INVESTIGATING THE FACTORS WHICH INFLUENCE THE CHILD'S CONCEPTION OF ANGLE

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ABSTRACT

The aim of the present study is to investigate the factors which influence the child's understanding of angle. Fifty-four students aged from 6 to 14, were set 92 activities to solve in three separate sections. The activities were elaborated according to six interwoven variables: (a) activities in static and dynamic perspectives carried out under (b) three different representational systems: oral (everyday life model), written (paper and pencil model), and body-syntonic (Logo model). These were inserted in three situations (c), rotation, navigation and comparison, using (d) different materials. The children were asked (e) to perform an action or to recognise differences and similarities between angles, followed by an explanation, or description of what they had done. All activities involved (f) different sizes of angle.

The findings were submitted to both quantitative and posteriori qualitative analysis. Cross-sectionally by age, the data indicate a strong trend of improved performance with age. This points to a developmental effect, but the school's influence has to be taken into account.

The results suggest that the child's acquisition of the conception of angle has a dynamic perspective as its starting-point. In particular, the children performed better within activities which involved rotation. This does not imply that every child used the dynamic perspective of angle consistently across all tasks. In fact the choice of perspective frequently changedaccording to the meaning of the situation, which could sometimes be depended on cultural influences. This was particularly apparent in the watch arena, the situation which the children were most successful. In a comparison of representational systems, the best performances were achieved in activities on Logo, while activities conducted with paper & pencil proved to be the most difficult. Performance was also enhanced in tasks which required action by the children.

These findings indicate that there exist various factors influencing a child's understanding of angle, and these factors are close interrelated.

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CHAPTER 1

INTRODUCTION

When I was at primary school in Brazil I used to listen to adults talking and I remember hearing the following expression "my life has turned 180 degrees." At that time I was being introduced to the topic of angle at school. To show that I knew about angles and degrees, I once said to a friend in front of the teacher:

- Nowadays my life has turned 360 degrees.

To which my teacher replied:

- So, your life did not change at all. I suppose you want to say 180 degrees, don't you?"

Then I asked myself "Why not 360 degrees? This is much more of a turn than 180 degrees."

On another occasion, I was watching a football match, which was being played between the boys in my neighbourhood among whom was my elder brother. He was quite near the opponent's goal, but on the left-hand side. Someone kicked the ball to him and I thought: "now, he is going to score a goal". To my surprise and disappointment he kicked the ball to another boy who was further from the goal, but positioned in front of it, and this boy scored the goal. I ran as fast as I could to my brother and asked him:

- Why didn't you score the goal?

And he replied:

- Because I didn't have the necessary angle. Couldn't you see?

Without having anything else to say, I said:

- Oh, yes, I saw...

In fact, I did not understand what he meant by the word 'angle'. "Which angle was he talking about?" I could not see any lines to form an angle, so how could he see the angle? So I concluded to myself that he was referring to some other kind of angle and not to the 'angle' I was learning about in school.

The above examples show how frequently the concept of angle is used, in different ways, within the everyday lives of people. On the other hand, the first example illustrates one of the possible misunderstandings that students might exhibit whilst forming the conception of angle. The expression "my life turned 180 degrees" is a metaphor suggesting that the life of a person has changed its direction. However I show in my example that I was only concerned with the magnitude of the numbers -- the larger the number the more I had changed -- rather than thinking of them in terms of what they represented from a geometrical viewpoint. The second example shows how difficult it is to contextualize, in everyday life, the mathematics content which is learnt in school. In this case I was only able to perceive an angle as it was drawn on a paper in my mathematics classroom.

The purpose of this thesis is to investigate the factors which influence a child's conception of angle: for example, as far as the children are concerned, their age and level of schooling, and as far as the tasks are concerned, the different situations into which they are inserted, the use of different materials and the meaning that they have for the children.

The thesis opens with an examination of the psychological literature concerned with concept formation. In Chapter 2, I discuss two psychological approaches, behaviourism and constructivism; The former because it seems to me to underpin much of the Brazilian school curriculum and the latter because it serves as the theoretical framework of this research. Later, I argue that it is important to consider knowledge and learning as composing two sides of the same coin. I also point to the necessity of including a theory of symbolisation in both a study of the learning process or of concept formation, since this, in my view, forms the basis of a child's representation. Further, I advocate that all interpretation of child's understanding must be underpinned from the viewpoint of his/her own experience. As well as the Piaget's view of constructivism, I also present the idea that interaction between a child and his/her social environment is essential for the acquisition of a concept. Finally, I give attention to the child's spontaneous and scientific concepts: spontaneous concept is acquired out a formal learning process; it comes from the child's interaction with his/her physical environment, whilst the scientific concept is acquired from a structured learning process. The power of the scientific concept is in its potential for generalising and it is this property which distinguishes it from spontaneous concepts.

In Chapter 3, I present a literature review concerning geometry in general and the conception of angle in particular. I start by giving a brief report of the origin of geometry. I demonstrate that angle does not have a single definition. The definitions that exist can be categorised into two perspectives: the static and the dynamic. I then go on to describe three types of geometry: Euclidean, Transformation and Turtle, identifying their main characteristics with regard to the conception of angle. I also look at different approaches of teaching geometry and distinguish two: the formal and the informal. By formal, I mean an approach using scientific concepts, and by informal I am referring to the geometry teaching which explores the child's spontaneous concepts. I point to evidence that suggests that it is important to start geometry with an informal approach, presenting objects and situations which have meaning for the children. Following on from this, I present two of the best known and respected studies of how children develop and learn geometry, namely those of Piaget and Van Hiele. I set out to highlight their contributions, but also what has been considered as their weaknesses and limitations. I go on to present how angle is introduced and developed in the Brazilian curriculum in order to give some background to assist in the interpretation of what the children, in my experiment, did and said. In this chapter, I also report research carried out about children's conception of angle. This is divided into three groups according to how the tasks were executed: one using paper and pencil, another in Logo, and the third in both. Finally, I discuss research which has looked at the formation of mathematical concepts in everyday life.

In Chapter 4, I present the design and the methodology of the study. From the basis of the psychological and mathematical theories discussed in Chapters 2 and 3, I describe and justify the way in which the activities of the study are arranged into the five categories through which a child's conception of angle is explored. The *perspective* of angle, whether it be the dynamic or the static, is one of these categories. Another category comprises the situations created in order to allow children to experience angle through navigation, rotation and comparison, where children were asked carry out activities such as to move and turn in a mini city, to turn objects conserving their Cartesian axis, or to compare open figures. This set of situations I categorised as *context*. These activities were also elaborated with the aim of exploring three different representational systems, which are related to the formality and informality of a learning environment. They are: paper and pencil, Logo and everyday life. I called this category *setting*. Paper and pencil was an obvious choice because of the school situation and previous research, but it also seemed to be a good place to explore the more static perspective of angle. Logo was chosen, again partly because of the considerable body of work previously undertaken in this computer language, but also because it has been considered as a good way to explore the dynamic perspective of angle. Everyday life has been referred to in the specialised literature as a good system to look at children's spontaneous concepts in both the dynamic and the static perspectives. For pragmatic reasons, I chose not use actual everyday life situations, but rather to develop models of activities from everyday life.

Next, I consider that the materials, which were manipulated by the children while they were carrying out experimental activities, were also relevant for the study. This category I called the *arenas*. Finally, in order to assist in the analysis of the data, the activities were divided into two groups: one in which children were asked to do something or to predict the results of their actions; the other in which they were asked to identify an angle of a particular type or to recognise whether angles were the same or different. These two categories were called *action* and *recognition* respectively. In almost all of the activities, the children were asked to describe or explain what they had done. This was called *articulation*. These three aspects -- recognition, action and articulation -- were called by *conditions*.

In the second part of Chapter 4, I go on to describe how the research was implemented. In separate sections I describe (1) how the pilot study (study 1) was carried out, (2) the changes to the research design after the pilot data had been analysed, and (3) the actual activities carried out in the main study (study 2) according to the five categories elaborated earlier. Also in this chapter, I talk about where the research took place and I give a profile of the children who took part in the research. The sample comprised 54 children, from 6 to 14 years of age, from Recife, North-East Brazil. Each child individually undertook 92 activities spread over three interviews.

The results are presented in two chapters. In Chapter 5, I present the quantitative results dividing the sample into two groups: Group 1, which is formed by children from middle school (11 to 14 years old) who are the oldest children of the sample; and Group 2 composed of children from the 6 to 10 years of age, who are in the elementary school. Chapter 6 comments on the qualitative analysis where the children's performance, in terms of action and recognition, are presented cross-sectionally by age and according to the first four categories described in Chapter 4. The way in which the children explain what they had done, that is, their articulation, is analysed qualitatively. From this qualitative analysis I draw out what seem to be the important explanatory variables for the children's performance, which I call the *references*. It was these which gave me significant clues to be used in the discussion and the interpretation of the research.

Finally, the concluding chapter presents the reflections on my journey through the thesis by highlighting the most central aspects obtained from the data, discussing the articulation between findings and theory, making explicit their theoretical significance, and giving attention to possible expanding, tendencies and aspects to be examined in future research.

CHAPTER 2

THE ACQUISITION OF CONCEPTS: A PSYCHOLOGICAL APPROACH

This research proposes to study how a child acquires the concept of an angle. It is necessary to discuss this question from at least three viewpoints: from the psychological viewpoint - referring to the formation of the concept; the mathematical viewpoint - concerning the definition of what an angle is; and from the teaching viewpoint - related to how the subject is taught in school.

This chapter will analyse the first of these three aspects, the psychological point of view, and the following chapter will deal with the mathematical perspective and the way it has been taught in Brazilian schools.

2.1 A PSYCHOLOGICAL THEORY

One of the key debates in cognitive psychology concerns the learning-knowledge and within this more general theme, one finds discussion of how concepts are formed. Many theories have been elaborated in an effort to explain how a child acquires a given concept. Here I will discuss only one theory which is the basis of the present study, namely constructivism.

The principal proposition of constructivism holds that it is the children who build their own version of reality through their own experiences. In the process of building their own knowledge, children play an active role in creating new relations between ideas which already exist, incorporating new pieces of information.

Piaget (1977), the best known constructivist, has focused on the developmental side of constructivism. He argues that the child builds his/her own constructs (or, 'schemes' in Piagetian terms[1]) out of his own experiences in his immediate world. Learning occurs from the starting point of action and the subsequent internalisation of this action (action and operations[2] of structures) by the child.

Vygotsky (1962), like Piaget, also took the developmentalist position, and, like Piaget, emphasised the importance of the action of the subject in the process of learning. The Vygotsky's concept of internalisation of action is similar to that proposed by Piaget, except that it incorporates additional social/cultural dimensions. In fact, while Vygotsky and Piaget start off from similar basic principles, their theories tend to draw apart from each other due to the emphasis Piaget gives to the biological/individual and Vygotsky to the social/cultural.

Thus, within constructivism, there are differing perspectives of the learning process. Ausubel (1968) argues that children need a guide in order to learn effectively. He defends the notion of meaningful verbal learning, where instead of discovering for themselves, children are introduced by the teacher to key concepts as an easier form of assimilation. For Ausubel, it is easier to learn

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1 - In Piagetian theory "Scheme" is an organised action which can be transferred or generalised by repetition in analogous situation. In other words, a scheme means the formation of a concept in a still-limited form, because it has only one meaning.
2 - The term "operation" is used by Piaget as meaning an action carried out in the mind, originated in processes such as combination, sorting, separating.

by means of language than by practical material. The danger of such a position is that it can easily slide into behaviourist practices.

Many mathematical educators (Cobb, 1990 ; Von Glasersfield, 1991 and so on) have adopted what they call 'radical constructivism'. This line of thinking is summarised by Von Glasersfield (1991) through two basic principles: 1) that knowledge is built up actively by the child, and (2) that the cognitive function is adaptive, in the biological sense of the term, serving as an organiser of the child's world of experience. Therefore, radical constructivism has assumed a position aligned to Piagetian's ideas, where the biological factor, takes on an important role in the children's concept formation.

One also finds, among mathematical educators, those who prefer 'trivial' or 'simple constructivism' (Davis, Maher and Noddings (1990) among others) which is based only on the first of the above two principles.

Returning to Vygotsky's and Piaget's work, it is clear that they have had considerable influence on recent research in the spheres of psychology and education. They have particularly influenced the work of psychologists such as Bruner, Vergnaud, Nunes, among others. Both Piaget and Vygotsky based their theories on a developmental view of constructivism. This means that the concepts that I will discuss here will basically be treated in the light of this perspective. However, my belief that adhering to the same theoretical infrastructure does not necessarily imply a complete hegemony of ideas and action between the various authors, and because I also believe that discussions are usually enriched by differing ideas, I will discuss major themes of this thesis by referring to the viewpoint of these authors, pointing out similarities and contrasts among their theories and also between their work and mine. With the intention of broadening and elucidating the discussion to the maximum, I will also seek, whenever possible, to bring into the discussion of the topics that follow, the perspective of other authors who have also dealt with the question of concept formation, and in particular, those who work in the field of mathematics education.

2.2 THE ROLE OF LEARNING

Learning is of greater or lesser importance on a child's development depending on the theoretical viewpoint. For some authors, learning is an independent process from development, for others is closely inter-related. According to Piaget, learning has a limited role within the wider process of knowledge. Learning how to do something, or learning a specific subject, can only take place within pre-existing cognitive structures. These structures are linked to the nature of knowledge and its function is in accordance with how the individual operates in the world.

From Piaget's viewpoint (1977), a child is not seen as a miniature adult, nor is his/her mind seen as a smaller version of an adult mind. The processes which the child goes through are qualitatively distinct from those of adults. Consequently, Piaget does not see any point in talking about learning in the sense of the adult transmitting his way of thinking to the child.

This differentiation between the mind of the child and the adult is also shared by Vygotsky (1962). He holds that an object can be identified both by the child and the adult -- which makes communication between the two possible -- however, the way the child thinks about the same object is different from the adult's thought processes and it is arrived at through different operations.

What seems substantial in the above mentioned Piagetian and Vygotskian ideas is the distinction they made between child and adult thinking, which lead me to be aware that if they, child and adult, have different cognitive processes, they must surely have different representational systems and they, consequently, will perceive a phenomenon differently.

The two authors diverge when Vygotsky, in contrast to Piaget, holds that learning is one of the principle sources of the concepts held by a child of school-age, and serves as a strong force in child development. For Vygotsky, learning establishes the direction of all of a child's mental development.

The elaboration of the *zone of proximal development* is the best example of Vygotsky's belief in the learning as interfering in the child's development. The zone of proximal development refers to the relationship between the level of a child's *effective development* and his/her level of *potential development.* The effective development is the psycho-intellectual function that a child has already reached as result of a specific development process already realised. Whereas the potential development refers to those processes that are still occurring inside the child and which are in the process of development and maturation. (Vygotsky, 1962; Sutherland, 1992; Wood, D.1992).

The close relation between learning and development is evident in Vygotsky's statement that the correct organisation of a child's learning should lead not only to his/her mental development, but to the activation of a whole group of developmental processes, which would remain inactive without learning.

In the relation between learning and development, the role of imitation is essential. Through imitation -- carried out within collective activities guided by adults -- a child can do much more than s/he could ever achieve independently. The difference between the level of tasks done with help of other[3], and the level of tasks developed as an independent activity is the *zone of proximal development*. Vygotsky explicitly says that: "Whatever a child is able to do today with the help of adults, s/he will be able to do so by him/herself tomorrow"[4] (1991, p.113).

Vergnaud (1987A, 1987B) belongs to the group of psychological authors who, like Vygotsky, also defends the possibility of learning by teaching. However, he makes it clear that learning depends fundamentally on the content of the knowledge which is to be taught. He argues that it is essential not to lose sight of the fact that every piece of knowledge refers to situations that the child needs to domain, and this domain arises through solving problems. If this is so, a developmental approach to learning requires definitions that permit us to deal with situations for which a concept is significant. In Vergnaud's words, "We need a theory of reference that refers concepts to situations".

Van Hiele (1986), a mathematics educator, also discusses the learning process. He states that during the learning process the child goes through five stages of thinking (visual, descriptive, theoretical, formal logic and laws of logic). The passage from one level to the next takes place through *structures* and *insight*. The structure can be one of two types. The first is Structure of action which is when the subject acts automatically, without thinking about what

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3 - Vygotsky (1991) uses the term "other" to refer to adults. I shall open this term to refer also to everything (and everyone) related to the child, such as colleagues, parents, guide-questions, meaningful situations (i.e., situations attained from child's environment) and so on.

4 - This quotation was a translation, made by myself, from a Brazilian book. The original text is "O que a criança pode fazer hoje com o auxílio dos adultos poderá fazê-lo amanhã por si só."

he is doing. For example in the case of a touch-typist copy-typing, or a pianist playing a melody from sheet music. In both cases, it is not necessary to think about the action. The second type of structure is mental structure which is when the subject acts in accordance with his/her thinking.

Van Hiele (1986) defines "Insight" as a process of acting adequately and deliberately in a new situation. The insight thus underlies the development of structures. A structure can always be extended or can be part of a larger structure. In the process of teaching-learning, it is up to the teacher to be aware of the level the child has reached and to set up situations where the structures can develop and the insights occur. Then the child will be able to progress from the level of simple thoughts to more complex levels.

From the Van Hiele proposal about insight and it role in the learning process, I can conjecture that if children are asked to solve tasks which are embedded in a suitable context in order to explore a specific topic, and If the tasks are also creative and stimulating enough to encourage children to look forward to solving a problem, then these tasks will probably provoke an insight process which, by its turn, will lead children to start a learning process.

2.2.1 SUMMARY

This section looks at the role of learning from different authors' perspectives. Piaget considers learning as having a widely limited role on child's knowledge, whilst Vygotsky, Vergnaud and Van Hiele give great importance to the learning process. In Vygotsky's point of view, the child's learning process takes place within what he calls *zone of proximal* *development.* For Vergnaud the learning process depends on the content to be taught. In this sense, it is essential to face a child with problem solving within which a concept is significant. Finally, Van Hiele assumes that the learning process takes place through what he calls *structure* (related to action) and *insight* (an adequate and deliberate act). He emphasises the necessity of having the contents to be learning inserted within context.

2.2.2 LEARNING AND CONTEXT

Many educational researchers (Nunes, 1991; Van Hiele, 1986; Saxe, 1991; Vygotsky, 1962, 1978) have claimed the importance of introducing any topic to children within context. For these authors, it is very hard to teach something detached from a situation in which it can be experienced.

In an anthropological approach, which is also suitable to be used in the educational field, Lave (1988), argues that the term "context" refers to two things: it both indicates a stable framework for activities, presenting properties which transcend the individual experience (in this sense, the context is beyond the individual control, once it exists prior to him); yet can be "experienced differently by different individuals" (p.151).

This apparent contradictory way of seeing context seems, in fact, a very good way. On the one hand, context can be viewed as existing because of its own properties, yet on the other hand it will only exist if the individual is able to recognise these properties.

In a rather similar way, but using a psychological perspective, Nunes[5] has given special attention to what she terms the *semantic situation*: a rich place of learning where a child can understand the semantics of the situation. Thus, a *semantic situation* refers to a situation that has meaning for a child. In this sense, the situation does not necessarily have to be inserted in the real world, but it must be a situation in which a child can make a parallel between this situation and his/her everyday life.

In summary, if one is either interested in teaching a child, or in knowing how much this child knows about a specific concept, more important than to insert a child into a real life context, it is to present to this child a semantic situation.

Van Hiele(1986) claims that both the context (the situation that the child is in) and the symbolisation (symbols, signals) are important elements in the formation of structures. The context according to Van Hiele is the subject; geometry, for instance, concerns the study of space. However, he states that the geometric context cannot be informed by explanation, rather it must be done (experienced) by children. Moreover, he observes that frequently, a teacher uses objects of context which have no meaning for the children. This is because "the teacher cannot give those objects sense by means of information" (p.60). Therefore, it is clear that for him context takes on an essential role on the children's learning process.

Further, Van Hiele (Ibid.) advocates that there is a inter-relation between context and symbols, i.e., context defines symbols as well as it is defined by them. Symbol is related here to a signification of the context. However, the

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5 - This approach was given by the author in a personally conversation with me. It is possible to find more about it in Nunes, 1991 and 1993 second part of this discussion (symbolisation) belongs much more to the semiotic field, which will be treated later in this chapter.

Although I agree with Van Hiele's position that learning plays an important part in acquisition of knowledge as well as that context is crucial in the learning process, nevertheless, epistemologically, it seems to me that this is not the end point of the discussion. In my view the question of child development - the psychological part of the question - should come together with the discussion of *how* the child learns.

Nunes (1991) referring to the results obtained from the Carraher, Carraher and Schliemann (1985) work, which involved the problem solving activities arithmetic with oral and written systems, states that the principles which controlled the process of calculation in both practices were the same but there were differences in the functional organisation of problem solving activities which depended on the use of the one or the other system. Nunes concludes that the representational systems, provided by the culture (social context is included in it), influence the functional organisation of the children's activities, but these systems may not be able to do it without the support of particular cultural systems of signs. This means that the same children perform differently when carrying out the same function with support of different systems. However, the representational systems were not able to change the basic psychological process of their users.

In this claim, Nunes is not only emphasising the relevance of the context but mainly the semiotic function as taking an important part in problem solving. Together with Piaget's, Vygotsky's, Van Hiele's and Vergnaud's semiotic ideas, Nunes' ideas will also be discussed in depth in a specific section. However, prior to this I would like to discuss first the role of knowledge, and thus discuss psycho-semiotic ideas both from the learning and the knowledge point of view.

2.3 THE ROLE OF KNOWLEDGE

There is interminable discussion among researchers concerning the source of knowledge, that is, how one can acquire the knowledge from the objective world to the subjective one. Piaget held that knowledge was found neither in the subject nor in the object, but rather in the interaction between the two. From this perspective, objects only become known through a series of successive approximations constructed by a subject during his various activities.

For Piaget (1977), the human being begins with a closed system of structures. The more interactions which exist between subject and object, the greater the possibility of enlarging this system. The result of this interaction brings a new structure into existence. Considering that a variety of structures are built up through individual experiences throughout our development, this implies an effective difference in terms of how a new structure is built up from one person to the next. For example, children who come from different backgrounds, and principally, those who have different interactions with objects (different types of experiences) will probably find themselves at different stages of development, even though they are the same age[6].

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6 - This example also explains that despite being a developmentalist theoretian and consequently writing extensively on stages of development (namely sensorimotor, operational and formal), Piaget's principal focus was not on the age at which certain stages occurred but on the order in which they occurred.

Following this Piagetian line of thought, Inhelder (1974) points out that "*concepts do not develop in closed systems, but are in constant interaction with each other... and it is this interaction that accounts for the child's progress during learning experiences* (P.15). This is a clear position of constructivism.

In Piagetian theory there is no sense in talking about the fragmentation of knowledge - for him, knowledge is situated within larger structures which characterise a stage. Thus, Piaget argues that what seems to be a fragmentation of knowledge is actually a *decalage process*, which is seen horizontally - when it occurs within the same stage of development - and vertically - when it refers to the connection between different stages, i.e., when it is related to the reconstruction of a structure that involves other mental operations.

The way in which a child in the sensory-motor phase understands the notion of space by its movements within his/her house is very different to the notion that an adolescent (supposedly in the formal stage) conceives this same space. Nevertheless both are capable of finding their way around the house. This is a typical example of vertical decalage, where the first child, while able to function in the environment, does not yet show a knowledge of the space and can easily get lost in his house, the second child, the adolescent, will not.

In this viewpoint, knowledge comes from logical operations, which are established during the course of cognitive growth. In turn, this cognitive growth, has a very close relationship with the structures - mental and biological - which develop out of four factors: maturation, physical environment, social environment and equilibration. Piaget (1977) is emphatic in stating that while he is aware of the influence that the social environment can exercise on the process of formation of concepts, his focus of attention lies far more on the development of the child in itself than on the influence of the social environment.

For Piaget (Ibid.), the most important of the four above factors is equilibration, because it embraces the other three factors. In equilibration, three processes are present: assimilation, accommodation and the adaptation between these two processes. Assimilation is described by Piaget as a continual evolution. This allows us to think that after the first assimilation, this process will always occur within already existent structures, making it possible to incorporate new structures to those already in existence.

In order for such incorporation to take place, there must be a link between the old and the new structure. The child creates this link through "theorems-in-action", that is, through the reciprocal interaction of the child and the new structure, which results in the formation of a scheme. The product of this interaction is a new world, in which the child plays a real role.

In relation to the process of accommodation, this is basically internal. Accommodation of a structure which has already been assimilated means changing the structure that already existed in the sense of adding the new characteristics of the object or event acquired form the assimilation that has been made. This is an internal process and as such happens in a different way from child to child and age group to age group.

In summary, according to the Piagetian theory of equilibration, concepts are originated primarily from action and thus from operations. In other words, a concept is the manifestation of an assimilation through the transformation of Piagetian schemes. Nevertheless, according to Piaget, to form a concept it is necessary that the action and the co-ordination of this action (the representation) both occur within a meaningful system. It was in order to explain how the child represents the world in which s/he operates that Piaget carried out a study on symbolic functions, more precisely, on the semiotic function. Due to the complexity of the theme, and the relevance it had for my study, I will discuss the semiotic function later, in a section devoted specially to it.

From what has been discussed above about Piagetian theory, I can point out two principal differences between his developmentalist position and that of Vygotsky. The first difference lies in the question of the influence of the social environment[7] on the child. While this is barely touched on in Piaget, Vygotsky holds that it is in the process of socialisation within his culture that the child learns to understand things that are common to his social experience. The tools, for example, are extremely important parts of the culture and of the social environment. And it is important that the child gains an understanding of the vital tools of our culture, such as the pencil, the rubber, books and the watch.

A second relevant difference between the approaches of the two authors lies in the emphasis they afford biological structures. In Piagetian theory, this is very significant, and is given a deterministic feature. In Vygotsky, on the other hand, this emphasis is reduced and, instead, the child has a far more active role throughout his or her development. Knowledge discussed through a deterministic logic - which is widely taken up by Piaget - is absent from Vygotsky's theory.

Concept formation, according to Vygotsky (1978), goes through three stages: the first stage is called "syncretism" or conglomeration of individual

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7 - I take social environment to include both the cultural environment and the child's individual context.

objects (heap), where the child groups objects without any objective criteria (often by some subjective reason) in a disorganised agglomeration.

In the second phase, the child goes on to operate through "thinking in complexes" in which the isolated objects become associated in the mind of the child not just through his subjective impressions, but also through the relations which really exist between the objects. The principal function of thinking in complexes is, therefore, to establish the relationship between them. This link and relationship, however, are unstable.

To illustrate this stage, the child engaged in thinking in complexes might, in comparing two equal angles, be drawn to different scales, s/he might say that the first angle is different because its sides are longer, or when comparing two different angles might say that they are also different, but this time, because the "gaps" between the two sets of sides are different. This sort of thinking is the starting point for the unification of disorganised impressions, i.e., in organising elements in groups, the child builds a basis for future generalisations.

The third stage in the concept formation is the phase of "potential concepts". This type of concept, while present, to a certain extent, in the phase of thinking in complexes will only solidify later (normally in adolescence). At this stage, children predominately use abstraction and they thus begin to abstract and unify the common characteristics in different objects in a uniform manner.

However, according to Vygotsky, a concept only appears when the abstracted characteristics are synthesised and the result of this synthesis becomes the main instrument of thinking.

Based on his experiments, Vygotsky holds that the social environment (context and culture) and communication (symbolic representations[8]) play a decisive role in concept formation, given that directed thought is social and is expressed through communication[9].

Again, on the question of concept formation, Vergnaud (1987B) did not consider the social aspects in this relationship, although he recognised their importance and interest. Of particular interest here is Vergnaud's focus on the relation between cognitive psychology and the epistemology of mathematics. According to Vergnaud, a concept cannot be isolated from problem-solving, because problem-solving is an integral part of concept formation. This idea can be organised thus: competencies are developed from the starting-point of students action in situations (problem-solving); while conceptions develops from students symbolic expressions (either verbal or other).

As competence is linked to implicit knowledge, it has a strict relationship with the formation of schemes. This leads to conceptions being analysed in terms of objects, their properties and relations. Conceptions can already be verbalised. "Scheme" is employed by Vergnaud in the same sense as it is in Piagetian theory: a scheme is an organised invariant in an action within a certain set of situations. It is the case that while it was Piaget who first introduced the question of the invariant, he himself did not recognise its importance in the development of the child's conceptions in mathematics and in physics. This was left to Vergnaud.

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8 - Following this discussion about the concept formation a need arises for a section dedicated to the question of symbolic representation. This will be dealt with in the next section.

9- The influence of social environment on the concept formation and learning processes, will be also discussed in a later section.

Vergnaud (1978B) argues that in concept formation, the invariants are also present, except that here, instead of appearing in an implicit (or intuitive) form, they are explicit. We might therefore talk of "invariants of schemes" (related to competence, where the child can correctly act on a certain situation) and "invariants of concepts" (related to the efficiency of the behaviour of the child, who is now able to generalise situations).

The connection between schemes and conceptions is built up from four types of elements: invariants of different levels, inferences, rules of action and expectation and predictions. Invariants are largely responsible for the efficiency of schemes, and because of this they are seen as an essential component of schemes. They also participate actively in the process of linkage between competence and conception. They can be described in terms of objects, properties and relations, and, finally, they can also be expressed through words and other symbolic representations.

Once again, in Vergnaud's studies, the question of symbolisation appears as an essential component in the concept formation. I will now go on to discuss this question.

2.3.1 SUMMARY

This section looks at the role of knowledge from different authors' viewpoints. Piaget considers knowledge is acquired from logical operations formed from the individual structures which, in turn, are developed from four factors: maturation, physical environment, social environment and equilibration. Vygotsky centralises his theory on the role of the socialisation. Like Piaget, Vygotsky also proposes a developmental theory, but where Piaget emphasises the equilibration process, Vygotsky gives attention to the social environment. In the Vergnaud viewpoint, knowledge arises from problem-solving. In this case a child, through the formation of schemes, acquires first competence and then the conception. All three authors consider the child's symbolic function to be integral part of the concept formation.

2.4 THE SEMIOTIC FUNCTION

2.4.1 A BRIEF HISTORY

Semiotics is the study of signs or signification. It refers to the analysis of systems of signalling. Its origin goes back to ancient Greece. According to Umberto Eco (1984), the sign was seen as being something that was not evident from the first view, which led us to some conclusion about the existence of something that, in turn, was also not immediately evident. Nevertheless, for the Greeks (with the exception of Aristotle) the theory of signs was not related to intentional or communicative signification. It was Kant who, for the first time, addressed the question of *mental representation* in the study of signs.

Kant (1929), held that the possibility of knowledge was to be found somewhere between intuition and judgement. And he goes further, affirming that it cannot be a product only of an image because an image can never embrace the totality of an object (whether the image is real or mental) which is comprised within a concept. Thus, it would be inadequate to talk of an image as a universal representation, as perception itself is relative. This argument brings us to two important conclusions: firstly that concepts are not images; secondly, that intuitions are not based upon images.

Thinking about the question of mental representation, Kant created the theory of the "schemata" of understanding. This theory separates the "scheme" from the image and suggests that concept formation is articulated within the process of imagination, by reason of the experience of the individual. Thus, one can consider Kant to be the inaugurator of constructivism, at least as regards the question of his epistemology. And there is no doubt that Kantian philosophy has had a profound influence on cognitive psychologists, not least on Piaget. Of modern thinkers, it was he who took on the role of elaborator and reformulator in modern times of the notion that scientific knowledge consists of going further than that which is apparently perceived, bringing to light the "deep structure" of the world.

2.4.2 THE SEMIOTIC FUNCTION AND ITS TERMINOLOGY

Before going further into the psychological questions of representation-signification as essential elements in concept formation, we need to acquaint ourselves with some terms which are central to the study of semiotics.

From the writings of the most modern semiotic authors (Frege, 1892; Pierce, 1940; and Saussure, 1966 among others), from whose work I tried to obtain the common meaning of their theory, one may consider that a *sign* is anything that brings us to a signification, and that *signification* is any expression that is linked to communication (verbal and non-verbal, such as words and gestures). Semiotics normally accepts as true the proposition that the relation between the sign and the signification is mediated by the concept.

Saussure (1966) defines signifier and signified as being the elements of the linguistic sign; the *signifier* being an acoustic or graphic "mark" of the sign, and the *signified*, the meaning (or the concept) that the signifier transmits. Thus, signifier and signified can be understood as the link between sign and signification. Thus, taking the sun as an example of sign, when a child draws a sun s/he is putting a signifier into the sign sun (a yellow circle is usually accepted as a "mark" for the sign sun). Even considering that this child knows that the sun is a star, the sun may signify brightness for this child. However, If this same drawing is shown to another child, s/he probably will recognise that it is a sun because it was drawn respecting the conventions, but perhaps because s/he came from a very hot place s/he may give a different signified for the sun, such as hotness. If both children report that the drawing is a sun this means that the first child could bring a signification for the sign sun, what makes the communication between both children easier, however sun may still has a different signified for each child.

In relation to *meaning*, Sinha (1988) considered it to be a general property of the sign-system and sign-usage. The meaning is formed both by the *representational meaning* defined in terms of the conditions on representation, i.e., related to the object, and by the *contextual meaning* which translates "all non-representational aspects of a signifying situation" (p.49), i.e., related to the idea of how the object is seen, by an individual, in a specific situation. Returning to the example used above, in terms of representational meaning the sun was the same for both children, nevertheless sun had different contextual meaning for them.

I also would like to define what is considered to be the *symbol* in the study of semiotics. Like the object, the icon, the index, the interpretant, the symbol is discussed as being something that belongs to the sign. Thus the *symbolic sign* seems to be related to the object, and is created through rules and/or conventions.

Peirce (1940), complementing the above assertion, holds that symbolic signs are arbitrary (for example, the word 'dog' bears no relation to the hairy, four-footed animal that barks). Sinha (1988) contributes to this argument by affirming that while the sign is arbitrary it is also motivated (i.e., it was chosen for personal reasons justified by considerations of a functional nature).

Up to here I have presented 'sign', 'signification', 'signifier', 'signified', 'meaning', and 'symbol' basically from the semiotic point of view. However, psychology has borrowed these terms as a helpful tool to explain the concept formation. In this case, the symbolic system is studied as a factor which has great influence upon cognitive development. This field of study is called psycho-semiotic. The next section discusses the semiotic function from the viewpoint of psychology.

2.4.3 THE PSYCHO-SEMIOTIC IN THE CONCEPT FORMATION

Piaget devoted three of his books ("Play, dreams and Imitation" (1962), "The Psychology of the Child" (1969) and "The Mental Image of the Child" (1971)) to analyse how children represent to themselves their own actions and the actions of others.

Before presenting and discussing this important part of Piagetian theory, I would like to first make clear how Piaget makes use of certain terms in the theory of signification. For Piaget the "meaning" is the relation between the *signified* (a real object, action or person) and the *signifier* (which is differentiated and specifically assists a representative purpose such as language, mental image, symbolic gesture, etc.). For Piaget signifier could fall into a three-fold classification: an *index* - an integral part of the signifier; a *sign* - a normally arbitrary convention, to which the signified is linked; or a *symbol*, which is "motivated"[10], and, being motivated, can be created by the individual himself. The sign is social, while the symbol may be either social or individual.

According to Piaget (1968), the semiotic function only appears after the sensorimotor phase when the child has become able to form a complete picture of a given situation thanks to the evolution of his capacity for representation[11] This capacity implies a representative evocation of an object or event which is not present. He argues that is possible to perceive the child's evolution through 5 behaviour patterns. While these behaviour patterns involve an increasing level of complexity, they appear almost simultaneously. *Imitation* is the starting-point of representation. The second is *symbolic play*, when the imitative gesture is becoming symbolic. The next step is *drawing*. The fourth behaviour pattern is the *mental image*. This appears as an internalised image. The fifth and last behaviour pattern described by Piaget is *verbal evocation*. This depends directly on the language - considering that the semiotic function separates thinking from action in order to produce mental representation, language has a key role in this formative process[12].

10- By "motivated" I am referring to that to which we are inspired, stimulated or influenced by an impression, feeling or reason.
11 - Representation is conceived here as the presentation, at the level of thought, of something which is absent perceptually.
12 - Representation is formed either by the differentiation of the signifier (which arises from signs that come from language) or by both language and mental image.

In summary, from Piaget's viewpoint (Piaget et al 1968, Furth 1969, 1977) knowledge involves more than simply a description of a thing, it is concerned with operating on this thing. The first aspect of knowledge -- describing things, which Piaget call figurative knowledge -- is present in any perception. It initially arises from imitation, in form of symbol formation, and thus transforms into a mental image "when the symbol becomes internalised" (Furth, 1977 pp. 70). The second aspect of knowledge -- operating on a thing, which Piaget named as operative knowledge -- is concerned with the transformation of the reality states. It involves a logical thought. Furth summarises operative knowledge as "the child's own activity on the outside world" (Ibid., pp. 70).

Two out of the five Piagetian behaviour patterns are relevant to my study. The first is the drawing behaviour pattern which is considered to be inseparable from the child's evolution of spatial concept. Drawing serves as a test of the child's intellectual development. For instance, at about the age of 4, the child draws squares, rectangles, circles etc., in the same way, representing them all with a closed curve, without taking straightness or angles into consideration. At this point, the child is only able to deal with the topological aspect of the drawing. At 7 or 8 years of age, the child is ready to deal with the projective properties. That is, the child becomes conscious that the properties of a given geometrical figure do not change in accordance with how it is projected on a plane (for example, the straight sides of a square always remain straight). Finally, around the age of 10, the child begins to consider the quantities and properties of space, such as angles and length of the sides.

The second is the mental image. Piaget identifies two types of images: the *reproductive image*, which is limited to representing things that have already been seen; and *anticipatory images*, where the child, even when it has not seen the figure previously, imagines movements and transformation and the results of these. While Piaget assumes that the reproductive image can include both static configurations and movements - and, indeed, even transformations - taking the results of his own studies into account, he concludes that the mental image of the child, on a pre-operational level, is basically static.

> *"It is not until the level of concrete operation (after seven to eight) that children are capable of reproducing movements and transformations, and by this stage the child can also anticipate in his imagery movements and transformations. This seems to prove (1) that imaginal reproduction even of well-known movements or transformations involves either anticipation or a reanticipation, and (2) that both reproductive and anticipatory images of movements or transformations depends on the operations that make it possible for the child to understand these processes. The formation of mental images cannot precede understanding".* (Piaget, J., 1969, pp. 71-72)

It seems that Piaget related the child's capacity to work with dynamic perspective with child's cognitive development without considering, for instance the possibility that a child has to learn from significant problem solving (Vergnaud), or from help of other (Vygotsky). Moreover, he did not test children by using activities which involved materials from their culture (from their everyday life). In my study about children's conception of the angle, I have endeavoured to set up activities where children (from 6 to 14 years of age) can assume both dynamic and static perspectives of the angle.

To sum up, in Piagetian theory, the importance of the semiotic function resides in making thinking possible. And it is through the differentiation of the signifier and the signified that representational thought becomes possible. This, in turn, permits the evocation of that which is absent (be it object or event).

Representational thought is, indeed, a necessary condition for the representational act as such. In the first instance, representation is the internalisation of an action (i.e. an effective action and an accommodation); after that it may be a hypothetical action and accommodation.

It is clear from his writings that Piaget believed that the "figurative" aspects of symbolic acquisition and their usage, including language, are subordinate to the child's 'operative' aspect of knowledge. In this sense, the path to child development, in Piagetian theory, takes the following sequence: first the figurative knowledge which starts from child's imitation and egocentric behaviour (the fruits of thinking and egocentric speech[13]), then socialised speech (where the child wants to be understood) and, finally, the appearance of the operative knowledge, when child is able to work with logical thought (if ... then).

This posture makes it clear that Piaget gave little (or no) importance to the influence of the social environment in the cognitive development of the child. Everything appears to take place in an intra-individual process. In this sense, the formation of concepts is considered as an isolated (personal) process in itself, from which it is expected that the development takes place only through internal processes (such as equilibration, schemes, representation, etc.). Consequently, the learning has nothing to do (or help, or interact) with it; the role of the teacher should be only to wait until child reaches the necessary stage of development to be, then, able to learn a specific concept.

Vergnaud (1984, 1987A), took Piaget's arguments further, in suggesting that if a cognitive psychologist wants to understand "what subjects do and what they say", s/he certainly needs to deal with more than a simply duality between

13 - Egocentric thought can be summarized briefly as that type of thinking in which the child is not concerned with being understood.

"representation/ represented". To handle this problem of representation/ symbolisation, Vergnaud makes use of three levels of entities and consequently two problems of correspondence.

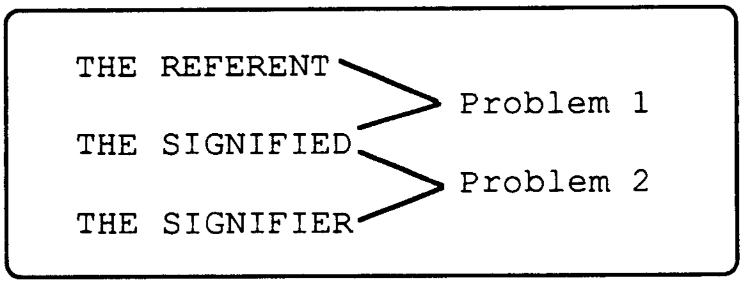

Figure 2.1: A semiotic problem proposed by Vergnaud

Before continuing further with this discussion, it is important to point out that although these terms (referent, signified, signifier) have been borrowed from linguistics, they are being used here within a psychological perspective.

Vergnaud (1987) defines the referent as if it is the real world. It emerges from a person's own experience; The signified is considered the centre of a theory of representation because it is at this level that "invariants are recognised, inferences drawn, action generated, and predictions made" (p.229) -- he refers to this level as being mainly cognitive. Finally, the signifier level is composed of different symbolic systems which present different organisations.

In any communication, for instance, we make use of symbols[14], the signifier level, but the meaning of the communication lies on the signified level. One obvious conclusion is that symbols are social (usually public), while meanings are individual (and very often they may not be the same from one person to another).

In order to clarify Vergnaud's ideas, lets consider a situation in which a child is dealing with a watch. The watch is the *referent* -- an object of the real

14 - Natural language is a symbolic system.

world, whose uses are presented in a child's life; the word "watch" is the *signifier* -- a child is able to relate the word with the object. Different types of watches (different shapes, different length, watches with or without hands, watches with or without numbers) means different signifiers for the same referent. The function of the watch, the way by which child comprehends it, is the *signified.*

Problem 1, referred to above by Vergnaud, seems to refer to an adequate adjustment between the signified level and the real world. This adjustment tends to be imperfect thanks to the subjective side of the representation made on the part of the individual. However, Vergnaud argues that, through some effective action, a person overcomes this problem by adequately representing part of the real world at the signified level.

The alignment between Vergnaud's belief and Piaget's position concerning the process of representation is clear, since both relate this process as having a close relation with child's effective action.

As regards the problem 2, the correspondence between signifier and signified involves the problem of unity of meaning. On the other hand, the correspondence between signified and signifier implies the existence, or non-existence, of a symbol which expresses a cognitive entity.

Thus, it is the number 1 problem, used as an interface and interaction between the referent (the real world) and the signified (individual representation), which will make possible the solution of the number 2 problem.

Although diSessa is not a psychologist, I would like to introduce his idea of "phenomenological primitives" which seems to me as another way of approaching representation and, at the same time, is also a complement to Vergnaud's idea. According to diSessa "representations are shaped by situations encountered and mastered by children"[15]. From the way children behave and then explain their behaviour, it is possible to discover that they have primitive conceptions which were acquired in their first attempt to deal with and master situations in their lives.

diSessa used this approach for conceptions concerning physical phenomena, but Vergnaud believes that it also applies to ideas of addition and subtraction, multiplication, fractions and ratios, functions, transformations and relationships. And in my view it is possible to include ideas of geometry in this range of subjects as well.

Returning to Vergnaud and considering symbolic systems as "conceptual amplifiers" (Lesh's expression, 1978), it is possible to posit three types of interactions producing "representation" in the child's mind:

> "*(1) The referent-signified interaction, in which action, chunks, and invariants of different levels, inferences, rules, and predictions play the main part;*
> *(2) The signified-signifier interaction, in which natural language and other symbolic systems provide aids for identifying invariants, for reasoning, for planning and controlling action;*
> *(3) The interaction between different symbolic systems...*" (1987A, p.232)

Such considerations regards the concept of "theorems-in-action", "schemes", "conceptions", "competence", and "symbolisation system" led

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15 - diSessa's view of Representation involves a mixture of conceptions, know-how, symbols, and signs. For more detail about diSessa's approach, see diSessa, A."Phenomenology and Evolution of Intuition" in Gentner, D. and Stevens, A.L. "Mental Models", Lawrence Erlbaum Associates, 1983, and diSessa, A. "Learning about Knowing", MIT, 1984.

Vergnaud to a comprehensive definition of what a concept consists of. It is a triplet of three sets:

S: *The set of situation, that make the concept meaningful;*
I: *The set of invariants (or theorems-in-action) that are progressively grasped by students, in a hierarchical fashion;*
$\mathcal{T}$: *The set of symbolic representations that can be used to represent these properties and the situations;* (Vergnaud, G. 1987, 1988).

Making a relation between the above scheme and the symbolic system, the S" (set of situation) is the *referent*, the "I" (set of invariants) is the *signified*, and the "$\mathcal{T}$" (set of symbolic representation) is the *signifier*.

Because a concept refers to more than one situation and a situation requires more than one concept, and because there exists a wider range of invariants involved in it, and a variety of symbolic expressions for these invariants, Vergnaud has based his theory on what he calls “conceptual fields”. A conceptual field has been described and illustrated by Vergnaud throughout his work (1984, 1987A, 1988A, 1988B). It can be defined as a set of situations, which requires a diversity of concepts, actions, and symbolic mastery representation, consistently linked to each other.

To sum up, symbolic representation is seen epistemologically, in Vergnaud's approach, as starting from the interaction between the referent and the signified, and thus obtaining the signifier.

Aligned with Vygotsky, I am also well conscious that teacher, colleagues, parents and the social environment could well assist a child in his/her process of learning as well as development.

For that very reason, I do not use solely the Piagetian and Vergnaud theories as my background, despite sharing the basic ideas with them, such as the equilibration and the representation processes, and the concepts of theorems-in-action, schemes, invariant, and internalisation.

2.4.4 PSYCHO-SEMIOTIC IN CONCEPTS FORMATION CONSIDERING THE INFLUENCE OF THE SOCIAL IN THE PROCESS

As I have already said before, Van Hiele's theory has also related the levels of child's thinking with the context and the symbolisation. He states that changing from one level of thinking to the next one involves the experience of context and the forming of symbols for this context. A context involves many different symbols as well as a symbol which is not exclusively used in one context.

According to Van Hiele, the starting-point of symbols is an image, in which the viewed properties and relations are for the time being projected. Through learning, the symbol loses its peculiarity of image and achieves a verbal significance, which means that the symbol becomes more useful for operations involving thinking. From this point of view Van Hiele (and always looking at the learning process) states that:

The first aim in developing the didactic of a certain topic is the formation of symbols belonging to the context of the topic involved. Afterward, those symbols will have to be developed in junctions of a network of relations that determines the second level of the topic. (1986 p.61)

In this sense, the symbol is seen as responsible for the direction of thinking in a topic. At the beginning, the symbol is perceived as representing the totality of the object properties. Later on, through learning process, a child becomes able to compare symbols and from these s/he is able to recognises properties embedded in it.

Van Hiele, however, does not clarify how this process occurs in terms of a child's thinking, and this permits one to analyse the symbolisation process as developing through both internal (intra-psycho) and external process (learning) For myself, it seems to be hard to believe in the idea that any concept can be acquired only via the external factor, such as learning.

Considering the influence of the social, Vygotsky states that each superior psycho-intellectual function shall appear twice during the child's development: first as an inter-psychic function (i.e., as the collective and social activities); and second as intra-psychic functions (i.e., as the individual activities related to the internal properties of child's thinking). The development of language, as described by Vygotsky, is a good example of the above statement. Language firstly arises as a possibility of communication between the child and people around him/her. It is only later that language (converted now into internal language) can be transformed into internal mental function, which will provide the fundamental mechanisms for the child's thought.

By looking at child development as characterised through a gradual internalisation process, the egocentric speech, which precedes the internal speech, is a phenomenon of transition between a child's collective and social activities (the speech for others, the social speech) and a child's individual activities (the speech for oneself, the internal speech). Considering that both internal language and thought arise from the inter-relations between the child and the people surrounding him/her, it is easy to suppose that these same inter-relations are also the source of the child's volitive process[16].

From these reasonable arguments, it seems to me a nonsense to speak about the tendency of child's thought to be absolutely independent from his/her knowledge (in terms of not considering his/her experience and culture).

2.5 FROM SPONTANEOUS TO SCIENTIFIC CONCEPT

To conclude this chapter, I would like to discuss how a spontaneous concept becomes a scientific concept. In other words, how does a child transform those concepts acquired from his/her everyday life into formal concepts. In fact, it was not by accident that I left this issue to the last part of the chapter. From my point of view it is a summary of the previous issues -- with regard either to the learning process or to the acquisition of knowledge -- in the sense that all of them were implicitly discussing how spontaneous and

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16 -Vygotsky refers that the child's ability forward controlling his/her own behaviour begins inside collective games. The volitive process, an internal power which is responsable for the control of voluntary behaviour, will only be developed later.

scientific concepts are acquired by a child. Therefore, the proposal of this section is to synthesise the theoretical background of the present study.

From the point of view of psychology, a concept can be described as an act of generalisation. It involves more than the sum of some associative networks, more than an act of using memory. Concept formation involves intra-psycho functions such as logical memory, abstraction, resolute attention, and capacity of making comparison as well as differentiation.

It is valid not only to spontaneous concepts, but also for those non-spontaneous ones. In this way, I completely agree with Piaget when he considers the first group of concepts as being the child's genuine ideas of reality which are developed through the child's own mental efforts, and the second as being those concepts which are effectively influenced by adults. On the other hand, he was not able to perceive the strong interaction between both type of concepts, which led him to give attention only to the child's spontaneous concepts.

Vygotsky (1962), on the contrary, assumes that the development of both concepts - spontaneous and non-spontaneous - are elements of a unique process, i.e., of concept formation. There is a close relationship between them, and they are continually influenced by each other.

However, both Vygotsky and Piaget stated that when a child is dealing with a spontaneous concept s/he is not aware of it, because his/her attentions is centred on the object instead of his/her own thought. For instance, when a child is riding a bicycle, s/he cannot explain how s/he is doing it, because his/her awareness is centred on the bicycle rather on his/her own motions. The child just knows *how* to ride a bike. This idea of spontaneous concept can also be related to the Van Hiele's idea of "structure" as well as to Vergnaud's ideas of 'theorems-in-action' and 'invariants of scheme'.

In the development of non-spontaneous (scientific) concepts, which are normally acquired in school, the relation between the child and the object is mediated, from the beginning, by previously acquired concepts[17]. This means that the notion of the formation of scientific concept implies a system of concepts (in sense that a scientific concept needs to be related with those acquired spontaneously by child and with those previous formed scientific concept). Vygotsky believes that the transference from spontaneous to scientific concepts is a two way road, i.e., early child's systematisation arises from the child's contact with scientific concepts, and thus they are transferred to the everyday life, as well as it is essential that the child forms spontaneous concepts (acquired from his/her everyday life) for a child to be able to make generalisations.

From the scientific to spontaneous concept there is an up-down change in the child's psychological structures. Spontaneous concepts are formed from concrete situations while scientific concepts are "mediated" by a conceptual field[18]. They present a reverse direction of development. In Vygotsky words: "the development of the child's spontaneous concepts proceeds upward, and the development of his scientific concepts downward" (1962, p.108).

The main difference between the concepts is the property of generalisation presents in scientific concepts. The more a concept can be generalised the more powerful it is. It requires a higher level of thinking, which implies that the child makes use of a system of relations, in which the perception and memory are absent.

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17 - Certainly, it is the teacher who brings to the classroom the scientific concepts, nevertheless for a concept to be acquired it is necessary that the child builds a bridge between what he/she is hearing from the teacher (or from the situation) and what he/she has already known (in an informal way) about it.
18 - The term "conceptual field" is used here as it is in Vergnaud theory, i.e., in terms of a group of inter-related concepts.

The ideas presented in this last section will have a decisive influence on my study. The research will present different settings which allow children to present spontaneous and non-spontaneous concepts.

2.6 SUMMARY

This chapter has discussed the importance of considering knowledge and learning processes as composing two faces of the same coin. It has also argued that either in the learning process or in the concept formation process, a theory of symbolisation, the basis of a child's representation, is necessary. In this sense, the chapter advocates that the meaning of a situation (for a child) has an important role in this process. Finally, the chapter defends the premise that the interaction between a child and his/her social environment also contributes a great deal to the acquisition of a concept. In this perspective the acquisition of both spontaneous and scientific concepts takes on an important role in the concept formation and the transition between one and the other concept should warrant special attention on the part of educational researcher.

CHAPTER 3

GEOMETRY AND THE CONCEPTION OF ANGLE

The purpose of this chapter is to discuss geometry in general and the concept of angle in particular. The chapter first presents, in brief, the origin of geometry and overview of how geometry and, in particular, how an angle is being classified today. Secondly, the chapter describes three types of geometry, as well as discussing the learning and the developmental processes of this subject as proposed by Van Hiele and Piaget. Next, taking into account that the purpose of studying how pupils acquire a concept is to know better how to improve their learning, this chapter also presents and discusses the way in which geometry, and in particular angle, is taught in Brazilian schools. Finally the chapter presents and discusses the findings of research and proposals of specialists in this field.

3.1 A BRIEF HISTORY OF GEOMETRY

According to Bourbaki (1976), the first people to make use of geometry were the ancient Babylonians, who used the notion of angle to calculate the positions of heavenly bodies and to draw up scientific and astrological tables. It was the Babylonians who introduced the unit of measurement of an angle -- the degree -- which we use today. After the Babylonians came the Egyptians who appeared to have restricted their use of geometry to the empirical application of the discipline, probably to solve everyday problems such as how to redraw the boundaries of farm properties which were periodically flooded by the River Nile, to compare areas, design and construct architectural projects and engineering works.

Finally came the Greeks, who not only assimilated the knowledge accumulated by the Egyptians but went further, seeking rigorous deductive proof of laws governing space. Some 300 years before the birth of Christ, the Greek philosopher Euclid wrote 'Elements of Geometry' in thirteen books, which is considered to be one of the classics that has most influenced Western geometrical thought from ancient times until the 19th century. The deductive features of Euclid's 'Elements' were considered until relatively recently to be the ideal model of scientific thinking. Only in the nineteenth century was Euclid's method challenged "when the discovery of non-Euclidean geometry...shattered infallibilist conceit" (Lakatos, 1976, p.139). Nevertheless, the Euclidean method has prevailed in the deductive approach to mathematical proof. What is more, perhaps no book other than the Bible can boast so many editions. Until this century the 'Elements' has been an important part of Western educational philosophy. To illustrate the influence of Euclid's work on education, it was only in the 1960's that Euclidean geometry was replaced by Transformation Geometry[1]. However, this change found some resistance among educationalists and as late as 1977 the Schools' Council stated that one aim of teaching geometry was as "an introduction to the deductive method", citing Euclid's work as the purest example of this (Wynne Willson, 1977, p.17). Up to now this approach is still adopted within the Brazilian geometry curriculum, that is, Euclidean geometry, as with emphasis on proof as needing to the more investigative work with concrete materials.

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1 - "Transformation Geometry" deals with transformations of rotation, reflection, translation and enlargement.

3.2 GEOMETRY AND THE CLASSIFICATION OF THE ANGLE

At the time of the Greeks, Geometry consisted of the study of space. For didactic convenience[2], geometry has been divided into plane geometry, where figures can be drawn on a sheet of paper, and spatial geometry.

Although geometry is seem as a powerful deductive way of thinking, the other side of the coin is that geometrical method also presents a weakness. When a figure has been imagined, or even drawn, for the purpose of guiding thought, it can mislead because it may not correspond to the real world. Freudenthal (1973) discussed three points of geometrical method, each of which has both strengths and weaknesses: firstly, "mastering degenerations", secondly, "the intuitivity of the orientation properties" and, thirdly, "operating with angles". For the purposes of this study, I will discuss only the third point raised by Freudenthal.

As has frequently been stressed in the literature (Heath, 1956, Freudenthal, 1973, Close 1982) no one definition of angle is universally accepted. Nevertheless, the more modern classifications generally includes angle into two categories: static or dynamic.

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2 By "didactic convenience" I am referring only to the teaching of mathematics in schools (primary and secondary), not to the many other types of geometry, such as: projective geometry, analytic geometry, algebric geometry, hyperbolic geometry, and so on.

The definitions that fall within the static category treat angle as a fixed dimension rather like height, weight or distance. In turn, these definitions can be sub-divided into 'ancient', where an angle is considered to be the difference in direction between two straight lines, and 'modern', where an angle is seen as part of a plane contained by two straight lines, also part of the plane, which meet at a given point. The definitions which fall within the dynamic category are considered 'modern' and treat angle as being "the quantity or amount (or the measure) of the rotation necessary to bring one of its sides from its own position to that of the other side without it moving out of the plane containing both." (Heath, 1956, p. 179).

Euclid, who defined a plane angle as being "the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line", is a typical example of an ancient static category. He did not include in his definition either the zero angle, or the straight angle or angles bigger than 180^0. Hilbert (1972) also excluded zero, straight and reflexive angles from his analysis. He previously defined points, lines and planes as 'sets of objects' and ray as the totality of all points of a line which lie on one side of a certain point on the line. He thus adopted a static definition of angle in terms of rays. His definition of angle is a good example of a modern static category. With reference to dynamic definitions, Choquet (1969), for example, treated angles in terms of rotations of the plane p: "For every 0 e p, a rotation about 0 is called an angle with vertex at 0. If (A_1,A_2) is a pair of half-lines whose origin is 0, the rotation about 0 taking A_1 to A_2 is called the angle formed by the pair; it is written A_1A_2." (p.79).

3.3 THREE GEOMETRIES

There are many types of geometry. For the propose of this study I will present only three: Euclidean geometry, Transformation geometry and an Alternative geometry (turtle geometry).

Euclidean geometry, also known as "traditional geometry", is used in nearly all primary and secondary schools in Brazil[3]. In an attempt to summarise Euclidean geometry, we can identify the following characteristics: (a) it presents a deductive character, by means of which it seeks to establish its conclusions with the rigour of absolute logical necessity; (b) it starts from five postulates, laws which refer specifically to geometrical questions which are not proven, but which are considered as true. These are used as basic premises for the elaboration of further geometrical laws.

The concept of angle in the Euclidean geometry is abstract. Its definition refers only to a plane surface. As with any other geometrical topic, the notion of angle is constructed using a logical system based on deductively. It is not constructed from everyday experiences.

Transformation geometry has been used in school in Great Britain, Russia, USA, Germany, Netherlands, and presumably in other countries too, as stated by Lesh (1976) and Willson (1977). Considering that the world about us is not static, when we look at it we notice movement as well as seeing objects. In two dimensions we may look not only at shapes drawn in a plane but also at the rigid motion of the plane - transformations and rotations for instance (Jegger, 1966). The Transformation geometry focuses on mappings, on a

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3 - By "didactic convenience" I am referring only to the teaching of mathematics in schools (primary and secondary), not to the many other types of geometry, such as: projective geometry, analytic geometry, algebric geometry, hyperbolic geometry, and so on.

plane, as the objects of study. These mappings or transformations change some properties of figures in the plane while preserving others. Authors, (Meserve, 1955, Edwards, 1989) argue that it is exactly the properties of the figures which are not changed by a particular set of transformations which allow us to form the geometric concepts. In this geometry, concepts are illustrated and justified through reflections, rotations, translations and enlargements of objects on the plane, which allow students to detect the variants and invariants of a given figure.

Turtle geometry was created from Logo, a computer language derived from the Lisp family, developed by Papert and Feurzig in the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology (MIT) at the end of the 1960's. Logo is specially known for its Turtle geometry, which can be manipulated through a few simple commands: FORWARD and BACK which "*change the turtle's position (the point on the plane where the turtle is located)*", and RIGHT and LEFT which "*change the turtle's heading (the direction in which the turtle is facing)*" (Abelson & diSessa, 1980, p.4). According to Papert (1980), Logo promotes a child's developmental thinking and problem-solving ability. In particular, Logo, through turtle geometry, can be a suitable way to present some geometrical concepts such as angle, similarity and properties of shapes (Papert et al., 1979; Noss, 1985; Kynigos, 1992).

Turtle geometry, unlike Transformation and Euclidean geometries, is not included in any formal school curriculum. Papert (1980) argues that Turtle geometry presents three important characteristics: position, orientation and dynamism, which differs from Euclidean geometry which was constructed upon the notion of the 'point' entity with position as its unique property. Papert further states that:

"Turtle geometry is a different style of doing geometry, just as Euclid's axiomatic style and Descartes' style are different from one another. Euclid's is a logical style. Decartes' is an algebraic style. Turtle geometry is a computational style of geometry" (1980, p.55).

Abelson (1980) lists as main characteristics -- and advantage -- of Turtle geometry its three properties: (a) Turtle geometry is *intrinsic,* that is, depends only on the figure in question, for instance, the four angles of a square is *intrinsic* to the square and Turtle draws a square in any orientation (depends on the turtle's initial heading); (b) Turtle geometry is *local* , that is, takes geometry up a little piece at a time. Drawing a circle with the turtle, for example, one can forget about the rest of the plane concentrating only with the small part of the plane which surrounds the turtle's current position; and (c) Turtle geometry is *procedural.* because it describes geometric objects in terms of procedures (such as interaction) rather than in terms of equations like in the traditional geometric formalism. Abelson completes this last property of turtle stating that: "*Moreover, the procedural descriptions used in turtle geometry are readily modified in many ways. This makes turtle geometry a fruitful arena for mathematical exploration*" (pp. 15).

Kieran (1986) suggested that turtle geometry as constructed through Logo, can be considered as an ideal vehicle to explore both a dynamic approach to angle and its measurement "the rotation for Logo turtle can be considered an example of a dynamic angle; the imput to the turtle turn, as a measurement of that angle." (pp. 99)

Hoyles (in press) stresses that children's action of naming procedures in Logo contributes to their mental construction of geometrical objects, that which is most likely to impel children to an eventual concept change. She is precise about the place occupied by the turtle geometry in geometry:

"In the turtle graphic subset of Logo, the primitives of the language provide tools to draw and to measure length and angle .... angles (are measured) in terms of rotation or turtle turns. Geometrical objects are constructed and modified in direct mode by turtle commands or at high level of abstraction through procedures which are symbolic and formalised."

After having described the above geometries, I consider it important to open this debate by presenting a section in which the learning process of geometry can be discussed.

3.4 LEARNING GEOMETRY

Geometry can be approached formally, that is, making use of logical-deductive thought, which requires strict notation and rigorous proof, or, informally, including the investigation of pattern, visualisation, properties of polyhedral and similarity, and manipulation of concrete materials as suggested by Shaughnessy and Burger (1985). Some authors (Dina Van Hiele, 1957; Freudenthal, 1971, Wirszup, 1976 among others) have stressed that the informal approach is not necessarily in conflict with the formal one and they have proposed using both. This perspective considers that children should start geometry through an informal approach, exploring material from their everyday life, and only after this should a logical deductive geometry be introduced to the students. At this point in time, the teacher should help the student to make a link between informal and formal geometries. In general, informal geometry is frequently used earlier than formal geometry (Van Hiele, 1980 and 1986). I will examine these two proposals later in more detail.

Informal geometry, also termed "experimental geometry", arises from the perspective that geometry is the understanding of space. "The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it" (Freudenthal, 1973, p.403). Mathematicians such as Tatiana Ehrenfest (in Freudenthal, 1973), Freudenthal (1973), and Dina Van Hiele (1957), and others, who hold that the teaching of geometry should start with this informal approach, do not reject deductive geometry but hold that this should be a later step. As Freudenthal (Ibid.) appropriately advocates:

> "*Geometry can only be meaningful if it exploits the relation of geometry to the experienced space...Geometry is one of the best opportunities that exists to learn how to mathematize reality*" (p.407).

Further on, he adds:

> "*..The initiating into geometry meant here is a bottom level activity to prepare the child for higher levels*" (p.408)

In fact, Dina Van Hiele (1957) proposed, and used in her teaching experiments, two stages in teaching geometry: an empirical stage, relating to practical applications, and a later stage involving the development of a logical structure.

It is clear that even those theorists who hold that geometry should be initially learnt by an informal method, also agree that this should serve as the basis for the later learning of formal deductive geometry, where the formal concepts and definitions must be properly learnt.

Finally, we must consider that, in terms of learning, geometrical figures present a dual nature: while geometry is a theory which refers to the physical space -- and in this way it is related to the visual perception as well as inductive thinking -- it is also an autonomous theory with its own axioms, which involves an abstract and deductive thinking. Laborde (in press) treats this duality tracing a differentiation between "drawing" -- which is related to the visual aspects and can expresses only some relevant properties for the problem to be solved -- and figure -- which whilst a material representation, is mainly related to the theoretical concepts.

But how can the transition from one geometry approach to the other be made? Piaget and Van Hiele both set out to study this question and how the child evolves in geometric thinking. They came from different disciplines and approached the question in different ways. The former investigated this issue from the perspective of a developmental psychology, whilst the latter, a mathematics educator, was more concerned with the learning process. I will start by discussing the ideas of Piaget and then go to Van Hiele's point of view.

3.4.1 PIAGET AND CHILDREN'S CONCEPTION OF ANGLE

Piaget contributed two important studies in the field of geometry which are included in his books: 'Child's Conception of Space' (1956) and 'Child's Conception of Geometry' (1960).

In the 'Child's Conception of Geometry', Piaget analysed the process whereby 4 to 10 year old children developed spontaneous measurement and metrical geometry. Piaget's experiment consisted of showing a drawing of two supplementary angles (see Fig 3.1) to the child, and asking him or her to copy it.

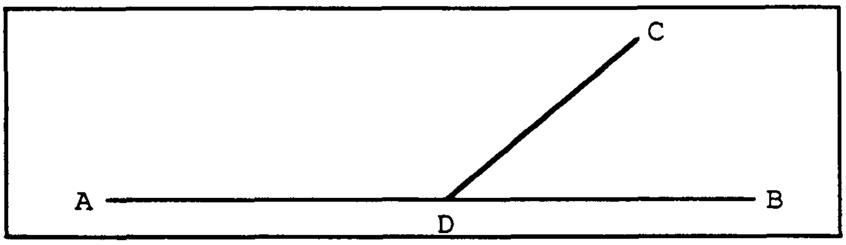

Figure 3.1: drawing used in Piaget's experiment

The children were not permitted to look at the example while they were drawing, but could study and measure it before starting to draw. A ruler, compass, string, etc. were provided. Piaget recognised five main distinct stages within this experiment: Stage IIA, (up to 6 years of age) the figure was drawn from visual estimate, without involving any attempt at measurement; Stage IIB, (about 7 years of age) the angles were still not measured; while the lines AB or CB, or both, were measured, no effort was made to measure the angular separation, except in terms of visual judgement; in Stage IIIA, (about 8 years of age) children were finally able to measure the lines AC and CD in order to fix the point D and to discover the correct inclination of CD; Stage IV, (about 10 years of age) was seen by Piaget as a mark towards the evolution of the formal operational stage. By measuring the perpendicular CK from point C to line AB, children were choosing the calculation of the tangent. In summary, Piaget et. al (1960) claims that the development of the child's conception of a angle involves the understanding of Cartesian geometry, where angles are measured through an imaginary system of axes.

There is no doubt that children from the last stage were using a higher level of imaginative construction. However, it seems that some important questions were not being asked. Is not the calculation of a tangent an accepted convention, which is, an adult convention? How much did the child's previous knowledge and experience from school contribute to dealing with this operation? Did the older children know much more about shapes than younger children? In other words, while the drawing may not have made sense to the younger children, it may have done so to the older ones, because of their previous experience in dealing with triangles, either in or out of school. If so, it would be natural for the older children to try to reproduce the drawing by measuring the triangle DCK.

In summary, from this experiment Piaget concluded that the development of the child's conception of a angle involves the understanding of Cartesian geometry, where angles are measured through an imaginary system of axes.

Although I do not deny the importance of cognitive development in children, it is clear that Piaget's experiment was not concerned with the meaning of the task (the semantic situation to be more precise), as well as the social experience of the child. In fact, his concern was to study children's logical thought, from the point of view of the formal knowledge.

Finally, the type of activity which was carried out in this experiment, does not appear to have been suitable, or sufficient, to investigate the children's conception of angle. In fact, this activity presents a strong similarity to those activities usually proposed by secondary schools when trigonometry is to be taught. From this, I suggest that Piaget was not really interested, or, it would appear, adequately prepared in terms of a research design, to either find the starting-point of the children's conceptions of angle, or to understand the development of it.

In 'The Child's Conception of Space', Piaget had already concluded that a child's evolution of spatial relations proceeds first from the perceptual ability, in which the knowledge of objects results from direct contact with them (figurative aspect of knowledge), and then the representation ability (or mental imagery, related to the operative knowledge), which implies the evocation of objects in their absence. Children develop the former ability around 2 years-old (the sensory-motor stage), while the last is only perfected from around 7 years of age (the operational stage). Within these periods of development Piaget distinguishes a progressive differentiation of geometrical properties, starting with those he calls topological, which in a sense is the child's first spatial intuition. Piaget refers to the topological geometry as being solely qualitative, retaining only the most general properties of space. The second group of properties that Piaget distinguishes are those he terms projective, which involves the ability to predict how an object will appear as viewed from different perspectives. Here the child takes into account the viewpoint which already conserves straight lines. This is, that the child is able to co-ordinate the observation of a straight line from different perspectives. The last group of geometrical properties acquired by child are Euclidean. This permits the generalisation of measurement properties into a system of co-ordinates. The child is now able to conserve distances, angles and parallel lines and, as in projective geometry, straight lines.

According to Piaget, it is at about the age of 4 that squares, rectangles, circles and other shapes are all represented by a closed curve without straightness or angles being taken into consideration. At 7 or 8 years of age the child is ready to deal with projective properties. Finally, at about 10, the child will start thinking in terms of the quantities and properties of space, such as angles and the length of the sides.

Because I believe that the formation of a concept is not simply a case of cognitive development[4], I am not convinced that a child's acquisition of angle necessarily follows this path. I would not feel comfortable agreeing with the Piagetian position before having investigated the influence of different perspectives of angle as described earlier within a range of different activities carried out in different situation by children of different ages.

3.4.1.1 Research Based on Piaget's work

Piaget's work on space has, in fact, been extensively criticised in two ways, from empirical findings and also from a more theoretical argument. I will first consider the theoretical argument by various people who do not agree with him.

Some authors (Weinzweig (1978), Fuson (1978) and Freudenthal (1983)) point out that Piaget's definitions of topological, projective and Euclidean geometries are not mathematically acceptable. The main point claimed by the mathematicians is that Piaget considers only very lowest level topological properties that has little relevance for geometrical concept.

As regards to the geometrical sequence proposed by Piaget (topology, followed by projective and finally Euclidean) Freudenthal (1983) argues that the projective properties referred to by Piaget, such as consideration of the straight line as a vision line, demands a high level of concept formation, which is often not found among adults, who in many cases do not know how to deal with this property.

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4 - It most probably involves many other factors, such as learning, social and cultural contexts, formation of, not just a concept but a conceptual field, as was argued in Chapter 2.

If even some adults do not understand some of the properties of projective geometry, and bearing in mind that according to Piaget it comes earlier than Euclidean geometry, I wonder how many children would be able to reach the former geometry. An empirical example which supports the Freudenthal statement is given below:

In an item on topological equivalence the APU (1980b) secondary survey found that less than 30% of the 15 year old children were able to select the figure which was topologically equivalent to a given figure, however 73% were able to successfully answer the Euclidean item. The researchers interpreted that the children's difficulty may have been due to the unfamiliarity of the terms and the concepts. These results indicate empirically that Piaget's claim that the sequence of the geometrical properties (topological, perspective and Euclidean) do not necessarily occur in this order and we should consider not only the child's cognitive development but also what a student learn from school.

Cosford (1978) argues that the child does not always follow the sequence proposed by Piaget, that is, topology followed by projective geometry and, finally, Euclidean geometry. For example, the topological equivalence might develop after some projective and Euclidean ideas have been acquired.

Freudenthal (1983) is also very firm in his evaluation of Piaget's work in the sphere of geometry. He starts the chapter on geometry in his book by saying:

> "*Even more than Piaget's other work that touches mathematics, it should have deserved from mathematicians serious criticism rather than mere shrugs of the shoulders*" (p.223)

This mathematician focuses his criticism on the Piagetian meaning of space and representation. Freudenthal argues that space, "whether as a mental object or as a concept", is to be seen as the final point of a development, since it will not gain a meaning before the student reaches an advanced mathematical level. The mathematician is clearly against the Piagetian argument which considers the mental object 'space' as responsible for the student's construction of the usual mental objects in geometry. According to Freudenthal, before a child arrives at space as a mental object, s/he must deal with mental objects which are first of all in a context, namely a geometrical context.

Finally, in my opinion, although Piaget gives attention to the child's symbolic representation, as stated in the previous chapter, in the case of the child's conception of angle he seems to consider only the child's acquisitions of angle in terms of invariants and static properties without contextualizing the situation in which the activity was inserted. Moreover, the Piagetian experiment seemed to be concerned much more with disclosing the development of children's effective strategy in the sense of their action, than to making significant progress in the children's symbolic system in terms of understanding the underlying components which were involved in the realisation of this action. Finally, I wonder why Piaget, who stated that the child's cognitive development continues until 15 years of age, should, in his study, consider only children up to 11 years of age? Why is this enough?

Turning now to Piaget's empirical findings, previous work carried out by the present author (Magina, 1988), tested 48 students from the 2nd and 6th grades, 8 and 12 years of age, divided into two experimental (Logo and game) and two control groups. One of their post-test activities included the replication of Piaget's experiment of angle[5] described earlier. Magina found that the best

5- Magina's study is related to the research for her M.A. The experiment involved 4 groups of 12 children. The first 2 groups were formed by children from 2nd grade who results in the post-test were obtained from the two experimental groups, which had received a training in using dynamic perspective of angle. Overall her results were very different from Piaget's. In fact, no students were able to copy the figure at all. However, in the following task in the experiment, when the children were asked to choose the correct definition of an angle from a multiple-choice selection, over half of the sample answered correctly.

From this, Magina drew some conclusions. First that the computer, as a way to explore the dynamic perspective, was an efficient way for children to acquire the concept of angle. The same can not be concluded from the static perspective utilised in Brazilian schools. However it was not clear whether the problem was the teaching of angle through the static perspective itself, or whether it was the quality of Brazilian public schools. Second, that to carry out the Piagetian task a child needed specific knowledge which could only be acquired from school teaching (such as how to use a protractor and rulers as well as understanding co-ordinate axis). So, the fact that a child was not successful in Piaget's task did not necessarily mean that he or she did not have a concept of angle, rather that the child had not yet learnt how to use geometric instruments for measuring (or reproducing) an angle. In conclusion, the author criticised Piaget's work by arguing that he was limited only to a static perspective and largely looking at the task within a framework of Euclidean geometry.

From her findings, Magina suggests that a static view of an angle, line segments drawn on a paper, is hard for students to perceive. It would therefore

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had been trained with LOGO: one group by learning LOGO programming and the other group by playing a game involving rotation, while the remaing groups, one from 2nd grade and the other from the 6th grade, were used only as control groups. The post-test consisted of a paper and pencil test, composed of 8 items which involved comparison between angles, estimation of angle measuremet, prediction of the number of sides needed to close figures of two sides, replication of Piaget's task, and a multi-choice item asking children to choose the correct definition of angle. The study presented as the correct definition that used within the dynamic category.

seem appropriate that more research comparing the dynamic and static perspective, both in a computer and non-computer environments should be carried out.

Fuson and Murray (1978) worked with 2 to 7 year old children studying their ability to recognise four shapes (circle, triangle, square and rhombus) by touch and by their ability to construct and draw them. Disagreeing with Piaget, who claimed that only around the age of 7 years-old a child would his/her representational ability be completely developed, the researchers found that the most of the children were able to identify all the shapes, after they have felt and manipulated each shape behind a screen, by the age of $3^1/_2$ years[6]. However, when these shapes were presented to the children to draw using paper and pencil, even on a generous criterion of success, it was only the 5 year old children who succeeded with the triangle and rhombus. These findings point in the opposite direction from what Piaget said about children's abilities in perception and representation which was that children would necessarily have to have acquired geometrical concepts. In fact, the Fuson and Murray findings suggest that many factors other than the child's cognitive development, such as the child's psychomotricity as well as the length of the objects used in the tasks, can interfere with the results of a given research. This leads the researcher to superficial or precipitated conclusions.

3.4.2 THE VAN HIELE MODEL OF LEARNING GEOMETRY

Van Hiele became known after he had presented a paper containing the main ideas of his thesis at a Mathematics Education conference in France, in

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6 - The authors replied to the Piaget and Inhelder study (1956). However the shapes used by Fuson and Murray were about 10cm longer of those used by Piaget.

1957. The focus of his work was on the different levels of thinking in geometry and the role of insight on learning geometry. Since then, the Van Hiele levels have gained more and more attention among the researchers in Mathematics Education, and nowadays his theory is mentioned in most of the articles concerning the teaching and learning of geometry. Moreover, in the last two decades there has clearly been an increase in geometry research which, in one way or another, is related to his proposal.

Van Hiele proposed that the student learned geometry through a developmental sequence which involved five levels. He argued that progression from one level to the next was more dependent on instructional experience than on age or maturation (in Piagetian terms). According to his theory the child at level 0 recognises names and compares geometric figures; at level 1, recognises the figures by its properties; at level 2 establishes the relation between the properties of a figure and between figures themselves; at level 3 is able to prove theorems deductively; and finally, at level 4 can establish theorems in different postulational systems and analyse and compare these systems.

The mental representation of a child is connected with the Van Hiele levels by making a relationship between the levels and the 'object thought', Nasser (1992) summarises it by given the following example:

> *"...at the Basic Level, the objects of thought are individual figures. At Level 1, the objects of thought are classes of figures, and the student discovers properties of these classes. At Level 2, these properties become the objects of thought, and the student can logically order these properties. At Level 3, the ordering relations become the objects of thought, on which students operate, and at Level 4, the objects of thought are the foundation of these ordering relations."* (p.21)

Wilson (1990), and Wilson and Admas (1992) in their proposal of teaching angle in a dynamic way, summarised the Van Hiele theory, in terms of student's progress in understanding angle, as follows: children may begin discriminating the existence of three angles (or corners) in a triangle, but they do not give attention to any particular properties of these angles. In the next step, the students comprehend that an angle can measure less than a right angle (acute) or more than a right angle (obtuse) and thus start to identify properties and relationships of angle. At the last stage they are able to operate with relationships such as "a triangle cannot have more than one obtuse angle because the three sides must form a close figure" (1990, p.7). In terms of the three type of geometries previously presented, it seems that higher levels of thinking proposed by Van Hiele are concerned with Euclidean geometry.

It is important to point out that although the Van Hiele's levels were directed to teaching, they were also concerned with learning processes, since according to him learning and teaching can be seen as composing the two sides of the same coin. There is no doubt that Van Hiele's work was of great value for the advancement of the investigations into the learning of geometry. His work has served as a guide for many other studies (Hoffer, 1983; Lovett, 1983; Hershkowitz, 1990; Nasser, 1992, among others): some confirm his result, others do not.

3.4.2.1 Research Based on the Van Hiele Model

The research project entitled "The Van Hiele Model of Thinking in Geometry Among Adolescent" (Fuys,D et al., 1988) funded by the National Science Foundation was one of the three Federally funded investigations of the

Van Hiele model from 1980-83. The focus of this research was the conduct and analysis of thirty-two sixth to ninth grade students, aiming to investigate how they learn geometry in light of the Van Hiele model. The project was carried out in three instructional Modules which treated the following geometric topics: in Module 1, basic geometric concepts (parallelism, angle, congruence, etc.) and the properties of quadrilaterals; in Module 2, angle measurement, angle sums of the angles of triangles, quadrilaterals and pentagons, and angle relationships in triangles and parallelograms (i.e., exterior and opposite angles); and in Module 3, area measurement. Clinical interviews were adopted with these modules. The general result supported the idea of a fixed sequence from levels 0, 1 to 2, as well as the consistency of student's level of thinking. The highest level of thinking attained by a student on one topic was also attained in the other topics. As this project reflects findings particularly relevant to my topic, I will further discuss it in a future section.

Hershkowitz (1990), in turn, reported irregularity within and between the Van Hiele's levels. According to her findings, a child operated at different levels according to the context in which the activity took place. Moreover, she noted that a child also could go from one level to another even within the same task. Hershkowitz's study seems to be more concerned with the problem of exploring children's conceptions in different situations rather than obtaining a child's profile of school knowledge and trying to improve this level within the school's curriculum. In other words, her study was oriented to teaching.

The study carried out by Nasser (1992) with Brazilian secondary school students from 13 to 16 years of age, showed that half the students in her sample were below Van Hiele's level 1. After a six month program with these children, Nasser found that half of the sample upgraded their levels. For Nasser this progress was probable because the instructional material developed in her study which gave "opportunity to students to cope more easily with tasks requiring higher levels than one attained" (p.272). Unlike Hershkowitz's study, this research was oriented to the teaching process.

De Villier (1987), based on his own findings, argues that hierarchical thinking is more dependent on the strategy of teaching used rather than the attained Van Hiele level. This line of argument is shared by Malan (1986) who, after an experiment involving 14 students who went through an alternative teaching program, hypothesised that instead of being seen as "prescriptive", Van Hiele levels should be seen more as "descriptive" of a given result. He concludes: "It is therefore possibly only 'prescriptive' in so far as the traditional approach" (p.20).

Whether the Van Hiele levels are a universal model of the way a child attains geometrical concepts, as believed by Nasser among others, or are only one of many possible approaches to learning geometry, as stated by De Villier and Malan, is an open question. What concerns us here is the contribution of Van Hiele's work to the process of learning geometry. Based on his experiments, Van Hiele discussed the importance of context[7]. He also proposed that the initiation into geometry should be based around objects from the real world, which would allow the students to grasp the space where they live. This appears not to happen if the students start to learn geometry through an abstract and deductive way as expressed in Euclidean geometry. However, perhaps because of Van Hiele's background, his model of thinking is directed to the school's curriculum. He does not discuss the formation of concepts, the student's spontaneous geometrical concepts, and their interrelationship with scientific concepts.

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7 He clearly states that the teacher has to create a geometrical context for the student.

3.4.3 COMPARISON PIAGETIAN AND VAN HIELE WORKS

When comparing Piagetian and Van Hiele's works, the first thing that we should take into consideration is the fact that while Vane Hiele is interested in the process of learning, Piaget is interested in knowledge. In the light of this postulation, everything falls into place, because Piaget, in his experiments, was only looking for absolute knowledge[8], which the child had at each age, without worrying about how it had been acquired (for example, at school), nor how the child could acquire new knowledge from a learning process.

One similarity between the two authors is the fact that both present stages which a child must follow. In this way, both advocate a theory of development. However, the transition between stages is differently proposed. Whilst Van Hiele distinguishes phases of a process of learning, which can be reached and even accelerated from teaching, Piaget's theory, whether of the child's conception of space or of any other acquisition of concepts, is described in terms of internal structures. The fact, as was stated in the previous chapter, that Piaget considered that concept formation arises from an action performed by the child, but this does not mean that the teacher can lead the child (either through teaching or through carrying out tasks) to develop faster.

Again, where Piaget and Van Hiele are similar is that neither took into account the influence of informal knowledge of the children's conceptions of angle. In other words, when they examined the child's performance, whether it was related to the learning process or to the knowledge process, they had as a

8 - By absolute knowledge I mean the understanding that the child had of the situation presented to him by Piaget. Piaget's usual method was to set up situations which were not related to everyday life so that the child would not make associations with tasks which he or she had already experienced previously. However, it is clear that older children (whom Piaget considered to be in the logic formal stage) would make use of previously acquired formal concepts.

referent the school geometry as opposed to informal geometry. As a consequence, valuable information is lost, because mathematics conceptions, at least in terms of schemes and the formation of invariants, can frequently arise from interaction with everyday life (or in the street, as it is referred to by Carraher and her group in the context of calculation).

3.5 SUMMARY

Up to this point, this chapter has shown how geometry has been classified, didactically, in spatial and plane geometries. In respect of the subject, the angle, I have shown that while there is no single definition for it, the definitions that exist can be categorised into: static and dynamic. In relation to the types of geometry - Euclidean geometry, Transformation geometry and Turtle geometry - were presented with their main characteristics. I have discussed the fact that geometry can be approached both in a formal way -- by using logical-deductive thinking -- and informally -- by exploring the physical environment -- pointing to the importance of starting geometry through the informal approach, by using objects which have meaning for the children. These three geometries defined the concept of angle in different ways: Euclidean geometry, considered the most traditional and formal, makes use of the static perspective and the concept of angle and is formulated from the paper and pencil environment; Transformation geometry, which can be viewed as a formal or an informal geometry, makes use of both static and dynamic perspectives of angle, and in which the concept of angle is to be built up from the concrete materials of the everyday life. Finally, Turtle geometry, most used in an informal geometry, takes the dynamic perspective where the concept of angle is acquired through the rotation of the turtle.

In conclusion, the chapter presented two of the best known and respected studies of how children develop and learn geometry -- those of Piaget and Van Hiele -- whose work has particularly influenced the design of this research.

3.6 THE BRAZILIAN CURRICULUM

This research was carried out in Brazil, which is culturally different from the United Kingdom as well as having a different educational system. Thus, before going on to discuss the place of geometry in the Brazilian curriculum, I will briefly describe the Brazilian educational system.

3.6.1. THE EDUCATIONAL SYSTEM IN BRAZIL

From the beginning of the 1970's, the Brazilian educational system began to be based on the US model. It starts with what is called the "first grade" -- eight academic years which are in turn divided into "elementary school" for the first 4 years, and "middle school" for the last 4 years. "Second grade" consists of three academic years similar to what the US system calls "high school". Before starting first grade, children must spend one year at literacy class where they are expected to learn how to read and write. Brazilian students are continually assessed by the class teacher. At the end of each year, it is the teacher who decides, based on his or her observations and on test marks, which students will be going up to the next schooling year and which will have to stay behind and take the academic year over again.

Despite the fact that Brazil and England have different educational systems, the best way to present a clear idea of what is meant by "elementary school", is to regard it as corresponding to an English Primary school. At this stage of Brazilian schooling, children have only one teacher who is responsible for introducing them to the following subjects: Mathematics, Portuguese, Science and Social Studies (Geography and History). It is only in the middle school that the different subject matters are taught by individual teachers.

During the years of the elementary school, children are introduced to the topic of angle although it is basically confined to shapes. The concept of measurement is only introduced, as an independent concept, in the later elementary grades. Neither in the first grade nor in the second grade can the students exercise any choice in the range of subjects they study -- all are obligatory.

Schooling is only compulsory until the end of the first grade. In state schools, the school day consists of four hours per day, five days a week. Free meals are provided half way through the four hour period. For poor children this is often the only meal they will get; many children continue going to school just in order to get the free meal.

At the present time, the education offered by the state system is of low quality due both to a shortage of teachers, who are very poorly paid, and to the scarcity of teaching materials. To have an idea of the extension of the crisis in the Brazilian educational system, it is only necessary to go in to anyone of the hundreds state schools placed in periphery of the Brazilian cities to be able to verify the absence of essential materials (such as chalk, water, school desk) through to auxiliary didactic materials (such as geographic maps, complementary exercise books for the students, auxiliary books to help teachers to elaborate their lessons). All of this has lead the educational system to a situation where strikes are a regular occurrence throughout the year. Brazil's adoption of a grading system in which the focus is on the grade rather than the age of the child has given rise to a serious problem in the state education system: the high number of children who fail the end of year assessment and are forced to repeat the year, often more than once. To illustrate how serious this problem is, out of 1,000 children who started school in the 1st year of the early elementary school, only 34 reached the 8th year of the later elementary school in eight years[9]. Even over a period of 20 years of study, only 435 of these 1,000 children managed to complete the compulsory eight year course continuum. Such a hopeless situation at school, combined with shortage of food and money at home, leads many poorer children to abandon school at an early age in order to beg or 'work' (in the informal economy, such as selling fruit and trinkets in the streets). And while school is, by law, compulsory, in practice, there is no control over truancy and when children drop out of school their parents are not even questioned about it.

3.6.2. ANGLE IN THE BRAZILIAN CURRICULUM

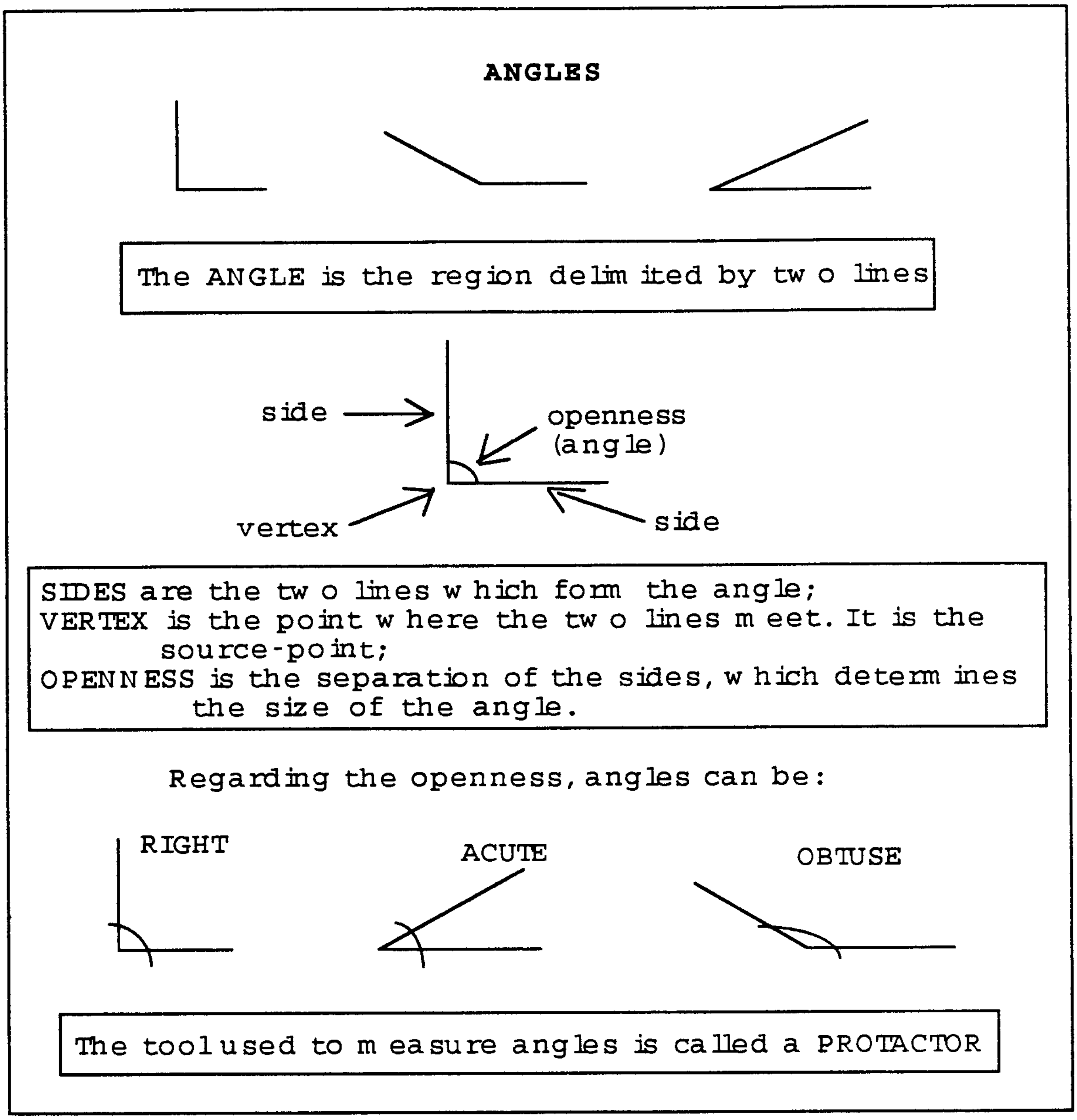
While Brazil does not have a national curriculum -- which means that there are differences in teaching from one geographical region to another and, within the same region, from one state to another -- it is possible to ascertain,

9 - These data were supplied by Klein, r & Ribeiro, S.C. In their report "Censo Educacional e o Modelo de Fluxo: o Problema de Repetência", ("The Educational Census and the Flow Model") produced for the National Laboratory of Computing Science - CNPq, and published by NUPES-USP (University of São Paulo) in January of 1992.

with a certain precision, what is taught during each school year through the textbooks which are specifically written for each year. In the elementary school, geometry appears as a subject in the textbooks but is taught non-systematically. Elementary school teachers do not receive special training in mathematics. They usually do not have sufficient knowledge of geometry to teach it properly and frequently express dislike of the subject. Consequently, geometry is often simply avoided, especially in state schools[10].

Nevertheless, the geometry programme which appears in the books to be taught to these school years concerns, basically, the topological geometry. In the first year, teachers introduce the ideas of open and closed curves, the recognition of the triangles, circles, squares and rectangles. In the second school year, the idea of a line segment, the recognition of polygons and three-dimensional figures (cone, cube, cylinder) are presented. In the third year definitions of lines, segments and angles, start to be taught as well as some geometric properties of figures (such as: a quadrilateral figure has four sides, has four angles and four vertexes). In the last year of the elementary school, geometry consists solely of revision of what has been taught in the three previous years, that is, geometric properties and definitions backed up with new exercises and examples. I have copied (and translated) the example below from a third year book in order to illustrate how angle is firstly presented to students.

10 - For more detail see Nasser (1992) who discusses this situation in depth.



EXAMPLE 3.1: Angle first presented to the students (Souza, 1986)

From the above example I would like to stress the way by which the figures are drawn, i.e., presenting one of the sides in the horizontal position, opening to the right and up side. Another important point to notice is the book only showed the static definition of angle.

From the first year of the middle school (i.e., the "fifth year of schooling"), the geometric notions taught are based on Euclidean geometry, introducing the notions of point, line and plane. And in the sixth year, the geometry course begins with the definition of the angle. The most usual one is:

> **An angle is the region between two lines which have the same origin and which are not opposite to each other** (Iezzi, G. et al, 6th grade, undated but in current use in schools, p. 198)

EXAMPLE 3.2: Usual definition of angle in the Brazilian textbooks at 6th grade.

Continuing to present how the definitions of angle are frequently transmitted to the Brazilian students, I'd like to complete the above example enumerating some of the definitions of angle found in the textbooks utilised for 7th grade. (The translation is mine).

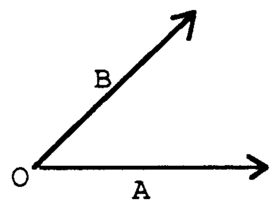
> **The angle is the region in a plane limited by two straight lines of the same origin.** (Volpino, 7th year, undated)
>
> **Considering OA and OB to be two distinct lines from the same origin O. The region of the plane delimited by these two lines is called the angle** (Domenico, 7th year, undated).
>
> **An angle is the geometric figure drawn by two lines from the same origin which are not coincident.** (Giovanni, 7th year, 1985).

EXAMPLE 3.3: Usual definition of angle in the Brazilian textbooks at 7th grade.

In the above examples all the notions of angle are classified in the category of static definitions. It is worth examining the definition that is given to children in the 4th year, "the angle is the region delimited by two lines" (example

1). Such a definition could well lead the child to the misconception that an angle is the area enclosed by two lines and that, therefore, if one increases the length of the lines of the figure, the region will grow and, consequently, the angle.

It is also in the sixth year that angle measuring operations begin to be tackled. Thus, the student is expected to learn about the unit of measurement, the "degree" (such as adding them up and diminishing them) and also learns how to use a protractor. The following example is to illustrate the type of exercise given to the student at this stage.

 AOB = ... (Iezzi, 6th year)	Complete: a) 1 degree has ...... minutes. b) 1 minute has ...... seconds. c) 1 degree has ...... seconds. (Iezzi 6th year)

EXAMPLE 3.4: Usual angle activities contained in the Brazilian textbook for 6th grade students.

In the seventh grade the geometry program is far more extensive, consisting of (a) angles: measurement, classification, vertically opposed and adjacent angles, etc.; (b) angles formed by parallel lines with a transversal; (c) polygons: nomenclature, sum of the internal angles, number of diagonals; (d) circle and circumference: arcs and angles.

To complete this overview of the Brazilian textbooks in current use in the state schools, I would like to show some examples of exercises that the students are asked to do in the seventh year of schooling:

Given the angle MON, answer 'true' or 'false', judging whether each point is internal or external the given angle:

a) Point A is internal to angle MON

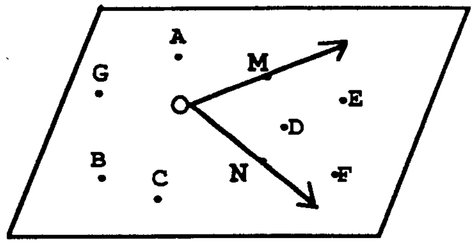
b) Point B is external to angle MON

c) Point C is external to angle MON

d) Point D is external to angle MON

e) Point E is internal to angle MON

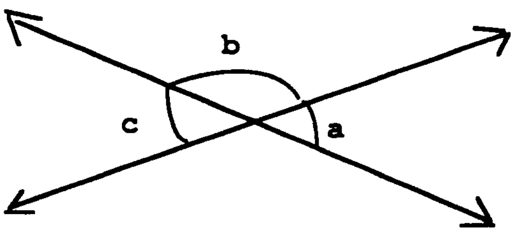
f) Point F is external to angle MON



(Volpino, 7th year)

EXAMPLE 3.5: Usual angle activities contained in the Brazilian textbook for 7th grade students.

Find out the measurement of angles **a**, **b** and **c** in the figure, given that **a** equals one sixth of **b** plus half of **c**.



SOLUTION:

a = b/6 + c/2	a = c
a+b = 180	
a = b/6 + a/2	b = 3a
a + 3a = 180	a = 45
a = c	c = 45
b = 3a	b = 135

(Goulart, 7th year)

EXAMPLE 3.6: Another usual angle activities contained in the Brazilian textbook for7th grade students.

The curriculum proposed in the eighth grade books is generally the following: (a) proportional segments and the Thales theorem; (b) similar triangles; (c) metric relations in a right angled triangle; Pythagorean' theorem; (d) metric relations in a circle; (e) area of plane figures.

From the above examples obtained from the textbooks, the lack of contextualization of the exercises can be noted. Angles appear, as do the rest of the school topics, without being related, in any way, to the child's daily experience of life. On top of that, it is possible to note (principally in example 6) that the main objective of the school when teaching the angle, seems to be to give the pupil as much practice as possible in solving algebraic problems and no visual dynamic dimension is presented. In other words, a clear trend towards the "algebraization" of geometry can be seen, where a pupil could solve the problem perfectly well without even looking at the figure, let alone trying to conceive it in space.

This is not a new debate. Freudenthal (1973) has exhaustively discussed the school inclination to be careless about geometry in favour of algebra. Hoyles (in press) goes further in this debate arguing that

> "*One consequence of the algebraisation of school geometry is that calculation and algebric manipulation become the focus of activity to the neglect of visual reasoning and the mobilisation of geometrical skills*".

The way that Brazilian school has been teaching angle, only working in a static perspective, may be leading the students to some well known misconceptions such as that the size of the angle is determined by the length of the rays of the angle and that one side of the angle must be in a horizontal position. I can also note a clear tendency for schools to work only with angles under 180 degrees "which probably gives the pupil the erroneous idea that every angle is delimited by a concave figure". (Magina,S. 1988, p.6).

It was through the things that I have been discussing up to this point, that my study was designed. In other words, I was wondering whether by allowing the students, from the begin of obligatory school until the end of it, to experience situations in which an angle could be seen not only in a static way but also in rotation, or turns, I would be likely obtain enough information, which could contribute to shedding some light on the understanding of the children's conceptions of angle, from its starting-point to its development.

3.7 RESEARCH INTO THE CONCEPT OF ANGLE

Up until the beginning of the 1980's, when one looked at the research carried out in the area of Mathematics Education, there were very few studies devoted to geometry. However, this picture has changed during the last decade. An example of this change is illustrated in the proceedings of the Annual Conferences on the Psychology of Mathematics Education (PME), where, in 1980, only three papers on Geometry were presented. By 1989 the number had risen to 20 and, by 1991, to 27. The majority of these papers were related to Van Hiele's theory, or investigations using Logo and microcomputers in the teaching and learning of Geometry. Concerned with my specific matter, angle, I found only works in which activities were carried out through the paper and pencil setting or through the Logo setting, or using both settings.

3.7.1 PAPER AND PENCIL ENVIRONMENT

In the United Kingdom, some of the most substantial items of research in the area of Mathematics Education were the surveys carried out between

1978 and 1982 by the Assessment Performance Unit Project (APU), which involved, every year, an average of 13,000 children, between 11 and 15 years, in England, Wales and Northern Ireland. The results obtained by the APU have been examined and discussed in later publications (DES (1980a, 1980b, 1981a, 1982b), Mason, K. (1987)).

The principal findings with regard to the question of angle were: (a) about 60% of 11 year-old pupils could recognise correctly a right angle, while only 15% of this age children were able to judge the correct size of a 120 degree angle; (b) only 43% of the pupils of this age were able to measure an angle of 60 degrees using a protractor; (c) only 4% of 15 year old defined an angle as an amount of turn or rotation.

From the reports of the APU Project, some important points should be noted: (1) in relation to the topic of angle, all the activities refer to written paper and pencil tests, in which the tasks of recognition, estimates, measures, and comparisons of angles were done in a static perspective and based on Euclid geometry; (2) only older children (15 years) were asked to explain in words what an angle is, and what these children answered was far from a dynamic perspective; (3) the younger children found it far more difficult to work with angles of more than 90 degrees than with acute angles and (4) there was a difference between the pupil's performances in recognising the value of an angle and knowing how to construct an angle, in that they were better able to recognise than they were to draw.

I wonder what APU survey would obtain if the project had also included tasks exploring the dynamic perspective as well as tasks related to students' everyday life, such as activities exploring the rotation of watch hands, or swinging doors, or walking around, or turning a turnstile arms and so on. Although surveys like this are important to show how far children's conceptions are from school knowledge, this survey could not present an alternative way of teaching angle, because it did not test children in the another perspective of angle, nor in any other situations in which children could have a conception, or at least scheme, of angle.

Another important research in this field comes from Close (1982). She applied her research in two stages, firstly with children of a primary school and then with students of a secondary school. The whole sample was again tested only with a paper and pencil test and subsequently some students were interviewed later. The interviews were used as a complementary method of investigation in which students were asked to explain their answers in the written test as well as to measure angle by using a full protract.

From the results, Close identified the attributes of angle affecting its sizes, "such as arm length and positioning on the page (including orientation)" (p.30) as the children's most serious misconceptions . She also noted that children perceived angle only by the static perspective. Close still identified one misconception which may have more cultural roots being related to spoken English. Many students named 180^0 as a full turn. Close interpreted that this misconception was caused by the general usage of the expression 'turned round'. 'Turned round' refers to a half turn, however the word seems to be referring to a full turn.

The research project "An Investigation of the Van Hiele Model of Thinking in Geometry Among Adolescents" carried out between 1980 and 1983 in USA, by Fuys et al (1988) also presented interesting findings from students' (mis)conceptions of angle. The project described factors that affected the performance of some students, limiting their progress within one level or to a higher level of thinking. The one factor was the students' geometric vocabulary, which appeared to be limited. For example, some students used the words 'point', 'vertex' or 'triangle' to refer to an angle; they also used the words 'straight' and 'right triangle' to refer to a right angle. The authors suggested the lack of use of the word 'angle' because of the influence of everyday language upon the students' vocabulary. A possible explanation of the students inclination to use the word 'triangle' instead of 'angle' was thought to be the preference for handling the gestalt of closed finite regions rather than open infinite space.

For twice, in Close and now in Fuys, children's misconceptions appeared related to the children' symbolic representation, where the signification of a sign was transformed by the influence of their cultural environment. This indicates that the children' spontaneous knowledge (the knowledge which is built up from the children's context and is related to their everyday experiences) emerged from the symbolic system (in both cases expressed in natural language) forming 'invariants of schemes' (as referred to by Vergnaud) which, in turn, was expressed by children's 'competence' whilst they were solving the problems. In summary, the children's behaviour in the above works seems to be referring to the Vergnaud semiotic problem (1987A) posed in Chapter 2.

Another fact that affected the performance of some students, was their misconceptions concerned with earlier learning. In that project, students said "an angle must have one horizontal ray", or "a right angle is an angle that points to the right", which the authors realised was the result of the students previous experiences with these figures, either in textbooks or in teachers' illustrations. This "may have been limited to specific orientations" (p.137).

I am inclined to interpret this result, once again, from the symbolic representation viewpoint, where the students' vocabulary, based on their everyday life, may have influenced their performance. Thus, the spontaneous knowledge, acquired from the context, was probably exerting more influence over children's actions, in terms of its present meaning, than what had been learnt in school. For example, a triangle is the shape of many things students see around them. On the other hand an angle for them represents an abstract definition. Pursuing this point of view, a student therefore can recognise correctly a triangle taking into account its angles by constructing a theorem-in-action, (it is needed to have three corners to be a triangle). In this case spontaneous concept was acting over the scientific concept (formal knowledge). However, the main question is: can one affirm that a student who behaves like this is not perceiving and understanding what an angle is? Putting it another way, does s/he not know angle or does s/he just not know the formal terminology and definition asked for in formal learning?

Students' misconceptions, it seems, according to the researchers of the project, are influenced by the school's teaching which is biasing the students' formation of the angle concept. According to Vergnaud (1982,1984), for students to form a conceptual field what is needed is a set of situations in which the set of invariants can be distinguished. In this case, if the student is presented only with angles in an upright orientation, this specific orientation becomes an invariant of angle, and s/he can only conclude that an angle has to point to the right as well as has to have one horizontal ray.

Finally, we also should consider that either in Close or Fuys research we are probably faced with a problem of students' symbolic system, where the way that students were looking at the experimental activities had to do with their representational system. We cannot forget that representation is the first step toward the symbolic system formation as was discussed in Chapter 2.

3.7.2 LOGO ENVIRONMENT

The review of literature has shown me that the children's learning process on the concept of angle has gained more attention in the last eight years through Logo. In fact, considering the many important projects and dissertations which have been carried out exploring geometry, the majority of them has included angle as a topic to be investigated (Noss, 1985; Magina, 1988; Kynigos, 1989). However, although it seems evident that Logo is a suitable vehicle to improve children's conception of angle, it is still not clear how much of the positive effect of microworlds in terms of making a link between 'drawing' and 'figure', i.e., how much a Logo microworld can provide aids for children to identify from their own schemes those invariants which will be the bridge for the acquisition of their scientific concepts.

In this way we find in Lehrer & Smith (1986), who stated that by an "adequate instruction in Logo", half of a third-grade class could spontaneously recognise angle as one of the important properties of a shape. We also have the Noss' study (1987) which showed that Logo significantly affects children's concept of angle, mainly in relation to measurement and conservation. In this same line, Kieran (1986), who investigated how children from 10 to 12 years of age developed their understanding of the angle through the use of Logo, concluded that children present both static and dynamic representations, depending on what sort of question they were tackling. According to Kieran, children seemed to keep these representations in different "mental compartments".

Some other researchers are not as optimistic as the previous authors. Hoyles and Sutherland (1989) for instance, noted from their findings in wider research project and, principally, from a detailed analysis of four children working in pairs, that the children made use of Logo's rotation commands without necessarily reflecting on what this represented in terms of angle. They held that the intervention of a teacher is necessary in order to enable the children to sort out their confusion as to what the turns of the turtle mean.

Carraher & Meira (1989) tested children after 15 and 30 hours of Logo training and found no evidence that Logo turn commands changed the students' semantic structure. Moreover, the authors claim that the changes in the students' semantic structures actually follow the same path as described by Piaget in his work "Child's Conception of Geometry" (1960). Setting aside the issue of 'efficiency' of Logo in exploring angle, it unquestionably offers a context in which to research a different view of angle.

Cope & Smith have stressed in several studies (1990, 1991, 1992) that with Logo children may acquire some angle misconceptions, such as confusion between internal and external angle, "which is not amenable to conventional teacher intervention..." (1990, p.16). They argue that the comprehension of the nature of turtle rotation as well as its effects is not a simple activity in Logo. From this, they suggest the construction of microworlds accomplished with effective teaching as a way to minimising the children's misconceptions.

Another important contribution comes from Clements and Battista works (1989, 1990). These author are clearly favourable to the use of Logo as a good tool to help children to understand angle. Aiming to investigate changes in children's mathematical knowledge, the authors (1990) tested 12 fourth grade students in two 40 minutes sessions per week for 40 sessions. The results indicated that children's geometrical conceptualisations and even their geometric thinking were enriched by the Logo environment

Kynigos' investigation (1992) of children's ideas of geometry through Logo microworlds, corroborates the affirmation of Papert (1980) and Lawler (1985) that the understanding that children have of turtle geometry is acquired through intuition. However, further on Kynigos draws our attention to the fact that while the use that children make of the intrinsic scheme (the use of turtle metaphor) "did not entail the use of geometrical notions, that is, the ideas embedded in the microworlds" (p. 116), this did not mean that the children had not made use of geometric concepts both before and during the research. Kynigos (1992) states that the Logo microworld:

> "*is the opportunity to form inductively developed understandings of geometrical ideas before they (children) are required to use these ideas in the deductive geometries of conventional curricula*". (p. 121)

Finally, In recent publications Hoyles (1992, in press), who has been working with Logo for the last decade, evaluates that the power of a Logo microworld has as much to do with which theory of learning are underlying its use, as with the aim of the teaching.

3.7.3 PAPER AND PENCIL AND LOGO ENVIRONMENT

As regarding the research which tested children in both paper and pencil and Logo settings tracing an comparison of their performances in relation to angle problem solving, there are also controversies among the researchers. On one hand we find works which point out that children who have previously had some training in Logo, obtain better results in a paper and pencil test compared with those children who did not have this experience. An example of this is Noss' work (1985) who compared the children's result from Logo tasks with the result of a paper and pencil test carried out by a control group. Both tests involved angle conservation and measurement. He found a significant difference in favour of the Logo children in both topics.

Another example which shows positive effect in favour of Logo comes from Magina's and Hoyles works (1991). They compared the results of a pilot study carried out in Brazil and in England which involved activities developed in the Logo setting as well as in the paper and pencil, and they found that the children performed better in Logo than in the paper and pencil setting for all the activities.

On the other hand, the recent findings of Simmons & Cope (1993) point to an opposite direction. The authors compared children's responses of rotation/angle solving problems carried out in both paper and pencil and Logo microworld, and they reported that the Logo turtle was better than paper and pencil only at the beginning, where children were able to solve problems at a low level. Nevertheless it inhibited children to move to a higher level of response as could verified in children's paper and pencil responses. Although my intention is not to disregard the results of these authors, I am not convinced that they actually created a microworld, on the contrary, as far as I could perceive they just presented, in brief, a few turtle commands and thus asked children to make specific angles and rotation. On the other hand, in the paper and pencil situation they gave protractors (a familiar tool) to these children and asked them to trace specific lines and angle on the paper. It seems obvious that children would be better in paper and pencil tasks (an environment which they were used to produce and reproduce angles) than in those carried out in the Logo situation.

3.7.4 EVERYDAY ENVIRONMENT

From the last decade one can find many important works based on the constructivism theory, which have been investigated using the individual's own, everyday setting in order to study mathematical conceptions (Fahrmeier, 1984; Scribner, 1986; Saxe, 1991; Nunes et al 1993). Although all these studies are, like mine, concerned with comparison between the formal and the informal approaches to mathematics, they explore the children's concept of calculations. In fact, I could not find in the literature any work in which the concept of angle has been studied using activities based on the everyday life of the individuals. I cite Bishop (1978, 1983) as one author who did research in geometry using for this the individuals' own, everyday life setting. He investigated spatial representation and the concept of area in geometry in Papua, New Guinea. I also found in Mukhopadhyay's study (1987) an example of an experiment exploring geometrical concepts.

However what is of more interest here is that all these studies suggest a strong influence of representational systems, culturally learnt from the everyday life of the people. It is probably because the difference among these systems that people from distinct cultures perform very differently one to the other when they are doing the same activity, or even, as it is well stated by Nunes (1991), it is for this reason that "the same people perform differently when carrying out the same function with the support of different systems."

The above works, which have studied mathematical concepts in different settings, present as a common point that they be carried out in the everyday life setting. Nunes et al (1993), for instance, giving a summary of their research which she and her group produced in the 1980's, report a study on proportionality carried out in a small community of fishermen, as well as the research on arithmetic carried out among professional carpenters in their local place of work, and the research on calculation carried out among seller children on the streets. The published research which was done by Nunes et all presents, as the main characteristics, an investigation involving both the spontaneous concepts, acquired from situations experienced in the everyday life setting, and the scientific concepts acquired from school knowledge. In these studies the author could compare subjects' performances when the tasks were related to their everyday setting with their performances when the tasks were presented following the traditional school situation (unprovided of any semantic situation). These studies were rooted in the constructivist approach, mainly in the social-constructivism as argued by Vygotsky and in the conceptual fields as stated by Vergnaud. Influenced by the studies of Nunes' group, I included in the root of my research the ideas of Vygotsky and Vergnaud.

To conclude this chapter, I would like to cite some of the considerations raised by Nesher and Kilpatrick in their book "Mathematics and Cognition: A Research Synthesis by the International Group for the Psychology of Mathematics Education" (1990), who propose a synthesis of research presented in PME over the last ten years. Chapter 4 ('Psychological Aspects of Learning Geometry', Hershkowitz 1990) is of special interest in that it examines the various studies in Geometry. Hershkowitz concludes the chapter by making the following points:

(1) "*Geometry learning begins when children start to 'see' and to 'know' the physical world around them, and it can continue to very high-level geometrical thinking through inductive processes or within deductive systems*" (p.93).

Despite this statement, which implies that the process of learning geometry begins very early, she was surprised to note that most of the research in this area has concentrated on 9 to 15 year old children. In fact, the majority of research is concerned with testing children who are expected to have learnt in school, the topic to be researched.

(2) "*Concerning geometry and the computer, we saw above that most software that serves research and instruction in geometry involves a high-level interaction with a computer*" (p.93).

On this point Hershkowitz argues the need to create more software capable of promoting and investigating the development of proof processes and strategies in children.

To finalise this chapter, I would express some considerations which led me to develop the present study. From the above authors, its seem quite clear that the static perspective is maybe not the easier way for children to learn angle. On the other hand, I had asked myself how much -- or even if at all -- can the dynamic perspective be superior to static one? The educators have been proposed to start geometry through the activities which involve concrete materials. However, I wonder in which way a contextualised task can present best meaning for children or how much the activities which are related to everyday life can help children to develop (or acquire) spontaneous (or intuitive) conceptions? And if so, how does the transition from this spontaneous conception to the formal one occur? Logo seems to be a good setting for developing children's conception of angle. If so, is it even better than paper and pencil environment or from everyday life experiences? Why? Can children present different understanding of one thing according to different situations? How does it happen?

These considerations together with those raised in works reviewed in the previous chapter influenced the design of the research reported here. The research design together with its research is presented in the next chapter.

CHAPTER 4

METHODOLOGY

This chapter describes the aim of the study: the model of the research and how it was carried out. It will first present the design, where the main argument of the thesis, the questions from which I initiated the study and the plan of the research on which the methodology and the analysis were based. The second part of the chapter, the method, describes the pilot study, after which it describes the changes that were carried out from the evaluation of it. However, the main focus of this part is on the description of the main study, from which the data were collected and analysed.

4.1 RESEARCH DESIGN

The design of the research was built up based on the main issues of the psychology and mathematics education theories discussed in Chapters 2 and 3 respectively, from which three fundamental questions arose:

1) Considering the Piaget and Vygotsky developmental perspectives, considering also Vygotsky and Vergnaud's ideas that firstly, a concept emerges spontaneously and thus is transformed into a scientific one and, still having in mind the Van Hiele's considerations about the learning process, I consider how the angle is understood by a child spontaneously, and to what extent this understanding varies with age and schooling?

2) From Van Hiele's statement about the importance of presenting a content inserted in a proper context plus Nunes' consideration about the influence of different representational systems over the functional organisation of people activity, my question is: does a child have a different perception of an angle in different situations?

If so, and thinking in terms of children's semiotic function, I finally ask:

3) How do situations make sense for child's understanding of an angle?

Bearing in mind the purpose of the study, and in particular the above questions, the research was set up from five variables: arenas, contexts, settings, angle perspectives, and activities' condition. In the centre of all there were the activities. The variables were planned to be in relationship to each other in such way that different contexts, for instance, were placed in different settings and arenas. These variables were also arranged in such an order that would allow the emergence of other possible variables which influence children's understanding of angle too. For example, thinking of the school and developmental possible effects over the child's acquisition of angle, it would be necessary to elaborate a plan of research which involved young children who were just starting school through to those who were completing their last obligatory school year; thinking of the child's both spontaneous and scientific concepts, it should be relevant for my proposal that the activities of the research were inserted in different representational systems, from which it was possible to have activities related to the school practice until the everyday life practice. Another example which shows my attempt to connect the theoretical background with my research questions throughout the design planing are demonstrated by the division of the activities which are categorised either into the dynamic or static classifications of angle. The activities involving the static classification were presented through asking the child to compare angles, while those involving the dynamic classification were implemented by asking the child to deal with the idea of rotation and navigation. The next section is to describe in depth the planning of the research.

4.1.1 DESIGN OF THE ACTIVITIES

When carrying out this design I had always to bear in mind the purpose and the questions of the study. Thus, the variables were set up in order to obtain as much information as possible from children. Because the variables were displayed in the research interwoven, it is not an easy task to describe them in isolation from the others. On the other hand they form the heart of this study and for this reason they must to be expressed very clearly.

The study included 92 activities, i.e., children were asked 92 times to give an answer. These activities were carried out inside arenas. Arenas are the concrete materials or the concrete situations which brought signification to the activities. It is the referent for the activities of this research.

From my attempt to classify the arenas, six distinct groups emerged. The arenas which share the same angle properties and relationships were grouped together. Because arenas and its activities were set up in function of the contexts, settings and angle perspectives, I shall first define these variables before describing the arena groups.

Context is defined as a situation in which a child experiences a given content. In this way, context gives meaning to the activities conducted inside the arenas, i.e., from the child's viewpoint, context is the semantic situation of the activities. In this case a context has an individual perspective, it is the signifier. On the other hand, context also establishes the properties of a given content and in this sense it is beyond the individual control, as stated by Lave (1988).

This apparent contradiction in fact reveals the strangeness of a context. On one hand context is public, presenting a universal, or at least cultural, meaning. Thus it is the signified given for a referent. On the other hand, because of the individual differences, context can result in different signifiers. The present research observed children making sense of angle throughout three different contexts: Navigation, rotation, and comparison. Navigation and rotation involved the idea of movement, but whilst in rotation children were asked to make and recognise turns around the same axis, in the navigation axis changed after each turn. In other words, navigation involved translations and rotations, and rotations were about different. In comparison, context children were 'invited' to deal with similarities and differences between figures and turns.

Children could experience these three contexts from three different practices: oral, written and virtual, which referred to three different representational systems. Oral practice is typically used in the everyday life of the people, written practice is the traditional way adopted by school in which children are systematically required to solve problems. Finally, problem solving in a computer environment inserts people in a virtual world framed by a screen and, in the specific case of Logo program, livens up a dynamic object (the Logo turtle). I called these representational systems settings. Therefore the present study had three settings: everyday, paper & pencil (p & p) and Logo.

The everyday *setting* is in fact a model of an everyday life situation rather than the situation itself such as "supermarket setting" in Lave's work (1988), or "street setting" (Caharrer & Schliemann (1985) and Nunes, Carraher

& Schliemann (1993)). The everyday *setting* I am referring to is one in which children can associate things which are common in their lives. This *setting* derives from a particular culture which, in my case, is the North-East of Brazil. With this *setting* I am hoping to explore the children' spontaneous concepts, using a oral practice.

Paper & pencil (p & p) *setting* was introduced in order to explore the children's conception of angle in a written practice. In fact, students tend to associate written activities with school exam, as if they were doing a test, because this is the traditional way used by school to test their knowledge. Although the study has tried to construct some activities exploring movement as well as to elaborate tasks which avoid similarities with those carried out in school, I cannot deny the close relation that this setting presents with the school setting. From this perspective, I believe that this setting is suitable to observe the children' scientific concept.

The Logo setting was included with the intention of exploring an alternative to Euclidean geometry, in which angle could be constructed in terms of movement. Because the study had no intention of teaching children how to program in Logo, activities carried out in this language aimed to form microworlds in which children could interact with the turtle geometry. The microworlds aimed to explore the conception of angle involving navigation, rotation and comparison contexts, through children's body-syntonic, i.e., when children identify themselves with the Logo turtle movement (Paper, 1980).

The study still included the static and dynamic perspectives of angle as a variable which may influence children's understanding of angle. Perspective can be said to define a particular way of thinking or seeing something, to refer to the way something can be understood. In the case of this study, *perspective* distinguishes two ways of seeing angle.

There is no universal consensus of a definition of angle, however as presented in Chapter 3, two ways of seeing angle can be distinguished: *static* or *dynamic* perspectives. By *static perspective* I have assumed Schotten's classification, found in Close, G. (1982) which includes most definitions of a plane angle "The angle is the portion of a plane included between two straight lines in the plane which meet in a point" (p.9). Whereas for *dynamic perspective*: I have also assumed the same author classification which says that "The angle is the quantity or amount (or measure) of the rotation necessary to bring one of its sides from its own position to that of the other side without its moving out of this plane contained both." (p.9).

Because *perspective* is understood as a particular way by which a person perceives something, thus I may perceive a given situation differently from another person. Therefore, it is possible that whereas some activities were included in this research as being dynamic, it will not be perceived like this by students. For this reason I will not refer to any activity as being dynamic or static, rather I will refer to it as being related to the dynamic or static perspectives.

However from the point of view of the concept formation, turn is an essential invariant to be considered in a dynamic situation. For example, If a student does not take the distance between the starting and end point of a given movement into consideration, the angle cannot be measured in this situation.

The last variable refers to the condition in which children were asked to solve the activities: by recognition, action, and articulation. These conditions are related to three different aspects to thinking about angle: perceiving similarities and differences between figures using angle as an invariant (recognition); building figures by reference to their angles (action); and clarifying if and how angle has been taken into account (articulation). Whilst recognition and action are concerned with children's behaviour, articulation was their explanation or description of their previous behaviour.

From the first two conditions I looked at children's operational invariants. These conditions formed the basis of the quantitative analysis. Whilst articulation, the third condition, was included in order to obtain information about children' symbolic function, since language, both oral and written, takes an active role in separating thinking from action. The qualitative analysis was carried out on the data from children's articulations.

It is also generally accepted that language is a powerful and essential tool for the emergence of a concept. However action (and in the case of this research, recognition as well) is an indispensable component in the construction of the scheme and children theorem-in-action development. Moreover, an action can both refer to a concept - presenting explicit invariants and theorems-in-conception - and to a scheme in which the operational invariants are implicit, or intuitive, and are presented in terms of theorem-in-action.

After having defined the five variables, I would like to return to the arenas in order to describe them in relation to the remaining variables. With regards to the settings, four different *arenas* were utilised in everyday setting (mini city, turnstile, watch, and stick game), whilst in the p & p setting there were five *arenas* : map, spirals, arrows, two angles, and four angles. Finally, in the Logo setting we had the same names for the *arenas* as in the p & p setting plus the watch arena. The next diagram shows the *arenas* distributed according the settings:

SETTING: Everyday --> ARENAS: a) Mini City
b) Watch
c) Turnstile
d) Stick game

SETTING: Paper & Pencil --> ARENAS: a) The Map
b) 2 Angles
c) 4 angles
d) Spiral
e) Arrow

SETTING: Logo --> ARENAS: a) The Map
b) 2 Angles
c) 4 angles
d) Spiral
e) Arrow
f) Watch

FIGURE 4.1: Arenas in relation to the settings

The different arenas of the study were not created at random; rather they formed a mathematical correspondence, from setting to setting, in accordance with the specific aspects of angles which I was interested in exploring[4]. Thus, the shape of mini city in the everyday setting and the shape of map arena in both p & p and Logo settings were the same, the tasks were also the same and all of them explore the navigation, rotation and comparison contexts, therefore I shall refer to these three arenas as group of 'map'. The 'two angles' and 'stick game' arenas, which were composed by shapes that presented the same-sized angle, formed another group of arenas which I called 'two angles'. The shapes

4 - In the method section a detailed description of all the Activities employed in the study will be given. At that stage, the correspondence discussed here can be verified.

and the questions used in 'four angles' arena were exactly the same for both p & p and Logo settings , thus these two arenas are referred to as the group of 'four angles'. It was also the case of 'spiral' arena in both p & p and Logo setting, which presented the same shapes and the amount of turns, they are consequently referred to as the arena group of 'spiral'. The arrow and turnstile arenas, where the number of turns made in both arenas, in terms of degrees, were the same and the activities included in them were similar, formed what I called by the group of 'arrow'. Finally we have two watch arenas, one carried out in the everyday setting and another in Logo setting. For the same principle as used to group the previous arenas, the two watches arena were put together in the arena group of 'watch'. To sum up, the research had six groups of arenas: map, two angles, four angles, spiral, arrow, and watch, which will be next presented one by one:

a) The Map: This group consists of (1) navigating a miniature car over a mini city in the everyday setting, (2) navigating an arrow by joining up the arrows on a map in the p & p setting and (3) navigating the turtle in a map drawn on the screen in the Logo setting.

This group of arenas was devised in order to explore the children's conception of angle in a dynamic perspective as well as in an informal way. Another reason to include this group of arenas was to explore angles bigger than 90^0. In fact, the literature has frequently stressed that children have difficulty in recognising angles bigger than 90^0 (Close, 1982; APU 1980a, 1981a). However, these studies tested children by asking them to recognise, or construct, an angle in a p & p setting only from the static perspective. In the map arena children were asked to make turns smaller than 90^0 (more precisely turns of 45^0) turns of 90^0, and turns bigger than 90^0 (in this case, turns of 120^0

and 180^0 respectively) all of them inserted in a dynamic perspective, after which children were asked to recognise these turns.

b) Two Angles: This group involves a comparison between two angles which were drawn in the p & p and in the Logo settings, and a comparison of two angles drawn in the stick game in the everyday setting.

The stick game is a popular game played mainly among pubescent boys in North-East of Brazil, although girls may play too. This game is played in pairs and the place to play it is in a back yard. Each child must have a tiny stick which is thrown. The first aim is to try to fix the stick in the sand. If the child succeeds in getting the stick into the sand, then a straight line is drawn from the point where the stick falls to the base of the opponent. The next point scored the child will connect the two points where the stick falls with a straight line, and so continue until the opponent is surrounded by a closed plane figure. One of the valuable features of this game is the variety of open figures drawn around the child during play. Using this game as a model, I made a 'stick game' from a geoboard with elastic bands representing the straight lines in the Stick Game. This I used with the children to make angles.

The value of the angles constructed by the elastic band in the stick game were the same as they were presented in the p & p and in the Logo settings. Because the geometry turtle was programmed to move and turn slowly while it was drawing a pair of figures on the screen, children had a chance to perceive the angle as a turn. The children did not have this same opportunity in p & p, where the figures appeared to them already drawn.

This group of arenas was also included in the study in order to have activities more closely related to those carried out in Brazilian schools which are much more concerned with the static perspective of angle.

Finally, through these two arenas I expected to be able to observe the children's possible misconceptions of angle such as the length of the rays of the angles as well as the shape of the figures in order to determine whether the pair of angles were the same or not as a false theorem-in-action, frequently described on the literature (APU, 1980a, 1981a; Close, 1982; Fuys, 1988; Magina, 1988).

c) Four Angles: This group of arenas were created in order to explore variables which are considered as factors which lead to children's misconceptions, such as internal versus external angles as well as different sized sides of angle.

Four angle arena consists of a comparison of four angles which were drawn in the p & p and Logo settings, in this way children could see the figures already drawn on the paper but they also could perceive turtle drawing the figures in a dynamic way. This group of arenas, together with the previous one, was used in which the activities were close to those used in Brazilian schools, however in two different representational systems. This will allow me to compare, for instance, what has been stressed in the literature (APU 1980a, 1981a; Close, 1982) that children tend to perceive only the internal angle of the figure even though an arrow is put on the figure to indicate the precise side of the angle which they should be focusing on.

d) The Spiral: This group consists of a comparison of two spirals which were drawn in the p & p setting and Logo settings. Spiral was the only arenas group which shows figures with more than 720^0. I was interested in comparing the children's performances in activities which dealt with figures showing more than two turns in a static perspective with their performances in a situation which involved dynamic perspective. Considering p & p as an excellent medium for static perspective and Logo as the ideal for dynamic perspective, I wondered whether this difference could bring about any contrast in the way children make sense of the activities carried out in this group of arenas.

e) The Arrow : This group consists of an arrow which was drawn inside of a square, using the p & p and in the Logo settings. These arenas were created with the aim of exploring the dynamic perspective even though in p & p setting. Using this group of arenas I wanted also to observe whether children make any relationship between the turns of the arrow and the turns of the watch, both of which have hands and turn to a right hand side. I was interested to know if an eventual invariant, such as the watch metric, acquired from their practice in manipulating a watch could be transferred to the arrow arena. And if so, how did the children carry out this transference of invariants.

f) The Watch: This group consists of exploring the ideas of rotation and comparison, using the hands of a watch, in an everyday setting; also the idea of rotation using a watch in a Logo Setting.

There was a variety of reasons for including the watch in my design. First it offers a good example of a dynamic perspective, which means that it is an ideal arena to observe how children cope with the idea of rotation as well as the ability to make comparisons in a dynamic way. Secondly the watch is a very familiar and a widely-used object all over the world. In some countries, the school take on the responsibility for teaching children about the watch, how a watch works or how to tell the time. In Brazil, however, schools do not feel obliged to include this in their curriculum, although a few private schools usually do so. This was not the case in the school used in my sample. In other words, I can assume that what the children in my sample knew about watches, had been learnt only from their experiences in the life outside classroom. We must bearing in mind that although digital watches have become the most popular kind of watch, analogue clocks still exist on a large scale and it is quite common to find these clocks in schools.

The third reason to include this group of arenas was the importance of the number 6 in Brazilian society. The number 6 is often associated with half a dozen, and is on the verge of receiving the nickname 'half'. Thus, one frequently hears expressions like "I would like half dozen eggs", or "one and a half dozen eggs" (in shopping), or even "my phone is: two, half, eight, one, half, three, zero" for referring to the phone number 268 - 1630. In the watch arena we have a dozen numbers and 6 could well be confused as the fixed point for half.

Finally, a watch provides a very precise form of measurement as when counting hours one by one, or counting minutes five by five, or understanding the importance of the number 60. For example 60 seconds = 1 minute, and 60 minutes = 1 hour. A watch has 12 numbers - 1 to 12. The distance from one number to the next number represents 5 minutes. Telling the time requires mathematical knowledge which involves a numerical operation. However the watch is of purely geometrical concepts, once everything is happening in a context of rotation.

Next figure shows, in summary, the variables set up for this study which are intricately related to each other and which have as the central point the 92 activities:

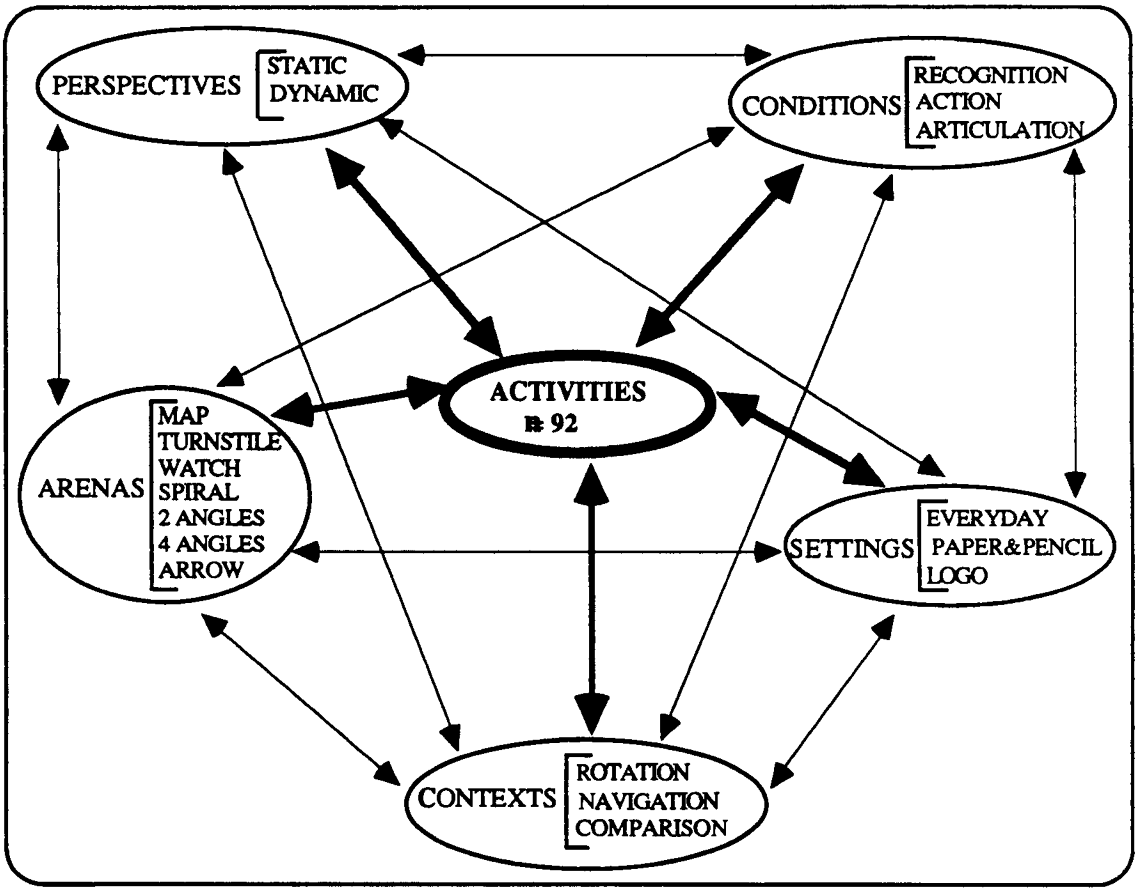


FIGURE 4.2: The Universe of the research

I would like to conclude this section presenting the two ways by which the activities can be classified. One way is by looking at the activities as they are presented to children, that is, considering the 92 activities with regards to the arrangement they are forming among arenas, settings, contexts, angle perspectives and conditions. Another way is by grouping these activities according to the size of the angle. The latter way is aligned with the point of view of school, where children's performances can be analysed in terms of value of angle, i.e., in terms of acute, right, obtuse, and RASO angles (the most common angles taught in school), as well as those angles involving more than one turn, such as one and half turn and two turns (hardly taught in math classroom as an angle). From this way 80 out of 92 activities were classified in seven distinct clusters:

1) Dealing with angles smaller than 90^0;
2) Dealing with angles of 90^0;
3) Dealing with angles of 180^0;
4) Dealing with angles of 540^0;
5) Dealing with angles of 720^0;
6) Dealing with angles larger than 720^0.

A further group, cluster 7, included the children's responses when comparing 4 and 6 angles. Thus this cluster did not refer to a measure of angle as the previous six clusters, rather it is related to the simultaneous comparison of several different sizes of angles.

The decision to have the activities classified from two different ways -- either considering the whole group of 92 activities which are cross-related to the previous described variables, or considering those 80 activities distributed inside the seven clusters -- allowed me to analyse the children's answers based on those most frequent values of angles taught in schools, such as acute and right angles and half turn, which were presented to children in both school and non-school similar situations, as well as to analyse children's responses in those infrequently values of angles such as 540^0, and 720^0, (clusters 1 to 5), from both school and non-school similar situations.

The last two clusters were also appropriate to look at the children's conception of angle as a dynamic perspective, since these clusters involved more than one turn. Finally, cluster 7 was important for me to bring about a close relationship between formal knowledge within the school context, and informal knowledge in everyday life. In fact, the simultaneous comparison between a group of 4 angles, involved the recognition of the biggest and the smallest angle among 4 open figures, when the activities clearly referred to the word 'angle'. The simultaneous comparison between a group of 6 angles involved the familiar arena of a watch. The diagram below summarises how elements which compose the universe of the study are distributed throughout the research.

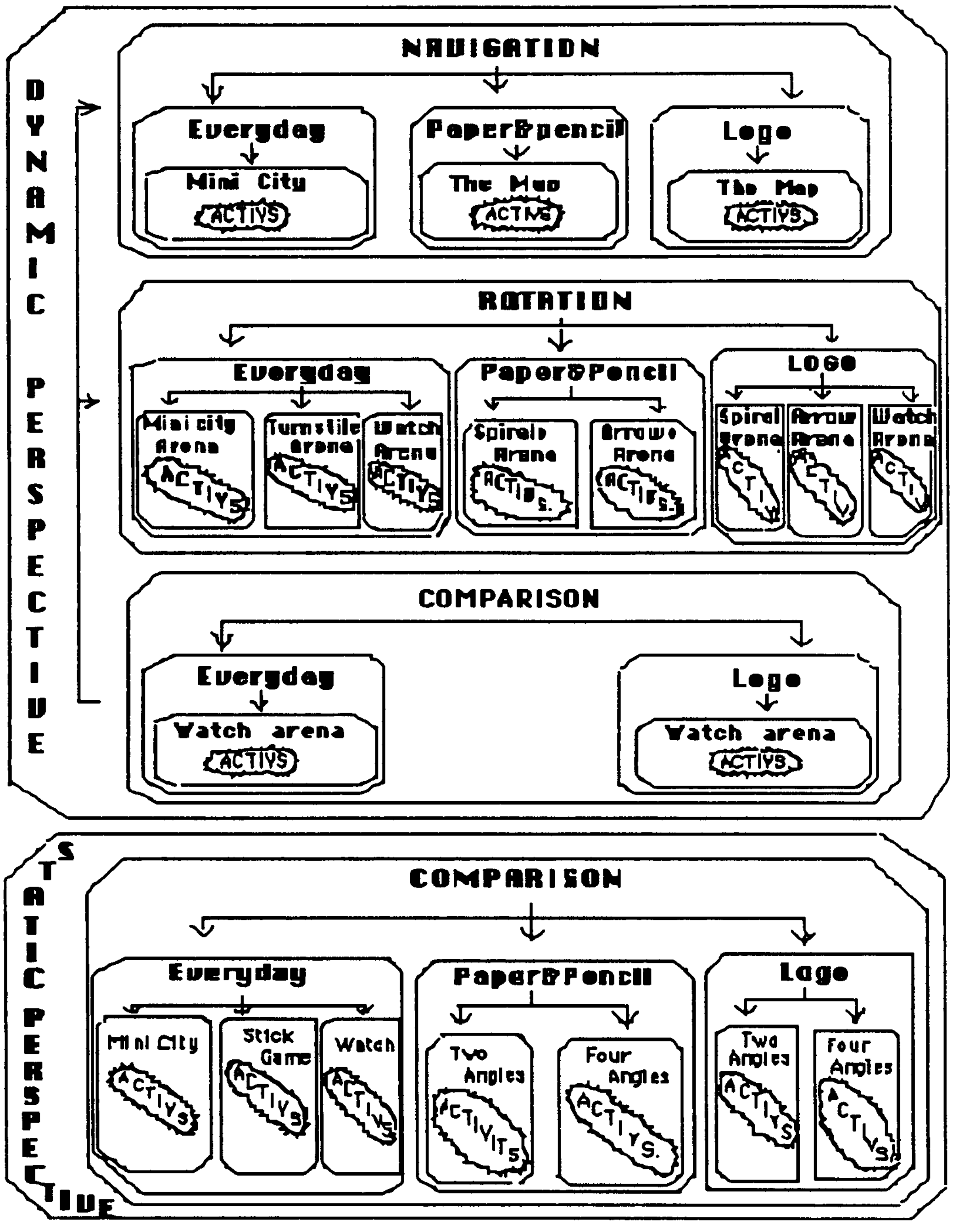


FIGURE 4.3: The Overview of the Study Design

Having described the plan to be used in the methodology and in the analysis, the following section will describe in detail the implementation of the present research.

4.2 METHOD

The research was divided into two studies: the outcome of the first study, the pilot study, was used to make adjustments to the design of the research, whilst the second study, the main study, is the one from which the results of the research will be analysed and the conclusion drawn.

4.2.1 PILOT STUDY

For pragmatic reasons, the pilot study was carried out partly in the North-East of Brazil and partly in England. As regards the Brazilian portion, 32 children (18 from public school, 14 from private school) from 5th to 8th grade of the later elementary school, and 1st year of the high school took part of the everyday setting, whereas the English pilot involved five 11 year-old children who were asked to do the Logo and p & p settings. The reason for having a small sample for Logo and p & p was that what of was concerned with here was to test the equivalence of tasks from setting to setting rather than to obtain a representative sample in this phase. Thus, although all the tasks were tried out, they were not applied as a set for each child as was the intention of the study 2; the children were submitted to a maximum of two settings.

The purpose of the pilot study was to find out what whether adjustments were required before the implementation of the main study. After the study, some arenas were excluded, new ones were included, some activities were changed, and even the conditions of the study were re-designed. Figure 4.4 below shows an overview of the planning for the pilot study taking into account settings and arenas.

SETTINGS	ARENAS	DESCRIPTION
	MINI CITY (to explore the idea of **navigation**)	A miniature city drawn on a rectangular wood (120 cm X 80 cm), with 2 mini cars to be driven by child.
	MECCANO (to explore the idea of **comparison**)	A toy which permitted the researcher to make shapes by fitting sticks to be recognised by child.
EVERYDAY	GEOBOARD (to explore the idea of **comparison**)	A 100-pin nail board distributed in 10 rows in which both researcher and child worked with elastic bands in order to build open figures.
	TURNSTILE (to explore the idea of **rotation**)	A miniature of a zoo entrance made in wood contained a wire turnstile. One of its hands was covered by a blue card to make a mark on it.
	WATCHES (to explore the idea of **rotation**)	2 blue card circular watches presenting different sizes. None showed numbers on their faces, only the minute and hour hands.
	FUTEBOL DE BOTÃO (to explore the idea of **imaginary angle**)	A small reproduction (120cm X 80cm) of a real football pitch, made in wood and painted green, played with buttons, palette and a very small ball.
	MINI SNOOKER (to explore the idea of **imaginary angle**)	A miniature of a snooker table (100cm X 70cm) made in wood and covered in green felt, played with 2 equal cues 80 cm long and 4 different colours of marble balls.
	MAP (to explore the idea of **navigation**)	A map was a reproduction of the mini city arena, presenting the same amount and value of mini city turns as well as the same questions.
	2 ANGLES (to explore the idea of **comparison**)	This arena was created to be in correlation to the geoboard. It consisted in comparing 2 angles.
P & P	4 ANGLES (to explore the idea of **comparison**)	This arena did not have a correspondence in everyday setting. It consisted in comparing 4 angles.
	SPIRALS (to explore the idea of **rotation**)	This arena was created to be in correlation to the watch. It consisted of comparing the turns of 2 spirals.
	ARROWS (to explore the idea of **rotation**)	This arena was created to be in correlation to the turnstile. It consisted of an arrow drawn inside of a square and the child was asked to predict where the arrow would be if it turned 'X' grades.
LOGO	MAP 2 ANGLES 4 ANGLES SPIRALS ARROWS	The arenas and activities of this setting were the same as in the paper & pencil.

FIGURE 4.4: An overview of the planning of the pilot study

With regard to the context, the pilot study used navigation, rotation, comparison and imaginary angle to examine the children's conception of angle.

The last context referred to the idea of drawing a mental line in order to form an angle in arenas such as snooker and futebol de botão. The classification of children's performances were based in five categories: recognition, building, predicting, description and explanation.

As was stated above, the pilot study did not present a sample statistically significant[5], and this fact led me to be wary about the consistency of the interpretations that I could make from the collected data. However, I would like to discuss some findings which I consider worthy of attention.

4.2.1.1 Findings From Pilot Study

A) The influence of setting:

I give two examples which serve to indicate the influence of setting on pupil responses:

A_1) When children were navigating the car in the mini city or the turtle around the map arena, the most difficult turn for them to recognise, implement or predict was less than 90^0. This contrasted with their responses in p & p which were more in line with previous research (for example APU, 1980; Close 1982; APU 1987), i.e., that children find acute angles comparatively easy but have greater problems in recognising angles greater than 180^0. This suggested an interpretation based upon 'figure and background'. In a p & p setting which had little semantic sense it was hard to perceive and angle bigger than 180^0 because it becomes the background of the smaller angle. Otherwise, in arenas such as map, the turn in a path is as much perceived as much bigger is the turn. The following figure illustrate this idea.

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5 - Either because the sample was small, either because the children were submitted to a maximum of two settings.

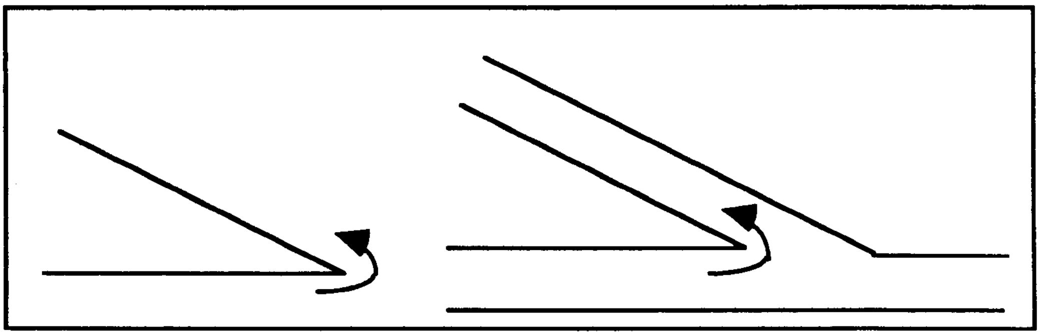

FIGURE 4.5: A turn in the p & p or in the screen and a turn in the mini city.

This finding was looked at carefully in the main study, where some more specific activities involving navigating context would be elaborated. Thus, specific turns involving less than 90^0, 90^0, between 90^0 and 180^0, and turns of 180^0 were systematically inserted in the activities.

A2) The results suggested that children performed better in the Logo setting than in p & p setting for all activities. Two interpretations could be given to this: first, the Logo activities took place after those with p & p, so children could have learnt something more about turn prior to their Logo work; second, the children developed their ideas of angle during their interactions with Logo that is, *while they were doing the tasks*. This was to be investigated further in the study 2.

B) The influence of presentation:

The recognition of similar angles in different orientations with rays of same size of sides, was not a problem for most of the children in all the three settings. However, when the size of the rays varied, this activity was the most difficult for English children whether in the p & p and Logo settings, while Brazilian children did not show the same difficulty. Nevertheless, there were significant differences in the way in which this situation was presented to the children: for English children the angles were presented one beside another (in both p & p and Logo settings), while for Brazilian children they were presented one inside another. Was the difference between Brazilian and English samples' results only caused by the way in which figures were displayed (beside or inside each other) or did the everyday arena represent a situation in which Brazilian children could find meaning? These differences in presentation were also to be further investigated in the main study.

C) Strategies:

Amongst English children, the most common way to make a comparison between a pair of angles was by looking at the 'openness' of the angles at the end of its sides. This was similarly the case within the Brazilian sample. Is 'openness' an operational invariant which is used independently of the setting? If so, does it change according to the age as well as school learning? At this point is important to stress that although the Brazilian sample was composed of children who were studying from 5th grade until 1st grade of high school, the children were from public school, which means, as stated in Chapter 3, that they had not necessarily learnt the topic of angle in school.

Also of interest here was that among the Brazilian sample only three (03) children attributed their answer to the angles of the figures. However, seven (07) other children referred to the openness at corners of figures. This can be seen as evidence that these children did not have a scientific knowledge of angle.

4.2.2 Changes from Pilot Study to Main Study

From the analysis of the pilot study I verified that the research design was not good enough and thus that it was necessary to make adjustments in order to reach the aim of the work. This adjustments can be described in terms of:

A) Arenas:

In the pilot study the 2 angles arena used different figures from the everyday setting to the Logo setting; In the everyday setting a pair of figures was placed next to each other, while in Logo setting a pair of figures was displayed one inside the other, i.e. a small figure inside a larger figure. In the subsequent analysis of these two arenas, it was demonstrated that Brazilian children performed better than their English counterparts. With the aim of exploring this result in depth , the study 2 utilised both types of display for the two angles arena, in the everyday, p & p, and Logo settings.

The watch arena, which in pilot study was only used in the everyday setting, was also introduced in the Logo setting for the main study. Watch was the only arena which used activities that involved quantification. A watch is a very precise object to measure time, and in its analogue form, a watch uses the metric of the angle (an hour is equal to a complete turn, an hour corresponds to a turn of 360^0, 90^0 represents a quarter turn, and when the minute hand moves 15 minutes it means that it has turned 90^0 and we refer to this movement as a 'quarter'). However, this arena has been modified and amplified from one to another study. In the case of the everyday setting, the main study used 9 watches[6] instead of only 2 as was used in the pilot study. Moreover, the main study included the oval shape for three of these watch as well as including some watches showing numbers on its face and some not.

To complete this list of changes, the snooker and futebol de botão arenas used in the everyday setting of the pilot study were excluded because the children's performances in the activities revealed more about the their ability in playing than their conception of angle.

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6 - The description of them will be given later on, when this study will be reported.

B) Contexts:

Because snooker and futebol de botão arenas were cut out, there was no reason to continue including the 'imaginary angle' as one of the contexts of the research. In fact, although I still believe that situations which involve 'imaginary angle', i.e., situations in which the angle cannot be concretely seen by the children, can be a good way to examine the children's conception of angle, unfortunately this did not happen in my study. As a proposition for further research, I would suggest the elaboration of accurate tasks using these two arenas in such a way that the results speak more about the children's concept formation than their skills. Another suggestion would be the elaboration of equivalent arenas to be used in other settings.

C) Conditions:

Conditions of the study also suggested the changes to be made for the main study. In the pilot study, building and prediction as well as description and explanation conditions were regarded as separate categories. In the main study these four conditions were considered as two, that is, building and prediction formed together 'action', and description and explanation formed 'articulation'. The reason for this was that prediction is a mental action, i.e. in order to predict something, a child has to build it in his/her own mind. Thus, I concluded that both building and prediction were referring to the child's action. In the same way, I realised from pilot study that when children were describing what they did, they were articulating their idea as much as when they were giving an explanation for another activity, i.e., both were referring to children's symbolic function.

4.2.3 MAIN STUDY

The main study was carried out in Recife city, situated in the North-East of Brazil between February and May 1991. It was undertaken in what can be considered a middle-class private school.

The whole study involved 92 activities altogether. 54 out of these activities were considered as involving the dynamic perspective of angle, while the 38 remaining activities as static; 37 out of the all activities were embedded in the everyday setting, 26 in the p & p, and 29 in the Logo setting. As regards the conditions, 61 out of these 92 activities explored recognition, and the 31 remaining activities explored the action condition[7]. In the case of arenas, the activities had the following distribution: mini city/map 33, watches 15, stick game/2 angles 17, 4 angles 4, turnstile/arrows 17, and spirals 6 activities. As described in the Analysis Design section, the study also included in its design 7 clusters, which were related to the size of angles in the tasks.

4.2.3.1. Sample

Fifty four (54) children took part in the study. They were divided into nine different age-groups of six children each. The first group consisted of 6 year-old children who were starting pre-school; the second group was composed of 7 year-old children from first grade; and the last group was composed of 14 year-old children who were in the 8th grade.

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7 - The articulation condition is omitted here because it does not belong to this stage of the analysis. What is important at this stage is the method of quantifing the number of correct and incorrect of children's answers.

The sample was divided into two groups: Group 1 was composed of 11-14 year-old children from middle school; and Group 2 was made up of 6-10 year-old children from the elementary school. The reason for this division was that in Brazil the teaching of geometry occurs differently from the elementary to middle school. As far as the curriculum relevant to this study is concerned, children have some contact with the topic of angle in the elementary school — although this is largely confined to 'playing' with shapes. More analytical activity including angular measurement is not introduced until the middle school.

All the subjects studied at a middle-class private school, this defined on the basis of tuition. This type of school has two important characteristics: (1) unlike schools for upper-class children, it lacks many educational technical resources[8] to offer to the students, and (2) this school offers a curriculum which is similar to that of the state schools. The main difference between this kind of school and state schools is that in the former, students are guaranteed a teacher for each subject and also a well-structured curriculum established by the Ministry of Education.

Two factors led to the decision to work with children from a private school. First was the fact that 'alfabetização' (pre-school) does not exist, officially, in State schools. Actually, by law, compulsory education is from 1st to 8th grades and children start school at the age of seven.

The second factor had an even greater influence on the decision. It concerns a specific problem that the Brazilian educational system has faced for many years with a very high number of students, because of their lack of progress, fail to go on to the next grade. These figures show that, in the case of state schools, only the most successful of the best students are able to finish

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8 - I.e. a library, science laboratory, computers, over-head projector, an art dept., music dept., etc...

their compulsory education within the period established by the educational system. In this case, if I had decided to work with students from state school maintaining my previous condition that each age-group of children should be at the same stage of schooling, I would not have a sample which represented the average of students, rather I would probably have only those clever students.

However, these alarming figures do not apply to private schools, where only a minority of students fail to reach the required level at the end of an academic year. Private schools may be viewed as providing the best educational standard available (which can mean that heavy demands are made on the children). The failure rate of students who attend this type of school is no higher than 15%.

The sample for the study was chosen on the basis of two main criteria: (1) the child's age i.e., a pre-school child had to be 6 years old, a first grade child 7 years old, and so on; and (2) the child's interest in taking part in the research. The teachers prepared a list of children who were at the right age to participate in a the study and from this list the children who were willing to participate in the study were selected.

The children were invited to participate in the research in their free school time. And before deciding whether they wanted to take part or not, they were made aware that they would be asked to come to three meetings, of one hour each, with the experimenter. In the case of children under 11 years old, a message was sent to the parents beforehand in order to explain the aim of the study, the way it would be carried out, and to ask them if they would allow their child to take part in it. Only those children who were able to bring back this note signed by one of the parents permitting his/her participation were accepted.

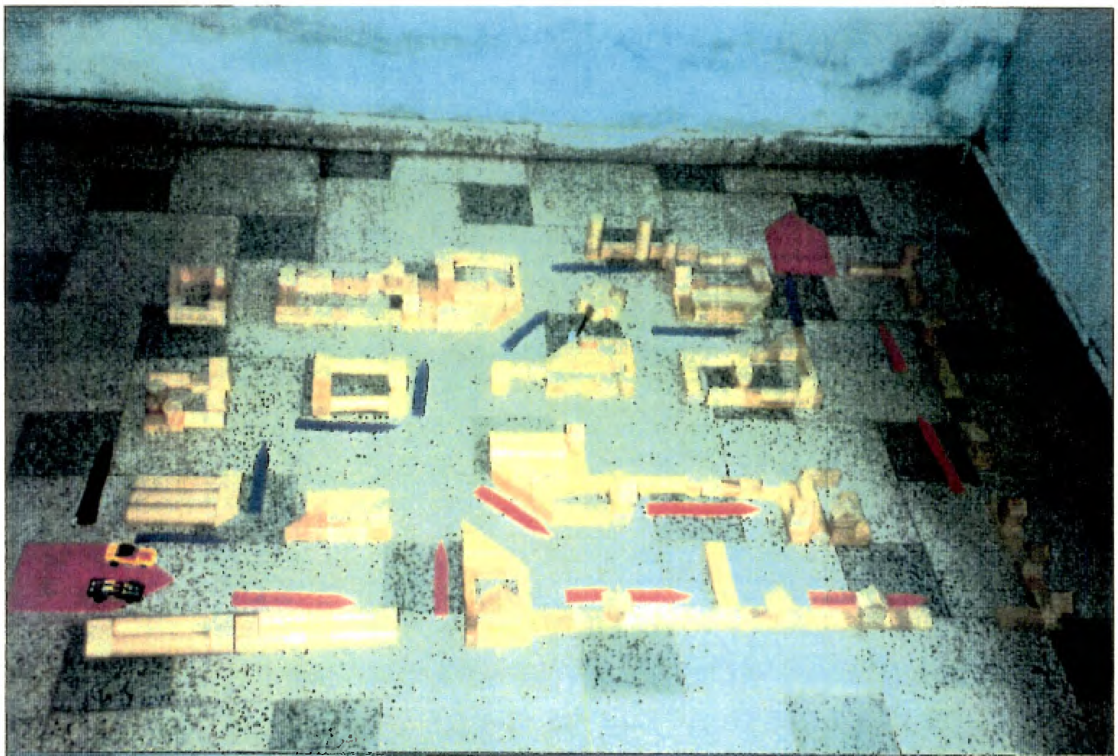
4.2.3.2 MATERIAL

The school was anxious to co-operate and help with the research and provided two special rooms in which the whole experiment could be carried out. This was useful as the materials could be kept in place throughout the whole research period. Because of the number of desks, one room was prepared to be the place where children were tested in the Logo and p & p settings The other room which had just one table was reserved for activities of the everyday setting. Next, I am presenting the materials according each setting:

4.2.3.2.1 The Everyday Setting

This Setting was not intended to include real-life situations or even pretend to reproduce them. Rather, it aimed to build arenas where the child could pretentially associate the activities with things and experiences from their world with which s/he might be already familiar. In other word, from this setting I aimed to investigate children's spontaneous concept about angle.

Mini City Arena This was constructed from 300 pieces of geometric wooden shapes (cubes, triangles, rectangles and cylinders), measuring 1-3 inches, arranged to form a miniature city. The Mini City was assembled in advance and occupied around 1 X 1.5 m^2 of the floor of the research room (see pictures 4.1 and 4.2 in next page). The starting and finishing points of the path to be followed by the child were indicated by a piece of cardboard in form of big arrow. Two miniature match-box cars were put on the starting point, and two different routes were indicated by means of cardboard tiny arrows along the way: 8 red arrows were placed along route A and 8 blue arrows along route B.




SAIDA

Watch Arena This consisted of different types of cardboard watches which varied according to size and shape and the presence or absence of numbers: three watches had a large circular shape, three a small circular shape and three an oval shape. Two of the watches in each set had numbers on their faces and one did not. Each set of watches were coloured differently to assist in distinguishing the pupils' responses; the large circular watches were blue, the small were red, and the oval watches were black.

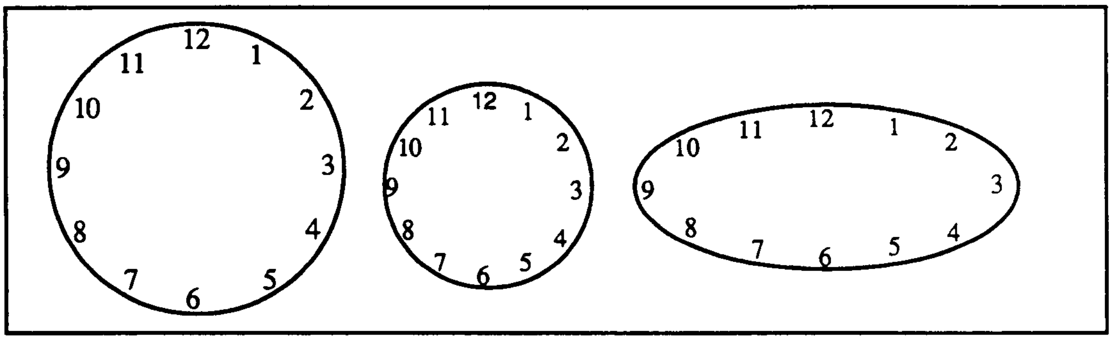


FIGURE 4.6: The three watch shapes used in the everyday

Stick Game Arena - This consisted of a 80 cm square, made of wood, containing 10 rows of 10 nails, two elastic bands, which were given to the child, and four elastic bands with which the researcher worked: two of the same size and two twice as big as the child's. The figure below shows the stick game with large and small elastic bands.

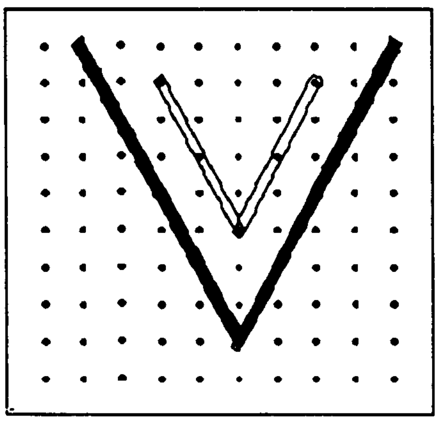


FIGURE 4.7: The stick game arena

Turnstile Arena -- This comprised a miniature turnstile, made of strong wire material, placed at the entrance of a miniature zoo made of wood (see Figure 4.7). A coloured cardboard arrow was placed over one of the turnstile arms, in order to mark the start of the turn.

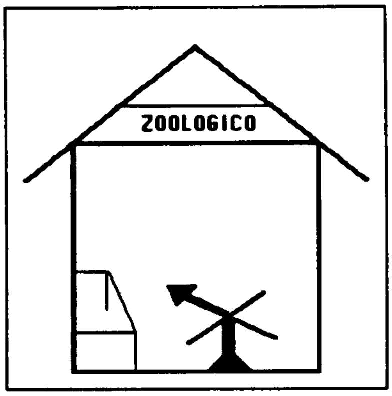


FIGURE 4.8 The Turnstile Arena

4.2.3.2.2 The P & P Setting

This setting consisted of a 10 pages test[9]. Compasses, rulers, protractor and pencils were put in front of the children but they were not encouraged to use them, unless they did so on their own initiative.

4.2.3.2.3 The Logo Setting

- An 8 bits microcomputer from Gradient;
- A 14" Colour TV;
- A disk drive;
- Two 5 1/4" floppy disks, one containing the Arena and Activity programmes,

9 - See annexe 1A (the original in Portuguese) and annexe 2B (translated to English).

and another disk to record the child's answers;

- Two colour pens (red and blue) which were used to mark the screen in order to indicate the routes A and B;
- Special liquid to clean the screen after the map and the arrow arenas.

It is important to point out that children did not have to learn anything about Logo. The programmes had been prepared beforehand. The reason for using this computer language was that it had a graphic function which could enable children to observe and make turns more easily.

4.2.3.3 <u>Procedure</u>

This section describes in detail the steps followed in the application of the study. It will be divided into three parts, each part corresponding to one interview with a child which was based around one setting.

However, before giving information about the procedure followed for each interview, a previous stage must be described, namely the introduction of the child to the study.

The first contact between the researcher and the child took place in the classroom. The researcher explained that she was doing research in order to try to understand how children of different ages think about things in general[10]. Continuing to explain about the research, the experimenter told the students that a game would be set up to be played in three different meetings. The students were told that the meetings would have to take place in the free

10 - Because mathematics uses to be viewed as the most difficult subject for students, and considering that the first interview would be based on activities in the everyday setting, the most informal one, I avoided making clear the specific subject of the research.

student hours and that each meeting would be on an individual basis lasting about an hour. In order to avoid the students being afraid that this research was used as a school test, the researcher made it very clear that the research had no relation to normal school activities, and the children were informed that no teacher, not even the director, would have access to any information about the meetings, and that these meetings could not be used by any teacher in assessing a school grade.

At the end of this initial contact in the classroom, the children were told that in the first meeting they would play with familiar objects such as miniature cars, watches and so on; the second meeting would be for them to answer some questions on paper; and then, in the last meeting they would play with some games on the computer.

The fact that the researcher had explained to the children what to expect at the outset was important because it ensured that children were aware of what they were going to do before deciding if they wanted to participate or not. Additionally it was very clear that the prospect of playing games on the computer was a major incentive to take part and the main reason why no children dropped out during the experiment. The children were very excited because it was the first time they had the chance to use a computer, and as the computer setting was the last one, they had to do the everyday and paper & pencil interviews first. The computer was so popular among the children that the researcher was very often approached by children who wanted to take part in the 'research of the computer'. This led the researcher to think of arranging something exciting for the children who could not take part in the research. So, I decided to give an introductory Logo class, in each classroom of the major first grade, at the end of the research - this was also a good way to say thanks for the warm hospitality of the school.

After a child had been chosen to take part in the research, a meeting was arranged at a time that suited him/her. The next meeting was arranged at the end of the session.

Before the child started the experiment, the researcher explained to him/her that there was no correct or incorrect answer to be given or expected, rather what concerned the researched was to find out how a child of a certain age thought in general. Since it was impossible to ask someone questions such as "how do you think" without offering something for the person to think about, the researcher devised some activities so as to observe how a child would handle them. The introductory remarks of the researcher were on the lines of the following quotation:

> *"If I ask a number of children of the same age to play with something and then ask him/her how he/she did or why he/she did it in a certain way, perhaps I can understand how children, of different ages think. This is because I do not have correct and incorrect answers, rather there is a certain way in which children from this or that age think. So, please don't worry about whether you are doing well or not because there is no right way. Just try to explain to me, as clearly as you can, the way in which you did the activities".* (translation of the introductory contact of the researcher in a classroom)

4.2.3.3.1 <u>The Everyday Setting</u>

Interviews based on activities in the everyday setting were conducted first. It consisted of an interview with one individual lasting 1 hour. Although the interview did not follow the pattern of a formal interview, guidelines were drawn up beforehand and used in the meeting to help the researcher not to forget essential questions or the need to follow a pre-established order[11].

The reason for starting the research with this setting was based on the premise that because of it's relationship with the children's everyday life, it would 'invite' children to solve a problem and express themselves from spontaneous concepts they had. In fact, although the concept of angle was embedded in all the activities, the child was much freer to perform them without thinking about or referring to it. This is because the arenas here were much more closely related to activities which the child had probably already met. Thus the tasks of navigating a miniature car in a mini city, for example, could well represent something which a child had done on many other occasions. However on this occasion there was an added stimulus because the mini car was foreign and very different from any other car played with before. Apart from some activities undertaken in the stick game arena (the later applied arena), the word 'angle' was not referred to in the section. The description of this setting will follow the same order as was adopted in the study, i.e. mini city, watch, turnstile, and stick game.

Mini City -- Three sets of 4 activities were included in this arena. The first set within activity 1 was related to the idea of navigation. In this set two tasks were carried out. In the first one, the child was asked to choose the 'best way' to navigate between two different routes (recognition), and afterwards s/he had to justify his/her choice (articulation). The intention here was to observe if the number of turns would be a criterion in the child's choice, i.e. whether this would be important enough in deciding the best way. In the second task, the child was asked to navigate the miniature car along both routes indicated (one after

11 - The guide-line of the Everyday setting interview can be found in the annexe 2A (the original in Portuguese) and in the annexe 2B (translated to English).

the other) and while doing this to count aloud each turn as it was made[12] - this activity involved an action from the child.

The second set in activity 2 was devised to explore the idea of rotation. It involved 9 tasks. First the child was asked if s/he had done any turn of 90^0 on route **A** and then on route **B** - if so, where. Then the child had to choose the largest turn s/he had made in both route **A** and route **B**. The answers were supplemented by the child's own explanation. In the fifth task the child was asked to compare the largest turn in **A** and in **B** and decide which was the larger turn between both. This choice was followed by the question "why" (articulation). In the sixth and seventh tasks the child was asked if s/he had done any quarter turns either in route **A** or **B**. Finally, the eighth and ninth tasks were about whether the child had made any half turn either in route **A** or **B**.

The third set in activity 3 involved a comparison between two turns of the same value, one done in a circle (in a roundabout) and the other in a corner (in a square). In this activity the child had to recognise and then explain the reason for his/her recognition.

The last set of tasks in activity 4 was a comparison between two similar curves showing different-sized streets. Once again, this activity was related to recognition. The child had to evaluate if s/he turned the same in both curves, and afterwards to explain why s/he had thought in that way.

<u>Watch</u> -- This arena was made up of 6 activities exploring the idea of comparison where the children were asked to work with recognition, and 5

12 - Besides the help of the tape recorder, the researcher was provided with a similar mini city, drawn on a piece of paper in which she could number, at the exact place, the turns that the child was counting (see annexe 2C).

activities involving mainly the idea of rotation where children were asked to deal with the activities by an action. In all the 11 activities the child was asked to explain his/her answers.

As regards the activities related to the comparison, in the first, 3 children were asked to compare between two watches. This was prefaced by an illustrative story:

> "*Let's imagine that a teacher has asked 3 students to do a classroom task. Unfortunately the teacher could not stay in the classroom. So, he put one watch in front of each child and asked him/her, when starting the task to press a button to start the watches as soon as they began to work. When the teacher returned to the classroom, he wanted to see from the 3 watches which student had finished his/her work first. When all three students started to do the task their watches were showing like this (12.00hs).*" (translation from the everyday guide-line interview)

Three different shaped watches (small and large circular watches, and an oval one) without numbers were shown simultaneously, each giving a starting time 12 o'clock. After telling this story, the researcher turned hands to the three finishing times and placed the watches in front of the child. S/he was asked to choose which of the three students had finished the classroom work first[13]. Figure 4.9 illustrates the first three activities.

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13 - This story as well as the turn of each watch could be repeated as often as was necessary until it was fully understood by the child.

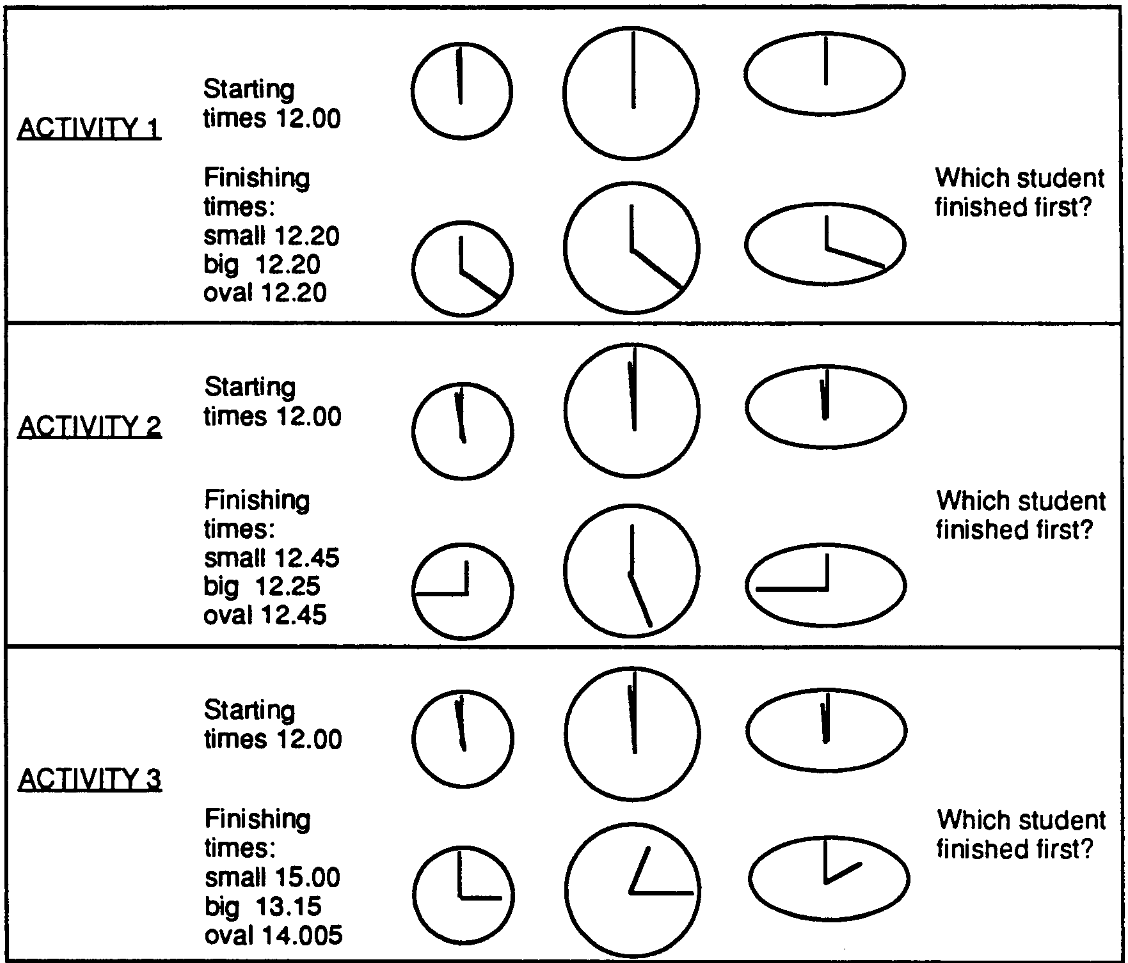


FIGURE 4.9: Description of the first three activities included in the watch arena.

Activity 4 made use of 6 watches: 2 small circular, 2 large circular, and 2 oval, none of which showed numbers. The researcher told the same story as before, but now related it to 6 children. The starting-point of all the watches was 12 hs and the end points are shown in the Figure 4.10 below:

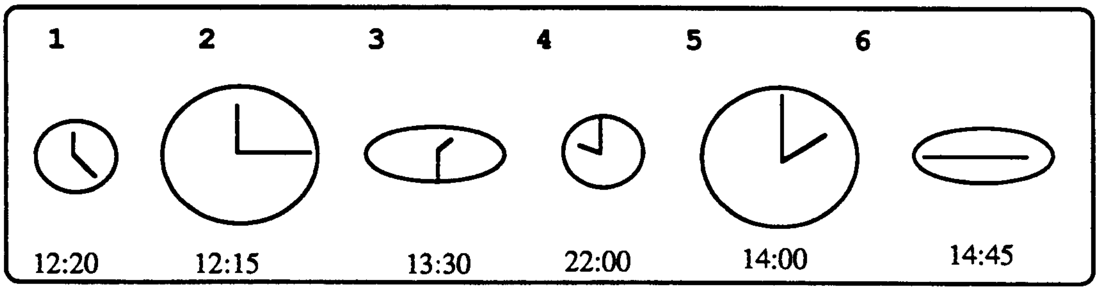


FIGURE 4.10: Description of the activity 4 in the watch arena

Activities 5 and 6 involved comparisons of half-hour time lengths starting from different times and using watches with numbers. Activity 5 used small circular and oval watches; activity 6 small and large circular watches. Setting the same context of doing homework as described previously, each child was asked whether or not the children had worked for the same length of time. If the answer to this question was no they were asked which child had worked longer and why. If they answered yes, they were asked to justify their answer.

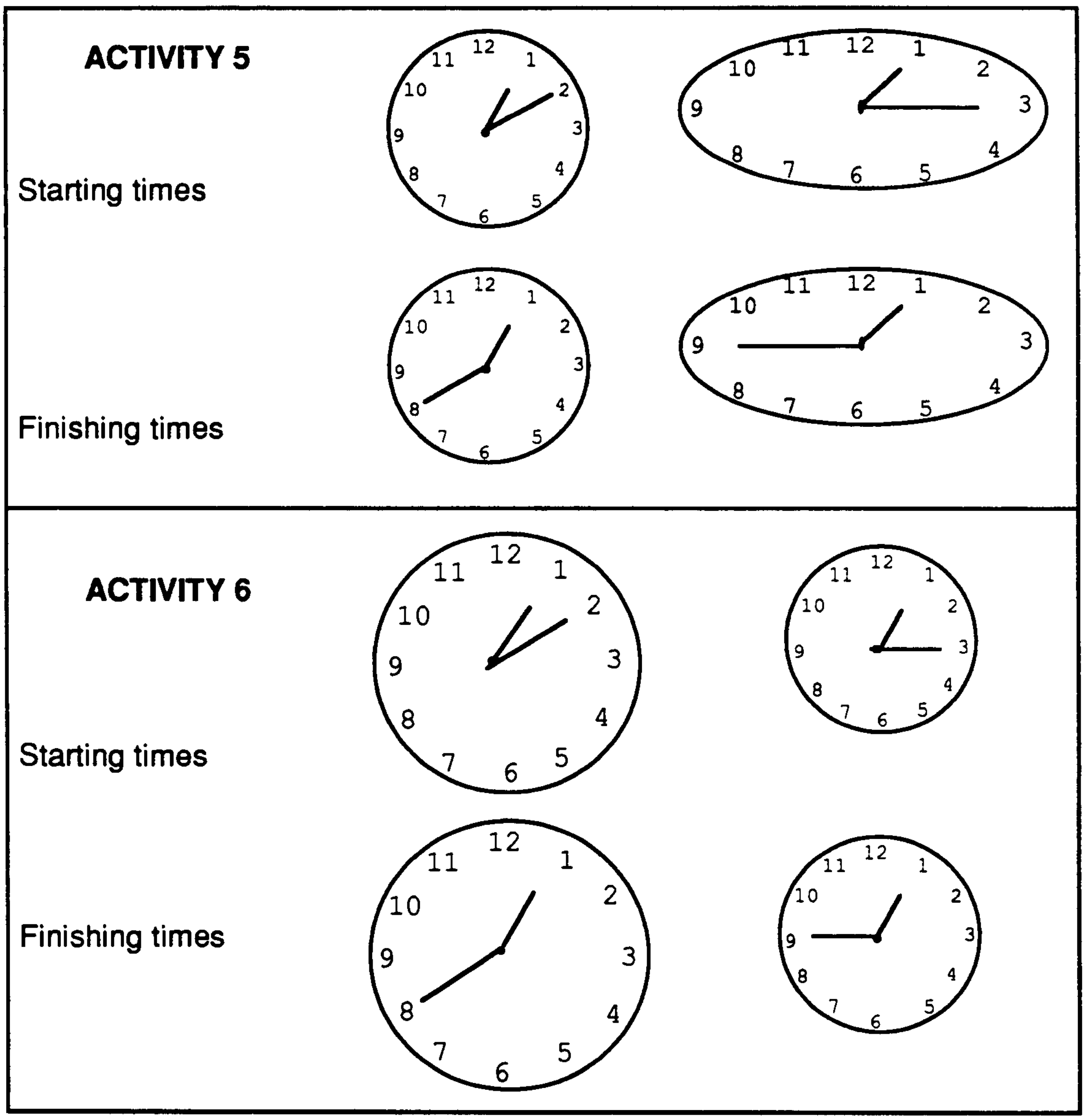


FIGURE 4.11: Description of the activities 5 and 6 in the watch arena

The last 5 activities explored the idea of rotation. The first three activities (activities 7, 8, and 9, shown in Figure 4.12) the child was asked to turn the minute hand through half a turn.

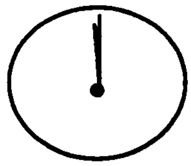
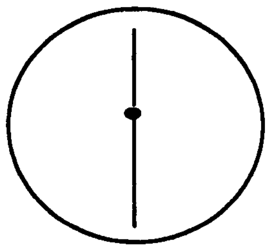
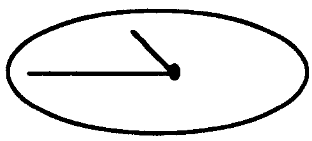
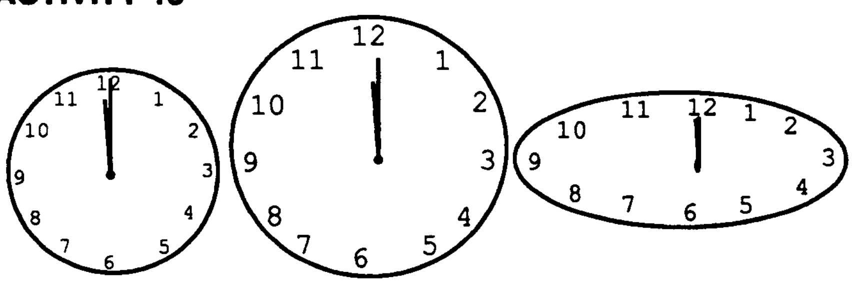
<u>Activity 7</u>	Starting time 12.00 Small Circle No numbers		Where would the minute hand be after it has turned through half a turn?
<u>Activity 8</u>	Starting time 12.30 Large Circle No numbers		Where would the minute hand be after it has turned through half a turn?
<u>Activity 9</u>	Starting time 11.45 Oval No numbers		Where would the minute hand be after it has turned through half a turn?

FIGURE 4.12: Description of activities 7, 8 and 9 in the watch arena

In activity 10 three watches, small and large circular and oval, all with numbers on their faces and all showing 12.00 o'clock were presented to the child simultaneously. At this time, the child was asked about half an hour.

ACTIVITY 10

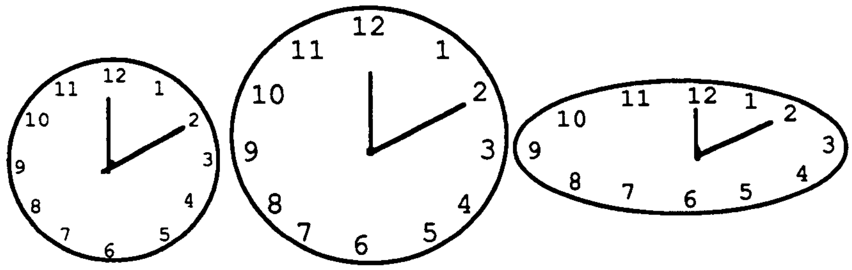


Where would the minute hand be half an hour later?

FIGURE 4.13: Description of the activity 10 in the watch arena

The last activity was exactly the same as activity 10 except the watches showed 12.10 as a starting time.

ACTIVITY 11



Where would the minute hand be half an hour later?

FIGURE 4.14: Description of the activity 11 in the watch arena

<u>Turnstile</u> -- This arena had only activities related to the idea of rotation. 4 activities were carried out in it.

The first activity aimed to ensure whether the child was really understanding what was being asked of him/her. Thus, the first question was: how many people can go into of the zoo if the turnstile does a turn? If the child understood a turn as just a quarter turn, then the next question would be: and if 4 people come into to the zoo, where will the arm of the turnstile stop? After asking these 2 questions, I was able to decide if I and the child were speaking about the same thing.

The second activity was devised to explore the value of half turn. This involved an action - by asking "where will the arm be after it had turned a half turn?" - and a recognition as well - by asking "how many people can come into the zoo?"

Third and fourth activities also involved action and recognition in the same order as the previous activity. In fact, the questions asked here were also the same, the only difference was that the third activity was regarding 2 complete turns, and the fourth was one and half turns. All activities required the child to articulate his/her answers.

Stick Game -- This arena was composed of 8 activities. 5 of them asked children to recognise, from a pair of opened figure, if the angles were the same. The figures were made by the researcher, one after the other, in front of the child.

Activity 1 - equal value of the angles (one next to the other)
- equal size of the rays
- different orientation

Activity 2 - different value of the angles (one next to the other)
- different size of the rays
- equal orientation

Activity 3 - equal value of the angles (one inside the other)
- different size of the rays
- equal orientation

Activity 4 - different value of the angles (one inside the other)
- different size of the rays
- equal orientation

Activity 5 - different value of the angles (one next to the other)
- different size of the rays
- different orientation

FIGURE 4.15: Description of the activities in the stick game arena

In all the activities, the child was asked to answer the same questions: "Are these angles the same?" and thus "Why?".

In activity 6 was carried out by the researcher making, with a large elastic band, an angle of 90^0. This angle occupied the whole first left vertical row as well as the whole lower horizontal row. Afterwards, the researcher gave a small elastic band to the child and asked him/her to make a similar angle to that. Finally the child was asked to explain how s/he knew that the angles were the same. In activities 7 and 8, the researcher repeated the same procedure and questions as in activity 5. The difference was that in activity 6 the researcher made an angle of 45^0, and in activity 7 she made an angle of 90^0. Both angles looked like the letter 'V' as illustrated in Figure 4.16.

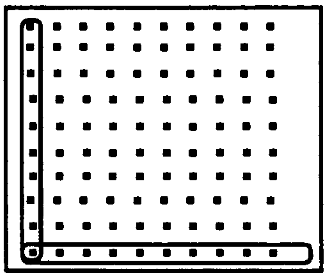
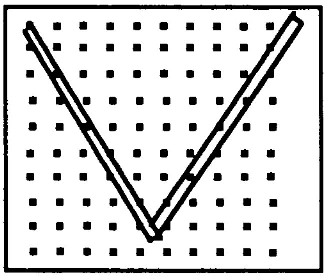
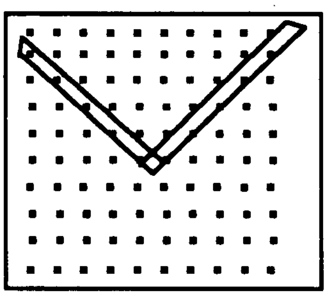
ACTIVITY 6		Could you draw an angle like this?
ACTIVITY 7		Could you draw an angle like this?
ACTIVITY 8		Could you draw an angle like this?

FIGURE 4.16: Description of activities 6,7, and 8 of the stick game arena.

4.2.3.3.2 <u>The P & P Setting</u>

Before distributing the test, the researcher explained to the child that s/he would answer some questions on a paper. Once more, the researcher emphasised that there was no right or wrong answer, and that the most important thing was the explanation that the child would give. So, it was essential that the child answered the questions as clearly as possible.

This setting was composed of 5 arenas distributed through the test in the following order: map, 2 angles, four angles, spiral, and arrow.

When the test was given to child, the researcher advised that there was no time limit and the child could ask if s/he had any doubt concerning the understanding of the questions.

In fact, some children asked for more explanation of some questions. In general, they told the researcher what they had understood from the question and asked for a confirmation whether they actually had understood correctly or not. The doubts changed from child to child, but the most common question, mainly from younger children, was about what is meant by an angle.

It is interesting to point out that no child used any of the available geometric instruments, but some of them asked for a ruler. Because a ruler was not provided, a few children used their pencil as an ruler in order to measure the sizes of the rays of the figures.

The map - this arena involved 12 activities. In the first one, the child was asked to choose one of the two presented routes to navigate. And afterwards s/he had to justify his/her choice. As in mini city, in the everyday setting, the intention here was to observe if the number of turns would be a criterion in the child's choice, i.e. whether this would be important in deciding the best way.

In the second activity, the child was first asked to navigate along both of the routes indicated by linking arrows (one after the other), after to number on the map the turns made and finally to state which of routes **A** and **B** had fewer turns. Thus these activities involved firstly an action and then a recognition from the child. In the third and fourth activities the child was first asked if s/he had made any turns of 90^0 on route **A** and then on route **B** - if so, to write down their number(s).

In the fifth and sixth recognitions the child had to choose the larger turn made in route **A** and in route **B**. The answers were supplemented by child's own explanation. In the seventh activities the child was asked to compare the largest turn in **A** and in **B** and then to choose which was greater, and give a reason "why". The eighth and ninth activities asked the child if s/he had made any quarter turns in route **A** or in route **B**. Finally, in the tenth and eleventh activities the child was asked if s/he had made any half turns in route **A** or in route **B**. If so, where.

Two angles - this arena comprised five activities involving comparison between a pair of angles. All five activities were carried out through multiple choice questions, but after the child has marked one of the four options, s/he was asked to justify their answers as is shown in the next Figure 4.17.

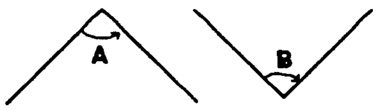
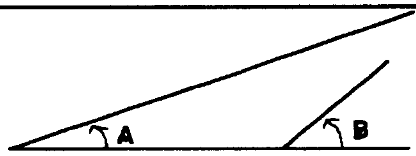
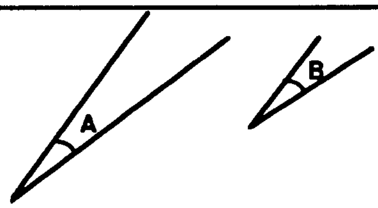
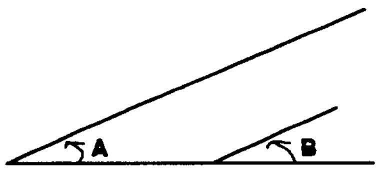
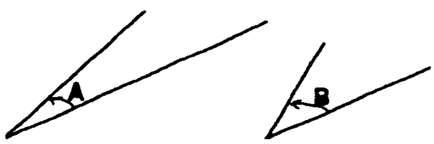
ACTIVITY 1	 	Compare angle **A** and **B**: **A** is the same as **B** (_) **A** is bigger than **B** (_) **B** is bigger than **A** (_) You **cannot** tell (_) Explain why you came to your answer __________
ACTIVITY 2	 	Compare angle **A** and **B**: **A** is the same as **B** (_) **A** is bigger than **B** (_) **B** is bigger than **A** (_) You **cannot** tell (_) Explain why you came to your answer __________
ACTIVITY 3	 	Compare angle **A** and **B**: **A** is the same as **B** (_) **A** is bigger than **B** (_) **B** is bigger than **A** (_) You **cannot** tell (_) Explain why you came to your answer __________
ACTIVITY 4	 	Compare angle **A** and **B**: **A** is the same as **B** (_) **A** is bigger than **B** (_) **B** is bigger than **A** (_) You **cannot** tell (_) Explain why you came to your answer __________
ACTIVITY 5	 	Compare angle **A** and **B**: **A** is the same as **B** (_) **A** is bigger than **B** (_) **B** is bigger than **A** (_) You **cannot** tell (_) Explain why you came to your answer __________

FIGURE 4.17: Description of the activities carried out in the 2 Angles arena.

Four Angles - In this arena there were two activities where the child was asked to compare four angles and choose the smallest angle (activity 1) and the biggest angle (activity 2) giving reason for each choice.

Spiral - This was composed of 3 activities, all concerned with comparing a pair of spirals. In each activity the child was asked to say whether the turn made in the construction of each spiral was the same or not.

In activity 1 both spirals presented the same amount of turns (3 and 1/2 turns), but spiral A was smaller than spiral B. In activity 2 spiral A was smaller than B, but it turned more than spiral B (spiral A did 3 turns and spiral B did 2 and 1/2 turns). Finally the activity 3 presented one spiral in circle and another in square, both figure presenting the same length. The circular spiral did 4 and 1/2 turns while the square spiral did 4 turns. Figure 4.18, shown below, describes how the activities were carried out in this arena.

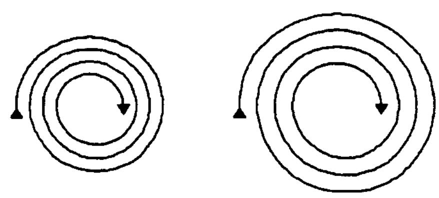
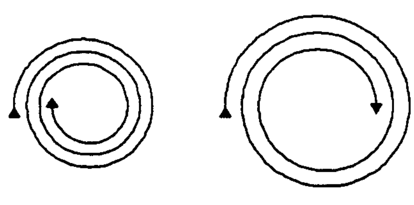
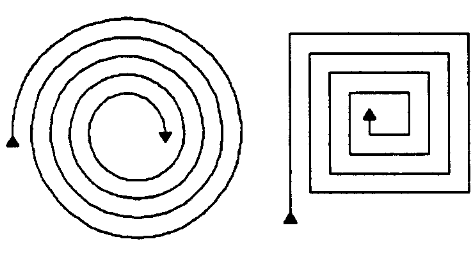
ACTIVITY 1		Compare the turn made in spiral **A** with the turn made in spiral **B** (The start and end points of spirals **A** and **B** are as indicated) Turn in **A** is the same as turn in **B** ( ): Turn in **A** is more than turn in **B** ( ) Turn in **B** is more than turn in **A** ( ) You **cannot** tell ( )
ACTIVITY 2		Compare the turn made in spiral **A** with the turn made in spiral **B** (The start and end points of spirals **A** and **B** are as indicated) Turn in **A** is the same as turn in **B** ( ): Turn in **A** is more than turn in **B** ( ) Turn in **B** is more than turn in **A** ( ) You **cannot** tell ( )
ACTIVITY 3		Compare the turn made in spiral **A** with the turn made in spiral **B** (The start and end points of spirals **A** and **B** are as indicated) Turn in **A** is the same as turn in **B** ( ): Turn in **A** is more than turn in **B** ( ) Turn in **B** is more than turn in **A** ( ) You **cannot** tell ( )

FIGURE 4.18: Description of activities carried out in the p & p spiral arena

<u>Arrow</u> This arena was comprised of 5 activities. In all of them the child was asked to predict where the arrow would be pointing after it had turned through a specific amount. This amount was given in degree and as a fraction of amount of turn: for example, 90^0 and as 1/4 turn.

The first 2 activities involved a rotation of 90^0. In the activity 1 the arrow was in the vertical position, pointing up. While in activity 2 it was presented in an inclined position pointing down.

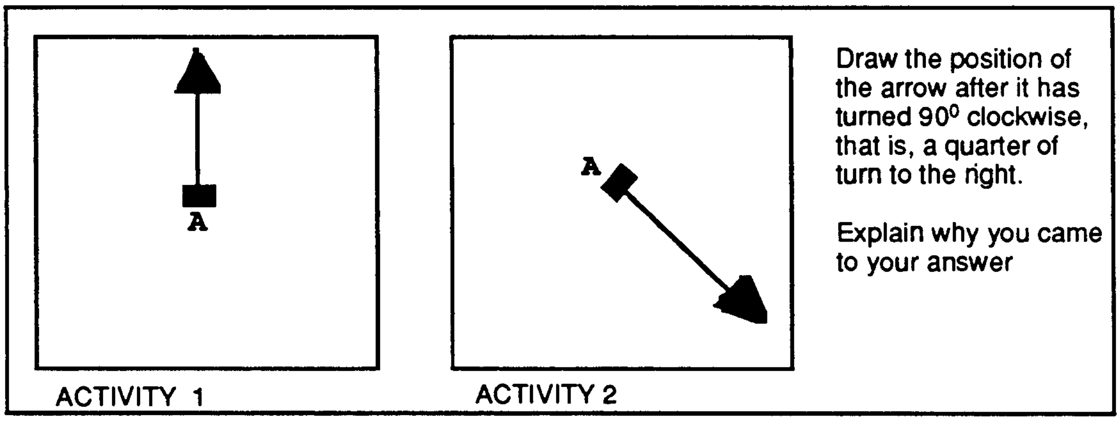


FIGURE 4.19: Examples of the arrow arena in p&p setting

The activity 3 involved a prediction of 1/2 turn (or 180^0). The arrow was shown in an inclined position pointing up. In the activity 4 children were asked to predict where the arrow will be after it turned 720^0 - 2 complete turns. The starting-point of the arrow was vertical facing down. Finally, in activity 5 children were asked to predict where the arrow would be after it had turned 540^0 - 1 $^1/_2$ turns. The starting-point of the arrow was horizontal pointing to the left.

4.2.3.3.3 <u>The Logo Setting</u>

Because children knew that the Logo setting involved playing on a computer, there is no doubt that this setting was the most antecipated for the children. It was common for the researcher to spend 5 to 10 minutes answering questions about the computer, disk drive, floppy disk and screen.

This setting was comprised of 6 arenas[14]. The first arena was the map. When the researcher ran the program, a map figure, similar to the ones used in the previous settings, was immediately drawn in the screen[15], with the Logo turtle placed in the horizontal position, inside of a square, in the right bottom side of this map. Thus, the researcher, using the special blue and red pens, drew the routes **A** and **B** on the screen. A blue pen was used to draw the route **A**, while the red pen used in the route **B**.

Under the picture of the map, the game asked the child to type his/her name. And then the first game instruction was given "*(name of child), bring the arrow (in this game, the turtle was called arrow) to the red house by the blue route*". Child was advised that s/he should use the keyboard arrow in order to make the turtle go forward or backward - as long as the child press this key the turtle would go forward. If the child wanted to bring the turtle backward, she should press the arrow key which was pointing to the left side. When the child wanted to turn the turtle to the left side, she had to press the arrow key which was pointing to the down up side. At this moment, the game asked "How many degree to the left do I turn?" And, when the child decided to turn the turtle to the right side, she had to press the arrow key which was pointing up down side. And now the game asked "How many degree to the right do I turn?" The child was allowed to correct his/her estimate of degree as much as s/he thought necessary.

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14 - In annexes 3A, 3B, 3C, 3D, 3E, and 3F there are illustrative example of each arena carried out in the Logo setting.
15 - See annexe 3A where the picture of the map as well as the activities embedded in it, together with english translation, are shown.

When the child reached the red house, the game gave an incentive sentence: "congratulations !!!" And then the turtle appeared again in the horizontal position, inside of a square, in the right bottom side of the map, and thus the game asked child to bring the turtle to the red house by the red way.

After the child had finished navigating along each route, the game asked the child to number each turn (using the colour pen over the screen) s/he had made. The researcher copied the child's numeration in a paper while s/he was doing it over the screen.

The arenas 2, 3, and 4 (2 and 4 angles, and spiral) were carried out exactly as it was in paper & pencil, i.e., it presented the same figures and questions[16]. However, the great difference from this setting to paper & pencil was that here child had the opportunity to observe the turtle slowly building the figures (walking and turning).

The way in which the arena 5 (arrow) was introduced to the child was not much different from the way it was in the paper & pencil setting. The presentation of the arrow was the same (an arrow inside of a square), and the child was asked to predict the place where the arrow would be after it had turned a certain amount, showing in both degrees and fractions of turn[17]. The child used a pen in order to draw his/her prediction on the screen and thus it was recorded on a paper by the researcher. Once again, the great difference between this setting and the paper & pencil setting was that in Logo the child saw (after each prediction) whether his/her action was correct or not. Only after the child had checked his/her prediction, was s/he asked to justify his/her action.

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16 - See annexes 3B, 3C, and 3D.
17 - See annexe 3E

Finally, the last arena to be carried out was the watch. A large circular watch with numbers in degrees was displayed on screen, i.e., in place of the number 1 in a real watch, appeared 30, for the number 2 appeared 60, for the number 3, 90 and so on. The 6 activities of which this arena was comprised were connected with half turn and half hour; the first 3 activities asked about half turn and the last 3 about half hour. The starting point of the watch showed (if the study had been using real watch numbers) the following hours: 12 o'clock, 12:30 hs, 11:45 hs, 2:10 hs, 12:45 hs, and 5:15 hs[19]. After each child's prediction the program turned the minute hand slowly in order to allow the child to observe what would happen. If the child had predicted correctly the following question appeared: "How did you know that it was here?", and if the child's prediction was wrong "Why did you think that it was (number chosen by child)?".

Before closing this chapter, I would like to report some of the children's comments made after most of the interviews. Frequently children asked the researcher about the study - what was it for, was it possible to say anything about children's thinking, how had they performed in general throughout the experiment. They also commented about which setting and activities they had liked the best. It was very clear that Logo was the favourite setting for all the children. Of all the arenas map and the watch were preferred. It is difficult to answer the effects of these influences but they cannot be ignored. Also after working in Logo, many children realised they had made mistakes in the paper & pencil setting mainly concerning turning the arrow, which they had only understood after they had performed on the Logo setting.

These comments were a bonus for the researcher. My feeling was that at the end of the fieldwork the majority of the children were, if not completely understanding angle, at least exhibiting interesting or doubts about it. Thus, in my opinion this awareness and questioning are the first and may be the most important step towards the acquisition of knowledge.

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19 - See annexe 3F

CHAPTER 5

QUANTITATIVE ANALYSIS

The analysis of this research is based on the data which are gathered from two groups: Group 1, composed of 11-14 year-old children from the middle school, and Group 2, made up of 6-10 year-old children from the elementary school. The reason for this division is that in Brazil the teaching of geometry varies very much from elementary to middle schools. Considering the great amount of data, the analysis will be presented in the next two chapters. The present chapter refers to the results which have been drawn from the quantitative data while chapter 6 will present the results from the qualitative analysis. Both chapters will report the results of both Groups 1 and 2.

The quantitative analysis is based on the average number of children's incorrect answers within settings, arenas, and ages, and in accordance with recognition and action conditions. The qualitative analysis is based on the children's articulation. In other words, the quantitative analysis refers to what the children did, while the qualitative analysis refers to the way in which these children described or explained the activities of their work.

The quantitative analysis looked at the children's performances first considering the set of 92 activities and then the set of 7 clusters (80 activities). Through this organising process it was possible to also analyse children's performances from the school point of view. So that the idea of angle, as viewed by children of different ages, remains the main focus of this research. The following Table 5.1 shows the distribution of both the 92 and 80 activities (the 7 clusters) in the different situations of the study. Moreover, Table 5.1.A (next page) presents, in detail, the activities which formed the 7 clusters.

		80 ACTIVITIES (7 clusters)			92 ACTIVITIES		
		STATIC	DYNAMIC	TOTAL	STATIC	DYNAMIC	TOTAL
SETTINGS	EVERYDAY	08	23	31	08	29	37
	P & P	18	05	23	21	05	26
	LOGO	04	22	26	04	25	29
	TOTAL	30	50	80	33	59	92
CONTEXTS	NAVIGATION	08	14	22	08	14	22
	ROTATION	0	25	25	0	28	28
	COMPARISON	18	15	33	21	21	42
	TOTAL	26	54	80	29	63	92
ARENAS	MAP/MINI CITY	08	16	24	11	22	33
	WATCH	0	15	15	0	15	15
	2 ANGLES	13	04	17	13	04	17
	4 ANGLES	02	02	04	02	02	04
	ARROW	0	14	14	0	17	17
	SPIRAL	03	03	06	03	03	06
	TOTAL	26	54	80	29	63	92
CONDITIONS	RECOGNITION	32	20	52	35	26	61
	ACTION	03	25	28	03	28	31
	TOTAL	35	45	80	38	54	92

Table 5.1: The distribution of the activities over the universe of the study, taking into account both 92 and 80 activities viewpoint

TABLE 5.1.A: Summary of the activities which formed the 7 Clusters[1]

CLUSTER 1 ANGLE < 90^0	**CLUSTER 2** ANGLE = 90^0	**CLUSTER 3** ANGLE = 180^0	**CLUSTER 4** ANGLE = 540^0
a) *Perspective*: Dynamic *Context*: Navigation *Setting*: Everyday, LOG *Arena*: Mini City, Map *Activity*:"How many turns did you make?" *Number of activities*: 4 *Condition*: Recognition a_1) *Perspective*: Static *Context*: Navigation *Setting*: P & P *Aren*: Map *Activity*:"How many turns did you make?" *Number of activities*: 2 *Condition*: Recognition b) *Perspective*: Dynamic *Context*: Comparison *Setting*: Everyday *Arena*: Mini City *Activity*:"Were the turns the same?" *Number of activity*: 1 *Condition*: Recognition c) *Perspective*: Static *Context*: Comparison *Setting*: Everyday, P & P, LOG *Arena*: Stick Game, 2 Angles *Activity*: Are these angles the same?" *Number of activities*: 11 *Condition*: Recognition d) *Perspective*: Static *Context*: Comparison *Setting*: Everyday *Arena*: Stick Game *Activity*: Can you construct a similar figure? *Number of activity*: 1 *Condition*: Action	a) *Perspective*: Dynamic *Context*: Navigation *Setting*: Everyday *Arena*: Mini City *Activity*: Did you turn 1/4 of a turn *Number of activities*: 2 *Condition*: Recognition a_1) *Perspective*: Static *Context*: Navigation *Setting*: P & P *Aren*: Map *Activity*: Did you turn 1/4 of a turn *Number of activities*: 2 *Condition*: Recognition b) *Perspective*: Dynamic *Context*: Navigation *Setting*: Everyday, LOGO *Arena*: Mini City, Map *Activity*: Did you turn 90^0? *Number of activities*: 4 *Condition*:Recognition b_1) *Perspective*: Static *Context*: Navigation *Setting*: P & P *Aren*: Map *Activity*: Did you turn 1/4 of a turn *Number of activities*: 2 *Condition*: Recognition c) *Perspective*: Dynamic *Context*: Rotation *Setting*: P & P, LOGO *Arena*: Arrows *Activity*: Where will the arrow be? *Number of activities*: 4 *Condition*: Action d) *Perspective*: Static *Context*: Comparison *Setting*: Everyday, P & P, LOGO *Arena*: Stick Game, 2 Angles *Activity*: Are these angles the same? *Number of activities*: 3 *Condition*: Recognition e) *Perspective*: Static *Context*: Comparison *Setting*: Everyday *Arena*: Stick Game *Activity*: Can you construct a similar figure? *Number of activities*: 2 *Condition*: Action	a) *Perspective*: Dynamic *Context*: Navigation *Setting*: Everyday, LOGO *Arena*: Mini City, Map *Activity*: Did you turn 1/2 of a turn *Number of activities*: 4 *Condition*: Recognition a_1) *Perspective*: Static *Context*: Navigation *Setting*: P & P *Aren*: Map *Activity*: Did you turn 1/2 of a turn *Number of activities*: 2 *Condition*: Recognition b) *Perspective*: Dynamic *Context*: Comparison *Setting*: Everyday *Arena*: Mini City *Activity*: Was the turning the same? *Number of activity*: 1 *Condition*: Recognition c_1) *Perspective*: Dynamic *Context*: Rotation *Setting*: Everyday, LOGO *Arena*: Watches *Activity*: Where will the hand be? *Number of activities*: 11 *Condition*:Action c_2) *Perspective*: Dynamic *Context*: Rotation *Setting*: P & P, LOGO *Arena*: Arrows *Activity*: Where will the arrow be? *Number of activities*: 2 *Condition*: Action d) *Perspective*: Dynamic *Context*: Comparison *Setting*: Everyday *Arena*: Watches *Activity*: Did they work the same? *Number of activities*: 2 *Condition*: Recognition	a) *Perspective*: Dynamic *Context*: Rotation *Setting*: Everyday P & P, LOGO *Arena*: Turnstile, Arrows *Activity*: Where will the arrow be after *X* turn(s)? *Number of activities*: 3 *Condition*: Action b) *Perspective*: Dynamic *Context*: Rotation *Setting*: Everyday *Arena*: Turnstile *Activity*: How many people can come into the Zoo? *Number of activity*: 1 *Condition*: Action

CLUSTER 5 ANGLE = 720^0	**CLUSTER 6** ANGLE > 720^0	**CLUSTER 7** comparing 4 and 6 angles	
a) *Perspective*: Dynamic *Context*: Rotation *Setting*: Everyday, P & P, LOG *Arena*: Turnstile, Arrows *Activity*: where will the arrow be after *x* turn(s)? *Number of activities*: 3 *Condition*: Action b) *Perspective*: Dynamic *Context*: Rotation *Setting*: Everyday *Arena*: Turnstile *Activity*: How many people can come into the Zoo? *Number of activity*: 1 *Condition*: Ation	a) *Perspective*: Static *Context*: Comparison *Setting*: P & P, LOGO *Arena*: Spiral *Activity*: Did the spirals make the same amuont of turns? *Number of activities*: 6 *Condition*: Recognition	a) *Perspective*: Static *Context*: Comparison *Setting*: P & P, LOGO *Arena*: 4 Angles *Activity*:Which is the smallest (biggest) angle? *Number of activities*: 4 *Condition*: Recognition b) *Perspective*: Dynamic *Context*: Comparison *Setting*: Everyday *Arena*: Watches *Activity*:Which watch has turned more (and less) from these 6? *Number of activity*: 2 *Condition*: Recognition	

1- The questions of the Activities are presented here in abbreviated sentences. The questions as they were done in the study are shown in the appendix. The "number of activity"indicates how many time children were asked to do the same activity

5.1 ANALYSIS OF GROUP 1: From 11 to 14 year-old children

Tables 5.2 and 5.3, presented in the next pages, provide an overview of the Group 1 results with regard to the whole set of activities distributed over the arenas, settings, and conditions (Table 5.2) and with regard to the whole set of activities distributed over the 7 clusters (Table 5.3). The Tables show, as was expected, that the older children made fewer mistakes than the younger ones. This means that 14 year-old children seem to be able to solve, on average, around 80% of a test involving angle, while 11 year-old children are able to solve only 50%.

Statistical tests were applied to these results in order to verify whether the differences among any of the sub-groups (activities, arenas, settings, the 7 clusters, and amongst the different age groups), were statistically significant.

A statistical test was selected which was appropriate to the nature of the research and the way in which the data were presented. The results of children's performances were arranged in the following different ways: (1) using an 'ordinal scale', i.e., managing averages, given in percentages, of the mistakes made in the experiment; (2) presenting "K related samples"[2] that is, the data were collected from the same children, in all three different settings of the study; (3) and finally using a "nominal scale" (i.e., using scale of categories) in order to compare whether there were differences in the children's performances either in the 92 activities or in the 7 clusters.

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2 - Siegel (1959) states this expression "K related samples" is used for researches which involve three or more samples which have been drawn from the same population. (Siegel, S."Nonparametric Statistics for the Behavioral Sciences", Internal Student Ed., 1959).

MIDDLE SCHOOL - FROM 11 TO 14 YEAR-OLD CHILDREN

TABLE 5.2: Total, with averages, of middle school children's incorrect answers over 92 activities[3]

ARENA	MINI CITY			WATCHES			STICK GAME / 2 ANGLES				4 ANGLES		TURNSTILE/ARROWS			SPIRALS		GENERAL RESULT
SETTING CONDITION / AGES	EVERY Recog	P & P Recog	LOGO Recog	EVERY Recog	EVERY Action	LOGO Action	EVERY Action	EVERY Recog	P & P Recog	LOGO Recog	P & P Recog	LOGO Recog	EVERY Action	P & P Action	LOGO Action	P & P Recog	LOGO Recog	
14 T/M	39/72	33/66	13/54	0/24	0/30	0/36	5/30	2/18	5/30	0/24	6/12	0/12	1/42	0/30	0/30	5/18	1/18	110/546
%	54.77	50	24.07	0	0	0	16.67	11.11	16.67	0	50	0	2.4	0	0	27.78	5.5	**20.15**
13 T/M	46/72	44/66	24/54	2/24	1/30	1/36	12/30	4/18	11/30	3/24	6/12	3/12	5/42	4/30	4/30	2/18	3/18	175/546
%	63.89	66.67	44.44	8.33	3.33	2.79	40	22.22	36.67	12.5	50	25	11.9	13.33	13.33	11.11	16.67	**32.05**
12 T/M	55/72	33/66	31/54	7/24	1/30	0/36	12/30	8/18	18/30	6/24	10/12	7/12	11/42	12/30	5/30	8/18	6/18	230/546
%	76.39	50	57.41	29.17	3.33	0	40	44.44	60	25	83.33	58.33	26.19	40	16.67	44.44	33.33	**42.12**
11 T/M	57/72	52/66	32/54	16/24	5/30	3/36	7/30	9/18	21/30	11/24	7/12	9/12	10/42	10/30	9/30	5/18	6/18	269/546
%	79.17	78.79	59.26	66.67	16.67	8.33	23.33	50	70	45.83	58.33	75	23.81	33.33	30	27.78	33.33	**49.27**
ALL AGES T/M	197/288	162/264	100/216	25/96	7/120	4/144	36/120	23/72	55/120	20/96	29/48	19/48	27/168	26/120	18/120	20/72	16/72	**784/2184**
%	**59.77**	**61.63**	**46.3**	**26.04**	**5.83**	**2.78**	**30**	**31.94**	**45.83**	**20.8**	**60.42**	**39.58**	**16.07**	**21.66**	**15**	**27.78**	**22.22**	**35.9**

TABLE 5.2A: Total, with average of middle school children's incorrect answers over the 6 Arenas

ARENAS	MINI CITY/MAP	WATCHES	STICK GAME/2 ANGLES	4 ANGLES	TURNSTILE/ARROWS	SPIRALS
T/M	459/768	36/360	134/408	48/96	71/408	36/144
%	**59.77**	**10**	**32.84**	**50**	**17.4**	**25**

3 - The numbers presented in all cells of this table, and of the tables2.a, 2.b, and 2.c as well, are showing firstly the **total numbers of incorrect answers (T)** divided to **the maximum allowed score (M)** and secondly result of this division presented in **percentage (%)**.

MIDDLE SCHOOL - FROM 11 TO 14 YEAR-OLD CHILDREN

TABLE 5.3: Total, with averages, of the middle school children's incorrect answers over the 7 Clusters

CLUSTERS / AGES	CLUSTER 1 ANG < 90^0	CLUSTER 2 ANG = 90^0	CLUSTER 3 ANG = 180^0	CLUSTER 4 ANG =540^0	CLUSTER 5 ANG = 720^0	CLUSTER 6 ANG > 720^0	CLUSTER 7 4 AND 6 ANG	TOT. INCOR MAX. SCORE	% INCORRECT
14 T/M	a) 14/36 b) 0/6 c) 7/66 d) 2/6	a) 14/24 b) 14/36 c) 0/24 d) 4/18 e) 0/12	a) 21/36 b) 2/6 c_1) 0/66 c_2) 0/12 d) 0/12	a) 0/18 b) 0/6	a) 0/18 b) 0/6	a) 6/36	a) 6/24 b) 0/12	89/480	**18.54**
%	**20.17**	**28.07**	**17.42**	**0**	**0**	**16.67**	**16.67**		
13 T/M	a) 20/36 b) 0/6 c) 20/66 d) 0/6	a) 24/24 b) 28/36 c) 5/24 d) 3/18 e) 2/12	a) 20/36 b) 4/6 c_1) 2/66 c_2) 1/12 d) 0/12	a) 2/18 b) 0/6	a) 0/18 b) 0/6	a) 5/36	a) 9/24 b) 2/12	160/480	**33.33**
%	**46.49**	**54.38**	**20.45**	**8.33**	**0**	**13.89**	**30.55**		
12 T/M	a) 28/36 b) 3/6 c) 32/66 d) 3/6	a) 24/24 b) 30/36 c) 11/24 d) 4/18 e) 5/12	a) 27/36 b) 6/6 c_1) 1/66 c_2) 3/12 d) 2/12	a) 3/18 b) 3/6	a) 1/18 b) 2/6	a) 14/36	a) 17/24 b) 3/12	222/480	**46.25**
%	**57.90**	**64.49**	**29.54**	**25**	**12.5**	**38.89**	**55.55**		
11 T/M	a) 27/36 b) 3/6 c) 39/66 d) 4/6	a) 24/24 b) 33/36 c) 7/24 d) 4/18 e) 4/12	a) 27/36 b) 5/6 c_1) 8/66 c_2) 6/12 d) 5/12	a) 6/18 b) 1/6	a) 1/18 b) 1/6	a) 10/36	a) 16/24 b) 8/12	242/480	**50.42**
%	**64.03**	**65.79**	**38.64**	**29.17**	**8.33**	**27.78**	**66.67**		
TOTAL T/M	a) 89/144 b) 6/24 c) 97/264 d) 9/24 201/456	a) 86/96 b) 105/144 c) 23/96 d) 15/72 e) 11/48 240/456	a) 95/144 b) 17/24 c_1) 11/264 c_2) 10/48 d) 7/48 147/528	a) 11/72 b) 4/24 15/96	a) 2/72 b) 3/24 8/96	a) 35/144	a) 48/96 b) 13/48 61/144	697/1920	**36.3**
%	**44.41**	**52.26**	**26.51**	**15.62**	**5.2**	**24.3**	**42.36**		

The use of both ordinal and nominal scales suggest the application of a non-parametric test. However, there are many non-parametric statistical tests, and my option to apply the χ^2 and Friedman tests was guided by the three factors that caracterise the study, which were previously described.

Because the χ^2 test is appropriate for nominal data, it was used to determine whether the differences between the total averages of mistakes for each age-group in Table 5.2 (general averages for 92 activities) and the total average of mistakes based on the seven clusters presented for each age in Table 4.3, were significant or not[4]. In this case, the χ^2 test is recommended because it is only based on one sample. Thus, in each age-group, the χ^2 test was applied to one sample, by comparing the total average of performances of an age-group in the 92 activities, (see Table 5.2) with the total average of performances of this same age-group in the 7 clusters (see Table 5.3).

From the χ^2 test, we were able to determine that there were no significant differences, within the same age, between children's performances in Tables 5.2 and 5.3. These findings are valid for all four ages of Group 1. With this information it was possible to proceed with the analysis of the data of the study by considering either all 92 activities, or using 80 activities (the 7 clusters) without any risk of bias or distortion.

It is important to emphasise that the activities carried out in the research were performed in arenas which were related to three different settings, three contexts and two perspectives of angle, and that the children's answers were examined under three conditions (recognition, action, and articulation) as well as in seven clusters. This means that the children's activity-based answers were inter-related because these same children also answered every other question

4 - According to Siegel "when the data of K mathed sample are in at least an ordinal scale, the Friedman two-way analysis of variance by rank is useful for testing the null hypothesis that the samples have been drawn from the same population". (Siegel, 1959, p.166) in the study as a whole. Considering this and having in mind that χ^2 refers to K Independent Samples it was rejected for this next stage of analysis. At the same time, it has been demonstrated that the Friedman test can deal with data which show exactly the same characteristics as those in this study[5]., that is, it was chosen because this test is to be applied for two or more related samples, which are presented in ordinal scale. Another advantage which justifies the use of this statistical test is that it is possible to find accurate Tables of probability for very small samples.

Table 5.2 is an overview of the children's performances across the 92 activities. Table 5.2A which was created from Table 5.2, shows, in detail, the children's performances (considering all 4 ages together) with relation to arenas. From the general results of each age-group (the 92 activities), shown in Table 5.2, the χ^2 was applied in order to verify whether the difference among the age-group performance was significant or not. It was also from the data of Table 5.2 that the Friedman test was applied to compare the different arenas, contexts, settings, and conditions. Accordingly Table 5.2.1 shows that the difference among the age-groups was statistically significant.

	AGES			
	11	12	13	14
AVERAGE	49.27	42.12	32.05	20.15

$df = 3, \quad 0.001 < p < 0.01$

Table 5.2.1: The difference of the middle school children's performances amongst the four age-groups according to χ^2 test.

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5 - The result of c^2 for one sample will just be discussed in the posterior sections, where the age-groups will be treated separately.

According to the Friedman test, shown in the Tables 5.2.2 and 5.2.3[6] below, the differences among the contexts (Table 5.2.2) and among the settings (Table 5.2.3) were both statistically significant at the level of p = 0.0046.

% OF INCORRECT RESPONSES 3 CONTEXTS			
CONTEXTS / AGES	1 NAVIGATION	2 ROTATION	3 COMPARISON
14	45.65	0	14.06
13	66.67	6.67	23.44
12	81.16	16	44.79
11	82.61	20	49.48
R_j	12	4	8

K = 3, N = 4, p = 0.0046

Table 5.2.2[7]: The result of the middle school children's performances in the 3 contexts according to the Friedman test.

% OF INCORRECT RESPONSES 3 SETTINGS			
SETTINGS / AGES	1 EVERYDAY	2 P & P	3 LOGO
14	21.76	31.41	8.04
13	32.41	42.95	21.84
12	43.52	51.92	31.61
11	48.15	60.90	40.23
R_j	8	12	4

df = 1, K = 3, N = 4, p = 0.0046

Table 5.2.3: The result of the middle school children's performances in the 3 settings according to the Friedman test.

6 - The following tables show, within cells, the percentage of incorrect answers of each age-group according to the condition described in the table. The tables also show the total of post in column j (Rj) which were used to obtain the level of significance according to Friedman test.

7 - I am adopting the following nomenclature: For Rj the sum of the posts in the j columns of the sample, for df degree of freedom, for letter K number of columns,, for letter N number of rows, and for letter p "the probability associated with the occurrence under h0 (null hypothesis) of a value as extreme as or more extreme than the observed value" (Siegel,S. 1959, p. xvi).

These results suggest that the children's performance changes, in a significant way, from context to context as well as from setting to setting. So far this implies that changes in conception of angle is open to question.

According to Table 5.2.2, children of all ages obtain the best results in the rotation context. If the activities involving the rotation contexts are selected from Table 5.3 (the average of the children's incorrect answers according the 7 clusters), it is noted that no age-group obtained an average of incorrect answers higher than 20% (20% was the average of the 11 years-old) in the rotation context. Furthermore, it was also found that 14 year-old children, surprisingly, made no mistakes at all in this context.

In contrast, performance in the navigation context, presented the highest average of the children's incorrect responses, no age-group obtained an average of incorrect answers lower than 45% (it happened with the age-group of 14) and 11 and 12 year-old children attained an average even higher than 80%.

It is interesting to note these contrasts between the children's performance in the rotation and navigation contexts because both contexts refer to the dynamic perspective. And, as we will see later, although children performed rather well in the rotation context, the incorrect responses in the navigation context serve to depress the general results in the dynamic perspective - at least as evident in this quantitative analysis.

From Tables 5.2, and 5.2.3, it is clear that the children performed best in the Logo setting (in all age-groups), and least well in the paper & pencil setting. Although the findings will be interpreted in great depth in chapter 6 (qualitative analysis chapter), I would like to make some brief comments of the data presented above. To analyse the differences amongst settings, we should take into account the possibility that working with a microcomputer is a pleasurable experience for children, whereas working with p & p has a stale familiarity about it and is rather an uninviting task. The children in the sample had never before had the opportunity to play with such a newfangled 'toy' as a computer, so it is likely that they were more interested in taking part in this part of the experiment than the other two settings.

The study also applied the Friedman's test in order to find out if the difference in children's performances among the arenas was random or not. The results are shown in the next Table 5.2.4

	% OF INCORRECT RESPONSES 6 GROUPS OF ARENAS					
ARENAS / AGES	GROUP 1 map/mini city	GROUP 2 watch	GROUP 3 2 angles	GROUP 4 4 angles	GROUP 5 Arrow/ turnstile	GROUP 6 spiral
14 year	44.27	0	11.76	25	0.98	16.67
13 year	59.37	4.44	29.41	37.5	12.74	13.89
12 year	61.98	6.67	43.14	70.83	27.45	38.89
11 year	73.44	22.22	47.06	66.67	28.43	30.55
R_j	23	4	15	21	8	13

df = 5, K = 6, N = 4 $p < 0.001$

Table 5.2.4: The results of the middle school children's performances in the 6 groups of arenas according to the Friedman test.

Table 5.2.4, supported by Table 5.2, suggests that angle questions embedded in the mini city/map and in the 4 angles arenas were poorly treated by all age-groups. This result was more strongly noted in the activities such as half turn and a quarter turn. Table 5.2.4 also shows that children gave better responses in the questions involved in the watch and turnstile/arrows arenas (in this order).

I would like to point out that the activities developed in the last two arenas mentioned above were also classified as making use of the dynamic perspective and the rotation context. In the arrow arena, it was not unusual for children to make a connection between the arrow and the minute hand of the watch.

When a comparison was made between the way children performed in the activities which took place in these two arenas, it was possible to demonstrate that children managed better in the watch arena.

Comparing the activities demanding a recognition with those involving action, it was found that children were able to deal better with those activities which involved action than with those requiring recognition. According to the Friedman test, this difference was significant (see Table 5.2.5 below)

% OF INCORRECT RESPONSES 2 CONDITIONS		
CONDITIONS / AGES	1 RECOGNITION	2 ACTION
14	29.72	1.61
13	43.33	10.21
12	53.61	19.98
11	61.94	24.73
R_i	4	8

df = 1, K = 2, N = 4, $0.02 < p < 0.05$

Table 5.2.5: The results of the middle school children's performance in the 2 conditions according to the Friedman test

Table 5.2.5, supported by Table 5.2, shows that, if just the everyday setting is considered, it is quite clear that it was easier for the children to act than to recognise - the difference in the average score between these two conditions being around 5 times in favour of action.

If special attention is paid to the results which come from the activities carried out in the rotation context, it is clear that the children presented, in general, best performance in the arenas and settings which were connected with this context such as watch, arrow and turnstile arenas.

Finally, the next Table 5.2.6, supported by Table 5.3 shown below, which compares the children's results with due regard to differences of ages across the 7 clusters, shows that the most difficult cluster of tasks was 'dealing with angle of 90' (cluster 2), followed by 'dealing with an angle smaller than 90', and 'comparing simultaneously 4 and 6 angles' (clusters 1 and 7 respectively). Differences between the clusters were also submitted to the Friedman test and a significant statistical result obtained.

% OF INCORRECT RESPONSES 7 CLUSTERS							
CLUSTERS / AGES	1 < 90^0	2 90^0	3 180^0	4 540^0	5 720^0	6 >720^0	7 4 & 6 ANGLES
14	20.17	28.07	17.42	0	0	16.67	16.67
13	46.49	54.38	20.45	8.33	0	13.89	30.55
12	57.90	64.49	29.54	25	12.5	38.89	55.55
11	64.03	65.79	38.64	29.17	8.33	27.78	66.67
R_j	23	27	14	7	5	14.5	21.5

df = 6, K = 7, N = 4, $0.001 < p < 0.01$

Table 5.2.6: The results of the children's performance in the 7 clusters according to the Friedman test

Before closing this part of the analysis and starting to look at the data age-group by age-group, I would like to list, in summary, the most interesting findings to be discussed later in Chapter 6.

Looking at children's responses with regards to the three contexts, it was in tasks which involved rotation that children from all ages presented fewer mistakes, while they showed great difficulties with activities which involved the idea of navigation.

Considering the settings, although the children's performances varied from age to age, all age-groups showed lowest average of mistakes in the Logo and highest in the p & p setting.

The results also show that the highest scores of children's incorrect responses were in the map arena. These results contrast with the ones from the watch and arrow arenas where none of the age-groups made more than 1/3 of mistakes either in one or another arena. The findings still showed that children from all age-groups obtained best results in those activities in which they were asked to do an action rather than a recognition.

Finally, activities involving angles equal to 90^0, less than 90^0, as well as activities of simultaneous comparison among 4 and 6 angles, proved to be very hard for all children. In contrast, the activities involving angles of 540^0 and 720^0, respectively, seemed to constitute no problem.

5.1.1 THE 14 YEAR-OLD CHILDREN

From Table 5.3.1 presented on the next page, the first interesting finding to note is that there were no mistakes at all in clusters 4 and 5. These clusters involved the turnstile and arrows arenas.

TABLE 5.3.1: Total, with averages, of the 14 years old children's incorrect answers over the 7 Clusters

CLUSTERS / CHILDREN		CLUSTER 1 ANG < 90^0	CLUSTER 2 ANG = 90^0	CLUSTER 3 ANG = 180^0	CLUSTER 4 ANG =540^0	CLUSTER 5 ANG = 720^0	CLUSTER 6 ANG > 720^0	CLUSTER 7 4 AND 6 ANGLES	TOT. INCOR / MAX. SCORE	% INCORRECT
CHILD 1	T/M	a)5/6 b)0/1 c)4/11 d)0/1	a)4/4 b)4/6 c)0/4 d)1/3 e)0/2	a)4/6 b)0/1 c_1)0/11 c_2)0/2 d)0/2	a)0/3 b)0/1	a)0/3 b)0/1	a)2/6	a)1/4 b)0/2	25/80	**31.12**
	%	**47.37**	**47.37**	**18.18**	**0**	**0**	**33.33**	**16.67**		
CHILD 2	T/M	a)2/6 b)0/1 c)1/11 d)0/1	a)4/4 b)2/6 c)0/4 d)1/3 e)0/2	a)1/6 b)1/1 c_1)0/11 c_2)0/2 d)0/11	a)0/3 b)0/1	a)0/3 b)0/1	a)0/6	a)1/4 b)0/2	14/80	**17.50**
	%	**15.79**	**36.84**	**9.09**	**0**	**0**	**0**	**16.67**		
CHILD 3	T/M	a)2/6 b)0/1 c)0/11 d)0/1	a)2/4 b)1/6 c)0/4 d)0/3 e)0/2	a)2/6 b)0/1 c_1)0/11 c_2)0/2 d)0/2	a)0/3 b)0/1	a)0/3 b)0/1	a)1/6	a)2/4 b)0/2	10/80	**12.50**
	%	**10.53**	**15.79**	**13.64**	**0**	**0**	**16.67**	**33.33**		
CHILD 4	T/M	a)2/6 b)0/1 c)0/11 d)1/1	a)0/4 b)1/6 c)0/4 d)0/3 e)0/2	a)6/6 b)1/1 c_1)0/11 c_2)0/2 d)0/2	a)0/3 b)0/1	a)0/3 b)0/1	a)2/6	a)0/4 b)0/2	13/80	**16.25**
	%	**15.79**	**5.26**	**31.82**	**0**	**0**	**33.33**	**0**		
CHILD 5	T/M	a)2/6 b)0/1 c)0/11 d)1/1	a)0/4 b)3/6 c)0/4 d)0/3 e)0/2	a)0/6 b)0/1 c_1)0/11 c_2)0/2 d)0/2	a)0/3 b)0/1	a)0/3 b)0/1	a)0/6	a)2/4 b)0/2	8/80	**10**
	%	**15.79**	**15.79**	**0**	**0**	**0**	**0**	**16.67**		
CHILD 6	T/M	a)1/6 b)0/1 c)2/11 d)0/1	a)0/4 b)3/6 c)0/4 d)2/3 e)0/2	a)6/6 b)0/1 c_1)0/11 c_2)0/2 d)0/2	a)0/3 b)0/1	a)0/3 b)0/1	a)1/6	a)0/4 b)0/2	19/80	**23.75**
	%	**15.79**	**47.37**	**27.27**	**0**	**0**	**16.67**	**0**		
TOTAL	T/M	a)14/36 b)0/6 c)7/66 d)2/6	a)14/24 b)14/36 c)0/24 d)4/18 e)0/12	a)21/36 b)2/6 c_1)0/66 c_2)0/12 d)0/12	a)0/18 b)0/6	a)0/18 b)0/6	a)6/36	a)6/24 b)0/12	89/480	**18.54**
	%	**20.17**	**28.07**	**17.42**	**0**	**0**	**16.67**	**13.89**		

The analysis of the 7 clusters shows that children did not make any mistakes in activity "B" of Cluster 1, in activities "C" and "E" of Cluster 2, in activities "C_1", "C_2", and "D" of cluster 3, in all activities of clusters 4 and 5, and in activity "B" of cluster 7. These activities involved all three settings and mini city, stick game, watches, arrows, and turnstile arenas. This means that the children did not make any mistakes in 11 out of the 21 activities of the 7 clusters, where 10 out of these 11 activities involved the dynamic perspective in different settings and arenas, 7 out of these 11 activities asked the child to do an action, and 7 activities were embedded in the rotation context.

Moreover, considering the whole set of activities (80 activities per child, which means 480 per age-group) the 14 year-old children did not make any mistakes in 192 out of the 480 activities. This is a very good result since it implies that this age-group did not make any mistake at all in 40% of the activities in the clusters.

Activities "C" and "D" of cluster 1, activities "D" and "E" of cluster 2, and activity "A" of cluster 7, refer to the "Two Angles" and "Four Angles" arena groups. These 21 activities compared angles of different sizes. When all the answers of these six children were taken into account, the total of possible incorrect answers was 126. However, only 19 incorrect answers were evinced, i.e., 15.1%. If in activity "B" of cluster 1, (a comparison between two equal turns in Mini City Arena) and in activity "A" of cluster 6 (a Comparison between the turns of two spirals), which are also related to the sizes of the angles, the number of questions increase to up 168, however the number of children's incorrect answers was still low (25 incorrect answers, i.e., 14.9%). These results indicate that these children did not experience problems with the size of angles.

The clusters in which children presented higher averages of mistakes were clusters 2 and 1. These clusters refer to angles of 90^0 (cluster 2) and angle < 90^0.(cluster 1). Looking at the arenas which composed these 2 clusters, mini city/ map arenas showed the highest number of children's mistakes. These arenas also had activities included in the third worst cluster (cluster 3 - angle of 180^0). 65 out of 89 of the children's mistakes, that is, 73% of the wrong answers, were given in the mini city/map arenas.

At this moment, an apparent contradiction seems to arise from the analysis of mini city/map arena, since it was referred to as both the best and worst arena. However, it is important to have in mind that six activities were carried out in the mini city arena (activities "A" and "B" of clusters 1, 2, and 3). From these, children only presented a good performance in activity "B" of cluster 1 - the activity about the size of angles.

From the average number of mistakes in both Tables 5.2 and 5.3.1 and bearing in mind the Brazilian educational system[8], it is possible to infer that at this age children have a clear notion of angle and also that they know how to deal with it in many different contexts, settings, and arenas, and under different conditions (recognition and action).

5.1.2 THE 13 YEAR-OLD CHILDREN

Table 5.3.2, presented on the next page, shows that the most difficult cluster of activities for children at this age was dealing with 90^0 (cluster 2), and within this cluster, it was the recognition of a 1/4 of turn (activity A) and of 90^0 (activity B), in mini city and map arenas respectively, which caused most of the children's difficulties

8 - In the Brazilian educational system, the academic year is composed by four units of two months each. Students are obligated to be argued through a pencil and paper test at least once in each unit. At the end of the fourth unit the student's average of correct answers in all the disciplines must be 70%. If the student does not reach this score in any of the disciplines, s/he is asked to do a final test and this time the student is asked to have a score of 50% of correct answers to be able to change to a higher level.

TABLE 5.3.2: Total, with averages, of the 13 years old children's incorrect answers over the 7 Clusters

CLUSTERS / CHILDREN	**CLUSTER 1** ANG < 90^0	**CLUSTER 2** ANG = 90^0	**CLUSTER 3** ANG = 180^0	**CLUSTER 4** ANG =540^0	**CLUSTER 5** ANG = 720^0	**CLUSTER 6** ANG > 720^0	**CLUSTER 7** 4 AND 6 ANGLES	TOT. INCOR / MAX. SCORE	% INCORRECT
CHILD 1 T/M %	a) 5/6 b) 0/1 c) 3/11 d) 1/1 **47.37**	a) 4/4 b) 6/6 c) 4/4 d) 1/3 e) 0/2 **78.95**	a) 2/6 b) 0/1 c_1) 0/11 c_2) 0/2 d) 0/2 **9.09**	a) 0/3 b) 0/1 **0**	a) 0/3 b) 0/1 **0**	a) 1/6 **16.67**	a) 1/4 b) 0/2 **16.67**	28/80	**35**
CHILD 2 T/M %	a) 4/6 b) 0/1 c) 4/11 d) 0/1 **52.63**	a) 4/4 b) 6/6 c) 0/4 d) 1/3 e) 1/2 **63.16**	a) 3/6 b) 1/1 c_1) 0/11 c_2) 0/2 d) 0/2 **18.18**	a) 0/3 b) 0/1 **0**	a) 0/3 b) 0/1 **0**	a) 0/6 **0**	a) 4/4 b) 0/2 **66.67**	30/80	**37.5**
CHILD 3 T/M %	a) 6/6 b) 0/1 c) 0/11 d) 0/1 **31.58**	a) 4/4 b) 1/6 c) 0/4 d) 0/3 e) 0/2 **26.31**	a) 4/6 b) 1/1 c_1) 0/11 c_2) 0/2 d) 0/2 **22.73**	a) 1/3 b) 0/1 **25**	a) 0/3 b) 0/1 **0**	a) 2/6 **33.33**	a) 1/4 b) 0/2 **16.67**	20/80	**25**
CHILD 4 T/M %	a) 6/6 b) 0/1 c) 2/11 d) 1/1 **47.37**	a) 4/4 b) 5/6 c) 0/4 d) 0/3 e) 0/2 **47.37**	a) 4/6 b) 0/1 c_1) 2/11 c_2) 0/2 d) 0/2 **27.27**	a) 1/3 b) 0/1 **25**	a) 0/3 b) 0/1 **0**	a) 0/6 **0**	a) 3/4 b) 1/2 **66.67**	29/80	**36.25**
CHILD 5 T/M %	a) 5/6 b) 0/1 c) 5/11 d) 0/1 **52.63**	a) 4/4 b) 4/6 c) 0/4 d) 1/3 e) 1/2 **52.63**	a) 4/6 b) 1/1 c_1) 0/11 c_2) 0/2 d) 0/2 **22.73**	a) 0/3 b) 0/1 **0**	a) 0/3 b) 0/1 **0**	a) 0/6 **0**	a) 0/4 b) 1/2 **16.67**	26/80	**32.50**
CHILD 6 T/M %	a) 5/6 b) 0/1 c) 4/11 d) 0/1 **47.37**	a) 4/4 b) 6/6 c) 1/4 d) 0/3 e) 0/2 **57.89**	a) 3/6 b) 1/1 c_1) 0/11 c_2) 1/2 d) 0/2 **22.73**	a) 0/3 b) 0/1 **0**	a) 0/3 b) 0/1 **0**	a) 2/6 **33.33**	a) 0/4 b) 0/2 **0**	27/80	**33.75**
TOTAL T/M %	a) 20/36 b) 0/6 c) 20/66 d) 0/6 **46.49**	a) 24/24 b) 28/36 c) 5/24 d) 3/18 e) 2/12 **54.38**	a) 20/36 b) 4/6 c_1) 2/66 c_2) 1/12 d) 0/12 **20.45**	a) 2/18 b) 0/6 **8.33**	a) 0/18 b) 0/6 **0**	a) 5/36 **13.89**	a) 9/24 b) 2/12 **30.55**	160/480	**33.33**

Analysing the activities in which children were asked to act and compare angles of different sizes, this age-group made 34 incorrect answers, i.e., 27.0% from the 126 activities (activities "C" and "D" of cluster 1, activities "D" and "E" of Cluster 2, and activity "A" of cluster 7). If activity "B" of Cluster 1 and activity "A" of cluster 6 (representing 168 activities)are included, the number of incorrect answers is 39, and the average of incorrect answers decreases to 23.21%. These percentages of children's mistakes are not lower than the previous age-group but they are still low, what allow us to state that like 14 year-old children this age-group does not experience problems with the size of angles, at least according to the quantitatively analysed data.

Table 5.3.2 again shows that activities which involve turns of 720^0 and 540^0 (clusters 5 and 4 respectively) were the easiest for all children. The activities of these clusters were related to the dynamic perspective, rotation context, turnstile/arrows arena group, and the action condition. In fact, taken together with the overall achievement of the children in each activity of each cluster, the activities which required children to act rather than to recognise, seemed, on the whole, to have been easier for them to handle.

The Table also demonstrates that these findings (the best and worst clusters, the most difficult activities for the children, and the condition in which they performed the best) were in line with those found in the quantitative analysis of the data from the 14 age-group children. However, we cannot forget that if we compare the Tables 5.3.1 and 5.3.2, the overall average of incorrect responses of the 13 year-old children was almost twice that of the 14 year-old children. This becomes evident if we note that the oldest group did not make any mistakes in 11 out of the 21 activities which compose the 7 clusters, while the 13 year-old children made no mistakes in only 5 out these 21 activities.

The profile of performance in these two age-groups was also different within the best and worst clusters. For instance, although these two age-groups achieved optimal results in clusters 5 and 4 (720^0 and 540^0 respectively), the 13 year-old age-group made a higher average of mistakes in cluster 4 than 14 year-old children. With regard to cluster 2 (90^0), the most difficult for both age-groups, the group of 14 years-old presented 28.07% incorrect responses in comparison with 54.38% presented by 13 year-old children.

Finally, looking at the 13 year-old children's results overall in Tables 5.2 and 5.3.2, and being successful in about 70% of the activities as an indication of understanding of angle, it seems reasonable to assert that this age-group had achieved a good level of competence. Nevertheless it must be pondered that this competence was most apparent in activities demanding an action rather than a recognition.

5.1.3 THE 12 YEAR-OLD CHILDREN

Although the differences between the general results as shown in Tables 5.2 (page 142) and 5.3.3 (next page) were non-significant, it was this age which revealed the greatest variation from one to another Table (there were 42.12% mistakes in Table 5.2, and in Table 5.3.3 46.25%, which means a difference of 4.13%).

TABLE 5.3.3: Total, with averages, of the 12 years old children's incorrect answers over the 7 Clusters

CLUSTERS / CHILDREN	**CLUSTER 1** ANG < 90^0	**CLUSTER 2** ANG = 90^0	**CLUSTER 3** ANG = 180^0	**CLUSTER 4** ANG =540^0	**CLUSTER 5** ANG = 720^0	**CLUSTER 6** ANG > 720^0	**CLUSTER 7** 4 AND 6 ANGLES	TOT. INCOR / MAX. SCORE	% INCORRECT
CHILD 1 T/M	a) 6/6 b) 0/1 c) 5/11 d) 0/1	a) 4/4 b) 6/6 c) 4/4 d) 1/3 e) 0/2	a) 3/6 b) 1/1 c_1) 0/11 c_2) 2/2 d) 0/2	a) 1/3 b) 0/1	a) 0/3 b) 0/1	a) 2/6	a) 4/4 b) 0/2	39/80	**48.75**
%	**57.89**	**78.95**	**27.27**	**25**	**0**	**33.33**	**66.67**		
CHILD 2 T/M	a) 4/6 b) 1/1 c) 6/11 d) 1/1	a) 4/4 b) 4/6 c) 2/4 d) 0/3 e) 2/2	a) 4/6 b) 1/1 c_1) 0/11 c_2) 0/2 d) 2/2	a) 1/3 b) 1/1	a) 1/3 b) 0/1	a) 2/6	a) 4/4 b) 0/2	42/80	**52.5**
%	**63.16**	**63.16**	**31.82**	**50**	**25**	**33.33**	**66.67**		
CHILD 3 T/M	a) 5/6 b) 1/1 c) 7/11 d) 0/1	a) 4/4 b) 6/6 c) 1/4 d) 0/3 e) 1/2	a) 6/6 b) 1/1 c_1) 0/11 c_2) 0/2 d) 0/2	a) 0/3 b) 1/2	a) 0/3 b) 1/1	a) 2/6	a) 3/4 b) 1/2	40/80	**50**
%	**68.42**	**63.16**	**31.82**	**25**	**25**	**33.333**	**66.67**		
CHILD 4 T/M	a) 3/6 b) 0/1 c) 4/11 d) 0/1	a) 4/4 b) 2/6 c) 1/4 d) 1/3 e) 0/2	a) 4/6 b) 1/1 c_1) 1/11 c_2) 0/2 d) 0/2	a) 0/3 b) 1/1	a) 0/3 b) 1/1	a) 1/6	a) 1/4 b) 1/2	27/80	**33.75**
%	**36.84**	**36.84**	**27.27**	**25**	**25**	**16.67**	**33.33**		
CHILD 5 T/M	a) 5/6 b) 0/1 c) 2/11 d) 1/1	a) 4/4 b) 6/6 c) 3/4 d) 0/3 e) 1/2	a) 4/6 b) 1/1 c_1) 0/11 c_2) 0/2 d) 0/2	a) 0/3 b) 0/1	a) 0/3 b) 0/1	a) 1/6	a) 1/4 b) 1/2	29/80	**36.25**
%	**36.84**	**73.68**	**22.73**	**0**	**0**	**16.67**	**33.33**		
CHILD 6 T/M	a) 5/6 b) 1/1 c) 8/11 d) 1/1	a) 4/4 b) 6/6 c) 0/4 d) 2/3 e) 1/2	a) 6/6 b) 1/1 c_1) 0/11 c_2) 1/2 d) 0/2	a) 1/3 b) 0/1	a) 0/3 b) 0/1	a) 6/6	a) 4/4 b) 0/2	47/80	**58.75**
%	**78.95**	**42.10**	**36.36**	**25**	**0**	**100**	**66.67**		
TOTAL T/M	a) 28/36 b) 3/6 c) 32/66 d) 3/6	a) 24/24 b) 30/36 c) 11/24 d) 4/18 e) 5/12	a) 27/36 b) 6/6 c_1) 1/66 c_2) 3/12 d) 2/12	a) 3/18 b) 3/6	a) 1/18 b) 2/6	a) 14/36	a) 17/24 b) 3/12	222/480	**46.25**
%	**57.90**	**64.49**	**29.54**	**25**	**12.5**	**38.89**	**55.55**		

Table 5.3.3, shows that activities in cluster 2 (angle of 90^0) were the most difficult for 12 year-old children. This fact had already been noted in the previous two age-groups (shown in Tables 5.3.1 and 5.3.2). with regards to cluster 1 (angles less than 90^0), the children's performances were almost the same as in cluster 2.

An examination of the activities included within these two clusters, showed that this poor performance was mainly due to the recognition of the turns in the mini city and map arenas. Additionally, the activities in cluster 3 (angle of 180^0), which involved recognition and comparison of half turns in the mini city and map arenas, seem to have been difficult for 12 years old.

The results given in Table 5.3.3, across the clusters, point to a tendency of 12 years old to deal better with those clusters involving angles of 540^0 and 720^0. Moreover, comparing children's performance in those activities in which they were asked to perform an action with those in which they were asked to perform a recognition task, the children performed better on the action task.

If we look at those activities which involve different sizes of angle, we find that out of 126 possible answers, this age group made 61 incorrect answer (48.41%). If the activities "B" of cluster 1, and "A" of cluster 6, are included, there is a slight decrease in the average of incorrect answers (46.43%). However, it is apparent that this age-group experienced problems in dealing with angles of different sizes and, in particular, confused the length of the rays of the angles with angular measurement. Based purely on the quantitative data and taking into account the Brazilian school average, we can affirm that the 12 year-old children do not have a clear conception of angle.

5.1.4 THE 11 YEAR-OLD CHILDREN

A careful examination of Table 5.3.4 shown on the next page, taken together with the information in Table 5.3.3, reveals something interesting when the performances of 11 and 12 year-old children are compared, that is, these two age-groups present similar results.

Although according to Table 5.2 it is possible to note a difference of up to 7.15% in the total averages between the 11 and 12 year-old children's mistakes (there was an average of 49.27% mistakes among 11 years old, and only 42.12% among 12 years old), this difference decreases to 4.17% when these ages are examined taking into account the 7 clusters (the average of 11 year-old mistakes was 50.42%, and 12 year-old 46.25%). This suggests that, considering only the quantitative data, it is difficult to distinguish between the knowledge of angle acquired when children are 11 and are 12 years old.

However it is important to point out that there were variations in the children's understanding of angle from one age to another. For the 11 year-old children the most difficult cluster was "comparing simultaneously 4 and 6 angles" (cluster 7) whilst for 12 year-old children it was 'dealing with an angle of 90^0' (cluster 2).

One possible reason for the difficulty experienced by 11 year-old children was the size and the openness of the internal angle[9] -- a difficulty not evident amongst the others 3 ages, 12, 13, and 14 year-old children. However this cluster was far from being the easiest one (in fact, it was the third most difficult cluster for these three ages).

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9 - This hypothesis will be discussed later in chapter 6 where the 11 year-old children performance will be analysed qualitatively.

TABLE 5.3.4: Total, with averages, of the 11 years old children's incorrect answers over the 7 Clusters

CLUSTERS / CHILDREN	**CLUSTER 1** ANG < 90^0	**CLUSTER 2** ANG = 90^0	**CLUSTER 3** ANG = 180^0	**CLUSTER 4** ANG =540^0	**CLUSTER 5** ANG = 720^0	**CLUSTER 6** ANG > 720^0	**CLUSTER 7** 4 AND 6 ANGLES	TOT. INCOR / MAX. SCORE	% INCORRECT
CHILD 1 T/M %	a) 4/6 b) 1/1 c) 4/11 d) 1/1 **57.89**	a) 4/4 b) 6/6 c) 2/4 d) 0/3 e) 0/2 **63.16**	a) 3/6 b) 1/1 c_1) 0/11 c_2) 0/2 d) 0/2 **22.73**	a) 0/3 b) 1/1 **50**	a) 0/3 b) 0/1 **0**	a) 2/6 **50**	a) 0/4 b) 0/2 **0**	29/80	**36.25**
CHILD 2 T/M %	a) 4.6 b) 1/1 c) 10/11 d) 0/1 **68.42**	a) 4/4 b) 6/6 c) 1/4 d) 1/3 e) 1/2 **57.89**	a) 4/6 b) 1/1 c_1) 2/11 c_2) 1/2 d) 2/2 **54.54**	a) 0/3 b) 0/1 **0**	a) 1/3 b) 0/1 **25**	a) 2/6 **33.33**	a) 3/4 b) 2/2 **83.33**	46/80	**57.5**
CHILD 3 T/M %	a) 4/6 b) 0/1 c) 10/11 d) 1/1 **73.68**	a) 4/4 b) 6/6 c) 1/4 d) 2/3 e) 1/2 **73.68**	a) 2/6 b) 0/1 c_1) 4/11 c_2) 2/2 d) 1/2 **45.45**	a) 2/3 b) 0/1 **50**	a) 0/3 b) 1/1 **25**	a) 3/6 **50**	a) 4/4 b) 1/2 **83.33**	49/80	**61.25**
CHILD 4 T/M %	a) 4/6 b) 1/1 c) 2/11 d) 1/1 **36.84**	a) 4/4 b) 4/6 c) 4/4 d) 0/3 e) 2/2 **73.68**	a) 6/6 b) 1/1 c_1) 1/11 c_2) 0/2 d) 0/2 **36.36**	a) 0/3 b) 1/1 **0**	a) 0/3 b) 0/1 **0**	a) 1/6 **16.6**	a) 1/4 b) 2/2 **50**	35/80	**43.75**
CHILD 5 T/M %	a) 5/6 b) 0/1 c) 8/11 d) 1/1 **68.42**	a) 4/4 b) 5/6 c) 0/4 d) 1/3 e) 0/2 **52.63**	a) 6/6 b) 1/1 c_1) 0/11 c_2) 1/2 d) 0/2 **36.36**	a) 1/3 b) 0/1 **25**	a) 0/3 b) 0/1 **0**	a) 0/6 **0**	a) 4/4 b) 2/2 **100**	39/80	**48.75**
CHILD 6 T/M %	a) 6/6 b) 0/1 c) 5/11 d) 0/1 **57.89**	a) 4/4 b) 6/6 c) 0/4 d) 1/3 e) 1/2 **57.89**	a) 6/6 b) 1/1 c_1) 1/11 c_2) 2/2 d) 2/2 **54.54**	a) 2/3 b) 0/1 **50**	a) 0/3 b) 0/1 **0**	a) 2/6 **33.33**	a) 4/4 b) 1/2 **83.33**	44/80	**55**
TOTAL T/M %	a) 27/36 b) 3/6 c) 39/66 d) 4/6 **64.03**	a) 24/24 b) 33/36 c) 7/24 d) 4/18 e) 4/12 **65.79**	a) 27/36 b) 5/6 c_1) 8/66 c_2) 6/12 d) 5/12 **38.64**	a) 6/18 b) 1/6 **29.17**	a) 1/18 b) 1/6 **8.33**	a) 10/36 **27.78**	a) 16/24 b) 8/12 **66.67**	242/480	**50.42**

Regarding activities which involve size of angle, the 11 year-old age-group obtained 65 out of a maximum of 126 incorrect answers (51.59%). If the activities "B" of cluster 1, and "A" of cluster 6, are included, the average of this age-group, as in the previous group decrease to 46.4% incorrect answers. However, this is still a high level of incorrect answers. This suggests that this age-group also experienced problems in handling angles which had different sizes of rays, as did the 12 year-old children.

Difficulties experienced with these activities seem to centre on a basis of uncertainty about whether it is required to recognise the length of the rays of the figure rather than the value of its angle: Is it the openness between the rays of the figure or the size of the rays? It is interesting to note that this confusion occurred in both static and dynamic perspectives, and in open and closed figures.

With regard to open figures, explored in a static perspective, this sort of problem is not surprising and, in fact, many authors have already reported similar results (see Close,G. 1982, Magina,S. 1988, among others). The surprising aspect of these activities was that we also found out that children who presented this same difficulty when they compared a spiral shape with another which did the same number of turns but in which one was larger than the other.

The last interesting finding from this age-group came from the result of the watch arena applied in the everyday setting. Although the activities embedded in this arena were characterised as related to a dynamic perspective, 2 out of the 6 children seemed to answer the first activity included in the letter of cluster 7 (by looking at these 6 watches, which one do you think turned more?) making use of the static perspective, and 4 out these same 6 children (including the 2 previous children plus 2 more children) seemed to have also answered by the same perspective in the second activity of the letter of this same cluster 7 (by looking at these 6 watches, which one do you think turned less?). Nevertheless, we will only probably be able to confirm this, after the qualitative analysis has been carried out.

5.1.5. SUMMARY OF EACH AGE-GROUP CHILDREN

Tables 5.4.1, 5.4.2, 5.4.3, and 5.4.4, shown in the next two pages, summarise the performances of the Group 1 children with regards to the best and worst performance taking into account the perspectives, contexts, settings, and conditions of the study.

Table 5.4.1: Summary of the 14 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♀	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 2 ♀	Static	Dynamic	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 3 ♂	Static	Dynamic	Rotation	Navigation	Logo	Peper&Pencil	Action	Recognition
CHILD 4 ♀	Static	Dynamic	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 5 ♂	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♂	Static	Dynamic	Rotation	Navigation	He performed the same in all 3 settings	———	Action	Recognition

Table 5.4.2: Summary of the 13 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♀	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 2 ♀	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 3 ♂	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 4 ♂	Static	Dyanmic	Comparison	Rotation	Logo	Paper&Pencil	Action	Recognition
CHILD 5 ♂	Static	Dyanmic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♀	Static	Dyanmic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition

Table 5.4.3: Summary of the 12 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♀	Static	Dynamic	Rotation	Navigation	Everyday	Paper&Pencil	Action	Recognition
CHILD 2 ♀	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 3 ♂	He Performed the same For Dynamic and Static		Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 4 ♀	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 5 ♂	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♂	Dynamic	Static	Rotation	Navigation	Everyday	Paper&Pencil	Action	Recognition

Table 5.4.4: Summary of the 12 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♂	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 2 ♀	Dynamic	Static	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 3 ♀	Dynamic	Static	Rotation	Navigation	Everyday	Paper&Pencil	Action	Recognition
CHILD 4 ♂	Static	Dynamic	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 5 ♂	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♀	Static	Dynamic	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition

5.1.5.1 <u>The 14 Year-old Children</u>

Table 5.4.1 shows that all six 14 year-old children performed similarly in the perspective, context, setting, and condition: the best perspective was static, the best context was rotation, the best setting was Logo, and the best condition was action.

When a comparison is made between the best performance of the 14 year-old children in perspective and in context, the results showed an apparent contradiction. At first sight it seems to be illogical that anyone could give, as his/her best performances, the static perspective together with the rotation context and the Logo setting.

However this apparent contradiction is clarified if we take into account two things: 1) The difference in children's performance between dynamic and static perspective was no higher than 10%; and 2) the children had an unsatisfactory performance in the navigation context, in which no child could solve more than 63% of the activities. Also the general average of incorrect responses for the six children was less than 50%, whereas in the rotation context children obtained 100% correct answers, and in the comparison context they obtained an average of 68% correct answers. As the activities in navigation context concerned dynamic perspective, it can be concluded that it was this context which pushed down the average for this perspective.

Navigation context is not easy. On the contrary, it is very difficult for children because they have to choose their own point of reference in order to complete it. In mini city arena, for example, a child controls the mini cars from outside, but if s/he wants to remember all the turns s/he makes along the way, s/he has to 'feel' as if s/he is inside the car. Another reason to consider why this context is difficult for children is the fact that navigation involves not only rotation but also translation. This requires the use of a mental operation more elaborate than which is required in the rotation context.

If we look at map arena in the p & p setting, we can see there is another factor which makes it hard for the child to carry out the activity of navigation. Navigating a car is something very different from joining up arrows along the way. What is more, in a p & p setting the child does not have to turn, s/he just has to push and pull her hand to the right or left, up or down. So in fact the child does not actually do any turn, although the map on the paper indicates turns.

Given these facts, I believe it would be fairer to the children's performances if I state that these children were able to solve the activities which were in the dynamic perspective better than those in the static perspective. Also when the children's references are taken into account, this assertion is strengthened.

Finally, if we look at map arena in a Logo setting related to the navigation context, we can also verify that it is not easy for a child to navigate a turtle on the screen, which is on a vertical plane, whilst the child's viewpoint is on the horizontal plane. Furthermore, it seems difficult to lose one's own point of reference in order to assume the point of reference of the turtle. This point is not new. There is a large debate about it in literature which will be discussed in a later chapter.

However, as we have already said, the Logo setting had a very special point in its favour. It was the first time these children had had an opportunity to have a hands-on experience with a computer. I believe that this is the reason for the children's great interest in, and attention to, all the activities related to this setting. I even believe that to play with the Logo was a strong incentive for the children, and it could well be the main reason why we did not lose any children during the experiment, as Logo was the last setting children did.

5.1.5.2 <u>The 13 Year-old Children</u>

Table 5.4.2 shows that the general performance of the 13 year-old age-group was not very different from 14 year-old children. That is, the average of correct answers, with regard to the perspectives, was better in static than in dynamic. The rotation context obtained the highest average of children's correct answers, followed by comparison and in third place navigation. These children also performed better in Logo than in everyday, which was the second, followed by the p & p setting. Finally, children found the right answers more easily in activities which asked them to act rather than in activities included in the recognition condition.

However, looked at individually, the results were not uniform for the whole group. The static perspective, for example, was better for five out of six children. The same thing happened in the context, where five out of six children were better in rotation while the one remaining child was better in comparison.

When considering the children's performance in Perspective and in context, the difference in the averages, between perspectives, and among the contexts, were higher in this age-group than in the previous one, although the children's results have not been as homogeneous as the 14 years old. With 13 year-old children we find they achieved a 16% higher performance in static than in dynamic perspective.

In navigation no child was able to find more than 27% of the right answers, and the group average was 21% correct answers. In comparison context, children had performed as well as the older age-group when they obtained 73% of correct answers, but in rotation the 13 year-old children were almost 10 points lower than 14 years old, although the former still obtained, on average, a score of 90.6%. Once again, it is noticeable how difficult the navigation context is for children. I still hold the view that the main reason for this is the changing of the children own point of reference.

As for the settings, the same explanation applies to the previous age-group. The opportunity to play any sort of 'game' in Logo, seems to be more interesting for children who have never been in front of a computer, than doing activities in p & p which can be less than interesting, to say the least.

I would like to discuss the performance 13 year-old children with regard to conditions. As with the older age-group, the children seemed to find it easier to do the activities which required an action rather than a recognition. In fact, in an action a person does not have any option, one just has to do something, whilst in a recognition there is more than one choice which can lead to some confusion.

This point will be discussed later on, when more age-groups have been analysed. This is because my prediction for this experiment was that children would be better in those activities which involved recognition rather than action. The reason for my prediction was that in recognition the figures are already prepared, whilst in action children have to use their imagination before they can draw or construct the figure. For this reason I thought that action could demand more, from the psychological point of view, than recognition.

5.1.5.3 The 12 Year-old Children

Table 5.4.3 shows the peformance of children from this age-group was similar to the two previous groups in both contexts and conditions of the study. Nevertheless the children's performance changes a lot when the settings and perspectives are taken into account.

Consider the settings. The 12 year-old children, like the 13 and 14 years old, performed worse in the p & p setting, whilst the children's best performance was in Logo. The everyday setting also happened to be one of the best for 2 out of the 6 children. In the perspectives, children did not perform homogeneously: 4 out of these 6 children were better in activities which involved the static perspective, whilst the other 2 children performed better in those activities which involved the dynamic perspective. These results are considered important because that is the first time the dynamic perspective, as well as everyday setting, appear in the columm of the best performance. However it would be premature to interpret anything from that at the moment.

5.1.5.4 The 11 Year-old Children

Table 5.4.4 shows that the general performance of this age-group, with regard to the best and worst context and condition of the study, was allied with the 12 years old age-group. The 11 age-group children were uniform in performing better in the rotation context as well as in acting over the activities. They showed more difficulty in activities embedded in the navigation context and in those activities which required children to recognise something.

The children of this age-group were divided according to the perspective, i.e., 3 of them performed better in those activities within the dynamic perspective and the other 3 in the static perspective. If we look at Tables 5.4.1, 5.4.2, 5.4.3, and 5.4.4, we note that in accordance with the decreasing age, the dynamic perspective was being introduced as the children's best performance.

When looking at the children's performance relating to the settings, it was in the Logo setting that 5 out of the 6 children performed better and the remaining child preferred the everyday setting. p & p was the worst for 4 out of the 6 children and everyday was the worst for the remaining two children. From all 24 children of the Middle school, we note that: 1) with regards to the contexts, all the 24 children performed better in those activities within rotation and all 24 had also more difficulty in the navigation context; 2) as for the settings, 20 children performed better in the Logo setting, one child performed equally in all three settings and the remaining 3 children performed better in everyday; and 3) all 24 children obtained best results from those activities which required them to act rather than to recognise.

5.1.5.5 <u>The Summary of Group 1 as a Whole</u>

Based on the previous Tables, but mainly on the former age-groups profile Tables, which present the performances of the middle school children with regards to their best and worst fulfilment considering the perspectives, contexts, settings, and conditions of the study, we can observe the following:

(a) From the viewpoint of perspective, all the 14 age-groups children presented their best performances in those activities included in the static perspective. 5

out of the 6 children of the 13 years old age-group perfomed better in those activities which were included in the static persperctive. In the 12 age-group, 3 children performed better in the activities which were involved in the static perspective, 1 child performed the same in the types of static and dynamic activities, and the remaining 2 in the dynamic activities. Finally, among the children of 11 year-old age-group the result was half and half, i.e., 3 children performed better in the activities considered dynamic, and 3 in the static. In summary, 17 out of the 24 children dealt better with the activities involving the static perspective, whilst 5 children (the youngest) performed better in the activities involving the dynamic perspective, and 1 child performed the same in both;

(b) from the viewpoint of the contexts, all 24 children performed best in the activities which were carried out inside the rotation context . All the activities of this context were classified as dynamic;

(c) from the viewpoint of the settings, 20 out of the 24 children presented their best performances in the activities developed inside the Logo setting. Because of the Logo turtle feature which allows children to see all its turns and moving around the screen, all the activities included in this setting were categorised as dynamic.

(d) from the viewpoint of the arenas, all the children's best performances were in the activities carried out in the watch arena (see Table 5.2.3). 25 out of the 29 activities realised in the Logo setting were included in the dynamic category;

(e) from the viewpoint of the conditions, the whole Group 1 children performed better in the action activities. 25 out of the 28 action activities were considered as dynamic.

After concluding this first part of the quantitative analysis, I would not feel confident to interpret or even state anything about the children's idea of angle. However, across the findings, except from the viewpoint of the perspective itself, the Group 1 children seems to deal better with the activities which were included in the dynamic perspective.

The next section will present the results from elementary school (Group 2). Because the quantitative analysis of the Group 1 children is already finished, the analysis of Group 2 is going to be carried out (a) by interpreting the Group 2 children's results themselves, and (b) by comparing the results from Group 2 with Group 1.

5.2 ANALYSIS OF GROUP 2: From 6 to 11 year-old children

Table 5.5, presented on the next page, provides an overview of the children's results regarding the whole set of activities distributed over the arenas, settings, and conditions. It shows - as was expected for the middle school children - that the older children made fewer mistakes than the younger. The difference between the results of the age groups was around 10%, except between the 9 and 8 year-old age-groups in which the difference was practically non existent. It means that 10 year-old children seem to be able to solve, on average, around 55% of any test involving angle, while children of 6 are able to solve only around 22%.

For the same reason already explained in the analysis of Group 1 -- middle school children -- the selected statistical test to determine whether the difference between the average of mistakes for each age-group in Table 5.4 (considering the 92 activities) and the average of mistakes based on the seven clusters presented for each age in Table 5.6, was χ^2.

Such as the middle school data, the χ^2 test determined that there was no significant difference, within the same age, between children's performances in the average of mistakes in 92 activities and in the average of mistakes in the seven clusters. This information allowed us to proceed with the data of the manor first grade by analysing either all 92 activities, or using the 80 activities (the 7 clusters) without being afraid to make bias.

The two next pages present Tables 5.5, 5.5.A, and 5.6. These Tables show the raw numbers of the children's mistakes and their averages in both the 92 activities (Tables 5.5, 5.5.A) and in the 7 clusters (Table 5.6). All the subsequent statistical tests obtained the data, total or partially, from these tables.

ELEMENTARY SCHOOL - FROM 6 TO 10 YEARS-OLD

Table 5.5: Total, with averages, of the elementary school children's incorrect answers over 92 activities

ARENA		MINI CITY			WATCHES			STICK GAME / 2 ANGLES				4 ANGLES		TURNSTILE/ARROWS			SPIRALS		GENERAL RESULT
SETTING CONDITION AGES		EVERY Recog	P & P Recog	LOGO Recog	EVERY Recog	EVERY Action	LOGO Action	EVERY Recog	EVERY Action	P & P Recog	LOGO Recog	P & P Recog	LOGO Recog	EVERY Action	P & P Action	LOGO Action	P & P Recog	LOGO Recog	
10	T/M	51/78	38/66	29/54	14/24	3/30	3/36	19/30	9/18	19/30	13/24	9/12	8/12	7/42	10/30	9/30	4/18	8/18	253/552
	%	65.38	57.57	53.70	58.33	10	8.33	63.33	50	66.33	54.17	75	66.67	16.67	33.33	30	22.22	44.44	**45.83**
09	T/M	55/78	51/66	28/54	17/24	7/30	7/36	18/30	10/18	23/30	14/24	12/12	11/12	13/42	14/30	9/30	7/18	6/18	302/552
	%	70.55	77.27	51.85	70.83	23.33	19.44	60	55.55	76.67	58.33	100	91.67	30.95	46.67	30	38.89	33.33	**54.71**
08	T/M	59/78	44/66	27/54	12/24	4/30	5/36	11/30	10/18	25/30	16/24	11/12	12/12	10/42	18/30	14/30	11/18	12/18	301/552
	%	75.64	66.67	50	50	13.33	13.89	36.67	55.55	83.33	66.67	91.67	100	23.80	60	46.67	61.11	66.67	**54.35**
07	T/M	57/78	47/66	40/54	19/24	21/30	12/36	21/30	7/18	24/30	16/24	11/12	12/12	13/42	14/30	9/30	7/18	6/18	385/552
	%	73.08	71.21	74.07	79.17	70	33.33	70	38.89	80	66.67	91.67	100	30.95	46.67	30	38.89	33.33	**69.74**
06	T/M	58/78	59/66	38/54	20/24	25/30	22/36	26/30	7/18	28/30	19/24	12/12	12/12	7/42	10/30	9/30	4/18	8/18	433/552
	%	74.36	89.39	70.37	83.33	83.33	61	86.67	38.89	93.33	79.17	100	100	16.67	33.33	30	22.22	44.44	**78.44**
ALL AGES	T/M	**280/390**	**239/330**	**162/270**	**82/120**	**60/150**	**49/180**	**95/150**	**43/90**	**119/150**	**78.120**	**55/60**	**55/60**	**87/210**	**93/150**	**80/150**	**48/90**	**49/90**	**1674/2760**
	%	**71.79**	**72.42**	**60**	**68.33**	**40**	**27.22**	**63.33**	**47.78**	**69.33**	**65**	**91.67**	**91.67**	**41.43**	**62**	**53.33**	**53.33**	**54.44**	**60.65**

TABLE 5.5.A: Total, with averages, of the elementary school children's incorrect answers over the 6 Arenas

ARENAS	MINI CITY/MAP	WATCHES	STICK GAME/2 ANGLES	4 ANGLES	TURNSTILE/ARROWS	SPIRALS
T/M	681/990	191/450	335/510	110/120	260/510	97/180
%	**68.79**	**42.44**	**65.69**	**91.66**	**50.98**	**53.89**

ELEMENTARY SCHOOL - FROM 6 TO 10 YEAR-OLD CHILDREN

TABLE 5.6: Total, with averages, of the elementary school Children's Incorrect Answers over the 7 Clusters

CLUSTERS / AGES		CLUSTER 1 ANG < 90^0	CLUSTER 2 ANG = 90^0	CLUSTER 3 ANG = 180^0	CLUSTER 4 ANG = 540^0	CLUSTER 5 ANG = 720^0	CLUSTER 6 ANG > 720^0	CLUSTER 7 4 AND 6 ANG	TOT. INCOR / MAX. SCORE	% INCORRECT
10	T/M	a)18/36 b)2/6 c)47/66 d)5/6	a)24/24 b)36/36 c)14/24 d)4/18 e)5/12	a)20/36 b)2/6 c_1)6/66 c_2)2/12 d)9/12	a)3/18 b)2/6	a)2/18 b)0/6	a)12/36	a)17/24 b)7/12	237/480	**49.37**
	%	**63.16**	**72.81**	**29.54**	**20.83**	**8.33**	**33.33**	**69.44**		
09	T/M	a)24/36 b)0/6 c)51/66 d)4/6	a)24/24 b)36/36 c)20/24 d)5/18 e)6/12	a)23/36 b)2/6 c_1)14/66 c_2)3/12 d)10/12	a)1/18 b)4/6	a)0/18 b)0/6	a)13/36	a)22/24 b)10/12	272/480	**56.67**
	%	**69.30**	**79.82**	**39.39**	**20.83**	**0**	**36.11**	**88.89**		
08	T/M	a)24/36 b)4/6 c)45/66 d)5/6	a)24/24 b)35/36 c)15/24 d)7/18 e)5/12	a)19/36 b)3/6 c_1)10/66 c_2)7/12 d)9/12	a)9/18 b)2/6	a)1/18 b)1/6	a)21/36	a)20/24 b)9/12	278/480	**57.92**
	%	**68.42**	**75.44**	**37.12**	**45.83**	**8.33**	**63.89**	**80.85**		
07	T/M	a)23/36 b)3/6 c)51/66 d)3/6	a)24/24 b)36/36 c)21/24 d)4/18 e)5/12	a)24/36 b)3/6 c_1)33/66 c_2)10/12 d)12/12	a)14/18 b)5/6	a)9/18 b)5/6	a)23/36	a)24/24 b)12/12	349/480	**72.71**
	%	**70.17**	**83.33**	**62.12**	**79.17**	**58.33**	**63.89**	**100**		
06	T/M	a)31/36 b)4/6 c)58/66 d)4/6	a)24/24 b)36/36 c)20/24 d)14/18 e)3/12	a)27/36 b)4/6 c_1)47/66 c_2)11/12 d)10/12	a)15/18 b)4/6	a)13/18 b)5/6	a)26/36	a)24/24 b)10/12	390/480	**81.25**
	%	**85.08**	**85.09**	**75**	**79.17**	**75**	**72.22**	**94.44**		
TOTAL	**T/M**	a)120/180 b)13/30 c)252/330 d)20/30 **405/570**	a)120/120 b)179/180 c)90/120 d)40/90 e)23/60 **452/570**	a)113/180 b)14/30 c_1)110/330 c_2)33/60 d)50/60 **321/660**	a)42/90 b)17/30 **59/120**	a)24/90 b)11/30 **35/120**	a) **97/180**	a)107/120 b)49/60 **156/180**	1525/2400	**63.54**
	%	**71.05**	**79.30**	**48.64**	**49.17**	**29.17**	**53.89**	**86.67**		

In spite of the quite small difference between the general result of 8 and 9 age-groups, Table 5.7.1, taking into account the all 92 activities and comparing the five different age-groups, shows that there was a significant difference in the children's performances.

	AGES				
	6	7	8	9	10
AVERAGE	78.44	69.74	54.35	54.71	45.83

df = 4, $0.02 < p < 0.05$

Table 5.7.1: The difference of the children's performances amongst the five age-groups according to χ^2 test.

Comparing the result of 8, 9, and 10 year-old children plus the results of the two lower age-groups of the middle school (11 and 12 year-old children), it was noted that, according to χ^2, no significant difference was found (see Table 5.7.2 below). In fact, children aged 10 made less mistakes than the 11 year-old age-group. Moreover the difference between 8 and 9 year-old children's scores was practically nothing. This is an interesting finding because the study has been considering the children's ages and their level of schooling as factors that influence children's conception of angle. In this case these children are from five different age-groups as well as different schooling stages (elementary and middle school). However it is still early to try an interpretation for this finding because up to this point only the quantitative results have been analysed. I expect that the qualitative analysis (next chapter) can provide more information in order to enrich this discussion .

	AGES					
	8	9	10	11	12	TOTAL
AVERAGE	54.35	54.71	45.83	49.27	42.12	246.28

df = 4, $0.5 < p < 0.7$

Table 5.7.2: The difference of the children's performances among the three age-groups according to χ^2 test.

c^2 test was also applied to compare the general result of middle school (Group 1) with the general result of elementary school (Group 2). In spite of the close results among 10, 11 and 12 year-old children as well as 8 and 9 year-old age-groups, the difference between both groups was statistically significant (see Table 5.7.3 below).

	post		
	GROUP 2 EARLY ELEMENTARY	**GROUP 1** LATER ELEMENTARY	TOTAL
AVERAGE	63.54	42.42	105.95

df = 1, $0.02 < p < 0.05$

Table 5.7.3: A comparison between children's performance in the elementary and middle schools according to c^2 test.

c^2 test showed a difference statistically significant among the results of age-group children from Group 1 (Table 5.2.1), it also presented a significant difference among age-group children from Group 2 (Table 5.7.1), the difference between Groups 1 and 2 (Table 5.7.2) was still statistically significant, but there were no significant difference when the results from 8 to 12 year-old children were compared. Crossing these results I can assume that it was the 6 and 7 year-old age-group children the responsible to push up the average of mistakes of Group 2 children; whilst it seems obvious that 13 and 14 year-old age-group children worked out in the inverse way, i.e., they pull down the average of mistakes of Group 1. The next two Tables are to verify this hypothesis. Table 5.7.4 shows the result of the c^2 test with regard to the comparison between 6 and 7 year-old children, while Table 5.7.5 refers to the comparison between 13 and 14 year-old children's results.

	post		
	6 years old	**7 years old**	TOTAL
AVERAGE	78.44	69.74	148.18

df = 1, $0.5 < p < 0.7$

Table 5.7.4: A comparison between 6 and 7 year-old children according to c^2 test.

	post		
	13 years old	**14 years old**	TOTAL
AVERAGE	32.15	20.15	52.20

df = 1, $0.1 < p < 0.05$

Table 5.7.5: A comparison between 13 and 14 year-old children according to c^2 test.

From the results shown in Tables 5.2.1, 5.7.1, 5.7.2, 5.7.3, 5.7.4 and 5.7.5 I felt confident to divide my sample, at least from the quantitative viewpoint, into three sub-groups: sub-group A including children from 6 to 7 years old who made about 70% of mistakes; sub-group B formed by children from 8 to 12 years old, who incorrectly answered about half of the study's questions; and sub-group C composed by children from 13 to 14 years old whose averages of mistakes were no higher than 30%.

In order to facilitate the visualisation of the difference of the children's performances from age-group to age-group, the next two figure show an overview of the elementary and middle school children taking into account the their fulfilment in the 7 clusters (Figure 5.1) and in the 92 activities (Figure 5.2).

FIGURE 5.1: THE AVERAGE, in %, OF THE ELEMENTARY AND MIDDLE SCHOOL CHILDREN'S INCORRECT RESPONSES OVER THE 7 CLUSTERS

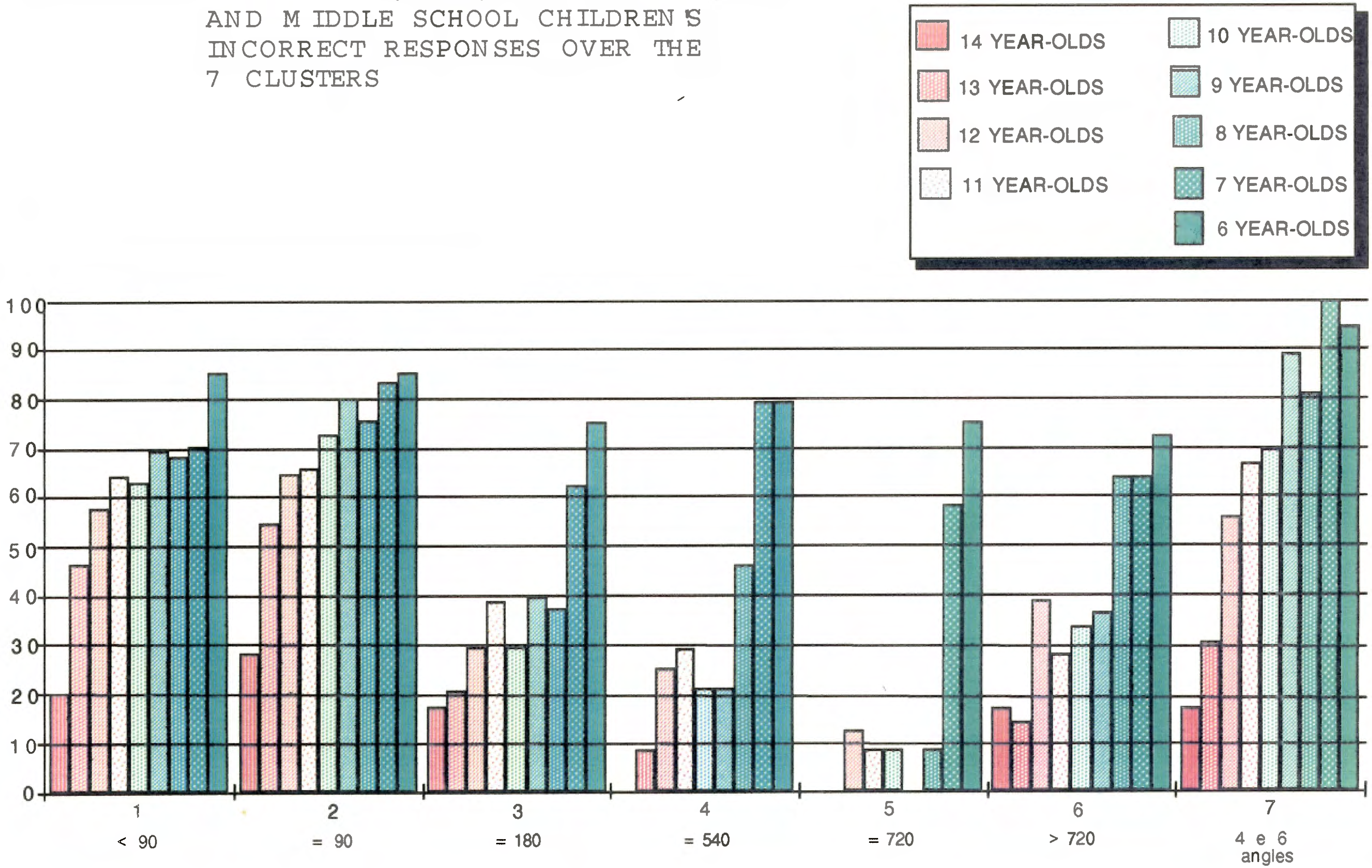
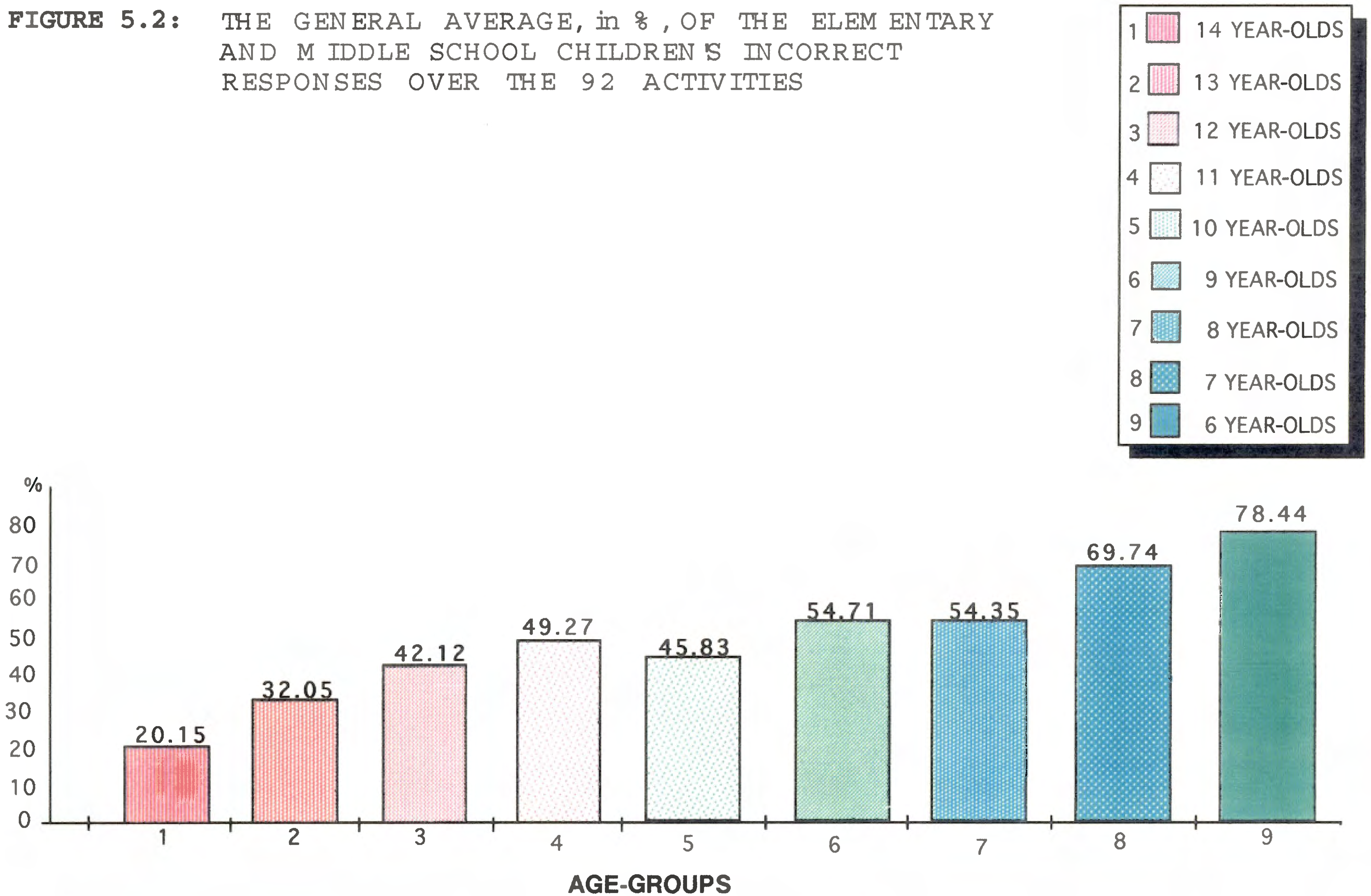


FIGURE 5.2: THE GENERAL AVERAGE, in %, OF THE ELEMENTARY AND MIDDLE SCHOOL CHILDREN'S INCORRECT RESPONSES OVER THE 92 ACTIVITIES



As regards to children's performance according to different contexts, settings, arenas and clusters, the Friedman test was used. The reason for using this test is based on the characteristics of our sample, which present a matched sample, i.e., the same sample was submitted to 3 different contexts, 3 different settings, and 6 different arena groups. This means, for instance, that there was no difference from one context to another in terms of who had taken part on them.

According to Friedman test, shown in the Tables 5.5.1 and 5.5.2 below, the difference among both the contexts (Table 5.5.1) and the settings (Table 5.5.2) were statistically significant.

% OF INCORRECT RESPONSES 3 CONTEXTS			
CONTEXTS / AGES	1 NAVIGATION	2 ROTATION	3 COMPARISON
10	74.24	19.44	55.55
09	81.06	28	62.12
08	77.27	30	64.65
07	81.06	64.67	70.71
06	89.39	76.67	80.30
R_j	15	5	10

K = 3, N = 5, p = 0.00077

Table 5.5.1: The result of the elementary school children's performances in the 3 contexts according to the Friedman test.

% OF INCORRECT RESPONSES 3 SETTINGS			
SETTINGS / AGES	1 EVERYDAY	2 P & P	3 LOGO
10	46.4	51.28	40.23
09	54.05	68.59	43.1
08	47.75	69.87	49.42
07	68.02	75	67.24
06	75.22	90.38	71.84
R_j	9	15	6

df = 1, K = 3, N = 5, p = 0.0085

Table 5.5.2: The result of the elementary school children's performances in the 3 settings according to the Friedman test.

The results shown in these two tables above are aligned with the analysis of the middle school group, i.e., that "children's performance change, in a significant way, from context to context as well as from setting to setting" (pp. 146). However so far the results implies changes in conception of angle continues to be an open question.

In fact, although the averages of children's mistakes in Group 2 have been, over all, higher than the averages of children from Group 1, the results of the Group 2 show that these children had , as was noticed in the Group 1, all the children who took part in this research performed better in those activities which were involved in the rotation context, they also obtained best performance in the activities carried out in the Logo setting. With regards to the children's poorest performances, they accurred on activities included in the natigation context and in the p & p setting.

Looking at the performance of Group 1 children with regards to their performances in the three contexts (Table 5.2.2), it was noticed that this group of children made, at minimum, four times more mistakes in the activities embedded in the navigating than in those involved in the rotation (the difference in children's performance is directly proportional to the age, i.e., as older the children were the bigger was this difference). Among children of Group 2 the difference between children's performance in the activities involved in the navigation and rotation contexts was, in maximum, 3.8 times more (as in Group 2, this difference increased in the older age groups, see Table 5.5.1). However, the most interesting finding shown by the comparison between Tables 5.2.2 and 5.5.1 was the enormous reduction of children's mistakes in the activities included in the rotation context at the same time as the age of the children were increasing (while 6 year-old children showed 76.67% of mistakes, the 14 year-old children made no mistake at all in this context). Regarding the difference between Group 1 and Group 2 children's performances in the activities included in the navigation context, the reduction of children's mistakes was not as big as it was in the rotation, that is, while 6 year-old children made 89.39% of mistakes, the 14 years old made 45.65%.

Looking in depth at the children's performances in activities embedded in the navigation context, it is possible to see that the groups of 8 and 10 year-old children performed better than did the 11 and 12 year-old children. Moreover, the 11 and 12 year-old children made as many mistakes as the 7 and 9 year-old age-groups. If we only look at the mistakes made by children in the activities embedded in navigating, it is possible to include the 7, 9, 11 and 12 year-old children in the same level of performance, whilst the 8 and 10 year-old children are in a higher level. However, if only the activities included in the rotation context are taken into account another picture emerges: the 7 year-old children would be in the lower level, the 8 and 9 year-old of age would form the next level, and the 10, 11 and 12 year-old children would be in the best level of performance among these 6 age-groups. In other words, whilst it was possible to observe an improvement in the children's performances in the activities embedded in the rotation context which were proportional to their ages, the same relationship could not be observed in those activities included in the navigation context.

As regards to the viewpoint of settings, from the results presented in Table 5.2.3 (Group 1) and Table 5.5.2 (Group 2) it is possible to observe that the differences between the children's mistakes, as regards the Logo and the p & p, as well as Logo and everyday activities, were less among children of Group 2 than among children of Group 1. If my previous interpretation is correct -- i.e., that children of middle school have performed better in the Logo setting activities because the work with the microcomputer was a pleasurable experience for them, whereas working with p & p had a stale familiarity about it and was rather an uninviting task -- the same interpretation cannot be used for the performance of elementary school children. In fact, even considering that it was through the Logo setting activities that children of Group 2 had their first opportunity to 'play' in a microcomputer environment, and although they had shown a great interesting in 'playing' in this environment, such a stimulus seemed not to be so strong as it was for children of Group 1.

Table 5.5.3, shown below, provides an overall of view children's performances taking into account the arenas in which the activities were carried out. The result of the Friedman test shows that the differences of children scores across the arenas were statistically significant.

% OF INCORRECT RESPONSES 6 GROUPS OF ARENAS						
ARENAS / AGES	**GROUP 1** map/mini city	**GROUP 2** watches	**GROUP 3** 2 angles	**GROUP 4** 4 angles	**GROUP 5** Arrow / turnstile	**GROUP 6** spiral
10	59.59	21.11	58.82	70.83	25.49	33.33
09	67.68	34.44	63.72	95.83	35.29	36.11
08	65.66	27.78	60.78	95.83	41.18	63.89
07	72.73	57.78	66.67	95.83	73.53	63.89
06	78.28	74.44	78.43	100	79.41	72.22
R_j	22	6	18	30	16	13

df = 5, K = 6, N = 5 $0.001 < p < 0.01$

Table 5.5.3: The result of the elementary school children's performances in the 6 groups of arenas according to the Friedman test.

Table 5.5.3 shows that the children's best performance was in the watch arena (in all the age-groups except for 6 year-old age-group). This result is in line with the analysis of Group 1. The 4 angles arena was poorly treated by children of all age-groups. In fact, it is important to bear in mind that activities developed in the watch arena were related to the dynamic perspective, whilst those activities included in 4 angles arena were related to the static perspective.

Comparing children's performance of Group 2 with children of Group 1 as regards the map/mini city arena, it was noted that 10 year-old children, surprisingly, made the same amount of mistakes as 13 year-old children. What is more, the 11 year-old children made even more mistakes in this arena than the group of 7 year-old children. In summary, considering both Groups 1 and 2, the children's performance was improving from 6 to 10 years old. At the age of 11 the children's performance fell to that of the 7 years old age-group's performance and then it started to be improved again, but no other age group performed the same as 10 year-old children until the age of 13 years old.

Looking in depth at the activities categorised in the map/mini city arena, it is clear that the recognising of 90^0 was the most difficult task for children of all ages (see the letters 'a' and 'b' of the cluster 2 in Tables 5.2 and 5.6). In fact, although 90^0 has been referred to as an angle that is easily by students, this was not observed in this arena. However, recognising of 90^0 in the 2 angles arena (see the letter 'd' of the cluster 2 in Tables 5.2 and 5.6) did not appear to be a difficult activity for these same children.

To understand the difference shown through the comparison between children's fulfilment in the map/mini city and in the 2 angles arenas, it is important to pay attention to the way in which the activities were presented to them. From this, two important factors must be taken into account in this analysis: (a) the cultural factor, and (b) learning process. In the map/mini city arena there were two question about 90^0: (1) did you turn any quarter of turn in any place along the way? and (2) did you turn any 90^0 in any place along the way? Regarding to the first activity carried out in the map/mini city, the terminology 'a quarter of turn' is completely unusual as referring to an amount of turn in the Brazilian culture, rather it has meaning only to refer to algebraic fractions -- no one in Brazil says 'a quarter to 2 o'clock' referring to the hour, we always say "fifteen to two o'clock", thus it would be very hard for Brazilian children identify 'quarter of turn' as a turn of 90^0. The second activity involving 90^0 in the map/mini city was related first to the learning process -- because 90^0 is a mathematical convention -- and thus to the need of contextualizing, in everyday life, the mathematics content which is learnt in school.. In the 2 angles arena the activities required from children much more the use of their perception than a recognising of the value of the angle.

Comparing once again the performance of the elementary school children group with the middle school children, it is noticed that the activities included in the watch arena were the easiest for children from both groups. As was expected, the elementary school chidden made more mistakes than the children from older group (Group 1), the children incorrect answers were proportional to their age, i.e. as young they were as much mistakes they made. However, this proportionality was not notice between 10 and 11 year-old children. Actually they presented a close performances to each another. this proximity between these two groups of children ages as regards to their performances was also noticed in the 4 angles, turnstile/arrow and spiral arenas.

Analysing the Group 2 children's fulfilment concerning the recognising and action situations, Table 5.5.4 below shows that these children performed better in the activities which involved action than those requiring a recognition. This finding was also found for children of Group 1.

% OF INCORRECT RESPONSES CONSIDERING TWO CONDITIONS		
CONDITIONS / AGES	1 RECOGNITION	2 ACTION
10	57.92	22.04
09	66.12	32.26
08	65.57	32.79
07	73.77	61.83
06	81.42	72.58
R_i	10	5

df = 1, K = 2, N = 5, $0.02 < p < 0.05$

Table 5.5.4: The result of the elementary school children's performances in the 2 conditions according to the Friedman test.

Finally Table 5.5.5 shows a summary of children's performances in the 7 clusters which presents a high level of significance.

% OF INCORRECT RESPONSES 7 CLUSTERS							
CLUSTERS / AGES	1 $< 90^0$	2 90^0	3 180^0	4 540^0	5 720^0	6 $>720^0$	7 4 & 6 ANGLES
10	62.28	72.81	29.54	20.83	4.17	33.33	69.44
09	69.3	79.82	39.39	20.83	0	36.11	88.89
08	68.42	75.44	37.12	45.83	8.33	63.89	80.55
07	70.17	83.33	62.12	79.17	58.33	63.89	100
06	85.08	85.09	75	79.17	75	72.22	94.44
R_i	24	31	13.5	16	6.5	15	34

df = 6, K = 7, N = 5, $p < 0.001$

Table 5.5.5: The results of the elementary school children's performances in the 7 clusters according to the Friedman test.

Children made less mistakes in cluster 5. This cluster was concerned the turn of 720^0 and included activities in the rotation context developed inside the turnstile and arrow arenas.

From the viewpoint of the distribution of the activities inside the arenas (Table 5.5.3) supported by the children general results taking in consideration the 7 clusters (Table 5.5.5), children's performances show that activities embedded in the 4 angles and in the map/mini city arenas were crudely responded by all the sample. Thus, although cluster 7 has included two activities carried out in the watch arena, there is no doubt that it was the children fulfilment in the activities carried out in the 4 angles arena the most responsible for the children poorest result in cluster 7.

5.2.1. SUMMARY OF EACH AGE-GROUP CHILDREN

Before closing the quantitative analysis considering the whole group of elementary school children and start to look at the children's performances from the viewpoint of the quality of the data, I would like to list, in summary, the most interesting findings described up to here. These findings were based on in the previous Tables, but mainly on the age-groups profile Tables (see in the next three pages the 5.8.1, 5.8.2, 5.8.3, 5.8.4, and 5.8.5 Tables, which present the performances of the elementary school children with regards to their best and worst fulfilment considering the perspectives, contexts, settings, and conditions of the study):

Table 5.8.1: The profile of the 10 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♂	Dynamic	Static	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 2 ♀	Static	Dynamic	Rotation	Navigation	Paper&Pencil	Everyday	Action	Recognition
CHILD 3 ♂	Dynamic	Static	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 4 ♀	Dynamic	Static	Rotation	Navigation	Everyday	Paper&Pencil	Action	Recognition
CHILD 5 ♀	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♂	Dynamic	Static	Rotation	Navigation	Logo	Everyday	Action	Recognition

Table 5.8.2: The profile of the 9 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♀	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 2 ♂	Dynamic	Static	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 3 ♀	Static	Dynamic	Rotation	Navigation	Logo	Peper&Pencil	Action	Recognition
CHILD 4 ♂	Dynamic	Static	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 5 ♀	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♀	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition

Table 5.8.3: The profile of the 8 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♀	Dynamic	Static	Rotation	Comparison	Logo	Paper&Pencil	Action	Recognition
CHILD 2 ♂	Dynamic	Static	Rotation	Comparison	Everyday	Paper&Pencil	Action	Recognition
CHILD 3 ♂	Static	Dynamic	Rotation	Navigation	Logo	Peper&Pencil	Action	Recognition
CHILD 4 ♀	Dynamic	Static	Rotation	Comparison	Logo	Paper&Pencil	Action	Recognition
CHILD 5 ♂	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♂	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition

Table 5.8.4: The profile of the 7 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♂	Static	Dynamic	Rotation	Navigation	Logo	Everyday	Action	Recognition
CHILD 2 ♂	Static	Dynamic	Comparison	Navigation	Logo	Everyday	Recognition	Action
CHILD 3 ♂	Static	Dynamic	Rotation	Navigation	Everyday	Logo	Action	Recognition
CHILD 4 ♂	Dynamic	Static	Rotation	Comparison	Everyday	Peper&Pencil	Action	Recognition
CHILD 5 ♂	Dynamic	Static	Rotation	Comparison	Logo	Everyday	Action	Recognition
CHILD 6 ♂	Static	Dynamic	Comparison	Navigation	Logo	Everyday	Recognition	Action

Table 5.8.5: The profile of the 6 year-old children's performances

	PERSPECTIVE		CONTEXT		SETTING		CONDITION	
	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE	BEST PERFORMANCE	WORST PERFORMANCE
CHILD 1 ♀	She performed the same for Dynamic and Static		Rotation	She performed the same in navigation and in comparison	Logo	Everyday	Action	Recognition
CHILD 2 ♂	Static	Dynamic	Rotation	Navigation	Logo	Peper&Pencil	Action	Recognition
CHILD 3 ♀	She performed the same for Dynamic and Static		Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 4 ♀	Static	Dynamic	Comparison	Navigation	Logo	Everyday	Action	Recognition
CHILD 5 ♀	Dynamic	Static	Rotation	Navigation	Logo	Paper&Pencil	Action	Recognition
CHILD 6 ♂	Dynamic	Static	Rotation	Navigation	Logo	Everyday	Recognition	Action

(a) From the viewpoint of the perspective, all the 10, 9, and 8 age-groups children presented their best performances in those activities included in the dynamic perspective. In the 7 age-group, 4 children performed better in the activities which were involved in the static perspective, and the remainding 2 in the dynamic activities. Finally, among the children of 6 year-old age-group the result was half and half, where 2 children performed better in the activities considered dynamic, 2 in the static, and 2 obtained the same score in both type of activities. In summary, 22 out of the 30 children dealt better with the activities involving the dynamic perspective, whilst only 6 children (the youngest) performed better in the activities considered static.

(b) from the viewpoint of the contexts, 27 out of these 30 children performed best in the activities which were carried out inside of the rotation context . All the activities of this context were classified as dynamic;

(c) from the viewpoint of the settings, 25 out of all the children presented their best performances in the activities developed inside of the Logo setting. Because of the Logo turtle feature which allows children to see all its turns and moving around the screen, all the activities included in this setting were categorised as dynamic;

(d) from the viewpoint of the arenas, the majority of the age-group children best result were in the activities carried out in the watch arena (see Table 5.5.3). 25 out of the 29 activities realised in the Logo setting were included in the dynamic category;

(e) from the viewpoint of the conditions, 27 children performed better in the action activities, while the remaining 3 in the recognising activities. 25 out of the 28 action activities were considered as dynamic.

From the all above, I can afirm that Group 2 children presented best performance in the acitivities which were included in the dynamic perspective. However it is necessary to consider that may be the children success in dealing with the dynamic activities was not due to the dynamism of the activities, rather this success was maybe due to the dynamic activities have been better elaborated than the statics and so that the dynamic activities were easier understood by the children. The answer for such doubt can only be given when the qualitative analysis will be proceeded in the next chapter. I hope that the qualitative analysis will clarify the many 'whys' arising from the results of the quantitative analysis. In other words, I believe that the qualitative analysis will help us to interpret the nature of the children's errors.

CHAPTER 6

QUALITATIVE ANALYSIS

This chapter analyses the children's responses when they were solving the tasks of the experiment. Thus, what concerns here are the types of articulations used by children, rather than their correct or incorrect responses. The qualitative analysis will be divided into two sections, each corresponding to the level of schooling, i.e., first section will report the findings of the middle school children (Group 1) and the second section, the findings of the younger children (children from Group 2). Each section will be treated firstly age-group by age-group and thus taking into account all the groups included in the section. Like in the quantitative analysis, comparisons between performances will be done.

In order to make sense of all the data, children's answers age-group by age-group were first summarised in tables (about 23 tables for each age-group) and thus transformed in diagrams in order to be presented in this chapter. The tables reported the children's answers on the 80 activities which form the 7 clusters. 53 out of the 80 activities required children to explain how they solved the problems and why they did in that particular way. In some activities these children's explanations were given in form of gestures. In the remaining 27, the children were asked to recognise or act out the tasks. It is important to make clear that these 27 activities were not less informative than the ones which required an explanation or description by the children. In fact, it is very useful to compare what the children said about the activities with what they had already done. It was also interesting to compare the way children explained similar activities bearing in mind their different backgrounds and schooling. In many instances, the children did not recognise similar activities because they were taking place in different situations.

The summary tables contain, in brief, the children's speech -- gestual and oral -- which could provide clues to the way in which children may have been thinking with regard to the children's action, recognition and articulation. For instance, activities which asked the children to recognise a quarter turn in the mini city led us to an interesting comparison with activities which asked children to recognise a turn of 90^0 also in the mini city. Indeed, one of the fascinating aspects of the study was the opportunity it gave me to look carefully at similar activities carried out in different arenas as well as settings and contexts.

The child's probable thought-processes were categorised as '*Reference.*' These *references* were not employed a priori in order to group children who seemed to be looking at angle in the same way. Rather, they emerged a posteriori from the data[1]. Thus, the precise terminology employed in this study was not pre-determined but formed on the basis of the children's own answers.

In my research I am interested in children's answers whether by the written or spoken word or by actions and gestures. Quantitative data shows trends but qualitative help me to understand the factors which are influencing the children acquisition of the concept of angle. The reason which justify my expectation is based on the data that I already have, which were obtained from semi-structured interviews with pre-defined questions to guide me. The children were asked to choose one of a number of options, or to do something specific, and then state the reason for their choice. In most of the activities the children were not asked directly about angle, nor were they even asked what they

1 - These data were collected froml the children's articulation.

thought the word angle meant, but instead the activities were embodied in a specific task. The data comprised of oral, gestual, or written language. My observations of the children's behaviour were also taken into account. In other words, the collected data emerged from what the children actually did and said.

I expect that the qualitative analysis will clarify and explain more clearly the reasons for the results shown in the quantitative analysis. From the children's utterances, I defined 17 categories of *Reference* as following:

1 Openness - This classification was used when the child referred to the openness of a figure. It seems that by doing this the child was either alluding to static perspective and consequently not attaching any importance to turns. This was the case with activities undertaken within 2 and 4 angle arenas[2]. Nevertheless the term 'Openness' was chosen to avoid making a wrong inference.

Example: In the stick game arena, a 11 year-old girl comparing this figure  explained that angle 'B' was bigger than angle 'A' because 'The openness of 'B' is bigger'.

2 Dynamic Perspective - This classification was applied when a child used a word indicating movement such as 'turn' or 'bend'.

Example: In the mini city arena a 13 year-old girl comparing two turns she explained that "They are the same because both are 1/2 turn".

3 Geometry Metric - This classification was used when the child's answer included a reference to value, in degrees, either perceiving the turn or the figure.

Example: when a child recognised that the two angles were the same "because both were 90^0".

2 - However it should not be forgotten that these two arenas were also applied in Logo, which means that in this setting the children were able to perceive the rotation of the turtle.

4 <u>The Word Angle</u> - This classification was applied whenever the child used the word 'angle' to refer to the figures. As I am interested in understanding the child's concept of angle, it was important to include this category in order to find out how many children were aware of the word angle, and in what way.

<u>Example</u>: In the stick game arena a 12 year-old girl comparing this figure  explained: "'A' looks bigger, but if you use a protractor you will see that 'B' is bigger in angle".

5 <u>Formal Learning</u> - This category was adopted when a child mentioned a particular term which suggested the use of school knowledge. For example, if a child used words such as 'parallelism', 'congruent angles', 'vertex', 'right angle', and so on.

<u>Examples</u>: A 14 year-old boy justified his correct prediction of where this arrow  would be after a turn of 90 by saying that "90^0 is a right angle". In the stick game arena, when a 14 year-old boy comparing this figure  explained that 'B' was bigger than 'A' "because 'B' is almost 180^0 and 'A' is an acute angle".

6 <u>Numerical Operation</u> - This classification was used when the child's answer referred to numerical operations, such as addition, subtraction, multiplication or division. In other words, it was used when the child relied on a numerical operation in order to explain the similarity or difference between figures or turns.

<u>Example</u>: A 10 year-old boy explained that if the turnstile did 2/1 turn 2 people could go into the zoo "because in a complete turn 4 people can come and the half of 4 is 2".

7 <u>Watch Metric</u> - This category was applied when a child's answer referred to the way in which a watch works, that is, when s/he mentioned the minutes, hours, half an hour, 30 minutes, and so on in his/her answer.

Example: A 13 year-old girl justified herself about where the arrow would stop after it turns half turn saying "1/2 is in the middle. I imagine as if it were the hand of the watch".

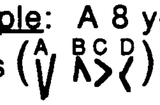
8 Shape - This classification was used when a child evaluated whether the angles of the figures were the same or not through a comparison of the shapes of these figures.

Example: In the stick game arena a 6 year-old boy was asked to do a similar figure to this () and he did, inside the researcher figure, a smaller figure which angle was bigger, like this () and explained that "they were the same because both are triangles".

9 Inclination - A child's answer was classified in the inclination reference when a child referred to the inclinations of the rays of the figures. This was the case whenever a child used the words 'inclination' or 'position' to justify his/her answer, or when s/he made a gesture with his/her hands to indicate the parallelism of the lines.

Example: A 11 year-old girl comparing whether two miniature cars did the same turn inthese turns () answered yes because "They (the cars) finished the curves in the same position. I can see by the inclination of the turns"

10 Internal Angle - This classification was used when a child looked only at the internal angle, even when there was an 'angle indicator'[3], pointing to the external angle of the figure.

Example: A 8 year-old boy comparing the smallest angle among these four angles () said that 'A' was the smallest because "it is the closest".

11 Angle Indicator - This classification was used when a child used the Angle Indicator of the figure as an invariant to decide whether the angles were the same or not. For example, some children used the distance between the angle

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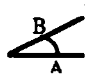
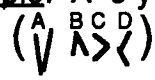
3 -  In this figure, the length of the curved line AB is what we call the "angle indicator". Many children referred to it as the "arrow", "line", or "vertex".

indicator and the corner of the angle as a factor to be considered in the comparison of two angles.

12 Length - This category was applied when the child's answer was related to the length of a line in the figures such as the lengths of the rays of an angle, or the lengths of the spiral.

Example: A 9 year-old boy was asked to choose the smallest angle among these 

in the 4 angle arena, he chose figure ‘C’ because “ it is the smallest triangle”, and in fact figure ‘C’ was the smallest turn among all the four.

13 Guess - It was classified as a guess when a child explicitly admitted that s/he had guessed his/her answer to the activity. This type of category was most used in the activities in which the child had to predict such as in the arrow, turnstile and watch arenas.

14 No Explanation - This classification was used when a child appeared to know something but could not give any explanation whatsoever and said things like “I don't know how to explain it”, or “It is because”, or “I just know it's like this”.

15 Not Known - This category was applied when a child explicitly said that s/he did not know what has been required in the activity, or when s/he said that s/he could not say anything about it, or when a child stated that s/he did not want to do it, or even when s/he expressed “I did not do this”.

Example: An 8 year-old boy asked if he had turn any 1/4 of turn in the mini city answerd: “I don't know what is 1/4 of turn”.

16 Imprecise Reference - This classification was used when a child did not say anything about the activity or when his/her explanation was obscure or insufficient to classify the answer in one of the above references.

Example: A 6 year-old girl asked to predict where the arm of the turnstile would stop after it turns 1 1/2 turn, predicted the correct place and justified by saying "because here is the right place"; How do you know, asked the researcher, and she answered "because you asked for 1 and a half." She could not give me one single reason for her prediction.

17 Irrelevant Explanation - It was classified as child's 'irrelevant explanation' when a child introduced in the answer her own feelings or preferences, or even when s/he introduced something not asked for in the activity.

Example: When a child used expressions such as "I know because I'm clever", or "this watch has turned more because the student isn't a good student", or "it comes to 6 people in half a turn of the turnstile by jumping". Or simply when the child makes use of any other reason irrelevant to the question being asked, simply in order to justify her answer.

After grouping the different types of responses into the above categories I noticed that some arenas 'enticed' most of the children to think in one or another particular way. For example, children in the 2 angles arena in the p & p setting, and also in the stick game in the everyday setting, frequently referred to the openness; whilst in the turnstile, watches, and arrow arenas children referred much more to the dynamic perspective.

The category of watch also needs additional discussion. Although this reference is very specific, it was included because many children used this metric to explain what activities they did in other arenas, i.e., in arrow and turnstile.

It was not unusual for a child to give answers to similar activities in a different way when in a new context. This phenomenon occurred not only among contexts, but also among settings and, within settings, from one arena to another, and even from one activity to another within the same arena. For instance, an 8 year-old boy after having correctly predicted the place where the minute hand would be after it turns 1/2 turn from an initial position of 12.00, he predicted that a watch which now showing 12:10 would 1/2 hour later be showing 12:30 "because here is the right place for 1/2 hour." Asked about his previous prediction of 1/2 turn and his new prediction he answered "There you asked me for 1/2 turn and 1/2 turn is in the middle of the whole turn, and now you asked me to turn 1/2 hour and 1/2 hour is when the hand arrived at the 6 (number 6)".

An initial reaction could easily be one of classifying this kind of behaviour as simply inconsistent. However it is important to take into account that many different types of associations may affect the way a child undertakes an activity. An example of this is the considerable number of children who made a relationship between the movement of the arrow in the arrow arena and the movement of the minute hand of a watch; or those children who failed to recognise a half turn in mini city but could easily recognise a half turn in the turnstile arena.

In some activities it was necessary to classify the children's answers in more than one category. This happened each time a child gave more than one explanation for the same activity. In fact, this occurred in most of the activities. Recalling Vergnaud's theory about the concept formation, previously discussed in Chapter 2, this is not surprising because a situation (in this case an activity) generally involves more than one concept, as well as a concept refers to more than one situation[4].

Findings like these may be regarded as an important milestone in mapping clearly a child's conception of angle. This will be discussed after the analysis of the individual ages has been carried out.

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4 - For more detail, see Vergnaud, G. (1983, 1988A, 1988B) and his theory on the formation of concepts.

6.1 PROFILE OF THE GROUP 1 CHILDREN

In this section the 11, 12, 13, and 14 year-old children are analysed, age by age, by looking at the refernces obtained through their explanations. I decided to start the analysis using diagram, because I believe that diagrams are a good manner to present data in an easy way of visualisation. It is also easier to compare the different age-groups when they are summarised diagramatically.

6.1.1 THE 14 YEAR-OLD CHILDREN

From the list of 17 references, it was possible to classify 277 references from this age-group. The reference to a dynamic perspective appears in 75 out of 277, which is 27.07% of all children's references. From the 17 references, dynamic perspective was the most frequent. The following diagram describes the most frequently cited references made by the children whilst they were dealing with the activities.

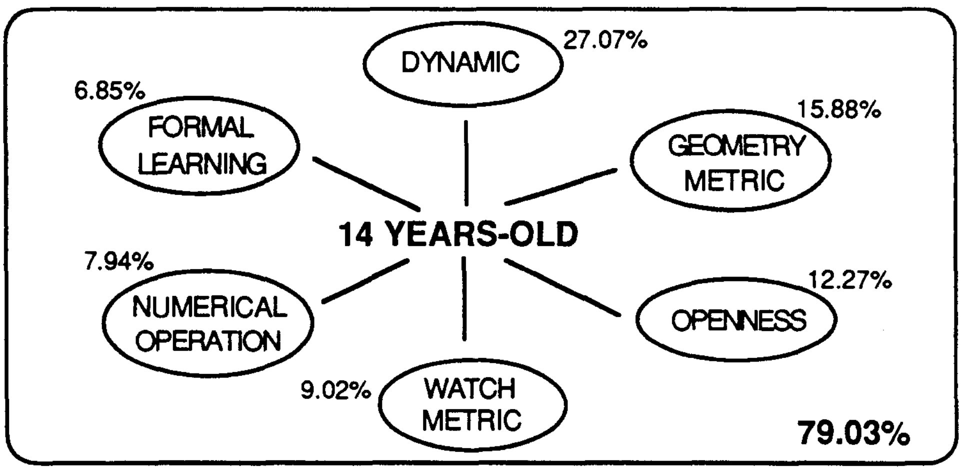

Diagram 6.1: The 6 most frequent references used by 14 age-group children, considering all the 3 settings.

The first interesting finding was that the children did not try to answer the questions just for the sake of saying something about them. They rarely guessed. Another interesting point was that references such as 'angle indicator' and 'length' were used in less than 1% of the answers. Moreover, references such as 'imprecise', 'irrelevant', ' shape', and 'no explanation', were infrequently used by the children (nothing higher than 2.2%). Finally, the six references shown in the above diagram indicates that children of this age-group actually used scientific concepts many times.

The references which showed that the child either lacked a clear knowledge of angle or misunderstood it altogether, were guessing, angle indicator, length, shape, imprecise, irrelevant and no explanation. These were less than 10% of all the answers. At the same time the most popular six references were those which were much more relevant to the acquisition of the conception of angle. This bears out that this age-group really does have a conception of angle, which is mainly in a dynamic perspective.

Before going on analysing another age-group, it must be noted that the seventh most cited reference was 'Not Known' which was used in 6.14% of all the references. However, most of the 'Not Known' came from those activities within the map arena group. This arena is thought of as the most difficult one, mainly because of the recognition of the number of turns.

So far I have been looking at the children's performances in general, without considering the differences between the settings. However, the children's performances from setting to setting may provoke a large difference to our understanding of the children's acquisition of the concept of angle. The next three diagrams show, in detail, all the references which appeared in each setting. A discussion of them follows:

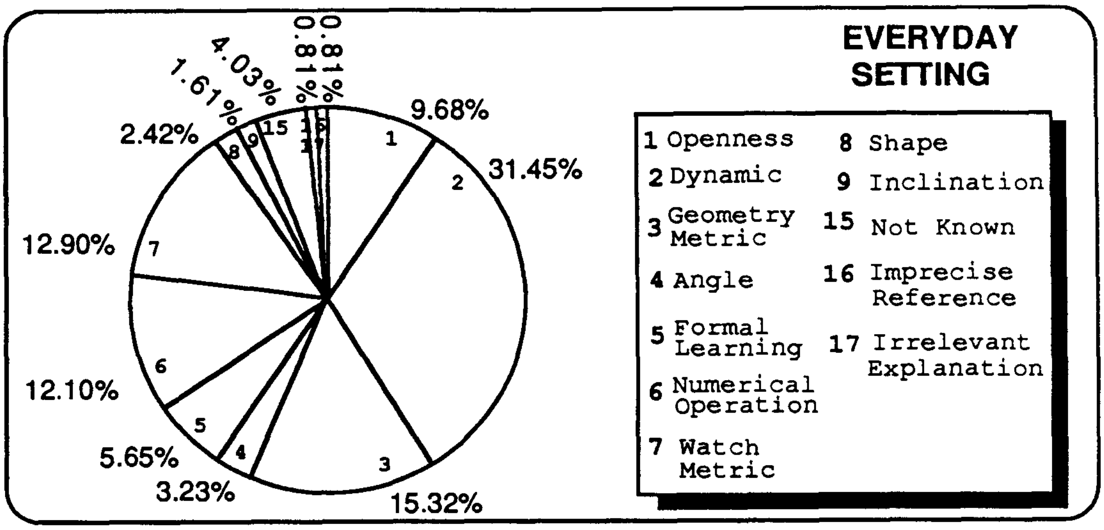

Diagram 6.2: The 14 age-group.children's references used in the everyday setting.

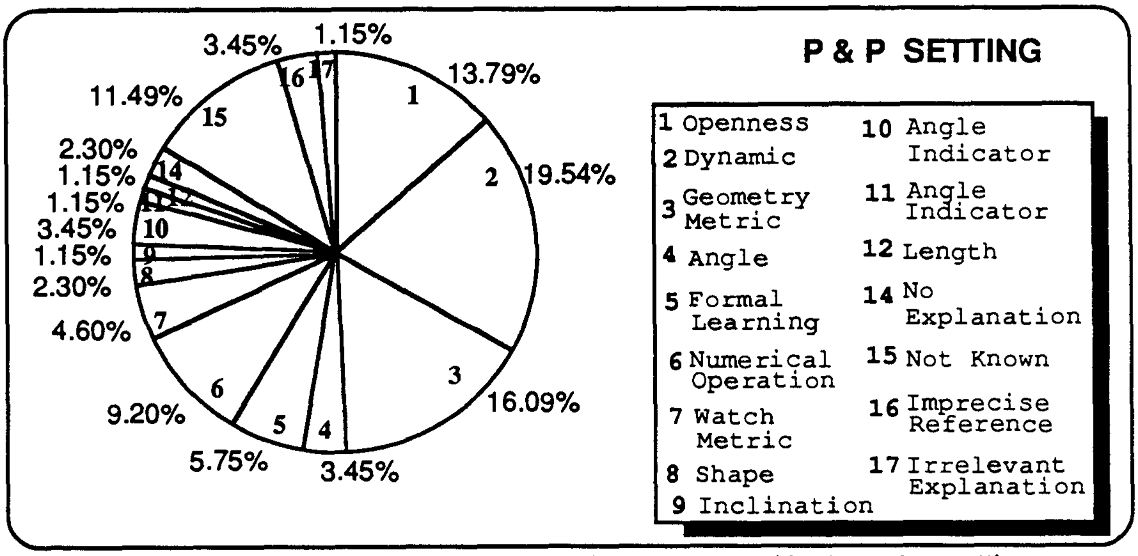

Diagram 6.3: The 14 age-group children's references used in the p & p setting.

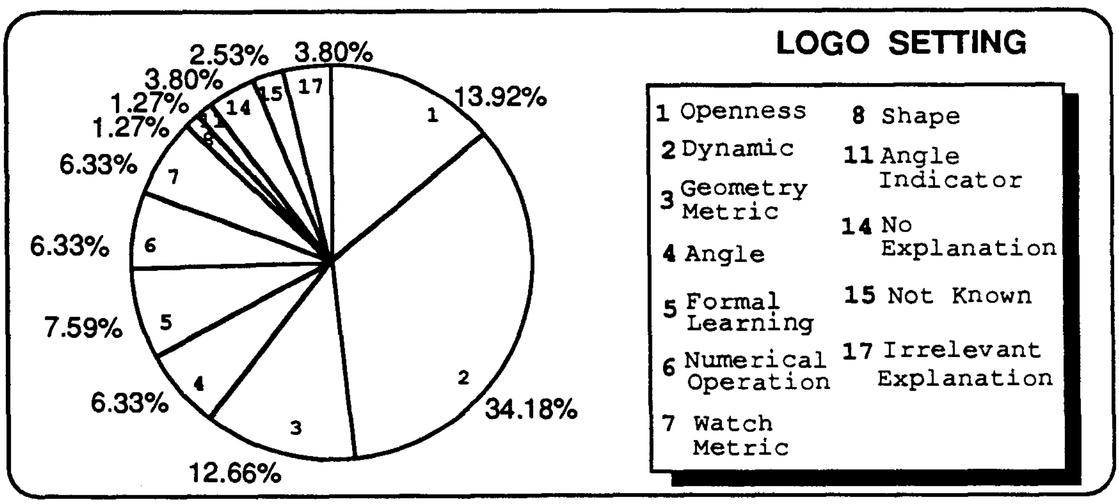

Diagram 6.4: The 14 age-group children's references used in the Logo setting.

The diagrams show that 14 year-old children used, not only different references from setting to setting, but also utilised these references in different proportions. The 'dynamic' reference is a good example: although it was the most cited reference in all three settings, there is clear decline in its use from Logo and everyday settings to the p & p setting. This was not a surprise since it is lined up with our previous expectation since the majority of the activities realised in the p & p setting were included in the static perspective.

Another interesting example is the use of 'not known' which appeared only, as one of the six most cited reference, in the p & p setting. An interpretation for this can be the fact that in this setting children did not have to answer to the researcher face to face, so that they were not ashamed to assume that they did not know something. Another interpretation may come from the own nature of the setting, in which activities were presented in absent of a semantic situation. By this way, althoguh I have been speaking about navigation context in the map arena, for instance, mapping on a p & p can have been experienced differently by these children.

It was noticeable that the children's answers using the watch metric as a reference fell sharply from the everyday to the Logo setting, finally disappearing from the group of the most six cited references, in the p & p setting. However, it must be remembered that this setting did not have the watch arena, which means that the references to this metric, found in that setting, came from arenas such as arrow and turnstile. Another point to take into account is the fact that in the everyday setting the watch arena had more activities than in the Logo setting.

It is also possible to state that the way by which dynamic perspective was explicit in the activities seems to be quite acceptable for these children.

Perhaps this way of thinking about angle -- in movement, rather than in form of figure, where its rays can wrongly influence the child's perception with regarding to the value of the angle -- makes more sense for children. In fact, through the movement of one of the two rays of a figure a child can realise that angle is directly related to the region between these two rays, and thus, by moving one of the rays this region will increase or decrease and consequently the value of the angle.

Another important point to be considered in the conclusion of the children of this age-group was the frequent mention made by these children of the value of the angle ('geometry metric' reference) in the activities in all three settings. This can be considered as a sign that they were using what they learnt in their math classroom outside of it. However this category was more used in the p & p setting (the school setting per excelence) than in the other two settings, although the difference between p & p and everyday setting was not remarkable. This hypothesis becomes stronger if we add the information that the 'formal learning' was not an unusual reference among the 14 year-old children. What was surprising was the fact of the most use of the school terminologies ('formal learning') in the Logo setting.

Finally, a finding which drew my attention was the children great use of the 'numerical operation' whilst they were explaining how they did the activities. Children used this explanation more in the everyday setting, specifically in the activities carried out in the turnstile arena, in which the complete turn of the turnstile meant that 4 people could come into the zoo. Thus, in this case number 4 was doubtless used as a mark of one complete turn and from it turns were multiplied or divided by 4. However children also used the 'numerical operation' in the p & p and Logo settings. This finding may be indicating that children tried to use their calculation knowledge to help them solve geometric problems.

I shall conclude the analysis of this age-group by giving a representative example of the 14 year-old children's performance across the experiment. My intention here is to show a typical 12 year-old child's responses in different arenas, settings and contexts:

Comparing two equal turns smaller than 90^0 in the mini city () a boy answered that "the only difference is the sizes of the streets, but this is not important to define their turns".

In the stick game, another arena also in the everyday setting, he was asked to compare again two equal angles smaller then 90^0 () and once more he did not present difficulty to answer correctly that they were the same "because of their openness....the openness of the vertex. The angles are the same, what is the difference is the length of the figures". He also gave a similar response when compared a pair of angle in the 2 angles arena in the p & p setting ().

When he was asked to predict where the arrow () would be after a turn of 180^0 he responded correctly in the p & p ("180 is twice 90....it is just turnning two right angles one after another... it will turn until the middle of the whole turn") as well as in the Logo setting ("I know it is; this is the correct place for 1/2 turn; if I made another 1/2 turn it (the arrow) would comeback to the same place").

In the watch arena he explained half turn as "half turn is in the middle of the watch, and in this case the middle of it is defined by the starting point of the minute hand. In a watch it is 1/2 hour".

In another occasion, he was asked to predict where the arm of the turnstile would be after the entrance of 8 people, and he said "in the same place because 1 turn is for 4 people, 4 X 2 = 8 ... after two turns it will stop in the same place").

Finally, comparing 6 watches in order to state which turned more and which turned less, he again responded correctly and explained "I can know which watch worked more and which worked less by looking firstly at the small hand and thus at the position of the big hand. It is because the first decision comes from the movement of the hour hand, looking at it and comparing its initial and final positions I can know how many turns the watch did...".

The above example shows that the 14 year-old children, in general, have a scientific conception of angle; they could work the activities in terms of invariants of conception rather than serely invariants of competence; the activities were understood by them from the operative aspects of knowledge.

6.1.2 THE 13 YEAR-OLD CHILDREN

As regards the references cited by this age-group, the following diagram shows the six most frequently used, from which I am going to start the analysis of these children age-group.

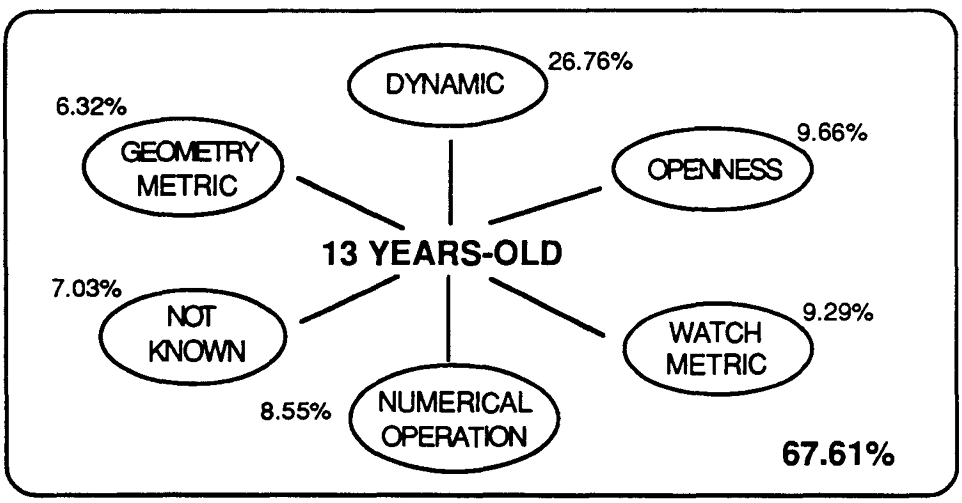

Diagram 6.5: The 6 most frequent references used by the 13 age-group children, considering to all 3 settings.

The classification of the 17 references was compiled on the basis of the 269 answers given by the six 13 year-old children. 5 out of the 6 responses used most frequently in this age-group were the same as for the previous age-group. One similarity between the 13 year-old and 14 year-old age-groups was the large number of dynamic answers; this was the most common reference in both groups.

However there were some important differences between the groups. For example, the 13 years old, cited less frequently the geometric terms ('geometry metric' reference) and terms usually learnt in school ('formal learning' reference) than the 14 year-old children. In fact 'formal learning' has not even apeared in the diagram.

Another important difference was the number of 'not known' in the 13 year-old age-group. In fact, this category was also cited in the previous age-group, but because it was in seventh place it did not appear in Diagram 6.1. However 'not known' occupied the fifth place in the general diagram of the 13 years of age children. I decided to point this out because of my interest in seeing whether the 'not known' percentage will increase after the other age-groups have been analysed.

References such as 'internal angle', 'irrelevant explanation', 'guess' and 'imprecise references' were lower cited by this age-group. Moreover, the sum of these 4 references was no higher than 6.7%. This would be a very good result for the 13 year-old age-group for the acquisition of the concept of angle if this group did not have a high average for 'length', 'angle indicator', and 'shape' (14.5% out of all the cited references). 13 year-old children seem to present more invariants of competence than a properly concept of angle, but this invariant changed according to the situation and did not always work out.

These results indicate that this age-group is more vunerable to misconceptions of the angle's invariants than the previous age-group. Also this group suffers less influence from formal knowledge probably learnt from school which can be noted by the clear decreasing in the children's responses of geometrical terminology, such as words like 'parallelism', 'vertex' etc.

Following the same method of presenting data adopted for the last age-group, the next step will be to show the three pie-charts, which wil give information on the average number of cited references according to each setting (everyday, Logo, and p & p). Afterwards, the findings will be discussed and compared with the age-group already analysed.

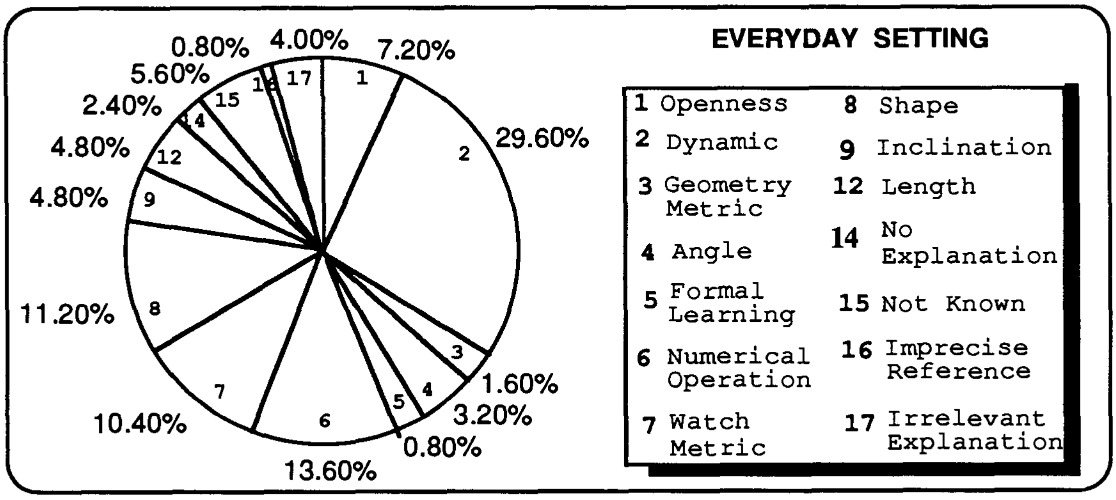

Diagram 6.6: The 13 age-group children's references used in the everyday setting.

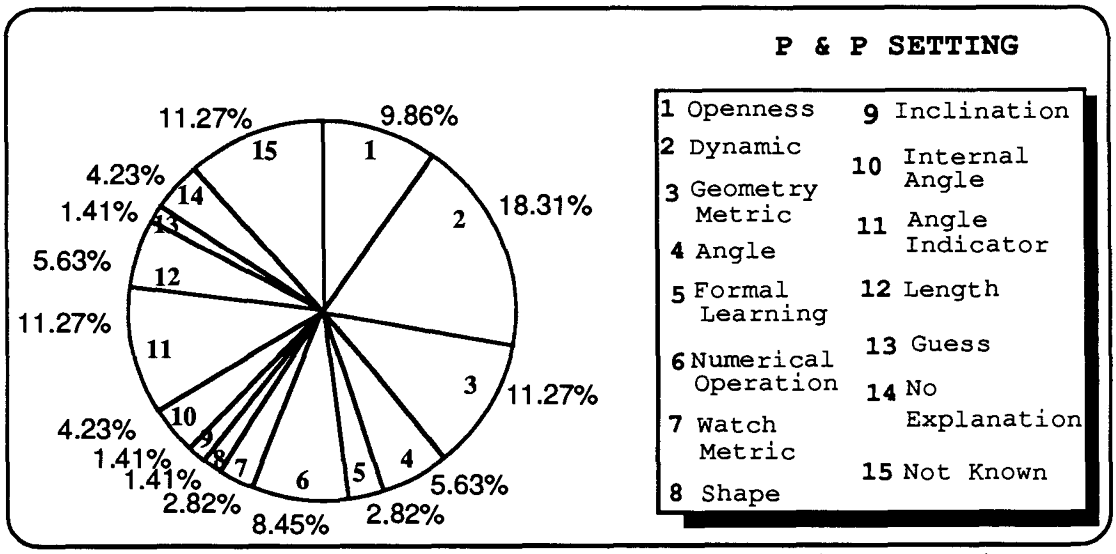

Diagram 6.7: The 13 age-group children's references used in the p & p setting.

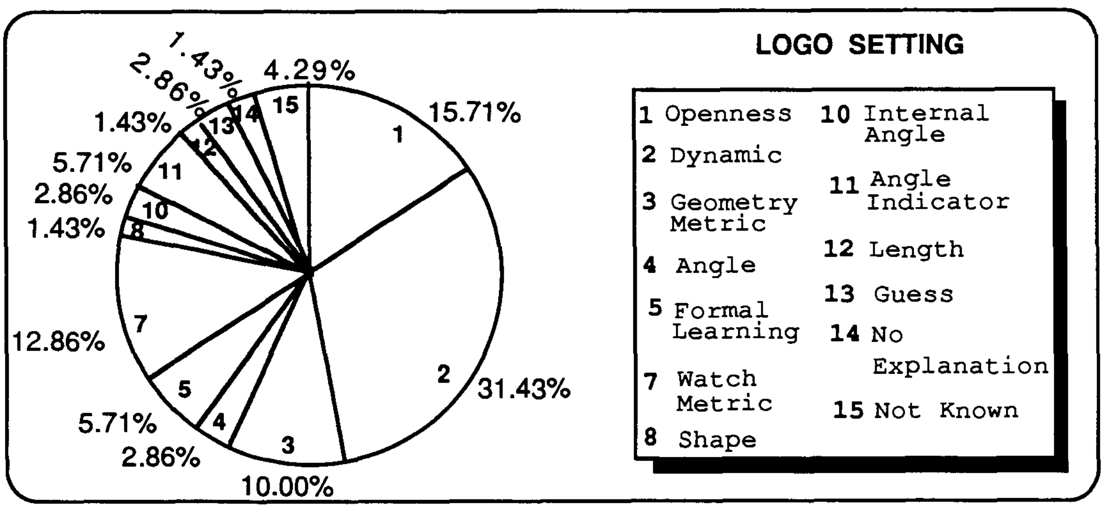

Diagram 6.8: The 13 age-group children's references used in theLogo setting.

From the Diagrams 6.6, 6.7 and 6.8 it can be seen that this group had different references, in terms of the average references used, from one setting to another.

Considered first is the reference to the dynamic perspective, which was the one most cited in all three settings. There was a small decline in the use of this reference from Logo to everyday setting, and from these two to p & p setting, which to me was not a surprise, since it follows the same path way found in the previous age-group.

A close look at the six most frequently cited references in all three settings, it is possible to note that only 'dynamic perspective' and 'openness' appeared in all the three settings. Actually, reference like 'numerical operation', which was the second most cited in the everyday and stayed in the fourth place in the p & p setting was not mentioned in the Logo setting at all. Numerical operation is, in fact, a good example to show the changes in children's responses from setting to setting. It appeared quite often in the most informal setting (everyday), which no numbers were presented in any arena (except those faced on watches), nevertheless children made use of this reference to explain activities carried out in turnstile and watch arenas. On the other hand, the Logo watch arena was faced with numbers referent to the angular measurement and children at this age are already able to make calculations (sum, subtraction) with angle, however they did not do so.

Another discrepancy among the settings occurred with regards to the children's use of 'geometry metric'. I discovered that 'geometry metric', which was frequently cited in the Logo and p & p settings was only poorly mentioned by the children in the everyday setting. In a inverse way, I noticed that 'shape', widely used by the children as a satisfatory explanation, in the everyday setting, was rarely explicit in the other two settings.

The above list of children's discrepance in the way of expliciting their answers amog settings are a clear evidence that situation is one factor which influence children's understanding of a concept, at least the concept of angle. In this particular, each setting means a different situation, involving different representational system. For instance, in the Logo and p & p settings, the children's way of expliciting the scientific concepts (references 3, 4, and 5) were very close to each other and together represented almost 1/5 of their answers. However, in the everyday setting scientific terminologies have a great decreasing. This results lead me to quest how much an informal situation can 'invite' children touse basicly an informal representational system.

With regards to the 'watch metric' reference, as was expected it was more cited in the everyday and Logo settings than in p & p setting. Comparing the two first setting, children used the metric of the watch to explain what they had done more in Logo than in the everyday. This finding is in opposition to what I noticed in the analysis of the previous age-group. Possibly, this occurred because the 14 year-old children did not use only the 'watch metric' to justify what they did in the activities carried out in the watch arena, rather they used the 'numerical operation to explain how the time is shown on it, i.e., multipling 5 by 5 up to 60.. Moreover, in p & p there was a sharper fall in the 'watch metric' than there was in the case with the 14 age-group. As was mentioned in the analysis of the previous age-group, p & p did not have the watch arena; this means that it was always cited in p & p. Was it done because the child transferred whatever s/he had learnt from the watch to another situation? If so, it is clear that this age-group children did less transfer of knowledge or learning from one to another situation than the older children.

6.1.3 THE 12 YEAR-OLD CHILDREN

When we consider the 290 answers given by this age-group in the activities, the following diagram shows the six most frequently cited references.

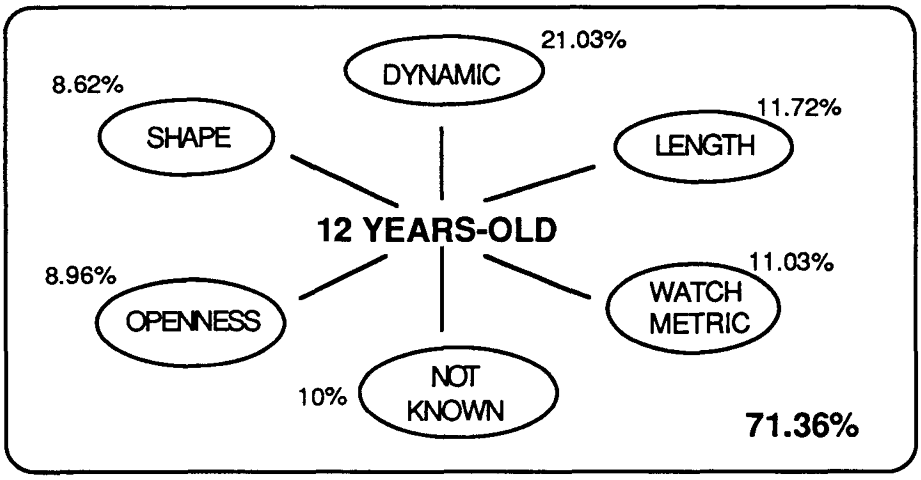

Diagram 6.9: The 6 most frequent references used by the 12 age-group children, considering all the 3 settings

The above diagram introduces two references, namely 'length' and 'shape', which were absent from previous diagrams for older children. At the same time two others were cut out, namely 'numerical operation' and 'geometry metric'. The absence of 'numerical operation' and 'geometry metric' references is an indication that children from this age made less use of formal learning. In fact, comparing the 6 most frequently used references found in 14 and 13 year-old age-groups with those most cited by 12 year-old age-group, it is possible to note a qualitative difference in content. These two older age-groups seem to be incorporating, or at least, in the case of 13 years old, beginning to incorporate, the scientific concept into their spontaneous responses.

Regarding to the 'length' and 'shape' references, the importance of their appearance is the fact that these two categories are misconceptions of angle. In the first case, which is 'length', child defined the size of an angle throughout by the length of its rays; whilst in the second case, 'shape', child appeared to be to be trying to make use of his/her early knowledge of geometry. In fact, geometry is first introduced in school with the study of shapes, such as the square, triangle, circle and rectangle to begin with. However what attracted my attention was the fact that at this age angle is thought in school by showing (in illustrations and in examples) open figures, as was also the case of this research. Nevertheless, children insisted to recall their early learning and referred to the figures as they were close. This type of children's behaviour was also found in Fuys et. al. (1988), previously disscussed in Chapter 3.

When I take into account the other references which did not appear in the above diagram, but is possible to perceive it through the summary tables in the appendix, I find this age-group made less use of 'guess' and 'imprecise reference' than 13 year-old children. Answers utilising references such as 'inclination', 'formal learning', 'irrelevant explanation', and 'internal angle' also obtained a very low cited (none of them was cited more than 2% by the children). It is advisable bear in mind that apart from 'irrelevant explanation' the other three references are concerning the formal learning which is normally acquired from school. Moreover, it is probable that when a child refers to the 'inclination' of the figure, s/he is using it as a helpfull invariant for s/he recognises the size of the angle.

After having presented the references cited by children in general, the settings will be displayed and analysed. Like in the two older age-groups, the analysis looking setting by setting will be carried out comparing what this age-group children gave as answer from each other setting, as well as comparing the 12 year-old children with the already previous age-groups.

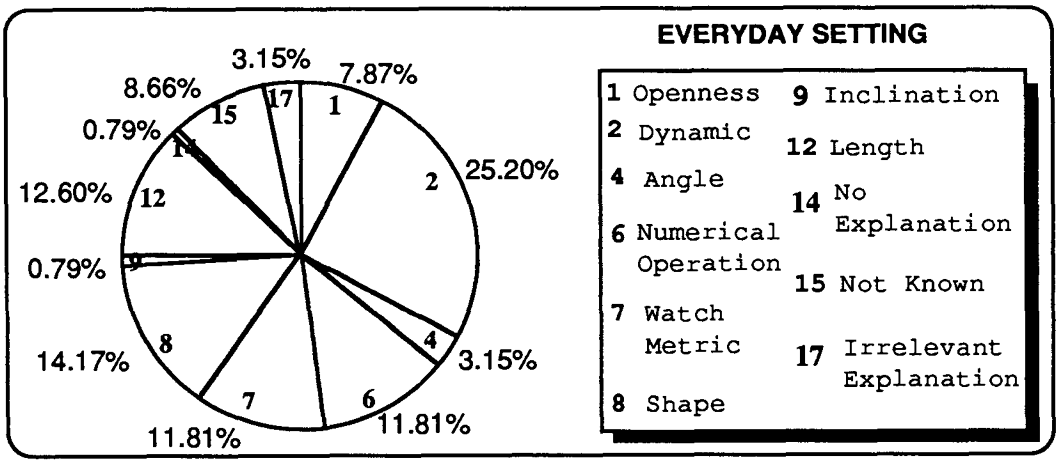

Diagram 6.10: The 12 age-group children's references used in the everyday setting.

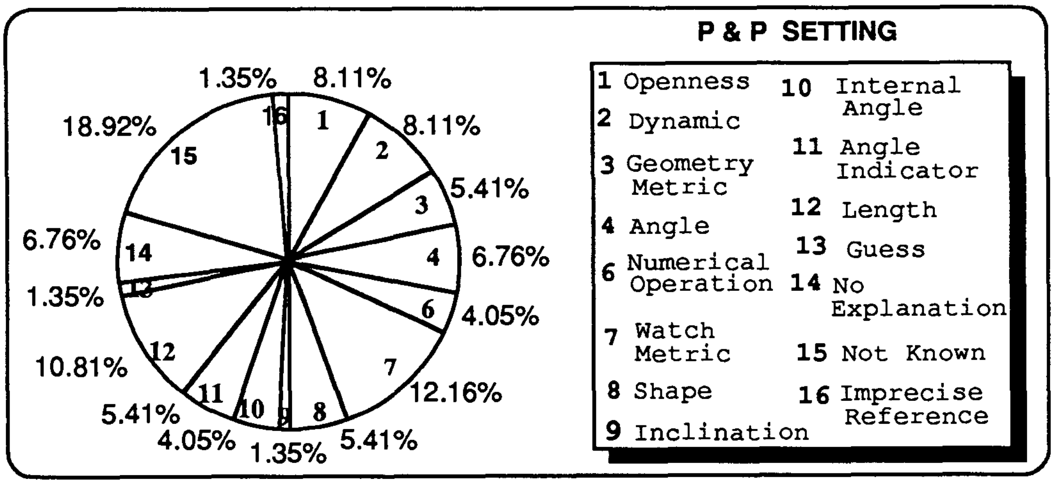

Diagram 6.11: The 12 age-group children's references used in the p & p setting.

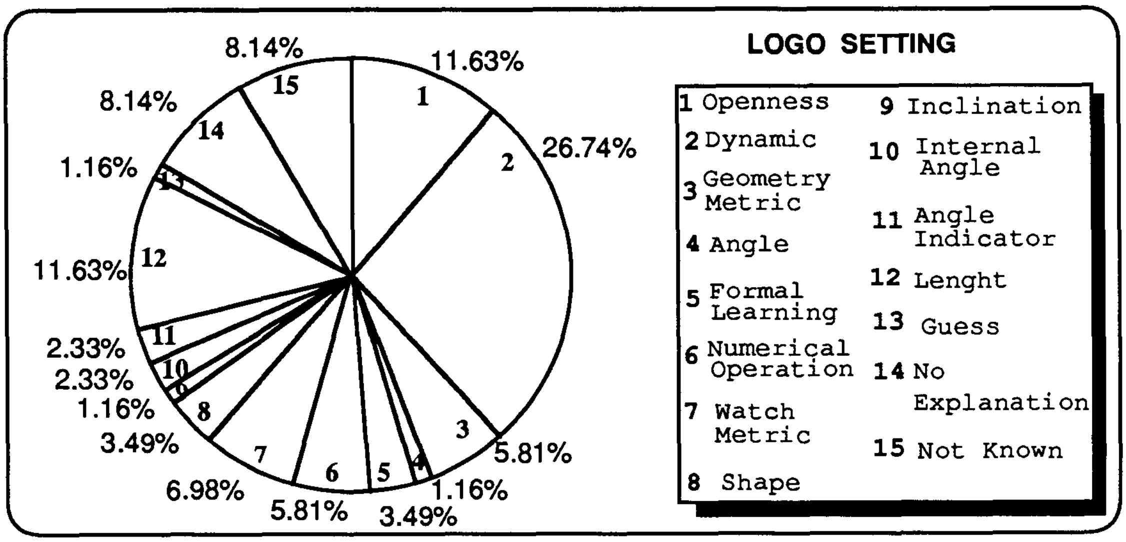

Diagram 6.12: The 12 age-group children's references used in the Logo setting.

The above diagrams show a great difference related to the chidren's answers over the three settings. However there were some similarities which are also important to discuss.

The first most obvious similarity of children's performance was plain to see when looking at the category of length. Although children have cited it more in the everyday than in the other two settings, 'length' was frequently referred by the children in the three settings. Comparing this age-group with the two older age-group children, we notice a great increase in the use of this reference from the two previous age-group to the 12 year-old children.

The reference to the 'dynamic perspective' was the most cited reference to everyday and Logo settings. However, when children were explaining what they did in p & p setting, the dynamic perspective fell sharply, occupying the fourth place together with the openness reference, representing only 8.11% of the children's answers. Although it was expected that this reference was less used in the p & p setting, what drew our attention was how much it fell in comparison with the other two age-group.

In an inverse way, the children's answers, concerning the 'watch metric' reference, in a p & p setting increased so much if we compare with the 13 and 14 year-old children. The 12 years old referred to the 'watch metric' in the p & p setting as much as in the everyday setting. Moreover, it was surprising that the frequency of the watch metric was mentioned twice as often in the p & p than in the Logo setting. It is good to remember once again that watch arena was not applied in p & p setting, which means that children who cited this reference in this setting-- and they cited it only in the arrow arena -- did it by transferring their knowledge from their experience with a watch. This made me wonder how much the figures utilised on the activities were could basicly be understood as a drawing by this age-group children.

Comparing the difference in the children's responses amog the three settings, it is interesting to point out that the 'imprecise' reference just appeared in the p & p setting. In fact, this age-group performed very different when we compared this setting with the other two, and the children performed also differently from what was expected. For instance, it was noted that 'geometry metric' was presented in the p & p setting as the sixth most frequently cited reference. Children did not make any reference at all to the 'geometry metric' in the everyday setting, and although in the Logo setting this reference did not appear among the six most common children's responses, it was as much used as in the p & p setting. It was also in the p & p setting that children made more reference to the word 'angle', which made this reference the fifth most cited. If we take into acount the the number of times in which children referred to the'angle' plus 'geometry metric' plus 'formal learning' in the p & p setting, it is possible to note a children clear tendency to adopt a more formal perspective of angle in this setting than in the other two. This result support my previous hypothesis that the nature of the p & p setting 'invites' children to, at least, try the use of scientific terminology, which means a children effort to solve activities throughout the scientif concepts perpective.

The use of the 'not known' reference was very high in the p & p setting. Actually it was the most cited reference, and it was twice as often cited in this setting than in the other two. The interpretation for this can be that children felt freer to answer what they wanted in this setting than in the other two settings. That is because they did not have to answer directly to the researcher, which when they did not know the answer, cause them no embarassment. Another important point to draw attention about 'not known' reference is the increase of this reference from the two previous age-groups to the 12 year-old children. In fact, from age-group to age-group children were assuming more and more that they did not know about the activity, mainly in the everyday and in the p & p settings. This was perhaps occurring because these two settings, unlike the Logo one, did not presenting clues to help children.

From further investigation of the differences among the settings, we note that children used 'shape' reference much more frequently in the everyday than in the other two settings. Moreover, when we consider only those references which can be related to children's misconceptions -- 'shape' and 'length' -- in the everyday setting these references totalled almost 27% out of all the children's answers, whereas in the other two settings this sum was much lower. Considering that the everyday setting was the most informal and less structured among all the three settings, it is good to be attempt to make a possible relationship between informal situations and children's misconceptions.

Finally, I would like to draw attention to the performance of children when using the reference to 'openness'. Taking into account the nature of the p & p setting, where the activities are presented to the children through a previously drawn figure, (i.e., where children are asked to work with static figures), and because I believed that a reference to an openness was related to the static perspective, I was expecting that this category would be, if not the most cited one in this setting, at least that the p & p would show a higher frequency of the 'openness' mentions than the other two settings. However it did not happen. 'Openness' appeared the same as 'dynamic', and behind 'not known', 'watch metric' and 'length' references in the p & p setting, and was even less cited in this setting than in the Logo. However, we cannot disregard that it was the p & p setting which presented the lowest average in 'dynamic' reference. Among the three settings, 'openness' was more frequently cited in Logo, although this setting also presented the highest frequency in citing the 'dynamic' reference. This may indicate that children were using both firstly dynamic and then static perspective in the Logo setting. This interpretation is also discussed by Kieran (1986).

6.1.4 THE 11 YEAR-OLD CHILDREN

The next diagram shows the 6 most cited references that this age-group presented which were classified from the 279 answers given by children.

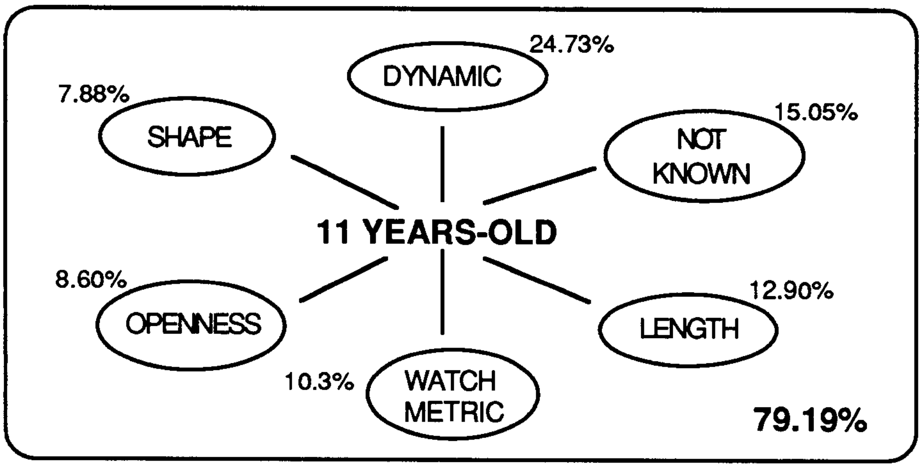

Diagram 6.13: The 6 most frequent references used by the 11 age-group children, considering all the 3 settings.

The shape of Diagram 6.13 is not very different from the 12 age-group diagram. In fact, it shows that, in general, both age-groups have the same references to explain their answers. The variation from one diagram to another is where the categories appear, as well as their frequency. For example, 'dynamic', 'openness' and 'shape' references appeared in the first, fifth and sixth places respectively in both age-groups, however children cited them in different frequencies. These differences between the two age-groups were much more tenuous in the other two references, i.e., in the 'openness' and in the 'shapes' references.

Continuing with the comparison of the performance of 11 year-old age-group with the 12 year-old children, an interesting result appeared from the reference of length. Although it was in the second place in 12 age-group and fell to the third place in the 11 age-group, the former group mentioned it much more than the previous age-group. It seems that the younger the children are the harder it is for them to realise which invariants are concerning the concept of an angle;. consequently the more misconception they present. However, whether this occurs because of the developmental factor, or because of the school factor, or even, what is most likely, because of both factors is an open question.

'Not known' showed a great increase from 12 to 11 year-old children's answers. It seems to be adequate to remember that the 11 age-group was composed of children who were just starting the middle school, therefore these children had not yet learnt angle in an analytical way. Taking into account the most cited references, 'Not known' did not appear for 14 year-old children, it appeared in the fifth place for 13 year-old children, representing 7.03% of the references by this age-group. Among the 12 year-old children 'not known' came in fourth place, obtaining 10% of all children's references. For 11 year-old age-group 'not known' reference was the second of the most six cited references, representing 15.05% of the children's answers. This increasing from age to age was not a surprise, rather it was expected that younger age-group children knew less than the older. What I was not expecting was that the children assumed this.

The next three diagrams are to show, in detail, all the references which appeared in each setting. A discussion of this then follows.

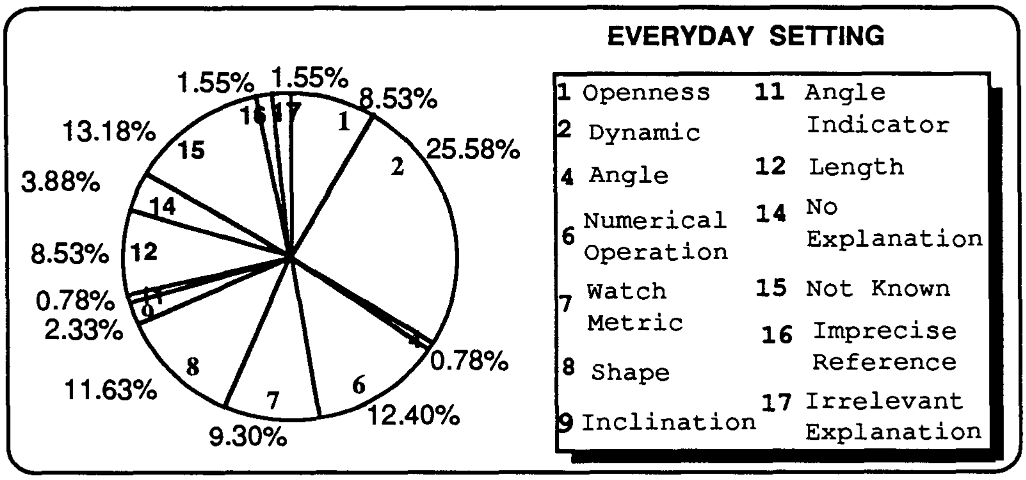

Diagram 6.14: The 11 age-group children's references used in the everyday setting.

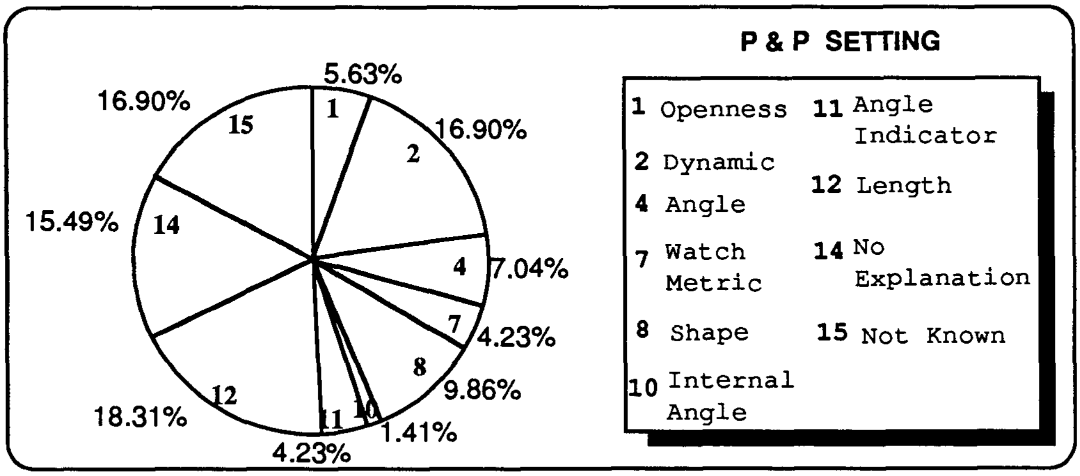

Diagram 6.15: The 11 age-group children's references used in the p & p setting.

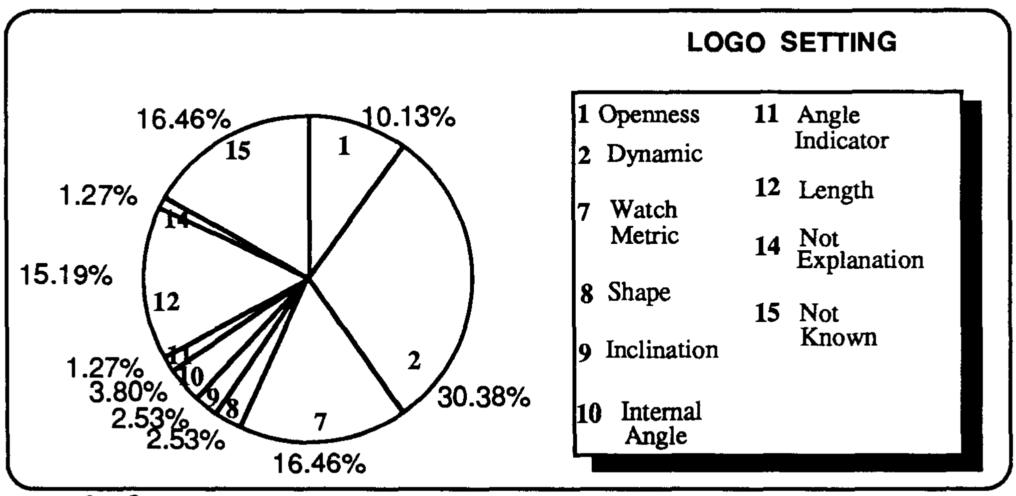

Diagram 6.16: The 11 age-group children's references used in the Logo setting.

In everyday setting, the 'dynamic' reference was by far the most cited one, as it was in the Logo setting. However such as for the 12 year-old age-group, it lost its privileged place, as the most cited reference, in the p & p setting, where 'dynamic' came after 'length' reference and beside 'not known' reference. However this age-group children cited the 'dynamic perspective', in the p & p, twice as often than the 12 year-old children did.

Looking at the references used in the settings, we note that 'numerical operations', unlike with the previous age-groups, just appears in the everyday. Children from the 11 age-group used 'numerical operations' reference in the watch and turnstile arenas. Looking at the everyday setting, it is interesting to note that the 11 year-old children made use of the 'numerical operation' almost as much as the three older age-groups. This means that while the previous age-group used 'numerical operation' to deal with activities included in the arrow, watch, and turnstile arenas, in all the settings, children of 11 years of age did not try to (or could not) make use of their arithmetic knowledge in order to help themselves in other than the everyday setting. In this way, for children of 11 years of age 'numerical operation' was useful in the activities included, for instance, in the watch arena, in the everyday setting, but it was not good enough to be used in the activities included in the watch arena in the Logo setting. In the particular case of watch, 'numerical operation' seems to be a much more appropriate tool to use in the Logo watch, where the numbers faced on it follows the already known angular metric, than in the everyday watch.

'Length' was the most used reference for the p & p setting which makes me conclude that because of the characteristics of this setting, which was presented uncontextualised, and in which the black lines of the figures were printed on a white paper, 'length' worked out as a satisfstory invariant for the children in the 11 year age-group, who tended to evaluate the angle through the lengths of its rays.

The 11 year-old children used the reference 'not known' much more than the previous age-group. The reason for this has already been justified by the fact that this younger children group had not properly learnt angle. However, the situation presents inverted when we compare these two groups in the p & p setting. In this case, 12 year-old children used the 'not known' reference more than the 11 year-old age-group. One explanation for this could be that the 12 year-old group tried to avoid admitting, as much as they could, that they did not know how to do some activities, but it was hard to do this in the p & p setting, whereas 11 year-old children, who knew that they were just starting a new stage of schooling, seemed not to be worried about not knowing. The consequence of this was that the 11 year-old children, whenever they did not know the activity did not hesitate in adimitting that they did not know about it, either in the everyday, or Logo, or in the p & p setting, whilst the 12 year-old children seemed to assume their lack of knowledge only in the absence of the researcher, i.e., when they answered the p & p test. Another possibile explanation, which does not exclude the previous, is that of the effect of schooling over these children.

The 'Openness' reference was less often used in 11 age-group than in the previous one. It appears as the fourth most cited reference by children in the Logo setting. In the other two settings it appeared in the sixth place. While the 'openness' was cited less in this age-group, the 'dynamic' reference presented the inverse way, that is, it was more cited by the 11 year-old children than by the 12 age-group, for all the three settings.

Comparing the 11 and 12 year-old children's way of making explicit 'openness' and 'dynamic' among the three settings, it is possible to note that there was no difference at all between these two categories as concerning the everyday setting. It is also perceived that in the Logo setting this difference is very small for 'openness' whilst there is a considerable difference in favour of 11 years of age-group for 'dynamic'. Finally, in the p & p setting there are clear differences between ages in both categories. Actually apart of 'not known' references, the 12 year-old children's responses were much more widely distributed among the remain ing 14 references[5] than they were in the 11 years of age-group, in which the responses were basicly concentrated on four references (numbers 2, 12, 14, and 15). From this comparison I can draw two interpretions: in relation to the settings, Logo and, mainly, p & p settings are experienced differently from age to age; in relation to the age-groups, and taking into account only the p & p setting, the 11 year-old children show that they are not able to deal with angle from the perspective of the scientific concept.

Another good point to be discussed is concerning the increase of the 'shape' reference in the children's answers in the two younger age-group in comparison with the 13 and 14 year-old children. In fact, among the 11, 12, and 13 year-old children, this reference was much more used in the activities included in the everyday. However among children of 11 and 12 years of age this reference was also cited with some frequency in the p & p setting. It is interesting to observe that while the 11 year-old children cited the 'shape' more than 12 age-group in the p & p setting, in the everyday setting the position is the inverse. This also happened with the 'length' reference. I am inclined to think that informally (in the everyday life situations) the 12 year-old children tend to commit more misconceptions and this tendency decline in the p & p and declines even more in the Logo setting (mainly as regards to the 'shape'). Among the 11 year-old children this picture changed and it was in the p & p

5 - This age-group did not refer to 'formal learning', nor 'irrelevant explanation'.

setting that I found more answers related to the length and the shape of the figures. Are the 11 year-old children less concerned with the difference of setting whilst they are much more influenced by type of activities which are inserted in the settings? In other words, is there a relationship between age and the influence of activities - the younger children are, the more influenced they are -- as well as age and the influence of setting -- the younger children are, the less influenced they are?

In Chapter 3 I had discussed a research project carried out by Fuys et al (1988) in which the authors suggested as a possible explanation for children inclination to use the word 'triangle' instead of 'angle' the children's "preference for handling the gestalt of closed finite regions rather than open infinite space" (pp. 76). According to my findings, in which I found that the younger the age-group the more the children looked at the figures as a closed finite region, I am inclined to agree with Fuy's suggestion, at least for the 11 and 12 year-old children.

Finally, it was noted that references such as 'geometry metric' and 'formal learning' were not cited, in all three settings, for this age-group. This makes a qualitative difference from the previous group which used 'geometry metric' as the sixth most cited reference for Logo and p & p settings and also used 'formal learning' in 3.49% of their answers in Logo setting.

6.1.5 SUMMARY OF GROUP 1 CHILDREN

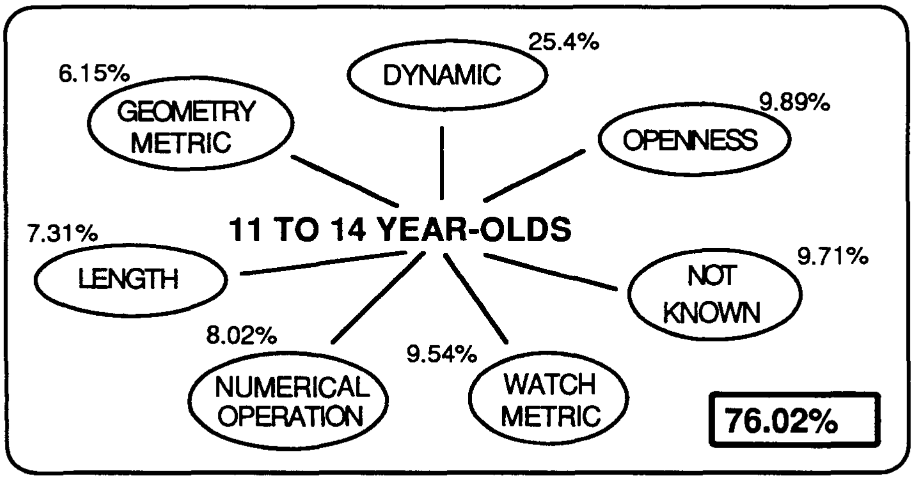

Diagram 6.17: The summary of the 7 most frequent references in the Middle School children, considering all the 3 settings.

The above diagram summarises the most quoted references by the children of the middle school. The first interesting finding comes from the difference between the most cited reference, the 'dynamic perspective', and the other five references. In fact, we can observe that about 1/4 of all the children's responses referred to the dynamic perspective, which confirms my previous conjecture that although the quantitative analysis has shown, at first glance, that the static perspective is the one in which the children seem to be able to think more easily of angle, when actually it was the children's worse performance in the navigation context which cause this misinterpretation.

The diagram also shows a high average of the 'Not known' reference. I can interprete this taking into account two important factors: the first is the cultural factor related to the linguistic meaning. This fact can be clearly noted, for instance, in two activities included in the map arena, where no children at all could correctly respond as to whether they had made any or if so how many 1/4 turns they had made. Moreover, most of them assumed that they did not know what it meant. In fact, for Brazilian society 'a 1/4 of a turn' is an unusual expression for a turn of 90^0. For Brazilians, fractions are only used to divide material things. I can divide a chocolate in four equal parts, or I can have 1/2 of an orange, but it is hard to imagine a turn divided into four other equal turns. Therefore, a 1/4 of turn is problably a good example to show a non-semantic situation for Brazilian students. Even the expression "half a turn" is not frequently used for a turn of 180^0 in the navigation context; Brazilian use to say "make a turn" instead of half a turn. Thus, if the expression "a turn" means half a turn, the expression "half a turn" may much likely represent a small turn.

The second factor to be considered was the level of formality imposed by the p & p setting in which children were faced with activities careless of contextual meaning. In fact, my intention for this setting was to present it as similar to the school test situation as possible. Students know that the school tests ask them about formal knowledge. In this way, they have to answer the question formally or leave it, assuming that they do not know about it. This I interprete as an evidence of the school effect.

I understand the high average of the 'watch metric' reference as a case of learning transfer made by children from the watch situation to the turnstile and arrow situations. This means that the signified of the watch metric was powerful enough (in sense of meaning) to be a reference to solving other activities inserted also in the rotation context. In this case watch arena, differently from 1/4 of turn is an example of a semantic situation, from which the children's operational invariants emerged allowing them to work in this arena by theorem-in-action. Further, children transfered their competence to another situation.

'Openness' reference, which were also often cited by children, is in my point of view, an indication that according to the situation, children use different operational invariants. In the case of this study, children seemed to be aware about the movement of the figures (dynamic perspective), but they also called attention to the final product of the figures solving the activities by the static perspective as well. This is a good example to show that a situation refers to many conceptions and a conception involves many operational invariants from different levels.

Finally, taking into account the 7 most cited references by these 4 age-groups (Diagrams 1, 5, 9, 13 and 17), it was noted that there is a big gap between the 14 age-group and the other three groups (11, 12 and 13 age-groups). The last groups are clearly in a lower stage of acquisition of the angle conception than the former group. However considering both the quantitative and the qualitative analysis I found that the 13 year-old age-group seemed to be in the half way between the group of 14 year-old children and the 12 and 11 age-groups. It is possible to note that the 14 age-group answered more consistently across the experiment. These children also showed more elaborate answers in terms of formal learning as well as in the formation concept.

Before stating the analysis of the Group 2, I still would like to present three diagrams which refer to the performance of Group 1 children according to each setting. These diagrams show only those reference which were the most cited either in the everyday, p & p or Logo settings.

Therefore, it is probable that some references appear showing a very low percentage of use in one setting but high in the other. In fact it is this idea: to show through the diagram of each setting not only the 6 most cited references of that setting but also the 6 most cited references of the other 2 settings. I believe that this will facilitate the observervations of the differences and similarities in the children's responses among the settings.

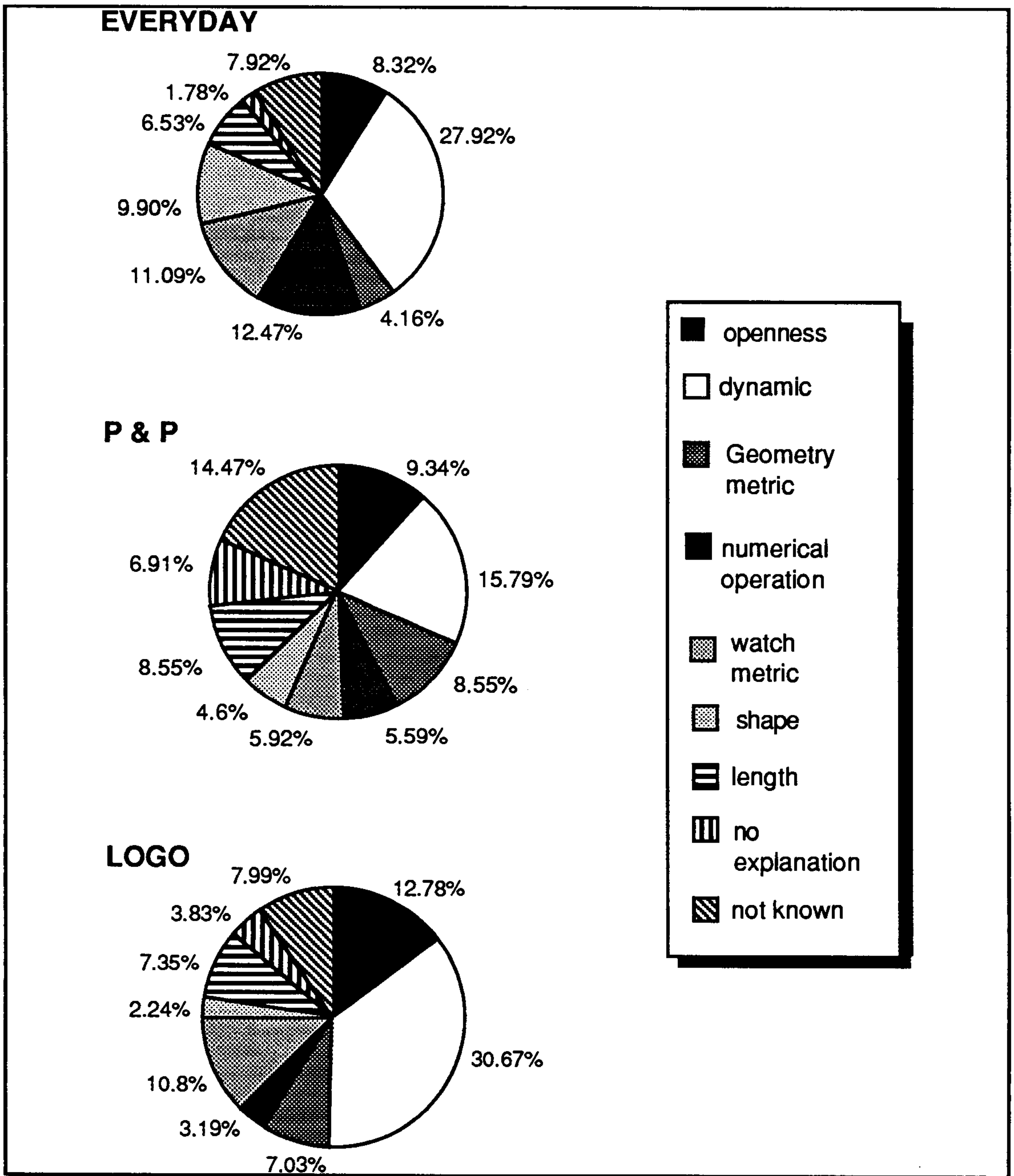

Diagram 6.18: The children's most cited references according each setting

From the above diagrams it is possible to note relevant differences and similarities in children's responses from one to the other settings. Let's start looking at these differences:

The first interesting difference is noted from the children's mention of the dynamic reference from p & p to the other two settings. This result confirms my previous statement that p & p is not a proper place to explore dynamic ideas.

Another difference is concerned with the reference to the geometry metric. In this turn it was in the everyday that children less referred to it. In fact, geometry metric is closely related to the formal learning whilst everyday is the most informal place I had in my research.

There are large differences in the averages of 'numerical operation' and in the 'watch metric' from setting to setting. It seems that because the feature of turnstile arena (four arms representing one turn) 'numerical operation' presented a high average in the everyday setting. In fact, the majority of the children made a relation between one turn and the number 4. However children also tried to use calculation to solve problems in the p & p setting as well (more than in the Logo). On one hand this is a surprise because no activity in the p & p involved any kind of number at all. On the other hand, p & p is known (in school) as a setting where students should solve problems through calculation.

With regards to the watch metric, it was a surprise to find a high average of it in the p & p setting because the watch arena was not applied in this setting. However we must not forget that the watch is a well-known tool, children can positively find meaning for it and thus select information in order to form their theorems-in-action. Since children formed a theorem-in-action to solve problems on watch, this theorem can be transferred to other activities which also involve indicators and rotation. This seems to have been the case of the activities realised in the arrow arena. An example of this is found in the arrow arena activities where children seem to have transferred their watch metric knowledge in order to solve activities in the arrow arena. In this case what seems to have happened was that the arrow tasks were perceived by the children through the same representational system as in watch. As we can see by the p & p this happened independently from the setting, i.e., it happened in the everyday as well as in the Logo and in the p & p setting.

As similarity, it is possible to observe that although children differences in the average of making explicit 'dynamic' category from one to the other settings. It was actually the most cited reference for all the three settings. The children's reference to the length of the figures was also quite constant among the settings. It seems that length actually is working as an operational invariant which leads children to form a false theorem-in-action which persists until later.

6.2 PROFILE OF THE GROUP 2 CHILDREN

The analysis of this group will be carried out in the same way as for children of the middle school group, i.e., in this section the 10, 9, 8, 7, and 6 year-old children are firstly analysed age by age. The children's articulation, classified in terms of references, will be the basis of this analysis. The second part of this section will be a summary of the Group 2 performance.

6.2.1 THE 10 YEAR-OLD CHILDREN

Considering the children's responses from the viewpoint of the 17 references, it was possible to classify 282 references. The following diagram shows the 6 most used references by this age-group, which embrace more than 4/5 of the classification.

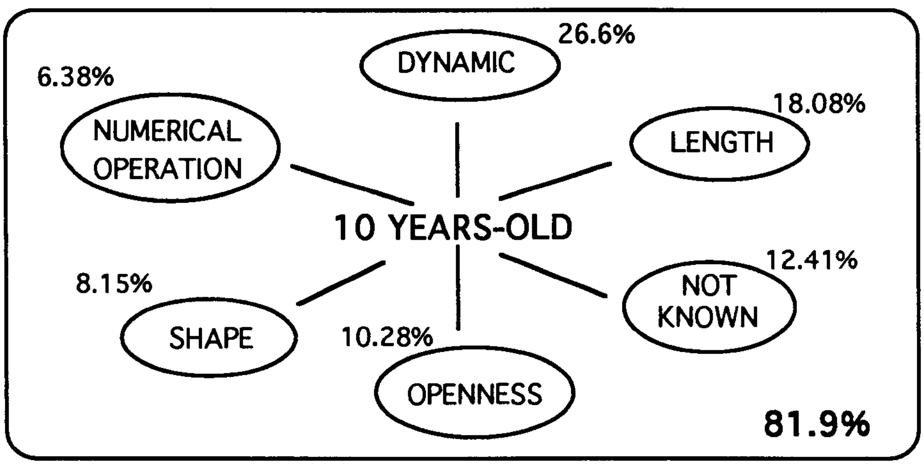

Diagram 6.19: The 6 most frequent references used by the 10 age-group children, considering all the 3 settings

The above diagram points to the dynamic reference as the most cited by these children. In fact, this reference was cited as much as it was for the 14 year-old children. However, there is a clear difference in qualitative terms between the 10 year-old answers to the 14 year-old children answers. An example of this can be noted when children were asked to compare two turns of 45^0 in the mini city, in the everyday setting and to answer if they were the same or not (activity "b" of the cluster 1):

A 14 year-old child correctly answered that the turns were the same because "the angles of their turns are the same: they are smaller than 90^0";
A 10 year-old child, who also answered correctly, justified: "one turn is like another. It is because the turns made by the cars, they were the same".

Although both children have correctly answered, it is possible to note a clear difference in terms of concept of angle: in the first case the 14 year-old child showed an understanding of what a turn means and she was able to explain it in terms of angular measure, while in the second case the child seems to have answered based only on his visual perception without making use of any scientific knowledge.

The mention of the length reference by this age-group was sharply increased when compared with the children from Group 1. This finding confirms what the literature (Close, 1982; Magina, 1988; among other) has already stated about children's most common misunderstandings in angle. In fact, if a child does not know what the angle of a figure is, the length or shape of this figure can easily gain a place of distinction in his/her evaluation of this figure. In the case of this age-group, we note that more than 1/3 of all the references cited by the children include either 'length', or 'not known', or 'shape'.

The influence of the length of the figures over the children's responses occurred even in activities carried out within arenas and settings which involved the idea of rotation. For example, when a boy of this age-group was asked to recognise among six watches which was the one which had turned less (activity "b" of cluster 7), he incorrectly chose an oval watch which was showing 1.30hs and justified: "This (watch) is narrow, it has small hands." In this case not only the length of the watch seems to have influenced the child's response but also the shape of the watch.

Another good point to discuss is the presense of 'numerical operation' as the fifth most cited reference. This is interesting because this reference was not frequently used either for the 12 or for 11 age-group children. In fact 'numerical operation', which was expected to be certainly cited in activities inside turnstile arena and perhaps in activities within spiral arena, was in fact used in many other arenas such as arrow in p & p and Logo settings, mini city in p & p, watch in everyday and, of course, in activities within turnstile and spiral arenas.

Comparing the way in which 10 year-old children used this reference with the way that 14 and 13 year-old children did, there is a great difference from one to another use: while the older children used it without loosing the idea of turns, i.e., having in mind the dynamic perspective, it was not the case for some 10 year-old children. Let's illustrate this giving examples of how 10 and 14 year-old children used this reference:

Example 1 - A 10 year-old boy, who correctly predicted in p & p setting where the arrow would stop after 1/4 of a turn, explained "I divided the square into four equal parts and thus I put the arrow on the edge of the first part." While a 14 year-old boy, who also predicted the correct place of the arrow after 1/2 turns in p & p setting, explained: "180 is twice 90....it is just turnning two right angles one after another... it will turn until the middle of the whole turn".

Example 2 - The same 10 year-old boy, when was predicting half a turn in the watch arena in everyday setting, explained his strategy saying "I divide it (watch) in two bounds...I break it in two bounds of the same size, when one finishes and another starts is the half". No children from 14 age-group used this reference in their explanations for activities within this arena.

Although these two activities are, from my perspective, dynamic per excelence, I am not sure if the younger boy gave any attention to the turns. Rather he seems to have solved the tasks as if he was dividing a chocolate in equal parts. In fact, fraction is taught in the Brazilian school in terms of dividing things (orange, chocolate, cake) and areas (square rectangle) and never in terms of dividing turns. On the other hand, the older child, as shown in the Example 1, used three different references to justify his answer: he considered the rotation of the arrow (dynamic perspective), he knew that half a turn was 180^0 (geometry metric), and finally he summed two right angles in order to obtain half turn (numerical operation). It is clear that the older children made use of three different conceptions in this situation. In other words, the system of representation for rotation of the 14 year-old child is much more extended than it is for the younger child.

Up to this point I have been looking at the 10 year-old children's performance in general. Nevertheless, I have noted from the analysis of the previous age-groups that the children's performance usually changes from setting to setting. I believe that the analysis of children's fulfilment in each setting and the changes which occurred from one to another setting can positively contribute to my understanding of the children's acquisition of the concept of angle. Thus, the following three diagrams show the children's performances according to each setting.

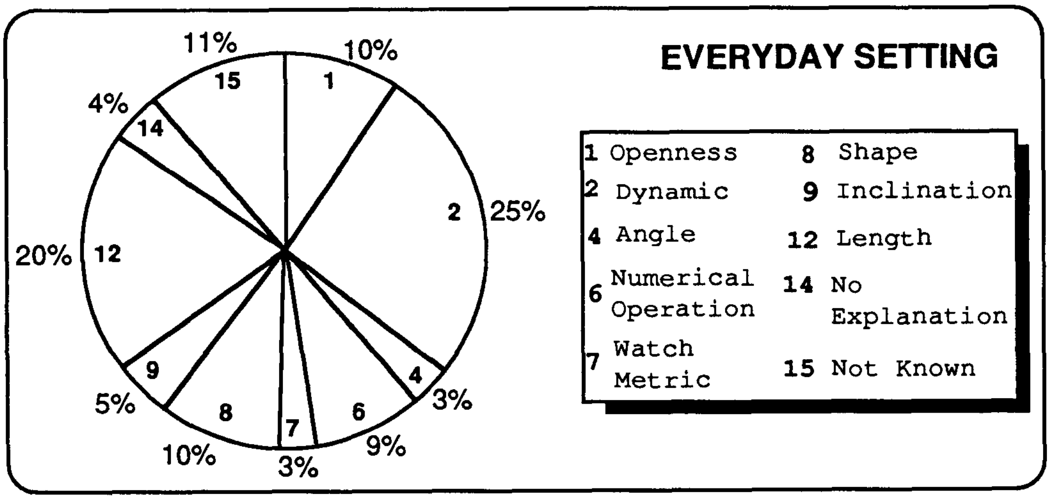

Diagram 6.20: The 10 age-group children's references used in the everyday setting.

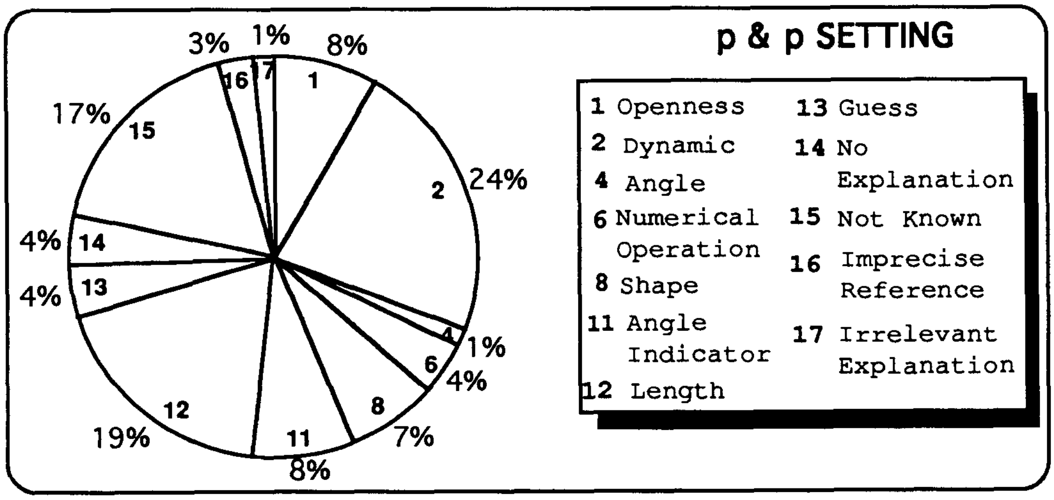

Diagram 6.21: The 10 age-group children's references used in the p & p setting.

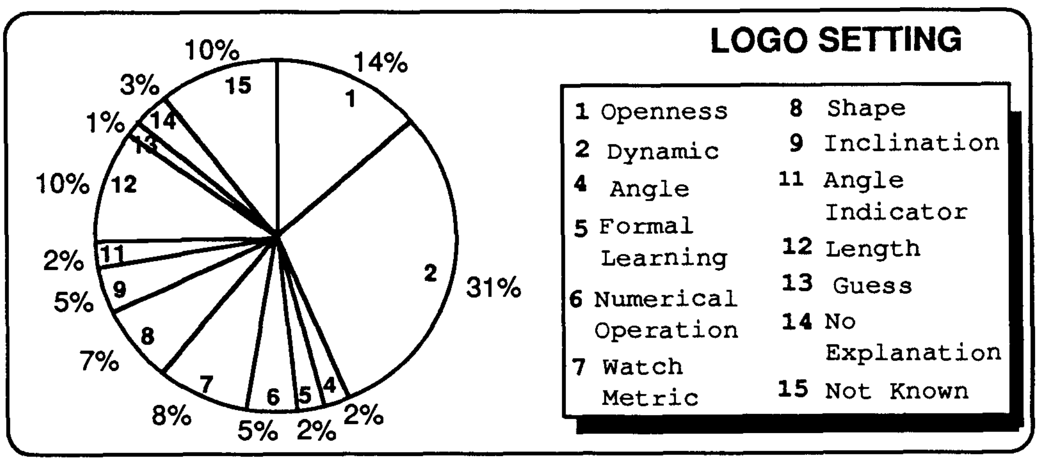

Diagram 6.22: The 10 age-group children's references used in the Logo setting.

The three diagrams show that the dynamic reference was largely used by children in all the three settings, although in Logo this use has been even more frequent than in the other two settings. The diagrams also show that there was a tenuous effort by the children to include the word angle in their answers. This may indicate that the children were trying to giving a scientific meaning for the tasks.

The influence of the length over the children's responses presents a great variation from one setting to another. It was in the everyday setting that children mostly justified their answers through this reference, whilst in the Logo they notably reduced its use.

As expected, no children justified their responses through 'geometry metric' or 'formal learning'. In fact, these two references have not appeared since the analysis of 12 year-old age-group children. As well as this observation I found that 'not known' was the third most cited reference for all the three settings. The three diagrams give evidence that the p & p setting was the one in which children showed more difficulty to deal with the concept of angle -- In this setting answers such as 'imprecise', 'irrelevant', 'not known', 'no explanation', 'guess', 'length' and 'shape' represented more than half of all the children's responses. This finding becomes more interesting if we compare p & p with Logo setting where the use of these references corresponded to around 1/4 of the all children's responses.

Comparing children's responses across the last three ages (10, 11 and 12 years of age) with relation to the reference to the inclination of the figures in different settings, I noted that in the everyday it was amply increased if compared with 11 and 12 year-old children; 'Inclination', in the everyday setting was slightly more cited by the 10 year-old children than even by the group of 13 years old. 'Inclination' was also referred to in the Logo setting and in this case the 10 year-old children used this reference more than any other older age-group children. It seems that 'Inclination' was very helpful for 10 year-old children when they were comparing two half turns in the mini city (∩ and ⊓). The majority of the children could correctly answer the question looking at the final position of the miniature cars by their visual perception. This was not the case of 14 year-old children as shows the following example:

A 14 year-old boy recognised that both turn were the same because "Both did a turn of 180"; while a 10 year-old boy said that the turns were the same because the cars "finished looking down". I am not sure if the latter child realised that both cars started to move from the same position, i.e., I am not sure whether the younger child took into account the starting point and the movement of the cars.

Another point to be noted in the everyday setting was the children's little mention of the watch reference. At this age I have no doubt that children know how to manage a watch. However most of them seem to have opted to use another reference to explain tasks included in the watch arena. The majority of the 10 year-old children also did not use 'watch metric' to explain activities carried out in the arrow arena as the previous age-groups children did. Rather, they mentioned the dynamic reference very much as the following example shows:

A 10 year-old boy who answered correctly on the five activities about prediction of half an hour and half a turn in the everyday watch, explained that "to complete half it (the hand) has to turn until the other side of the watch". Although this explanation does not say anything about whether he knows the watch metric or not, he clearly preferred to explain half a turn through the dynamic perspective only.

Looking more carefully at the children's responses in the p & p setting, I could observe that 'angle indicator', which was not cited in the everyday setting and which was referred to very little in the Logo, was the most cited in the p & p setting. The differences in the use of references made by children from setting to setting is evidence that settings are involving different representational systems.

6.2.2 THE 9 YEAR-OLD CHILDREN

When the 278 answers given by this age-group in the activities are considered, the following diagram shows the six most frequent references.

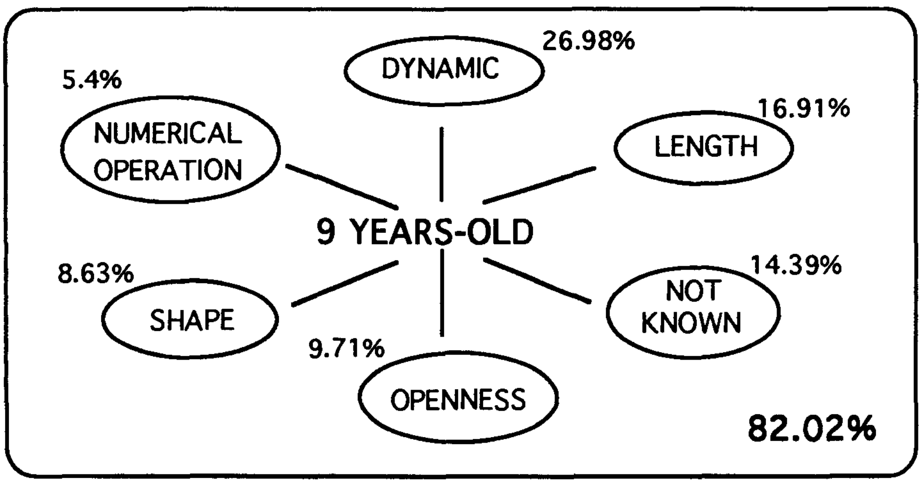

Diagram 6.23: The 6 most frequent references used by the 9 age-group children, considering all the 3 settings

The above diagram presents many similarities with the 10 year-old age-group diagram: the total number of cited references was almost the same, the six most cited references by this age-group are the same as the previous group, the first four most cited references are displayed in the same order as they were for 10 year-old children, and the percentage of the first two are very close to those presented by the anterior age-group.

Regarding the differentiation from this diagram to the previous age one, it is possible to note that there was an increase in the number of times that the 9 year-old children assumed (or clearly demonstrated) that they did not know the activity. 'Not known' reference had constantly increased from 14 to 11 year-old age-group and thus it dropped down when the data for 10 year-old children were analysed. Now it starts to increase again.

However the main difference between this diagram and the diagram from the previous age-group is the increasing of the 'shape' and the decreasing of 'numerical operation'. In fact, 9 year-old children cited these two references in an inverse way to the 10 year-old age-group and it is possible to perceive qualitative changes from the older group's responses to the younger. For example in the recognising of half a turn in the mini city, while no 10 year-old child referred to the 'shape' of the figures and two of them made use of the 'numerical operation', among the 9 year-old children none referred to 'numerical operation' and two out the 6 children did refer to the shape of the turn. It is good to point out that fraction is a subject taught when a child is 10 years old.

So far, I have been looking at the children's performances in general, without considering the differences between the settings. However, the analysis of previous age-groups has demonstrated that it is possible to obtain more accurate information if the children's responses are looked at setting by setting. The next three diagrams show, in detail, all the references which appeared in each setting. A discussion of them follows:

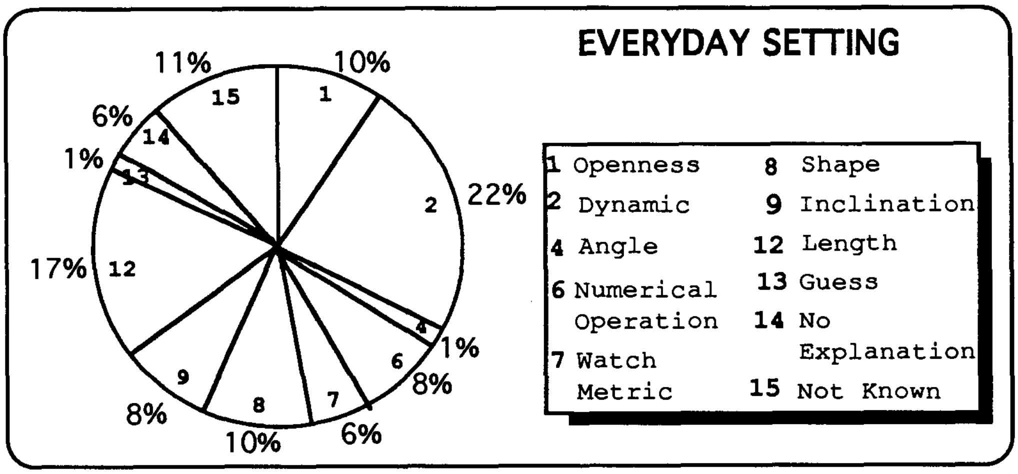

Diagram 6.24: The 9 age-group children's references used in the everyday setting.

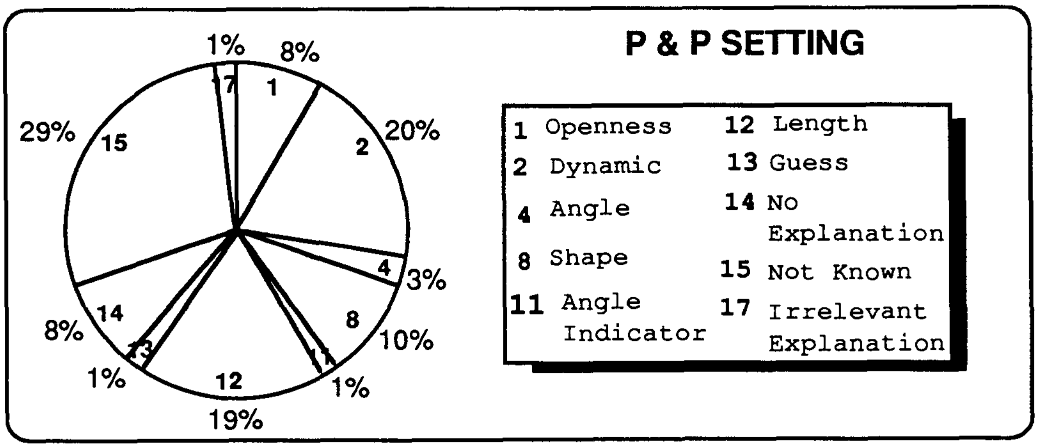

Diagram 6.25: The 9 age-group children's references used in the p & p setting.

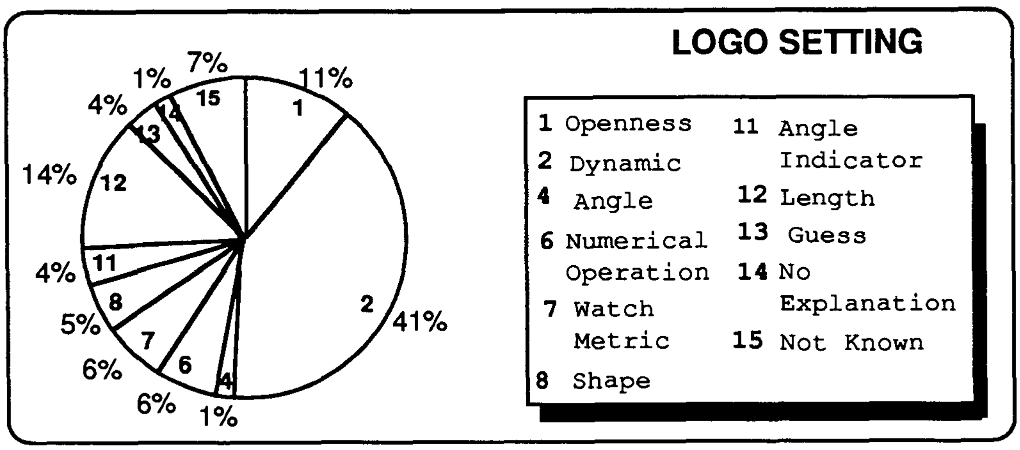

Diagram 6.26: The 9 age-group children references used in the Logo setting.

Considering the references which indicate that children did not know the activity (references 13, 14, 15 and 17) we note that they corresponded to more than 1/3 of children's responses in the p & p setting while in the everyday they were less than 1/5 and in Logo setting these responses were some where around 12% of the children's responses.

With regard to the references related to misconception ('shape' and 'length') children made a great deal of use of them in the everyday and p & p settings. However in the Logo setting these references are considerably less cited by children which allows us to conjecture that 'length', and mainly 'shape', were not such important factors for children to solve the tasks as they were in the other two settings.

Drawing attention only to children's responses in the everyday setting, I note that it was only in this setting that children referred to the inclination of the figure. Moreover, comparing 9 year-old children's responses with the 10 year-old age-group in the everyday setting, I observe that 'inclination' was more cited here than in the previous age-group. Such as happened with 10 year-old children, 'inclination' was cited by the 9 age-group children when they were explaining the difference between the initial and final position of the cars in the mini city, i.e. when children were comparing turns in mini city. The following examples illustrates children's use of this reference:

Example 1 - A 9 year-old girl, comparing these two turns (___/ _/) made by the cars that she was navigating, correctly answered that the turns were the same "because I can see by looking at the position in which the cars stopped".
Example 2 - Later on, when this same child was asked to compare these two turns (∩ and ⊓), still in mini city arena, she again said that they were the same "because the cars are in the same position after have done the turn. They started like this (she does with her hand the inclination of the cars initial position) and thus they are now like this (again she does inclination with her hands)".

Although she seems to be speaking about the parallelism of the cars at the final position, she does not use any scientific concept to explain her responses, rather she seems to refer from her visual perception only. An important factor to be considered here is that she did not have any drawn line to guide her in the comparison, i.e., she dealt with imaginary lines. Thinking of it in terms of learning and in terms of how could one helps children in the passage from the spontaneous to scientific knowledge, we must consider that activities like these two (which are full of meaning and carried out dynamically) may be able to help children to make this link.

Looking at the children's responses in the Logo setting and comparing 9 year-old children with 10 year-old age-group children, I note that the group of younger children justify their answers much morethrough dynamic reference than the older ones. Moreover, children from the younger age-group clearly used less 'openness' in their explanation than the older children. Although these findings are pointing to the 'dynamic perspective' as representing children's intuitive thought, it is early to draw this conclusion before examining the three remain age-group children.

To conclude the analysis of this age-group, I would like to emphasise that in no setting 'formal learning' or 'geometric metric' references were used by these children at all. Moreover, in general, children from this group referred less to the word angle than the older age-groups. This finding is confirming what seem to be obvious: the younger the age-group the less scientific are their explanations. However, it is good to be attentive to the fact that this age-group made less reference to the length and to the angle indicator of the figures in their answers than the previous age-group. And these two references are considered as children's misconception. What have to be examined here by future careful investigation is whether these misconception are intuitives or are they built up from school learning?

6.2.3 THE 8 YEAR-OLD CHILDREN

The next diagram shows the 6 most cited references of the 8 year-old age-group classified from the 302 answers given by children.

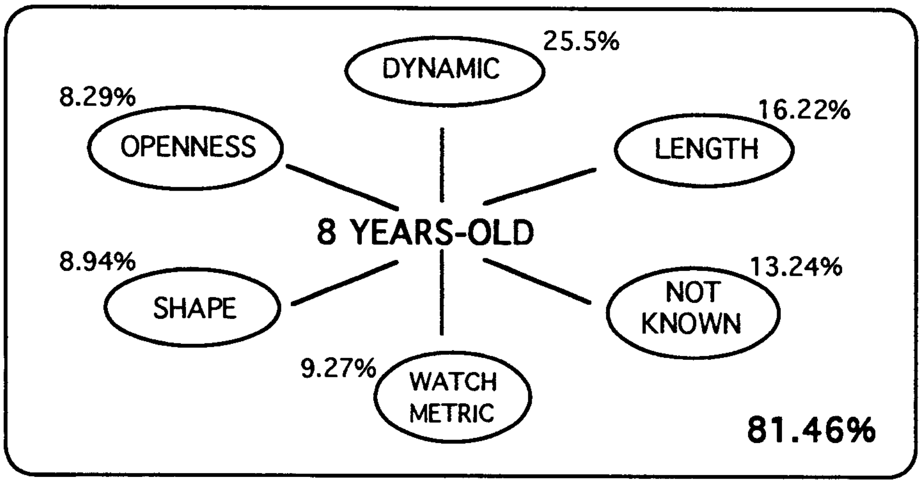

Diagram 6.27: The 6 most frequent references used by the 8 age-group children, considering all the 3 settings

As the two previous age-groups, 'dynamic', 'length' and 'not known', in this order, were also the most mentioned references by the 8 year-old children. Considering that this result is true for three different age-groups (10, 9 and 8 year-old age-group children) which are the older age-groups of elementary school group, I feel confident to interpret that (1) children under 10 years old do not know angle, at least from the scientific (formal) knowledge viewpoint, (2) their responses were intuitively dynamic and (3) they take the length (sizes) of the figures as a determinant factor to solve problems involving angle.

The appearance of 'watch metric' as one of the most cited reference by this age-group was a surprise, because it is the first time that this reference obtained a prominent place among children of the elementary school group. Children cited watch metric in the watch and arrow arenas, either in everyday, p & p and Logo setting (see the next three diagrams). From the viewpoint of classification, up to here there is nothing different from the way in which this age-group used watch metric to the way that the older children used it. However, from the point of view of children' strategies the analysis of children's performances differed widely from that of the older groups. The next examples illustrate three different ways of thinking and solving problems involving 1/2 turn in the watch arena. They come from 8, 10 and 13 year-old children respectively.

Example 1 - After having correctly predicted the place where the minute hand would be after it turns 1/2 turn from an initial position of 12.00, an 8 year-old boy predicted that a watch which was showing 12:10 would 1/2 hour later be showing 12:30 "because here is the right place for 1/2 hour." Asked about his previous prediction of 1/2 turn and his new prediction he answered "There you asked me for 1/2 turn and 1/2 turn is in the middle of the whole turn, and now you asked me to turn 1/2 hour and 1/2 hour is when the hand arrived at the 6 (number 6)".

Example 2: A 10 year-old boy correctly answered to all the questions about half a turn in the everyday setting and explained: "I divide it (watch) in two bounds...I break it in two bounds of the same size, the half is when one (bound) finishes and another starts". For the next two questions, which involved half an hour, he could only answer correct the first of them and his explanation was: "Last time I jumped 7 number to stop at 6 (points to number 6), now I did the same". This same boy answering the three questions of half an hour in Logo setting made a mistake only in the first activity, he explained: "I had counted 6 numbers but I started to count from the wrong number, I didn't jump."

Example 3: For the activities which involved half a turn in the everyday setting, a 13 year-old boy, who correctly answered all the three questions, justified: "1 turn will be in the same place and 1/2 turn stops in the middle like in the 1/2 hour". Asked about half an hour, he said: "the hand will move 5 by 5 until 30". He used this same explanation of counting 5 by 5 in the activities carried out Logo watch arena saying: "I counted 5 by 5 until 30. I don't care with numbers of this crazy watch".

From the above examples it is possible to determine three models of children representations: in the first example the child was not able to realise half a turn and half an hour as doing the same amount of rotation; he also perceived '6' as a fix point; the second child answered the questions by numerical operation - dividing the shape - and also by watch metric but in this case he did not know the correct metric; finally, the representation of the third child was clearly dynamic, making relationship between turn and hour, but he also was able to use the metric of the watch (even when the face numbers were not those traditionally found in a watch).

Continuing through the analysis of this age-group, it possible to note the reference to the openness of the figures was less used here than in the two previous age-groups as well as the reference to the numerical operation which even appear among the six more cited references. To refer to the numerical operation seems to be a children's expertise of introducing a precise form of measurement, i.e., they are trying to find a way out in order to quantify turns so that they may show and prove they are right in their answers. However the 8 year-old children are just learning how to operate with numbers and perhaps they do not feel confident to make use of this strategy as frequently as the older children.

Nevertheless, in order to have a more accurated comprehension of these age-group children it is necessary to look at their performances separately, that is, setting by setting. The three diagrams shown in the next page are built precisely to help me to understand better the 8 year-old children's fulfilment

EVERYDAY SETTING

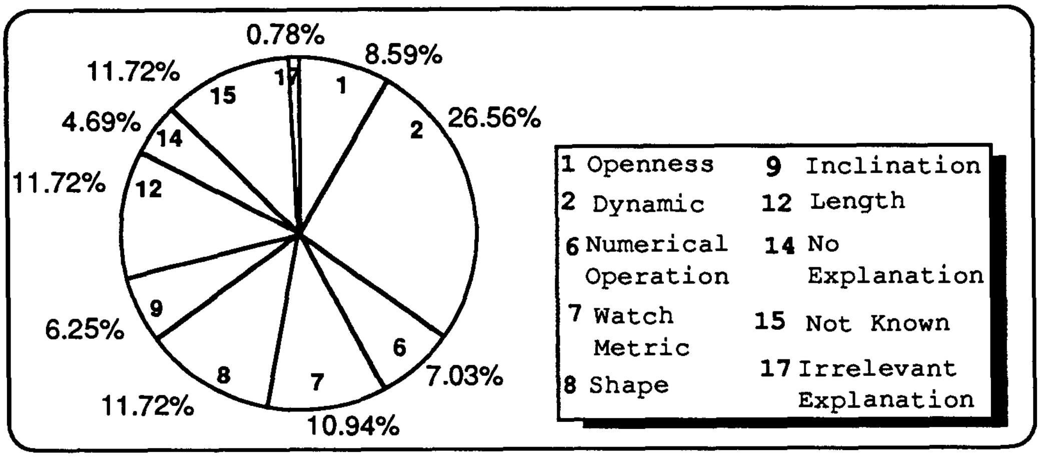

Diagram 6.28: The 8 age-group children's references used in the everyday setting.

PAPER & PENCIL SETTING

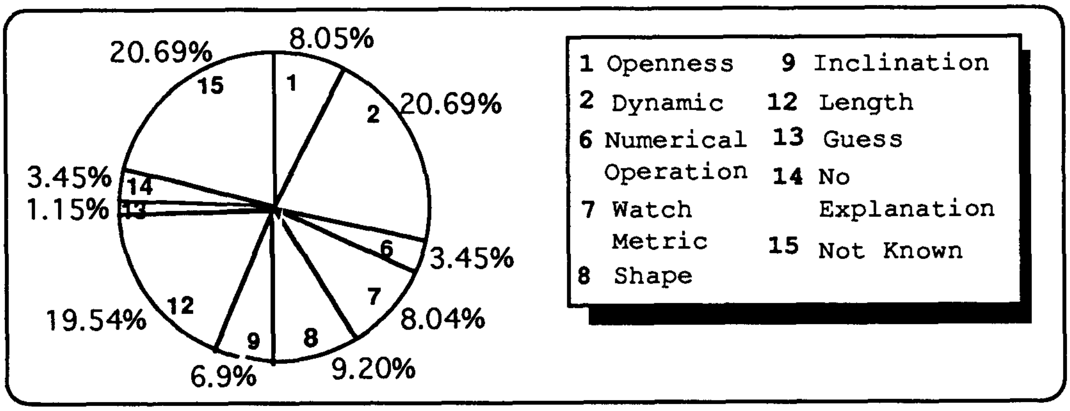

Diagram 6.29: The 8 age-group children's references used in the p & p setting.

LOGO SETTING

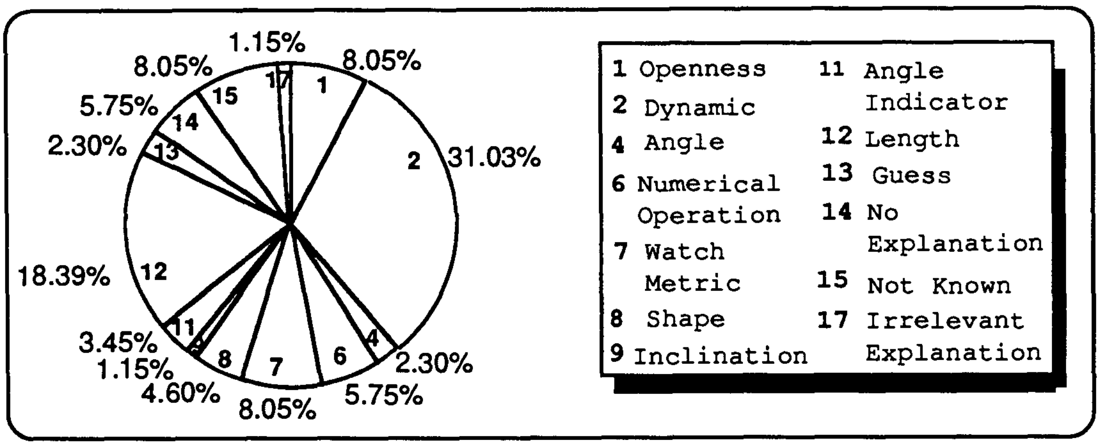

Diagram 6.30: The 8 age-group children's references used in the Logo setting.

The diagrams show that dynamic was the most cited reference by children in all three settings, and that the Logo continues to be the setting which children thaught more dynamically. 'Not know' and 'length' come in the second and third places. This result confirms my previous statement that " (1) children under 10 years old do not know angle at least from the scientific (formal) knowledge viewpoint, (2) their responses were intuitively dynamic and (3) they take the length (sizes) of the figures as a determinant factor to solve problems involving angle." (page 241).

Although 'inclination' does not appear very cited among the 8 year-old children general diagram (Diagram 6.26) in the everyday and p & p settings it was often referred. However, 'inclination' seem to have a less precise representation for children from age of 8 years old than it had for the older children. Here, inclination appears related to a fix point (as the case for the hands of watch or the 1/2 turn in the arrow) or as an referencetial for comparing two lines (in the case of comparison of turns in the mini city as well as comparing pair of angle in the stike game). If in the first case 'inclination' was a misconception, much likely influenced by the cultural meaning of the watch, in the second case, it was a good sign of the children's acquisition of a operational invariant.

Comparing children's performances among settings, we note that they used fewer numbers of references in the everyday and p & p settings (only 10 in each setting) than in the Logo. In fact, we can observe that, up to here, the elementary school children's responses have not exceding to up 11 references in the everyday setting. Moreover, taking into account the last three age-groups (10, 9, and 8), about 2/3 of the children's responses in the everyday setting has been restricted to only four references. 3 out of them are constantly used in all the three age-groups: 'dynamic', 'length', and 'not known' references. Moreover, while for the older age-groups the fourth reference was 'openness', for 8 year-old children it was 'shape'.

Looking at the everyday and p & p settings we observe that no children referred to any category related to the scientific knowledge, such as 'geometry metric', 'angle', and 'formal learning' at all; and in the Logo setting chidlren only use the word 'angle' but in a quit insignificant average.

These results confirm my obvious expectation i.e., that the more younger children are, less undesrtood they have about angle. However what seems of importance here is the younger group effort in order to bring the knowledge they have into the activities. In other words, although the younger children did not present a specific concept in those references, they solve the activities using operational invariants which, in many cases were a misconception.

6.2.4 THE 7 YEAR-OLD CHILDREN

The below diagram shows that in about 1/3 of the activities children had no idea of what were being asked to them. In fact, this was mainly noted in those activities which asked children about precise size of angle, such as 1/4 of turn, 90^0, 180^0, and even 1/2 a turn.

The diagram also shows that more than 1/3 of children's explanations were based on the length, shape and inclination of the figures. This demonstrates an increasing of these categories in comparison to the previous age-groups. Moreover, I noted that for these children 'inclination' were basically referred as a fixed position of figures.

Another information given by the diagram is concerning the 'dynamic' reference, which has presenting a continue decreasing from age to age, but now fell sharply if compared with 9 and 10 year-old children. Therefore, I can assume that these children were not taking into account the movement. Although they have said words like 'turn', 'bend' I am not sure whether they were really take it into account The general diagram of this age-group led me to quest whether this age-group has less (if any) conception of angle.

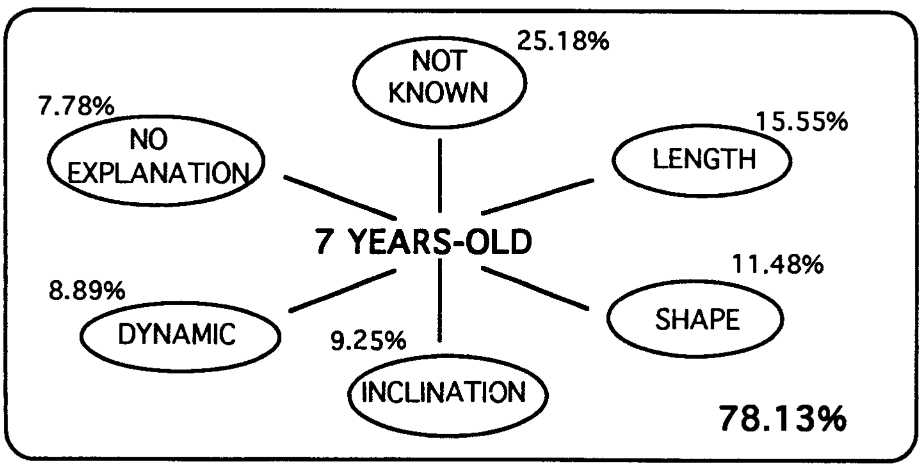

Diagram 6.31: The 6 most frequent references used by the 7 age-group children, considering all the 3 settings

The below three diagrams give detailed information about children's responses from which I intend to draw a better profile of this age-group

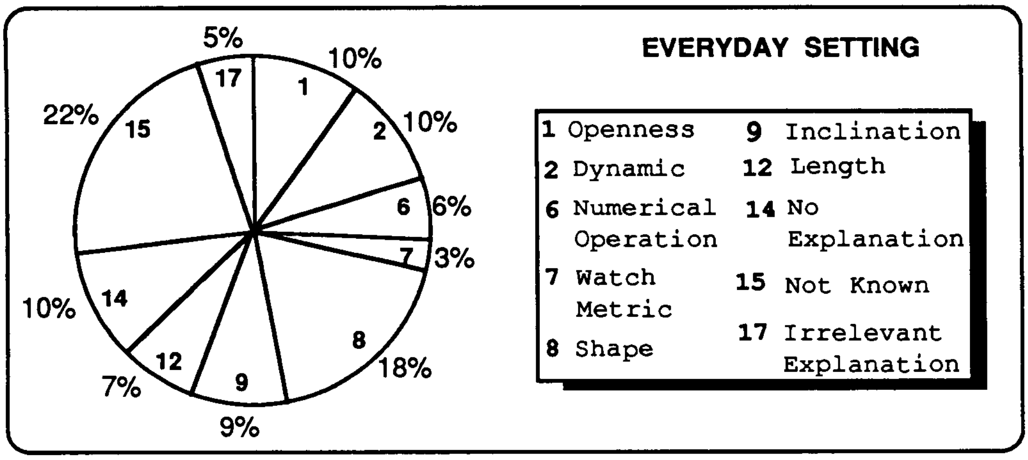

Diagram 6.32: The 7 age-group children's references used in the everyday setting.

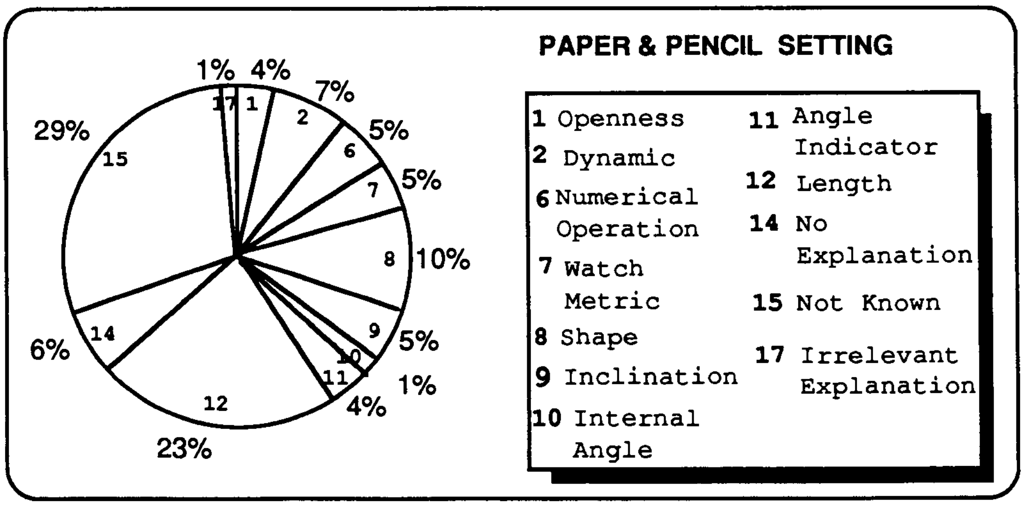

Diagram 6.33: The 7 age-group children's references used in the p & p setting.

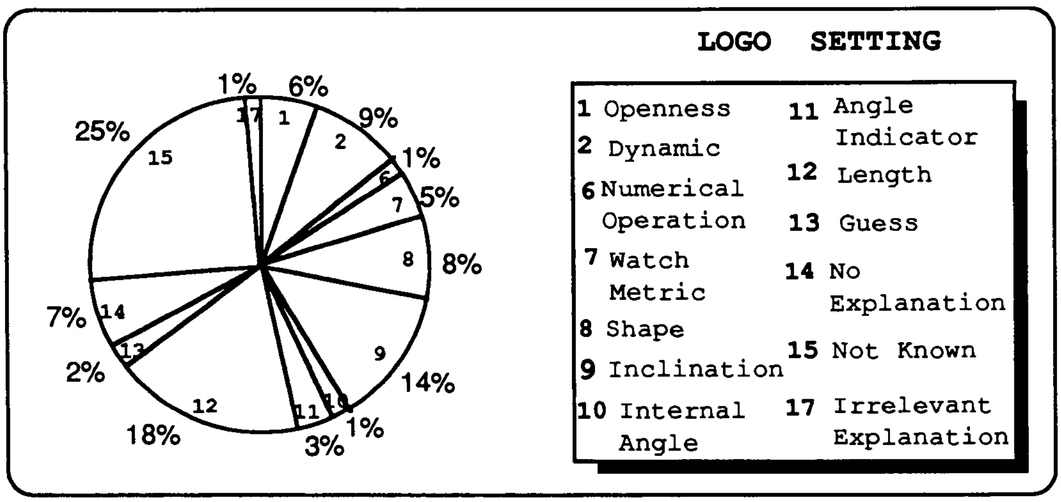

Diagram 6.34: The 7 age-group children's references used in the Logo setting.

A first observation from children's responses across the settings is the fall in the numbers of references used by children from everyday to the other two setting. This has been occuring since the analysis of 10 year-old children. Although the average of children's reference to 'not known' was very high in this setting, it was also in the everyday that children less assumed that they did not know about the activity, . Another important point to be noted in the comparison between this setting and the other two is that children gave much more irrelevant explanation here than in the others, even though the average of this reference was very low, . Finally, apart 'not known' reference, 'shape' seems to have been the greatest influence over children's responses in the everyday setting, whilst in the Logo and p & p setting it was 'length'.

It is clear, from the quantitative analysis and confirmed by the analysis of children's articulations, that children at this age have not formed operational invariants yet, or at least eficient invariants, to cope with angle. However from the observations described in the oprevious paragraph, I can state that children are clearly taking 'shape' and 'length' as as the meaning of the activities, and they were, finally, dealling with the figurative knowledge in sense of preceiving the figures as a drawing. Therefore, I can assume that angle was not their concern, and much less the properties (invariants) involved in it could be perceived by them.

6.2.5 THE 6 YEAR-OLD CHILDREN

The below diagram shows that the numbers of the activities in which the 6 year-old children could not explain ('no explanation' reference) or could not know ('not known' reference) were over 1/3 of the whole experiment. This means that 1/3 of the tasks did not make sense for this age-group.

It is interesting to observe that for the first time 'dynamic' category does not apear as one of the six most cited reference. This fact becomes more important if we draw attention for the increasing of 'openness' and 'inclination' references. I shall consider that children made reference to the inclination of the figure only to designate a fix point.

The absence of 'dynamic' reference, the increasing of 'openness' and the 'inclination' references, the high average of 'not known', 'no explanation' and 'length' show me that these children were not thinking of angle and much less in this as a movement.

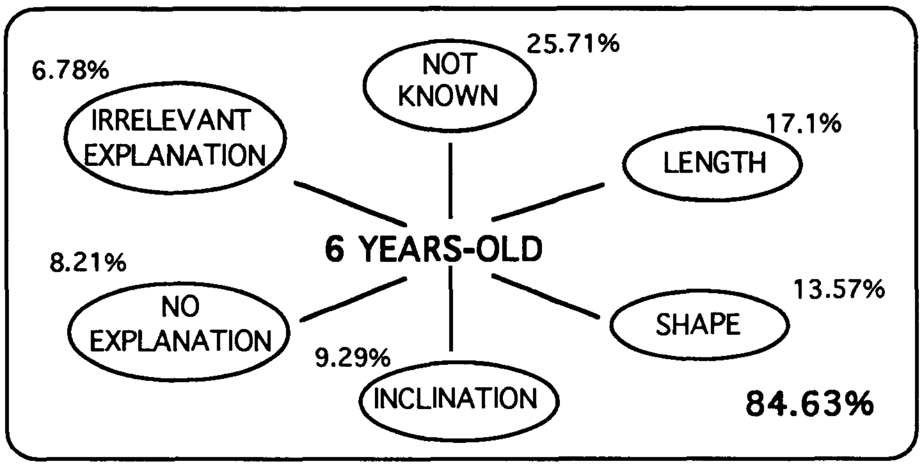

Diagram 6.35: The 6 most frequent referencesused by the 6 age-group children, considering all the 3 settings

The next three diagrams, which present the profile of children's performances across the three settings, will allow me to have more information in order to analyse this age-group.

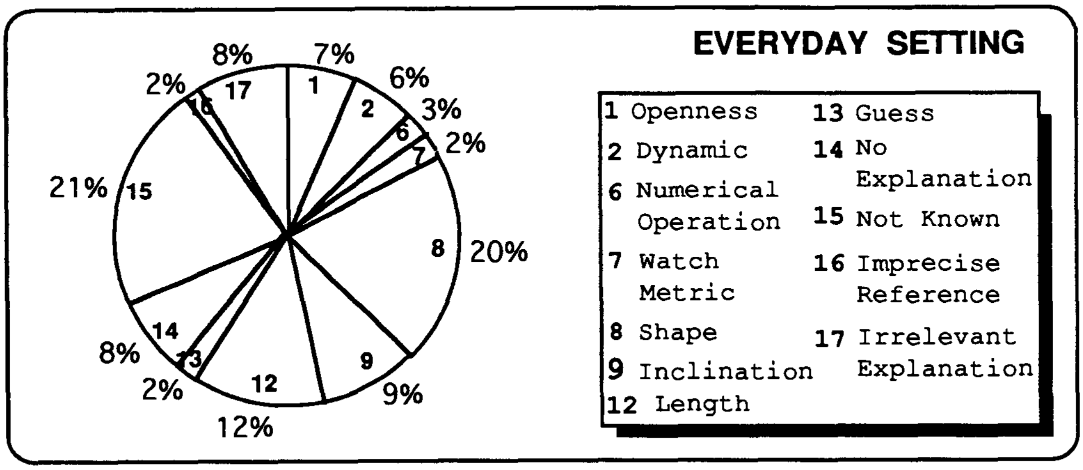

Diagram 6.36: The 6 age-group children's references used in the everyday setting.

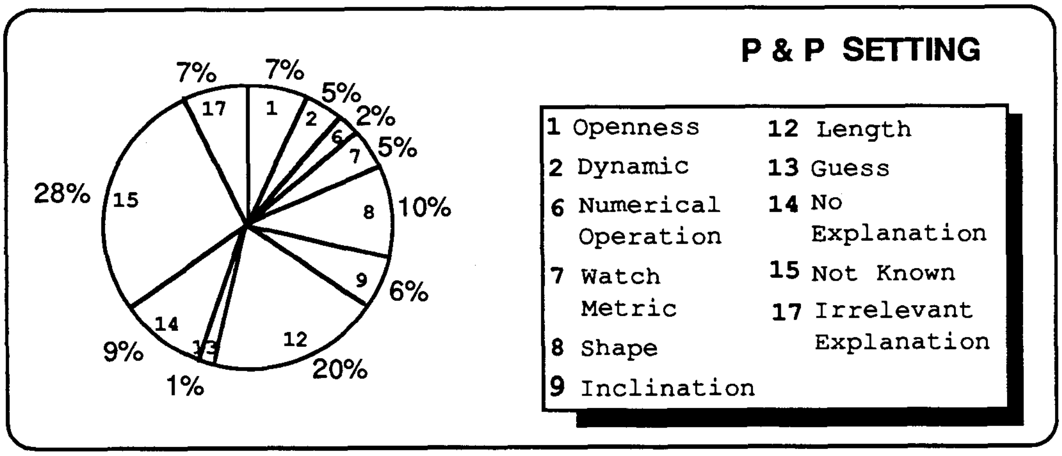

Diagram 6.37: The 6 age-group children's references used in the p & p setting.

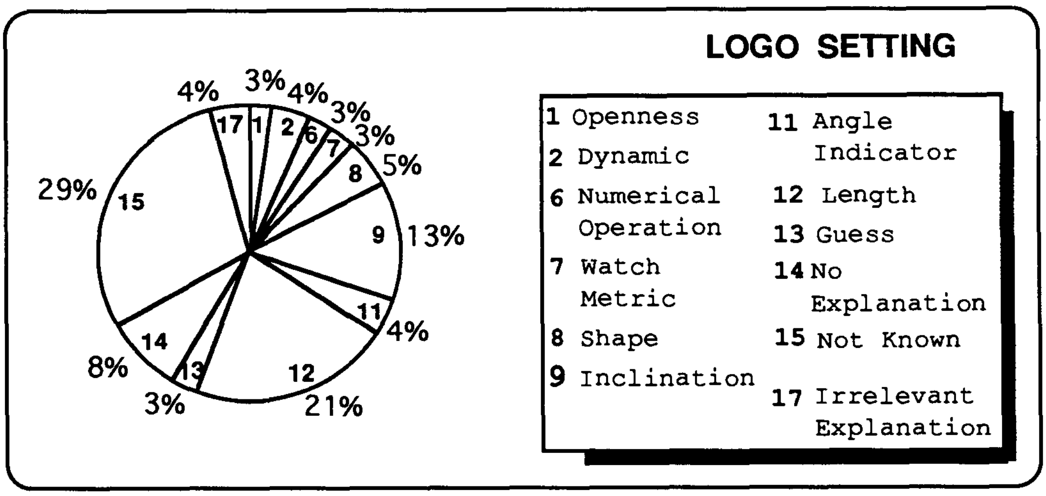

Diagram 6.38: The 6 age-group children's references used in the Logo setting.

The above diagrams clearly show that this age-group had no idea of what was being asked for them most of the time. This is valid for all the three settings. Moreover, when children tried to give an explanation, they basically did by referring to the length or to the shape of the figures.

Looking at the differences among settings, what seems interesting is that while in the everyday setting children were more concerned with the shape of the figures, in the p & p and Logo settings it was length which play this role. This behaviour was also noted in the previous age-group.

Another good point to be discussed is the low averages of the 'dynamic' reference in all the settings. This makes me wonder how much are these children able to perceive (or to predict) movement or, at least, how much did children take the movement of the figures into account. Piaget (1968) has stated that the children's mental image, on a pre-operational level, is basically static. As far as my findings are concerned, I agree with him at least with regarding to the relevance children give to the movement.

Finally, the results of this age-group, led me to state that, such as the previous age-group, 6 year-old children were basically dealing with the 'shape' and 'length' as the invariants of the activities. They were tackling the figurative knowledge in sense of perceiving the figures as a drawing. Therefore, as I stated in the analyses of the 7 year-old children I can assume again that angle was not their concerning. Children at these ages do not have a conception of angle definitely.

6.1.6 SUMMARY OF GROUP 2 CHILDREN

The below diagram shows that, except the 'dynamic' and 'openness' references, the remain references are related to misconception or lack of knowledge. Special attention must be given for 'inclination'. This reference did not denote a misconception among 10, 9, and 8 year-old children, however it was for the younger ages.

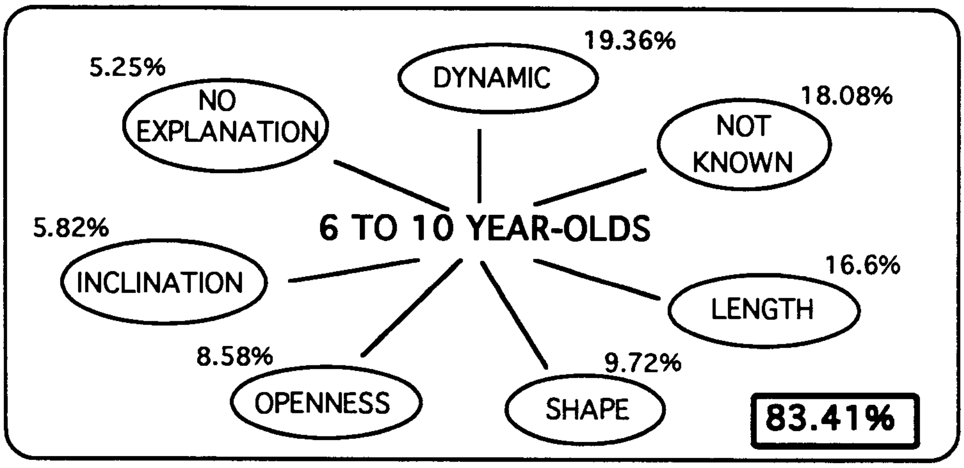

Diagram 6.39: The summary of the 7 most frequent references in the Elementary School children, considering all the 3 settings.

Following the same way in which Group 1 was analysed, I can also divide this group into two sub-groups: the lower sub-group is composed by 6 and 7 year-old children, who definitely do not present yet the conception of angle, either from static or dynamic perspective, and who did not show even a spontaneous knowledge of this matter. This sub-group was responsable for the high averages in the references which were related to the lack of knowledge or misconceptions of angle within Group 2. The higher sub-group level is formed by 8, 9, and 10 year-old children, which answers fluctuated between the use of spontaneous knowledge and the absense of knowledge. For this sub-group the changes of contexts, settings and arenas influenced in the children's competences (or understanding) to deal with angle so much.

6.3 COMPARING THE PERFORMANCES BETWEEN GROUP 1 AND 2

I would like to conclude this chapter by making a comparison between those seven most frequent references used by children of Group 1 and those used by children of Group 2.

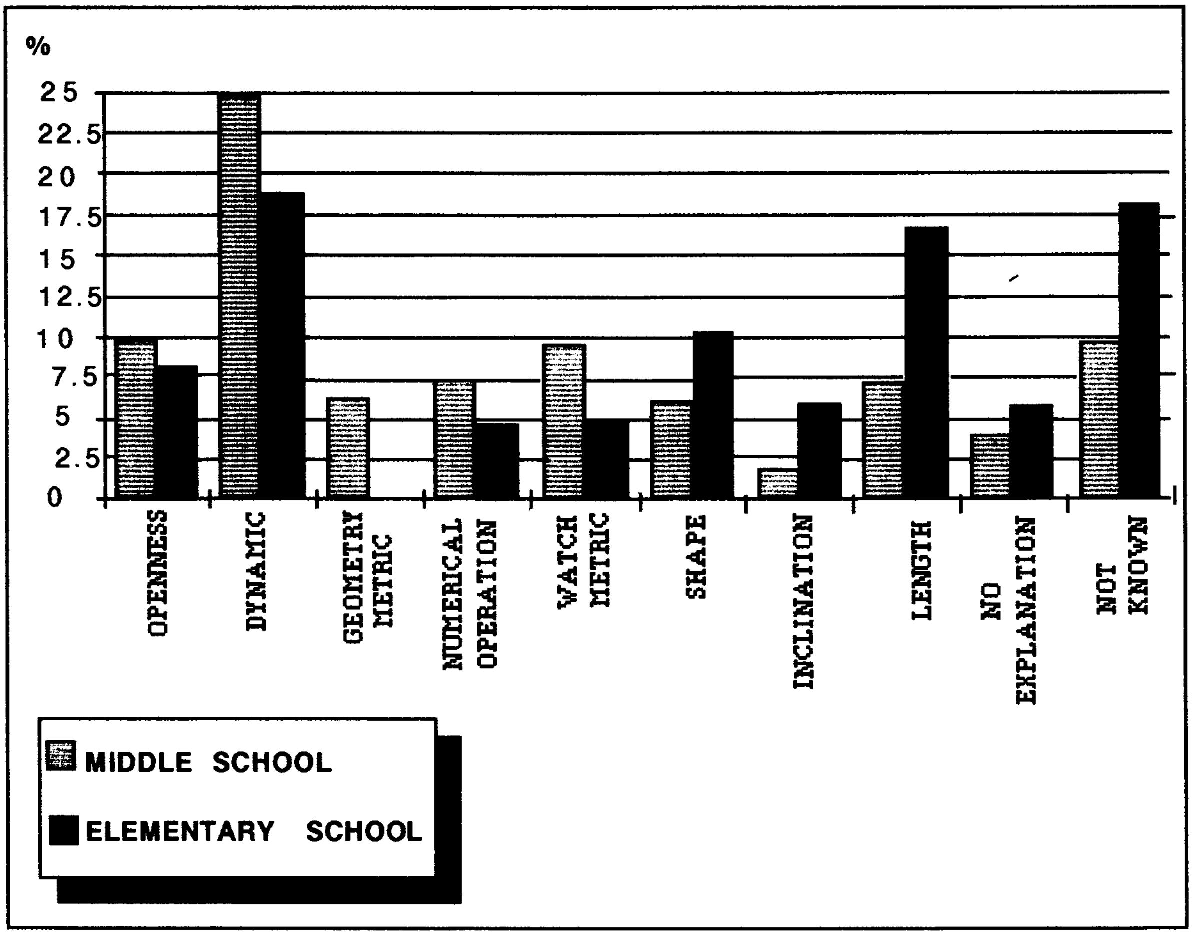

Figure 6.1: Comparing elementary and middle school children in relation to their most frequent references.

The above Figure shows that the Group 1 children obtained higher averages in the categories related to the scientific concept (such as geometry metric or numerical operation), as well as in the categories related to the cultural meaning (such as watch metric). It also possible to note that children from middle school were really concerned with the movement involved in the activities. The responses of the Group 2 children, in opposition, was mainly classified as meaning lack of knowledge or as misconceptions. However, I cannot forget that 6 and 7 age-groups, which presented a very fragile (and many times, a complete absence) of conception of angle, were pushing up the averages of these types of categories inside the Group 2. The 13 and 14 age-groups, on the other hand, seem to have exactly the same role, but in the opposite way, inside the Group 1, i.e., they pushed up the averages of the categories which were related to the children's scientific concepts. Therefore, it is possible to divide the nine age-groups of children, at least, into three sub-groups, where 6 and 7 year-old children was the sub-group which presented the lower performances, children between 8 and 12 years of age compose the sub-group of the intermediate level of performances, and the 13 and 14 form the highest level of performances in terms of understanding the concept of angle.

CHAPTER 7

DISCUSSION AND CONCLUSIONS

The central focus of this research Is to identify the factors which influence a child's conception of angle. Theories from psychology and mathematics education were interwoven to form a basis for designing the study and the interpretation of the results.

The study was carried out in the North-East of Brazil, where fifty-four children aged from 6 to 14 were asked to undertake 92 activities. These activities involved the concept of angle, set within a variety of different situations. Some of them served as a subset of the variables of the study, which will be described, in summary, later in this Chapter.

The general conclusions below summarise and integrate the analyses and the partial conclusions developed throughout the six chapters. These findings are re-visited, discussing them against the backdrop of the broader themes and the theoretical issues of the research.

The chapter begins with a review of the child's concept formation. It is followed by a re-assessment of the variables which underpinned the design of this study in relation to learning and the concept of angle. The chapter presents a summary of the quantitative and qualitative findings considered to be the most relevant factors in the child's acquisition of the concept of angle. These issues are discussed in the light of the theoretical background. Finally, indications for future research are suggested.

7.1 SUMMARY OF THE THEORETICAL FRAMEWORK

This study is guided by constructivism. The principal proposition of constructivism is that children build a version of reality through their own experiences. In this process they play an active role in creating new relations between their ideas, from which new structures emerge. The theories of the main authors who guided this study are summarised in Chapters 2 and 3. Here I refer to certain key points of their work which support the interpretation of my findings.

Piaget contended that learning is the result of the twin processes of action and internalisation of action, and evolving within a developmental framework. From Piaget (Piaget et al 1968, Furth 1969, 1977) I embraced the perspective in which knowledge can be understood from two viewpoints: the first aspect of knowledge -- describing things, which he called figurative knowledge -- is present in any perception. It initially arises from imitation, starting the symbol formation. Later, it is transformed by the equilibration process into a mental image, i. e., an internalised symbol (Furth, 1977). The second aspect of knowledge -- operating on a thing, which Piaget named operative knowledge -- is concerned with the transformation of reality states. It involves a logical thought. The 'figurative' aspects of symbolic acquisition and their usage, including language, are subordinated to the child's 'operative' aspect of knowledge.

Vygotsky's theory (1962, 1991) also presents a developmentalist vein, but additionally guided by the social/cultural aspects of knowledge, from which I borrowed two main ideas. The first is that the learning process takes place within what he calls a zone of proximal development, which allows children to reach higher stage (level) with the help of 'others'. The second is his distinction between spontaneous and scientific concepts and how both are elements of the same process, i.e., of the concept formation. They are continually influenced by each other. The spontaneous concept arises from the child's everyday life experience, whilst the scientific concept is usually acquired at school, with the help of the teacher.

Vergnaud[1] makes a similar distinction between spontaneous (called 'ordinary') and scientific concepts, which illuminate the understanding of the results of this research. He argues that ordinary concept has much to do with a person's level of competence. This competence is shown by the operational invariants which emerge from schemes acquired from a child's interaction with the situation. Thus the operational invariants will constitute theorems-in-action as well as the theorem-in-conception. He emphasises that ordinary and scientific concepts can co-exist in harmony, depending on the situation in which each concept, or combination of concepts, might be applied. Thus, it is essential to confront a child with problem solving which puts him/her in a position of understanding the meaning of the concept .

Nunes (1992, 1993) stresses the importance of inserting activities in semantic situations. Semantic situations are defined by Nunes as 'rich' places for learning, although not necessarily in the real world. In such an environment,

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1 - Part of Vergnaud's theory was further developed in his presentation at the Institute of Education, University of London, on 27 March 1994. The clarification of some points of his theory was made through a personal appointment, given to me at the same day. For a more comprehensive review see Vergnaud, 1983, 1985, 1987A, 1988A, and 1988B.

children can appreciate the meanings and purposes of their activity. Nunes also stresses the importance of the functional organisation of problem-solving. According to her approach, functional organisation is fundamentally influenced by the sign-system involved in the activity itself, whether oral or written. Her view was also helpful for the interpretation of my findings. Nunes argues that the representational systems influence the functional organisation of the children's activities However, these systems may not be able to influence the functional organisation without the support of particular cultural sign systems . This means that the same children may perform differently when carrying out the same function supported by different systems.

When looking for a model of learning mathematics, Van Hiele (1986) found that the level of thinking the child achieves is influenced by the context in which the activities are inserted. The way children symbolise the context is also important. He states that the change from one level of thinking to the next involves the child's experience within a context and the way s/he cope with symbols inserted in this context. A context involves many different symbols, and any given symbol is not restricted to one context only. For Van Hiele, the starting-point of symbols is an image, in which the properties and relations are projected. Through learning, the symbol loses its peculiarity of image and achieves a verbal significance. Therefore, the symbol becomes more useful for operations involving thinking.

These theories were the bases of this research. They also oriented its design, as follows.

7.2 SUMMARY OF THE DESIGN OF THE STUDY

This section focus on the variables used for the elaboration and implementation of the activities of this study. Up to six sets of variables were used in different combinations.

The first set of variables was the way the angle 'appeared' within an activity. The angle could be categorised either as dynamic or static from a priori analysis of the requirements of the activity. These variables were used for two reasons. Firstly because static and dynamic perspectives have been identified in the literature as a reasonable classification for angle (Choquet, 1969; Close, 1982; Hilbert, 1972). Secondly because of the controversy surrounding the question of whether children tend to assume the static perspective of angle (as stated by Piaget, 1960; Close ,1982)[2], or whether children perceive and work better in the dynamic perspective. The latter is argued by Noss (1985), Kynigos (1989), and Magina (1988) among others, who worked with children within the turtle geometry of Logo microworld. In this research children's performance on activities in which the angle appeared in both perspectives is used.

Another set of variables was the settings in which the experiment took place. The settings are models of environments. Different setting features define different representational systems. Three settings were investigated: everyday, p & p and Logo. It was anticipated based on Nunes (1993), that the everyday setting would afford an oral practice, and p & p a written practice. The Logo setting would afford the practice of the body syntonic[3] through a virtual world delimited by the computer screen. Within these three settings the angle could be approached formally and informally, as well as through the dynamic and static perspectives. This study also reflects upon the importance of exploring

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2 - Both authors worked with children in a paper & pencil environment.
3 - The expression 'body syntonic' was coined by Papert (1980).

the activities within different combinations of settings and contexts. Thus, a set of contexts was included as another set of variables.

Context is defined here as a situation which allows a person to experience a given content. Based on Van Hiele's definition (1986) of context and Vegnaud's definition (1985) of representation, this research takes contexts as playing two roles in the symbolic function. Context is the signifier of a content. At the same time context is the signified given by a person in a specific situation. This research includes three contexts: navigation, rotation and comparison. They involve either the static or dynamic perspective of angle. Contexts were also present in all three settings. Based on this design, can children draw on the same invariants in different situations (contexts) or do they rely on different representational systems (settings)?

The arena where the activities were carried out formed the fourth set of variables. Arena is the concrete material, or an objective situation. For example, the watch arena in the everyday setting was 9 cardboard analogue watches, some with face numbers and all of them having two hands which could make turns around. Whereas in the Logo setting this arena was a drawing of an analogue watch, i.e., a circle with an angular measure instead of watch metric measure. Therefore, the watch arena involved the rotation context. This study had six different group of arenas: map, 2 angles, 4 angles, spiral, arrow/turnstile, and watch.

The way children were asked to solve the activities was another set of variables called conditions. There were three conditions: recognition, action and articulation. When children were asked to compare figures or turns, the condition was recognition; when they were asked to construct a figure or to predict a turn, it was an action condition. The third condition was when the children were asked to explain, or describe, what they had done; and this condition was called articulation. Piaget (1977) has stressed the distinction between an action (in this study, a recognition too) and the internalisation of this action. The bridge between both processes is constructed firstly by a child's theorem-in-action followed by his/her theorem-in-conception (Vergnaud, 1985, 1988B). The first two conditions, action and recognition, formed the bases of the quantitative analysis, where children were studied through their performances. The third condition, articulation, formed the qualitative part of the study. When these variables are analysed together, they demonstrate the child's symbolic function as a whole.

The last set of variables was the size of angle. This was approached by grouping the activities into seven clusters. 80 out of the 92 activities of the study were grouped according to the value of the angle involved. Therefore, cluster 1 included activities in which angles measured less than 90^0; cluster 2, angle of 90^0; cluster 3, angle of 180^0; cluster 4, 540^0; cluster 5, 720^0; cluster 6, angle wider than 720^0; and cluster 7, a comparison of 4 or 6 figures with different angle values, which were presented to the children at the same time. The activities were grouped inside the clusters independently of other variables, i.e., activities from different perspectives, settings, contexts, arenas and involving different conditions were classified inside the same cluster. The reason for grouping by angle value was that the principle concern in this set of variables was the relationship between spontaneous and scientific concepts. For example, 90^0 was presented to children in many different situations, both similar to and different from the way this angle is presented in the math classroom. The question to be answered was "do these approaches make a difference in children's understanding of angle?"

7.3 DISCUSSION OF FINDINGS AND IMPLICATIONS

This section discusses the main results of both the children's responses and their articulation, i.e., how they described what they had done or explained how they solved the activities. The interpretation of the results is enriched by the discussion of the children's articulations. From their speech or gestures, it was possible to categorise what were termed 17 references. These categories concerned the different ways children explained (or, for some children, described) how they solved the activities. The references, in the first instance, were the way I found to group children's operational invariants. These invariants emerged when they were solving the activities and represented important pieces of information selected by children to characterise the activities. These invariants were expressed in the form of symbolic function (articulations). Moreover, these 17 references also allowed me to look at children's articulations in term of spontaneous and scientific concepts.

The research design was based on a priori quantitative categories, whilst a posteriori qualitative categories emerged from children's articulations and were grounded in the data from the interviews. Therefore, although children's articulations were not taken into account in the quantification of data, *what* children gave as answer (in terms of recognition and action) in the test situations, as well as *how* they explained their responses (articulation) are discussed together in order to build a picture of the factors involved in the children's understanding of angle. The summary of the findings will now be presented in terms of issues:

7.3.1 DEVELOPMENTAL FACTOR

The results pointed to the presence of the developmental factor as influencing the children's performances. A progressive increase, in the averages of correct answers, was noted as the child matured (from 6 to 14), although this difference was less in the performances of 8 to 12 year old children. From the viewpoint of performance, the sample can be divided into three sub-groups. First, the group of higher level performances, formed by 13 and 14 year-old children who showed a good performance, solving more than 70% of the activities correctly. Second, the group of intermediary level performances, composed by children of 8 to 12 years of age, who were able to solve about the half of the experimental activities. Third, the lower level performances group, formed by 6 and 7 year-old children, who could not solve the great majority of the activities. Within these three sub-groups there was a tendency indicating that older children made fewer mistakes than the younger ones. Moreover, whilst the lower level group said explicitly that they did not know about half of the activities, among the children of the higher level group the 'not known' reference was not very frequent

However, these differences were not very marked and age did not necessarily lead to improvement, as shown the difference between the results of 10 and 11 year-old children. I shall take two points into consideration. Firstly, 10 and 11 age-group children were classified in the same sub-group. Secondly, the difference in favour of the 10 year-old children was small. Therefore I do not consider that this divergence refutes the developmental factor. Instead I understand that this difference shows that there were other factors influencing children's performances.

I can interpret the developmental factor from my theoretical framework, either from the perspective of knowledge, as is referred by Piaget, or from the learning process, as advocated by Van Hiele, or even from both knowledge and learning processes, as discussed by Vygotsky. However, this was not a longitudinal study. The results of this research did not allow an evaluation of whether these children's developmental factor corresponded to the Piagetian model. According to Piaget's perspective, the developmental factor is a consequence of the relationship between a child's structures and his/her interaction with the object (a situation, or a person, or an object). This process require biological maturation. Neither can I that the developmental factor follows the model proposed by Vygotsky. He argues that the developmental factor emerges from a child's social process which comes together with the learning process, enlarging the 'zone of proximal development'. I cannot be sure whether the children followed the model advocated by Van Hiele in which development is guided by another a more important factor, i.e., the learning process.

The findings show that the most successful children were the older ones. It was also found that the older children came up with more complex responses to the activities. Moreover, the older children were more consistent in their responses from situation to situation[4]. However, it must be remembered that the older the children the higher level of schooling. Based on this argument, it is hard to separate the developmental factor from the school factor. Thus, I shall now consider the school factor which, together with the developmental factor, has influence over children's performances.

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4 - I consider as a 'situation' each variable of the study, such as the different arenas, settings, contexts, angle perspectives, and the conditions (action and recognition).

7.3.2 SCHOOL FACTOR

The main fact which led me to state that the school was one of the factors responsible for the children's performances was based on the evidence that middle school children were able to solve at least half of the activities, whilst among the elementary school children this only occurred with the 10 year-old children.

The influence of school could also be discerned from the children's articulations. For instance, 14 year-old children often referred to formal learning as well as to the geometry metric, that is, even though they did not use a protractor they often estimated the angles, referred to them in degrees, and incorporated in their speech the school terminology such as 'parallelism', 'acute angle', 'vertex' words. These terms were used less among 13 year-old children, and continued to fall sharply until complete disappearance at the age of 11.

On the other hand, the children's results were not wholly consistent. In fact, younger age-group children presented better performances than older ones depending on which value of angle was involved in the activity. For instance, 12 year-old children made more mistakes than the 9 year-olds in those activities which involved angles of 720^0 and wider (clusters 5 and 6 respectively). The same also occurred between 8 and 11 year-old children for activities involving 180^0 and 540^0 angles (clusters 3 and 4 respectively), where the younger group obtained better results. Although these differences in the results of younger and older age-groups were not large in terms of percentage, the question raised by these results was that from the point of view of school learning, how could children, who were three levels bellow the other, present better results solving the same activities?

These findings are in contrast to what has been emphasised by the literature, which shows that young children (up to age of 11) have difficulty coping with angles wider than 90^0 (APU, 1981A; APU, 1981B; APU, 1987). It is also generally held that the younger the children are the more mistakes they make in problems involving angles (Close, 1982; Fuys et al, 1988). However there are some important differences between the design of the present research and those cited above. Whilst in those studies children were tested only in a paper & pencil environment, here children were presented with the idea of angle in two environments other than paper & pencil. Moreover, in contrast to the previous studies, even in the p & p setting there were activities which involved not only the static but also the dynamic perspective of angle. In addition, this research elaborated, for both perspectives of angle, activities similar to those applied inside the maths classroom as well as activities related to everyday life. For example, children were asked to deal with angles larger than 90^0 by moving a minute hand of a watch, by turning a miniature car on a map, and by comparing open figures in a paper & pencil environment. I could continue listing more examples, but my concern here is to point out that factors other than developmental and school effects must be considered in order to make sense of children's results. To take just one example, we can point to the influence of the settings upon children's spontaneous and scientific concepts. In fact, the children's articulations showed that the everyday was the one in which children least referred to the 'geometry metric' and 'formal learning' categories.

7.3.3 THE ANGLE PERSPECTIVE FACTOR

The quantitative findings evidenced a diminution, from 8 to 14 year-old children, with regards to the number of the correct solutions in activities inserted in the dynamic perspective of angle[5]. Moreover, the 12, 13 and, above all, the 14 year-old age-groups, as well as 6 and 7 year-old children, presented better performances in the static activities. The relevance of this finding is to understand why middle school children were better in this perspective while the elementary school children were better in the dynamic perspective: did the middle school children have difficulty in perceiving the movement of figures? How can the similarity between the older age-group (12, 13 and 14 year-old) profiles and the younger groups (6 and 7 years of age) be explained?

In Piaget's work about children's symbolic function (1968), I can find a developmental explanation for the 6 and 7 year-old results. According to Piaget, this is a problem related to the child's capacity to represent movement, which is not complete about these ages. The reproduction and anticipation of movement emerge from a child's mental image and this does not occur before child's understanding of these processes, which takes place according to the evolution of child's capacity of representation. However this explanation does not confer with the 12, 13, and 14 year-old results. These children have been expected to be able to reproduce as well as anticipate movement without any difficulty.

In fact, when the qualitative data are taken into account we note that children between 8 and 14 referred quite often to the turns of the figures,

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5 - This result is not a statement that 8 year old children solved more dynamic questions correctly than the 14 year old children. Rather, it refers to a comparison, within groups, between the average of correct responses taking into account the dynamic and static perspectives.

principally when the activities were executed in the everyday and Logo settings. The same was not observed among 6 and 7 year-old children, who referred to this category very little in comparison to the older age-groups. Moreover these younger age-groups continued to used the static reference as much as the older children. This is an evidence that for younger children were concerned with the figure itself as it was in a given moment, i.e., before or after it had moved. In this way, young children were only using the figurative aspect of knowledge.

The contrast between quantitative and qualitative children's results with regards to the angle perspective factor can only be understood by reference to other variables of the research. When Vergnaud (1987A) stressed that every piece of knowledge must be related to situations already mastered by children, we must think of the relationship between children' schemes and the signified of their operational invariants. When Nunes (1992) argued that the meaning of a situation is provided by culture, I can presume that the cultural factor mediated by the representational systems helps to provide meaning for a given situation.

In this study the activities within the dynamic perspective were designed through navigation or rotation contexts. Moreover, whilst in the navigation context children could only make sense of angle through the activities carried out inside the map arena (mediated by the three settings), in the rotation they had more opportunity for understanding because the angle could be experienced from three different arenas (watch, turnstile and arrow) and also in different settings. The results of the middle school children showed that they solved many more activities inserted in the rotation context than in the navigation.

If we look further inside navigation, we realise that some activities did not present meaning for children because of their culture. It was the case when children were asked if they made a 1/4 turn while navigating, and if so, where and how many. No age-group responded correctly in this activity. In fact, for Brazilians the meaning of 1/4 is related to quantified things (orange, apple, chocolate, etc...) where the product can be touched and distributed among people. Even in school, examples of fraction are only given in relation to counting things. Still in navigation, children were asked if they had turned half a turn. Although 'half a turn' is a known value of angle used in the middle school, this expression is not usual in the Brazilian society, at least not in this context. For example, if I was driving a car forward the northwards thinking that I was going South and if I asked someone whether I was going in the right way, the person would say that I was not and I should "turn and thus go ahead". In other words, in the Brazilian popular sense, when I say that I made a turn I mean that I made half a turn in the navigation context. If I say that I made a 'half a turn' a Brazilian would understand this expression to mean I had only turned a little. The same is not true if the context is rotation.

From the above, I conclude that depending on the situation and objects in which the activities were carried out children tended to consider one or another perspective of angle. This finding indicates that children (and adults, as well) have not only one conception of a given content. Moreover, beside the conception children also maintain their spontaneous concepts about the content, which allow them to solve problems not only through their knowledge but also throughout their competence (in different levels). This explains, for instance, why children did not recognise half a turn in the map arena, but could recognise half a turn in the watch arena, or were able to relate half a turn and an angle of 180^0 in the arrow arena. The way children solve a task depends, therefore, on how they understand the problem, the meanings they ascribe to it. This understanding, in turn, arises from the signified attributed for the situation from which children will form their operational invariants, such as the fact that "a quarter" is used to divide concrete things.

7.3.4 THE SETTING FACTOR

The majority (and in some age-groups, all) children presented their best performances in the Logo setting. This result was true for all the age-groups, independent of school level. My first explanation for this result does not relate to any of the theories mentioned, but to the children's motivation to play with Logo, since the computer had never been used before by these children.

However although motivation is a good starting point to form a concept, the motivational factor per se is not sufficient for the appearance of a conception, nor is it strong enough to develop a child's competence in this setting, since the children were asked to solve only a few problems in just one contact with Logo. The motivational factor could, perhaps, be responsible for the child's attention while they were solving the activities. Thus, in my view, children present a better performance in the Logo setting for three main reasons: first, children received feedback. In this sense the Logo turtle played the role of the 'other' who helped children to improve their conception formation, or at least their competence, through the zone of proximal development process. Thus, the turtle feedback, which showed the correct response by drawing it step by step, may induce children to an eventual concept change (Hoyles, in press) or, at least, may help children to form new operational invariants from the observation of turtle behaviour which will later be used in different activities.

The second reason is that the turtle drew the figures and showed the solutions to the tasks in a dynamic way. In this setting children were moving or watching the turtle movement around the screen, and the turtle turns were made in slow motion on the screen. This interaction between child and turtle, enable the children to assume the role of the turtle (the idea of 'body syntonic', Papert, 1980). Moreover it was an opportunity to understand geometrical ideas by solving the activities inductively i.e., using their spontaneous concepts (Kynigos, 1992). Children could observe every single movement made by the turtle.

My last reason is a cultural speculation. The act of 'driving' the turtle was not very different from driving a car in electronic games for the children. In fact, although the children in this sample did not have a computer at home, they had had the opportunity to play video game either in friend's homes or on the hundreds of machines available in video arcades. From this perspective the turtle represented a tool which was already internalised as a symbol of contemporary culture (an object moving on screen which interacts with children by playing with him/her). These three reasons seems to be enough to justify why the Logo turtle is a helpful tool in bringing meaning to a situation.

In contrasts, most of the children, independent of age, presented their poorest performances in the p & p setting. This setting was similar to a school test, i.e., a multiple choice test, to be solved individually, with many questions absent of meaning to the children. I am not the first to show the poor responses of children performance in the p & p setting when compared with their performances in another representational system. Carraher, Carraher, & Schliemann (1985) reported a similar result from their research which compared children's performances in solving oral (from everyday setting) and written (from p & p setting) tests. From this and similar works, Nunes et al (1993) concluded that a child's functional organisation was influenced by the representational systems, mediated by the cultural systems of signs.

Comparing Logo and p & p settings it is possible to note the use of different symbolic systems, body syntonic and written practices, by the same child. The principal differences observed between these two practices were:

1) P & p sets the meaning of the situation aside. The concern in this setting was to find the specific solution which the children knew the teacher (or in the case of this study, the researcher) had in mind a priori; to understand the activity was not the point because it 'came from and goes nowhere'. In contrast, the Logo turtle preserves (or brings) the meaning of the situation: it is the turtle which asks the child, establishing an 'conversation' between the two; independent of child's responses (whether these were correct or not) the turtle continued to interact with the child, showing not only the answer but, most importantly the path it used in order to reach this result;

2) Because of its own features, p & p presents arenas and activities in a static perspective. Even when children were asked to predict the future position of the arrow in the arrow arena, children will only 'see' movement using symbolisation and mental image. Logo follows the opposite perspective. The turtle moves around the screen all the time and the child's representation of the arrow movement will be positively tested later on. At this moment children can assume the turtle position.

3) Because of the absence of meaning plus the close relation between p & p and school tests, children are more likely to use their scientific concepts (even if they are not clear about it in their mind or if they not have any at all). This tendency seem to be lost in Logo and children could feel freer to explore their spontaneous -- or intuitive, in the Abelson sense (1980), or intrinsic in Kynigos work (1988) -- concepts.

More evidence about the differences between a child's performances from setting to setting can be gleaned from the qualitative findings. In fact, I found differences in the children's explanations for the same type of activity, presented in the same arena and context but in different settings. For instance, some children explained their predictions of a quarter of a turn in the arrow arena of the p & p setting differently from their predictions in the arrow arena of the Logo setting. In the first setting, two 10 year-old children divided the square where the arrow was inserted into four equal parts, whilst in the second setting these same children made a link between the arrow movement and the watch hand movement.

The above example demonstrates that children experienced activities in p & p setting from a static perspective; the prediction of half a turn was solved by dividing the square into equal parts (calculation) instead of representing a specific turn to be made by the arrow (rotation). In the Logo setting this same activity had a different representation and the signified of the activity was actually the rotation of the arrow. In this last case, children could bring another representation into the activities namely rotation. In other words, instead of representing a quarter of turn as a fixed part of the square, (without considering the movement of the arrow) children represented this quarter of a turn as a place where the arrow would stop after making a turn, as occurs with the hands of a watch. The turns of the turtle in the previous arena helped children to form a successful theorem-in-action of rotation which could be used in situations which involved a pointer, such as the arrow and watch arenas.

I will present a better example to show that child's symbolic function emerges from his/her representations after dealing with tasks inserted in semantic situations. I found that some children used the same explanation for different activities which were explored in different arenas, but in the same context and setting. This happened in the children's prediction of half a turn in a watch arena and one and half turns in the arrow arena, both in the Logo setting. For these children both the arrow and the minute hand on the watch would stop on the bottom because that was number 6 which means half. It seems that children who used this type of representation were taking their first steps toward concept formation. However, up to this point the cultural meaning for '6' had been strongly represented by having a fixed place (bottom of the watch) and representing a fixed quantity outside the watch. From this viewpoint, depending on how elaborate their level of representation is, plus how much they are able to symbolise from the cultural sign systems, plus, of course, the knowledge they have about the content, children will be able to generalise schema, construct theorems-in-action and form conceptions. The cultural meaning of the number 6 within Brazilian society is, therefore, a good example of the extent that a symbol from a particular culture provides meaning for the situation, influencing the functional organisation of children's activities (Nunes, 1992)

Moreover, taking into account only the difference between the children's explanation with regards to the settings, the dynamic perspective was less cited in the p & p setting than in the other two (except for 6 year-old children); the shape of the figures attracted the children attention much more in the everyday setting; whilst the length of the figures was far more frequently used to solve activities in the p & p setting than in the everyday or Logo settings; finally the setting in which the movement of the figures was most often referred to was Logo. Shape in the everyday setting, length in the p & p setting, and turn in the Logo setting were the parts of the situation which were selected by children at different moments in order to represent a feature of the angle. All of them represented invariants which emerged from the signified applied by children for the situations experienced in different representational systems (settings). Moreover, the first two invariants, shape and length reflect a child's static perspective, whilst the third invariant shows turn (dynamic perspective) as representing an important feature to be retained from the situation. Therefore the above report shows that an operational invariant can change from one setting to another because it is not yet a conception.

7.3.5 THE CONTEXT FACTOR

The great majority of the children presented their best performances when the activity was part of the rotation context. In contrast, a large number of children could not solve activities in the navigation context. The first fact to be taken into account from this result is the mathematical properties involved in the contexts: whilst rotation involves turning around the same point (same axis), navigation presupposes translations and rotations and rotations occur in different axis. Therefore, in the mathematical sense, the context of navigation is more complex than rotation, since rotation is one of the steps involved in navigation, i.e., children need to know (or at least, to carry out) rotation in tasks involving navigation, but the contrary is not true. From the psychological perspective it is also possible to note differences between the two contexts which were probably influencing the children's experience.

Van Hiele (1986) has stated that a learning process takes place within contexts which allow children to experience a given content; context sets the properties of this content. This study complements this statement and places context in a broader perspective. In this way a learning process will take place depending on the context which 'set' the properties (invariants) of a given content and depending on the child's capacity of representation. According to the result of this combination a child's will or will not be able to deal with the meaning which underlies the activities of this context.

In the case of the navigation context, the children were asked to recognise specific values of turns that they made whilst they were navigating in the map arena. Such activities were beyond the capacity of elementary school children, who had never learnt how to measure an angle. Moreover, because of the absence of formal learning, children from this level of school were not successful in activities in which they were asked to compare the angles of figures (comparison context), whether in the everyday, p & p, or Logo setting. Comparing children's articulations for the activities carried out in the navigation and comparison contexts, I noted that while in the navigation activities they could escape the question by simply not making the turn in question, in the comparison they had to choose one of the figures as the answer. This forced children to select any information from the figures in order to represent the situation and form the invariants from it. Nevertheless, elementary school children could simply take into account the perceptual aspects of the figure, i.e., they could solve the activities through the figurative aspects of knowledge. Among the middle school children, I noted that they did not connect a 1/4 of turn with a turn of 90^0 and also had difficulty in recognising half a turn in the map arena, for the reason discussed in the previous sections. However, they could solve many activities in the comparison context, and they presented a very good overall performance in solving activities in the rotation context.

Half a turn is a good example for showing that context sets the properties of a given content depending on the meaning of the situation for children: whilst middle school children did not recognise the half a turn they made in the map arena, most of them were able to correctly recognise and predict half a turn in the watch, turnstile and arrow arenas.

7.3.6 THE ARENA FACTOR

In spite of the difference in relation to the number of mistakes, made by middle to elementary school children in favour of the former, all the age-groups performed better in activities in the watch, arrow and turnstile arenas (in this order). The middle school children's poorest performance was in activities carried out inside the map arena, whilst we find the elementary school children's worst performances were in activities in the 4 angles arena.

How can these differences be explained? Concerning the children's difficulty to solve problems within the map arena, I believe I have little to add to the points made earlier with regards to the influence of context and of the child's symbolic function. The poor performance of the elementary school children in those activities realised inside the 4 angles arena was to some extent an expected finding, which can be understood by reference to the role of school. Children from elementary school had not yet learnt how to measure an angle, thus they clearly could not apply formal procedures in these activities. At this point one might ask: lower case I anticipated that there was no way that younger children could correctly carried out these sort of activities, why did I include them in my study? The answer is simple: I wondered how these children would deal with the sort of situation which they had not yet (formally) learnt: could they approach these activities using spontaneous conceptions? Which other metric could they use? And also, how different would their answers be from those of the older children -- would they present misconceptions? if so, which? Would these misconceptions still persist after school teaching, becoming a false theorems-in-action?

In the qualitative analysis I found that younger children actually presented three main misconceptions: they were influenced by the shape, length and the position (inclination) of the figures. This is not a new finding; many studies have referred to one or more of these invariants to demonstrate children's misconceptions, (Close, 1982; APU, 1987; Magina, 1988). However none of them used a sample which took in children from their first year at primary school until the end of secondary school. Moreover these studies examined children's results in the light of scientific conception, that is, from the school perspective and this was not the aim of the present study. Continuing with the discussion about children's misconception, we noted that length was the one which remained to be present in children's mind for longest, forming a false theorem-in-action, whilst shape tended to disappear and the meaning for inclination, in terms of symbolic function expressed by language changed from one age to the next until being used first as a theorem-in-action (when the way in which the figures were displayed, or the turn stopped, was taken into account) , then as a theorem-in-conception (when children referred, by gesture or by using words like 'position', to the inclination of one figure in relation to the other) and finally as invariant of the concept of angle (when the parallelism of the sides of the figures was explicitly used to justify whether the angle was similar angle or not).

Turning now to discuss the children's best performances in relation to the arenas, I would like to start by looking at the activities from the point of view of the watch arena, in which the children had so far shown their best performance. The watch seems to have provided a signifier from which children could make sense of rotation as it is transacted by the hands on the watch face. Moreover, from a mathematical viewpoint, the numbers on the watch face allowed children to exploit a metric. Thus, the watch arena allowed children to work with more than one representation, number and angle, at the same time. In addition, numbers provided support, in terms of both metric and symbolic function, since the number is a powerful cultural system sign, known to the children before starting school.

With regards to the turnstile/arrow arena group, children were asked to predict the same value of turn in both turnstile and arrow arenas. However whilst the turnstile arena was only carried out in the everyday setting, the arrow arena was presented in the p & p and Logo settings. Comparing all the children's age-groups results in activities in the arrow arena we note that they performed better when the activities were carried out in the Logo setting -- a setting from which children received feedback after each prediction. This is not a new result, since I have previously discussed the probability that the interaction between children and Logo could be working as a factor for the enlargement of their zone of proximal development. Comparing middle school children's turnstile and Logo arrow results I found no difference in performance between the two activities. However, there was a difference in favour of the turnstile when the elementary school children's performances are considered. What was causing the difference? In my point of view, the difference probably consisted of the strategies children were using in order to solve the activities in each arena. In the arrow arena children were probably thinking only in terms of rotation, even though they sometimes saw a relationship between the turns of the arrow and the turns of the watch hand. In the turnstile, children thought not only in terms of turns but also in terms of counting. That is, it seems that in activities with the turnstile children were working with more than one invariant for the same situation, as in watch arena The difference lay in the fact that while there were actual numerals (the referent) in the watch arena, in the turnstile they were counting actions (numbers were a representation). Therefore, it seems that for children these two representations of numbers could not come together. Future research could be undertaken in order to study this point more carefully.

7.3.6 THE FACTOR OF AN ACTION VERSUS A RECOGNITION

From 8 to 14 years of age all the children performed better when they were asked to act in an activity rather than to recognise figures and turns. From this a question is posed: why do children produce better results in activities in which they were, in most cases, asked to anticipate an action than in activities where they were basically asked to use their perception and memory (elements of the figurative aspect of knowledge, in Piaget's theory)? To investigate this question we have to look at it together with the other variables involved in the present research. With regards to the arenas, children were asked to carry out an action in the watch, arrow, turnstile and stick game arenas. It was in activities in three out of these four arenas that children presented their best performances. Moreover, the majority of the action activities were exploring the dynamic perspective of angle in the rotation context (a context in which children from all the age-groups presented their best performances). On the other hand, activities in which children were required to use recognition were carried out in the map, 2 angles, 4 angles arenas (from which children showed their poorest performances) plus some activities inserted in the stick game and watch arenas. In addition, in the p & p setting (the setting in which most of the sample presented their poorest performance) children were required to recognise figures rather than to act upon them.

The results therefore showed that children were more successful when they were acting than when they were recognising figures. In my view, this result shows that when children are recognising figures, in a static perspective, they are more likely to simply mobilise their figurative knowledge. On the other hand, activities involving an action seemed to have a stronger effect on the child's operative aspects of knowledge, relying on perception rather than mathematical analysis

7.3.7 THE CULTURAL FACTOR

Since I have been pointing to the relevance of having activities inserted in semantic situations, from where the signified for a content embedded in the activity will be constructed, allowing the emergence of a child's operational invariants, I would like to conclude this list of factors emphasising the cultural factors in a child's understanding of angle.

It was possible to note the influence of culture over children's conceptions in many activities and arenas. For the purpose of this thesis, which is not an anthropological study, I selected only one example which I believe to be enough to demonstrate how strong cultural factors can be as an element of influence over child behaviour. The example I am going to discuss refers to the meaning of the number 6 for Brazilian society. This influenced the children's responses not only in the everyday and Logo watch arenas but also in the everyday turnstile arena and in the p & p and Logo arrow arenas.

I am going to open the discussion via the watch arena, since it was from this that the cultural factor became evident in my study. The analogue watch was included as an arena for a series of tasks in this research largely because of its potential to carry semantic sense in everyday life. Moreover, the notion of measuring angles by rotation is an integral part of the way time can be represented on a watch. Furthermore, a watch presents a very precise metric provided by the numbers on is face. This means that a duration of time can be assessed by angle size and measured by rotation; or by counting through the numbers on the watch face. The efficacy of the latter approach would of course depend on some appreciation of the semiotic function of the 12 numbers on the clock face: that is, the significance of the number 60 in measuring time and the fact that the distance from one number to the next represents 5 minutes.

The findings showed that the number 6 assumed different representations depending on the children's symbolic function. For some children number 6 was the place for the 1/2 hour, for others it was the amount of numbers, and, finally, number 6 was related to group of 6 numbers because 6 is half of 12. The way in which children from different ages represented number 6 showed the importance of the symbolic formation. In this way, I identified three stages in children's behaviour and articulation: in the first, the number 6 was related to the 1/2 hour but it has a fixed place. Half an hour should also have a fixed position on the watch. This is a representation related to the figurative aspect of the conception. Children here pay attention only to the perceptual mnemonic aspects of the situation. This idea was strong enough to be transferred from everyday to the Logo setting, which showed different numbers in the watch as well as the arrow and turnstile arenas. In the p & p arrow arena, for instance, when children of this stage were asked to predict where the arrow would be after one and half turns they simply pointed to the bottom of the square because one turn was to be in the same place and half was on the bottom. Therefore, children were forming schema and finding out invariants, which seem to be strongly influenced by their culture. In the second stage children realised that the number 6 was actually related to the 1/2 hour but that in the case of the watch situation it meant jumping 6 numbers. However, in this stage children seemed not to understand why they have to jump 6 numbers and this led them to be not unsure of whether they had to consider the starting-point or not. It is possible to observe that at this stage the children were acting and making some relations and propositions, but without consistency. Finally the children created a relationship between the number 6 and the 1/2 hour in the sense of thinking about the 1/2 hour as a single 'leap' or group of 6 numbers and not as the number 6. At that moment the referent was not the number 6 but six numbers; the signifier for six continued to be half but in sense of middle; and the signified changed from a static view to a dynamic perspective, where the rotation of the hand became the main focus. Some children at this stage correctly explained their prediction in both arrow and turnstile arenas making an analogy between these and the watch hand. We are faced here with a true child's theorem-in-conception born from a strong cultural tool.

From the findings presented in this study, I conclude that there is no sense in speaking about a child's acquisition of the angle concept from a single factor. In fact, from my interpretation of the findings leads me to suggest that: (1) various factors underlie the children's conception of angle; (2) this conception is a result of the interaction between both spontaneous and scientific conceptions; (3) the starting-point of a spontaneous conception seems to be the figurative aspects of knowledge which do not require an internalised action, but from the signified given by children to phenomena experienced from semantic situations, information will be selected in order to form operational invariants; (4) from these processes (operation and transformation) children internalise symbols by mental image, from which the operative aspects of knowledge emerge forming scientific conceptions; (5) all of these are related to each other in an intricate "spider's web".

7.4 REFLECTIONS ON THE STUDY AND SUGGESTIONS FOR FUTURE RESEARCH

In this section I would like to make some remarks which are the result of reflections on my journey through this study. Such reflections have led me to think of future work which may contribute to shedding some more light on our comprehension of the factors which influence the understanding of angle. For this reason my reflections will be presented together with suggestions for further research.

I started this study with many questions in mind about how children understand angle -- this was the focus of my master dissertation. From the literature I found that works on this theme have been approached from one of the two perspectives, either static or dynamic. And depending on which perspective is assumed, research has been carried out either in paper & pencil or in the computer setting (mainly using Logo) respectively. Some of the latter have also considered to applying a static test, using paper & pencil setting, in a control group or in the same experimental group. However, no research seems to have considered both setting as experimental and, as far as I know, no research has included everyday life as another possibility of setting which would compose, together with the other two, a rich environment set to explore the child's understanding of angle. Thus, the starting-point I had in mind for this study took in both dynamic and static perspectives, as well as the everyday, paper & pencil, and Logo settings.

After the analysis of the data, specific findings called my attention to some important factors which were influencing children's understanding and which deserve to be studied more carefully in future research.

The first finding was the cultural factor. In fact, although the everyday setting was included in order to explore the children's spontaneous concept, this was experimental research and the everyday setting was less pervasive than it would probably be in anthropological or cross-cultural research. Therefore, arena such as the watch could give much more information if explored in depth. For example, this arena could be included not only in the everyday and Logo settings but also in the paper & pencil setting. In the everyday setting the watch was presented to the child in three different ways: without numbers, with traditional watch face numbers, and in the Logo, where the watch face numbers followed the angular metric. I wonder how children would cope with these three metrics from setting to setting. I also think that it would be productive to have activities exploring, for instance, a quarter of hour (an uncommon expression in Brazilian society).

A second surprising finding which was a consequence of the cultural factor was the effect, sometimes positive, sometimes negative, that an arena full of semantic meaning could bring into a similar arena which was devoid of lacking cultural meaning or which presented a different meaning. In the case of this study, many children referred to the watch metric whilst they were solving problems in the arrow or turnstile arenas. The common factor among the arenas consisted of asking children to turn a pointer. I believe that a research design which combined cultural and non-cultural arenas involving different metrics, would certainly make a great contribution to this debate.

My last reflection concerns the dynamic and static perspectives. The finding demonstrated that the older children, the more they referred to the dynamic perspective. It was also demonstrated that the paper & pencil setting is the hardest one for children of all ages. Paper & pencil was the setting which, unlike the other two, basically explored static activities. This happened because

I was interested in exploring the relationship between paper & pencil and school, as I did between the spontaneous concept and the everyday setting, and also between dynamic and the Logo setting. However, thinking only in terms of dynamic and static perspectives, I realised that the arenas could be better balanced intra and inter settings. In this way the everyday setting should involve other static arenas besides the stick game, the one in which children did not make association with their daily life. And the paper & pencil setting should involve dynamic arenas other than the arrow in which children can make associations with their everyday life. For future research I would propose a design which included, in each setting, a balanced number of arenas, in terms of dynamic and static as well as in terms of cultural and non-cultural meaning.

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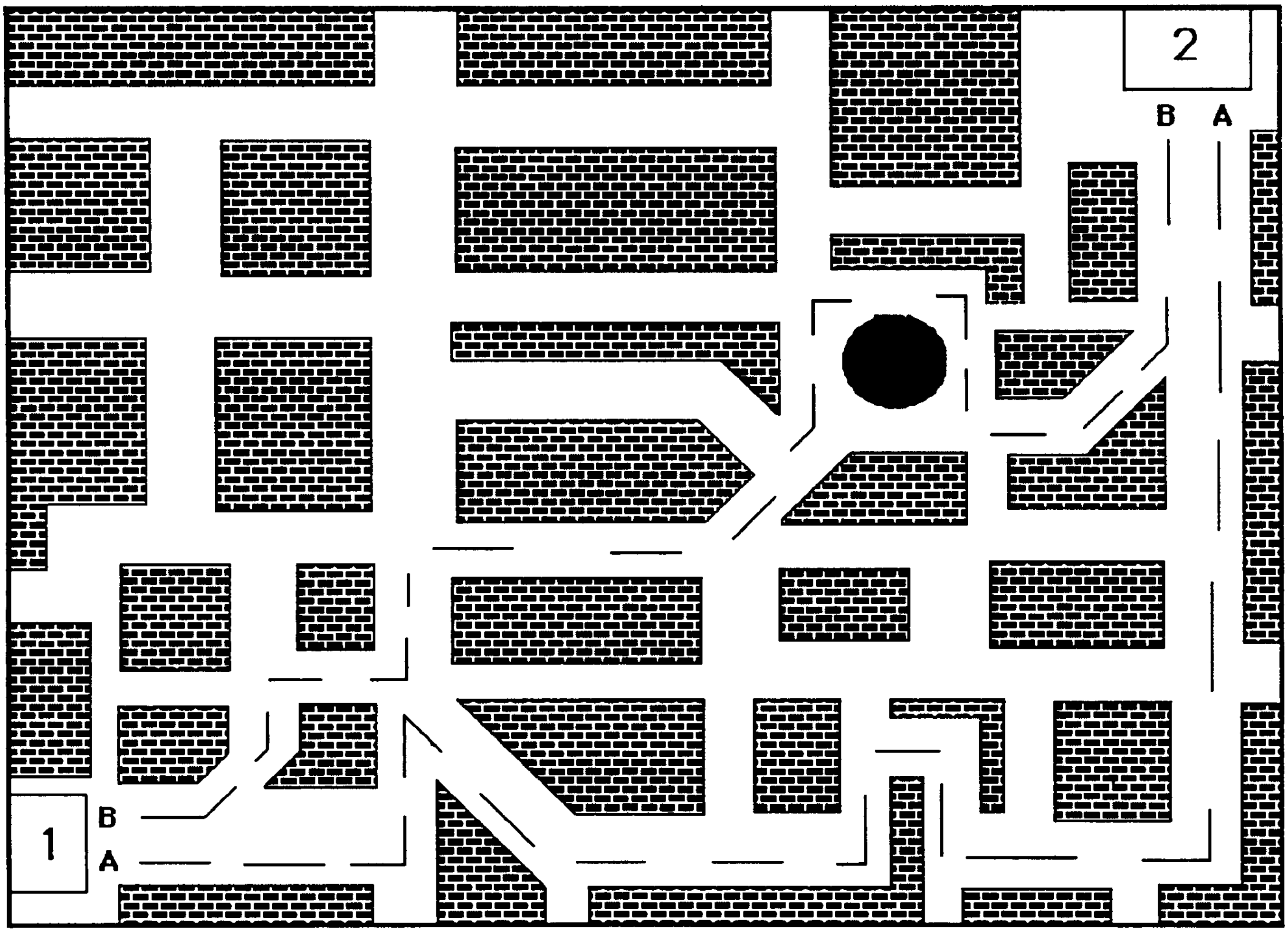
PAPEL & LAPIS

DATA: ________________
NOME: ____
IDADE: ___________ SÉRIE: ________________

1. O MAPA

O mapa abaixo indica que existem dois caminhos para se ir do ponto 1 até o ponto 2, que são a rota **A** e a rota **B**.



a) Entre a rota **A** e **B**, qual a que você escolheria para ir de **1** até **2**?
RESP: ____________________
POR QUE? ____
____
____

No mapa, ligue as setas de cada rota e enumere cada um dos seus giros.
b) Quantos giros tem a rota **A** e a rota **B**?

RESP: Rota **A** = ________ giros
Rota **B** = ________ giros

c) Qual delas faz menos giros?
RESP:__________________________

D) Você girou 90^0 em algum ponto do rota **A**?
RESP: ____________
SE SIM Escreva o número (ou números) do giro (ou giros) onde você girou 90^0?
RESP: _______________________________

E) Você girou 90^0 em algum ponto do rota **B**?
RESP: ____________
SE SIM Escreva o número (ou números) do giro (ou giros) onde você girou 90^0?
RESP: _______________________________

F) Qual foi o maior giro que você fez na rota **A**?
RESP: ____________
Explique sua resp.____

G) Qual foi o maior giro que você fez na rota **B**?
RESP: ____________
Explique sua resp.____

H) Comparando o maior giro da rota **A** e o maior da rota **B**, qual deles é o maior?
RESP: ____________
Explique sua resp.____

I) Você girou um quarto (1/4) de volta em alguma giro da rota **A**?
RESP:_____________
Se **SIM** Escreva o número (ou números) do giro (ou giros) onde você girou um quarto de volta
RESP:_____________
COMO VOCE SABE?____

J) Você girou um quarto (1/4) de volta em alguma giro da rota **B**?
RESP:_____________
Se **SIM** Escreva o número (números) do giro (giros) onde você girou um quarto de volta
RESP:_____________

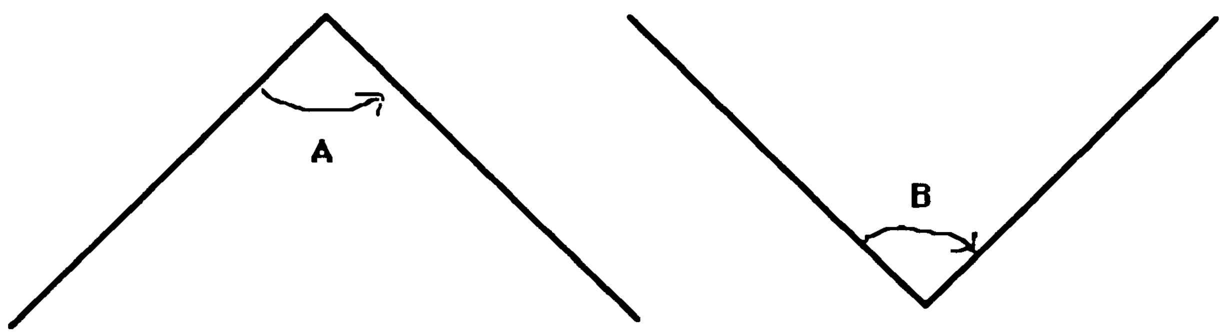
COMO VOCE SABE?____
____
____

L) Voce girou meia volta em algum giro da rota **A**?
RESP:_____________
Se **SIM** Escreva o número (números) do giro (giros) onde você girou meia volta
RESP:_____________
COMO VOCE SABE?____
____
____

M) Voce girou meia volta em algum giroda rota **B**?
RESP:_____________
Se **SIM** Escreva o número (números) do giro (giros) onde você girou meia volta
RESP:_____________
COMO VOCE SABE?____
____
____

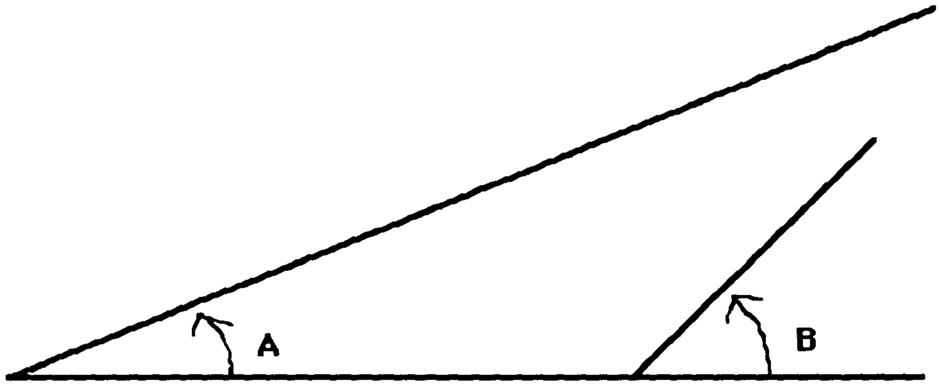
2. PAR DE ANGULOS

Compare os pares de ângulos e marque com um "x" (x) a resposta correta



a) Compare os ângulos **A** e **B**
O ângulo **A** é igual ao ângulo **B** (__)
O ângulo **A** é maior que o ângulo **B** (__)
O ângulo **B** é maior que o ângulo **A** (__)
Voce **não pode** dizer nada (__)

EXPLIQUE COMO VOCE CHEGOU A EST RESPOSTA____________________
____
____

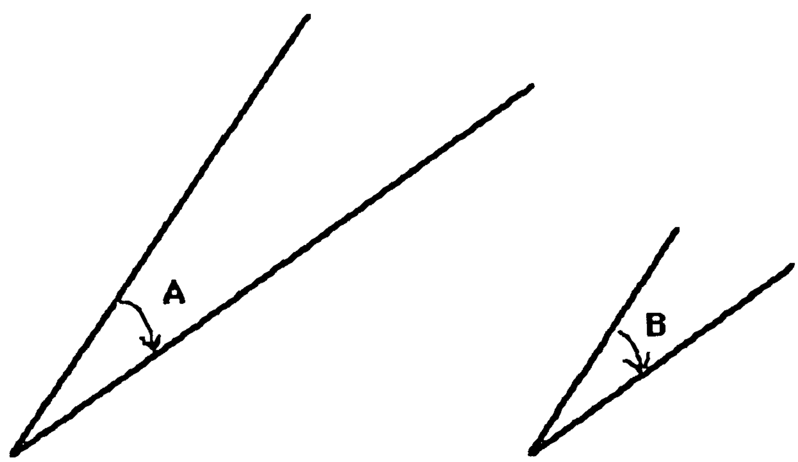


b) Compare os ângulos **A** e **B**

O ângulo **A** é igual ao ângulo **B** (__) O ângulo **A** é maior que o ângulo **B** (__)

O ângulo **B** é maior que o ângulo **A** (__) Você **não pode** dizer nada (__)

EXPLIQUE COMO VOCE CHEGOU A ESTA RESPOSTA__________________

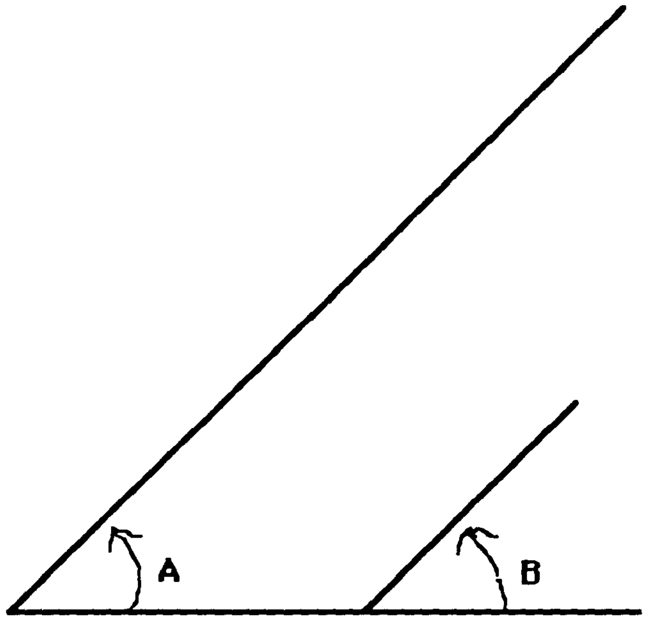


c) Compare os ângulos **A** e **B**

O ângulo **A** é igual ao ângulo **B** (__) O ângulo **A** é maior que o ângulo **B** (__)

O ângulo **B** é maior que o ângulo **A** (__) Você **não pode** dizer nada (__)

EXPLIQUE COMO VOCE CHEGOU A ESTA RESPOSTA__________________

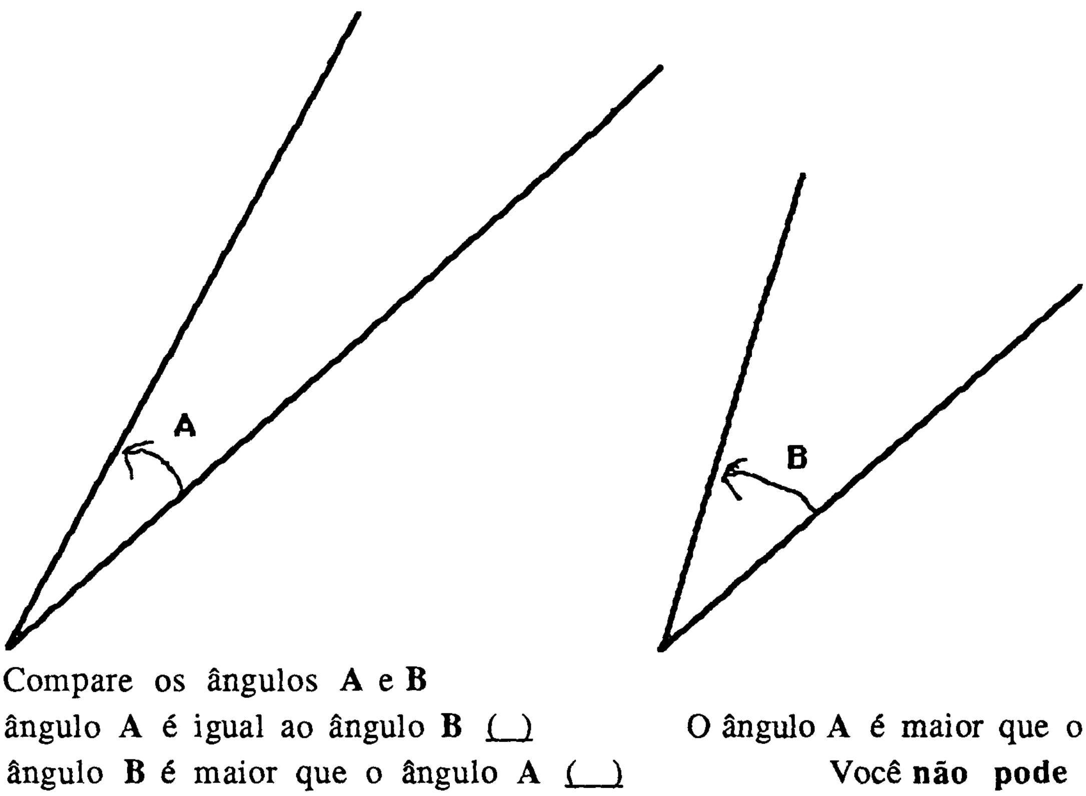


d) Compare os ângulos **A** e **B**

O ângulo **A** é igual ao ângulo **B** (__) O ângulo **A** é maior que o ângulo **B** (__)

O ângulo **B** é maior que o ângulo **A** (__) Você **não pode** dizer nada (__)

EXPLIQUE COMO VOCE CHEGOU A ESTA RESPOSTA________________



e) Compare os ângulos **A** e **B**

O ângulo **A** é igual ao ângulo **B** (__) O ângulo **A** é maior que o ângulo **B** (__)

O ângulo **B** é maior que o ângulo **A** (__) Você **não pode** dizer nada (__)

EXPLIQUE COMO VOCE CHEGOU A ESTA RESPOSTA________________

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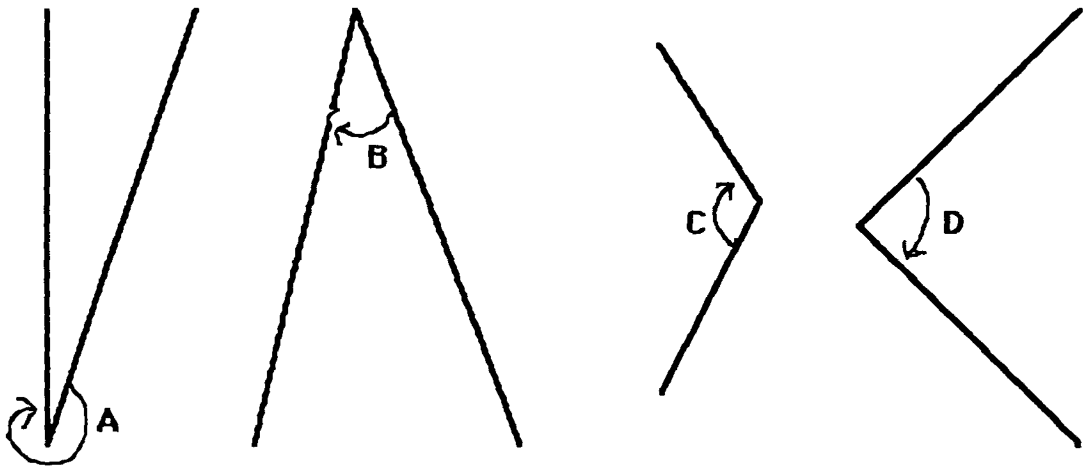
3. QUATRO ANGULOS

Observe atentamente os ângulos **A**,**B**,**C** e **D**.

a)



Compare os ângulos acima A, B, C e D e diga qual deles é o MENOR ANGULO?

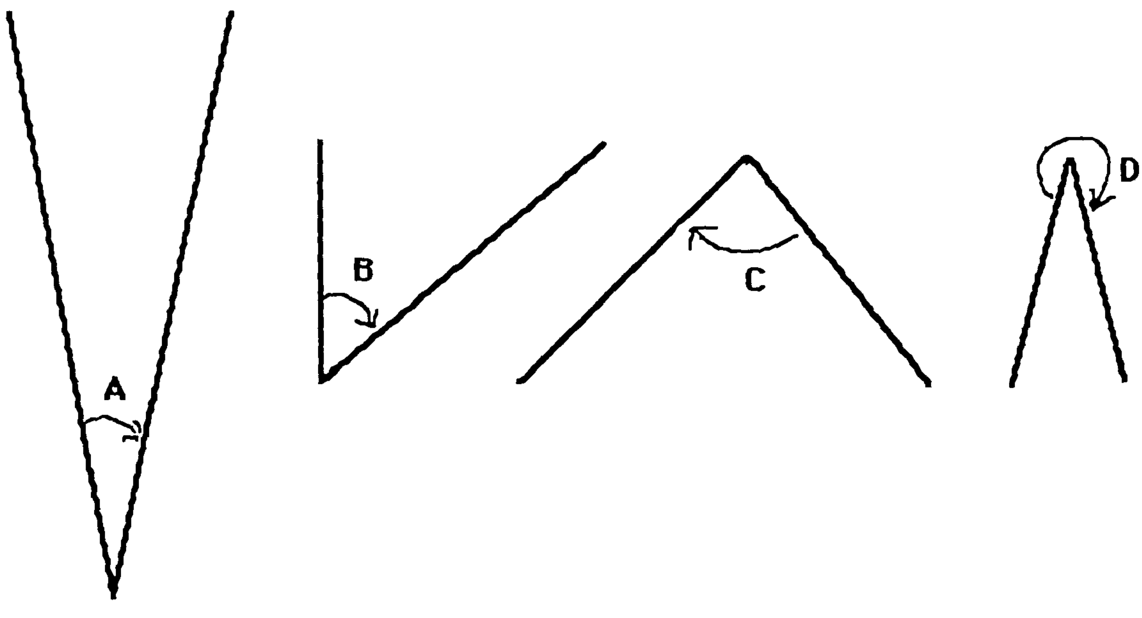
RESP. ___________________

Como você sabe que ele é o MENOR?______________________________

____

____

b)



Compare os ângulos acima A, B, C e D e dige qual deles é o MAIOR ANGULO?

RESP. ___________________

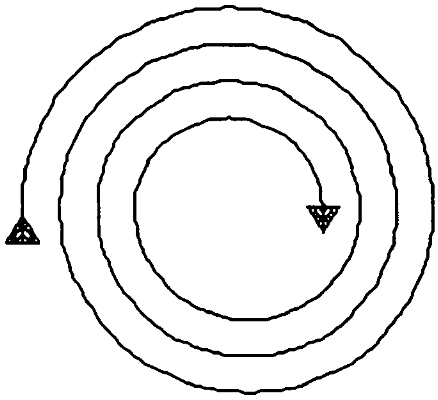
Como você sabe que ele é o MAIOR?______________________________

____

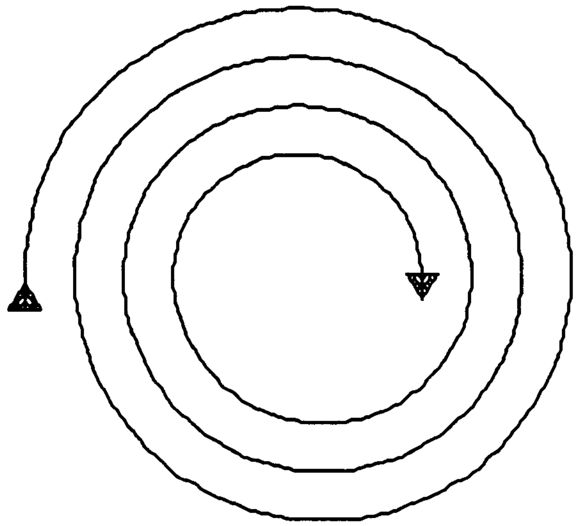
____

4. ASPIRAL

Compare os pares de aspiral e marque com um "x" (x) a resposta corr



A



B

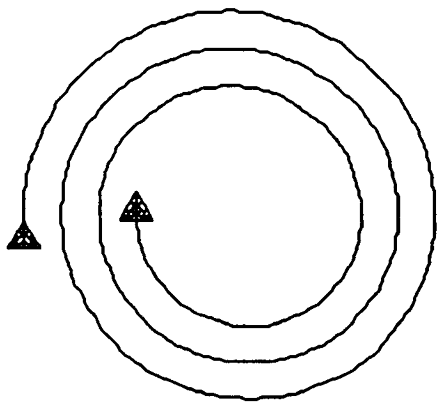
A) Compare os giros no aspiral **A** e no aspiral **B**

A e **B** tem a mesma quantidade de giros (__):

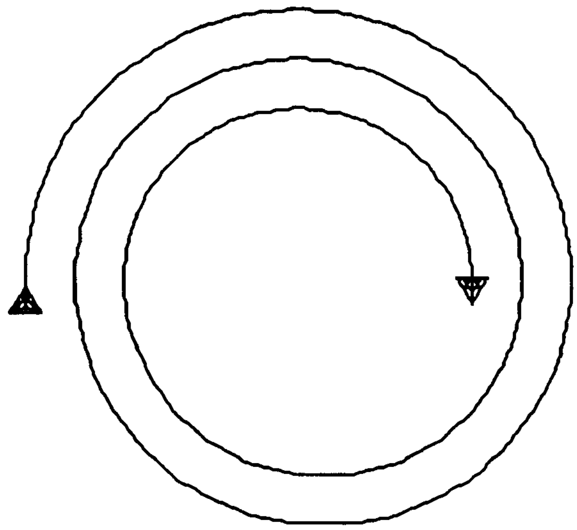
A tem mais giros que **B** (__)

B tem mais giros que **A** (__)

Você **não pode** dizer nada (__)



A



B

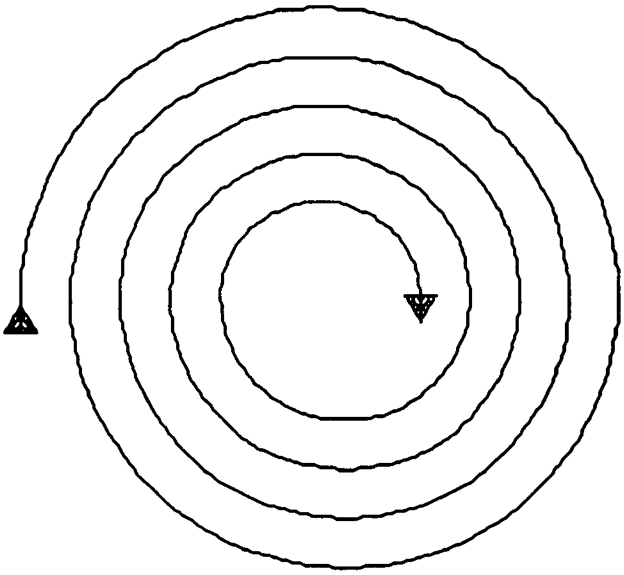
B) Compare os giros no aspiral **A** e no aspiral **B**

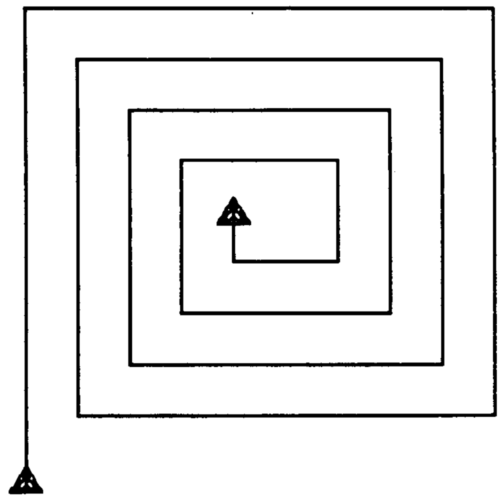
A e **B** tem a mesma quantidade de giros (__):

A tem mais giros que **B** (__)

B tem mais giros que **A** (__)

Você **não pode** dizer nada (__)





A **B**

A) Compare os giros no aspiral **A** e no aspiral **B**

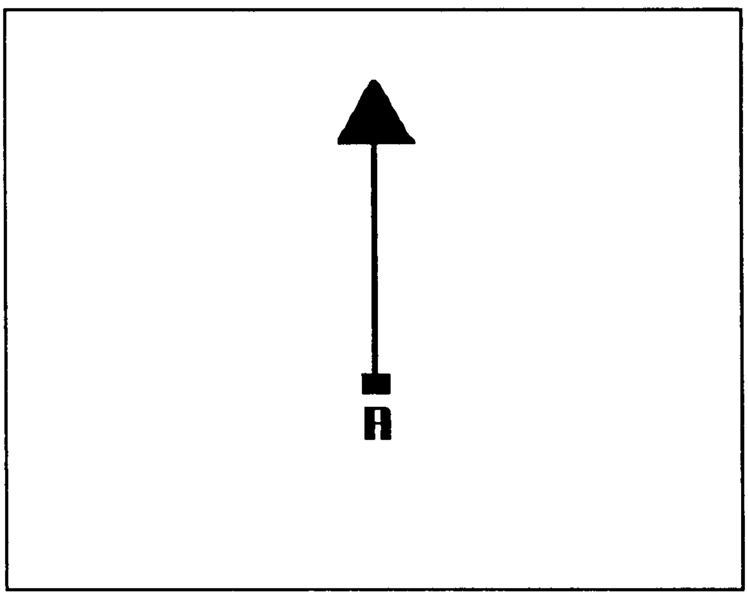
A e **B** tem a mesma quantidade de giros (__) **A** tem mais giros que **B** (__)

B tem mais giros que **A** (__) Você **não pode** dizer nada (__)

5. MOVIMENTO DA SETA

O desenho abaixo mostra uma seta presa num quadrado. Desenhe a posicao que ela ficara depois que ela girar:

A)



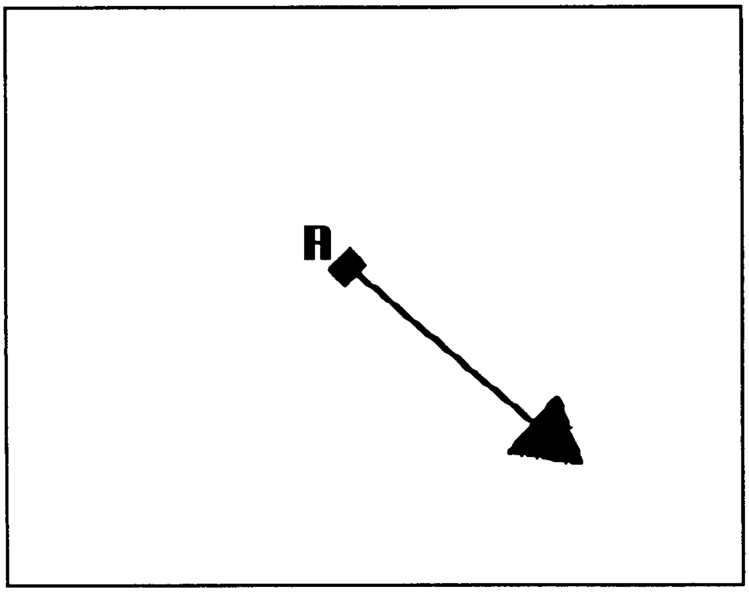
Uma seta está fixa no ponto **A** como mostra o desenho acima.
Desenhe dentro do quadro a posição que a seta ficará depois dela ter girado 90^0 no sentido horário - isto é, um quarto de volta para a direita.

Explique como você chegouu a esta resposta ________________________

____

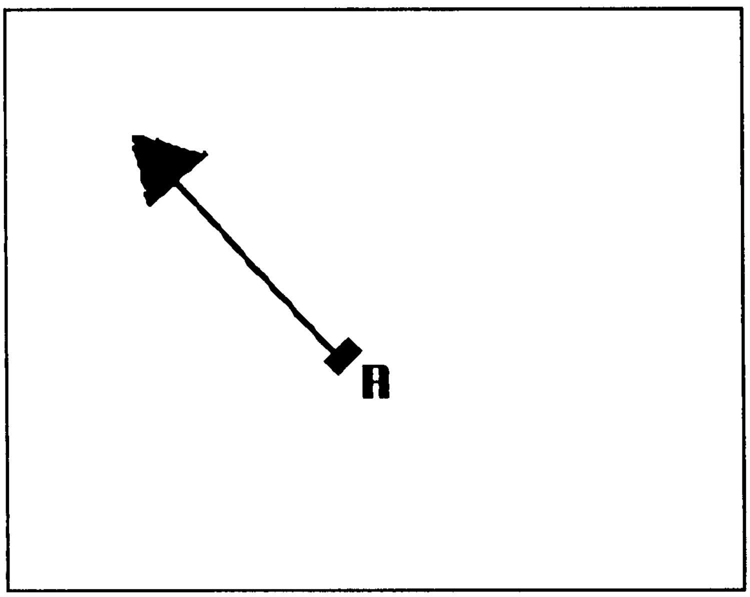
____

B)



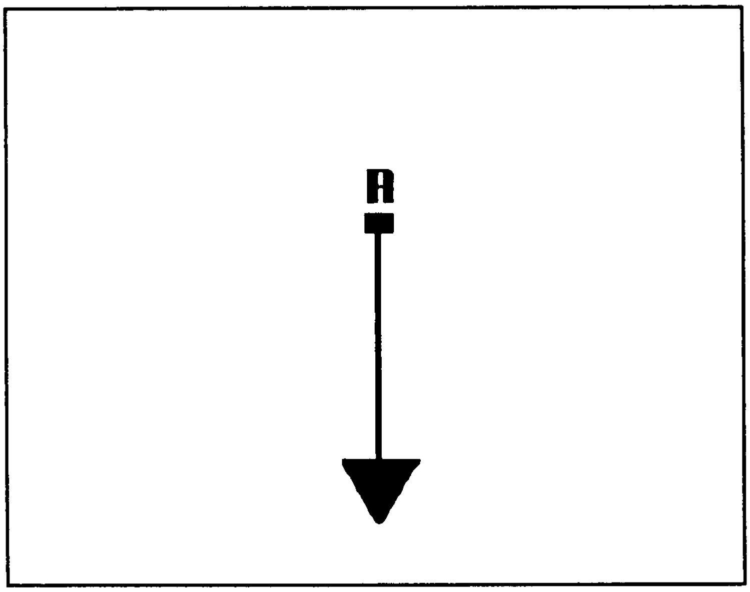
Uma seta está fixa no ponto A como mostra o desenho acima.
Desenhe dentro do quadro a posição que a seta ficará depois dela ter girado 90^0 no sentido anti-horário - isto é, um quarto de volta para a esquerda
Explique como você chegouu a esta resposta ______________________
____
____

C)



Uma seta está fixa no ponto A como mostra o desenho acima.
Desenhe dentro do quadro a posição que a seta ficará depois dela ter girado 180^0 no sentido horário - isto é, meia volta para a direita
Explique como você chegouu a esta resposta ______________________
____
____

D)

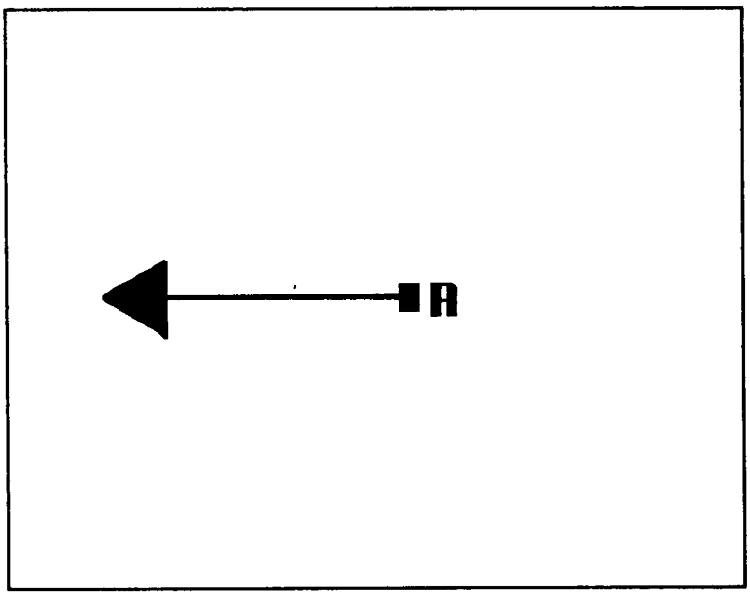


Uma seta está fixa no ponto A como mostra o desenho acima.
Desenhe dentro do quadro a posição que a seta ficará depois dela ter girado 720^0 no sentido anti-horário - isto é, duas voltas completas para a esquerda
Explique como você chegouu a esta resposta ______________________

____

____

E)



Uma seta está fixa no ponto A como mostra o desenho acima. Desenhe dentro do quadro a posição que a seta ficará depois dela ter girado 540^0 no sentido horário - isto é, uma volta e meia para a direita
Explique como você chegouu a esta resposta ______________________

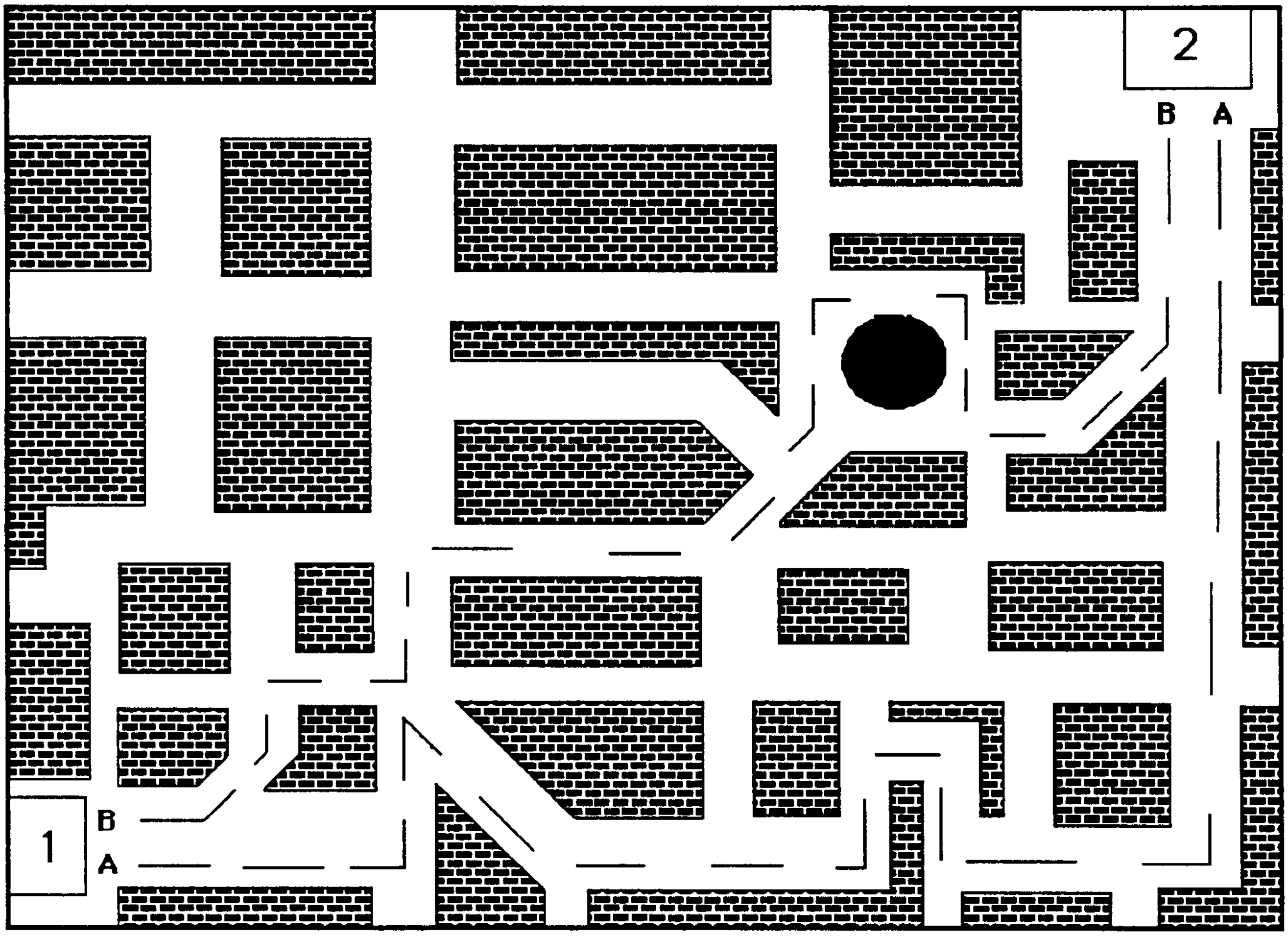
____

____

PAPER & PENCIL

DATE: ______________
NAME:____
AGE: ____________ YEARS OF SCHOOLING:_________________

1 THE MAP
On the map below there are two ways indicated to go from **1** to **2**, route **A** and route **B**.



a) Which of route **A** or route **B** would you choose to go from **1** to **2**?
ANSWER: ___________________
Why?:____
____

On the map, link up the arrows along each route and number each turn in order.
b) How many turns there are in route **A** and route **B**?
ANSWER: Route **A** has________turns
Route **B** has________turns

c) Which of route **A** and route **B** has fewer turns?
ANSWER:__________________________

d) Did you turn 90^0 at any point along route **A**?
ANSWER: ___________

IF YES write down the number (or numbers) of the 90^0 turn (or turns)?
ANSWER: ____

e) Did you turn 90^0 at any point along route **B**?
ANSWER: ___________

IF YES write down the number (or numbers) of the 90^0 turn (or turns)?
ANSWER: ____

f) Write down the number of the largest turn you made along route **A**?
ANSWER: _____________

Explain your answer: ____
____

g) Write down the number of the largest turn you made along route B?
ANSWER: _____________

Explain your answer? ____
____

h) Compare your answer to **f)**, the largest turn along route **A** with your answer to **g)**, the largest turn along route **B**, which of them is largest?
ANSWER: _____________
WHY?____
____

i)Did you turn a quarter of complete turn at any point along route **A**?
ANSWER:__________
Write down the number (or numbers) of the quarter turn (or turns)?____________________________

j) Did you turn a quarter of complete turn at any point along route **B**?

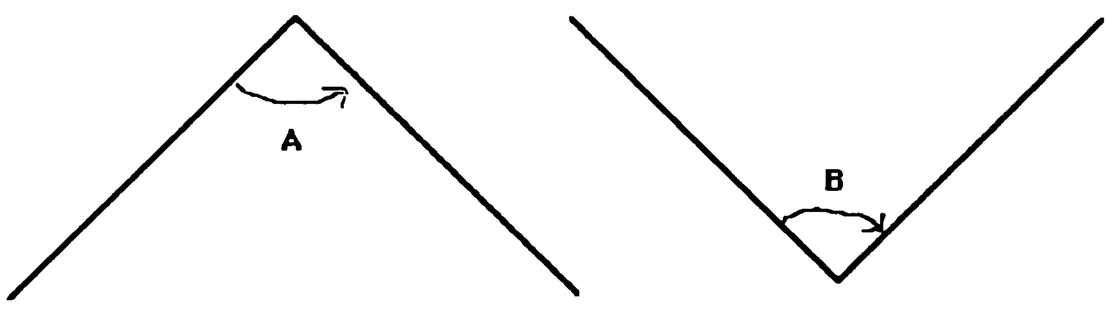
ANSWER:__________
Write down the number (or numbers) of the quarter turn (or turns)?____________________________

k) Did you turn a half of complete turn at any point along route **A**?
ANSWER:__________
Write down the number (or numbers) of the half turn (or turns)? ______________________________

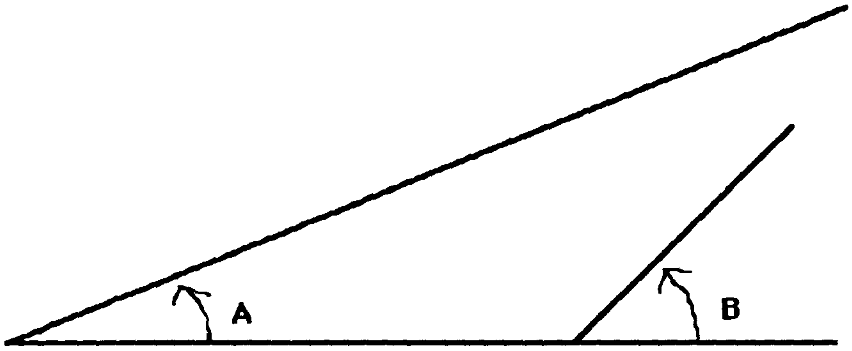
l) Did you turn a half of complete turn at any point along route **B**?
ANSWER:__________
Write down the number (or numbers) of the half turn (or turns)? ______________________________

2. TWO ANGLES

In each of the following questions, compare each angle in a pair and tick (√) the correct answer



a) Compare angle **A** and **B**
A is the same as **B** ( ):   **A** is bigger than **B** ( )
B is bigger than **A** ( )   You **cannot** tell ( )
EXPLAIN WHY YOU CAME TO YOUR ANSWER ____

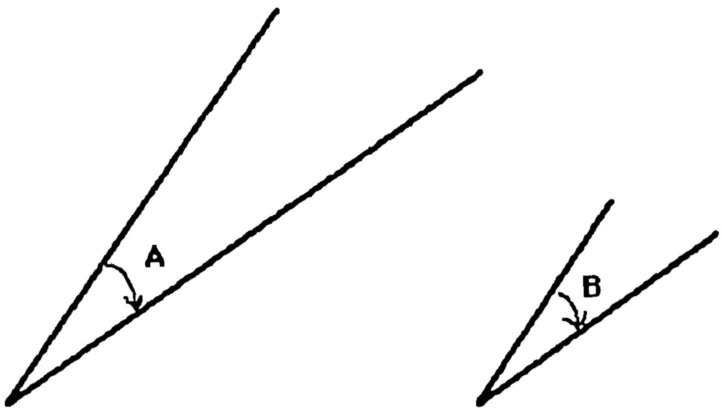


b) Compare angle **A** and **B**

A is the same as **B** (__): **A** is bigger than **B** (__)

B is bigger than **A** (__) You **cannot** tell (__)

EXPLAIN WHY YOU CAME TO YOUR ANSWER ____

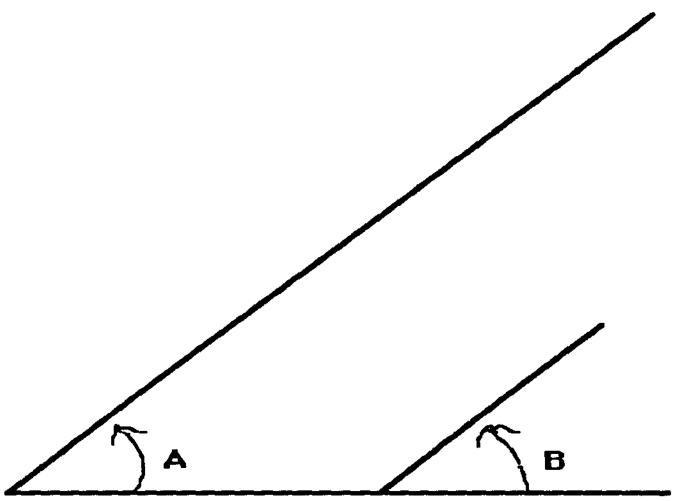


c) Compare angle **A** and **B**

A is the same as **B** (__): **A** is bigger than **B** (__)

B is bigger than **A** (__) You **cannot** tell (__)

EXPLAIN WHY YOU CAME TO YOUR ANSWER ____

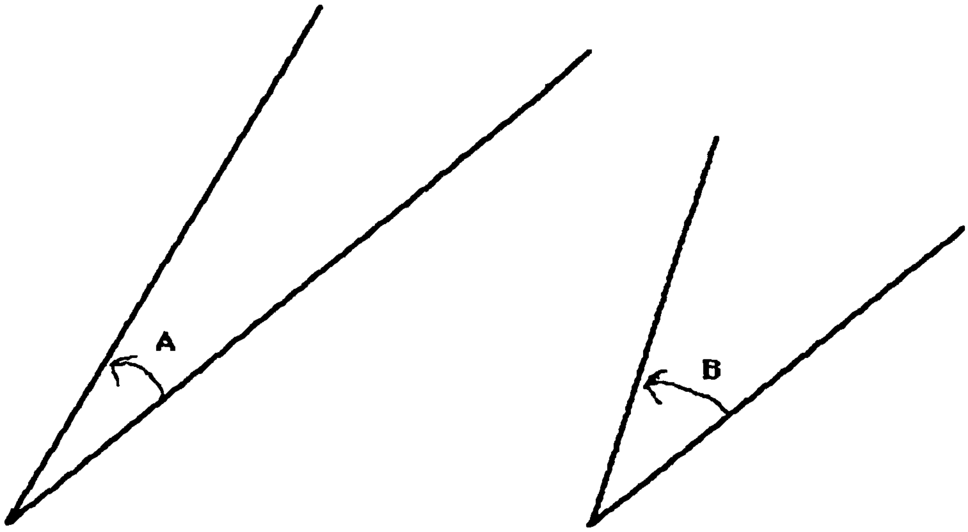


d) Compare angle **A** and **B**

A is the same as **B** (__): **A** is bigger than **B** (__)

B is bigger than **A** (__) You **cannot** tell (__)

EXPLAIN WHY YOU CAME TO YOUR ANSWER ____



e) Compare angle **A** and **B**

A is the same as **B** ( ): **A** is bigger than **B** ( )

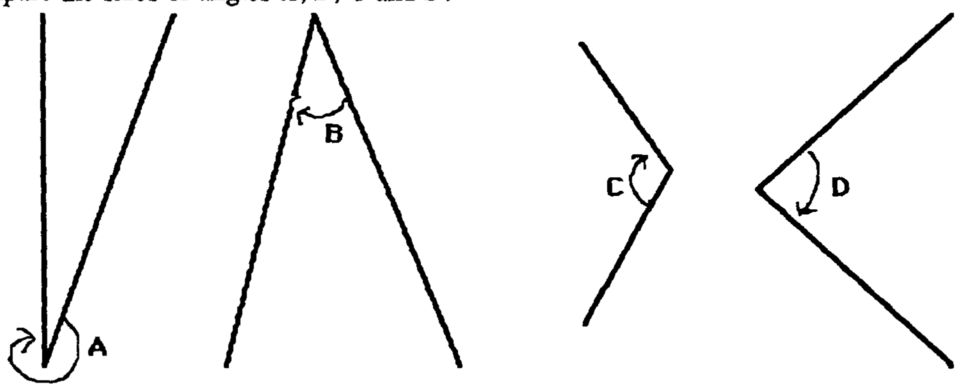
B is bigger than **A** ( ) You **cannot** tell ( )

EXPLAIN WHY YOU CAME TO YOUR ANSWER ______________________________

----

3. FOUR ANGLES

a)Compare the sizes of angles **A**, **B**, **C** and **D**.

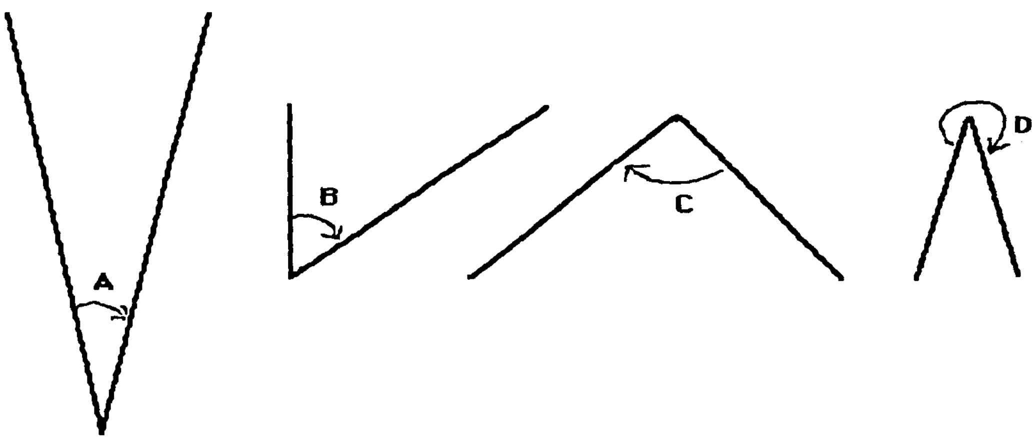


Which of them is the **smallest**? ____________

EXPLAIN WHY YOU CAME TO YOUR ANSWER ______________________________

----

b)Compare the site of angles **A**, **B**, **C** and **D**.



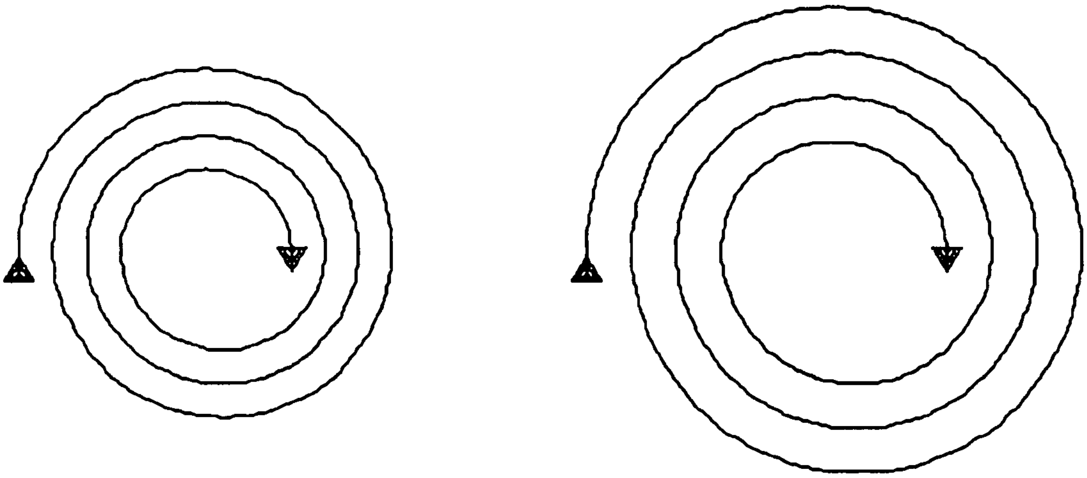
Which of them is the **largest**?____________

EXPLAIN WHY YOU CAME TO YOUR ANSWER ______________________________

----

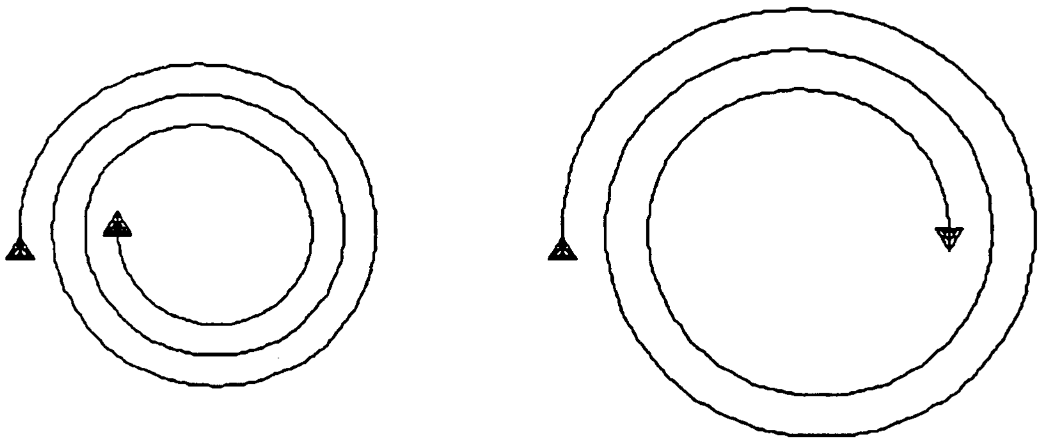
4. SPIRAL

In each of the following questions, compare each spiral in a pair and tick (√) the correct answer



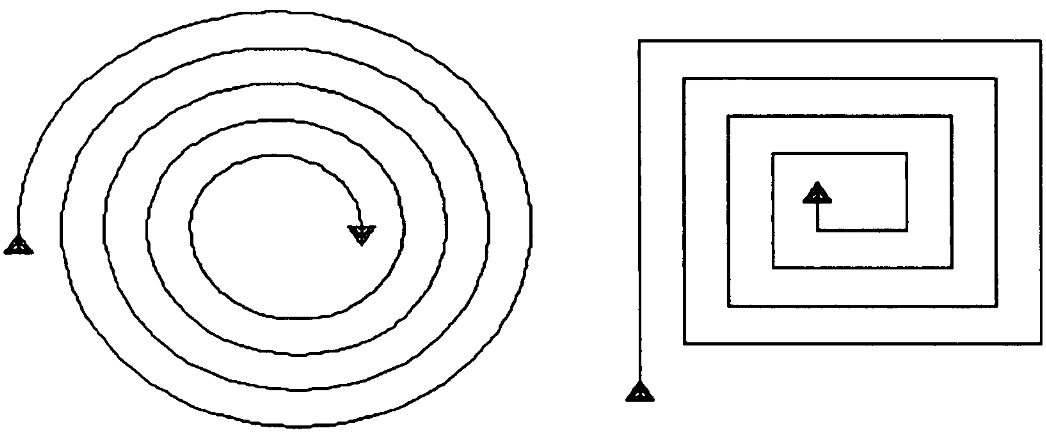
A B

a) Compare the turn made in spiral **A** with the turn made in spiral **B**
(The start and end points of spirals **A** and **B** are as indicated)
Turn in **A** is the same as turn in **B** ( ): Turn in **A** is more than turn in **B** ( )
Turn in **B** is more than turn in **A** ( ) You **cannot** tell ( )



A B

b) Compare the turn made in spiral **A** with the turn made in spiral **B**
(The start and end points of spirals **A** and **B** are as indicated)
Turn in **A** is the same as turn in **B** ( ): Turn in **A** is more than turn in **B** ( )
Turn in **B** is more than turn in **A** ( ) You **cannot** tell ( )



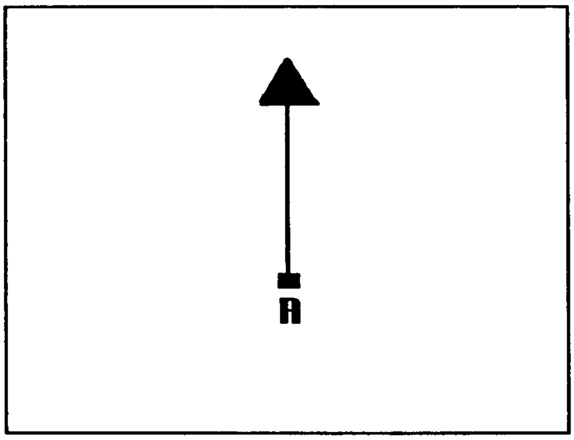
A B

c) Compare the turn made in spiral **A** with the turn made in spiral **B**
(The start and end points of spirals **A** and **B** are as indicated)
Turn in **A** is the same as turn in **B** ( ): Turn in **A** is more than turn in **B** ( )
Turn in **B** is more than turn in **A** ( ) You **cannot** tell ( )

5. ARROW

In each of the following questions draw the position of the arrow after the specified turn about A.

a)

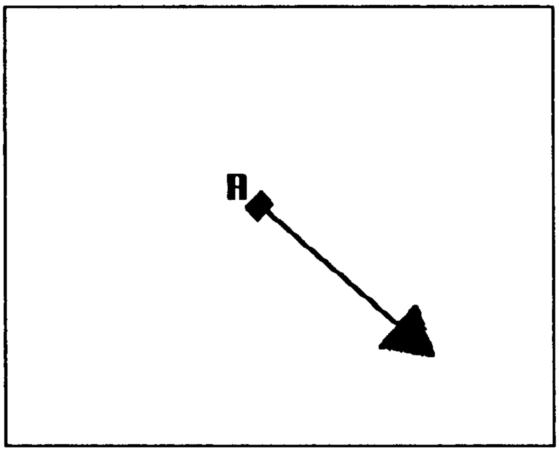


An arrow is fixed at point A as shown above. Draw the position of the arrow after it has turned 90^0 clockwise - that is, a quarter turn to the left side

Explain why you came to your answer?____

---

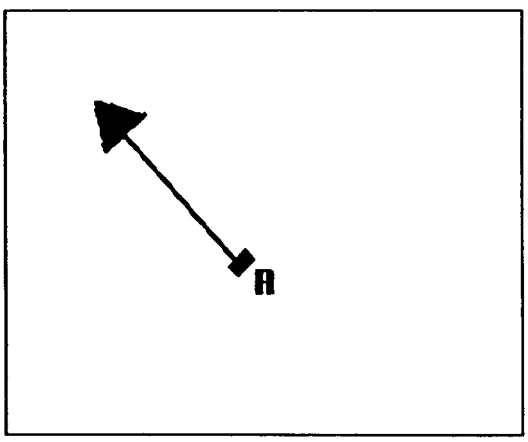
b)



An arrow is fixed at point A as shown above. Draw the position of the arrow after it has turned 90^0 anticlockwise - that is, a quarter turn to the left side.

Explain why you came to your answer?____

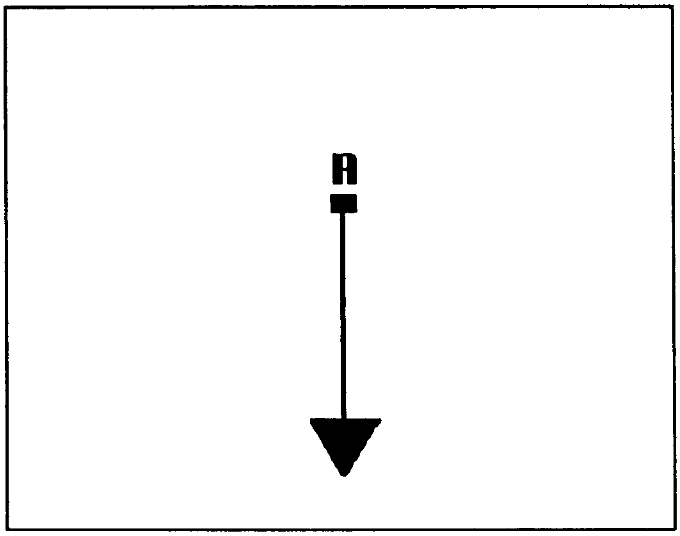
C)



An arrow is fixed at point A as shown above. Draw the position of the arrow after it has turned 180^0 clockwise - that is, a quarter turn to the right side.

Explain why you came to your answer?____

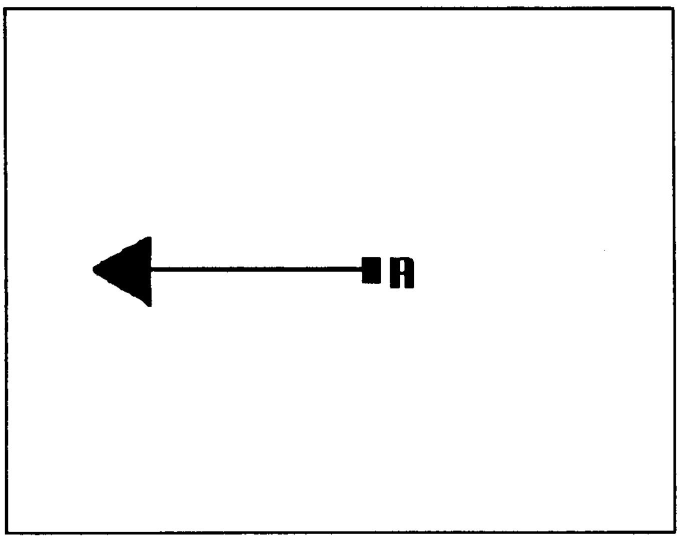
D)



An arrow is fixed at point **A** as shown above. Draw the position of the arrow after it has turned 720^0 anticlockwise - that is, a quarter turn to the left

Explain why you came to your answer?____

E)



An arrow is fixed at point **A** as shown above. Draw the position of the arrow after it has turned 540^0 clockwise - that is, a quarter turn to the right

Explain why you came to your answer?____

ROTEIRO DE APLICAÇÃO DO SETTING DO DIA-A-DIA

ARENA: **MINI CITY**

CONTEXTO DE NAVEGAÇÃO:

ATIVIDADE 1:

RECONHECIMENTO: Existem 2 caminhos para se ir do ponto 1 para o ponto 2, que é a rota **A** e a rota **B**. Qual das duas voce acha melhor para navegar seu carrinho?

ARTICULAÇAO: Por que?
AÇÃO: Você poderia dirigir o carrinho pela rota **A** e ir contando em voz alta quantas curvas ela tem e depois fazer a mesma coisa na rota **B** ?

RECONHECIMENTO; Qual das duas rotas tem menos curvas ?

CONTEXTO DE ROTAÇÃO

ATIVIDADE 1: (Falando sobre valor de giro)

RECONHECIMENTO$_1$: Você virou 90^0 alguma vez na rota **A**? Onde? Você virou 90^0 alguma vez na rota **B**? Onde?

RECONHECIMENTO$_2$: Qual a curva que você virou mais na rota **A**?

ARTICULAÇAO: Explique sua resposta

RECONHECIMENTO$_3$: Qual a curva que você virou mais na rota **B**?

ARTICULAÇAO: Explique sua resposta.
RECONHECIMENTO$_4$: Comparando a maior curva da rota **A** com a maior da rota **B**, qual das duas é a maior de todas?

ARTICULAÇAO: Por que?
RECONHECIMENTO$_5$: Vocé girou um quarto de volta em algum ponto da rota **A**?
RECONHECIMENTO$_6$: Você girou um quarto de volta em algum ponto da rota **B**?
RECONHECIMENTO$_7$: Você girou meia volta em algum ponto da rota **A**?

RECONHECIMENTO$_8$: Você girou meia volta em algum ponto da rota **B**?

CONTEXTO DE COMPARAÇÃO

ATIVIDADE 1 (Comparando entre um giro no círculo e em quina)

RECONHECIMENTO: Você acha que o carrinho vira a mesma quantidade de giros aqui (círculo) e aqui (quina)?

ARTICULAÇAO: Por que você acha isso?

ATIVIDADE 2 (Comparando dois ângulos iguais quando o carrinho está andando em ruas de tamanhos diferentes)

RECONHECIMENTO: Você acha que você virou o mesmo tanto nessa (apontar a curva) e nessa (apontar a curva) curva?

ARTICULAÇAO: Por que você acha isso?

TAREFA DOS RELÓGIOS

CONTEXTO DE COMPARAÇÃO

ESTÓRIA: Vamos fazer de conta que você e mais dois amiguinhos seus iam fazer um dever de classe e que o professor de vocês quisesse saber qual dos três iria terminar o dever primeiro. Mas acontece que o professor foi chamado pelo diretor e tinha que sair da classe. Ele então decidiu colocar um relógio na frente de cada um de vocês para depois saber quem terminou primeiro. Quando vocês começaram a fazer o dever, todos os 3 relógios estavam nesta posição (12 hs.)

ATIVIDADE 1

RECONHECIMENTO: Quando você terminou seu relógio (circular pequeno) estava nessa posição (12:20 hs), e o dos seus amiguinhos estavam nessa (12:20 hs.) e nessa (12:20 hs.) posição. Olhando para os relógios, quem você acha que terminou primeiro?

ARTICULAÇÃO: Por que?

(SE A CRIANÇA DER A RESPOSTA CORRETA, ISTO É: "NÓS TERMINAMOS AO MESMO TEMPO")

EXAMINADORA: Mas olhe, eu pensei que fosse você quem terminou primeiro porque isso aqui (area do ângulo interno) é menor do que a dos outros não é ?

(SE A CRIANÇA DER QUALQUER EXPLICAÇÃO ATRAVÉS DA ESTIMATIVA DE TEMPO)

EXAMINADORA: Como você sabe da hora? Eu não estou vendo nenhum número nos relogios, e aí, você adivinha é? Explica isso pra mim melhor.

ATIVIDADE 2

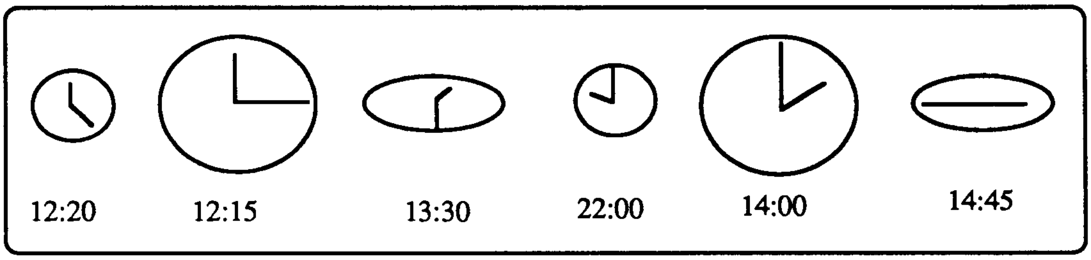
(DEPOIS DE CONTAR A MESMA ESTORIA, COM TODOS OS 3 RELOGIOS COMEÇANDO EM 12 HS. E O PEQUENO E O OVAL TERMINANDO EM 12:45 Hs., O GRANDE EM 12:25)

RECONHECIMENTO:Olhando para os relogios, quem você acha que terminou primeiro?

ARTICULAÇÃO: Por que?

ATIVIDADE 3
(DEPOIS DE CONTAR A MESMA ESTORIA, COM TODOS OS 3 RELOGIOS COMEÇANDO EM 12 Hs. E O PEQUENO TERMINANDO EM 15 Hs., o OVAL TERMINANDO EM 14 Hs., O GRANDE EM 13:15 Hs)

ATIVIDADE 4 (USANDO OS 6 RELOGIOS QUE NÃO NÚMERO)
A FIGURA ABAIXO MOSTRA A POSIÇÃO FINAL DOS RELOGIOS



RECONHECIMENTO: 6 pessoas começaram a trabalhar quando os relogios estavam assim (mostrar os 6 relogios em 12 hs.). Eles marcaram o tempo que cada um gastou trabalhando e quando eles terminaram o trabalho, o relogios de cada um estava assim (mostrar a posição final de cada relogio). Qual foi a pessoa que trabalhou mais? E qual foi deles que trabalhou menos?

ARTICULAÇÃO: Por que você acha isso?

ATIVIDADE 5 (USANDO UM OVAL E UM PEQUENO, OS 2 COM NUMEROS)

GRANDE: INICIO 13:10 PEQUENO: INICIO 13:15
TÉRMINO 13:40 TÉRMINO 13:45

RECONHECIMENTO$_1$: Quanto tempo cada um trabalhou?

ARTICULAÇÃO$_2$: Como você sabe?

RECONHECIMENTO$_1$: Quem trabalhou mais?

ARTICULAÇÃO$_2$: Como você sabe?

ATIVIDADE 6 (USANDO UM GRANDE E UM PEQUENO, OS 2 COM NÚMEROS)

GRANDE: INICIO 13:10 PEQUENO: INICIO 13:15
TÉRMINO 13:40 TÉRMINO 13:45

RECONHECIMENTO$_1$: Quanto tempo cada um trabalhou?
ARTICULAÇÃO$_2$: Como você sabe?
RECONHECIMENTO$_1$: Quem trabalhou mais?
ARTICULAÇÃO$_2$: Como você sabe?

CONTEXTO DE ROTAÇÃO:

ATIVIDADE 1
(USANDO APENAS UM RELOGIO PEQUENO SEM NÚMERO)

AÇÃO: (Posição inicial 12 hs.) Se eu trabalhasse o tanto de meia volta, onde estariam os ponteiros do relogio quando eu já tivesse trabalhado esse tanto?

ARTICULAÇÃO$_1$: Por que?

(SE A CRIANÇA DER QUALQUER EXPLICAÇÃO ATRAVÉS DA ESTIMATIVA DE TEMPO)

ARTICULAÇÃO$_2$: Como você pode saber sobre o tempo se eu apenas falei sobre volta e o relogio não tem números?

ATIVIDADE 2
(USANDO APENAS UM RELOGIO GRANDE SEM NÚMEROS)

AÇÃO: (POSIÇÃO INICIAL DE 12:30 Hs). Se eu começasse a trabalhar quando o relogio estivesse nessa posição e eu trabalhasse o tanto que equivale a meia volta, onde os ponteiros estariam quando eu terminasse de trabalhar esse tanto?

ARTICULAÇÃO$_1$: Por que?

(SE A CRIANÇA DER QUALQUER EXPLICAÇÃO ATRAVÉS DA ESTIMATIVA DE TEMPO)

ARTICULAÇÃO$_2$: Como você pode saber sobre a hora, se eu todo o tempo so lhe falei sobre a quantidade de volta? Eu achei que quando o ponteiro parava aqui era porque passou uma hora, não é?

ATIVIDADE 3: (USANDO APENAS O RELOGIO OVAL SEM NÚMEROS)

ACÃO: (POSIÇÃO INICIAL 12:45 Hs.) Se eu começasse a trabalhar quando o relógio estivesse nessa posição e eu trabalhasse um tanto equivalente a meia volta, onde os ponteiros estariam quando eu terminasse de trabahar esse tanto?

ARTICULAÇÃO: Por que?

ATIVIDADE 4: (USANDO AS 3 FORMAS DE RELÓGIOS, TODOS COM NÚMEROS E NA POSIÇÃO INICIAL DE 12 Hs.)

AÇÃO: Você pode me mostrar onde os ponteiros dos 3 relogios estarão depois de ter passado meia hora?

ARTICULAÇÃO: Por que?

ATIVIDADE 5: (USANDO AINDA OS 3 RELOGIOS, TODOS COMEÇANDO AS 14:10 Hs.)

AÇÃO: Você pode me mostrar onde os ponteiros dos 3 relogios estarão depois de ter passado meia hora?

ARTICULAÇÃO: Por que? Eu achava que meia hora era aqui (no número 6 do relogio), não é?

TAREFA DE BORBOLETA

Contexto de Rotação

ATIVIDADE 1
RECONHECIMENTO$_1$: Se a borboleta der uma volta quantas pessoas podem entrar?

ARTICULAÇÃO$_1$: Por que?

(SE A CRIANÇA DISSER UMA PESSOA)
RECONHECIMENTO$_2$: Se entrar 4 pessoas onde vai parar o braço da borboleta? Como você chama esse giro que a borboleta deu: uma volta, 2 voltas, 3 voltas, 4 voltas, uma volta completa?

ARTICULAÇÃO$_2$: Como é isso, explique-me.

ATIVIDADE 2

RECONHECIMENTO: Se a borboleta girar meia volta a começar deste ponto aqui (apontar para a seta que está colada em um dos braços da borboleta) quantas pessoas vão entrar?

ARTICULAÇÃO: Como é que você sabe?

(se a criança disser 2 pessoas)

ATIVIDADE 3

RECONHECIMENTO$_1$: Veja se o braço da borboleta parando aqui não poderia ter entredo 6 pessoas? Voçê pode explicar isso? Vamos ver se pode mesmo (rodar de um em um até 6).

(Ê, entraram 6 pessoas.)
RECONHECIMENTO$_2$: A borboleta fez a mesma coisa para entrar 2 e 6 pessoas?

ARTICULAÇÃO: Como ê isso, explique-me.

ATIVIDADE 4
ACÃO: Se a borboleta girar 2 voltas completas onde ela vai parar?

RECONHECIMENTO$_1$: Quantas pessoas entraram?
ARTICULAÇÃO$_1$: Como é isso?
(Se a criança tiver respondido corretamente 8 pessoas)
RECONHECIMENTO$_2$: Eu pensei que se ela parasse aqui iam entrar 4 pessoas, não é não?
ARTICULAÇÃO$_2$: Por que?

ATIVIDADE 5
AÇÃO: Quando a borboleta rodar 1 volta e meia onde é que ela vai parar?
RECONHECIMENTO: Quantas pessoas vão entrar?
ARTICULAÇÃO: Por que?

STICK GAME

ATIVIDADE 1: ÂNGULO =
TAMANHO=
ORIENTAÇÃO≠
RECONHECIMENTO: Esses ângulos são iguais?
ARTICULAÇÃO: O que é que tem de iguais (OU DIFERENTES) neles?

ATIVIDADE 2: ÂNGULO≠ (one beside another)
TAMANHO≠
ORIENTAÇÃO=

RECONHECIMENTO: Esses ângulos são iguais?
ARTICULAÇÃO: O que é que tem de iguais (OU DIFERENTES) neles?

ATIVIDADE 3: ÂNGULO=
TAMANHO≠
ORIENTAÇÃO=
RECONHECIMENTO: Esses ângulos são iguais?
ARTICULAÇÃO: O que é que tem de iguais (OU DIFERENTES) neles?

ATIVIDADE 4: ÂNGULO≠ (one inside another)
TAMANHO≠
ORIENTAÇÃO=
RECONHECIMENTO: Esses ângulos são iguais?
ARTICULAÇÃO: O que é que tem de iguais (OU DIFERENTES) neles?

ATIVIDADE 5: ÂNGULO=
TAMANHO≠
ORIENTACÃO≠
RECONHECIMENTO: Esses ângulos são iguais?
ARTICULAÇÃO: O que é que tem de iguais (OU DIFERENTES) neles?

ATIVIDADE 6:
ACÃO: Construa um ângulo igual a esse (90^0 vertic/horizont)
ARTICULAÇÃO: Como é que você sabe que eles são iguais? Eu estou achando eles diferentes, não é?

ATIVIDADE 7:
ACÃO: Construa um ângulo igual a esse (45^0 em forma de "V")
ARTICULAÇÃO: Como é que você sabe que eles são iguais? Eu estou achando eles diferentes, não é?

ATIVIDADE 8:
ACÃO: Construa um ângulo igual a esse (90^0 em forma de "V")
ARTICULAÇÃO: Como é que você sabe que eles são iguais? Eu estou achando eles diferentes, não é?

CHECK-LIST FOR THE EVERYDAY SETTING

ARENA: **MINI CITY**

NAVIGATION CONTEXT:

ACTIVITY 1: (Introduction)

RECOGNITION$_1$: There are 2 ways to go from point 1 to point 2. Which of route **A** and the route **B**. would you choose to go from 1 to 2?

ARTICULATION: Why?

ACTION: Could you drive the small car through the route **A** and also counting loud the turns you are doing, and do the same for route **B**?

RECOGNITION$_2$: Which of route **A** and **B** has fewer turns?

ROTATION CONTEXT

ACTIVITY 1: (concerning to the value of the turn)

RECOGNITION$_1$: Did you turn 90^0 at any point along route **A**? Where? Did you turn 90^0 at any point along route **B**? Where?

RECOGNITION$_2$: Name the largest turn you made along route **A**?

ARTICULATION: Explain your answer.

RECOGNITION$_3$: Name the largest turn you made along route **A**?

ARTICULATION: Explain your answer.

RECOGNITION$_4$: Comparing your answer to the largest turn along route **A** with your answer to the largest turn along route **B**, which of them is the largest?

ARTICULATION Why?

RECOGNITION$_5$: Did you turn 1/4 of turn at any point along route **A**? Where?

RECOGNITION$_6$: Did you turn 1/4 of turn at any point along route **B**? Where?

RECOGNITION$_7$: Did you turn 1/2 of turn at any point along route **A**? Where?
Did you turn 1/2 of turn at any point along route **B**? Where?

COMPARISON CONTEXT

ACTIVITY 1: (comparing between a turn done in a circle and a turn done in a square)

RECOGNITION: Do you think that the cars did the same amount of turn in here (circle) and here (square), or not?

ARTICULATION: Why?

ACTIVITY 2: (comparing between two equal angles, but presenting different ray-sizes)
RECOGNITION$_1$: Do you think that the cars turned the same or not?

ARTICULATION: Why?

WATCH ARENA

COMPARISON CONTEXT

STORE: "Lets imagine that a teacher asked 3 children to do a classroom task. Unfortunately the teacher could not stay in the classroom. So, he put one watch in front of each child and asked them, when they started the task, to press a button to start the watches as soon as they began to work. When the teacher came back to the classroom, he wanted to see the 3 watches to decide which student had finished his work first. when all three students started to do the task their all watches were showing like this (12.00hs)".

ACTIVITY 1

RECOGNITION: When you finished your watch (small circular) was like this (12.20hs), and your colleagues watches were like this (12.20hs) and like this (12.20hs). By looking at the watches, who do you think that finished first?

ARTICULATION: Why?

(If the child answer that all finish at the same time)
EXAMINER: But look, I thought that it were you because this (point to the internal angle) is smaller than the other, don't you think?

(If the child answer the question by estimating the time)

EXAMINER: How do you know about the time? I cannot see any numbers on the watches, so how can you know?

ACTIVITY 2
(After tell a similar store like before, and with the 3 watches starting at 12.00hs. and the small circular and the oval finishing at 12.45hs and the big circular watch finishing at 12.25hs)

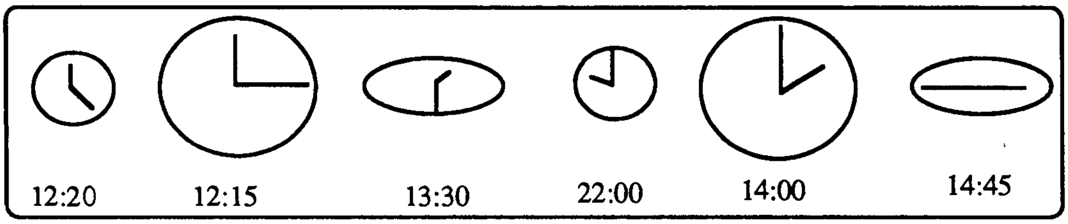
RECOGNITION: By looking at the watches, who do you think that finished first?

ARTICULATION: Why?

ACTIVITY 3
(After tell a similar store like before, and with the 3 watches starting at 12.00hs. and the small circular FINISHING AT 15.00hs, the oval finishing at 14.00hs and the big circular watch finishing at 13.15hs)
RECOGNITION: By looking at the watches, who do you think that finished first?

ARTICULATION: Why?

ACTIVITY 4 (Using 6 non numbered watches at the same time)



6 people started to work when all these watches were like this (the watches have to be at 12.00hs).Each of these 6 people used one watch in order to show how long each one had worked:
RECOGNITION$_1$: By remembering the position everybody start and looking at the final positions, who do you think that worked more?

ARTICULATION: Why
RECOGNITION$_2$: By remembering the position everybody start and looking at the final positions, who do you think that worked less?

ARTICULATION: Why

ACTIVITY 5 (Using one oval and one small circular watches, both facing numbers)

SMALL: Starting-point 13:10 End-point 13:40
OVAL: Starting-point 13:15 End-point 13:45

RECOGNITION: Did they work the same or different?

ARTICULATION: How do you know?

ACTIVITY 6 (Using one big and one small circular watches)

BIG: Starting-point 13:10 End-point 13:40
SMALL: Starting-point 13:15 End-point 13:45

RECOGNITION: Did they work the same or different?
ARTICULATION: How do you know?

ROTATION CONTEXT:

ACTIVITY 1 (USING ONLY ONE SMALL WATCH WITHOUT NUMBERS)
ACTION: (initial position 12 Hs.).If I started to work whenthe watch was like this and I worked as 1/2 turn, where would be the hands when I finished the work?

ARTICULATION$_1$: why?

(IF THE CHILD ANSWER THROUGH ESTIMATING THE HOUR)
ARTICULATION$_2$: How can you know the hour if I had just spoken about turns and this watch has no numbers?

ACTIVITY 2 (USING ONLY ONE BIG WATCH WITHOUT NUMBERS)
ACTION: (initial position 12:30 Hs.).If I started to work when the watch was like this and I worked as 1/2 turn, where would be the hands when I finished the work?

ARTICULATION$_1$: why?

(IF THE CHILD ANSWER THROUGH ESTIMATING THE HOUR)
ARTICULATION$_2$: How can you know the hour if I had just spoken about turns and this watch has no numbers?

ACTIVITY 3:(USING ONLY ONE BIG WATCH WITHOUT NUMBERS)
ACTION: (initial position 12:45 Hs.).If I started to work when the watch was like this and I worked as 1/2 turn, where would be the hands when I finished the work?

ARTICULATION: why?

ACTIVITY 4: (USING THE 3 SHAPES OF WATCHES, ALL WITH NUMBERS AND SHOWING 12 Hs.)
ACTION: Can you show me where will be the hands after 1/2 hour?

ARTICULATION: Why?

ACTIVITY 5: (STILL USING THE 3 WATCHES, ALL STARTING AT 14:10 Hs.)
ACTION: Can you show me where will be the hands after 1/2 hour?
ARTICULATION: Why? I thought that 1/2 hour would be here (at the position of number 6), isn't it?

TURNSTILE ARENA

Rotation Context

ACTIVITY 1

RECOGNITION$_1$: If the turnstile does 1 turn how many people can entry?

ARTICULATION$_1$: Why?

(IF THE CHILD ANSWER 1 PERSON)
RECOGNITION$_2$: If 4 people come into where will the arm of the turnstile stop? how do you call this turn made by the turnstile: 1 turn, 2 turns, 3 turns, 4 turns, one complete turns?

ARTICULATION$_2$: Why?

ACTIVITY 2

RECOGNITION$_1$: If the turnstile turns <u>1/2 turn</u> starting from this point (to point to the arrow fix over one of the turnstile arm) how many people can come into the zoo?

ARTICULATION: How do you know?

(IF THE CHILD ANSWER 2 PEOPLE)

ACTIVITY 3

RECOGNITION$_2$: Starting to turn in this point and finishing here (to show the previous situation) could 6 people come into the zoo? (if the child say NO) Let's see if it is possible (to turn slow and to count loud).

RECOGNITION$_3$: Did the turnstile the same thing as before?

ARTICULATION: Why?.

ACTIVITY 4

ACTION: If the turnstile to turn <u>2 complete turns</u> where will it stop?

RECOGNITION: How many people can come into the zoo?

ARTICULATION$_1$: Why?

(If the child answer correct)

ARTICULATION$_2$: Why were not 4 people?

<u>ACTIVITY 5</u>

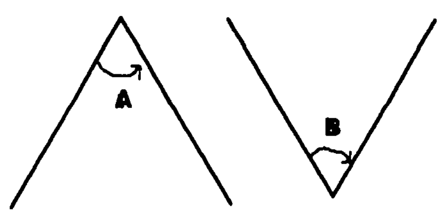
ACTION: If the turnstile to turn 2 complete turns where will it stop?

RECOGNITION: How many people can come into the zoo?

ARTICULATION: Why?

STICK GAME

ACTIVITY 1:
ANGLE =
SIDES =
ORIENTATION ≠
RECOGNITION: Are these angles the same?
ARTICULATION: Why?



ACTIVITY 2:

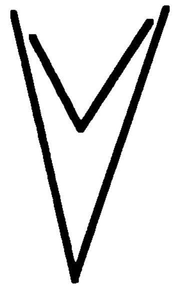
ANGLE ≠

SIDES ≠

ORIENTATION =

RECOGNITION: Are these angles the same?

ARTICULATION: Why?



ACTIVITY 3:

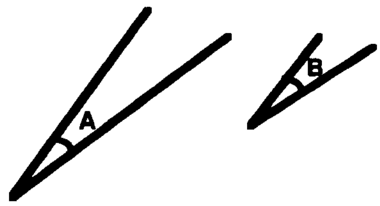
ANGLE = (one inside the other)

SIDES ≠

ORIENTATION =

RECOGNITION: Are these angles the same?

ARTICULATION: Why?



ACTIVITY 4:

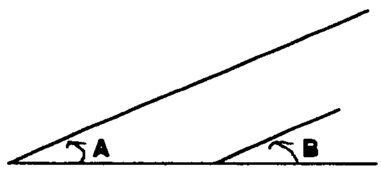
ANGLE ≠ (one inside of the other)

SIDES ≠

ORIENTATION =

RECOGNITION: Are these angles the same?

ARTICULATION: Why?



ACTIVITY 5:

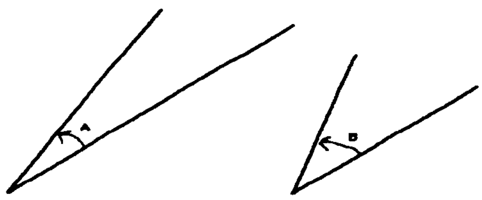
ANGLE ≠ (one next to the other)

SIDES ≠

ORIENTATION ≠

RECOGNITION: Are these angles the same?

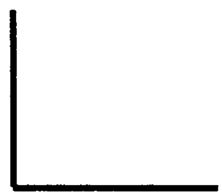
ARTICULATION: Why?



ACTIVITY 6:

ACTION: Draw a similar angle to this

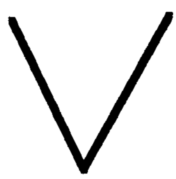
ARTICULATION: How do you know that they are similar to each other?



ACTIVITY 7:

ACTION: Draw a similar angle to this

ARTICULATION: How do you know that they are similar to each other?



ACTIVITY 8:

ACTION: Draw a similar angle to this

ARTICULATION: How do you know that they are similar to each other?

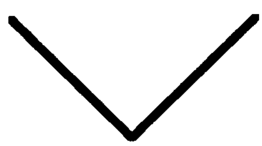
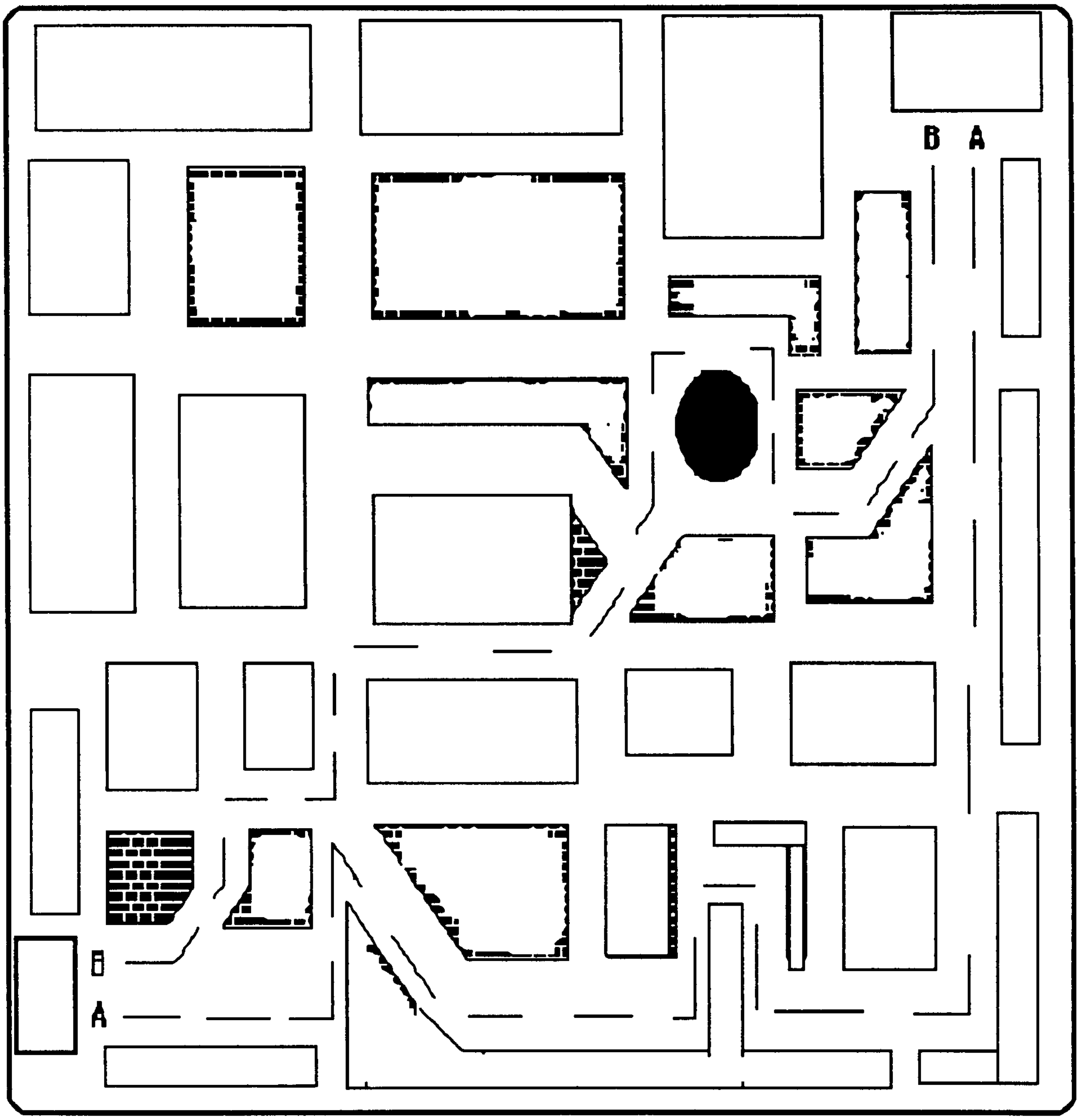
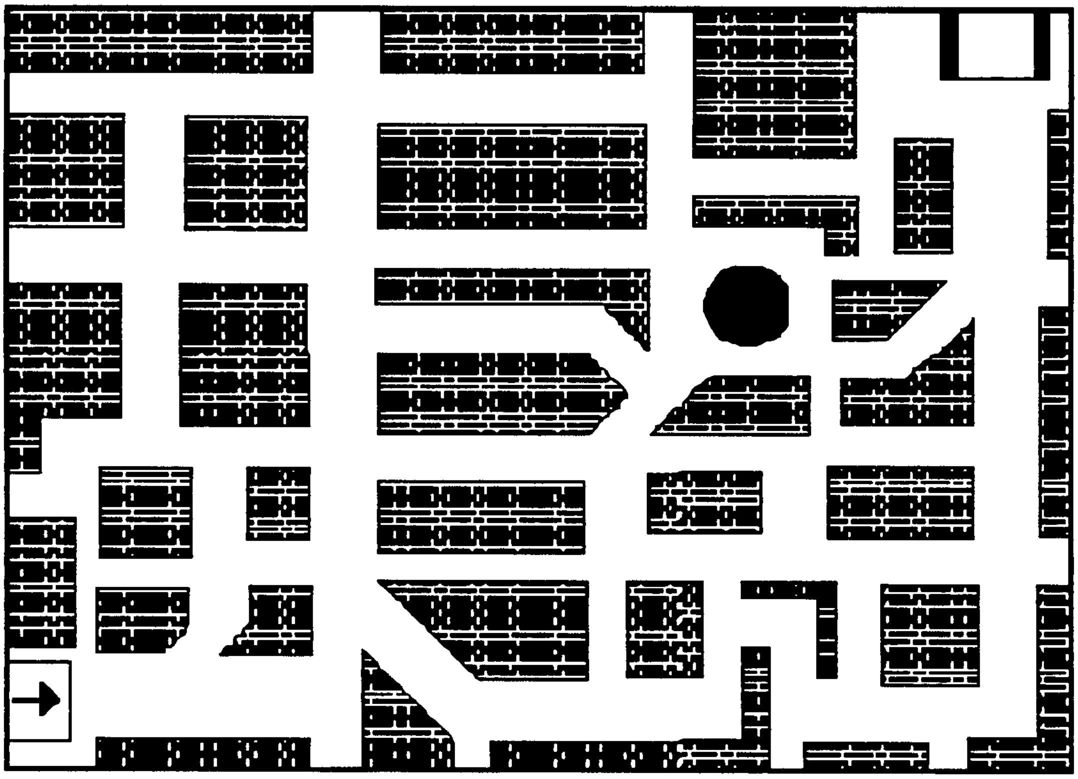


DIAGRAMA PARA MARCAR RESPOSTAS DAS CRIANÇAS NA MINI CITY
(DIAGRAM TO MARK CHILDREN'S RESPONSES IN THE MINI CITY)



ARENA DO MAPA NO LOGO | **THE MAP ARENA IN THE LOGO SETTING**



QUAL O SEU NOME?
Sandra

SANDRA, LEVE A SETA PARA A CASA VERMELHA PELO CAMINHO AZUL

GIRAR PARA ESQUERDA QUANTOS GRAUS?
90

(DEPOIS QUE A CRIANÇA COMPLETOU A NAVEGAÇAO PELO CAMINHO "A")
MUITO BEM !!!

SANDRA, ENUMERE AS CURVAS QUE A SETA FEZ PARA CHEGAR NA CASA VERMELHA E ESCREVA O TOTAL DE CURVAS FEITAS.
8

A SETA GIROU 90 GRAUS EM ALGUMA CURVA?
Sim
ONDE?
1, 4, 5, 6, 7, 8

EM QUAL CURVA A SETA GIROU MAIS?
Na 2

WHAT IS YOUR NAME?
Sandra

SANDRA, BRING THE ARROW TO THE RED HOUSE THROUGH THE BLUE WAY

HOW MANY DEGREES TO THE LEFT SIDE DO YOU WANT TO TURN?
90

(AFTER THE CHILD'S NAVIGATION THROUGHOUT THE WAY "A")
VERY WELL !!!

SANDRA, NAMBER ALL THE CURVES YOU DID FROM THE STARTING POINT UNTIL THE RED HOUSE AND WRITE DOW THE TOTAL OF THE CURVES.
8

DID THE ARROW TURN 90 DEGREE IN ANY TURN?
Sim.
WHERE?
1, 4, 5, 6, 7, 8

IN WHICH CURVE DID THE ARROW DO THE BIGEST TURN?
In the 2

A SETA DEU MEIA VOLTA EM ALGUMA CURVA?
Não

SANDRA, LEVE A SETA PARA A CASA VERMELHA PELO CAMINHO VERMELHO.

GIRAR PARA ESQUERDA QUANTOS GRAUS?
90

(DEPOIS QUE A CRIANÇA COMPLETOU A NAVEGAÇAO PELO CAMINHO "A")
MUITO BEM !!!

SANDRA, ENUMERE AS CURVAS QUE A SETA FEZ PARA CHEGAR 'A CASA VERMELHA E ESCREVA O TOTAL DE CURVAS FEITAS.
12

A SETA GIROU 90 GRAUS EM ALGUMA CURVA?
Sim
ONDE?
3, 4, 5, 6, 7, 9, 10, 11

EM QUAL CURVA A SETA GIROU MAIS?
Na 9 e 10

A SETA DEU MEIA VOLTA EM ALGUMA CURVA?
Sim
ONDE?
Nas curvas 9 e 10 juntas

EM QUE CAMINHO ESTÁ A MAIOR CURVA DE TODAS?
No B

COLOQUE O DISQUETE DO ALUNO NO DRIVE E TECLE EM QUALQUER TECLA

DID THE ARROW TURN HALF TURN IN ANY CURVE?
No

SANDRA, BRING THE ARROW TO THE RED HOUSE THROUGH THE RED WAY

HOW MANY DEGREES TO THE LEFT SIDE DO YOU WANT TOI TURN?
90

(AFTER THE CHILD'S NAVIGATION THROUGHOUT THE WAY "A")
VERY WELL !!!

SANDRA, NAMBER ALL THE CURVES YOU DID FROM THE STARTING POINT UNTIL THE RED HOUSE AND WRITE DOWN THE TOTAL OF THE CURVES.
12

DID THE ARROW TURN 90 DEGREE IN ANY TURN?
Yes.
WHERE?
3, 4, 5, 6, 7, 9, 10, 11

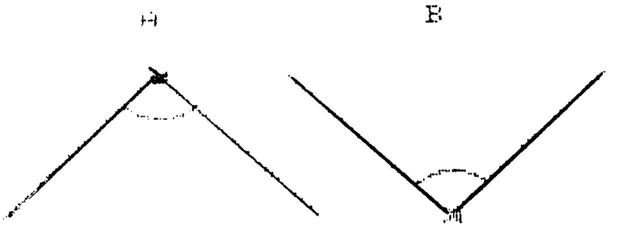
IN WHICH CURVE DID THE ARROW DO THE BIGEST TURN?
In the 9 and 10

DID THE ARROW TURN HALF TURN IN ANY CURVE?
Yes
WHERE?
9 and 10 together

IN WHICH WAY IS THE BIGGEST TURN OF ALL?
In B

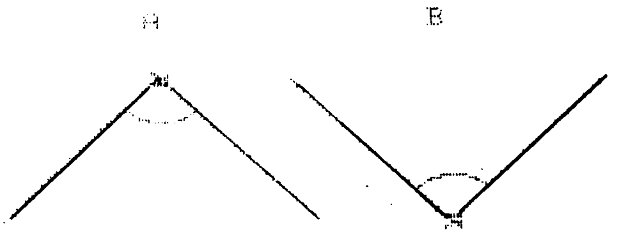
PUT THE STUDENT'S DISK INTO THE DRIVE AND PRESS IN ANY KEY

P R O G R A M A   P A R



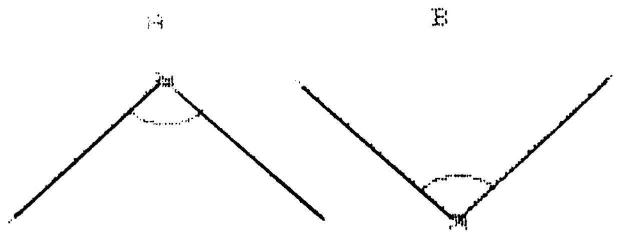
Qual e o seu nome ?

Sandra



Qual e a sua serie ?

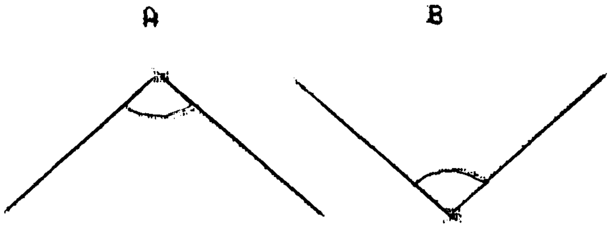
3.serie B



Voce acha que esses angulos

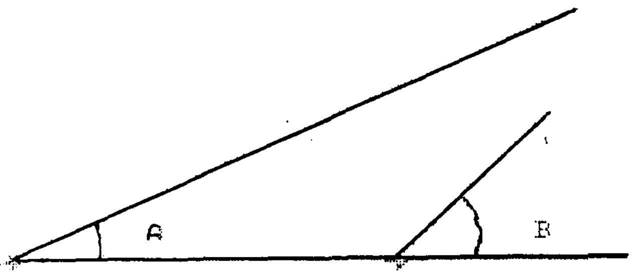
sao igua

ou diferentes ?



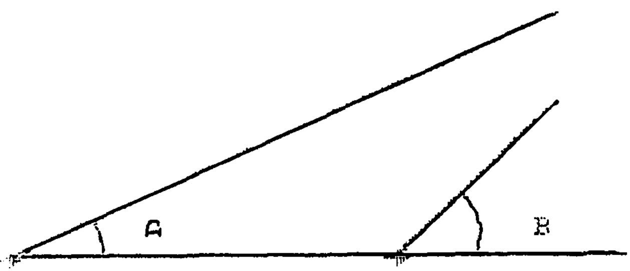
Como voce sabe que eles sao diferentes ?

Porque está virado.



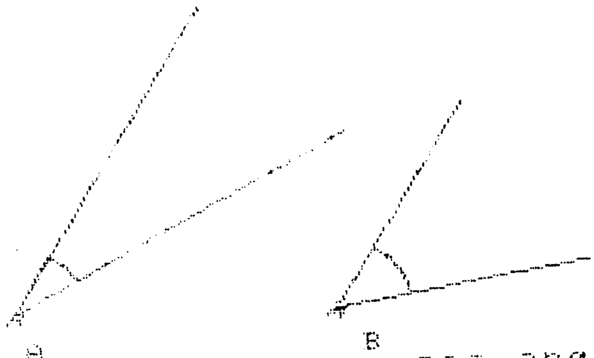
Voce acha que esses angulos → sao iguais ou diferentes ?

diferentes também.



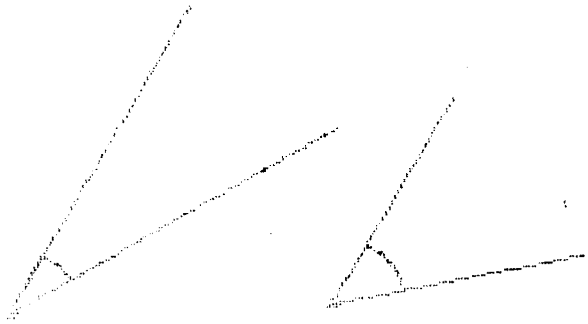
Como voce sabe que eles sao → diferentes ?

O B está mais aberto que A.

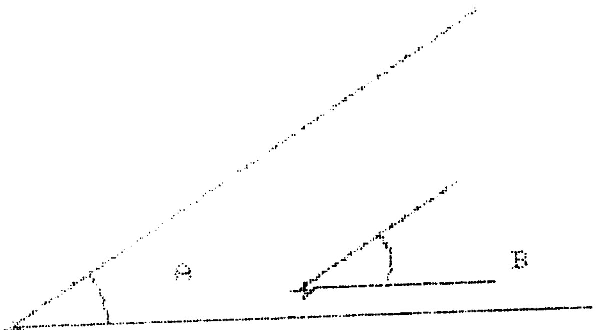


Voce acha que esses angulos→
sao iguais ou diferentes ? →

Iguais

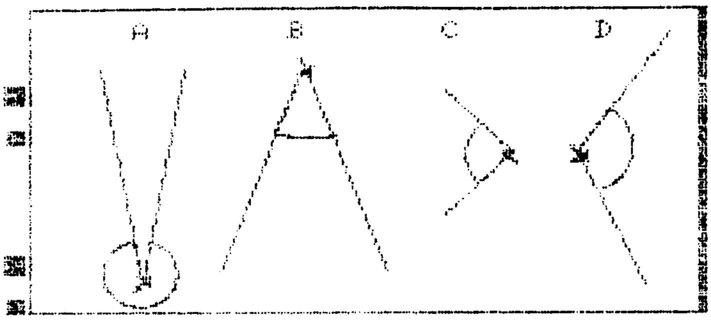


Como voce sabe que eles sao →
Iguais ?
Porque estão do mesmo lado.



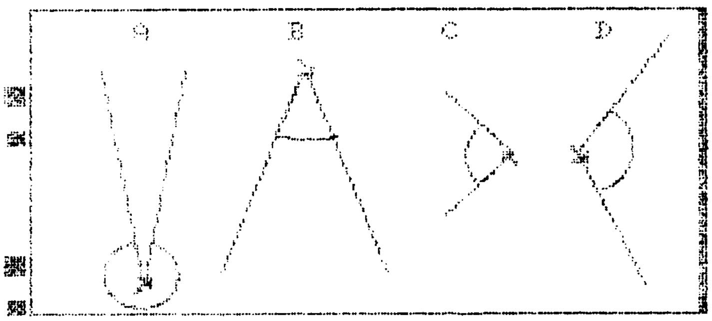
Voce acha que esses angulos→
sao iguais ou diferentes ?

PROGRAMA ANGULO



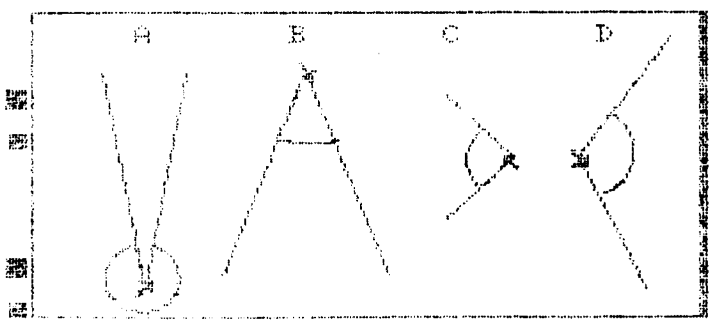
Escreva o seu nome

Sandra



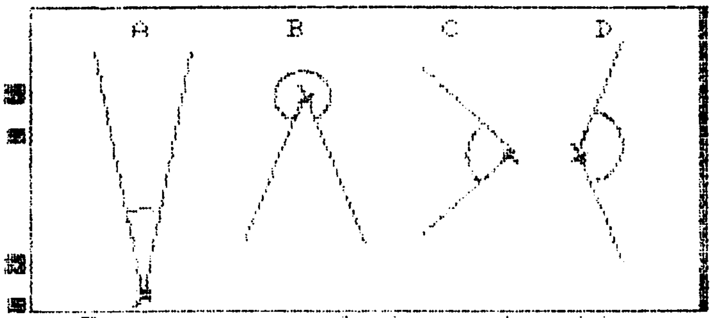
Escreva a letra da figura
que tem o MENOR ANGULO

B



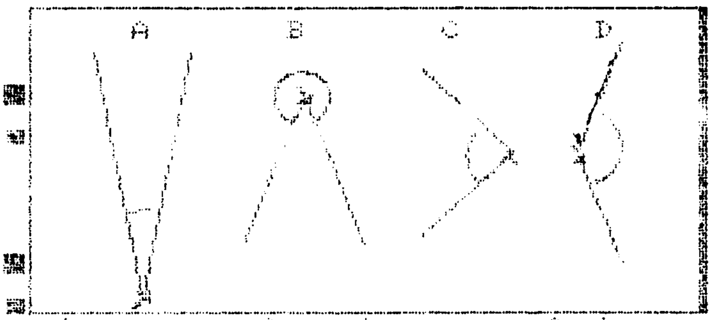
Como você sabe que foi a
letra " B

Foi a seta que girou menos.



Escreva a letra da figura que tem o MAIOR ângulo

D

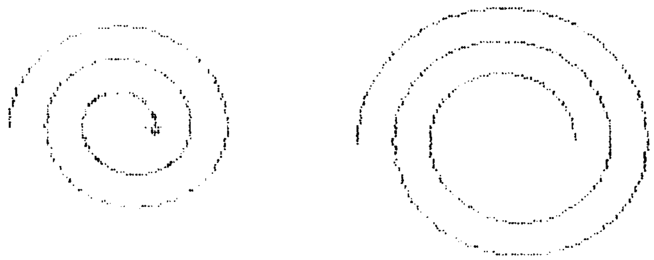


Como você sabe que foi a letra " D

Porque é o mais aberto.

P R O G R A M A   E S P I R A L

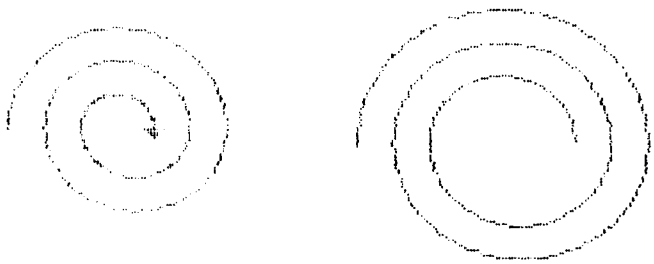
A B



Qual o seu nome?

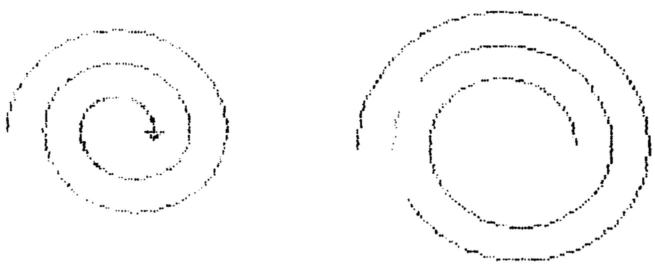
Sandra

A B



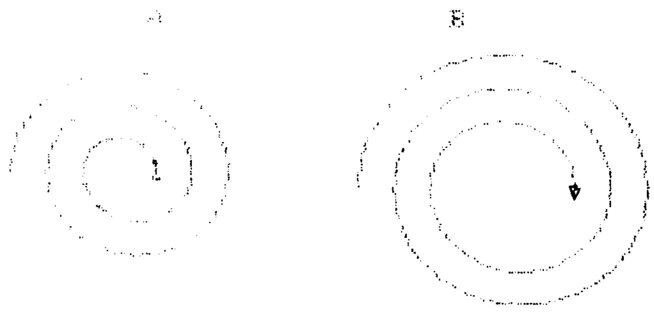
Sandra , Compare bem as espirais A e B.

A B



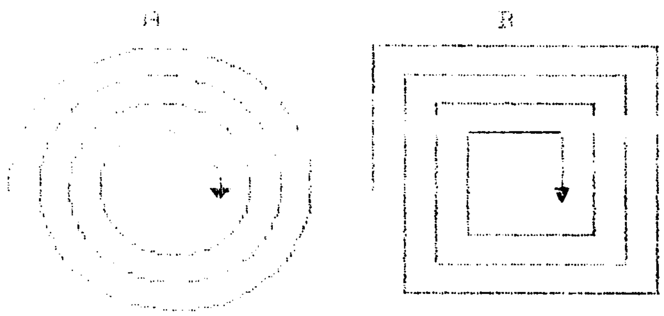
A quantidade de giros delas →
são IGUAIS ou DIFERENTES ?

Iguais

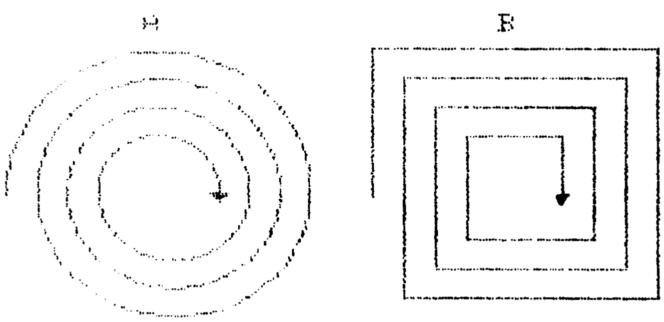


como você fez para saber que→ elas são iguais

+ muitas.



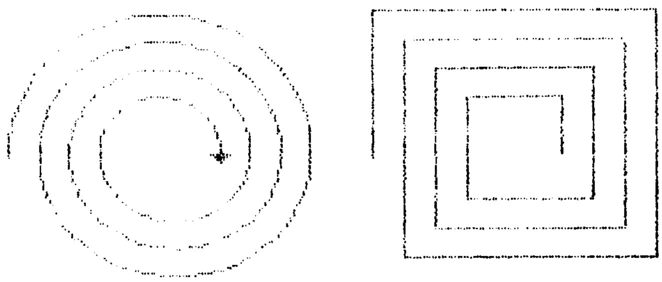
Sandra , Compare com atenção as espirais A e B. →



A quantidade de giros delas → são IGUAIS ou DIFERENTES ?

Diferentes

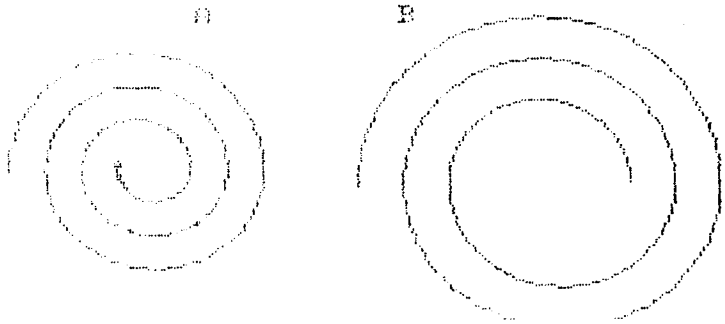
A B



Como você fez para saber que→
elas são biferentes

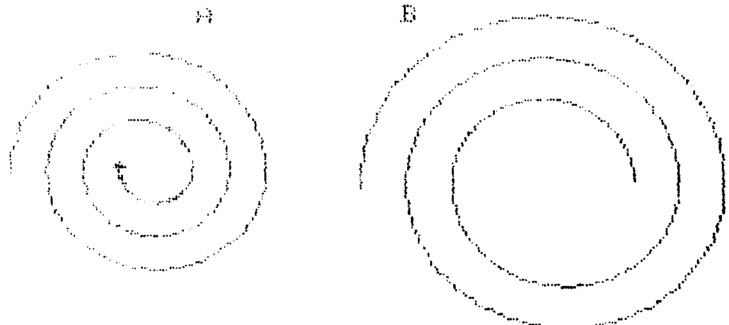
A espiral B girou menas veze→
s.

A B



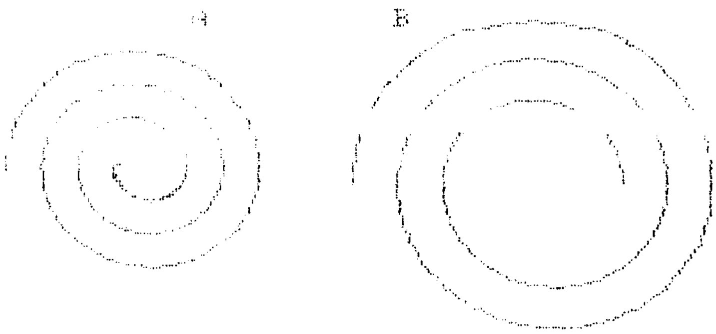
as espirais A e B.

A B



A quantidade de giros delas →
são IGUAIS ou DIFERENTES ?

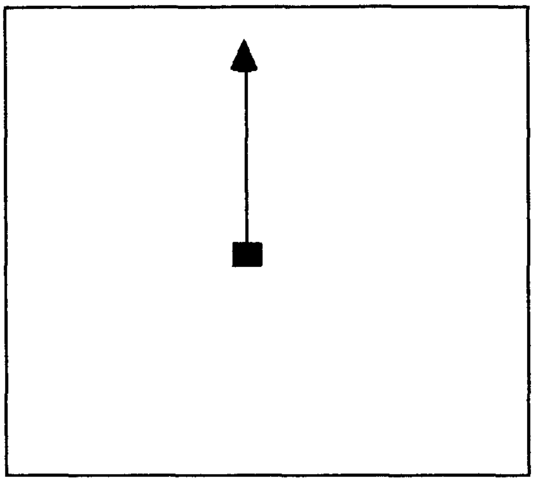
iba.



como você fez para saber que→
elas são biferentes

a B deu menas voltas.

PROGRAMA SETA

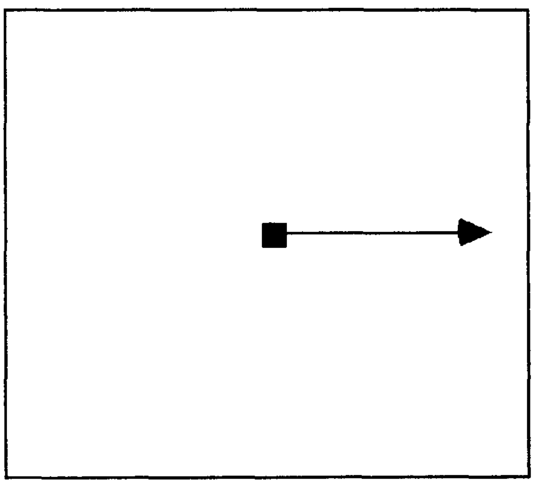


Qual e o seu nome?

Sandra

Sandra, onde voce acha que a seta vai parar depois que
ela girar 90 - ou seja, 1/4 de volta no sentido horario?
DESENHE SUA RESPOSTA, COM O PINCEL, DIRETO NA TELA

Para conferir sua resposta basta voce apertar no ENTER



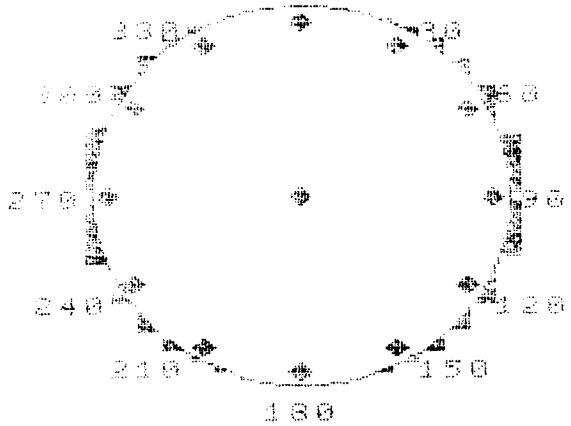
E entao, o lugar onde a seta parou foi o mesmo lugar do
seu desenho?

Foi.

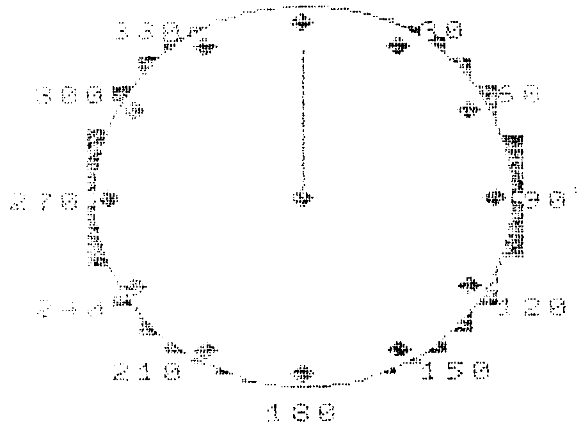
Por que voce achou que a seta iria parar ai?

porque ai e o lugar do 90.

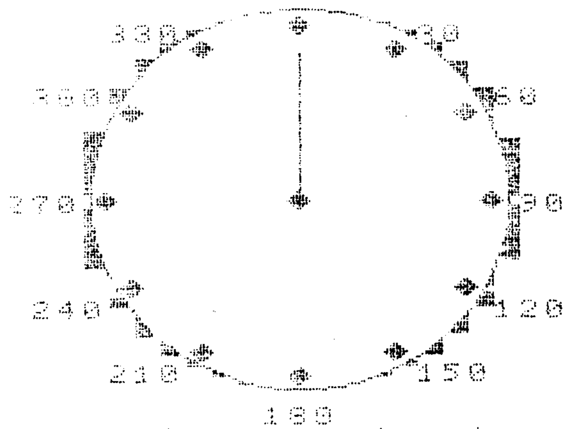
PROGRAMA RELOGIO



QUAL O SEU NOME ?
Sandra

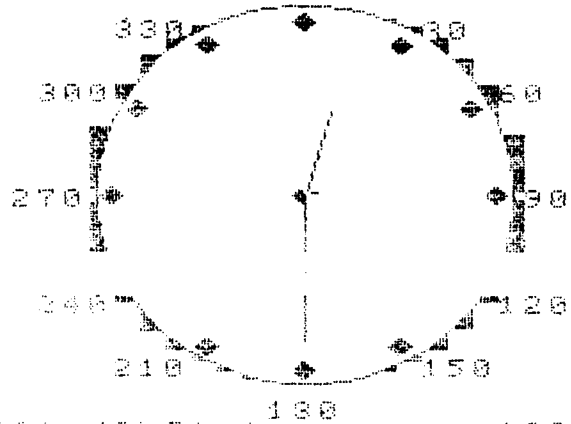


HORAS: 12
MINUTOS: 00



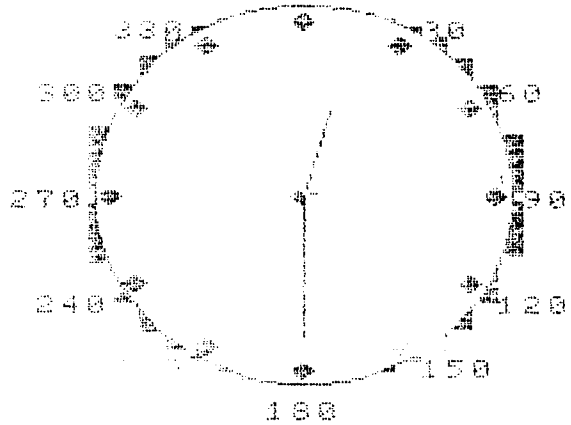
maior vai parar depois
que ele rodar MEIA VOLTA .

180

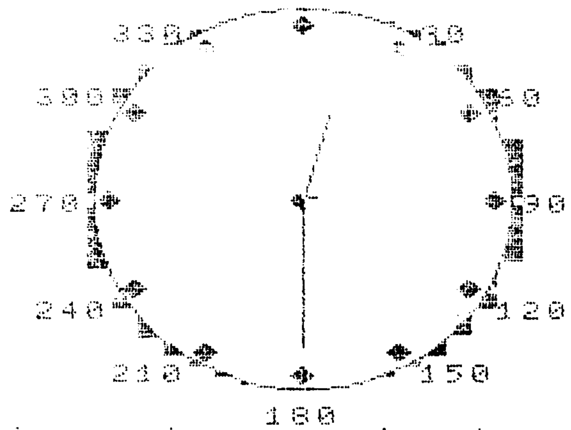


MEIA VOLTA ia ser em 180

Porque é 180 graus.

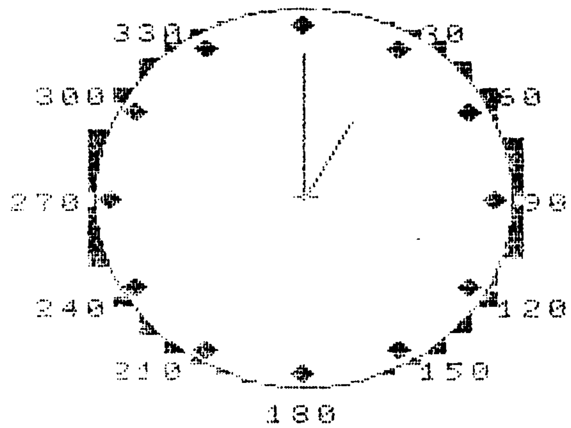


HORAS: 12
MINUTOS: 30

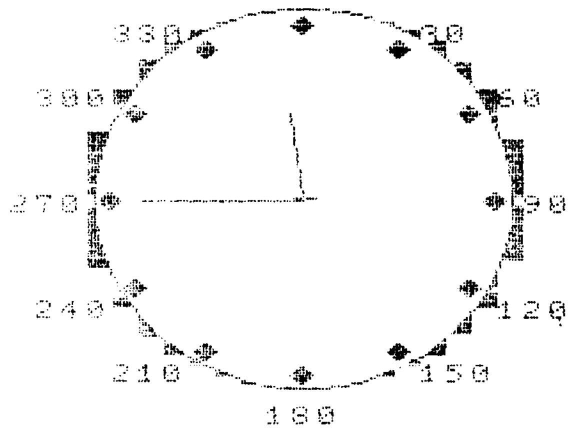


maior vai parar depois
que ele rodar MEIA VOLTA .

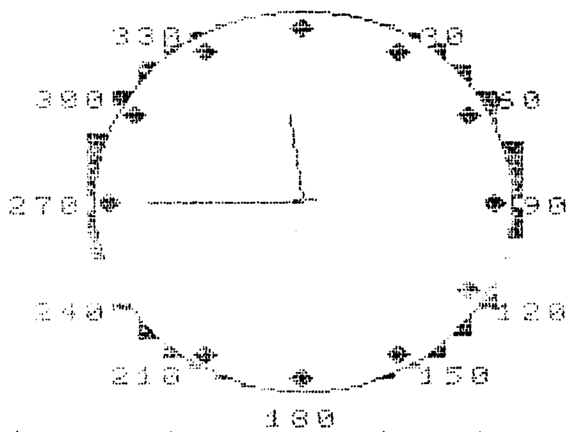
0



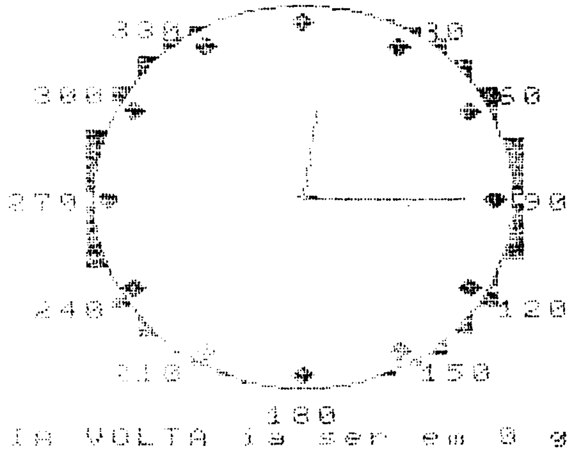
180 + 180 = 360 mesmo que 0. →



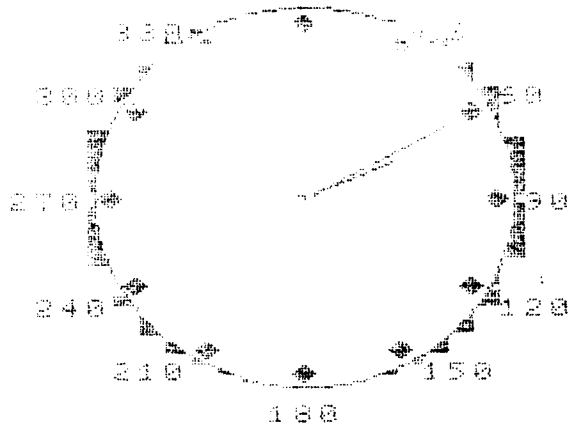
HORAS: 11
MINUTOS: 45



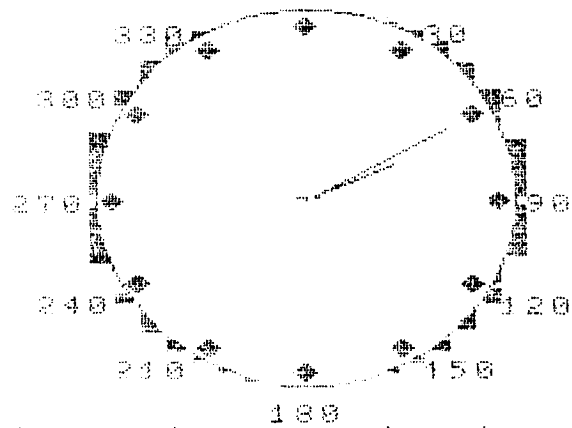
maior vai parar depois
que ele rodar MEIA VOLTA .
0 graus



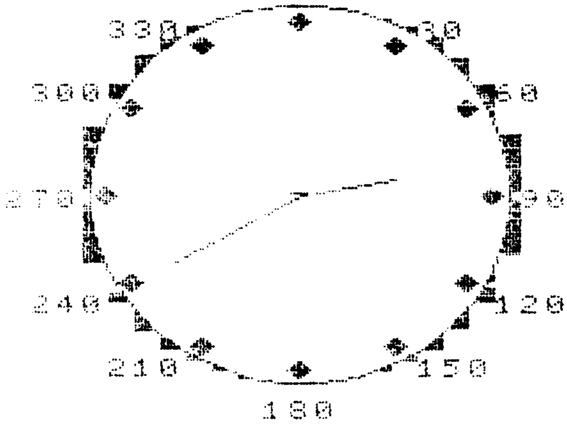
MEIA VOLTA ia ser em 0 graus→
?
Me confundi.



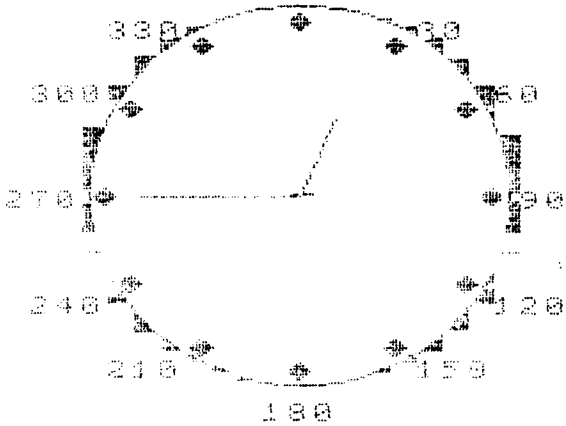
HORAS: 2
MINUTOS: 10



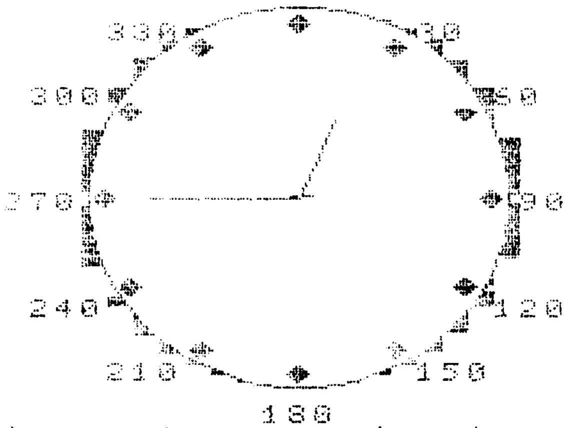
maior vai parar depois
que ele rodar MEIA HORA .
240



10 min. + 30 min = 40 ou 240→
graus.



HORAS: 12
MINUTOS: 45



maior vai parar depois
que ele rodar MEIA HORA .
90