Understanding Variation in Primary School Children's Arithmetical Ability: The Contributions of Social, Environmental, and Cognitive Factors

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ABSTRACT

The major purpose of the thesis is to attempt to understand some of the reasons for children's differential achievement in arithmetic. Research has associated various factors with arithmetic performance, however, usually in isolation. The present study examines a combination of social, environmental, and cognitive factors as related to arithmetic achievement, based on a sample of 91 8-9-year-old Greek children who were identified as belonging into one of three levels of arithmetic ability, above average, average, and below average, and a group of children with mild reading difficulties. Children in the math ability groups had at least average reading performance. Social and environmental factors included self-concepts, attitudes and home practices, parental help and encouragement, and parent-school relations and academic status. Cognitive components included knowledge and skill in formal and informal arithmetic and working memory efficiency. As part of the study, children were interviewed on the social and environmental factors and went through a battery of tests on the cognitive factors. Children's parents filled out a questionnaire. From the total of social and environmental factors, children's attitudes to arithmetic, parents' beliefs of children's attitudes, and mothers' academic status were associated with children's arithmetic achievement. From the total of cognitive factors, knowledge and skill in informal arithmetic and base ten system, knowledge of addition facts, problem-solving skills, speech articulation, and speed of reciting even numbers predicted children's arithmetic achievement. When both social and environmental and cognitive factors regressed on children's performance, mothers' beliefs of their child's attitudes, mothers' academic level, knowledge of informal arithmetic and base ten system, and problem-solving skills predicted children's achievement in arithmetic.

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To My Dad
and Everything
He Represents

CHAPTER 1

CHILDREN'S VARIATION IN ARITHMETIC ACHIEVEMENT: AN INTRODUCTION

"... for many pupils this is precisely the characteristic of mathematics which gives them so much motivation - they get things absolutely right, ten out of ten, red ticks abound, they experience frequent success, and this is a very satisfying experience. But for those pupils at the other end of the spectrum, the constant failure, the repeated judgement that their responses are wrong, and the red crosses proliferating in their exercise books, all add up to a depressing and frustrating experience." (Haylock, 1991, p. 35)

1.1 Introduction

One of the recurrent themes in educational research involves the attempt to unravel the complex determinants of children's academic attainment. Children differ in their academic achievement and this is particularly true in the case of reading and arithmetic. The implications of doing particularly well or experiencing severe difficulties in arithmetic are explicit in every culture; from the simple day-to-day situations one is called to face to major decisions as to a future career, all make the issue of individual differences in early arithmetic achievement a crucial topic in educational and psychological research. Despite educators' and psychologists' acknowledgement of the phenomenon of individual differences in arithmetic achievement, research on the topic is scanty compared to research on children's reading attainment and related disabilities.

In Greece, the issue of children's achievement in school arithmetic has only very recently gained some recognition. Research conducted by the University of Thessaloniki in co-operation with the Greek Association for Mental Health and Child Neuropsychiatry showed that *severe* math difficulties are more common in children in the third grade than in any other early school grade (Tzouriadou, 1990). Using a sample of 1,038 children, they found that 49% of children in Grade 3 (8 years old) face severe difficulties in arithmetic, while the corresponding figures for Grades 1, 2, and 4 are 3%, 13%, and 26%, respectively. However, *mild* difficulties in arithmetic are found mostly in Grade 2 (29%), and figures drop in Grades 1 (10%), 3 (14%), and 4 (9%). The reasons for children's underachievement have not yet been researched (For a comparison between the Greek and the British educational systems, see Appendix 1.1).

In attempting to answer the question "What accounts for children's different performance in arithmetic ?", and, therefore, discuss variation, three main issues emerge. First, despite the cultural importance attached to arithmetic, research on children's individual differences in the subject has mainly focused on children's arithmetic difficulties in the primary school. Giftedness in mathematics has largely been ignored, with the exception of a few researchers who have attempted to examine some aspects of mathematical precocity, such as domain specificity (Dark & Benbow, 1991; Robinson, Abbott, Berninger, & Busse, 1996), strategy use (Geary & Brown, 1991), and self-concepts and attitudes (Chen & Stevenson, 1995; Crystal & Stevenson, 1991; Miserandino, 1996; Stevenson & Lee, 1990).

Second, studies have identified correlates of children's performance that come from individual fields of research. For example, studies in social psychology saw the significance of children's attitudes, children's self-concepts, parents' help with the homework and their encouragement - to name only a few, on children's arithmetic achievement (Aiken, 1970; Blatchford, 1997a, 1997b; Chen & Stevenson, 1995; Crystal & Stevenson, 1991; Schunk, 1990; Stevenson & Lee, 1990; Tizard, Blatchford, Burke, Farquhar, & Plewis, 1988; Young-Loveridge, 1991). Cognitive studies have identified children's learning style - field independence (Saracho, 1995), their knowledge of addition facts and dealing with large numbers (Russell & Ginsburg, 1984), and their working memory storage capacity (Baddeley, 1990; Hitch & McAuley, 1991; Siegel & Ryan, 1989) as factors contributing to children's performance in arithmetic. A basic assumption in psychology, however, is that children may perform at a high level for many reasons, not solely out of a desire to learn or because of a particular interest in the subject. Ability, although necessary, is not sufficient for learning.

Third, general aspects, like general intelligence, memory capacity, or educational experiences at home, have been associated with children's arithmetic performance. However, recent research indicates higher correlations between children's arithmetic ability and math-specific factors like children's self-concepts in arithmetic (Blatchford, 1992, 1997b; Marsh, 1990; Schunk, 1990), their attitudes to arithmetic and their numerical activities at home (Miserandino, 1996; Tizard et al., 1988; Young-Loveridge, 1991), domain specificity of arithmetic difficulties as compared to reading disabilities (Rourke & Finlayson, 1978; Share, Moffitt, & Silva, 1988; Siegel & Ryan, 1989). Research on cognitive math-specific factors, however, has been limited to factors related to difficulties rather than talent in arithmetic; with the exception of a few studies (Dark & Benbow, 1991; Robinson et al., 1996), there is hardly any evidence of cognitive factors specific to arithmetic which might relate to children's arithmetical precocity.

The current study accounts for all three issues. It examines children of different arithmetic ability on a combination of math-specific elements coming from two distinct research areas. The aim is to uncover the complex mechanism that underlies variation, focusing on social and environmental and cognitive factors related to arithmetic that may characterise different levels of arithmetic achievement of children sharing normal intelligence and reading ability.

1.2 Social, Environmental, and Cognitive Factors Related to Children's Arithmetic Achievement

The majority of the literature reviewed for this purpose refers to studies on math-specific elements associated with children's arithmetic disabilities, both from a cognitive and social and environmental point of view. Explanatory variables come from different areas of psychology and education, from non-intellectual elements, such as children's self-concepts, home practices, or parental help and encouragement, to cognitive characteristics of the child, such as working memory and prior arithmetic knowledge. Thus, the factors to be described are more likely to reflect deficiencies rather than exceptional abilities, they come from two distinct areas of research, and they are specific to mathematics rather than general to cognitive functioning.

Social and Environmental Variables

The contribution of non-intellectual components to children's attainment has been the centre of a significant body of research under the realms of social psychology. Factors within the child, such as self-assessment and attitudes, as well as factors related to the child's home environment, such as their numerical experiences at home, parental help and encouragement, parents' contact with the school, and parent education, have often been found to exert an individual as well as a combined influence on children's achievement. The majority of studies to be reviewed have examined a variety of factors in relation to children's achievement. The review is based on topics, while studies and the variables they examined are outlined on Appendix 1.2.

First, strong and positive correlations have been found between children's beliefs about their perceived ability (self-concepts) in school arithmetic and their actual performance in the subject: children having more positive views about their performance tend to perform better in the subject (Marsh, 1990; Schunk, 1990). This holds true independently of the accuracy of their self-evaluations (Blatchford, 1997b). Also, the reasons that children believe they achieve at any level is of importance: attribution theory postulates that achievement behaviours are mediated by ability perceptions which in turn are based on the perceived causes for success or failure (Weiner, 1979). Children's perceived performance and their attributions for this level of attainment are critical in determining not only present but also future success or failure (Blumenfeld, Pintrich, Meece, & Wessels, 1982).

Another area which has been well researched is children's attitudes and their relation to school achievement. Early work by Aiken (1970, 1972, 1976) has suggested that attitudes to arithmetic and performance in the subject are strongly related, further more in a reciprocal way: attitudes towards arithmetic may affect achievement in the subject, and achievement may influence children's attitudes. More recent work (Chen & Stevenson, 1995; Crystal & Stevenson, 1991; Stevenson & Lee, 1990; Young-Loveridge, 1991) also points to the direction of a strong correlation. In addition, children's numerical activities at home have been found to associate with children's arithmetic at school. Evans and Goodman (1995) have argued that the importance of children's arithmetic activities at home lies not only to its influence on children's knowledge of mathematics, that is, through practice and experience, but also to its impact on children's disposition to mathematical experiences.

The role of parents on children's learning and school performance is a central one. Studies have identified a number of parental variables associated with children's arithmetic performance (Chen & Stevenson, 1995; Crystal & Stevenson, 1991, Stevenson & Lee, 1990; Tizard et al., 1988). The majority of these studies are cross-cultural and have also included children who are doing very well in arithmetic. Parents' views, especially those of mothers, on children's ability and performance, children's attitudes and numerical experiences at home, the amount of help they provide with the homework and encouragement, their relations with the school, and their education have been researched. Mothers of children who are better in arithmetic (Chinese and Japanese) are better informed about their child's ability and performance, nurture a more helping environment, and have better contact with the school than mothers of children who do less well (American); the latter also seem to be less aware of children's difficulties and do not convey any significant educational messages to their children. Finally, mothers' academic status is a significant factor related to children's achievement in school, mainly through their involvement with the child's schooling (Stevenson & Baker, 1987).

Cognitive Components

Up to the present time, the most significant contributions to the comprehension of the cognitive bases for children's underachievement in arithmetic come from two distinct areas in cognitive psychology, namely, understanding of basic arithmetic concepts and computational skill and working memory storage capacity.

Evidence on what "math difficulty" children know and have the ability to do in arithmetic has added a lot to our understanding of arithmetic difficulties. Based on previous findings of studies conducted in Africa and in America (Ginsburg, 1982; Ginsburg, Posner, and Russell in Russell & Ginsburg, 1984), Russell & Ginsburg (1984) investigated the issue of "essential cognitive normality" in children with mathematics difficulties, by comparing fourth-grade math difficulty children with their normal and third-grade peers, on measures of formal and informal arithmetic knowledge and skill. In specific, Russell and Ginsburg examined whether math difficulty children "exhibit distinctive or deficient concepts and processes of mathematical thought" (p. 218), in other words, whether they differ fundamentally from their peers in mathematical performance, concepts, and skills. They hypothesised that math difficulty children would not be seriously deficient in knowledge and skill in informal arithmetic, however, they would lag behind their peers in knowledge of base ten system and skill in written arithmetic.

Russell and Ginsburg did not find any evidence for serious deficiencies in children with mathematical difficulties. Children experienced difficulties in calculation, especially when large numbers were involved, and in remembering number facts, however, they displayed only immature procedures. Russell and Ginsburg's work has shown that children with specific difficulties in arithmetic are significantly behind their peers in arithmetic skill. However, they do not suffer from serious deficiencies.

Children's working memory storage efficiency has been found to vary as a function of arithmetic achievement. Working memory refers to the temporary storage of information while other cognitive tasks are being performed (Baddeley, 1990). After introducing the digit span concurrent memory tasks, Baddeley and Hitch (1974) conceptualised a working memory model according to which working memory is a system with limited capacity, a "work space" in which two systems operate simultaneously; this work space is allocated to either storage or control processing demands.

Work on specific subtypes of learning difficulties has shown that working memory efficiency may vary with the type of the learning difficulty. Siegel and Ryan (1989) examined children's concurrent memory spans by crossing type of disability (arithmetically disabled and reading disabled) with nature of concurrent span task (Working Memory - Counting span task, adapted from Case, Kurland, & Goldberg, 1982, and Working Memory - Sentences span task, adapted from Daneman and Carpenter, 1980). Children with specific arithmetic difficulties were found to have shorter spans when they had to retain concurrently numerical information, whereas their spans were unaffected when they had to recall sentences. Children with reading difficulties, on the other hand, showed a more generalised deficit, with shorter spans than children with arithmetic learning difficulties both when the information to be processed involved words and numbers.

The work of Hitch and McAuley (1991) has shed more light on the issue of working memory capacity of children with specific arithmetical difficulties. They replicated Siegel and Ryan's (1989) findings, where arithmetic difficulty was found to be associated with limited spans only when numerical information was processed, further independent of the modality of the stimulus (visual or auditory). However, it was significantly slow counting procedures and low auditory digit spans that were thought to be the main cause of the impairment. Children's articulation rate resembled that of their normal peers. Hitch and McAuley explained the impairment in concurrent counting spans in terms of children's slower access of digits in long term memory.

1.3 The Present Study: Aim and Hypotheses

The main aim of the present study is to try and explain variation in children's arithmetic achievement, by examining the independent contribution of two sets of factors - social and environmental and cognitive. In recognition of the fact that a child's performance on a task at any particular point in time is a complex function of many interacting factors (Evans & Goodman, 1995), and that cognitive as well as social and environmental elements are significantly related to achievement, any attempt to understand the complete causal chain associated with arithmetic attainment must include the effects of both sets of variables on children's performance. While the two sets of factors are thought to be mutually exclusive, they may coexist within children of particular levels of arithmetic ability.

The investigation will begin by measuring correlations between individual sets of factors and children's performance, based on group analyses. It will proceed to explore the individual contribution of those social and environmental and cognitive factors found to associate with performance, to children's achievement, using a series of multiple regression analyses. Last, an attempt will be made to combine the significant correlates and explore the total contribution of social and environmental and cognitive factors to variation in children's arithmetic achievement.

Individual steps and related hypotheses are:

Social and Environmental

The relationship between children's arithmetic achievement and some social and environmental factors is investigated, the main emphasis being on beliefs about achievement, attitudes and home practices, parental help and encouragement, and parent-school relations and parental academic status.

The purpose is to examine how children's and parents' reports vary with children's performance. Children who belong to three different arithmetic levels, that is, above average, average, and below average, but who nevertheless possess at least average reading abilities, are interviewed and compared on their self-concepts, attitudes and home practices, and reports of parental help. Children's parents describe their own beliefs about the child's achievement in arithmetic, the child's attitudes and home practices, the amount of help and encouragement they provide the child with, their relationship with the school, and their academic status.

Significant differences are expected to be observed between children of different arithmetic ability, as well as between their parents, in most of the measures under investigation. Self-concepts and attitudes to arithmetic, as well as children's reports of parental help with the homework are expected to discriminate between children (Aiken, 1970, 1972, 1976; Blatchford, 1992, 1997a, 1997b; Marsh, 1990; Miserandino, 1996; Schunk, 1990; Tizard et al., 1988; Young-Loveridge, 1991). Parents are expected to differ in their reports of help they provide the child with, in their involvement with the child's schooling, and in their academic status (Chen & Stevenson, 1995; Crystal & Stevenson, 1991; Stevenson & Baker, 1987; Stevenson & Lee, 1990; Tizard et al., 1988).

Last, regression analyses investigate the independent contribution of social and environmental variables to children's variation in arithmetic performance.

Cognitive

The relationship between children's cognitive abilities and their arithmetic performance is explored next. This investigation focuses to two major cognitive components of arithmetic performance, namely, arithmetic knowledge and computational skill and working memory storage capacity.

Children's formal and informal arithmetic knowledge and skill are examined as a function of children's mathematical group. This examination involves comparing the performance of three groups which differ in mathematical performance, that is, children with excellent arithmetic abilities, children who are average in mathematics, and children with arithmetic difficulties, but who show at least average reading ability on a number of measures. The tasks to be employed cover five major mathematical areas: knowledge of informal mathematical concepts and informal calculational skills, understanding of base ten concepts and related enumeration skills, error strategies in written addition and subtraction and other calculational procedures, knowledge of number facts, and problem-solving skills. The tasks have been adapted from Russell and Ginsburg (1984).

It is hypothesised that skill in mental and written addition, accuracy in problem solving, ability to deal with large numbers, and knowledge of addition facts will discriminate between children with arithmetic difficulties and their normal peers. Overall, children are not expected to vary in their understanding of informal arithmetic concepts and base ten concepts, nor in their use of principles to solve written problems. Children who are particularly good in arithmetic are expected to be accurate in all tasks.

Following the group comparisons, regression analyses examine the independent contribution of knowledge in formal and informal arithmetic to children's variation in arithmetic achievement.

Children's working memory efficiency is investigated by comparing children with arithmetic difficulties to children with above average arithmetical ability on concurrent memory tasks, as well as measures of digit span, recitation of number sequences, speech articulation, and speed of counting. In addition, a group of children with average maths but below average reading ability are compared to the math-competent children, since both groups had better arithmetic skill than their below average peers, and differed only in their reading performance. All tasks are adapted from Hitch and McAuley (1991), with the exception of a word span task which is constructed by the author and employed for the purpose of examining whether arithmetic ability is associated with recall of specific numerical information.

It is hypothesised that children with arithmetic difficulties will be impaired only on span tasks involving counting. Also, math-competent children and those with mild reading difficulties but average math skill are expected to resemble each other in counting spans. Finally, children with arithmetic difficulties and those of above average arithmetic skill are not expected to differ in their word spans.

After group differences are examined, regression analyses investigate the independent contribution of working memory processes to variation in children's attainment in arithmetic.

Social and Environmental and Cognitive Combined

The last step in the present study is to explore the degree to which variance in children's arithmetic achievement is explained by the total of factors found to associate significantly with performance. A combined multiple regression analysis is employed for that purpose.

In the absence of previous findings upon which to support and construct parallel hypotheses, the last analysis constitutes an exploratory inquiry. To the author's knowledge, there have been no studies combining social and environmental correlates of children's arithmetic performance with cognitive counterparts. The current study is the first attempt to explain variation in children's arithmetic ability through a combination of factors.

Chapter 1 has presented the general theoretical framework of the present research. The aim and individual hypotheses have also been pronounced.

Chapter 2 describes the process of selecting the sample. Issues discussed include the measures used, the selection procedure, and the definition of the groups involved in each investigation. A summary of the research design concludes the chapter.

Chapter 3 explores the relationship between social and environmental factors and children's arithmetic achievement. First, the studies which have associated such factors to school and arithmetic performance are described, and the corresponding hypotheses are stated. A section on methodology describes the materials that were used for the collection of the data as well as the actual process of collecting the information. The findings of the study are reported in detail, first describing the group comparisons and then the results of the regression analyses. A discussion of the issues raised by the results completes the investigation.

Chapters 4 and 5 investigate the relationship between children's arithmetic achievement and knowledge of formal and informal arithmetic and working memory storage efficiency, respectively. Within each chapter, the major studies that the present research has been based on are described, followed by the individual hypotheses. A section on the methods employed follows, including a brief summary of the sample and a detailed description of the tasks and the procedure. An analysis of the findings on group comparisons and the regression analyses of each investigation precedes a short discussion which relates the findings of the present study to the existing literature.

Chapter 6 summarises the findings of the individual chapters and further examines the independent variation accounted by all factors found to associate with performance. For that purpose, a combined version of multiple regression analyses is employed, including both social and environmental and cognitive variables. A general discussion explains children's variation in arithmetic ability, describes the major limitations of the current design, and offers suggestions.

CHAPTER 2

SAMPLE SELECTION

2.1 Design

The main purpose of the thesis is to explain children's variation in arithmetic ability through the contribution of social and environmental and cognitive factors associated with arithmetic performance.

Individual aims and corresponding methods of investigation include:

i. to explore whether and how children's and their parents' views on school achievement, attitudes and home practices, parental help and encouragement, parent-school relations, and parental academic status vary with children's arithmetic ability, further identifying those variables that predict children's arithmetic performance,

ii. to examine whether and how children's performance on different measures of formal and informal arithmetical knowledge and skill varies with their arithmetic performance, and identify the specific components that contribute to children's variation in performance,

iii. to investigate whether and how working memory storage efficiency relates to children's arithmetic performance, further identifying the components that predict significantly children's performance, and

iv. to assess the variation accounted by the factors combined. Those factors include the social and environmental as well as cognitive variables found to vary and associate with children's arithmetic performance.

The first aim involves comparing the reports of children - and the corresponding reports of their parents - belonging to three levels of arithmetic achievement, namely, above average, average, and below average. The focus of the investigation will be on personal beliefs about performance, corresponding attributions, and aspirations, attitudes and home practices, and parental help. Children's parents will provide additional information on their beliefs of the easiness of arithmetic for their child, their own numeracy difficulties, their beliefs about the child's performance as opposed to ability, their way of encouraging their child to do well in arithmetic, their contact with the school and their academic background. Regression analyses will further identify those variables that might explain variation in children's arithmetic performance.

The second aim entails comparing the performance of those children who differ in arithmetic ability on measures of knowledge and skill in informal arithmetic, understanding of base ten concepts and related enumeration skills, error strategies in written addition and subtraction and other calculational procedures, knowledge of number facts, and skill in problem solving. Regression analyses will explore the independent contribution of these components to children's variation in arithmetic achievement.

The third aim involves comparing the performance of children with arithmetic difficulties to that of children with above average arithmetic ability on concurrent memory tasks as well as measures of digit span, word span, speed of reciting number sequences, and speech articulation rate. A group of children with mild reading difficulties but with at least average mathematical strengths will also be compared to children with above average maths ability to obtain further evidence on the nature of the relation between children's arithmetic difficulties and (specific) working memory deficits. Regression analyses will examine the independent contribution of working memory spans and more basic component skills to children's variation in arithmetic.

Finally, to examine the total contribution of both the social and environmental and the cognitive factors to children's variation in arithmetic, those variables found to associate with children's arithmetic achievement will be the entries in regression analyses, and the amount of variation in children's achievement accounted by the total of these factors will be assessed.

For the purpose of the present study, four groups of children were identified: three differed in arithmetic performance (above average, average, and below average) and one in reading (mild reading difficulties with average mathematical ability). In what follows, the process of selecting the sample is described, along with a detailed account of the measures and methods used. Children's age and ability in arithmetic and reading are described as a function of ability group and study.

2.2 Sample Selection

The current research was conducted in primary schools in the area of Athens. The schools' mathematics programs were similar. The sample consisted of children in Grade 3, an age range which matched that of previous studies.

2.2.1 Measures Used

Arithmetic

Two mathematical tests were used to identify children with poor, average, and excellent arithmetic abilities. Since there were no standardised mathematical achievement tests in Greek, British standardised equivalents were employed. These were the "Y" Mathematics Series Y2 Forms A and B by Young (1979) and the Basic Mathematics Test B by the National Foundation for Educational Research in England and Wales (NFER, 1971). These tests were matched for both age range and their assessment purpose. Also, a comparison of the material covered in third-grade arithmetic textbooks with the items in the above tests showed similar arithmetic levels. Finally, to ensure these tests were equivalent to Grade 3 mathematics curriculum in Greece, the two tests were piloted with 50 fourth-grade children in Greek schools. This process led to eliminating the items that the majority of the older children could not solve.

The "Y" Mathematics Series Y2 test is a standardised measure designed to assess children's understanding of number. More specifically, it measures children's ability to handle basic number operations as well as their knowledge of fractions, ratio, and proportion. It also examines children's ability to measure using a ruler, their knowledge of time (using clocks), and their ability to use graphs. Overall, the test consisted of an untimed oral section and a timed section. The untimed oral section originally involved 20 items; after piloting the test, three items which involved reading a table, dividing numbers, and measuring in centimetres (using a ruler) were eliminated. The timed section (25 mins.) involved solving multiple-digit operations in numeric form, as well as word problems. The piloting process showed that all operation items could be included (20), however, the number of word problems should be limited to 8 (compared to 15 in the original version). The seven word problems that had to be eliminated involved dealing with division, fractions, calendar, and time. Maximum score for the test was 45. There are two parallel Forms (A and B) of

Young's Y2 test, both of which were used in order to ensure children seated next to each other would not copy. A copy of each form can be found in Appendices 2.1 and 2.2 respectively. The items that were excluded are marked with a star.

The Basic Mathematics Test B is also a standardised measure of children's understanding of the fundamental relationships and processes that form the basis of all mathematical work. The test includes a wide range of operations that apply to the topics of shape, relations, interpretation of a pictogram, volume, size, length, area, fractions, permutation, approximation, place value, weight, and time. There were no time constraints for completion of this test. The original version of the test consisted of 40 items presented orally which covered the following mathematical operations: equating and ordering, adding and subtracting, counting, dividing, multiplying, and classifying. After piloting the test, three items were eliminated: these involved equation, shape, and division. Maximum score was 37. A copy of the test can be found in Appendix 2.3. As with the previous test, the items that were excluded are marked with a star.

Reading

Children's reading ability was examined using two tests that were constructed by the author for the purpose of the present research: a reading comprehension test and a sequence test. The reading comprehension test involved the standard procedure of reading a text and answering ten questions based on it. Having first handed out the text written on A4 sheets, the experimenter read the text to the pupils. After the text was read, the children were instructed to answer the questions without making any mistakes and were encouraged to have a look at the text if needed. Some questions were inferential and some were based on direct retrieval of information from the text (see Appendix 2.4).

The sequence test involved putting nine sentences of a text into order. Each sentence was printed on a different index card (0.12 x 0.20 cm), without punctuation or capital letters (see Appendix 2.5). Each child had his own set of cards to put in order.

Both texts were taken from a children's anthology reading book. Bearing in mind that reading ability does not imply a single underlying process but refers to a combination of activities, these tests were used as a means by which to screen the children only and not to investigate their reading abilities in depth.

Age

The children's date of birth was recorded, to ensure groups would not differ in mean age.

In sum, five measures were gathered for each child.

2.2.2 Selection Procedure

The selection process began with the administration of the four test measures to third-grade pupils in ten different day-schools located in five eastern suburbs of Athens, Greece. Letters of consent were sent to parents and only those children whose parents agreed were included in the testing. The children were tested in their classrooms in two main sessions. Each session consisted of a reading and a math test, the order of which was always counterbalanced. The second session followed the first one usually with a day difference, however the class schedule did not always permit this agenda. The maximum "break" was a calendar week.

The teacher was always present at the testing, refraining, however, from any major involvement. She would prepare the list of birth dates of the participating pupils, assign some work to those children not participating, and keep children quiet when necessary.

The sample that was initially examined consisted of 327 pupils. However, a large number of children were automatically excluded because of the following limitations: incomplete data sets (due to absences or dropouts, date of birth not registered, etc.), two teachers withdrawing from the study (i.e., massive elimination of children), cases of dyslexia or severe learning difficulties, and foreign nationality.

The four groups were identified based on two major screening levels, after frequencies and means for each measure were analysed. First, children's data were sorted out based on their reading scores: this process discriminated between children who were below average in reading (reading difficulty with at least average arithmetic abilities) and those who were above average in reading (prospective candidates for the mathematical groups). Following this distinction, children with above average reading abilities were discriminated on the basis of their mathematical performance. This procedure involved sorting out children's data on the two arithmetic tests, splitting the scores into three levels.

In screening children, a series of *t* tests, Analyses of Variance, and Student-Newman-Keuls were employed. The procedure involved comparing group means on all five measures, until groups were clearly defined: the tests were repeatedly conducted while eliminating children until equality (e.g., age) and significant differences (e.g., reading or math scores) among groups were achieved.

2.2.3 Groups Defined According to Selection Criteria

Four groups of children were identified. Three groups of children varied significantly in arithmetic performance, ranging from below average, to average, and above average. All children had the same reading ability. A fourth group differed from the rest in that they had significantly lower reading ability, however, they possessed at least average arithmetic skills.

The sample for the screening tests consisted of 293 children. Appendix 2.6 gives a detailed account of children's scores in arithmetic and reading and how these varied as a function of gender in the initial sample (n = 293) and final groups (n = 91). It was observed that boys scored higher than girls on both maths tests. Girls scored higher on the reading comprehension test, however, there were no gender differences in performance on the sequence task. Table 2.1 shows the distribution of boys and girls as a function of final group.

TABLE 2.1

Frequencies of Children as a Function of Group and Gender

	Above Average	**Average**	**Below Average**	**Reading Difficulty**	**Total**
Gender					
Male	23	5	3	14	**45**
Female	13	15	14	4	**46**
Total	**36**	**20**	**17**	**18**	**91**

As can be seen on Table 2.1, a similar pattern of gender differences was observed when individual groups were selected: boys were overrepresented in the above average math group and the reading difficulty group, while girls were more common in the average and below average math groups. Gender differences in arithmetic achievement in the primary years are not uncommon (Geary, 1994), with research suggesting that they tend to further increase as students go higher in the secondary and GCSE levels (Shuard, 1983; Walden & Walkerdine, 1985; Walkerdine, 1998). The present study, however, does not attempt to examine any hypotheses on gender differences in children's variation in arithmetic achievement.

Table 2.2 shows the distribution of children as a function of group and individual study.

TABLE 2.2

Distribution of Children as a Function of Group and Study

	Above Average	Average	Below Average	Reading Difficulty	Total
Study					
R&G, H&M, Both parents	14	-	9	-	**23**
R&G, H&M, One parent	-	-	2	-	**2**
R&G, H&M	1	-	2	-	**3**
R&G, Both parents	11	12	2	-	**25**
R&G, One parent	3	5	1	-	**9**
R&G	1	3	-	-	**4**
H&M, Both parents	6	-	-	-	**6**
H&M	-	-	1	18	**19**
Total	**36**	**20**	**17**	**18**	**91**

Note. *R&G* = Number of children participating in the replication of Russell and Ginsburg's (1984) study. *H&M* = Number of children participating in the replication of Hitch and McAuley's (1991) study.

In total, 91 children (45 boys and 46 girls) participated in the present study: 36 were identified as above average in maths, 20 were average in maths, 17 were below average in maths, and 18 children were in the reading difficulty group.

Social and Environmental

The investigation of social and environmental factors relating to arithmetic achievement included all children belonging to the three mathematical groups (n = 73): 36 were above average, 20 were average, and 17 were below average. Children's parents also participated. Arithmetic performance varied from below average, to average, and above average. All children had at least average reading ability. These children further participated in either one or both studies on cognitive factors associated with arithmetic achievement. Table 2.3 shows the total number of children in the mathematical groups and the corresponding number of fathers and mothers who returned the questionnaires.

TABLE 2.3

Frequencies of Participating Children and Parents as a Function of Mathematical Group

	Children	**Fathers**	**Mothers**
Group			
Above Average	36	31	35
Average	20	13	16
Below Average	17	11	14

While the majority of parents had agreed to participate in the study, some did not return the questionnaire. Furthermore, some items had not been answered. To account for such changes in the number of responses, reference to sample size will be made constantly in the analysis of parental variables.

Table 2.4 shows children's scores on the tests that were used for sample selection. A series of t tests examined mean differences and similarities.

TABLE 2.4

Means (Standard Deviations) and Statistical Comparisons Among the Three Mathematical Groups on the Pre-Test Measures ($n = 73$)

	Above Average ($n = 36$)	Statistical Comparison (AA - A)	Average ($n = 20$)	Statistical Comparison (A - BA)	Below Average ($n = 17$)	Statistical Comparison (BA - AA)
Age (mos.)	99	ns	97	ns	97	$t = 2.22^{*}$
	(2.8)		(3.8)		(3.7)	
Read Comp	6.44	ns	6.50	ns	6.12	ns
	(0.9)		(1.1)		(1.1)	
Sequence	32.42	ns	32.80	ns	31.94	ns
	(3.5)		(3.2)		(1.1)	
Young	26.86	$t = 12.55^{**}$	20.55	$t = 7.72^{**}$	15.00	$t = 15.04^{**}$
	(2.6)		(1.1)		(2.8)	
NFER	22.28	$t = 10.43^{**}$	12.05	$t = 9.17^{**}$	6.53	$t = 14.36^{**}$
	(5.7)		(1.1)		(2.3)	

$^{*}p < .05$. $^{**}p < .001$.

It can be observed that the three mathematical groups differed significantly in their math scores, but not in their reading scores. An age difference was observed between above average and below average children, however, it was not of great magnitude (two months).

Cognitive

Formal and Informal Arithmetic Knowledge and Skill

For the replication of Russell and Ginsburg's (1984) study on children's formal and informal arithmetic knowledge and skill, children from the three mathematical groups were selected. Table 2.5 shows the score limits that discriminated children on the four pre-test measures.

TABLE 2.5

Score Limits of Children's Performance on the Four Measures Used to Identify the Three Mathematical Groups for Russell and Ginsburg's Tasks

	Test[a]			
	Read Comp	**Sequence**	**Young**	**NFER**
Group				
Above Average	5-8	26-36	23-31	15-36
Average	5-8	26-36	19-22	10-14
Below Average	5-7	27-36	9-18	1-9

[a]Maximum score on each test in the order presented above: 10, 36, 45, 37.

Table 2.6 shows the mean levels of performance of each group on the five pre-test measures, as well as the results of the statistical comparisons among them. Again, a series of t tests examined differences and similarities in group means.

TABLE 2.6

Means (Standard Deviations) and Statistical Comparisons Among the Three Mathematical Groups on the Pre-Test Measures (n = 66)

	Above Average (n = 30)	**Statistical Comparison (AA - A)**	**Average (n = 20)**	**Statistical Comparison (A - BA)**	**Below Average (n = 16)**	**Statistical Comparison (BA - AA)**
Age (mos.)	98	ns	97	ns	97	ns
	(2.6)		(3.8)		(3.8)	
Read Comp	6.40	ns	6.50	ns	5.90	ns
	(1.0)		(1.1)		(0.9)	
Sequence	32.03	ns	32.80	ns	32.00	ns
	(3.5)		(3.2)		(3.7)	
Young	26.67	$t = 11.34$*	20.55	$t = 7.83$*	14.81	$t = 14.31$*
	(2.6)		(1.1)		(2.8)	
NFER	22.07	$t = 9.12$*	12.05	$t = 8.75$*	6.50	$t = 12.77$*
	(5.9)		(1.1)		(2.3)	

*$p < .001$.

It can be observed that the three groups of children differed significantly in their scores on both arithmetic tests; they did not, however, differ in their scores on either reading test, nor did they differ in their age means.

In sum, 66 children participated in the replication of Russell and Ginsburg (1984): 30 were above average, 20 were average, and 16 were below average in mathematics. These children did not differ in terms of age, nor did they differ in reading ability. The only significant differences among the three groups were in their mathematical performance.

Working Memory Processes

For the purpose of examining children's working memory efficiency and its relation to arithmetic achievement, children were selected from the below average and above average mathematical groups. To further test for the specificity hypothesis, that is, whether children's arithmetic difficulties are associated with specific deficits in working memory processes, an additional group of children with mild reading difficulties were compared with children belonging to the above average mathematical group. Table 2.7 shows children's score limits on the pre-test measures.

TABLE 2.7

Score Limits of Children's Performance on the Four Measures Used to Identify the Three Groups for Hitch and McAuley's Tasks

	Test[a]			
	Read Comp	**Sequence**	**Young**	**NFER**
Group				
Above Average	5-8	26-36	23-31	15-36
Below Average	5-9	27-36	9-18	1-9
Reading Difficulty	0-5	9-36	18-30	10-27

[a]Maximum score on each test in the order presented above: 10, 36, 45, 37.

Table 2.8 shows the mean age and levels of performance of each group on the pre-test measures, along with similarities and significant differences among the three testing groups. The findings are based on a series of t tests.

TABLE 2.8

Means (Standard Deviations) and Statistical Comparisons Among the Three Groups on the Pre-Test Measures (n = 53)

	Below Average (n = 14)	Statistical Comparison (BA - AA)	Above Average (n = 21)	Statistical Comparison (AA - RD)	Reading Difficulty (n = 18)	Statistical Comparison (BA - RD)
Age (mos.)	97	ns	98	ns	98	ns
	(3.8)		(3.4)		(5.3)	
Read Comp	6.07	ns	6.43	t = 8.38*	3.89	t = 5.69*
	(1.2)		(0.9)		(1.0)	
Sequence	31.64	ns	33.05	t = 4.65*	22.83	t = 3.80*
	(3.6)		(2.9)		(8.9)	
Young	15.00	t = 13.52*	27.33	t = 4.97*	22.56	t = - 6.74*
	(2.7)		(2.6)		(3.4)	
NFER	6.36	t = 11.11*	23.33	t = 4.14*	15.83	t = - 7.58*
	(2.3)		(6.4)		(4.6)	

*$p < .001$.

It can be observed that children did not differ in terms of age. Children in the above average group and those belonging to the below average group differed significantly only in their math scores. Children belonging to the above average group and those in the reading difficulty group differed significantly in their reading ability. While they also differed in mean mathematical performance, both scored significantly higher than children in the below average group.

The sample that participated in the study of working memory processes consisted of 53 children: 14 were below average in maths, 21 were above average in maths, and 18 were identified as having mild reading difficulties.

Reliability

Since children's math scores were considered the most critical measure in the selection process, reliability data on these measures were collected. Children (n = 33) who had answered Form A of the "Y" Mathematics Series (Young, 1979) were also examined on Form B. There were 11 children from the above average group, 7 children from the average group, 8 children from the below average group, and 7 children from the reading difficulty group. The selection of the subjects was random.

It was found that children's scores on Form A and Form B of the "Y" Mathematics Series were highly correlated ($r = .94$, $p < .005$).

2.3 Summary of Procedures in the Main Study

The entire study extended over the academic year 1994 - 1995 (September - June). Children were interviewed in a quiet room at school, during ordinary school hours (8:30 a.m. to 1:30 p.m.). During testing, the tasks on formal and informal arithmetical knowledge were always given first, with memory tasks presented later. The social and environmental factors were examined at the concluding part of each testing session.

Each child was seen twice and each session usually lasted an hour. The reading group received only the working memory tasks (one session).

2.4 Introduction to the Next Chapter

The association between children's performance in arithmetic and the two major sets of factors is examined next. Social and environmental factors, arithmetic knowledge and skill, and working memory processes are described separately in the following three chapters. The investigation begins with the examination of social and environmental factors and their relation to children's arithmetic ability.

CHAPTER 3

SOCIAL AND ENVIRONMENTAL FACTORS RELATED TO CHILDREN'S ARITHMETIC ACHIEVEMENT

3.1.1 Introduction

Achievement has been known to be subject to a variety of socio-psychological influences. The present chapter examines the relationship between children's arithmetic achievement and a combination of process and status social and environmental variables. The purpose is to examine whether and how these measures varied with children's achievement in arithmetic, further identifying those variables which explained variation in children's arithmetic performance.

The theoretical background of the study is presented first, followed by the methods employed to collect the data from the children and their parents. Children's and parents' responses are then analysed as a function of children's arithmetic achievement. The chapter concludes with a short discussion on the factors found to be significantly associated with children's performance.

3.1.2 Review of Studies Relating Social and Environmental Factors to Arithmetic Achievement

Research has identified an increasing number of social and environmental factors that are linked to children's academic achievement, with some of these factors also relating to children's performance in arithmetic. To select the factors to be examined in the present study, it was necessary not only to consider the strength of the associations observed elsewhere, but also to account for associations which might exist with relatively under-researched topics.

Factors residing within the child have been researched early on. For example, there is much evidence on the relation between children's academic and math self-concepts and their achievement in school and arithmetic (Blatchford, 1997a, 1997b; Marsh, 1990; Schunk, 1990; Shavelson & Bolus, 1982; Young-Loveridge, 1991). Also, the relation between children's attitudes to school and school performance in the subject has interested researchers as early as 1960s (Aiken & Dreger, 1961) and 1970s (Aiken, 1970, 1972, 1976). Recent attempts have further tried to build up a scientific model, focusing on how children's attitudes to specific school subjects affect their attainment in those subjects (Schofield, 1982; Stevenson & Lee, 1990; Tizard et al., 1988; Young-Loveridge, 1991). Finally, children's academic activities at home have been related to their school achievement (Chen & Stevenson, 1995; Stevenson & Lee, 1990), with the role of numeric activities being further investigated in relation to children's performance in arithmetic (Young-Loveridge, 1991).

The role of home environment on children's school learning and school performance has also been the focus of a considerable body of research. Different research traditions exist, each of which is characterised by the criteria used to examine possible links and by the strength of the association between the variables being studied and children's arithmetic performance. One research tradition which has prevailed in the literature on home environments and school learning emphasise the effect of process variables on children's academic achievement (Iverson & Walberg, 1982). As opposed to status variables which label or characterise families and which are relatively unchangeable (e.g., socio-economic measures, amount of education, parental experiences and aspirations, etc.), process variables refer to specific, direct, and changeable measures of the environment, including what people actually do and what they think, feel, and value. Studies which emphasise process variables report higher correlations with children's achievement than studies focusing on status variables. In a quantitative synthesis of 18 studies on the influence of home environment on school learning, Iverson and Walberg used Marjoribanks' distinction between the Chicago and British schools of research on home environments and compared the findings of studies focusing on process and status variables, respectively. Iverson and Walberg concluded that measures of socio-psychological environment correlate better with children's achievement than status variables. In other words, specific social-psychological or behavioural processes thought conducive to learning and stimulating growth, such as parental behaviours (e.g., reading to the child, etc.) and direct parent-child interactive behaviours, correlate better with achievement and ability than material conditions at home or parent occupation.

Further more evidence on the effect of home environment comes from a comprehensive study conducted by Reynolds and Walberg (1992). They constructed a model of mathematics achievement in Grades 7 and 8 according to which performance is affected indirectly by the children's home environment, the child's motivation, prior achievement (Grade 7), and peer environment. Home environment was measured by parents' expectations, parents' education, and number of resources. Reynolds and Walberg found that 78% of the effect of home environment was transmitted through prior achievement which would be expected since the influence of family on children's schooling is strong and has been exerted over a number of years. In addition, children's mathematical performance was found to be directly affected by children's out of school reading and instructional time (measured by textbook coverage and new material covered).

Influential sources can be far beyond our knowledge. Within the limited number of determinants examined so far, research in education and social psychology has provided strong evidence on the effect of factors within the child and process variables focusing on parent-child interaction, on children's school attainment. Some of those factors are also associated with children's achievement in arithmetic in specific. Emphasis has thus been placed on the relation between children's arithmetic achievement and their self-assessments in maths, their attitudes to arithmetic and numerical home practices, as well as their parents' support with the homework, and contact with the school. Parents' academic status, although a status variable, has been also found to strongly relate to children's academic and math attainment. These relations are examined in detail next.

3.1.2.1 Evaluation of Performance, Attributions, Aspirations, and the Relation Between Performance and Ability

Evaluation of Performance

Self-Concept

Much of the importance of studying children's self-concepts is borne out of evidence suggesting a strong positive relationship with school achievement. A considerable amount of research in education and psychology has stressed the importance of motivational factors in the learning process and the role of self-image held by students as potential determinant of performance. In many cases, the link between self-appraisals and later achievement related behaviours and school performance may hold even when earlier experiences are contradictory (Assor & Connell, 1992).

Self-concept, broadly defined by Shavelson and Bolus (1982), refers to one's collective self-perceptions of him- or herself "that are formed through experiences with and interpretations of one's environment and that are influenced especially by reinforcements, evaluations by significant others, and one's attributions for one's own behaviour" (p. 3).

A number of attributes are attached to self-concept (for a review, see Schunk, 1990; Shavelson & Bolus, 1982). Self-concept is considered *multi-dimensional*. It may refer to self-esteem (whether one accepts and respects oneself or one's sense of self-worth) or to self-confidence (the extent to which one believes he can perform competently). The two facets can be related, in that self-confidence increases if one holds high levels of self-esteem, tries a difficult task, and subsequently succeeds. In the same way, self-esteem increases if one believes he can perform competently, and succeeds in a difficult task.

Second, self-concept is considered to be *hierarchically organised*. A general self-concept is located at the top of the hierarchy and specific sub-area self-concepts are located towards its base. A general self-concept, for example, is formed by self-perceptions in the academic areas combined with those in non-academic domains (e.g., social, emotional, and physical). Accordingly, the academic concept is formed by combining one's sub-area self-concepts (e.g., arithmetic and reading). These sub-area self-concepts are in turn influenced by one's self-perceptions of specific behaviours.

Self-concept *stability* refers to the flexibility of the self-concept, that is how easy or difficult it is to change. General self-concept is stable; however, as one goes down in the hierarchy, self-concept becomes situation dependent and thus less stable. Stability depends in part on how crystallised or structured are one's beliefs, with beliefs becoming crystallised with repeated similar experiences. It is believed that by adolescence people have relatively well-structured perceptions of themselves with respect to such characteristics as general intelligence, sociability, and honesty. Brief experiences providing evidence that conflicts with individuals' beliefs do not have much impact. In contrast, self-concept is readily modified in areas where people have ill-formed notions about themselves, usually because they have little if any experience.

Variability, as Bandura (in Schunk, 1990) also contends, is an attribute of self-concept which does not allow for general predictions as to how a person might act on specific situations. A person's perceptions of his competencies (self-confidence) and his self-worth (self-esteem), that is, the self-concept, may vary for performances in different domains (e.g., a person might judge herself highly capable in intellectual endeavours and feel a high sense of self-worth, moderately competent in social activities but feel inadequate, and low in competence in sports but not feel inadequate) or for different activities within the same domain (e.g., within the intellectual domain, a student may evaluate her performance as low in athletics, moderate in English, and high in mathematics). Finally, self-confidence and self-worth may be completely unrelated in that a person may feel highly capable in areas from which he derives no pride or may maintain high levels of self-worth despite acknowledging his poor performance.

Differentiation is another attribute of self-concept. Self-concept becomes increasingly multifaceted as the individual develops from infancy to adulthood. It is said to proceed from a concrete view of oneself to a more abstract one. Young children hold diffused and loosely organised views of themselves, defining themselves concretely (e.g., in terms of appearance or name). With development, and especially schooling, they come to view themselves in a more abstract way and acknowledge that behaviours do not always match capabilities, since they develop separate conceptions of underlying traits and abilities. At that point, self-concept is better organised and more complex.

The *working* self-concept is another conceptualisation which allows for a relatively stable core general self-concept, surrounded by domain-specific self-concepts. Working self-concept refers to those self-schemas that are mentally active at the moment - one's presently accessible self-knowledge. It clearly suggests that not all representations are equally active at the same time. Domain-specific self-concepts are thus activated by task circumstances and are considered to be more easily altered, than is the general self-concept.

Finally, self-concept researchers view self-concept as being in *dynamic* interplay with the environment - reacting to it while simultaneously influencing it. Self-concept is not passively formed through environmental interactions, rather it is believed a dynamic structure that mediates significant intrapersonal and interpersonal processes. The self-schemas which comprise self-concept are formed through experiences, they process personal and social information, and they vary in elaboration (some contain more information than others), reference point (some may refer to the present self while others to the future self), or direction (some may be positive while others may be negative).

Research Evidence

There is enough evidence which suggests a positive relation between children's academic self-concept and school achievement. Studies have shown, however, that *young* children are not always accurate in their self-assessments. Accuracy is found to improve with children's age (Blatchford, 1997b; Tizard et al., 1988), with more specific domains (Blatchford, 1997b; Marsh, 1990; Schunk, 1990; Shavelson & Bolus, 1982; Wylie, 1979), and with more general reference group (Blatchford, 1997b; Marsh, 1990).

Among the first to examine this relation, Bernstein (1964) argued that if feelings of success or failure in mathematics are experienced for some time, they will lead to a particular self-image held by the pupil (e.g., "I'm not much good") which will in turn affect both their expectations of future confidence and their actual performance. Recent attempts have also pointed out that primary children's perceptions of their own abilities are likely to play a very important part in their progress at school (Blumenfeld, Pintrich, Meece, & Wessels, 1982).

Tizard, Blatchford, Burke, Farquhar, and Plewis (1988) found that 7-year-old children were generally inaccurate in their self-assessments. Children's beliefs about their mathematical and reading ability did not correspond to their test scores. Overall, children overestimated their achievement in each subject; children who rated themselves as *above average* in maths were no more likely to have above average scores than those who rated themselves as *average* and vice versa. Only a few children who rated themselves as *below average* did indeed tend to score below average on the tests. Gender differences were further observed, where girls despite doing slightly better than boys on the maths tests tended to underestimate their achievement: 83% of boys rated themselves as above average in maths, while only 63% of girls thought they were above average.

In their longitudinal study of American, Chinese, and Japanese mothers and almost 1,500 children in Grades 1 and 5, Stevenson and Lee (1990) investigated the relative disadvantage of American children compared to their Asian peers in academic achievement, especially in maths and reading. First, Stevenson and Lee found fifth-grade children were inaccurate in their self-ratings for both arithmetic and reading. Correlations between children's self-ratings of how good they believed they were in maths and their actual performance on achievement tests were similar for children of all three cultures (.41 for American, .46 for Chinese, and .49 for Japanese children), despite significant mean differences in performance on the achievement test. American children were more likely to overestimate their performance both in maths and reading, also reporting maths was easy. Self-ratings in reading were even less accurate: correlations for American children were .36, for Chinese .36, and for Japanese .05. Japanese children in specific found it more difficult to rate their reading ability, perhaps due to the fact that since four different writing systems are operating in Japan one may learn to read one but not the other and thus it is quite difficult to rate reading ability in general.

Accordingly, Miserandino (1996) found that third- and fourth-grade children may vary in their self-assessments despite the similar levels of achievement. By examining children of above average mathematical competence, Miserandino (1996) found that some children were certain of their ability and experienced the encouragement yielded by such beliefs, while others did not evaluate their achievement as being so high and suffered from lack of motivation. Also, of the children with above average mathematical competence, those who perceived their ability with uncertainty reported feeling anxious, angry, and bored in school and reported avoiding, ignoring, and faking schoolwork. By contrast, those who felt certain of their ability reported feeling more curious and participated in, enjoyed, and persisted more at school tasks.

At about 9 years of age, children start to be increasingly more accurate in their self-evaluations. Young-Loveridge (1991) examined a group of 9-year-old children who scored high and low in maths. She found that children doing well in maths would be more likely to believe they are good in maths, while children in the low scoring group would be equally likely to say they are good or bad. Gender differences were again an astonishing finding: more boys than girls would think that they are good at maths and that was true for both low and high scoring groups. In the low scoring group, 1 boy out of 15 but 7 girls out of 19 thought they were not good in maths, while 6 boys and 3 girls thought they were good. In the high scoring group, no boy and only 2 girls thought they were not good; the rest believed they were good at maths.

Further supporting the hierarchical organisation of self-concept, as suggested by Shavelson and Bolus (1982), research has shown that the relationship between achievement and self-assessment is stronger when domain-specific self-concepts are examined. Children become more accurate as they get older and evaluate performance on specific subjects. Blatchford (1997b), for example, observed a school subject effect on self-assessment of children from 7 to 16 years. More specifically, he found that children's self-assessments and attainment in both mathematics and English were not related in the age of 7 years, but a growing accuracy in children's assessment was observed at 11 and 16 years of age. While overall levels of self-assessment fell from 7 to 16 years, different trends were found for mathematics and English: self-ratings in English were higher than those in mathematics, and children's ratings of their arithmetic performance suffered the highest drop from 7 to 11 but they remained static from 11 to 16 years. In other words, pupils at 11 and 16 years had higher self-ratings in English than mathematics, while at 7 years, self-ratings were higher in mathematics.

Accordingly, by reviewing seventy-eight studies examining the relationship of self-concept to academic ability and achievement from age 6, Wylie (1979) found that the average correlation between achievement measures (grade point averages) and overall measures of self-concept was .30. Higher correlations (.50) were found between achievement (grade point averages) and measures of academic self-concept (self-concept of ability) than between achievement and overall self-concept. Wylie observed that the highest correlations with academic achievement had in fact been obtained with subject-area self-concepts (e.g., English and mathematics). More recent evidence (Marsh, 1990; Schunk, 1990) also suggests that higher correlations with school attainments are achieved with narrowly defined, domain-specific academic self-concept.

Research has shown, however, that even older children are not always accurate in their self-assessments. Chen and Stevenson (1995) attempted to explain the differences in arithmetic achievement between Chinese, Japanese, Asian-American, and Caucasian-American 17-year-old students. Arithmetic performance ranged from highest to lowest in the order cited above, with Asian-American students not differing significantly from their Japanese peers. Children were asked how good they thought they were in maths and how hard math was for them. While there was a relation between achievement, self-concepts, and beliefs about difficulty, correlations between achievement and self-concepts showed that Asian-American and Caucasian-American had a more realistic view of their performance and their difficulty in maths. The views of Japanese and especially Chinese students, however, did not correlate highly with their math scores. It could be argued that because East Asian culture encourages modesty, the students would refrain from evaluating their performance as highly as they ought to; or the emphasis on hard work to always achieve higher goals would compel the students to underestimate their current level of performance. Such issues should always be considered in the interpretation of research findings in cross-cultural studies.

Finally, children's accuracy of self-assessments is subject to the reference or comparison group against which children compare their performance. Blatchford (1997b) argued that children "move from absolute or individual comparisons, generated by self comparison, when assessing performance, to normative or group standards, generated by social comparisons" (p. 355). In other words, with age, children develop a normative conception of their ability, that is, they view their performance relative to that of others.

Marsh (1990), accordingly, argued that children's self-appraisals vary depending on the frame of reference against which children have to evaluate their academic achievement. The main reason academic self-concepts are thought to differ from the corresponding academic achievement is because they are based on two different frames of reference that are used to evaluate the two constructs. Blatchford (1997b) used Marsh's (1990) external frame of reference model to examine differences in children's beliefs about their arithmetic and reading performance in three different age levels, that is, 7, 11, and 16 years of age. Accordingly, he used a conception of self-appraisal which focuses on judgements about attainments relative to an external (e.g., "I am better, not as good, or the same as others") rather than internal standard (e.g., "I am able or unable to do a task"). Based on earlier evidence (Blatchford, 1992) suggesting that at 11 years children maintained a higher estimation of their attainment when the reference group was a general one and that group differences may vary with the type of reference group, Blatchford (1997b) used two social comparison or reference groups, a general reference group consisting of children in general and a group consisting of other children in one's class in school.

Children's ratings varied as a function of reference group. More specifically, children were more likely to hold positive beliefs (i.e., "better than") for their achievement when compared to children of the same age in general rather than when compared to their classmates. This lowering of ratings was possibly due to the fact that with age children tend to base their assessments on normative social comparisons and the classroom environment does indeed suggest more immediate comparisons of this kind. Another possibility is that pupils may draw ideal self-judgements (i.e., how they would like to be) or judgements based on how much they could achieve (*the possible selves* as described by Anderman and Maehr, 1994, in Blatchford, 1997b). Ratings of achievement in English in particular suffered the highest drop, with beliefs about math ability being less affected. This was attributed possibly to the fact that by the age of 16 maths is more likely to be taught in mainstream, that is, mixed ability, classes.

Parents' Evaluation of Children's Performance

Parents' beliefs about the child's performance in arithmetic have often been related to children's actual performance. Stevenson and Lee (1990) found Grade 1 and Grade 5 American children to be significantly behind their Chinese and Japanese peers in arithmetic. In their attempt to explain these differences in the context of home environment, they examined mothers' beliefs about the child's general school and arithmetic performance and how these may vary as a function of children's arithmetic achievement. Mothers used a 9-point rating scale, from *much below average* to *much above average*.

In terms of general scholastic achievement, American mothers were found to be biased positively in their evaluations: they were satisfied with their child's performance even though it was low. The ratings given to American children were as high as those given to Chinese children, while being further higher than those given by Japanese mothers.

In terms of children's achievement in arithmetic, however, mothers were able to accurately evaluate their child's performance in arithmetic, with their beliefs varying as a function of children's arithmetic achievement. All correlations between mothers' ratings and children's scores in arithmetic in Grades 1 and 5 were significant.

The relationship between children's and their parents' beliefs often make parents' role a central one. Parents' beliefs, independent of their accuracy, may be affecting children's performance, by perpetuating, if not creating, differences in their self-concepts.

For example, studies with children from Grades 5 through 11 (Parsons, Adler, & Kaczala, 1982) have shown that parental beliefs about their children's abilities are related to and predictive of children's own beliefs. Parents' beliefs were found to be stronger influences than children's own past performances in maths. More specifically, Parsons et al. found that children's self-perceptions, expectancies, and task difficulty related consistently to both their perceptions of their parents' beliefs and to their parents' actual estimates of their children's abilities: in other words, parents who think that math is hard for their children and who think their children are not very good in math have children who also possess a low self-concept of their math ability, see math as difficult, and have low expectancies for their future performances in math. No gender differences were observed.

Examining children from Grades 6 to 11, Jacobs (1991) also found that parents' ability perceptions affected children's ability perceptions. Path analyses showed that mothers' perceptions were influenced to a great extent by children's previous mathematics grade (.41) and to a lesser though substantial degree by the child's gender (.29). Significant interactions were also observed: mothers' gender stereotypes about maths ability interacted with the child's gender to directly influence their own beliefs about their child's math ability, including the likelihood of future success in maths. Mothers' gender stereotypes also interacted with the child's gender to indirectly influence the child's own ability perceptions and performance in maths. Children's performance was found to be influenced by both their self-perceptions and their parents' stereotypes.

Attributions for Performance

It is natural to ask why did one fail a test or do well on a task. The major assumption of attribution theory is that achievement behaviours are mediated by ability perceptions; these perceptions are in turn based on interpretations of the causes of success or failure (Weiner, 1979; Blumenfeld et al., 1982). As Weiner (1979) postulates "the search for understanding is the basic 'spring of action' " (p. 3).

Most dominant main, frequent, and reasonable causes of achievement performance include ability, amount of effort that was expended, and difficulty of the task. However, there is a wider range of events or processes which are sufficient, necessary, or both, reasons for achievement performance, such as physiological processes (e.g., mood, maturity, health), others (e.g., teachers, peers, family), acquired characteristics (e.g., habits, attitudes), which also comprise the central determinants of success and failure. Other reasons include luck, experience, bias, and so forth. There is also a cultural element in attributional causes: patience has been attached to Greek and Japanese individuals, while tact and unity have prevailed in India (Triandis, 1972, in Weiner, 1979).

Weiner's attribution model involves some central assumptions. The first assumption is that rather general values are assigned to factors. The second assumption is that the task outcome is differentially ascribed to the causal sources. The third assumption is that future expectations of success and failure would be based upon one's perceived level of ability in relation to the perceived difficulty of the task (which is formally labelled as *Heider's can*) as well as an estimation of the intended effort and anticipated luck.

Inasmuch as the list of conceivable causes of success and failure is infinite, Weiner postulates it is essential to create a classification scheme or a taxonomy of causes. In doing so, similarities and differences are delineated and the underlying properties of the causes are identified. This is an indispensable requirement for the construction of an attributional theory of motivation. The three dimensions of causality are locus, stability, and controllability. Based on this argument, causes theoretically can be classified within one of eight cells (2 levels of Locus x 2 levels of Stability x 2 levels of Control). For example, ability is internal, stable, and uncontrollable, while typical effort is internal, stable, and controllable. Task difficulty is external, stable, and uncontrollable. Luck, in that effect, is external, unstable, and uncontrollable.

While further and different empirical procedures, such as factor analytic methods, multidimensional scaling methods, and cluster studies, have identified more dimensions, for example, intentionality and globality, the three dimensions mentioned above emerged most often and steadily. Weiner stresses the importance of the psychological consequences of properties, contending that each of the three dimensions of causality has a primary psychological function or linkage as well as a number of secondary effects.

One dimension of causality is *stability* which defines causes on a stable (invariant) or unstable (variant) continuum. Relatively fixed characteristics include ability, typical effort, or family. Unstable factors refer to immediate effort, attention, or mood. Effort and attention may be augmented or decreased from one episode to the next, while mood is conceived as a temporary state. However, the perceived properties of a cause may vary among experimenters, for example, mood may be thought of as a temporary state or as a permanent trait and task difficulty may be considered a stable characteristic despite being thought of as unstable in sales territory. The stability dimension relates to the magnitude of expectancy change following success or failure. For example, if one attains success or failure and if the conditions or causes of that outcome are perceived as remaining unchanged (e.g., ability), then success or failure will be anticipated with a greater degree of certainty. But if the conditions or causes are subject to change (e.g., luck, mood), then there is some doubt that the prior outcome will be repeated.

Another dimension of causality is *controllability*, where causes are categorised as controllable or uncontrollable. Effort, for example, is considered a controllable cause since it is perceived as subject to volitional control, while mood is perceived as an uncontrollable cause of success or failure. The control dimension has implications for decisions for helping, evaluation and liking. For example, students would not lend their notes to an unknown classmate if he did not try to take notes himself (internal and controllable). Given success, high effort is rewarded more often than high ability; given failure, lack of effort is punished more than lack of ability. Accordingly, a teacher will not particularly like a student who does not try.

A major dimension widely examined which is also of special interest in the present study, is that of *locus*. The locus of causality can be either internal (within the pupil) or external (outside the pupil). Internal or personal causes include ability, effort, mood, maturity, or health. External sources of causality include teacher, task, and family. However, the relative placement of a cause on this dimension is not invariant over time or between people: for example, health as a cause for failure (i.e., "I'm a sickly person" - internal) or (i.e., "The flu bug got me" - external). Consequently, the taxonomic placement of a cause depends upon its subjective meaning. Nonetheless, in spite of possible individual variation, there is general agreement when distinguishing causes as internal or external.

The locus dimension has implications for self-esteem, one of the emotional consequences of achievement performance, with affect being also a secondary association for causal stability. Weiner reports three major findings. First, there are emotions that are outcome-dependent; for example, success is always associated with "good" feelings (e.g., pleasure, happiness, satisfaction) while failure is always associated with "bad" feelings (e.g., displeased, uncheerful, upset). These are independent of the attributions given. However, and this is the second argument, there are some attribution-affect linkages; that is, there are some more distinct emotions accompanying these general feelings. For example, success or failure due to ability is linked to feelings of confidence and competence or incompetence respectively; failure or success due to others is associated with hostility or gratitude respectively; luck is likely to elicit surprise, and so forth. Third, there are particular affects which cluster with the internal causes, which in turn relate to self-esteem, such as competence, pride, confidence, satisfaction, and shame. These are more frequently reported with internal or self-ascriptions rather than external attributions.

Research Evidence

Internal attributions have been found to be mostly associated with higher performance in arithmetic. Ability and effort are very often the reasons children offer for their success in maths.

Young-Loveridge (1991) found that the majority (two-thirds) of 9-year-old children who thought they were good in maths attributed their performance to effort (i.e., trying hard). Young-Loveridge found that children doing well in mathematics would be more likely to believe they were good in mathematics, while children in the low scoring group would be equally likely to say they were good or bad. Children were further asked why they thought they were good or bad in mathematics. The major reason for believing they were either good or bad was either getting many answers right or wrong respectively on worksheets or tests. Of those children who thought they were good at maths ($n = 22$, 13 high-scoring, 9 low-scoring), thirteen children thought they were naturally good, that is, they attributed their success to their ability (6 low-scoring and 7 high-scoring; there were more boys than girls; 30% and 12%

respectively). Two-thirds of those who thought they were good at maths attributed it to effort (i.e., trying hard), with no differences being observed between boys and girls (59% and 68% cf.). Very few children attributed their success to luck (6%) and only six children thought they were good because maths was easy. Eight children attributed not being good at maths to lack of ability (i.e., not naturally good at maths), with no differences being observed between boys and girls (11% and 12% cf.). Fifteen children believed they were not good because they did not try hard (more boys than girls, 33% and 15% cf.). Ten percent attributed failure in maths to luck (fairly similar pattern for boys and girls). Almost half of the children (51%) thought they failed in maths because it was difficult (more girls than boys, 59% and 41% cf.). Young-Loveridge argued that boys tended to attribute their success to ability more strongly than girls (30% and 12% cf.), while girls tended to attribute failure in maths more strongly to task difficulty (59% and 41% cf.).

Miserandino (1996) examined third- and fourth-grade children of above average arithmetic competence and found that not all children believed they were good in arithmetic. As mentioned earlier, those who were certain of their ability participated in and persisted more at school tasks, while those who were unsure did not evaluate their achievement as being so high and suffered from lack of motivation. Overall, children believed ability was more important for success in mathematics, whereas other factors such as effort may be more important for success in reading and spelling. Miserandino related those findings to self-determination theory and the motivational model of engagement, according to which the motivation behind the engagement may in fact be more important (than ability) in understanding and predicting subsequent engagement and learning.

Internal attributions are significant determinants of maths achievement even in higher grades. In their attempt to construct a model of maths achievement in Grade 8, Reynolds and Walberg (1992) found that children's academic motivation, along with home and peer environment and prior achievement, had a significant indirect effect on their maths performance. Children's motivation was measured through their emphasis on effort, which focused on issues like "trying hard to my best" and "trying harder if my grades are bad".

Finally, internal attributions for success are common even in children in 11th grade. In their cross-cultural study on math achievement, Chen and Stevenson (1995) examined Japanese, Chinese, Asian-American, and Caucasian-American children and found that children differed in arithmetic performance, with Japanese and Chinese students being better than their American peers. Chen and Stevenson investigated students' beliefs on the four most important factors affecting performance in arithmetic: a good teacher, innate intelligence, home environment, and studying hard. They found that the majority of Chinese and Japanese students believed effort (e.g., the road to success is through studying hard) was the most important factor that may influence achievement in mathematics. American students, however, would think that having a good teacher was the most important factor affecting performance. Even within the sample of American students, effort was stressed more often by the Asian rather than the Caucasian students.

Attributions to internal reasons may be related to higher performance, however, a further distinction between ability and effort may also affect performance levels: this relates to another dimension of attributions, namely controllability. Effort is controllable since it is thought of as being subject to volitional control, while ability is innate and an uncontrollable cause of success or failure.

Attributions to effort have been associated with higher levels of achievement than have attributions to ability. For example, given success, high effort is rewarded more often than high ability. As Miserandino (1996) found, third- and fourth-grade above average in math children who were uncertain of their arithmetic performance attributed their achievement to lack of ability.

Also, one of the major reasons for American children's underachievement in school, including maths, compared to that of their Asian peers was the emphasis their parents placed on innate abilities (Stevenson & Lee, 1990). Parents of American children in Grades 1 and 5 emphasised innate abilities as a determinant of performance, which as a consequence distinguished between children of higher and lower ability. Children not doing so well in turn would not be encouraged to work harder because they would not achieve regardless of how hard they tried. Tracking was popular in school, where children belonged to either faster or slower learning group. Chinese and Japanese mothers, on the other hand, emphasised effort and hard work and were not pro-nativists. Asian teachers accordingly shared the same beliefs, while tracking did not exist in their schools.

The attributions parents make, however, may be subject to parents' own biases. In turn, internal attributions may often raise self-esteem and enhance performance, yet they may have the reverse outcome in failure. Attributions to effort do not contribute to a stable notion of one's ability in a particular domain, so attributing one's success to effort is not as ego enhancing as attributing it to ability. In their examination of parents' attributions for children's math achievement, Parsons, Adler, and Kaczala (1982) found that parents of children in Grades 5 to 11 conveyed their own expectancies which eventually were associated with children's performance in the subject. While parents of daughters did not rate their child's math abilities as significantly lower than did parents of sons, parents of daughters reported that mathematics was harder for their child and that their child had to work harder to do well in math. Attributing one's success to effort does not reflect a stable idea of one's ability and may also leave doubt about one's future performance on increasingly difficult tasks. If one is having to try very hard to do well now and one expects next year's math course to be even harder, one may not expect to do well next year. Parsons et al. argued that perceptions of how hard one is trying in the present are negatively correlated with future expectancies, with one's estimates of one's ability, and with the difficulty of the task, being also causally related to children's self-concepts of their math ability one year later.

Children's Aspirations for Performance

The type of motivation has been found to affect arithmetic achievement throughout school. Stevenson and Lee (1990) found that external sources of motivation are more likely to be associated with lower rather than higher arithmetic and general school performance. American mothers were found to emphasise external source of motivation, while mothers of Asian children would stress the importance of being highly educated. Chinese and Japanese children performed better in maths and school than their American peers.

Chen and Stevenson (1995) showed that those students who set high standards for themselves scored the highest, while those who studied hard to get a better job in the future scored the lowest. Japanese, Chinese, Asian-American, and Caucasian-American students were compared on their arithmetic performance and were found to differ between them from highest to lowest in the above order. The students were further asked on the most critical motivational factor, that is, the most important reason for them to work hard, having to choose from nine common reasons: to gain more knowledge, to get good grades, to go to college, to please parents, to please teachers, to get a better job in the future, or because they set high standards for themselves, they had no other choice, or they did not know what to do with their time.

Overall, high standards were observed in Asian and Asian-American children and their parents. The three most frequent choices, in order of importance, were to get a better job, to go to college, and to gain more knowledge. Finding a better job was the most common response of Asian-American students (40%); gaining knowledge (20%) and going to college (19%) followed. Caucasian-American, Chinese, and Japanese students were equally likely to mention going to college the main reason for studying hard (25%, 25%, and 28% cf.) as to mention getting a better job (31%, 28%, and 30% cf.).

Parents' Beliefs About Easiness of Arithmetic for Children

Studies with children in Grades 1 and 5 have shown that parents', especially mothers', beliefs about the child's difficulties may vary with children's performance, with lower mathematical performance being associated with reduced awareness of the child's problems in the subject.

In their cross-cultural study of mathematical achievement, Stevenson and his colleagues (Crystal & Stevenson, 1991; Stevenson & Lee, 1990) found that Japanese and Chinese children were better in arithmetic than their American peers. Children's mothers were further interviewed on their beliefs about their child's difficulties in mathematics, the kinds of problems they thought the child might experience, and what they did about those problems. Stevenson and his colleagues found significant differences between mothers as a function of children's arithmetic achievement.

More specifically, they found that while American children's performance in maths was below that of their Asian peers, American mothers perceived their children as having fewer, less serious, and more transitory problems with mathematics than did Asian parents. The nature of children's difficulties also differentiated mothers: American mothers perceived their children as having significantly more problems with multiplication tables, calculation, and computation, while Asian mothers reported more problems with applied problems. That, the authors argued, may have been due to Asian teachers spending more time in working with applied problems, following the curriculum. Stevenson and his colleagues argued that American mothers' lack of awareness of children's math problems reduced their effectiveness as a source of help to the child, consequently they were less likely to provide assistance to their child.

3.1.2.2 Attitudes and Home Practices

Attitudes

Children's motivation to study a subject and subsequent performance on that subject are influenced by affective variables such as their attitudes about what they are studying. Although not always significant or high (Aiken, 1970; Neale, 1969), a positive relationship between attitudes and achievement has been suggested. Research has further observed variation as a function of children's age and gender.

Thurstone and Chave (1951) refer to attitude as the degree of positive or negative affect associated with some psychological object. According to Aiken (1970), attitude is a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept, or another person.

Children's Attitudes to School

Blatchford (1996) examined children's attitudes to school and how these varied with age, from ages 7, 11, and 16. The relationship between children's views on school and their attainment in arithmetic and reading was also investigated. First, variation was observed in children's attitudes to school during the primary but not the secondary years: children's attitudes varied between ages 7 and 11, while children who liked school at 11 years were also likely to like school at 16 years. Second, no variation was observed in the relationship between attitudes and achievement: there was no association between liking for school and attainment in arithmetic at 7, 11, or 16 years.

The association between attitudes to school and arithmetic achievement was less clear in Stevenson and Lee (1990). They examined the relative disadvantage of American children in Grades 1 and 5 compared to their Asian peers with regards to arithmetic. They found Chinese children liked school the most, while the American and Japanese children were less positive.

Children's Favourite School Subject

Young-Loveridge (1991) found that 9-year-old children's likelihood of choosing arithmetic as their favourite school subject varied as a function of children's arithmetic ability. When asked about their favourite school subject, 32% of high scoring children chose arithmetic as their first most favourite subject, while only 18% of low scoring children did so.

Blatchford (1996) asked children at 11 and 16 years to name three favourite school subjects. He found that at age 11 the majority of students (66%) chose maths as their favourite subject, while at age 16 maths was the second most popular subject (37%). Reading was students' first choice (44%). Blatchford also found that the primary reason for students' liking for maths was enjoyment, with interest in the subject and ability in the subject following.

Children's Attitudes to Arithmetic

Evidence suggests that from an early age children have already formed some opinion about arithmetic. Tizard et al. (1988), for example, found 7-year-old children to have lucid opinions about arithmetic, further being able to justify them. They asked children how they felt about maths, based on a series of faces ranging from a broad smile to a deep frowning. The majority of boys (80%) and 62% of girls chose one of the smiling faces. Children were further able to justify their responses: the most popular reason for liking maths was a liking for doing sums, while they would not like maths mostly because it was difficult.

Early studies of children in Grades 1 to 8 have shown that correlations between student attitudes and achievement were higher for arithmetic than for reading, spelling or language (Brown & Abell, 1965). Children's liking for arithmetic was further related to pupils' general learning ability.

Stevenson and Lee (1990) found a strong positive relation between attitudes and achievement in children in Grades 1 and 5. American, Chinese, and Japanese children were found to differ in arithmetic achievement, in that American children were significantly behind their Asian peers in accuracy in calculations and word problems. While all children expressed either neutral or positive attitudes towards math, Chinese and Japanese children in both grades expressed more positive attitudes to arithmetic than American children. The relation was also significant in the case of reading, despite differences not being culture dependent.

In addition, Stevenson and Lee found that attitudes to arithmetic did not correlate only with children's *actual* performance, but also with their *perceived* performance. Some of the strongest correlations obtained in that study were in fact between the children's ratings of how good they thought they were in a subject and how much they liked the subject. In reading, the correlations were

.55 for the American children, .40 for the Chinese children, and .68 for the American children. In mathematics, the correlations were even higher (.58, .61, and .77 cf.). Children clearly liked the subjects in which they thought they were doing well and disliked subjects in which they thought they were doing poorly. The question of why American children liked mathematics and believed that they were good at it pointed to the direction of an easier American math curriculum than those of Asian countries. This was further supported by fifth graders' ratings of the difficulty of mathematics: American and Chinese children rated maths as significantly less difficult than did the Japanese children. Stevenson and Lee attributed this to the fact that mathematical concepts tended to be introduced somewhat earlier in Japanese than in American textbooks.

Accordingly, Young-Loveridge (1991) found that mathematics was rather unpopular with 9-year-old low scoring children than with their high scoring peers: 44% of low scoring children categorised maths as one of top three favourite subjects, while the corresponding figure for the high scoring group was 62%. The overwhelming majority (91%) believed maths was important, because of its usefulness for specific occupations such as shopkeepers, teachers, and bank personnel, to name a few. The next most common reason was maths usefulness in getting or doing a job, shopping, understanding particular processes in maths (especially when dealing with money), managing school work, being able to construct buildings and boats, and child rearing.

Low scoring children enjoyed maths less than did high scoring students. When asked whether they enjoyed maths, children who were doing well in maths were slightly more positive about maths than low scoring children (65% and 50% cf.): high boys were the most positive (92%) and low girls the least positive (42%). Young-Loveridge found that low scoring boys were more positive than high scoring girls (60% and 50% cf.). Addition was the most common thing children liked about maths (35%), with subtraction (21%), multiplication (15%), and division (13%) coming next. A few children mentioned tables, money, counting, written problems, square roots, and writing on sheets or the blackboard. Many children liked maths when it was easy, and a few mentioned preferring it when it was difficult. The most common aspects of arithmetic that children did not like was division (16%), multiplication (12%), and subtraction (6%). Other less frequent responses included tables, fractions, and thousands. Real life problems such as cutting into shapes and measuring were mentioned by only two children, which suggests that children's work mainly involved writing and solving exercises.

Schofield (1982) found, furthermore, that the relation between attitudes and achievement may vary with children's gender, grade level, type of achievement test used (tests assessing conceptual vs. computational skills), and testing occasion (early vs. late in school year). Schofield examined children from Grades 3 to 6 and found that the relation between attitudes and achievement in girls was very low, unstable in significance and at time (Grade 5) negative. The exact relationship within boys was strong, positive, and consistently significant across grade levels. Schofield interpreted these findings in terms of Fennema and Sherman's (1977) argument that stereotyping maths as a male subject may be a mediating variable affecting sex differences, for example, favouring males, on a variety of relevant attitudes such as confidence in learning maths or perception of its usefulness. Schofield also found that the relationship between attitudes and achievement was stronger when tests assessing computational skills were used rather than with tests measuring conceptual skills. That, Schofield argued, may well have been due to the fact that at elementary level computational skills are more easily observed and rewarded and consequently children appreciate it more. Finally, Schofield found that the relationship between attitudes and achievement in maths was stronger later during the school year rather than early.

Studies with older children have also shown how a liking or dislike for arithmetic is related to arithmetic achievement. In his review of studies examining the relation between attitudes and performance, Aiken (1970) showed how sixth-grade pupils' attitudes were consistently related to their performance in a reciprocal way, that is, their attitudes affected their achievement and achievement in turn affected their attitudes. Aiken described how students who did not like arithmetic showed a rigidity when dealing with difficult arithmetic tasks and resorted to rote memory or dishonest means to pass the task. Those who liked arithmetic, however, tended to persevere toward the solution of arithmetic problems.

A positive relation between attitudes and achievement has further been evidenced in studies with 13-year-old children (Aiken, 1972). Students with positive attitudes towards maths tended to have higher grades in maths and in school work in general. Attitudes to maths usually related to an interest in the topic and a liking for word problems, computations, use of terms, and use of symbols.

Fennema and Sherman (1977) examined the relationship between attitudes and achievement as a function of children's gender in Grades 9 and 12. Significant age by gender interactions were observed. Affective variables, such as confidence in learning maths, attitude toward success in math, parents' attitudes, and perceived usefulness of maths, had an effect on math achievement in girls. Girls experienced lower confidence levels, which could not be explained in terms of poorer performance. With age, greater increases for girls than for boys in positive attitudes toward mathematics were observed as students advanced in the maths sequence. Correlations between maths achievement and affective variables were higher in girls in Grade 12.

Chen and Stevenson (1995) compared 11th-grade Caucasian-American, Asian-American, Chinese, and Japanese students on their attitudes about mathematics. These four groups differed in their arithmetic achievement from lowest to highest in the above order. Children were asked how much they liked mathematics and how interesting they thought the subject was. The four groups differed in their attitudes, with Asian-American children being more positive and interested than any other group and Japanese students being the least positive and less interested, based on both questions. In general, correlations between attitudes and math scores were not high, however, the correlations between Asian-American students' achievement and their interest in maths were significantly lower than those for the other three groups. Chen and Stevenson attributed this to parental pressure which made students work hard regardless of any interest in the subject.

Parents' Attitudes to Arithmetic

In his review of factors affecting children's attitudes to mathematics, Aiken (1970) postulated that parents affect children's attitudes and achievement in three main ways, namely, by their own attitudes, by their expectations regarding the child's achievement, and by their encouragement. Parents' attitudes to arithmetic have been found to affect children's performance indirectly through influencing children's own attitudes. Reporting on the negative attitudes towards math, Aiken and Dreger (1961) argued that these result from experiences specific to the learning of mathematics, especially the manner in which significant others, such as teachers and parents, instruct children in the subject.

Aiken (1972) found that the influence of parents is best pictured by the fact that pupils' attitudes and achievement in maths are positively related to the attitudes of their parents. He examined the attitudes of students in Grade 8, college freshmen, and graduate students in education, investigating the interaction of factors which affect attitudes and achievement in maths. While there were some interactions with age and gender, it was generally found that father variables ("My father likes maths" and "He made high grades when he was in school") were a significant factor in determining male students' attitudes towards the subject. The reported attitudes and achievement of the mother ("My mother likes maths" and "My mother made high grades when she was in school") were associated especially with female students' attitudes toward maths.

Studies in the 1950s, 1960s, and 1970s have seen the significance of parents' attitudes on students' own attitudes. Poffenberger and Norton (1959), for example, conducted a research on the development of attitudes toward mathematics, testing the hypothesis that the family conditions the attitudes of children. They gave university freshmen questionnaires on their attitudes toward mathematics and their parents' attitudes and expectations. Poffenberger and Norton concluded the following: (a) parental attitudes toward mathematics, especially those of fathers, were related to students' attitudes toward the subject matter, (b) parental expectations regarding students' math achievement were positively related to students' attitudes toward the subject, and (c) students' attitudes were influenced by parental encouragement in math courses.

Poffenberger (1959) further found that children who see themselves as negatively perceived by their parents may perceive their parents as being negatively oriented to other aspects of life. While college students with a close relationship with their fathers were similar in their ratings of their fathers' attitudes toward mathematics (50% reported "Father likes maths" and 50% reported "Father dislikes maths"), students who reported having a distant relationship with their fathers tended to consider them as disliking mathematics (27% reported "Father likes maths" while 73% reported "Father dislikes maths").

It has been found, however, that parents do not always influence their children's attitudes and beliefs through role modelling. Parsons et al. (1982) found that parental division of who did the math tasks at home did not affect children's self-concept, task-concept, and performance measures. Also, children's perceptions of their parents attitudes to arithmetic did not have an effect on children's self-perceptions and perceived importance of maths.

Parents' Beliefs About the Academic Importance of Arithmetic

Stevenson and Lee (1990) found significant differences in mothers' beliefs about the importance of doing well at school, as a function of children's arithmetic performance. Grade 1 and Grade 5 American children were found to be behind their Chinese and Japanese peers in arithmetic achievement. One of the factors Stevenson and Lee investigated in relation to children's differences in arithmetic performance was the importance parents placed on academic achievement. Variation in parents' beliefs was observed as a function of children's arithmetic and school achievement. Education, the authors argued, was highly prized in Chinese and Japanese cultures. Going to school and getting good grades were supposed to be children's two main responsibilities. The family was also committed to the child's schooling, communicating messages on the importance of doing well. On the other hand, American mothers, while placing some emphasis on arithmetic and school achievement, they would nevertheless be satisfied if the child performed average, would not be alarmed unless children showed noticeably low levels of achievement, and considered school as a source of general cognitive development. Accordingly, they would not encourage academic activities outside school hours.

Parsons et al. (1982) further found that parents' beliefs of the importance of maths varied with children's gender. Overall, maths was more important for boys than for girls; parents of boys believed it was more important that the child did well in maths than parents of girls. However, it was mothers' reports who were more characteristic of gender differentiation; mothers believed maths was significantly more important for boys than for girls, while fathers' reports did not differ significantly as a function of the child's gender.

Home Activities

To better understand the influence of home environment on children's arithmetic performance at school, it is necessary not only to investigate children's and parents' beliefs, but also to consider more proximal factors such as daily arithmetic routines that might improve performance in various ways. Just like reading activity at home has been identified as a significant positive influence on measures of students' reading achievement and attitudes towards reading (Rowe, 1991), children's numerical activities at home have been found to serve as promoters of learning and achievement in arithmetic.

Tizard et al. (1988) examined the effect of education experiences at home, such as experience with books and home teaching, among others. Engagement in such activities was found to associate with arithmetic achievement upon entry to school, however, it was not significantly associated with progress in maths in the three year infant-school period.

In their longitudinal study of differences in arithmetic achievement between American, Chinese, and Japanese children in Grades 1 and 5, Stevenson and Lee (1990) found a relation between children's academic activities at home and their achievement in arithmetic. Chinese children were more likely to engage in academic rather than nonacademic activities at home. While Japanese children were less likely than Chinese children to engage in academic activities, still they were much more likely than American children to do so. Chinese and Japanese children had been found to perform higher than their American peers in mathematics.

Young-Loveridge (1991) asked 68 9-year-old pupils to think of anything they do at home that uses numbers. While 31% of the children could not give any examples of times or places where numbers are used at home (37% of boys and 27% of girls), the most frequent responses of those children who could think of such occasions were counting or establishing cardinality (22%), telling the time (22%), playing games (19%), and operating household appliances such as video, microwave, television, and radios (12%).

In constructing a model of maths achievement in Grades 7 and 8, Reynolds and Walberg (1992) found that home activities such as out of school reading affected directly performance in arithmetic. Also, home environment measures, for example, number of resources, parents' expectations, parents' education, were also found to influence children's arithmetic performance, however, indirectly.

In another cross-cultural study, Chen and Stevenson (1995) examined 17-year-old children's motivation and achievement in arithmetic. They found Asian-American students scored higher than their Caucasian-American peers, but lower than Chinese and Japanese students. In their examination of factors related to such differences in maths achievement, Chen and Stevenson found that a significant factor that differentiated East Asian children from Caucasian-American and Asian-American children was the achievement-related behaviours that children engaged in at home. Such activities referred to the amount of time they spent studying, and their extra-curricular activities. Chen and Stevenson argued that such behaviours were found to facilitate learning and improve performance.

3.1.2.3 Parental Help With Homework

Ideally, parents create an environment which allows the child to carry on his academic responsibilities at home. Parents can help with the homework in different ways. For example, they can help indirectly, by organising the child's time and space. Or they can help directly, by instructing the child or studying together.

Child-parent interaction is important from a young age. As early as in 1967, Freeberg and Payne acknowledged that a dimension that can be considered fairly distinctive for the cognitive-intellectual phases of child rearing is the degree of parent willingness to spend time with the child and to interact with him in a variety of situations. They identified the age range from 2 to 6 years of age - that is, before children enter school - as the critical period.

Newson and Newson (1977) found that eighty-one percent of parents would read at their 7-year-old child at home. Moreover, a significant relationship was found between the amount of parents' reading to the child at the age of 4 and the child's reading ability at age 7. That was true independent of the social class of the family or the gender of the child. While parents reported being uncertain about the method they should employ in helping children with their reading homework, they would nevertheless provide this help. In the case of arithmetic, rates of help dropped dramatically because parents were not confident enough to provide this help mainly because of their restricted arithmetic knowledge and because of their lack of awareness of appropriate teaching methods.

Recent work focusing on children from 6 to 10 years of age points out that parental involvement with the child's homework is related to children's achievement. In their cross-cultural study, Stevenson and Lee (1990) found that children's educational achievement was important to parents in all three cultures, namely, Chinese, Japanese, and American, despite differences in children's arithmetic achievement. As mentioned earlier, Chinese and Japanese children were better in arithmetic than their American peers. Despite the common interest in children's school achievement and the importance they placed on it, Chinese and Japanese mothers showed a greater commitment to their children's academic achievement than did American mothers. Both groups of Asian mothers dedicated themselves and their time to their children's schoolwork and gave children more benefits if they were good at school. American mothers were not as much committed in terms of time and effort to their children's success, as were Chinese and Japanese mothers. American mothers were involved in the child's difficulties mainly in order to maintain their child's interest in schoolwork and emphasised external sources of motivation. American mothers were dedicated to their children's education up to the moment children entered the school.

Children of 8 to 10 years of age (Grades 3 to 6) also seem to benefit from parental, especially mothers', involvement. Grolnick and Ryan (1989) found that mother versus father involvement was important in the prediction of children's competence and school self-regulation. More involved mothers had children who evidenced higher achievement and were better adjusted according to teachers. Mothers were found to be more involved than fathers in child rearing and they would spend more time actively interacting with their children. Parental involvement was, furthermore, significantly and positively related to socio-economic status.

Gottfried, Fleming, and Gottfried (1994) examined the effect of mothers' motivation on 9- and 10-year-old children's academic success. In sum, they found that parental motivational practices, assessed by mothers' practices, had significant direct effects on academic intrinsic motivation. Task endogeny had a positive impact on academic intrinsic motivation; task-extrinsic consequences had a negative impact on academic intrinsic motivation. Second, academic intrinsic motivation significantly and positively predicted subsequent motivation and achievement. Third, parental motivational practices had significant indirect effects on subsequent motivation and achievement, with task endogeny having a positive effect and task-extrinsic consequences having a negative effect. The two parental motivational practices dimensions appear to be theoretically similar to what Deci and Ryan described in 1985 as autonomy-supporting versus controlling parent styles. The promotion of children's task endogeny can be considered as encouraging children's autonomy and self-determination in learning tasks; the provision of task-extrinsic consequences may be considered as controlling children's behaviour. Parental motivational practices indirectly but significantly influence subsequent achievement through their effect on earlier academic intrinsic motivation. Hence, children with higher academic intrinsic motivation at age 9 have higher motivation and achievement at age 10 and conversely, children with lower academic intrinsic motivation at age 9 have lower academic intrinsic motivation and achievement at age 10.

3.1.2.4 Parent - School Relations and Parent Education

Another literature emphasises the relation between the parents and the school, as well as parents' educational status, with regards to children's achievement at school.

Parent - School Relations

In their longitudinal study of home factors affecting attainment in mathematics, Tizard et al. (1988) found that the level of parents' contact with the school was significantly related to children's progress in reading and writing in the early years of schooling. Children of parents who reported greater contact with the school and greater knowledge of the school showed greater progress than children whose parents did not have such knowledge. In the case of mathematics, the amount of parental knowledge of and contact with the school was significantly related to children's numerical skills in the nursery level. It did not relate, however, to children's progress over the period of the first three years at school.

Tizard et al. also found that parents would usually learn about the curriculum by looking at their children's work. Written information from school was very unusual. In addition, the majority of parents (80%) reported meeting with the teacher to discuss the child's progress at least once every year, however, those meetings were not very informative. While parent-teacher communication has indeed increased in recent years, Tizard et al. argued that there is a need to find ways to extend it.

Hughes, Wikeley, and Nash (1994) have provided further evidence on the limited communication between parents and teachers. In their longitudinal study of parents' views about the National Curriculum from Year 1 to Year 3, Hughes et al. found that parents knew very little of what was going on at school, that is, what children learned in maths, English, and science, despite their strong interest in getting more information about those issues to be able to assist their child with the homework. While there was enough formal and informal contact with the school, through newsletters, parents' evenings and casual conversations with the teachers, what was communicated with the teachers was limited: parents would not be effectively informed about what the children were doing in class and how well they were doing. Parents would rather rely on what the child said and did. Furthermore and despite the limited communication, the majority of parents thought the teachers were doing a good job and were happy with the child's school.

Newson and Newson (1977) examined the relation between the school and parents of 7-year-old children. They found that children's mothers were very satisfied with the school, despite the limited communication and knowledge they had of it. In specific, 82% percent of mothers were very satisfied with the school, while only 11% had some reservations; these referred mostly to the teaching methods and the general organisation of the teaching. Also, the majority of parents acknowledged the significance of the teacher for the child's education and well-being. They further expressed some concern about not knowing much about the teaching methods used at school that would enable them to better assist the child with the homework, yet they felt the teacher would disapprove of their involvement with the teaching of school subjects. Accordingly, there were no instances of a school introducing parents to the means by which the teaching of reading was approached in order to enlist their informed help.

Parental opinions on the suitability of the curriculum has also been found to vary with children's performance in Grades 1 and 5. Stevenson and Lee (1990) found that Chinese, Japanese, and American mothers shared the same beliefs about the curriculum, despite children's different levels of school achievement. As has been mentioned earlier, Chinese and Japanese children were better in school and maths than their American peers. While the majority of mothers (80%) believed the difficulty of the curriculum was "about right", some variation was observed in the opinions of the rest of the mothers as a function of children's achievement: for example, relatively more Chinese or Japanese mothers than American mothers believed the curriculum was too difficult (14% of Chinese mothers, 13% of Japanese mothers, and only 2% of American mothers). Also, of those mothers who believed the curriculum was too easy, American mothers were somewhat more numerous than Chinese or Japanese mothers (10%, 3%, and 2% cf.). As children progressed to Grade 5, more significant variation was observed in mothers' opinions about the curriculum: 28% of Chinese mothers and 30% of Japanese mothers now believed that the curriculum had become too hard, while American mothers did not change their opinions on the difficulty of the curriculum from Grades 1 to 4. The optimism most American mothers expressed - that is, approving the curriculum despite knowing the children were not so good - was thought to be due to the lower academic standards they held for their children.

Studies with children from age 5 to age 17 have also evidenced a relation between parental involvement in schooling and children's performance. Stevenson and Baker (1987) investigated the relation between parental involvement in school activities and the child's school performance, mother's education and parental involvement in school activities, and the strength of these relations for children of different ages. The researchers found that children of parents who were more involved in school activities did better in school than children with parents who were less involved. In addition, they found that the higher the educational status of the mother the greater the degree of parental involvement in school activities and that the younger the age of the child the greater the degree of parental involvement.

Parent Education

Research has suggested that the influence of parents' academic status on children's school performance can be direct or indirect, and varying in degree. Tizard et al. (1988), for example, found that children's numerical, reading, and writing skills were significantly related to mother's education upon entry to school; that is, children with mothers with higher qualifications had higher scores on entry to school than did the rest of the children. In the three year infant-school period, however, none of the home variables examined was significantly related to *progress* in maths.

In their cross-cultural study of children's school achievement in Grades 1 to 5, Stevenson and Lee (1990) examined American children's underachievement in school compared to their Asian peers. They found that approximately 10% of the variability in children's achievement scores could be accounted for by variability in parental education. Correlations between the average of the mothers' and fathers' level of education and their children's reading and mathematics scores ranged between .22 and .34 ($p < .001$).

Studies with older children further support the positive relationship between parental academic background and children's performance in arithmetic. In their attempt to construct a model of mathematics achievement in Grades 7 and 8, Reynolds and Walberg (1992) found that parents' education had an indirect effect on children's achievement. Home environment, which was measured through parents' expectations, parents' education, and number of resources, was found to influence achievement indirectly. Reynolds and Walberg reported that 78% of the indirect effect of home environment was transmitted through prior achievement. That was not surprising given the strong influence of the family environment on children's schooling over the years. Prior achievement, in turn, was found to have a direct effect on mathematics achievement.

In a study on eighth graders' transition from middle school to high school, Baker and Stevenson (1986) found that the educational level of the mother affected the strategies which parents employed to manage their child's school career. These strategies were considered to further have a direct effect on children's educational achievement. The number and types of strategies employed did not vary among mothers, however, the implementation of strategies related to mothers' educational status via their knowledge about the child's school performance and the frequency of contact with the teacher. More specifically, mothers with at least college education had more contact with the teacher, knew more about their child's performance, and were more likely to actively manage their child's academic career.

Finally, there are studies with older children which evidence a significant though low influence of parents' education on children's achievement. Chen and Stevenson (1995) examined math achievement in Chinese, Japanese, Asian-American, and Caucasian-American high school children (17 years old). They found significant differences in children's arithmetic performance, with Chinese and Japanese children being better in maths than their American peers. Within each culture group, there was a positive relation between children's scores and fathers' years of education, where children with fathers having postgraduate education would be more likely to score higher than children whose fathers had finished junior high school. However, there was no overall difference between culture groups; parents' amount of education did not vary with children's achievement across cultures. In other words, within each level of education, the rank order of children's scores did not vary.

3.1.3 Aim and Hypotheses

The aim of the present investigation is to identify the social and environmental factors which might account for children's variation in arithmetic performance. Based on previous research findings, the social and environmental factors that are investigated hereby include factors that relate to the child and to the child's parents and are associated with children's general school or math achievement in specific. These variables are categorised into four major thematic sections: evaluation of performance and other issues related to achievement, attitudes and home practices, parental help and encouragement, and parent - school relations and parent education. Information on children's and parents' beliefs and practices in reading are also of interest, for the purpose of relating them to beliefs and practices in arithmetic.

Evaluation of Performance and Other Issues Related to Achievement

Children's Self-Concepts and Parents' Evaluation of Performance

Evidence has suggested a positive relationship between self-concept and achievement (Schunk, 1990; Shavelson & Bolus, 1982). While very young children are not accurate in their self-assessments (Blatchford, 1997b; Miserandino, 1996; Stevenson & Lee, 1990; Tizard et al., 1988), accuracy has been found to improve with age (Blatchford, 1997b; Chen & Stevenson, 1995), with more general reference group (Blatchford, 1997b; Marsh, 1990), and with more domain specific self-concepts (Blatchford, 1997b; Schunk, 1990). Some research, however, has identified 9-year-old children who were quite able to evaluate their performance (Young-Loveridge, 1991). Based on that evidence, children in the present study who do better in arithmetic would be expected to hold more positive views about their performance than do children who do less well in arithmetic.

Following the evidence that children may be more accurate in their self-assessments when an external frame of reference is used (Marsh, 1990; Blatchford, 1997b), children are asked to compare their performance to that of two other math ability groups in general, instead of the rest of the children in their classroom.

Based on evidence suggesting that parents' perceptions about children's math performance may influence the child's own ability perceptions as well as their future performance (Jacobs, 1991; Parsons et al., 1982), the present study further examines similarities and differences between children's self-concepts and parents' evaluations of the child's performance in school and in arithmetic. Based on evidence suggesting that mothers were inaccurate in their ratings of children's general academic performance in Grades 1 and 5 (Stevenson & Lee, 1990), mothers' ratings of children's general school achievement would not be expected to relate to children's achievement in arithmetic, in the present study.

However, based on evidence that mothers were accurate judges of children's arithmetic achievement in Grades 1 and 5 (Stevenson & Lee, 1990), children who do better in arithmetic would be expected to be given higher ratings than those given to children who do less well in the subject.

Attributions for Performance

The reasons we give for our achievement are important determinants of future performance (Weiner, 1979). Research has shown that higher mathematical performance is more likely to be attributed to internal, for example, ability and effort, rather than external reasons (Chen & Stevenson, 1995; Reynolds & Walberg, 1992; Weiner, 1979; Young-Loveridge, 1991). Based on that evidence, children who think they are above average in arithmetic would be more likely than children who think they are less good in the subject to attribute their performance to internal reasons.

In light of evidence suggesting that parents' attributions for the child's performance indirectly influence the child's subsequent performance - through directly influencing the child's own perceptions (Parsons et al., 1982), the present study also examines parents' attributions for children's performance. Research has shown that higher levels of arithmetic performance have been associated with parents' internal attributions for children's arithmetic performance (Stevenson & Lee, 1990). In the present study, accordingly, parents' internal attributions would be expected to be associated with children's higher levels of achievement in arithmetic.

Children's Aspirations for Performance

In their examination of students in Grade 11, Chen and Stevenson (1995) found that children who set higher standards for themselves (internal reasons) do better in arithmetic than children who want to be good in the subject for external gains, for example, simply to get a good job. The present study examines young children's aspirations. Of those children who want to be better in arithmetic, those who mention internal gains would be more likely to belong to high ability groups than do those who mention external benefits.

Parents' Beliefs About Easiness of Arithmetic for Children

Studies on children in Grades 1 and 5 have shown that mothers of children who do less well in arithmetic are less aware of the child's problems in the subject than do mothers of children who are better in arithmetic (Crystal & Stevenson, 1991; Stevenson & Lee, 1990). More specifically, mothers of children who were not doing well in arithmetic thought the child found arithmetic easy. The present study examines the exact relationship between parents' beliefs of the easiness of arithmetic for the child and the child's actual performance in the subject. Parents of children who do well in arithmetic would be expected to be less likely than parents of average and below average children to believe the child finds most topics in arithmetic easy to understand.

Further Topics Explored

In addition to the issues discussed so far, the present study extends the investigation to the relationship between children's arithmetic attainment and (a) parents' own numerical difficulties and (b) parents' beliefs about the child's performance as opposed to their ability in arithmetic.

Attitudes and Home Practices

Children's Attitudes to School

Blatchford (1996) found that children's liking for school did not relate to their achievement in arithmetic in years 7, 11, and 16 years. Stevenson and Lee (1990) did not find any relationship between children's attitudes to school and their performance in arithmetic, either. Based on this evidence, no variation would be expected in children's attitudes to school as a function of their arithmetic performance.

Children's Favourite School Subject

Based on evidence suggesting a positive relationship between arithmetic achievement and young children choosing arithmetic as their favourite school subject (Young-Loveridge, 1991), children who do better in arithmetic would be more likely to choose arithmetic as their favourite school subject than do children who do less well in the subject.

Children's Attitudes to Arithmetic

Based on research findings suggesting a positive relation between attitudes to arithmetic and achievement in early primary school (Aiken, 1970; Schofield, 1982; Stevenson & Lee, 1990; Young-Loveridge, 1991), children's attitudinal beliefs would be expected to relate to their arithmetic performance in that children with higher levels of performance would hold more positive attitudes to arithmetic, while children who do less well in arithmetic would be expected to express more negative attitudes to the subject.

While children's attitudes have usually been examined using a single index, for example, a liking for the subject (Tizard et al., 1988), the present study investigates children's attitudes to different measures, such as the textbook, the homework, and feelings about missing an arithmetic class. This would enhance accuracy in measurement by allowing detection of any variation between measures.

Children's Favourite Topic in Arithmetic

Young-Loveridge (1991) found that 9-year-old children's favourite topics in arithmetic were addition, subtraction, multiplication, and division, while children's least favourite topic was division, multiplication, and subtraction.

The present study examines children's most and least favourite topics in arithmetic as a function of their arithmetic performance.

Parents' Attitudes to Arithmetic

Parents' attitudes are also examined, based on evidence suggesting a significant association between children's and parents' attitudes (Aiken, 1972; Aiken & Dreger, 1961; Poffenberger, 1959; Poffenberger & Norton, 1959). Parents' attitudes are measured by asking them on their favourite school subject at age 8 to 9 years. Their preferences are thus examined as a function of children's arithmetic performance. We would expect parents of children who do better in arithmetic to report arithmetic as their favourite school subject more often than do parents of children doing less well in the subject.

Parents' Beliefs About the Academic Importance of Arithmetic

Based on evidence on the effect of parents' beliefs about the academic importance of arithmetic and children performance on the subject (Stevenson & Lee, 1990), parents of children who do better in arithmetic would be expected to hold higher views of the importance of arithmetic than parents of children doing less well in the subject.

Home Activities

Primary and high school children's arithmetic performance has been found to be influenced by the amount of engagement in academic (Stevenson & Lee, 1990), reading (Reynolds & Walberg, 1992), and achievement-related (Chen & Stevenson, 1995) activities at home. Based on this evidence, children who perform higher in arithmetic would be expected to engage more in numerical activities at home than do children who do less well in arithmetic.

Further Topics Explored

The present study further examines the relationship between children's arithmetic achievement and (a) parents' beliefs of the child's attitudes to arithmetic and (b) parents' reports of children's home activities.

Parental Help and Encouragement

Parental Help With Homework

Based on evidence suggesting a positive relation between parental involvement with the child's homework and children's academic motivation (Gottfried et al., 1994) and achievement in arithmetic and reading (Grolnick & Ryan, 1989; Stevenson & Lee, 1990), children who do better in arithmetic would be expected to report receiving more direct and indirect help than children who are average and below average in arithmetic. Parents' reports would be expected to vary accordingly.

Further Topics Explored

The present study further examines some topics which are relatively under-researched. More specifically, the study further examines the relationship between children's arithmetic attainment and (a) children's satisfaction with parental help, (b) parents' confidence in helping with the homework in arithmetic, (c) the amount of time parents spend with their child per day, and (d) parents' way of encouraging children to do well in school arithmetic.

Parent - School Relations and Parent Education

Relatively little is known about the relationship between the parents and the school, however, a few researchers have identified a positive relation between parental involvement in the child's schooling and children's school and math achievement. In general, children of parents who are more involved in school activities have been found to do better in school than children with parents who are less involved (Stevenson & Baker, 1987).

Curriculum Opinions

Stevenson and Lee (1990) found that while 80% of mothers believed the arithmetic curriculum in Grades 1 and 5 was "about right" relatively more mothers of children doing better in arithmetic than mothers of children who did poorly in the subject believed the curriculum in arithmetic is too difficult for the child. Accordingly, in the present study, more mothers of children who do well in arithmetic than mothers of children doing poorly in the subject would be expected to believe the curriculum is unsuitable for the child's age.

Information on the Curriculum

Tizard et al. (1988) found that the majority of parents would usually learn about the curriculum covered in class in the first three grades by looking at children's work. While the majority of parents, in the present study, would be expected to learn about the curriculum in arithmetic classes by looking at their child's work, the study further explores how the way parents are informed about the curriculum might vary as a function of children's arithmetic achievement.

Contact With Teacher

Tizard et al. (1988) found that children of parents who had greater contact with the school showed more progress in reading and writing than children with parents who did not have such contact. Also, parents' contact with the school was found to relate to children's numerical skills upon entry to school. In the present study, parents of children who do better in arithmetic would be expected to meet more with the teacher to discuss the child's progress in the arithmetic than do parents of children who do less well in the subject.

Further Topics Explored

In addition to the topics discussed so far, the present study examines the relationship between children's arithmetic attainment and parents' evaluation of the teacher's help with the child's difficulties in arithmetic.

Parent Education

Evidence has suggested a positive relation between parents' academic status and children's school (Baker & Stevenson, 1986; Stevenson & Lee, 1990), reading (Stevenson & Lee, 1990), and arithmetic (Chen & Stevenson, 1995; Reynolds & Walberg, 1992; Stevenson & Lee, 1990; Tizard et al., 1988) achievement. Based on this evidence, in the present study, parents of children who do better in arithmetic would be expected to be more academically affluent than parents of children who are doing less well in the subject.

METHODOLOGY

3.2.1 Introduction

This section describes the methods used to collect the information on social and environmental factors. It includes the design of the current investigation, a brief description of the sample (see chapter 2 for a detailed account), a summary of the procedures used to collect the data from the children and their parents, and a description of the instruments used.

3.2.2 Design

One purpose of the present thesis was to detect any association between children's achievement in arithmetic and some social and environmental factors, such as self-evaluations and other issues related to performance, attitudes and home practices, parental help and encouragement, and parent - school relations and parent education.

The examination involved comparing the reports of children who differed in arithmetic performance, that is, children of above average mathematical abilities, children of average arithmetic skill, and children with arithmetic difficulties. All children had at least average reading strengths. As the focus of the research is arithmetic performance, data on children with reading difficulties were not collected. Information on children's beliefs and practices was collected during structured interviews, using a questionnaire consisting of both open-ended and closed questions. The interviews provided data on children's self-concepts, attributions, and aspirations, attitudes and home practices, and reports of parental help with the homework.

In addition to children's reports, data from children's parents were also collected. The intention was to compare the reports of the parents whose children had been interviewed; those were parents of children belonging in one of the mathematical groups, that is, above average, average, or below average. Their reports were examined accordingly as a function of children's arithmetic achievement. Data from parents were collected based on a questionnaire comprising both open-ended and closed questions. The questionnaire included some of the items children had been asked to respond to (i.e., their beliefs about the child's performance and corresponding attributions, their beliefs about the child's attitudes and home practices, and their reports of helping the child with the homework), as well as an additional set of items on their beliefs about the easiness of arithmetic for their child, their own numeracy difficulties, their beliefs of the child's performance as opposed to ability, their own attitudes, the methods they employed to encourage the child to do well in arithmetic, their relation with the school, and their academic background.

As domain specific self-concepts may vary across academic subjects (Schunk, 1990) and as there is a need to differentiate between mathematical and verbal schemata (Marsh, 1990), information on self-assessment and other topics was collected separately for arithmetic and reading. While information was collected for both arithmetic and reading for the purpose of examining variation, however, the main focus was on arithmetic.

"Reading" was meant to refer to "Greek" or "Language". In children's and parents' questionnaires, the wording of the questions included "Language"; respondents, however, used any of the three words. Studies have shown it is not unusual to use these words interchangeably (Stevenson & Lee, 1990). In translating and coding the responses for the present thesis, however, the word "Reading" was used.

3.2.3 Sample

The investigation of social and environmental factors relating to arithmetic achievement included children whose arithmetic performance varied from below average, to average, and above average. All children had at least average reading ability. These children further participated in either one or both studies on cognitive factors associated with arithmetic achievement (see Table 2.2 in chapter 2).

Children belonging to the three arithmetic groups participated in the current examination. In total, there were 73 children: 36 above average, 20 average, and 17 below average in arithmetic. The three groups differed significantly in their math scores but not in their reading scores. There was a slight difference in mean age between below average and above average children, however, it was of small magnitude (two months). For children's scores on the five pre-test measures, as well as the statistical comparisons between groups on those measures, see Table 2.4 (chapter 2).

Children's parents also participated in the study for the purpose of investigating their beliefs and practices as related to children's arithmetic performance. The majority of parents (55 fathers and 65 mothers) returned the questionnaires (for frequencies per group, see Table 2.3 in chapter 2), however, not every item had been answered. As a result, the number of mothers' and fathers' responses varied with item. To account for such changes in frequencies of responses, reference to sample size will be constantly made in the analysis of parental variables. As it will be observed in the next section, some results are treated with caution due to low response frequencies (N.B. Frequencies of responses are always reported in the analysis of each item).

3.2.4 Procedure

Children belonging to the three arithmetic groups were seen twice. Information on social and environmental variables was collected after the completion of the cognitive tests at the end of each session. All children received the questions in the same order, half of the questions being introduced at the end of the first session, and the rest at the end of the second session. All children were interviewed by the author, who coded the responses on the spot or wrote them down verbatim and coded them later.

Parents received the questionnaire at home via the child. At the end of the first session, each child was given two envelopes, each of which clearly indicated the name of the father and the mother. Each envelope included a questionnaire and a letter with instructions. The envelopes were returned to school via the child again and were collected by the teacher or the author herself. Envelopes were asked to be sealed to ensure discretion. Parents were asked not to collaborate in answering the items.

3.2.5 Materials

Two questionnaires were constructed for the purpose of examining children's and parents' beliefs and practices individually. While some questions were common, parents had to respond to an additional set of items on further issues.

Children's Questionnaire

The children's questionnaire comprised 26 items examining three main topics: children's self-concepts (4 items: 2 structured, 2 open-ended), their attitudes and home practices (16 items: 11 structured, 5 open-ended), and their reports of parental help (6 items: 2 structured, 4 open-ended). For a copy of the items, see Appendix 3.1.

Table 3.1 shows the main areas of interest in the questionnaire. Pupils' self-assessments in arithmetic and reading were collected using an adapted version of the methods employed by Tizard et al. (1988). In addition, following evidence suggesting that children may be more accurate in their self-assessments when an external frame of reference (reference group) is used (Blatchford, 1997b; Marsh, 1990), children were shown a picture of three groups of children whose expressions varied from smiling to frowning and were told: "These are three groups of children. This group (pointing to the one on the left) is very good in arithmetic. This group (pointing to the middle one) is so-so in arithmetic. This one (pointing to the one on the right) is bad in arithmetic. Which group do you think you belong to ?" Children were also asked to justify their responses.

Children were also asked about their aspirations for their performance on arithmetic (e.g., "Do you want to be better in arithmetic ?") and justified their answers.

The procedure was repeated for children's self-concepts and aspirations in reading. All justifications were analysed based on Weiner's (1979) classification of internal and external causes of performance.

TABLE 3.1

Summary of Items in Children's Questionnaire

Topic	Summary of Items
Evaluation of Performance	* Children's self-concepts in arithmetic and reading; attributions * Children's aspirations for their performance in arithmetic and reading; justifications
Attitudes & Home Practices	* Children's attitudes towards school, arithmetic, and reading; justifications * Children's home practices in arithmetic and reading
Parental Help	* Children's reports of indirect help, general help with homework, and specific help in arithmetic and reading; levels of satisfaction

Information on children's attitudes was mostly collected based on a series of ratings which indicated their attitudes about school, arithmetic, and reading. The rating procedure was demonstrated by asking the children to indicate how much they liked school, arithmetic, and reading by pointing to one of four faces with expressions ranging from a broad smile to a deep frown. This method has been used in studies with young children (Tizard et al., 1988). Children were shown the index card on which the faces were printed, and were then told:

"Here we have a very happy face, a happy one, a sad one, and a very sad one. I want you to tell me which face best shows how you feel about ____ ?"

Questions on school attitudes included children's degree of liking school. The investigation of children's attitudes to arithmetic and reading, however, included children's opinions on separate individual measures, to ensure accuracy. Thus, children's attitudes toward arithmetic included opinions about the textbook, the homework, feelings about missing an arithmetic class, most and least favourite topics in the subject, and favourite school subject. Attitudes toward reading included feelings about the reading textbook, the reading homework, and missing reading on a regular day, as well as feelings about reading alone at home, to the parents, and to the teacher.

Children's home practices in arithmetic were investigated by asking the children whether they engaged in the following numerical activities: grouping, dealing with money, playing number games, helping with the cooking, time telling, and counting or doing operations. Children's reading activities at home were investigated through items on whether parents read to them and the number of books for reading at home. Whether children read alone at home or not was noted when asked about their attitudes toward reading alone. Children were asked to give examples of the behaviours they would engage in.

Children's reports of the help they received in doing their homework were examined through direct and indirect assistance with the homework. First, children were asked whether and who helped them indirectly with their school homework (e.g., whether anyone tidies up their room in order for them to study, prepares their meals, and keeps quiet); they were also asked whether and who helped them with their school homework in general; last, children were asked whether and who helped them with their specific homework in arithmetic and reading (e.g., whether anyone helps them with the homework by teaching, coaching, or studying together). All questions were open-ended. In the event of reporting being helped, children were further asked how satisfied they felt with this help; children rated their satisfaction based on the four faces whose expression ranged from a broad smile to a deep frown.

Parents' Questionnaire

Table 3.2 shows a summary of the main topics discussed in the parents' questionnaires. It consisted of 36 items in total. For a copy of the items, see Appendix 3.2.

Parents' evaluation of their children's performance were examined separately for their general scholastic, arithmetic, and reading achievement. The scale ranged from 1 (*above average*) to 4 (*cause for concern*). Parental attributions for children's arithmetic and reading performance were examined in the light of Weiner's (1979) classification of internal and external causes of performance.

In addition, parents were asked to discuss some further issues. These referred to parents' beliefs about the easiness of arithmetic and reading for the child (i.e., whether the child finds most topics in arithmetic easy to understand), personal reports of any numeracy or literacy problems they might have faced in the past (i.e., whether they had experienced any related difficulties at work, in getting jobs, in household management, in doing courses, in leisure, or other domains), and personal beliefs of children's performance as opposed to their ability in arithmetic and reading (i.e., whether the child is doing as well as she is capable of). Parents were welcome to use the *Don't Know* category when necessary. In total, 11 items examined parents' beliefs about issues related to children's achievement; all were structured.

Parents' views of children's attitudes toward school arithmetic and reading were collected separately and ranged from 1 (*strongly likes*) to 4 (*strongly dislikes*). *Don't Know* was also available as an option. In addition, parents were asked about their favourite school subject when they were at the age of 8, as well as about the importance of doing well in either subject; the latter involved rating from 1 (*most important*) to 8 (*least important*) a variety of school subjects, including arithmetic and reading. Information on children's home practices was collected by asking parents whether children engaged in any of the following activities: grouping, dealing with money, playing number games, helping with the cooking, telling the time, and counting (the same group of activities the child had already been asked to report in the case of arithmetic) and whether the child engaged in reading alone or whether they read to him (in the case of reading). There were 7 items in the investigation of parents' beliefs of children's attitudes and practices, 3 of which were open-ended.

Parents' involvement and encouragement were measured by the amount of help they provide the child with and the way of encouraging the child to do well at school. Parents were asked whether they helped children with their school homework in an indirect way, that is, by tidying up their room, preparing their meals, and keeping everyone quiet, as well as directly by teaching them, helping them with their difficulties, or studying together. Justifications were always provided. Parents' levels of confidence in providing help were examined using a scale from 1 (*very confident*) to 5 (*not confident at all*).

In addition, parents were asked to report how many hours they spent with the child per day; fixed response patterns involved sets of two hours from 1 (*0-2 hours*) to 5 (*8+ hours*). Finally, parents were asked whether and how they encouraged the child to do well at school, in arithmetic, and in reading; open-ended questions referred to each case separately. In total, 9 items examined parents' reports of help and encouragement, 6 of which were open-ended.

TABLE 3.2

Summary of Items in Parents' Questionnaire

Topic	Summary of Items
Evaluation of Performance	* Children's general school, arithmetic, and reading performance; attributions * Easiness of arithmetic and reading for the child * Numeracy and literacy problems * Children's performance as opposed to ability in arithmetic and reading
Attitudes & Home Practices	* Children's attitudes toward arithmetic and reading * Favourite school subject * Academic importance of arithmetic and reading * Children's home practices in arithmetic and reading
Parental Help & Encouragement	* Indirect help with school homework and direct help with homework in arithmetic and reading; levels of confidence * Time (hours per day) spent with child * Method of encouragement to do well at school, in arithmetic, and reading
Parent - School Relations & Parent Education	* Curriculum opinions * Way of being informed about the material covered in arithmetic and reading class * Contact with the teacher to discuss progress in arithmetic and reading * Evaluation of teacher's help in arithmetic and reading * Academic Status

Parents' relations with the school were examined based on the following items: opinions about the curriculum covered in class (e.g., whether it is suitable for the child's age), the way of being informed about the material covered in class (e.g., options included the child's book, what the child said, written information from school, or any of the above), frequency of meetings with the teacher to discuss the child's progress, and evaluation of the teacher's help with the child's difficulties; the latter involved a scale from 1 (*helps a lot*) to 3 (*not helps at* *all*). *Don't Know* was also available as an option. Finally, background information on parents' academic status was collected, based on six options: *primary*, *high school*, *lykion*, *technical*, *university*, and *other*. In total, 9 items covered parent - school relations and academic status: 5 structured and 4 open-ended.

Piloting

Both children's and parents' questionnaires were piloted using third-grade pupils and their parents from a Greek school in the south area of London. Overall, there were no alterations in the phrasing of items in either children's or parents' questionnaires; all items were clear to the respondents and were answered accurately.

The only change introduced was the elimination of one of the faces in children's response scale. Initially, there were five faces denoting respectively *very happy*, *happy*, *so-so*, *not happy*, and *not happy at all*. However, none of the children used the *so-so* category. Based on their own reports, when they wanted to express average feelings, they would either choose the *happy* or *not happy* category. This led to reducing the number of response categories to four.

RESULTS

3.3.1 Introduction

This section reports the findings of the examination of social and environmental variables in relation to children's arithmetic achievement. The sample consisted of children belonging to three ability groups: above average, average, and below average in arithmetic. Children's parents also participated and their responses were examined as a function of children's arithmetic performance. The investigation focuses first on group comparisons and then on regression analyses employed to predict children's achievement.

In this section, the findings are reported based on the following four thematic categories: evaluation and other issues related to performance, attitudes and home practices, parental help and encouragement, and parent - school relations and parent education. Some major aspects of children's beliefs, attitudes, and views were investigated both through questionnaires and interviews of the children and their parents. These included children's beliefs about their performance in arithmetic and reading, attitudes to reading and arithmetic, views of home practices, and amount of parental help with homework. In addition, information was obtained from parents about the child's ease with arithmetic and reading, their view of how to encourage their child's progress, the academic importance of arithmetic and reading, their knowledge of the school, and their academic background.

3.3.2 Group Comparisons on Social and Environmental Measures

In describing parents' reports, many χ^2 tests will be reported that involve a large proportion of small expected frequencies. This is because some response categories were uncommon. The chi-square value is the difference between observed and expected frequencies. In cases where expected frequencies are quite low, however, the power of the chi-square test drops dramatically, rendering the test statistic meaningless (Delucchi, 1983). In these cases, an alternative test will be reported. The Kruskal-Wallis 1-Way Anova examines possible differences among (three or more) independent groups based on combining the cases, ranking them, and calculating the average of those ranks. It is not affected by small sampling. No discrepancies were observed between the results of the chi-square and the alternative test value.

3.3.2.1 Evaluation of Performance, Attributions, Aspirations, Easiness, Relation Between Performance and Ability, and Parents' Numeracy and Literacy Difficulties

Children were asked how they saw their performance in arithmetic and reading and what they attributed it to. In addition, they were asked whether and why they would like to be better in each of these subjects. Children's parents were asked about their own beliefs about the child's general scholastic, arithmetic and reading performance. Their own attributions, as well as their beliefs about the child's ability and ease with the two subjects, were also examined. Finally, parents' own difficulties in arithmetic and reading were investigated. The aim was to determine whether children's and parents' beliefs varied with the child's actual arithmetic performance.

Evaluation of Performance

Children

The importance of examining children's self-concepts lies in their strong association with achievement (Schunk, 1990). Studies have shown that children between 7 and 9 years are generally not very accurate in their self-assessments (Blatchford, 1997b; Miserandino, 1996; Tizard et al., 1988). However, accuracy increases with age (Blatchford, 1997b; Chen & Stevenson, 1995), with a more general reference group as opposed to the rest of the children in the class (Blatchford, 1997b; Marsh, 1990), and with domain specific self-concepts (arithmetic or reading) as opposed to a general academic self-concept (Blatchford, 1997b; Schunk, 1990). Some studies have shown that 9-year-old children doing well in arithmetic are more likely to believe they are good in the subject than do their peers who do less well (Young-Loveridge, 1991). The present study investigated children's accuracy of self-assessments in arithmetic and reading.

Children were asked what they believed about their performance in arithmetic and reading. They were shown three groups of pupils printed on an index card, one of which was above average, the other was average, and the third was below average in arithmetic, and were asked which group they thought they belonged to and why. The same procedure was used to investigate children's self-concepts in reading.

Children classified themselves as belonging to one of three groups in arithmetic: above average, average, or below average. Table 3.3 shows the distribution of classifications according to children's maths ability group.

TABLE 3.3

Children's Perceived Performance in Arithmetic as a Function of Their Actual Performance

	ACTUAL PERFORMANCE		
	Above Average (*n* = 36)	**Average** (*n* = 20)	**Below Average** (*n* = 17)
PERCEIVED PERFORMANCE			
Above Average	30	14	12
Average	6	6	5
Below Average	0	0	0

One can see that no child judged themselves to belong to the below average group. A comparison between children who thought they were above average and the rest of their peers showed that the overwhelming majority of children claimed to be above average (χ^2 (1, 73) = 20.84, $p < .01$). There was no association between self-perception and maths ability (χ^2 (2, 73) = 1.74, ns).

Children were also asked which group they thought they belonged to in reading: above average, average, or below average. As already reported, children in all the math groups were at least average in reading. Based on their scores on the reading comprehension and the sequence tasks, children were categorised as average or above average in reading. As some children were not consistent in their performance on both tests, the following results will be based on children's performance on reading comprehension, while reference will be made to the pattern of results based on their performance on the sequence task.

TABLE 3.4

Children's Perceived Achievement in Reading as a Function of Their Reading Performance on the Reading Comprehension Test

	ACTUAL PERFORMANCE	
	Above Average (n = 33)	Average (n = 40)
PERCEIVED PERFORMANCE		
Above Average	23	33
Average	10	7
Below Average	0	0

Table 3.4 shows how children rated themselves on how well they were doing in reading. It can be observed that no child categorised themselves as below average in reading. The majority believed they possessed above average reading abilities (χ^2 (1, 73) = 20.84, $p < .01$). The same pattern of results was found with performance on the sequence task (see Appendix 3.3). Overall, there was no association between reading ability and self-perception (Reading Comprehension: χ^2 (1, 73) = 1.66, ns; Sequence: χ^2 (1, 73) = 1.42, ns).

Parents

Parents' beliefs about the child's performance were also examined, in view of evidence suggesting that parents' perceptions about the child's performance may influence children's actual achievement (Jacobs, 1991; Parsons et al., 1982). Research has shown that parents of children in Grades 1 and 5 are accurate in their ratings of children's arithmetic achievement but not children's general school performance (Stevenson & Lee, 1990). The present study investigated parents' beliefs about the child's general scholastic, arithmetic, and reading performance. Their answers were based on a scale from 1 (*above average*) to 4 (*cause for concern*). Table 3.5 shows parents' beliefs about their child's performance in school, arithmetic, and reading as a function of children's math group.

Only one mother thought her child's achievement at school was a cause for concern. The rest of the parents held more positive views. Overall, both parents' views varied according to children's maths group (Kruskal-Wallis 1-Way Anova, Fathers χ^2 (2, 55) = 10.16, $p < .01$; Mothers χ^2 (2, 64) = 8.18, $p < .05$). As Table 3.5 shows, parents' views did show some correspondence with their children's performance in maths in that above average children were more likely to be rated as above average and below average children were more likely to be rated as average.

Parents' beliefs about the child's arithmetic performance also varied with children's math group (Kruskal-Wallis 1-Way Anova, Fathers χ^2 (2, 53) = 14.24, $p < .01$; Mothers χ^2 (2, 64) = 11.62, $p < .01$). As in the case of general scholastic achievement, the majority of parents of above average children believed their child's performance was above average and the majority of parents of below average children believed their child was average in arithmetic. There was only one father who acknowledged his child's below average arithmetic skill, however, he did not believe it was a cause for concern.

TABLE 3.5

Parents' Evaluation of Children's General Scholastic, Arithmetic, and Reading Achievement as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
General Scholastic						
Answered	**31**	**13**	**11**	**34**	**16**	**14**
Above Average	25	8	3	30	9	8
Average	6	5	8	4	7	5
Below Average	0	0	0	0	0	0
Cause for Concern	0	0	0	0	0	1
Arithmetic						
Answered	**31**	**12**	**10**	**34**	**16**	**14**
Above Average	29	7	4	30	9	6
Average	2	5	5	4	7	8
Below Average	0	0	1	0	0	0
Cause for Concern	0	0	0	0	0	0
Reading						
Answered	**31**	**13**	**11**	**33**	**16**	**14**
Above Average	23	8	6	24	12	10
Average	8	5	5	9	4	4
Below Average	0	0	0	0	0	0
Cause for Concern	0	0	0	0	0	0

No parent believed their child was below average in reading. Parents' beliefs about the child's reading performance were not found to vary significantly with children's arithmetic performance (Kruskal-Wallis 1-Way Anova, Fathers χ^2 (2, 55) = 1.65, ns; Mothers χ^2 (2, 63) = 0.05, ns). As Table 3.5 suggests, they all thought the child was at least average in reading. Since one of the selection criteria was that children should be at least average in reading, parental beliefs about children's reading performance were quite accurate.

Using a series of Mann-Whitney U-Wilcoxon Rank Sum W tests, parents' beliefs about the child's reading ability were examined as a function of children's scores on the reading comprehension and the sequence tasks. There was no association between fathers' beliefs about the child's reading skills and children's scores on the sequence task (z = - 0.92, ns); however, children with higher scores on the reading comprehension task were more likely to be rated by their fathers as above average in reading (z = - 2.11, $p < .05$). No association was found between mothers' beliefs and children's scores on the reading comprehension (z = - 1.84, ns) and the sequence (z = - 1.33, ns) tasks.

Reliability

Children's and parents' reports were compared for reliability purposes. The reliability measures included the percentage of perfect agreement between reports, Cohen's *kappa*, and the correlation coefficient of Spearman (r_s) because the data was in ordinal scale. Table 3.6 shows the agreement between children's self-concepts and parents' appraisal of children's performance in arithmetic and reading, following the *above average, average*, and *below average* categorisation. Sometimes *kappa* could not be computed because the values or number of categories of one respondent did not equal those of the other respondent.

TABLE 3.6

Percentage of Perfect Agreement, Cohen's *kappa*, and Spearman r_s Between Children's and Parents' Reports of Children's Achievement in Arithmetic and Reading

	Arithmetic				*Reading*			
	n	%	*k*	r_s	*n*	%	*k*	r_s
Child - Father	53	75%	-	.37*	55	65%	.13	.14
Child - Mother	64	75%	.36	.37*	63	75%	.33	.33*
Father - Mother	52	79%	-	.50**	54	78%	.47	.48**

*$p < .01$. **$p < .001$.

Parents' beliefs about children's performance in arithmetic and children's self-evaluations correlated moderately. In reading, only mothers' views correlated moderately children's views. Agreement between parents' reports was high and correlations were moderate for both subjects.

Attributions for Performance

Children

Attributions for performance exert a significant influence on academic achievement (Weiner, 1979). Studies with young children (Young-Loveridge, 1991) and older students (Chen & Stevenson, 1995; Reynolds & Walberg, 1992) have shown that high arithmetic performance is more likely to be attributed to internal rather than external reasons. The present study examined this hypothesis.

In explaining why they belonged to a particular group, some children simply referred to their attainments. Others did not offer any explanation. Most understood the question as requiring an attribution. Weiner's (1979) distinction between internal and external attributions was used to categorise the reasons given. Table 3.7 shows the frequencies of reasons given by children and how these varied with self-perception. Some children gave more than one attribution.

Children who considered themselves above average in arithmetic were more likely to attribute this performance to internal rather than external factors (Wilcoxon Matched-Pairs Signed-Ranks, $z = -2.78$, $p < .01$). Internal factors usually referred to ability (e.g., "... because I'm very good in maths"), a liking for the subject (e.g., "... because I like maths a lot"), prior knowledge (e.g., "... because now that we are doing subtraction, I was already ahead"), and home practice (e.g., "... because I do repetitions, I look into old books"). Other internal attributions involved effort (e.g., "... because I try to do them as best as possible") and attention (e.g., "... I think it's because I'm paying attention to my teacher").

External factors most commonly referred to help from family members (i.e., father, mother, and sister), either in the form of checking the homework (e.g., "... because dad is checking my homework"), teaching (e.g., "... because dad is very good in maths and he teaches me a lot"), or providing practice (e.g., "... because mum gives me additions to solve"). Other external factors involved teachers' attributions (e.g., "... because the teacher says I'm good") and task difficulty (e.g., "... because maths is very easy !").

TABLE 3.7

Children's Attributions for Their Perceived Performance in Arithmetic

	PERCEIVED PERFORMANCE	
	Above Average (n = 56)	**Average (n = 17)**
Number of children making no attributions	17	5
Internal		
Ability	13	12
Liking	10	0
Prior Knowledge	6	0
Solitary Home Practice	8	0
Effort	2	0
Attention	2	0
Total Internal[a]	**32**	**12**
External		
Help from Father	6	0
Help from Mother	7	0
Help from Sister	1	0
Teacher's Attributions	2	0
Task Difficulty	2	0
Total External[a]	**14**	**0**
Total Internal and External[a]	**7**	**0**

[a]Number of children making an internal, an external, or both internal and external attributions.

Children who thought they were simply average in maths attributed their performance only to their ability, for example, being slow in counting (Wilcoxon Matched-Pairs Signed-Ranks, $z = -3.06$, $p < .01$).

There was no evidence of an association between children's attributions for their arithmetic achievement and their actual level of math performance (Mann-Whitney U-Wilcoxon Rank Sum W tests not significant).

Table 3.8 shows the frequencies of reasons given by children and how these varied with self-perceptions in reading. Children who thought they were above average in reading were more likely to attribute it to internal rather than external reasons (Wilcoxon Matched-Pairs Signed-Ranks, $z = -4.88$, $p < .01$). Internal reasons most frequently involved ability, where children reported "being good" or "accurate", giving further perceived indices of good reading skills, such as speed (e.g., "... because I can read fast" or "... because I read slowly"), reading with feeling (e.g., "... because I can colour my voice"), fluency (e.g., "... because I can read easily and fluently" or "... because when I read, it's as if I know it word by word"). Liking for the subject (e.g., "... because I like reading a lot"), prior knowledge (e.g., "... because I'm ahead"), and practice out of school, that is, reading books other than the textbook at home, were also common, along with practising the homework many times at home.

External attributions involved help from the mother (e.g., "... because I read to mum") and teachers' attributions (e.g., "... because when I read, the teacher says 'Bravo, you read well' ").

TABLE 3.8

Children's Attributions for Their Perceived Performance in Reading (Reading Comprehension)

	PERCEIVED PERFORMANCE	
	Above Average (n = 56)	**Average (n = 17)**
Number of children making no attributions	8	2
Internal		
Ability	32	12
Liking	3	1
Prior Knowledge	3	0
Solitary Home Practice	5	0
Homework Practice	6	2
Total Internal[a]	**44**	**15**
External		
Help from Mother	2	1
Teacher's Attributions	5	0
Total External[a]	**6**	**1**
Total Internal and External[a]	**2**	**0**

[a]Number of children making an internal, an external, or both internal and external attributions.

Also, children who thought they were average readers were more likely to attribute it to internal rather than external reasons (Wilcoxon Matched-Pairs Signed-Ranks, z = - 3.30, p < .01). Internal factors related to ability (e.g., "... because when I read sometimes I forget words... sometimes if I read only once, I change the tone"), a liking for reading (e.g., "... because I can't stand Reading"), and practising the homework (e.g., "... because I don't read it many times; if I did, I'd be in Group 1"). Only once was external help mentioned (e.g., "... because if mum helps more, I'll be in the first group").

There was no association between children's reading performance on either task and the attributions they offered for their perceived achievement. A series of Mann-Whitney U-Wilcoxon Rank Sum W tests were conducted but were not found to be significant.

Parents

The positive relation between arithmetic ability and internal attributions (Chen & Stevenson, 1995; Reynolds & Walberg, 1992; Weiner, 1979; Young-Loveridge, 1991) was further examined based on parents' reports. Parents in the present study were asked what they believed accounted for their child's performance in arithmetic and reading. Parents' responses were also categorised and examined based on Weiner's (1979) internal versus external locus of causal attributions. Internal causes referred to ability, effort, persistence, fear of failure, obedience, nervousness, motivation, interest, and hyperactivity, among others. External attributions referred to the desire to please others, teaching quality, nature of the topic, peer influence, parental help, and parental encouragement, among others. Parents had to choose from a list of perceived causes. Some parents mentioned both internal and external causes. Table 3.9 shows the frequencies of reasons given by parents and how these varied with perceived performance.

Parents who thought their child was above average in arithmetic were more likely to attribute their child's performance to internal rather than external reasons (Wilcoxon Matched-Pairs Signed-Ranks, Fathers $z = -3.07$, $p < .01$; Mothers $z = -2.69$, $p < .01$).

Parents who thought their child was average in arithmetic were equally likely to attribute their child's performance to internal as well as external causes (Wilcoxon Matched-Pairs Signed-Ranks, Fathers $z = -0.40$, ns; Mothers $z = -1.83$, ns).

There was only one father who believed his child was below average in arithmetic; he attributed her arithmetic performance to both internal (viz., fear of failure, hyperactivity, laziness, and nervousness) as well as external reasons (viz., parental encouragement and parental help).

TABLE 3.9

Variation in Attributions According to Type of Attribution and Parents' Perception of Their Children's Performance in Arithmetic

	PERCEIVED PERFORMANCE				
	FATHER			MOTHER	
	AA (n = 40)	A (n = 12)	BA (n = 1)	AA (n = 45)	A (n = 19)
Number of parents making no attributions	5	2	0	2	3
Internal					
Ability	26	5	0	31	4
Interest	20	4	0	31	5
Motivation	8	1	0	10	2
Confidence	16	2	0	18	5
Persistence	13	1	0	19	5
Fear of Failure	2	0	1	4	4
Effort	17	4	0	26	10
Dis/Obedience	1	0	0	3	1
Hyperactivity	2	2	1	4	2
Nervousness	0	3	1	0	5
Laziness	0	1	1	1	3
Total Internal[a]	**34**	**10**	**1**	**42**	**16**
External					
Parental Encouragement	15	4	1	22	7
Parental Help	15	4	1	22	10
Please Parents	7	2	0	13	1
Peer Influence	1	0	0	3	2
Teaching Quality	14	3	0	21	4
Nature of Subject	3	1	0	9	3
Adverse Home Conditions	0	0	0	0	1
Total External[a]	**20**	**8**	**1**	**31**	**12**
Total Internal and External[a]	**19**	**5**	**1**	**30**	**12**

[a]Number of parents making an internal, an external, or both internal and external attributions.

There was some evidence of an association between children's actual performance in arithmetic (based on their scores on Young's and NFER tests) and parents' attributions for the child's performance. A series of Mann-Whitney U-Wilcoxon Rank Sum W tests showed that parents of children who were doing very well in arithmetic were more likely to attribute the child's performance to ability (Young: Fathers z = - 2.79, p < .01; Mothers z = - 2.52, p < .02; NFER: Fathers z = - 2.25, p < .05; Mothers z = - 2.16, p < .05). Fathers of above average children would attribute performance to the child's high interest in the subject (Young: z = - 2.15, p < .05; NFER: z = - 2.10, p < .05) and high levels of confidence (Young: z = - 2.01, p < .05).

Parents were also asked to give reasons for their child's performance in reading. Also using Weiner's (1979) distinction between internal and external causes, the same list that was used in the question on arithmetic was further used in this question. Table 3.10 shows parents' attributions for the child's perceived performance in reading.

Parents who thought their child was above average in reading were more likely to attribute the child's performance to internal rather than external causes (Wilcoxon Matched-Pairs Signed-Ranks, Fathers $z = -3.30$, $p < .01$; Mothers $z = -2.82$, $p < .01$). Parents who thought their child was average in reading would be equally likely to mention internal and external reasons (Wilcoxon Matched-Pairs Signed-Ranks, Fathers $z = -1.26$, ns; Mothers $z = -1.83$, ns).

TABLE 3.10

Variation in Attributions According to Type of Attribution and Parents' Perception of Their Children's Performance in Reading

	PERCEIVED PERFORMANCE			
	FATHER		MOTHER	
	AA (n = 37)	A (n = 18)	AA (n = 46)	A (n = 17)
Number of parents making no attributions	4	4	4	1
Internal				
Ability	24	6	23	6
Interest	19	2	28	3
Motivation	9	1	9	6
Confidence	14	2	16	5
Persistence	8	5	14	5
Fear of Failure	2	1	2	2
Effort	18	7	22	13
Dis/Obedience	1	0	1	0
Hyperactivity	1	1	3	3
Nervousness	2	1	0	3
Laziness	0	1	2	5
Total Internal[a]	**33**	**12**	**41**	**16**
External				
Parental Encouragement	13	4	21	8
Parental Help	11	6	19	8
Please Parents	9	1	9	3
Peer Influence	1	0	3	2
Teaching Quality	11	2	17	3
Nature of Subject	2	1	7	4
Adverse Home Conditions	0	0	0	1
Total External[a]	**19**	**8**	**29**	**12**
Total Internal and External[a]	**19**	**6**	**28**	**12**

[a]Number of parents making an internal, an external, or both internal and external attributions.

A series of Mann-Whitney U-Wilcoxon Rank Sum W tests examined the relation between parents' attributions and the child's actual performance. It was found that fathers of children who were doing very well in reading would be more likely to attribute their child's performance to ability rather than fathers of children who were doing less well in the subject (Sequence: $z = -2.40$, $p < .05$). No other attributional cause was found to relate to children's reading attainment.

Reliability

Children's attributions for their perceived performance in arithmetic and reading and parents' corresponding attributions were compared for the purpose of examining reliability of the reports. Reliability measures included the percentage of perfect agreement between children's and parents' reports, Cohen's *kappa*, and Pearson's *r* because the data was in nominal scale. Table 3.11 shows the agreement between children's and parents' attributions for the child's performance. These involved internal and external attributions.

TABLE 3.11

Percentage of Perfect Agreement, Cohen's *kappa*, and Pearson's *r* Between Children's and Parents' Attributions for Children's Achievement in Arithmetic and Reading

	Arithmetic				*Reading*			
	n	%	*k*	*r*	*n*	%	*k*	*r*
Internal								
Child - Father	47	51%	.00	.01	47	72%	.06	.11
Child - Mother	59	59%	.05	.15	59	78%	.11	.24
Father - Mother	45	91%	-.03	.04	46	93%	-.03	.03
External								
Child - Father	47	34%	-.10	.18	47	47%	.04	.07
Child - Mother	59	32%	-.06	.10	59	34%	.00	.00
Father - Mother	45	78%	.50	.52*	46	83%	.63	.66*

* $p < .001$.

Children's and parents' reports of internal and external attributions for children's arithmetic performance did not correlate significantly. The same was true for children's reading performance. Moreover, parents did not agree with each other in their reports of internal attributions for children's performance for either subject. There was only a moderate correlation between parents' reports of external attributions for the child's performance in arithmetic and reading.

Children's Aspirations for Performance

Studies have shown that children who set higher standards for themselves (internal reasons) do better in arithmetic than students who work hard for external gains, for example, simply to get a better job (Chen & Stevenson, 1995). Primary children in the present study were asked whether and why they would like to be better in arithmetic and reading separately. Children's reasons were categorised based on Weiner's (1979) distinction between internal and external locus of causality. Children's frequencies of justifications for their aspirations in arithmetic can be found in Appendix 3.4.

Despite their belief that they were above average in arithmetic, the vast majority of children wanted to be better in maths (χ^2 (1, 73) = 65.22, $p < .01$). The three math groups did not differ in their aspirations (χ^2 (2, 73) = 2.11, ns). There were only 2 children who deviated from this pattern: both were above average in maths and the reasons they provided referred to their ability: i.e., "... because I can't be better than what I am now" and "... because I'm very good".

Children offered both internal and external reasons for wanting to be better in arithmetic (Wilcoxon Matched-Pairs Signed-Ranks, z = - 0.12, ns). Internal factors related to children's liking for the subject (e.g., "... because I like arithmetic"), their competitive feelings (e.g., "... because I don't want other children to exceed me but me to exceed them"), their academic aspirations to be a good student (e.g., "... because I want to be a good student" or "... because I want to be good in all lessons"), as well as to the importance of learning, with special reference to arithmetic (e.g., "... because I will learn more things" or "... because I want to learn more operations").

External factors were equally frequent and related to thoughts about future employment and adult role in a more general way. For example, children wanted to be better in arithmetic to ensure entrance to the university to do a degree, to be better qualified for a job, and to be better equipped to teach and help their own children. Other external factors referred to children's desire to get a better grade on the report, to please others (e.g., father, mother, and teacher), and to be able to answer accurately to arithmetic problems presented orally by others on occasion (e.g., "... because when somebody asks how much is 1000 + 1000, I will know the answer"). A few children referred to arithmetic as being useful or to the enjoyment of verbal rewards when they would do well (e.g., "... because I will get 'Bravo' !").

The relationship between children's reasons to be better in arithmetic and their actual math performance was examined. It was found that children who thought about future employment and adult role had significantly higher scores on the NFER math test than children who did not refer to it (Mann-Whitney U-Wilcoxon Rank Sum W, $z = -2.24, p < .05$).

Children were also asked whether and why they would like to be better in reading. The majority of children wanted to be better (χ^2 (1, 73) = 61.49, $p < .01$). There was no difference between groups, that is, independent of their actual scores on reading comprehension (χ^2 (1, 73) = 0.58, ns) and the sequence (χ^2 (1, 73) = 0.27, ns) tasks. Only 3 children responded negatively and their reasons varied from "... because it (reading ability) is good enough" to "... because how much better can I read ?" and "... I can't answer".

Appendix 3.5 shows children's reasons for wanting to improve their reading skills as a function of their actual reading performance on the reading comprehension task. The frequencies of children's aspirations based on their reading scores on the sequence task can be found in Appendix 3.6.

Children who were above average on the reading comprehension test gave both internal and external reasons for wanting to be better in reading (Wilcoxon Matched-Pairs Signed-Ranks, $z = -1.03$, ns). Internal reasons involved the desire to read better (e.g., "... because I want to read better"), a liking for reading (e.g., "... because I like reading"), competitive feelings (e.g., "... because I'd like to be the 'first' in my class"), the desire to be a good student (e.g., "... because I want to be a much better student"), and an opportunity to learn more (e.g., "... because I want to learn more things"). The external reasons offered by children usually involved a desire to be better because of the usefulness of the subject (e.g., "... because reading is a very useful subject"), a desire to please others (e.g., "... because I don't want my parents to feel sad"), a desire to get a better grade (e.g., "... because I want to get an A"), while there would also be concerns about the future or employment (e.g., "... because when I grow up I want to read easier"), and criticism from their friends (e.g., "... because my friends call and say I didn't read the page well").

Children whose reading comprehension scores were average were more likely to mention internal rather than external reasons (Wilcoxon Matched-Pairs Signed-Ranks, $z = -2.39, p < .05$). Internal reasons were identical to those mentioned by the above average children, however, identification with others was also reported (e.g., "... because I want to be like my dad"). External reasons were again common to those mentioned by the above average children, except from usefulness (no average reader mentioned it) and praise (e.g., "... because I'll get Bravo") that no above average reader reported it.

Another series of Wilcoxon Matched-Pairs Signed-Ranks tests showed that children scoring above average on the sequence task wanted to be better more for internal rather than external reasons ($z = -2.64$, $p < .01$), while average readers (based on the sequence task) would offer both internal and external reasons ($z = -0.93$, ns).

A series of Mann-Whitney U-Wilcoxon Rank Sum W tests showed some association between children's reasons for wanting to be better in reading and their actual reading achievement. The total number of children who mentioned external reasons had significantly lower scores on the sequence task than children who did not mention external reasons ($z = -2.19$, $p < .05$). Accordingly, those children who wanted to be better in order to please others had significantly lower scores on the sequence task than children who did not mention pleasing others as motivating them to be better ($z = -2.30$, $p < .05$). Finally, there was a tendency for children who simply wanted to learn to read better and with less mistakes to be better on the sequence task than children who did not mention that reason; however, it did not reach significance.

Parents' Beliefs About Easiness of Arithmetic for Their Child

Studies conducted by Stevenson and his colleagues (Stevenson & Lee, 1990; Crystal & Stevenson, 1991) have shown that mothers of children doing less well in arithmetic are less aware of the child's problems with the subject. The present study investigated parents' knowledge of the child's difficulties in arithmetic and reading. More specifically, parents were asked whether the child found most topics in arithmetic and reading easy to understand. Table 3.12 shows parents' responses for each subject separately.

TABLE 3.12

Parental Beliefs About Easiness of Arithmetic and Reading for Their Child as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Arithmetic						
Answered	**31**	**13**	**11**	**34**	**16**	**14**
Yes	29	10	7	33	14	8
No	2	1	3	1	1	5
Don't Know	0	2	1	0	1	1
Reading						
Answered	**31**	**13**	**11**	**34**	**16**	**14**
Yes	29	11	9	32	16	12
No	1	1	2	2	0	0
Don't Know	1	1	0	0	0	2

No association was found between fathers' beliefs and children's arithmetic performance (χ^2 (4, 55) = 8.66, ns). While some fathers did not know whether the child faced any difficulties or not, the majority of those who did said the child found most topics in arithmetic easy to understand (χ^2 (1, 52) = 30.77, p < .01).

Mothers, however, varied in their beliefs as a function of children's math group (Kruskal-Wallis 1-Way Anova, χ^2 (2, 64) = 6.11, p < .05). As Table 3.12 also suggests, that was due to some mothers of below average children who thought their child did not find most topics in arithmetic easy to understand.

Only a few parents reported not knowing whether the child found most topics in reading easy. Of those who did know, the majority believed the child found most reading easy (Fathers χ^2 (1, 53) = 38.21, p < .01; Mothers χ^2 (1, 62) = 54.26, p < .01). Parents did not differ in their views (Fathers χ^2 (4, 55) = 3.67, ns; Mothers χ^2 (4, 64) = 9.07, ns).

Parents' Numeracy and Literacy Difficulties

In addition to children's difficulties, the present study further investigated numeracy and literacy problems that children's parents might have experienced and how these might vary with children's arithmetic and reading performance. Parents were asked whether and where they had faced any numeracy or literacy problems, further specifying the domain(s): at work, getting jobs, doing courses, household management, leisure, or other. Table 3.13 shows the frequencies of parents facing literacy or numeracy problems.

The majority of parents reported not having any numeracy (Fathers χ^2 (1, 38) = 17.79, $p < .01$; Mothers χ^2 (1, 44) = 32.82, $p < .01$) or literacy problems (Fathers χ^2 (1, 37) = 14.30, $p < .01$; Mothers χ^2 (1, 44) = 36.36, $p < .01$). There was no association between parents' arithmetic difficulties and children's math group (Fathers χ^2 (2, 38) = 0.88, ns; Mothers χ^2 (2, 44) = 0.87, ns). Accordingly, there was no association between parents' literacy problems and children's math group (Fathers χ^2 (2, 37) = 0.16, ns; Mothers χ^2 (2, 44) = 1.75, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 37) = 0.03, ns; Mothers χ^2 (1, 44) = 0.02, ns; Sequence: Fathers χ^2 (1, 37) = 0.00, ns; Mothers χ^2 (1, 44) = 2.10, ns).

TABLE 3.13

Frequencies of Parents Reporting Numeracy and Literacy Problems as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Arithmetic						
Answered	**19**	**11**	**8**	**25**	**10**	**9**
Problems	3	1	2	2	1	0
Reading						
Answered	**19**	**11**	**7**	**24**	**10**	**10**
Problems	4	2	1	2	0	0

Appendix 3.7 shows the frequencies of problems some parents had faced as a function of area, subject, and children's math group. The most frequent domains in which parents had faced both numeracy and literacy problems were in finding a job, at work, and in doing courses. Less often they would face problems in household management or in leisure. In facing problems with reading in specific, two fathers said they had never been good in reading, so they never learned how to spell accurately.

Parents' Beliefs About the Relation Between Performance and Ability

The present study further investigated a topic which is relatively unexamined in the literature on maths achievement, namely, parents' beliefs about the relationship between the child's performance and his ability. Parents were asked whether children were doing as well as they could in arithmetic and reading, their responses being categorised as *can do better* or *cannot do better.* Some parents had no clear idea about their child's performance and reported not knowing whether they could do better or not (*Don't Know*). Table 3.14 shows parents' beliefs about the relationship between children's ability and performance on arithmetic and reading, based on children's math group. The analysis will be based on those parents who knew about children's performance as opposed to their ability.

TABLE 3.14

Parental Beliefs About the Relation Between Children's Performance and Their Ability in Arithmetic and Reading as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Arithmetic						
Answered	**31**	**13**	**11**	**34**	**16**	**14**
Can Do Better	6	3	5	4	6	9
Cannot Do Better	23	8	5	27	9	4
Don't Know	2	2	1	3	1	1
Reading						
Answered	**31**	**13**	**11**	**34**	**16**	**14**
Can Do Better	6	2	3	7	5	3
Cannot Do Better	23	10	8	25	11	10
Don't Know	2	1	0	2	0	1

No differences among fathers were observed (χ^2 (2, 50) = 3.17, ns). The majority believed their children were doing as well as they could in arithmetic (χ^2 (1, 50) = 9.68, $p < .01$). Mothers, however, varied in their appraisals as a function of children's math group (Kruskal-Wallis 1-Way Anova, χ^2 (2, 59) = 13.64, $p < .01$). As Table 3.14 also suggests, mothers of above average children differed significantly in their beliefs from mothers of average (χ^2 (1, 46) = 4.36, $p < .05$) and below average children (χ^2 (1, 44) = 13.96, $p < .01$), in that above average children were thought of as doing their best (i.e., could not do better in arithmetic), while average and below average children were thought of as being able to do better in arithmetic. Mothers of the latter two groups did not vary in their beliefs (χ^2 (1, 28) = 2.39, ns).

Furthermore, Table 3.14 shows parents' evaluation of children's reading performance as opposed to their reading ability, based on children's math group. Most parents believed their children were doing as well as they could (Fathers χ^2 (1, 52) = 17.30, $p < .01$; Mothers χ^2 (1, 61) = 15.75, $p < .01$). The three groups of parents did not differ in their appraisals (Fathers χ^2 (2, 52) = 0.40, ns; Mothers χ^2 (2, 61) = 0.53, ns).

The same pattern of results was found when parents' beliefs were examined as a function of children's reading group (i.e., average and above average). Table 3.15 shows parents' beliefs based on children's performance on the reading comprehension test. Appendix 3.8 shows the corresponding beliefs as a function of children's performance on the sequence task.

TABLE 3.15

Parental Beliefs About the Relation Between Children's Performance and Their Ability in Reading as a Function of Children's Reading Group (Reading Comprehension)

	FATHER		**MOTHER**	
	AA	**A**	**AA**	**A**
Answered	**26**	**29**	**31**	**33**
Can Do Better	7	4	5	10
Cannot Do Better	18	23	24	22
Don't Know	1	2	2	1

The majority thought their children were doing as well as they could in reading. Parents of average and above average in reading children did not differ in their appraisals (Reading Comprehension: Fathers χ^2 (1, 52) = 1.35, ns; Mothers χ^2 (1, 61) = 1.61, ns; Sequence: Fathers χ^2 (1, 52) = 0.12, ns; Mothers χ^2 (1, 61) = 3.45, ns).

3.3.2.2 Attitudes and Home Practices

The second aim of the present investigation was to determine whether children's attitudes and home practices, and parents' corresponding views, varied with children's arithmetic achievement. Children were asked about their attitudes towards school, arithmetic, and reading, further reporting on their favourite school subject and the topics they like most and least in arithmetic. Parents reported whether the child liked arithmetic and reading, their preference regarding the subjects taught in school, and their views on the academic importance of arithmetic and reading.

Children's arithmetic and reading activities at home were investigated through reports of engaging in a variety of numerical and reading activities. Book availability was also examined as part of the investigation of children's reading activities at home.

Attitudes

Children' s Attitudes to School

Research has shown that children's attitudes to school are not associated with performance in arithmetic (Blatchford, 1996; Stevenson & Lee, 1990). In the present study, children were asked how much they liked school. Their attitudes ranged from 1 (*like very much*) to 4 (*not like at all*). These in turn corresponded to four faces varying in degree of happiness, printed on an index card. Children had simply to choose the face that best described how they felt about school. Table 3.16 shows children's attitudes to school, as a function of their arithmetic achievement.

TABLE 3.16

Frequencies of Children as a Function of Their Attitudes Toward School and Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Like Very Much	25	14	10
Like	10	6	7
Not Like Much	1	0	0
Not Like At All	0	0	0

Most children said that they liked school very much. This was more common than any other attitude (χ^2 (1, 73) = 8.56, $p < .01$). Liking for school was found to be unrelated to mathematical performance; the three groups of children did not differ in their liking for school (χ^2 (2, 73) = 0.7, ns).

Children who said they liked school very much gave varied reasons for their attitude. The overwhelming majority (86%) mentioned academic gains, such as learning more about the world and acquiring reading and writing skills. The next most common reason for liking school was the opportunity it provided for social interaction (37%), such as playing with friends and making new friends. A few children (10%) compared their being at school favourably with being at home, where they would be bored or interfered with by siblings. No other reason was mentioned by 5 or more children. The full set of reasons and their frequencies are given in Appendix 3.9.

While all the reasons for liking school very much were understandably positive about school, the reasons for liking school were more mixed. Some children seemed to be explaining why they did not like school very much; for example, they complained about the homework or having to get up early. The positive reasons for liking school corresponded to those given by children who said they liked school very much, that is, reasons concerning academic gains were most common (mentioned by 39%) and opportunity for social interaction came next (mentioned by 26%). For a detailed account of children's reasons for simply liking school, see Appendix 3.10.

Children's Favourite School Subject

Research has suggested that children who do better in arithmetic are more likely to choose arithmetic as their favourite school subject (Young-Loveridge, 1991). Accordingly, children in the present study were further asked to name their favourite school subject. Table 3.17 shows how many children chose arithmetic as opposed to the rest of the subjects. Appendix 3.11 has a more detailed account of children's preferences.

TABLE 3.17

Frequencies of Children as a Function of Choosing Arithmetic as Their Favourite School Subject and Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Arithmetic	18	6	1
Other	18	13	14

While three children claimed to like all subjects equally, the rest had specific preferences. Of the children showing a preference for a specific subject, there was an association between picking arithmetic as the most favourite school subject and mathematical ability (χ^2 (2, 70) = 8.85, p < .05). Children with above average arithmetic ability were more likely than the rest of the children to mention arithmetic as their favourite school subject.

Children's Attitudes to Arithmetic

A positive relation exists between attitudes to arithmetic and performance in school arithmetic (Aiken, 1970). Studies have shown that young children who do well in arithmetic hold more positive views about the subject than do children who do less well in the subject (Schofield, 1982; Stevenson & Lee, 1990; Young-Loveridge, 1991). The relationship has been found to hold even when children's perceived performance is investigated (Stevenson & Lee, 1990). Children's attitudes towards arithmetic were examined through a set of individual measures, namely, opinions about the textbook and the homework and feelings about missing an arithmetic class. Table 3.18 shows children's responses (for gender differences, see Appendix 3.12).

Textbook in Arithmetic

Overall, the groups differed in their liking for their school arithmetic textbook (Kruskal-Wallis 1-Way Anova, χ^2 (2, 73) = 8.0, p <.05). As Table 3.18 also suggests, this was largely due to the small number of children who expressed mild and strong dislike for the textbook. These children were in the below average group.

Only a few children did not offer any explanation why they liked the textbook. The majority of children who liked the textbook, as opposed to those who did not, referred to its interesting and pleasant content (45%): nice exercises, new material, different problems from the ones the teacher gave, and so forth. The second most common reason was liking maths (26%). Other reasons involved the opportunity to learn more (13%) and ease or difficulty of the textbook (7%). No other reason was mentioned by 5 or more children.

On the other hand, the reasons most often provided for not liking the textbook related to children's negative attitudes towards the subject (e.g., "... I don't like these new additions-subtractions" and "... I don't like math, nor doing it; only sometimes) and to the difficulty often encountered because of the complexity of the textbook (e.g., "... it has difficult things to solve" and "... it has very difficult exercises").

TABLE 3.18

Frequencies of Children as a Function of Attitudes to Individual Measures in Arithmetic and Mathematical Group

	Answered	Like Very Much	Like	Not Like Much	Not Like At All
Arithmetic Textbook					
Above Average	36	29	7	0	0
Average	20	14	6	0	0
Below Average	17	8	5	3	1
Arithmetic Homework					
Above Average	36	26	9	1	0
Average	20	13	5	1	1
Below Average	17	10	5	2	0
Miss Arithmetic Class[a]					
Above Average	36	0	6	14	16
Average	20	0	3	11	6
Below Average	17	4	4	3	6

[a]The exact phrasing was *Very Happy*, *Happy*, *Sad*, and *Very Sad* respectively.

Homework in Arithmetic

The three groups of children did not differ in their attitudes towards arithmetic homework (Kruskal-Wallis 1-Way Anova, χ^2 (2, 73) = 1.3, ns). The majority of children said they liked the homework in arithmetic very much, as opposed to any other response category (χ^2 (1, 73) = 8.56, p < .01). Some children said they simply liked it and only a few children held negative feelings towards it.

The most common reason for liking the homework in arithmetic related to the issue of ease or difficulty (38%): some children liked it because the exercises were easy, while other pupils enjoyed the difficult ones more. Children's liking for maths, both in general as well as doing specific operations, was the second most common reason for liking the arithmetic homework (28%). Finally, 21% of the children who liked the homework recognised its advantage of learning more, practising, and expanding their knowledge. Other reasons related to the benefits of doing it at home (e.g., more time to solve them or bring them correct to school), a sense of being amused, or even in contrast to not having anything to do; however none of these reasons were mentioned by 5 or more children.

Children's (n = 4) reasons for not liking the arithmetic homework related to ability (e.g., "... because sometimes I don't know most of them and I sit and think how to do them" and "... because I'm not doing very well"), difficulty of the homework, and lack of free time to do other things.

Missing Arithmetic Class

Last, the three groups did not differ in their attitudes towards missing an arithmetic class (Kruskal Wallis 1-Way Anova, χ^2 (2, 73) = 3.39, ns). Children were more likely to say they were sad or very sad if they missed an arithmetic class than to say they were happy or very happy (χ^2 (1, 73) = 20.84, p < .01).

The majority of children who reported feeling a little or very sad if they missed an arithmetic class attributed it to their liking for arithmetic (70%). The next most common reason was their disappointment over not progressing, that is, not learning more maths (16%). No other reason was mentioned by 5 or more pupils.

There were only a few children who said they would be a little or very happy if they missed an arithmetic class. Those children referred to maths as being difficult (e.g., multiplication, division), complained about the homework being too much, expressed a mild dislike for the subject or a liking for other subjects, and expressed happiness for leaving school an hour earlier.

Children's Favourite Topic in Arithmetic

In addition to their attitudes toward the textbook, the homework, and missing a class, children were asked to name their most and least favourite topic in arithmetic. Young-Loveridge (1991) found that children's favourite topic in arithmetic was addition, subtraction, multiplication, and division (in order of preference); children's least popular topics were division, multiplication, and subtraction. The present study examined children's preferences, further more as a function of their performance in arithmetic. Table 3.19 shows children's responses as a function of their arithmetic achievement (for a more detailed account of children's preferences, see Appendix 3.13).

Some topics in arithmetic were found to be more popular than others. For example, operations were children's favourite part of arithmetic. Operations involved doing addition, subtraction, multiplication, or division, all of which would be performed either in isolation or as part of solving word problems. A few children preferred measuring, which referred to dealing with time, money, weight, length, or using metre rulers. Only a couple said they liked everything they had done in arithmetic.

TABLE 3.19

Frequencies of Children as a Function of Most and Least Favourite Topic in Arithmetic and Mathematical Group

	BEST			WORST		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Answered	**36**	**20**	**17**	**36**	**20**	**17**
All Best/None Worst	**2**	**1**	**0**	**13**	**6**	**3**
Operations	**33**	**18**	**15**	**20**	**14**	**13**
Addition	11	5	8	1	2	1
Subtraction	0	2	0	8	6	5
Multiplication	6	4	3	5	2	1
Division	10	4	1	5	4	4
Operations	0	0	1	0	0	0
Word Problems	3	2	1	0	0	2
Tables	2	1	1	1	0	0
'Epalithefsi'[a]	1	0	0	0	0	0
Measure	**1**	**1**	**1**	**3**	**0**	**1**
Decimal	**0**	**0**	**1**	**0**	**0**	**0**

[a]Re-doing an operation using a different method, as a way of checking whether the initial outcome was accurate.

All but two children justified their choice. The most common reason for choosing operations as the most favourite topic in arithmetic related to their ease or difficulty (52%): some children liked them because they were easy, others because they were difficult. The next most common reason for choosing operations referred to the procedures applied (30%): children reported being amused doing them and thought they were interesting, new, and very useful (e.g., "... because in addition, you add, you don't sit there counting"). Some children reported liking operations because they knew them well (12%), either from present or past practice. Last, a few children chose operations as their favourite topic in arithmetic because they learned more (8%) or because they exercised their mind by thinking a lot (8%).

Furthermore, Table 3.19 shows that operations were children's least favourite topic, too. Subtraction, in specific, was the most common topic mentioned by all three groups. Children reported not liking subtraction usually because it was difficult and time consuming; borrowing, in specific, was considered very confusing. The second least favourite subject was division; children in the above average group also mentioned multiplication as the least favourable topic in arithmetic.

Children's Attitudes to Reading

In addition to children's attitudes toward arithmetic, the present investigation also examined children's feelings about reading. Children's attitudes towards reading were examined based on the same measures employed in the examination of children's attitudes towards arithmetic (i.e., feelings about the textbook, the homework, and missing a class), as well as some additional questions on their feelings about reading alone, to their parents, and to their teacher. Appendix 3.14 shows children's responses to all measures. Children's attitudes towards reading were examined on the basis of both their arithmetic and reading scores. The latter included separate analyses for the reading comprehension and the sequence tasks, as it has been reported earlier that children were not consistent on both tests. All, however, were at least average.

Table 3.20 shows children's attitudes toward the textbook and the homework, as well as their feelings about a day at school without any reading. While opinions on the textbook and the corresponding homework would refer to the book used in the Language course, feelings about missing reading would generalise to all taught subjects that would normally require it, such as History, Religion, or Geography, among others.

The most common attitude was liking the textbook very much (χ^2 (1, 73) = 3.96, $p < .05$). Children did not differ in their attitudes as a function of their arithmetic group (χ^2 (2, 73) = 0.95, ns) or reading group (Reading Comprehension: χ^2 (1, 73) = 2.61, ns; Sequence: χ^2 (1, 73) = 0.08, ns).

The most popular reason for this almost unanimous positive view of the textbook used in Language was its attractive content; children referred to the exciting stories, the beautiful pictures, and the nice exercises which were also considered to be easy. The next most common reasons referred to children's liking for reading and the benefit of learning new things by reading the textbook.

There were only two children who reported not liking the textbook: one believed the textbook was very easy and the other thought it was boring.

TABLE 3.20

Frequencies of Children as a Function of Attitudes to Individual Measures in Reading and Mathematical Group

	Answered	Like Very Much	Like	Not Like Much	Not Like At All
Textbook in Language					
Above Average	36	22	12	1	1
Average	20	11	9	0	0
Below Average	17	12	5	0	0
Reading Homework					
Above Average	36	24	10	1	1
Average	20	13	6	0	1
Below Average	17	11	4	1	1
Miss Reading Class[a]					
Above Average	36	1	5	14	16
Average	20	2	1	10	7
Below Average	17	4	3	2	8

[a]The exact phrasing was *Very Happy, Happy, Sad,* and *Very Sad* respectively.

Homework in Reading

The majority of children liked their homework in reading very much (χ^2 (1, 73) = 7.25, $p < .01$). They did not differ in their attitudes as a function of their arithmetic (χ^2 (2, 73) = 0.03, ns) or reading performance (Reading Comprehension: χ^2 (1, 73) = 0.42, ns; Sequence: (χ^2 (1, 73) = 0.25, ns).

The reasons children liked doing their reading homework varied from a general liking for reading to having more time to practise at home, realising that work and practice at home facilitated reading in class, as well as being pleased to show the teacher and the parents how well they could read. Some children also reported having fun doing the grammar and comprehension exercises which accompanied each text, as well as learning a lot from them.

Very few children reported not liking the homework. While some referred to their inadequate reading skills (e.g., "... because I'm not very good at it"), others thought the homework was difficult or a waste of time.

Missing Reading

Children were more likely to say they would feel sad or very sad if they did not do any reading at school rather than happy or very happy (χ^2 (1, 73) = 23.03, $p < .01$). They did not differ in their attitudes towards missing reading as a function of their arithmetic (χ^2 (2, 73) = 4.76, ns) or reading performance (Reading Comprehension: χ^2 (1, 73) = 0.02, ns; Sequence: χ^2 (1, 73) = 2.29, ns)

The most common reason for feeling sad was because they liked doing some reading in every subject (including Language). Children also mentioned that school would be boring and tiring without reading, that they would not learn much that day, or that they would have difficulty learning the lesson by heart at home (if they were not first introduced to it at school). Some children would feel sad because they would miss the chance to read to the teacher or to say the lesson to the teacher after having learned it by heart, or because they would forget the lesson by the time they did reading again. Those children who felt simply sad would also feel a little relieved because they did not read well, got confused with long words, and because the lesson might have been very difficult.

Although infrequent, some children would feel happy not doing any reading; that was either because of their negative attitudes towards reading or because of performance anxiety (e.g., they explained that they could not read well, so the teacher would not catch them not knowing the lesson or the homework or she would not reprimand them if they did not read well).

The investigation of children's attitudes to reading further included some questions on how children felt about different reading habits. More specifically, children were asked on their attitudes towards reading to themselves, to their parents at home, and to the teacher in class. Appendix 3.14 shows children's attitudes to the measures mentioned. Table 3.21 shows children's responses as a function of their mathematical performance.

Reading Alone

In the case of reading alone, children were more likely to say they simply liked it or liked it very much rather than not liked it or not liked it at all (χ^2 (1, 73) = 65.22, $p < .01$). Children did not differ in their attitudes toward reading alone at home as a function of their mathematical group (Kruskal-Wallis 1-Way Anova, χ^2 (2, 73) = 1.07, ns) or reading group (Reading Comprehension: χ^2 (2, 73) = 0.47, ns; Sequence: χ^2 (2, 73) = 0.09, ns).

The most common reason for children's positive feelings toward reading alone at home was the joy of the learning experience. Most children reported taking pleasure from learning new things, each with their own preference (e.g., Hercules, Christ, fairy tales, and comics). In other words, there was an element of interest and amusement associated with the content of the books they read at home. Also, children frequently felt this was good reading as well as writing practice: they were introduced to new words, phrases, and ideas, all of which were thought of as useful in writing compositions at school. Few children mentioned they liked it because it made time pass quickly or because they enjoyed the quiet home environment.

There were two children who reported not liking reading alone: one preferred others reading to him and the other felt bored because he had already read his out of school books "more than a hundred times".

Reading to Parents

Children belonging to the three mathematical groups did not differ in their attitudes toward reading to their parents (Kruskal-Wallis 1-Way Anova, χ^2 (2, 47) = 0.38, ns). From those children who reported reading to them, the majority tended to like it very much (χ^2 (1, 47) = 3.60, $p < .10$). Children differed, however, in their attitudes towards reading to their parents as a function of their reading group based on their scores on reading comprehension (χ^2 (1, 47) = 5.35, $p < .05$): above average readers simply liked reading to their parents, while average readers liked it very much. No differences in children's attitudes were observed between children who were average and above average on the sequence task (χ^2 (1, 47) = 1.04, ns).

TABLE 3.21

Children's Attitudes Towards Reading Alone, to Their Parents, and to Their Teacher as a Function of Mathematical Group

	Answered	Like Very Much	Like	Not Like Much	Not Like At All
Reading Alone					
Above Average	36	23	13	0	0
Average	20	10	10	0	0
Below Average	17	10	5	2	0
Reading to Parents					
Above Average	24	16	8	0	0
Average	14	8	6	0	0
Below Average	9	6	3	0	0
Reading to Teacher					
Above Average	36	26	8	2	0
Average	20	14	6	0	0
Below Average	17	13	4	0	0

The most common reason for children's positive attitudes was the desire to please their parents, by showing them how good they were in reading, while receiving praise and encouragement in return. Some children liked being listened to, while others realised this was good practice for school reading and an invaluable learning experience. Children reported enjoying it even though sometimes they may have got confused or reprimanded for not having read well.

Reading to Teacher

No differences were observed between children in their attitudes towards reading to the teacher, whether in terms of their math (Kruskal-Wallis 1-Way Anova, χ^2 (2, 73) = 0.22, ns) or reading performance (Reading Comprehension:

χ^2 (2, 73) = 1.74, ns; Sequence: χ^2 (2, 73) = 2.05, ns). The most common attitude was liking to read to the teacher in class very much (χ^2 (1, 73) = 14.92, $p < .01$).

Apart from taking pleasure from reading, the fact that they were being listened to by other children and especially by the teacher accounted for their gratification. Children enjoyed showing the teacher and the rest of the students how good they were in reading or what a good student they were in general. There was also some pride in being asked to read by the teacher or in putting their hand up when the teacher asked who would like to read. A few children said that was good practice which would improve their reading skills; they would occasionally be corrected if they made a mistake. Last, some mention would be made to the grades children would get or to the fact that if they read to the teacher, they would spend less time reading at home. While many children would experience performance anxiety, they considered the above advantages more important.

Parents' Attitudes to Arithmetic

The present study also examined parents' beliefs of the children's attitudes, that is, whether the child liked arithmetic and reading. Their beliefs were measured based on a scale ranging from 1 (*likes very much*) to 4 (*not likes at all*). *Don't Know* was also available as an option. Since some parents did not have a clear idea about the child's attitudes, the following analysis will be based on those parents who knew about the child's feelings. Table 3.22 shows parents' responses.

There were some differences between parents in their beliefs about the child's attitudes to arithmetic as a function of children's arithmetic group (Kruskal-Wallis 1-Way Anova, Fathers χ^2 (2, 51) = 9.86, $p < .01$; Mothers χ^2 (2, 62) = 15.76, $p < .01$). As Table 3.22 also suggests, while the majority of fathers of below average children thought the child held moderate feelings (i.e., *quite likes* or *not likes*), the majority of fathers of above average children thought the child held more positive attitudes (i.e., *quite likes* or *likes very much*). Also, the majority of mothers of above average children believed the child liked arithmetic very much, while mothers of below average children were equally likely to report any attitude.

TABLE 3.22

Parental Beliefs About Children's Attitudes Towards Arithmetic and Reading as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Arithmetic						
Answered	**30**	**13**	**11**	**34**	**16**	**14**
Likes Very Much	17	3	1	24	4	3
Quite Likes	11	8	4	9	10	3
Not Likes	2	1	4	1	2	4
Not Likes At All	0	0	0	0	0	2
Don't Know	0	1	2	0	0	2
Reading						
Answered	**30**	**13**	**11**	**34**	**16**	**14**
Likes Very Much	16	5	4	16	10	4
Quite Likes	9	6	6	16	5	8
Not Likes	2	0	1	2	1	0
Not Likes At All	0	0	0	0	0	0
Don't Know	3	2	0	0	0	2

Parents were more likely to report the child quite liked or liked reading very much rather than not liked or not liked at all (Fathers χ^2 (1, 49) = 37.73, p < .01; Mothers χ^2 (1, 62) = 50.58, p < .01). Parents' beliefs did not vary with children's arithmetic (Kruskal-Wallis 1-Way Anova, Fathers χ^2 (2, 49) = 1.48, ns; Mothers (χ^2 (2, 62) = 1.64, ns) and reading performance (Reading Comprehension: Fathers χ^2 (2, 49) = 0.40, ns; Mothers χ^2 (2, 62) = 0.78, ns; Sequence: Fathers χ^2 (2, 49) = 3.39, ns; Mothers χ^2 (2, 62) = 1.49, ns).

Reliability

While children's attitudes were assessed through a number of individual measures, parents' beliefs about the child's feelings referred to a single attitude. Despite the difference in the number of items, and possibly in the variability of attitudes within multiple measures, an attempt was made to examine the similarities between children's and parents' reports. Those parents who did not have a clear idea about the child's attitudes were not included in the following analysis.

TABLE 3.23

Percentage of Perfect Agreement, Cohen's *kappa*, and Spearman r_s Between Children's and Parents' Reports of Children's Attitudes Towards Arithmetic and Reading

	Arithmetic				Reading			
	n	%	*k*	r_s	*n*	%	*k*	r_s
Attitudes								
Father - Mother	48	60%	-	.56***	46	65%	.34	.40**
Attitudes to Textbook								
Child - Father	51	45%	-	.25	49	45%	-	.14
Child - Mother	62	53%	.15	.19	62	42%	-	.07
Attitudes to Homework								
Child - Father	51	51%	-	.33*	49	49%	-	.06
Child - Mother	62	48%	.09	.18	62	47%	-	.18
Attitudes to Missing a Class								
Child - Father	50	46%	-	.20	49	51%	-	.29*
Child - Mother	61	34%	.00	.07	62	48%	-	.39**

$^*p < .05$. $^{**}p < .01$. $^{***}p < .0001$.

Table 3.23 shows the agreement between children's attitudes towards arithmetic and reading and parents' beliefs about the child's attitudes. Measures include percentage of perfect agreement, Cohen's *kappa,* and Spearman r_s because the data was in ordinal scale. Kappa would not be computed when categories or values of one respondent did not equal those of the other respondent.

Children's attitudes referred to the textbook, the homework, and feelings about missing a class, while parents had to rank their child's general like or dislike for each subject. Both responses were based on a four-point scale. Although parents agreed moderately with each other, there was no overall agreement between children's attitudes to various measures and parents' beliefs about children's general attitude.

Parents' Favourite School Subject

Research has shown that parents' beliefs of children's attitudes to arithmetic are related to students' attitudes and subsequent performance (Aiken, 1972; Aiken & Dreger, 1961; Poffenberger, 1959; Poffenberger & Norton, 1959). The present study, accordingly, examined parents' favourite school subject when they were at their child's age, that is, 8-9 years old. Some parents mentioned more than one subject. Table 3.24 shows the proportion of parents reporting arithmetic and reading as their favourite school subject.

TABLE 3.24

Proportion of Parents Reporting Arithmetic and Reading as Their Favourite School Subject(s) as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Arithmetic	.52	.69	.63	.33	.31	.23
Reading	.11	.08	.25	.42	.25	.39

Parents' preference for arithmetic did not vary with children's arithmetic performance (Fathers χ^2 (2, 48) = 1.16, ns; Mothers χ^2 (2, 62) = 0.47, ns). Arithmetic was not mothers' favourite subject; they were more likely to report other subjects rather than maths (χ^2 (1, 62) = 9.29, $p < .01$). Fathers were equally likely to choose arithmetic as their favourite subject as any other subject (χ^2 (1, 48) = 1.33, ns).

Reading was not parents' favourite subject, either (Fathers χ^2 (1, 48) = 27.00, $p < .01$); Mothers χ^2 (1, 62) = 4.13, $p < .05$). There was no association between parents' preference for reading and children's math group (Fathers χ^2 (2, 48) = 1.47, ns; Mothers χ^2 (2, 62) = 1.42, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 48) = 0.76, ns; Mothers χ^2 (1, 62) = 0.62, ns; Sequence: Fathers χ^2 (1, 48) = 0.01, ns; Mothers χ^2 (1, 62) = 2.11, ns).

Parents' Beliefs About the Academic Importance of Arithmetic

Research has shown that parents who believe doing well in arithmetic is important have children who do better in the subject (Stevenson & Lee, 1990). The present study examined the hypothesis, by asking parents to rate eight school subjects in order of importance. The analysis was conducted based on whether arithmetic and reading were rated as one of the three most important school subjects. Table 3.25 shows parents' responses.

The majority of parents reported it was very important that their child did well in arithmetic and reading; that is, they believed arithmetic (Fathers χ^2 (1, 52) = 48.08, $p < .01$; Mothers χ^2 (1, 61) = 57.07, $p < .01$) and reading (Fathers χ^2 (1, 51) = 7.08, $p < .01$; Mothers χ^2 (1, 60) = 38.40, $p < .01$) were among the three most important subjects children were at school. Parents' beliefs about the importance of arithmetic did not vary with children's arithmetic performance (Fathers χ^2 (2, 52) = 3.40, ns; Mothers χ^2 (2, 61) = 0.86, ns).

TABLE 3.25

Frequencies of Parents as a Function of Personal Beliefs About the Importance of Arithmetic and Reading and Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Arithmetic						
Answered	**29**	**12**	**11**	**33**	**16**	**12**
Among First 3	29	11	11	32	16	12
Reading						
Answered	**29**	**12**	**10**	**32**	**16**	**12**
Among First 3	19	8	8	28	14	12

Parents' beliefs about the importance of reading did not vary with children's arithmetic (Fathers χ^2 (2, 51) = 0.75, ns; Mothers χ^2 (2, 60) = 1.67, ns) or reading performance (Reading Comprehension: Fathers χ^2 (1, 51) = 0.45, ns; Mothers χ^2 (1, 60) = 0.60, ns; Sequence: Fathers χ^2 (1, 51) = 0.26, ns; Mothers χ^2 (1, 60) = 2.41, ns), either.

Home Practices

Children

Research has shown that education experiences at home have a positive effect on children's achievement from entry to school (Tizard et al., 1988) to primary level (Stevenson & Lee, 1990; Young-Loveridge, 1991) and high school (Chen & Stevenson, 1995). Accordingly, the present study examined children's numerical and reading activities at home as a function of children's performance in each subject.

Children were asked on their arithmetic and reading practices at home. They were first asked whether they engaged in some numerical activities that form part of everyday life, for example, grouping, dealing with money, playing number games, help with cooking, telling the time, and counting. For gender as well as group differences, see Appendix 3.15.

More specifically, grouping would refer to putting things into groups, such as toys, books, and so forth. Dealing with money would include sorting or counting, as part of a game or a real situation (e.g., shopping). Number games would refer to playing games such as monopoly, cards, chess, and backgammon, among others. Cooking would refer to children's involvement in the process of cooking by measuring, weighing, and setting or keeping time. Time would refer to children's being conscious of or telling the time. Finally, counting would involve all the instances where the child might engage in the process of counting, whether at home or in any other setting outside school.

Table 3.26 shows the proportion of children involved in the numerical activities described above. While some practices were common amongst the children of all ability groups, others did show variation. For example, children varied in their likelihood of telling the time (χ^2 (2, 73) = 25.38, $p < .01$). As Table 3.26 also suggests, that was due to average and above average children being more likely to tell the time than children in the below average group. In addition, there was some variation in the reports of playing number games at home, but only in the degree: the majority of above average, average, and below average children reported playing number games at home (92%, 95%, and 71% cf.).

TABLE 3.26

Proportion of Children Engaging in Numerical Activities at Home as a Function of Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Activities			
Grouping	.86	.90	.88
Money	.64	.60	.65
Number Games	.92	.95	.71
Cooking	.03	.00	.00
Time Telling	.94	.75	.29
Counting	.67	.75	.77

Children's reading activities at home focused on children's involvement in any solitary reading which did not form part of school homework, and their parents' habit of reading to them. In each case, children were asked to report how often those activities took place. Children's answers were categorised as *Often* or *Not Often*. For the purpose of the present investigation, *Often* denotes from everyday to once a week; *Not Often* refers to any lower frequency, that is, from twice a month to reading over the holidays (e.g., Christmas and summer). Some children did not specify the frequency of the above activities; they were either vague (e.g., "sometimes") or did not mention any frequency. Table 3.27 shows how many children reported engaging in reading activities at home, and the corresponding frequencies.

Child Reading Alone

All children said they read alone at home, apart from their homework or school related reading. The majority said they read at home often, which is at least once a week. Usually, it would be bedtime reading to relax (evenings, afternoons, or both) or reading for recreation during the day or on the week-end.

TABLE 3.27

Frequencies of Children Engaging in Reading Activities at Home as a Function of Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Child Reads Alone	**36**	**20**	**17**
Often	35	20	13
Not Often	1	0	3
Not Specify	0	0	1
Parents Read to Child	**13**	**8**	**8**
Often	7	4	6
Not Often	4	3	2
Not Specify	2	1	0

Parents Reading to Child

While parents were more often portrayed as not reading to their children rather than reading to them, the difference was not significant (χ^2 (1, 73) = 3.08, ns). There was no association between children's reports of parents' reading to them and children's mathematical (χ^2 (2, 73) = 0.58, ns) and reading performance (Reading Comprehension: χ^2 (1, 73) = 1.03, ns; Sequence: χ^2 (1, 73) = 2.20, ns). As Table 3.27 further suggests, of those children being read to, the majority of children reported doing so often, from everyday to at least once a week. The rest of the children reported very low frequencies, from once or twice a month up to once in four months.

Appendix 3.16 shows the frequencies of reasons children gave when asked why their parents read to them. They believed parents read to them because they wanted to please them, to help them learn more things, to improve their reading skills (through modelling), as well as to pass their time (both the child's and their own), and to provide some relaxation for their child before they slept (bedtime). Very few children said their parents read to them because they could not do so themselves (because they were either tired or the text was too long), or that parents read to them to prevent them from watching television. One child said his parents were interested in the story they had been reading to him.

Parents

Parents, in turn, were asked on children's numerical and reading activities at home. Parents' reports of children's arithmetic activities were based on the same set of activities children's reports were based, namely, grouping, dealing with money, playing number games, helping with cooking, time telling, and counting. Table 3.28 shows the proportion of parents who reported their child engaged in the numerical activities mentioned above.

TABLE 3.28

Proportion of Parents Reporting Children's Engagement in Numerical Activities at Home as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Activities						
Grouping	.43	.46	.60	.35	.31	.39
Money	.43	.54	.30	.56	.56	.54
Number Games	.86	.54	.90	.85	.81	1.00
Cooking	.25	.39	.40	.47	.31	.39
Time Telling	.68	.69	.50	.82	.81	.62
Counting	.64	.69	.50	.62	.44	.54

Overall, parents' reports did not vary with children's arithmetic performance. The only significant difference between fathers was in their reports of children's involvement with number games; as in the case of children, however, their difference was only in the degree of engagement, with more than half of fathers within each group saying their child did play number games at home.

Parents were further asked whether and how often the child read alone at home, apart from their homework or school related books. Parents' responses were categorised as *Often* (i.e., the child read everyday, once a week, or very regularly) or *Not Often* (i.e., the child did not read very often, only on holidays, rarely, or never). Table 3.29 shows parents' reports as a function of children's arithmetic performance.

TABLE 3.29

Parents' Reports of Children's Reading Activities at Home as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Parents Reporting Child Reading Alone						
Answered	**30**	**13**	**8**	**34**	**16**	**13**
Yes	**28**	**11**	**6**	**33**	**16**	**12**
Often	12	8	4	19	12	5
Not Often	7	3	2	9	3	7
Not Specified	9	0	0	5	1	0
No	**1**	**0**	**2**	**1**	**0**	**1**
Don't Know	**1**	**2**	**0**	**0**	**0**	**0**
Parents Reporting Reading to Child						
Answered	**28**	**13**	**9**	**34**	**16**	**13**
Yes	**8**	**4**	**4**	**19**	**10**	**9**
Often	0	0	3	6	6	3
Not Often	5	3	1	13	4	5
Not Specified	3	1	0	0	0	1
No	**20**	**9**	**5**	**15**	**6**	**4**

Some fathers did not know whether the child read alone at home because they usually came home late from work. The majority of parents, however, provided complete information on the child's activities. Most parents said the child engaged in solitary reading at home (Fathers χ^2 (1, 48) = 36.75, $p < .01$; Mothers χ^2 (1, 63) = 55.25, $p < .01$). Parents' reports did not vary with children's arithmetic (Fathers χ^2 (2, 48) = 5.92, ns; Mothers χ^2 (2, 63) = 1.39, ns) or reading group (Reading Comprehension: Fathers χ^2 (1, 48) = 0.36, ns; Mothers χ^2 (1, 63) = 2.13, ns; Sequence: Fathers χ^2 (1, 48) = 0.45, ns; Mothers χ^2 (1, 63) = 2.42, ns). Of those parents who said the child did not do any reading apart from that related to school, only two parents explained why: one attributed it to lack of time and the other to the child's preference for playing.

Table 3.29 also shows how often the child read alone, based on parents' reports. While some parents did not specify or were rather vague about the frequency of the activity, the majority gave specific information. Overall, parents were more likely to report the child read often rather than not often (Fathers χ^2 (1, 36) = 4.00, $p < .05$; Mothers χ^2 (1, 55) = 5.25, $p < .05$). Parents did not vary in their reports as a function of children's mathematical (Fathers χ^2 (2, 36) = 0.29, ns; Mothers χ^2 (2, 55) = 4.48, ns) or reading group (Reading Comprehension: Fathers χ^2 (1, 36) = 0.06, ns; Mothers χ^2 (1, 55) = 0.31, ns; Sequence: Fathers χ^2 (1, 36) = 0.51, ns; Mothers χ^2 (1, 55) = 1.72, ns).

Children were portrayed as reading quite often, usually 3 to 4 times per week, after finishing their homework or during bedtime (afternoon or evening). A few parents who explained why the child engaged in that activity mostly referred to a liking for reading, an interest in books, and a desire to learn. A couple of parents said the child was imitating them or that they themselves had no time to read to the child. The most popular books were fairy tales, adventures, mysteries, mythology, and comics. Only one parent mentioned an encyclopaedia.

When asked whether they read to the child, the majority of fathers said they did not do so (χ^2 (1, 50) = 3.92, p < .05). Fathers did not vary in their reports as a function of children's math group (χ^2 (2, 50) = 0.43, ns) and reading group (Reading Comprehension: χ^2 (1, 50) = 0.64, ns; Sequence: χ^2 (1, 50) = 1.98, ns). Mothers, however, were equally likely to read or not read to the child (χ^2 (1, 63) = 3.57, ns), despite a tendency (p = .0588) to report reading to the child rather than not. Mothers did not vary in their reports as a function of children's math group (χ^2 (2, 63) = 0.44, ns) or scores on the sequence task (χ^2 (1, 63) = 1.66, ns), however, they did show some variation with children's scores on the reading comprehension test (χ^2 (1, 63) = 8.62, p < .01): while the majority of mothers of average readers reported reading to the child, mothers of above average in reading children were equally likely to read or not read to the child.

Table 3.29 also shows that only a few parents reported how often they read to the child. Due to many low expected frequencies, it was unreliable to conduct statistical analysis; however, all fathers of average and above average children reported not reading often to the child and 3 out of 4 fathers of below average children reported reading to the child often. There was no variation in fathers' reports of frequency as a function of children's reading scores (Reading Comprehension: Fathers χ^2 (1, 12) = 0.44, ns; Sequence: χ^2 (1, 12) = 1.03, ns).

Only one mother did not specify how often she read to the child. The rest were equally likely to read to the child often as well as not often (χ^2 (1, 37) = 1.32, ns). No variation was observed in mothers' reports as a function of children's math scores (χ^2 (2, 37) = 2.23, ns) and reading scores (Reading Comprehension: χ^2 (1, 37) = 1.78, ns; Sequence: χ^2 (1, 37) = 1.34, ns).

Two main reasons were offered by parents for not reading to the child: one related to the child's age (the child was old enough to read alone) and the other to parents' own schedule (they were too busy working or looking after younger siblings). The very few parents who explained why they read to the child focused mainly on the benefits of being read to as a means of improving children's reading ability and expand their vocabulary (i.e., accurate intonation, precise pronunciation, or acquisition of new words). Only one father and one mother gave examples of what they read to the child; those included newspaper articles, a story book, and a book on animal life.

Reliability

Table 3.30 shows the degree of agreement between children and parents in their reports of children's home activities in arithmetic and reading. Percentage of perfect agreement, Cohen's *kappa*, and Pearson's *r* (nominal data) were used for that purpose.

Children's numeric activities at home were examined based on a list of activities including counting, grouping, cooking, playing number games, and telling the time. Children's reading activities involved solitary reading and parents reading to the child.

Parents agreed highly with each other in terms of children's involvement with number games, helping with the cooking, and telling the time, but not so high in terms of children's engagement in grouping and counting. The same was true for children's and parents' reports.

High levels of agreement were observed between children and parents as well as between parents in their reports of children's solitary reading. Agreement between reports on parents' reading to the child was moderate.

TABLE 3.30

Percentage of Perfect Agreement, Cohen's *kappa*, and Pearson's *r* Between Children's and Parents' Reports of Children's Numerical and Reading Activities at Home

	n	%	*k*	*r*
Arithmetic				
		Grouping		
Child - Father	51	45%	-.05	-.08
Child - Mother	62	45%	.10	.18
Father - Mother	48	56%	.13	.13
		Number Games		
Child - Father	51	69%	-.16	-.17
Child - Mother	62	77%	-.13	-.13
Father - Mother	48	90%	.64	.66***
		Counting		
Child - Father	51	43%	-.31	-.33*
Child - Mother	62	42%	-.24	-.27*
Father - Mother	48	65%	.26	.26
		Cooking		
Child - Father	51	71%	.08	.21
Child - Mother	62	60%	.04	.15
Father - Mother	48	73%	.42	.43**
		Telling Time		
Child - Father	51	59%	.04	.04
Child - Mother	62	71%	.17	.17
Father - Mother	48	73%	.32	.34*
Reading				
		Child Reading Alone		
Child - Father	48	94%	-	-
Child - Mother	63	97%	-	-
Father - Mother	47	98%	.79	.81***
		Parents Reading to Child		
Child - Father	50	60%	.14	.14
Child - Mother	63	65%	.33	.35**
Father - Mother	49	57%	.17	.19*

*$p < .05$. **$p < .01$. ***$p < .00001$.

Number of Resources

In their attempt to construct a model of maths achievement, Reynolds and Walberg (1992) found that number of resources, that is, books at home to read, had a significant indirect effect on children's math achievement in Grades 7 and 8. The present study, accordingly, examined the availability of books in children's homes. Children were asked on the amount of books at home to read, with special reference to books that were not part of the third-grade school curriculum. Table 3.31 shows children's frequencies as a function of number of books and mathematical group.

TABLE 3.31

Frequencies of Children as a Function of Book Availability at Home and Mathematical Group

	Above Average (n = 36)	**Average (n = 20)**	**Below Average (n = 17)**
Less than 10	7	5	7
10 to 30	19	13	5
More than 30	9	2	5
Other	1	0	0

Children's reports of the amount of books available at home did not vary with children's arithmetic (Kruskal-Wallis 1-Way Anova, χ^2 (2, 73) = 1.87, ns) or reading performance (Reading Comprehension: χ^2 (3, 73) = 1.09, ns; Sequence: χ^2 (3, 73) = 0.96, ns). The majority of children reported having more than 10 and less than 30 books to read at home.

3.3.2.3 Parental Help and Encouragement

The third aim of the present investigation was to determine whether reports of parental help and encouragement varied with children's arithmetic performance. Parental help was examined through a set of measures indicating direct and indirect involvement with the child's homework. More specifically, parents' indirect help with the homework (i.e., preparing meals and tidying up the room), their direct help with school homework, and their help with the homework in arithmetic and reading (i.e., tutoring or coaching) was the focus of both children's and parents' reports. Parents also mentioned the amount of time spent with the child every day. Children's satisfaction levels and parents' confidence levels were further examined.

Parents were asked whether and how they encouraged their child to do well at school, in arithmetic, and in reading. A distinction was made between motivational support and tuition or help.

Help With Homework

Children

Studies have suggested a positive relationship between children's achievement and help with their homework, in that children who receive more help with their homework in arithmetic are more likely to do better in the subject (Grolnick et al., 1989; Stevenson & Lee, 1990). The present study examines this hypothesis, further distinguishing between two types of help with the homework, namely, direct and indirect.

Indirect Help With Homework

Children were asked who prepared their meals, kept everybody quiet, and tidied up their room in order for them to do their homework. They were then asked who helped them with the school homework in general, as well as specifically in arithmetic and reading. Children's responses as a function of their arithmetic performance can be observed on Table 3.32.

Children mentioned their mother, their father, both parents, another member of the family (i.e., grandmother, siblings) or reported receiving no help at all (alone). Table 3.32 shows the frequencies of children as a function of mathematical performance and agent of help, where the categories presented are not mutually exclusive. Appendices 3.17 and 3.18 show children's responses as a function of their performance on the reading comprehension and sequence tasks respectively.

The majority of children reported receiving help from the mother in tidying up their room, keeping everybody quiet, and preparing meals. Children's reports of indirect help did not vary with their math group (χ^2 (6, 77) = 5.78, ns) and reading group on reading comprehension (χ^2 (3, 77) = 1.62, ns). Some association was found between children's reports and their reading scores on the sequence task (χ^2 (3, 77) = 43.39, $p < .01$; Cramer's $V = .75$, $p < .01$). As Appendix 3.18 also suggests, above average readers (based on the sequence task) would unanimously rely on others for help, while average readers would be more likely to rely on themselves. Not many children were able to explain why that help was provided. Those who did, however, referred to their being able to study in peace, better, and quicker, all of which would help them understand better what they read and become excellent students.

TABLE 3.32

Frequencies of Children as a Function of Agent of Help and Mathematical Group

	No Help (Alone)	Help from Mother	Help from Father	Help from Other Member
Indirect Help				
Above Average	15	17	2	4
Average	8	8	2	3
Below Average	3	13	1	1
General Help				
Above Average	3	29	16	0
Average	2	14	5	2
Below Average	0	17	5	0
Help with Arithmetic				
Above Average	6	21	11	1
Average	2	10	7	4
Below Average	0	14	7	0
Help with Reading				
Above Average	21	9	1	1
Average	10	7	0	1
Below Average	10	5	0	2

Note. Above Average $n = 36$. Average $n = 20$. Below Average $n = 17$.

General Help With School Homework

Children's reports of the help they received with their school homework in general can be observed on Table 3.32. There was no association between children's reports and their arithmetic group (χ^2 (6, 93) = 9.57, ns) and reading group on reading comprehension (χ^2 (3, 93) = 3.14, ns). Most children reported receiving help with their school homework instead of doing it alone. Mother was, again, the person most frequently helping with children's school homework, while father was frequently mentioned by children in the above average group. Some variation was found with children's scores on the sequence task (χ^2 (3, 93) = 15.85, $p < .01$; Cramer's $V = .41$, $p < .01$); some average readers reported receiving no help with their homework. The rest would mention mother as the primary source of help and only a few would mention their father. Help with the homework would typically involve checking and correcting mistakes.

Specific Help With Homework in Arithmetic and Reading

Children did not differ in their reports of the help they received with their arithmetic homework (χ^2 (6, 83) = 11.57, ns). The majority of children reported receiving help in doing their homework, rather than doing it alone. As Table 3.32 also suggests, mothers were most frequently reported as helping the child, and fathers were also mentioned by above average children. Children reported receiving help primarily because of the difficulty they faced with word problems and exercises, usually by means of explanation and encouragement.

Children's reports of the help they got with their reading homework did not vary with children's arithmetic group (χ^2 (6, 67) = 3.15, ns) or reading group based on reading comprehension (χ^2 (3, 67) = 2.32, ns). While children were more likely to do the reading homework alone, they would still rely on their mothers' help. There was some association, however, between children's reports and their performance on the sequence task (χ^2 (3, 67) = 37.07, $p < .01$; Cramer's $V = .74$, $p < .01$). As Appendix 3.18 also shows, above average readers would rely entirely on others for help.

Satisfaction From Help With Homework

The present study further examined children's satisfaction from the help they received in doing their homework in arithmetic and reading, as a function of their achievement in the two subjects. Table 3.33 shows children's level of satisfaction from the help they received with their homework in arithmetic:

apart from two children in the above average group who did not answer this question (despite having said they were being helped by their mother), the majority of children reported feeling very satisfied from the help they received (χ^2 (1, 63) = 17.29, $p < .01$). Children did not differ in the level of satisfaction as a function of their arithmetic group (χ^2 (2, 63) = 2.28, ns). The most common reason for being so satisfied was because they managed to understand them and learn them better. This in turn made them better students and enabled them to get higher grades.

TABLE 3.33

Frequencies of Children as a Function of Satisfaction From Help in Arithmetic and Reading and Mathematical Group

	Helped	Very Satisfied	Satisfied	Not Very Satisfied	Not Satisfied At All
Arithmetic					
Above Average	28	21	7	0	0
Average	18	12	6	0	0
Below Average	17	15	2	0	0
Reading					
Above Average	15	10	5	0	0
Average	9	7	2	0	0
Below Average	7	5	2	0	0

Those children who reported receiving help with their reading homework were further asked how satisfied they felt from this help. Table 3.33 shows children's responses. From those who received help, there was only one child in the average group who did not respond to the question. The majority of the children who responded reported being very satisfied from the help they got (χ^2 (1, 31) = 5.45, $p < .05$). Children did not differ in the degree of satisfaction as a function of their math group (χ^2 (2, 31) = 0.34, ns) or reading group (Reading Comprehension: χ^2 (1, 31) = 0.55, ns; Sequence: χ^2 (1, 31) = 0.08, ns). The most common reasons for feeling so satisfied was because they learned more, they understood better the material, and they enjoyed being cared for.

Parents

Parents were asked whether and how often they helped their child indirectly, by organising some routines, such as preparing meals, tidying up the child's room, and keeping everybody quiet. They were also asked whether and how often they assisted the child directly with their homework in arithmetic and reading. Parents were asked to justify their responses, and report how confident they felt with the help they provided with children's homework. Table 3.34 shows the frequencies of parents' responses as a function of children's math group.

Indirect Help With Homework

There was some association between fathers' reports of indirect help and children's math group (χ^2 (2, 44) = 6.94, $p < .05$; Cramer's $V = .40$, $p < .05$). As Table 3.34 also suggests, this was largely due to the majority of fathers of below average children being more likely to provide indirect help, while only half of the fathers of above average children reported doing so. The difference between fathers of above average children and fathers of below average children was statistically significant (χ^2 (1, 33) = 6.86, $p < .01$; Fisher's Exact test, $p < .01$). Fathers of average children did not differ from fathers of above average children (χ^2 (1, 36) = 0.75, ns) and below average children (χ^2 (1, 19) = 3.68, ns). There was no association between fathers' reports of indirect help and children's reading group (Reading Comprehension: χ^2 (1, 44) = 2.40, ns; Sequence: χ^2 (1, 44) = 2.76, ns).

The overwhelming majority of mothers reported helping their child by organising things, i.e., preparing their meal, keeping everybody quiet, cleaning up their room, and organising their free time (χ^2 (1, 62) = 54.26, $p < .01$). Only two mothers said they did not provide these. There was no association between mothers' reports of indirect help and children's math group (χ^2 (2, 62) = 0.88, ns) and reading group (Reading Comprehension: χ^2 (1, 62) = 0.00, ns; Sequence: χ^2 (1, 62) = 0.02, ns).

For the purpose of the present investigation, *Often* denoted everyday up to once a week, and *Not Often* referred to rarely, almost never, or very few times. Some parents did not specify how often they assisted the child with these activities. Of those who did, fathers seemed to be equally likely to say they did those often or not often (χ^2 (1, 18) = 0.89, ns), while mothers were more likely to do those often (χ^2 (1, 48) = 44.08, ns). *Often* for mothers, in specific, usually referred to everyday or almost everyday, based on their reports. There was no association between the frequency of indirect parental help and children's math group (Fathers χ^2 (2, 18) = 0.64, ns; Mothers χ^2 (2, 48) = 0.86, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 18) = 0.75, ns); Mothers χ^2 (1, 48) = 0.67, ns; Sequence: Fathers χ^2 (1, 18) = 1.04, ns; Mothers χ^2 (1, 48) = 1.31, ns).

Not all parents explained why they assisted the child with these activities. Appendix 3.19 shows the number of parents who did, as a function of their reasons for providing that help. The majority of mothers said those routines were a prerequisite for the child to study properly, that they liked doing them because they saved time for the child to either rest or play, that the child was too young to do them alone, and that it was their obligation. The rest would give a variety of reasons, such as " ... because I forget my troubles ... ", " ... because she'll learn later how to organise her life ... ", or " ... I show her my love this way ... ". Fathers would occasionally say they helped so that the child had more free time or that it was their obligation.

Help With Homework in Arithmetic and Reading

Parents were further asked whether they helped their child directly with their homework, by coaching or tutoring, i.e., answering their questions, or studying together. In terms of frequency of help, *Often* would refer to everyday up to once a week, and *Not Often* denoted rare instances of help. Some parents did not specify how often they helped the child: they either did not refer to the frequency or were rather vague, that is, "... when is needed... " or " ... when she is in great difficulty ... ". Table 3.34 shows parents' responses for each subject separately.

The majority of parents reported helping their child by answering their questions, studying together, and helping with their difficulties in general (Fathers χ^2 (1, 54) = 12.52, $p < .01$; Mothers χ^2 (1, 64) = 49.00, $p < .01$). Parents' reports of direct help with the homework in arithmetic did not vary with children's arithmetic group (Fathers χ^2 (2, 54) = 5.17, ns; Mothers χ^2 (2, 64) = 1.32, ns).

A large percentage of parents who reported helping their child with the homework in arithmetic did not specify how often they did so. Due to many low expected frequencies of responses, it was unreliable to run statistical tests. However, as Table 3.34 also suggests, the majority of mothers reported helping at least once a week (if not more often), while fathers would be equally likely to help often and not often.

Parents did not explain *why* they helped the child with their homework in arithmetic. They would rather describe *what* they did when they saw the child was in difficulty or when the child asked for help. Accordingly, they reported answering the child's questions or helping the child to find the solution.

TABLE 3.34

Frequencies of Parents as a Function of Type and Frequency of Help With Homework and Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Indirect Help						
Answered	**25**	**11**	**8**	**34**	**16**	**12**
Help	**12**	**7**	**8**	**33**	**15**	**12**
Often	5	3	3	25	10	12
Not Often	3	3	1	1	0	0
Not Specified	4	1	4	7	5	0
Help with Arithmetic						
Answered	**30**	**13**	**11**	**34**	**16**	**14**
Help	**21**	**8**	**11**	**31**	**15**	**14**
Often	5	1	4	10	6	12
Not Often	7	4	2	8	2	2
Not Specified	9	3	5	13	7	0
Help with Reading						
Answered	**28**	**12**	**9**	**34**	**15**	**13**
Help	**17**	**7**	**6**	**33**	**13**	**13**
Often	4	1	0	16	4	5
Not Often	8	3	2	7	4	3
Not Specified	5	3	4	10	5	5

While the majority of mothers reported helping the child with their homework in reading (χ^2 (1, 62) = 50.58, $p < .01$), fathers were equally likely to help or not (χ^2 (1, 49) = 2.47, ns). There was no association between parents' reports of help with the homework in reading and children's math scores (Fathers χ^2 (2, 49) = 0.16, ns; Mothers χ^2 (2, 62) = 3.28, ns). Appendices 3.20 and 3.21 show parents' frequencies as a function of help with the child's reading homework and children's reading group based on reading comprehension and sequence tasks respectively. Some association was found between fathers' reports and children's reading group based on the sequence task (χ^2 (1, 49) = 5.30, $p < .05$). As Appendix 3.21 suggests, the majority of fathers of children who were above average readers reported helping their child, while fathers of average readers would be equally likely to report helping or not helping. No other associations were observed (Reading Comprehension: Fathers χ^2 (1, 49) = 1.50, ns; Mothers χ^2 (1, 62) = 0.35, ns; Sequence: Mothers χ^2 (1, 62) = 0.18, ns).

As in the case of arithmetic, a large percentage of parents who reported helping the child with the homework in reading did not specify how often they did so. The few parents who did were equally likely to report helping often or not often.

Also, parents explained *how* they helped the child rather than *why* they did so. Responses would typically include helping with the intonation, explaining the meaning of new words or helping when the child needed it or asked for help.

Confidence in Helping With Homework

The study further examined parents' degree of confidence in helping children with their homework in arithmetic and reading. Parents' responses ranged from 1 (*very confident*) to 5 (*not confident at all*). Table 3.35 shows the frequencies of parents who reported helping their child with their arithmetic and reading homework as a function of group and level of confidence. Appendices 3.22 and 3.23 show the frequencies of parents as a function of confidence levels and children's performance on the reading comprehension and sequence tasks respectively.

As Table 3.35 also suggests, the majority of fathers and mothers were more likely to feel quite and very confident rather than moderately confident, a little confident, or not confident at all. Parents who helped their child with the homework in arithmetic did not vary in their confidence levels as a function of children's math group (Kruskal-Wallis 1-Way Anova, Fathers χ^2 (2, 38) = 4.35, ns; Mothers χ^2 (2, 59) = 1.76, ns).

TABLE 3.35

Frequencies of Parents as a Function of Help With Reading and Arithmetic, Levels of Confidence, and Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Arithmetic						
Helped	**21**	**8**	**11**	**31**	**15**	**14**
Not Mention	**1**	**1**	**0**	**1**	**0**	**0**
Mean Confidence Levels	**4.2**	**4.0**	**3.6**	**4.2**	**3.9**	**3.6**
Not Confident At All	0	1	1	2	1	1
A Little Confident	0	1	1	1	0	3
Moderately Confident	4	0	3	3	3	2
Quite Confident	1	0	2	8	6	2
Very Confident	15	5	4	16	5	6
Reading						
Helped	**17**	**7**	**6**	**33**	**13**	**13**
Not Mention	**1**	**0**	**0**	**1**	**0**	**0**
Mean Confidence Levels	**4.0**	**4.0**	**3.0**	**4.1**	**4.2**	**4.5**
Not Confident At All	0	1	1	2	1	0
A Little Confident	2	1	1	2	0	0
Moderately Confident	4	0	2	3	2	2
Quite Confident	2	0	1	9	2	2
Very Confident	8	5	1	16	8	9

Parents who helped their child with the homework in reading were also more likely to be quite or very confident rather than moderately confident, a little confident, or not confident at all. Parents' confidence levels did not vary with the child's math group (Kruskal-Wallis 1-Way Anova, Fathers χ^2 (2, 29) = 2.76, ns; Mothers χ^2 (2, 58) = 1.40, ns). Some association was found between fathers' reports of confidence and children's reading group based on performance on the sequence task (Mann - Whitney U - Wilcoxon Rank Sum W test, Sequence: $z = -3.34$, $p < .01$). As Appendix 3.23 also suggests, fathers of children in the above average reading group were more likely to feel quite or very confident, while fathers of average readers would be more likely to feel moderately confident on the average. No other associations between parental confidence in helping with children's reading homework and children's reading group were observed (Mann - Whitney U - Wilcoxon Rank Sum W test, Reading Comprehension: Fathers $z = -1.09$, ns; Mothers $z = -0.31$, ns; Sequence: Mothers $z = -1.74$, ns).

Children's reports of the help they received with their school homework and parents' reports of the help they provided the child with were compared for the purpose of examining the extent to which the two sources agreed with each other. Table 3.36 shows that the measures used included percentage of perfect agreement, Cohen's *kappa*, and Pearson's *r* (for nominal data).

TABLE 3.36

Percentage of Perfect Agreement, Cohen's *kappa*, and Pearson's *r* Between Children's and Parents' Reports of Indirect and Direct Help With Homework

	n	%	*k*	*r*
Parental Indirect Help with School Homework in General				
Child - Father	44	36%	-.05	-.19
Child - Mother	62	48%	-.06	-.18
Child - Parents	62	63%	-.03	-.09
Father - Mother	42	64%	.08	.20
Parental Direct Help with Homework in Arithmetic				
Child - Father	54	59%	.28	.37*
Child - Mother	64	67%	.13	.21
Child - Parents	64	89%	.18	.22
Father - Mother	53	77%	.17	.24
Parental Direct Help with Homework in Reading				
Child - Father	49	41%	.01	.03
Child - Mother	62	35%	.00	.00
Child - Parents	63	43%	.02	.11
Father - Mother	47	60%	-.02	-.03

*$p < .01$.

That involved comparing children's and fathers' reports, as well as children's and mothers' reports. In addition, a comparison between children reporting being helped and parents as a unit (a single source of help) was conducted. Table 3.36 shows the agreement between reports of the children and those of the father, the mother, and both parents. Agreement of reports was examined for indirect help with the homework, as well as for help with each subject separately. It was found that only children's reports of father's help with the homework in arithmetic and fathers' corresponding reports correlated moderately. No other significant correlations were observed between children's and parents' reports.

Child - Parent Interaction

The present study also examined the amount of time parents spent with their child. More specifically, parents were asked, on the average, how many hours per day they spent with the child. Their responses were categorised into the following: *0 to 2 hours, 2 to 4 hours, 4 to 6 hours, 6 to 8 hours,* and *at least 8 hours* per day. Table 3.37 shows the frequencies of parents as a function of time spent with the child and children's mathematical group.

TABLE 3.37

Frequencies of Parents as a Function of Hours Spent With Children (per day) and Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Answered	**31**	**13**	**11**	**34**	**16**	**14**
0 - 2 hours	10	6	5	2	0	1
2 - 4 hours	12	6	1	2	0	2
4 - 6 hours	6	1	2	10	6	5
6 - 8 hours	1	0	2	11	6	4
8 + hours	2	0	1	9	4	2

The majority of fathers reported spending a maximum of 4 hours per day with the child (χ^2 (1, 55) = 11. 36, $p < .01$), while the majority of mothers reported spending at least 4 hours with them on a daily basis (χ^2 (1, 64) = 39.06, $p < .01$). Parents' involvement with the child, as measured by the amount of time spent together every day, did not vary with children's mathematical group (Fathers χ^2 (8, 55) = 9.46, ns; Mothers χ^2 (8, 64) = 4.59, ns).

Parental Encouragement

In the present study, information on whether and how parents encouraged their child to do well at school, in arithmetic, and in reading was also collected. Table 3.38 shows parents' frequencies as a function of method of encouraging their child to do well at school, in arithmetic and in reading and children's mathematical group.

Parents' ways of encouragement were categorised as motivational, that is, through advice, suggestions, praise, reward, and giving examples or as instructional, that is, through help, teaching, coaching, and practising together. There were some parents who mentioned both ways and others who did not specify how they provided encouragement (e.g., "... in different ways... " or " ... I'm trying to make her feel comfortable in the society... "). The categories viewed on Table 3.38 are mutually exclusive.

First, parents were asked to report whether and how they encouraged their child to do well at school. The overwhelming majority of parents reported encouraging the child to do well at school (Fathers χ^2 (1, 52) = 48.08, $p < .01$; Mothers χ^2 (1, 62) = 58.06, $p < .01$). There was no association between parents' reports and children's math group (Fathers χ^2 (2, 52) = 0.81, ns; Mothers χ^2 (2, 62) = 0.84, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 52) = 1.10, ns; Mothers χ^2 (1, 62) = 1.02, ns; Sequence: Fathers χ^2 (1, 52) = 1.02, ns; Mothers χ^2 (1, 62) = 0.84, ns). Only two parents said they did not offer any encouragement; that was because the child was very efficient and thus did not need extra encouragement.

Of those parents who reported encouraging the child to do well at school, some did not specify how they did it or were rather vague (e.g., " ... I help her to be a decent person in society... " or " ... I try to make him confident... "). Based on the reports of parents who said they encouraged their child and specified the way, motivational support was significantly more common than tuition or help (Fathers χ^2 (1, 39) = 35.10, $p < .01$; Mothers χ^2 (1, 53) = 41.68, $p < .01$). Parents' method of encouragement did not vary with children's math group (Fathers χ^2 (2, 39) = 2.98, ns; Mothers χ^2 (2, 53) = 1.28, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 39) = 1.20, ns; Mothers χ^2 (1, 55) = 0.04, ns; Sequence: Fathers χ^2 (1, 39) = 1.20, ns; Mothers χ^2 (1, 54) = 0.53, ns). Overall, fathers and mothers would stress to the child the importance of doing well at school for later in life, whether as a result of good grades or learning in general, further giving relevant examples. They would encourage the child through reward, praise, discussion, building their confidence through telling them they have got a lot of potential and that they could do even better, showing them how satisfied they were with their performance or their success, and by challenging their competitive feelings.

TABLE 3.38

Frequencies of Parents as a Function of Method of Encouraging Children and Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
School						
Answered	**29**	**12**	**11**	**34**	**16**	**12**
Encourage	**28**	**12**	**11**	**33**	**16**	**12**
Motivational Support	22	9	7	27	14	9
Tuition / Help	0	1	0	2	0	1
Unspecified	6	2	4	4	2	2
Arithmetic						
Answered	**27**	**10**	**10**	**33**	**16**	**12**
Encourage	**26**	**10**	**10**	**29**	**16**	**11**
Motivational Support	9	5	7	14	6	5
Tuition / Help	8	4	2	6	7	5
Unspecified	9	2	2	9	5	1
Reading						
Answered	**27**	**9**	**10**	**33**	**16**	**12**
Encourage	**25**	**8**	**9**	**28**	**14**	**10**
Motivational Support	12	7	6	20	8	4
Tuition / Help	4	0	0	2	1	5
Unspecified	10	1	2	6	5	1

The majority of parents reported encouraging the child to do well in arithmetic (Fathers χ^2 (1, 47) = 43.09, $p < .01$; Mothers χ^2 (1, 61) = 42.64, $p < .01$). Parents' reports did not vary with children's mathematical group (Fathers χ^2 (2, 47) = 0.76, ns; Mothers χ^2 (2, 61) = 2.10, ns).

Based on those parents who reported encouraging the child and specified the way, motivational support and tuition or help with the homework were equally likely to be reported (Fathers χ^2 (1, 35) = 1.40, ns; Mothers χ^2 (1, 43) = 1.14, ns). There was no association between parents' way of encouragement in arithmetic and children's math group (Fathers χ^2 (2, 35) = 1.61, ns; Mothers χ^2 (2, 43) = 2.20, ns). As Table 3.38 also suggests, the majority of parents would rely a lot on motivation, while helping also with the homework in arithmetic, when the child was in difficulty or had questions. Overall, parents were equally likely to encourage the child by highlighting the importance of arithmetic in life, by suggesting repetitions, by reward and praise, as well as by helping the child with their homework and by providing a lot of practice. Suggestions were also popular, including guidelines for more efficient study methods and encouragement to study more, among others.

The overwhelming majority of parents reported encouraging the child to do well in reading, too (Fathers χ^2 (1, 46) = 31.39, $p < .01$; Mothers χ^2 (1, 61) = 30.31, $p < .01$). Parents' reports did not vary with children's math group (Fathers χ^2 (2, 46) = 0.14, ns; Mothers χ^2 (2, 61) = 0.10, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 46) = 0.00, ns; Mothers χ^2 (1, 61) = 3.45, ns; Sequence: Fathers χ^2 (1, 46) = 1.10, ns; Mothers χ^2 (1, 61) = 0.55, ns). Appendices 3.24 and 3.25 show parents' method of encouragement as a function of children's reading group based on their performance on the reading comprehension and sequence tasks respectively.

Fathers' way of encouraging the child to do well in reading did not vary with children's math group (χ^2 (2, 29) = 3.77, ns) and reading group (Reading Comprehension: χ^2 (1, 29) = 0.74, ns; Sequence: χ^2 (1, 29) = 0.51, ns). The most common method fathers employed for encouraging the child to do well in reading was through motivational support (χ^2 (1, 29) = 15.21, $p < .01$).

Mothers' reports of how they encouraged the child to do well in reading varied with children's math group (χ^2 (2, 40) = 9.19, $p < .05$; Cramer's $V = .48$, $p < .05$). As Table 3.38 also suggests, that was due to mothers of average and above average children being more likely to give motivational support in the form of buying books and encouraging them to read a lot, while mothers of below average children would also provide help with the homework and practice by reading to the child.

However, mothers' way of encouraging the child to do well in reading did not vary with children's reading group (Reading Comprehension: χ^2 (1, 40) = 0.94, ns; Sequence: χ^2 (1, 40) = 3.68, ns). The most common way was through motivational support (χ^2 (1, 40) = 14.40, ns).

3.3.2.4 Parent - School Relations and Parent Education

In the present study, parent - school relations were examined through parents' evaluation and knowledge of the curriculum in arithmetic and reading classes, contact with the teacher, and evaluation of the teacher's help with the child's difficulties in arithmetic and reading. The aim was to determine whether specific components of the parent - school relation varied with children's arithmetic achievement. Parents were also asked to report the highest academic degree they had completed. The aim was to examine whether parents' academic status varied with children's mathematical achievement.

Parent - School Relations

Curriculum Opinions

Stevenson and Lee (1990) found that Grade 1 and Grade 5 Chinese and Japanese children were better in arithmetic than their American peers. While there were no significant differences between mothers in their opinions on the curriculum, relatively more Asian than American mothers believed the curriculum in arithmetic was too difficult for the child. The present study examined parents' views on whether the curriculum covered in the arithmetic and reading classes was suitable for the child's age or not. The reasons parents offered for their opinions were also examined. Table 3.39 shows the frequencies of parents' responses. It can be observed that some parents did not have a clear opinion on the topic. The analysis focused on those parents who responded in either direction.

The majority of fathers believed the material covered in the arithmetic class was suitable for the child's age (χ^2 (1, 46) = 25.13, $p < .01$). The three groups of fathers did not differ in this respect (χ^2 (2, 46) = 4.86, ns).

Mothers, however, differed in their beliefs about the math curriculum (Cramer's $V = .44$, $p < .01$): mothers of above average children differed in their views from mothers of average children (Fisher's Exact test, $p < .05$) and those of below average children (Fisher's Exact test, $p < .01$). Mothers of average children and mothers of below average children did not differ in their views (χ^2 (1, 25) = 0.33, ns). As Table 3.39 suggests, while above average mothers reported unanimously the curriculum was appropriate, some average and below average mothers held opposite views.

TABLE 3.39

Frequencies of Parents as a Function of Their Opinion About the Arithmetic and Reading Curriculum and Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Arithmetic						
Answered	**31**	**13**	**11**	**34**	**16**	**12**
Suitable	24	10	6	32	10	8
Not Suitable	3	0	3	0	3	4
Don't Know	4	3	2	2	3	0
Reading						
Answered	**24**	**10**	**8**	**32**	**16**	**11**
Suitable	15	7	7	27	14	10
Not Suitable	3	1	1	4	2	1
Don't Know	6	2	0	1	0	0

Parents who held negative views about the curriculum were more likely to offer reasons for their beliefs rather than parents who held more positive views. Appendix 3.26 shows the frequencies of reasons parents gave for the perceived unsuitability of the curriculum taught in arithmetic classes. It can be observed that such beliefs were based on two major issues: one referred to the nature of the material and the other referred to the way the material was presented in books or by the teacher (method of instruction). Complaints about the nature of the material would include " ... it's too much and too difficult ... ", " ... it's too much ahead for his age ... ", or " ... my child went to school at the age of five-and-a-half ... "; comments on the method would include " ... it's a tiring method of teaching arithmetic ..." or "... there are no graphic representations which would make it easier for the child to find the solution ... ". It was found that parents (fathers) of above average children believed the material covered in class was too easy (e.g., one father argued that "... children at this age should be taught Algebra"), while parents of average and below average children would typically stress the difficulty of the material and the complexity of the method of presenting it to the children.

The majority of parents believed the reading curriculum was appropriate for the child's age (Fathers χ^2 (1, 34) = 16.94, $p < .01$; Mothers χ^2 (1, 58) = 33.38, $p < .01$). There was no association between parents' beliefs about the curriculum in reading and children's math group (Fathers χ^2 (2, 34) = 0.12, ns; Mothers χ^2 (2,

58) = 0.12, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 34) = 1.38, ns; Mothers χ^2 (1, 58) = 0.09, ns; Sequence: Fathers χ^2 (1, 34) = 0.04, ns; Mothers χ^2 (1, 58) = 1.98, ns).

While some parents did not specify why they thought the reading material covered in class was inappropriate (e.g., " ... it could be better ... "), the majority would give specific examples of either poor content, ineffective teaching, or confusing organisation of the material in the textbook. Appendix 3.26 shows the frequencies of reasons parents gave for their disapproval. In the first case, parents would be likely to say " ... the content is outdated, it is completely indifferent to the child, it does not correspond to children's present interests... " or " ... this change in the language finds me completely opposite (then gives example of a new grammatical rule)... ". In the latter case, parents would typically say " ... it should be better organised in the textbook... " or " ... the rules should be put in a more analytic way, so that children understand them better... ".

Information on Curriculum Covered in Class

Tizard et al. (1988) found that parents of children would usually learn about the curriculum covered in class during the first three years of school by looking at their child's work. Written information from school was scarce. The present study examined parents' way of being informed about the curriculum covered in arithmetic and reading, further more as a function of children's performance in arithmetic and reading.

Parents were asked how they were usually informed about the arithmetic and reading material covered in school, by selecting from the following options: *the child's textbook, the child's reports, school material*, or *some other source*. It could also be any combination of the above. Table 3.40 shows the proportion of parents reporting each source of information on the curriculum.

It can be observed that the most common source of information on the material covered in arithmetic class was the child's textbook. Some parents relied on what the children said about the material coverage, while very few parents mentioned written material sent from school. There was only one father who reported checking the child's homework everyday as a way of keeping in touch with what was being taught (categorised as *Other*). No association was found between children's math group and parents' way of being informed about the material covered in arithmetic classes (Child's textbook: Fathers χ^2 (2, 51) = 0.25, ns; Mothers χ^2 (2, 62) = 0.88, ns; Child's reports: Fathers χ^2 (2, 51) = 3.28, ns; Mothers χ^2 (2, 62) = 2.95, ns; School material: Fathers χ^2 (2, 51) = 0.26, ns; Mothers χ^2 (2, 62) = 2.48, ns; Other: Fathers χ^2 (2, 51) = 0.91, ns).

TABLE 3.40

Proportion of Parents as a Function of Source of Information on the Curriculum and Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Arithmetic						
Answered	**27**	**13**	**11**	**34**	**16**	**12**
Child's Textbook	.89	.85	.91	.97	.94	1.0
Child's Reports	.44	.39	.73	.32	.56	.50
School Material	.15	.15	.09	.15	.06	0
Other	.04	0	0	0	0	0
Reading						
Answered	**27**	**13**	**11**	**34**	**16**	**13**
Child's Textbook	.82	.85	.91	.97	1.0	1.0
Child's Reports	.44	.39	.73	.47	.63	.69
School Material	.15	.15	.09	.15	.06	.23
Other	.04	0	0	0	0	0

The textbook used in Language was the far commonest source of parents' being informed on the reading material covered at school. The next most common source was children's own reports. Very few parents mentioned the school as the source of information. One father said he would be informed by his child's homework (the same father as in the case of arithmetic; also categorised as *Other*). No association was found between the way parents were informed about the reading material covered in class and children's math group (Child's textbook: Fathers χ^2 (2, 51) = 0.53, ns; Mothers χ^2 (2, 63) = 0.87, ns; Child's reports: Fathers χ^2 (2, 51) = 3.28, ns; Mothers χ^2 (2, 63) = 2.29, ns; School material: Fathers χ^2 (2, 51) = 0.26, ns; Mothers χ^2 (2, 63) = 1.67, ns; Other: Fathers χ^2 (2, 51) = 0.91, ns).

Accordingly, the way parents were informed about the reading material covered in class did not vary with children's reading group, based on their performance on the reading comprehension test (Child's textbook: Fathers χ^2 (1, 51) = 0.35, ns; Mothers χ^2 (1, 63) = 1.05, ns; Child's reports: Fathers χ^2 (1, 51) = 0.98, ns; Mothers χ^2 (1, 63) = 1.27, ns; School material: Fathers χ^2 (1, 51) = 0.33, ns; Mothers χ^2 (1, 63) = 1.28, ns; Other: Fathers χ^2 (1, 51) = 1.15, ns) and the sequence task (Child's textbook: Fathers χ^2 (1, 51) = 0.35, ns; Mothers χ^2 (1, 63) = 0.87, ns; Child's reports: Fathers χ^2 (1, 51) = 1.57, ns; Mothers χ^2 (1, 63) = 0.20, ns; School material: Fathers χ^2 (1, 51) = 0.06, ns; Mothers χ^2 (1, 63) = 0.01, ns; Other: Fathers χ^2 (1, 51) = 0.91, ns).

Contact With the Teacher

In examining children from nursery to top infant level, Tizard et al. (1988) found that children of parents who reported having greater contact with the school had greater progress in reading and writing than children whose parents did not have such contact. Parents' contact with the school was also found to significantly relate to children's numerical skills in the nursery level. In the first three years of schooling, the majority of parents met with the teacher to discuss the child's progress in arithmetic at least once a year; they further reported those meetings were not very informative.

The present study examined parental contact with the school as a function of children's arithmetic and reading performance. Contact with school was measured through the occurrence and frequency of meetings with the teacher to discuss the child's progress in arithmetic and reading. Parents were asked whether and how often they met with the teacher to discuss the child's progress. Parents' responses were categorised as *Often* and *Not Often*. For the purpose of the present investigation, *Often* refers to informal meetings or scheduled meetings once a month or on the reports day once a semester. *Not Often* denotes very scarce meetings. Some parents did not mention how often they met or, if they did, they were not explicit about the frequency of meetings. Table 3.41 shows parents' responses.

While mothers were significantly more likely to say they met with the teacher rather than not meeting with her (χ^2 (1, 63) = 41.29, $p < .01$), fathers were not (χ^2 (1, 47) = 0.19, ns). There was no association between children's math group and parents' reports of meeting with the teacher to discuss children's progress in the subject (Fathers χ^2 (2, 47) = 1.62, ns; Mothers χ^2 (2, 63) = 2.40, ns).

Of those parents who had contact with the teacher and specified the frequency of the meetings, the majority reported meeting with her often (Fathers χ^2 (1, 21) = 5.76, $p < .05$; Mothers χ^2 (1, 55) = 33.62, $p < .01$). The frequency of parent - teacher meetings to discuss the child's progress in arithmetic did not vary with children's math group (Fathers χ^2 (2, 21) = 2.95, ns; Mothers χ^2 (2, 55) = 4.58, ns).

TABLE 3.41

Frequencies of Parents' Meetings With the Teacher About Arithmetic and Reading as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Arithmetic						
Answered	**25**	**13**	**9**	**34**	**16**	**13**
Contact	**10**	**8**	**4**	**30**	**16**	**11**
Often	6	7	3	28	14	7
Not Often	4	1	0	2	1	3
Not Mention	0	0	1	0	1	1
Reading						
Answered	**27**	**13**	**10**	**33**	**16**	**13**
Contact	**11**	**7**	**5**	**30**	**15**	**10**
Often	8	6	4	27	13	6
Not Often	3	1	0	3	1	3
Not Mention	0	0	1	0	1	1

Table 3.41 shows the same pattern of parent - teacher meetings to discuss the child's reading achievement. The majority of mothers reported meeting rather than not meeting with the teacher (χ^2 (1, 62) = 37.16, $p < .01$), while fathers did not (χ^2 (1, 50) = 0.32, ns). No association was found between parents' reports of meeting with the teacher to discuss the child's progress in reading and children's math group (Fathers χ^2 (2, 50) = 0.69, ns; Mothers χ^2 (2, 62) = 2.37, ns). and reading group (Reading Comprehension: Fathers χ^2 (1, 50) = 2.01, ns; Mothers χ^2 (1, 62) = 1.24, ns; Sequence: Fathers χ^2 (1, 50) = 1.15, ns; Mothers χ^2 (1, 62) = 0.46, ns).

Of those parents who reported seeing the teacher to discuss progress in reading and specified the frequency of the meetings, the majority reported meeting with the teacher often rather than not often (Fathers χ^2 (1, 22) = 8.91, $p < .01$; Mothers χ^2 (1, 53) = 28.70, $p < .01$). There was no association between parents' frequency of meetings with the teacher to discuss the child's progress in reading and children's math group (Fathers χ^2 (2, 22) = 1.57, ns; Mothers χ^2 (2, 53) = 3.90, ns) and reading group (Reading Comprehension: Fathers χ^2 (1, 22) = 0.17, ns; Mothers χ^2 (1, 53) = 0.21, ns; Sequence: Fathers χ^2 (2, 22) = 0.04, ns; Mothers χ^2 (1, 53) = 0.72, ns).

Evaluation of Teacher's Help

As a part of the examination of parent-school relations, the present study further examined the parents' views of the teacher's level of helpfulness. For that purpose, parents were asked to evaluate the teacher's help with children's difficulties in arithmetic and reading. Evaluations ranged from 1 (*helps a lot*) to 3 (*not helps at all*). Table 3.42 shows parents' responses as a function of subject and children's math group. As there were some parents who could not provide an answer, the following findings are based on those parents who were able to assess teacher's help.

No association was observed between fathers' beliefs and children's math group (χ^2 (4, 42) = 8.46, ns). The majority of fathers believed the teacher helped the child with their difficulties in arithmetic very much rather than simply helped or not helped at all (χ^2 (1, 42) = 21.43, $p < .01$).

TABLE 3.42

Parental Evaluation of Teacher's Help as a Function of Subject and Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Arithmetic						
Answered	**30**	**13**	**11**	**34**	**16**	**13**
Helps A Lot	22	10	4	32	13	6
Helps	1	1	3	0	2	3
Not Helps At All	1	0	0	1	0	0
Don't Know	6	2	4	1	1	4
Reading						
Answered	**30**	**13**	**11**	**34**	**16**	**13**
Helps A Lot	21	11	5	32	12	6
Helps	1	0	2	0	3	3
Not Helps At All	1	0	1	1	0	0
Don't Know	7	2	3	1	1	4

Mothers, however, varied in their beliefs (Kruskal-Wallis 1-Way Anova, χ^2 (2, 57) = 6.53, p <.05). As Table 3.42 also suggests, while the overwhelming majority of mothers of above average children reported the teacher was very helpful, mothers of average and below average children felt the teacher was moderately to very helpful.

Accordingly, the majority of fathers believed the teacher was very helpful with the child's difficulties in reading (χ^2 (1, 42) = 24.38, $p < .01$). There was no association between fathers' beliefs and children's math group (χ^2 (4, 42) = 6.95, ns) and reading group (Mann-Whitney U-Wilcoxon Rank Sum W test: Reading Comprehension: $z = -0.56$, ns; Sequence: $z = -0.56$, ns).

Some variation was found in mothers' evaluation of teacher's help in reading and children's math group (Kruskal Wallis 1-Way Anova, χ^2 (2, 57) = 6.57, $p < .05$). As Table 3.42 also suggests, while the majority of mothers of above average children thought the teacher was very helpful, some mothers of average and below average children believed the teacher was moderately helpful. However, there was no association between mothers' beliefs about the teacher's help in reading and children's reading group (Mann-Whitney U-Wilcoxon Rank Sum W test: Reading Comprehension: $z = -0.29$, ns; Sequence: $z = -0.10$, ns). The majority believed the teacher was very helpful (χ^2 (1, 57) = 32.44, $p < .01$).

Parent Education

Studies have shown that children of parents with higher academic qualifications do better in arithmetic (Chen & Stevenson, 1995; Reynolds & Walberg, 1992; Tizard et al., 1988) and in school in general (Baker & Stevenson, 1986; Stevenson & Lee, 1990) than do children of parents who are less educated. The present study examined the relationship between children's performance in arithmetic and reading and their parents' academic status.

Today, the Greek Educational System consists of 12 years of obligatory education (6 years of primary education, 3 years of high school education, and 3 years of lykion education). However, when the parents in the present study were at school (about 40 years ago), compulsory education was restricted to high school years (i.e., 9 years). Getting a degree, however, was very popular as a means by which to ensure future wealth and success.

Table 3.43 shows the frequencies of parents completing each academic level, based on children's mathematical group. The first level, *Primary Only,* shows the frequencies of parents whose formal education was limited to the first 6 years of school. *High School Compulsory* shows the number of parents who finished the basic education, that is, both primary and high school. *Lykion* reports those parents who finished the complete range of available school education, that is up to 18 years of age. Finally, the number of parents who pursued a degree can be found under *Graduate.* This category involved parents who completed 2-year courses (e.g., Teaching Academy, Technical Institutions, etc.) and formal university degrees (e.g., law, medicine, etc.).

TABLE 3.43

Frequencies of Parents as a Function of Academic Status and Children's Mathematical Group

	FATHER			MOTHER		
	AA	**A**	**BA**	**AA**	**A**	**BA**
Answered	**31**	**13**	**11**	**33**	**16**	**14**
Primary Only	2	0	2	1	0	3
High School Compulsory	4	3	0	2	2	2
Lykion	4	2	1	16	6	5
Graduate	21	8	8	14	8	4

As Table 3.43 also suggests, more than half of the fathers in each group had been awarded an undergraduate degree. It also suggests that mothers in each group had predominantly either completed Lykion or also held a university degree. There was no evidence that parents' educational level varied with children's math group (Fathers χ^2 (6, 55) = 5.57, ns; Mothers χ^2 (6, 63) = 8.87, ns) and reading group (Reading Comprehension: Fathers χ^2 (3, 55) = 0.37, ns; Mothers χ^2 (3, 63) = 4.64, ns; Sequence: Fathers χ^2 (3, 55) = 4.37, ns; Mothers χ^2 (3, 63) = 3.48, ns).

3.3.3 Multiple Regressions of Social and Environmental Factors on Children's Arithmetic Achievement

A series of multiple regression analyses explored the independent contribution of variables that might explain variation in arithmetic achievement. According to this method, the values of one variable, the dependent variable, are predicted from the values of other variables, the independent variables, by utilising the presence of an association between the three or more variables (Kinnear & Gray, 1995). Backward regression analyses were conducted, a method that removes individual variables whose probability of F is greater than .10, until a model is reached in which no more variables are eligible for removal. Variables are entered as a group and then removed individually (Norusis, 1993).

Entries in the regression analyses included those social and environmental variables that had been found to discriminate between children of different arithmetic performance (children variables) and between parents of children differing in performance (mother and father variables) and which were significantly correlated with children's achievement. Children's scores on the two mathematical tests, the "Y" Mathematics Series Y2 test (Young, 1979) and the Basic Mathematics Test B (NFER, 1971) were the dependent variables in the current analyses. In the individual analyses so far, the dependent variable was children's performance, however, in the form of ability groups: children's scores on the two mathematical tests were categorised into three mathematical groups. In the present analysis, raw scores were used as the dependent variable and two sets of analyses were conducted, one for children's scores on either test.

The prediction procedure involved identifying the regressors, summarising - when necessary - for the purpose of controlling for the number of variables to be entered in the multiple regression, and conducting the analyses. When outliers were observed, that is, cases outside 3 standard deviations, they were excluded and the analyses were repeated.

Data on social and environmental variables were collected by all children belonging to the three mathematical groups (n = 73). Children's parents filled out a questionnaire, however, the number of responses would vary with item, being at times very limited; that was especially true in the case of fathers. Thus, to control for and increase the size of the sample, separate analyses were conducted for the children, their mothers, and their fathers.

The independent variables or predictors were those children and parent variables which referred to children's general scholastic and arithmetic performance (responses referring to reading did not serve as regressors in the present analysis) and which were significantly correlated with arithmetic achievement ($p < .05$).

For the purpose of controlling for the number of variables to be entered in the regression analyses some variables were summarised. First, children's attitudes to arithmetic were assessed through three different measures: children's opinions about the textbook used in arithmetic, their opinions about the homework, and their feelings towards missing an arithmetic class. All three measures were assessed using a scale ranging from 1 (*like very much*) to 4 (*not like at all*); in the case of children's feelings towards missing a class, the wording of the scale was modified and the scale ranged from 1 (*very sad*) to (*very happy*). An average value was calculated for every child by adding the ratings of the three measures and dividing the total by the number of measures; thus, each child had a score from 3 to 12 as an average attitude to arithmetic.

Accordingly, children's involvement in numerical activities at home were measured through their engagement in six different activities related to arithmetic: grouping, dealing with money, playing number games, helping with the cooking, telling the time, and counting on several occasions. For each child, a single value was calculated, which represented the total number of activities the child reported engaging in at home (maximum score of 6). The same procedure was applied for parents' reports of children's involvement in numerical activities at home.

Last, children were asked to report whether they received any direct and indirect help with the homework, further naming the person who provided this help. A summarised variable, for example, Parental Indirect Help, was constructed to indicate the total indirect help a child received from neither parent (score 0), from the mother or the father (score of 1), or from both parents (score of 2). Combined variables were constructed for parents' corresponding reports, as well as for reports of direct help with the homework.

Appendix 3.27 shows Pearson's correlation coefficients of the main social and environmental variables examined in the present study (including the summaries) and children's performance on Young's test and the NFER test. Some variables were found to associate with performance on both tests, while others were significantly related to performance on only one test. Those variables that were not associated with performance on any one of the math tests were excluded from the prediction analyses. Appendix 3.28 shows intercorrelations among variables.

Table 3.44 shows the children and parent variables that correlated significantly with children's performance on at least one of the mathematical tests and further regressed on performance. Children variables included children's reports of their attitudes to arithmetic, their numerical activities at home, and the indirect help they received from both the mother and the father with the homework (total indirect help). Mother variables included their beliefs about the easiness of arithmetic for their child, the child's performance as opposed to ability, the child's attitudes to arithmetic, the numerical activities the child engaged in at home, the suitability of the curriculum, and their academic level. Fathers' beliefs about the easiness of arithmetic for their child, the child's performance as opposed to ability, and the child's attitudes to arithmetic correlated with and regressed on children's achievement.

TABLE 3.44

Social and Environmental Variables Associated With Children's Arithmetic Achievement

Variables	Young only	NFER only	Both
Children			
Attitudes to arithmetic	-	-	√
Numerical activities at home	√	-	-
Parental indirect help (total)	-	-	√
Mothers			
Ease of arithmetic for the child	-	-	√
Child's performance vs. ability	-	-	√
Child's attitudes to arithmetic	-	-	√
Child's numerical activities at home	-	√	-
Curriculum opinions	-	-	√
Academic level	√	-	-
Fathers			
Ease of arithmetic for the child	-	-	√
Child's performance vs. ability	-	√	-
Child's attitudes to arithmetic	-	-	√

There were some social and environmental variables that were significantly associated with children's arithmetic performance, yet, they were not included in the analysis for theoretical and statistical purposes. For example, mothers' and fathers' beliefs about the child's general scholastic and arithmetic performance correlated with children's actual performance, but it was rather unlikely to establish a causal relationship or interpret the relation, unless a model was constructed and employed (which was not the case in the present study). In other cases, a single score would suffice to render an association statistically significant, without, however, reflecting the overall pattern of responses: for example, the significance of the association between children's arithmetic performance and their aspirations to be better in arithmetic was based on one score only: 72 out of 73 children said they wanted to be better in arithmetic. Also, only one mother believed arithmetic is not among the three most important subjects taught in school; one score that deviated from the overall pattern of responses made the association between mothers' beliefs about the academic importance of arithmetic and children's arithmetic achievement significant. Last, a significant correlation would be disregarded due to small sample size; for example, parents' reports of indirect help with the child's homework correlated with arithmetic performance, however, that was based on a restricted number of parental responses (n = 42).

Children

As Table 3.45 suggests, children's attitudes to arithmetic (averaged) and their reports of total parental indirect help with the homework (from both the mother and the father) were significant predictors of their performance on both mathematical tests.

TABLE 3.45

Summary of Backward Multiple Regression Analyses of Children Variables on Arithmetic Achievement

Dependent Variable	**Predictor Variable**	**B**	**SE B**	**Beta**	**T**	**Sig T**
Young[a]	Child's attitudes	3.74	1.01	.39	3.70	< .01
	Parental indirect help	1.94	0.96	.21	2.01	.05
NFER[b]	Child's attitudes	4.38	1.53	.32	2.86	< .01
	Parental indirect help	2.85	1.46	.22	1.95	.06

Note. Parental Indirect Help involved 3 categories: 0 = Neither parent, 1= One parent, 2= Both parents.
[a]$df = 72, R^2 = .23, F = 10.37, p < .01$. [b]$df = 72, R^2 = .17, F = 7.08, p < .01$.

The two variables together explained 23% of the variance in children's achievement on Young's test. The two variables together explained 17% of the total variance in children's performance on the NFER test. No outliers were observed in either analysis.

Mothers

Five mother variables regressed on children's performance on Young's test (see Table 3.44). Mothers' beliefs about the easiness of arithmetic for their child, their beliefs about the child's performance as opposed to ability, their beliefs about the child's attitudes to arithmetic, and their academic level were significant predictors of children's performance. As Table 3.46 also suggests, the four variables together explained 57% of the total variance in achievement on Young's test. No outliers were observed.

TABLE 3.46

Summary of Backward Multiple Regression Analyses of Mother Variables on Children's Arithmetic Achievement

Dependent Variable	Predictor Variable	B	SE B	Beta	T	Sig T
Young[a]	Child's ease	3.55	1.93	.23	1.84	.07
	Child's performance	3.07	1.64	.27	1.88	.07
	Child's attitudes	2.21	1.21	.31	1.97	.05
	Academic level	1.47	0.71	.22	2.07	.04
NFER[b]	Child's attitudes	5.48	1.15	.57	4.77	< .01

Note. Child's Attitudes involved 3 categories: 1= Likes very much, 2= Quite likes, 3= Rest (Not likes much & Not likes at all).
[a] $df = 50$, $R^2 = .57$, $F = 15.28$, $p < .01$. [b] $df = 49$, $R^2 = .32$, $F = 22.72$, $p < .01$.

Five mother variables regressed on children's performance on the NFER test (see Table 3.44). From those variables, only mothers' beliefs about the child's attitudes to arithmetic remained in the equation. As Table 3.46 also suggests, 32% of the variance in performance on the NFER test was explained by mothers' beliefs about the child's attitudes to the arithmetic. No outliers were found.

Fathers

Two father variables regressed on performance on Young's test (see Table 3.44): only fathers' beliefs about the child's attitudes to arithmetic predicted performance on Young's test, explaining 26% of the total variance in children's scores. No outliers were observed.

TABLE 3.47

Summary of Backward Multiple Regression Analyses of Father Variables on Children's Arithmetic Achievement

Dependent Variable	Predictor Variable	B	SE B	Beta	T	Sig T
Young[a]	Child's attitudes	3.75	0.92	.51	4.07	< .01
NFER[b]	Child's attitudes	5.07	1.42	.47	3.57	< .01

Note. Child's Attitudes involved 3 categories: 1= Likes very much, 2= Quite likes, 3= Not likes much.
[a] $df = 49$, $R^2 = .26$, $F = 16.57$, $p < .01$. [b] $df = 47$, $R^2 = .22$, $F = 12.74$, $p < .01$.

Table 3.47 shows the results of the analysis. From the three father variables that regressed on performance on the NFER test, only fathers' beliefs of the child's attitudes to arithmetic were significant predictors of children's performance, explaining 22% of the total variance in achievement. No outliers were observed.

Total of Social and Environmental Variables

To examine how much variance in children's performance could be explained by the *total of* social and environmental elements, two more backward regression analyses were conducted. They examined the predictive value of all those social and environmental components that had been found in the individual regression analyses (per respondent) to predict children's performance on Young's test and the NFER test. Table 3.48 shows a summary of the findings of the two analyses.

The individual regression analyses conducted so far on children, their mothers, and their fathers have identified seven predictors of children's performance on Young's test: children's attitudes to arithmetic and the corresponding beliefs of their mothers and fathers, children's reports of indirect help (total from both parents) with the homework, mothers' beliefs about the easiness of arithmetic for their child, mothers' beliefs of the child's performance as opposed to ability, and mothers' academic status.

When all these variables regressed on performance on Young's test, three of them remained in the equation: as Table 3.48 also shows, children's attitudes to arithmetic, mothers' beliefs of their child's attitudes, and mothers' academic status together explained 54% of the total variance in performance on Young's test. No outliers were observed.

TABLE 3.48

Summary of Backward Multiple Regression Analyses of Social and Environmental Variables on Children's Arithmetic Achievement

Dependent Variable	Predictor Variable	B	SE B	Beta	T	Sig T
Young[a]	Child's attitudes (mother)	2.43	0.84	.35	2.87	< .01
	Child's attitudes (child)	3.44	1.04	.42	3.30	< .01
	Academic level (mother)	1.45	0.69	.24	2.09	.04
NFER[b]	Child's attitudes (child)	3.67	1.79	.29	2.05	.05
	Child's attitudes (father)	3.63	1.51	.34	2.41	.02

[a]$df = 43$, $R^2 = .54$, $F = 15.66$, $p < .01$. [b]$df = 47$, $R^2 = .28$, $F = 8.63$, $p < .01$.

Accordingly, previous analyses had identified four children and parent variables which had predicted children's performance. Those variables, then, regressed on performance on the NFER test: children's attitudes to arithmetic, the corresponding beliefs of their mothers and fathers, and children's reports of parental indirect help with the homework.

As Table 3.48 also suggests, two variables were found to predict children's performance: children's attitudes to arithmetic and fathers' beliefs of their child's attitudes together explained 28% of the total variance in achievement on the NFER test. No outliers were observed.

DISCUSSION

3.4.1 Introduction

Research in psychology has identified some social and environmental factors as potential determinants of children's academic attainment. Studies have associated some of those variables with children's arithmetic performance in specific; children in those studies, however, would sometimes experience difficulties in other school subjects as well.

The present study specifically accounted for this issue. It examined factors already found to be associated with school and arithmetic performance, the purpose being, however, to examine their relation to children's achievement in arithmetic in specific having controlled for performance in all other academic subjects. Thus, children in the present study varied only in their arithmetic performance, being below average, average, or above average, while their reading and general school performance was satisfactory.

Further effort was made to examine the math-specific nature of some of these factors. For example, while academic activities at home have been found to relate to arithmetic achievement (Stevenson & Lee, 1990), the present study focused on children's numerical activities at home and how these might explain variation in children's arithmetic achievement. The present study also explored the relation between arithmetic achievement and a number of factors which remain relatively unexamined. The purpose was to identify further more factors external or residing within the child which may account for differences in arithmetic attainment. Finally, information on beliefs and practices in reading were also collected. The purpose was to construct a profile of reading achievement in children varying in arithmetic performance.

In this section, the results of the group comparisons are discussed first, followed by a description of the new findings on the prediction analyses that add to our current knowledge of the relationship between social and environmental factors and children's achievement in arithmetic.

3.4.2 Understanding Variation in Social and Environmental Factors

First, the results of group comparisons are discussed in the light of previous literature, following the section areas examined so far.

3.4.2.1 Evaluation of Performance, Attributions, Aspirations, Easiness, Relation Between Performance and Ability, and Parents' Numeracy and Literacy Difficulties

Arithmetic

The importance of examining children's self-concepts lies in their strong association with achievement (Schunk, 1990). In the present study, however, there was no association between arithmetic ability and self-perception; the majority of 8-year-old children believed they were above average in arithmetic. While some studies have identified accurate 9-year-old children (Young-Loveridge, 1991), more studies have shown that children between 7 and 9 years are generally not very accurate in their self-assessments (Blatchford, 1997b; Miserandino, 1996; Tizard et al., 1988). Accuracy has been found to increase with age (Blatchford, 1997b; Chen & Stevenson, 1995). It could be argued that children were inaccurate because of the format of the question; that is, they may have been inclined to choose the smiling faces more often because of their attractive nature. A response bias while understandably possible is highly improbable given that this method has been used effectively in research with young children (Dowker, 1998; Tizard et al., 1988) and that in the present study children's justifications were soundly based and in accordance with the level implied by the perceived level of achievement.

Parents' beliefs about the child's performance in arithmetic and in school in general were also examined, in view of evidence suggesting that parents' perceptions about the child's performance may influence children's actual achievement (Jacobs, 1991; Parsons et al., 1982). Children's parents varied in their evaluation of their children's arithmetic and general scholastic achievement as a function of children's arithmetic performance; parents of above average children thought the child was above average in math and school, while parents of below average children thought the child was simply average. Accordingly, research has shown that parents of children in Grades 1 and 5 were accurate in their ratings of children's arithmetic achievement, but not when children's general school performance was investigated (Stevenson & Lee, 1990); mothers of children who did less well in arithmetic and in school gave their child much higher ratings than did mothers of children who did well in school and arithmetic. In line with the optimism that those mothers showed, parents in the present study were also found to overestimate children's performance.

Attributions for performance exert a significant influence on academic achievement (Weiner, 1979). Studies with young children (Young-Loveridge, 1991) as well as older students (Chen & Stevenson, 1995; Reynolds & Walberg, 1992) have shown that high arithmetic performance is more likely to be attributed to internal rather than external reasons. The present study used Weiner's (1979) distinction between internal and external attributions to categorise the reasons children offered for belonging to a particular group. Both children who thought they were average and those who thought they were above average in arithmetic attributed their performance to internal reasons (e.g., ability). While both groups held positive views about their performance, children's attributions were further examined as a function of children's actual performance; again, there was no association between attributions and children's performance.

The positive relation between arithmetic ability and internal attributions (Chen & Stevenson, 1995; Reynolds & Walberg, 1992; Weiner, 1979; Young-Loveridge, 1991) was further examined based on parents' reports. Stevenson and Lee (1990) have found a positive relation between parents' internal attributions and children's high levels of performance. Accordingly, parents varied in their attributions for their children's arithmetic achievement; those who thought the child was above average in arithmetic attributed it to internal reasons, while those who thought the child was simply average would attribute it to both internal and external reasons. Furthermore, there was some association between parents' attributions and children's actual performance in arithmetic; parents of children who were doing better in arithmetic were more likely to attribute the child's performance to ability, interest, and confidence.

Studies have shown that high school children who set higher standards for themselves (internal reasons) do better in arithmetic than students who work hard for external gains, for example, simply to get a better job (Chen & Stevenson, 1995). Children in the present study were asked whether and why they would like to be better in arithmetic and their reasons were categorised based on Weiner's (1979) distinction between internal and external locus of causality. The majority of children wanted to be better in arithmetic for both internal and external reasons. Some association was found between children's actual arithmetic performance and reasons for wanting to be better in arithmetic: those who did better on the NFER (1971) test were more likely than children doing less well on the test to think about future employment or adult role. Both of these observations show that young children place more emphasis on external rather than internal gains; it is possible that internal gains are more powerful determinants of older children's performance (Chen & Stevenson, 1995), while external gains are more applicable to young children.

Studies (Crystal & Stevenson, 1991; Stevenson & Lee, 1990) have shown that mothers of children doing less well in arithmetic are less aware of the child's problems with the subject; mothers of children doing less well in arithmetic thought the child found arithmetic easy. In the present study, the majority of fathers believed the child found most topics in arithmetic easy to understand. Mothers' beliefs, however, varied as a function of their child's arithmetic achievement; while the majority of mothers of children doing well in arithmetic thought the child found most topics easy, some mothers of below average children thought the child did not find most topics in arithmetic easy to understand. The present relation between mothers' awareness of their child's problems and children's arithmetic performance is opposite to that found in studies with children in Grades 1 and 5 (Crystal & Stevenson, 1991; Stevenson & Lee, 1990); mothers of children who were not doing well thought the child found arithmetic easy.

Last, no marked differences were observed between children's arithmetic performance and parents' numeracy problems; the majority of parents reported not having faced any difficulties in arithmetic. Also, while the majority of fathers believed their children were doing their best in arithmetic, mothers varied as a function of children's arithmetic attainment; mothers of above average children believed their children were doing their best, while mothers of average and below average children believed the child could do better.

Reading

Children in the present study were either average or above average in reading. The majority of children believed they were above average in reading and the majority of mothers believed the child was at least average in reading. Some fathers' beliefs of children's reading ability varied with children's reading performance on the reading comprehension test: children scoring higher on the test were more likely to be rated by their fathers as above average in reading. Parents agreed moderately between them in their beliefs about children's reading skills and mothers agreed more with children than fathers did.

There was no association between children's perceived ability in reading and their attributions for their performance; both those who thought they were average and those who thought they were above average in reading attributed their performance to internal reasons (e.g., ability). There was no association between children's actual performance in reading and their attributions, either. Parents varied in their attributions for children's reading performance; parents of children who were doing well in reading would be more likely to attribute it to ability. Children and parents, as well as parents between them, did not agree significantly in their reports.

Most children wanted to be better in reading, for both internal and external reasons. The majority of parents believed the child found most topics in reading easy to understand, reported not having faced any difficulties in reading, and believed their children were doing their best in reading.

3.4.2.2 Attitudes and Home Practices

School

Liking for school was unrelated to mathematical performance; the majority of children liked school very much. Research has also suggested that attitudes to school are unrelated to arithmetic performance (Blatchford, 1996; Stevenson & Lee, 1990). In addition, there was an association between picking arithmetic as the most favourite school subject and arithmetic achievement; of the children showing a preference for a specific subject, children with above average arithmetic ability were more likely than the rest of the children to mention arithmetic as their favourite school subject. Studies have shown that children who did better in arithmetic were more likely to choose arithmetic as their favourite school subject (Young-Loveridge, 1991).

Arithmetic

A positive relation exists between attitudes to arithmetic and performance in school arithmetic (Aiken, 1970). Studies have shown that young children who did well in arithmetic held more positive views about the subject than children who did less well in the subject (Schofield, 1982; Stevenson & Lee, 1990; Young-Loveridge, 1991). The relationship has been found to hold even when children's perceived performance was investigated (Stevenson & Lee, 1990). In the present study, children's attitudes towards arithmetic were examined through opinions on individual measures, namely, opinions about the textbook and the homework, and feelings about missing an arithmetic class.

Children varied in their liking for the textbook as a function of their arithmetic performance; some children with arithmetic difficulties expressed mild and strong dislike for the textbook. As children reported themselves, that was mainly due to a dislike for arithmetic and to the difficulty they experienced due to its complexity (i.e., difficult things to solve or difficult exercises). The majority, however, liked the homework very much and would feel a little or very sad if they missed an arithmetic class.

Young-Loveridge (1991) found that children's favourite topic in arithmetic was addition, subtraction, multiplication, and division (in order of preference); children's least popular topics were division, multiplication, and subtraction. The present study examined children's preferences, further more as a function of their performance in arithmetic. Children did not differ in their favourite topics in arithmetic; operations were children's best and least favourite topic. While addition, multiplication and sometimes division were children's most favourite topics, subtraction and division were the least favourite ones.

The present study further explored parents' beliefs of the child's attitudes to arithmetic. Parents' reports showed some relative accuracy, in that fathers of above average children believed the child quite liked or liked arithmetic very much, while fathers of below average children believed the child quite liked or not liked arithmetic. Accordingly, mothers of above average children believed the child liked arithmetic very much, while mothers of below average children were equally likely to report any attitude. Although parents agreed moderately with each other, there was no overall agreement between children's attitudes to different measures in arithmetic and parents' beliefs of children's general attitude to the subject.

Research has shown that parents who believe doing well in arithmetic is important have children who do better in the subject (Stevenson & Lee, 1990). In the present study, there was no association between parents' beliefs of the academic importance of arithmetic and children's achievement in the subject: arithmetic invariably featured as one of the three most important subjects taught in school. In the aforementioned study, Chinese and Japanese parents placed emphasis on academic attainment, including arithmetic and reading; Asian children did better than their American peers in arithmetic, reading, and school in general. Accordingly, Greek parents place arithmetic from an early age in the top three important school subjects, next to reading and writing, as they do with academic achievement in general. Some parents believed History is more important than learning to write; however, practically no parent placed arithmetic in less than third in importance. That was independent of what their favourite subject was: mothers were more likely to choose another subject, while fathers were equally likely to choose arithmetic (but not reading) or any other subject as their favourite. The present study examined parents' favourite school subject when they were at their child's age, that is, 8-9 years old, based on research suggesting that parents' beliefs of children's attitudes to arithmetic are related to students' attitudes and subsequent performance (Aiken, 1972; Aiken & Dreger, 1961; Poffenberger, 1959; Poffenberger & Norton, 1959).

Research has shown that education experiences at home have a positive effect on children's achievement from entry to school (Tizard et al., 1988) to primary level (Stevenson & Lee, 1990; Young-Loveridge, 1991) and high school (Chen & Stevenson, 1995). Children's arithmetic achievement, in specific, has been found to be influenced by the amount of engagement in academic (Stevenson & Lee, 1990), reading (Reynolds & Walberg, 1992), and achievement-related (Chen & Stevenson, 1995) activities at home. The present study explored the relation between numerical home activities and children's arithmetic achievement at school. While some practices were common amongst the children of all ability groups, others did show variation. For example, children differed in their likelihood of telling the time: children in the average and above average groups were more likely to tell the time than children in the below average group. There was also some variation in playing number games, however, the difference was in the magnitude (at least half of the children in every group reported playing number games). Based on parents' reports, only reports of involvement with number games varied with children's math group; as in the case of children, however, their difference was only in the degree of engagement, with more than half of fathers in each group saying their child played number games at home. High levels of agreement between children and parents, as well as between parents, were observed in children's playing number games, helping with the cooking, and telling the time.

Reading

Children in the present study were either average or above average in reading. Most children liked the homework in reading and the textbook used in Language very much and would feel sad or very sad if they did not do any reading at school. They also liked reading alone and to their parents and liked reading to their teacher even more. The majority of parents believed the child quite liked or liked reading very much. Although parents agreed moderately with each other, there was no overall agreement between children's attitudes and parents' beliefs of children's attitudes. Most parents believed it was very important that their child did well in reading: reading featured as one of the three most important subjects at school. Mothers were more likely to choose any other subject except reading as their favourite, while fathers were equally likely to choose arithmetic or any other subject except reading as their favourite.

Children did not vary in their reports of reading activities at home; the overwhelming majority said they read alone at home, their parents would read to them or not, and possessed between 10 and 30 books at home to read. The majority of parents said the child read at home alone, with most fathers reporting not reading to the child and most mothers being equally likely to say they read to the child or not (some variation was observed in mothers' reports based on children's scores on the reading comprehension; the majority of mothers of children in the average group reported reading to the child, while mothers of children in the above average group were equally likely to say they read or not to the child). High levels of agreement between children and parents, as well as between parents, were observed only in children's reading alone.

3.4.2.3 Parental Help and Encouragement

Arithmetic

Studies have suggested a positive relationship between children's achievement in arithmetic and the help they receive with their homework, in that children who receive more help are more likely to do better in the subject (Grolnick et al., 1989; Stevenson & Lee, 1990). The present study examined this hypothesis, further distinguishing between two types of help with the homework, namely, indirect and direct.

The majority of children reported receiving help in tidying up their room; mother was the most frequently reported person. The majority of mothers reported helping the child. Fathers varied in their reports of indirect help as a function of children's arithmetic achievement; while the majority of fathers of below average children reported helping the child by tidying up their room or keeping everybody quiet, only half of fathers of above average children did so.

The majority of children reported receiving help in doing their general school homework, as well as their homework in arithmetic; mother was again the most frequently reported person. Most children felt very satisfied with the help they got with their homework in arithmetic. Most parents reported helping the child and feeling quite and very confident in doing so. Overall, there was no agreement between children's and parents' reports on direct and indirect help, except for children's reports of fathers' help with the homework in arithmetic which correlated moderately with fathers' corresponding reports.

No variation was observed in the amount of time spent with the child as a function of children's arithmetic achievement. The majority of fathers would spend up to 4 hours with the child per day, while the majority of mothers would spend at least 4 hours per day. Also, the overwhelming majority of parents reported encouraging the child to do well at school and in arithmetic; the most common method of encouraging for school was motivating the children, while motivation as well as tuition and help was also provided in the case of arithmetic.

Reading

The children in the present study were either average or above average in reading, based on performance on both the reading comprehension test and the sequence task. The findings on parental help, however, showed some variation as a function of reading test.

Based on children's performance on the reading comprehension test, there was no variation in children's and their parents' reports on any measures as a function of reading group. Both average and above average children reported receiving indirect and direct help in their general school and reading homework, mainly from their mother. Most children were very satisfied from this help. Most mothers reported helping their children both directly and indirectly with their homework in reading. Fathers would be less likely than mothers to help, being equally likely to help or not. Of those parents who reported helping the child with the homework in reading, the majority felt quite confident and very confident in doing so.

Based on children's performance on the sequence task, some variation was observed between average and above average children and their parents. Children in the above average group reported relying entirely on others' indirect (their mother), direct general school (mother and father) and direct reading (mother) help with the homework; average children would be more likely to rely on themselves for indirect, direct general school, and direct reading homework. Most children were very satisfied with the help they received. Mothers unanimously reported helping the child both directly and indirectly and felt quite confident and very confident in providing help. Most fathers were equally likely to help indirectly or not and fathers of above average in reading children reported helping the child, while fathers of average children would be equally likely to help or not. Accordingly, fathers of above average children would feel quite confident and very confident in helping the child, while fathers of average children would feel moderately confident on the average. Overall, most parents reported encouraging the child to do well in reading. The most common method was motivational support.

3.4.2.4 Parent - School Relations and Parent Education

Arithmetic

The present study examined parents' opinions on the suitability of the curriculum in arithmetic. Stevenson and Lee (1990) found that mothers of Grade 1 and Grade 5 children did not vary in their curriculum opinions as a function of children's arithmetic achievement; however, relatively more mothers of children doing well in the subject than mothers of children doing less well believed the curriculum in arithmetic was too difficult for the child. In the present study, fathers' opinion about the curriculum did not vary with children's arithmetic achievement; the majority believed it was suitable for the child's age. Mothers' opinions, however, showed some variation, in that the majority of mothers of above average children believed the curriculum was suitable, while some mothers of average and below average in arithmetic children believed the arithmetic curriculum was not suitable for the child's age. The two most common reasons for the negative views on the curriculum were the nature of the material (too easy or too difficult) and the method of presentation (in the textbook or by the teacher).

Tizard et al. (1988) found that parents of children in the first three years of school would learn about the curriculum covered in class by looking their child's work. Accordingly, in the present study, the child's textbook was the most common source of information. The second most common answer would be children's own reports. Written information was, as in Tizard et al., scarce. No variation was observed in the way parents were informed about the curriculum as a function of children's achievement in arithmetic.

Tizard et al. found that children whose parents had more contact with the teacher had greater progress in reading and writing in the first three years of schooling than did children of parents who did not have such contact. Moreover, parents' contact with the teacher related to children's numerical skills upon entry to school. In the present study, no marked differences were observed between parents in their contact with the teacher as a function of children's arithmetic attainment: mothers would be generally more likely to report meeting with the teacher to discuss the child's progress in arithmetic than fathers did. In terms of frequency, Tizard et al. found parents contacted the teacher once every year, while parent-teacher meetings in the present study were found to occur more often, that is, frequent informal meetings or scheduled meetings once a month or on the reports day once a semester.

The present study further explored parents' evaluation of the teacher's help with the child's difficulties in arithmetic. Overall, fathers evaluated the teacher as very helpful. Some variation was observed between mothers, in that the majority of mothers of above average in arithmetic children considered the teacher very helpful, while some mothers of average and below average children rated the teacher as moderately helpful.

Research has shown that children's school achievement in Grades 1, 5, and 8 is influenced by parents' academic status (Baker & Stevenson, 1986; Stevenson & Lee, 1990). Some relationship between academic status and children's arithmetic performance has been also observed in Grades 7 to 8 (Reynolds & Walberg, 1992), however, it decreases with age, for example, 17 years and over (Chen & Stevenson, 1995). In the present study, no marked differences were observed between parents in their academic status, as a function of children's arithmetic achievement. Fathers were predominantly holding a university degree, while mothers had either simply finished school or also held a university degree.

Reading

Children in the present study were either average or above average. No marked differences were observed in their parents' relation to school or academic background, as a function of children's reading performance. More specifically, the majority of parents believed the reading curriculum was suitable for the child's age, were informed about the curriculum mainly through the children's textbooks, kept regular contact with the teacher to discuss the child's progress in reading (informal meetings or scheduled meetings once a semester), believed the teacher was very helpful with the child's difficulties in reading, and held a university degree (fathers) or had either simply finished school or also held a university degree (mothers).

3.4.3 The Contribution of Social and Environmental Factors to Children's Variation in Arithmetic Achievement

The present study further examined the degree to which variation in children's performance could be explained independently from social and environmental factors. Separate analyses for each respondent were first conducted.

Children's attitudes to arithmetic and their reports of total parental indirect help with the school homework explained variation in children's achievement on Young (1979) and the NFER (1971) tests (23% and 17% cf.). Mothers' beliefs of the child's ease with arithmetic, the child's performance as opposed to ability, the child's attitudes to arithmetic, and their own academic background together explained 57% of children's variation in Young's test. Only mothers' beliefs of the child's attitudes to arithmetic explained 32% of the total variance on the NFER test. Fathers' beliefs of the child's attitudes to arithmetic explained variation on both Young's and the NFER tests (26% and 22% cf.).

A combined regression analysis saw the significance of children's personal reports of their attitudes to arithmetic in accounting for variance on both tests, while mothers' corresponding beliefs and their academic background also explained variation on Young's test, and fathers' beliefs of the child's attitudes also explained variation on the NFER test.

3.4.4 Introduction to the Next Study

Some social psychological factors have been identified as correlates of children's performance in school arithmetic. The next chapter describes children's formal and informal arithmetic knowledge and skill, as related to their differential achievement in arithmetic. This is the first part of the investigation of cognitive factors and their role in children's arithmetic attainment.

CHAPTER 4

ARITHMETIC KNOWLEDGE AND SKILL RELATED TO CHILDREN'S ARITHMETIC ACHIEVEMENT

4.1.1 Introduction

The present chapter examines the relationship between children's formal and informal arithmetic knowledge and skill and their performance in arithmetic. The purpose is to detect how cognitive factors, such as knowledge and skill in arithmetic, varied with children's achievement and how differences in this knowledge and skill might explain variation in children's achievement. First, the theoretical background on which this study was based is presented, followed by the sample used and the methods employed. A description and discussion of the findings concludes this chapter.

4.1.2 Review of Research on Maths Difficulty Children's Formal and Informal Arithmetic Knowledge and Skill

To select the cognitive factors which may account for children's variation in arithmetic, the present study has borrowed its theoretical framework from studies on the cognitive bases for children's difficulties in arithmetic. Children with mathematics difficulties are basically children who have normal levels of intelligence and exhibit average levels of achievement in all school subjects, except arithmetic. In attempting to examine the specific nature of arithmetic difficulties, research explored children's strengths and weaknesses in different areas of mathematical thinking. The aim was to identify the specific mathematical components in which children are severely deficient and which constitute the underlying causes of such difficulties.

Research has examined these children's formal and informal arithmetic knowledge and skill; the corresponding areas investigated include informal concepts and related calculational skills, knowledge of base ten concepts and calculational skills, error strategies in written calculation, knowledge of number facts, and problem solving skills. The present study examined these hypotheses, further including children of particularly good arithmetic skills. The evidence coming from this research is presented next.

4.1.2.1 Informal Concepts and Calculational Skills

The examination of children's informal knowledge and skills involves children's understanding of informal concepts such as "relative magnitude" and "more", as well as children's strengths and strategies in mental addition. Research has shown that "mathematics difficulty" children in Grades 2 to 6 possess an adequate knowledge of more and relative magnitude and have adequate skills in mental addition algorithmic calculations (Ginsburg, 1982; Ginsburg, Posner, & Russell, in Russell & Ginsburg, 1984). Clearly, they enter school with the same informal knowledge of numerical inequality and relative magnitude as normal children do. Such children appreciate informal concepts and employ meaningful strategies in dealing with nonwritten calculations.

The finding that children with arithmetic difficulties do not suffer from inadequate informal knowledge and skills is, however, not surprising, given that such knowledge appears at a very early age. Informal concepts do not derive from schooling and they refer to the knowledge of basic mathematical concepts that children possess already before they enter school. These concepts usually derive from the child's everyday experience with the environment, spontaneous counting, etc. Although children may understand the concepts, they may nevertheless not know their definition; for example, children may know that 8 is more than 5, without however knowing that the concept of "more" involves lack of one-to-one correspondence between items of sets. Evidence on numerical inequality suggests that preschool children know how to determine which of two numbers is more or larger than the other. Ginsburg (1982) has shown that from years two to six, young children's informal skills yield a surprising degree of accuracy with respect to judgements of "more" and "less".

Another research on children's informal understanding of numerical concepts has suggested that preschool children already know how to use analog representations on number line. Resnick (1983) found that children from 2 to 5 years seem to possess a mental number line involving notions of relative magnitude: first, they understand that numbers represent positions on this line and that they are linked together by a successor (Next) relationship; then, they appreciate the existence of a directional marker which signifies that positions further on the line are larger. Young children use this mental number line for both counting (e.g., to establish quantities, thus, cardinalities, by the operation of counting) and directly comparing quantities (e.g., to make accurate magnitude comparisons, such as more or less). Resnick argues that as children enter school, they already have a representation of number in the form of a mental number line.

Although several studies suggest some competence in mental arithmetic is common among preschool children, this competence is extremely limited, that is, to very small addends (Starkey, 1992). Once one uses numbers above four, performance declines substantially. It is therefore possible that children with mathematics difficulties may be less competent at even simple mental addition, that is, involving addition and subtraction of numbers less than 300.?

Dowker (1989) examined the strategies 5-9-year-old children use in dealing with estimation. Children were presented with good and bad estimates and were asked whether the answer was from *very good* to *very silly*. Children were divided into 4 levels depending on their performance on addition problems and had then to judge estimation problems depending on their level of performance; from simple (e.g., 5 + 2) to more difficult additions (e.g., 217 + 285). Dowker (1989) found that young children were not accurate in their judgements, even though the problems were quite easy: many of the estimates were less than one of the addends or more than twice the exact answer. Older children were more accurate and used more efficient strategies, like rounding. Dowker found that children may use appropriate strategies in problems just above their level, however, they do not do so when the problems are too difficult.

Furthermore, Dowker (1998) found that 5-9-year-old children's ability to judge whether the answer provided to a problem is right or wrong was significantly related to calculation and the use of principles in derived fact strategies, especially when addition problems are involved. Dowker argued that both exact unknown fact derivation (derived fact strategy use) and approximate unknown fact derivation (estimation) are important components of unknown fact derivation.

4.1.2.2 Base Ten Concepts and Related Enumeration Skills

Resnick (1983) has suggested that children's calculation difficulties may be due to a limited understanding of the base ten system. Clinical studies conducted by Ginsburg and his colleagues (Ginsburg, 1982; Ginsburg, Posner, & Russell, 1981a) have also shown that children's mathematical difficulties often stem from their inability to fully comprehend base ten concepts. In dealing with the base ten system, we basically deal with children's decimal knowledge.

The introduction of this knowledge at school is a significant stage in the development of arithmetic understanding and number representation. According to Resnick (1983), in preschool years, children possess a mental number line which enables them to understand first that numbers represent quantities and second, that numbers further on the line are larger. Children are thus able to compare quantities for relative size. During early school years, however, they learn to interpret numbers in terms of part and whole relationships (the part-whole schema): that is, they understand that numbers are compositions of other numbers.

Resnick (1983) describes the development of this knowledge in terms of three stages. During the first stage, *the unique partitioning of multidigit numbers into units and tens*, the child understands that two-digit numbers are a composition of a tens value and a units value (e.g., 47 equals 4 tens plus 7 units, or the Next schema). During the second stage, *the multiple partitionings of multidigit numbers*, the child realises the equivalence of multiple partitionings and the noncanonical representations of quantity, or the Trade schema. Through counting or experimentation (exchange), the child understands that 1 block can represent 10 blocks, and realises that he can exchange quantities and still maintain equivalence of the whole (e.g., 40 + 7 = 30 + 17). During the third stage, the child applies the part-whole schema to the conventions of written arithmetic. This is a formal arithmetic stage in which exchange principles are applied to written numbers to produce a rationale for algorithms involving carrying and borrowing. In sum, Resnick argues that part-whole is available to children from a very early age and that its systematic application to quantity characterises the early years of school. A first elaboration of the basic part-whole schema is its attachment to procedures for counting up and taking away. The schema in turn allows numbers to be interpreted both as positions on the mental number line and, simultaneously, as compositions of other numbers. This interpretation of number appears to underlie both story-problem solution and invented mental arithmetic procedures for small numbers that characterise the earliest school years.

Children, although capable of structural knowledge from an early age, may nevertheless fall short of correct calculations (Resnick, 1983). The hypothesis behind this is that children learn algorithms for written addition and subtraction, without linking them to their decimal knowledge. Rather, they conform to the procedures learned at school, lacking a rationale that makes the procedure sensible. Research (Brown & Burton, 1978) has shown that children's errors result from the systematic application of wrong "buggy" algorithms. Or that children ignore the part-whole schema. VanLehn's theory suggests that the correct algorithm has been learned but is incomplete for certain problems, either because an incomplete algorithm was taught or because certain steps have been forgotten. In sum, Resnick's (1983) work has also shown that children may be confused with multidenominations or share Brown and Burton's bugs such as smaller-from-larger, borrow-from-zero, 0 - N = 0, etc.

4.1.2.3 Error Strategies in Written Arithmetic

Research has shown that children's difficulties in written arithmetic are often a result of using systematic error strategies (Ginsburg, 1982) or bugs (Brown & Burton, 1978). Studies with African children conducted by Ginsburg, Posner and Russell (Russell & Ginsburg, 1984) also provided evidence that children's errors in written calculations are primarily due to correct procedures used improperly.

Ginsburg (1982) argued that children's mathematical behaviour is not capricious and that their errors are based on systematic rules. In examining American children in Grades 2 to 6, he found that children's errors stemmed from organised strategies and rules. For example, children may subtract the smaller from the larger as this is sound, however, they may forget to borrow if the larger number is at the bottom. Ginsburg contends that these organised strategies or rules may be introduced to children, but are either not taught well or are taught incompletely. Children, therefore, use derivations of these rules which may be objectively illogical but psychologically make sense to them.

Brown and Burton (1978) dealt with children's errors in a systematic and more sophisticated way. They tried to construct, use, and infer a diagnostic model for procedural skills in mathematics, that is, a model that captures a student's common misconceptions or faulty behaviour as simple changes to or mistakes in a correct model of the underlying knowledge base. Contrary to teachers who believe that children do not follow the procedures very well, Brown and Burton argued that children may simply follow the wrong procedures.

Each skill is composed of subprocedures. Incorrect implementations related to these subprocedures are called "bugs". According to Brown and Burton, bugs involve incorrect actions taken in place of the correct ones, that is, they may call upon other correct subprocedures but they are either used in a wrong way or at inappropriate times. Bugs can be found in combinations, for example, a child may have a borrowing bug as well as a bug in his subtraction facts table (14 - 6 = 7). Each subprocedure of a skill may have many buggy versions associated with it. And several distinct bugs can generate the same answer !

In other words, it is not always obvious how bugs in any particular subprocedure or set of subprocedures will be manifested on the surface (i.e., the answer). Some of the complicating factors are that a single buggy subprocedure can be used by several high-order procedures in computing an answer or that two bugs can have interactions with each other. These factors are further complicated by the fact that not all sample problems will manifest all of the possible symptoms. People usually determine symptoms by considering the skills or subprocedures used in solving one particular sample problem. They often miss symptoms generated by other procedures that can, in principle, use the given buggy subprocedure but which, because of the characteristics of the particular problem, were not called upon. As Brown and Burton suggest, if a different sample problem had been chosen, it might have caused the particular faulty subprocedure to have been used for a different purpose, thereby generating different symptoms.

Examples of subtraction bugs include: subtracting the smaller digit in each column from the larger digit regardless of which is on top; in borrowing, adding 10 to the top digit of the current column without subtracting 1 from the next column to the left; always subtracting all borrows from the leftmost digit in the top number, and so forth.

4.1.2.4 Some More Evidence on Children's Mathematical Difficulties

Russell and Ginsburg (1984) investigated the issue of "essential cognitive normality" of children with mathematics difficulties. Based on the clinical studies conducted by Ginsburg and his colleagues in the United States and Africa, Russell and Ginsburg tested the hypotheses that "math difficulty" (MD) children do not suffer from a general cognitive deficiency rather from immature mathematical knowledge, for example, a difficulty in dealing with large numbers and faulty induction of standard rules. Based on the hypotheses examined so far, they focused on fourth-grade children of normal intelligence who nevertheless experience mathematical difficulties and compared their formal and informal mathematical knowledge with that of their fourth-grade peers whose math performance was adequate as well as to that of younger, third-grade pupils. The battery of tests they employed firstly involved tasks on children's informal concepts and calculational skills, base ten concepts and related enumeration skills, and error strategies and other calculational procedures.

The first hypothesis was that children's difficulties did not derive from inadequate knowledge and skill in informal arithmetic. It was hypothesised that children who suffer from mathematical difficulties have inadequate knowledge of informal concepts and substantial strengths in mental calculations. Indeed, Russell and Ginsburg found that math difficulty children showed an adequate understanding of the concepts of "more" and relative magnitude and their skills in estimation were proficient. In fact, they did not differ from their third- and fourth-grade peers. In mental addition, math difficulty children performed significantly lower than their fourth-grade peers, but at about the same level as the third-grade pupils. The three groups did not differ in the strategies they used to solve mental addition problems and they all showed an adaptive deployment of strategies depending on the magnitude of the problems. When dealing with small numbers, math difficulty children used appropriate strategies but did not execute them properly. When dealing with large numbers, the same children made increasingly more mistakes which were due to minor execution errors, despite using mental algorithms to solve the problems.

Russell and Ginsburg argued that math difficulty children possessed adequate informal knowledge of mathematics, which is at the same level as their peers. They further concluded that children with mathematics difficulties do experience minor difficulties in the execution of adequate strategies, but their major difficulty lies in the application of concepts and skills to problems involving large numbers.

The second hypothesis referred to children's knowledge of base ten system. It was hypothesised that children's difficulties in arithmetic were due to inadequate knowledge of the base ten system. Five measures were employed to examine this hypothesis: first, children's accuracy and strategy in counting rows of ten dots, where it was found that math difficulty children did not differ from their normal or younger peers in accuracy and strategy use (i.e., wide use of counting or enumeration by tens). Second, children engaged in counting different sums of money: math difficulty children were significantly less accurate than their fourth-grade peers, but at the same level as third-grade children. Third, children had to calculate how many Xs are in Y, which involves decomposing large numbers into smaller ones: math difficulty children could barely do half of the trials and were significantly behind their normal peers. Fourth, children had to judge which of two large numbers was more to show their knowledge of place value in the written notation system: math difficulty children seemed to appreciate key aspects of written notation, with no differences being observed between children. Finally, children's understanding of the representation of place value was examined by having children dividing a pile of poker chips into quantities representing the value of each digit on a given number: no significant differences in performance were observed among the three groups.

Russell and Ginsburg concluded that mathematically disabled children possess some elementary base ten concepts and related skills, however, they lack fluency in dealing with larger numbers, whether in counting them or breaking them down into smaller numbers: while all children showed the same proficiency in enumeration by tens, identification of larger written numbers, and representation of place value, math difficulty children as well as their third-grade peers were severely impaired in counting large numbers or decomposing large numbers into their smaller components. There was no proof for Resnick's (1983) assertion that math difficulty children's inaccurate calculation stems from inadequate understanding of the base ten system; simply a reflection of children's difficulty with large numbers.

The third hypothesis was that "bugs" or systematic strategies wrongly applied accounted for math difficulty children's errors in written calculations; children's skills in written addition and subtraction, as well as their ability to identify errors were examined for that purpose. Math difficulty children exhibited as immature arithmetic knowledge as their third-grade peers, both in the amount and nature of errors; both groups erred significantly more often than fourth-graders who nevertheless made more "sophisticated" errors. Math difficulty children's errors indeed derived from common bugs, such as misalignment, writing numbers as they sound, doing the wrong operation, and subtracting the upper digit from the bottom one when the upper is smaller. They also engaged in frequent miscalculations.

Furthermore, math difficulty children were able to identify as many errors as their third-grade peers, being significantly fewer than the those identified by their fourth-grade peers. Success in identification also varied with the severity of the errors, that is, simple miscalculation was easier to identify and monitor, while carrying and alignment errors were more difficult. Math difficulty children did not attend carefully to more complex problems and consequently identified fewer errors of calculation than their fourth-grade peers.

Having examined the above hypotheses on the cognitive bases for children's arithmetic difficulties, Russell and Ginsburg made a further step in the investigation of children's arithmetic knowledge. In addition to the topics discussed so far, they also examined those children's knowledge of number fact and problem solving skills.

4.1.2.4.1 Knowledge of Number Facts

Relatively unexplored, maths difficulty children's knowledge of number facts was investigated. It was hypothesised that lack of factual knowledge may underlie children's difficulties. The task employed was simple: children had to respond to addition facts, as quickly as possible without counting. Russell and Ginsburg found that math difficulty children's knowledge of addition facts was severely limited; they knew fewer addition facts than their normal fourth-grade and even their third-grade peers.

That finding was surprising given that math difficulty children's performance on mental addition and written calculation, which both presuppose some knowledge of number facts, was moderate. Russell and Ginsburg gave two possible explanations for this paradox; first, they argued that children may have used counting to solve the problems on both tasks, while on addition facts they had to respond quickly without counting; and, second, based on an examination of children's errors on those tasks, they found that the majority were calculational errors involving faulty number facts. Russell and Ginsburg contended that children's poor knowledge of number facts is related to either a dysfunctional short-term memory capacity or insufficient knowledge of principles from which number facts can be deduced.

4.1.2.4.2 Problem - Solving Skills

Russell and Ginsburg (1984) further explored math difficulty children's problem-solving skills. They hypothesised that difficulties in problem solving may underlie children's difficulties in arithmetic. While the area of problem solving is indeed vast, the authors focused on two key aspects: knowledge and use of principles or of strategies as a shortcut to mental labour and ability to solve word problems involving the four basic operations.

Math difficulty children showed a satisfactory understanding of commutativity - that the order in which items are added does not affect the sum of the operation - and of reciprocity - that the solution to 19 - 8 is one of the addends in 11 + 8, thus showing abstract modes of thinking. In fact, they resembled their normal peers in employing insightful solutions to shortcut the process of adding and subtracting. Baroody and his colleagues (Baroody & Ginsburg, 1986; Baroody, Ginsburg, & Waxman, 1983) have also shown that children appreciate the principle of commutativity from an early age. Cowan and Renton (1996) and Ganetsou (1993) showed that young children appreciate commutativity even before they learn how to do sums.

Solving word problems discriminated between children of different levels of achievement. Children were able to solve addition and simple subtraction problems using insightful calculational routines, however, math difficulty children's performance was severely limited in more complex problems (e.g., subtraction using irrelevant information and complex subtraction). Russell and Ginsburg (1984) concluded that children with mathematics difficulties are able to comprehend simple forms of story problems and select appropriate calculational routines for solving them, while their low performance in complex story problems was attributed to the semantic complexity of these problems, as defined by Riley, Greeno, and Heller (1983).

4.1.2.5 Conclusions

The tasks that have been found to account considerably for children's difficulties in arithmetic mainly refer to dealing with large numbers, reproducing number facts, employing bugs or error strategies in written calculations, and solving word problems. Russell and Ginsburg (1984) concluded that mathematics difficulty children are ordinary children who simply exhibit immature mathematical knowledge. They possess many cognitive strengths such as strategies for mental addition, understanding of base ten notions, insightful solutions in problem solving, and ability to interpret elementary story problems. Furthermore, their difficulties result from factors such as immature mathematical knowledge, poor execution of adequate strategies, inattention, and lack of facility in dealing with large numbers. There was no support for a general cognitive deficiency, conceptual incapacity, or a general developmental lag.

4.1.3 Aim and Hypotheses

The aim of the present investigation is to identify the cognitive factors that are specific to mathematics which may account for children's different levels of mathematical performance. These factors relate to tasks that examine children's knowledge of and skill in formal and informal arithmetic.

Russell and Ginsburg (1984) examined a wide range of math-specific factors which may account for children's arithmetic difficulties, by comparing fourth-grade math difficulty children to their normal peers. They used a large number of tasks which covered children's knowledge of and skill in five major arithmetic areas: informal concepts and calculational skills, base ten concepts and related enumeration skills, error strategies and other calculational procedures, knowledge of addition facts, and problem-solving skills. They found that math difficulty children suffer from a limited knowledge of addition facts and difficulties in dealing with large numbers. However, they have a satisfactory knowledge of informal arithmetic concepts, they apply insightful solutions to written arithmetic (principles), and they have adequate problem-solving skills. Russell and Ginsburg concluded that math difficulty children suffer from "essentially normal, if immature and inefficient, mathematical knowledge" (p. 242).

Accordingly, the present study explored the domains in which Greek children with arithmetic difficulties lack sufficient knowledge or skill. However, it extended the investigation to children who are doing particularly well in arithmetic. Unlike previous studies that compared mathematics difficulty children with their normal peers, the present study compared the arithmetic knowledge and skills of children with arithmetic difficulties to children who were average in maths as well as to children with above average mathematical performance. This design would give evidence of the math specific limitations that may account for children's arithmetic difficulties and it would also allow a precise appreciation of mathematical knowledge and skill which characterises other arithmetic ability levels, namely above average.

The present study focused on third-grade rather than fourth-grade children. Research in Greek primary schools has suggested that *severe* arithmetic difficulties are mostly evident in children in the third grade (Tzouriadou, 1990); by including third-grade children therefore the underlying causes of those difficulties - and the areas of excellent skill - would be most striking. Also, it would be possible to observe whether some relations observed in Russell and Ginsburg (1984) exist in younger children. Thus, two sets of comparisons would be allowed: one between third-grade children of different arithmetic ability and another between groups across studies, taking into account the age difference.

The tasks that were employed for this investigation are those used in the study conducted by Russell and Ginsburg. They cover a wide spectrum of arithmetic concepts and skills and enable an extensive evaluation of children's strengths and weaknesses. Direct comparisons with the pattern of results (i.e., math difficulties) in Russell and Ginsburg would also be facilitated.

Given the age of the children in the present study and the findings of Russell and Ginsburg, we would expect knowledge of addition facts and ability to deal with large numbers to discriminate children with arithmetic difficulties from their peers. Children with arithmetic difficulties would be expected to be less accurate in mental addition, written addition and subtraction problems, and complex word problems.

However, the following would not be expected to differentiate third-grade children with math difficulties from their peers, since they have been observed in children even before they enter school. For example, knowledge of informal arithmetic concepts (e.g., "relative magnitude" and "more") as well as base ten concepts would be expected to be widespread among children. Children would also be expected to employ the same error strategies ("bugs") in dealing with written calculations as their normal peers and not differ in their use of insightful solutions (principles) to solve written problems.

Children of above average mathematical performance would be expected to be particularly successful in all tasks.

METHODOLOGY

4.2.1 Introduction

This section describes the design, the sample (for a detailed account of the sample, see chapter 2), and the methods used to collect data on children's arithmetic knowledge and skill.

4.2.2 Design

One purpose of the present thesis was to examine whether children's performance on measures of formal and informal arithmetical knowledge and skills varied according to their mathematical ability.

The hypothesis was examined by comparing the performance of three groups which differed in mathematical performance, that is, children with excellent arithmetic abilities, children who are average in mathematics, and children with arithmetical difficulties. All children showed at least average reading abilities. The tasks employed covered five major mathematical areas: knowledge of informal mathematical concepts and informal calculational skills, understanding of base ten concepts and related enumeration skills, error strategies in written addition and subtraction and other calculational procedures, knowledge of number facts, and problem-solving skills.

While previous research has been limited to comparing arithmetically disabled children with their normal peers, the present study extended the investigation to children of excellent arithmetic abilities. It appears that relatively little is known about the skills that discriminate children in terms of mathematical performance. Thus, depending on the tasks on which performance would vary significantly among groups, a framework of cognitive arithmetic abilities could be constructed.

Also, compared to previous research examining fourth-grade pupils, the present study focused on third-grade Greek children, based on evidence suggesting that severe arithmetic difficulties in Greek primary children are mostly evident in Grade 3.

A brief description of the sample is followed by a detailed description of the methods used to collect the data.

4.2.3 Sample

The investigation of knowledge and skill in arithmetic relating to mathematical achievement included children who were above average in maths, children of average arithmetic strength, and children who performed low in arithmetic. All children had to be at least average in reading ability. For a detailed account of the process of sample selection, see chapter 2 (Sample Selection).

For the replication of Russell and Ginsburg's (1984) study on children's formal and informal arithmetic knowledge and skills, 66 children were selected: 30 were above average, 20 were average, and 16 were below average in arithmetic. The children did not differ in terms of age or in reading ability. The only significant difference among the three groups was their mathematical performance. Table 2.6 (see chapter 2) shows the mean performance levels and statistical comparisons between groups on the five pre-test measures, respectively.

4.2.4 Procedure

Children received the tasks on formal and informal arithmetical knowledge in three groups:

Group 1
Addition facts (Task 12), Larger written numbers (Task 8), Representation of place value (Task 9), Estimation (Task 4), Accuracy and bugs in written addition and subtraction (Task 10), and Monitoring errors (Task 11). All children received these tasks in this order.

Group 2
Mental addition (Task 3), Use of principles (Task 13), Enumeration by tens (Task 5), Counting large numbers (Task 6), Which is closer to X (Task 2), Which number is more (Task 1), and Multiples of large numbers (Task 7). Half of the children received these tasks in this order and half in the reverse order.

Group 3
Word problems (Task 14). They were always presented in the same order.

The order in which the three groups were presented was balanced. Following Russell and Ginsburg's (1984) pattern, there were four orders of presentation: half of the children received Group 1 first and Group 2 second; the other half received Group 2 first and Group 1 second. Within each order, half of the children received the tasks in Group 2 in the order described above and half of them in the reverse order. The word problems were always presented last. The children were equally distributed in each order of presentation (for frequencies distribution, see Appendix 4.1).

4.2.5 Tasks

Children went through a battery of tasks. This section describes the tasks used to examine children's formal and informal arithmetic knowledge and skills. The tasks used to investigate children's arithmetic knowledge were adapted from Russell and Ginsburg (1984). There were 14 tasks covering five major areas in arithmetic:

I. Informal Concepts and Calculational Skills

1. Which Number Is More ?

Children had to identify which of two numbers presented orally is more. The exact instructions in this task were: "I will tell you some numbers and I want you to tell me which one is more. For example, if I asked you which is more 10 or 5 ? 49 or 19 ? Now I will give you some bigger ones." There were two test trials and four formal trials involving the following pairs of numbers: *(9000 vs. 3200)*, *(365 vs. 701)*, *(1500 vs. 4000)*, and *(602 vs. 542)*. A score was granted for each correct response. Half of the children received the trials in this order, and half in the reverse.

2. Which Is Closer to X ?

Children were asked to imagine a number line. They were then asked to judge which of two numbers is closer to a target number. More specifically, the instructions were: "On this ruler, can you tell me which is closer to 6, 4 or 9 ? Which is closer to 7, 2, or 10 ? Now, imagine that this ruler or number line extends over to the hundreds and thousands." There were two test trials. The main trials involved the following sets of numbers: *(200: 99 or 400)*, *(5000: 1000 or 8000)*, *(700: 300 or 900)*, and *(5000: 2000 or 9000)*. Half of the children received the sets in the order listed, while the rest of the children received them in the reverse order. There was an accuracy score for every trial correctly solved.

3. Mental Addition

This task is one of the measures used to assess children's skills in informal arithmetic. As in Ginsburg, Posner, and Russell (1981b), the child had to solve addition and subtraction problems without using paper and pencil. The exact instructions were: "I will give you some addition and subtraction problems and I want you to solve them in your head or using your fingers. For example, how much is x plus y ? How did you figure it out ?" Children's strategies were noted, and further explanations were asked when necessary (e.g., if a child said "I did it in my head", he would be asked "So, what exactly did you do in your head ?"). The problems were always presented in the following order, however, half of the children received the first six problems at the beginning of the interview and the second six at the end, while the other half received the second six first and the first set of six trials at the end. The test trials were: *12 + 7, (19 - 7), 220 + 110, 35 + 14, (14 + 35), 63 + 31, 11 + 8, (8 + 11), 210 + 140, 39 - 12, (27 + 12),* and *32 + 24*. The problems in parentheses were not included in the scoring; they were included only for the purpose of Task 13 (Use of Principles, to be described later). Children were given an accuracy score for every problem correctly solved.

4. Estimation

This is the second measure of children's informal arithmetical skills, which involved orally presented problems. Children had to judge if the miscalculated answer to a problem is close or far away from the correct one. The exact instructions were: "I will give you some problems and their answers. The answers, however, are wrong, yet I don't want you to give me the right one; I just want you to tell me if the given answer is close to or far away from the correct one. For example, if I told you 2 plus 2 equals 5, you know it's not the right answer, but is it close or far away from the right one ? Now, if I told you 2 plus 2 equals 100 ?" The six test trials involved the following problems: *(91 + 24 = 50), (53 + 28 = 926), (340 + 570 = 8000), (32 + 43 = 70), (433 + 510 = 900),* and *(210 + 530 = 300)*. Half of the children received the problems in this order, while the other half in the reverse order. An accuracy score was given if the child made a correct judgement (i.e., correct approximation).

II. Base Ten Concepts and Related Enumeration Skills

5. Enumeration by Tens

The stimuli in this task were dots on four A4 (manila) cards. Each card contained ten horizontal rows of spots (radius 0.5 in.), in alternating rows of red and blue dots. The set sizes were: *100, 50, 120,* and *80*. All children received the trials in the above order. An accuracy score was given for each set size correctly counted. Instructions were simple and presented orally: "Here I have some cards. I just want you to tell me how many dots there are on each card." Children were further asked on the strategy they used to count the dots. Children's strategy in each trial was categorised according to the following list:

i. Enumeration by ones: Dots counted one by one.

ii. Enumeration by larger numbers: Dots counted by twos, fives, or another number except tens.

iii. Enumeration or multiplication by tens: Dots counted by tens or the number of rows of dots in the matrix counted and multiplied by ten.

iv. Other: Any other strategy.

6. Counting Large Numbers

This task is an adaptation from a task that was originally devised by Weinstein in 1978 (Russell & Ginsburg, 1984). In this task, children had to count money (in this case, Greek Drachmas), which was presented into piles. It involved both notes and coins. Each pile consisted of the following denominations: *430 drs.* (4-100s, 3-10s), *660 drs.* (6-100s, 1-50, 1-10), *1,530 drs.* (3-500s, 3-10s), and *3,020 drs.* (5-500s, 5-100s, 2-10s). Each child was presented with the same order of trials. An accuracy score was given for each pile correctly counted. At the end of each trial, children were asked about the strategy they used to count the money.

7. Multiples of Large Numbers

The exact instructions for this task were: "Now, I will give you some more problems. If I asked you how many 'twos' are in four, you would say 'two' because two plus two makes four. Now, can you tell me how many 'twos' are in eight ? O.k., now, how many 'threes' are in nine ?" After it was ensured the child knew what she was supposed to do, testing began. There were six trials in this task, which involved the following pairs: *(10 * 100), (2 * 10), (100 * 1000), (20 * 100),* and *(500 * 1000).*

8. Larger Written Numbers

The stimuli in this task were pairs of large numbers with the same number of digits, printed on index cards (0.12 x 0.20 cm), the one number directly above the other. Children were shown each card and were asked to point to the larger number. There were four trials, containing the following pairs of numbers: *(799999 vs. 811111), (522222 vs. 288888), (833333 vs. 177777),* and *(944444 vs. 499999).* The order of trials was always the same. Children were awarded one point for each correct answer.

9. Representation of Place Value

In this task, a pile of beans was used, along with two cards (0.13 x 0.20 cm) each having a number printed on them. There were two trials, *25* and *37*. The instructions for this task were: "Here I have a pile of beans. I want you to take away this number of beans (showing one card)." The experimenter waited until the child did the sorting. "Now, I want you to split this pile into two parts, one 'showing' *this* part of the number (pointing to the tens digit), and the other 'showing' *that* part of the number (pointing to the units digit)." There was no penalty score if the child miscalculated in sorting the initial pile of beans. They scored one point if they split correctly the pile into tens and units. In other words, children were successful if they separated the pile correctly into tens and units, even if they miscalculated in the initial split.

III. Error Strategies and Other Calculational Procedures

10. Accuracy and Bugs in Written Addition and Subtraction

As in Ginsburg, Posner, and Russell (1981a), children had to write down and solve addition and subtraction problems which were presented orally. All the children were presented with the problems in the following order: *(21 + 37), (28 - 7), (49 - 32), (12 + 6), (57 + 25), (185 + 72), (64 - 28), (234 + 43), (179 + 153),* and *(252 - 198).* Four of these problems involved alignment difficulties, four involved renaming (borrowing or carrying) difficulties, and two did not involve any of the above mentioned difficulties. Children's responses were examined for evidence of underlying strategies, including bugs, such as misalignment and defective "renaming". An accuracy score was given for the number of problems solved correctly.

11. Monitoring Errors

Children had to identify common buggy errors in written calculations. They were presented with addition problems displayed in vertical form with their solutions (incorrect) on index cards (0.13 x 0.20 cm) and were asked whether the solution was right or wrong. There were six incorrect problems with common bugs such as misalignment, miscalculation, and miscarrying. In between, there were two correct problems which were not included in the scoring. The sums were the following:

10	6	14	21	100	12	13	25	18
+ 10	+ 11	+ 10	+ 12	+ 1	+ 7	+ 12	+ 17	+ 13
20	71	34	33	200	89	25	32	211

All the children received the problems in the order above. The exact instructions were: "I am going to show you some problems. Some of the answers are right, however, some are wrong. I want you to tell me whether you think an answer is right or wrong. Look at this problem (10 + 10 = 20). Here is how it was worked out. Zero plus zero is nothing, so I put down the zero. One plus one is two, so I write down the two. Is that right or wrong ? Why do you think it is _____ ?" No child failed this test trial. Success score was the number of problems correctly identified.

IV. Knowledge of Number Facts

12. Addition Facts

Children's knowledge of number facts was examined by asking them to answer to ten addition facts as quickly as possible, in order to prevent counting and encourage retrieval. The exact instructions were: "I am going to give you some addition problems. I want you to tell me the answer right away without figuring it out. If I asked you how much 'two plus two' is, you would say 'four' because you just know it. But if you don't know the answer, just tell me you need to figure it out. How much is x plus y ?" Each correct answer was given a point. The addition facts were the following: *(2 + 5)*, *(6 + 3)*, *(7 + 8)*, *(9 + 3)*, *(7 + 5)*, *(9 + 8)*, *(7 + 6)*, *(4 + 9)*, *(4 + 3)*, and *(6 + 2)*. Half of the children were presented with the facts in this order and half of them in the reverse order.

V. Problem-Solving Skills

Children's problem-solving skills were measured by two tasks, one involving understanding and use of principles and the other examining comprehension and calculation procedures in solving story problems.

13. Use of Principles

Knowledge of two fundamental principles was examined in the present study: commutativity of addition, and reciprocity of addition and subtraction. Children had to solve the following four pairs of addition and subtraction problems:

(12 + 7) and *(7 + 12)*   *(35 + 14)* and *(14 + 35)*

where addends in each set of problems were reversed (commutativity), and

(11 + 8) and *(19 - 8)*   *(39 - 12)* and *(27 + 12)*

where the solution to the first problem is part of the second problem (reciprocity). What is characteristic about those pairs is that the second problem can be solved without calculation, given that the child understands commutativity and reciprocity.

The problems were part of the Mental Addition task (Task 3). Strategies were noted. The child was granted knowledge of the principles if (a) she verbalised the principle (e.g., "... because 12 + 7 = 19 and so does 7 + 12 because they are the same ... !"), (b) she referred to the previous problem (e.g., "... because it is the same as the previous problem !"), or (c) she explained that the second problem involved a change in the order of the numbers in the first problem (e.g., "... it is the reverse !").

14. Story Problems

Children were asked to solve eight word problems which involved addition, subtraction, multiplication, and division. They were told that they could think in their heads, use their fingers for counting, or use paper and pencil which were provided. Each problem was printed separately on an index card (0.13 x 0.20 cm, letter size 14) which was placed in front of the child. The experimenter read out the problem and waited until the child responded. Then the child was asked about the strategy she used to solve the problem, for example, "How did you find it ?" All children were presented with the eight problems in the same order. The content and order of the problems are the following:

i. Simple addition: "Anna had 11 drachmas. When she was walking to school, she found seven more drachmas. How many drachmas did she have altogether ?"

ii. Simple subtraction: "Maria liked to eat cookies. She baked 14 cookies and ate seven of them. How many cookies did she have left ?"

iii. Addition with several addends: "Chris collected eggs from the chicken coop. On Monday he got seven eggs, on Tuesday 10 eggs, and on Wednesday six eggs. How many eggs did he have in all ?"

iv. Complex subtraction: "There are 23 children in the lunchroom. Seven are boys and the rest are girls. How many girls are in the lunchroom ?"

v. Subtraction with irrelevant information: "Costas and Sissy were playing cards. After seven turns, Costas had 21 points and Sissy had eight points. How many points was Costas ahead after seven turns ?"

vi. Addition with irrelevant information: "For three days, Helen did the housework for the neighbour to make money. She got 5 drs. for washing dishes, 9 drs. for painting the kitchen, and 7 drs. for raking leaves. How much money did Helen get for three days work ?"

vii. Multiplication: "At Stelios' garage there were 10 cars. Each car had three flat tyres. How many tyres did Stelios have to fix ?"

viii. Division: "Nadia had 20 pieces of banana bubble gum. She wanted to give the same number of pieces of gum to four friends. How many pieces should each friend get ?"

RESULTS

4.3.1 Introduction

This section reports the results of the examination of the children's formal and informal arithmetical knowledge and skills. The sample consisted of children belonging to three ability groups: below average, average, and above average. The present investigation first focuses on the pattern of results from three comparisons: one between below average and above average children, another between below average and average children and the other between above average and average children. The findings are presented as a function of mathematical area. Furthermore, the study examines the independent contribution of those factors in children's variation in arithmetic.

4.3.2 Group Comparisons on Measures of Formal and Informal Arithmetic

Performance among groups was compared statistically through a series of *t* tests. When score distributions were not normal, nonparametric tests were employed and their results are reported. Within tasks, variation between trials was examined through a series of McNemar and Cochran tests. Table 4.1 shows the mean performance scores of the three groups in all tasks, as well as the results of statistical comparisons between them.

4.3.2.1 Informal Concepts and Calculational Skills

1. Which Number Is More ?

In this task, children had to judge which of two large numbers is more. Significant differences among groups were observed (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 9.33, p < .01). As Table 4.1 also suggests, children with arithmetic difficulties did not differ significantly from their average peers. Above average children were significantly better than their peers in identifying bigger from smaller numbers.

TABLE 4.1

Mean Number (Standard Deviations) of Correct Responses of the Three Groups on Tasks of Formal and Informal Arithmetic Knowledge and Skill

Task	Maximum Score	Below Average (BA) $n = 16$	Statistical Comparison (BA - A)	Average (A) $n = 20$	Statistical Comparison (A - AA)	Above Average (AA) $n = 30$	Statistical Comparison (AA - BA)
I. Informal Concepts and Calculational Skills							
1. Which number is more ?	4	3.0 (1.1)	ns	3.3 (0.9)	$t = 2.45^*$	3.8 (0.4)	$t = 2.95^{**}$
2. Which is closer to X ?	4	2.4 (0.7)	ns	2.9 (0.9)	$t = 1.95^*$	3.3 (0.6)	$t = 4.31^{***}$
3. Mental addition	8	4.5 (2.8)	$t = 2.18^*$	6.4 (2.3)	$t = 2.91^{**}$	7.9 (0.4)	$t = 4.77^{***}$
4. Estimation	6	3.7 (1.8)	$t = 2.39^*$	4.9 (0.9)	$t = 2.34^*$	5.4 (0.8)	$t = 3.67^{**}$
II. Base Ten Concepts and Related Enumeration Skills							
5. Enumeration by tens	4	2.8 (1.1)	ns	3.2 (1.0)	$t = 2.82^*$	3.8 (0.4)	$t = 3.94^{**}$
6. Counting large numbers	4	1.9 (1.2)	$t = 2.37^*$	2.8 (1.0)	$t = 3.68^{**}$	3.7 (0.6)	$t = 5.65^{***}$
7. Multiples of large numbers	6	2.6 (1.7)	$t = 2.71^*$	3.9 (1.3)	$t = 4.88^{***}$	5.6 (1.0)	$t = 6.47^{***}$
8. Larger written numbers	4	3.1 (1.1)	ns	3.4 (1.0)	ns	3.7 (1.1)	ns
9. Representation of place value	2	1.1 (1.0)	ns	1.7 (0.7)	ns	1.9 (0.5)	$t = 2.72^*$
III. Error Strategies and Other Calculational Procedures							
10. Accuracy and bugs in written addition & subtraction	10	4.6 (2.3)	$t = 2.90^{**}$	6.7 (2.1)	$t = 3.15^{**}$	8.4 (1.4)	$t = 5.98^{***}$
11. Monitoring errors	6	4.5 (1.4)	ns	5.2 (0.8)	ns	5.6 (0.8)	$t = 2.98^{**}$
IV. Knowledge of Number Facts							
12. Addition facts	10	2.9 (1.9)	$t = 3.32^{**}$	4.7 (1.3)	$t = 5.03^{***}$	6.8 (1.6)	$t = 7.56^{***}$
V. Problem-Solving Skills							
13. Use of principles							
a. Commutativity	1	0.8 (0.4)	ns	0.8 (0.4)	ns	0.8 (0.4)	ns
b. Reciprocity	1	0.1 (0.3)	$t = 2.62^*$	0.5 (0.5)	ns	0.4 (0.5)	ns
14. Story problems	8	3.1 (1.6)	ns	4.1 (1.8)	$t = 7.29^{***}$	7.4 (1.1)	$t = 9.62^{***}$

$^*p < .05.$ $^{**}p < .01.$ $^{***}p < .001.$

While all children were moderately to highly accurate, performance varied with trial (Cochran Q (3, 66) = 12.92, $p < .005$). As Table 4.2 shows, the trials involving hundreds tended to be solved more often than those involving thousands.

TABLE 4.2

Success Frequencies per Trial on Which Number Is More ? (Task 1)

Success	62	61	55	51
Trial	365 * 701	602 * 542	9000 * 3200	1500 * 4000

However, the only significant differences were between (1500 * 4000) and (365 * 701) and (602 * 542) respectively, according to McNemar tests.

2. Which Is Closer to X ?

Children were asked which of two numbers was closer to a target number. Both below average and average pupils were less successful than the above average group. The difference between below average and their average peers was not significant.

TABLE 4.3

Success Frequencies per Trial on Which Is Closer to X ? (Task 2)

Success	62	53	49	29
Trial	700: 300*900	5000: 2000*9000	5000: 1000*8000	200: 99*400

A Cochran test (Q (3, 66) = 42.38, $p < .001$) showed significant differences in success in the four trials; the identification of which number is closer to 200, 99 or 400 was the hardest to pass. There were no differences between (5000: 2000 or 9000) and (5000: 1000 or 8000), according to McNemar tests.

3. Mental Addition

In this task, children had to solve addition and subtraction problems without using paper or pencil. Significant differences between groups were observed (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 27.75, $p < .01$). While above average students solved almost all trials successfully, below average children were more likely to pass only half of them. Below average children had significantly lower levels of performance than both their average and above average peers. The latter two groups also differed in success levels.

TABLE 4.4

Success Frequencies per Trial on Mental Addition (Task 3)

Success	60	60	58	55	55	50	50	47
Trial	12 + 7	11 + 8	35 + 14	63 + 31	32 + 24	220 + 110	210 + 140	39 - 12

A Cochran test (Q (7, 66) = 35.36, $p < .001$) showed that success levels varied with trial. As Table 4.4 shows, performance on the two trials involving additions with three-digit numbers (hundreds) and the subtraction trial did not vary, according to McNemar tests; they were passed less often than additions with one- or two-digit addends.

Strategies

Children's strategies to solve the addition problems were analysed using Russell and Ginsburg's (1984) stratification of strategies. After giving the solution to each problem, each child was asked "How did you find it ?". Answers fell into the following categories:

i. <u>Counting</u>

The child calculated the sum by counting on from one addend through the next (e.g., 12 + 7 is 12, 13, 14, 15, 16, 17, 18, 19) or by using another overt counting procedure (e.g., the child said "I counted with my fingers" and had already been observed doing it silently).

ii. Regrouping

The child broke the addends down into more manageable units which were then added by using a number fact, counting, or some other procedure. Examples involve breaking down numbers into units of hundreds, tens, and ones, for example 220 + 110 is [(200 + 100) + (20 + 10)] or any other kind of breaking down with the purpose of facilitating the solution, for example, 63 + 31 is [(63 + 30) + 1].

iii. Mental Algorithm

The child calculated the sum by using the written addition algorithm as a mental strategy in which the numbers are operated upon as digits (not as tens and hundreds). The child could then use counting to determine the sum of pairs of digits, for example, 32 + 24 = [(2 + 4 is 4, 5, 6) + (3 + 2 is 3, 4, 5)].

iv. Other

This category included various instances that did not fit into any of the above categories. For example, there were cases of retrieval, where the child simply gave the solution, either saying "I knew it !", "It was easy !", or without giving any reason at all for his answer. In other instances, children's explanations were not clear as to the strategy used; for example, a child would repeat one or both pairs of addends (e.g., in 63 + 31, the child would justify his answer "because 63 + 31 makes 94" or "because 3 + 1 = 4"), making it difficult for the author to determine how the sum was calculated. Instances of ambiguity also include "I did it in my head", "I did it vertically", or "Big numbers up, small numbers down". Finally, no response or no attempt to solve the problem was coded under the above category when the child did not give any solution to the problem.

Children's strategies were also coded by a second researcher, using no more than one sum per child (66 sums) (reliability test, Cohen's $k = .96, p < .05$).

Russell and Ginsburg reported two major findings: first, their groups resembled each other in their use of strategies and second, children showed a systematic deployment of strategies according to sum size (i.e., as sum sizes increased, children were using mental algorithm to solve the problems). Table 4.5 shows the proportional use of each strategy as a function of group and sum size, as evidenced in the present study. Counting was mainly used in sums under 20, except for two children who used it in trials between 20-50. Also, Mental

Algorithm declined in use dramatically in sums over 100 for all three groups. It was Regrouping that was used more than any other strategy by all groups in all trials (except for the average group on trials of 0-20 sum size where they used Mental Algorithm slightly more). Regrouping reached its highest frequency in trials of 100 and above, especially for above average children (75% of trials). There also seems to be a switch from Retrieval to Regrouping as sum sizes increase.

TABLE 4.5

Proportional Use of Different Strategies on Informal Addition and Subtraction Problems (Task 3) as a Function of Sum Size and Mathematical Group

Problem	**12 + 7 11 + 8**			**35 + 14 39 - 12**			**63 + 31 32 + 24**			**220 + 110 210 + 140**		
Sum Size	**(0 - 20)**			**(20 - 50)**			**(50 - 100)**			**(100 - above)**		
Group[a]	**AA**	**A**	**BA**	**AA**	**A**	**BA**	**AA**	**A**	**BA**	**AA**	**A**	**BA**
Strategies												
Counting	.08	.13	.34	-	.05	.03	-	-	-	-	-	-
Regrouping	.43	.30	.31	.58	.43	.22	.48	.45	.38	.75	.53	.34
MA	.22	.33	.16	.37	.28	.16	.37	.33	.16	.10	.10	.03
Other	.27	.18	.13	.05	.08	.09	.15	.08	.09	.12	.08	.16
Retrieval	.18	.13	-	.03	-	-	.07	-	-	.05	-	-
Ambiguity	.08	.05	.13	-	.08	.09	.08	.08	.09	.07	.08	.16
No Response	-	.08	.06	-	.18	.50	-	.15	.38	.03	.30	.47
Total Wrong	.02	.13	.19	.02	.23	.53	-	.18	.47	.03	.30	.56

[a]Based on 60/40/32 responses for each group for each sum size (30 / 20 / 16 x 2 problems).

Table 4.5 also shows that average and above average children's frequencies of strategies used were rather similar, whereas below average children differed from their peers. Moreover, below average children relied a lot on Counting, and when sum sizes increased - and they could not count - they refrained from giving any answer.

Table 4.6 shows the frequencies and corresponding success rates of each strategy as a function of trial and mathematical group.

TABLE 4.6

Frequencies and Corresponding Success Rates of Strategies as a Function of Trial (Task 3) and Mathematical Group

	Counting	**Regrouping**	**MA**	**Other**	No Response	**Sum**
Trial 1: 12 + 7						
AA	3/3	**13/14**	7/7	6/6	0	30
A	2/2	**7/7**	7/7	3/3	1	20
BA	5/5	**4/6**	2/2	1/1	2	16
Trial 2: 220 + 110						
AA	0	**23/23**	3/3	3/3	1	30
A	0	**11/11**	2/2	1/1	6	20
BA	0	**5/6**	0	2/3	7	16
Trial 3: 35 + 14						
AA	0	**18/18**	11/11	1/1	0	30
A	0/1	**11/11**	7/7	0	1	20
BA	0	**5/5**	3/3	2/3	5	16
Trial 4: 63 + 31						
AA	0	**16/16**	11/11	**3/3**	0	30
A	0	**9/9**	5/5	**3/3**	3	20
BA	0	**5/6**	2/2	**1/3**	5	16
Trial 5: 11 + 8						
AA	2/2	**12/12**	6/6	10/10	0	30
A	3/3	**4/5**	5/6	4/4	2	20
BA	5/6	**4/4**	3/3	2/3	0	16
Trial 6: 210 + 140						
AA	0	22/22	3/3	4/4	1	30
A	0	10/10	2/2	2/2	6	20
BA	0	4/5	1/1	2/2	8	16
Trial 7: 39 - 12						
AA	0	16/17	11/11	2/2	0	30
A	1/1	5/6	4/4	3/3	6	20
BA	1/1	2/2	2/2	0	11	16
Trial 8: 32 + 24						
AA	0	**13/13**	11/11	6/6	0	30
A	0	**8/9**	8/8	0	3	20
BA	0	**6/6**	**3/3**	0	7	16
All Trials[a]						
AA	5/5	**133/135**	**63/63**	**29/29**	**2**	**240**
A	6/7	**65/68**	**40/41**	**16/16**	**28**	**160**
BA	11/12	**35/40**	**16/16**	**10/15**	**45**	**128**
All Trials - All Groups[b]						
	22/24	**233/243**	119/120	**55/60**	**75**	**528**

[a]Based on 240 / 160 / 128 responses for each group (30 / 20 / 16 children x 8 trials). [b]Based on 528 responses (66 children x 8 trials).

Table 4.7 shows children's frequencies and types of errors in mental addition. The examination of children's errors showed that below average children made more errors (56 errors) than any other group, while average children made 33 errors only. The above average group was highly accurate (only 3 errors). Also, children's errors tended to increase with sum size, that is, in sums of 50 and above.

Children's errors were categorised as follows:

Memory errors were those in which a unit or tens was lost during calculation. For example, in 210 + 140, the child said "(200 + 100) + 40 makes 340".

Miscalculations included cases of calculation errors. For example, in 12 + 7, the response was "2 + 7 = 8 + 10 = 18".

Miscellaneous / Not Categorisable referred to those errors that could not fit into any of the above categories, including instances of ambiguous responses.

TABLE 4.7

Frequencies (Percentages) of Errors on Mental Addition as a Function of Type of Error and Mathematical Group

	AA (n = 30)	A (n = 20)	BA (n = 16)
Number of Errors	3	33	56
Type of Error			
Memory	1 (33)	1 (3)	5 (9)
Miscalculation	0	1 (3)	3 (5)
Miscellaneous / Not Categorisable	0	3 (9)	3 (5)
No Response	2 (66)	28 (85)	45 (80)

As Table 4.7 also suggests, no attempt to solve the problem accounted for the majority of children's errors, independent of the mathematical group the children belonged to. The rest of the errors involved memory, miscalculations, and a few other sorts.

4. Estimation

In this task, the children had to judge if the answer provided to a problem was close or far away from the correct solution. Performance varied significantly with mathematical group (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 14.03, p <

.001). Children with math difficulties scored significantly lower than both children who are average and those who are above average in maths. Above average and average children also differed in their performance.

TABLE 4.8

Success Frequencies per Trial on Estimation (Task 4)

Success	58	57	57	51	49	46
Trial	(53+28=926)	(92+24=50)	(340+570=8000)	(32+43=70)	(210+530=300)	(435+510=900)

Performance also varied significantly with trial (Cochran *Q* (5, 66) = 14.21, $p < .05$). Table 4.8 also shows the trials on which performance did not vary, according to McNemar tests. It also shows that children were more likely to fail those trials involving hundreds than the trials involving two-digit numbers.

4.3.2.2 Base Ten Concepts and Related Enumeration Skills

5. Enumeration by Tens

In this task, the children had to count rows of dots, with their accuracy and strategy used being noted. Significant differences were observed among groups (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 17.06, $p < .001$). An examination of the mean scores of the three groups (see Table 4.1) showed that all of the children were moderately accurate in this task. Below average pupils were significantly less successful than their above average peers. The children in the average group did not differ from their below average peers, however, they differed significantly from children in the above average group.

TABLE 4.9

Success Frequencies per Trial on Enumeration by Tens (Task 5)

Success	61	58	54	49
Trial	50	80	100	120

Table 4.9 shows the children's success per trial. A Cochran test (Q (3, 66) = 11.30, $p < .05$) showed that the four trials differed in success levels; the larger the number, the more significant the differences in performance.

TABLE 4.10

Proportion (Frequencies) of Success in Using the "Enumeration by Tens" Strategy as a Function of Trial on Task 5 and Mathematical Group

	Group			
	Average Average (*n* = 30)	**Average** (*n* = 20)	**Below Average** (*n* = 16)	**All Groups**
Trial				
100	.88 (25/26)	.80 (14/16)	.75 (10/12)	.82
50	.93 (28/28)	.90 (17/18)	.69 (11/11)	.86
120	.87 (26/26)	.70 (11/14)	.56 (8/9)	.74
80	.93 (28/28)	.90 (15/18)	.75 (11/12)	.88

A few children in the below average group counted in ones or larger numbers, for example, in twos. As can be inferred from Table 4.10, the most common strategy used by children in every mathematical group was the enumeration or multiplication by tens. While children with math difficulties used it relatively less frequently than the rest of the groups, still it was the most widely used strategy within that group.

Enumeration or multiplication by tens was also the most popular strategy across trials, with the exception of below average children who would use another method in counting the 120 trial. Table 4.10 further shows that enumeration by tens was highly successful.

6. Counting Large Numbers

Children were asked to count different sums of money (Greek Drachmas), another task which presupposes an ability to deal with tens and hundreds. Significant differences among groups were observed (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 30.40, p < .01). Several t tests showed that below average children scored significantly lower than both average and above average children. Also, children in the average group differed significantly from the other two groups. Table 4.1 also shows that while children in the above average group had almost perfect scores, the below average group could solve barely half of the trials. Was it a difficulty with large numbers or a specific difficulty in "counting money", as some children complained ?

TABLE 4.11

Success Frequencies per Trial on Counting Large Numbers (Task 6)

Success	59	59	44	37
Trial (Greek Drs.)	430	660	1530	3020

Table 4.11 shows how performance varied with trials. A Cochran test showed that there were significant differences between trials (Cochran Q (3, 66) = 40.38, p < .001). There were no differences between the trials involving hundreds or those involving thousands, according to McNemar tests. Moreover, it was observed that the trials involving the hundreds were passed more often than the trials involving the thousands. That could have been due to some children reporting knowing how to count *only* up to 1,000".

7. Multiples of Large Numbers

In this task, the children had to say how many Xs (small numbers) are in Y (large number). Table 4.1 shows the statistical comparisons between the groups. Performance varied as a function of mathematical group (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 34.25, $p < .01$).

TABLE 4.12

Success Frequencies per Trial on Multiples of Large Numbers (Task 7)

Success	58	54	49	47	43	35
Trial	50 * 100	500 * 1000	2 * 10	10 * 100	100 * 1000	20 * 100

A Cochran test (Q (5, 55) = 37.09, $p < .001$) showed there was variation in performance among trials. As can be observed on Table 4.12, some sets of trials involving multiples of the same number (50 * 100 and 500 * 1000) showed no significant differences. However, (2 * 10) and (20 * 100) may reflect a greater difficulty that the children had with these two number pairs.

The examination of children's errors have shown that despite acknowledging the importance of the left-most digit and doing the decomposing, children would often make mistakes like responding 50 to (20 * 100).

8. Larger Written Numbers

In this task, the children had to compare two large numbers shown in a vertical form and decide which is the larger of the two. The three groups did not differ in their performance: they all showed a clear understanding of the concept of "more".

TABLE 4.13

Success Frequencies per Trial on Larger Written Numbers (Task 8)

Success	61	60	59	48
Trial	833333 177777	944444 499999	522222 288888	799999 811111

As Table 4.13 also shows, significant differences among trials were observed (Cochran Q (3, 66) = 24.44, p < .001), with no differences being observed between the three trials, according to McNemar tests. Children were the least successful on (799999 * 811111).

9. Representation of Place Value

Children had to count a number of beans out of a pile and then separate this pile into tens and units. The groups varied in their performance (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 9.69, p < .01). Significant differences were observed between children in the two extremes of arithmetic performance. Children with math difficulties resembled their average peers in their understanding of units and tens concepts (see Table 4.1). The two trials did not show any significant variation in performance, according to McNemar tests.

4.3.2.3 Error Strategies and Other Calculational Procedures

10. Accuracy and Bugs in Written Addition and Subtraction

Children were asked to write and solve addition and subtraction problems. While the majority of above average children were highly accurate, children in the below average group were barely able to solve half of the problems. Performance varied as a function of mathematical group, with significant differences being observed between children of all ability groups (see Table 4.1).

TABLE 4.14

Success Frequencies per Trial on Accuracy and Bugs in Written Addition and Subtraction (Task 10)

Success	64	62	57	56	52	46	42	35	28	16
Trial	21+37	12+6	28-7	49-32	234+43	57+25	185+72	64-28	179+153	252-198

Performance varied with trials (Cochran Q (9, 66) = 192.02, $p < .001$). Table 4.14 shows variation between trials, according to McNemar tests. Also, the trials involving no difficulties (e.g., 21 + 37) and those with alignment difficulties (e.g., 28 - 7) were passed more often than trials involving renaming difficulties (involving carrying or borrowing, e.g., 252 - 198). Furthermore, children's answers were examined on the basis of error strategies which fell into the following categories:

i. Representation Problems

This category included two major types of errors: (a) writing numbers as they sound and (b) misalignment.

(a) In the first case, the children wrote the numbers as they heard them and not based on place value. For example, "2034" would stand for "two hundred and thirty-four" (234).

(b) Misalignment involved errors in the position of the addends. They were either explicit, that is, children wrote "27 - 7 = 50" or "12 + 6 = 70" by placing the second addend or subtractant on the far left side (vertical form) or implicit evidenced by the solution provided (horizontal form).

ii. Faulty Procedures

(a) Addition Bugs: Three subcategories of addition bugs were observed.

First, a child would fail to carry, that is, he would not carry the unit to the tens (e.g., 179 + 153 = 322) or hundreds (e.g., 185 + 72 = 157) column. Or, the child would write both digits (e.g., 57 + 25 = 712) without adding the unit. Another example of the latter is "185 + 72 = 1157".

Second, a child would carry to the wrong column. For example, "179 + 153 = 422".

Third, a child would write "179 + 153 = 2113", where it is rather unclear how the operation was performed.

(b) Subtraction Bugs: Three subcategories were formed in dealing with subtraction errors.

First, some children failed to borrow, that is, they did not subtract from the tens column the unit that they had borrowed for the units column. For example, "64 - 28 = 46".

Second, some children subtracted the smaller from the larger, or the upper from the lower, without engaging in the process of borrowing. For example, "252 - 198 = 146".

Third, some children used rather ambiguous subtraction procedures. For example, "49 - 32 = 47", "49 - 32 = 07", and "28 - 7 = 12".

iii. Faulty Mental Arithmetic

This category involved instances of simple miscalculation, where the child would solve the problem making a mistake that was clearly due to inaccurate addition or subtraction facts. For example, "57 + 25 = 81".

iv. Other

This category included erroneous procedures that did not relate to any of the above categories of errors.

Table 4.15 shows children's error strategies (frequencies) in calculating the addition and subtraction problems as a function of mathematical group (see Appendix 4.2 for a more detailed account of children's errors).

TABLE 4.15

Number of Children (Frequencies of Errors) as a Function of Type of Error in Written Addition and Subtraction (Task 10) and Mathematical Group

	Group		
	AA (*n* = 30)	**A** (*n* = 20)	**BA** (*n* = 16)
Total	**23 (42)**	**16 (54)**	**16 (88)**
Type of Error			
Representation Problems	-	3 (5)	4 (12)
Writing numbers as they sound	-	3 (5)	4 (7)
Misalignment	-	-	2 (5)
Faulty Procedures	16 (26)	10 (30)	13 (39)
Buggy addition algorithm	8 (8)	5 (9)	7 (12)
Buggy subtraction algorithm	12 (18)	9 (21)	11 (27)
Faulty Mental Arithmetic *(miscalculation)*	12 (16)	10 (12)	12 (26)
Other	-	5 (7)	7 (11)

Children with arithmetic difficulties were more likely to make an error than children who were average or above average in maths. Overall, the most common errors involved buggy algorithms, especially in dealing with subtraction. That held true independent of the mathematical group the children belonged to. Faulty mental arithmetic, that is, miscalculations, were less common but frequent. Errors in representing the numbers were very few among the average children, but substantially more in children with mathematical difficulties.

11. Monitoring Errors

In this task, the children were given written addition problems in a vertical form and had to judge whether they were correctly calculated. If they thought they were not, they were asked to identify the error (e.g., "Why do you think it is not right ?"). In between trials, there were some correctly solved problems that were not included in the scoring. Overall, performance varied with mathematical group (Kruskal-Wallis 1-Way Anova, χ^2 (2, 66) = 12.73, $p < .01$).

Children with math difficulties were significantly behind their above average peers in detecting simple calculational, alignment, and carrying errors. As Table 4.1 suggests, however, below average children were similar to their average peers. There were no differences between average and above average children.

TABLE 4.16

Success Frequencies per Trial on Monitoring Errors (Task 11)

Success	64	63	62	54	53	47
Trial	Miscalculation 14 + 10 —— 34	Miscalculation 12 + 7 —— 89	Misalignment 6 + 11 —— 71	Miscarry 18 + 13 —— 211	Miscarry 25 + 17 —— 32	Misalignment 100 + 1 —— 200

Performance varied with trials (Cochran Q (5, 66) = 34.37, $p < .001$). As can be observed on Table 4.16, miscalculations were detected more often than defective carrying and alignment errors. There was only one child who responded negatively to the "correct" trials; however, she had responded negatively to all trials.

Justifications

Children were asked to justify their responses. Apart from some cases of erroneous answers (i.e., saying the problem is *correct* when it was not) where children did not offer any justification, all correct answers (i.e., saying the problem is incorrect) were justified. Justifications varied among trials, however, a general difficulty pointing out the error was observed: instead of explicitly *verbalising* the principle, children would rather engage in "doing" (*performing*) the problem as a means to communicate their judgement about the error. *Other* justifications usually referred to "no response" or some irrelevant comments. Table 4.17 shows the frequencies of children's justifications as a function of error and mathematical group.

i. Misalignment

There were two trials involving misalignment: (100 + 1 = 200) and (6 + 11 = 71). Out of all the children who responded, only six trials were explicitly justified (e.g., "... because 6 should be under the units, on the right side..." or "... because 1 is in the wrong place; it should be on the right, under the last zero, with the units..."). None of these children was in the below average group. The rest of the trials were justified by "doing" the operation: "... because 100 + 1 makes 101..." or "... because 6 + 1 makes 7 and 1 is tens...". It was clear that children treated the units as such, yet, could not verbalise the alignment error. They would rather give the correct solution to the problem, either by doing it step by step or simply by providing the answer. Finally, there were some children who did not spot the misalignment error, for example, "... because 6 + 1 = 8, so it should be 18 instead of 71..." or "... because 6 + 1 does not make 1 but 7...".

TABLE 4.17

Patterns (Frequencies) of Justification of Errors as a Function of Type of Error on Task 11 and Mathematical Group

	Type of Error																	
	Misalignment						Miscarry						Miscalculation					
	(6+11=17)			(100+1=200)			(25+17=32)			(18+13=211)			(12+7=89)			(14+10=34)		
	AA	A	BA	AA	A	BA	AA	A	BA	AA	A	BA	AA	A	BA	AA	A	BA
Justification																		
Verbalise	2	-	-	4	-	-	1	1	1	4	3	2	7	8	6	-	-	-
Perform	27	16	8	19	16	8	24	6	4	22	5	7	22	10	7	30	20	14
"Correct"	-	-	3	6	4	8	3	7	3	2	4	4	-	1	1	-	-	2
Other	1	4	5	1	-	-	2	6	8	2	8	3	1	1	2	-	-	-

Note. Above Average *n*= 30. Average *n*=20. Below Average *n*=16.

ii. Miscarry

It was equally difficult for the children to articulate the carrying error. Table 4.17 shows that in the majority of trials the children engaged in doing the addition, carrying successfully (e.g., "... because 8 + 3 = 11, we keep 1 and add 1 + 1 + 1 = 3..."). There were also some instances where the children would "solve" the problem using another technique (e.g., regrouping: 20 + 10 + 5 + 7 = 42, retrieval, etc.) or would "identify" an error which did not refer to the carrying (e.g., "... because 5 + 7 = 12, not 2 ... 2 + 1 = 3, o.k. ...", "... because 5 + 7 = 11, not 2...", or "... because these numbers are too small to make 211..."). A few children articulated the carrying error (e.g., "... because they forgot to add the carrying unit to the tens...") or pointed to the tens as the source of the error (e.g., "... this should be 1 tens more...").

iii. Miscalculation

In the problems (14 + 10 = 34) and (12 + 7 = 89), children should identify the error being 1 + 1 = 2 and not 3 or should comment on the presence of 8. When asked why they thought the trials were wrong, the majority of children performed the operation: "... because 4 + 0 = 4 and 1 + 1 = 2..." or "... because 2 + 7 = 9 and 1 is the tens...". Many used retrieval (e.g., "... because 12 + 7 makes 19 !"). Those who verbalised the error said explicitly that "8 should be 1" and that "... because 1 + 1 = 2...". Erroneous justifications (*Other*) involved "because 2 + 7 = 8", "because 1 + 1 = 3", and so forth. Across groups, the most popular way to justify the miscalculation was to provide the correct answer immediately, while "re-doing" the problem.

4.3.2.4 Knowledge of Number Facts

12. Addition Facts

Children had to respond to ten addition facts as quickly as possible, without counting. Children with math difficulties were severely hindered in this task. As Table 4.1 also suggests, below average children, on average, could barely recall one-third of simple addition facts. Significant differences in success levels were observed between all groups.

TABLE 4.18

Success Frequencies per Trial on Addition Facts (Task 12)

Success	60	52	52	49	36	21	20	19	18	17
Trial	2+5	6+3	6+2	4+3	9+3	9+8	7+8	7+6	7+5	4+9

A Cochran test (Q (9, 66) = 184.1, $p < .001$) showed that children's performance also varied with trial. As can be observed on Table 4.18, children, generally, did not differ significantly in solution success between additions that yielded a two-digit sum (i.e., more than 10): the majority were equally hard to solve. However, the examination of success rates per sum size showed that children with arithmetical difficulties erred more often on large addition facts than on small ones.

4.3.2.5 Problem - Solving Skills

13. Use of Principles

Children had to count one problem and then solve its commuted version. A Cochran test (Q (3, 66) = 89.62, $p < .001$) showed that the use of principles varied with trial. Overall, the children were more likely to use commutativity rather than reciprocity to solve the corresponding problems.

TABLE 4.19

Success Frequencies per Trial on Use of Principles (Task 13)

Success	52	48	24	10
Trial	Commutativity (12 + 7) (7 + 12)	Commutativity (35 + 14) (14 + 35)	Reciprocity (11 + 8) (19 - 8)	Reciprocity (39 - 12) (27 + 12)

Table 4.19 shows the frequencies of using the commutativity and reciprocity principles as a function of mathematical group and trial. Within each group, the use of the commutative principle did not vary between trials; within each group, children were more likely to use it on both trials.

If we analyse the results in terms of children's use of the principle on *at least one* trial (i.e., giving one point to those children who used it on one or both trials, for example 01, 10, or 11), there were no significant differences among the groups (t tests not significant). This can also be observed on Table 4.1.

TABLE 4.20

Frequencies of Use of Principles as a Function of Trial (Task 13) and Mathematical Group

		Group	
	Above Average (*n* = 30)	**Average (*n* = 20)**	**Below Average (*n* = 16)**
Trial			
Commutativity			
both	21	14	11
one	3	3	2
none	6	3	3
Reciprocity			
both	8	2	0
one	4	8	2
none	18	10	14

Table 4.20 shows that use of the reciprocity principle, on the other hand, varied with trial as well as with mathematical group. While above average and below average children resembled each other, that is, they did not use it on any trial (both McNemar tests not significant), their average peers were more likely to use it on at least the first trial (Rec. 1). Group mean comparisons based on consistency in using the principle on one or both trials *(at least once)* showed that average and above average children were more likely to use it on at least one trial, compared to their below average peers who would not use it at all. This can also be observed on Table 4.1.

To sum, while use of commutativity did not vary between trials for any group, use of reciprocity, however, varied among groups. More specifically, while below average children would use commutativity at least once - yet not use reciprocity at all, their average and above average peers were more likely to use both principles at least once.

14. Story Problems

Children had to solve eight word problems, involving addition, subtraction, multiplication, and division, using paper and pencil if necessary. T-tests showed significant differences in performance among all groups. Children in the above average group were highly accurate, while those with arithmetic difficulties could barely solve half of the problems (see Table 4.1). Average and below average children, however, did not differ significantly in their accuracy. Table 4.21 shows the significant differences in performance among trials (Cochran Q (7, 66) = 85.97, $p < .001$).

TABLE 4.21

Success Frequencies per Trial on Story Problems (Task 14)

Success	61	53	52	48	38	37	35	28
Trial	1	3	2	6	7	4	8	5

Trial Specifications:
1 = *Simple Addition*
2 = *Simple Subtraction*
3 = *Addition with Several Addends*
4 = *Complex Subtraction*
5 = *Subtraction with Irrelevant Information*
6 = *Addition with Irrelevant Information*
7 = *Multiplication*
8 = *Division*

Table 4.22 shows children's success rates as a function of type of problem and mathematical group. Children with arithmetic difficulties were severely hindered when dealing with multiplication and division. Those children were also impaired when the operation involved subtraction, both in its complex form and the one involving irrelevant information.

TABLE 4.22

Frequencies of Success on Word Problems as a Function of Type of Problem and Children's Mathematical Group

	Group		
	Above Average (n = 30)	**Average (n = 20)**	**Below Average (n = 16)**
Word Problem			
Simple Addition	30	18	13
Simple Subtraction	29	12	11
Addition with Several Addends	30	14	9
Complex Subtraction	27	6	4
Subtraction with Irrelevant Information	23	3	2
Addition with Irrelevant Information	28	13	7
Multiplication	27	11	0
Division	27	5	3

Problems involving division and subtraction with irrelevant information constituted a source of difficulty for average children, too. Table 4.22 also shows that the majority of above average children had extremely efficient problem-solving skills, with only a few children showing some confusion when they had to subtract while accounting for irrelevant information.

4.3.3 Multiple Regressions of Measures of Formal and Informal Arithmetic on Children's Arithmetic Achievement

The present study further explored the independent contribution of formal and informal arithmetic knowledge and skill in explaining variation in arithmetic achievement. A series of multiple regression analyses were used for that purpose. According to this method, the values of one variable, the dependent variable, are predicted from the values of other variables, the independent variables, by utilising the presence of an association between the three or more variables (Kinnear & Gray, 1995). Backward regression analyses were conducted, a method that removes individual variables whose probability of F is greater than .10, until a model is reached in which no more variables are eligible for removal. Variables are entered as a group and then removed individually (Norusis, 1993).

Data on formal and informal arithmetic knowledge and skill (Russell & Ginsburg, 1984) were collected for the majority of children belonging to the three mathematical groups (n = 72), including six children that were not examined in the group comparisons. The use of the maximum number of children, however, would ensure an adequate sample size for the regression analysis.

Children's scores on the two mathematical tests, the "Y" Mathematics Series Y2 test (Young, 1979) and the Basic Mathematics Test B (NFER, 1971) were the dependent variables in the current analyses. In the group comparisons so far, the dependent variable was children's performance in the form of ability groups: children's scores on the two mathematical tests were categorised into three mathematical groups. In the present analysis, raw scores were used as the dependent variable and two analyses were conducted, one for children's scores on either test.

The prediction procedure involved identifying the regressors, summarising for the purpose of controlling for the number of variables to be entered in the multiple regression and conducting the analyses. When outliers were observed, that is, cases outside 3 standard deviations, they were excluded and the analyses were repeated. Entries in the regression analyses included those cognitive variables which had been found to differentiate between children, and which were significantly correlated with children's achievement.

The first step was to identify the independent measures or predictors, that is, the cognitive variables that were significantly correlated with arithmetic achievement ($p < .05$). Appendix 4.3 shows the correlations between all measures and their association with arithmetic performance. Table 4.23 shows that all measures on formal and informal arithmetic knowledge and skill were significantly associated with children's performance on both mathematical tests.

TABLE 4.23

Measures of Formal and Informal Arithmetic Knowledge and Skill Associated With Children's Arithmetic Achievement

Variable	Young only	NFER only	Both
Informal Concepts & Calculational Skills			
Which number is more ?	-	-	√
Which is closer to X ?	-	-	√
Mental addition	-	-	√
Estimation	-	-	√
Base Ten Concepts & Enumeration Skills			
Enumeration by tens	-	-	√
Counting large numbers	-	-	√
Multiples of large numbers	-	-	√
Larger written numbers	-	-	√
Representation of place value	-	-	√
Error Strategies & Calculational Procedures			
Accuracy and bugs in written calculation	-	-	√
Monitoring errors	-	-	√
Knowledge of Number Facts			
Addition facts	-	-	√
Problem-Solving Skills			
Use of principles	-	-	√
Story problems	-	-	√

To control for the number of variables to be entered in the regression, the variables had to be reduced to a manageable set that would still be conceptually consistent with the theoretical formulations. The process of summarising the data generally involved averaging test scores based on meaningful categories. The summarised categories were the ones already defined by Russell and Ginsburg (1984). More specifically, these included informal concepts and calculational skills, base ten concepts and related enumeration skills, error strategies and other calculational procedures, knowledge of number facts, and problem-solving skills. Table 4.24 shows how the data were summarised.

The process of getting a single index of children's abilities involved calculating a single value which represented the mean performance level of a particular child on all tests within a particular mathematical area. As tasks differed between them in the maximum score children could get, scores were first standardised and z scores were provided for each measure. Then, a mean value of z scores for each category or mathematical area was calculated. Scores on sub-tests within category correlated significantly with each other (see Appendix 4.4).

TABLE 4.24

Summarised Categories of Formal and Informal Arithmetic Knowledge and Skill to Be Used in Prediction of Children's Arithmetic Performance

Summarised Variable	Explanatory Variable
Informal	Task 1 Which number is more ? Task 2 Which is closer to X ? Task 3 Mental addition Task 4 Estimation
Base ten	Task 5 Enumeration by tens Task 6 Counting large numbers Task 7 Multiples of large numbers Task 8 Larger written numbers Task 9 Representation of place value
Errors	Task 10 Accuracy and bugs in written addition and subtraction Task 11 Monitoring errors
Addition facts	Task 12 Addition facts
Problem solving	Task 13 Use of principles Task 14 Story problems

Since children's performance on all tasks was associated with their performance on both Young's and the NFER tests, only one set of summaries was produced. This was commonly used in the analyses predicting performance on each mathematical test.

Two backward multiple regressions were conducted, one predicting children's performance on Young's test and another predicting performance on the NFER test. Table 4.25 shows the predictive value of the measures of formal and informal arithmetic knowledge and skill on children's arithmetic achievement.

TABLE 4.25

Summary of Backward Multiple Regression Analyses of Summarised Tasks on Formal and Informal Arithmetic Knowledge and Skill on Arithmetic Achievement

Dependent Variable	Predictor Variable	B	SE B	Beta	T	Sig T
Young[a]	Addition facts	1.47	0.55	.28	2.65	.01
	Base ten	3.08	0.61	.42	5.07	< .01
	Informal	2.13	0.79	.27	2.68	< .01
NFER[b]	Addition facts	2.40	0.86	.30	2.80	< .01
	Base ten	2.70	1.04	.25	2.58	.01
	Informal	2.49	1.32	.22	1.89	.06
	Problem solving	1.87	1.01	.19	1.85	.07

[a] $df = 71$, $R^2 = .70$, $F = 53.69$, $p < .01$. [b] $df = 71$, $R^2 = .67$, $F = 33.58$, $p < .01$.

It was found that children's knowledge of addition facts and their knowledge and skill in informal arithmetic and base ten system together explained 70% of the total variance in performance on Young's test. In addition to those variables, problem solving skills were also found to be a significant predictor of children's performance on the NFER test. The four variables together explained 67% of the total variance in children's performance on the NFER test.

DISCUSSION

4.4.1 Introduction

The analysis of children's scores on measures of formal and informal arithmetic knowledge and skills has shown that performance varied with arithmetic group. More specifically, it was found that third-grade below average children differed significantly from their above average peers in performance on almost all tasks. They resembled, however, their average peers in more than half of the tasks.

Prediction analyses further showed that knowledge and skill in informal arithmetic and base ten system, along with knowledge of addition facts predicted children's performance on both mathematical tests used initially for sample selection (NFER, 1971; Young, 1979). Children's skill in problem solving also predicted performance on the NFER test.

In what follows, the results of the group comparisons and their relation to previous studies are discussed first, followed by the evidence from the prediction analyses.

4.4.2 Understanding Variation in Children's Formal and Informal Arithmetic

The present study examined children's individual differences in arithmetic by investigating their knowledge and skill in formal and informal arithmetic. The hypotheses examined came from research evidence on children's math difficulties, summarised in Russell and Ginsburg's (1984) study on the cognitive bases for children's math difficulties. Compared to that study, however, which examined knowledge and skill of math difficulty fourth-grade children in relation to that of their normal peers as well as to third-grade children, the present study compared younger third-grade children with math difficulties to their average peers, further including a group of children with exceptional arithmetic abilities.

A major point of departure in discussing the findings of the present investigation is the awareness that arithmetical ability is not unitary. Arithmetical ability is a composite entity, consisting of several components related to different areas in arithmetic. Although some components may and very often are found to relate, each may exist on its own right. Variability may exist between ability groups, where marked differences are observed between above average and below average children in their performance on different tasks examining different areas in arithmetic. Individual differences in arithmetic performance, however, exist even between children of the same level of arithmetic ability (Dowker, 1998). Accordingly, the present study has shown that marked discrepancies may exist between different domains of arithmetic within children experiencing mathematics difficulties; those children showed limited knowledge and skill in some areas of arithmetical ability, like word problems and addition facts, but not others (e.g., understanding of informal concepts and commutativity). The issue of variability is a crucial one and should always be considered in research on children's differences in arithmetic performance.

The results from group comparisons gave evidence of the tasks which discriminated between children of below average, average, and above average arithmetic achievement. Comparisons were also made between the present findings and those in Russell and Ginsburg to examine pattern of variation, further accounting for age differences. Overall, children with arithmetic difficulties in the present study showed the lowest levels of performance across both studies. Average children in the present study performed better than third-grade and fourth-grade math difficulty children, but were lower than fourth-grade normal children in Russell and Ginsburg. A comparison of Russell and Ginsburg's normal third-grade group and the present average third-grade suggests that on the average Greek children are better than American children on measures of relative magnitude, estimation, mental addition, understanding and use of base ten concepts, and principled knowledge. However, Greek children fall behind their American peers in accuracy in written arithmetic and word problems, further performing more errors which are nevertheless based on sound procedures (bugs).

Finally, above average children in the present study showed the highest performance levels in most tasks across both studies, with the exception of addition facts and use of reciprocity. In what follows, children's performance in each task is discussed as a function of group and study. The issue of "essential cognitive normality" is also discussed.

4.4.2.1 Informal Concepts and Calculational Skills

The examination of children's knowledge and skill in informal arithmetic involved comparing children on tasks which measured understanding of concepts that children have already acquired before they enter school (e.g., "more" and "relative magnitude"), and skill in mental addition and estimation. Children with math difficulties showed similar levels of understanding of relative magnitude and "more" compared with their average peers but both were "outdone" by their peers with excellent math abilities.

Ability in mental addition and estimation, however, discriminated between all children. Although the Estimation task also required going further along or further back in the mental number line as did the previous tasks, both children with arithmetic difficulties and their average peers were impaired by contrast to above average children. It is suggested that the difficulty of this task compared to the previous two may lie in the extra effort children make to deal with solutions to addition problems. Until now, children had to move on the mental number line identifying and comparing distances between numbers (conceptual); in this task, however, even though they were not asked to provide the correct solution to the addition problems, the children have nevertheless tried a "quick" solution to the problems in their heads (the "correct" one) in their attempt to be even more accurate. In this case, children who did not solve the problems accurately were likely to make mistakes in estimating the distance between the "correct" answer (their own faulty solution) and the one provided.

Research has shown that computational estimation, that is, the process of "estimating the result of a computation by performing some mental calculation on approximations of the original numbers" (Sowder, 1992, p. 371) is highly correlated with mental arithmetic. Sowder (1992) argued that mental calculation is a significant component of and facilitates computational estimation, by enabling the invention of procedures which may be idiosyncratic but effective for a particular problem. Both, however, require and are further related to an understanding of the number system (or number sense). In her study on children's differences in arithmetic development, Dowker (1998) found that estimation, referred to as the approximate unknown fact derivation, correlated highly with calculation and furthermore with derived fact strategy use in 5- to 9-year-old children.

Above average children in the present study were highly accurate in mental addition, reaching the highest levels of performance on all tasks. Average children performed moderately yet significantly less accurately. Children with math difficulties were far behind compared with the rest of the children and their low success levels were largely due to "not responding" as sum sizes increased. An examination of children's strategy use further showed that this difficulty with large numbers was based on the fact that below average children relied a lot on counting as a means to solve the problems. When sum sizes increased - and in the absence of more sophisticated strategies, arithmetically disabled children chose to remain silent. Apart from those who did not respond, those who did try the sums over 50 usually made a memory error, that is "forgetting" to add tens or units (e.g., 63 + 31 = 60 + 30 = 90 + 3). In sums under 50, below average children showed adequate calculational strengths.

Average and above average children, on the other hand, were comparatively more accurate which can be largely due to the adaptive deployment of strategies for different sized sums. They resembled each other in the use of more sophisticated strategies to solve the problems, switching from retrieval to regrouping as sum sizes increased. Regrouping, that is, breaking large numbers into more manageable units, was in fact the most common strategy used to solve mental addition problems. Work done by Siegler (1998) has provided significant evidence on children's individual differences in strategy choice, especially on strategy use shifting with experience. More specifically, children may at first use multiple strategies which vary in effort and probably in accuracy, however, with experience they come to switch to retrieval from memory. Siegler (1988) has further identified three types of first graders: the not-so-good students, the good students, and the perfectionists. The latter included children who despite being equally likely as the good students to be accurate and retrieve number facts from memory with success, they would nevertheless engage in checking more often than any other group. Higher standards, Siegler argued, is what characterises those children.

As in the previous studies, children with math difficulties in the present study have been found to suffer from a limited procedural knowledge of informal arithmetic, while they show a fluent understanding of informal concepts such as "relative magnitude" and "more". Russell and Ginsburg (1984) had also found that such concepts are abundant in fourth-grade math difficulty children as well as their normal fourth- and third-grade peers. Further evidence (Ginsburg, 1982; Resnick, 1983, among others) also suggests that children have a sufficient grasp of informal arithmetic concepts even before they come to school.

The impairment observed when task requirements switched from conceptual to procedural was also observed in fourth-grade children in Russell and Ginsburg:

math difficulty children differed from their normal peers in performance on mental addition problems, but resembled each other in the strategies they used, even though math difficulty children used the same strategies in a less accurate manner. Third-grade below average children in the present study did not show this fluency with sophisticated strategies, thus their ability to deal with mental addition problems was restricted to sums of small size (0-20) where they could apply basic counting procedures. This in turn has led to erring (usually not responding) more often when sum sizes increased.

4.4.2.2 Base Ten Concepts and Related Enumeration Skills

The examination of children's knowledge and application of base ten system concepts involved children's identifying larger numbers, enumeration skills, representing place value, counting large numbers, and decomposing large numbers.

Ability to identify the larger of two written numbers did not discriminate between children of different arithmetic performance. Overall, children in the present study were more accurate than both third- and fourth-grade children in Russell and Ginsburg (1984); below average children in the present study did even better than normal fourth-grade children. Children in that study showed similar levels of accuracy. It is a striking finding that children with math difficulties showed high ability in making accurate judgements of pairs of multi-digit numbers. However, there is evidence suggesting that the ability to compare numbers may be independent of the ability to read or write numbers. Research (Donlan, 1998) on the development of arithmetical skill in children with specific language impairments has pointed to the direction of a dissociation of verbal and nonverbal arithmetical skills in the case of place-value knowledge. More specifically, children were successful in judging the relative magnitude of double-digit numbers, despite failing to read the numbers during a transcoding task.

Average and below average children did not differ in their understanding of units and tens; they showed similar accuracy in enumeration. Children in math difficulties in the present study were as accurate as fourth-grade math difficulty and third-grade children in Russell and Ginsburg. Average children in the present study did slightly better than normal fourth graders. Children with excellent arithmetic abilities did better than all children in either study. The children in the present study did not differ in the strategies they used to count the dots: the majority used the enumeration of multiplication by tens, that is, a strategy that reduces calculational effort. According to this strategy, children count the dots on one row (e.g., 10) and then add (e.g., 10, 20, 30, etc.) or multiply (e.g., 10 x 5, etc.) by the number of the rest of the rows.

Ability to represent numbers by splitting piles of beans into tens and units did not discriminate between below average and average children, either. Below average in the present study were more accurate than fourth-grade math difficulty children in Russell and Ginsburg; however, their math difficulty children were even less accurate than their third-grade peers. Average children in the present study were better than both third-grade and fourth-grade math difficulty in Russell and Ginsburg. Children with exceptional arithmetic skill were more accurate than their below average peers in representing place value, further scoring the highest of children in either study.

Variation was observed in children's ability to count large amounts of money and decompose large quantities (numbers) into smaller components, as a function of children's arithmetic achievement. More specifically, in counting money, below average children were less accurate than all their third-grade peers in the present study and in Russell and Ginsburg; yet, they were more accurate than fourth-grade math difficulty children in that study. The average children in the present study did better than third graders in Russell and Ginsburg but less well than normal fourth graders.

Two hypotheses are proposed, as to what may have caused below average children's impairment. The first comes from children's comments on counting large numbers, for example, "I know how to count up to 1,000 only..."; it was indeed found that the trials most often misjudged by children were the ones over 1,000, that is, 1,530 and 3,020. The second refers to children's exposure to the materials: there were children who complained about "not knowing much about money". There is no doubt, it can be a combination of both, that is, children may have less experience with the thousand drachmas notes than with smaller (hundreds) drachmas notes that are used widely.

Accordingly, in decomposing numbers, below average children could barely solve half of the problems. Apart from calculational errors, children were often confused with the process of decomposition; that is, on (20*100), they would answer 50, instead of 5. This shows that in calculation, they give priority to the first digit but task demands limit their accuracy. It is nevertheless suggested that children with arithmetic difficulties do experience confusion when decomposing large quantities into smaller components. In addition, average children were more accurate than both third-grade and fourth-grade math difficulty children in Russell and Ginsburg. Above average children in the present study showed the highest levels of accuracy compared to children in either study.

To conclude, children with arithmetic difficulties have a sufficient understanding of the base ten system and exhibit strengths such as counting by tens or identifying the left-most digit as crucial in determining the relative magnitude of numbers. They also show a clear understanding of the part-whole schema, that is, that numbers are composed of other numbers (Resnick, 1983), but they can get easily confused depending on the demands of the task. While they apply their knowledge efficiently in small numerosities, they exhibit severe weaknesses when dealing with large numbers, especially the ones over the hundreds. Children of above average math ability, on the other hand, show a mature understanding of base ten concepts and an extraordinary fluency in calculating accurately numbers of any size.

4.4.2.3 Error Strategies and Other Calculational Procedures

Another significant area of mathematical knowledge is the ability to perform written calculations. Skill in written addition and subtraction was examined by asking the children to write down and solve addition and subtraction problems. Their errors constituted significant evidence for the examination of strategies children use to solve written calculations. A second task investigated children's ability to detect errors like misalignment, miscarry, and miscalculation.

Skill in written addition and subtraction discriminated between children of different arithmetic performance. Children with arithmetic difficulties solved less than half of the problems, average children were moderately accurate, and children with exceptional arithmetic performance solved 8 out of 10 problems on the average. Average and below average children were less accurate than third-grade children in Russell and Ginsburg. The children with exceptional arithmetic abilities did almost as well as normal fourth-grade children.

Apart from "not solving the problem", the most common errors of children with math difficulties were miscalculations and buggy subtraction algorithms, such as subtracting the upper digit from the lower when the upper is smaller. These errors were also prominent in their average peers' performance, as well as in the relatively infrequent instances of errors made by above average children. Overall, the children in the present study made more "sophisticated" errors, compared to third and fourth-grade children in Russell and Ginsburg (1984). Fourth-grade math difficulty children and their third-grade peers in that study made more "primitive" errors compared to both the average and below average third-grade children in the present study; such errors would include writing numbers as they sound, performing the wrong operation, employing buggy subtraction procedures, or performing simple miscalculations.

Miscalculation and bugs were a common source of difficulty for all children, however, in a different degree. Miscalculations were found to be due to a limited knowledge of addition facts (will be described in the next section). Bugs or systematic error strategies have also been described as commonly accounting for children's errors. Ginsburg (1982) contends, accordingly, that Grade 2 to Grade 6 children's errors are not capricious but based on systematic rules that children have been taught at school. For example, subtracting the upper from the lower digit when the upper is smaller, instead of borrowing. Ginsburg (1982) argues that children eventually learn to perform such procedures correctly, but until they reached that level of accuracy, their errors are based on distorting or misinterpreting the sound rules they have learned. Brown and Burton (1978) also define bugs as incorrect implementations of subprocedures of a skill used to solve a particular problem. They argue that bugs can be manifested in isolation or combination and that many bugs may even generate a correct answer. Examples of bugs include continuing to borrow from all columns, once borrowed, subtracting all borrows from the left-most digit in the top number, subtracting the upper from the lower digit when upper is smaller, always answering 0 when the top digit in the column is 0, among others.

Average and below average children did not differ in their ability to detect errors in written calculations: they both showed an adept ability to identify misalignment, miscalculation, and miscarrying errors. Justifications did not vary with mathematical group, either. Overall, children did not verbalise the principles underlying each error; when asked why they thought the problem was wrong, they engaged in "doing" the problem as a means of responding, showing how it should be done. Compared to Russell and Ginsburg (1984), below average children in the present study were as good as normal fourth-grade children, being further more accurate in detecting errors than third-grade and fourth-grade math difficulty children. Children with exceptional arithmetic skill did better than any other group in either study.

In sum, while children with arithmetic difficulties show limited accuracy in written addition and subtraction, their errors are as sophisticated as those of their average and above average peers; that is, they include buggy algorithms and systematic error strategies. Math difficulty children in Russell and Ginsburg performed more primitive errors. In identifying errors, below average children in the present study were as accurate as fourth-grade normal children.

4.4.2.4 Knowledge of Number Facts

Knowledge of number facts, especially large addition facts, discriminated children of different arithmetic performance. The task involved simply responding to addition facts without counting. A dramatic drop in performance was observed, with children exhibiting the poorest achievement levels compared to the rest of the tasks. Children with math difficulties, in particular, suffered the most severe impairment, being accurate to barely one-third of the trials, while average children answered roughly half of the trials and above average children were accurate in seven out of ten trials. Below average children scored the lowest of all children in both the present study and that of Russell and Ginsburg (1984). Average children in the present study were as accurate as math difficulty fourth-grade children in the cited study.

However, it was limited knowledge of *large* addition facts that was responsible for children's low performance: children were quite successful with smaller numerosities. Proof supporting the size effect hypothesis is ample. First, children were more likely to be accurate on sums of less than 10 ("small sums", e.g., 2 + 5) than on sums of more than 10 ("large sums", e.g., 4 + 9). Children with math difficulties, in particular, erred more often on large sums: 75% (n = 12) of these children solved *none* of the large sums, while the same proportion (75%) solved at least 3 out of 4 small sums successfully. Below average children's difficulty with large number facts can also be traced back to their inability to cope with large problems in mental calculation (Task 3). When dealing with sums under 20, children used counting and error frequencies were low; as sum sizes increased, children did not even attempt to do the problems. In written addition and subtraction (Task 10), below average children's most common errors were simple miscalculations and buggy subtraction algorithms.

This finding is not surprising, since large numbers constitute a constant source of difficulty for children. In the case of number facts, it is possible that children are less frequently exposed to larger-number facts than smaller facts. Ashcraft and Christy (1995) found that small number facts, that is, those involving operands with 2 to 5, are presented twice as often as large facts (i.e., those involving operands more than 5) in school arithmetic textbooks in Grades 1 to 6. The implications are obvious and point towards the direction of less frequent encounters leading to less practice which in turns leads to poor learning.

There is evidence from the area of memory suggesting that the efficiency of retrieving information (representations of facts, in this case) from long-term memory is associated with performance on arithmetic tasks. Siegler and

Shrager (1984), among others, have proposed a model of associations, according to which the more times a problem and an answer are associated (encountered), the stronger their trace in long term memory, and the greater the probability of retrieving them correctly during arithmetic calculation. This is also examined in detail in the following chapter.

4.4.2.5 Problem - Solving Skills

The investigation of children's problem-solving skills focused on two main abilities in dealing with mathematical problems: knowledge and use of principles to shortcut the labour of calculation, and skill in solving word problems.

Children were invariably fluent in their use of commutativity: children with math difficulties were equally as likely to use commutativity to solve commuted versions of sums as any other third-grade child. Use of reciprocity showed some variation, where children with math difficulties used the principle less often than their average peers, yet as frequently as their above average peers. Based on below average children's reports, they relied on working out the solution to the "reverse" version of the problem, using counting. Russell and Ginsburg (1984) also found commutativity being used widely. However, children in that study also used reciprocity quite extensively. The widespread use of commutativity among third-grade children is not surprising. Research on children's knowledge of mathematical principles has shown that children from a very young age show a clear understanding of the commutative law of addition, that is, the order in which items are added does not affect the sum of the operation (Baroody & Gannon, 1984; Baroody & Ginsburg, 1986; Baroody, Ginsburg, & Waxman, 1983). Some children appreciate the principle even before they can do sums (Cowan & Renton, 1996; Ganetsou, 1993).

Average and below average children showed similar strengths in solving word problems. Both groups resembled fourth-grade math difficulty children in Russell and Ginsburg (1984). In other words, below average and average third-grade children show a procedural knowledge and skill which closely resembles that of fourth-grade below average children. Children with exceptional arithmetic skills, however, were more accurate than any child in either study.

A closer look into the relation between level of problem difficulty and success rates shed some light in the difficulties faced by the mathematically disabled children. It is suggested that children's difficulties arise from the specific type of the problem and its semantic structure. Riley, Greeno, and Heller (1983) have distinguished different types of problems, each varying in complexity.

Indeed, a common source of confusion for children with arithmetic difficulties in either study was the subtraction and the multiplication problems. Below average children in the present study were impaired when dealing with subtraction, both the complex and the one involving irrelevant information. It was further observed that no child in the below average group was able to do the multiplication problem correctly. The fact that the commonest way of responding was to perform another operation (addition and subtraction) means that those children had a problem in performing the particular operation. Children also experienced difficulties with division; half of them did not attempt to do it. What is being suggested is that since both operations were introduced late in the second grade, children were not as confident in dealing with them as they were with the rest of arithmetic operations.

In sum, children with arithmetic difficulties showed some form of abstract thinking using insightful solutions to problems as well as substantial strength in solving elementary problems. They experienced difficulties in doing complex operations or operations which they had only recently been taught.

4.4.3 The Contribution of Knowledge and Skill in Formal and Informal Arithmetic to Children's Variation in Arithmetic Achievement

The present study further examined the independent contribution of knowledge and skill in formal and informal arithmetic to children's mathematical performance. Tasks were grouped together and a single value indicated a child's ability in each mathematical area examined. When the grouped variables regressed on children's performance on both math tests, knowledge and skill in informal arithmetic and base ten system and knowledge of addition facts predicted performance on both tests. The variables that were excluded, despite being associated with achievement, were correlated with other variables. No independent variation is accounted by these abilities.

The three grouped variables accounted for 70% of variation in children's performance on the Young (1979) test. The specific test indeed examines children's informal concepts, base ten concepts, and addition facts.

Children's problem-solving skills also predicted performance on the NFER (1971) test; the four variables together accounted for 67% of variance in children's performance on the NFER test.

4.4.4 Introduction to the Next Study

This chapter examined variation in formal and informal arithmetic knowledge and skill as a function of children's arithmetic ability. Also, the independent contribution of these variables in children's variation in arithmetic was investigated. Chapter 5 describes the second path of examination of cognitive factors related to arithmetic achievement, namely, working memory storage capacity. Children's concurrent memory spans and simple memory and counting tasks are examined as a function of children's arithmetic achievement. The theoretical framework of the study is first introduced, followed by the methods used, the results, and a short discussion of the findings.

CHAPTER 5

WORKING MEMORY PROCESSES RELATED TO CHILDREN'S ARITHMETIC ACHIEVEMENT

5.1.1 Introduction

Another area of research which has attempted to explain children's arithmetical difficulties is human memory. The purpose of the current investigation is to examine how children's concurrent memory spans and other memory and basic counting tasks varied with children's arithmetic performance, and how variation in spans and other tasks might explain variation in children's achievement in arithmetic. First, the theoretical background of the study is described, followed by the methods used to collect the data, the results of the analyses, and a short discussion on the findings.

5.1.2 Review of Studies on Children's Working Memory Processes

Efficiency of working memory has systematically been related to performance on cognitive operations or tasks such as reasoning (Johnson-Laird, 1982), reading (Baddeley, 1990; Daneman & Carpenter, 1980; Siegel & Ryan, 1989), and arithmetic (Baddeley, 1990; Healy & Nairne, 1985; Hitch, 1978; Hitch & McAuley, 1991). Researchers, accordingly, have investigated the length of memory spans in children suffering from different types of learning disabilities (Share, Moffitt, & Silva, 1988; Siegel & Ryan, 1989; Hitch & McAuley, 1991). As the present study focuses on arithmetic achievement, emphasis is placed on studies that have examined memory spans of children who suffer from arithmetic difficulties despite having normal intelligence and normal levels of attainment in other academic subjects.

First, the construct and function of working memory is outlined, followed by common applications to different cognitive areas, such as reasoning, reading, and arithmetic. Major studies which shed light on the relationship between working memory and children's arithmetical difficulties are reviewed next. These studies also constitute the basis for the current investigation.

5.1.2.1 The Working Memory Hypothesis: Structure and Evidence

Baddeley (1990) gives an account of the early debate over the nature of short term or working memory. He argues that the first theoretical construct which received satisfactory levels of acceptance was that of Atkinson and Shiffrin in the 60s.

Short Term Store as an Operational or Working Memory

The Atkinson and Shiffrin model, or modal model, placed short-term store in the most critical place in central information processing, being responsible for co-ordinating the subroutines that are responsible for both acquiring new material and retrieving old. According to this model, environmental input, perceived by sensory buffers (visual, auditory, and haptic), enter a short-term store or temporary working memory where traces are held for a limited time only. Through specific control processes, such as rehearsal and coding, traces can be stored in long-term memory, the permanent memory store. The longer the trace is in the short-term store, the greater the probability that it will be transferred to long-term. Information can then be retrieved (another control process) from long-term store, be processed in short-term memory and exit as a response (output).

The Working Memory Model

Studies using more sophisticated techniques, however, gave evidence of a concept of working memory that could be more scientifically examined and better consolidated. Baddeley and Hitch (1974) introduced the digit span concurrent memory tasks, which shed more light to the structure of working memory. Eventually, they came up with the working memory model, drawing upon studies in verbal reasoning, language comprehension (prose), and free recall (long term memory). Their major findings were: (a) reasoning time would increase with concurrent memory load of digits, (b) the level of comprehension would lower with concurrent memory load of digits, and (c) free recall remained intact if material (digits) was coded at the beginning (learning) of the words to be learned.

According to this model, working memory refers to the temporary storage of information while other cognitive tasks can be performed. The researchers conceptualised it as a working memory system with limited capacity which can be conceptualised as a "work space" in which two systems operate simultaneously. This work space is allocated to either storage or control processing demands. More specifically, Baddeley (1990) argued that working memory consists of a central executive system which co-ordinates other subsidiary or "slave" subsystems. These subsystems may be responsible for manipulating speech-based information (e.g., the articulatory or phonological loop) or visual images (e.g., the visuospatial scratchpad). The articulatory loop consists of the phonological store which temporarily holds speech-based items as a phonological buffer, and an articulatory control system which rehearses overtly, covertly, or may even recall overtly, the items (i.e., inner speech).

The articulatory loop has been studied both extensively and systematically and has been found to play a critical role in cognitive operations (Baddeley, 1990). Evidence that it exists comes primarily from the effects it has on recall. According to the phonological similarity effect, items that are phonologically (acoustically or phonemically) similar are more difficult to be recalled than items that are unsimilar. For example, the letter sequence *DTBC* is more difficult to recall than the letter sequence *UARX*. This is because the phonological store is based on a (speech) phonological code. Items that are similar in sound have similar traces that are more difficult to discriminate during recall.

The irrelevant speech effect occurs when irrelevant material gains access to the phonological store and disrupts its operation. For example, it is observed that when repetition of a series of digits is accompanied by white noise, nonsense words, or spoken words, levels of correct repetitions drop. Since the store operates in a phonological and not semantic level, that is, it represents items as codes and not as words, the effect is hypothesised to increase when meaningful material is attended; studies, however, have been inconclusive.

Word length has been also found to be important in recall. The word length effect suggests that longer words or speech, referring to duration, are more difficult to remember than shorter ones. Since subvocal rehearsal maintains items or traces in the phonological store, the quicker it runs (i.e., short words), more items are rehearsed and thus maintained. Spans decrease, on the other hand, when longer words are involved.

Finally, articulatory suppression denotes the absence of the subvocal rehearsal process which in turn disturbs the operation of the phonological loop; it occurs when articulation of an irrelevant item is required during presentation and recall. The articulation of an irrelevant item (e.g., *the*) during recall dominates the articulatory control process, hence preventing it from being used to either maintain material already in the phonological store or convert visual material into a phonological code. Suppression of the articulatory processes has different influence on each effect described above: it abolishes the word-length effect, since the length of the word is crucial only if the subject rehearses, as well as the phonological similarity and irrelevant speech effects, if the information is presented visually (because in both situations auditory materials are stored in the phonological buffer).

5.1.2.2 The Role of Working Memory in Cognitive Tasks: Reasoning, Reading, and Arithmetic

The efficiency of working memory has been considered the single biggest factor in *reasoning*. Johnson-Laird (1982), in his analysis of the formation of mental models, related the process of deduction of inferences in syllogisms to the efficiency of working memory storage. He outlined three stages that a human reasoner goes through in order to solve correctly a simple or complex syllogism: first, he constructs a mental model of the situations that are described in the premises; second, he searches for alternative models to check the validity of the inference, by switching around the interpretations; third, he has to put into words the common characteristic of a set of mental models (deduction). Working memory, he claimed, is crucial in construction of a model, in that a representation of one premise should be held in working memory whilst information of the other premise is combined with it. Johnson-Laird and his colleagues concluded that deducing an inference and inferring the valid inference both depend on the following factors which relate to memory (a) the amount of combinations or interpretations (i.e., the greater load on working memory, the harder it is to make an inference), (b) the timing (i.e., the shorter time needed, the more efficient the storage and output), and (c) the sequencing of these operations (*B-A, B-C* is more difficult than *A-B, B-C*).

During *reading*, the executive may be conceptualised as retrieving information about syntax, word meanings, phonological rules, or all three of them while the subsidiary system retains the words, phrases, or sentences while they are being processed and for brief periods in order that longer units of text can be comprehended (Baddeley, 1990).

Daneman and Carpenter (1980) developed a reading span test, which was used in many studies to be described later. The subjects had to read aloud sentences and then recall the last word. Set sizes of sentences increased until performance was impaired on the basis of 3 out of 3 sentences per set. Span was taken as the largest set that performance was correct. The size of this sentence-based working memory span has been found to relate to reading comprehension. As the number of words that the individual was required to remember was increased, the demands on working memory were assumed to increase. The researchers argued that one of the contributing factors to reading difficulties may be relatively poor working memory when language is involved.

Working memory has also been related to performance on different *arithmetic* operations. For example, Healy and Nairne (1985) describe a model of the counting process according to which subjects keep track of their location in the counting sequence by monitoring phonologically coded short-term memory representations of the numbers.

Solution of arithmetic problems similarly relates to working memory efficiency. Applying Baddeley's (1990) model, during the process of solving word problems, the executive monitors and retrieves information about the operation to be used (e.g., multiplication facts), while the subsidiary system stores the specific numbers involved in the calculation.

As written calculations serve as a permanent working storage, mental arithmetic involves the extra process of temporarily holding initial information (e.g., addends) until the operation is completed. Hitch (1978) examined the role of working memory in mental arithmetic. He gave addition problems (3-digit addends) orally with the instruction to write the answer and explain the strategy that was used. He found that, as subjects do the calculation in stages, some form of storage occurs (e.g., retaining hundreds and tens as units are calculated). Next, instructions about strategy use were manipulated by imposing limitations which put more constraints on the subjects with storage. A comparison between performance on fully mental addition and fully written addition showed that forgetting the addends is an important limitation in mental calculations. It was observed that errors increased in frequency from units through to hundreds, which suggests that traces in working storage undergo some form of decay. In sum, Hitch argued that both initial information (addends) and interim information (results of adding tens, units, and hundreds) are held in working storage.

5.1.2.3 Working Memory Deficits Related to Children's Arithmetic Difficulties: A Review of Research

There is enough evidence suggesting a need to differentiate between subtypes of learning difficulties in the examination of working memory deficits. In a study on gifted adolescents, Dark and Benbow (1991) showed that mathematically precocious youth were better able to store and manipulate numerical (digits) and spatial (stimuli location) information, while their verbally precocious peers were better with words. This was further attributed to differences in item identifiability in long term memory.

A contributing factor to difficulties with arithmetic is relatively poor working memory. Children's computational errors and slow counting procedures have been associated with memory deficits. For example, Geary (1990) argued that such errors and immature procedures found in children with arithmetic difficulties are attributable to the weak representation of arithmetic facts in long term memory.

Evidence that children with mathematical difficulties suffer from deficits in short term memory comes from a wide range of studies. A brief account of the variability of the findings is given first, along with the underlying theories that have been hypothesised to account for such deficits. Some of these studies are discussed later in greater detail.

It has been suggested that different subtypes of learning disabilities relate to different working memory deficits. Siegel and Ryan (1989) argued for the specificity of deficits, depending on the kind of information (verbal or numerical) to be processed. More specifically, they hypothesised that children with reading difficulties would be impaired on language-related working memory tasks, while children with arithmetic difficulties would be impaired on a counting working memory task.

They examined children suffering from different subtypes of learning disabilities, that is, arithmetic and reading, using two concurrent memory tasks; the Working Memory - Counting task (Case, Kurland, & Goldberg, 1982) and the Working Memory - Sentences task. The latter was adapted from Daneman and Carpenter (1980) and involved supplying the missing word of incomplete sentences presented orally and recall all supplied words at the end of each set.

Siegel and Ryan (1989) found that arithmetically disabled children experienced difficulties only with the Working Memory - Counting task, while reading disabled children had problems with both the Working Memory - Sentences and the Working Memory - Counting task. They attributed this impairment to the verbal requirement of both the reading and the arithmetic tasks. Due to research constrains, their reading group mostly consisted of children with reading and arithmetic difficulties. A separate analysis, however, showed the same pattern of results.

In sum, Siegel and Ryan argued that children who suffer from both reading and arithmetic difficulties suffer from a generalised-purpose working memory, while children with mathematical difficulties suffer from a deficit in a special type of working memory which specialises in arithmetic operations.

Hitch and McAuley (1991) further found that arithmetically disabled children show impaired memory spans when they deal with numerical information, but their spans are normal when dealing with nonarithmetic information. Also, they found that arithmetically disabled children have difficulty with counting (show slow counting procedures) and have short spans for digits. Hitch and McAuley explained these deficits in terms of slow access of number representations in long term memory.

As the studies described above show, different speculations about working memory deficits in arithmetically disabled children have been made. However, each received different degrees of support. While the investigation continues, there are two hypotheses that have gained substantial recognition: the rate of articulation (speech rate) which relates to working memory and ability to access items in long-term memory. Each is discussed in isolation.

Articulation Rate (Speech Rate)

One of the subsidiary systems most extensively studied, is the articulatory or phonological loop. It consists of two components which serve each other: the *phonological store* which works as a buffer, holding speech-based information for 2 seconds, and the *articulatory control process* which involves rehearsing overtly (recall) or covertly (inner speech) the items to be remembered (Baddeley, 1990). The assumption is that this subvocal rehearsal maintains items in the phonological store by refreshing their traces. Memory traces in the phonological store are assumed to fade after about 2 seconds, unless they are refreshed by the subvocal rehearsal of the articulatory control process. Thus, the articulatory process rehearses the trace and feeds it back to the store.

Studies have shown that the faster the articulatory process runs, the more items will be maintained, and the longer the memory span will be. Speech rate is a measure of the rehearsal rate of this process. Baddeley, Thompson, and Buchanan (1975) examined university students' memory spans for words with different phonemes and number of syllables. They found a significant correlation between memory span and articulation rate, further claiming that spans equal the number of words the subjects could read in 2 seconds. Reading rate also correlated with memory spans.

Hitch, Halliday, and Littler (1993), accordingly, found a linear relationship between mean articulation time and mean spans of children of different ages, while the same relationship was not observed in the case of mean item identification time.

A step forward in this investigation was made by Cowan (1992). Based on the timing of spoken recall of 4-year-old children, he found that articulation rate during interword pauses, that is, the rate of reactivating items during pauses, has a stronger relationship to memory span than word length. Research done by Henry (1991) has shown that 5-year-old children do not show word length effects in recall, which presuppose the existence of rehearsal, when the modality of input is auditory. Word length effects may be present in 5-year-olds, however, they are not due to rehearsal. Seven-year-old children do show such effects, independent of the modality of the input. Further research focusing on college students (Cowan, Day, Saults, Keller, Johnson, & Flores, 1992), however, showed that the relationship between articulation rate, word length, and immediate recall is rather unclear. By manipulating both the length of words in the first half and the second half of the lists as well as using both forward and backward directions, they found that the length of words recalled first influence the recall of words output subsequently.

The hypothesis that speech rate is a determinant of memory span has also gained strong support in the area of arithmetic. Baddeley (1990), for example, describes the relationship between speech rate and counting speed. Kail (1992) examined the relation between speech rate and memory span for digits. He found that articulation rate correlated negatively with measures of digit span and letter span; that is, the faster the counting speed, the longer the short term memory span for digits. Moreover, Naveh-Benjamin and Ayres (Baddeley, 1990) showed there is a relationship between memory span and the time it takes to articulate the digits one to ten.

Retrieving Information from Long Term Memory

An alternative explanation of differences in concurrent memory spans has been found in the role of long term memory; it refers to the process of accessing items in the long term store.

The most sustained hypothesis related memory spans to the speed of item identification, that is the speed with which an item is accessed in long term memory. Case, Kurland, and Goldberg (1982) found that developmental increases in span relate to what they called "operational efficiency". They argued that short-term memory consists of a hypothetical *storage space*, which refers to the amount of space for storing information, an *operating space*, which refers to the amount of space for executing intellectual operations, and a *total processing space* which is the total amount of processing resources (i.e., the sum of storage and operating space). They explained working memory capacity (in specific, memory spans) in terms of a trade-off between storage and operating spaces, according to which a decrease in the operating space would lead to an increase in the amount of space used for storage, and thus to longer spans. Operating space, however, would depend on the speed of executing the operations (operating speed).

Evidence supporting this hypothesis came from children's and adults' performance on a task Case et al. devised, the Counting Span task. According to this task, the subjects have to count the green spots from a field of green and yellow spots (printed on cards) and recall the products of all their counts. Span is taken as the largest number of counting operations in which performance was accurate. Case and his colleagues found a linear relationship between counting efficiency and counting spans, where increases in counting speed led to longer spans for digits. In other words, children who counted more quickly (operating speed) were able to recall more products of their counts. The same linear relationship was found between speed of word repetition and span for these words: increases in operation speed (i.e., more rapid repetition of words) led to longer word spans.

Thus, Case et al. argued that developmental increases in memory span are not due to increases in storage capacity, rather they are due to the availability of more memory resources for storage which is a result of the decrease in the capacity taken up by identification.

Accordingly, differences in memory spans of gifted adolescents have been explained in terms of the item identification hypothesis, further relating it to the strength of the representations in long term memory. Dark and Benbow (1991) compared verbally and mathematically precocious adolescents, on measures of coding and manipulating information in working memory, as well as storing information in long term memory. The stimuli used were digits, letters, words, and location stimuli. Overall, they found that capacity varies as a function of group and type of stimulus; that is, mathematically precocious adolescents were better able to handle digits and location stimuli, while their verbally precocious peers were better with words. Dark and Benbow suggested that these differences in working memory enhancement levels are attributed to differences in identifiability; that is, mathematically gifted subjects had stronger representations of digits, whereas verbally gifted subjects had stronger representations of verbal materials. This leads to a more rapid identification of the items, which in turn leads to longer spans for those items.

In the same way, poor representations of arithmetic facts in long term memory have been suggested to account for many arithmetic difficulties. For example, Geary (1990) explained arithmetically disabled children's frequent counting and retrieval errors in terms of a theory of representations of arithmetic facts, namely the distributions of associations model of strategy choices, proposed by Siegler and Shrager (1984).

According to this model, performance on arithmetic problems depends on retrieval from long term memory. In sum, the probability of correctly retrieving (from long term memory) the answer to a problem indicates the associative strength between the problem and its solution. Practice and experience lead to stronger representations of arithmetic facts in long term memory which in turn strengthen the associations between problem and solution. The latter suggests more accurate performance.

A major study which related arithmetical difficulties to speed of accessing number representations in long term memory was conducted by Hitch and McAuley (1991). They conducted two experiments which disproved the speech rate hypothesis of limited memory spans while giving more importance to the role of long term memory. They compared 8-9-year-old children with arithmetical learning difficulties (ALD) but who nevertheless possessed average reading abilities to their normal peers.

In the first experiment, they attempted to replicate Siegel and Ryan's (1989) findings, using more efficient methods. As it has been mentioned earlier, Siegel and Ryan (1989) compared children's performance on a visual counting task and a listening sentence task; however, this method overlooked significant implications of the visual versus auditory modality. Given the evidence that children with arithmetical learning difficulties are significantly impaired on measures of visuospatial abilities (Rourke & Finlayson, 1978) and on visuospatial memory tasks (Fletcher, 1985), Hitch and McAuley (1991) used four concurrent tasks, by crossing type of operation (counting vs. comparison) and type of modality (visual vs. auditory).

The Visual Counting Span was based on Case et al.'s (1982) original version. Children had to count the green dots from a field of green and yellow dots, and recall the products of all their counts. Set sizes increased until performance was impaired which was taken as the child's span for digits presented visually. They further introduced a non-visual task, the auditory counting span, where children were asked to count the tappings of a tin held out of sight, and recall all counts at the end of each set of counts.

Instead of the listening task, they introduced the comparison span task, which would give evidence of children's non-arithmetic concurrent task abilities. It consisted of a visual and an auditory version. In the Visual Comparison Span, children had to choose the odd card out of a triad. Two of the cards contained 3 spots in the same pattern, while the spots on the third one were in a different pattern. Children had to remember the location of the "different" card and recall the position of all cards at the end of each set. The Auditory Comparison Span engaged children in judging which nonsense word is different from a set of 3 nonsense words, repeat it, and remember the different word of each set.

Hitch and McAuley (1991) found that ALD children's performance on tasks involving counting was impaired, independent of the visuospatial nature of the information they processed (visual or auditory). Overall, they found that spans involving visual presentation were higher than spans involving auditory input. In addition, spans were higher when the operation was counting rather than comparison.

Thus, they confirmed Siegel and Ryan's (1989) observation that children with arithmetic learning difficulties are impaired on the visual counting span, further extending it to non-visual counting span. Also, their spans were unaffected in the comparison span tasks. That is enough evidence that they did not suffer from a generalised working memory deficit. However, in order to claim for the existence of a deficit in a specific subtype of working memory that specialises in arithmetic operations, it was necessary to prove that it is the *combination* of arithmetical processing and temporary information storage which is responsible for ALD children's impaired performance, and not individual problems with counting or working memory capacity per se.

In a second experiment, Hitch and McAuley (1991) tested this hypothesis. The investigation focused on two options with regards to the lower counting spans of children with arithmetic learning difficulties: difficulties with counting and a deficit in temporary information storage. Hitch and McAuley employed a battery of tests which would allow the examination of these processes in isolation.

First, they examined children's auditory digit spans and spot counting time. In the first task, children had to repeat sequences of digits read by the experimenter; in the second, children had to count as quickly as possible the green spots from a field of green and yellow spots. As mentioned above, the rationale was to measure children's counting speed in the absence of concurrent memory load, and children's short-term memory storage capacity in the absence of concurrent cognitive processing such as counting. These two tasks were also subtasks involved in performance on the Visual Counting Span in the first experiment.

Children also went through two other counting tasks, where they had to recite the number sequences from 1 to 20 and from 2 to 20 as quickly as possible. These tasks measured children's knowledge of number sequences as well as their counting ability in the absence of the one-to-one correspondence requirement in a visuospatial arrangement. Finally, an articulation task gave evidence of children's articulatory fluency, since, as described earlier, speech rate has been found to relate to memory span.

Children with arithmetical learning difficulties were significantly behind compared to their peers in almost all measures. More specifically, they had slower spot counting times (and made more errors) and lower digit spans than their normal peers. A multiple regression showed that both of these subtasks explained variability in Visual Counting Span. In addition, children with arithmetic learning difficulties were found to be significantly slower in reciting numbers from 1 to 20 and 2 to 20. However, their articulation rates did not differ from that of their peers.

Clearly, these findings did not support the hypothesis of a memory subsystem being responsible for arithmetic operations. ALD children's low visual counting spans were due to difficulties with counting itself and lower spans for digits. However, the relationship between deficits in counting, counting span, and digit span was unclear. Since articulation rate was normal, the hypothesis of low speech rate leading to poorer recall could not be substantiated.

What Hitch and McAuley argued for is that slower access to digit representations in the long term memory may account for these deficits. More specifically, they interpreted ALD children's short memory spans in terms of a decrease in operation (i.e., counting) efficiency, proposed by Case et al. (1982). They argued that ALD children's shorter digit spans are due to their slower access to digit representations in long term memory.

Accordingly, difficulties in reciting the number sequences (from 1 to 20 and 2 to 20) were partly attributed to slower access to numbers in long term memory (probably due to low practice and familiarity) and to a particular difficulty with learning sequences.

5.1.2.4 Conclusions

It has been shown that children's arithmetic difficulties relate to working memory processes. There are many manifestations of such deficits, including short memory spans, especially for digits, ineffective counting procedures, poor representation of arithmetic facts, among others. It cannot be safely concluded what underlies these deficits; however, slow speech rate and problems with accessing numerical information in long term memory (whether slow speed or weak representations) have been found systematically to relate to arithmetically disabled children's short memory spans.

5.1.3 Aim and Hypotheses

The aim of the present investigation is to identify working memory processes and simple memory and basic component abilities which may specifically account for children's different levels of arithmetic performance.

The investigation of working memory processes involves tasks which are referred to as concurrent span tasks; these tasks usually involve children retaining information while performing another cognitive operation. Case et al. (1982) developed the counting span task in which the child must count the green spots from a field of green and yellow spots and recall the products of all of his counts after a set of cards. Daneman and Carpenter (1980) designed the listening span task, where the child had to provide the missing word from sentences and recall all the missing words from a set of sentences.

Previous research has shown that children with arithmetic difficulties do not suffer from a generalised working memory deficit. Siegel and Ryan (1989), for example, found that arithmetically disabled children have lower spans only when the operation to be performed involved counting as opposed to supplying the missing word of incomplete sentences.

More evidence about this specific working memory deficit in arithmetically disabled children came from Hitch and McAuley (1991). They compared children with specific arithmetic learning difficulties with children matched for

IQ and reading. The children with specific arithmetic learning difficulties were impaired only when they had to retain information while performing *arithmetic* operations, whether these involved counting visible objects or sounds. They did not differ from the rest of the children when they dealt with non-arithmetic information (e.g., identifying the card which contains a pattern of spots that is different from the rest and recall its location in every triad). Hitch and McAuley did not attribute the impairment in counting spans to any deficit in a specific arithmetic component of working memory. They suggested that the selective impairment of these children is simply due to lower digit spans and slower counting. These in turn might be due to slower access to digits in the long-term memory.

The present study replicated Hitch and McAuley's comparison of children with different maths ability on concurrent span tasks and more basic component tasks, that is, digit span and counting. It included, however, three groups that varied in their maths ability; that is, children with below average maths ability, children with average maths ability but below average reading performance, and children with above average arithmetic ability. It also included another task; word span.

The rationale for these was as follows: by including above average maths children further evidence on the nature of the association between maths ability and span task performance would be obtained (above average and below average differed in maths ability). Above average children were also compared to children with average maths but below average reading ability: as both these groups had better arithmetic skills than their below average peers and they differed significantly in their reading scores, the specificity of the span deficits to children with arithmetic difficulties could be further assessed.

By including a word span task, the suggestion that children with arithmetic learning difficulties have a specific problem in remembering numerical information would be examined.

Based on the findings mentioned above, children with arithmetic difficulties would be expected to be impaired only on span tasks involving counting. However, we would expect children with average math and below average reading ability to have the same counting spans as their above average peers.

Finally, we would expect children with arithmetic difficulties to have the same spans for words as their above average peers.

METHODOLOGY

5.2.1 Introduction

This section describes the sample (for a detailed account of how the sample was selected, see chapter 2), the tasks employed, and the process of collecting data on children's working memory processes and other counting and speed measures.

5.2.2 Design

Another purpose of the present thesis was to investigate whether specific limitations in working memory storage capacity are related to children's arithmetic difficulties.

The investigation of this hypothesis involved comparing performance of children with below average arithmetic abilities to that of children with above average arithmetic skills on concurrent memory tasks as well as measures of digit span, word span, reciting number sequences, and speech articulation rate. In addition, a group of children with average maths but below average reading ability were compared to children with above average maths ability on the above measures; this comparison would give further evidence on the nature of the relation between children's arithmetic difficulties and specific working memory deficits.

A brief summary of the sample is followed by a detailed description of the tasks employed to collect the data.

5.2.3 Sample

For the purpose of the examination of the specificity hypothesis, i.e., whether children's arithmetic difficulties are associated with specific deficits in working memory processes, children who were below average in maths were compared to children with above average mathematical ability. In addition, children with below average reading performance but who possessed at least average math ability were compared to children who are above average in maths. The sample consisted of 53 children: 14 below average in maths, 21 above average in maths, and 18 reading difficulty children.

Tables 2.7 and 2.8 (see chapter 2) show the children's score limits on the pre-test measures, and the mean levels (and statistical comparisons) of performance of each group on the five pre-test measures, respectively. Children did not differ in their age means. The above average and below average groups differed only in their math scores. The above average and reading difficulty groups differed in their reading ability; while they also differed in their maths scores, both were significantly better than their below average peers.

5.2.4 Procedure

The tasks measuring children's processes of working memory were presented after the Russell and Ginsburg (1984) tasks. Those children belonging to the reading difficulty group received only this set of tasks. Tasks were administered in the pattern specified by Hitch and McAuley (1991). Children were always tested on the concurrent memory tasks first. The order of the concurrent memory tasks was counterbalanced, except that type of operation (counting vs. comparison) was always blocked. The rest of the measures were examined in the order presented below.

5.2.5 Tasks

Children's working memory efficiency was examined using Hitch and McAuley's methods. This section describes the concurrent memory span and other basic component tasks that were employed. In sum, children's performance on ten tasks was coded.

I. Concurrent Memory Span Tasks

1. Visual Counting Span

This task is an adaptation from an earlier version devised by Case et al. (1982). The child was shown index cards (0.12 x 0.20 cm) on which there were green and yellow dots (radius 0.01 cm) (see Appendix 5.1). Each card had from one to ten green spots and from one to ten yellow spots in a random pattern. The exact instructions were: "I will show you some cards on which there are both yellow and green dots. I want you to count only the green ones and tell me how many there are. But I want you to remember how many there were on each card, because I will ask you to repeat all of them at the end." Children were reminded they should recall them in the order shown, however, they were not penalised for order mistakes. A test trial of two cards took place to make sure that the child understood the instructions. If the child erred on a particular set size, the procedure was repeated. If she erred again, that size was taken as the span. If she was correct, then set size increased. Span was taken as the largest set size on which performance was correct. The entire procedure was repeated and the mean span was calculated.

2. Auditory Counting Span

Children were asked to count a set of tappings (from 1 to 10) on a tin held out of sight. At the end of each set, the child had to recall all his counts. The exact instructions were: "I will be tapping on this (showing the tin) and I want you to count how many times I'm tapping. But I also want you to remember how many times I tapped each time, because I will ask you at the end. And try to remember them in the order I will show them." A test trial followed to make sure the child understood the instructions. As in the previous task, set size increased with correct performance. If the child erred, the same set size of tappings was repeated. If again the child made a mistake in recall, that was taken as his span. The entire procedure starting from set size two was repeated a second time, and the mean span was calculated. Again, scoring was without regard to order.

3. Visual Comparison Span

In this task, the child had to point to the odd card out of a triad. Each card (0.12 x 0.20 cm) contained three spots (radius 0.01 cm) in a specific pattern, however, on one of the cards the pattern was different (see Appendix 5.2). The cards were turned upside down one at a time at a rate of about one second per card and the child pointed to the card which contained the odd pattern. The exact instructions were: "Here are some cards. Each one has a special pattern or design on it; however, two of them have the same design, and one has a different one. I will be showing you each card, one at a time, and having shown you all three, I want you to point to the one that was different. At the end, I want to see if you remember where each different card was (in each triad).

Remember, I want you to recall them in the order I will show them to you. O.k. ? Now, let's start with these two rows." After it was ensured the child understood what he was supposed to do, testing commenced. Set size increased with every set correctly recalled. Span was taken as the largest size on which performance was correct. The procedure was repeated and the mean span was calculated. Although children were encouraged to recall the locations of the odd cards in the order they were presented, they were not penalised if they made an error.

4. Auditory Comparison Span

In this task, the stimuli were triads of nonsense "consonant-vowel-consonant" (CVC) words, with one of them differing in two phonemes; for example, bac-bim-bac. The words were presented orally by the experimenter (one word per second) and extra care was taken to ensure children listened to the words clearly. The triads of words were: *(μαφ μαφ μην), (ταν ταν τικ), (ποκ ποκ παρ), (λαν λαν λεκ), (βελ βελ βαν), (κατ κατ κερ), (γιρ γιρ γακ), (νατ νατ νεκ), (ρομ ρομ ραν), (σαλ σαλ σερ), (δαμ δαμ δικ), (ζεκ ζεκ ζιπ), (θιχ θιχ θεμ), (φαδ φαδ φιπ), (χιν χιν χατ), (πισ πισ πεκ), (βοπ βοπ βικ), (ψακ ψακ ψεμ), (γαπ γαπ γοφ),* and *(ξαν ξαν ξιμ).* The triads were always randomly assigned.

The exact instructions were: "I know you will like this one. I will tell you three words, two of which are the same and one is different. When I finish, I want you to repeat the word that you thought was not like the other two. After I finish with all triads, I want you to try and recall the different words in each triad, and remember, I want you to recall them in the order I showed them to you." A test trial of size two followed. If the child remembered both "odd" words, set size was increased. If not, size 2 was repeated. If the child failed again to recall the odd ones, that was taken as her span. The procedure was repeated and a mean span was calculated. As in the previous tasks, children were not penalised for not recalling the words in the order presented. It was later observed that children enjoyed this task much more than the rest.

II. Other Memory and Counting Tasks

In the following tasks, a stop-watch was used to measure children's time to complete each subprocedure. The tasks were always presented in the order presented below.

5. Spot Counting Task

In this task, the stimuli were the cards (0.12 x 0.20 cm) containing both yellow and green spots (radius 0.01 cm) that were used in the Visual Counting Span task. However, only those containing from seven to ten green spots were used. There were eight trials in this task, each time the number of green spots varying. All children went through the same order of trials. The instructions were: "Remember these cards with the yellow and green dots ? Now, I want you to count only the green ones very clearly but as quickly as possible, because I will be timing you. Don't forget to point to each dot you count, and please don't make any mistakes." There was no test trial since the procedure was very simple. Each card was turned when the child was ready. The experimenter signalled "Go !" while starting the stop-watch and completion time was noted.

6. Recitation Counting 1-20

In this task, children were instructed to count from one to twenty as quickly as possible, though very clearly. They were also reminded that they should make no mistakes, but if they did, they should continue. Errors were noted. This procedure was repeated three times. The time for each trial was noted and means were calculated.

7. Articulation Task

The stimuli in this task were four multisyllabic words. Hitch and McAuley (1991) used the following: *butterfly*, *caterpillar*, *motorbike*, and *helicopter*. Since the corresponding words in Greek are also multisyllabic, they were included, except for *caterpillar*: it translates into *kolona* which is a short (three-syllable) word. Instead, the word *skou-li-ka-de-ra* was used (which refers to a well known kind of worm) which is multisyllabic. Children were asked to repeat each word as quickly as possible, without any mistakes or garbling, until instructed to stop. Time was noted for eight repetitions. The same procedure was used for each of the four words.

8. Auditory Digit Span

In this task, the experimenter read out a sequence of digits (from 1 to 9) at a rate of one digit per second, and the child had to repeat the sequence immediately. The exact instructions were: "I will read you some numbers and I want you to repeat them immediately in the order I read them out. Try not to make any mistakes." The first trial was of set size two. If the child did not recall correctly, the same size procedure was repeated. If the child was correct, set size was increased by one. When the child erred two consecutive times on the same set size, that was taken as span. This procedure was repeated and mean span was calculated.

9. Recitation Counting 2-20

In this task, the child was asked to count from two to twenty by twos, as quickly as possible, clearly, and without making any mistakes. Errors were noted. The procedure was repeated and mean completion time was calculated.

10. Auditory Word Span

Following Hitch and McAuley's (1991) hypothesis that children's memory span would be unimpaired if words rather than digits were accessed in long term memory, the present investigation included the auditory span for words. This task was always presented at the end, so as not to disrupt the sequence of tasks as presented in Hitch and McAuley's study. Children's word span was measured using 8 two-syllable (in Greek) words. The words, which are of everyday use, were the following:

νερο (ne-ro) = water
ματι (ma-ti) = eye
παιδι (pe-di) = child
γομα (go-ma) = rubber
ψωμι (pso-mi) = bread
ξυλο (xy-lo) = wood
χαρα (ha-ra) = happiness
λεφτα (le-fta) = money

The child had to repeat the words that were presented orally, in the order they were presented. Testing began with set size two. With every correct performance, set size increased. Span was taken as the largest set size on which the child's performance was correct. Children were not penalised for order errors. They were, however, strongly encouraged to follow the order of presentation. The exact instructions were: "I will tell you some words and I want you to repeat them in the order I present them. You don't have to hurry; just make sure you remember all of them and in this order." The procedure was repeated and mean span for each child was calculated.

RESULTS

5.3.1 Introduction

This section reports the findings of the examination of children's performance on short term memory and other memory and basic counting measures. The sample consisted of above average and below average in mathematics children, as well as children with average maths ability but below average reading scores (reading difficulty).

First, the present investigation focuses on the pattern of results from two different group comparisons: one between above average and below average children, and the other between above average and reading difficulty children. The findings are presented as a function of task complexity: concurrent memory spans are presented first, followed by basic component tasks. Second, multiple regression analyses examine the independent contribution of concurrent memory spans and the rest of the measures on children's arithmetic achievement. A combined multiple regression analysis of all cognitive factors on children's achievement will complete this section.

5.3.2 Group Comparisons on Concurrent Memory Measures and Other Memory and Counting Tasks

Each task is described separately. Performance among groups was compared statistically through a series of *t* tests, Three-Way Analysis of Variance, and Student-Newman-Keuls.

5.3.2.1 Concurrent Memory Span Tasks

In all four concurrent memory span tasks, distributions were normal. In the tasks which dealt with auditory information, distributions were either slightly negatively skewed (counting span task) or positively skewed (comparison span task). Table 5.1 shows the mean memory spans for each group on each of the four tasks.

1. Visual Counting Span

Children had to count the green spots on an index card which contained both green and yellow spots, and be able to recall the products of increasing sets of counts. Span was defined as the largest set size where performance was correct. Table 5.1 shows that children with arithmetic difficulties had shorter counting spans from children with above average mathematical abilities. The difference was significant.

TABLE 5.1

Mean Spans (Standard Deviations) and Statistical Comparisons Among the Three Groups on the Counting and Comparison Concurrent Memory Tasks ($n = 53$)

	Below Average (n = 14)	Statistical Comparison BA-AA	Above Average (n = 21)	Statistical Comparison AA-RD	Reading Difficulty (n = 18)
Visual Counting Span	4.00	t = 3.09**	4.74	ns	4.42
	(0.7)		(0.7)		(1.0)
Auditory Counting Span	3.57	ns	3.76	ns	3.50
	(0.7)		(0.7)		(0.7)
Visual Comparison Span	3.11	t = 2.37*	3.81	ns	3.39
	(0.9)		(0.8)		(0.8)
Auditory Comparison Span	1.50	ns	1.76	ns	1.81
	(0.5)		(0.4)		(0.6)

*$p < .05$. **$p < .005$.

There was no significant difference between children with above average math performance and those with average math but below average reading ability.

2. Auditory Counting Span

In the Auditory Counting Span, children had to count the number of tappings that the experimenter produced and recall all the products of their counts at the end of each set of taps. Span was taken as the largest set size where performance was correct.

Across groups, auditory counting spans were shorter than visual counting spans. As Table 5.1. also suggests, however, no significant variation in spans were observed among groups; there were no significant differences among the three groups' mean auditory counting spans.

3. Visual Comparison Span

In this task, children had to identify the odd card from a set of three, based on the pattern of the dots they contained, while remembering the location of the card in the triad. Span was taken as the largest set size on which performance was correct.

The variation in children's mean visual comparison spans resembled that of their visual counting spans. Table 5.1 shows that children with math difficulties had significantly shorter spans for nonarithmetic information presented visually than their above average peers. However, there was no significant difference between children with average math but below average reading ability and those in the above average math group.

4. Auditory Comparison Span

Children had to repeat the odd word from a set of three nonsense words (presented orally by the author), and recall all three after the end of each set of triads. As sets increased in size, children had to recall increasingly more nonsense words. Span was taken as the mean set size on which performance was correct.

As Table 5.1 also suggests, no significant differences in mean spans were observed among the groups. The auditory comparison memory spans were the shortest of all spans.

Comparison of the Four Concurrent Memory Spans

It was found that the two math ability groups differed in mean spans when they had to recall information presented visually, independent of the operation involved (counting or comparison). A three-way mixed design Analysis of Variance was conducted, using group (below average vs. above average vs. reading difficulty) as between-subjects factor, and type of operation (counting vs. comparison) and modality of stimulus presentation (visual vs. auditory) as within-subjects factors. Table 5.2 shows the results of the Analysis of Variance.

TABLE 5.2

Three-Way Analysis of Variance on Performance on Concurrent Memory Span Tasks by Crossing Type of Operation With Modality and Group (Above Average, Below Average, and Reading Difficulty, $n = 53$)

Source	df	F
	Between subjects	
***Modality* (M)**	1	397.37**
***Operation* (O)**	1	176.97**
M x O	1	39.01**
S within-group error	50	(0.99)
	Within subjects	
***Group* (G)**	2	3.86*
G x M	2	0.28
G x O	2	2.27
G x M x O	2	0.99
G x S within-group error	50	(0.31)

Note. The values enclosed in parentheses represent the mean square errors. S Subject.
*$p < .05$. **$p < .01$.

Significant main effects of group, operation, and stimulus modality were found. The examination of group means showed that children with arithmetic difficulties had shorter spans than children of above average mathematical abilities, and that, overall, spans were longer when the operation involved counting (rather than comparison) and presented visually (rather than orally).

Table 5.2 also suggests a significant interaction between operation and modality: spans on some concurrent (a combination of operation with modality) tasks were longer than on others. An inspection of means showed that visual counting spans were the largest memory spans, while auditory comparison spans were the shortest spans.

5.3.2.2 Other Memory and Counting Tasks

5. Auditory Digit Span

Children had to repeat a sequence of numbers read out by the author. The initial set size was two and it increased with every successful response. As Table 5.3 also suggests, a significant difference was observed between the two math groups in their mean digit span, that is the maximum number of digits to be retained in short term memory: children with arithmetic difficulties recalled significantly less digits than their above average peers. No significant differences were observed in mean digit spans between above average in maths children and their reading difficulty peers.

TABLE 5.3

Means (Standard Deviations) and Statistical Comparisons Among the Three Groups' Digit Spans and Spot Counting Completion Time (in seconds)

	Below Average (n = 14)	Statistical Comparison BA-AA	Above Average (n = 21)	Statistical Comparison AA-RD	Reading Difficulty (n = 18)
Auditory Digit Span	5.18	t = 3.49**	5.88	ns	5.83
	(0.5)		(0.7)		(0.9)
Spot Counting Time	2.70	t = - 2.96*	2.28	t = - 3.63**	2.75
	(0.5)		(0.3)		(0.5)

*p < .05. **p < .005.

The relationship between digit span and performance on the two math tests used for sample selection was examined. Table 5.4 shows Pearson's r. Performance on the two math tests correlated with performance on the auditory digit span task.

TABLE 5.4

Correlation of Children's Digit Span and Performance on the Two Pre-Test Mathematical Tests (n = 53)

	Young	NFER
Auditory Digit Span	.38*	.45**

*p < .05. **p < .005.

6. Spot Counting Task

Children had to count as quickly as possible the green spots from a field of green and yellow spots printed on cards. The number of green spots differed on each card and the mean time for ten trials was calculated. Children's errors were also recorded.

Table 5.3 shows that children with arithmetic difficulties were significantly slower than children with above average math abilities in spot counting speed. Reading difficulty but average in maths children were found to be significantly slower than above average in maths children. There were 3 outliers in this task, however, there were no changes in the pattern of results when they were excluded from the analysis.

Table 5.5 shows the number of children erring on the Spot Counting Time task, as well as the frequencies of these errors. There were no significant differences in the amount of errors performed by each group.

TABLE 5.5

Number of Children Erring on the Spot Counting Task and Frequencies of Errors as a Function of Group

	Group		
	Below Average (n = 14)	**Above Average (n = 21)**	**Reading Difficulty (n = 18)**
Spot Counting Errors	5 (10)	7 (17)	7 (11)

It was observed that the three groups did not differ in the kind of errors they made, either. In general, children's errors were not cases of severe miscalculations, rather they were due to the children's eagerness to give an answer as quickly as possible (speed requirement). For example, a child would rush at the beginning of the counting, get stuck, and start all over again (e.g., "1, 2, 3, 4 ... oh, no, ... 1, 2, 3, 4, 5 ..."); or he would get carried away as his pointing overrode his verbal counting (e.g., a boy was pointing to the seventh green spot while he was still reciting number five). There were, however, cases where a child would repeat the same number (e.g., "...6, 7, 7, 8") or where counting would not match the child's pointing at all (i.e., a boy was reciting without pointing to any particular spot).

Auditory Digit Span and Spot Counting Task

Hitch and McAuley (1991) found that digit span and spot counting, the two subtasks involved in the visual counting span, together explained 42% of the total variance in that task. A multiple regression was conducted, with digit span and spot counting time as the independent variables, and visual counting span as the dependent variable.

Table 5.6 shows the results of a multiple regression using data from the three groups. It was found that digit span and spot counting time together explained 30% of the total variance in the visual counting span. Digit span, however, was the only significant predicting variable. Digit span alone explained 29% (R^2) of variation in Visual Counting Span ($p < .001$).

TABLE 5.6

Standard Multiple Regression Analysis of Digit Span and Spot Counting Time on Visual Counting Span ($n = 53$)

Predictor	B	SE B	Beta
Auditory Digit Span	0.59	0.13	.53*
Spot Counting Time	- 0.15	0.21	- .09

Note. R^2 = .30.
*$p < .001$.

Table 5.7 shows the correlation coefficient of digit span, spot counting time, and visual counting span.

TABLE 5.7

Pearson's Correlation Matrix for Digit Span, Spot Counting Time, and Visual Counting Span ($n = 53$)

	Auditory Digit Span	Spot Counting Time
Visual Counting Span	.54*	ns
Auditory Digit Span	-	ns

*$p < .001$.

There was a high positive correlation between digit span and visual counting span. Spot counting time was not significantly correlated with either digit span or visual counting span.

7. Auditory Word Span

In this task, children had to repeat a sequence of words read out by the experimenter. The sequence increased in size with every correct repetition. The last set size in which performance was correct was taken as the word span of the child. It was observed that across groups word spans were shorter than spans for digits.

TABLE 5.8

Means (Standard Deviations) and Statistical Comparisons Among the Three Groups' Spans for Words and Measures of Speed in Articulation and Recitation (in seconds)

	Below Average (n = 14)	Statistical Comparison BA-AA	Above Average (n = 21)	Statistical Comparison AA-RD	Reading Difficulty (n = 18)
Auditory Word Span	4.61	t = 3.47**	5.31	ns	5.22
	(0.6)		(0.6)		(0.9)
Articulation Task	6.45	t = - 3.03*	5.63	t = - 2.74*	6.28
	(1.0)		(0.6)		(0.9)
Recitation Counting 1-20	6.33	ns	5.71	ns	6.18
	(1.0)		(1.0)		(0.9)
Recitation Counting 2-20	11.77	t = - 3.01*	4.62	t = - 2.11*	6.33
	(8.8)		(1.6)		(3.1)

*$p < .05$. **$p < .005$.

Table 5.8 shows that above average and below average children differed significantly in the amount of words to be retained, with children with arithmetic difficulties recalling significantly less words than their above average peers. Reading difficulty children did not differ in mean word spans from above average in maths children.

8. Articulation Task

In this task, children had to repeat as quickly as possible a multisyllabic word until they were asked to stop. The author marked the time for eight repetitions. This procedure was repeated with four different words. A mean articulation time was calculated for each child.

Table 5.8 shows that children with arithmetic difficulties were significantly slower than children with above average maths ability. Also, above average children were significantly faster than reading difficulty children in repeating the multisyllabic words. There was one outlier in the reading difficulty group but no changes in the relationships were observed when it was not included in the analysis.

Finally, apart from a general observation and "complaint" on the part of the children that they "got mixed up repeating these multisyllabic words as fast as they could", they did not make errors and seemed to enjoy it.

9. Recitation Counting 1-20

In this task, children had to repeat as quickly as possible the number sequence from 1 to 20. This was repeated three times and a mean recitation time (in seconds) was calculated for each child. As Table 5.8 also suggests, the three groups did not differ in their mean speed to recite from 1 to 20. In total, there were 5 outliers in this task: 2 in the below average group and 3 in the reading difficulty group. However, excluding these outliers did not change the pattern of results reported above.

The examination of children's errors in reciting the sequence from 1 to 20 showed that children did not differ in the type of errors they made, either. Compared to reciting from 2 to 20, counting from 1 to 20 involved less errors for all groups, as can also be seen on Table 5.9.

TABLE 5.9

Number of Children Erring on the Recitation Counting 1-20 and 2-20 Tasks and Frequencies of Errors

	Group		
	Below Average (n = 14)	**Above Average (n = 21)**	**Reading Difficulty (n = 18)**
Recitation 1-20	2 (2)	3 (4)	3 (3)
Recitation 2-20	10 (14)	7 (8)	7 (12)

These types of errors were common in all three groups. Repetitions (e.g., "... 11, 12, 13, 13, 14...") as well as omissions (e.g., "... 15, 17, 18...") were frequent, along with some instances of stopping and starting from the beginning. In general, children "garbled" or "tried to catch their breath".

10. Recitation Counting 2-20

As in the previous task, children had to recite as quickly as possible a number sequence which, in this case, was from 2 to 20, counting the even numbers only. The author calculated the mean time for two trials. Table 5.8 shows that the three groups differed significantly in their speed of reciting from 2 to 20. Children with arithmetic difficulties, in particular, had the longest mean completion time, which was almost three times slower than that of their above average peers. Reading difficulty children also differed from the above average group. While below average and reading difficulty children were slower in reciting 2 to 20 than 1 to 20, the reverse was true for their above average peers. There were two outliers in this task: one among below average children and one among reading difficulty children. Their exclusion did not bring any changes in the relations among groups, only in their means.

Table 5.9 shows that children with arithmetic difficulties made more errors than their above average peers in reciting the number sequence 2 to 20. The same was true for the reading difficulty group, however, the difference was of small magnitude. Overall, errors typically involved counting the odd numbers (e.g., "... 2, 5, 7, 9..."), reciting both even and odd numbers (e.g., "... 8, 9, 10, 12, 13, 14, 16..."), repeating the same number (e.g., "... 16, 16, 18..."), omitting some of them (e.g., "... 16, 20"), or "getting stuck" on a particular number and continuing after thinking.

Concurrent Span Tasks and Basic Component Tasks

A series of backward regression analyses were conducted in order to examine which of the individual tasks predicted performance on the concurrent memory span tasks. For the counting span tasks, the predictive value of digit span, spot counting, and recitation from 1 to 20 and 2 to 20 was examined. For the comparison span tasks, entries included word span and speech articulation. Table 5.10 shows which variables remained in the equation.

TABLE 5.10

Summary of Backward Regression Analyses of the Individual Measures Predicting Concurrent Memory Spans

Variable	Predictor	B	SE B	Beta
Visual Counting Span[a]	Auditory Digit Span	0.50	0.13	.46**
	Recitation Counting 2-20	-0.04	0.02	- .29*
Auditory Counting Span	(F was undefined)			
Visual Comparison Span	(F was undefined)			
Auditory Comparison Span[b]	Auditory Word Span	0.29	0.08	.45**

[a] $df = 52$, $R^2 = .37$. [b] $df = 52$, $R^2 = .20$.
*$p < .05$. **$p < .001$.

As Table 5.10 suggests, digit span and recitation in twos together explained 37% of variation in visual counting span. Word span explained 20% of variance in auditory comparison span. None of these variables predicted auditory counting and visual comparison spans.

5.3.3 Multiple Regressions of Concurrent Memory Spans and Basic Component Tasks on Children's Arithmetic Achievement

A series of multiple regression analyses explored the independent contribution of working memory processes in children's arithmetic achievement. The multiple regression involves predicting the values of one variable, the dependent variable, from the values of other variables, the independent variables, by utilising the presence of an association between the three or more variables (Kinnear & Gray, 1995). In backward regression analyses, variables are removed until a model is reached in which no more variables are eligible for removal (i.e., all variables whose probability of F was less than .10 remained). Variables are entered as a group and then removed individually (Norusis, 1993).

Data on memory spans and speed (Hitch & McAuley, 1991) were collected for children belonging to the above average and below average mathematical groups, as well as the reading difficulty group (n = 53).

Children's raw scores on the two mathematical tests, the "Y" Mathematics Series Y2 test (Young, 1979) and the Basic Mathematics Test B (NFER, 1971) were the dependent variables in the current analyses; two analyses were thus conducted, one for children's scores on either test.

Entries in the regression analyses included those concurrent spans and more component processes which had evidenced differences between children and which were significantly correlated with children's arithmetic achievement. First, regressors were identified, then variables were summarised for the purpose of controlling for the number of variables to be entered in the multiple regression, and finally the analyses were conducted. When outliers were observed, that is, cases outside 3 standard deviations, they were excluded and the analyses were repeated.

The first step was to identify the independent measures or predictors, that is, the cognitive variables that were significantly correlated with arithmetic achievement ($p < .05$). Appendix 5.3 shows Pearson's correlation coefficients of working memory and other speed measures and their association with arithmetic performance. Table 5.11 shows the association of all measures with performance on either or both math tests. From all measures examined, recitation in ones could not enter the regression because children's speed of reciting numbers from 1 to 20 was not associated with performance on any of the two mathematical tests.

TABLE 5.11

Measures of Working Memory Capacity and Speed in Counting and Speech Associated With Children's Arithmetic Achievement

Variable	Young only	NFER only	Both
Span			
Visual counting span	-	-	√
Auditory counting span	-	√	-
Visual comparison span	-	-	√
Auditory comparison span	-	-	√
Digit span	-	-	√
Word span	-	-	√
Speed			
Spot counting time	-	√	-
Speech articulation	√	-	-
Recitation counting 2-20	-	-	√

All other measures were associated with performance on at least one mathematical test. Since performance on some measures was associated with children's achievement on only one of the two arithmetical tests, two separate summaries were produced for the purpose of using each summary as a predictor of children's performance on the mathematical test that the variables were significantly associated with.

Table 5.12 shows the sets of summaries that were used to predict performance on Young's test and those used on the NFER test. The two summarised categories included span and speed. These were the two main dimensions also explored by Hitch and McAuley (1991). An effort was made to ensure every variable within each summarised category correlated with at least one other variable in that category. Appendix 5.4 shows Pearson's correlation matrix of performance on all measures. While children's auditory counting spans did not correlate with any other span measure, they were nevertheless included in the regression analysis since they were significantly associated with performance on the NFER test. In summarising variables, neither span nor speed measures needed to be standardised; they were simply averaged.

TABLE 5.12

Summarised Categories of Memory Span and Other Speed Measures to Be Used in Prediction of Children's Arithmetic Achievement

Summarised Variable (Young)	Explanatory Variable	Summarised Variable (NFER)	Explanatory Variable
Span1	Visual counting span Visual comparison span Auditory comparison span Digit span Word span	**Span2**	Visual counting span Auditory counting span Visual comparison span Auditory comparison span Digit span Word span
Speed1	Speech articulation Recitation counting 2-20	**Speed2**	Spot counting time Recitation counting 2-20

Two multiple regression analyses were conducted, one predicting children's performance on Young's test and another predicting performance on the NFER test. Table 5.13 shows the predictive value of memory capacity and speed measures on children's arithmetic performance.

TABLE 5.13

Summary of Backward Multiple Regression Analyses of Summaries of Memory Spans and Speed on Arithmetic Achievement

Dependent Variable	Predictor Variable	B	SE B	Beta	T	Sig T
Young[a]	Span1	2.27	1.25	.22	1.83	.07
	Speed1	1.06	0.24	.54	4.48	< .01
NFER[b]	Span2	7.15	2.15	.43	3.33	< .01
	Speed2	0.80	0.38	.27	2.12	.04

[a]$df = 52, R^2 = .46, F = 21.03, p < .01$. [b]$df = 52, R^2 = .38, F = 15.02, p < .01$.

The analyses showed that both span and speed measures predicted performance on the two mathematical tests. More specifically, span1 (mean visual counting, visual comparison, auditory comparison, digit span, and word span) and speed1 (mean speech rate and recitation in twos) together explained 46% of the variance in performance on Young's test. The analysis also showed that all span measures (averaged) and counting speed measures (mean spot counting and recitation in twos) together explained 38% of the total variance in achievement on the NFER test. No outliers were observed.

Total of Cognitive Variables

To examine the amount of variance in achievement explained by the total of cognitive variables, two backward multiple regressions were conducted of the summarised cognitive variables on achievement. Summarised knowledge and skill in informal arithmetic, base ten knowledge and skill, accuracy and bugs in written calculation, knowledge of addition facts, problem solving skills, span1, and speed1 regressed on performance on Young's test. Table 5.14 shows that knowledge of addition facts, understanding and skill in the base ten system, problem solving skills, and speed1 together explained 77% of the total variance in children's performance on the test. No outliers were observed.

TABLE 5.14

Summary of Backward Multiple Regression Analyses of Summaries of All Cognitive Variables on Arithmetic Achievement

Dependent Variable	Predictor Variable	B	SE B	Beta	T	Sig T
Young[a]	Addition facts	1.85	0.64	.34	2.89	< .01
	Base ten	2.03	1.13	.25	1.81	.08
	Problem solving	1.92	1.05	.24	1.83	.08
	Speed1	0.46	0.25	.23	1.87	.07
NFER[b]	Addition facts	2.21	1.19	.28	1.86	.07
	Informal	3.66	1.90	.32	1.92	.06
	Problem solving	4.58	1.48	.38	3.11	< .01

Note. When only those variables that were found significant in the individual analyses (formal and informal arithmetic and working memory) regressed on Young's test, problem solving was no longer a significant predictor. There was no difference in the predictors of performance on the NFER test.
[a]$df = 33, R^2 = .77, F = 24.38, p < .01$. [b]$df = 33, R^2 = .76, F = 31.58, p < .01$.

Knowledge and skill in informal arithmetic, base ten knowledge and skill, accuracy and bugs in written calculations, knowledge of addition facts, problem solving skills, span2, and speed2 regressed on performance on the NFER test. As Table 5.14 suggests, knowledge of addition facts and informal arithmetic, along with problem solving skills, together explained 76% of the total variance in performance on the NFER test. No outliers were observed.

DISCUSSION

5.4.1 Introduction

The purpose of the present investigation was to examine the relation between arithmetic ability and working memory efficiency. Evidence for the specific nature of this relation was provided by two different group comparisons: one involved children who differ in arithmetic ability and the other involved children who are closer in arithmetic ability but differ in reading performance. Both were equally necessary to substantiate the specificity hypothesis (Hitch & McAuley, 1991; Siegel & Ryan, 1989). The present study also examined the independent contribution of working memory processes to children's differences in arithmetic achievement.

The majority of above average and reading difficulty children were boys, while most below average children were female. While not investigated in the present study, in considering for group differences, children's gender should be acknowledged.

The findings on those groups and tasks that are common to both the present study and that by Hitch and McAuley (1991) are reviewed first. Then, the additional findings that the present study has suggested are outlined. Based on these, the relation between arithmetic difficulties and working memory deficits is discussed.

5.4.2 Understanding Variation in Working Memory and Other Memory and Counting Tasks

Studies on human memory processes have related working memory efficiency to children's performance in arithmetic. According to the working memory model, working memory refers to the temporary storage of information while other cognitive tasks are performed (Baddeley, 1990; Baddeley & Hitch, 1974). While a common assumption in the literature is that a trade-off between storage and processing takes place in working memory, recent findings point towards the significance of the temporal dimension of working memory spans (i.e., retention time) rather than resource sharing (Towse, Hitch, & Hutton, 1998).

Arithmetic performance has been related to working memory processes. Hitch (1978), for example, examined the importance of working memory in mental calculations. In doing the calculations, units or tens are stored while hundreds are calculated. Hitch (1978) found that limited performance in mental calculations was very often due to forgetting an addend. Siegel and Ryan (1989) further found that working memory storage capacity may vary as a function of children's difficulty, that is, whether in reading or arithmetic. Further observed in Hitch and McAuley (1991), children with arithmetic learning difficulties were impaired only when the information to be processed, that is, concurrently held and then recalled, involved counting.

The pattern of differences Hitch and McAuley (1991) found was that the children with lower arithmetic ability had lower counting spans, but not comparison spans. They confirmed Siegel and Ryan's (1989) deficit in math disabled children, further extending it to nonvisual information. Some studies have suggested restricted concurrent spans in dealing with visuospatial information in mathematically disabled children (Fletcher, 1985; Rourke & Finlayson, 1978), while others have shown that spatial difficulties do not necessarily lead to nonverbal calculation problems in kindergarten and first-grade children (Jordan & Montani, 1996). Hitch and McAuley (1991), nevertheless, attributed the unimpaired visual comparison spans to either the nature of the tasks (i.e., that they may differ from the visuospatial elements examined in other studies) or to the fact that children were matched on a largely non-verbal intelligence test, while in other studies children were not matched for intelligence at all.

The limited counting concurrent spans found in arithmetically disabled children were further related to limited performance in spot counting, shorter digit spans, and slower recitation rate of the counting list. Arithmetically disabled children, however, resembled their normal peers in speech articulation. Hitch and McAuley proposed that the difficulties on counting span tasks might be attributable to slower counting procedures and lower digit spans which in turn are due to problems in retaining and accessing numerical information.

In the present study, however, it was ability to process visual information that discriminated between children of different arithmetic achievement. More specifically, the pattern of results of the two comparisons on children's concurrent spans is that children with specific arithmetic difficulties had shorter visual spans, whereas children with at least average maths ability seemed to be unaffected by the visual nature of the information to be processed. These findings are by no means surprising; the association between arithmetic difficulties and reduced performance levels on tasks which involve visuospatial stimuli has been cited elsewhere. Rourke and Finlayson (1978), for example, found that arithmetically disabled children are impaired on tasks using visuospatial information. Also, Fletcher (1985) has shown that children with math difficulties face problems in dealing with nonverbal information, for example dot patterns (as used here in the visual comparison span task).

Children with arithmetic difficulties further differed from their above average peers on simpler counting tasks, such as spot counting and counting in twos, and speech articulation. When the difference in word spans is also taken into account, the present results indicate that the children in the below average group had general difficulties in retrieving information from memory and maintaining both numerical and non numerical information in memory.

As well as these two groups, there also were a group of children with average mathematical skills but who had lower reading ability. In considering the findings based on those children, it is necessary to recognise the exact nature of those difficulties: due to the orthographic transparency of the Greek language, those children had more difficulties in comprehension and inference rather than in phonological or orthographical decoding. The findings with these children and what extra information they offer are reviewed now.

Children with average maths but below average reading ability did not differ from their above average in maths peers on any concurrent memory span task. The two groups that were closer in maths ability had similar spans for words and digits, and were equally fast in reciting number sequences, both in ones and twos. Even though they differed in counting speed and speech rate, still they were significantly faster than arithmetically disabled children.

These additional findings shed more light into the specific nature of deficits that characterise children with arithmetic difficulties. It seems that arithmetically disabled children suffer from difficulties in retaining numerical information, however, they suffer from further impairment in basic counting skills and simple memory spans. While these limitations were also observed in

Hitch and McAuley (1991), the fact that arithmetically disabled children have also limited spans for words constitutes evidence that their deficit generalises to retaining non numerical information.

In sum, the pattern of differences found in the two separate comparisons is that children with arithmetic difficulties are impaired on most measures, whereas children with at least average maths ability are not. When we take into account that the latter group of children differed in reading ability, we may conclude that differences in reading comprehension and in sequencing were not associated with performance on most measures.

5.4.3 The Contribution of Working Memory and Basic Component Tasks to Children's Variation in Arithmetic Achievement

The present study further examined the degree to which variation in children's arithmetic achievement could be explained by differences in concurrent spans and other counting or basic component tasks. The analysis showed that the majority of span and speed measures predicted performance on both math tests used initially for sample selection. It was found that all measures predicted children's performance, except auditory counting spans and spot counting on Young's (1979) test, and speech articulation on the NFER (1971) test. Recitation in ones was not included since it did not discriminate between children of different arithmetic performance.

The analysis of the independent variation explained by the total of cognitive factors examined in this study showed that knowledge of addition facts and base ten system along with problem solving and speed (speech articulation and recitation in twos) explained 77% of variation in children's performance on the Young's test. As mentioned in earlier sections, Young's test included items on understanding of base ten concepts, as well as actual word problems. The test also consisted of a timed section.

Knowledge of addition facts, informal arithmetic and problem solving explained 76% of total variance on the NFER test. An investigation of the items on the test showed a correspondence in topics examined.

5.4.4 Introduction to the Last Chapter

The examination of children's working memory and other basic components of arithmetic skill completes the exploration of cognitive factors and their relationship to children's arithmetic performance. It also signals the end of individual analyses of factors related to children's arithmetic performance.

The next chapter examines the contribution of both social psychological and cognitive factors associated with performance to children's variation in arithmetic. A general discussion on the findings of the present thesis, along with implications and suggestions are considered.

CHAPTER 6

CHILDREN'S VARIATION IN ARITHMETIC ACHIEVEMENT: OVERVIEW AND CONCLUSIONS

6.1 Introduction

The purpose of the present study was to identify some social and environmental and cognitive math-specific factors that might explain variation in children's arithmetic performance. Each set of factors - social and environmental, and cognitive - has been examined first in isolation for the purpose of establishing their association with children's arithmetic achievement and their contribution to children's variation in achievement. Next, the total variance explained by both the social and environmental and the cognitive variables that were found to associate with children's performance is explored.

This chapter is the final part of this research. First, it describes the process of identifying the total variance explained by both sets of factors, followed by the findings of the last three chapters, where emphasis was placed on those variables that were found to vary significantly with children's arithmetic attainment. The chapter concludes with a general discussion of how variation in children's arithmetic achievement can be explained by the factors analysed hereby. Interpretations, limitations, and suggestions for future research are considered.

6.2 The Contribution of Social, Environmental, and Cognitive Factors to Children's Variation in Arithmetic Achievement

Having examined the independent contribution of cognitive and social and environmental factors to children's arithmetic ability, an attempt was made to explore how much of the variance in children's performance could be explained by the two sets of factors combined.

A major statistical consideration was the number of entries, that is, the variables to enter the regression analysis. To control for the number of those variables, the social and environmental components were limited to those variables that had been found to individually predict children's performance on the Young (1979) and the NFER (1971) tests (see Table 3.48 in chapter 3). From the cognitive domain, all the summaries of variables (summarised categories) were included.

Thus, to predict performance on Young's test, children's attitudes to arithmetic, mothers' beliefs of children's attitudes, and mothers' academic level were the social and environmental regressors; the cognitive components included informal arithmetic, base ten system, accuracy and bugs, addition facts, problem solving, span1 (visual counting span, visual comparison span, auditory comparison span, digit span, and word span), and speed1 (speech articulation and recitation 2-20).

To predict performance on the NFER test, the social and environmental variables included children's attitudes to arithmetic and fathers' beliefs of children's attitudes; the cognitive variables included informal arithmetic, base ten system, accuracy and bugs, addition facts, problem solving, and the summarised variables span2 (visual counting span, auditory counting span, visual comparison span, auditory comparison span, digit span, and word span), and speed2 (spot counting and recitation 2-20). Table 6.1 shows a summary of the factors remaining in the regression equation.

TABLE 6.1

Summary of Backward Multiple Regression Analyses of Social, Environmental, and Cognitive Variables on Arithmetic Achievement

Dependent Variable	Predictor Variable	B	SE B	Beta	T	Sig T
Young[a]	Base ten	1.99	1.08	.24	1.85	.08
	Informal	3.25	1.00	.41	3.24	< .01
	Child's attitudes (mother)	2.38	0.96	.28	2.48	.02
	Academic level (mother)	1.68	0.62	.27	2.70	.01
NFER[b]	Informal	5.95	1.29	.56	4.60	< .01
	Problem solving	4.79	1.43	.41	3.36	< .01

[a]$df = 29$, $R^2 = .77$, $F = 21.32$, $p < .01$. [b]$df = 26$, $R^2 = .76$, $F = 38.75$, $p < .01$.

Children's knowledge and skill in informal arithmetic and the base ten system, along with mothers' beliefs of the child's attitudes to arithmetic and their academic status together explained 77% of children's performance on Young's test. Children's knowledge and skill in informal arithmetic and their problem solving skills together explained 76% of children's performance on the NFER test.

Although a considerable amount of variance in children's arithmetic achievement was found to be explained by both sets of factors, the size of the sample indicated that the findings should be treated with caution. As can also be seen on Table 6.1, the number of children who participated in both cognitive studies and on whom data on parental variables was collected was significantly limited.

Table 6.2 provides a summary of the findings of the series of backward multiple regression analyses conducted at the concluding part of each chapter, including the regression just conducted. It shows the social and environmental and cognitive factors that were found to predict children's arithmetic performance both individually and combined.

TABLE 6.2

Social, Environmental, and Cognitive Factors Predicting Children's Performance in Arithmetic

	n	Predictor Young	*n*	Predictor NFER
SOCIAL AND ENVIRONMENTAL				
Children	73	Child's attitudes Parental indirect help	73	Child's attitudes Parental indirect help
Mothers	51	Child's ease Child's performance Child's attitudes Academic level	50	Child's attitudes
Fathers	50	Child's attitudes	48	Child's attitudes
TOTAL	44	Child's attitudes (child) Child's attitudes (mother) Academic level (mother)	48	Child's attitudes (child) Child's attitudes (father)
COGNITIVE				
Russell & Ginsburg	72	Addition facts Base ten Informal	72	Addition facts Base ten Informal Problem solving
Hitch & McAuley	53	Span1 Speed1	53	Span2 Speed2
TOTAL	34	Addition facts Base ten Problem solving Speed1	34	Addition facts Informal Problem solving
SOCIAL, ENVIRONMENTAL, AND COGNITIVE				
TOTAL	30	Informal Base ten Child's attitudes (mother) Academic level (mother)	27	Informal Problem solving

6.3 Understanding Variation in Achievement: Summary of Findings

To summarise the factors or variables found to vary as a function of children's performance and those contributing to children's variation in arithmetic performance:

Social and Environmental

Group Comparisons:

From the social and environmental elements that were examined as a function of children's arithmetic performance, children's attitudes to the textbook used in arithmetic, their numerical activities at home, and their reports of indirect help from both the father and the mother in doing their homework were significantly associated with children's performance in arithmetic. No marked differences were observed between children in their self-concepts in arithmetic and reading, their attitudes to school, and the amount of help with their homework in arithmetic and reading: the majority believed they were very good in arithmetic and reading, liked school very much, and reported receiving help in doing their homework.

From the parental variables examined, those which varied as a function of children's math ability group were: parents' beliefs about the child's school and arithmetic performance, mothers' beliefs about the easiness of arithmetic for their child, mothers' beliefs about the child's performance as opposed to ability, parents' beliefs about the child's attitudes to arithmetic, fathers' reports of indirect help with the child's homework, mothers' opinions on the suitability of the curriculum, and mothers' beliefs about the helpfulness of the teacher. On the other hand, no association was observed between children's performance and parental numeracy or literacy problems, parental beliefs about the academic importance of arithmetic, parental reports of children's numerical practices at home, parental help and encouragement, parental contact with the teacher for arithmetic, or parental academic status. There was no overall agreement between children's and parents' reports.

Prediction Analyses:

The present study further investigated the degree to which variation in children's performance might be explained by differences in the social and environmental factors examined hereby. Separate analyses for each respondent were conducted and have shown that: (a) children's attitudes to arithmetic and their reports of total parental indirect help with the school homework explained variation in both math tests, (b) mothers' beliefs about the easiness of arithmetic for their child, their beliefs about the child's performance as opposed to their ability, their beliefs about the child's attitudes to arithmetic, and their academic status together explained variation in Young's (1979) test; from the mother variables, only mothers' beliefs about the child's attitudes to arithmetic explained variation on the NFER (1971) test, and (c) fathers' beliefs about the child's attitudes to arithmetic explained variation in both math tests.

A combined regression analysis on Young's test saw the significance of children's own reports of their attitudes to arithmetic and mothers' corresponding beliefs as well as academic status. Variation in performance on the NFER was explained by children's own reports of attitudes to arithmetic and fathers' corresponding beliefs.

Cognitive

The second path of investigation focused on the relation between cognitive skills and underlying components and children's achievement in arithmetic. Those skills and components referred to measures of formal and informal arithmetic knowledge and skill, and working memory efficiency. The individuality of those measures suggested a separate examination of each set.

Group Comparisons:

The association between formal and informal arithmetic knowledge and children's arithmetic achievement focused on five major mathematical areas adapted from Russell and Ginsburg (1984). Variation between below average and above average children was found in every mathematical area, except for their use of commutativity and their accuracy in identifying larger written numbers. Average and below average children resembled each other in their understanding of informal and base ten concepts, use of commutativity, and ability to solve word problems.

The association between working memory and children's arithmetic achievement focused on concurrent spans and some more basic component tasks. Working memory measures comprised counting and comparison spans, both visual and auditory. Simple span measures included both digit and word spans, and simple speed measures included counting and speech rate. With the exception of word span, all measures were adapted from Hitch and McAuley (1991). Variation was observed as a function of children's arithmetic achievement, where poor arithmetic performance was associated with impaired visual spans but not auditory spans. Difficulties in simple counting procedures and accessing both numerical and non numerical information from long term memory were found to underlie that variation. The specificity hypothesis was also examined: children of above average arithmetic skill did not differ from children with mild reading difficulties and average arithmetic ability on most of the measures.

Prediction Analyses:

From the examination of children's formal and informal arithmetic knowledge, all five mathematical areas were summarised and regressed on achievement on both math tests. Knowledge of addition facts, informal knowledge and skill, and base ten knowledge and skill predicted performance on Young's (1979) test. Problem-solving skill also predicted performance on the NFER (1971) test.

From the examination of children's working memory processes, all measures were summarised into span and speed. Mean spans (visual counting, visual comparison, auditory comparison, digit span, and word span), and mean speeds (mean speech rate and recitation in twos) explained variation in performance on Young's test. All span measures (averaged) and counting speed measures (mean spot counting and recitation in twos) explained variation in achievement on the NFER test.

A combined regression analysis showed that knowledge of addition facts, understanding and skill in the base ten system, problem solving skills, and speed (speech articulation and recitation in twos) explained variation in children's performance on Young's test. Knowledge of addition facts, informal arithmetic, and problem solving skills explained variation in performance on the NFER test.

The next section includes a discussion of how children vary in arithmetic ability, considering the contribution of cognitive and social and environmental factors examined in the current study.

6.4 Towards an Explanation of Children's Variation in Arithmetic: General Discussion

Introduction

While it is common for children to differ in school achievement, a growing concern revolves around their individual differences in arithmetic. The question that cognitive psychologists and educators constantly strive answer is why and how children who share normal levels of intelligence and possess the same strengths in reading and all other school subjects, vary in arithmetic achievement.

In order to keep the main emphasis on ways of thinking about arithmetic achievement and on ways of improving children's performance on the subject, research has largely been restricted to arithmetic difficulties observed in children in the early years of primary school. A considerable body of research in cognitive psychology has focused on the cognitive components of children's arithmetic abilities mainly by examining children's difficulties in the subject. Two explanations that dominate the literature focus on children's understanding of major mathematical concepts and computational skills (Russell & Ginsburg, 1984) and working memory storage capacity (Hitch & McAuley, 1991). Yet few investigators have attempted to examine the cognitive bases of children's arithmetic precocity. Dark and Benbow (1991), for example, examined the domain specificity of mathematical precocity in adolescents: they found that it is associated with more efficient storage and manipulation of numerical and spatial information, rather than words. Also, Robinson et al. (1996), in their examination of preschool and kindergarten arithmetically precocious children, found that those children are characterised by advanced visual-spatial skills as well as high performance levels on quantitative measures, such as the Stanford-Binet and Key Math, among others.

Excellence in arithmetic has been researched more extensively by investigators in the area of social psychology. In their attempt to provide some explanations on the underlying social and environmental causes of children's underachievement in arithmetic, researchers have compared children of different arithmetic ability on various psychological constructs. Accordingly, a number of those constructs have been found to associate with children's performance in arithmetic: children's self-concepts, their attitudes to arithmetic, their home practices, parental support, parent education, to name only a few (Aiken, 1970, 1972; Chen & Stevenson, 1995; Crystal & Stevenson, 1991; Marsh, 1990; Schunk, 1990; Stevenson & Lee, 1990; Tizard et al., 1988; Young-Loveridge, 1991).

What is common to those studies is the information they provide on children's specific difficulties in arithmetic, either by explaining them or simply by relating them to possible influences. While those studies may be rich in findings, however, two questions remain open to investigation.

The first one refers to differentiation in arithmetic ability. Within the literature on early mathematical development, nearly all research has focused on the typical pace and sequence of skill acquisition, to the exclusion of individual differences other than those labelled disabilities. We seem to know some of the cognitive bases for children's arithmetic difficulties, however, we know abysmally little about the cognitive processes underlying arithmetic precocity. This investigation would eventually give evidence of potential determinants of arithmetic achievement. For that purpose, children with arithmetic difficulties should be compared and contrasted to children who are doing particularly well in the subject on different measures.

The second question refers to the nature of correlates of arithmetic performance. How much do cognitive or social and environmental factors contribute to or account for children's arithmetic performance individually ? The intent would not be to judge which set of factors would be more influential, rather the degree of the influence. For that purpose, children's arithmetic ability should be examined across domains.

The present study attempted to answer both questions: it examined variation in children's arithmetic achievement in the light of both social and environmental and cognitive factors. The aim was to identify, understand, and explain the underlying components of arithmetic performance, possible correlates and constituents which characterise three distinct levels of arithmetic performance: below average, average, and above average.

Social and Environmental

From the total of social and environmental factors examined hereby, a significant association was observed between children's arithmetic ability and their attitudes to arithmetic, their parents' beliefs about the child's attitudes, and their mothers' academic background. Some additional variables were also found to relate to ability, however, they did not show on the final regression analysis of the total of social and environmental factors: for example, children's reports of parents' indirect help with the homework and mothers' beliefs about the easiness of arithmetic for their child as well as children's performance as opposed to their ability were also associated with performance on Young's (1979) test.

The relation between attitudes and achievement has been the centre of a considerable amount of research. Since the 1970s, Aiken (1970, 1972) discussed the reciprocal nature of the relationship between attitudes and performance, in that attitudes may influence performance and performance may in turn affect children's attitudes. More recent research has also shown that children's attitudes correlate with children's actual performance (Stevenson & Lee, 1990; Tizard et al., 1988; Young-Loveridge, 1991), in that, children may like the subjects they are good in. Studies have further shown that attitudes may relate to children's perceived performance, in that, children may like particularly the subjects they think they are good in (Stevenson & Lee, 1990). Gender differences in the relation between attitudes and achievement have also been observed, where correlations are low and unstable for girls but strong, positive, and significant in boys (Schofield, 1982). It has been cited, however, that children may work hard and perform well despite a relatively low interest they may have in the subject, simply because of parental pressure (Chen & Stevenson, 1995).

In the present study, some association was observed between primary school children's arithmetic performance and their attitudes to the subject. A general understanding of children's attitudes to arithmetic was achieved through their feelings toward individual measures, for example, their liking for the arithmetic homework and the textbook used in arithmetic, and their feelings towards missing an arithmetic class. Variation was observed in children's attitudes to the textbook, where children with arithmetic difficulties expressed a mild or strong dislike, mainly because of a dislike for the subject and of the difficulty they would encounter from the complexity of the book. Some variation was also observed in children's favourite school subject: children with particularly good arithmetic skills were more likely than the rest of the children to choose arithmetic as their favourite school subject. Otherwise, the majority of children liked the homework, mainly because of its ease or difficulty or of a liking for maths and the satisfaction from learning more and expanding their knowledge. Also, most children would feel sad if they missed an arithmetic class, mainly because they liked the subject (70%).

The present relation between children's attitudes to the textbook and their achievement can be explained by evidence relating children's attitudes to their ability (Stevenson & Lee, 1990; Tizard et al., 1988; Young-Loveridge, 1991). Despite their belief they were good in arithmetic, some children with arithmetic difficulties expressed negative feelings towards the textbook; a closer look at the children's own reports shows, however, what largely accounted for their dislike was the difficulty they experience in doing exercises, due to the complexity of the book. Difficulty is an indicator of actual ability and does account for those children's dislike.

Parents' beliefs about their child's attitudes to arithmetic were also associated with children's performance in the subject: parents of children who were particularly good in arithmetic believed the child held positive views about the subject, while parents of children with average and below average arithmetic ability believed the child was rather negatively oriented towards arithmetic. However, there was not much correspondence between children's own attitudes to arithmetic and parents' beliefs about the child's attitudes. It has been observed that children varied only in their attitudes to the textbook used in arithmetic class, in that children with arithmetic difficulties would express mild and strong dislike for the textbook. However, the majority of children liked the homework in arithmetic very much and would feel sad or very sad if they missed an arithmetic class. Parents' inaccuracy with regard to their children's beliefs is not uncommon in psychological research.

Last, the educational level of the mother has been found to associate with children's achievement in school arithmetic: the majority of mothers of mathematically talented children had completed the full range of obligatory schooling or also held a university degree, while some mothers of children with arithmetic difficulties had discontinued their education after the first six or nine years of schooling. The presence of an association between mothers' education and children's achievement is not surprising, since mothers' education has been associated with performance even upon entry to school (Tizard et al., 1988).

A well documented explanation of how the educational level of the mother may affect children's school performance is through mothers' involvement with the child's schooling (Stevenson & Baker, 1987). The educational level of the mother indicates the mother's experiences and knowledge of how one can progress through the educational system. So, involvement of a more educated mother in the school career of the child may be more effective. Mothers, accordingly, reported meeting with the teacher at least once a month to discuss the child's progress in arithmetic and reading, some would express judgement about the teacher's help with the child's difficulties in arithmetic, the majority were informed on what the children did in the arithmetic class from what the child said and from checking the child's textbook, and reported helping the child with their homework. In addition to mothers' high involvement with the child's day-to-day schooling activities, they were also more involved than fathers with the child's life: the majority reported spending at least 4 hours per day with the child, whether preparing meals for them, cleaning and tidying up, or helping them directly with the homework.

Cognitive

From the total of cognitive factors that were found to associate with arithmetic ability, the following skills and underlying components predicted children's arithmetic performance: knowledge of addition facts, understanding of base ten concepts and ability to deal with large numbers, knowledge and skill in informal arithmetic, problem solving ability, speech rate, and speed of reciting even numbers. With the exception of addition facts, these cognitive skills and underlying components remained significant even when both social and environmental and cognitive factors were examined in combination. Each measure will now be discussed separately.

Knowledge of addition facts was found to contribute significantly to children's variation in arithmetic performance. Children with difficulties in arithmetic could solve barely 3 out of 10 addition facts, while average children succeeded on 5 out of 10 facts. Children of excellent arithmetic ability showed remarkable accuracy on almost 8 out of 10 addition facts.

Knowledge of number facts referred to addition facts, that is, solutions that the child eventually remembers (retrieves from long term memory) without having to count (e.g., "2 plus 2 equals 4"). Russell and Ginsburg (1984) also found that children with math difficulties suffer from severely limited knowledge of addition facts: fourth-grade children with mathematical difficulties were even behind their third-grade peers in terms of accuracy. In the present study, however, it was *large* number facts that constituted a difficulty to children with arithmetic difficulties. A possible explanation is lack of practice. Ashcraft and Christy (1995) report that small number facts, that is, facts involving operands with 2 to 5 are presented twice as often as large number facts in school arithmetic textbooks, consequently, it is possible to assume that less frequent encounters lead to less practice which in turn leads to poor learning. This explanation, however, would suggest that math able children may have had more practice; a definite answer should await further research.

Ability to solve word problems was another significant predictor of children's arithmetic ability. Problem-solving ability referred to children's appreciation of the principles of commutativity and reciprocity, as well as their skill in solving word problems. Knowledge of commutativity was found to be common to children of all ability levels, yet, this was not surprising since research indicates that children appreciate the principle long before they enter school or even before they can do sums (Baroody & Gannon, 1984; Baroody & Ginsburg, 1986; Baroody, Ginsburg, & Waxman, 1983; Cowan & Renton, 1996). Russell and Ginsburg (1984) also found that understanding of commutativity was common among math difficulty children and their average peers. Knowledge of reciprocity, however, was less common, a finding that was also observed in Russell and Ginsburg. In the present study, children with arithmetic difficulties were significantly less likely than their average and above average peers to use reciprocity to solve written problems.

Ability to solve story problems discriminated between children of different arithmetic ability. While children of exceptional arithmetic ability solved almost all the word problems accurately, children with less arithmetic strength solved only half of the problems. It was found, however, that difficulties related to specific word problems. For example, those children experienced particular difficulties with complex subtraction, multiplication, and division problems. Since the two operations are introduced late in Grade 2, it is speculated that children are not confident in dealing with them. Riley, Greeno, and Heller (1983) also distinguish among different types of problems, each varying in complexity.

Knowledge of base ten system also accounted for children's variation in arithmetic ability. Children of above average math ability showed a mature understanding of base ten concepts and an extraordinary fluency in calculating accurately numbers of any size, while children with arithmetic difficulties exhibited severe weaknesses when dealing with large numbers, especially the ones over one hundred. They did show, however, a sufficient understanding of the base ten system and the part-whole schema, that is, that numbers are composed of other numbers (Resnick, 1983), and exhibited strengths, such as counting by tens or identifying the left-most digit as crucial in determining the relative magnitude of numbers.

Ginsburg and his colleagues (Ginsburg, 1982; Ginsburg, Posner, & Russell, 1981a) had attributed much of children's arithmetic difficulties to their inability to fully comprehend base ten concepts. Russell and Ginsburg (1984), however, did not find evidence for an inadequate knowledge of the base ten system in children with math difficulties. Also evidenced in the present study, children with mathematical difficulties suffer from lack of fluency with large numbers, while they possess some elementary base ten concepts and related enumeration skills.

Children's knowledge and skill in informal arithmetic predicted significantly their achievement in the subject. Children's understanding of relative magnitude and their accuracy in mental addition were the two main measures investigated in this part; these are considered to be part of the child's repertoire even before they enter school. It was found that knowledge of "more" and "closer", both of which require the use of a mental number line, was common to all children; however, it was mental addition and estimation that discriminated between children of different arithmetic ability. While above average children showed an exceptional ability in mental addition and estimation, children in the lower ability range clearly lacked procedural skill.

The same pattern of results has been observed in previous research. Ginsburg and his colleagues (Ginsburg, 1982; Ginsburg, Posner, & Russell, 1981a) argued that children's disabilities in arithmetic do not stem from inadequate knowledge of informal concepts or skill in informal arithmetic. Russell and Ginsburg (1984), accordingly, did not find evidence for an inadequate knowledge of informal arithmetic in fourth-grade children with arithmetic difficulties; they clearly resembled their normal peers. However, they did report an impairment when task requirements switched from conceptual to procedural. While their math difficulty children resembled their normal peers in understanding of relative magnitude, they fell behind in mental addition. Also, an examination of children's strategies to solve addition problems showed that above average children used more sophisticated strategies, such as regrouping and mental algorithm, while children with arithmetic difficulties had a limited repertoire of strategies, using some basic counting procedures, which enabled them to be accurate in sums under 50, yet not responding in larger sums.

The investigation of working memory in relation to children's arithmetic ability saw the significance of speech rate and speed of recitation in twos. The articulation task involved children repeating four multisyllabic words as quickly as possible. In recitation by twos, the child was asked to count from two to twenty by twos as quickly as possible. Both tasks discriminated between children of different arithmetic ability, that is, below average and above average arithmetic ability, as well as above average arithmetic ability and those with average arithmetic but below average reading ability.

Hitch and McAuley (1991) found that children with lower arithmetic ability have lower counting spans, a deficit they attributed to problems retaining and accessing specifically numerical information. Children had shorter digit spans and were slower in counting, but they did not differ in speech articulation. In the present study, children with arithmetic difficulties had shorter visual spans, but not auditory spans. While difficulties in dealing with visuospatial and generally nonverbal information are common in arithmetically disabled children (Fletcher, 1985; Rourke & Finlayson, 1978), the present study showed a more generalised deficit in retrieving information from memory and maintaining both numerical and non numerical information in memory: children were slower in counting, recitation in twos, and speech rate, while their word spans were also short. The comparison between children with more similar arithmetic ability further substantiated the contribution of speech rate and recitation to arithmetic ability: children with above average arithmetic ability and those with average arithmetic but below average reading ability were equally fast in reciting number sequences. While they differed in counting speed, they were still significantly faster than arithmetically disabled children.

Speech rate is related to memory spans and arithmetic performance. Speech rate is a measure of trace rehearsal rate. Based on the working memory model, one of the subsidiary systems that the central executive is responsible to coordinate is the articulatory or phonological loop (Baddeley, 1990). This in turn consists of the phonological store - which retains speech-based information for 2 seconds, and the articulatory control process - which involves overtly (recall) or covertly (inner speech) rehearsing the items to be remembered. The articulatory process rehearses the traces and feeds them back to the store.

The faster the articulatory process runs, the more items will be maintained, and the longer the memory span will be. Based on the findings of the present research, the speed with which items were rehearsed may have accounted for children's different memory spans and performance on the rest of counting tasks. Below average children had shorter spans both for numeric and non numerical information than did their above average peers. Also, they were slower in other counting tasks, such as spot counting, recitation in twos, and digit span. Speech rate, accordingly, has been related to counting speed (Baddeley, 1990). Naveh-Benjamin and Ayres (Baddeley, 1990) showed there is a relationship between memory span and the time it takes to articulate the digits one to ten. In the present study, children did not differ in recitation in ones, however, this counting sequence is used extensively. Kail (1992) found that articulation rate correlated negatively with measures of digit span and letter span; that is, the faster the counting speed, the longer the short term memory span for digits.

To sum, the present research has identified some major cognitive math-specific skills and underlying components as powerful determinants of arithmetic performance. Strength in informal arithmetic, base ten system, and solving word problems, along with knowledge of addition facts, and speed of speech and recitation discriminated between children of different arithmetic ability. In addition, some social psychological elements were found to relate to performance, however, did not explain variation to a great extent: children's attitudes to arithmetic and the corresponding beliefs of their parents, along with mothers' academic level correlated significantly with and related to how children come to perform in the subject.

On the other hand, children's knowledge of informal arithmetic concepts and base ten concepts, their appreciation of principles, and the errors (bugs) they make in written calculations were found to be common to children of all levels of arithmetic ability. A large number of social and environmental factors examined hereby did not discriminate between children either: it seems that the majority of children held a positive view of their performance which they attributed mainly to internal reasons, they enjoyed the benefits of parental help and support, and lived in an environment where messages on the importance of arithmetic and generally academic success for future prosperity are constantly communicated either directly in the form of advice and suggestions or indirectly through parental involvement with the child's academic and non academic affairs.

A More Theoretical Perspective: Synthesis

Having identified the main social, environmental, and cognitive elements which contribute to children's arithmetical ability, an attempt will now be made to understand how these processes interact to enable or handicap arithmetical competence. Arithmetical ability is thought to be influenced by both environmental factors as well as factors residing within the child. The latter can be social, cognitive, or both. Overall, these factors are independent in that they do not always co-exist within children of specific arithmetical skills; however, they are often found to interact in ways that promote or hinder arithmetic performance. The exact nature and power of interactions between influences is yet far from clear. The present research has attempted to examine patterns of interactions and how these varied with arithmetical ability.

On one hand, social and environmental elements have been found to contribute abysmally little to children's differences in arithmetic performance. Despite the large number of variables examined, only mothers' academic status and their beliefs of the child's attitudes to arithmetic would continue to be significantly related to achievement at the completion of the analysis. Possible explanations for the absence of further more associations are given in the next section on evaluation and shortcomings of the present study. The prevalence of mothers' education and their beliefs about the child's attitudes, however, suggests a significant relationship that should be investigated.

It is common sense that a parent who is more involved with the child's life is more likely to influence the child on several occasions than a parent who meets with the child less frequently. Mothers' high academic qualifications should be evident in an educationally supportive environment they might create for the child to live in and the messages they carry on to their child about the importance of doing well at school. Performance in arithmetic may be one of the areas where children are encouraged to do well, given that mothers acknowledge its academic importance.

In addition, mothers of children who did well in arithmetic believed the child held positive views about the topic, while mothers of children doing less well believed the child held rather negative views. It is unclear whether mothers' beliefs about the child's attitudes predict children's performance; they may affect children's performance via their help with the homework. However, this possibility was not supported in the present study; reports of help with the homework did not differentiate between children or between mothers. It is possible that the nature of help may affect children's performance and this will be discussed later in this section. Improved performance in turn may affect mothers' beliefs about the child's attitudes to the subject.

In research on social and environmental factors and their influence on achievement, parents' academic status has been repeatedly singled out as a significant predictor. This is true in younger as well as older children than those examined in the present study. While parents' education continues to affect children's performance throughout school, the degree of influence may decrease during adolescence. During that time, social interactions and peer influences become more significant at the expense of home influences. Children benefit from an educationally supportive environment which constantly nurtures the support and encouragement to achieve academic success. Poor performance at school, however, does not always suggest limited material or spiritual conditions at home; factors residing within the child very often prove more powerful in both determining and predicting children's performance in arithmetic and generally at school.

On the other hand, cognitive processes have been found to contribute largely and directly to children's arithmetical ability. A major assumption in the current literature is that arithmetical ability is not unitary. Accordingly, one of the striking findings of the present research is that children with arithmetic difficulties show some strengths in some areas of arithmetic but not in others. This supports the componential nature of arithmetic which implies that arithmetical ability consists of different components, such as understanding of concepts, knowledge of number facts, basic number knowledge, and ability to follow procedures. Dowker (1998) gives a detailed account of the structure of arithmetical ability and how these components consist of further subcomponents. Furthermore, within-children discrepancies have been found in normally achieving children as well as adults (Dowker, 1994, in Dowker, 1998).

The present study has identified similarities and differences in performance between children of different arithmetical ability. It has also considered discrepancies in math difficulty children's performance on different areas of arithmetic. The study saw the overall significance of knowledge and skill in the areas of informal arithmetic, base ten system, and problem solving. These were areas, or skills, that constantly predicted children's achievement throughout the analysis and which remained in the final regression equation when both social and environmental and cognitive factors were examined for their predictive value.

Each of these areas consisted of further subcomponents where children's performance also varied. For example, knowledge and skill in informal arithmetic included understanding of relative magnitude (where math difficulty children had sufficient strengths) and *mental addition* and *estimation* (where math difficulty children showed limited skills). Understanding and skill in base ten system included knowledge of place value and accuracy in applying base ten concepts (where math difficulty children's performance was adequate), and *dealing with larger numbers* (where math difficulty children were impaired). Finally, children with math difficulties showed an appreciation of the commutative law of addition, while their *use of reciprocity* and *problem solving skill*s were limited.

It is difficult to identify a single aspect of arithmetic skill that may account for children's variation in performance. A striking observation, however, is math difficulty children's difficulty with large numbers, both in written and mental form, which seems to restrain many of those children's arithmetic skills. Children with math difficulties faced severe difficulties, often refraining from attempting the problem, when large numbers were involved, while they were as accurate as their average and above average peers when they had to deal with small numerosities. What is also being suggested, however, is that more mature procedural skills and more efficient fact retrieval may have alleviated much of this impairment.

The difficulty in dealing with large numbers was observed in children's performance on different arithmetic tasks. In mental addition, children with mathematics difficulties could solve accurately simple addition problems using counting, while their performance dropped dramatically with trials of sums over 50, where they would not attempt to solve the problems. However, if those children had acquired appropriate strategies like mental algorithm (Russell and Ginsburg, 1984) where numbers are treated as digits and not as tens and hundreds (e.g., 35 + 14 could be solved as 5 + 4 [is 5, 6, 7, 8, 9] and 3 + 1 [is 3, 4]), they would have been able to solve some of the problems. Also, limited knowledge of addition facts may have contributed to this drop in performance; miscalculations were children's most common error, especially when three-digit numbers were involved.

In estimation, children were less successful in trials involving hundreds than those involving two-digit numbers. Both mental addition and estimation, which are strongly related based on the calculation process that they both require, showed the same pattern of results. Again, children would benefit from more number facts being available.

The difficulty imposed by the magnitude of numbers was mostly evident in children's application of base ten concepts. While children's understanding of place value, their awareness that the left-most digit is important in establishing numerosity, and their ability to count in tens was adequate, they could not apply any of this knowledge when they had to count large amounts of money or decompose numbers into their smaller components.

In solving word problems, math difficulty children were quite successful in problems involving simple addition or subtraction with numbers up to just above 20. They had major difficulties, however, when they had to apply multiplication and division, which were both introduced later in Grade 2, or when the wording of the problems became more complex.

Finally, a direct relationship between number magnitude and fact retrieval was observed; performance was severely restricted to number facts whose sum was less than 10.

The issue of fact retrieval refers to the relation between working memory and children's arithmetical difficulties. Another striking finding was math difficulty children's relatively limited performance on measures of working memory. Speech rate is another candidate for children's limited working memory spans and their difficulties in other basic component measures; it may also account for difficulties in further more areas of arithmetic. Speech rate is a measure of trace rehearsal rate. Based on the working memory model (Baddeley, 1990), the articulatory or phonological loop consists of the phonological store - which retains speech-based information for two seconds, and the articulatory control process - which involves overtly or covertly rehearsing the items to be remembered. The articulatory process rehearses the traces and feeds them back to the store. One well documented approach is that the faster the articulatory process runs, the more items will be maintained, and the longer the memory span will be.

The present findings point to the direction of speech rate as another major cognitive subcomponent associated with children's achievement. The slow speed with which items are rehearsed may have accounted for children's limited concurrent and simple spans, as well as delays in speed; below average children had shorter spans for numerical and non numerical information as well as for words and digits than did their above average peers. Also, they were slower in simple counting tasks, such as spot counting and recitation in twos.

Speech rate and recitation in twos together predicted children's performance on Young's test; a test which consisted of a timed section. Speech rate has been related to counting speed: for example, there is a relationship between memory span and the time it takes to articulate the digits one to ten (Baddeley, 1990); also, articulation rate correlates negatively with measures of digit span and letter span; that is, the faster the counting speed, the longer the short term memory span for digits (Kail, 1992).

While none of these measures showed on the final regression, it is nevertheless important to acknowledge children's generalised deficit in retrieving and retaining information in working memory and its relation to slow speech rate. One major reason is that these limitations may underlie difficulties in further more arithmetic skills. For example, they may relate to difficulties in fact retrieval, which may further relate to difficulties in overt recall of number facts, estimation, and written and mental calculation. Or, they may relate to limitations in procedural knowledge, which may further relate to efficiency in calculation, availability of counting knowledge during computations, and attention allocation.

The effect of parental involvement, mainly in the form of practice and teaching, is of prime importance in the interpretation of the present findings. Mothers' academic status was found to account for children's variation in arithmetic achievement; mothers' years of schooling were significantly related to children's achievement in arithmetic. In chapter 3, it was also observed that mothers spent at least 4 hours per day with their child, while fathers would spend a maximum of 4 hours per day with the child. The frequent mother-child interaction and the high academic level of mothers, for example, is a combination that promotes the development of arithmetic skills. Furthermore, mothers of children who did well in arithmetic thought the child held positive feelings towards the subject. Mention has been made earlier that the exact direction of this relationship is unclear; a reciprocal effect, however, is quite reasonable. A mother who sees her child being positively oriented to arithmetic may provide more assistance with the subject with the subsequent result of improved arithmetic performance at school. Improvement may in turn affect mothers' beliefs that the child does well in arithmetic and holds positive feelings about it.

The amount of help with the homework in arithmetic was not found to discriminate between children (or their parents) of different arithmetical ability; reports of help with the homework in arithmetic were common. It is possible, however, that the nature of this assistance is more important in determining children's ability; children may benefit more from a tutoring mother (with the subsequent long term effects of learning) than from a mother who is simply helping the child solve a particular exercise (with assistance being limited to the situation).

The nature of interaction at home may suggest significantly less or more exposure to arithmetic, which in turn may promote or impair arithmetical ability. Apart from the messages communicated from the parent to the child, the quality of mother-child interaction at home relates to practice, where differences in exposure may to a certain degree account for differences in children's performance. For example, an academically enriched environment may involve greater practice of a number of skills such as more instances of mothers asking the child about number facts. Alternatively, mothers of children doing less well, while still assisting with the child's homework, may not be as effective as mothers who have more advanced arithmetic knowledge.

Another element of mothers' involvement that relates to the cognitive processes present in the child is teaching. Given the academic importance attached to arithmetic by highly educated mothers, it is possible that those mothers would attempt to introduce their child to written arithmetic sooner than the rest of the mothers. This could account for the immature procedural skills in children with math difficulties. Or, better educated mothers may place more emphasis on teaching and learning to follow the rules and principles than on simple informal arithmetic.

Finally, speech rate could also improve by more practice at home. A significant indicator of such an improvement due to practice has been counting speed; children with math difficulties had normal speed in counting the number sequence from 1 to 10, while they were significantly slower in counting from 2 to 20. Parents should also emphasise verbal communication in both arithmetic and non arithmetic contexts (e.g., reading aloud). Given the links between speech rate, memory measures, and procedural skills, more practice at home may be suggestive of improvements in arithmetic performance.

Geary (1993) argued that children's performance on any arithmetical test depends on the efficiency of both their procedural and fact retrieval skills. Those skills are further based on children's counting knowledge, working memory, and counting speed. Accordingly, in his examination of specific mathematical deficits in children with mathematical disabilities, Geary argued that those children suffer from two basic functional numerical deficits, one related to procedural skills and the other to fact retrieval from long-term memory. The first deficit is mostly evident in children's immature arithmetic procedures and procedural errors, usually a result of developmental delays in the acquisition of conceptual knowledge underlying procedural use. The second deficit refers to the representation and retrieval of arithmetic facts from long-term memory, which however, does not seem to disappear with development.

In the present study, the deficits observed in math difficulty children are more of a developmental delay nature. Children shared the same strengths as their normal peers in many areas of arithmetical ability, for example, knowledge of informal concepts, small number facts, addition principles, place value, as well as accuracy in simple addition and subtraction problems. Their difficulty with large numbers in both written and mental calculations, their use of immature counting procedures and bugs, and their restricted knowledge of multiplication and division point to the direction of some developmental lag rather than severe cognitive deficits. Speech rate, as in the case of counting speed, could also further improve with practice. While some of these abilities may not necessarily improve with age, for example, approximate unknown fact derivation (Dowker, 1998), it is likely that children's performance in most of the problem areas mentioned above will improve dramatically with age and more practice at home.

6.5 Evaluation and Shortcomings of the Present Research

The present research was an attempt to identify some math-specific factors that might explain children's variation in school arithmetic. It is true that influential sources can be far beyond our knowledge. And that we are biased as to the factors we examine. As Tizard et al. (1988) point out "no researcher is free of prejudice or unexamined assumptions; these influence both one's choice of questions to study and one's method of tackling them" (p. 21). In the present study, a wide range of measures were examined, being either cognitive or social and environmental in nature. The purpose for their selection was their significant association with children's performance in arithmetic, based on previous research findings. The present study combined the two sets of factors, further exploring the association between more relatively under-researched topics and children's arithmetic achievement.

The present findings suggested that variation in arithmetic achievement was explained through most of the cognitive skills and underlying cognitive components examined hereby and a small number of social and environmental factors. Of those cognitive and social and environmental elements significantly related to achievement, some were more strongly associated with performance than others. All factors that were found to be linked to achievement were, however, elements that discriminated between children of different arithmetic ability. In the final stage, some of those factors were further identified as significant predictors of children's achievement in arithmetic.

Overall, the present research employed a correlational approach to examining children's variation in arithmetic. Associations, however, do not suggest causality. The present research did not attempt to identify causes of children's performance; nor were the factors under exploration perceived or assumed to be causes of children's differences in arithmetic. The nature of the investigation was correlational, with variation being explained in terms of strong associations between individual factors and achievement levels. To examine causality would necessitate the introduction and application of an intervention. Rigorous manipulation of significant correlates of achievement would be expected to potentially show some degree of change in children's achievement.

Whilst the present research successfully addressed the issue of social and environmental and cognitive correlates of arithmetic achievement, it could have been strengthened by more extensive sampling. The present design allowed for hypotheses and predictions as to the degree of individual contribution of each set of factors to arithmetic performance, however, it could not answer with certainty how much variation could be explained by both social and environmental and cognitive factors; the number of children who participated in both cognitive studies and on whom information on parental variables was collected was limited. To answer with confidence how much both sets of factors contribute to children's arithmetic ability, information on all parental variables should be collected. In the present study, questionnaires were sent home via the child and replies were left to the respondents' consideration; it was later observed that parents had not responded to every item featuring on the questionnaire. Structured interviews is a safe alternative which would ensure responses to every item, further allowing for clarification of any uncertainties which may have caused parents not to respond to some items.

In turn, measuring the extent of variation accounted by both sets of factors would allow further speculations relating to potential determinants of arithmetic performance, for example, classroom interaction, teaching style, or children's personality.

The present study would be further strengthened by the use of dependent measures designed and used in the specific setting, that is, in Greece. In the present study, children's arithmetic ability - and differences for that matter - was measured based on children's performance on two mathematical tests: the "Y" Mathematics Series Y2 by Young (1979) and the Basic Mathematics Test B by the National Foundation for Educational Research in England and Wales (NFER, 1971). These tests distinguished between children of different arithmetic ability, based on scores which were used as cut-off points to categorise children into three ability groups. While the sample consisted of Greek children, however, the tests were originally British. The present study has thus identified Greek children differing in arithmetic ability using British tests which have not been standardised in Greece. As has also been mentioned in the chapter on sample selection (chapter 2) to this day there are no Greek standardised measures of arithmetic ability.

The rationale was to employ standardised measures which would enable a valid categorisation of children's ability. To account for any threats to reliability, the tests have been subjected to extensive piloting and repetitive evaluation at different stages; their diagnostic value and their equivalence in terms of age range, however, were the prime reasons for selecting them. Effort was made to ensure equivalent levels between the Greek arithmetic curriculum and the content of the tests: as Greek and British curricula are not the same, the age of children would be a significant factor suggesting equivalent levels of mathematical mastery. The content of the tests was then compared to the contents of Grade 3 Greek arithmetic textbooks. Furthermore, the two mathematical tests were given to Grade 4 children in Greek schools to eliminate those items that even older children could not solve successfully; an item being too difficult for fourth-grade children would not help understand younger children's difficulty. Finally, during group comparisons, statistical tests were run repeatedly to ensure that each group was significantly different in arithmetic from the other two groups; no child near cut-off points was included. After the groups were defined, a reliability test showed that performance on Forms A and B of Young's (1979) test correlated highly ($r = .94$, $p < .005$).

Overall, the present research has evidenced significantly more and stronger associations with cognitive rather than non-intellectual elements. Despite being selected on the basis of their significance observed in previous studies and being more related to math than to other scholastic activities, the majority of social and environmental factors were nevertheless found not to associate with children's achievement in arithmetic. With the exception of some variation observed between children in reports of their attitudes to arithmetic or between mothers in their curriculum opinions and their evaluation of the teacher's help, there was no variation in most of the social and environmental measures as a function of children's attainment in arithmetic.

The present study does not challenge previous findings. It is being argued that evidence of significant links between some social and environmental variables and children's performance in arithmetic may come from studies focusing on children whose both arithmetic and general school performance was poor (Stevenson & Lee, 1990). The present research, however, attempted to explain variation in arithmetic performance based on a sample of children who varied between them in arithmetic achievement only while their reading and overall school performance was satisfactory. It is being suggested that the significance of some social and environmental variables that were found elsewhere to relate to achievement and that were examined in the present study would not apply in the case of arithmetic only.

Alternatively, a sensitive reappraisal of the way some social psychological variables were measured needs to be considered. For example, given the evidence of a reciprocal relationship between children's academic self-concepts and school achievement, it would be unlikely that a relationship does not exist; it is possible that this relationship may be weak in very young children (i.e., before the age of 9 years). However, a further consideration of the absence of strong associations between a number of well-researched social and environmental factors (e.g., home activities or parental help with the homework, among others) and children's achievement, points to the direction of a thorough re-evaluation of the measures used to collect this information; this in turn necessitates a critical review of the validity and use of questionnaires in psychological research.

6.6 Suggestions for Future Research

Geary (1993) hypothesised that performance on ability tests is based on cognitive skills, like procedural and fact retrieval, which are influenced in turn by underlying cognitive components, like counting knowledge, working memory, and counting speed. The procedural and memory-retrieval components are functional skills which manifest themselves *during* the process of problem solving; the rest three components are skills that underlie or contribute to the procedural and memory-retrieval components. In his attempt to explain the cognitive deficits in mathematically disabled children, Geary suggested that the lower order deficits of those children may reside in those five component skills: procedural, memory retrieval, conceptual, working memory, and counting speed.

Based on findings from studies on addition, Geary concluded that mathematically disabled children experience two distinct functional deficits: procedural and memory retrieval, which nevertheless follow different developmental directions. Procedural (or computational) deficits follow the developmental-delay model; MD children usually exhibit immature skills closely resembling those of their younger normal peers. Eventually they come to resemble their normal peers in computational skills by the end of 2nd grade. Memory-retrieval deficits, however, follow the developmental-difference model; MD children show a pattern of performance which is qualitatively different from that of their normal peers and which persists for many MD children throughout the elementary years. Thus, while computational skills are more likely to improve with time, some sort of intervention would be needed for memory-retrieval skills; counting speed, for example, could be improved, despite research suggesting mixed results with regards to the speed-of-processing differences between MD and normal children.

The present study has, accordingly, found an association between underlying cognitive components and arithmetic achievement; for example, knowledge of addition facts (whether defective retrieval or lack sufficient knowledge) discriminated between children of different arithmetic ability. This could be further related to the finding that children with math difficulties experience problems in accessing information from the long-term memory and were significantly slower in processing both numerical and non-numerical information. More specifically, there was an association between counting speed and performance; if knowledge of addition facts is related to speed, then increase in speed would result in increase in performance.

Since children varied in their speed of counting from 2 to 20, it could be argued that increases in counting speed would result in faster recognition of facts in long-term memory and faster retrieval times. The rationale behind this is that improving the efficiency of underlying components would result in improvement in performance per se. Given that math difficulty children did not differ in their speed in counting from 1 to 20, it could be suggested that below average children's speed would improve with practice. Geary (1993) also found that studies on counting speed have provided mixed findings on how consistent counting-speed deficits are in MD children.

Future research on math-specific factors related to arithmetic achievement could focus on yet another combination of children's abilities: children doing poorly at school and in reading, but significantly better in arithmetic. Studies have focused on children doing badly in school, in maths, and in reading (Stevenson & Lee, 1990). Most social and environmental factors examined in the present study have already been found to relate to school achievement; some have been found to associate to arithmetic attainment, too. However, most of the previous studies have examined children who did poorly at school in general. The present research accounted for that issue, in that it examined achievement in arithmetic having controlled for performance in all other academic subjects. Children in the present study varied only in their arithmetic performance, while their reading and general school performance was satisfactory. In a further attempt to examine the math-specific factors related to arithmetic achievement, future research should consider children who do poorly at school but well in arithmetic. That would be another indication of specific arithmetic factors related to mathematical performance in specific.

Lastly, based on experience, we acknowledge that children do not always catch up from one year to the other. As with computational deficits which follow a developmental-delay trajectory (Geary, 1993), some deficits may disappear after some specific age; others, however, may persist over time. In the same way, ability and performance are not thought to be consistent. In considering children's differences in arithmetic ability, it is possible that differences in this ability become less apparent in the next grades, as difficulties may be overcome or precocity may level up with the more advanced material. It is indeed worth examining how consistent these differences are over the primary school years.

An Epilogue

The current study draw attention to the complexity of interactions which combine to enable or handicap the acquisition of desirable skills and abilities. It based its hypotheses on a combination of research questions which dominate the current literature, yet remain unclear. Considering the implications of doing well or badly in arithmetic, however, along with the cultural importance of the subject and the notion that knowledge builds up as children move on to higher grades, makes the issue of children's variation in arithmetic a crucial one. A sensitive re-appraisal of possible determinants of arithmetic ability by all concerned will do much to reduce the anxiety and stress that many young children face when presented with arithmetic tasks.

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APPENDICES

APPENDIX 1

Appendix 1.1 Greek and British Educational Systems: A Comparison of the Structure, the Arithmetic Curriculum, and Specific Reading Difficulties

General Structure of the Educational Systems

The Greek Educational System consists of 12 years of obligatory education, six of which comprise the primary school years (6-12 years old), three comprise the high school years (12-15 years old), and three years comprise the lykion (15-18 years old). School education begins at 6 and ends at 18 years of age. At the age of 18, students may go through the University Entrance Exams.

The British Educational System consists of 11 years of obligatory education, six of which comprise the primary school years (Key Stage 1, 5-7 years old; Key Stage 2, 7-11 years old), and five of which comprise the secondary school years (Key Stage 3, 11-14 years old; Key Stage 4, 14-16 years old). At the age of 16, students take the General Certificate of Secondary Education exams, and those who wish to continue their education may do A levels for two years.

The National Curriculum and Instruction Methods in Primary School

Throughout the primary years, Greek children do basic language (including reading, writing, and composition) and arithmetic courses, as well as physical science, music, and art. In Grade 3, children are introduced to history, science, geography, and English. The remaining grades are characterised by more advanced levels of already existing courses, for example, introduction of geometry in arithmetic courses, technology in science courses, etc. There is a textbook for each course taught in the curriculum. Each textbook is accompanied by a teacher's manual which specifies the instruction method of each topic within every course. There are no final exams on any grade in the primary level. Teachers occasionally administer tests but only for the purpose of assessing children's progress.

In Key Stages 1 and 2, children are taught English, mathematics, science, technology (design and technology, and information technology), history, geography, art, music, and physical education. Instead of textbooks, a variety of schemes constitute the main source of information on what children should be taught. This suggests that pedagogy, including the organisation of the class and how the material is to be taught, relies entirely on the teachers' individual interpretation of the curriculum. At the end of each Key Stage, that is, at age 7 and 11 years, children go through exams.

Arithmetic Curriculum: 8 Years Old

In terms of the arithmetic curriculum, Greek children in Grade 3 already have sufficient knowledge of informal arithmetic concepts, skill in both mental and written calculation (in the case of operations, multiplication and division are introduced later in Grade 2), as well as understanding of place value and basic addition and subtraction principles. In addition, children show some ability to deal with fractions, shape, grouping, time, and money.

During Key Stage 2, children are taught to develop an understanding of number (understanding of place value and extending the number system, understanding relationships between numbers and developing methods of computation, solving numerical problems), understanding of shape, space (position and movement), and measures, handling data (collecting, representing and interpreting data, understanding and use of probability).

The arithmetic curriculum taught in Year 4 which corresponds to Grade 3 in Greece is quite similar, only with wider coverage of multiplication, division, graphs, measuring, fractions, and shape.

Specific Reading Difficulties

As Greek and English differ with respect to orthography and pronounciation, some major difficulties in reading are specific to each language. On one hand, Greek is an orthographically transparent language, where letter-sound correspondence is clearly established. Reading difficulties in Greek children would understandably not refer to decoding problems, but rather to difficulties in comprehension of a text despite being adequately read. In other words, inferential problems, that is, making sense of what they read, are children's most common source of difficulty.

English language, on the other hand, is more irregular. One is required to move from phonological to orthographical decoding (of words), further taking into account both the grammar and syntax (of the sentence). Reading difficulties could result from difficulties in decoding, which are further enhanced by having to consider the rest of the sentence. For example, the sentences *The teacher instructed the class to read the text* and *Having read the text, the students moved on to answering the questions,* involve the verb *read* in different forms (present and past participles). To be able to read the verb correctly, one should comprehend the sentence taking into account the grammar and syntax.

The exact nature of reading difficulties should be emphasised and considered when comparing the results from studies using reading difficulty children as controls. As these difficulties may vary with language, direct comparisons may not always be possible.

Appendix 1.2 Research Evidence on the Relation Between Social and Environmental Variables and Children's Arithmetic Performance

Study	Age	Factors Related
Aiken (1970)	6y.o.-college	attitudes
Aiken (1972)	13, 18, graduates (college)	attitudes parents' attitudes (child's attitudes)
Aiken (1976)	school[a]	attitudes
Aiken & Dreger (1961)	freshmen (college)	attitudes parents' attitudes (child's attitudes)
Assor & Connell (1992)	school[a]	self-concepts
Baker & Stevenson (1986)	Grade 8	mothers' education
Bernstein (1964)	school[a]	attitudes
Blatchford (1992)	7 & 11 y.o.	self-concept
Blatchford (1996)	7, 11, 16 y.o.	attitudes
Blatchford (1997a)	7, 11, 16 y.o.	self-concept
Blatchford (1997b)	7, 11, 16 y.o.	self-concept
Blumenfeld, Pintrich, Meece, & Wessels (1982)	primary	self-concept attribution
Brown & Abell (1965)	Grades 1-8	attitudes
Chen & Stevenson (1995)	17 y.o.	values regarding education standards & aspirations importance of effort achievement attitudes
Crystal & Stevenson (1991)	Grades 1 & 5	mothers' view of child's problems mothers' view of nature of problems
Evans & Goodman (1995)	primary	self-concept numerical acitivities at home
Fennema & Sherman (1977)	Grades 9-12	attitudes (i.e. confidence)
Freeberg & Payne (1967)	2-6 y.o.	parental involvement

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(Continued)

Study	Age	Factors Related
Gottfried, Fleming, & Gottfried (1994)	9-10 y.o.	mothers' involvement
Grolnick & Ryan (1989)	Grades 3-6	parental involvement
Hughes, Wikeley, & Nash (1994)	Years 1-3	parent-school relations
Iverson & Walberg (1982)	5-11 y.o.	Chicago school: social psychological activeness of family academic guidance family work habits language models home intellectuality
Jacobs (1991)	Grades 6-11	self-concept parents' beliefs (child's self-concept)
Marsh (1990)	sophomore (high school)	self-concept
Miserandino (1996)	Grades 3 & 4	perceived competence autonomy
Neale (1969)	Grade 6	attitudes
Newson & Newson (1977)	7 y.o.	parent-school relations parents reading to child
Parsons, Adler, & Kaczala (1982)	Grades 5-11	self-concept parents' beliefs (child's self-concept)
Poffenberger (1959)	freshmen (college)	attitudes
Poffenberger & Norton (1959)	freshmen (college)	attitudes
Reynolds & Walberg (1992)	Grades 7-8	home environment (indirect): parents' expectations parents' education number of resources academic motivation (indirect) out-of-school reading (direct)
Schofield (1982)	Grades 3-6	attitudes

(Continues Next Page)

(Continued)

Study	Age	Factors Related
Schunk (1990)	school[a]	self-concept
Shavelson & Bolus (1982)	12-13 y.o.	self-concept
Stevenson & Baker (1987)	5-17 y.o.	mothers' involvement in school mothers' education (involvement)
Stevenson & Lee (1990)	Grades 1 & 5	parental involvement mothers' academic aspirations mothers' academic expectations self-concepts & confidence easiness/difficulty of subject school variables
Tizard, Blatchford, Burke, Farquhar, & Plewis (1988)	nursery top infant	education home experience parent-school relationship parents' attitudes parents' educational beliefs parents' education
Weiner (1979)	school[a]	attributions
Wylie (1979)	school[a]	self-concept
Young-Loveridge (1991)	9 y.o.	favourite subject favourite topics in arithmetic attitudes self-concept

[a] *School* refers to all academic levels from primary school to college.

APPENDIX 2

Appendix 2.1 "Y" Mathematics Series Y2 Form A (Young, 1979)

'Y' MATHEMATICS SERIES Y2

D. Young

Form A

NAME ____________ **TODAY'S DATE** ____________

CLASS ____________ **DATE OF BIRTH** ____________

SCHOOL ____________ **AGE: yr** _____ **mth** ____________

Oral section pages 1—2 ________ **/20**

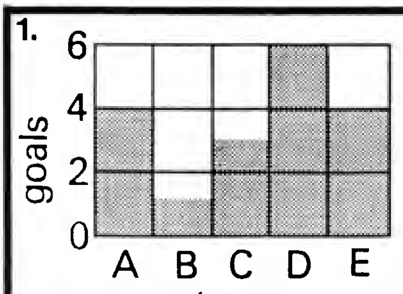
Computation section page 3 ________ **/20**

Total Score X (pages 1—3) ________ **/40 Quotient** ____ **(X)**

Written section page 4 ________ **/15**

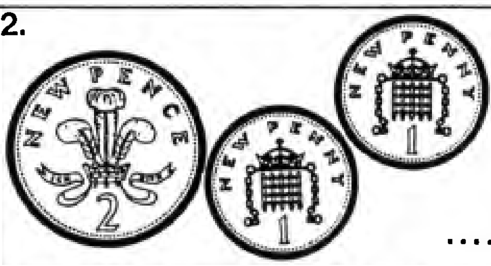
Total Score F (pages 1—4) ________ **/55 Quotient** ____ **(F)**

1. M



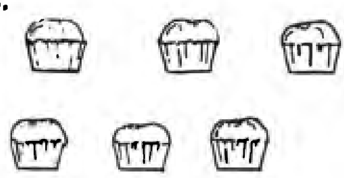
......and......

2.



.........p

3.



......cherries

4. M

SUN	MON	TUE	WED	THU	FRI	SAT
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

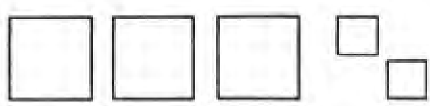
....

5.

32, 36, 40, [ ], 48

....

6.

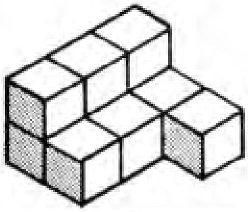


....

page 2

M		M

7.

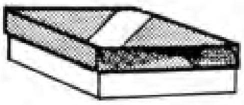


....

8.

40 p £......

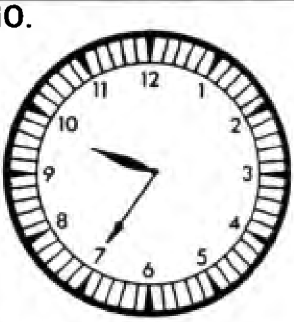
9.



28 chocolates

.... children

10.

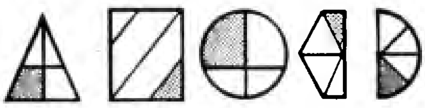


....

11.

59 29 45 71 32 ....

12.

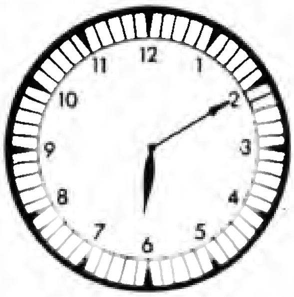


A B C D E ....

13.

624 ....

14.



....

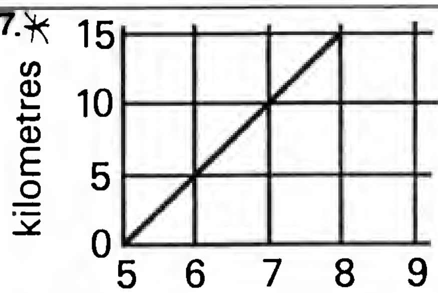
15.

4895 ....

16.

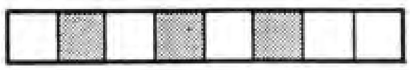
$\frac{1}{2}$ $\frac{3}{10}$ $\frac{1}{10}$ $\frac{7}{10}$ $\frac{1}{5}$ ....

17. *



......km

18.



....

19. *

36 18 24 ....

20. *

......cm

pages 1—2, Oral (20)

page 3

		M
1.	15 + 8 = ...	
2.	15 − 7 = ...	
3.	5 × 3 = ...	
4.	9 ÷ 3 = ...	
5.	32 + 47 = ...	
6.	23 − 6 = ...	
7.	6 × 6 = ...	
8.	16 ÷ 4	
9.	69 + 59 = ...	
10.	59 − 25 = ...	
11.	8 × 4 = ...	
12.	35 ÷ 5 = ...	
13.	53 + 158 = ...	

		M
14.	63 − 15 = ...	
15.	9 × 6 = ...	
16.	49 ÷ 7 = ...	
17.	$\begin{array}{r} 447 \\ 685 \\ +798 \\ \hline \\ \hline \end{array}$	
18.	$\begin{array}{r} 406 \\ -\ \ 56 \\ \hline \\ \hline \end{array}$	
19.	$\begin{array}{r} 39 \\ \times\ \ 7 \\ \hline \\ \hline \end{array}$	
20.	$6\overline{)744}$	
	page 3, Comp. (20)	

page 4

		M
1. If I turn from facing north to facing south, through what fraction of a complete circle have I turned?	......	
2. Brian has 90 foreign stamps and half as many British stamps. How many has he altogether?	......stamps	
3. How long is it from 10pm on Tuesday to 4am on Wednesday?	......h	
4. A clock loses 5 minutes per day. If it was correct 48 hours ago, how slow is it now?	......min	
5. A girl has saved £2 more than her sister. Together they have saved £10. Write down what each has saved.	£...... and £......	
*6. A newspaper said that a cricketer had missed his half-century by 2 runs. How many runs had he scored?	......runs	
7. A concert started at 6.45pm and lasted for 2½ hours. At what time did the concert end?	......pm	
8. When eight was taken from a number 5 times, the final number was 3. What was the number?	......	
9. A coach tour covered 1000 kilometres. On the first day the coach travelled 200 kilometres and, on the second day, 240 kilometres. What was the remaining distance?	......km	
*10. At a supermarket, 5 cashiers checked out 200 customers in one hour. How many cashiers would be needed to deal with 280 customers in one hour?	......cashiers	
*11. If a train travels at a steady speed of 57 kilometres per hour how far will it travel in 30 minutes?	......km	
*12. A woman won a quarter-share of a prize. The prize was 6 dozen eggs. How many eggs did she get?	......eggs	
*13. Half of the number b is 7. One third of the number c is also 7. Find $b + c$.	......	
*14. My alarm clock is set for 7.30. If I go to bed at 11.15 how long will it be before the alarm rings?	......h......min	
*15. Four-fifths of a group of 35 new houses are occupied. How many houses are still vacant?	......houses	
page 4, Written problems (15)		

Appendix 2.2 "Y" Mathematics Series Y2 Form B (Young, 1979)

'Y' MATHEMATICS SERIES Y2

D. Young

Form B

NAME ____________________ **TODAY'S DATE** ______________

CLASS ____________________ **DATE OF BIRTH** ______________

SCHOOL ____________________ **AGE: yr** _____ **mth** ______________

Oral section pages 1—2 __________ **/20**

Computation section page 3 __________ **/20**

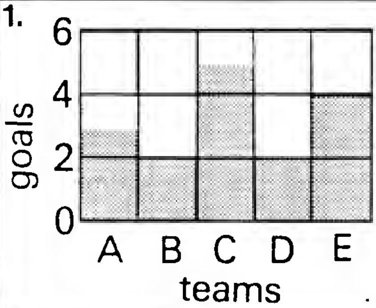
Total Score X (pages 1—3) __________ **/40 Quotient** ____ **(X)**

Written section page 4 __________ **/15**

Total Score F (pages 1—4) __________ **/55 Quotient** ____ **(F)**

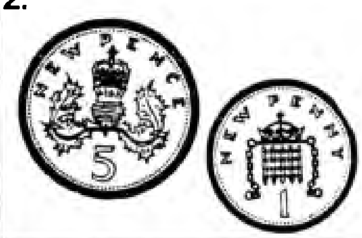
M

1.



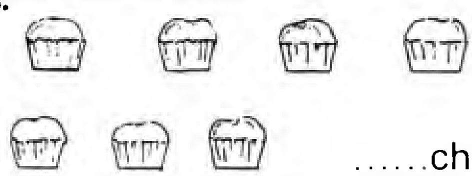
......and......

2.



.........p

3.



......cherries

M

4.

SUN	MON	TUE	WED	THU	FRI	SAT
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

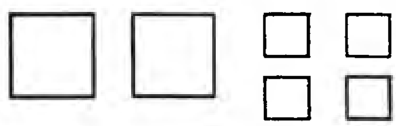
....

5.

34, 37, 40, ☐, 46

....

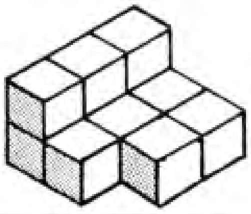
6.



....

page 2

7.	M

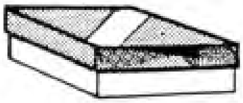


... .

8.

60 p £......

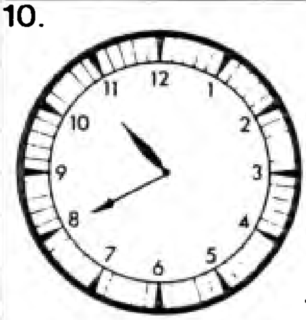
9.



24 chocolates

.... children

10.

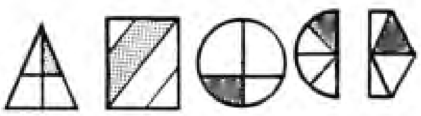


....

11.

32 29 71 46 59 ....

12.

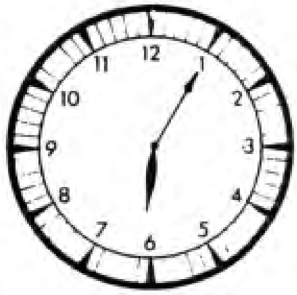


A B C D E ....

13.

735 ....

14. M



....

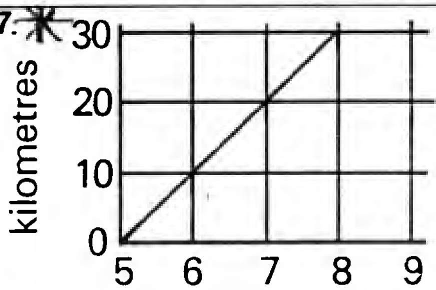
15.

4976 ....

16.

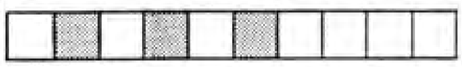
$\frac{1}{2}$ $\frac{5}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{8}$ ....

17.✳



......km

18.



....

19.✳

24 20 36 ....

20.✳

......cm

pages 1—2, Oral (20)

page 3

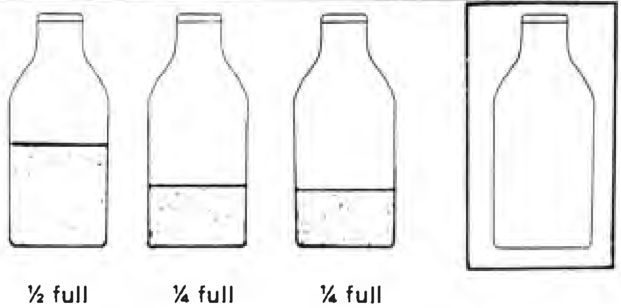
1.	M
17 + 8 = ...	
2. 14 − 8 = ...	
3. 7 × 2 = ...	
4. 6 ÷ 3 = ...	
5. 31 + 34 = ...	
6. 30 − 7 = ...	
7. 5 × 6 = ...	
8. 18 ÷ 6 = ...	
9. 91 + 28 = ...	
10. 99 − 36 = ...	
11. 4 × 9 = ...	
12. 28 ÷ 4 = ...	
13. 55 + 88 = ...	

14.	M
105 − 8 = ...	
15. 8 × 7 = ...	
16. 42 ÷ 7 = ...	
17. 698 357 +849 ____ ____	
18. 607 − 57 ____ ____	
19. 26 × 8 ____ ____	
20. 5)715	
page 3, Comp. (20)	

page 4

	M
1. How much altogether is needed to give 15 pence to Bill and to each of his four brothers? ......p	
2. A boy throws three darts. He gets treble-eight, double-seven and thirteen. What is his score?	
3. A settee and two chairs cost £170. If the settee costs £90 what is the price of one chair? £......	
4. These boys were born on the dates given below their names Alan 3.1.1975 Ben 3.9.1975 Colin 3.6.1975 Dave 3.8.1976 Write the names of the two oldest boys. .... and............	
5. Two women both spent half of the money in their purses. One began with £20, the other with £10. What was the difference between the amounts they had left? £......	
*6. What number is covered up? 8 + 7 = ▢ + 5	
7. There were 95 cars in a car park. If the cars were in 5 equal lines how many were there in each line? ......cars	
8. A football match lasts 90 minutes. If it begins at 3.30 what time is it when the whistle blows for half-time?	
9. How many two figure numbers begin with 7?	
*10. Mark bought a second-hand bicycle for £22. He spent £2.50 on repairs and then he re-sold the bicycle for £27. What profit did he make? £......	
*11. A clock gains 4 minutes per day. It it is correct at noon on Monday, how fast will it be by noon the following Thursday? ......min	
*12. A train left station A at 14.56 and arrived at station B at 17.49. To the nearest hour, how long did it take? ......h	
*13. If 7 bottles of squash will make 56 drinks, how many drinks can be made from 11 bottles of squash? ......drinks	
*14. If 18 is subtracted from a number and the difference is 7, what is the number?	
*15. A dealer bought 4 cameras for £50 each and sold them for a total of £280. What was the profit on each camera? £......	
page 4, Written problems (15)	

Appendix 2.3 Basic Mathematics Test B (NFER, 1971)



1..................

3.

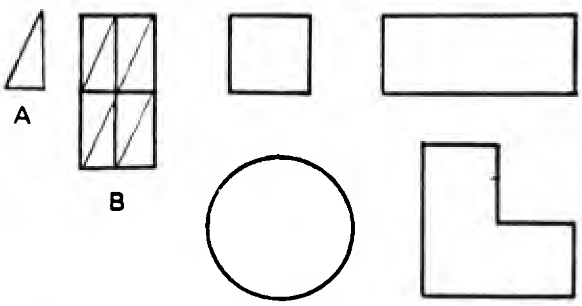
$$7 + 2 = 9$$

2. $\square + \square = 9$

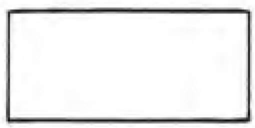
3. $\square + \square = 9$

2..................

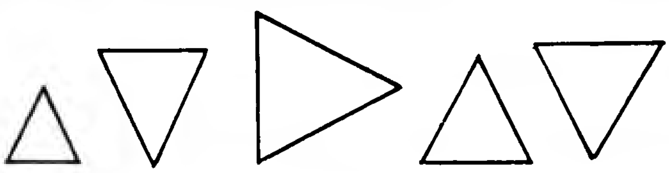
3..................



4..................



5..................

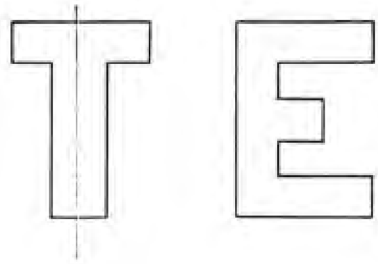


6..................

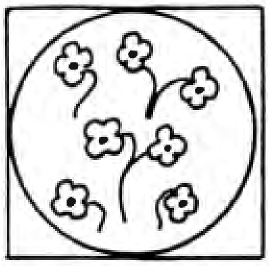
12, 10, 8, 6, $\square$

7..................

R	W



8................



9. [ ]

9.................

),11.

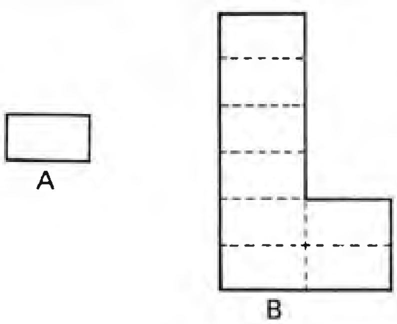
10. [years]

10...............

11. [years]

11...............

2.



12. [cards]

12...............

R	W

(5)

14.

$$13 - 8 = 5$$

13. $\square - \square = 5$ 13...............

14. $\square - \square = 5$ 14...............

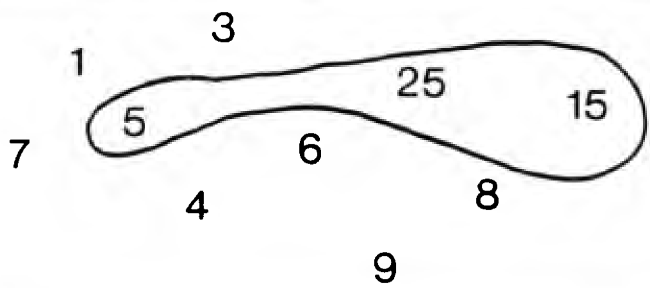
$$11 + 4 + \square = 21$$

15...............

7777	7877	8777	7778	7787

16...............

17...............



18...............

,20.

$+ \quad - \quad \times \quad \div$

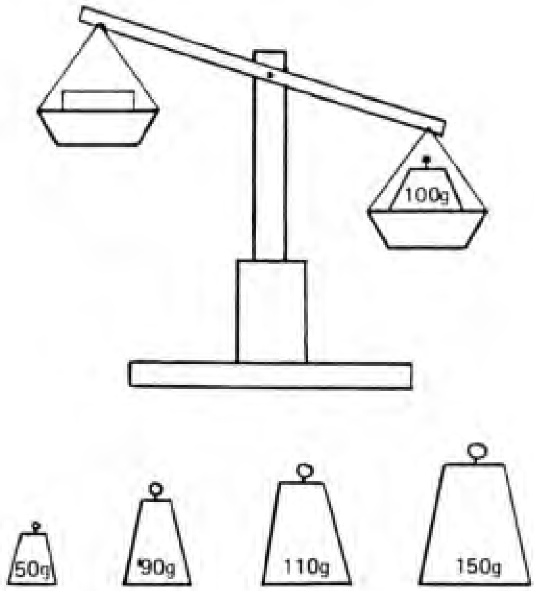
19. $4\tfrac{1}{2} \; \square \; \tfrac{1}{2} = 5$ 19...............

20. $18 \; \square \; 6 = 3$ 20...............

R	W

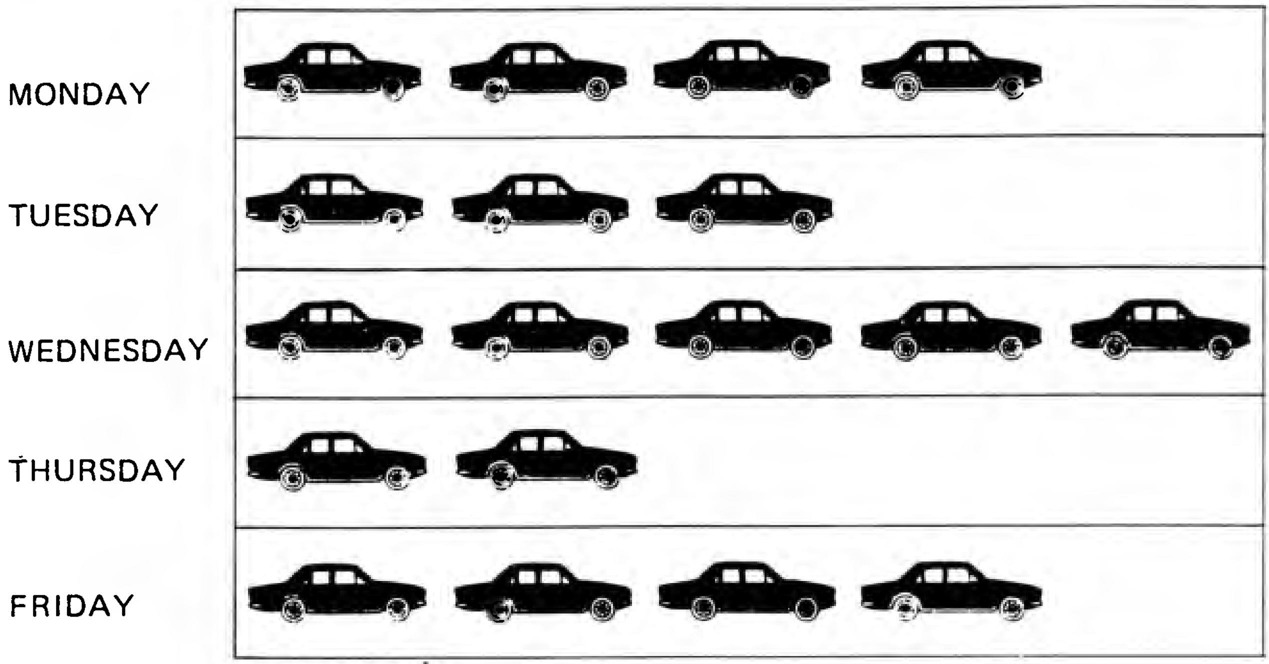
(8)

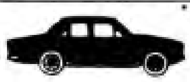
1.



21...............

2-25.



 stands for 5 cars

22. [ ]

22...............

23. [ ]

23...............

24. [ ] and [ ]

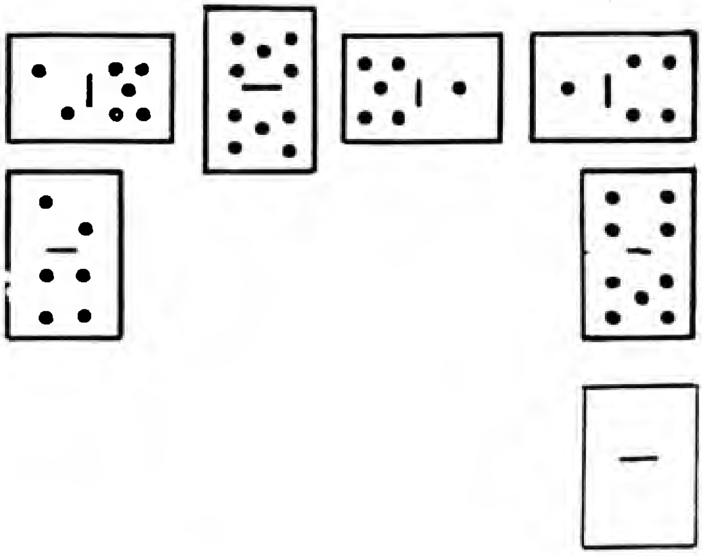
24...............

25. [cars]

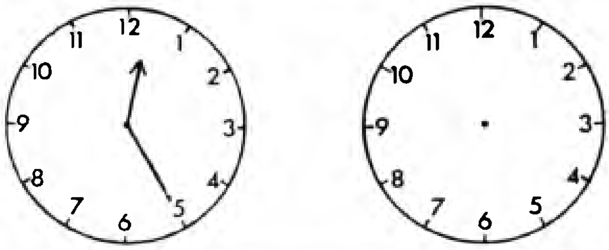
25...............

R	W

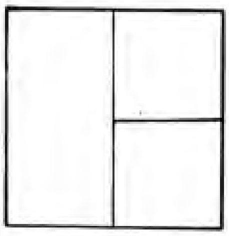
(5)



26...............



27...............

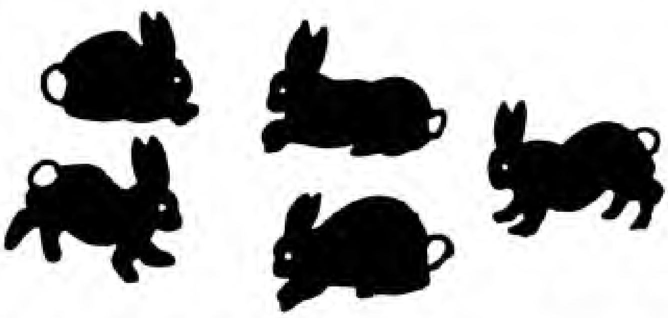


28. [of the cake]

28...............

30.

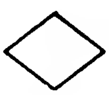
29. [grammes]



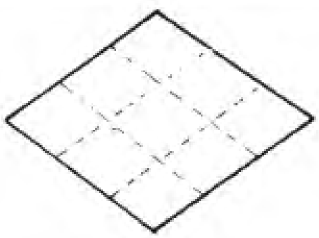
29...............

30. [days]

30...............



A



B

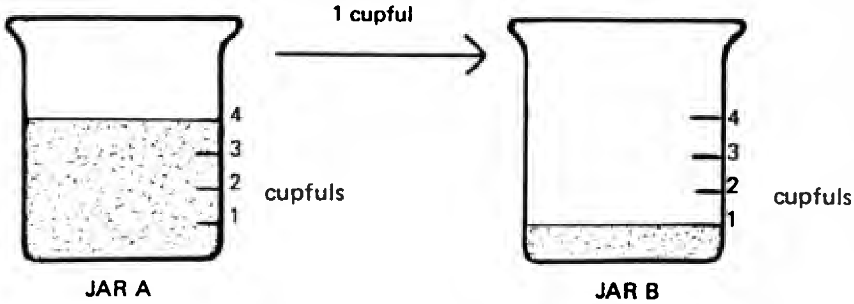
31. [tiles]

31...............

R	W

(6)

33.

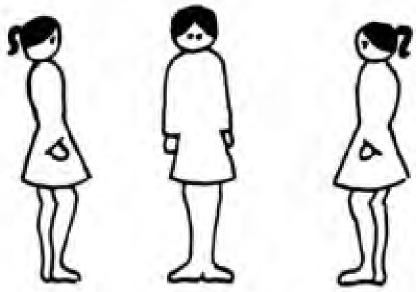


32..........

32. cupfuls

33. cupfuls

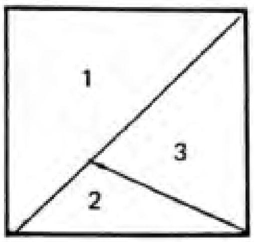
33..........



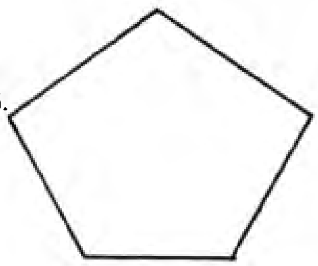
34.

34..........

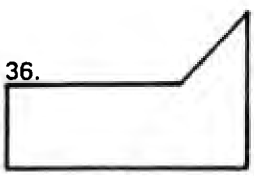
36.



35.

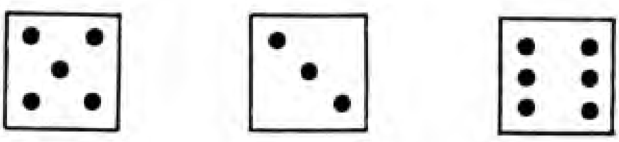


36.



35..........

36..........



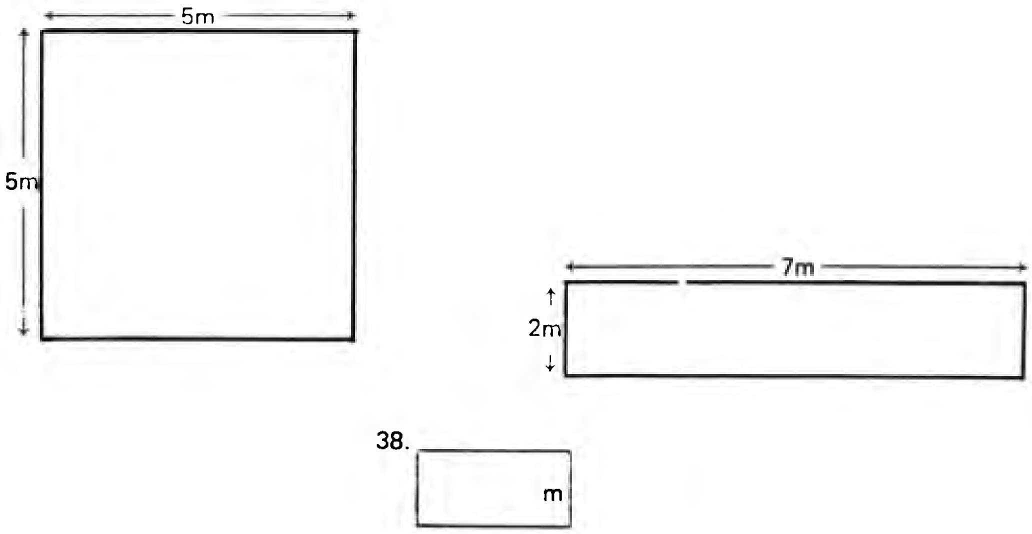
37.

my score

37..........

R	W

(6)



38................

---

*

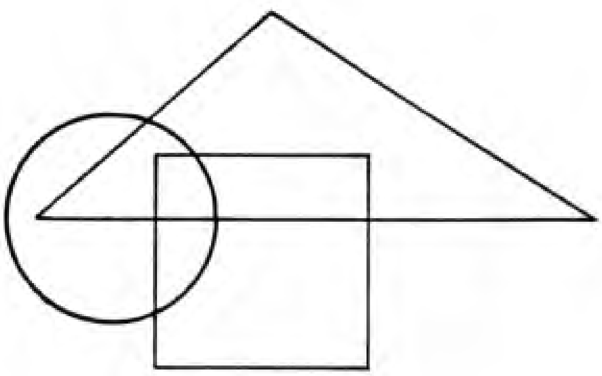
2 3 4 5 6 7 8 9 10

16

39................

---

*



40................

R	W

(3)

Appendix 2.4 Reading Comprehension Test

"Mother decided to do some washing that day. Because she wanted to keep an eye on Johnny, she took him by the hand, let him sit in the small garden nearby, and suggested that he should sit quietly. Little Johnny sat quietly for some time, because he wanted to keep his promise to mummy. He played with the ants and then decided to go to the cow. "Good Morning !" he said, but the cow did not pay any attention to him. He then went to find the little pigs, with whom he played in the mud for some time.

When mother saw him, she took him home, bathed him, and put him to bed. But little Johnny could not sleep; he waited until mother left the room and he got up and went to the window. In seconds, he was out in the garden. After climbing over the fence, he started chasing Max, his dog, who was running towards the lake. They played for a long time, until they got tired and Johnny was hungry. "So, Max" Johnny said, "it's time to go back home." However, Johnny soon discovered that, because it was already dark, he could not find his way back home. He got scared and started crying.

Fortunately, his mother went up to his room to check if he was well-covered, and was shocked when she saw the bed empty. After looking around in the house, she went out into the garden. When she did not find him there, she started running towards the lake in terror. After a careful search, she found Johnny, who was still crying. On the way home, poor Johnny kept on telling her that he would never do that again."

<u>Reading Comprehension Questions (including valid responses)</u>

1. What did mother decide to do that day ?
(*She decided to do the washing*.)

2. What did Johnny promise, when his mother let him sit in the little garden nearby ?
(*He promised to sit quietly*.)

3. What did the cow do ?
(*The cow did not pay any attention to him* or *The cow did not reply*.)

4. Why did mother bath Johnny ?
(*Because he was dirty* or *Because he had mud all over*.)

5. What did Johnny do when mother put him to bed and left ?
(*He got up and went to the window* or *He went to the window*.)

6. How did Johnny get out in the garden ?
(*Through the window*.)

7. Where were Johnny and Max playing ?
(*Near the lake, Next to the lake,* or *In the lake*.)

8. Why did Johnny start crying ?
(*Because he was lost* or *Because he could not find his way back home*.)

9. How did mother understand Johnny was missing ?
(*She went to his room* or *She went to check if he was well-covered*.)

10. What lesson did Johnny learn ?
(*Never to leave the house secretly*.)

Appendix 2.5 Sequence Task

once upon a time there was a very good man who lived with his wife and son

one day he decided to go to the big city

he left for the big city

one day he arrived there

he knocked on the door of a very big house

he asked the landlord to take him as a servant

he worked hard and made a lot of money

after some time he decided to go back home to his family

he thanked the landlord he packed and left

Appendix 2.6 Distribution of Math and Reading Scores as a Function of Gender in the Initial Sample and Final Groups

The sample for the screening tests consisted of 293 children, of whom 128 were boys. Their ages ranged from 7 years 6 months to 9 years 7 months (X = 8 years 2 months, SD = 4 months). The mean ages of the boys and girls were the same.

Arithmetic Tests

On the Young's test, the overall mean raw score was 20.1 and the standard deviation was 6.0. Table A shows the distributions of scores according to gender. The three central intervals divide the sample into three roughly equal groups.

TABLE A

Distribution of Scores on Young's Test According to Gender

	Raw score					
	< 9	**9 - 18**	**19 - 22**	**23 - 31**	**> 31**	**X (SD)**
Boys	7	29	29	62	1	20.9 (6.4)
Girls	2	67	43	53	0	19.4 (5.5)

On the NFER test, the overall mean raw score was 13.5 and the standard deviation was 7.4. Table B shows the distribution of scores according to gender. The three main intervals divide the sample into three roughly equal groups.

TABLE B

Distribution of Scores on NFER Test According to Gender

	Raw score				
	0	**1 - 9**	**10 - 14**	**15 - 36**	**X (SD)**
Boys	1	33	32	62	15.1 (8.0)
Girls	0	63	57	45	12.3 (6.6)

On both maths tests, boys had significantly higher scores: Young's, t (291) = 2.12, $p < .05$, NFER, t (245.06) = 3.23, $p < .001$.

Reading Tests

The raw scores on the reading comprehension test ranged from 0 to 10. The mean score was 5.8 and the standard deviation was 1.85. Table C shows the distributions of scores according to gender. Approximately half the sample scored 6 or more.

TABLE C

Distribution of Scores on Reading Comprehension Test According to Gender

	Raw score					
	0	**1 - 4**	**5**	**6 - 9**	**10**	**X (SD)**
Boys	2	32	21	72	1	5.5 (2.0)
Girls	0	27	32	105	1	6.0 (1.7)

The raw scores on the sentence sequencing task ranged from 0 to 36. The mean score was 28.6 and the standard deviation was 6.8. Table D shows the distribution of scores according to gender. Roughly two thirds of the sample scored 26 or more.

TABLE D

Distribution of Scores on Sentence Sequencing Test According to Gender

	Raw score				
	< 9	**9 - 25**	**26 - 35**	**36**	**X (SD)**
Boys	1	42	63	22	28.3 (7.1)
Girls	1	35	89	30	28.8 (6.5)

The girls scored higher on the reading comprehension test but there were no gender differences on the sentence sequencing task: Reading comprehension, t (252) = 2.08, $p < .05$, Sentence sequencing, t (261) = 0.63, ns.

Formal and Informal Arithmetic

For the Russell and Ginsburg (1984) replication, three pools were formed of children who met the following criteria:
- they scored 5 or more on reading comprehension and 26 or more on sentence sequencing.
- they were in corresponding intervals on the two mathematics tests.

The three pools differed in mathematical ability: a below average pool, an average pool, and an above average pool. The reading test score requirements resulted in even fewer boys in the below average and average pools. Table E shows the gender ratios in these pools and in the final groups.

TABLE E

Gender Ratios in Mathematical Ability Pools and Groups in Replication of Russell and Ginsburg (proportions of boys in parentheses)

		Below average	**Average**	**Above average**
	Young's	9 - 18	19 - 22	23 - 31
	NFER	1 - 9	10 - 14	15 - 36
Pools Boys: Girls		5: 18 (22)	7: 19 (27)	36: 26 (58)
Groups Boys: Girls		3:13 (19)	5: 15 (20)	19: 11 (63)

As Table E shows, girls are overrepresented in the below average and average groups and less common in the above average group. The ratios of boys to girls in the pools and in the actual groups are, however, very similar.

Working Memory

For the replication of Hitch and McAuley (1991), the below average and above average mathematics groups were selected from the corresponding pools. The reading difficulty group was selected from a pool constructed of children who met the following criteria:
- scores between 18 and 30 on the Young's test, and between 10 and 27 on NFER.
- scores between 1 and 5 on reading comprehension and between 9 and 36 on sentence sequencing. Table F shows the gender ratios in these pools and in the final groups.

TABLE F

Gender Ratios in Pools and Groups in Replication of Hitch and McAuley (proportions of boys in parentheses)

	Below average	Above average	**Reading difficulty**
Pools Boys: Girls	5: 18 (22)	36: 26 (58)	26: 14 (65)
Groups Boys: Girls	1: 13 (7)	12: 9 (57)	14: 4 (78)

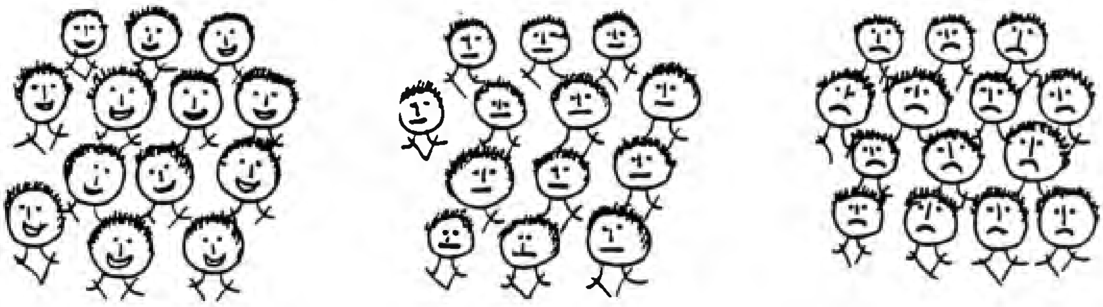
The gender imbalances in the groups match those in the pools and the gender ratios are quite similar given the small samples.

APPENDIX 3

3.1 Children's Questionnaire (Sample)

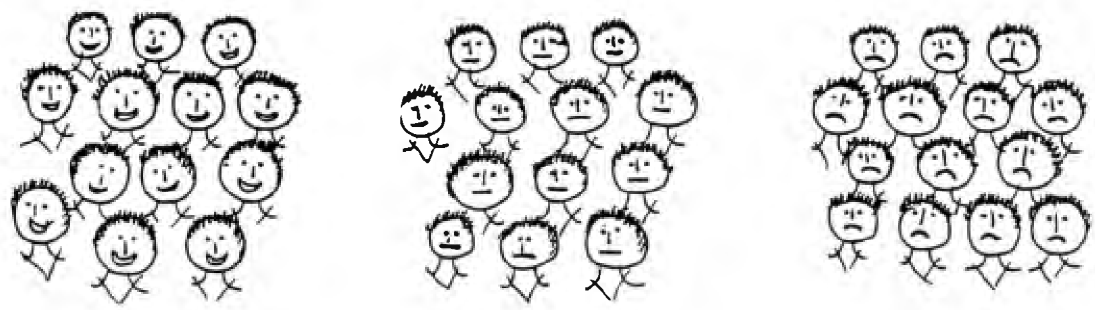
I. EVALUATION OF PERFORMANCE AND ASPIRATIONS

1. These are three groups of children. This group (pointing to the one on the left) is very good in arithmetic. This group (pointing to the middle one) is so-so in arithmetic. This one (pointing to the one on the right) is bad in arithmetic. Which group do you think you belong to ? Why ?



2. Do you want to be better in arithmetic ? Why ?

3. These are three groups of children. This group (pointing to the one on the left) is very good in reading. This group (pointing to the middle one) is so-so in reading. This one (pointing to the one on the right) is bad in reading. Which group do you think you belong to ? Why ?



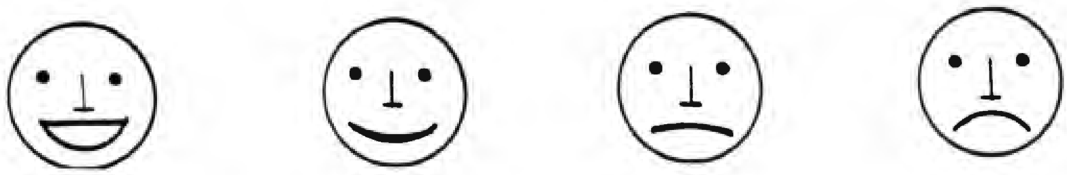
4. Do you want to be better in reading ? Why ?

II. ATTITUDES AND HOME PRACTICES

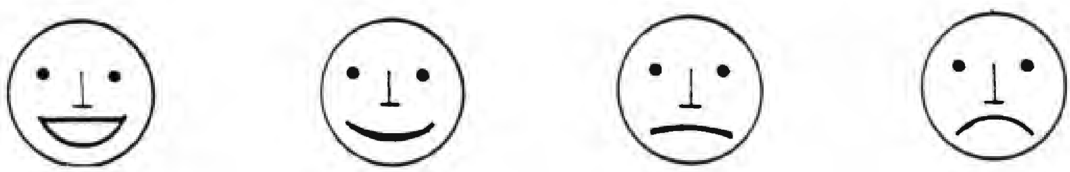
5. Here we have a very happy face, a happy one, a sad one and a very sad one. I want you to tell me which of these faces best shows how you feel about school. Why ?

6. What is your favourite school subject ? Why ?

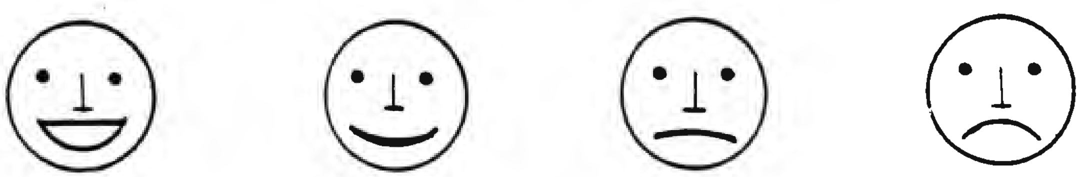
7. Which of these faces shows how you feel about your textbook in arithmetic ? Why ?



8. Which of these faces shows how you feel about doing your homework in arithmetic ? Why ?



9. Now, imagine that one day you don't do arithmetic at school. Which face shows how you feel about missing an arithmetic class ? Why ?



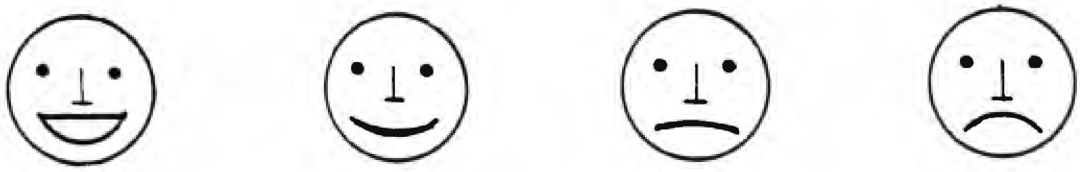
10. What did you like most in arithmetic till now (a topic) ? Why ?

11. What did you like the least or not at all ? Why ?

12. Which face shows how you feel about the (reading) textbook you use in Language ? Why ?

13. Which face shows how you feel about doing your homework in reading ? Why ?

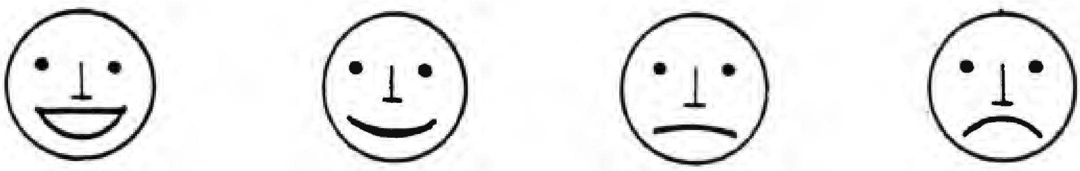
14. Now, imagine a whole day at school without reading at all. Which face shows how you feel about missing reading ? Why ?



15. Which face shows how you feel about reading to yourself at home ? Why ?

16. Which face shows how you feel about reading to your parents ? Why ?

17. Which face shows how you feel about reading to the teacher ? Why ?



18. At home, do you like doing any of the following ?

* ____ sorting out (e.g., putting or storing books together, clothes, etc.)
* ____ dealing with money (e.g., coins, shopping, etc.)
* ____ playing number games (e.g., cards, chess, backgammon, monopoly, etc.)
* ____ cooking (e.g., helping with measuring, weighing, etc.)
* ____ time telling
* ____ counting things (e.g., doing operations, etc.)

19. Do your parents read to you ? Why ?

20. How many books do you have for reading at home ?

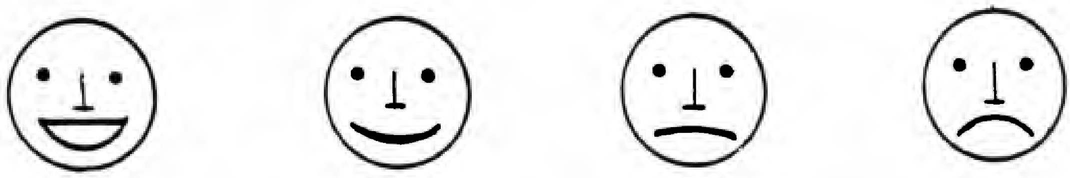
III. CHILDREN'S REPORTS OF PARENTAL HELP

21. At home, when you have to study, does anybody clean your room, prepare your meals, organise your free time, keep everybody quiet, etc. ? Why ?

22. Who helps you with your school homework ?

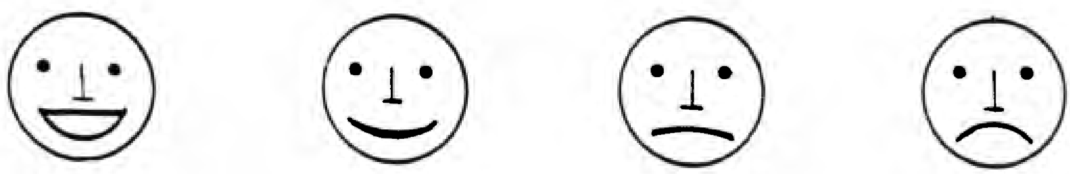
23. Does anybody help you with your homework in arithmetic, by tutoring or coaching, e.g., answering your questions, helping you with difficulties, studying together, etc. ? Why ?

24. Now, look at these faces and tell me which face best shows how you feel about this help. Why ?



25. Does anybody help you with your homework in reading, by tutoring or coaching, e.g., answering your questions, helping you with difficulties, studying together, etc. ? Why ?

26. Which face shows how you feel about this help ? Why ?



Appendix 3.2 Parents' Questionnaire (Sample)

I. EVALUATION OF CHILD'S PERFORMANCE

1. What do you think about your child's general scholastic achievement ?

cause for concern *below average* *average* *above average*

2. What do you think about your child's performance in arithmetic ?

cause for concern *below average* *average* *above average*

3. What do you think about your child's reading performance ?

cause for concern *below average* *average* *above average*

4-5. What do you think accounts for your child's performance in these subjects ? (please discuss each subject separately)

ARITHMETIC		READING
________	fear of failure	________
________	ability	________
________	interest	________
________	motivation	________
________	confidence	________
________	effort	________
________	persistence	________
________	laziness	________
________	dis/obedience	________
________	nervousness	________
________	hyperactivity	________
________	violence	________
________	easily upset	________
________	nature of topic	________
________	peer pressure	________
________	adverse home conditions	________
________	desire to please parents	________
________	parental encouragement	________
________	parental help	________
________	absenteeism	________
________	teaching quality	________
________	Other (please specify)	________

6. Does she find *most* topics in school arithmetic easy to understand ?

yes *no* *don't know*

7. Does she find *most* topics in school reading easy to understand ?

yes *no* *don't know*

8-9. *If* you had any numeracy problems, literacy problems, or both, how did they affect your everyday life ? (please discuss each subject separately)

ARITHMETIC		READING
________	at work	________
________	getting jobs	________
________	household management	________
________	doing courses	________
________	leisure	________
________	no specific context	________
________	other (please specify)	________

10. Is she doing as well as she is capable of in arithmetic ?

yes *no* *don't know*

11. Is she doing as well as she is capable of in reading ?

yes *no* *don't know*

II. ATTITUDES AND HOME PRACTICES

12. Do you think your child likes school arithmetic ?
not likes at all   *not likes*   *quite likes*   *likes very much*   *don't know*

13. Do you think she likes reading at school ?
not likes at all   *not likes*   *quite likes*   *likes very much*   *don't know*

14. What was your favourite school subject at that age ?

15. How important is for you that the child does well in the following school subjects: (please rate them in order of importance, giving 1 to the *most important* and 8 to the *least important*)

* _____ sports
* _____ reading
* _____ music
* _____ art
* _____ arithmetic
* _____ history
* _____ writing
* _____ science

16. At home, has your child shown any involvement in :

* ____ sorting out (e.g., putting or storing books together, clothes, etc.)
* ____ dealing with money (e.g., coins, shopping, etc.)
* ____ playing number games (e.g., cards, chess, backgammon, monopoly, etc.)
* ____ cooking (e.g., helping with measuring, weighing, etc.)
* ____ time telling
* ____ counting things (e.g., doing operations, etc.)

17. Does your child do any reading alone at home ?

18. Do you read to your child ?

III. PARENTAL HELP AND ENCOURAGEMENT

19. How much time (hours per day) do you usually spend with your child ?

0 - 2 *2 - 4* *4 - 6* *6 - 8* *8 - over*

20. Parents often help the child with her homework by organising things, such as preparing their meal, organising their free time, keeping everybody quiet, cleaning up their room, etc. Do you do these things ? Why ? How often ?

21. Do you help your child with his homework in arithmetic by tutoring or coaching, e.g., answering his questions, helping with his difficulties, studying together, etc. ? Why ? How often ?

22. Do you help your child with her homework in reading by tutoring or coaching, e.g., answering her questions, helping with her difficulties, studying together, etc. ? Why ? How often ?

23. How confident do you feel in helping your child with her arithmetic homework ? Please circle the number which best characterises you, 1 meaning *very confident* and 5 meaning *not conffident at all*.

1 2 3 4 5

24. How confident do you feel in helping your child with her homework in reading ? Please circle the number which best characterises you, 1 meaning *very confident* and 5 meaning *not confident at all.*

1 2 3 4 5

25. Do you encourage your child to do well at school ? If yes, how ?

26. Do you encourage your child to do well in school arithmetic ? If yes, how ?

27. Do you encourage your child to do well in reading at school ? If yes, how ?

IV. PARENT-SCHOOL RELATIONS AND PARENT EDUCATION

28. Do you think the arithmetic curriculum is suitable for your child's age ? Why ?

29. Do you have the same beliefs about the reading curriculum ? Why ?

30. How are you informed about the material covered in arithmetic class ?
child's textbook *what child says* *written information from school* *other*

31. How are you informed about the reading material covered in class ?
child's textbook *what child says* *written information from school* *other*

32. Have you discussed your child's progress in arithmetic with the teacher this year ? How often ?

33. Have you discussed your child's progress in reading with the teacher this year ? How often ?

34. How helpful is the teacher with your child's difficulties in arithmetic ?

helps a lot *helps* *not helps at all* *don't know*

35. How helpful is the teacher with your child's difficulties in reading ?

helps a lot *helps* *not helps at all* *don't know*

36. What is your academic background ?

primary *high school* *lykion* *technical* *university* *other*

Appendix 3.3 Children's Perceived and Actual Performance in Reading as a Function of Their Actual Reading Performance on the Sequence Task

	ACTUAL PERFORMANCE Above Average (n = 35)	**Average** (**n = 38**)
PERCEIVED PERFORMANCE		
Above Average	29	27
Average	6	11
Below Average	0	0

Appendix 3.4 Motivational Factors Associated With Children's Aspirations to Be Better in Arithmetic

	Above Average (**n = 34**)	Average (n = 20)	Below Average (n = 17)
Internal			
Liking	7	5	3
Competitiveness	2	1	0
Good Student	4	0	2
Knowledge	3	3	5
Total Internal[a]	15	9	9
External			
Future job/Adult role	8	3	1
Better Grades/Pass	2	3	4
Please Others	4	1	1
Answer Others	1	3	1
Usefulness	1	1	1
Praise	1	0	1
Total External[a]	**15**	**11**	**8**

[a]Number of children wanting to be better in arithmetic for internal or external reasons.

Appendix 3.5 Motivational Factors Associated With Children's Aspirations to Be Better in Reading (Reading Comprehension)

	Above Average (n = 33)	Average (n = 40)
Internal		
Read Better	7	11
Liking	4	5
Competitiveness	1	2
Good Student	3	4
Knowledge	1	3
Identification	0	2
Total Internal[a]	**16**	**27**
External		
Usefulness	2	0
Praise	0	2
Please Others	1	2
Grade	3	2
Future	3	4
Criticism	1	1
Total External[a]	**10**	**11**

[a]Number of children wanting to be better in reading for internal or external reasons.

Appendix 3.6 Motivational Factors Associated With Children's Aspirations to Be Better in Reading (Sequence Task)

	Above Average (n = 35)	Average (n = 38)
Internal		
Read Better	12	6
Liking	4	5
Competitiveness	1	2
Good Student	3	4
Knowledge	2	2
Identification	1	1
Total Internal[a]	**23**	**20**
External		
Usefulness	0	2
Praise	1	1
Please Others	0	3
Grade	2	3
Future	4	3
Criticism	0	2
Total External[a]	**7**	**14**

[a]Number of children wanting to be better in reading for internal or external reasons.

Appendix 3.7 Frequencies of Parents as a Function of Problem Area and Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Arithmetic						
Problems[a]	**3**	**1**	**2**	**2**	**1**	**0**
At Work	2	1	2	1	0	0
Find a Job	2	1	1	1	0	0
Doing Courses	2	1	1	0	1	0
Household Management	1	0	1	1	0	0
In Leisure	1	0	1	1	0	0
Reading						
Problems[a]	**4**	**2**	**1**	**2**	**0**	**0**
At Work	1	1	1	2	0	0
Find a Job	2	1	1	0	0	0
Doing Courses	3	1	1	1	0	0
Household Management	1	0	1	0	0	0
In Leisure	2	0	1	0	0	0
Spelling	1	1	0	0	0	0

[a]Number of parents reporting having faced numeracy and literacy problems respectively.

Appendix 3.8 Parental Beliefs About the Relation Between Children's Performance and Ability in Reading (Sequence Task)

	FATHER		MOTHER	
	AA	A	AA	A
Answered	**27**	**28**	**35**	**29**
Can Do Better	6	5	5	10
Cannot Do Better	20	21	28	18
Don't Know	1	2	2	1

Appendix 3.9 Children's Reasons for Liking School Very Much as a Function of Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Liked Very Much	**25**	**14**	**10**
Academic Profits	22	11	9
Social Interaction	10	6	2
Contrast With Home	2	2	1
Teacher	2	2	0
Nice Environment	1	0	0
Parents Being Teachers	1	0	0

Appendix 3.10 Children's Reasons[a] for Liking School as a Function of Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Liked	**10**	**6**	**7**
Academic Issues	6	3	4
Social Interaction	4	3	3
Contrast With Home	1	0	2
Teacher	0	2	0
Early Wake Up	1	1	0
Not Enjoy Breaks	1	0	0

[a]Total of positive and negative reasons.

Appendix 3.11 Children's Favourite School Subject as a Function of Gender and Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Arithmetic	**18**	**6**	**1**
Girls	*6*	*5*	*1*
Boys	*12*	*1*	*0*
Language	**1**	**4**	**6**
Girls	*0*	*3*	*6*
Boys	*1*	*1*	*0*
History	**8**	**3**	**3**
Girls	*2*	*1*	*1*
Boys	*6*	*2*	*2*
Religious Education	**6**	**5**	**2**
Girls	*3*	*5*	*2*
Boys	*3*	*0*	*0*
Composition	**1**	**0**	**0**
Girls	*1*	*0*	*0*
Boys	*0*	*0*	*0*
Gym	**1**	**0**	**2**
Girls	*1*	*0*	*1*
Boys	*0*	*0*	*1*
The World & Us	**1**	**1**	**1**
Girls	*0*	*0*	*1*
Boys	*1*	*1*	*0*
All	**0**	**1**	**2**
Girls	*0*	*1*	*1*
Boys	*0*	*0*	*1*

***Subject Description*:**

Arithmetic: solution of oral and written operations and word problems

Language: reading, grammar, syntax, dictation

History: textbook, read lesson in class, learn it at home, to be examined by teacher

Religious Education: textbook, read lesson in class, learn it at home, to be examined by teacher

Composition: writing short essays on a topic specified by the teacher, usually in class

Gym: athletic exercises

The World and Us: textbook, read lesson in class, learn it at home, to be examined by teacher

Appendix 3.12 Children's Attitudes Towards Arithmetic as a Function of Gender, Individual Measures, and Mathematical Group

	Answered	Like Very Much	Like	Not like Much	Not like At All
ARITHMETIC TEXTBOOK					
Above Average	36	29	7	0	0
Girls	13	10	3	0	0
Boys	23	19	4	0	0
Average	20	14	6	0	0
Girls	15	12	3	0	0
Boys	5	2	3	0	0
Below Average	17	8	5	3	0
Girls	14	8	3	2	1
Boys	3	0	2	1	0
ARITHMETIC HOMEWORK					
Above Average	36	26	9	1	0
Girls	13	10	2	1	0
Boys	23	16	7	0	0
Average	20	13	5	1	1
Girls	15	10	5	0	0
Boys	5	3	0	1	1
Below Average	17	10	5	2	0
Girls	14	9	3	2	0
Boys	3	1	2	0	0
MISS ARITHMETIC CLASS[a]					
Above Average	36	0	6	14	16
Girls	13	0	3	4	6
Boys	23	0	3	10	10
Average	20	0	3	11	6
Girls	15	0	0	9	6
Boys	5	0	3	2	0
Below Average	17	4	4	3	6
Girls	14	3	3	2	6
Boys	3	1	1	1	0

[a]The exact phrasing was *Very Happy, Happy, Sad,* and *Very Sad* respectively.

Appendix 3.13 Proportion of Children as a Function of Most and Least Favourite Topic in Arithmetic and Mathematical Group

	Above Average (n = 36)		Average (n = 20)		Below Average (n = 17)	
	Best	Worst	Best	Worst	Best	Worst
Addition	.31	.03	.25	.10	.47	.06
Subtraction	-	.22	.10	.30	-	.29
Multiplication	.17	.14	.20	.10	.18	.06
Division	.28	.14	.20	.20	.06	.24
Operations	-	-	-	-	.06	-
Problems	.08	-	.10	-	.06	.12
Tables	.06	.03	.05	-	.06	-
Weight	-	.03	-	-	-	.06
Time	-	.03	-	-	-	-
Metre	-	.03	.05	-	-	-
Money	.03	-	-	-	-	-
Length	-	-	-	-	.06	-
Units & Hundreds	-	-	-	-	.06	-
"Epalithefsi"[a]	.03	-	-	-	-	-
Liked All	.06	.36	.05	.30	-	.18

[a]Re-doing an operation using a different method, as a way of checking whether the initial outcome was accurate

Appendix 3.14 Children's Attitudes Towards Reading as a Function of Gender, Individual Measures, and Mathematical Group

	Answered	Like Very Much	Like	Not like Much	Not like At All
TEXTBOOK in LANGUAGE					
Above Average	36	22	12	1	1
Girls	*13*	*10*	*2*	*0*	*1*
Boys	*23*	*12*	*10*	*1*	*0*
Average	20	11	9	0	0
Girls	*15*	*9*	*6*	*0*	*0*
Boys	*5*	*2*	*3*	*0*	*0*
Below Average	17	12	5	0	0
Girls	*14*	*11*	*3*	*0*	*0*
Boys	*3*	*1*	*2*	*0*	*0*
READING HOMEWORK					
Above Average	36	24	10	1	1
Girls	*13*	*9*	*3*	*1*	*0*
Boys	*23*	*15*	*7*	*0*	*1*
Average	20	13	6	0	1
Girls	*15*	*12*	*3*	*0*	*0*
Boys	*5*	*1*	*3*	*0*	*1*
Below Average	17	11	4	1	1
Girls	*14*	*10*	*2*	*1*	*1*
Boys	*3*	*1*	*2*	*0*	*0*
MISS READING CLASS[a]					
Above Average	36	1	5	14	16
Girls	*13*	*0*	*3*	*4*	*6*
Boys	*23*	*1*	*2*	*10*	*10*
Average	20	2	1	10	7
Girls	*15*	*0*	*0*	*8*	*7*
Boys	*5*	*2*	*1*	*2*	*0*
Below Average	17	4	3	2	8
Girls	*14*	*2*	*3*	*2*	*7*
Boys	*3*	*2*	*0*	*0*	*1*

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	Answered	Like Very Much	Like	Not like Much	Not like At All
READING ALONE					
Above Average	36	23	13	0	0
Girls	13	8	5	0	0
Boys	23	15	8	0	0
Average	20	10	10	0	0
Girls	15	7	8	0	0
Boys	5	3	2	0	0
Below Average	17	10	5	2	0
Girls	14	9	4	1	0
Boys	3	1	1	1	0
READING TO PARENTS					
Above Average	24	16	8	0	0
Girls	7	4	3	0	0
Boys	17	12	5	0	0
Average	14	8	6	0	0
Girls	10	5	5	0	0
Boys	4	3	1	0	0
Below Average	9	6	3	0	0
Girls	7	4	3	0	0
Boys	2	2	0	0	0
READING TO TEACHER					
Above Average	36	26	8	2	0
Girls	13	10	2	1	0
Boys	23	16	6	1	0
Average	20	14	6	0	0
Girls	15	11	4	0	0
Boys	5	3	2	0	0
Below Average	17	13	4	0	0
Girls	14	10	4	0	0
Boys	3	3	0	0	0

[a]The exact phrasing was *Very Happy, Happy, Sad,* and *Very Sad* respectively.

Appendix 3.15 Frequencies of Children Engaging in Numerical Activities at Home as a Function of Gender and Mathematical Group

	Above Average (n = 36)	Average (n = 20)	Below Average (n = 17)
Grouping	**31**	**18**	**15**
Girls	*13*	*14*	*12*
Boys	*18*	*4*	*3*
Money	**23**	**12**	**11**
Girls	*10*	*9*	*10*
Boys	*13*	*3*	*1*
Number Games	**33**	**19**	**12**
Girls	*12*	*15*	*10*
Boys	*21*	*4*	*2*
Cooking	**1**	**0**	**0**
Girls	*0*	*0*	*0*
Boys	*1*	*0*	*0*
Time Telling	**34**	**15**	**5**
Girls	*11*	*11*	*5*
Boys	*23*	*4*	*0*
Counting	**24**	**15**	**13**
Girls	*10*	*10*	*10*
Boys	*14*	*5*	*3*

Appendix 3.16 Distribution of Parents as a Function of Reason for Reading to Children at Home (Children's Reports) and Children's Mathematical Group

	Above Average	Average	Below Average
Number of children whose parents read to	13	8	8
To improve my reading	3	1	0
To please me	3	0	3
To pass our time	2	3	0
To sleep/bedtime	2	2	2
Not to watch T.V.	0	0	1
To learn things	2	1	1
Can't read alone	1	1	0
Parents interested in the story	0	0	1

Appendix 3.17 Frequencies of Children as a Function of Agent of Help and Reading Achievement (Reading Comprehension)

	No Help (Alone)	Help from Mother	Help from Father	Help from Other Member
Indirect Help				
Above Average	15	17	2	3
Average	11	21	3	5
General Help				
Above Average	3	29	16	2
Average	2	31	10	0
Help with Reading				
Above Average	21	9	1	1
Average	20	12	0	3

Appendix 3.18 Frequencies of Children as a Function of Agent of Help and Reading Achievement (Sequence Task)

	No Help (Alone)	Help from Mother	Help from Father	Help from Other Member
Indirect Help				
Above Average	0	30	3	7
Average	26	8	2	1
General Help				
Above Average	0	46	21	2
Average	5	14	5	0
Help with Reading				
Above Average	0	14	1	2
Average	41	7	0	2

Appendix 3.19 Frequencies of Parents as a Function of Reasons for Helping Children Indirectly With Homework and Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Help	**12**	**7**	**8**	**33**	**15**	**12**
Study better	1	0	0	4	5	1
Save time	2	0	1	4	4	4
Young: can't read alone	0	0	0	6	1	1
Obligation	1	0	1	3	2	4
Other	1	2	2	5	1	2

Appendix 3.20 Frequencies of Parents as a Function of Help and Children's Reading Achievement (Reading Comprehension)

	FATHER		MOTHER	
	AA	A	AA	A
Indirect Help				
Answered	**22**	**22**	**30**	**32**
Help	**16**	**11**	**29**	**31**
Often	7	4	19	28
Not Often	3	4	0	1
Not Specified	6	3	10	2
Help with Reading				
Answered	**23**	**26**	**31**	**31**
Help	**12**	**18**	**29**	**30**
Often	3	2	10	15
Not Often	3	10	8	6
Not Specified	6	6	11	9

Appendix 3.21 Frequencies of Parents as a Function of Help and Children's Reading Achievement (Sequence Task)

	FATHER		MOTHER	
	AA	A	AA	A
Indirect Help				
Answered	**19**	**25**	**34**	**28**
Help	**9**	**18**	**33**	**27**
Often	4	7	27	20
Not Often	1	6	0	1
Not Specified	4	5	6	6
Help with Reading				
Answered	**23**	**26**	**34**	**28**
Help	**18**	**12**	**32**	**27**
Often	3	2	15	10
Not Often	8	5	6	8
Not Specified	7	5	11	9

Appendix 3.22 Parental Levels of Confidence With Helping With Children's Homework in Reading as a Function of Children's Reading Achievement (Reading Comprehension)

	FATHER		MOTHER	
	AA	A	AA	A
Helped	**12**	**18**	**29**	**30**
Not Mention	**0**	**1**	**0**	**1**
Mean level	**3.5**	**4.0**	**4.2**	**4.2**
Not Confident At All	1	1	2	1
A Little Confident	2	2	1	1
Moderately Confident	3	3	2	5
Quite Confident	2	1	7	6
Very Confident	4	10	17	16

Appendix 3.23 Parental Levels of Confidence With Helping With Children's Homework in Reading as a Function of Children's Reading Achievement (Sequence Task)

	FATHER		MOTHER	
	AA	A	AA	A
Helped	**18**	**12**	**32**	**27**
Not Mention	**0**	**1**	**0**	**1**
Mean level	**4.4**	**2.7**	**4.5**	**3.9**
Not Confident At All	0	2	1	2
A Little Confident	1	3	0	2
Moderately Confident	3	3	3	4
Quite Confident	1	2	7	6
Very Confident	13	1	21	12

Appendix 3.24 Frequencies of Parents as a Function of Method of Encouraging Children and Children's Reading Achievement (Reading Comprehension)

	FATHER		MOTHER	
	AA	A	AA	A
School				
Answered	**25**	**27**	**31**	**31**
Encourage	**24**	**27**	**31**	**30**
Motivational Support	17	21	28	23
Tuition / Help	1	0	2	2
Unspecified	6	6	2	6
Reading				
Answered	**23**	**23**	**30**	**31**
Encourage	**21**	**21**	**23**	**29**
Motivational Support	13	12	14	18
Tuition / Help	3	1	2	6
Unspecified	4	8	5?	5?

Appendix 3.25 Frequencies of Parents as a Function of Method of Encouraging Children and Children's Reading Achievement (Sequence Task)

	FATHER		MOTHER	
	AA	A	AA	A
School				
Answered	**26**	**26**	**34**	**28**
Encourage	**26**	**25**	**33**	**28**
Motivational Support	17	21	28	23
Tuition / Help	1	0	1	2
Unspecified	8	4	5	3
Reading				
Answered	**23**	**23**	**34**	**27**
Encourage	**20**	**22**	**30**	**22**
Motivational Support	11	14	16	16
Tuition / Help	1	3	7	1
Unspecified	8	4	7	5

Appendix 3.26 Frequencies of Reasons for Parents' Beliefs About the Unsuitability of the Curriculum as a Function of Children's Mathematical Group

	FATHER			MOTHER		
	AA	A	BA	AA	A	BA
Arithmetic						
Unsuitable[a]	**3**	**0**	**3**	**0**	**3**	**4**
Content/Nature	2	0	2	0	2	2
Instructional Method	0	0	1	0	1	2
Not Specified	1	0	0	0	0	0
Reading						
Unsuitable[a]	**3**	**1**	**1**	**4**	**2**	**1**
Content/Nature	1	0	1	1	1	1
Instructional Method	1	0	0	2	1	0
Not Specified	1	1	0	1	0	0

[a]Number of parents reporting the curriculum in arithmetic and reading respectively is not suitable for the child's age.

Appendix 3.27 Correlations Between Social and Environmental Variables and Children's Arithmetic Achievement

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Performance, Attrbutions, Aspirations, Easiness, and Relation Between Performance and Ability						
Children's self-concepts	gself	1= Above average 2=Average 3=Below average	No	73	-.2009*	-.1931*
Fathers' perceived performance	gfa	1=Above average 2=Average 3=Below average 4= Cause for concern	Yes	53	-.5252	-.4808
Mothers' perceived performance	gmo	1=Above average 2=Average 3=Below average 4=Cause for concern	Yes	64	-.4757	-.4096
Fathers' perceived school performance	fgrsch	1=Above average 2=Average 3=Below average 4=Cause for concern	Yes	55	-.3953	-.4367
Mothers' perceived school performance	mgrsch	1=Above average 2=Average 3=Below average 4=Cause for concern	Yes	64	-.3419	-.3478

* nonsignificant

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Children's attributions	cmattr1	1=Internal or external 2=Both internal and external	-	73	-.1622*	-.1351*
	cmattrin	1=No (Internal) 2=Yes (Internal)	-	73	.0090*	-.1626*
	cmattrex	1=No (External) 2=Yes (External)	-	73	-.0465*	.0651*
Fathers' attributions	fattrm1	1=Internal or external 2=Both internal and external	-	47	-.1531*	-.0827*
	fattrmi	1=No (Internal) 2=Yes (Internal)	-	47	.1870*	.2316*
	fattrme	1=No (External) 2=Yes (External)	-	47	-.2670*	-.2200*
Mothers' attributions	mattrm1	1=Internal or external 2=Both internal and external	-	59	-.2951	-.2340*
	mattrmi	1=No (Internal) 2=Yes (Internal)	-	59	-.1479*	-.2136*
	mattrme	1=No (External) 2=Yes (External)	-	59	-.2577	-.1763*

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Children's aspirations	cbetter1	1=No 2=Yes	No	72	-.1691*	-.2456
Children's reasons for wanting to be better	cmbetter	1=Internal or external 2=Both internal and external	No	60	.0193*	-.0647*
	cmbetint	1=No (Internal) 2=Yes (Internal)	-	71	-.0135*	-.1083*
	cmbetext	1=No (External) 2=Yes (External)	-	71	-.1011*	-.0799*
(Children's reasons for not wanting to be better: ability, n = 2)						
Fathers' beliefs about child's ability	fexpm1	1=Child can do better 2=Child is doing his best	No	50	.2327*	.2857
Mothers' beliefs about child's ability	mexpm1	1=Child can do better 2=Child is doing his best	Yes	59	.5364	.4811
Fathers' beliefs about child's ease	feasym1	1=No 2=Yes	No	52	.3376	.2757
Mothers' beliefs about child's ease	measym1	1=No 2=Yes	Yes	62	.5000	.3540

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Children's Attitudes and Home Activities						
Children's attitudes to school	cattsch	1=Like very much 2=Like 3=Not like much 4=Not like at all	No	73	-.0672*	-.0910*
Children's favourite school subject (not including "All"; only those with preference)	cfavsubj	1=Arithmetic 2=Other	Yes	70	-.1697*	-.1483*
Children's attitudes to arithmetic (summarised: txbk, hw, miss class)	clikem3	1=Like very much 2=Like 3=Not like much 4=Not like at all	Yes (textbook) No (homework) (miss class)	73	-.3454	-.2955
Fathers' beliefs about child's attitudes	flikem3	1=Likes very much 2=Likes 3=Not likes much 4= Not likes at all	Yes	51	-.5089	-.4652
Mothers' beliefs about child's attitudes	mlikem3	1=Likes very much 2=Likes 3=Not likes much 4=Not likes at all	Yes	62	-.5935	-.5141

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Children's numeric activities at home (summarised 6 activities)	chomeact	interval: score 1-6	Yes (time) (games)	73	.2419	.2161*
Fathers' reports of child's activities (summarised 6 activities)	fhomeact	interval: score 1-6	Yes (games)	51	.0888*	.0518*
Mothers' reports of child's activities (summarised 6 activities)	mhomeact	interval: score 1-6	No	62	.1952*	.3293
Parental Involvement						
Fathers' time spent with child	fhours	1=0-2hrs/day 2=2-4hrs/day 3=4-6hrs/day 4=6-8hrs/day 5=8+ hrs/day	No	55	-.0777*	.0050*
Mothers' time spent with child	mhours	1=0-2hrs/day 2=2-4hrs/day 3=4-6hrs/day 4=6-8hrs/day 5=8+ hrs/day	No	64	.0258*	.1478*
Child's min time spent with parents	chrsmin	interval	No	65	.0599*	.1500*
Child's max time spent with parents	chrsmax	interval	No	64	.1276*	.2125*

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Indirect Help						
Children's reports of parents' help	cindhprs	0=none of parents 1=one parent 2=both parents	No	73	-.2792	-.2672
Children's reports of fathers' help	cindhelf	1=No 2=Yes	No	73	-.1399*	-.0832*
Children's reports of mothers' help	cindhelm	1=No 2=Yes	No	73	-.2607	-.2751
Parents' reports of indirect help	prsheind	0=none of parents reported helping 1=one parent reported helping 2=both parents reported helping	Yes	42	-.3171	-.2025*
Fathers' reports of help	fhelpind	1=No 2=Yes	Yes	44	-.3367	-.2216*
Mothers' reports of help	mhelpind	1=No 2=Yes	No	62	.0559*	.0497*
Help with Homework in General						
Children's reports of general help	chwheprs	0=none of parents 1=one parent 2=both parents	No	73	.0842*	.0946*

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Direct Help with Arithmetic Homework						
Children's reports of parents' help (0=Alone n=8 and Other n=5)	chelpprs	0=none of parents 1=one parent 2=both parents	No	73	-.2365	-.0985*
Children's reports of fathers' help	chelpmf	1=No 2=Yes	No	73	-.1196*	.0324*
Children's reports of mothers' help	chelpmm	1=No 2=Yes	No	73	-.1555*	-.1450*
Parents' reports of help	prshelpm	0=none of parents reported helping 1=one parent reported helping 2=both parents reported helping	No	53	-.2598*	-.1550*
Fathers' reports of help	fhelpm	1=No 2=Yes	No	54	-.1641*	-.0986*
Mothers' reports of help	mhelpm	1=No 2=Yes	No	64	-.1935*	-.0969*
Children's levels of satisfaction (Not Alone or Other, only parental)	csatm	1=Very satisfied 2=Satisfied 3=Not very satisfied 4=Not satisfied at all	No	58	.1069*	.1220*

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Fathers' level of confidence (those who help)	fconfim	1=Not at all confident 2=A little confident 3=Average 4=Quite confident 5=Very confident	No	38	.3147*	.3116*
Mothers' level of confidence (those who help)	mconfim	1=Not at all confident 2=A little confident 3=Average 4=Quite confident 5=Very confident	No	59	.1572*	.1314*
Parental Encouragement						
Fathers' encouragement to do well at school	fencs	1=No 2=Yes	No	52	-.1572*	-.0766*
Mothers' encouragement to do well at school	mencs	1=No 2=Yes	No	62	-.1223*	-.1632*
Fathers' way of encouraging	fencsw	1=Motivational support 2=Tuition/help	No	39	-.0718*	-.1127*
Mothers' way of encouraging: motivational support	mencsw1	1=No 2=Yes	No	53	-.0914*	-.0392*
Mothers' way of encouraging: tuition / help	mencsw2	1=No 2=Yes	No	53	-.1348*	-.2024*

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Fathers' encouragement to do well in arithmetic	fencm	1=No 2=Yes	No	47	-.1654*	-.0857*
Mothers' encouragement in math to do well in arithmetic	mencm	1=No 2=Yes	No	61	-.0720*	-.1084*
Fathers' way of encouraging: motivational support	fencmw1	1=No 2=Yes	No	33	-.0453*	-.1390*
Mothers' way of encouraging: motivational support	mencmw1	1=No 2=Yes	No	41	.1535*	.1231*
Fathers' way of encouraging: tuition / help	fencmw2	1=No 2=Yes	No	33	-.0334*	-.0037*
Mothers' way of encouraging: tuition / help	mencmw2	1=No 2=Yes	No	41	-.2030*	-.1758*
Fathers' beliefs of academic importance of arithmetic	feducthm	1=Math not among first 3 2=Math among first 3	No	52	.0591*	.0900*
Mothers' beliefs of academic importance of arithmetic	meducthm	1=Math not among first 3 2=Math among first 3	No	61	-.1235*	-.3063

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Parental Contact with School						
Fathers' contact with teacher	fcontm	1=No 2=Yes	No	47	.0000*	.0187*
Mothers' contact with teacher	mcontm	1=No 2=Yes	No	63	-.0291*	-.1376*
Fathers' evaluation of teacher's help	fthelpm2	1=Helps a lot 2=Helps 3=Not helps at all	No	42	-.1338*	-.1709*
Mothers' evaluation of teacher's help	mthelpm2	1=Helps a lot 2=Helps 3=Not helps at all	Yes	57	-.2134*	-.2038*
Parental Curriculum Opinions						
Fathers' evaluation of the curriculum	fcuropm1	1=Not suitable 2=Suitable	No	46	.1672*	.1106*
Mothers' evaluation of the curriculum	mcuropm1	1=Not suitable 2=Suitable	Yes	57	.3584	.3239
Fathers being informed by textbook	finfmbk	1=No 2=Yes	No	51	-.0467*	.1071*
Mothers being informed by textbook	minfmbk	1=No 2=Yes	No	62	.0342*	.0621*

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Fathers being informed by child rep.	finfmrep	1=No 2=Yes	No	51	-.0678*	-.0636*
Mothers being informed by child rep.	minfmrep	1=No 2=Yes	No	62	-.0533*	-.0416*
Fathers being informed by school	finfmsch	1=No 2=Yes	No	51	.0328*	.1137*
Mothers being informed by school	minfmsch	1=No 2=Yes	No	62	.1566*	.2352*
Parental Attitudes						
Fathers' favourite school subject	ffavm	1=Arithmetic 2=Other	No	48	-.2311*	-.0895*
Mothers' favourite school subject	mfavm	1=Arithmetic 2=Other	No	62	.1823*	.1471*
Fathers' numeracy problems	fprom	1=No 2=Yes	No	38	-.0222*	-.0456*
Mothers' numeracy problems	mprom	1=No 2=Yes	No	44	.0034*	-.0802*

* ns

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Parental Academic Status						
Fathers' academic level	facad1	1=Primary Only 2=High School Compulsory 3=Lykion 4=Graduate	No	55	.0730*	.1446*
Mothers' academic level	macad1	1=Primary Only 2=High School Compulsory 3=Lykion 4=Graduate	No	63	.3282	.2288*

* ns

Appendix 3.28 Pearson's Correlation Matrix of Social and Environmental Variables Associated With Children's Arithmetic Achievement

Variables	1	2	3	4	5	6	7	8	9	10	11
1. Fathers' perceptions (math)	1.000 (53) p=.	.5016 (52) p=.000	.7140 (53) p=.000	.7060 (52) p=.000	.1306 (49) p=.371	.1114 (49) p=.446	-.3734 (50) p=.008	-.5250 (50) p=.000	-.5657 (48) p=.000	.4929 (48) p=.000	.760 (53) p=.588
2. Mothers' perceptions (math)		1.000 (64) p=.	.3605 (54) p=007	.4576 (64) p=.000	.0514 (59) p=.699	.0291 (59) p=.827	-.2054 (51) p=.148	-.4455 (62) p=.000	-.3362 (49) p=.018	-.5342 (59) p=.000	.0819 (64) p=.520
3. Fathers' perceptions (school)			1.000 (55) p=.	.5741 (54) p=.000	.0476 (50) p=.743	.0194 (50) p=.893	-.2616 (52) p=.061	-.2636 (52) p=.059	-.4927 (50) p=.000	-.3273 (50) p=.020	.0989 (55) p=.473
4. Mothers' perceptions (school)				1.000 (64) p=.	-.0744 (59) p=.575	-.0957 (59) p=.471	-.1318 (51) p=.356	-.1822 (62) p=.156	-.2119 (49) p=.144	-.3955 (59) p=.002	.0680 (64) p=.593
5. Mothers' attributions (int/ext vs both)					1.000 (59) p=.	.9588 (59) p=.000	-.2299 (48) p=.116	-.1148 (57) p=.395	-.2600 (46) p=.081	-.2102 (55) p=.123	.2064 (59) p=.117
6. Mothers' attributions (ext vs no)						1.000 (59) p=.	-.2188 (48) p=.135	-.1022 (57) p=.449	-.2387 (46) p=.110	-.2102 (55) p=.123	.2153 (59) p=.102
7. Fathers' beliefs of ease							1.000 (52) p=.	.6119 (49) p=.000	.1809 (50) p=.209	.3336 (47) p=.022	-.0506 (52) p=.722
8. Mothers' beliefs of ease								1.000 (62) p=.	.0932 (47) p=.533	.3403 (57) p=.010	-.0457 (62) p=.724
9. Fathers' beliefs of ability									1.000 (50) p=.	.3510 (45) p=.018	-.0891 (50) p=.538
10. Mothers' beliefs of ability										1.000 (59) p=.	. (59) p=.
11. Children's aspirations											1.000 (72) p=.

Variables	12	13	14	15	16	17	18	19	20	21	22	23	24
1. Fathers' perceptions (math)	.4940 (53) p=.000	.6401 (49) p=.000	.6089 (50) p=.000	-.2679 (53) p=.052	-.2091 (51) p=.141	-.0272 (53) p=.847	.0103 (53) p=.941	.1775 (42) p=.261	.1547 (44) p=.316	.2688 (53) p=.052	.0727 (50) p=.616	-.5515 (46) p=.000	-.0964 (51) p=.501
2. Mothers' perceptions (math)	.1489 (64) p=.240	.6334 (50) p=.000	.7065 (62) p=.000	-.2411 (64) p=.055	-.1167 (62) p=.366	.2102 (64) p=.096	.1306 (64) p=.304	.0694 (42) p=.662	.1960 (43) p=.208	.3761 (64) p=.002	.0835 (61) p=.522	-.3403 (57) p=.010	.0561 (63) p=.663
3. Fathers' perceptions (school)	.3076 (55) p=.022	.6326 (51) p=.000	.3059 (52) p=.027	-.1671 (55) p=.223	-.1602 (52) p=.257	-.0686 (55) p=.619	-.0250 (55) p=.856	.1822 (42) p=.248	.2069 (44) p=.178	.0876 (55) p=.525	.1000 (51) p=.485	-.3301 (47) p=.023	-.2018 (53) p=.147
4. Mothers' perceptions (school)	.2595 (64) p=.038	.6317 (50) p=.000	.2952 (62) p=.020	-.1121 (64) p=.378	-.1160 (62) p=.369	.0807 (64) p=.526	.0516 (64) p=.686	.1875 (42) p=.234	.2032 (43) p=.191	.2132 (64) p=.091	.0770 (61) p=.555	-.2776 (57) p=.037	-.1556 (63) p=.223
5. Mothers' attributions (i/e)	.2293 (59) p=.081	.3067 (47) p=.036	.2483 (57) p=.063	.0699 (59) p=.599	.0967 (58) p=.470	-.1777 (59) p=.178	-.2299 (59) p=.080	.1014 (40) p=.533	.1620 (41) p=.312	-.0572 (59) p=.667	-.0799 (57) p=.555	-.2037 (53) p=.143	.1429 (58) p=.285
6. Mothers' attributions (e)	.2655 (59) p=.042	.2665 (47) p=.070	.2686 (57) p=.043	.0353 (59) p=.791	.1087 (58) p=.417	-.1558 (59) p=.239	-.1980 (59) p=.133	.1451 (40) p=.372	.2070 (41) p=.194	.0126 (59) p=.925	-.0762 (57) p=.573	-.2037 (53) p=.143	.1368 (58) p=.306
7. Fathers' beliefs of ease	-.3460 (52) p=.012	-.3264 (50) p=.021	-.2803 (49) p=.051	.0852 (52) p=.548	.2711 (49) p=.060	.0243 (52) p=.864	.1204 (52) p=.395	-.2115 (40) p=.190	-.2545 (42) p=.104	-.1262 (52) p=.373	-.0551 (48) p=.710	.3704 (44) p=.013	.2389 (50) p=.095
8. Mothers' beliefs of ease	-.3366 (62) p=.007	-.3816 (48) p=.007	-.5946 (61) p=.000	.1842 (62) p=.152	.2646 (60) p=.041	-.2366 (62) p=.064	-.1301 (62) p=.313	-.1254 (40) p=.441	-.1812 (41) p=.257	-.2727 (62) p=.032	-.0482 (59) p=.717	.2182 (56) p=.106	.3151 (61) p=.013
9. Fathers' beliefs of ability	-.2332 (50) p=.103	-.5106 (48) p=.000	-.3454 (47) p=.017	.0141 (50) p=.923	.1052 (47) p=.481	-.0125 (50) p=.931	.0250 (50) p=.863	.0445 (40) p=.785	-.0932 (42) p=.557	-.1161 (50) p=.422	-.0936 (46) p=.536	.3148 (42) p=.042	-.1104 (48) p=.455
10. Mothers' beliefs of ability	-.2045 (59) p=.120	-.4849 (47) p=.001	-.7111 (57) p=.000	.2740 (59) p=.036	.1140 (57) p=.398	-.0431 (59) p=.746	-.0012 (59) p=.993	-.0673 (38) p=.688	-.1845 (39) p=.261	-.2545 (59) p=.052	-.0945 (57) p=.484	.3289 (53) p=.016	-.0503 (59) p=.705
11. Children's aspirations	.1210 (72) p=.311	.1491 (51) p=.296	.1086 (62) p=.401	.1062 (72) p=.374	-.2053 (62) p=.109	.1163 (72) p=.330	.1220 (72) p=.307	-.1177 (42) p=.458	-.1210 (44) p=.434	-.2193 (72) p=.064	-.0167 (61) p=.899	-.0500 (57) p=.712	. (63) p=.

Variables	1	2	3	4	5	6	7	8	9	10	11
12. Children's attitudes	.4940 (53) p=.000	.1489 (64) p=.240	.3076 (55) p=.022	.2595 (64) p=.038	.2293 (59) p=.081	.2655 (59) p=.042	-.3460 (52) p=.012	-.3366 (62) p=.007	-.2332 (50) p=.103	-.2045 (59) p=.120	.1210 (72) p=.311
13. Fathers' beliefs of attitudes	.6401 (49) p=.000	.6334 (50) p=.000	.6326 (51) p=.000	.6317 (50) p=.000	.3067 (47) p=.036	.2665 (47) p=.070	-.3264 (50) p=.021	-.3816 (48) p=.007	-.5106 (48) p=.000	-.4849 (47) p=.001	.1491 (51) p=.296
14. Mothers' beliefs of attitudes	.6089 (50) p=.000	.7065 (62) p=.000	.3059 (52) p=.027	.2952 (62) p=.020	.2483 (57) p=.063	.2686 (57) p=.043	-.2803 (49) p=.051	-.5946 (61) p=.000	-.3454 (47) p=.017	-.7111 (57) p=.000	.1086 (62) p=.401
15. Children's home activities	-.2679 (53) p=.052	-.2411 (64) p=.055	-.1671 (55) p=.223	-.1121 (64) p=.378	.0699 (59) p=.599	.0353 (59) p=.791	.0852 (52) p=.548	.1842 (62) p=.152	.0141 (50) p=.923	.2740 (59) p=.036	.1062 (72) p=.374
16. Mothers' beliefs of activities	-.2091 (51) p=.141	-.1167 (62) p=.366	-.1602 (52) p=.257	-.1160 (62) p=.369	.0967 (58) p=.470	.1087 (58) p=.417	.2711 (49) p=.060	.2646 (60) p=.041	.1052 (47) p=.481	.1140 (57) p=.398	-.2053 (62) p=.109
17. Children's reports of total parental indirect help	-.0272 (53) p=.847	.2102 (64) p=.096	-.0686 (55) p=.619	.0807 (64) p=.526	-.1777 (59) p=.178	-.1558 (59) p=.239	.0243 (52) p=.864	-.2366 (62) p=.064	-.0125 (50) p=.931	-.0431 (59) p=.746	.1163 (72) p=.330
18. Children's reports of mother's indirect help	.0103 (53) p=.941	.1306 (64) p=.304	-.0250 (55) p=.856	.0516 (64) p=.686	-.2299 (59) p=.080	-.1980 (59) p=.133	.1204 (52) p=.395	-.1301 (62) p=.313	.0250 (50) p=.863	-.0012 (59) p=.993	.1220 (72) p=.307
19. Parents' reports of total indirect help	.1775 (42) p=.261	.0694 (42) p=.662	.1822 (42) p=.248	.1875 (42) p=.234	.1014 (40) p=.533	.1451 (40) p=.372	-.2115 (40) p=.190	-.1254 (40) p=.441	.0445 (40) p=.785	-.0673 (38) p=.688	-.1177 (42) p=.458
20. Fathers' reports of indirect help	.1547 (44) p=.316	.1960 (43) p=.208	.2069 (44) p=.178	.2032 (43) p=.191	.1620 (41) p=.312	.2070 (41) p=.194	-.2545 (42) p=.104	-.1812 (41) p=.257	-.0932 (42) p=.557	-.1845 (39) p=.261	-.1210 (44) p=.434
21. Children's reports of total parental help in maths	.2688 (53) p=.052	.3761 (64) p=.002	.0876 (55) p=.525	.2132 (64) p=.091	-.0572 (59) p=.667	.0126 (59) p=.925	-.1262 (52) p=.373	-.2727 (62) p=.032	-.1161 (50) p=.422	-.2545 (59 p=.052	-.2193 (72) p=.064
22. Mothers' academic importance of arithmetic	.0727 (50) p=.616	.0835 (61) p=.522	.1000 (51) p=.485	.0770 (61) p=.555	-.0799 (57) p=.555	-.0762 (57) p=.573	-.0551 (48) p=.710	-.0482 (59) p=.717	-.0936 (46) p=.536	-.0945 (57) p=.484	-.0167 (61) p=.899

Variables	12	13	14	15	16	17	18	19	20	21	22	23	24
12. Children's attitudes	1.000 (73) p=.	.4128 (51) p=.003	.3126 (62) p=.013	-.2517 (73) p=.032	-.1659 (62) p=.197	.1241 (73) p=.295	.1538 (73) p=.194	.2134 (42) p=.175	.2379 (44) p=.120	.3202 (73) p=.006	.1280 (61) p=.325	-.4605 (57) p=.000	-.2023 (63) p=.112
13. Fathers' beliefs of attitudes		1.000 (51) p=.	.6420 (48) p=.000	-.0752 (51) p=.600	-.1046 (48) p=.479	-.0293 (51) p=.838	-.0648 (51) p=.651	.1835 (40) p=.257	.3098 (42) p=.046	.1346 (51) p=.346	.1464 (48) p=.321	-.4299 (43) p=.004	-.0569 (49) p=.698
14. Mothers' beliefs of attitudes			1.000 (62) p=.	-.3102 (62) p=.014	-.2177 (60) p=.095	.1307 (62) p=.311	.0666 (62) p=.607	-.0475 (40) p=.771	.1205 (41) p=.453	.3706 (62) p=.003	.1129 (59) p=.394	-.4122 (55) p=.002	-.0689 (61) p=.639
15. Children's home activities				1.000 (73) p=.	-.0093 (62) p=.943	.0115 (73) p=.923	-.0078 (73) p=.948	.1489 (42) p=.347	.2191 (44) p=.153	-.0370 (73) p=.756	-.1414 (61) p=.277	.0338 (57) p=.803	.0689 (63) p=.592
16. Mothers' beliefs of activities					1.000 (62) p=.	-.2019 (62) p=.115	-.1535 (62) p=.234	.1511 (41) p=.346	.1046 (42) p=.510	-.1129 (62) p=.382	-.2085 (60) p=.110	.1430 (56) p=.293	.1759 (61) p=.175
17. Children's reports of total parental indirect help						1.000 (73) p=.	.9071 (73) p=.000	-.1704 (42) p=.281	-.0098 (44) p=.950	.2792 (73) p=.017	.1208 (61) p=.354	-.0060 (57) p=.964	-.1920 (63) p=.132
18. Children's reports of mother's indirect help							1.000 (73) p=.	-.0443 (42) p=.780	.0467 (44) p=.764	.2724 (73) p=.020	.1312 (61) p=.313	-.0600 (57) p=.657	-.1673 (63) p=.190
19. Parents' reports of total indirect help								1.000 (42) p=.	.9605 (42) p=.000	.1383 (42) p=.382	-.1254 (40) p=.441	-.3245 (36) p=.053	.1734 (41) p=.278
20. Fathers' reports of indirect help									1.000 (44) p=.	.2246 (44) p=.143	-.1265 (41) p=.431	-.3633 (37) p=.027	.1699 (42) p=.282
21. Children's reports of total parental help in maths										1.000 (73) p=.	-.0036 (61) p=.978	-.3838 (57) p=.706	-.0536 (63) p=.676
22. Mothers' academic importance of arithmetic											1.000 (61) p=.	-.0520 (55) p=.706	.0450 (60) p=.733

Variables	1	2	3	4	5	6	7	8	9	10	11
23. Mothers' opinions about the arithmetic curriculum	-.5515 (46) p=.000	-.3403 (57) p=.010	-.3301 (47) p=.023	-.2776 (57) p=.037	-.2037 (53) p=.143	-.2037 (53) p=.143	.3704 (44) p=.013	.2182 (56) p=.106	.3148 (42) p=.042	.3289 (53) p=.016	-.4605 (57) p=.000
24. Mothers' academic level	-.0964 (51) p=.501	.0561 (63) p=.663	-.2018 (53) p=.147	-.1556 (63) p=.223	.1429 (58) p=.285	.1368 (58) p=.306	.2389 (50) p=.095	.3151 (61) p=.013	-.1104 (48) p=.455	-.0503 (59) p=.705	-.2023 (63) p=.112

Variables	12	13	14	15	16	17	18	19	20	21	22	23	24
23. Mothers' opinions about the arithmetic curriculum	-.4605 (57) p=.000	-.4299 (43) p=.004	-.4122 (55) p=.002	.0338 (57) p=.803	.1430 (56) p=.293	-.0060 (57) p=.964	-.0600 (57) p=.657	-.3245 (36) p=.053	-.3633 (37) p=.027	-.3838 (57 p=.003	-.0520 (55) p=.706	1.000 (57) p=.	.0243 (56) p=.859
24. Mothers' academic level	-.2023 (63) p=.112	-.0569 (49) p=.698	-.0614 (61) p=.639	.0689 (63) p=.592	.1759 (61) p=.175	-.1920 (63) p=.132	-.1673 (63) p=.190	.1734 (41) p=.278	.1699 (42) p=.282	-.0536 (63) p=.676	.0450 (60) p=.733	.0243 (56) p=.859	1.000 (63) p=.

APPENDIX 4

Appendix 4.1 Frequencies of Children as a Function of Order of Presentation of Formal and Informal Arithmetic Tasks and Mathematical Group

Patterns of Order	AA (*n* = 30)	A (*n* = 20)	BA (*n* = 16)	Total (*n* = 66)
Group I: **First** 12, 8, 9, 4, 10, 11 **Second** 3, 13, 5, 6, 2, 1, 7 **Third** 14	7	5	4	16
Group II: **First** 12, 8, 9, 4, 10, 11 **Second** 7, 1, 2, 6, 5, 13, 3 **Third** 14	7	5	4	16
Group III: **First** 3, 13, 5, 6, 2, 1, 7 **Second** 12, 8, 9, 4, 10, 11 **Third** 14	8	5	4	17
Group IV: **First** 7, 1, 2, 6, 6, 13, 3 **Second** 12, 8, 9, 4, 10, 11 **Third** 14	8	5	4	17

Task Specifications:

1. *Which number is more ?*
2. *Which is closer to X ?*
3. *Mental addition*
4. *Estimation*
5. *Enumeration by tens*
6. *Counting large numbers*
7. *Multiples of large numbers*
8. *Larger written numbers*
9. *Representation of place value*
10. *Accuracy and bugs in written addition and subtraction*
11. *Monitoring errors*
12. *Addition facts*
13. *Use of principles*
14. *Word problems*

Appendix 4.2 Number of Children Erring on Written Addition and Subtraction Problems and Frequencies of Different Types of Errors (in detail)

	AA (n = 30)	A (n = 20)	BA (n = 16)
Total	23 (42)	16 (54)	16 (88)
Type of Error:			
Writing numbers as they sound	-	3 (5)	4 (7)
Misalignment	-	-	2 (5)
Buggy addition algorithm			
failure to carry	2 (2)	4 (5)	3 (5)
carry to wrong column	4 (4)	2 (3)	1 (1)
other addition bug	2 (2)	1 (1)	4 (6)
Buggy subtraction algorithm			
failure to deduct	7 (8)	5 (7)	4 (6)
subtract upper from lower	7 (7)	5 (11)	7 (17)
other subtraction bug	2 (3)	1 (3)	3 (4)
Simple miscalculation	12 (16)	10 (12)	12 (26)
Other	-	5 (7)	7 (11)
No answer	10 (13)	12 (29)	9 (31)

Appendix 4.3 Correlations Between Performance in Formal and Informal Arithmetic and Children's Arithmetic Achievement

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Informal Concepts and Calculational Skills						
Which number is more ?	Task 1	interval: scores (max. score 4)	Yes:2	72	.4522	.4423
Which is closer to X ?	Task 2	interval: scores (max. score 4)	Yes:2	72	.4007	.4700
Mental addition	Task 3	interval: scores (max. score 8)	Yes:3	72	.6526	.5501
Estimation	Task 4	interval: scores (max. score 6)	Yes:3	72	.5299	.5533
Base ten concepts and related enumeration skills						
Enumeration by tens	Task 5	interval: scores (max. score 4)	Yes:2	72	.5221	.4632
Counting large numbers	Task 6	interval: scores (max. score 4)	Yes:3	72	.7019	.6367
Multiples of large numbers	Task 7	interval: scores (max. score 6)	Yes:3	72	.7403	.6906
Larger written numbers	Task 8	interval: scores (max. score 4)	Yes:1	72	.3044	.3325
Representation of place value	Task 9	interval: scores (max. score 2)	Yes:1	72	.4582	.3768
Error strategies and other calculational procedures						
Accuracy and bugs in written addition and subtraction	Task 10	interval: scores (max. score 10)	Yes:3	72	.6103	.6367
Monitoring errors	Task 11	interval: scores (max. score 6)	Yes:2	72	.4936	.4218
Number facts						
Addition facts	Task 12	interval: scores (max. score 10)	Yes:3	72	.7148	.7100
Problem-solving skills						
Use of principles	Task 13	interval: scores (max. score 4)	No	72	.2449	.2657
Story problems	Task 14	interval: scores (max. score 8)	Yes:2	72	.7866	.7533

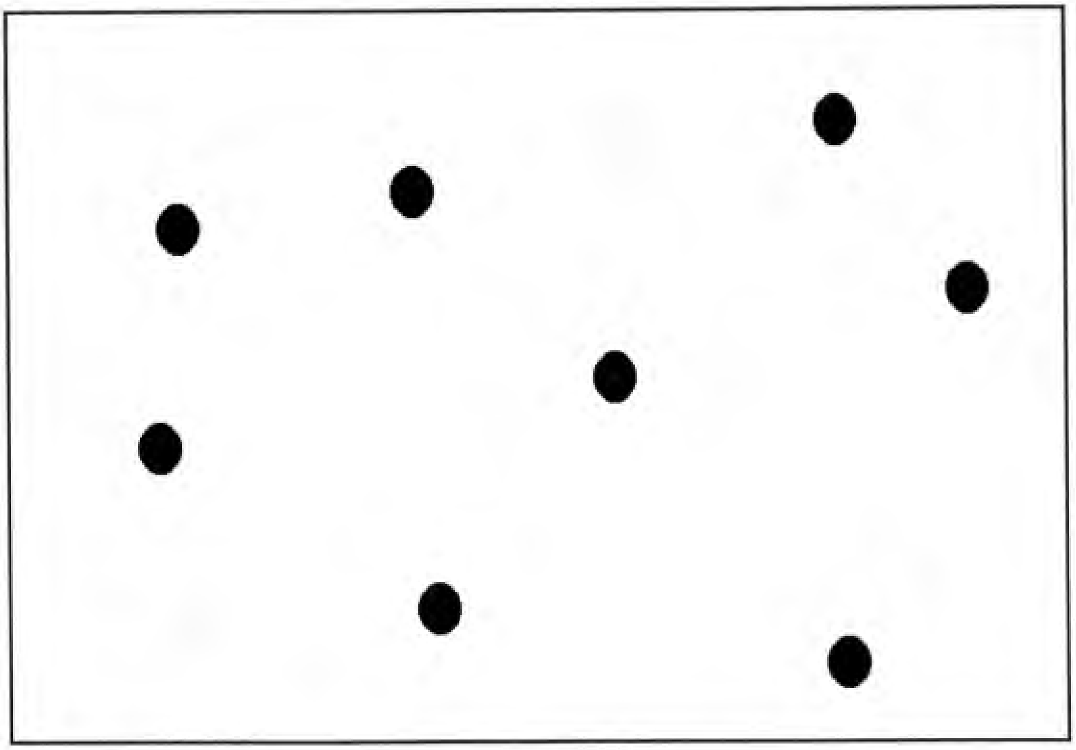
* nonsignificant

Appendix 4.4 Pearson's Correlation Matrix of Children's Peformance on Tasks of Formal and Informal Arithmetic Knowledge and Skill ($n = 72$)

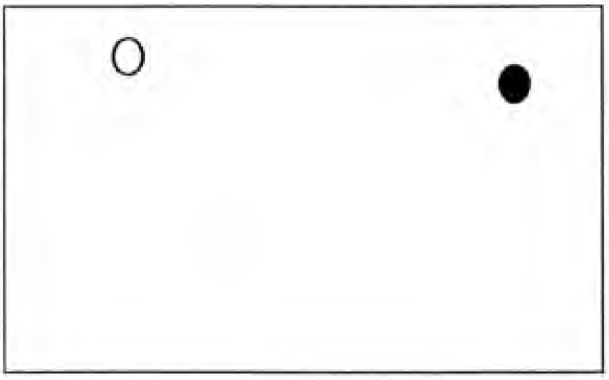
Tasks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Key
Task 1	1.000 p=.	.2855 p=.015	. 2988 p=. 011	.3025 p=.010	-.0363 p=.762	.3065 p=.009	.4046 p=.000	.2895 p=.014	.0076 p=.949	.4921 p=.000	.2484 p=.035	.4644 p=.000	.2023 p=.088	.3913 p=.001	*Which Number Is More ?*
Task 2		1.000 p=.	.3976 p=.001	.3211 p=.006	.1584 p=.184	.2325 p=.049	.3087 p=.008	.2833 p=.016	.0231 p=.847	.3148 p=.007	.1831 p=.124	.4699 p=.000	.1666 p=.162	.4173 p=.000	*Which Is Closer to X ?*
Task 3			1.000 p=.	.3796 p=.001	.4315 p=.000	.5896 p=.000	.5404 p=.000	.4788 p=.000	.4075 p=.000	.5533 p=.000	.4712 p=.000	.5651 p=.000	.3163 p=.007	.6909 p=.000	*Mental Addition*
Task 4				1.000 p=.	.1325 p=.267	.4794 p=.000	.4260 p=.000	.2995 p=.011	.2590 p=.028	.3718 p=.001	.4732 p=.000	.6032 p=.000	.0990 p=.408	.5320 p=.000	*Estimation*
Task 5					1.000 p=.	.3462 p=.003	.4821 p=.000	.2320 p=.050	.4525 p=.000	.3774 p=.001	.3043 p=.009	.3497 p=.003	.2153 p=.069	.4636 p=.000	*Enumeration by Tens*
Task 6						1.000 p=.	.7855 p=.000	.3385 p=.004	.5321 p=.000	.4017 p=.000	.5736 p=.000	.5464 p=.000	.2499 p=.034	.6994 p=.000	*Counting Large Numbers*
Task 7							1.000 p=.	.3931 p=.001	.4764 p=.000	.5038 p=.000	.5272 p=.000	.5729 p=.000	.2733 p=.020	.6896 p=.000	*Multiples of Large Numbers*
Task 8								1.000 p=.	.3665 p=.002	.4313 p=.000	.4159 p=.000	.4021 p=.000	.1111 p=.353	.4385 p=.000	*Larger Written Numbers*
Task 9									1.000 p=.	.3170 p=.007	.4426 p=.000	.2211 p=.062	.1680 p=.158	.4270 p=.000	*Representation of Place Value*
Task 10										1.000 p=.	.4502 p=.000	.6022 p=.000	.1128 p=.346	.6092 p=.000	*Written Addition and Subtraction*
Task 11											1.000 p=.	.4545 p=.000	.2187 p=.065	.5346 p=.000	*Monitoring Errors*
Task 12												1.000 p=.	.1724 p=.148	.6379 p=.000	*Addition Facts*
Task 13													1.000 p=.	.2427 p=.040	*Use of Principles*
Task 14														1.000 p=.	*Word Problems*

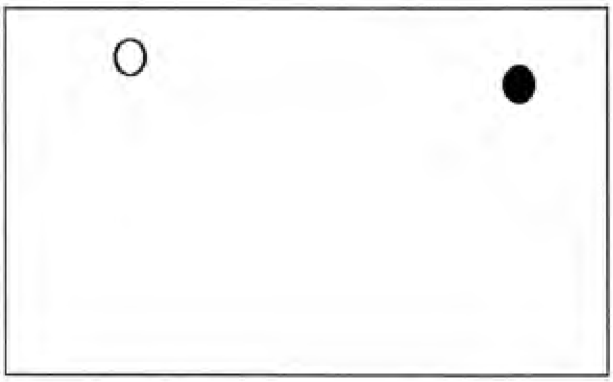
APPENDIX 5

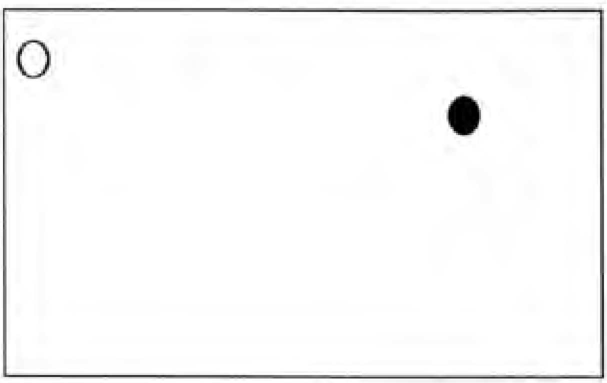
Appendix 5.1 Sample Material Used in the Visual Counting Span Task



Appendix 5.2 Sample Material Used in the Visual Comparison Span Task







Appendix 5.3 Correlations Between Working Memory Measures and Children's Arithmetic Achievement

Description	Variable	Coding	Sig Differences among groups	Number of Responses	Young (1979)	NFER (1971)
Span						
Visual counting span	vcngs	interval: scores	Yes:1	53	.4399	.4565
Auditory counting span	acnts	interval: scores	No	53	.2353*	.2842
Visual comparison span	vcomps	interval: scores	Yes:1	53	.3468	.2994
Auditory comparison span	acomps	interval: scores	No	53	.3117	.3053
Digit span	digitsp	interval: scores	Yes:1	53	.3784	.4549
Word span	wordsp	interval: scores	Yes:1	53	.3284	.5025
Speed						
Spot counting	spotcnt	interval: secs.	Yes:2	53	-.2008*	-.3608
Speech rate	spart	interval: secs.	Yes:2	53	-.3527	-.2586*
Recitation 1-20	reccntsm	interval: secs.	No	53	-.1286*	-.1662*
Recitation 2-20	reccntl	interval: secs.	Yes:2	53	-.6155	-.4646

* nonsignificant

Appendix 5.4 Pearson's Correlation Matrix of Children's Performance on Working Memory Spans and Other Counting Tasks (n = 53)

Tasks	1	2	3	4	5	6	7	8	9	10	Key
Task 1	1.000 p=.	.3229 p=.018	.6187 p=.000	.5233 p=.000	.5434 p=.000	.5084 p=.000	-.1538 p=.272	-.1950 p=.162	-.4237 p=.002	-.1941 p=.164	*Visual Counting Span*
Task 2		1.000 p=.	.2043 p=.142	.1805 p=.196	.1674 p=.231	.0971 p=.489	-.0984 p=.483	-.1243 p=.375	-.2105 p=.130	-.2008 p=.149	*Auditory Counting Span*
Task 3			1.000 p=.	.2272 p=.102	.2974 p=.031	.1428 p=.308	-.2741 p=.047	-.0652 p=.643	-.4267 p=.001	-.0933 p=.506	*Visual Comparison Span*
Task 4				1.000 p=.	.4973 p=.000	.4455 p=.001	-.0499 p=.722	-.0878 p=.532	-.3392 p=.013	-.0738 p=.599	*Auditory Comparison Span*
Task 5					1.000 p=.	.6414 p=.000	-.1237 p=.377	-.2984 p=.030	-.2973 p=.031	-.3041 p=.027	*Digit Span*
Task 6						1.000 p=.	-.0350 p=.803	-.1553 p=.267	-.2935 p=.033	-.1069 p=.446	*Word Span*
Task 7							1.000 p=.	.3840 p=.005	.1636 p=.242	.5115 p=.000	*Spot Counting*
Task 8								1.000 p=.	.1381 p=.324	.6779 p=.000	*Speech Articulation*
Task 9									1.000 p=.	.1300 p=.353	*Recitation 1 20*
Task 10										1.000 p=.	*Recitation 2-20*