1977).

In the algebra domain pupils sometimes need to use variables to represent generalised numbers and sometimes to represent specific unknowns (when solving equations for example). This research did not address the question of whether or not understanding that a variable can represent a range of numbers would be an obstacle when needing to use a variable as a specific unknown. Although the author suspects that this would not be the case further research needs to be carried out in this area.

A limitation of the present study was that the author had no influence on the "paper and pencil" algbera work of the longitudinal case study pupils. More research needs to be carried out in situations in which the pupils' Logo experience and the "paper and pencil" algebra experience can be integrated. In addition research needs to be carried out to investigate whether pupils with Logo experience of variable can more easily use and manipulate objects in the algebra domain than pupils who have had no experience of variable in Logo.

The author does not suggest that an understanding of all the categories (e.g understanding that a variable represents a range of numbers) outlined at the beginning of this chapter will imply that a pupil has a comprehensive understanding of variable. These categories have only provided a way to analyse the data. What is important is that a pupil is able to use algebra to solve problems. More work needs to be carried out on the analysis of the use of algebra within a range of problem solving situations in order to identify which understandings are likely to be derived from which uses.

There is evidence from this study that considerable variation exists between pupils in their "hands on" use of the computer to negotiate a problem solution. Obviously the Logo syntax is an essential tool during this negotiation phase. More research should be carried out on the nature of this interaction with the computer and how this may relate to individual pupil differences and also to pupil's use of natural language.

At the beginning of the period of research the author's view of herself was that she allowed pupils the freedom to work as they wished. Evidence from the transcript data shows that this was not the case and pupils clearly attempted to take on the author's "hidden agenda" on variable thus indicating that a "didactical contract" existed between the author and the pupils working at the computer. The nature of the "didactical contract" does, however, appear to be changed by the computer environment and this needs to be investigated further.

As mentioned several times throughout this thesis it is beyond the scope of this study to extend psychological theories on learning. The evidence is that pupils do construct their own meaning from a situation. Continued analysis of pupil's language which initially appeared to make no sense to the author could almost always be traced to a meaningful (from the pupil perspective) previous situation. This is the value of longitudinal data. Further analysis of this longitudinal data from differing theoretical perspectives could be invaluable in contributing to existing theories on learning.

9.7 SUMMARY

This study was almost all carried out in the "normal" classroom and within this context certain elements of the research were not within the control of the author. However the author believes that it is only by carrying out research in the classroom situation that it is possible to provide results which have any validity for classroom practice.

The overall conclusion of this research is that Logo experience does enhance pupils' understanding of variable in an algebra context, but the links which pupils make between variable in Logo and variable in algebra depend very much on the nature and extent of their Logo experience. The present algebra curriculum will need to be adapted to suit the needs of these Logo experienced pupils and this of course is another topic for future research. It is ironical that the present trend is such that for many secondary school pupils their introduction to algebra is both being delayed and restricted, when at the same time this thesis has shown that pupils' use of variable in Logo programming is likely to make algebra more meaningful and accessible to them.

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APPENDIX 1: Concepts in Secondary Mathematics and Science (C.S.M.S.) Algebra Test

As part of the research programme "Concepts in Secondary Mathematics and Science" just under 1000 secondary pupils aged 14 + were tested on their understanding of algebra (generalised arithmetic) (Küchemann, 1981). This project is often referred to as the C.S.M.S project. The full test is presented overleaf, although only a subset of this algebra test has been used for this study. The facility rates for the 14 year old sample are known for each item.

Algebra 1

Name .... *School* .... *Class* ......

Date .... *Date of Birth* ....
day *month* *year*

Boy or Girl ....

---

1. Fill in the gaps:

$x \longrightarrow x + 2$ $\qquad$ $x \longrightarrow 4x$

$6 \longrightarrow .$ $\qquad$ $3 \longrightarrow .$

$r \longrightarrow .$

---

2. **Write down** the smallest and the largest of these: **smallest** **largest**

n + 1, n + 4, n − 3, n, n − 7. ....

---

3. **Which is** the larger, 2n or n + 2 ? ....

Explain: ....

---

4. 4 added to n can be written as n + 4. **Add 4 onto each of these:**

8 n + 5 3n

....

n multiplied by 4 can be written as 4n. **Multiply each of these by 4:**

8 n + 5 3n

....

---

5. **If** a + b = 43

a + b + 2 = ...

If n − 246 = 762

n − 247 = ...

If e + f = 8

e + f + g = ...

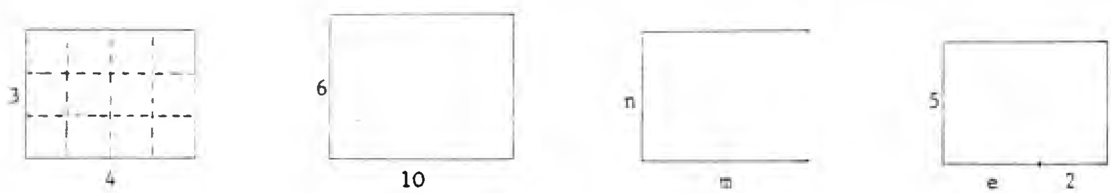
---

6. **What** can you say about a **if** a + 5 = 8 ....

What can you say about b if b + 2 is equal to 2b ....

---

7. What are the areas of these shapes?

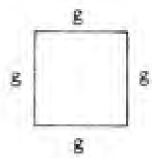


A = ... A = ... A = ... A = ...

8. The perimeter of this shape is equal to 6 + 3 + 4 + 2, which equals 15.

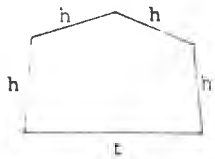
Work out the perimeter of this shape. p = ...

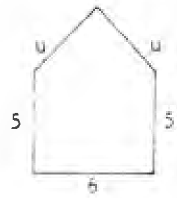
9.



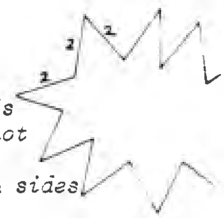
This square has sides of length g.
So, for its perimeter, we can write p = 4g.

What can we write for the perimeter of each of these shapes?





Part of this figure is not drawn.
There are n *sides altogether, all of length 2.*



p = ... p = ... p = ... p = ...

---

10. Cabbages cost 8 pence each and turnips cost 6 pence each.

If c stands for the *number* of cabbages bought and t stands for the *number* of turnips bought, what does

8c + 6t stand for? ....

What is the total number of vegetables bought? ....

---

11. What can you say about u if u = v + 3 and v = 1 ....

What can you say about m if m = 3n + 1 and n = 4 ....

---

12. If John has J marbles and Peter has P marbles, what could you write for the number of marbles they have altogether? ....

13. a + 3a can be written more simply as 4a.

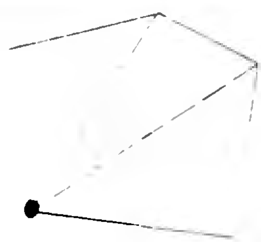
Write these more simply, where possible:

$2a + 5a =$	....		
$2a + 5b =$	....	$3a - (b + a) =$	....
$(a + b) + a =$	....	$a + 4 + a - 4 =$	....
$2a + 5b + a =$	....	$3a - b + a =$	....
$(a - b) + b =$	....	$(a + b) + (a - b) =$	....

---

14. What can you say about r if $r = s + t$
and $r + s + t = 30$ ....

15.



In a shape like this
you can work out the number of diagonals by
taking away 3 from the number of sides.

So, a shape with 5 sides has 2 diagonals;

a shape with 57 sides has .... diagonals;

a shape with k sides has .... diagonals.

---

16. What can you say about c if $c + d = 10$
and c is less than d ....

---

17. Mary's basic wage is £20 per week.
She is also paid another £2 for each hour of overtime that she works.

If h stands for the number of hours of overtime that she works, and
if W stands for her total wage (in £'s)
write down an equation connecting W and h: ....

What would Mary's total wage be if she
worked 4 hours of overtime? ....

18. When are the following true -always, never, or sometimes?
Underline the correct answer:

$A + B + C = C + A + B$ Always. Never. Sometimes, when ....

$L + M + N = L + P + N$ Always. Never. Sometimes, when ....

---

19. $a = b + 3$. What happens to a if b is increased by 2? ....

$f = 3g + 1$. What happens to f if g is increased by 2? ....

---

20. Cakes cost c pence each and buns cost b pence each.
If I buy 4 cakes and 3 buns,
what does

$4c + 3b$ stand for? ....

---

21. If this equation $\longrightarrow$ $(x + 1)^3 + x = 349$
is true when $x = 6$,

then
what value of x
will make this equation $\longrightarrow$ $(5x + 1)^3 + 5x = 349$
true?

$x =$ ....

---

22. Blue pencils cost 5 pence each and red pencils cost 6 pence each.
I buy some blue and some red pencils and altogether it costs me 90 pence.

If b is the number of blue pencils bought, and
if r is the number of red pencils bought,
what can you write down about b and r? ....

---

23. You can feed any number into this machine: $\rightarrow \boxed{-10} \rightarrow \boxed{\times 5} \rightarrow$

Can you find another machine that has the same overall effect? $\rightarrow \boxed{\times \ldots} \rightarrow \boxed{+ \ldots} \rightarrow$

APPENDIX 3.1: Overview of Logo Commands

The Longitudinal Case Study

Throughout this study the pupils used RML Logo. This Logo does not possess infix operators (e.g +, *). Instead the pupils had to use prefix operators (e.g. ADD, MUL).

The turtle starting postion for this Logo is pointing **horizontally to the right.**

This RML version of Logo is no longer in common use and it has been decided to present the pupils' Logo procedures in a more standard form. However it has been necessary to maintain the prefix operators. The following list of Logo commands are the ones which have been used throughout this thesis:

FD n	Moves turtle forward n steps
BK n	Moves turtle backwards n steps
RT n	Turns turtle n degrees anticlockwise
LT n	Turns turtle n degrees clockwise
CS	Clears the graphics screen
CT	Centres the turtle
PU	Lifts the turtle pen
PD	Drops turtle pen
PE	Activates the turtle eraser
HT	Hides the turtle
ST	Shows the turtle
SETX n	Moves turtle horizontally to x-coordinate at n
SETY n	Moves the turtle vertically to y-coordinate at n
SETXY n m	Moves turtle to x-coordinate at n and y-coordinate at m
SETH p	Sets the turtle heading to p degrees (0 vertically up the screen)
ARCL a b	Draws an arc to the left (radius a and size b (in degrees))
ARCR a b	Draws an arc to the right (radius a and size b (in degrees))
ADD p q	Outputs p added to q
SUB p q	Outputs q subtracted from p
MUL p q	Outputs p multiplied by q
DIV p q	Outputs p divided by q
GRQ p q	Tests to see if p is greater than q and outputs True or False
LRQ p q	Tests to see if p is less than q and outputs True or False

REPEAT n [a b c d....] This repeats the list of commands in the square brackets n times

TO *name inputs*	Signals start of title line of defined procedure
END	Indicates end of procedure definition
OP *object*	Returns control to calling procedure, with *object* as output.

The Primary Study

The primary pupils used LCSI Logo for the BBC computer. These pupils did not have to use the prefix operators ADD, SUB, MUL and DIV. Apart from this the Logo commands given above are identical to LCSI Logo. The main difference between the primary study pupils' procedures and the longitudinal study pupils' procedures is that in LCSI Logo the turtle starts pointing **vertically upwards**. In addition in LCSI Logo the name of the variable in the title line of a procedure can be preceededby a colon (:) as opposed to a quote mark (").

APPENDIX 3.2: The Scaling Letters Task

Write a procedure to draw a letter ....

```
TO L
LT 90
FD 40
BK 40
RT 90
FD 40
BK 40
END
```

Then change your procedure by multiplying each distance command by a scaling input.

```
TO L "SCALE
LT 90
FD MUL :SCALE 40
BK MUL :SCALE 40
RT 90
FD MUL :SCALE 40
BK MUL :SCALE 40
END
```

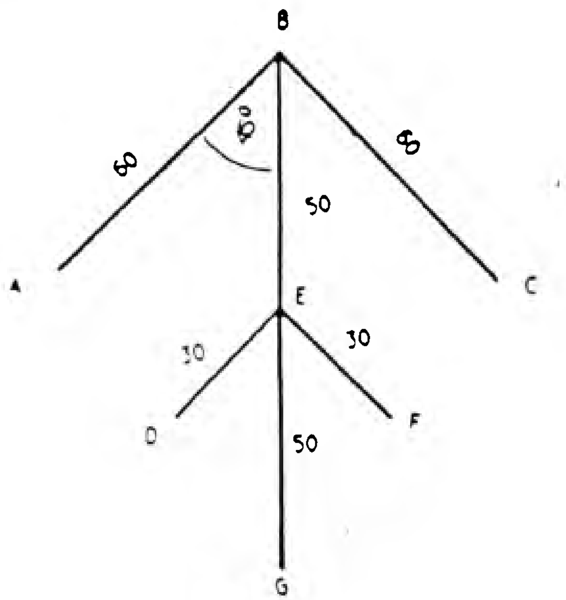
What is happening to your letter?

Now try:

```
L 1.0
L 0.5
L 2.7
L -1.9
```

How big can you make it?
How small can you make it?

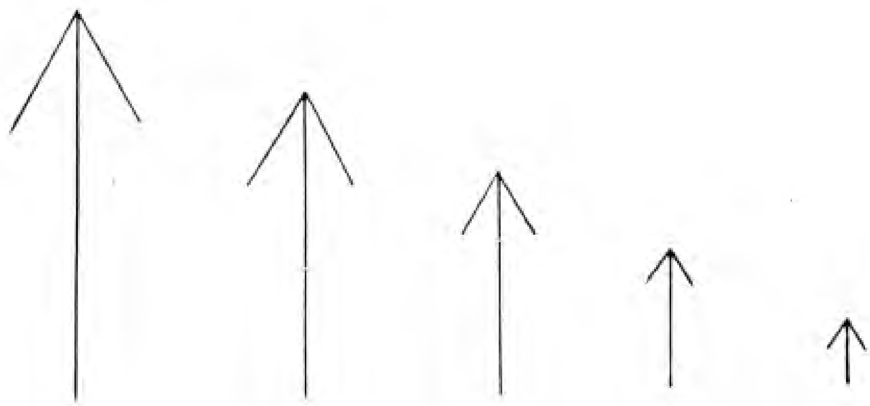
APPENDIX 3.3: The Arrowhead Task



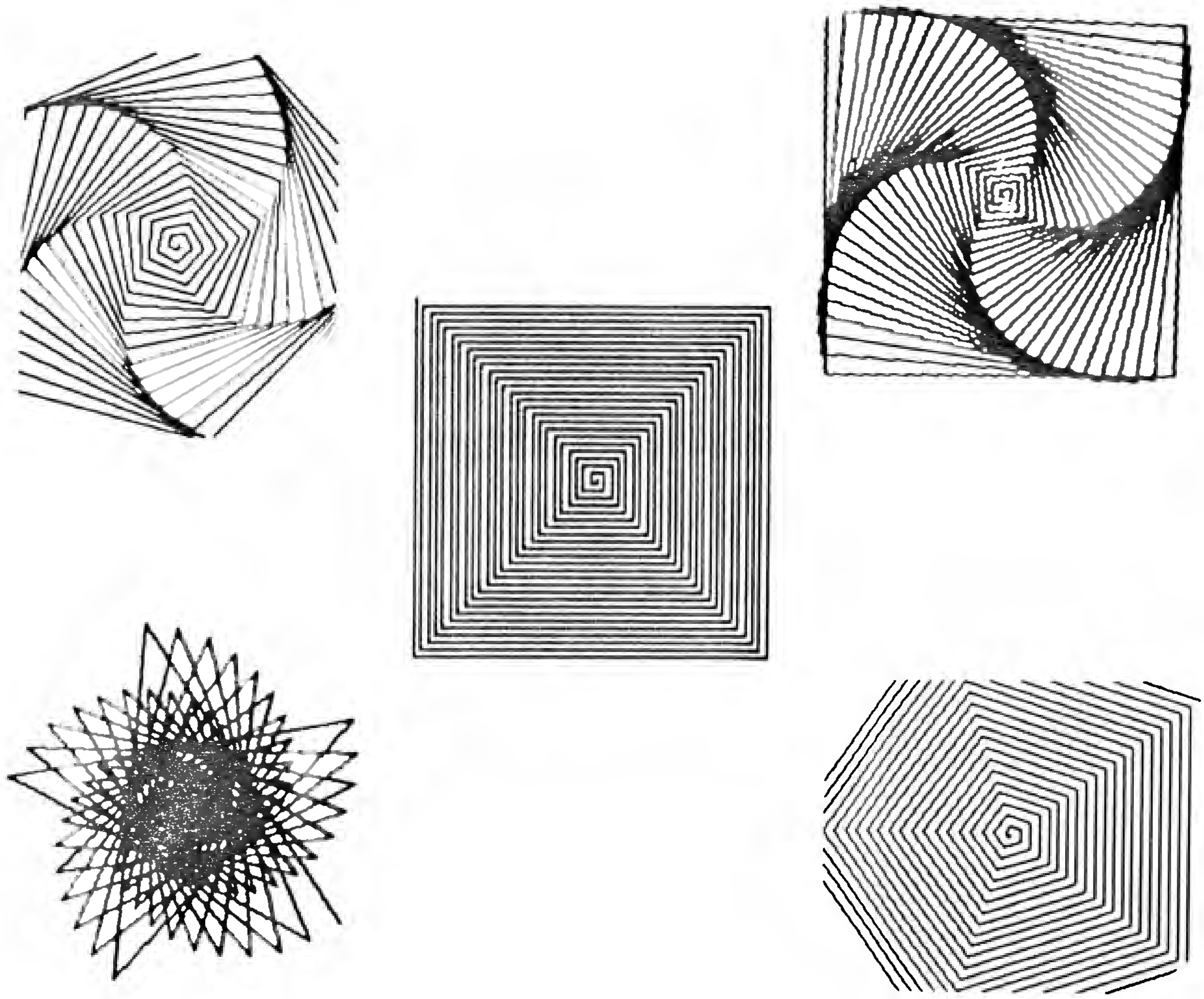
(Note: The lengths and letters were not included on the original task).

MAKE A PROCEDURE TO DRAW THIS SHAPE
AS BIG OR AS SMALL AS YOU WISH.

APPENDIX 3.4: The Row of Pines Task



APPENDIX 3.5: The Spiral Task



APPENDIX 4.1: The SMILE Curriculum

The mathematics curriculum of the longitudinal case study pupils was SMILE (Secondary Mathematics Individualised Learning Experiment). In this scheme pupils work at their individual levels and the teachers does not usually teach the class as a whole. "With SMILE children learn to organise the work for themselves. They choose the order in which they do their work and often discuss with their teacher the new work which will be appropriate for them. The materials used are stored around the classroom and the children are responsible for fetching what they need and returning it. They learn to use a filing system and reference books. SMILE encourages children to work effectively in an independent way" (SMILE - A guide for parents). The pupils work is set from a matrix of all the 1500 SMILE tasks. These tasks are arranged in topics and levels of difficulty. Each task will have a level assigned to it, although the pupil does not necessarily know this level. A complete record of all the pupils work is kept. For the pupils engaged in this study the teacher regularly calculated a SMILE level for all the pupils in the class by averaging out the level of the pupil's previous ten tasks. It is these SMILE levels which have been used to rank the pupils in the class of the longitudinal case study pupils and the comparison group of pupils.

APPENDIX 4.2 INTERVIEWS FOR PUPIL PROFILES

A detailed profile was built up of all the case study pupils. This data collected towards the pupil profile included:

Written Task

All the pupils in the research class were given the following written task.

Imagine that you are writing to a friend to tell her/him about your maths lessons since you have been at School A.

I want you to describe to your friend a really good time in your maths lessons, a time that "sticks in your memory". Describe to your friend what happened in this good time and how you felt about it. In other words, explain why it was a good time for you.

Then I want you to describe to your friend a really bad time in your maths lessons. I want you to explain why this was a bad time for you and again describe how you felt about it.

I would like you to write all about this on one or two sheets of paper.

Structured Interview

A structured interview was carried out with each of the four case study pairs individually. The interviewer encouraged the pupil to recall critical incidents in her/his mathematical experience.

The aim of the interview was:-

- to obtain information about the pupil's attitude to mathematics
- to obtain information about the pupil's attitude to Logo
- to obtain information about how the pupil views Logo in relationship to mathematics.

Teacher Interviews

Discussions with the mathematics teacher was on-going. The class teacher was interviewed towards the end of the research in order to elicit her:

APPENDIX 4.2 contd.

(a) overall impression of the Logo activities in the classroom in relation to:

- cognitive outcomes
- affective outcomes
- social outcomes
- classroom management

(b) specific comments on the case study pupils in terms of:

- her general view of the pupils
- her view of their mathematical aptitude
- her view of the effect of the Logo activities on the pupils' mathematical learning attitude and motivation.

The form tutor was interviewed toward the end of the the research in order to elicit her:

specific comments on the case study pupils in terms of:

- general ability
- general attitude to work
- general behaviour in school
- her view of their personality
- any discussion about Logo she has observed the pupils having during tutorial sessions.

APPENDIX 4.3: CATEGORIES OF TEACHER INTERVENTION

The following categories of intervention were used as a basis for analysis:

MOTIVATIONAL
Reinforcement (R) e.g. "That's good"
Encouragement (E) e.g. "Try it"

REFLECTION
Looking Forward (F)
a) Process (P) Encouraging pupils to reflect on and discuss the process
b) Goal (G) Encouraging pupils to reflect on their ultimate goal.

Looking Back (LB)
a) Process (P) as above
b) Goal (G) as above

DIRECTIONAL Influencing and/or changing the focus of the pupil's attention
Nudge (N) e.g. "Do you want to clear the screen?" or "How about doing your square?"

Method (M) Encouraging pupils to use suitable methods of problem solving (which are already familiar to them).

Building (B) Encouraging pupils to apply a particular piece of previously learned material or knowledge.

Factual (F) a) NEW (F.N) Supplying a particular piece of new information which is necessary to enable the pupil to continue.

b) RECALL (F.R) Reminding pupils of a piece of information (referring them to the handbook).

Powerful Idea (P.I) Introducing a "new Powerful Idea" or concept, such as procedure, the Repeat statement or the idea of a Variable.

Mathematical Idea (M.I) Introducing a new Mathematical idea.

(Note (R) indicates requested by pupil).

APPENDIX 4.4: Case Study Pupils Algebra Experience.

From the records of SMILE tasks engaged in by the case study pupils it has been possible to get a picture of the SMILE algebra tasks which these pupils engaged in as part of their "normal' school mathematics work. No attempt has been made to find out the pupils' performance on these tasks.

The following is a summary of this algebra work for each of the longitudinal case study pupils.

	Year 1	Year 2	Year 3
Sally	DIME Number machines DIME Simple Mappings DIME Mappings & Graphs	DIME Quadratic Mappings Simple Equations Inverse Mappings	Algebraic Identities
Asim	DIME Number machines DIME Simple mappings DIME Mappings & Graphs	DIME Quadratic Mappings Simple Equations Inverse mappings	Algebraic Identities
George	None	DIME Number Machines Simple Mappings	DIME Simple Mappings DIME Mapping & Graphs Simple Algenraic Identities
Janet	None	Dime Number Machine Simple Mappings	Simple Mappings
Ravi	None	None	None
Jude	None	None	None
Linda	None	None	None
Shahidur	None	None	None

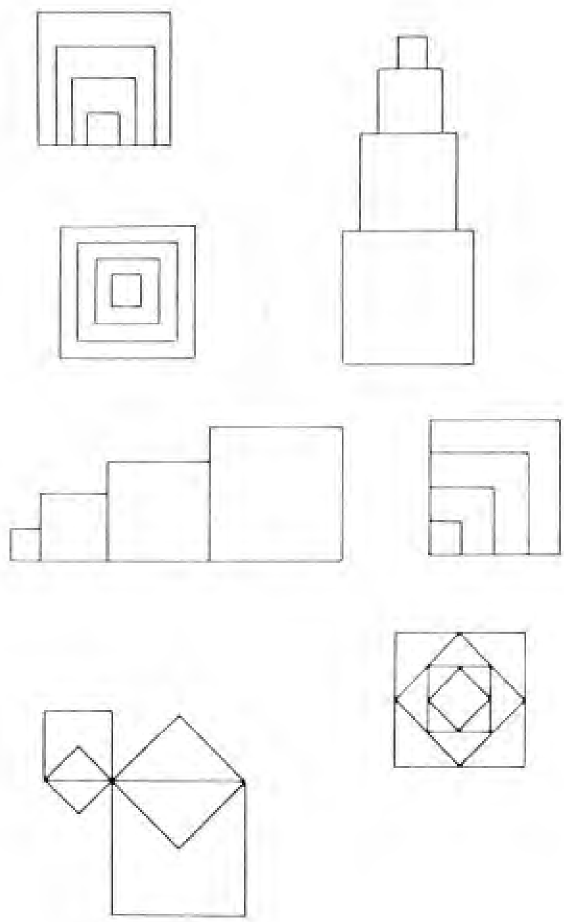
The DIME material (Giles1984) are published by Tarquin Publications, Stradbroke, Diss, Norfolk

APPENDIX 5.1 STRUCTURED TASK: PATTERN OF SQUARES

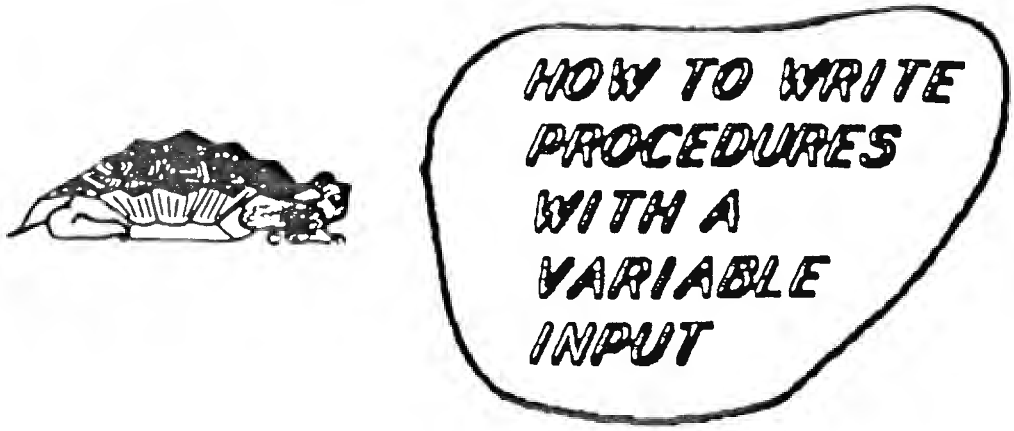
On this day the case study pupils again visited the University. They first of all carried out the Four Squares Task (Appendix 4a) individually.

They also carried out the Variable Squares Task (based on the idea in Rouchier, Samurçay 1985).

This task took the form of a game which was played between two of the pairs. The purposed of the task was to prompt the pupils into seeing the need for the use of variable and to use this in their programming and also to see whether the pupils would show an understanding of process by being able to follow through a program written by another pair. Both pairs of pupils were given a handout on which were drawn the 7 figures (fig. 1). They were told that each figure was made up of the same 4 squares and that we wanted them to be able to draw all the figures in the 'easiest' possible way. The game consisted of each pair building a program for one of the figures, with the other pair being required to guess from their program which figures the program drew. Communication between the pairs, who were in separate rooms was allowed in the form of written messages.



APPENDIX 5.2: Handout on Variable



Choose a name for your variable input
e.g. WHATEVER then define a procedure :

```
TO SHAPE :WHATEVER
FD :WHATEVER
RT 30
END
```

You now type

SHAPE $\boxed{\text{ANY NUMBER}}$

Try typing SHAPE followed by different numbers
What happens?

APPENDIX 5.3: INDIVIDUAL LABORATORY "PAPER AND PENCIL" TASKS

(a)

This is a procedure
which draws a rectangle.....

```
TO RECTANGLE "FUN "SUN
    FD :FUN
    RT 90
    FD :SUN
    RT 90
    FD :FUN
    RT 90
    FD :SUN
    RT 90
    END
```

If I type RECTANGLE 50 100 what
will I draw?

Draw the rectangle and label the lengths
of the sides.

(b)

Home **School**

This is a procedure which
draws rectangles.....

```
TO RECTANGLE 'C
    FD :C
    RT 90
    FD MUL 5 :C
    RT 90
    FD :C
    RT 90
    FD MUL 5 :C
    RT 90
    END
```

If I type RECTANGLE 30 draw the
rectangle that will be drawn and
label its' lengths.

(c)

Name **School**

```
TO PUZZLE "BIT
   LT 90
   FD MUL :BIT 30
   RT 90
   FD MUL :BIT 20
   BK MUL :BIT 20
   LT 90
   FD MUL :BIT 30
   RT 90
   FD MUL :BIT 40
   BD MUL :BIT 40
   END
```

Can you trace out the shape drawn by this procedure, marking all the lengths.

(d)

Name **School**

```
TO SURPRISE "SCALE
   LT 90
   FD MUL :SCALE 60
   BK MUL :SCALE 30
   RT 90
   FD MUL :SCALE 20
   LT 90
   FD MUL :SCALE 30
   BK MUL :SCALE 60
   END
```

Can you trace out the shape drawn by this procedure, marking all the lengths.

(e)

If I type into the computer PUZZLE can you draw out the pattern which the turtle makes marking all the lengths ?

```
TO PUZZLE
CHALLENGE 50
BOX
CHALLENGE 40
BOX
CHALLENGE 30
END
```

```
TO CHALLENGE "WOT
REPEAT 3 [FD :WOT LT 120]
END
```

```
TO BOX
FD 55
END
```

(f)

```
TO PAT "NUM
CHALLENGE :NUM
BOX
CHALLENGE SUB :NUM 10
BOX
CHALLENGE SUB :NUM 20
END
```

```
TO CHALLENGE "WOT
REPEAT 3 [FD :WOT LT 120]
END
```

```
TO BOX
FD 55
END
```

If I type into the computer PAT 50 can you draw out the pattern which the turtle makes marking all the lengths. ?

(g)

Name

School

```
TO MYSTERY "NUM
CHALLENGE :NUM
BOX
CHALLENGE SUB :NUM 10
END
```

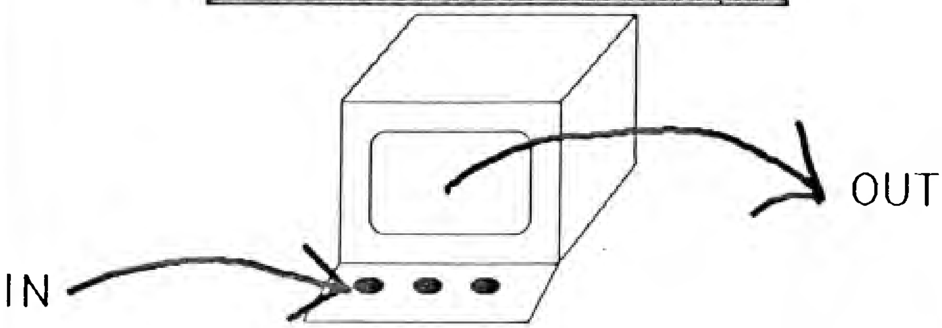
```
TO CHALLENGE "WOT
REPEAT 3 [FD :WOT LT 120]
END
```

```
TO BOX
FD 55
END
```

If I type into the computer MYSTERY 50 can you draw out the pattern which the turtle makes marking all the lengths. ?

(a)

A FUNCTION MACHINE



You can use the computer to make a function machine.
For example to make an ADDFOUR machine type

```
TO ADDFOUR "NUM
OP ADD 4 :NUM
END
```

Type in the commands for ADDFOUR

Don't forget to type END.

Now try

```
PRINT ADDFOUR 3
PRINT ADDFOUR 12.5
PRINT ADDFOUR -5
```

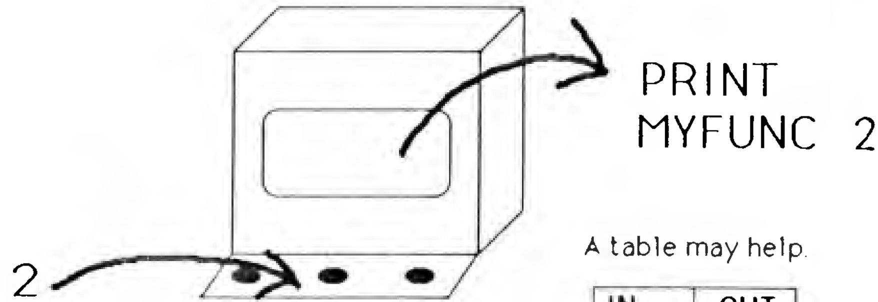
Now make your own function machine.
You might like to use

ADD + , SUBTRACT -,
MULTIPLY * , DIVIDE /.

(b)

GUESS MY FUNCTION

1. Choose a partner.
2. Build a function machine without letting your partner see the function.
3. Ask your partner to put some numbers in your function machine so that she/he can guess the function.



A table may help.

IN	OUT
2 →	?
5 →	
1 →	
4 →	
-1.5 →	

4. When your partner thinks she has guessed the function she must use it to build her function machine (e.g. called YOURFUNC)

5. Use the computer to test if YOURFUNC is the same as MYFUNC

PRINT MYFUNC $\boxed{?}$

PRINT YOURFUNC $\boxed{?}$

(c)

UNDOING A FUNCTION.

Build a function e.g.

```
ADDFOUR "Z
OP :Z + 4
END
```

Complete the table
ADDFOUR FUNCTION

IN	OUT
1	
6	
5	
-4	

You can now build a function to undo the ADDFOUR function:

```
UNDOADDFOUR "Y
OP SUB :Y 4
END
```

Try

```
PRINT UNDOADDFOUR ADDFOUR 5
PRINT UNDOADDFOUR ADDFOUR 9
```

What function will undo the UNDOADDFOUR function ?

Build the MULTEN function

```
MULTEN "X
OP MUL :X 10
END
```

Can you build the UNDOMULTEN function ?

(You might want to call it a shorter name)

Build the SUBFIVE function

```
SUBFIVE "Z
OP SUB :Z 5
END
```

Can you build the UNDOSUBFIVE function ?

(d)

FUN WITH FUNCTIONS

Build two function machines

e g

ADDFOUR

```
ADDFOUR "X
OP SUB :X 4
END
```

MULTEN

```
MULTEN "Y
OP MUL :Y 10
END
```

What happens when you type

PRINT ADDFOUR MULTEN 5 ?

Try with some other numbers

Try to complete the table without using the computer and then check your results

ADDFOUR MULTEN MACHINE

IN	OUT
1	
2	
5	
9	
$^{-}7$	
$^{-}3$	

Is ADDFOUR MULTEN 3

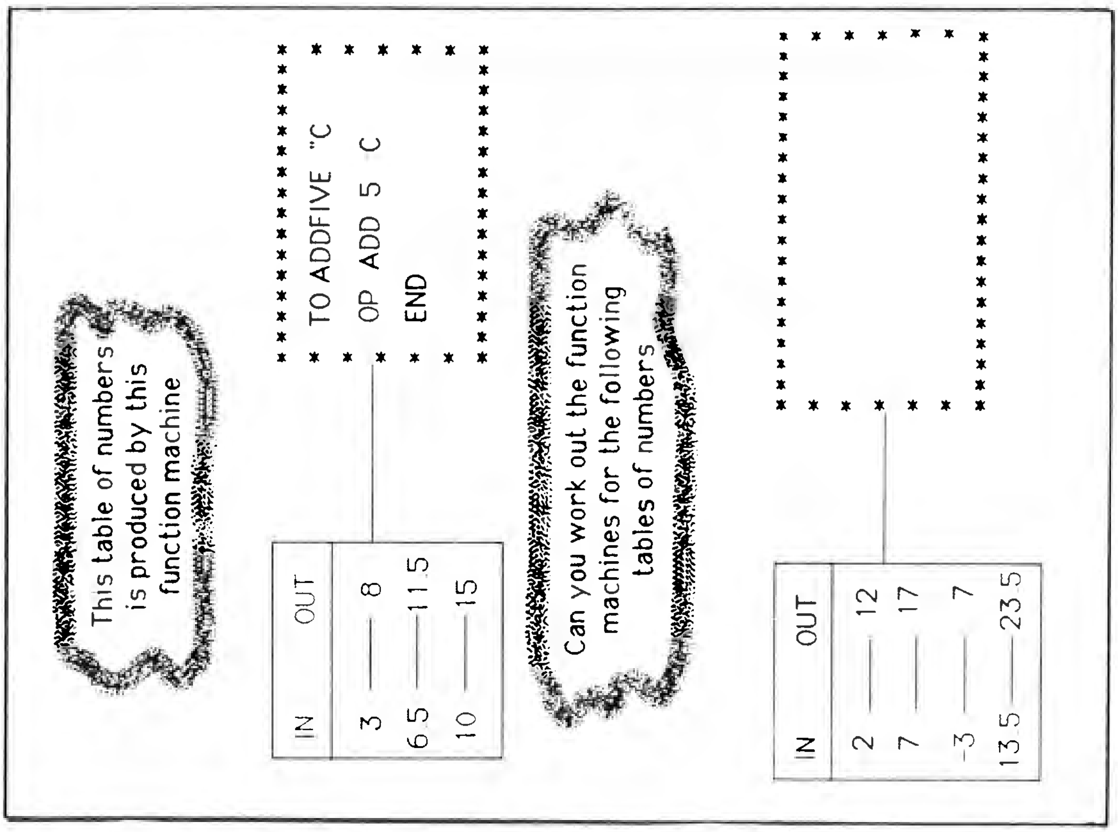
the same as

MULTEN ADDFOUR 3 ?

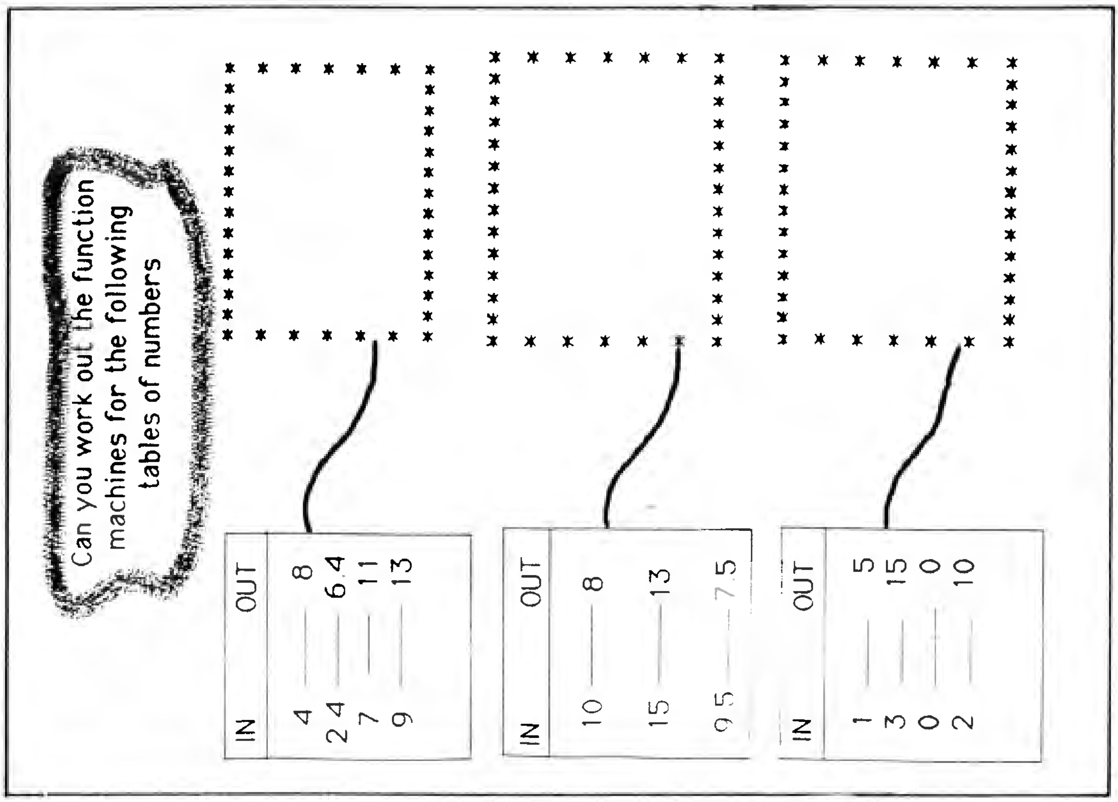
Give a reason for your answer.

APPENDIX 6.2: "Paper and Pencil" Based Function Material

(a)



(b)



(c)

Can you work out the function machines for the following tables of numbers

IN	OUT
10	3
-4	-11
20.4	13.4
15	8

IN	OUT
1	4
2.5	10
-2	-8
3	12

IN	OUT
2	6.5
7	11.5
-2	2.5
12.3	16.8

(d)

Can you work out the function machines for the following tables of numbers

IN	OUT
10	5
-4	-2
3	1.5
18	9

IN	OUT
2	10
0	0
7	35
3	15

IN	OUT
3	7
5	5
-1	11
2.4	7.6

(e)

Can you work out the function machines for the following tables of numbers

IN	OUT
1	16
4	4
8	2
12	1.75

(f)

LOGO IS VERY LIKE ALGEBRA. IN ALGEBRA WE USUALLY USE SINGLE LETTERS.............

.......Here are some statements which are similar.

LOGO	ALGEBRA
ADD 5 :N	5 + N
ADD :X 3	X + 3
DIV :W 2	W / 2
MUL 4 :Z	4Z
SUB :A 2	A − 2
SUB 3 :B	3 − B

(g)

CAN YOU COMPLETE THE FOLLOWING TABLE ?

LOGO	ALGEBRA
ADD 4 :M	
SUB :W 10	
	P + 2
	X/5
MUL 2 :N	
DIV 10 :A	
	8 − R
	4Y

(h)

THESE ARE ALL WAYS OF WRITING THE SAME ADDSIX FUNCTION

```
**************
* TO ADDSIX "W  *
* OP  ADD  :W6  *
* END           *
**************
```

IN	OUT
2 ——	8
9 ——	15
4 ——	10

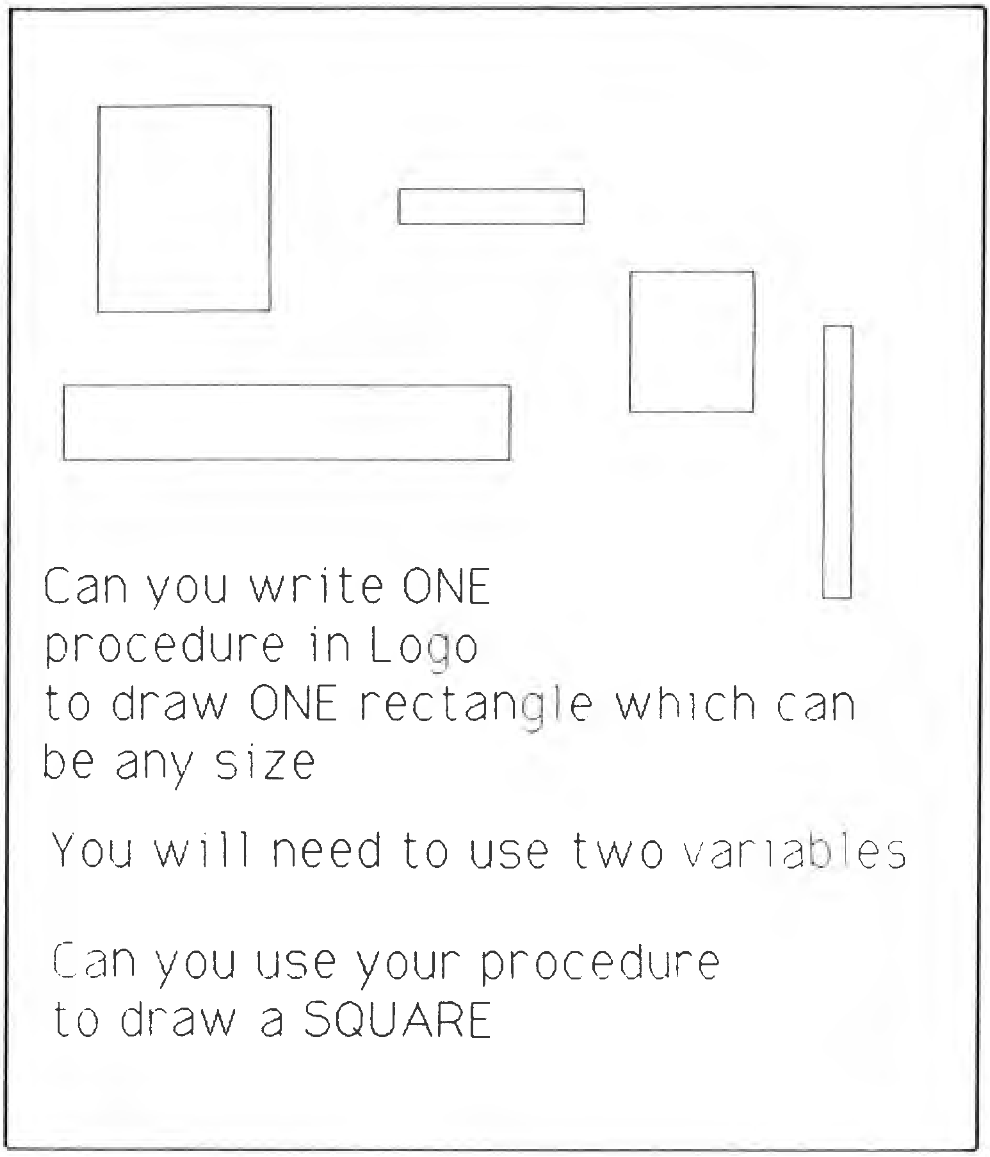
W —— W + 6

Can you complete the following tables for a MULFOUR function

```
**************
* TO MULFOUR "A  *
* OP  MUL  4 :A  *
* END            *
**************
```

IN	OUT
3 ——	12
2 ——	
5 ——	

A ——


Can you write ONE
procedure in Logo
to draw ONE rectangle which can
be any size
You will need to use two variables
Can you use your procedure
to draw a SQUARE

INVESTIGATION

Write procedures for the following two functions

```
TO FUNNY :X
OP 5 + X
END
```

```
TO SUNNY :Y
OP 5 * Y
END
```

Try PRINT FUNNY 3 and PRINT SUNNY 3

FUNNY

IN	OUT
-3 →	→
-1 →	→
0 →	→
0.2 →	→
0.5 →	→
0.8 →	→
1 →	→
3 →	→
7 →	→

SUNNY

IN	OUT
-3 →	→
-1 →	→
0 →	→
0.2 →	→
0.5 →	→
0.8 →	→
1 →	→
3 →	→
7 →	→

Try some more numbers and fill in the two tables

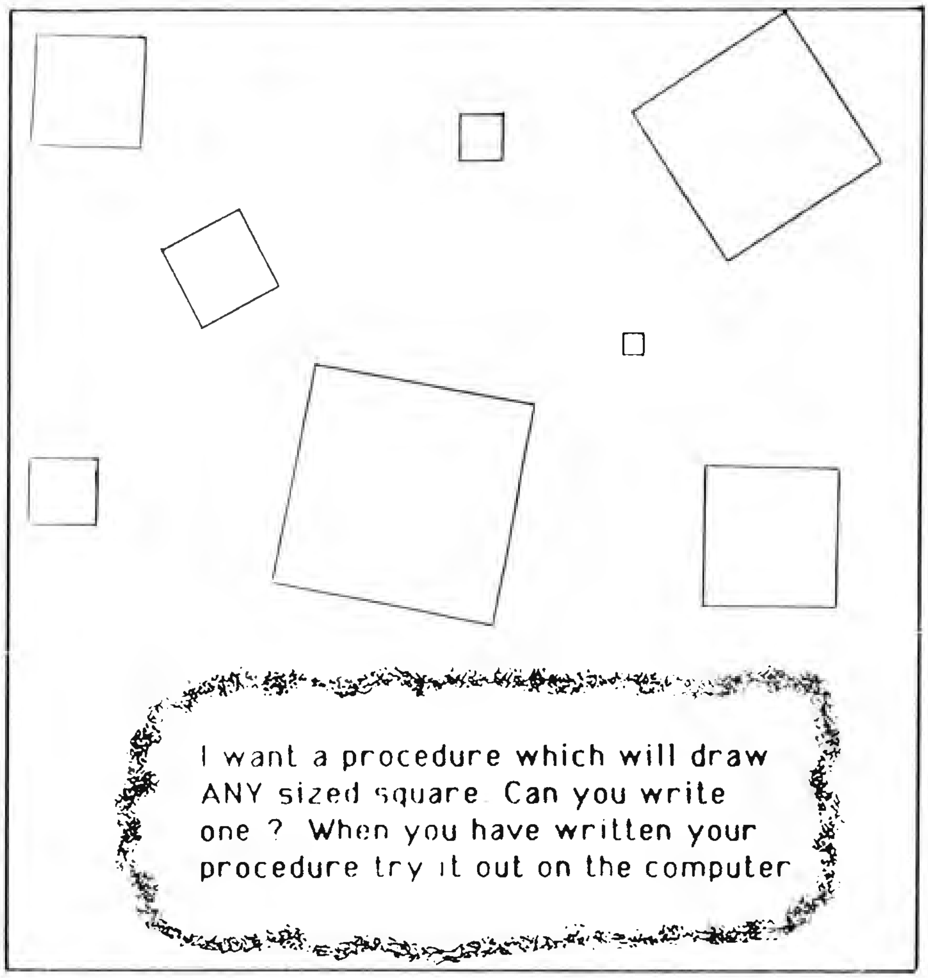
When does FUNNY output a larger number than SUNNY?

When does FUNNY output a smaller number than SUNNY?

When do FUNNY and SUNNY output the same number?

APPENDIX 8.3: Pre-algebra Study - Structured Interview Programming Tasks

(a)



(b)