

# **Essays in Labour Economics**

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in partial fulfillment of the requirements for  
the degree of Doctor of Philosophy

by

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# Declaration

I, Michael Reinhold Graber, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Signed:

Date: London, August 2017

# Abstract

In this thesis I study the nature of labour income risk and labour market dynamics.

The first chapter attempts to enhance our understanding of the interplay between labour and financial markets. We develop a simple general equilibrium model with labour market frictions and an imperfect financial market. When calibrating the model to the Great Recession and its aftermath, we find that the lack of an improvement in the financial sector's effectiveness to intermediate resources played a crucial role in the slow recovery of the labour market.

The second chapter uses rich Norwegian population panel data to provide new evidence on labour income risk over the life cycle. We find that the income processes differ systematically by age, skill level and their interaction. Our findings suggest that the redistributive nature of the Norwegian tax–transfer system plays a key role in attenuating the magnitude and persistence of income shocks, especially among the low skilled.

The third chapter presents an estimated income process that is consistent with recent evidence on income risk over the life and business cycle. Using Norwegian population panel data, we estimate an income process that allows income risk within each skill group to depend on the previous income level, calendar time and experience.

The fourth chapter attempts to bridge the gap between the literature on income dynamics and the search and matching literature. We develop a frictional model of the labour market in which inequality in earnings and consumption results from the interaction between heterogeneity of workers, heterogeneity of jobs, shocks to human capital, and labour market frictions.

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# Introduction

A desire to understand the nature of labour market risk is the driving force behind this thesis. The research reported here relates to two strands in the literature and attempts to bridge the gap between them. The first concerns the specification and estimation of labour income processes. Using population panel data from Norway, this thesis reveals new insights into the risks households face from fluctuating incomes. Labour income risk has many dimensions and some of the underlying primitive sources of risk are associated with unemployment, job-mobility and promotions. This naturally leads to a second set of literature which concerns the modeling of the dynamics in labour markets following the tradition of search and matching models.

The first chapter, co-authored with Carlos Carrillo-Tudela and Klaus Wälde, attempts to enhance our understanding of the interplay between labour and financial markets. We develop a simple general equilibrium model with labour market frictions and an imperfect financial market. Our aim is to analyse the transitional dynamics of unemployment and vacancies when financial constraints are in place. We model the financial sector as a monopolistically competitive banking sector that intermediates financial capital between firms. This structure implies a per period financial resource constraint which has a closed form solution and describes the transition path of unemployment and vacancies to their steady state values. We show that the transition path crucially depends on the degree of wage flexibility. When wages do not depend on the unemployment rate, the transition path is always downward sloping. This implies unemployment and vacancies adjust in opposite directions as observed in the data. When calibrating the model to the Great Recession and its aftermath, we find that the lack of an improvement in the financial sector's effectiveness to intermediate resources played a crucial role in the slow recovery of the labour market.

The second chapter, co-authored with Richard Blundell and Magne Mogstad, addresses the following questions: What do labour income dynamics look like over the life-cycle? What is the relative importance of persistent shocks, transitory shocks and heterogeneous profiles? To what extent do taxes, transfers and the

family attenuate these various factors in the evolution of life-cycle inequality? In this chapter, we use rich Norwegian population panel data to answer these important questions. We let individuals with different education levels have a separate income process; and within each skill group, we allow for non-stationarity in age and time, heterogeneous experience profiles, and shocks of varying persistence. We find that the income processes differ systematically by age, skill level and their interaction. To accurately describe labour income dynamics over the life-cycle, it is necessary to allow for heterogeneity by education levels and account for non-stationarity in age and time. Our findings suggest that the redistributive nature of the Norwegian tax–transfer system plays a key role in attenuating the magnitude and persistence of income shocks, especially among the low skilled. By comparison, spouse’s income matters less for the dynamics of inequality over the life-cycle.

The third chapter studies the distributional dynamics of individual income over the life and business cycle. Building on the quantile panel data framework recently developed by Arellano, Blundell, and Bonhomme (2016), we model the income process as a first-order quantile-autoregressive process. We let individuals with different education levels have a separate income process, and within each skill group and cohort we allow the conditional quantile functions to vary unrestrictedly over time. Our approach therefore represents a tractable way of allowing income risk within each skill group to depend on the previous income level, calendar time and experience. We estimate the income process using population panel data from Norway from 1979 to 2013. We find that income dispersion (i) follows a U-shape over the life cycle, (ii) follows a U-shape with the previous income level, and (iii) is procyclical for those at the bottom, acyclical for those in the middle, and countercyclical for those at the top of the distribution of previous income. In addition, we find that skewness (i) follows an inverted U-shape over the life cycle, (ii) declines with the previous income level, and (iii) is procyclical.

The final chapter, co-authored with Jeremy Lise, attempts to bridge the gap between the literature on income dynamics and the search and matching literature. In this chapter we develop a frictional model of the labour market in which inequality in earnings and consumption results from the interaction between heterogeneity of workers, heterogeneity of jobs, shocks to human capital, and labour market frictions. Specifically, we allow for permanent differences across individuals in productivity, the ability to acquire human capital, as well as the ability to find and keep jobs. We consider jobs which differ in both productivity and the extent to which they facilitate further human capital accumulation for workers. Shocks to human capital are permanent, they are carried by the worker even when she

changes jobs. Labour market frictions induce transitory shocks as workers stochastically move up the job ladder. They are transitory in the sense that the direct productivity effect of the job on the worker's wage disappears when the worker leaves the job, either through a spell of unemployment or by a move directly to another job. However, depending on the extent to which jobs differ in the degree to which they facilitate human capital accumulation, and the severity of frictions, there may be permanent effects on the workers human capital and hence earnings arising from differential labour market histories. We demonstrate how the key distributions and parameters of our model are identified in the presence of matched employer-employee data. In a simulation exercise we show that the model is able to reproduce the linear age profile for the variance of log earnings and consumption, as well as the negative skewness and excessive kurtosis of the distribution of earnings growth, including the increasing negative skewness of earnings growth conditional on the previous earnings level.

# Chapter 1

## Unemployment and Vacancy Dynamics with Imperfect Financial Markets

### 1.1 Introduction

The Great Recession has highlighted the importance financial markets can have on the performance of the labour market. Most countries that were affected by the 2007/2008 financial crisis saw a burst of layoffs that made their unemployment rates increase dramatically and stay stubbornly high during the recovery period. Since the financial crisis particularly affected the ability of firms to expand and create new vacancies due to the lack of available investment funds, it has been argued that the credit crunch played an important role in slowing down the recovery of unemployment. In this paper we investigate to what extent the adjustment of unemployment and vacancies depends on the effectiveness of financial markets to intermediate resources.

We construct a simple general equilibrium model with labour market frictions and an imperfect financial market. Our focus is to analyse the transitional dynamics of unemployment and vacancies in response to unexpected job displacement and financial shocks. We are particularly interested in assessing the effects of these shocks on unemployment and vacancies when financial constraints are in place.

Our framework extends the canonical search and matching model (as described in Pissarides, 2000) by adding a monopolistically competitive banking sector that firms must visit in order to finance job creation. Once jobs are filled and firms become productive, they service their debts over time until the job is exogenously destroyed. This simple structure implies that at any point in time firms flow

profits must be used to cover the cost of posting vacancies. The resulting per period financial resource constraint is the key element of our analysis. It shows that the number of vacancies is positively related to firms' flow profits. Since in the search and matching framework, the latter is directly related to the level of unemployment, the resource constraint then describes the relation unemployment and vacancies must satisfy to guarantee equilibrium in the banking sector.<sup>1</sup> Furthermore, this constraint has a closed form solution and describes the transition or saddle path of the economy towards its steady state.

These features allow us to characterize the out-of-steady-state dynamics of our model. In particular, we are able to characterize how the out-of-steady-state dynamics of unemployment and vacancies depend on unexpected changes to the variables of interest. A decrease in labour productivity or in the productivity of the banking sector, for example, shifts the transition path downwards and decreases the rate at which unemployment and vacancies adjust towards the new steady state. Changes in the rate at which employed workers become unemployed or changes in the parameters governing the matching technology generate movements along the transition path without affecting the rate at which unemployment and vacancies adjust towards the new steady state.

We show that the dynamics of unemployment and vacancies along the transition path crucially depends on the degree to which labour market tightness and, in particular, the unemployment rate affects wages. When firms and workers Nash bargain over the expected match surplus (as is traditionally assumed), wages depend on unemployment because agents' outside options and disagreement payoffs equal their respective values of search. In this case unemployment and vacancies adjust following a non-monotonic relationship. For low levels of unemployment and vacancies, they both adjust in the same direction. For larger values they adjust in opposite directions. The resource constraint implies that as unemployment increases the number of productive firms decreases, reducing the available funds for job creation and generating a negative relation between unemployment and vacancies. However, the non-monotonicity arises due to the feedback effect of the job finding rate on wages and ultimately profits. An increase in unemployment reduces workers' outside options and increases firms' flow profits, which in turn implies there are more funds per productive firms to finance vacancies, generating a positive relation between unemployment and vacancies. At low levels of unemployment and vacancies the latter force dominates, while for larger values the former force dominates. As wages become less dependent on the unemployment

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<sup>1</sup>Benmelech, Bergman, and Seru (2011) document, using data for the US and Japan, a negative relation between firms' cash flows and employment as well as a negative relation between the extent of credit availability to firms and local unemployment rates.

rate due to some form of wage rigidity, this feedback effect diminishes.<sup>2</sup> We show that when there is no interaction between unemployment and wages, such as when firms and workers Nash bargain over the *flow* surplus (see Marcusse, 2016), one always obtains a negative relation between unemployment and vacancies along the transition path.

Being able to generate such an adjustment process is important as a main feature of the canonical search and matching model, as described in Pissarides (2000), is that unemployment and vacancies always adjust in the same direction along the transition path. Blanchard, Diamond, Hall, and Yellen (1989) document, however, that unemployment and vacancies adjust in opposite directions after a shock to output or to the rate at which workers and firms separate. Furthermore, Shimer (2005) and many others have shown that when the canonical search and matching model is calibrated to the US, shocks to the job destruction rate generate a counterfactual positive correlation between unemployment and vacancies. Given that we are able to solve for the transition path in closed form, we provide an analytical characterization of how unexpected changes to aggregate output, the job destruction rate or to banking sector parameters affect the rate by which unemployment and vacancies adjust between steady states.

In the quantitative section of the paper we analyze whether the out-of-steady-state dynamics implied by our model can replicate the observed dynamics of the unemployment and vacancy rate in the US economy for the period 2007-2014. Through the lenses of our model, we interpret a sequence of observed unemployment and vacancy points as movements along a saddle path towards a new steady state. We first calibrate the model to match the transition of the unemployment and vacancy rates from the beginning to the end of the Great Recession. We then explore changes in vacancy costs, the rate of job destruction and in the parameters governing the financial sector that can account for the observed unemployment and vacancy dynamics during the recovery period. We find that the calibration favours wages that are isolated from the unemployment rate. This property delivers a downward sloping transition path that replicates the observed sequence of unemployment and vacancy points very well. The main message of this exercise is that the observed slow recovery in the labour market was due to the lack of a significant improvement in the effectiveness of the banking sector in intermediating resources to fund job creation.

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<sup>2</sup>In the sequential bargaining protocol proposed by Hall and Milgrom (2008), for example, outside options are still described by the agents' values of search but are no longer equal to their disagreement payoffs. In this case, unemployment affects wages through the risk of negotiation breakdown, which in the Hall and Milgrom (2008) calibration, drastically reduces the effects of unemployment on wages.



Our approach to model the banking sector using a monopolistic competitive structure is consistent with the evidence presented by Bikker and Haaf (2002). These authors evaluate the degree of competition and concentration of the banking sector in 23 OECD countries, including the US, for the period 1988 to 1998. They find that across all these countries the banking sector can be best characterised by monopolistic competition. Kadir, Habibullah, Hook, and Mohamed (2015) show that our approach to model the banking sector is also in accordance with the vast majority of studies that evaluate the degree of competition and concentration of the banking sector across developed and developing economies.<sup>3</sup> Gerali, Neri, Sessa, and Signoretti (2010) and La Croce and Rossi (2015) followed this approach and embed a monopolistic competitive banking sector into a dynamic general equilibrium framework. They, however, assume a perfectly competitive labour market and hence cannot explore the interaction between imperfect competition in the banking sector and search frictions in the labour market. Recently a large body of work has appeared which studies the interaction between financial and labour market frictions. To be best of our knowledge, none of these studies formulates the banking sector using a monopolistically competitive market structure. For example, Carlstrom and Fuerst (1997), Chugh (2013) and Petrosky-Nadeau (2014), among many others, assume a competitive financial market where frictions arise due to costly state verification. In this environment, the current (idiosyncratic) state of the firm (the borrower) is private information to the firm, but the lender can learn it by paying a verification cost. Firms and lenders sign one-period debt contracts that specify the size of the loan and a liquidation threshold. Jermann and Quadrini (2012), Garin (2015) and Buera, Jaef, and Shin (2015), among others, use a somewhat related approach and assume frictions arise because the ability of firms to borrow is limited by an enforcement constraint which is subject to random shocks. Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2013) propose an alternative approach and model the financial market as a frictional market governed by a meeting function that brings together lenders and vacant firms, taking the interest rate as given.

Most of the aforementioned papers also develop their theories in the context of a dynamic stochastic general equilibrium environment or in the context of a stochastic version of the search and matching model (see Andolfatto, 1996, and Shimer, 2005, among others). Instead we propose a model without aggregate uncertainty (as in Pissarides, 2000). In our model the evolution of unemployment and vacancies is studied after an unexpected shock to aggregate variables using the out-of-

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<sup>3</sup>Kadir, Habibullah, Hook, and Mohamed (2015) present an extensive list of studies that find that across developed and developing countries the banking sector can be characterised by monopolistic competition.

steady-state dynamics implied by our model, characterised by the transition path between steady states. In reality the dynamics of unemployment and vacancies are probably driven by a combination of a sequence of random shocks and movements along a transition path that shifts when these shocks get realised. Our parsimonious approach allows us to solve and explore the model's mechanism analytically. We emphasize the resource constraint, imposed by the financial sector, as an important determinant of the relationship between unemployment and vacancies in an economy's adjustment process.

The rest of the chapter is outlined as follows. In the next section we present the search and matching model that describes the aggregate labour market and the monopolistically competitive banking model that describes the financial sector. In Section 1.3 we characterise the equilibrium and discuss the steady state and out-of-steady-state dynamics. Here we analyse the main implications of the financial resource constraint on the transition path of vacancies and unemployment and show how parameters governing the financial market affect this transition path. Section 1.4 presents the calibration procedure and describes its main results. Section 1.5 concludes discussing briefly the main results of the chapter. All proofs and tedious derivations are relegated to a technical Appendix.

## 1.2 The Model

### 1.2.1 Basic Framework

The labour market setup follows Pissarides (2000, ch. 1). Since our objective is to understand the transitional dynamics of the economy we consider out-of-steady-state analysis. Time is continuous with infinite horizon. There is a unit mass of workers and a mass of firms. Both agents discount the future at a potentially time-dependent interest rate  $r(t)$ . Workers can be either employed or unemployed. Unemployed workers receive constant benefits  $z$  per unit of time. An employed worker receives a wage rate of  $w(t)$  per unit of time. Each firm has only one job that can be either vacant or filled. A filled job generates a constant flow of output  $p > z$ . A firm with a vacant job pays a cost measured in terms of productivity units of  $k > 0$  per unit of time. Jobs are destroyed at an exogenous Poisson rate  $s > 0$ . Once destroyed, the firm's job becomes vacant and the worker becomes unemployed.

Agents must search for each other to find a match. The search process is sequential and random and we assume that only vacancies and unemployed workers search. Meetings are governed by a meeting or "matching function"  $m(u(t), v(t))$  which

gives the number of meetings that take place per unit time as a function of the number of unemployed workers  $u(t)$  and the number of vacancies  $v(t)$ . Assume that  $m(\cdot)$  is increasing and concave in both arguments and exhibits constant returns to scale. Let  $\theta(t) \equiv v(t)/u(t)$  denote the labour market tightness. Constant returns to scale then imply that the job filling rate is given by  $q(\theta(t)) \equiv m(u(t), v(t))/v(t)$ , while the job finding rate is then  $\lambda(\theta(t)) = \theta(t)q(\theta(t))$ . These rates govern the Poisson processes by which agents meet in this labour market.

### 1.2.2 Bellman Equations for Workers and Firms

Workers and firms are risk neutral. The workers' objective is to maximise the expected present value of their lifetime income  $\mathbb{E}_0 \int_0^\infty e^{-\int_0^s r(s)ds} y(t) dt$  with flow income  $y(t) = \{z, w(t)\}$ . Let  $U$  denote the expected value of an unemployed worker and let  $W$  denote the expected value of a worker employed at some net wage  $w$ . Dynamic programming arguments imply that the following  $U$  and  $W$  satisfy the Hamilton-Jacobi-Bellman equations

$$r(t)U(t) = z + \dot{U}(t) + \lambda(\theta(t))[W(t) - U(t)], \quad (1.1)$$

$$r(t)W(t) = w(t) + \dot{W}(t) + s[U(t) - W(t)]. \quad (1.2)$$

The firm's flow profit from a filled job is

$$\pi(t) = p - w(t). \quad (1.3)$$

An unfilled job yields a flow profit of  $-k$  with  $k > 0$ . Firms are infinitely lived and their objective is to maximise the expected present value of total profits  $\mathbb{E}_0 \int_0^\infty e^{-\int_0^s r(s)ds} \varpi(t) dt$  with flow profits  $\varpi(t) = \{-k, \pi(t)\}$ . Let  $V$  denote the expected value of holding a job vacant. Let  $J$  denote the expected value of a filled job paying  $w$ . We then obtain the Hamilton-Jacobi-Bellman equations by dynamic programming arguments,

$$r(t)V(t) = -k + \dot{V}(t) + q(\theta(t))[J(t) - V(t)], \quad (1.4)$$

$$r(t)J(t) = \pi(t) + \dot{J}(t) + s[V(t) - J(t)]. \quad (1.5)$$

The interpretation of the above equations is identical to the canonical search and matching model only that discounting takes place at an endogenous interest rate  $r(t)$ .

### 1.2.3 Free Entry and Wage Determination

For given tightness and wage rate, the number of vacancies is determined by a free entry condition. As long as the value  $V$  of opening a vacancy is positive, firms will create vacancies and enter the labour market. Firms will stop entering only when there are no more (intertemporal) profits to be made, i.e.  $V = 0$ . Using the Bellman equation (1.4), we obtain that

$$J(t) = \frac{k}{q(\theta(t))}. \quad (1.6)$$

When an unemployed worker and a vacant firm meet,  $p > z$  ensures that they immediately form a productive match. It has been standard to use the generalised Nash bargaining solution as a way to determine wages. In this case, it is typically assumed that the worker's and firm's outside options and disagreement payoffs are the same and given by  $U(t)$  and  $V(t)$ , respectively. This protocol implies that agents receive a constant fraction of the expected match surplus and yields a fully flexible wage that is a linear function of  $p, z$  and  $\theta$ . On the other extreme, Hall (2005) proposed an alternative wage determination mechanism motivated by his observation that wages do not seem to behave as spot wages in the data. He uses a Nash demand game in which wages are fixed within the bargaining set. In this setup, wages are not renegotiated until they lie outside the bargaining set and hence prevent inefficient separations.

Other wage determination mechanisms that lie somewhere in between the above two cases have also been studied in the literature. In particular, the sequential bargaining protocol proposed by Hall and Milgrom (2008) generates a form of wage rigidity by partially isolating wages from the influence of  $\theta$ . The crucial aspect of their bargaining protocol is that agents' disagreement payoffs are no longer equal to the agents' outside options,  $U(t)$  and  $V(t)$ . The disagreement payoffs are independent of  $\theta$  and agents receive their outside options only when the negotiations break down, which happens with some probability every period.<sup>4</sup>

In this paper we follow an agnostic approach to wage determination and use the following relation:

$$w(t) = (1 - \beta)z + \beta[p + \tau\theta(t)k]. \quad (1.7)$$

In this wage equation  $\beta$  is the worker's exogenous bargaining power standard in the Nash bargaining protocol. The crucial parameter, however, is  $\tau \in [0, 1]$  which determines the extent to which  $\theta$  affects wages. When  $\tau = 1$ , for example, we

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<sup>4</sup>See also the staggered wage setting protocol proposed by Gertler and Trigari (2009) as another example of a wage determination protocol that delivers wage rigidity.

are back to the well known Nash bargaining solution of the canonical search and matching framework. When  $\tau \in (0, 1)$  we have an outcome that partially isolates the wage from the influences of labour market tightness. This case captures, in reduced form, the *spirit* of Hall and Milgrom's (2008) wage determination protocol. When  $\tau = 0$  the outcome is the same as the one obtained when workers and firms Nash bargain over the *flow* surplus,  $p - z$ . In this case, wages are fully isolated from the influence of labour market tightness.<sup>5</sup>

The main reason for the choice of the functional form presented in equation (1.7) is that we are interested in understanding the role wage rigidity plays in the interaction between the financial and labour markets. Equation (1.7) presents a specification that allows us to do this in a simple and tractable way. In the quantitative section we recover  $\tau$  and  $\beta$  from our calibration procedure.

### 1.2.4 Equilibrium without a Financial Sector

Given a time-dependent interest rate, equilibrium can then be described by the evolution of the unemployment rate  $u(t)$  and the evolution of labour market tightness  $\theta(t)$ . Inflows to unemployment amount to  $s[1 - u(t)]$ , while  $\lambda(\theta(t))u(t)$  unemployed individuals find a job at each instant. For a path of labour market tightness, the unemployment rate  $u(t)$  in this economy evolves over time according to

$$\dot{u}(t) = s[1 - u(t)] - \lambda(\theta(t))u(t). \quad (1.8)$$

The evolution of labour market tightness can be determined by the value of a filled vacancy (1.6) and by an equation describing its evolution over time. After some steps (see Appendix A.1) we obtain a differential equation describing the evolution of labour market tightness

$$\dot{\theta}(t) = \left[ \frac{q(\theta(t))}{q'(\theta(t))} \right] \left[ \frac{(1 - \beta)}{k} (p - z)q(\theta(t)) - r(t) - s - \tau\beta\lambda(\theta(t)) \right]. \quad (1.9)$$

Equations (1.8) and (1.9) determine the paths of  $u(t)$  and  $\theta(t)$  as a function of  $r(t)$ .

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<sup>5</sup>Marcusse (2016) argues that Nash bargaining over the flow surplus can be thought of as a similar bargaining protocol as that of Hall and Milgrom (2008) under the conditions that offers arrive instantaneously and there is no risk of negotiation breakdown during bargaining. Mortensen and Nagypal (2007) also argue that one can obtain the wage equation  $w = z + 0.5 \times (p - z)$  as the solution to a symmetric sequential bargaining game under the assumption that a worker obtains flow utility  $z$  and the firm a zero flow payoff while bargaining continues and that agents negotiate over  $p$ .

### 1.2.5 The Financial Sector

We now close our matching model by including a financial sector. We consider a banking sector that is the only source for financing the vacancy costs and to which all profits of productive firms flow. In this environment, a potential market entrant that wants to finance a vacancy must visit a bank and ask for a flow of resources allowing to cover the vacancy costs  $k$  to be paid at each point in time until a worker is found. In order to get these resources, the firm needs to sign a contract that says that the entrant commits to repay the bank by the flow of profits it makes once the vacancy is filled and until the next separation takes place. The bank bears all the risk and diversifies across all entrants and productive firms such that the bank behaves as if the world was deterministic.

Suppose that the banking sector consists of  $n(t)$  different types of banks offering each one single banking service  $i$  at any time  $t$ . Banks operate under monopolistic competition. Financial services are aggregated to one big “financing package for opening a vacancy” by a technology of the Dixit-Stiglitz type

$$Y(t) = \left[ \int_0^{n(t)} x(i, t)^\gamma di \right]^{1/\gamma}, \quad (1.10)$$

where  $x(i, t)$  is the amount of services provided by bank  $i$  at time  $t$  and  $\gamma \in (0, 1)$  determines the degree of substitution between financial services. The elasticity of substitution between financial services is given by  $(1 - \gamma)^{-1}$ .

Banking service  $i$  is produced by the technology

$$x(i, t) = by(i, t) - \phi, \quad (1.11)$$

where  $b$  is a productivity parameter,  $y(i, t)$  is the input of the final good produced and consumed in this economy, and  $\phi$  describes the fixed costs to be paid by monopolistic competitors. Just as with vacancy costs, fixed costs in the banking sector are measured in units of the output good.

Service providers maximise profits by choosing output  $x(i, t)$  optimally at each point in time. As all firms use the same technology, service provision will be symmetric and the usual steps (see Appendix A.2.1) imply aggregate output of the banking sector (1.10) amounts to

$$Y(t) = n(t)^{1/\gamma} x(t) = n(t)^{1/\gamma} \left[ b \frac{\pi(t)[1 - u(t)]}{n(t)} - \phi \right]. \quad (1.12)$$

A crucial assumption here is that we require that all resources available for financ-

ing vacancies must actually be used for financing vacancies. Resources must not be lost or allowed to enter the model. Making such a market-clearing assumption for the banking sector implies that the aggregate banking output (1.12) equals the total cost of financing vacancies. The latter is given by the cost  $k$  per vacancy times the number of vacancies,  $\theta(t)u(t)$ , such that

$$n(t)^{1/\gamma} \left[ b \frac{\pi(t)[1-u(t)]}{n(t)} - \phi \right] = k\theta(t)u(t). \quad (1.13)$$

Economically speaking, market clearing for financial services (1.13) determines the number of vacancies  $v(t)$ . Technically, as vacancies are already determined in (1.9), this additional market fixes the endogenous interest rate  $r(t)$ .

While the resource constraint described in (1.13) makes sure that resources used for financing vacancies can only come from profits made by firms, it does not guarantee that there are no resources left unused. As monopolistic service providers make a profit, this profit needs to go somewhere. It can actually not be ruled out at this point that firms would even make negative profits, given that there are fixed costs  $\phi$  to be paid per period. To guarantee that all resources supplied by firms making a profit are used either for covering fixed costs for the provision of services or for financing vacancies, we apply the standard assumption here as well and assume that there is free entry into and exit from the banking sector. This implies (see Appendix A.2.2) that the number of services is given by

$$n(t) = (1 - \gamma)b \frac{\pi(t)[1-u(t)]}{\phi}. \quad (1.14)$$

Substituting (1.14) into (1.13) and some algebra (see Appendix A.2.3) establishes that

$$\frac{\theta(t)^\gamma}{(1 - \beta)^{\frac{p-z}{k}} - \tau\beta\theta(t)} = b \left[ (1 - \gamma) \frac{k}{\phi} \right]^{1-\gamma} \frac{1-u(t)}{u(t)^\gamma} \gamma^\gamma. \quad (1.15)$$

Equation (1.15) describes the resource constraint that is consistent with free entry in the banking sector. Under this specification, all profits made by firms are used for financing vacancies and all costs of vacancies are financed by firms' profits. Profits made by banks are used to pay their fixed costs. This makes sure that the financial market is in equilibrium, no resources leave or enter the model and we have specified a general equilibrium matching model.

## 1.3 General Equilibrium

Equations (1.8) and (1.9) provide the basis to understand the goods and labour market dynamics by describing  $\dot{u}(t)$  and  $\dot{\theta}(t)$ . Equation (1.15) describes equilibrium in the financial market. These three equations simultaneously solve for  $u(t)$ ,  $\theta(t)$  and  $r(t)$ . Before we describe the out-of-steady-state dynamics of this system, we analyse its steady state.

### 1.3.1 Zero-motion Lines and Steady State

From equation (1.8) we obtain that the zero-motion line for  $u$  is given by

$$\lambda(\theta) = s \frac{1-u}{u} \Leftrightarrow u = \frac{s}{s + \lambda(\theta)}, \quad (1.16)$$

which describes a negative relationship between  $u$  and  $\theta$ . The zero-motion line for  $\theta$  is implicitly given, from (1.9), by

$$(1 - \beta) \frac{p-z}{k} q(\theta) - s - r(t) - \tau \beta \lambda(\theta) = 0. \quad (1.17)$$

What is special about this zero-motion line is that the interest rate is a function of time which means that the zero-motion line shifts in the  $(u - \theta)$  space. What is standard is that the zero-motion line for  $\theta$  is not a function of  $u$ , i.e. it is horizontal in the  $(u - \theta)$  space.

In standard descriptions of phase diagrams, the equilibrium path is to be inferred from the zero-motion lines and laws of motions subsequently. In our system, however, the equilibrium path towards the steady state is described in closed-form by (1.15). Using the latter equation it is easy to verify that  $\theta$  falls with  $u$ : The right-hand side unambiguously falls in  $u$  while the left-hand side rises in  $\theta$ . Our general equilibrium matching model therefore provides an explicit expression for the transition path in terms of unemployment and vacancies, which we discuss below.

In a steady state, the unemployment rate, labour market tightness and the interest rate are constant. Denote their steady state values as  $u^*$ ,  $\theta^*$  and  $r^*$ . To show the existence of a steady state note from (1.16) that as  $u$  goes to zero,  $\theta$  grows unboundedly; and while as  $u$  goes to one,  $\theta$  goes to zero. From (1.15), however, we have that as  $u$  goes to zero,  $\theta$  goes to  $(1 - \beta)(p - z)/\tau \beta k$ ; while as  $u$  goes to one,  $\theta$  goes to zero. Hence these functions intersect at  $u = 1$  and  $\theta = 0$ . Further, since these functions are continuous and decrease monotonically, they can intersect at most once at some  $u \in (0, 1)$  and  $\theta \in (0, \infty)$ . Given the steady state values of  $u$



and  $\theta$ , the interest rate  $r$  then adjusts such that (1.17) holds. Since in the case in which  $u = 1$  and  $\theta = 0$  (1.17) implies  $r$  is undetermined, in what follows we focus on characterising the transition dynamics towards the interior steady state,  $(u^*, \theta^*, r^*)$ , given that one exists.

In Appendix A.3 we provide a sufficient condition under which a unique interior steady state exists for any CRS matching function. Further, we show that under a Cobb-Douglas matching function  $M(u, v) = Au^\alpha v^{1-\alpha}$ , the parametric restriction  $\gamma + \alpha \geq 1$  is sufficient (but not necessary) to guarantee existence of an interior steady state equilibrium.

### 1.3.2 Transitional Dynamics

An insightful way to analyse the transition path described in (1.15) is to consider it in Beveridge space; i.e.  $v - u$  space. It is well documented that unemployment and vacancies move in opposite directions. Blanchard, Diamond, Hall, and Yellen (1989) and, more recently, Shimer (2005) and Sniekers (2016) show that unemployment and vacancies move in opposite directions during the adjustment process of the US economy; and similar results have been obtained for European countries (see Elsby, Hobijn, and Şahin, 2013). It is of interest to understand the conditions under which the interaction between the labour market and the financial sector, as modeled in this chapter, has the potential to generate such a negative relation.

Re-writing equation (1.15) in  $v - u$  space, we obtain that the sign of the slope of the transition path is determined by (see Appendix A.4)

$$\text{sign} \left[ \frac{dv}{du} \right] = \text{sign} \left[ \frac{\partial \pi}{\partial u} (1 - u) - \pi \right].$$

To understand this condition, note that equation (1.13) implies that aggregate firm profits and the number of vacancies must move in the same direction along the equilibrium path. Aggregate firms' profits, however, depend positively on (i) the number of jobs filled and negatively on (ii) the wage paid to workers; and both are inversely related to the unemployment rate. The slope of the transition path then depends on how responsive wages are to changes in the unemployment rate.

Using the expression for  $\partial \pi / \partial u$ , we find that

$$\frac{dv}{du} < 0 \Leftrightarrow \tau < \tau^* \equiv \frac{u^2(1 - \beta)(p - z)}{\beta k v}, \quad (1.18)$$

where  $\tau^*$  describes the threshold value of our wage rigidity parameter such that when  $\tau < \tau^*$  the transition path is downward sloping and when  $\tau > \tau^*$  the tran-

sition path is upward sloping. Note that for values  $\tau \in (0, 1]$ , the transition path is typically non-monotonic as  $\tau^*$  changes with  $u$  and  $v$  satisfying (1.15).<sup>6</sup> When  $\tau = 0$ , however, this non-monotonicity disappears. In this case the feedback effect between unemployment and profits disappears,  $\partial\pi/\partial u = 0$ , and the transition path is downward sloping for all values of  $u$  and  $v$ .

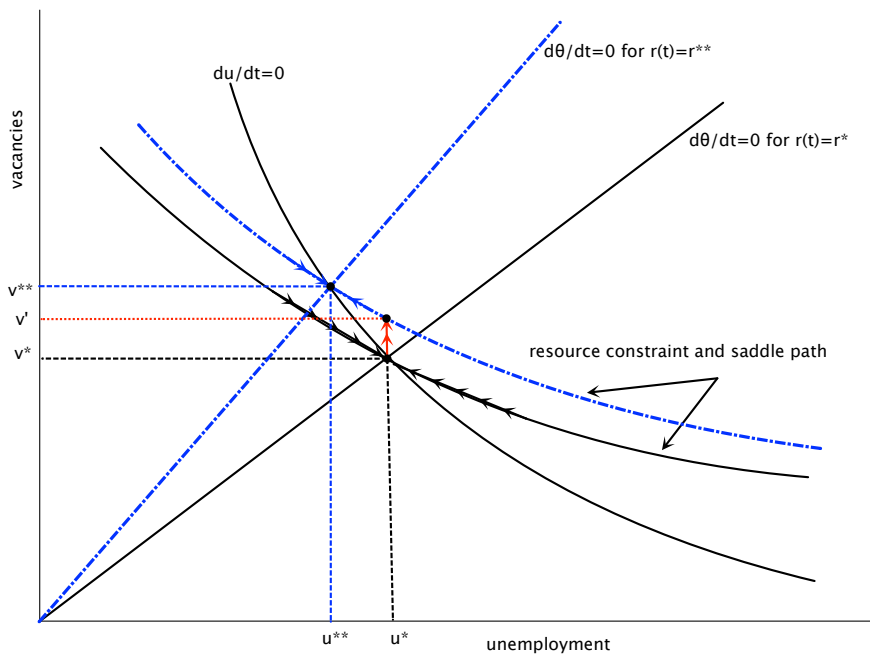
This feature incorporates an important dimension to the canonical search and matching model. In the latter, with a constant interest rate, the transition path towards the steady state is given by the zero-motion line for  $\theta$  for any value of  $\tau$ . This implies that during adjustment, vacancies and unemployment move in the same direction irrespectively of the degree of wage rigidity as modeled in (1.7). Here the above arguments imply that during adjustment vacancies and unemployment can move in *opposite* directions.

### 1.3.3 Changes in Output and the Job Destruction Rate

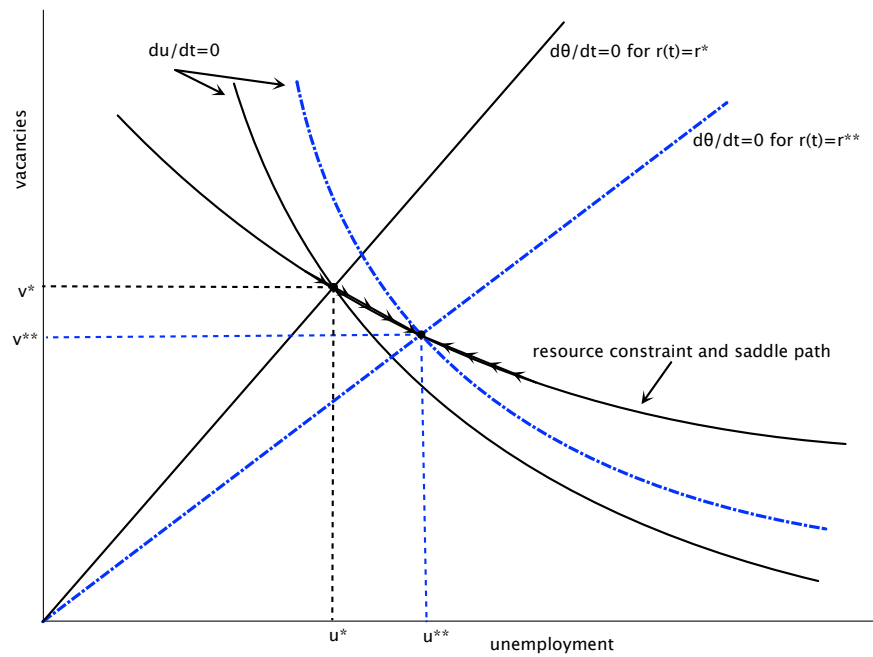
To illustrate this difference consider a one time unexpected increase in aggregate productivity  $p$ . Figure 1.1a depicts this exercise in  $v - u$  space assuming the existence of an interior steady state and a range of values for  $v$  and  $u$  within which the transition path is downward sloping. In this section we want to show the qualitative workings of the model under the latter conditions. In Section 1.4.3 we explore quantitatively how much insulation from  $\theta$  is required to obtain a downward sloping transition path on the relevant range of values for  $v$  and  $u$ .

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<sup>6</sup>In all our numerical simulations we find that in this case the transitions path first increases and then decreases as we increase  $u$ .



(a) Increase in aggregate productivity



(b) Increase in the job destruction rate

Figure 1.1: Transitional dynamics after a one time unexpected shock to  $p$  and  $s$

An increase in  $p$  generates in both models an upward rotation of the zero-motion line for  $\theta$ . In our model, in addition, the resource constraint shifts outwards. In the figure, the new curves are depicted as dashed curves. The increase in  $p$  makes labour market tightness jump upwards as firms create new vacancies up to the point in which the economy is on the new transition path at the original unemployment rate. In our model the initial jump of vacancies (shown by the solid arrowed line from  $v^*$  to  $v'$ ) is smaller than in the canonical search and matching model as our transition path lies below the zero-motion line for  $\theta$ , over the relevant range. Further, along this path the unemployment rate decreases, while the vacancy rate increases. These transitional dynamics then yield counter-clockwise movements of  $u$  and  $v$  and a new steady state that is characterised by a higher vacancy rate and a lower unemployment rate. These features are consistent with the evidence in Blanchard, Diamond, Hall, and Yellen (1989), who show that the counter-clockwise movements of  $u$  and  $v$  around the zero-motion line for  $u$  after a productivity shock involve these rates moving in opposite directions.

As a second example consider a one time unexpected increase in the job destruction rate,  $s$ . Figure 1.1b shows this exercise. Here the difference between the two models is starker. After an increase in  $s$ , both models imply that the zero-motion line for  $u$  shifts to the right, while the zero-motion line for  $\theta$  rotates downwards (in  $v - u$  space). Once again, in the figure, these are depicted by the dashed curves. In the canonical model, however,  $v$  jumps downwards, while  $u$  stays constant immediately after impact. As the economy adjusts, both variables then increase along the new zero-motion line for  $\theta$  until the new steady state is achieved. In our model, the transition path does not depend on  $s$  (see 1.15), which implies that  $v$  does not jump. Instead  $v$  decreases and  $u$  increases smoothly along the transition path until the new steady state is achieved. Furthermore, an increase in the job destruction rate will always imply a new steady state with a lower vacancy rate and a higher unemployment rate, while in the canonical model the new steady state can be characterised by a higher vacancy and unemployment rates. These features are also consistent with the evidence presented in Blanchard, Diamond, Hall, and Yellen (1989) and Shimer (2005) on the effects of reallocations shocks on  $v$  and  $u$ .

Note also that in our model changes in the matching function parameters will have similar effects as changes in the job destruction rate, although in the opposite direction. For example, consider a Cobb-Douglas matching function,  $M(u, v) = Au^\alpha v^{1-\alpha}$ . Changes in  $A$  and  $\alpha$  will shift the zero-motion line for unemployment along the transition path of the economy, generating different dynamics relative to the canonical model.

In addition since in our model changes in  $p$  affect both the position and the slope of the transition path, output affects the rate at which unemployment and vacancies adjust from one steady state to another. On the contrary, since the job destruction rate and the matching function parameters do not determine the transition path, the rate at which unemployment and vacancies adjustment along the transition path to the new steady state is independent of these variables.

### 1.3.4 Changes in the Financial Sector

We now consider how the market power of banks, measured by the price mark-up,  $1/\gamma$ , the productivity of each bank,  $b$ , and the fixed cost,  $\phi$ , affect the transitional dynamics of this economy and the steady state equilibrium. For this purpose, re-write (1.15) in implicit form as

$$\Psi(\theta, u) \equiv \theta u - \frac{\gamma\phi}{(1-\gamma)k} \left[ \frac{b(1-\gamma)[(1-\beta)(p-z) - \tau\beta k\theta][1-u]}{\phi} \right]^{1/\gamma} = 0. \quad (1.19)$$

Note that its slope is given by

$$\frac{d\theta}{du} = - \frac{n[\theta(1-\gamma)k(1-u) + \phi n^{\frac{1}{\gamma}}]}{k(1-\gamma)(1-u)[nu + n^{\frac{1}{\gamma}}\tau(1-u)b\beta]} < 0, \quad (1.20)$$

and recall that when  $u = 0$ ,  $\theta = (1-\beta)(p-z)/\tau\beta k$  and that when  $u = 1$ ,  $\theta = 0$ . Therefore, since the intersections of  $\Psi$  with the axis are independent of  $\gamma$ ,  $\phi$  or  $b$ , to analyse the impact of these variables on the transition path it is sufficient to analyse their impact on (1.20).

First consider an increase in the banks' fixed cost. Differentiation of (1.20) with respect to  $\phi$  implies that the transition path experiences a leftward expansion and becomes flatter for all values of  $u$  when  $(1-\beta)(p-z)u > \tau\beta\theta k$ . Note that this is the same condition required to guarantee a downward sloping transition path in Beveridge space. Since the zero-motion line for unemployment is independent of  $\phi$ , in those cases in which the latter condition is satisfied, the new steady state unemployment rate increases, while labour market tightness decreases which, by virtue of (1.17), implies that the interest rate increases. Because the transition path becomes flatter, the rate at which unemployment and vacancies arrive to the new steady state decreases.

Now consider an increase in bank's productivity. Differentiation of (1.20) with respect to  $b$  implies that the transition path experiences a rightward expansion and becomes steeper for all values of  $u$  when  $(p-z)(1-\beta)u > \tau\beta\theta k$ . In these cases and given that the zero-motion line for unemployment is independent of  $b$ , the new

steady state is characterised by a lower level of unemployment, a higher labour market tightness and, by virtue of (1.17), a lower interest rate. Furthermore, the rate at which unemployment and vacancies arrive to the new steady state increases.

Finally consider an increase in  $\gamma$ , such that the elasticity of substitution between financial products increases and banks' mark-ups decrease. In this case differentiation of (1.20) shows that there is an ambiguous impact of  $\gamma$  on the slope of the transition path. In Appendix A.5 we show conditions under which an increase in  $\gamma$  has the same effects as an increase in  $b$ , at least for the cases in which  $\gamma \rightarrow 1$  and  $\gamma \rightarrow 0$ . We will turn to these comparative statics in more detail in the next section, where we quantitatively evaluate the model.

## 1.4 Quantitative Analysis

The objective of this section is to analyse whether the transition path implied by our model can replicate the dynamics of the unemployment and vacancy rates in the US economy for the period 2007-2014. To do so, we consider two sub-periods: (i) The Great Recession (November 2007 -- August 2009) and (ii) the Recovery (September 2009 -- December 2014). Through the lenses of our model, we interpret a sequence of  $(u, v)$  points within a given sub-period as movements along the transition path towards a new steady state. This section proceeds by calibrating the model to match the unemployment and vacancy dynamics during the Great Recession. That is, we calibrate the model to match the transition from the beginning of the Great Recession period to the end of the Great Recession. We then explore changes in vacancy costs and changes in the financial sector that can account for the observed unemployment and vacancy dynamics during the recovery period.

### 1.4.1 Parametrisation

The length of a period in the model is set to one month. We use information on the number of vacancies from the Job Openings and Labor Turnover Survey (JOLTS) and seasonally adjusted monthly series on the stock of employed, unemployed and short-term unemployed workers provided by the Bureau of Labor Statistics.<sup>7</sup> From these series we construct monthly series of job-finding-, unemployment- and vacancy rates (see Figures A.4 and A.6 in Appendix A.6).<sup>8</sup>

<sup>7</sup>We use the BLS series LNS13000000, LNS13008396 and LNS12000000.

<sup>8</sup>Let  $U_t$  denote the number of unemployed in month  $t$ , and let  $U_{t+1}^s$  correspond to the number of short-term unemployed with unemployment durations of less than 5 weeks in month  $t + 1$ .

We use a Cobb-Douglas specification for the matching function,  $M(u_t, v_t) = Au_t^\alpha v_t^{1-\alpha}$ , which implies a job finding rate of  $\lambda(\theta_t) = A\theta_t^{1-\alpha}$ . The parameters  $A$  and  $\alpha$  are then obtained from regressing the (log) job finding rates on a constant and (log) labour market tightness using data for the pre-crisis period December 2000 -- October 2007. Given our estimates of  $A$  and  $\alpha$ , we set the job destruction rate such that we match the steady-state unemployment rate  $u^*$  at the end of the Great Recession, i.e.  $s = \frac{u^*\lambda(\theta^*)}{1-u^*}$ . Furthermore we set the interest rate  $r^* = 0.0027$  such that it corresponds to the (annual) bank prime loan rate of 3.25% at end of the Great Recession.<sup>9</sup>

After normalising the productivity parameters to unity,  $p = b = 1$ , we are then left with  $\mathbf{x} = \{k, z, \beta, \tau, \phi, \gamma\}$  parameters to recover. For this we exploit the variation in the observed values of  $u$  and  $v$  during the Great Recession. In particular, we minimise the squared relative distance between the observed vacancy rates and the ones implied by our transition path taking the observed unemployment rates as given. That is, we choose

$$\mathbf{x} = \arg \min \sum_t \left( \frac{v_t - \hat{v}(\mathbf{x}; u_t)}{v_t} \right)^2 \quad \text{subject to (1.17) and } v^* = \hat{v}(\mathbf{x}; u^*), \quad (1.21)$$

where  $\hat{v}(\mathbf{x}; u_t)$  denotes the vacancy rate that solves (1.15) given the vector of parameters  $\mathbf{x}$  and the observed unemployment rate  $u_t$ . The first restriction is given by the zero-motion line for  $\theta$ , while the second restriction requires that the transition path must go through the steady state  $(u^*, v^*)$  at the end of the Great Recession. In addition, we impose two further restrictions to pin down  $\mathbf{x}$ . First, we interpret  $z$  to represent unemployment benefits and set the replacement ratio to one half. Second, we follow Silva and Toledo (2009) and Petrosky-Nadeau and Wasmer (2013) and require that the total vacancy cost amount to 3.6% of the wage rate. In Appendix A.7 we present further details on the implementation of the optimisation problem.

Table 1.1 shows the parameter values obtained from our calibration procedure as well as the steady state targets. Note that the elasticity of the matching function is close to Shimer (2005) and Hall (2005). Also note that to generate a sufficiently downward sloping resource constraint, the calibration procedure yields a value of  $\tau$  very close to zero. In Section 1.4.3 below, we discuss this result further. Further, since the calibration gives workers a high bargaining power, the implied

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Following standard practice in the literature, the job-finding rate in month  $t$  is then given by  $U2E_t = 1 - (U_{t+1} - U_{t+1}^s)/U_t$ . Similarly, the job-destruction rate in month  $t$  is given by  $E2U_t = U_{t+1}^s/E_t$ , where  $E_t$  denotes the number of employed workers in month  $t$  (see Figure A.5 in Appendix A.6).

<sup>9</sup>The data on bank prime loan rates are taken from the online data base of the Federal Reserve Bank St. Louis: <https://research.stlouisfed.org/fred2>

Labour Market Parameters		Financial Sector Parameters		Steady state (8/2009)	
Matching efficiency $A$	0.430	Fixed costs financial sector $\phi$	0.028	$u^*$	0.096
Elasticity of matching function $\alpha$	0.709	Elasticity of substitution $\frac{1}{1-\gamma}$	1.097	$\theta^*$	0.155
Vacancy cost $k$	2.320	Banks' productivity $b$	1.000	$r^*$	0.0027
Worker's bargaining weight $\beta$	0.919			$s^*$	0.027
Wage rigidity $\tau$	9.12e-8			$\lambda(\theta^*)$	0.250
Unemployment benefits $z$	0.479				
Labour productivity $p$	1.000				

Table 1.1: Calibrated Parameters

wage equals 0.96 similar to the one obtained by Hall (2005). Although the value of  $k$  seems high, this value is calculated as  $v^*k = 0.036w = 0.0346$  and hence the low vacancy rate observed in August 2009 ( $v^* = 0.015$ ) implies a  $k = 2.32$ . Also note that the value of the elasticity of substitution between financial services,  $1/(1 - \gamma) = 1.097$ , implies that the banking sector in our calibration is far from competitive and enables banks to command high monopoly rents.

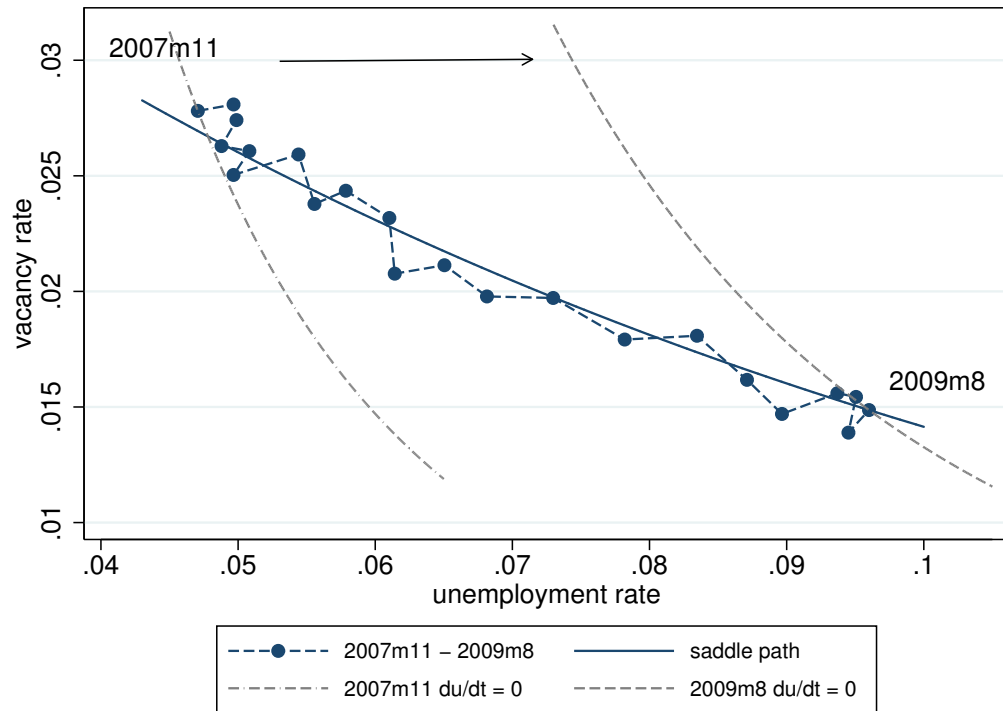


Figure 1.2: Unemployment and vacancy dynamics during the Great Recession

Figure 1.2 shows the model implications for the observed unemployment and vacancy dynamics during the Great Recession. The dots depict the unemployment and vacancy rate pairs observed during this period, which moved from low unemployment-high vacancy rates to high unemployment-low vacancy rates. In November 2007, immediately before the crisis, the US economy experienced an unemployment rate of 4.7% and a labor market tightness of 0.59. We assume that this was the steady state of the economy before the Great Recession with a job



destruction rate of 0.018 that is consistent with the observed unemployment rate at that time. As discussed by Elsby and Smith (2010), the Great Recession in the US was characterised by a sharp increase in the job destruction rate. In our model the increase in  $s$  shifts the zero-motion line of unemployment such that the post-crisis steady state implies a higher unemployment level. As one can observe from Figure 1.2,  $v$  decreased and  $u$  increased sharply during the Great Recession and our saddle-path tracks these movements closely. Given the unexpected nature of the financial crisis (see Caballero and Kurlat, 2009), Figure 1.2 can then be interpreted as the empirical counterpart to Figure 1.1b shown in Section 1.3.3 when discussing the effects of an unexpected shock in  $s$ .

### 1.4.2 The Recovery

During the period September 2009 -- December 2014 the economy underwent a slow recovery, where the unemployment and vacancy rates slowly reverted to their pre-recession levels (see Figure A.6 in Appendix A.6). We now analyse what change in the parameters  $\{p, k, \gamma, b\}$  are required to match the observed transition to the new steady state during the recovery.<sup>10</sup> This exercise informs us whether the model requires drastic changes to the parameters governing the financial sector to explain the transitional dynamics of unemployment and vacancies in the aftermath of the Great Recession. This exercise also informs us about the magnitude of the change the model requires in output per worker and vacancy costs to fuel firm entry and converge to the new steady state.

For this exercise we take the new steady state to be December 2014 (the end period of our window of observation). This steady state is characterised by  $u^{**} = 0.058$ ,  $\theta^{**} = 0.54$  and  $r^{**} = 0.0027$ . Since by December 2014 the job destruction rate decreased relative to August 2009 (see Figure A.5 in Appendix A.6), the zero-motion line for the unemployment rate shifted to the left. As before, we set the job destruction rate such that we match the steady-state unemployment rate  $u^{**}$ , i.e.  $s = \frac{u^{**}\lambda(\theta^{**})}{1-u^{**}} = 0.22$ . We then calibrate  $\{p, k, \gamma, b\}$  by solving the optimisation problem in (1.21) using data on vacancies and unemployment during the Recovery. All other parameters are held fixed. As before we require that the transition path goes through the steady state  $(u^{**}, v^{**})$ .

Figure 1.3 shows the model's implications for the observed unemployment and vacancy dynamics during the Recovery under this exercise. Relative to the Great

<sup>10</sup>We keep the banks' fixed cost of entry,  $\phi$ , constant in this exercise, as a joint minimisation with respect to  $b$  and  $\phi$  exhibits many local minima. Further, holding  $\phi$  constant is consistent with the evolution of the ratio between banks operational expenses and the employment in the financial sector as obtained from the OECD Banking Statistics: Financial Statements of Banks. This series shows basically no change during the period 2007 – 2014.

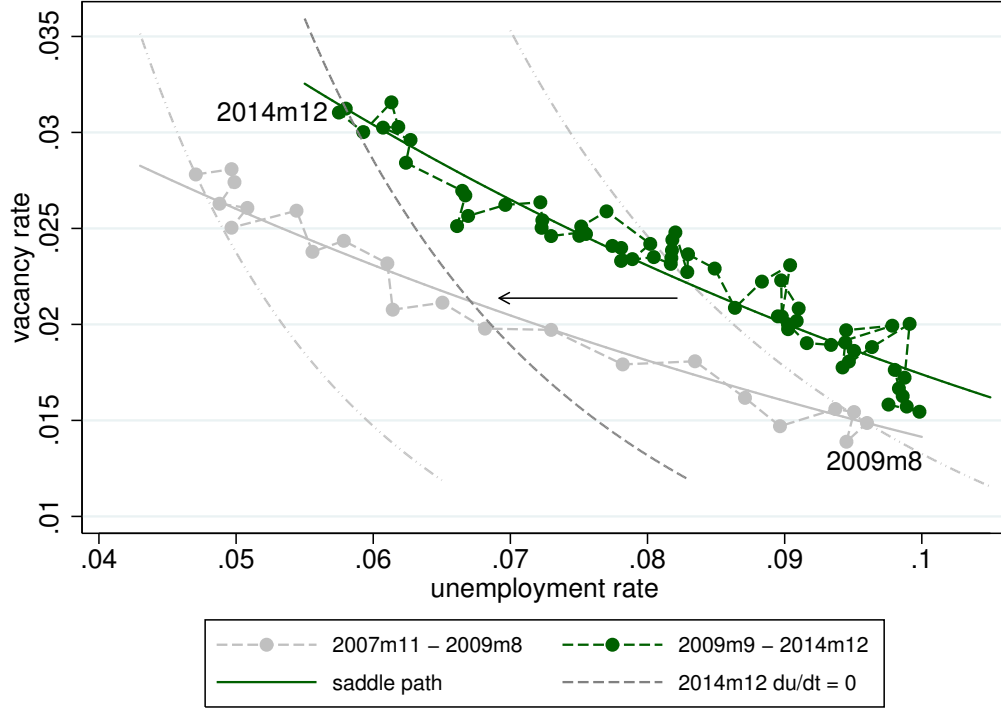


Figure 1.3: Unemployment and vacancy dynamics in the aftermath of the Great Recession

Recession, the model's transition path shifted to the right with a slight upward rotation. However, note that the transition path during the Recovery period is still relatively flat, implying that the unemployment and vacancy rates converge slowly to the new steady state. The new values for the calibrated parameters are  $p = 0.98$ ,  $k = 1.08$ ,  $b = 0.97$  and  $\gamma = 0.078$ , where the latter implies an elasticity of substitution between financial products of 1.085.

The first implication of this exercise is that, from the lenses of our model, the observed recovery in the labour market during the 2010 – 2014 period was not due to improvements in the effectiveness of banks to intermediate financial resources,  $b$ , or due to an increase in the degree of competition among banks  $\gamma$ . When compared to the values in Table 1.1, the value of these parameters hardly changed. Indeed,  $b$  only dropped by 3 percentage points and the elasticity of substitution dropped by about one percentage point. Compared to the Great Recession period the number of banks in the financial sector decreased from  $n_{GR} = 1.25$  to  $n_R = 1.23$  and the aggregate output of banks remained unchanged  $Y_{GR} = Y_R = 0.034$ . This implication seems to have some support in the data. For example, Figure A.7 in the Appendix A.6 shows that the money multiplier, a measure related to the productivity of banks, had a large drop during 2008 and then stayed essentially flat through the rest of the period.

The second implication of this exercise is that convergence to the new steady state was propelled by a lower vacancy cost and job destruction rate, which led to an increase in firm entry.<sup>11</sup> However, as opposed to the canonical search and matching model firm entry is not a jump variable. In our model job creation needs to be financed by the profit of existing firms using the banking sector to intermediate the financial resources. Given that the effectiveness of the banking sector to undertake such a task hardly changed during this period, the increase in firm entry developed slowly over time and hence produced a slow recovery in the unemployment and vacancy rates.

### 1.4.3 The Role of Wage Rigidity

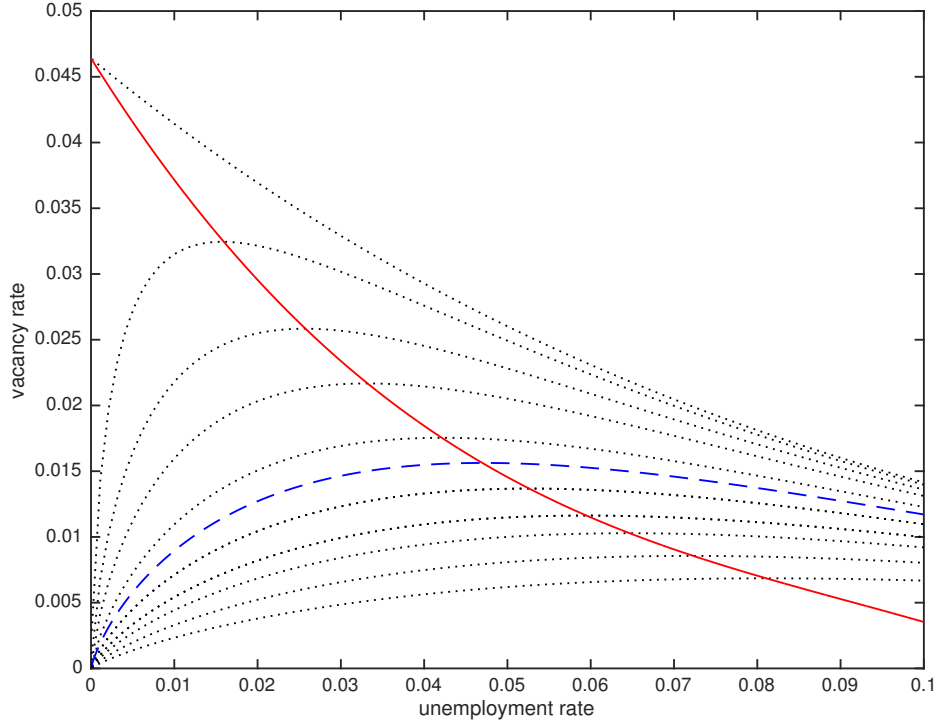
Table 1.1 shows that to replicate the dynamics of unemployment and vacancies as observed during the Great Recession, the calibration requires workers and firms to be essentially (Nash) bargaining over the flow surplus,  $p - z$ , such that wages become nearly isolated from  $\theta$  and hence  $u$ . Given that this form of wage rigidity is important to match the data, we now investigate how much isolation wages require in our calibrated model in order to guarantee a downward sloping transition path in the observed range of  $v$  and  $u$  values.

Figure 1.4 shows a collection of transition paths (dotted curves) generated by assuming different values of  $\tau \in [0, 0.019]$ , but maintaining the rest of the parameters at the values shown in Table 1.1. The higher transition path is obtained when  $\tau = 0$ , while the lowest transition path is obtained when  $\tau = 0.019$ . For  $\tau > 0$ , each transition path starts at the origin and slopes upwards until condition (1.18) is satisfied. At the implied inflection point the transition paths become downward sloping. The solid line in Figure 1.4 trace the negative relation between the different values of  $\tau$  and the inflection points of the transition paths. The dash line shows the transition path generated by  $\tau = 0.003$ , the highest value of  $\tau$  that guarantees a downward sloping transition path for all the observed values of the unemployment rate in our data, 4.7% to 10%. The main message from this exercise is that, holding constant the rest of the parameter values, our model requires wages to be quite strongly isolated from labour market tightness to be consistent with the observed negative relationship between unemployment and vacancies.<sup>12</sup>

The need for wages to be strongly isolated from  $\theta$  in order to replicate the data is consistent with the finding of Marcusse (2016). In particular, this author analyses

<sup>11</sup>As an alternative calibration we restricted the value of  $p$  to stay constant at one and obtained that  $k = 1.13$ ,  $\gamma = 0.078$  and  $b = 0.93$ , confirming that changes in  $p$  are of second order importance for our results.

<sup>12</sup>The latter conclusion holds also in the presence of a larger difference between  $p$  and  $z$ , as in Shimer (2005), or with a lower value for workers' bargaining power,  $\beta = 0.5$ .

Figure 1.4: Transition paths for different values of  $\tau$ 

whether Nash bargaining over flow surplus allows the canonical search and matching model to better explain the observed relationship between unemployment and vacancies in the US labour market relative to other forms of wage determination: Nash bargaining over the match surplus, Hall and Milgrom (2008) sequential bargaining and Kalai and Smorodinsky (1975) bargaining. Marcusse (2016) consistently finds that Nash bargaining over the flow surplus improves the ability of the canonical search and matching model in replicating the observed Beveridge curve under productivity ( $p$ ) and job destruction ( $s$ ) shocks. Here we obtain a similar result. In the presence of job destruction shocks, our model essentially requires workers and firms to Nash bargain over the flow surplus in order to replicate the negative co-movement of  $v$  and  $u$  observed in the data.<sup>13</sup>

Given this result, one could question whether it is necessary to have an imperfect financial market in order to generate a downward sloping transition path in the presence of a  $\tau$  close to zero. Note that our banking sector market-clearing assumption implies that the resource constraint, equating aggregate banking output to total vacancy costs, must be satisfied at any point in time. Equation (1.15) describes such a resource constraint under monopolistic competition. Irrespective of the market structure imposed on the banking sector, however, the resource constraint will imply that aggregate banking output will be increasing in aggre-

<sup>13</sup>Hall and Milgrom (2008) also calibrate the risk of breakdown during bargaining to be 0.0055% a day, suggesting a need to strongly isolate wages from labour market tightness in order for their sequential bargaining model to replicate the data.

gate firm profits and total vacancy cost will be increasing in unemployment. This generates a relationship between  $v$  and  $u$ . As discussed in Section 1.3.2, the role of  $\tau$  is to influence this relationship by isolating the impact of  $u$  on wages and ultimately on firms' profits. Therefore, it is possible to have a different market structure describing the banking sector and have a downward sloping transition path when  $\tau$  is close to zero. In this paper we have assumed that the banking sector is characterised by monopolistic competition because this is inline with a large body of work that studies the observed degree of competition and concentration in the banking sector across countries and time periods (see Bikker and Haaf, 2002, Kadir, Habibullah, Hook, and Mohamed, 2015, and the reference within). In turn, this structure allows us to study, in a parsimonious way, the rate at which our economy recovers from unexpected shocks to the productivity of and the degree of competition in the banking sector.

## 1.5 Conclusion

In this chapter we have constructed a simple general equilibrium matching model with an imperfect financial market in the form a monopolistically competitive banking sector. The role of the financial sector is to fund job creation through the firms' profits. The critical element of our model is the per period financial resource constraint that determines the transitional dynamics of vacancies and unemployment towards the steady state. The resource constraint adds a new dimension to the canonical search and matching model. It makes it potentially consistent with the fact that vacancies and unemployment adjust in opposite directions. We show that this feature is readily obtained when wages are Nash bargained over the flow surplus,  $p - z$ . To illustrate some of the quantitative implications of our model we calibrated to match the transitional dynamics of the Great Recession and its aftermath. We find that observed slow recovery in the labour market was due to the lack of a significant improvement in the effectiveness of the banking sector in intermediating resources to fund job creation.

The model we developed is very parsimonious as our goal was to understand its main mechanism using analytical solutions, rather than numerical simulations. Clearly this comes at the cost of presenting a perhaps too simplistic model. In particular, an important assumption made here is that firms always required external funding to finance job creation. This assumption might be reasonable among small firms, but it is somewhat more difficult to defend among bigger firms with large internal financial reserves. Indeed it has been argued that some firms were not short of funds but were just reluctant to spend some of it to finance invest-

ment and hence job creation (see Monacelli, Quadrini, and Trigari, 2011). Adding this feature is an important extension to the model developed here. However, we leave this extension for future research.

## Chapter 2

# Labour Income Dynamics and the Insurance from Taxes, Transfers, and the Family

### 2.1 Introduction

The aim of this paper is to examine the dynamics of labour income over the working life and to explore the impact of two mechanisms of attenuation or insurance to labour income shocks. The first is the tax and transfer system; the second is spouse's income. We focus on three dimensions of inequality: individual market income, individual disposable income, and family disposable income; and explore the links between them over the life-cycle. Our objective is to provide a detailed picture of the dynamics of inequality over the life-cycle, following individuals from many different birth cohorts across their working lifespan. By linking up individuals with other family members, we are able to examine the impact of spouse's income and the role of the tax-transfer system as mechanisms to smooth shocks to individual market income.

There are a number of key questions addressed. What do labor income dynamics look like over the life-cycle? What is the relative importance of persistent shocks, transitory shocks and heterogeneous profiles? To what extent do the tax and transfer system attenuate these various factors in the evolution of life-cycle inequality? What happens when we add in income sources of spouses? Answering these questions has proved to be quite difficult. One problem that is often argued to hinder analysis is data availability. While the ideal data set is a long panel of individuals, this is somewhat a rare event and can be plagued by problems such as attrition and small sample sizes. An important exception is the case where

countries have available administrative data sources. The advantages of such data sets are the accuracy of the income information provided, the large sample size, and the lack of attrition, other than what is due to migration and death.

To investigate the above questions, we exploit a unique source of population panel data containing records for every Norwegian from 1967 to 2006. Norway provides an ideal context for this study. It satisfies the requirement for a large and detailed data set that follows individuals and their family members over long periods of their working career. It also has a well developed tax-transfer system, and our data provides us with a measure of income pre and post the payment of taxes and the receipt of transfers. To understand the role of taxes, transfers and the family in attenuating shocks to labor income requires a model that allows for key aspects in the evolution of labour income over the life-cycle. The extensive literature on the panel data modeling of labor income dynamics points to three ingredients of potential significance: shocks of varying persistence; age and time dependence in the variance of shocks; and heterogeneous age profiles.<sup>1</sup> The size and detailed nature of the data we are using allow us to explore the importance of these three ingredients for labor income dynamics. Additionally, by following many different birth cohorts across their working lifespan we are able to allow a flexible structure for time effects in deriving our life-cycle profiles.

Our key findings on the labor income dynamics of males are three-fold. First, the magnitude of permanent and transitory shocks vary systematically over the life-cycle. Indeed, we may strongly reject the hypothesis of age-independent variance of shocks. Second, there is essential heterogeneity in the variances of permanent shocks across skill groups. For low skilled, the magnitude of permanent shocks is monotonically increasing in age. For example, a permanent shock of one standard deviation implies a 35 percent change in individual market income for a low skilled 30 year old; the corresponding number for a low skilled 55 year old is 50 percent. High skilled, on the other hand, experience large permanent shocks early in life; these shocks decrease in magnitude until age 35, after which they are relatively small and fairly stable. Third, the variance of transitory shocks exhibits a decreasing profile over the life-cycle. While this findings holds for all skill groups, high skilled tend to experience relatively large transitory shocks early in life.

The evidence of heterogeneity in the dynamics of labor income by age, skill level, and their interaction motivates and guides our analysis of the insurance from taxes,

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<sup>1</sup>See, for example, the recent review by Meghir and Pistaferri (2011), and the extensive list of studies referenced therein. These studies build and extend on the original papers by MaCurdy (1982) who developed the permanent transitory framework, Moffitt and Gottschalk (2012) who show the potential importance of time dependence in the variance components, and Lillard and Weiss (1979) and Guvenen (2009a) who established a role of heterogeneous profiles.



transfers and the family. We find that the tax-transfer system reduces both the level and persistence of shocks to labor income. In particular, taxes and transfers lead to a remarkable flattening of the age profiles in the variances of permanent and transitory shocks for the low skilled. At age 55, for example, a permanent shock of one standard deviation implies a 50 percent change in annual market income for a low skilled; the corresponding number for annual disposable income is only 31 percent. After taking taxes and transfers into account, spouse's income matters little for the dynamics of inequality over the life-cycle.

Taken together, our results suggest that a progressive tax-transfer system could be an important insurance mechanism to labour income shocks, especially for low skilled. These results may have implications for both policy and a large and growing literature on consumption inequality and the overall ability of families to insure labour income shocks (see e.g. Blundell, Pistaferri, and Saporta-Eksten, 2012). Economic theory predicts that consumption responds strongly to permanent shocks, and empirical evidence suggests little if any self-insurance in response to permanent shocks among individuals with no college education (see e.g. Blundell, Pistaferri, and Preston, 2008). Our study points to the importance of understanding the nature of risk that families face over the life-cycle, and the extent to which taxes and transfers crowd out or add to the insurance available in financial markets, the family or other informal mechanisms.

Our paper also contributes to the literature on modeling of labor income dynamics. Identification of credible income processes is key for answering a number of important economic questions, including life-cycle consumption and portfolio behavior (see e.g. Gourinchas and Parker, 2002), the sources of inequality (see e.g. Huggett, Ventura, and Yaron, 2011), and the welfare costs of business cycles (see e.g. Storesletten, Telmer, and Yaron, 2001, 2004b). The conclusions reached about these questions likely depend on the specification of the labor income process used to calibrate the models. The relatively small scale of the available U.S. panel surveys has forced researchers to rely on simple models that impose economically implausible restrictions (see the discussions in Baker and Solon, 2003; Meghir and Pistaferri, 2011). Using rich Canadian data from 1976 to 1992, Baker and Solon (2003) reject several of these restrictions, including no life-cycle variation in the variance of transitory shocks.<sup>2</sup> DeBacker, Heim, Panousi, Ramnath, and Vidangos

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<sup>2</sup>Like Moffitt and Gottschalk (2012) and Haider (2001), Baker and Solon (2003) go beyond earlier models by allowing for changes over calendar time in both the persistent and transitory components of income. However, Baker and Solon (2003) do not allow for life-cycle variation in the variance of permanent shocks. Ostrovsky (2010) extends the analysis in Baker and Solon (2003) to the period from 1985 to 2005. See also Ostrovsky (2012), who uses Canadian data to estimate a model which allows for separate correlation of spouse's permanent and transitory components.

(2013) use a large panel of tax returns to study income dynamics in the U.S. over the period 1987-2009.<sup>3</sup> Their estimates point to the importance of allowing for time dependence in the variance components of income.

Our study complements these studies by bringing new evidence on several issues pertinent to modeling of income processes. One key finding is that allowing for both age and time dependence in the variance components is essential to accurately describe labor income dynamics. In particular, when restricting the variances of the error components to be constant across the life-cycle, we miss the large permanent shocks that occur late (early) in life for the low (high) skilled. Another key finding is that allowing for heterogeneity by education levels is necessary to capture labor income dynamics of young and old workers. When we restrict the income processes at the variance level to be the same across skill groups, we find a U-shaped age profile in the variances of permanent shocks; however, this pattern is at odds with the age profiles of both high and low skilled.<sup>4</sup> By way of comparison, the dynamics of income over the life-cycle change little when restricting the transitory shocks to be uncorrelated over time or allowing for heterogeneous experience profiles within each skill group. Indeed, only for the high skilled, there is evidence of significant unobserved heterogeneity in the income growth rates. Accounting for this heterogeneity lowers the persistence of permanent shocks somewhat, but barely moves the age profiles in the variances of permanent and transitory shocks.

The remainder of this chapter proceeds as follows. Section 2.2 presents our data and discusses institutional details. Section 2.3 describes our panel data specification for income dynamics and presents our findings on the labor income process of males. Section 2.4 explores the degree of insurance provided by taxes and transfers as well as the income of the spouse. Section 2.5 offers evidence on several issues pertinent to modeling of income processes. Section 2.6 concludes.

## 2.2 Data and Institutional Details

### 2.2.1 Data and Sample Restrictions

Our analysis employs several registry databases maintained by Statistics Norway that we can link through unique identifiers for each individual. This allows us to construct a rich longitudinal data set containing records for every Norwegian

<sup>3</sup>See also Kopczuk, Saez, and Song (2010).

<sup>4</sup>Using PSID, Karahan and Ozkan (2013) also find a U-shaped age profile in the variances of permanent shocks.

from 1967 to 2006. The variables captured in this data set include individual demographic information (including gender, date of birth, and marital status) and socioeconomic information (including years of education, market income, cash transfers). The data contains unique family identifiers that allow us to match spouses and parents to their children.

The coverage and reliability of Norwegian registry data are considered to be exceptional (Atkinson, Rainwater, and Smeeding, 1995). Educational attainment is reported by the educational establishment directly to Statistics Norway, thereby minimizing any measurement error due to misreporting. More importantly, the Norwegian income data have several advantages over those available in many other countries. First, there is no attrition from the original sample because of the need to ask for permission from individuals to access their tax records. In Norway, these records are in the public domain. Second, our income data pertain to all individuals and all jobs, and not only to jobs covered by social security. Third, we have nearly career-long income histories for certain cohorts, and do not need to extrapolate the income profiles to ages not observed in the data. And fourth, there are no reporting or recollection errors; the data come from individual tax records with detailed information about the different sources of income.

We study income dynamics for the 1925-1964 annual birth cohorts during the period 1967-2006. The reason for this selection of cohorts is to ensure fairly long records on earnings for each individual. We restrict the sample to males, to minimize selection issues due to lower labor market participation rates for women in the early periods. In line with much of the previous literature, we exclude immigrants and self-employed. We further refine the sample to be appropriate for our analysis of labour income dynamics. In each year, we select males who are between the ages of 25 and 60. These individuals will likely have already completed most of their schooling and are too young to be eligible for early retirement schemes. In our baseline specification, we further restrict the sample to individuals with at least four subsequent observations with positive market income. This restriction gives us the largest possible sample, given that transitory shocks are assumed to follow a first-order moving average process.

Applying these restrictions provides us with a panel data set with 40 time periods and 934,704 individuals. We will refer to this as the baseline sample. On average, this sample consists of 23,368 individuals per birth cohort. Our model estimates age-specific variance components from age 26 to 58. By following many birth cohorts, we are able to allow a flexible structure for calendar time effects in deriving our life-cycle profiles. For the 1942-1946 cohorts, we observe income at every age. For the cohorts born earlier (1925-1941), we miss one or more income observations

between the ages of 25 and 41. For the cohorts born later (1947-1964), income is no longer observed at some point after age 42. As a result, our age-specific estimates are based on an unbalanced panel of income. Appendix Figure B.3 shows the sample size by age. The number of observations declines late (early) in the working lifespan because we are not observing the labor income of younger (older) cohorts at these ages. It is therefore reassuring that the mean and variance of income display similar shapes over the life cycle across cohorts.

The income variables that we consider are defined as follows. The first variable is *individual market income*, defined as the annual pretax earnings.<sup>5</sup> The second variable is *individual disposable income*, incorporating annual earnings and cash transfers net of taxes.<sup>6</sup> The third variable is *family disposable income*. Our measure of family disposable income pools the individual disposable income of the spouses (if the male has a spouse).

Throughout the analysis, we partition the baseline sample into three mutually exclusive groups according to educational levels. The reason is that previous studies point to heterogeneity in the dynamics of labor income by educational levels. Low skilled is defined as not having completed high school (32 percent of the baseline sample), medium skilled includes individual with a high school degree (48 percent of the baseline sample), and the high skilled consists of individuals who have attended college (20 percent of the baseline sample).

## 2.2.2 Institutional Details and Descriptive Statistics

Before turning to the estimation of the income processes, we describe a few important features of our data and the Norwegian setting.

We first consider the pattern of labor market participation over the life cycle. Appendix Figure B.4 shows the population share of males with positive market income by age. We see that the labor market participation rate starts out at around 90 percent when individuals are young. The participation rate remains at this high level until individuals reach their 50s, at which point they start exiting the labor market at an increasing rate. In particular, low skilled individuals are relatively likely to exit the labor market before they can receive (early) retirement benefits.

Appendix Figure B.5 shows the levels and growth rates in market income by the

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<sup>5</sup>Unfortunately, we are unable to measure hourly wages because we do not have data on working hours.

<sup>6</sup>Our measure of disposable income excludes income from financial assets and subtracts taxes on earnings and cash transfers. In every year, we compute taxes using a tax simulation program available at Statistics Norway. Due to data availability, our measure of cash transfers omits certain short-term and work-related benefits, including sickness pay and unemployment benefits.

age at which individuals exit the labor market. We see that early exits from the labor market are associated with low and declining market income in the years prior to exit. Given the sample restriction of non-zero market income, our baseline sample will therefore be of higher quality towards the end of the life-cycle (especially among the low educated). This should put downward pressure on the magnitude of shocks late in life, and most likely give us a lower bound on the insurance from taxes and transfers at these ages.

Next, we consider how individual market income varies over the life-cycle in our baseline sample. Figure 2.1 shows the age profiles in the different measures of income by education levels. Each market income profile displays the familiar concave shape documented and analyzed by Mincer (1974), but the college-educated workers experience more rapid market income growth early in the working lifespan. Figure 2.2 shows the variance of log market income over the life-cycle according to education levels. In line with the prediction of the Mincer model, the variances of medium and high skilled have a U-shaped profile.<sup>7</sup> Among low skilled, the variance of log market income is weakly increasing until they are in their mid 40s, after which it rises rapidly.

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<sup>7</sup>Using Census data from the U.S., Heckman, Lochner, and Todd (2003) also find that the variance of log labor income over the life has a U-shaped pattern.

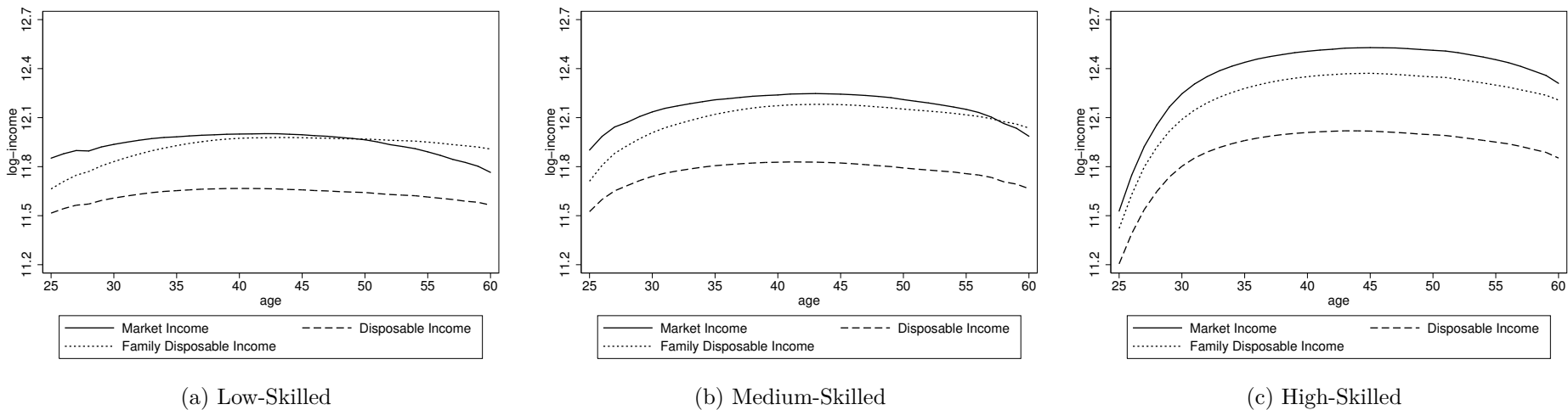


Figure 2.1: Age profiles in the log of income

**Notes:** This figure uses the baseline sample to show the age profiles in the log of income by educational levels. The age profiles are adjusted for education-specific calendar time effects. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college.

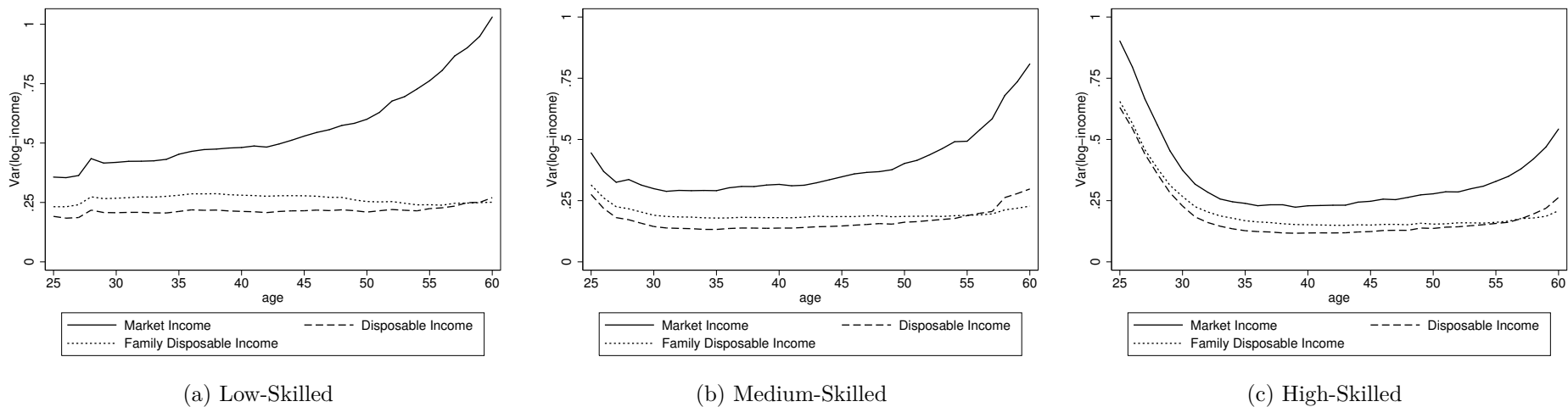


Figure 2.2: Age profiles in the variances of log income.

**Notes:** This figure uses the baseline sample to show the age profiles in the variances of log income by educational levels. The age profiles are adjusted for education-specific calendar time effects. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college.

We then examine the extent to which the tax and transfer system affects the mean and the variance of log individual income over the life-cycle. Figure 2.1 show how the progressive nature of the tax system dampens the income differentials between high skilled and low skilled after age 35.<sup>8</sup> At the same time, low skilled are more likely to receive cash transfers while working (such as partial disability benefits), especially towards the end of the working lifespan. Figure 2.2 shows how the tax-transfer system eliminates the increase over the life-cycle in the variance of log market income among the low educated. By comparison, taxes and transfers do less to the large income variance among the high skilled early in their careers.

Lastly, we consider how family disposable income varies over the life-cycle in our baseline sample. Figure 2.1 compares the age profiles in log family disposable income and log individual disposable income. In the beginning of the working lifespan, relatively few males are married and individual disposable income is therefore quite similar to family disposable income.<sup>9</sup> When the males are in their mid 30s, the vast majority are married and the income of the spouse plays a more important role. At this point, about 80 percent of the spouses are participating in the labor market, thus contributing significantly to family income.<sup>10</sup> Figure 2.2 shows the life-cycle variation in the variance of log family disposable income. The family income measure displays quite similar variance over the working lifespan as compared to the measure of individual disposable income.

## 2.3 Labour Income Dynamics

### 2.3.1 A Panel Data Specification for Labour Income Dynamics

To understand the role of the tax and transfer system in attenuating shocks to income for individuals and families over the life-cycle requires a model that allows for key aspects in the evolution of labour market income over the working life. As we noted in the introduction, the extensive literature on the panel data modeling of income dynamics has pointed to three key ingredients of potential significance: shocks of varying persistence; age and time dependence in the variance of shocks;

<sup>8</sup>The tax system is progressive through deductions and surtaxes. Appendix Figures B.1 and B.2 show the tax rates on market income in different years.

<sup>9</sup>Appendix Figure B.6 shows the share of the sample that is married by age and educational levels.

<sup>10</sup>Appendix Figure B.7 shows the labor market participation rate of the spouses by age and educational levels.



and heterogeneous age profiles. The size and detailed nature of the Norwegian population panel allow us to combine all three of these components and let the degree of persistence and the variance of the shocks vary in a quite unrestricted way by age and calendar time.

Let  $Y_{i,a}^c$  denote the market income of individual  $i$  from birth cohort  $c$  at age  $a$ . To obtain log income net of observable characteristics and common aggregate time trends, denoted  $y_{i,a}^c$ , we run cross-sectional first-stage regressions of  $\log Y_{i,a}^c$  on a set of covariates.<sup>11</sup> A panel data specification that encompasses many of the ideas in the literature is:

$$y_{i,a}^c = \alpha_i^c + \beta_i^c p_a + v_{i,a}^c + \tau_{i,a}^c, \quad (2.1)$$

where  $\alpha_i^c$  is an individual initial condition, while  $\beta_i^c$  allows for an idiosyncratic experience profile in the deterministic trend variable  $p_a$  (e.g.  $p_a = a - 25$  with a linear experience profile). Taken together, these two terms capture individual-specific unobserved heterogeneity in the levels and growth rates of labor income. We allow for correlation between  $\alpha_i^c$  and  $\beta_i^c$ .

Income shocks are decomposed into a permanent (or persistent) component  $v_{i,a}^c$ ,

$$v_{i,a}^c = \rho^c v_{i,a-1}^c + u_{i,a}^c,$$

where  $u_{i,a}^c$  is a serial uncorrelated mean-zero shock, and a transitory component  $\tau_{i,a}^c$ , which is assumed to follow a MA(1) process.

$$\tau_{i,a}^c = \varepsilon_{i,a}^c + \theta^c \varepsilon_{i,a-1}^c,$$

where  $\varepsilon_{i,a}^c$  is a serial uncorrelated mean-zero shock. The permanent and transitory innovations are assumed to be independent of each other and independent of  $\alpha_i^c$  and  $\beta_i^c$ . Some examples of permanent innovations are associated with job mobility, long-term unemployment, health shocks, and promotions. Transitory shocks to individual labor income typically include overtime labor supply, piece-rate compensation, bonuses, etc.; in general, such shocks are mean reverting and their effect does not last long.

In Appendix B.1, we describe every step of the estimation procedure for the income

<sup>11</sup>In each year, we perform a separate OLS regression of  $\log Y_{i,a}^c$  on a quadratic polynomial in age and dummies for education, region, family size and marital status. We allow for interactions between family size and marital status as well as interactions between the quadratic polynomial in age and the education dummies. From these regressions, we predict log market income of individual  $i$  from birth cohort  $c$  at age  $a$ ,  $\log \hat{Y}_{i,a}^c$ . The residual log income  $y_{i,a}^c$  is given by  $\log Y_{i,a}^c - \log \hat{Y}_{i,a}^c$ .

process given in (2.1). There are, however, three important features to notice. First, we allow for the permanent component  $v_{i,a}^c$  to have a  $\rho^c$  coefficient less than unity. Since we have long enough panels for individuals in each of the cohorts, the parameters of this process together with those for the transitory process and the heterogeneous profiles can be separately identified.

Second, the overall persistence of shocks to the net log income measure  $y_{i,a}^c$  depends on the weighted sum of the permanent and transitory processes  $v_{i,a}^c$  and  $\tau_{i,a}^c$  respectively, where the weights reflect the variance share of each of these components. To see this, consider our baseline specification of the income process which imposes homogenous experience profiles (i.e.  $\beta_i^c = 0$ ). Assuming that  $\text{var}(y_{i,a}^c) \simeq \text{var}(y_{i,a+1}^c)$ , the first order autocorrelation at age  $a$

$$\varrho_a^c = \frac{\text{cov}(y_{i,a}^c, y_{i,a+1}^c)}{\sqrt{\text{var}(y_{i,a}^c)}\sqrt{\text{var}(y_{i,a+1}^c)}}$$

can be expressed as

$$\varrho_a^c \simeq \frac{\text{var}(\alpha_i^c) + \rho^c \sum_{s=0} (\rho^c)^{2s} \text{var}(u_{i,a-s}^c) + \theta^c \text{var}(\varepsilon_{i,a}^c)}{\text{var}(\alpha_i^c) + \sum_{s=0} (\rho^c)^{2s} \text{var}(u_{i,a-s}^c) + \text{var}(\varepsilon_{i,a}^c) + (\theta^c)^2 \text{var}(\varepsilon_{i,a-1}^c)}.$$

This illustrates that by allowing the variances of each component to differ by age, we are in effect, allowing the autocorrelation of income shocks to vary quite unrestrictedly over the life cycle (even though  $\rho$  does not depend on age).

Lastly, the use of data that follows actual cohorts over the life cycle allows us to accurately measure their true earnings pattern and estimate the labor income dynamics experienced by individuals. The model given in (2.1) is estimated separately by education levels using an equally-weighted minimum distance approach applied to second order moments. At every age, we average the moments across cohorts before estimating the income process.<sup>12</sup> Without further restrictions, the estimates could be interpreted as an average (or a typical) labor income dynamics experienced by these cohorts over their working lifespan. To determine the relative contribution of age and calendar time effects to the labor income dynamics require further restrictions. If one were to assume no cohort effects, we would effectively control for calendar time effects by averaging the moments across cohorts. Heathcote, Storesletten, and Violante (2005) argue that time effects are required to account for the observed trends in inequality. One might, however, suspect

<sup>12</sup>We have also estimated model (2.1) separately for each cohort, and then computed the weighted average of the parameters across the cohorts. Because this procedure is computationally quite costly, and the estimates are very similar, we only report results for which the moments are averaged across the cohorts before estimation.

that cohort effects should play some role in the distribution of fixed effects. For example, rising college enrollment rates may have changed the level of permanent wage dispersion of younger relative to older cohorts. We incorporate this source of heterogeneity across cohorts by estimating the income processes separately by education levels.

### 2.3.2 Baseline Estimates

We begin by considering the labor income dynamics of males. The model given in (2.1) is estimated separately by education levels.<sup>13</sup> For now, we impose homogenous experience profiles ( $\beta_i = 0$ ) in the estimation.<sup>14</sup> Instead of presenting the labor income dynamics of each cohort, we average the moments across the cohorts before estimating the income process. As a result, the estimates should be interpreted as an average (or a typical) labor income dynamics experienced by these cohorts over their working lifespan.

The first column of Table 2.1 reports the parameter estimates for individual market income of males. For each skill group, we find that the persistence parameter ( $\rho$ ) is either one or close to one. This suggests that the shocks to (log) labour income can be described as the sum of a transitory shock and an highly persistent process. Because of the unit root, we cannot identify the variance of initial conditions ( $var(\alpha_i)$ ) for the low or medium skilled.

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<sup>13</sup>Standard errors are based on nonparametric bootstrap (of each estimation stage) with 70 bootstrap replications.

<sup>14</sup>As shown in Section 2.5, the persistence, magnitude, and age profiles of the permanent and transitory shocks change little when we allow for heterogenous profiles ( $\beta_i^c \neq 0$ ).

	Individual Market Income	Individual Disposable Income	Family Disposable Income
Low-Skill			
$\rho$	1.00 (0.000000)	0.87 (0.006287)	0.87 (0.003763)
$var(\alpha_i)$	- -	0.035360 (0.001234)	0.034113 (0.001018)
$\theta$	0.238500 (0.004793)	0.215220 (0.004638)	0.207820 (0.004811)
Medium-Skill			
$\rho$	1.00 (0.000000)	0.89 (0.005500)	0.89 (0.004781)
$var(\alpha_i)$	- -	0.030796 (0.001162)	0.027141 (0.000899)
$\theta$	0.258840 (0.0028949)	0.238450 (0.003982)	0.243650 (0.003916)
High-Skill			
$\rho$	0.98 (0.013981)	0.94 (0.029910)	0.85 (0.010652)
$var(\alpha_i)$	0.000152 (0.000043)	0.000447 (0.014922)	0.030992 (0.000868)
$\theta$	0.294650 (0.0049582)	0.270220 (0.005652)	0.278160 (0.005665)

Table 2.1: Parameter estimates from the model of income dynamics

**Notes:** This table presents the parameter estimates from the model of income dynamics described in Section 2.3.1. We use the baseline sample and estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Standard errors (in parentheses) are based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.

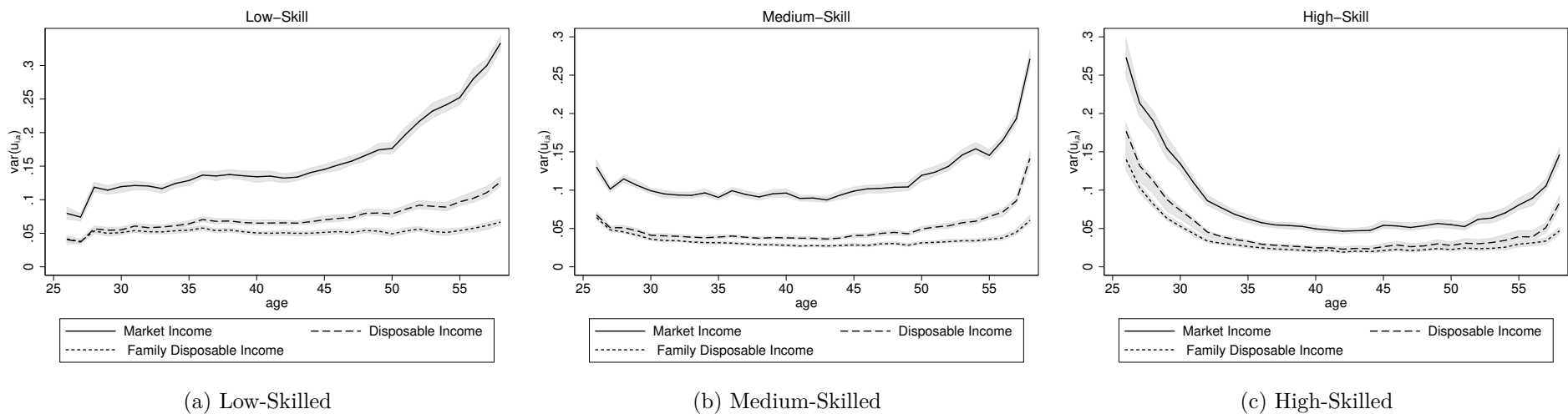


Figure 2.3: Age profiles in the variances of permanent shocks to income

**Notes:** This figure graphs the age profiles in the variances of permanent shocks to income. The age profiles are based on the model of income dynamics described in Section 2.3.1. We use the baseline sample and estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. The age profiles are adjusted for education-specific calendar time effects. The 95 percent confidence interval is based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.

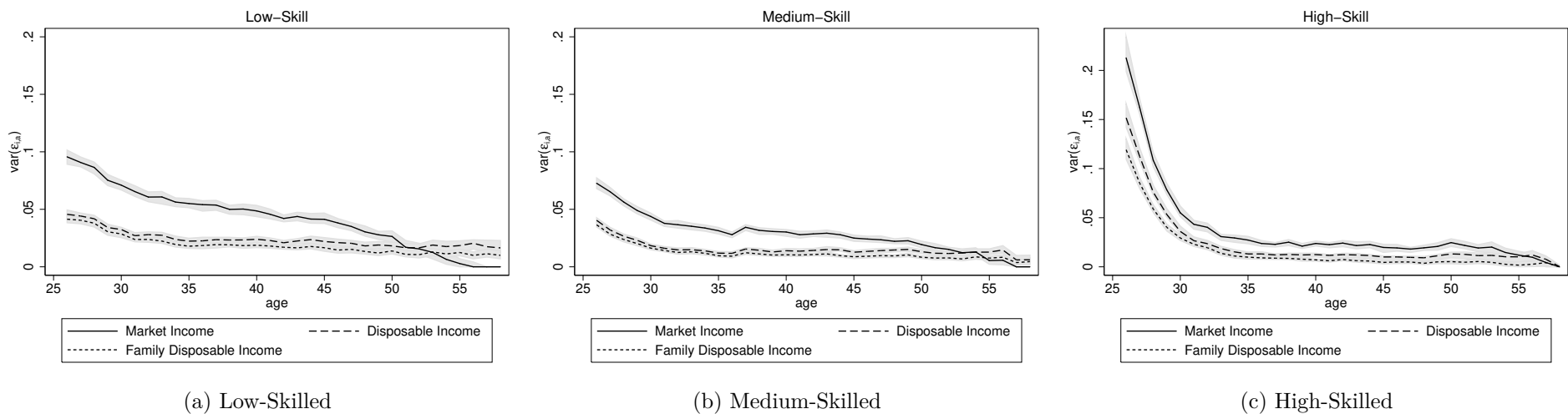


Figure 2.4: Age profiles in the variances of transitory shocks to income

**Notes:** This figure graphs the age profiles in the variances of transitory shocks to income. The age profiles are based on the model of income dynamics described in Section 2.3.1. We use the baseline sample and estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. The age profiles are adjusted for education-specific calendar time effects. The 95 percent confidence interval is based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.

The more persistent the shocks, the more important it is to know whether workers at different ages face the same variance of permanent shocks, or if the magnitude changes systematically over the life-cycle. Figure 2.3 examines this by showing the age profile in the variance of permanent shocks ( $var(u_{i,a})$ ) according to education levels. The magnitude of permanent shocks varies systematically over the life-cycle. Indeed, we may strongly reject the standard specification with age-independent variance components.

Another key finding is the heterogeneity in the variances of permanent shocks by education levels. For low skilled, the magnitude of permanent shocks is monotonically increasing in age. For example, a permanent shock of one standard deviation implies a 35 percent change in individual market income for a low skilled 30 year old; the corresponding number for a low skilled 55 year old is 50 percent. High skilled, on the other hand, experience large permanent shocks early in life; these shocks decrease in magnitude until age 35, after which they are relatively small and fairly stable. For example, a permanent shock of one standard deviation implies a 28 percent change in individual market income for a high skilled 55 years old; the corresponding number for a high skilled 28 (40) years old is 44 (22) percent.

The variance of transitory shocks, shown in Figure 2.4, exhibits a decreasing profile over the life-cycle. While this findings holds for all skill groups, high skilled tend to experience relatively large transitory shocks early in life. At the same time, the MA parameter differs by skill group. A larger proportion of the transient shocks persist for another period for high skilled workers than for low skilled workers.

To see the importance of low incomes in determining the age profiles of labour market shocks, we present results in Appendix Figures B.8 and B.9 where we exclude observations with low market incomes.<sup>15</sup> The profiles are much flatter. The presence of low market incomes early and late in life is mirrored in hours of work over the life-cycle. When looking at decennial Norwegian Census data over the period of study, we find that mean hours across the life-cycle is inverse U-shaped. There is an increase until individuals are in their early 30s, then a flattening, and eventually a decrease toward retirement. The opposite is true for the variance of log hours, which is U-shaped. In particular, there is a sharp downward trend in the dispersion of hours worked before age 35.<sup>16</sup>

<sup>15</sup>In these figures, we exclude observations with market income lower than the basic amount threshold of the Norwegian Social Insurance Scheme; market income above this threshold gives eligibility for unemployment benefits and matters for old-age pension payments.

<sup>16</sup>Kaplan (2012a) documents patterns in the US. He argues that labor market frictions are important in accounting for the patterns of inequality in consumption and hours over the lifecycle.

### 2.3.3 Model Fit

We now examine the performance of the model given in (2.1) in fitting the variance of (residual) income growth rates as well as one-lag covariances. For each income measure and every skill group, we conclude that the baseline specification with homogenous profiles ( $\beta_i = 0$ ) and a MA(1) achieves a very good fit of these key moments over the life-cycle.

The theoretical moments for the baseline model are given by

$$\begin{aligned} \text{var}(\Delta y_{i,a}^c) &= (\rho^c - 1)^2 \text{var}(v_{i,a-1}^c) + \text{var}(u_{i,a}^c) + \text{var}(\varepsilon_{i,a}^c) \\ &+ (\theta^c - 1)^2 \text{var}(\varepsilon_{i,a-1}^c) + \theta^2 \text{var}(\varepsilon_{i,a-2}^c) \end{aligned}$$

$$\begin{aligned} \text{cov}(\Delta y_{i,a}^c, \Delta y_{i,a+1}^c) &= (\rho^c - 1)^2 \rho^c \text{var}(v_{i,a-1}^c) + (\rho^c - 1) \text{var}(u_{i,a}^c) \\ &+ (\theta^c - 1) \text{var}(\varepsilon_{i,a}^c) - \theta^c (\theta^c - 1) \text{var}(\varepsilon_{i,a-1}^c) \end{aligned}$$

Appendix Figures B.12 -- B.14 show the model fit for the variance of the growth rate, while Appendix Figures B.15 -- B.17 displays the match for the one-lag covariance profile of the growth rate. We find that the model matches the variance of the growth rate observed in the data almost perfectly. When  $\rho = 1$ , we effectively target the variance of the growth rate in the estimation. As a result, the age dependence of the variances shocks allows us to match the age profile very well. When  $\rho < 1$ , the moments used in the estimation differ from those shown in the figure. It is therefore reassuring to find that the model also in this case fits the data very well.

## 2.4 Insurance from Taxes, Transfers and the Family

The evidence of heterogeneity in the dynamics of labor income by age, skill level, and their interaction raises a number of important questions. To what extent does the tax and transfer system attenuate or insure the shocks to market income at different parts of the life-cycle? Does the addition of income sources from the spouse offset or enhance labour market shocks? In this section, we investigate these questions.

### 2.4.1 Taxes and Transfers

The second column of Table 2.1 reports the estimation results for individual dis-



posable income of males. Importantly, the tax-transfer system reduces the level and persistence of both the permanent and the transitory shocks. The estimated persistence parameter falls the most for low skilled; when  $\rho = 0.87$ , the effect of an income shock is reduced to 25 percent of its initial value in ten years. At the same time, Figures 2.3 and 2.4 show that taxes and transfers lead to a remarkable flattening of the age profiles in the variances of permanent and transitory shocks for the low skilled. At age 55, for example, a permanent shock of one standard deviation implies a 50 percent change in annual market income for a low skilled; the corresponding number for annual disposable income is only 31 percent.

Shifting attention to the high skilled, we can see that taxes and transfers do little to the age profile in the variance of transitory shocks. As shown in Figure 2.4, it exhibits a decreasing and convex profile also in individual disposable income; indeed, the magnitudes of the transitory shocks are only slightly lower for disposable income than for market income. The impact of taxes and transfers is somewhat larger for the variance of permanent shocks. Early in life, the permanent shocks to market income of high skilled are attenuated substantially, although they remain large. Towards the end of the life-cycle, the tax-transfer system reduces the magnitude of the permanent shocks somewhat.

Taken together, our results suggest that the progressive nature of the Norwegian tax-transfer system plays a key role in attenuating the magnitude and persistence of income shocks, especially among the low educated. This finding could have important implications for consumption inequality and the overall ability of families to insure labour income shocks. Economic theory predicts that consumption responds strongly to permanent shocks, and empirical evidence suggests little if any self-insurance in response to permanent shocks among individuals with no college education (see e.g. Blundell, Pistaferri, and Preston, 2008).

### 2.4.2 Family Income

We now shift attention to examining whether the addition of income from the spouse offsets or enhances labour market shocks. There are competing forces at play when going from individual to family income (see e.g. Blundell, Pistaferri, and Saporta-Eksten, 2012). The first is that the variance of market income is relatively large among females, reflecting considerable dispersion in hours worked. The second is that the stochastic component of labor income processes are likely to be correlated across spouses. If spouses were adopting perfect risk sharing mechanisms, they would select jobs where shocks are negatively correlated. Alternatively, assortative mating can imply that spouses work in similar jobs, similar industries, and even in the same firm; as a consequence, their shocks could be pos-

itively correlated. The third is that family labour supply is a possible insurance mechanism to market income shocks. For example, the wife's labor supply could increase in response to negative income shocks faced by the husband (see e.g. Lundberg, 1985). By comparing the dynamics of individual and family income over the life cycle, we are able to assess the overall impact of these three factors, but not identify the individual contribution of each factor.<sup>17</sup>

Figures 2.3 and 2.4 display the age profiles in the variances of transitory and permanent shocks to family disposable income. By comparing these profiles to the ones for individual disposable income, we can see that the magnitude of the shocks change little when we add the income of the spouse. This suggests that risk sharing through (negative) assortative mating or labor supply offset the high variance in income of wives and any positively correlated shocks. Table 2.1 shows that permanent shocks remain highly persistent for low and medium skilled, while falling for the high skilled when we add spouse's income. We can further see that the persistence of transitory shocks change little when including the income of the spouse.<sup>18</sup>

## 2.5 Investigation of Income Processes

This section takes advantage of the size and detailed nature of the Norwegian data and brings new evidence on several issues pertinent to the modeling of income processes.

### 2.5.1 Nonstationarity in Age and Time

Our rich panel data allows us to let the variances components depend on age in an unrestricted way, while controlling flexibly for calendar time effects. This raises questions such as: What is missed by the standard specification in the literature with age-independent variance components? How important is it to account for calendar time effects such as the business cycle or tax reforms?

Appendix Table B.1 investigates the implications of assuming age-independent

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<sup>17</sup>We have also estimated a model which allows for separate correlation of spouse's permanent and transitory components. To this end, we restrict the baseline sample to couples where both spouses have at least four subsequent observations with positive market income. The estimates from this sample of dual earner couples suggest weak negative correlation across spouses in the shocks to disposable income over most of the life-cycle. However, we need to be cautious in interpreting these estimates because the labor force participation of women is relatively low and unstable; as a result, the dual earner couples are not representative for our baseline sample of working males.

<sup>18</sup>Our results are not sensitive to whether we adjust for economies of scale by employing the usual equivalence scales. To see this, note that the log of family income is equal to the log of the incomes of the husband and the wife, subtracted the log of the equivalence scale (e.g. the square root of family size); the former term will clearly dominate the latter term.

shocks. We display the parameter estimates from a model in which the variances of the error components in equation (2.1) are restricted to be constant across the life cycle. For the high skilled, the estimated persistence parameter falls from almost one with age-dependent variance of shocks to .75 with age-independent variance of shocks. By comparison, the age-independent specification does not affect the estimates of the persistence parameter for the low and medium skilled. Figures 2.5 and 2.6 show the misspecification bias from restricting the variances of transitory and permanent shocks to be constant over the life cycle. These figures highlight the importance of allowing for age nonstationarity to capture the labor income dynamics of low and high skilled workers.

What features of the data give rise to the misspecification bias we observe? Recall that high skilled have a U-shaped age profile in the variance of individual market income. We argue that targeting these moments with an age stationary model puts a downward pressure on  $\rho$ . With a persistent parameter close to one, it becomes difficult to match the U-shaped profile with an age-independent specification because the permanent shocks would then accumulate over the life cycle, generating an increasing and convex profile in the variance of individual market income.

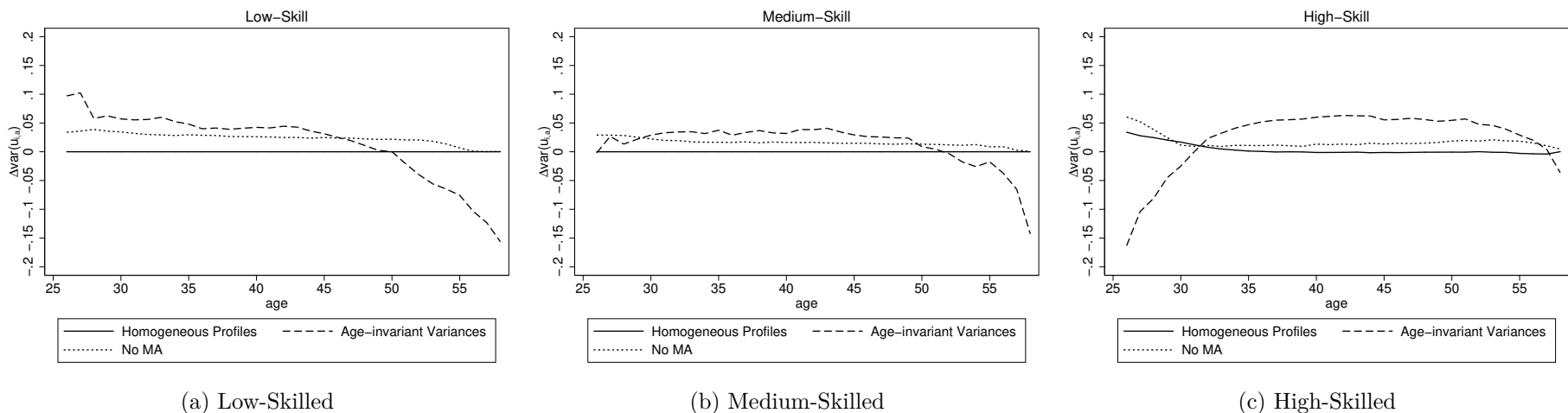


Figure 2.5: Misspecification bias in the variance of permanent shocks to individual market income

**Notes:** This figure graphs the differences in the estimated variances of permanent shocks to individual market income by age between (i) a specification without and with linear heterogeneous profiles, (ii) a specification without and with age-dependent variances of shocks and (iii) a specification without and with a MA(1) process in transitory shocks to income. The age profiles are based on the model of income dynamics described in Section 2.3.1. We use the baseline sample and estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. The age profiles are adjusted for education-specific calendar time effects.

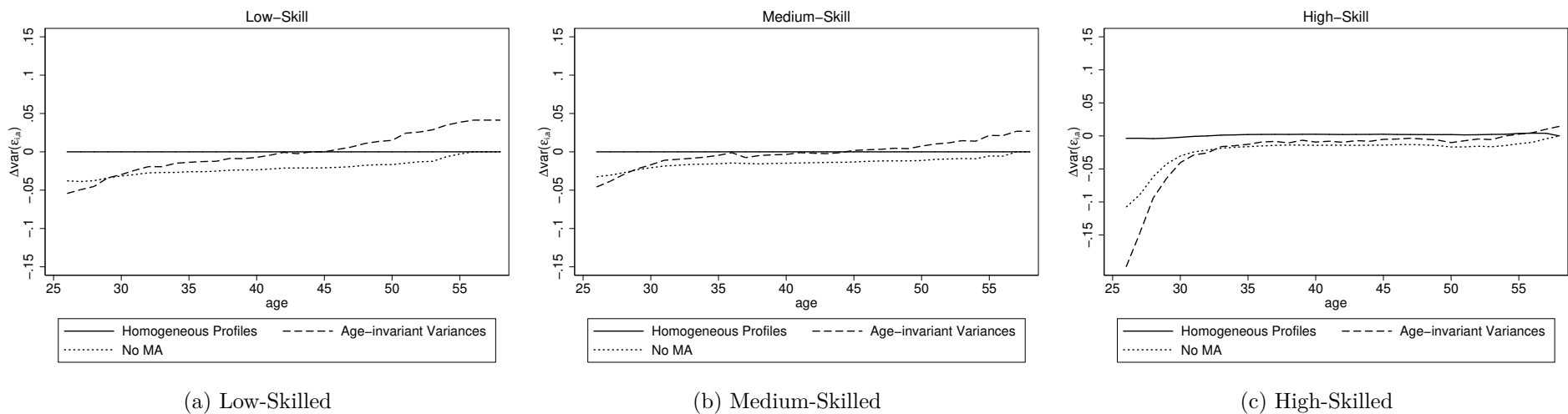


Figure 2.6: Misspecification bias in the variance of transitory shocks to individual market income

**Notes:** This figure graphs the differences in the estimated variances of transitory shocks to individual market income between (i) a specification without and with linear heterogenous profiles, (ii) a specification without and with age-dependent variances of shocks and (iii) a specification without and with a MA(1) process in transitory shocks to income. The age profiles are based on the model of income dynamics described in Section 2.3.1. We use the baseline sample and estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. The age profiles are adjusted for education-specific calendar time effects.

In Figure 2.7, we illustrate the importance of allowing for time nonstationarity to get a clear picture of the typical income dynamics over the life cycle. We estimate the model given in (2.1) separately by cohort, and graph the age profiles in the variance of transitory shocks to disposable income for different cohorts; for brevity, we do not split the sample by education. The 5 year interval between the cohorts allows us to clearly see the impact of a tax reform: For each cohort, we observe a spike in the variance of transitory shocks at that the time of the change in tax policy.<sup>19</sup> By comparison, our baseline results control for such calendar time effects by averaging the moments across the cohorts before estimating the income process. After taking out calendar time effects, the variance of transitory shocks exhibits a smooth and decreasing profile over the life-cycle.

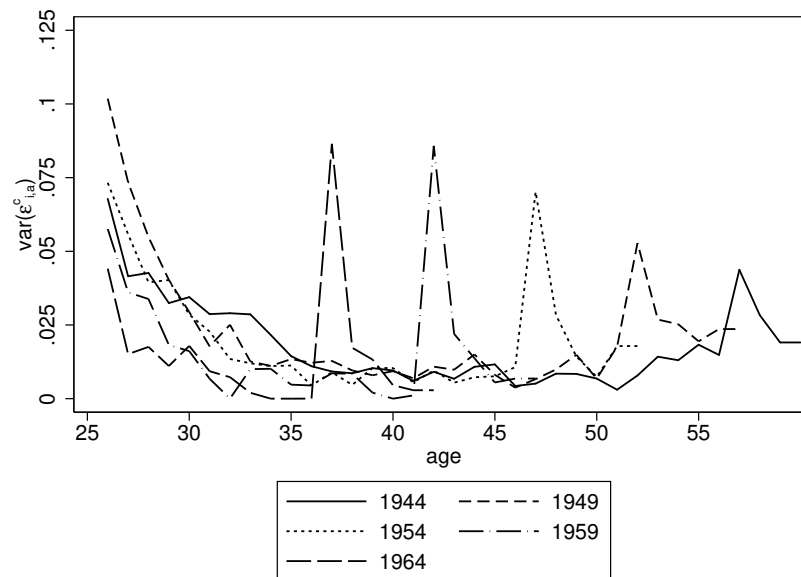


Figure 2.7: Age profiles in the variances of transitory shocks to individual disposable income by birth cohort

**Notes:** This figure graphs the age profiles in the variances of transitory shocks to individual disposable income separately by cohort. We consider cohorts born in 1944, 1949, 1954, 1959, and 1964. For each cohort, we estimate the model of income dynamics described in Section 2.3.1.

## 2.5.2 Heterogeneous profiles

Our findings suggest important heterogeneity in labor income dynamics by age, skill level, and their interaction. This raises questions such as: What happens if we do not allow for the possibility that individuals with different education levels

<sup>19</sup>The tax reform affected both the timing of income reporting and the incentives to shift and reclassify income.

face different income processes at the variance level? How important is it to allow for unobserved heterogeneity in the income growth rates within skill groups?

Appendix Table B.2 displays parameter estimates from the baseline model of income dynamics when we do not split the sample by education. The persistence parameter in the pooled sample is one, suggesting that the shocks to (log) labour income can still be described as the sum of a transitory shock and an highly persistent process. Figures B.10 and B.11 show the age profiles in the variances of shocks when we restrict the income processes at the variance level to be the same across skill groups. Because the results from the pooled sample mixes the income processes of low and high skilled, we obtain an inverse U-shaped age profile in the variances of permanent shocks. However, this pattern is at odds with the age profiles of both high and low skilled: While the former group experience large permanent shocks early in life, the latter group faces the largest shocks at older ages. These findings point to the importance of allowing for heterogeneity by education levels to capture the labor income dynamics of young and old workers.

So far, we have imposed homogenous experience profiles (i.e.  $\beta_i = 0$ ) within each skill group. We now relax this assumption and allow for a linear experience profile in the model given by equation (2.1). Appendix Table B.3 displays the parameter estimates for individual market income, while Figures 2.5 and 2.6 show the misspecification bias from imposing homogenous experience profiles. The results suggest education levels do a good job in capturing heterogeneity in the dynamics of labor income over the life-cycle. Only for the high skilled, there is evidence of significant unobserved heterogeneity in the income growth rates; accounting for this heterogeneity lowers the persistent parameter from .98 to .90, but barely moves the age profiles in the variances of permanent and transitory shocks.

Appendix Figure B.18 illustrates the heterogeneity in market income profiles for the high skilled. There is a non-negligible fanning out of the income profiles. At the same time, there is a negative correlation between the initial conditions and the individual-specific income growth rate. This means that high skilled workers with relatively low market income at age 25 (the initial age) tend to have stronger income growth over the life cycle, offsetting some of the fanning out displayed in Figure B.18.

### 2.5.3 Serially correlated transitory shocks

Because we have long panel of individuals, we can separately identify a transitory process with serially correlated shocks and a permanent process which allows for a persistence parameter less than unity. In many cases, however, this is difficult because the panel of individuals is too short (or plagued by problems such as

attrition and small sample sizes).

In Figures 2.5 and 2.6, we examine the implications of restricting the transitory component in the model given by equation (2.1) to be uncorrelated over time. In the simple case of serially uncorrelated transitory shocks, all the persistence in the income data is attributed to the permanent income component. By comparison, the transitory component is assigned a larger share of the total variance in our baseline model, because the process capture short-duration persistence in the data. However, our estimates suggest the misspecification bias from assuming serially uncorrelated transitory shocks is relatively small compared to the biases from ignoring heterogeneity in labor income dynamics by age and skill level.

## 2.6 Conclusion

What do labor income dynamics look like over the life-cycle? What is the relative importance of persistent shocks, transitory shocks and idiosyncratic trends? To what extent do taxes, transfers and the family attenuate these various factors in the evolution of life-cycle inequality? In this paper, we used rich Norwegian panel data to answer these important questions. We estimated a process for income dynamics that allows for key aspects in the evolution of labour income over the life-cycle, including non-stationarity in age and time, shocks of varying persistence, and heterogeneous profiles.

Our estimates of the labor income dynamics of males showed that the magnitude of permanent and transitory shocks vary systematically over the life-cycle, and that there is essential heterogeneity in the variances of these shocks across skill groups. We found that the progressive nature of the Norwegian tax-transfer system plays a key role in attenuating the magnitude and persistence of income shocks, especially among the low skilled. Spouse's labour market income, on the other hand, matters less for the dynamics of inequality over the life-cycle.

The size and detailed nature of the data we are using also allowed us to bring new evidence on several additional issues pertinent to modeling of income processes. One key finding was that restricting the age and time dependence of the variance of income shocks can lead to quite misleading conclusions about the income process. Another key finding was that allowing for heterogeneity by education levels is necessary to capture the labor income dynamics of young and old workers. By way of comparison, the dynamics of income over the life-cycle change little when restricting the transitory shocks to be uncorrelated over time or allowing for heterogeneous experience profiles within each skill group. Indeed, only for the high skilled, there is evidence of significant unobserved heterogeneity in the



income growth rates. Accounting for this heterogeneity lowers the persistence of permanent shocks somewhat, but barely moves the age profiles in the variances of permanent and transitory shocks.

## Chapter 3

# Labour Income Dynamics over the Business Cycle

### 3.1 Introduction

The distributional dynamics of individual income over the business cycle is a key ingredient in models that address a number of important questions, including the distribution of the welfare costs of business cycles (Lucas, 1987; Storesletten, Telmer, and Yaron, 2004b; Krebs, 2007; De Santis, 2007; Krueger, Mitman, and Perri, 2016) and the implications of cyclical income risk for aggregate output and consumption (Bayer, Luetticke, Pham-Dao, and Tjaden, 2017; McKay, 2017).

The objective of this paper is to present an estimated income process that is consistent with a variety of evidence on income risk over the life and business cycle. There is an extensive literature that studies the impact of aggregate shocks on the variance and skewness of individual labour income shocks. Storesletten, Telmer, and Yaron (2004b) estimate an income process allowing the variance of the persistent income shock to depend on the aggregate state. They conclude that the variance of persistent income shocks is highly countercyclical in the US. Busch and Ludwig (2016), using survey data from Germany, extend their framework by allowing the skewness of the distribution of persistent income shocks to be state-contingent. Their estimates suggest a countercyclical variance and left-skewness. In contrast, Guvenen, Ozkan, and Song (2014) and Busch, Domeij, Guvenen, and Madera (2016) analyse second and higher order moments of the distribution of income growth rates using large administrative datasets from various countries. They provide empirical evidence for a countercyclical left-skewness of income growth rates, but conclude that their variance is largely acyclical. The literature on income dynamics has also pointed out that income risk varies sys-

tematically over the life cycle (see for example Chapter 2 in this thesis and the references therein) and with the previous earnings level (Guvenen, Karahan, Ozkan, and Song, 2016).

In this chapter we specify and estimate an income process that builds on the quantile panel data framework recently developed by Arellano, Blundell, and Bonhomme (2016). In our model idiosyncratic income follows a general first order Markov process. Specifically, we model the income process as a first-order quantile-autoregressive process. The stochastic process for income specifies any quantile of the income distribution conditional on the previous income level. Modeling the conditional distribution of income has the key advantage that it represents the most comprehensive measure of income risk (under the Markov assumption). We let individuals with different education levels have a separate income process, and within each skill group and cohort we allow the conditional quantile functions to vary unrestrictedly over time. Our approach therefore represents a tractable way of allowing income risk within each skill group to depend on the previous income level, calendar time and experience.

We estimate the income process using population panel data from Norway from 1979 to 2013. We find that income dispersion (i) follows a U-shape over the life cycle, (ii) follows a U-shape with the previous income level, and (iii) is procyclical for those at the bottom, acyclical for those in the middle, and countercyclical for those at the top of the distribution of previous income. In addition, we find that skewness (i) follows an inverted U-shape over the life cycle, (ii) declines with the previous income level, and (iii) is procyclical.

The layout of the remainder of this chapter is as follows. The next Section presents our income panel data and our choice of the business cycle indicator. Section 3.3 describes our panel data specification for the income process and presents the measures of conditional and average income uncertainty and skewness. Section 3.4 presents the main results and Section 3.5 concludes.

## 3.2 Data Description

### 3.2.1 Norwegian Registry Data

The empirical analysis employs several registry databases maintained by Statistics Norway that we can link through unique identifiers for each individual. This allows us to construct a rich longitudinal data set containing individual records for the entire resident population of Norway from 1979 to 2013. The panel data

set contains individual demographic information (including gender, date of birth, marital status, family size and composition), socio-economic data (including income from various sources and education), and exact geographical identifiers. The data also contains unique family identifiers that enable us to measure income at the household level by matching individuals with their spouses, and parents to their children.

Registry data are ideal for our purpose of studying income dynamics, in particular because of the long time dimension and the large number of cross-sectional observations. Furthermore, the coverage and reliability of Norwegian registry data are considered to be outstanding (Atkinson, Rainwater, and Smeeding, 1995). There is no attrition from the original sample because of the need to ask for permission from individuals to access their tax records. In Norway, these records are in the public domain. The income data also pertain to all individuals and all jobs, and not only to jobs covered by social security and measures of incomes are recorded without any top- or bottom coding. In addition, there are little reporting or recollection errors, which is a common problem with survey-based micro datasets. The income data come from individual tax records with detailed information about the different sources of income, while educational attainment is reported by the educational establishments directly to Statistics Norway.

The income variables that we consider are defined as follows. The first variable is annual *individual market income*. This includes all income received as a worker during a given calendar year, including acquired benefits during unemployment, sickness allowance and maternity/paternity leave benefits. The second variable is *individual disposable income* measured as individual market income plus cash transfers net of taxes on earnings.<sup>1</sup> The third income variable is *family disposable income*, defined as the sum of the individual disposable income of the spouses. Throughout the empirical analysis we treat cohabiting couples identical to married couples.<sup>2</sup> Finally, all income measures are deflated to the base year 2005 using the Consumer Price Index (CPI) provided by Statistics Norway and converted to US Dollars by using the average of the daily exchange rates in 2005.<sup>3</sup>

<sup>1</sup>As seems appropriate in this context, this income measure does not include income from financial assets. In every year, we compute taxes using a tax simulation program available at Statistics Norway.

<sup>2</sup>Cohabitants are identified from the National Censuses of Population and Housing in 1980 and 2001, and the National Survey of Population and Housing in 1990, as well as from Statistics Norway's own datasets identifying cohabiting couples after 2004. Further, cohabiting couples are roughly identified from annual registry data as men and women living at the same address who at some point have children together or become cohabitating couples, or who get married at a later point in time.

<sup>3</sup>The CPI is available here: <http://www.ssb.no/en/inntekt-og-forbruk>. The daily exchange rates between USD and NOK are available here: [http://www.norges-bank.no/en/Statistics/exchange\\_rates/](http://www.norges-bank.no/en/Statistics/exchange_rates/). The average daily exchange rate (NOK/USD) in 2005

The life cycle is measured by *potential labor market experience*, defined as age minus years of education minus 6. To minimize the impact of heterogeneity in the initial transition from education to the labor market, we focus on the range of potential experience from 4 to 40 throughout the empirical analysis. We define a *cohort* as a group of individuals that have completed education in a given year.

Throughout the empirical analysis we focus on households with a male earner as household head. In every year, we restrict the panel to include non-immigrant men and their spouse (if they have one) who filed a tax return and who have non-missing information on key demographic characteristics. To ensure fairly long records on earnings for each individual, we focus on cohorts that enter the labour market between 1950 and 2005. In line with much of the previous literature on income dynamics, we exclude individuals whose primary source of income comes from self-employment. We also exclude observations in a given year if they fall below a time-varying minimum threshold. This minimum threshold is taken to be 0.1 basic amounts, which amounts to roughly 1,300 USD in 2013.<sup>4</sup> Such a minimum threshold condition is fairly standard in the income dynamics literature and ensures that we select individuals with a reasonably strong labor market attachment.<sup>5</sup>

Applying these restrictions leaves us with an unbalanced panel of 27,272,712 household-year observations. To account for heterogeneity by educational attainment, we partition the sample into three mutually exclusive groups according to educational levels of the household head. Low skilled is defined as not having completed high school (20% of the sample), medium skilled includes those with a high school degree (52% of the sample), and the high skilled consists of those who have attended college (29% of the sample). Table 3.1 displays some summary statistics for the three subsamples and Appendix Figure C.1 graphs the average log income over the life cycle.

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was 6.4450.

<sup>4</sup>One basic amount is the so called threshold of substantial gainful activity. The nominal level of the threshold varies year-by-year according to the development of wages in the Norwegian economy.

<sup>5</sup>From a technical perspective a minimum threshold also sidesteps the issue of taking the logarithm of small numbers.

variable	N	Mean	Std. Dev.	10pctile	25pctile	Median	75pctile	90pctile
age	5,377,227	36.27	9.87	23.00	28.00	36.00	44.00	50.00
p. experience	5,377,227	20.62	10.21	7.00	12.00	20.00	29.00	35.00
married	5,377,227	0.40	0.49	0.00	0.00	0.00	1.00	1.00
children	5,377,227	0.96	1.14	0.00	0.00	1.00	2.00	3.00
market income	5,377,227	40,348.74	22,613.57	13,191.46	28,731.58	39,192.58	50,131.80	63,967.12
disposable income	5,377,227	28,629.56	13,304.12	13,336.87	21,699.78	27,449.69	34,757.82	43,719.57
family disposable income	5,377,227	36,947.15	20,783.43	14,363.49	23,244.61	33,395.98	47,234.95	63,629.99

(a) Low-skilled

variable	N	Mean	Std. Dev.	10pctile	25pctile	Median	75pctile	90pctile
age	14,106,136	38.94	9.81	26.00	31.00	38.00	47.00	53.00
p. experience	14,106,136	20.35	10.00	7.00	12.00	20.00	28.00	35.00
married	14,106,136	0.53	0.50	0.00	0.00	1.00	1.00	1.00
children	14,106,136	1.07	1.14	0.00	0.00	1.00	2.00	3.00
market income	14,106,136	52,115.60	28,691.23	27,126.93	37,740.87	47,904.05	61,733.43	80,657.37
disposable income	14,106,136	35,038.03	15,862.77	20,855.57	26,204.25	32,488.98	41,371.34	52,342.48
family disposable income	14,106,136	47,165.48	24,072.34	22,758.58	30,771.19	42,853.25	59,133.27	78,108.02

(b) Medium-skilled

variable	N	Mean	Std. Dev.	10pctile	25pctile	Median	75pctile	90pctile
age	7,789,349	41.55	9.94	29.00	33.00	40.00	49.00	56.00
p. experience	7,789,349	19.00	9.87	6.00	11.00	18.00	27.00	34.00
married	7,789,349	0.62	0.49	0.00	0.00	1.00	1.00	1.00
children	7,789,349	1.13	1.16	0.00	0.00	1.00	2.00	3.00
market income	7,789,349	69,530.29	50,329.06	33,259.02	46,711.06	60,742.78	82,063.64	112,832.84
disposable income	7,789,349	43,888.58	26,645.44	23,984.41	30,721.20	39,220.31	51,645.95	68,881.73
family disposable income	7,789,349	61,405.41	36,076.08	27,278.28	38,646.39	55,275.75	77,351.39	102,316.48

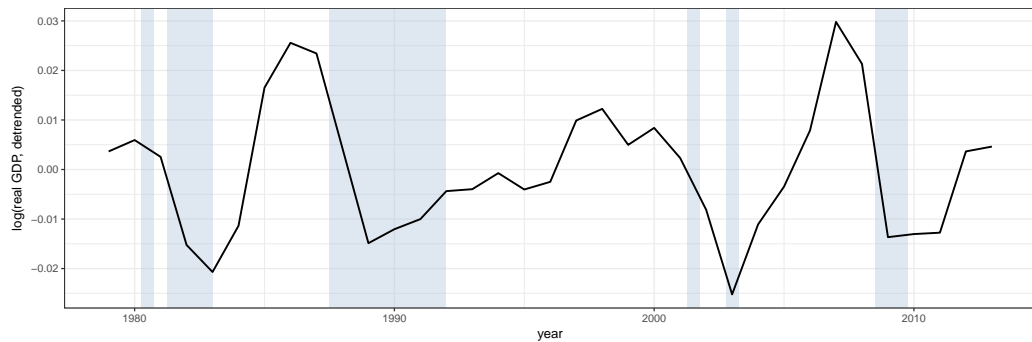
(c) High-skilled

Table 3.1: Summary Statistics

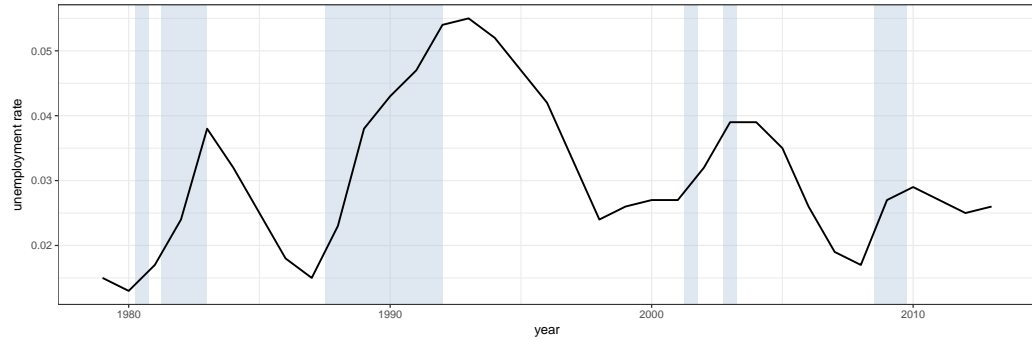
**Notes:** This table shows summary statistics of key variables by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

### 3.2.2 Business Cycle Conditions

The primary business cycle indicator used in this chapter is the annual real Gross Domestic Product (GDP) of mainland Norway.<sup>6</sup> To capture the business cycle, we apply a Hodrick-Prescott filter to the natural logarithm of annual real GDP of mainland Norway, using a smoothing parameter of 6.25 for annual data as suggested by Ravn and Uhlig (2002). Hence the business cycle conditions are expressed in terms of the log deviation of the real GDP from its trend. For robustness checks, I consider the the annual unemployment rate as an alternative business cycle indicator.<sup>7</sup> Figure (3.1) shows the evolution of the two business cycle indicators over time.<sup>8</sup>



(a) HP-filtered natural logarithm of annual real GDP



(b) Unemployment rate

Figure 3.1: Business cycle indicators

**Notes:** This figure shows the evolution of business cycle indicators over time. Panel (a) shows the time-series of the HP-filtered natural logarithms of annual real GDP. Panel (b) shows the time-series of the unemployment rate. The grey bars indicate recessions in Norway as dated by Aastveit, Jore, and Ravazzolo (2016) according to their preferred method (MS-FMQ).

<sup>6</sup>The GDP series relates to mainland Norway and therefore excludes offshore activity, namely oil and gas extraction and international shipping. The main reason for the exclusion of these sectors is the fact that their production may show large fluctuations that have very small short term effects on the Norwegian labor market (see Aastveit, Jore, and Ravazzolo, 2016).

<sup>7</sup>The time-series for the unemployment rate is published by the Norwegian Labour and Welfare Administration and are available to download from <https://www.nav.no/no/NAV+og+samfunn/Statistikk/Arbeidssokere+og+stillinger+-+statistikk/Historisk+statistikk>.

<sup>8</sup>The cross-correlation between the two time-series over the sample period is  $-0.55$ .

The Norwegian economy experienced 6 recessions (peak to trough) over the sample period (see Aastveit, Jore, and Ravazzolo, 2016).<sup>9</sup> Norway experienced a double-dip recession at the beginning of the 1980s, followed by a deep and long-lasting recession at the end of the 1980s. This particular recession turned into a banking crisis that was accompanied by a large increase in the unemployment rate by 4 percentage points until the early 1990s. The early 2000s were characterised by two relatively short-lived recessions. Finally, the Great Recession had a relatively moderate impact on the Norwegian labour market with an already relatively low unemployment rate raising by only 1.2 percentage points.

### 3.3 Empirical Approach

#### 3.3.1 The Income Process

Following the mainstream approach in the income dynamics literature, we decompose log income of household  $i$  in year  $t$  into two components,

$$\ln Y_{it} = f_t(X_{it}) + y_{it}. \quad (3.1)$$

The component  $f_t(X_{it})$  is a function that depends on calendar time  $t$  and on observable household characteristics  $X_{it}$ , and  $y_{it}$  represents the idiosyncratic component of income.

**Aggregate Variation in Income.** The first component of decomposition (3.1) captures the deterministic cross-sectional variation in income attributable to observable household characteristics such as marital status, household size, region of residence, potential experience and cohort. Aggregate variation in log income is captured by the functional dependence on calendar time. Ideally one would like to control for the effects from potential experience ( $h$ ), cohort ( $c$ ) and calendar time ( $t$ ) in an unrestricted way. Recall, however, that we have defined a cohort as a group of individuals who have completed education in a given year, so that potential experience, year and cohort are perfectly collinear, i.e.  $c = t - h$ . Thus the level effects of these three factors cannot be separately identified.<sup>10</sup>

<sup>9</sup>Note that there is no authoritative dating of business cycles for the Norwegian economy. Aastveit, Jore, and Ravazzolo (2016) provide dates of business cycles according to 4 different methods. We follow here their preferred method (MS-FMQ) and date business cycles according to Table 1 in their paper.

<sup>10</sup>Even if we were to define cohort by the year of birth, we would still have to deal with a substantial problem of multicollinearity.



An extent literature has pointed out several ways to deal with this fundamental identification problem. The standard approach involves imposing one or more restrictions. Deaton (1997) for example suggests a normalisation that makes the year effects average to zero over the sample period and be orthogonal to a time trend, such that all growth is attributed to experience and cohort effects. Another widespread approach is to assume that either cohort or time effects are zero, so that secular trends appear only in time or cohort effects respectively.<sup>11</sup> We follow an approach suggested by Heckman and Robb (1985) instead. The general idea is to treat experience, cohort and time effects as proxy variables for underlying variables which are not themselves linearly dependent. Hence we interpret time effects as proxy variables for underlying aggregate economic conditions that in principle can be measured by business cycle indicators. Conveniently, this approach allows us to measure the average cyclical variation of log income. Hence we specify

$$f_t(X_{it}) = x'_{it}\varphi + \psi_h + \lambda_c + \xi \ln Z_t, \quad (3.2)$$

where  $\psi_h$  and  $\lambda_c$  denote experience and cohort fixed effects, and  $x_{it}$  captures the deterministic income effects attributable to household size, marital status and region. Of particular interest in the context of our analysis is the coefficient  $\xi$  which measures the average elasticity of income with respect to the contemporaneous cyclical conditions  $Z_t$ . To allow for the possibility that the cyclical variation of income changes over the life cycle, we implement an alternative specification,

$$f_t(X_{it}) = x'_{it}\varphi + \psi_h + \lambda_c + \sum_h \xi_h \mathbb{1}_h \ln Z_t, \quad (3.3)$$

where  $\mathbb{1}_h$  is an indicator variable that takes on the value one if the individual has potential experience  $h$ . The series  $\{\xi_h\}$  measures the average elasticity of income with respect to the contemporaneous cyclical conditions at any stage in the life cycle.

**Dynamics of Residual Income.** We now shift attention to the dynamics of the idiosyncratic income component  $y_{it}$ . Motivated by the quantile-based framework recently proposed by Arellano, Blundell, and Bonhomme (2016), we model its law of motion as a first-order quantile autoregressive process. For *a given cohort* of

<sup>11</sup>As highlighted by Heathcote, Storesletten, and Violante (2005), the age profiles of inequality can look very different depending on whether one assumes cohort or time effects to be at work. In a recent paper, Schulhofer-Wohl (2013) proposes an estimation method for structural life cycle models that avoids the documented problems of the standard approach.

households, the idiosyncratic income is assumed to evolve over time according to

$$y_{it} = Q_t(y_{i,t-1}, u_{it}), \quad (3.4)$$

where  $Q_t(y_{i,t-1}, \tau)$  denotes the  $\tau$ -th conditional quantile of  $y_{it}$  given  $y_{i,t-1}$ . The innovation  $u_{it}$  given  $y_{i,t-1}, \dots, y_{i,1}$  is uniformly distributed on the unit interval, and the distribution of the initial condition  $y_{i,1}$  is left unrestricted.

Some discussion of our modeling choice is warranted. In contrast to Arellano et al. (2016), we do not retain the typical permanent-transitory framework and instead restrict ourselves to a single-error formulation. This choice is made solely for computational reasons in view of the large sample size we are dealing with in the estimation. Provided that we have estimates for the idiosyncratic income component, a single-error formulation has the advantage that the estimation only involves a series of quantile-autoregressions as opposed to an iterative estimation procedure as in Arellano et al. (2016). Considering also that a transitory income component often represents a mixture of transitory income shocks and measurement error, we hope that the single-error formulation is less problematic in the presence of high-quality administrative data.

As highlighted by Arellano et al. (2016), nonlinearities in the persistence of the income process are a key feature of the nonlinear income process (3.4). In this framework, the partial derivative of the  $\tau$ -th conditional quantile function,

$$\rho_t(\tau, y_{i,t-1}) = \frac{\partial Q_t(y_{i,t-1}, \tau)}{\partial y_{i,t-1}}, \quad (3.5)$$

measures the persistence of  $y_{i,t-1}$  when the process is hit by a shock  $u_{i,t}$  that has rank  $\tau$ . This allows for the possibility that both the rank of the income shock as well as the household's current position in the income distribution can affect the persistence of  $y_{i,t-1}$  in rather general ways. Allowing for nonlinear persistence, as highlighted by these authors, is key to capture empirically the effects of a variety of income shocks. Appendix Figures (C.2) - (C.5) show that nonlinear persistence is a key feature of the data, and that average persistence varies systematically over the life cycle. Our empirical results, however, suggest that persistence does not appear to change over the business cycle in any systematic and economically relevant way. For the rest of the paper, we therefore abstract from analysing it in more detail.

### 3.3.2 Conditional Uncertainty and Skewness

A key advantage of the quantile autoregressive process (3.4) is that it allows us to characterise any quantile of the conditional distribution. Clearly, the conditional distribution as a whole is the most comprehensive measure of income risk. It combines all aspects of income uncertainty by its nature. In the interest of an insightful empirical analysis, however, we restrict ourselves to a few characteristics of the conditional distribution. In line with the recent literature on income risk over the business cycle (e.g. Guvenen, Ozkan, and Song, 2014), we focus on two key characteristics of a distribution: dispersion and asymmetry.

Typically, the dispersion of a distribution is measured by its standard deviation. A corresponding quantile-based measure for some  $\tau \in (0.5, 1)$  is

$$\sigma_t(y_{i,t-1}, \tau) = Q_t(y_{i,t-1}, \tau) - Q_t(y_{i,t-1}, 1 - \tau). \quad (3.6)$$

From the perspective of household  $i$  in period  $t - 1$ , the measure  $\sigma_t(y_{i,t-1}, 0.75)$  represents the interquartile range of the household's possible (residual) income realisations one period ahead in  $t$ . Put differently, it measures the width of the range which holds 50% of the household's (residual) income next period and as such can be interpreted as a measure of conditional income uncertainty. Average income uncertainty is given by

$$\bar{\sigma}_t(\tau) = \mathbb{E} [Q_t(y_{i,t-1}, \tau) - Q_t(y_{i,t-1}, 1 - \tau)], \quad (3.7)$$

where the expectation is taken with respect to the distribution of  $y_{i,t-1}$ .

The degree of asymmetry of a distribution is measured by its skewness. For some  $\tau \in (0.5, 1)$ , a quantile-based measure of conditional skewness is

$$\gamma_t(y_{i,t-1}, \tau) = \frac{Q_t(y_{i,t-1}, \tau) + Q_t(y_{i,t-1}, 1 - \tau) - 2Q_t(y_{i,t-1}, 0.5)}{Q_t(y_{i,t-1}, \tau) - Q_t(y_{i,t-1}, 1 - \tau)}, \quad (3.8)$$

and its average is given by

$$\bar{\gamma}_t(\tau) = \mathbb{E} \left[ \frac{Q_t(y_{i,t-1}, \tau) + Q_t(y_{i,t-1}, 1 - \tau) - 2Q_t(y_{i,t-1}, 0.5)}{Q_t(y_{i,t-1}, \tau) - Q_t(y_{i,t-1}, 1 - \tau)} \right]. \quad (3.9)$$

For the interpretation of some of the empirical results discussed further below, it is useful to note the link between the conditional measure of dispersion and skewness. Rearranging (3.8) yields,

$$\frac{Q_t(y_{i,t-1}, \tau) - Q_t(y_{i,t-1}, 0.5)}{\sigma_t(y_{i,t-1}, \tau)} = \frac{\gamma_t(y_{i,t-1}, \tau) + 1}{2}, \quad (3.10)$$

which implies that less than half of the overall dispersion is explained by the length of the upper tail if the distribution is skewed to the left ( $\gamma_t(y_{i,t-1}, \tau) < 0$ ).

### 3.3.3 Empirical Implementation

The estimation strategy involves two steps. In the first step, we estimate the income process (3.1) by pooled OLS separately for each skill group and for each income measure, using log deviations of real GDP from its trend as business cycle indicator. In the second step, we then work with the residuals from these regressions,  $y_{i,t} \equiv \ln Y_{i,t} - \hat{f}_t(X_{it})$ , where  $f_t(\cdot)$  is specified according to (3.3).

For the empirical implementation of (3.4), we need a flexible, yet parsimonious specification of a continuous function. A straightforward way to approximate a continuous and smooth function is accomplished by the use of orthogonal polynomials (see for example Judd, 1998, ch. 6). For each cohort and time period, we specify the  $\tau$ -th conditional quantile function as,

$$Q_t(y_{i,t-1}, \tau) = \sum_{k=0}^4 \kappa_{t,k}(\tau) H_k(y_{i,t-1}), \quad (3.11)$$

where  $H_k(\cdot)$  denotes the Hermite polynomial of degree  $k$  and  $\kappa_{t,k}(\tau)$  denotes its corresponding polynomial coefficient. For any given cohort and year, we obtain the polynomial coefficients corresponding to any given  $\tau \in (0, 1)$  by quantile-autoregression. Based on these estimates, we calculate the corresponding time and cohort specific measure of dispersion and skewness, where we evaluate expressions (3.6) and (3.8) at values of lagged income corresponding to selected percentiles of the distribution income in the previous period.

Note that for any given cohort, the  $\tau$ -th conditional quantile function (3.4) is allowed to vary with time, and thus, experience in an unrestricted way. In other words, we allow for unrestricted variation in the conditional quantile function over time, experience and across cohorts. The same applies to the measures of uncertainty and skewness by construction. Of course, and as already pointed out in Section 3.3.1, not all three effects can be identified without additional restrictions.

As before, we interpret time effects as proxy variables for underlying aggregate economic conditions that are measured by business cycle indicators (Heckman and Robb, 1985). Specifically, we calculate correlations between the various measures of conditional uncertainty and skewness, and the change in the business cycle indicator, whilst controlling for potential experience and cohort effects. To fix

ideas, consider the following regression framework:

$$m_{ct} = \alpha + \psi_h + \lambda_c + \xi \Delta \ln Z_t + \varepsilon_{ct}, \quad (3.12)$$

where  $\psi_h$  and  $\lambda_c$  denote experience and cohort fixed effects respectively. The dependent variable  $m_{ct}$  corresponds to one of the cohort and time specific estimates of conditional or average dispersion and skewness. The coefficients  $\xi$  then capture the cyclicity of these measures, while the coefficients on the experience dummies trace out the average life cycle profiles.

To calculate confidence intervals, we apply a block bootstrap procedure in which we resample households, but take the residuals from the first stage as given. Sampling at the household level ensures that the autocorrelation structure from the original sample is preserved.

## 3.4 Main Results

### 3.4.1 Aggregate Variation in Income

I estimate the income process (3.1) together with specification (3.2) and (3.3) by pooled OLS separately for each skill group and for each income measure, using log deviations of real GDP from its trend as business cycle indicator. Table 3.2 reports the results of these regressions.

The results reported in columns (1) represent the estimates for specification (3.2). For each skill group, I find that individual market income is more cyclical than individual disposable income, which in turn appears to be more cyclical than family disposable income. For instance, a one-percent negative deviation of GDP from its trend implies that market income for low-skilled males is reduced by around 1.44% on average. This is in contrast to around 1.14% for individual disposable income and 1.08% for family disposable income. Another key finding is that, low-skilled households have the largest exposure to aggregate cyclical income risk, while high-skilled households are the least exposed to cyclical fluctuations. For example, high skilled households experience on average a 0.51% drop in their family disposable income in response to a one percent deviation of GDP from its trend. The corresponding number for medium skilled households is 0.81% and 1.08% for the low skilled. Taken together, these results suggest that (i) the cyclicity of income varies substantially across skill groups, and (ii) the tax-transfer system as well as spouse's income play an important role in reducing aggregate income variation.

	market income		disposable income		family disposable income	
	(1)	(2)	(1)	(2)	(1)	(2)
ln(real GDP, det.)	1.438*** (0.019)		1.142*** (0.014)		1.080*** (0.014)	
1{exp.=5}×ln(real GDP, det.)		3.560*** (0.144)		2.789*** (0.118)		2.804*** (0.117)
1{exp.=10}×ln(real GDP, det.)		1.925*** (0.110)		1.330*** (0.086)		1.210*** (0.084)
1{exp.=20}×ln(real GDP, det.)		1.267*** (0.111)		0.976*** (0.080)		1.005*** (0.075)
1{exp.=30}×ln(real GDP, det.)		0.977*** (0.100)		0.788*** (0.065)		0.718*** (0.058)
1{exp.=40}×ln(real GDP, det.)		1.021*** (0.201)		1.086*** (0.103)		0.959*** (0.092)
$R^2$	0.16	0.16	0.26	0.26	0.49	0.49
Num. Obs.	5,377,227	5,377,227	5,377,227	5,377,227	5,377,227	5,377,227
(a) low-skilled						
	market income		disposable income		family disposable income	
	(1)	(2)	(1)	(2)	(1)	(2)
ln(real GDP, det.)	1.127*** (0.009)		0.880*** (0.007)		0.814*** (0.006)	
1{exp.=5}×ln(real GDP, det.)		2.280*** (0.075)		1.697*** (0.060)		1.671*** (0.059)
1{exp.=10}×ln(real GDP, det.)		1.317*** (0.054)		0.952*** (0.040)		0.871*** (0.039)
1{exp.=20}×ln(real GDP, det.)		1.224*** (0.047)		0.888*** (0.034)		0.793*** (0.030)
1{exp.=30}×ln(real GDP, det.)		0.699*** (0.057)		0.613*** (0.039)		0.573*** (0.034)
1{exp.=40}×ln(real GDP, det.)		0.867*** (0.103)		0.837*** (0.062)		0.762*** (0.051)
$R^2$	0.17	0.17	0.28	0.28	0.54	0.54
Num. Obs.	14,106,136	14,106,136	14,106,136	14,106,136	14,106,136	14,106,136
(b) medium-skilled						
	market income		disposable income		family disposable income	
	(1)	(2)	(1)	(2)	(1)	(2)
ln(real GDP, det.)	0.815*** (0.013)		0.542*** (0.010)		0.505*** (0.009)	
1{exp.=5}×ln(real GDP, det.)		1.963*** (0.097)		1.492*** (0.080)		1.368*** (0.077)
1{exp.=10}×ln(real GDP, det.)		0.836*** (0.061)		0.514*** (0.049)		0.421*** (0.046)
1{exp.=20}×ln(real GDP, det.)		0.870*** (0.060)		0.483*** (0.045)		0.446*** (0.042)
1{exp.=30}×ln(real GDP, det.)		0.377*** (0.073)		0.320*** (0.058)		0.268*** (0.049)
1{exp.=40}×ln(real GDP, det.)		-0.376 (0.224)		-0.487** (0.062)		-0.159 (0.133)
$R^2$	0.22	0.22	0.30	0.30	0.55	0.55
Num. Obs.	7,789,349	7,789,349	7,789,349	7,789,349	7,789,349	7,789,349
(c) high-skilled						

Table 3.2: Aggregate variation in income

**Notes:** This table present coefficients of pooled OLS regressions. The results from specifications (3.2) and (3.3) are presented in column (1) and (2) respectively. The business cycle indicator is the HP-filtered natural logarithm of real GDP of mainland Norway. Other controls include potential experience fixed effects, cohort fixed effects, region fixed effects, a dummy for marital status and dummies for the number of children in the household. Standard errors are presented in parentheses, and are clustered at the individual-level. \*\*\*, \*\*, and \*, represent statistical significance at 0.1%, 1%, and 5% levels, respectively.

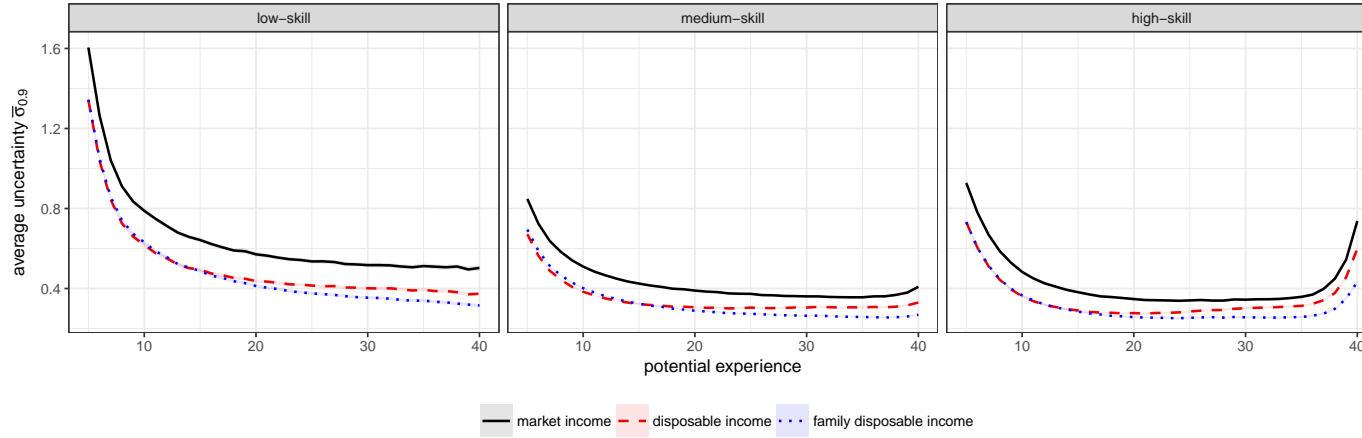
We now shift attention to the results of specification (3.3) presented in columns (2). We find that income is more cyclical for workers early in their careers. For example, a one-percent negative deviation of GDP from its trend implies on average a reduction of around 3.56% in market income among low skilled males five years after leaving education. Twenty and thirty years after having completed education, this number is reduced to around 1.27% and 0.98% respectively. It is only at the very end of the working life, where income becomes more cyclical again. With forty years of potential experience, the average elasticity amounts to 1.02. Generally, the cyclical income exhibits a U-shape for both low- and medium skilled households, whereas for high skilled it appears to be monotonically decreasing with experience. The same life cycle pattern is also observed after accounting for taxes and transfers and the income of the spouse. These results suggest that the average exposure to cyclical risk differs substantially across different points in the life cycle, with younger individuals experiencing a more cyclical income stream.

Finally, I examine whether the previous results remain economically and statistically significant if I use the demeaned unemployment rate as business cycle indicator. I also investigate whether including individual fixed effects instead of cohort fixed effects significantly changes the previous results. The prior results remain robust against these different specifications and are reported in Appendix Table C.1 and Appendix Table C.2 respectively.

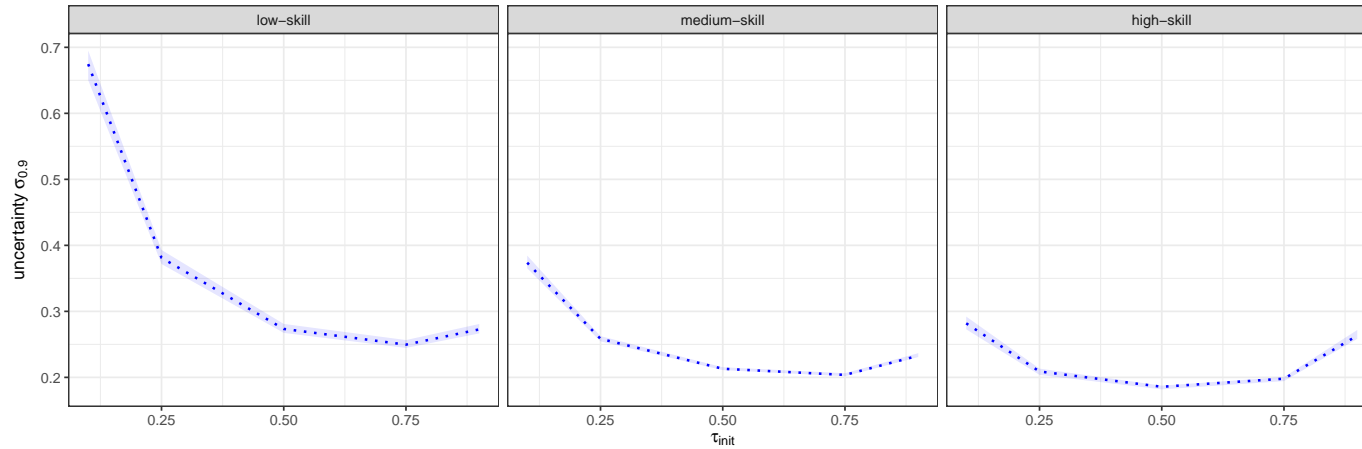
### 3.4.2 Idiosyncratic Income Risk

In this section we investigate the distributional dynamics of (residual) income over the life and business cycle.

Figure (3.2a) shows the estimated life cycle profiles of average income dispersion, defined in (3.7) and evaluated at  $\tau = 0.9$ . We find that the magnitude of income dispersion varies systematically with potential experience and across skill groups. Households are exposed to a substantial amount of income uncertainty early in their careers. We also observe that at almost any stage in the life-cycle the low skilled are exposed to more income uncertainty than the medium skilled, who in turn face more uncertainty than the high skilled. A comparison across the different measures of income reveals that the tax- and transfer system leads to a remarkable compression of incomes. A pooling of resources at the household level leads to a further reduction in dispersion after around 20 years of potential experience. These results suggest the tax- and transfer system and the family provide a significant amount of insurance against income shocks.



(a) Average dispersion over the life cycle



(b) Conditional dispersion in family disposable income

Figure 3.2: Average and conditional dispersion

**Notes:** This figure shows (a) the life cycle profile of the average dispersion, defined in (3.7) with  $\tau = 0.9$ , (b) conditional dispersion in family disposable income, defined in (3.6) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{init}$  percentile of the distribution of  $y_{i,t-1}$ ; 20 years of potential experience. The estimates are net of cohort and business cycle effects based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



We also observe that exposure to income uncertainty varies systematically in the cross-section. As shown in Figure (3.2b) conditional dispersion follows a U-shape. Households at the top and bottom of the cross-sectional income distribution in  $t - 1$  face a larger dispersion of their income realisations in  $t$  compared to those in the middle of the distribution. Appendix Figures (C.8) and (C.9) confirm that the same pattern holds for other income measures and at different points in the life cycle. Taken together, these results suggest that income uncertainty varies systematically over the life cycle, within the cross-section and across skill groups.

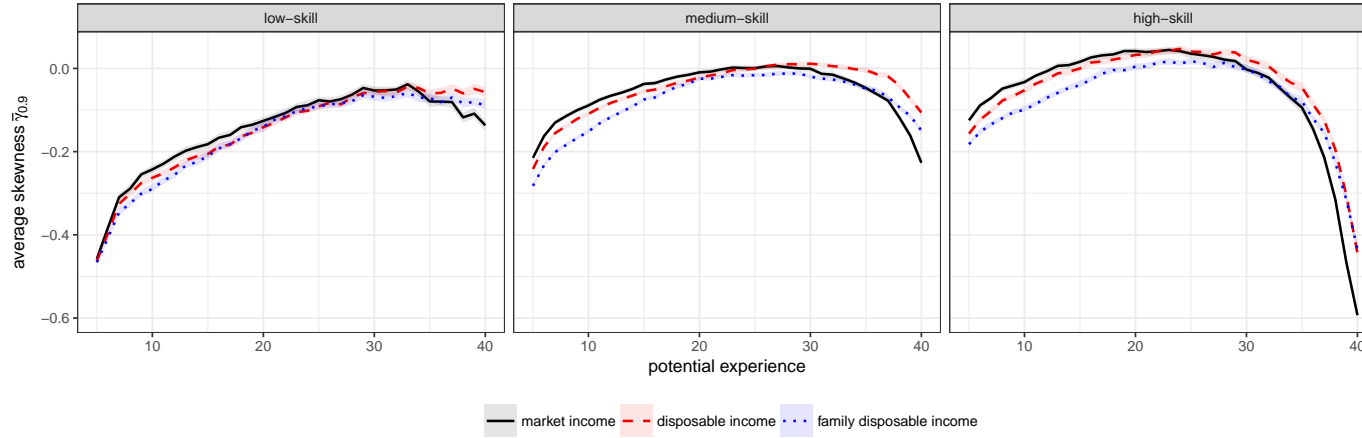
In Table (3.3) we report the cyclicalities of conditional income uncertainty. Overall, the effects are quantitatively small. This is consistent with the empirical evidence reported in Guvenen, Ozkan, and Song (2014) and Busch, Domeij, Guvenen, and Madera (2016), who investigate the dispersion in income growth rates over the business cycle for the US, Germany and Sweden. However, we observe an interesting asymmetry in the cyclicalities of conditional income dispersion. Households currently at the bottom of the cross-sectional income distribution experience an increase in the dispersion of their possible income realisations next period, while income realisations for those at the top becomes more compressed. This is in contrast to the median household, who faces an income uncertainty that remains largely unaffected by the expansion. For example, consider an increase in the growth rate of GDP by two standard deviations. For the medium-skilled, this implies that the log 90-10 market income differential decreases by around 0.0146 at the 90th percentile, it essentially remains unchanged for the median household ( $-0.004$ ) and it increases for the household at the 10th percentile by 0.0188. To investigate the reason behind these asymmetric changes, we take a closer look at the movements in the tails of the conditional distribution. During an expansion, our estimates of the conditional quantile functions reveal that the upper tail increases while the bottom tail shrinks. For the household at the 10th percentile, the upper tail increases relatively more than the bottom tail shrinks, which naturally leads to more dispersed income realisations in the following period. At the top of the distribution, however, it is the shrinkage of the bottom tail that outweighs the expansion of the upper tail, which therefore results in a compression of income realisations next period. Appendix Table (C.3) reveals that we find the same pattern using the unemployment rate as an alternative business cycle indicator.

We now shift attention to the asymmetry of the conditional distribution. Figure (3.3a) graphs the life cycle profile of the average skewness measure, defined in (3.9) and evaluated at  $\tau = 0.9$ .

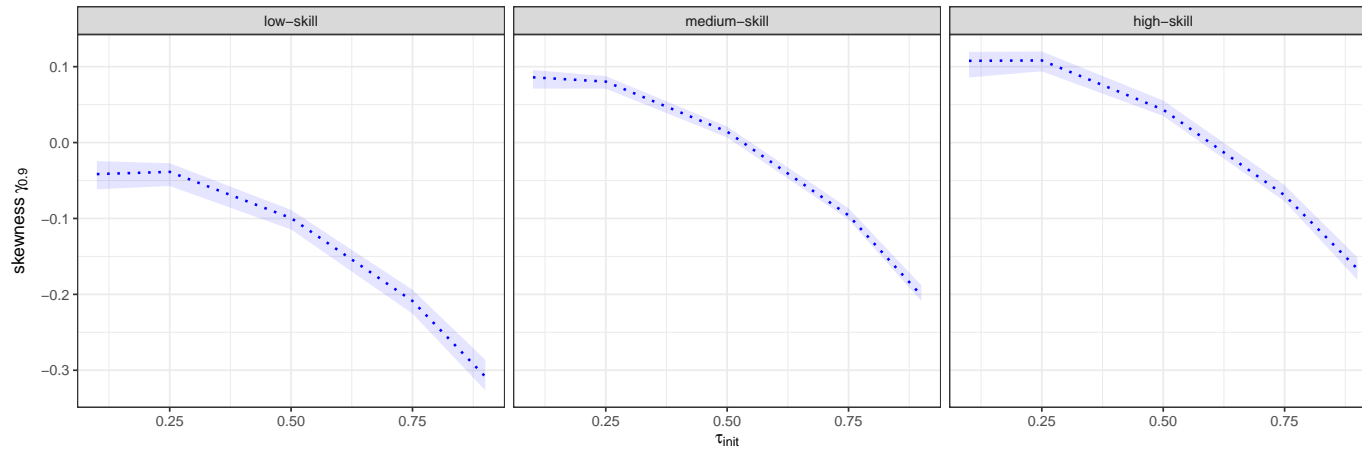
$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \ln(\text{real GDP, det.}),$ standardised	0.0092* (0.0018)	-0.0001 (0.0005)	-0.0082* (0.0006)	0.0124* (0.0013)	0.0002 (0.0004)	-0.0047* (0.0005)	0.0091* (0.0012)	0.0003 (0.0004)	-0.0045* (0.0005)
(a) low-skilled									
$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \ln(\text{real GDP, det.}),$ standardised	0.0094* (0.0007)	-0.0020* (0.0002)	-0.0073* (0.0003)	0.0110* (0.005)	-0.0011* (0.0002)	-0.0047* (0.0002)	0.0093* (0.0005)	-0.0006* (0.0001)	-0.0040* (0.0002)
(b) medium-skilled									
$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \ln(\text{real GDP, det.}),$ standardised	0.0025* (0.0009)	-0.0019* (0.0003)	-0.0042* (0.0004)	0.0047* (0.0006)	-0.0012* (0.0002)	-0.0039* (0.0003)	0.0046* (0.0005)	-0.0010* (0.0002)	-0.0031* (0.0003)
(c) high-skilled									

Table 3.3: Cyclicalities of conditional dispersion

**Notes:** This table presents the results of model (3.12). Cyclicalities are captured by the coefficient on the change in HP-filtered log real GDP of mainland Norway, normalised to have mean zero and unit standard deviation. The dependent variables are the cohort and time specific measures of conditional dispersion, defined in (3.6) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{\text{init}}$  percentile of the distribution of  $y_{i,t-1}$ ; Bootstrap standard errors are presented in parentheses. \* represents statistical significance at 5% level.



(a) Average skewness over the life cycle



(b) Conditional skewness in family disposable income

Figure 3.3: Average and conditional skewness

**Notes:** This figure shows (a) the life cycle profile of average skewness, defined in (3.9) with  $\tau = 0.90$ , (b) conditional skewness in family disposable income, defined in (3.8) with  $\tau = 0.90$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{init}$  percentile of the distribution of  $y_{i,t-1}$ ; 20 years of potential experience. The estimates are net of cohort and business cycle effects based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

First, note that there are only minor differences in the skewness profiles across the different measures of income. This is in stark contrast to the findings on average income dispersion, where we found a remarkable compression due to the tax- and transfer system and spouse's income. We observe that the tax- and transfer system leads to a slightly more left-skewed distribution early in the life cycle, while the opposite effect can be observed towards the end of the life cycle. When we add spouse's income, we find that this tends to further decrease skewness compared to individual disposable income. However, the effects are quantitatively small. Consider for example low-skilled households 10 years after completing education. Skewness in family disposable income is around  $-0.29$  compared to  $-0.26$  for market income. According to equation (3.10), this implies that the upper to lower tail ratio is around  $36/64$  for family disposable income compared to  $35/65$  for market income. More importantly, we find a more pronounced variation in skewness with potential experience. With 30 years of potential experience, the average upper to lower tail ratio in family disposable income is  $47/53$ . Overall we find that average skewness roughly follows an inverted U-shape over the life cycle. Finally, a comparison across skill groups reveals that skewness risk decreases with educational attainment - leaving aside the comparably low skewness measure for the high skilled after 35 years of experience which most likely is a retirement effect.<sup>12</sup> With 20 years of experience, the upper to lower tail ratio is  $43/57$  for the low skilled,  $49/51$  for the medium-skilled and  $50/50$  for the high-skilled.

Consistent with the empirical evidence in Guvenen, Karahan, Ozkan, and Song (2016) and Arellano, Blundell, and Bonhomme (2016), we also find substantial variation in exposure to skewness risk in the cross-section. Figure (3.3b) plots estimates for the conditional skewness (3.8) for households in the middle of the life cycle. Households who are at the top of the income distribution today are exposed to more skewness risk regarding their income realisations of tomorrow. Put differently, the likelihood of large downward movements is higher for those at the top compared to those at the bottom of today's cross-sectional distribution. Appendix Figures (C.10) and (C.11) show that roughly the same pattern can be observed for other income measures and at different points in the life cycle. Taken together, these results suggest that conditional skewness varies systematically over the life cycle, within the cross-section and across skill groups. In contrast to the findings on income dispersion, however, the tax and transfer system and the pooling of income at the household level does not appear to have a major impact on the exposure to skewness risk.

<sup>12</sup>Note, in particular, that high skilled with more than 35 years of experience are on average 60 years or older, while the low-skilled at this point in the life cycle are only in their early to mid 50's.

$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \ln(\text{real GDP, det.}),$ standardised	0.0133* (0.0015)	0.0219* (0.0011)	0.0157* (0.0014)	0.0161* (0.0014)	0.0277* (0.0010)	0.0130* (0.0013)	0.0151* (0.0014)	0.0287* (0.0010)	0.0141 (0.0012)
(a) low-skilled									
$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \ln(\text{real GDP, det.}),$ standardised	0.0165* (0.0009)	0.0247* (0.0006)	0.0165* (0.0008)	0.0189* (0.0009)	0.0254* (0.0005)	0.0124* (0.0008)	0.0152* (0.0009)	0.0225* (0.0006)	0.0110* (0.0008)
(b) medium-skilled									
$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \ln(\text{real GDP, det.}),$ standardised	0.0157* (0.0017)	0.0235* (0.0010)	0.0186* (0.0014)	0.0191* (0.0015)	0.0261* (0.0010)	0.0156* (0.0013)	0.0151* (0.0014)	0.0222* (0.0008)	0.0173* (0.0013)
(c) high-skilled									

Table 3.4: Cyclicity of conditional skewness

**Notes:** This table presents the results of model (3.12). Cyclicity is captured by the coefficient on the change in detrended, real GDP of mainland Norway, normalised to have mean zero and unit standard deviation. The dependent variables are the cohort and time specific measures of conditional dispersion, defined in (3.6) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{\text{init}}$  percentile of the distribution of  $y_{i,t-1}$ ; Bootstrap standard errors are presented in parentheses. \* represents statistical significance at 5% level.

Finally, we shift attention to the cyclical properties of the conditional skewness measure reported in Table (3.4). Overall, we find that conditional skewness decreases during downturns, consistent with the empirical evidence in Guvenen, Ozkan, and Song (2014). Accordingly, in contractions negative income realisations are more likely than positive ones. Consider for example the low skilled with 20 years of experience. As shown in Figure (3.3b), the average conditional skewness in family disposable income for the median household is around  $-0.1$ , implying an upper to lower tail ratio of 45/55. A drop in GDP growth by 2 standard deviations leads to a reduction in skewness by 0.0574 which corresponds to an upper-to-lower tail ratio of 42/58. The magnitude of this effect is comparable to the estimates reported in Busch and Ludwig (2016) for Germany. Overall, cyclicity however does not appear to differ substantially across the different measures of income and across skill groups. We do, however, observe an asymmetry when we look at the effects in the cross-section. Conditional skewness appears to be more cyclical for households in the middle of the distribution compared to those at the top or bottom of the distribution, i.e. cyclicity appears to follow an inverted U-shape across the income distribution.

## 3.5 Conclusion

In this chapter we have presented an estimated income process that is consistent with a variety of evidence on income risk over the life and business cycle. In our model, idiosyncratic income follows a general first order Markov process. Specifically, we model the income process as a first-order quantile-autoregressive process building on the recent work by Arellano et al. (2016). Modeling the conditional distribution of income has the key advantage that it represents the most comprehensive measure of income risk. We let individuals with different education levels have a separate income process, and within each skill group and cohort we allow the conditional quantile functions to vary unrestrictedly over time. Our approach therefore represents a tractable way of allowing income risk within each skill group to depend on the previous income level, calendar time and experience.

However, the model presented here also has several drawbacks. For example, we follow the traditional approach in the income dynamics literature and remove deterministic and aggregate variation of income in a preliminary step. It therefore implicitly assumes that aggregate shocks have an immediate and homogenous impact on everyone. A more satisfactory approach would probably be to work directly with log income instead of residual income and allow the conditional quantile functions to also depend on observable household characteristics. In the future it will be interesting to use this income process in a standard life cycle model of consumption and savings with aggregate shocks to investigate the distribution of the welfare costs of business cycles.

## Chapter 4

# Labour Market Frictions, Human Capital Accumulation, and Consumption Inequality

### 4.1 Introduction

Two well established facts in the literature which analyses the life cycle patterns of income and consumption is that both the variance of log wages and the variance of log consumption increase approximately linearly with age, with the slope for consumption rising less than for wages or earnings. In their review, Meghir and Pistaferri (2011) argue that this is a robust feature of the data that any model must confront. The linear rise in the variance of log wages is consistent with both the accumulation of substantial permanent shocks to wages (MaCurdy, 1982) or heterogeneity across workers in the rate of human capital accumulation (Lillard and Weiss, 1979). Within the lifecycle permanent income hypothesis model, the linear rise in the variance of log consumption requires substantial permanent uncertainty on the part of the workers. This is consistent with substantial permanent shocks to wages but not with heterogeneous income growth across workers, unless workers do not know their growth rates and must learn about them, leading them to behave as if they faces substantial permanent uncertainty (Guvenen, 2007).

Recently, Arellano, Blundell, and Bonhomme (2016) and Guvenen, Karahan, Ozkan, and Song (2016) have highlighted that year-to-year changes in earnings are far from symmetric and are certainly far from Normally distributed. Of particular interest for our work is the fact that earnings changes display substantial negative skewness and substantial kurtosis. In other words, earnings changes tend to be very small on average and the large changes tend to be negative. In addition, Gu-

venen et al. (2016) find that the skewness of year-to-year changes becomes more negative as they condition on higher quantiles of the earnings distribution. This increasing negative skewness, as the authors of both papers highlight, is consistent with the earnings dynamics that come from a standard job ladder model.

The asymmetry of earnings shocks implied by the job ladder model, and the implications for consumption and savings decisions has previously been explored by Lise (2013). He finds that this increasing negative skewness of wage shocks implies that workers at the top of the job ladder increase their savings rate substantially to insure against the increasing down-side risk of falling off the job ladder. The model in Lise (2013) does well in replicating the cross-sectional dispersion in wages, assets and consumption, however, it is inconsistent with the well established linear increase in log earnings and consumption over the life cycle. The job ladder model, combined with the empirical rates at which workers change jobs, implies that the distribution of earnings for a cohort of workers will look stationary after roughly 10 years, implying that the variance of earnings becomes flat after this period as does the variance of consumption. The basic job ladder model implies far too much stationarity to be consistent with the facts about the age profile of the variance of earnings or consumption.

In this chapter we develop a model in which inequality in earnings and consumption results from the interaction between heterogeneity of workers, heterogeneity of jobs, shocks to human capital, and labour market frictions. Specifically, we allow for permanent differences across individuals in productivity, the ability to acquire human capital, as well as the ability to find and keep jobs. We consider jobs which differ in both productivity and the extent to which they facilitate further human capital accumulation for workers. Shocks to human capital are permanent, they are carried by the worker even when she changes jobs. Labour market frictions induce transitory shocks as workers stochastically move up the job ladder. They are transitory in the sense that the direct productivity effect of the job on the worker's wage disappears when the worker leaves the job, either through a spell of unemployment or by a move directly to another job. However, depending on the extent to which jobs differ in the degree to which they facilitate human capital accumulation, and the severity of frictions, there may be permanent effects on the workers human capital and hence earnings arising from differential labour market histories.

Exiting work combining job search and human capital accumulation includes Bunzel, Christensen, Kiefer, and Korsholm (1999); Rubinstein and Weiss (2006); Barlevy (2008); Yamaguchi (2010); Burdett, Carrillo-Tudela, and Coles (2011); Veramendi (2011); Bowlus and Liu (2013) and Bagger, Fontaine, Postel-Vinay, and



Robin (2014). While some of these models include features not included here, they all focus on the contribution of human capital accumulation and job search (and in some cases learning about own ability) for explaining the average age profile of log wages. None of these papers has a model of consumption and savings, and none focus on the facts about the linear increase in the variance of log earnings and consumption with age or the asymmetry of earnings changes discussed above.

Our model is closest in spirit to Bagger, Fontaine, Postel-Vinay, and Robin (2014), and as such the model touches on many of the same questions in the literature such as the effect of experience verses job tenure for wage growth (see, for example, Abraham and Farber, 1987; Altonji and Shakotko, 1987; Topel, 1991; Buchinsky, Fougère, Kramarz, and Tchernis, 2010); on earnings dynamics, a topic for which there is a very large existing literature (see, for example, MaCurdy, 1982; Abowd and Card, 1989; Topel and Ward, 1992; Meghir and Pistaferri, 2004; Browning, Ejrnaes, and Alvarez, 2010; Altonji, Smith, and Vidangos, 2013); as well as the recent literature on the asymmetry of earnings changes (Arellano, Blundell, and Bonhomme, 2016; Guvenen, Karahan, Ozkan, and Song, 2016).

We demonstrate how the key distributions and parameters of our model are identified in the presence of matched employer-employee data. In a simulation exercise we show that the model is able to reproduce the linear age profile for the variance of log earnings and consumption, as well as the negative skewness and excessive kurtosis of the distribution of earnings growth, including the increasing negative skewness of earnings growth conditional on the previous earnings level.

This chapter proceeds as follows. We present the model in Section 4.2. In Section 4.3 we establish non-parametric identification of the model. Section 4.4 describes the numerical method to solve the model and presents the results of a simulation exercise. Section 4.5 concludes.

## 4.2 The Model

### 4.2.1 The Environment

**Agents.** Time is continuous and indexed by  $t$ . The economy is populated by a unit mass of workers characterized by their human capital  $\mathbf{h}_t = (h_0, h_{1t})$ . The first component  $h_0 \in \mathbb{R}^+$  is a constant worker heterogeneity parameter that reflects permanent differences in ability between workers. The second component  $h_{1t} \in \mathbb{R}^+$  is a worker's accumulated human capital at time  $t$ . We denote by  $L(h_0)$  the exogenous measure of workers with a type weakly below  $h_0$ . Workers

are risk averse and maximise expected lifetime utility, ordering consumption paths according to

$$\mathbb{E}_0 \int_0^\infty e^{-[\rho+\xi]t} u(c_t) dt,$$

where  $\rho$  is the subjective rate of time preference and  $c_t$  is instantaneous consumption at time  $t$ . Workers leave the labour force at exogenous rate  $\xi$  and are replaced by an equal mass of unemployed labour market entrants, so the effective discount rate is equal to  $\rho + \xi$  (see Blanchard, 1985). There is a single riskless asset  $a$  with interest rate  $r$  which allows workers to transfer resources over time. The change of wealth over time is then given by the return on assets,  $ra_t$  plus income,  $i_t$  less consumption expenditure,

$$da_t = (ra_t + i_t - c_t)dt \quad \text{subject to } a_t \geq \underline{a},$$

where  $\underline{a}$  is the lower bound on assets. Workers can either be employed and earning an endogenous wage, unemployed and receiving benefits, or out of the the labour force and receiving an exogenous flow of income.

On the other side of the labor market there is a continuum of firms indexed by their productivity type  $y \in [\underline{y}, \bar{y}] \subset \mathbb{R}^+$ . The measure of firms with productivity weakly below  $y$  is exogenous and denoted by  $\Gamma(y)$ . Firms are risk-neutral and maximize the present value of profits.

**Matching and Production.** Workers search for jobs when unemployed and for better jobs when they are employed. Search is random and all workers sequentially sample from the same exogenous firm type distribution  $\Gamma(y)$  at type-dependent Poisson rate  $\lambda(h_0)$ . A match between a firm of type  $y$  and a worker with skill  $\mathbf{h}_t$  produces output  $f(\mathbf{h}_t, y)$  given by,

$$f(\mathbf{h}_t, y) = h_0 h_{1t} y.$$

A match becomes unprofitable when it is hit by an adverse idiosyncratic productivity shock. The Poisson rate at which these shocks occur depends on the fixed worker type  $h_0$  and is denoted by  $\delta(h_0)$ .

When unmatched, workers produce home production  $b(\mathbf{h}) = h_0 h_{1t} b$ . Given the assumed proportionality of market and home production to a worker's human capital, no unemployed worker would ever accept a job from a firm with productivity below  $y < b$ , and thus the lower bound of the support of the effective sampling distribution is  $\underline{y} = b$ .

**Human Capital Accumulation.** On the worker side, a match between a firm  $y$  and a worker  $\mathbf{h}$  contributes to human capital development. Human capital  $h_{1t}$  evolves stochastically over time according to a diffusion process on a bounded interval  $[\underline{h}_1, \bar{h}_1] \subset \mathbb{R}^+$  with reflecting barriers:

$$dh_{1t} = \mu(\mathbf{h}_t, y)dt + \sigma(\mathbf{h}_t)d\mathcal{B}_t, \quad (4.1)$$

where  $d\mathcal{B}_t \equiv \lim_{\Delta t \rightarrow 0} \sqrt{\Delta t} \varepsilon_t$  with  $\varepsilon_t \sim \mathcal{N}(0, 1)$  is the increment of a standard Brownian motion with  $\sigma(\mathbf{h}) \geq 0$  governing the diffusion of the process. The drift rate  $\mu(\cdot, \cdot)$  is a continuous function that depends on both the worker and the firm type. We further assume that the drift rate is non-decreasing in the firm type, i.e.  $\frac{\partial}{\partial y} \mu(\mathbf{h}_t, y) \geq 0$ .<sup>1</sup>

### 4.2.2 Wage Contracts

We will restrict our attention to wage contracts that are of the piece rate form, implying that at any point in time  $t$  the wage paid to the worker is restricted to be a share of the total output of the worker-firm match. Let  $0 \leq \theta_t \leq 1$  denote the contractual piece-rate at time  $t$ . A worker  $\mathbf{h}_t$  employed at firm  $y$  then receives a wage given by

$$w_t = \theta_t f(\mathbf{h}_t, y) = \theta_t h_0 h_{1t} y.$$

Next consider the very same worker receiving an outside offer from a firm of type  $y'$ . The incumbent firm  $y$  and the firm  $y'$  Bertrand compete over the worker's services similar to the piece rate version of the sequential auction framework of Postel-Vinay and Robin (2002) implemented in Bagger, Fontaine, Postel-Vinay, and Robin (2014). Since the human capital accumulation rate is assumed to be non-decreasing in the firm type, the worker will move to firm  $y'$  whenever  $y' > y$ . The most the current firm can offer to the worker is a wage that equals the total output of the current match, equivalent to a piece-rate equal to one. We assume that the firm  $y'$  is willing to offer the worker a contract that matches this wage offer.<sup>2</sup> On the other hand a worker at firm  $y$  who is being poached by a less productive firm  $y' \leq y$  stays with the current employer. The worker's piece-rate only gets updated if the outside firm is productive enough to be able to offer the worker a higher wage, all other things being equal. Specifically this implies that

<sup>1</sup>We treat the relationship between the speed of human capital accumulation and the productivity of a job as an exogenous technology to be estimated. Recently, in a closely related model, Lentz and Roys (2015) show that with heterogeneous firms the optimal employment contract features more productive firms providing more training.

<sup>2</sup>Note here we depart from Bagger, Fontaine, Postel-Vinay, and Robin (2014) since we compare flow output as opposed to present values to determine the updated piece-rate.

the piece-rate offered to the worker receiving an outside offer from firm  $y'$  at time  $\tau > t$  is given by

$$\theta_\tau = \begin{cases} \frac{y}{y'} & \text{if } y' \geq y \\ \max \left\{ \theta_t, \frac{y'}{y} \right\} & \text{if } y' < y \end{cases}.$$

Some discussion of our choice of wage setting is warranted. The main benefit of our chosen wage determination is that identification and interpretation is transparent. Theoretically, our wage setting process satisfies individual rationality since it guarantees that match formation and separations are efficient and that the participation constraint of both the worker and firm are always satisfied at the prevailing piece rate contract. However, our restricted contracts are unlikely to be optimal for at least two reasons. First, since workers are risk averse firms may be able to extract more of the match surplus by offering a contract that is smoother than that implied by the piece rate.<sup>3</sup> The extent to which the worker values this additional smoothness would depend on how close she is to the borrowing constraint, implying that any such contract would need to condition on the worker's asset position. Second, in the case where a firm with higher productivity also provides faster human capital accumulation, it could earn higher profit by offering a wage lower than that implied by the piece-rate as the worker will also value the higher rate of human capital accumulation. Again, how much the worker values wages today versus more human capital tomorrow will depend on her asset position, which the firm would need to know to make the profit maximizing offer. Wage contracts that condition on the asset position of a worker strike us as a poor representation of reality. While we do give up on optimality, our piece-rate contracts ensure wages reflect worker productivity, firm productivity, and the competition for workers' services, and satisfy all participation constraints. We also have the additional benefit of transparent identification and interpretation of the model structure.

### 4.2.3 Worker Value

Let  $W(a, \mathbf{h}, y, \theta)$  denote the value of employment for a worker of type  $\mathbf{h}$  with assets  $a$  employed by a type- $y$  firm under a contract that specifies a piece-rate  $\theta$ . We assume that unemployment is equivalent to employment at the lowest

<sup>3</sup>For example, Burdett and Coles (2003) characterize the optimal wage-tenure contract in a wage posting setting and Lamadon (2014) characterizes the optimal contract when search is directed, there are shocks to both worker and firm productivity. In both cases the optimal wage contract is substantially smoother than would be the case with a share of output. However, workers have no access to borrowing or savings technology so cannot do any consumption smoothing on their own, which is an important feature of our model.

productive firm when the worker receives the total output as wage, i.e. the value of unemployment is given by  $W(a, \mathbf{h}, \underline{y}, 1)$ . The Hamilton-Jacobi-Bellman equation can then be written as (see Appendix D.1 for a formal derivation)

$$\begin{aligned} [\rho + \lambda(h_0) + \delta(h_0) + \xi]W(a, \mathbf{h}, y, \theta) = & \max_{a - \underline{a} \geq c \geq 0} \left\{ u(c) + \frac{\partial}{\partial a} W(a, \mathbf{h}, y, \theta) [ra + \theta f(\mathbf{h}, y) - c] \right. \\ & + \mu(\mathbf{h}, y) \frac{\partial}{\partial h_1} W(a, \mathbf{h}, y, \theta) + \frac{\sigma(\mathbf{h})^2}{2} \frac{\partial^2}{\partial h_1^2} W(a, \mathbf{h}, y, \theta) + \delta(h_0)W(a, \mathbf{h}, \underline{y}, 1) + \xi R(a, \mathbf{h}) \\ & \left. + \lambda(h_0) \int \max\{W(a, \mathbf{h}, y', \frac{y}{y'}), W(a, \mathbf{h}, y, \max\{\theta, \frac{y'}{y}\})\} d\Gamma(y') \right\}. \end{aligned} \quad (4.2)$$

The first-order necessary condition for optimal consumption choices states that the marginal utility from consumption equals the shadow price of wealth:

$$u'(c) = \frac{\partial}{\partial a} W(a, \mathbf{h}, y, \theta), \quad (4.3)$$

In addition, recall that the stochastic process for human capital is bounded by reflecting barriers. In Appendix D.1 we show that this gives rise to the following boundary conditions:

$$\frac{\partial}{\partial h_1} W(a, \underline{\mathbf{h}}, y, \theta) = \frac{\partial}{\partial h_1} W(a, \bar{\mathbf{h}}, y, \theta) = 0, \quad (4.4)$$

where  $\underline{\mathbf{h}} = (h_0, \underline{h}_1)$  and  $\bar{\mathbf{h}} = (h_0, \bar{h}_1)$ .

When the worker makes a transition into retirement, he receives a constant flow of payments  $q(\mathbf{h})$ . The flow value of retirement is given by

$$\rho R(a, \mathbf{h}) = \max_{a - \underline{a} \geq c \geq 0} \{u(c) + \frac{\partial}{\partial a} R(a, \mathbf{h}) [ra + q(\mathbf{h}) - c]\}, \quad (4.5)$$

with first-order condition

$$u'(c) = \frac{\partial}{\partial a} R(a, \mathbf{h}).$$

#### 4.2.4 Cross-Sectional Distributions

The following flow equations define the stationary equilibrium distributions:

**Unemployment.** Let the distribution of unemployed of type  $h_0$  be  $u(h_0)$ . In the stationary equilibrium,

$$[\lambda(h_0) + \xi]u(h_0) = \delta(h_0)[1 - u(h_0)] + \xi \ell(h_0),$$

where the flows out of unemployment come from transitions into employment and retirement, and the inflow comprises separations from employment and newly born workers. These flows produce the stationary unemployment rate by worker type:

$$u(h_0) = \frac{\delta(h_0) + \xi \ell(h_0)}{\delta(h_0) + \xi + \lambda(h_0)}.$$

**Employment.** The stationary flows in and out of employment are given by

$$[\delta(h_0) + \lambda(h_0)\bar{\Gamma}(y) + \xi][1 - u(h_0)]G(y|h_0) = \lambda(h_0)\Gamma(y)u(h_0),$$

which, after substituting for  $u(h_0)$ , defines the stationary relationship between the sampling distribution and the cross sectional distribution:

$$\begin{aligned} G(y|h_0) &= \frac{[\delta(h_0) + \ell(h_0)\xi]\lambda(h_0)\Gamma(y)}{[\delta(h_0) + \xi + \lambda(h_0)\bar{\Gamma}(y)][\lambda(h_0) + [1 - \ell(h_0)]\xi]} \\ \Leftrightarrow \Gamma(y) &= \frac{[\delta(h_0) + \xi + \lambda(h_0)]G(y|h_0)}{\lambda(h_0)[[\delta(h_0) + \xi\ell(h_0)][\lambda(h_0) + [1 - \ell(h_0)]\xi]^{-1} + G(y|h_0)}. \end{aligned} \quad (4.6)$$

### 4.3 Identification

This section lays out the formal identification arguments for the key distributions and parameters of the model.

**Worker type distribution  $L(h_0)$ .** We normalize the location of the firm type distribution,  $\underline{y} = 1$  and we normalize a worker's human capital  $h_{1t_0} = 1$  at the time when she takes up her first job in  $t_0$ . This implies that the worker's skills upon taking up the first job is  $\mathbf{h}_{t_0} = (h_0, 1)$ . The worker's initial wage contract as specified by the model implies a piece-rate  $\theta_{t_0} = y^{-1}$  when the worker is hired by firm  $y$ . Since unemployment is equivalent to working with the lowest productivity firm ( $h_0 h_{1t} b = h_0 h_{1t} \underline{y}$ ), her initial wage is

$$w_{t_0} = h_0,$$

and our estimate of a worker type is given by  $\hat{h}_{0i} = w_{it_0}$ . Thus, the distribution of fixed-worker types is identified by the distribution of initial wages. A nonparametric estimate for the distribution of fixed worker types is then the empirical cumulative distribution function,

$$\hat{L}(h_0) = \frac{1}{N} \sum_{i=1}^N \mathbb{1}(w_{it_0} \leq h_0).$$

**Sampling distribution of firm types**  $\Gamma(y) = \Gamma(y(p))$ . With matched employer-employee data we have a way to directly estimate the rank of firm types based on their ability to hire from other firms. Since all workers prefer higher  $y$  firms, the ability of a firm to poach workers from other firms is increasing in  $y$ .<sup>4</sup>

Let  $p(y)$  be the rank, normalized to lie in  $[0,1]$ , of the share of all hires that a firm of type  $y$  poaches from other firms:

$$p(y) = \text{rank} \left[ \frac{\int \lambda(h_0)(1 - u(h_0))G(h_0, y)dh_0}{\int \lambda(h_0)u(h_0) + \lambda(h_0)(1 - u(h_0))G(h_0, y)dh_0} \right] \in [0, 1],$$

which is monotonically increasing in  $y$ . This relationship can be inverted allowing us to write  $y(p)$ , and thus  $G(h_0, y(p)) = G_p(h_0, p)$ .

We can estimate the type of each firm  $j$  by

$$\hat{p}_j = \text{rank} \left[ \frac{\text{number of workers hired from other firms}}{\text{total number of workers hired}} \right] \rightarrow [0, 1].$$

A direct nonparametric estimate of the sampling distribution  $\Gamma(y(p))$  is available from the distribution of jobs accepted out of unemployment.<sup>5</sup>

**Poisson arrival rates.** Given estimates for the ranking of firms, worker fixed effects and a non-parametric estimate of the sampling distribution  $\Gamma$ , the Poisson arrival rates are identified from observed job durations. Consider the following sampling frame in which we include the first job spell upon entering the labor market for each worker in our sample to construct the following observation for

<sup>4</sup>This is a robust implication of the job ladder model that holds both in steady state (Burdett and Mortensen, 1998) and out of steady-state (Moscarini and Postel-Vinay, 2013). Bagger and Lentz (2014) use this monotonicity to help identify firm types in a related model without human capital accumulation where wages are not fully informative about how workers rank firms.

<sup>5</sup>An alternative estimator is available by first estimating the cross-sectional distribution of worker-types across job types:

$$\hat{G}(h_0, y(p)) = \hat{G}(h_0, p) = \frac{1}{N} \sum_{i=1}^N \mathbb{1}(h_{0i} \leq h_0) \times \mathbb{1}(p_i \leq p),$$

where  $p_i$  is the firm type a worker is employed at and  $N$  is the number of workers in a cross sectional sample, and the obvious estimator for  $\hat{G}(y(p)|h_0)$ . Using the equilibrium steady-state relationship between the sampling distribution and the cross sectional distribution (4.6) provides an estimate of the sampling distribution  $\Gamma(y; \cdot)$  conditional on  $\delta(h_0)$ ,  $\lambda(h_0)$ ,  $\xi$  and  $\ell(h_0)$ :

$$\hat{\Gamma}(y; \delta(h_0), \lambda(h_0), \xi) = \frac{[\delta(h_0) + \xi + \lambda(h_0)]\hat{G}(y|h_0)}{\lambda(h_0) \left[ [\delta(h_0) + \xi\ell(h_0)] [\lambda(h_0) + [1 - \ell(h_0)]\xi]^{-1} + \hat{G}(y|h_0) \right]}.$$

We prefer to use the direct estimate of  $\Gamma(y(p))$  based on jobs accepted out of unemployment as it does not impose the steady-state relationship (4.6), although it does, of course, assume  $\Gamma$  is constant over our sample period.

each worker  $i$ :

$$\mathbf{x}_i = (h_{0i}, p_{it_0}, d_i, \tau_{EEi}, \tau_{EUi}, \tau_{ERi}, c_i),$$

where  $h_{0i}$  is the worker type,  $p_{it_0}$  is the firm type of this job,  $d_i \leq T$  is the duration in the initial job spell before either the first transition or the end of the sample window,  $c_i = 1$  if  $d_i = T$  and zero otherwise,  $\tau_{EEi} = 1$  an worker is observed making a job to job change,  $\tau_{EUi} = 1$  if an worker makes a transition into unemployment, and  $\tau_{ERi} = 1$  if a worker is observed transiting into retirement.

Conditional on worker and firm type  $(h_{0i}, p_{it_0})$  the likelihood contributions for initial job spells are:

$$\begin{aligned} \ell(\mathbf{x}_i) = & \gamma(p_{it_0}) \times [\delta(h_{0i}) + \xi + \lambda(h_{0i})\bar{\Gamma}(p_{it_0})]^{1-c_i} \\ & \times e^{-[\delta(h_{0i}) + \xi + \lambda(h_{0i})\bar{\Gamma}(p_{it_0})]d_i} \times \left( \frac{\delta(h_{0i})}{\delta(h_{0i}) + \xi + \lambda(h_{0i})\bar{\Gamma}(p_{it_0})} \right)^{\tau_{EUi}} \\ & \times \left( \frac{\lambda(h_{0i})\bar{\Gamma}(p_{it_0})}{\delta(h_{0i}) + \xi + \lambda(h_{0i})\bar{\Gamma}(p_{it_0})} \right)^{\tau_{EEi}} \times \left( \frac{\xi}{\delta(h_{0i}) + \xi + \lambda(h_{0i})\bar{\Gamma}(p_{it_0})} \right)^{\tau_{ERi}}. \end{aligned}$$

The likelihood is a function of the transition rate functions  $\lambda(h_0)$  and  $\delta(h_0)$  and the retirement rate  $\xi$ , where we plug in our first stage non-parametric estimate of the sampling distribution for  $\Gamma$ .

### Human capital accumulation and the contribution of firm type to wages

In order to identify the deterministic and stochastic components of the human capital accumulation function, as well as the contribution to wages of firm type, we require wage observations on two consecutive job spells, plus information on where the worker transited from. An individual observation is given by

$$\mathbf{x}_{i,j,j',j''} = (h_{0i}, \log w_{ij_1}^s, \log w_{ij_2}^s, d_{ij_1}, p_{ij_1}, p_{ij_2}, p_{ij_0}),$$

where  $h_{0i}$  is the estimated fixed worker type;  $w_{ij_1}^s$  and  $w_{ij_2}^s$  are the starting wages for the worker at jobs  $j_1$  and  $j_2$ ;  $d_{ij_1}$  is the duration of the job spell at firm  $j_1$ ;  $p_{ij_1}$  and  $p_{ij_2}$  are the estimated firm types for the jobs corresponding to starting wages  $w_{ij_1}^s$  and  $w_{ij_2}^s$ ; and  $p_{ij_0}$  is the firm type corresponding to the job held prior to  $p_{ij_1}$ .

The protocol for setting the piece-rate implies that the starting wages at jobs  $j_1$  and  $j_2$  are given by:

$$\begin{aligned} \log w_{ij_1t}^s &= \log y(p_{ij_0}) + \log h_{0i} + \log h_{1it} \\ \log w_{ij_2t+d}^s &= \log y(p_{ij_1}) + \log h_{0i} + \log h_{1it+d}, \end{aligned}$$

where we note that the starting wage in job  $j_1$  depends on the firm productivity of



job  $j_0$  and, similarly, the starting wage of job  $j_2$  depends on the firm productivity of job  $j_1$ . Now, if we condition on  $p_{ij_0} = p_{ij_1}$ , the change in starting wages between job  $j_1$  and  $j_2$  reflects only human capital accumulated at job  $j_1$ :

$$\log w_{ij_2t+d}^s - \log w_{ij_1t}^s = \log h_{1it+d} - \log h_{1it}.$$

Suppose for simplicity that human capital follows a geometric Brownian motion,

$$dh_{1t} = \mu(h_0, y)h_{1t}dt + \sigma h_{1t}d\mathcal{B}_t.$$

Applying Itô's lemma gives us the following stochastic differential equation for the logarithm of human capital:

$$d \log h_{1t} = \left[ \mu(h_0, y) - \frac{\sigma^2}{2} \right] dt + \sigma d\mathcal{B}_t.$$

Thus the difference in starting wages is Normally distributed with mean given by

$$\mathbb{E}_t \left[ \log w_{ij_2t+d}^s - \log w_{ij_1t}^s | p_{ij_0} = p_{ij_1} \right] = \left[ \mu(h_{0i}, y(p_{ij_1})) - \frac{\sigma^2}{2} \right] d_{ij_1},$$

and variance

$$\text{var}_t \left[ \log w_{ij_2t+d}^s - \log w_{ij_1t}^s | p_{ij_0} = p_{ij_1} \right] = \sigma^2 d_{ij_1}.$$

Taken together, the drift rate and the diffusion parameter are identified from the mean and the variance of the conditional distributions of the log difference in starting wages.

**Preference parameters** Given first stage estimates of the distribution of worker heterogeneity, the sampling distribution, contact rates, separation rates, and the technology of human capital accumulation, we can estimate the rate of time preference and the coefficient of relative risk by indirect inference. Specifically, we solve and simulate from the workers' problem (4.2) to match the mean and variance of the age profile for consumption.

## 4.4 Simulation

This section lays out the numerical method to solve the dynamic programming problem and then illustrates some key features of the model on simulated panel data.

### 4.4.1 Numerical Solution

We apply and extend the finite difference method proposed by Achdou, Han, Lasry, and Moll (2015) to solve the dynamic programming problem numerically. We solve for the value functions at  $N$  discrete points using linearly spaced grids for  $(a, \mathbf{h}, y, \theta)$ . Derivatives are approximated by finite differences, where  $\Delta_x$  denotes the forward difference operator and  $\nabla_x$  the backward difference operator in dimension  $x$ .

**Retirement value.** In the first step, we iteratively solve for the value of retirement (4.5). Let  $R^i(a, \mathbf{h})$  denote a guess for the value function at iteration step  $i$ . The updated guess  $R^{i+1}(a, \mathbf{h})$  is then implicitly defined by

$$\frac{1}{\Delta} [R^{i+1}(a, \mathbf{h}) - R^i(a, \mathbf{h})] + \rho R^{i+1}(a, \mathbf{h}) = u(c^i(a, \mathbf{h})) + \frac{\partial}{\partial a} R^{i+1}(a, \mathbf{h}) [ra + q(\mathbf{h}) - c^i(a, \mathbf{h})], \quad (4.7)$$

where  $\Delta$  denotes the step size. We approximate the partial derivative with respect to wealth using an upwind scheme. The basic idea is to use a forward (backward) difference approximation whenever the drift of the state variable  $a$  is positive (negative). Thus we calculate savings using both forward and backward difference approximations

$$\begin{aligned} s_{\Delta}^i &= ra + q(\mathbf{h}) - c_{\Delta}^i(a, \mathbf{h}) \\ s_{\nabla}^i &= ra + q(\mathbf{h}) - c_{\nabla}^i(a, \mathbf{h}), \end{aligned}$$

where consumption is implicitly defined by the first order condition given the current guess for the value function,

$$\begin{aligned} c_{\Delta}^i(a, \mathbf{h}) &= (u')^{-1}(\Delta_a R^i(a, \mathbf{h})) \\ c_{\nabla}^i(a, \mathbf{h}) &= (u')^{-1}(\nabla_a R^i(a, \mathbf{h})). \end{aligned}$$

The approximation of the derivative of the value function with respect to wealth is then given by

$$\begin{aligned} \frac{\partial}{\partial a} R^{i+1}(a, \mathbf{h}) &\approx \Delta_a R^{i+1}(a, \mathbf{h}) \mathbf{1}\{s_{\Delta}^i > 0\} + \nabla_a R^{i+1}(a, \mathbf{h}) \mathbf{1}\{s_{\nabla}^i < 0\} \\ &\quad + u'(ra + q(\mathbf{h})) \mathbf{1}\{s_{\Delta}^i \leq 0 \leq s_{\nabla}^i\}, \end{aligned} \quad (4.8)$$

where the last term captures situations where the sign of the savings is undetermined for some grid points.<sup>6</sup> At these grid points we set savings to zero, so that the derivative of the value function is equal to the marginal utility from consuming the flow income. Similarly, consumption  $c^i(a, \mathbf{h})$  is then defined by

$$c^i(a, \mathbf{h}) = c_{\Delta}^i(a, \mathbf{h})\mathbf{1}\{s_{\Delta}^i > 0\} + c_{\nabla}^i(a, \mathbf{h})\{s_{\nabla}^i < 0\} + [ra + q(\mathbf{h})]\mathbf{1}\{s_{\Delta}^i \leq 0 \leq s_{\nabla}^i\}. \quad (4.9)$$

Substituting (4.8) and (4.9) into (4.7) yields a linear system of equations which can be efficiently solved even in the presence of a large number of grid points by exploiting the sparse nature of the problem. Finally we continue iterating on the value function until convergence.

**Worker value.** Given the retirement value solved in the previous step, we can solve for the worker value. Given a guess for the worker value  $W^i(a, \mathbf{h}, y, \theta)$ , the update  $W^{i+1}(a, \mathbf{h}, y, \theta)$  is implicitly defined by

$$\begin{aligned} & \frac{1}{\Delta} [W^{i+1}(a, \mathbf{h}) - W^i(a, \mathbf{h}, y, \theta)] + [\rho + \lambda(h_0) + \delta(h_0) + \xi] W^{i+1}(a, \mathbf{h}, y, \theta) = u(c^i(a, \mathbf{h}, y, \theta)) \\ & + \frac{\partial}{\partial a} W^{i+1}(a, \mathbf{h}, y, \theta) [ra + \theta f(\mathbf{h}, y) - c^i(a, \mathbf{h}, y, \theta)] + \mu(\mathbf{h}, y) \frac{\partial}{\partial h_1} W^{i+1}(a, \mathbf{h}, y, \theta) \\ & + \frac{\sigma(\mathbf{h})^2}{2} \frac{\partial^2}{\partial h_1^2} W(a, \mathbf{h}, y, \theta) + \delta(h_0) W^{i+1}(a, \mathbf{h}, \underline{y}, 1) + \xi R(a, \mathbf{h}) \\ & + \lambda(h_0) \int \max\{W^{i+1}(a, \mathbf{h}, y', \frac{y}{y'}), W^{i+1}(a, \mathbf{h}, y, \max\{\theta, \frac{y'}{y}\})\} dF(y'), \quad (4.10) \end{aligned}$$

where  $\Delta$  denotes the step size. Once again we approximate the first order partial derivatives using an upwind scheme. Specifically, we approximate the first order partial derivative with respect to human capital  $h_1$  as follows:

$$\begin{aligned} \frac{\partial}{\partial h_1} W^{i+1}(a, \mathbf{h}, y, \theta) & \approx \Delta_{h_1} W^{i+1}(a, \mathbf{h}, y, \theta) \mathbf{1}\{\mu(\mathbf{h}, y) \geq 0\} \\ & + \nabla_{h_1} W^{i+1}(a, \mathbf{h}, y, \theta) \mathbf{1}\{\mu(\mathbf{h}, y) < 0\}, \quad (4.11) \end{aligned}$$

while the second order partial derivative with respect to human capital  $h_1$  is approximated by a central finite difference. We also have to take into account that the diffusion process is reflected at the boundaries. The reflection at the barriers of the human capital process implies the following boundary conditions:

$$\frac{\partial}{\partial h_1} W^{i+1}(a, \underline{\mathbf{h}}, y, \theta) = \frac{\partial}{\partial h_1} W^{i+1}(a, \bar{\mathbf{h}}, y, \theta) = 0, \quad (4.12)$$

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<sup>6</sup>Note that the concavity of the value function implies that  $\Delta_a R(a, \mathbf{h}) < \nabla_a R(a, \mathbf{h})$  and therefore  $s_{\Delta} < s_{\nabla}$ . Thus it can arise in some situations that  $s_{\Delta} \leq 0 \leq s_{\nabla}$ .

where  $\underline{\mathbf{h}} = (h_0, \underline{h}_1)$  and  $\bar{\mathbf{h}} = (h_0, \bar{h}_1)$ . To approximate the partial derivative with respect to wealth, we proceed as follows. Given the current guess for the value function, savings based on the forward and backwards difference approximations are given by

$$\begin{aligned} s_{\Delta}^i &= ra + \theta f(\mathbf{h}, y) - c_{\Delta}^i(a, \mathbf{h}, y, \theta) \\ s_{\nabla}^i &= ra + \theta f(\mathbf{h}, y) - c_{\nabla}^i(a, \mathbf{h}, y, \theta), \end{aligned}$$

where consumption is implicitly defined by the first order condition,

$$\begin{aligned} c_{\Delta}^i(a, \mathbf{h}, y, \theta) &= (u')^{-1}(\Delta_a W^i(a, \mathbf{h}, y, \theta)) \\ c_{\nabla}^i(a, \mathbf{h}, y, \theta) &= (u')^{-1}(\nabla_a W^i(a, \mathbf{h}, y, \theta)). \end{aligned}$$

This yields the following approximation for the shadow value of wealth:

$$\begin{aligned} \frac{\partial}{\partial a} W^{i+1}(a, \mathbf{h}, y, \theta) &\approx \Delta_a W^{i+1}(a, \mathbf{h}, y, \theta) \mathbf{1}\{s_{\Delta}^i > 0\} + \nabla_a W^{i+1}(a, \mathbf{h}, y, \theta) \mathbf{1}\{s_{\nabla}^i < 0\} \\ &\quad + u'(ra + \theta f(\mathbf{h}, y)) \mathbf{1}\{s_{\Delta}^i \leq 0 \leq s_{\nabla}^i\}, \quad (4.13) \end{aligned}$$

where the last term captures situations where the sign of the savings is undetermined for some grid points. At these grid points we set savings to zero, so that the derivative of the value function is equal to the marginal utility from consuming the flow income. Similarly, consumption  $c^i(a, \mathbf{h}, y, \theta)$  is then defined by

$$c^i(a, \mathbf{h}, y, \theta) = c_{\Delta}^i(a, \mathbf{h}, y, \theta) \mathbf{1}\{s_{\Delta}^i > 0\} + c_{\nabla}^i(a, \mathbf{h}, y, \theta) \mathbf{1}\{s_{\nabla}^i < 0\} + [ra + \theta f(\mathbf{h}, y)] \mathbf{1}\{s_{\Delta}^i \leq 0 \leq s_{\nabla}^i\}. \quad (4.14)$$

Finally, the integral in (4.10) is approximated by a finite sum,

$$\begin{aligned} &\int \max\{W^{i+1}(a, \mathbf{h}, y', \frac{y}{y'}), W^{i+1}(a, \mathbf{h}, y, \max\{\theta, \frac{y'}{y}\})\} dF(y') \\ &\approx \sum_{y'} \max\{W^{i+1}(a, \mathbf{h}, y', \frac{y}{y'}), W^{i+1}(a, \mathbf{h}, y, \max\{\theta, \frac{y'}{y}\})\} \hat{f}(y'), \quad (4.15) \end{aligned}$$

where  $\hat{f}(\cdot)$  is an appropriate probability mass function. Substituting (4.11), (4.12), (4.13), (4.14) and (4.15) into (4.10) yields a linear system of equations which can be efficiently solved even in the presence of a large number of grid points by exploiting the sparse nature of the problem. Finally we continue iterating on the value function until convergence.

### 4.4.2 Model Features

For the simulation we assume that human capital follows a Geometric Brownian motion with a drift rate that depends positively on the worker type and the firm type in an additive separable way,

$$\mu(h_0, y) = \mu_0 + \mu_1 \log h_0 + \mu_2 \log y.$$

The distribution of fixed worker types  $L(h_0)$  is assumed to be log-normal, and the distribution of firm productivities is assumed to be bounded Pareto with lower support equal to one,

$$y(p) = \left( -\frac{p\bar{y}^\alpha - p - \bar{y}^\alpha}{\bar{y}^\alpha} \right)^{-\frac{1}{\alpha}}.$$

We further set the retirement rate  $\xi$  such that it implies an average working life of 40 years. The job-contact and job-destruction rates are assumed to depend linearly on the fixed worker type,

$$\lambda(h_0) = \exp(\lambda_0 + \lambda_1 \log h_0) \quad \delta(h_0) = \exp(\delta_0 + \delta_1 \log h_0).$$

They are set such that the stationary unemployment rate for the median worker type is around 6 percent. The subjective rate of time preference and the interest rate are set to 5 and 3 percent annual, and the coefficient of relative risk aversion is set to 2. All parameter values used for the simulation are summarised in Appendix Table D.1.

We simulate the model for 100,000 households at a monthly frequency over 40 years and construct a panel of annual consumption and annual earnings. In Figure (4.1) we present the model simulated age profiles for the variance of log annual earnings and the variance of log annual consumption. The model simulated profiles have the well known features highlighted by Meghir and Pistaferri (2011): the level of the variance of log wages is higher than for consumption, and the age profiles are linear.

In Figure (4.2) we plot Kelly's skewness measure of year-to-year changes in the log of annual earnings, conditional on the percentile of the earnings distribution in the initial year. The skewness becomes increasingly negative as we condition on higher and higher percentiles of the initial earnings distribution. The model produces a shock process in which negative shocks tend to be larger than positive shocks, and they become increasingly larger as we move up the percentiles of the initial earnings distribution. In the language of the model, workers tend to move gradually up the job ladder in small or moderate steps, while separating from a

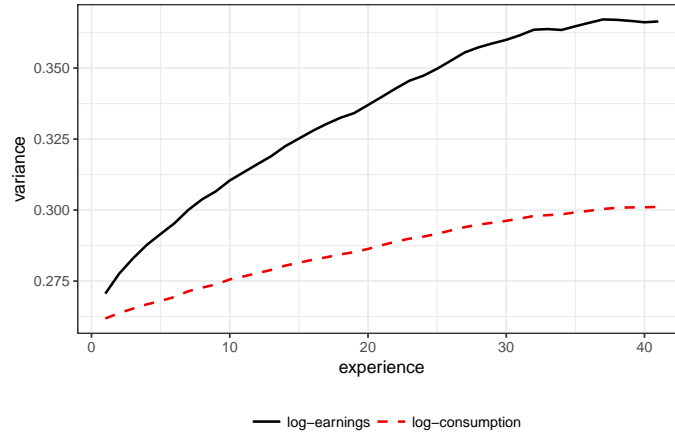


Figure 4.1: Simulated age profile for the variance of log earnings and consumption

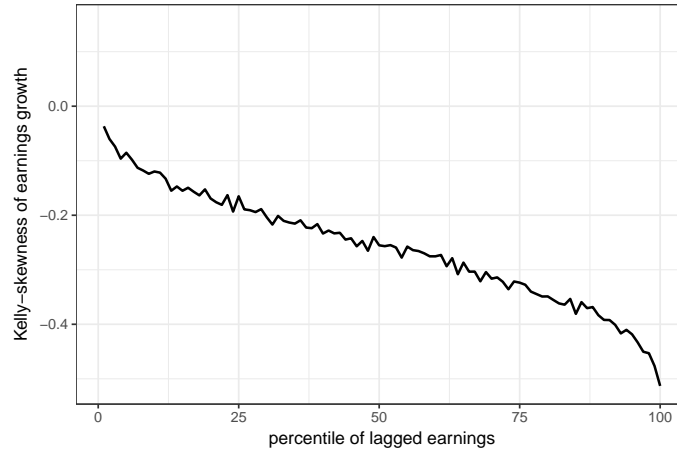


Figure 4.2: Kelly-skewness of simulated log earnings changes conditional on percentiles of initial earnings distribution

job leads to fall back to the bottom of the ladder. The negative change associated with falling off the job ladder is larger the higher up the ladder the worker was employed.

We plot the kurtosis of year over year changes in the log of annual earnings, conditional on the percentile of the earnings distribution in the initial year in Figure (4.3). The model produces a distribution of earnings growth rates that is substantially more peaked than what would arise from log-normal shocks (which would produce a kurtosis of 3).

In summary, the model produces a stochastic process for wage changes that is consistent with the patterns highlighted by Arellano et al. (2016) and Guvenen et al. (2016). Most year-to-year changes in annual earnings tend to be small, and the large changes tend to be negative rather than positive. This pattern becomes exaggerated the higher a worker is on the job ladder.



Figure 4.3: Kurtosis of simulated log earnings changes conditional on percentiles of initial earnings distribution

## 4.5 Conclusion

In this chapter we have developed a model in which inequality in earnings and consumption results from the interaction between heterogeneity of workers, heterogeneity of jobs, shocks to human capital, and labour market frictions. We laid out the formal identification arguments for the key distributions and parameters of the model and illustrate some key features of the model via simulations. In particular we showed that the model is able to reproduce the linear age profile for the variance of log earnings and consumption, as well as the negative skewness and excessive kurtosis of earnings growth rates, including the increasing negative skewness of log earnings changes conditional on the previous earnings level. The latter features are consistent with the patterns highlighted by Arellano et al. (2016) and Guvenen et al. (2016). Our model can be estimated on matched-employer employee data together with data on consumption. This would then allow us to decompose the inequality in earnings and consumption resulting from the interaction between heterogeneity of workers, heterogeneity of jobs, shocks to human capital, and labour market frictions.

# Appendix A

## A.1 Deriving Equation (1.9)

The equation describing the evolution of  $J$  over time results from equation (1.5) with  $V = 0$ ,

$$\dot{J}(t) = [r(t) + s]J(t) - \pi(t). \quad (\text{A.16})$$

With profits and the wage being substituted out from the profit equation (1.3) and wage equation (1.7), i.e. with

$$\pi(t) = p - w(t) = p - [(1 - \beta)z + \beta[p + \tau\theta(t)k]] \quad (\text{A.17})$$

$$= (1 - \beta)(p - z) - \beta\tau\theta(t)k \quad (\text{A.18})$$

we get

$$\dot{J}(t) = [r(t) + s]J(t) - (1 - \beta)[p - z] + \tau\theta(t)k. \quad (\text{A.19})$$

Using (1.6) to compute  $\dot{J}(t) = -\frac{k}{q(\theta(t))^2}q'(\theta(t))\dot{\theta}(t)$  and substitute  $J(t) = \frac{k}{q(\theta(t))}$  into (A.19), we find (1.9).

## A.2 The Financial Sector

In this section we derive in more detail the financial sector assuming that the banking sector is described by a monopolistically competitive industry.

### A.2.1 Monopolistic Competition

A service provider  $i$  maximises

$$\pi_s(i, t) = \hat{p}(i, t)x(i, t) - c(x(i, t)).$$



Given the parameter  $\gamma$ , this implies a mark-up pricing of

$$\hat{p}(i, t) = \frac{c'(x(i, t))}{\gamma}. \quad (\text{A.20})$$

We now consider the costs of providing  $x(i, t)$ . Given the technology (1.11),  $x(i, t) = by(i, t) - \phi$ , the costs to produce output  $x(i, t)$  is given by

$$c(x(i, t)) = y(i, t),$$

where  $y(i, t)$  is the input of the final good whose price is normalised to one. The cost function therefore reads

$$c(x(i, t)) = \frac{\phi + x(i, t)}{b}.$$

From (A.20), this implies that the price  $\hat{p}(i, t)$  of one unit of service is given by the usual mark-up pricing rule

$$\hat{p}(i, t) = \frac{b^{-1}}{\gamma}. \quad (\text{A.21})$$

Marginal costs to provide one unit of  $x(i, t)$  are given by the price of the output good (which is normalised to one) divided by the productivity parameter  $b$  from (1.11). The mark-up  $1/\gamma$  is determined by the price-elasticity of demand for services  $x(i, t)$  implied by (1.10).

Profits therefore can be computed to amount to

$$\pi_s(i, t) = \hat{p}(i, t)x(i, t) - \frac{\phi + x(i, t)}{b} = \hat{p}(i, t)x(i, t) - \frac{\hat{p}(i, t)\gamma}{b^{-1}} \frac{\phi + x}{b},$$

where the last equality used (A.21). Hence,

$$\pi_s(i, t) = \hat{p}(i, t)x(i, t) - \hat{p}(i, t)\gamma\phi - \hat{p}(i, t)\gamma x(i, t) = \hat{p}(i, t)[(1 - \gamma)x(i, t) - \gamma\phi].$$

As all firms use the same technology, the banking sector is symmetric and input per banking services is given by

$$y(i, t) = y(t) = \frac{\pi(t)[1 - u(t)]}{n(t)}.$$

The second equality shows that the input is given by total real profits of active firms divided by the number of banking services. Note that the second equality is the first crucial component of our general equilibrium setup. Resources available at each point in time are given by real profits  $\pi(t)$  per active firm times the number of

active firms, which is given by the number of employed workers  $1 - u(t)$ . Equation (1.11) then implies that output per service provider is given by

$$x(t) = by(t) - \phi = b \frac{\pi(t)[1 - u(t)]}{n(t)} - \phi. \quad (\text{A.22})$$

Given symmetry and (A.22), we obtain (1.12) in the text.

### A.2.2 The Number of Banks

Monopolistic service providers  $i$  choose output  $x(t)$  such that profits are maximised.<sup>7</sup> This yields markup pricing (A.21) and implies flow profits per service provider are given by  $\pi_s(t) = \hat{p}(t)[(1 - \gamma)x(t) - \gamma\phi]$ . After substituting output  $x$  from (A.22) in  $\pi_s(t) = \hat{p}(t)[(1 - \gamma)x(t) - \gamma\phi]$ , we obtain

$$\begin{aligned} \pi_s(t) &= \hat{p}(t) \left[ (1 - \gamma) \left( b \frac{\pi(t)[1 - u(t)]}{n(t)} - \phi \right) - \gamma\phi \right] \\ &= \hat{p}(t) \left[ (1 - \gamma)b \frac{\pi(t)[1 - u(t)]}{n(t)} - (1 - \gamma)\phi - \gamma\phi \right] \\ &= \hat{p}(t) \left[ (1 - \gamma)b \frac{\pi(t)[1 - u(t)]}{n(t)} - \phi \right]. \end{aligned}$$

Given free entry of banks, profits  $\pi_s(t)$  are driven to zero and we get (1.14).

### A.2.3 Deriving the Resource Constraint

Noting that  $b \frac{\pi(t)[1 - u(t)]}{n(t)} = \frac{\phi}{1 - \gamma}$  and  $\frac{\phi}{1 - \gamma} - \phi = \frac{\gamma}{1 - \gamma}\phi$ , the resource constraint (1.15) can be re-written as

$$\left[ (1 - \gamma)b \frac{\pi(t)[1 - u(t)]}{\phi} \right]^{1/\gamma} \frac{\gamma}{1 - \gamma}\phi = k\theta(t)u(t). \quad (\text{A.23})$$

---

<sup>7</sup>We suppress the provider index  $i$  as (A.22) has established symmetry.

Using (1.3) and (1.7) to substitute out for profits and wages yields (A.17). Substituting this into (A.23) yields

$$\begin{aligned}
 & \left[ (1 - \gamma)b \frac{((1 - \beta)(p - z) - \beta\tau\theta(t)k) [1 - u(t)]}{\phi} \right]^{1/\gamma} \frac{\gamma}{1 - \gamma} \phi = k\theta(t)u(t) \\
 & \Leftrightarrow \left[ (1 - \gamma)b \frac{k}{\phi} \left( (1 - \beta) \frac{p - z}{k} - \beta\tau\theta(t) \right) [1 - u(t)] \right]^{1/\gamma} \gamma = (1 - \gamma) \frac{k}{\phi} \theta(t)u(t) \\
 & \Leftrightarrow (1 - \gamma)b \frac{k}{\phi} \left( (1 - \beta) \frac{p - z}{k} - \beta\tau\theta(t) \right) [1 - u(t)] \gamma^\gamma = \left( (1 - \gamma) \frac{k}{\phi} \right)^\gamma \theta^\gamma(t) u^\gamma(t) \\
 & \Leftrightarrow b \left[ (1 - \gamma) \frac{k}{\phi} \right]^{1-\gamma} \left( (1 - \beta) \frac{p - z}{k} - \beta\tau\theta(t) \right) [1 - u(t)] \gamma^\gamma = \theta^\gamma(t) u^\gamma(t) \\
 & \Leftrightarrow b \left[ (1 - \gamma) \frac{k}{\phi} \right]^{1-\gamma} \frac{1 - u(t)}{u^\gamma(t)} \gamma^\gamma = \frac{\theta^\gamma(t)}{(1 - \beta) \frac{p - z}{k} - \beta\tau\theta(t)}.
 \end{aligned}$$

The last expression is (1.15).

### A.3 Existence of an Interior Steady State

We characterise conditions under which a unique interior steady state exists. To do so we analyse how does the equilibrium path (1.15) behave with respect to labour market tightness after substituting out for unemployment using (1.16). Let the left-hand side of (1.15) be described by

$$T_1(\theta) = \frac{\theta}{[(1 - \beta)(p - z) - \tau\beta k\theta]^{1/\gamma}}.$$

Similarly, substituting out for  $u$  using (1.16), the right-hand side of (1.15) can be described by

$$T_2(\theta) = C \frac{(s + \lambda(\theta))}{s} \left[ \frac{\lambda(\theta)}{s + \lambda(\theta)} \right]^{\frac{1}{\gamma}},$$

where

$$C = \frac{\gamma}{(1 - \gamma)^{-\frac{1-\gamma}{\gamma}}} \frac{\phi^{-\frac{1-\gamma}{\gamma}}}{k} b^{\frac{1}{\gamma}}$$

describes a constant. Note that both  $T_1$  and  $T_2$  take the value of zero when  $\theta = 0$ . Differentiation of  $T_1$  and  $T_2$  with respect to  $\theta$  implies that both functions are

increasing in  $\theta$ .

$$\begin{aligned}\frac{dT_1}{d\theta} &= \frac{1}{\gamma} \left[ \frac{\gamma(1-\beta)(p-z) + \theta\tau\beta k(1-\gamma)}{[(1-\beta)(p-z) - \tau\beta k\theta]^{\frac{1-\gamma}{\gamma}}} \right] > 0, \\ \frac{dT_2}{d\theta} &= \frac{C\lambda'(\theta)}{s} \left( \frac{\lambda(\theta)}{s + \lambda(\theta)} \right)^{\frac{1}{\gamma}} \left[ 1 + \frac{s}{\gamma\lambda(\theta)} \right] > 0.\end{aligned}$$

Further differentiation implies that  $d^2T_1/d\theta^2 > 0$ , while

$$\frac{d^2T_2}{d\theta^2} = \frac{C}{s} \left( \frac{\lambda(\theta)}{s + \lambda(\theta)} \right)^{\frac{1}{\gamma}} \frac{1}{(\lambda(\theta)\gamma)^2} \left[ \lambda''(\theta)\lambda(\theta)\gamma(\lambda(\theta)\gamma + s) + \frac{\lambda'^2 s^2}{s + \lambda(\theta)}(1-\gamma) \right].$$

The sign of  $d^2T_2/d\theta^2$  is then determined by the sign of the term in the squared brackets. Note that the first term inside the squared brackets is negative (as the job finding rate is concave in  $\theta$ ), while the second term is positive. Given that  $d^2T_2/d\theta^2 < 0$  for all  $\theta$ , continuity of  $T_1$  and  $T_2$  imply that in this case there exists a unique interior steady state equilibrium. If  $d^2T_2/d\theta^2 > 0$  or non-monotone, we could also have multiple interior steady state equilibria or no equilibria at all.

Given that in the quantitative section of the paper we focus on a Cobb-Douglas matching function,  $M(u, v) = Au^\alpha v^{1-\alpha}$ , it is instructive to analyse the conditions for existence under such a parametrisation. Noting that under this matching function the job finding rate and its derivatives are given by:  $\lambda(\theta) = A\theta^{1-\alpha}$ ,  $\lambda'(\theta) = (1-\alpha)A\theta^{-\alpha}$  and  $\lambda''(\theta) = -\alpha(1-\alpha)A\theta^{-(1+\alpha)}$ , the term in the squared bracket in the above expression for  $d^2T_2/d\theta^2$  is given by

$$-\frac{A^2\theta^{-2\alpha}(1-\alpha)}{s + A\theta^{1-\alpha}} \left[ s^2(\gamma - (1-\alpha)) + \alpha\gamma A\theta^{1-\alpha}[s(1+\gamma) + A\theta^{1-\alpha}\gamma] \right].$$

Inspection shows that  $\gamma + \alpha \geq 1$  provides a sufficient (but not necessary) condition for  $d^2T_2/d\theta^2 < 0$  and hence for existence of a unique interior steady state equilibrium.

## A.4 The Slope of the Resource Constraint

### A.4.1 Preliminaries

Re-writing equation (1.15) in  $v - u$  space using an implicit formulation yields

$$G(v, u) \equiv v - \frac{\gamma\phi}{(1-\gamma)k} \left[ \frac{(1-\gamma)b[(1-\beta)(p-z) - \tau\beta k \frac{v}{u}][1-u]}{\phi} \right]^{\frac{1}{\gamma}} = 0. \quad (\text{A.24})$$

It follows from (A.24) that when all workers are unemployed,  $u = 1$ , there are no vacancies  $v = 0$ . In this case no firm is producing and hence firms profits are zero implying that there are no available funds to pay for vacancies.

Now consider the slope of  $G(v, u)$ . Note that the resource constraint shows that aggregate firms' profits and the number of vacancies must move in the same direction along the equilibrium path. Aggregate firms' profits, however, depend positively on (i) the number of jobs filled and negatively on (ii) the wage paid to workers; and both are inversely related to the unemployment rate. The slope of  $G(v, u)$  then depends on how responsive are wages to changes in the unemployment rate. Employing the implicit function theorem, we obtain (see Appendix A.4.2)

$$\frac{dv}{du} = \frac{\partial n}{\partial u} \left[ \left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \left( \frac{1}{\gamma n} \right) \right]^{-1} + \frac{(1-\gamma)b\tau\beta k(1-u)}{u\phi} \right]^{-1},$$

where the slope of the equilibrium path in Beveridge space is determined by

$$\text{sign} \left[ \frac{\partial n}{\partial u} \right] = \text{sign} \left[ \frac{\partial \pi}{\partial u} (1-u) - \pi \right], \quad (\text{A.25})$$

which in turn depends on how the unemployment rate affects firms flow profits via wages.

#### A.4.2 Total Differentiation of $G(v, u)$

Total differentiation of  $G(v, u)$  implies  $\frac{dv}{du} = -\frac{\frac{\partial G(v, u)}{\partial u}}{\frac{\partial G(v, u)}{\partial v}}$ . Originally, equation (A.24) reads

$$G(v, u) \equiv v - \frac{\gamma \phi}{(1-\gamma)k} \left[ \frac{(1-\gamma)b[(1-\beta)(p-z) - \tau\beta k \frac{v}{u}][1-u]}{\phi} \right]^{\frac{1}{\gamma}}.$$

As from (1.14) and (A.17),

$$n = \frac{(1-\gamma)b[(1-\beta)(p-z) - \tau\beta k \frac{v}{u}][1-u]}{\phi}, \quad (\text{A.26})$$

we can express it more compactly as

$$G(v, u) = v - \frac{\gamma \phi}{(1-\gamma)k} n^{\frac{1}{\gamma}} = 0.$$

It follows that

$$\begin{aligned}\frac{\partial G(v, u)}{\partial u} &= - \left[ \frac{\gamma \phi}{1 - \gamma} k \frac{n^{\frac{1}{\gamma}}}{\gamma n} \frac{\partial n}{\partial u} \right] = - \frac{\partial n}{\partial u} \left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1 - \gamma) k \gamma n} \right] \\ \frac{\partial G(v, u)}{\partial v} &= 1 - \frac{\partial n}{\partial v} \left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1 - \gamma) k \gamma n} \right].\end{aligned}$$

Thus

$$\frac{dv}{du} = - \frac{- \frac{\partial n}{\partial u} \left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1 - \gamma) k \gamma n} \right]}{1 - \frac{\partial n}{\partial v} \left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1 - \gamma) k \gamma n} \right]} = \frac{\frac{\partial n}{\partial u}}{\left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1 - \gamma) k \gamma n} \right]^{-1} - \frac{\partial n}{\partial v}} = \frac{\partial n}{\partial u} \left[ \left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1 - \gamma) k \gamma n} \right]^{-1} - \frac{\partial n}{\partial v} \right]^{-1}.$$

Note that from (A.26)

$$\frac{\partial n}{\partial v} = - \frac{(1 - \gamma) b \tau \beta k (1 - u)}{\phi u},$$

so that

$$\frac{dv}{du} = \frac{\partial n}{\partial u} \left[ \left[ \frac{\gamma \phi n^{\frac{1}{\gamma}}}{(1 - \gamma) k \gamma n} \right]^{-1} + \frac{(1 - \gamma) b \tau \beta k (1 - u)}{\phi u} \right].$$

#### A.4.2 The Slope of $G(v, u)$

The resource constraint is falling by (A.25) iff

$$\frac{dv}{du} < 0 \Leftrightarrow \text{sign} \left[ \frac{\partial \pi}{\partial u} (1 - u) - \pi \right] < 0.$$

As  $\partial \pi / \partial u = \beta \tau v k / u^2 > 0$  from (A.17), this holds iff, using (A.17),

$$\begin{aligned}\frac{\beta \tau v k}{u^2} (1 - u) - \pi < 0 &\Leftrightarrow \frac{\beta \tau v k}{u^2} (1 - u) < \pi \Leftrightarrow \frac{\beta \tau v k}{u^2} (1 - u) < (1 - \beta)(p - z) - \beta \tau \theta k \\ &\Leftrightarrow \beta \tau \frac{v}{u} \frac{1 - u}{u} < (1 - \beta) \frac{p - z}{k} - \beta \tau \frac{v}{u} \Leftrightarrow \frac{\beta \tau v}{u^2} < (1 - \beta) \frac{p - z}{k} \\ &\Leftrightarrow \tau < \frac{1 - \beta}{\beta v} \frac{p - z}{k} u^2.\end{aligned}$$

## A.5 Comparative Statics for the Slope of $\Psi$

We want to analyse how changes in  $\phi$  and  $b$  affect  $d\theta/du$ , the slope of  $\Psi$ , as described in (1.20). To do this let

$$\begin{aligned}\Psi_1 &= -n[\theta(1-\gamma)k(1-u) + \phi n^{\frac{1}{\gamma}}] \\ \Psi_2 &= k(1-\gamma)(1-u)[nu + n^{\frac{1}{\gamma}}(1-u)\tau b\beta].\end{aligned}$$

**Changes in  $\phi$ .** Noting that  $dn/d\phi = -n/\phi$ , we have that

$$\begin{aligned}\frac{\partial \Psi_1}{\partial \phi} &= \frac{n}{\phi \gamma} [\theta(1-\gamma)k(1-u)\gamma + \phi n^{\frac{1}{\gamma}}], \\ \frac{\partial \Psi_2}{\partial \phi} &= -\frac{k(1-\gamma)(1-u)}{\phi \gamma} [nu\gamma + n^{\frac{1}{\gamma}}(1-u)\tau b\beta].\end{aligned}$$

Since the

$$\frac{\partial[d\theta/du]}{\partial \phi} = \frac{1}{\Psi_2^2} \left[ \frac{\partial \Psi_1}{\partial \phi} \Psi_2 - \frac{\partial \Psi_2}{\partial \phi} \Psi_1 \right],$$

the sign of the change is determined by the expression in the squared bracket. Substituting the corresponding expressions and some algebra establishes that

$$\frac{\partial[d\theta/du]}{\partial \phi} = \frac{1}{\Psi_2^2} \frac{nk(1-\gamma)^2(1-u)n^{\frac{1}{\gamma}}}{\phi \gamma} [nu\phi - \theta(1-\gamma)k(1-u)^2\tau b\beta].$$

Noting that  $n = (1-\gamma)b^{\frac{[(1-\beta)(p-z)-\beta\theta k][1-u]}{\phi}}$ , the above expression can be simplified to

$$\frac{\partial[d\theta/du]}{\partial \phi} = \frac{1}{\Psi_2^2} \frac{nbk(1-\gamma)^3(1-u)^2n^{\frac{1}{\gamma}}}{\phi \gamma} [(1-\beta)(p-z)u - \tau\beta\theta k].$$

The slope of  $\Psi$  increases with  $\phi$  when  $(1-\beta)(p-z)u > \tau\beta\theta k$ . Since  $\Psi$  is downward sloping, an increase in its slope implies it becomes flatter, which in turn implies that  $\Psi$  shifts to the left towards the origin.

**Changes in  $b$ .** In this case we have that  $dn/db = n/b$  and

$$\begin{aligned}\frac{\partial \Psi_1}{\partial b} &= -\frac{n}{b\gamma} [\theta(1-\gamma)k(1-u)\gamma + \phi n^{\frac{1}{\gamma}}(1+\gamma)], \\ \frac{\partial \Psi_2}{\partial b} &= \frac{k(1-\gamma)(1-u)}{b\gamma} [nu\gamma + n^{\frac{1}{\gamma}}(1-u)\tau b\beta(1+\gamma)].\end{aligned}$$

Since the

$$\frac{\partial[d\theta/du]}{\partial b} = \frac{1}{\Psi_2^2} \left[ \frac{\partial \Psi_1}{\partial b} \Psi_2 - \frac{\partial \Psi_2}{\partial b} \Psi_1 \right],$$

the sign of the derivative is determined by the expression in the squared bracket. Substituting the corresponding expressions and some algebra establishes that

$$\frac{\partial[d\theta/du]}{\partial b} = -\frac{1}{\Psi_2^2} \frac{nk(1-\gamma)(1-u)n^{\frac{1}{\gamma}}}{b\gamma} \left[ nu\phi - \theta(1-\gamma)k(1-u)^2\tau b\beta \right],$$

where the term in the squared bracket is the same as in the case of changes in  $\phi$ . Using the expression for  $n$  we obtain

$$\frac{\partial[d\theta/du]}{\partial b} = -\frac{1}{\Psi_2^2} \frac{nk(1-\gamma)^2(1-u)^2n^{\frac{1}{\gamma}}}{\gamma} [(1-\beta)(p-z)u - \tau\beta\theta k].$$

The slope of  $\Psi$  decreases with  $b$  when  $(1-\beta)(p-z)u > \tau\beta\theta k$ . Since  $\Psi$  is downward sloping, a decrease in its slope implies it becomes steeper, which in turn implies that  $\Psi$  shifts to the right, away from the origin.

**Changes in  $\gamma$ .** In this case we have that  $dn/d\gamma = -n/(1-\gamma)$  and that

$$\frac{\partial(n^{\frac{1}{\gamma}})}{\partial \gamma} = -\frac{n^{\frac{1}{\gamma}}}{\gamma^2(1-\gamma)} [(1-\gamma)\ln(n) + \gamma].$$

These expressions together imply

$$\begin{aligned} \frac{\partial\Psi_1}{\partial\gamma} &= 2n\theta k(1-u) + \frac{\phi n^{\frac{1}{\gamma}}}{\gamma^2(1-\gamma)} [\gamma(1+\gamma n) + (1-\gamma)\ln(n)], \\ \frac{\partial\Psi_2}{\partial\gamma} &= -2nuk(1-u) - \frac{k(1-u)^2n^{\frac{1}{\gamma}}\tau b\beta}{\gamma^2} [\gamma(1+\gamma) + (1-\gamma)\ln(n)]. \end{aligned}$$

Since the

$$\frac{\partial[d\theta/du]}{\partial\gamma} = \frac{1}{\Psi_2^2} \left[ \frac{\partial\Psi_1}{\partial\gamma}\Psi_2 - \frac{\partial\Psi_2}{\partial\gamma}\Psi_1 \right],$$

once again the sign of the change is determined by the expression in the squared bracket. Substituting the corresponding expressions and some algebra establishes that the sign of  $\partial[d\theta/du]/\partial\gamma$  equals the sign of

$$\begin{aligned} &\phi u [n\gamma^2 + (1-\gamma)\ln(n)] - \\ &\left[ \tau b\beta\theta k(1-u)^2(1-\gamma)^2(\gamma + \ln(n)) + \phi u\gamma + \phi n^{\frac{1-\gamma}{\gamma}}(1-u)\tau b\beta(n-1)[\gamma + (1-\gamma)\ln(n)] \right]. \end{aligned}$$

Consider the sign of  $\partial[d\theta/du]/\partial\gamma$  as  $\gamma \rightarrow 1$ . Since in this limit  $n \rightarrow 0$ , we find that  $\partial[d\theta/du]/\partial\gamma < 0$  when  $\tau b\beta\lambda(\theta) - s > 0$ . On the other hand, when  $\gamma \rightarrow 0$ , we find that in this limit  $\partial[d\theta/du]/\partial\gamma < 0$  when  $n > 1$ . In both cases a decrease in the slope of  $\Psi$  implies it becomes steeper, which in turn implies that  $\Psi$  shifts to the



right, away from the origin.

## A.6 The Impact of the Financial Crisis: Time-Series

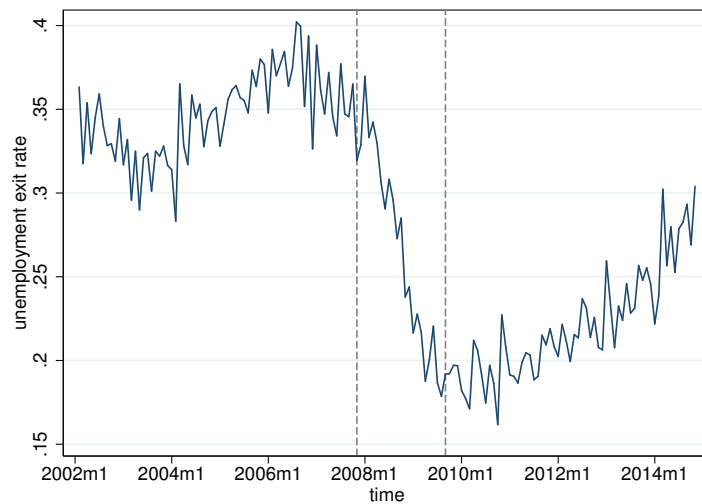


Figure A.4: Unemployment exit rate

**Notes:** This figure shows the monthly unemployment exit rate for the US. The exit rate was calculated using BLS data on the seasonal adjusted number of unemployed and unemployed with durations less than 5 weeks. The Great Recession is the period between the two dashed lines.

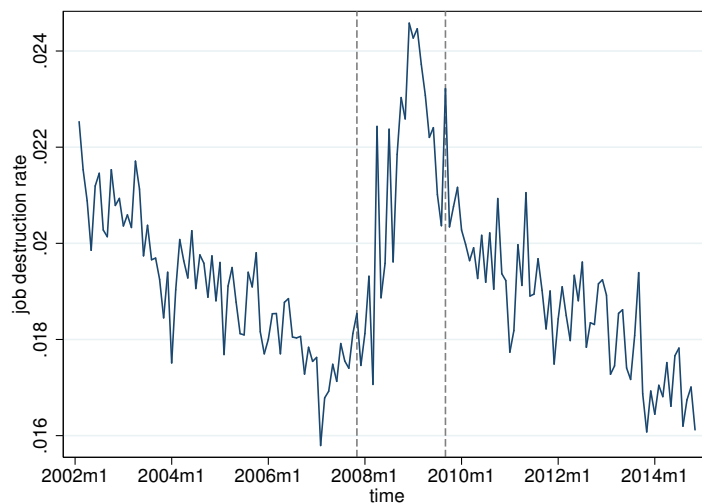


Figure A.5: Job destruction rate

**Notes:** This figure shows the monthly employment exit rate for the US. The exit rate was calculated using BLS data on the seasonal adjusted number of employed and unemployed with durations less than 5 weeks. The Great Recession is the period between the two dashed lines.



Figure A.6: Unemployment and vacancy dynamics

**Notes:** This figure shows the monthly unemployment and vacancy rate for the US. The data is taken from the BLS and the JOLTS. Rates are calculated by dividing the number of vacancies and unemployed by the sum of the employed and unemployed in a given month. The Great Recession is the period between the two dashed lines.

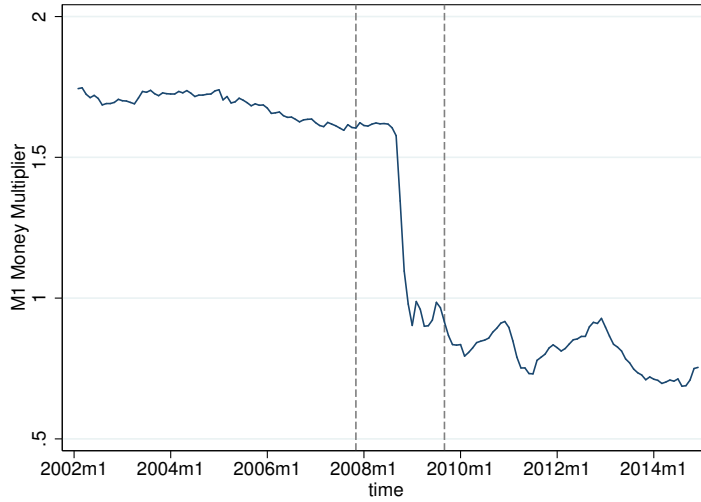


Figure A.7: Money multiplier

**Notes:** This figure shows the evolution of the M1 money multiplier over time. The data is provided by the Federal Reserve Bank of St. Louis (<https://research.stlouisfed.org/fred2/series/MULT>). The Great Recession is the period between the two dashed lines.

## A.7 Details on the Calibration

To calibrate the parameters  $\mathbf{x} = \{k, z, \beta, \tau, \phi, \gamma\}$ , we minimise the squared relative distance between the observed vacancy rates and the ones implied by our transition

path taking the observed unemployment rates as given. That is, we choose

$$\mathbf{x} = \arg \min \sum_t \left( \frac{v_t - \hat{v}(\mathbf{x}; u_t)}{v_t} \right)^2, \quad (\text{A.27})$$

where  $\hat{v}(\mathbf{x}; u_t)$  denotes the vacancy rate that solves (1.15) given the vector of parameters  $\mathbf{x}$  and the observed unemployment rate  $u_t$ . The minimisation problem is subject to the following equality constraints:

$$\begin{aligned} (1 - \beta) \frac{p - z}{k} q(\theta^*) - s - r^* - \tau \beta \lambda(\theta^*) &= 0 \\ v^* - \hat{v}(\mathbf{x}; u^*) &= 0 \\ 0.5w - z &= 0 \\ 0.036w - v^*k &= 0. \end{aligned}$$

In practice, we use the `fmincon` function from the Optimization Toolbox (Version 7.2) in Matlab (Version 8.5.0.197613 (R2015a)) designed to find the minimum of a function  $f(x)$  with linear and nonlinear inequality and equality constraints. Note that at every evaluation of the objective function, we also have to solve for the series of vacancy rates  $\hat{v}(\mathbf{x}; u)$ . Given a guess for the parameter vector  $\mathbf{x}$  and the observed unemployment rates, we can numerically solve equation (1.15) to obtain the series of vacancy rates implied by our model. In practice, we use the `lsqnonlin` function from the Optimization Toolbox - a nonlinear least square solver - to perform this step and check that the value of the objective function is close to 0. The advantage of the non-linear least square solver compared to Matlab's built-in solver for nonlinear systems, `fsolve`, is that we can impose a non-negativity constraint ensuring the stability of the optimisation procedure.

# Appendix B

## B.1 Estimation Details

**Quasi-differences.** Let  $\Delta^\rho y_{i,a}^c \equiv y_{i,a}^c - \rho^c y_{i,a-1}^c$ . Specification (1) implies that

$$\Delta^\rho y_{i,a}^c = \alpha_i^c(1-\rho^c) + \beta_i^c \Delta^\rho p_a + u_{i,a}^c + \Delta^\rho \varepsilon_{i,a}^c + \theta^c \Delta^\rho \varepsilon_{i,a-1}^c, a = a_{\min}^c + 1, \dots, a_{\max}^c, \quad (\text{B.1})$$

where the youngest and oldest age at which we observe cohort  $c$  is denoted by  $a_{\min}^c$  and  $a_{\max}^c$  respectively. With  $A^c \equiv a_{\max}^c - a_{\min}^c$  we can re-write (B.1) in vectorised form as

$$\Delta^\rho y_i^c = \alpha_i(1 - \rho^c)\iota + \beta_i \Delta^\rho p + u_i^c + \Delta^\rho \varepsilon_i^c + \theta^c \Delta^\rho L \varepsilon_i^c, \quad (\text{B.2})$$

where  $\iota$  is a  $A^c \times 1$  vector of ones and  $L$  represents the lag-operator. The  $A^c \times A^c$  auto-covariance matrix is then given by

$$\text{var}(\Delta^\rho y_i^c) = [(1 - \rho^c)\iota, \Delta p] \text{var}(\gamma_i^c) [(1 - \rho^c)\iota, \Delta p]' + \text{var}(u_i^c) + \text{var}(\Delta^\rho \varepsilon_i^c + \theta^c \Delta^\rho L \varepsilon_i^c), \quad (\text{B.3})$$

where we have used the notation  $\gamma_i^c \equiv [\alpha_i^c, \beta_i^c]'$ . Note that the  $(a, a + s)$  element of  $\text{var}(\Delta^\rho y_i^c)$  is given by

$$\begin{aligned} \text{cov}(\Delta^\rho y_{i,a}^c, \Delta^\rho y_{i,a+s}^c) &= [(1 - \rho^c), \Delta p_a] \text{var}(\gamma_i^c) [(1 - \rho^c), \Delta p_{a+s}]' \\ &+ \begin{cases} \text{var}(u_{i,a}^c) + \text{var}(\varepsilon_{i,a}^c) + (\theta^c - \rho^c)^2 \text{var}(\varepsilon_{i,a-1}^c) + (\theta^c \rho^c)^2 \text{var}(\varepsilon_{i,a-2}^c) & \text{if } s = 0 \\ (\theta^c - \rho^c) (\text{var}(\varepsilon_{i,a}^c) - \theta^c \rho^c \text{var}(\varepsilon_{i,a-1}^c)) & \text{if } s = 1 \\ - (\theta^c \rho^c \text{var}(\varepsilon_{i,a}^c)) & \text{if } s = 2 \\ 0 & \text{if } s > 2 \end{cases} \end{aligned}$$

**Estimation.** For a given value of  $\rho$  and for each cohort  $c$ , we calculate the empirical counterpart to expression (B.3). We then average the  $(a, a')$ -cell of these matrices across all cohorts that we jointly observe at age  $a$  and age  $a'$ . Let the resulting empirical auto-covariance matrix be denoted by  $\widehat{\text{var}(\Delta^\rho y_i)}$  and define

the stacked vector of its unique elements by

$$\hat{M} = \text{vech}(\widehat{\text{var}(\Delta^\rho y_i)}).$$

Let the parameters to be estimated be denoted by  $\Theta$ . The equally-weighted minimum distance estimator  $\tilde{\Theta}$  is then given by

$$\tilde{\Theta} = \arg \min_{\Theta} [M(\Theta; \rho) - \hat{M}]' [M(\Theta; \rho) - \hat{M}],$$

where  $M(\Theta; \rho)$  are the corresponding stacked vector of theoretical moments. Since we have averaged the empirical moments across cohorts,  $\Theta$  contains the average (or typical) profiles of variances of permanent and transitory shocks,  $\text{var}(u_i)$  and  $\text{var}(\varepsilon_i)$ , the moving average parameter  $\theta$ , an estimate for the typical variance of initial conditions  $\text{var}(\alpha_i)$ , the variance of the growth rate  $\text{var}(\beta_i)$  and the correlation between  $\alpha_i$  and  $\beta_i$  denoted by  $\rho_{\alpha\beta}$ . Once we have solved for  $\tilde{\Theta}$  for each value of  $\rho$  belonging to a grid, we select the estimator  $\tilde{\Theta}$  together with  $\rho$  that minimises the distance between the empirical and theoretical moments.

In practice, we find  $\tilde{\Theta}$  for a given value of  $\rho$  by solving a constrained nonlinear optimization problem. The inequality constraints imposed ensure positive values for variances at any age. We further normalize the variance of transitory shocks to be constant from age 24 to age 26 and the variance of permanent and transitory shocks to be constant between age 59 and 60. Note that in our baseline specification we also impose  $\beta_i = 0$ . Finally, we perform this estimation separately by education and income measures.

## B.2 The Tax System

The Norwegian tax system is progressive through deductions and surtaxes. Figure B.1 shows the marginal tax rates for single earner couples and for single persons (or dual earner couples) at the end of 2006. There is a 7.8 % social security contribution on market income. The market income is taxed at a flat rate of 28 percent; on top of that, there are two surtax brackets adding an additional 9 and 12 percent to the marginal tax rates. Single earner couples and single persons (or dual earner couples) are taxed differently: The latter type of households only gets 50 percent of the standard deduction. Over time, the the Norwegian tax system has become less progressive through a series of policy changes. Figure B.2 summarizes these changes by displaying the average tax rates on market income over time.

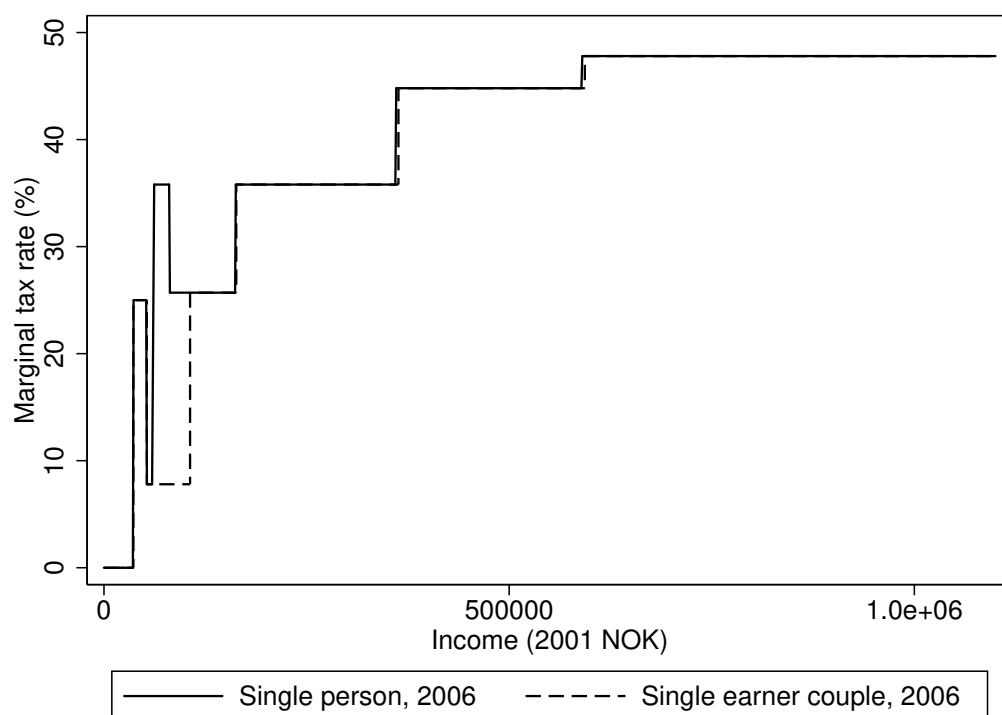


Figure B.1: Marginal tax rates on market income in 2006

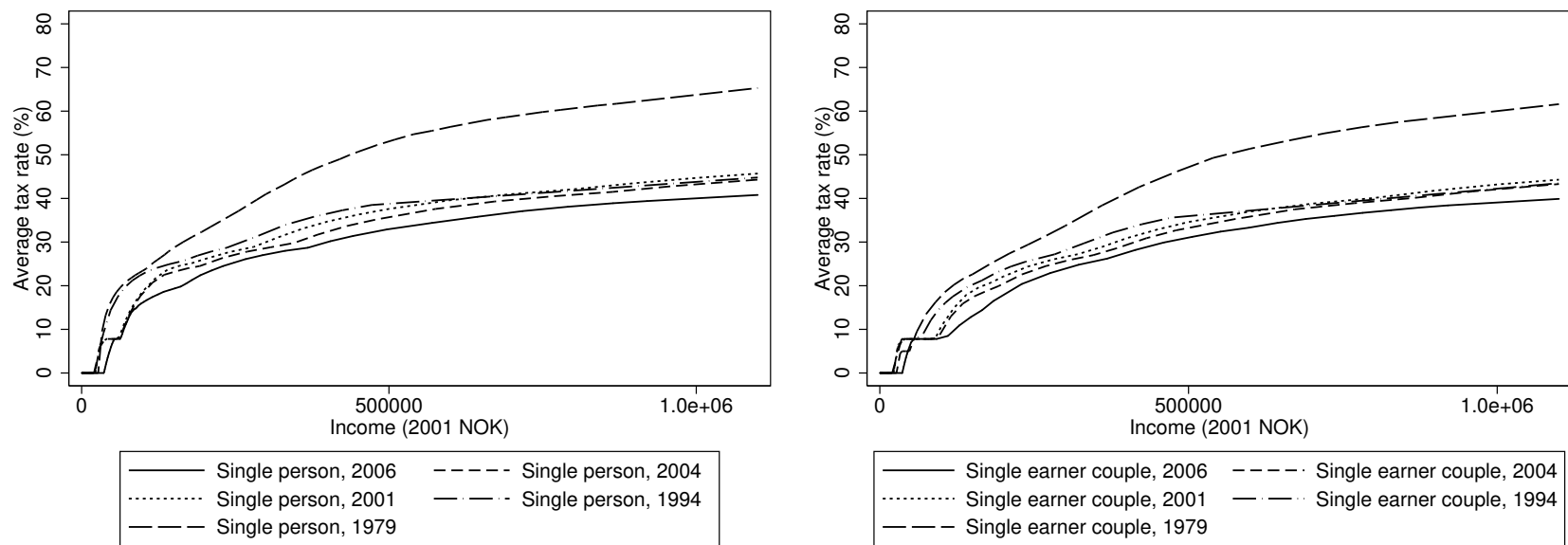


Figure B.2: Average tax rates on market income in different years

## B.3 Additional Tables and Figures

	Low-Skilled	Medium-Skilled	High-Skilled
$\rho$	1.00 (0.000000)	1.00 (0.000000)	0.75 (0.000000)
$var(\alpha_i)$	- -	- -	0.077078 (0.000753)
$var(u_i)$	0.176590 (0.000786)	0.128030 (0.000530)	0.109590 (0.000736)
$var(\varepsilon_i)$	0.041355 (0.000470)	0.026831 (0.000332)	0.014493 (0.000500)
$\theta$	0.289020 (0.003764)	0.295160 (0.003180)	0.306340 (0.009686)

Table B.1: Parameter estimates from the model of income dynamics with age-independent variance of shocks

**Notes:** This table presents the parameter estimates from the model of income dynamics. We estimate the model described in Section 2.3.1, except for imposing age-independent variances of transitory and permanent shocks. We use the baseline sample and estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Standard errors (in parentheses) are based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.

	Individual Market Income	Individual Disposable Income	Family Disposable Income
	Pooled		
$\rho$	1.00 (0.000000)	0.85 (0.004133)	0.86 (0.004962)
$var(\alpha_i)$	- -	0.035644 (0.000483)	0.031509 (0.000473)
$\theta$	0.271470 (0.001784)	0.250870 (0.002389)	0.251930 (0.002673)

Table B.2: Parameter estimates from the model of income dynamics in the pooled sample

**Notes:** This table presents the parameter estimates from the model of income dynamics. We estimate the model described in Section 2.3.1. We use the baseline sample, but do *not* estimate the model separately by educational levels. Standard errors (in parentheses) are based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.



	Low-Skilled	Medium-Skilled	High-Skilled
$\rho$	1.00 (0.000000)	1.00 (0.000000)	0.90 (0.045857)
$var(\alpha_i)$	- -	- -	0.026887 (0.032780)
$var(\beta_i)$	0.000000 (0.000000)	0.000000 (0.000000)	0.0002773 (0.000098)
$\rho_{\alpha\beta}$	- -	-	-0.998930 (0.067835)
$\theta$	0.238500 (0.003588)	0.258830 (0.002895)	0.293430 (0.004917)

Table B.3: Parameter estimates from the model of income dynamics with heterogeneous profiles in market income

**Notes:** This table presents the parameter estimates from the model of income dynamics. We estimate the model described in Section 2.3.1 with heterogeneous profiles. We use the baseline sample and estimate the model for individual market income separately by educational levels with. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Standard errors (in parentheses) are based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.

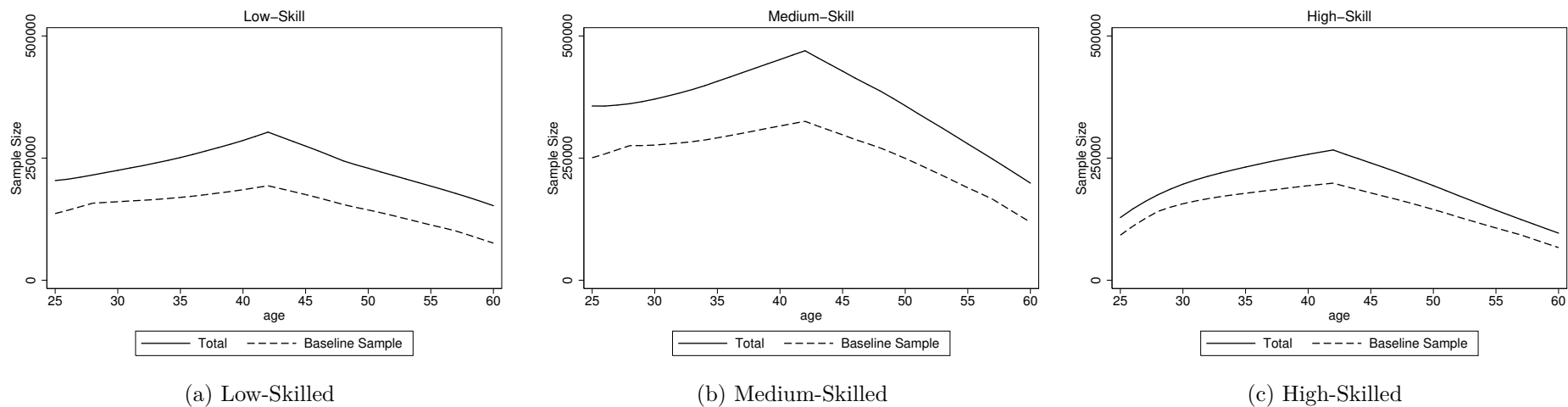


Figure B.3: Population and sample size by age

**Notes:** Using data from the period 1967-2006, this figure shows the number of observations by age for (a) all males born between 1925 and 1964 and (b) the baseline sample. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college.

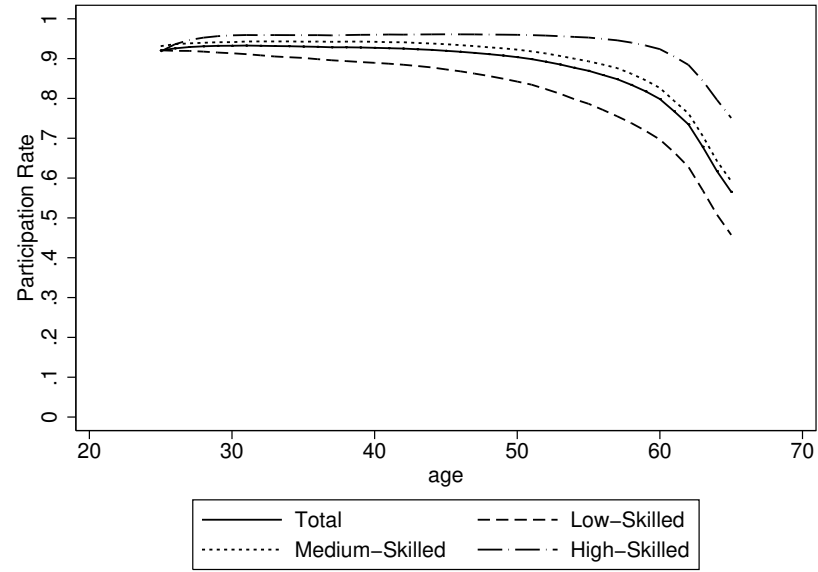


Figure B.4: Labor force participation rate of males by age

**Notes:** This figure shows the population share of males with positive market income by age, using data from the period 1967-2006. The sample consists of males born between 1925 and 1964. In each year, we exclude immigrants and self-employed. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college.

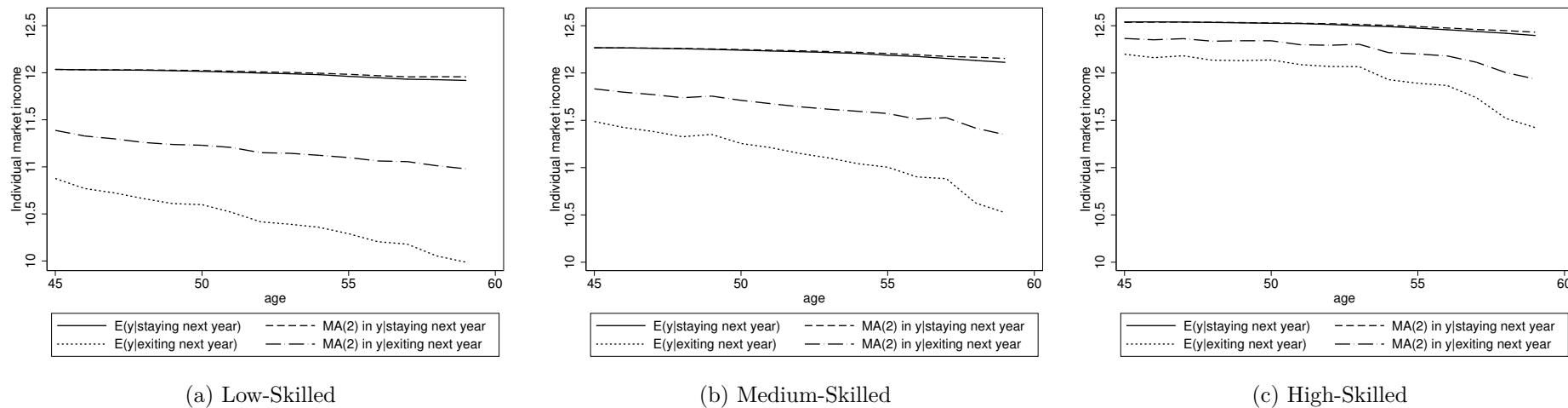
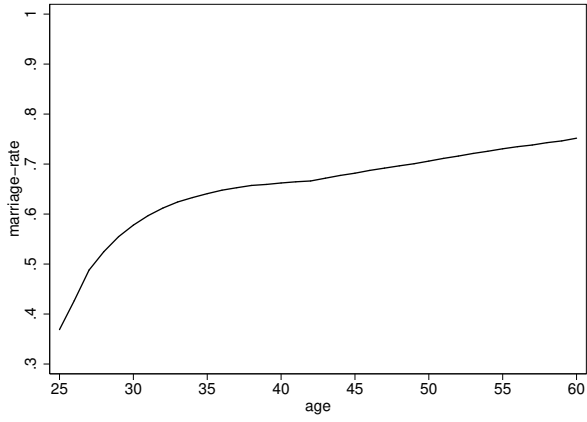
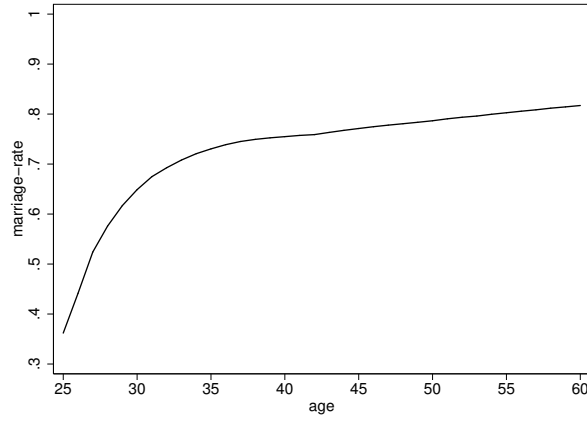


Figure B.5: Levels and growth in log market income for early and late exits from the labor force

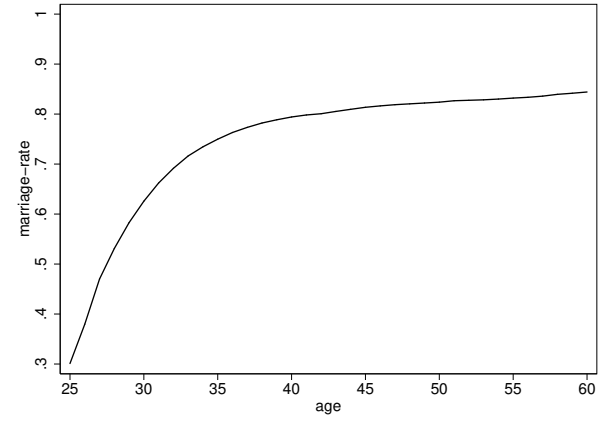
**Notes:** This figure uses the baseline sample to show the levels and growth rates in log market income by age; we show this separately for individuals who exit and stay in the labor market in the subsequent year. The growth rate of earnings comes from a simple moving average over the three previous years (*i.e.*  $y_{i,a} = \frac{1}{3} \sum_{l=0}^2 y_{i,a-l}$ ). Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college.



(a) Low-Skilled



(b) Medium-Skilled



(c) High-Skilled

Figure B.6: Proportion of baseline sample that is married by age and educational levels

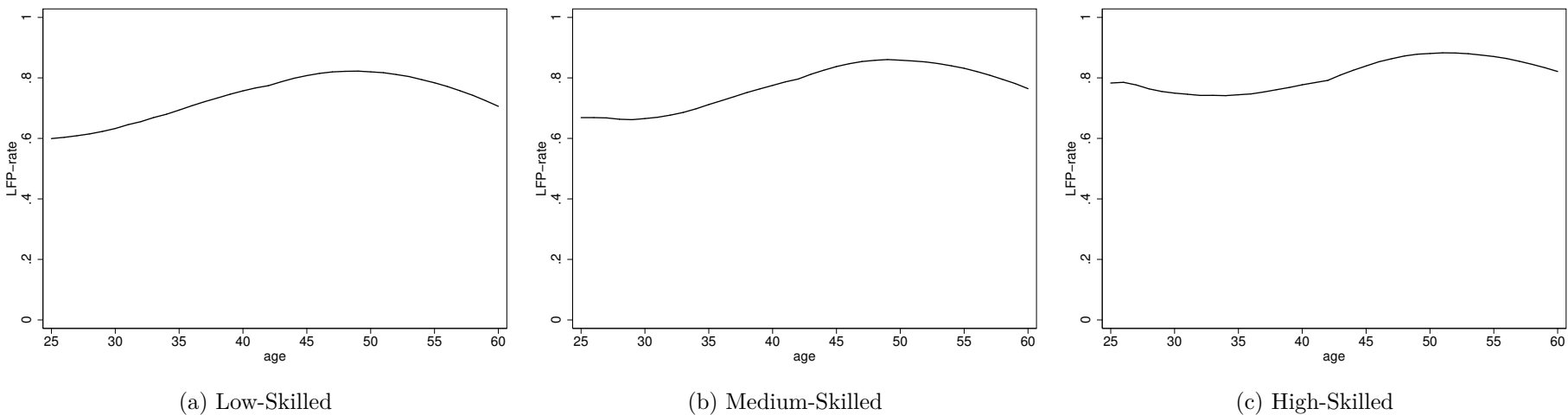


Figure B.7: Labour force participation rate of spouses by age and educational levels

**Notes:** This figure shows the labor force participation rate (LFP) of the spouses by age and educational level of the husbands. Participation is equal to one if the wife has positive labor income in a given year. This figure uses the baseline sample and, in each year, excludes individuals who are not married.

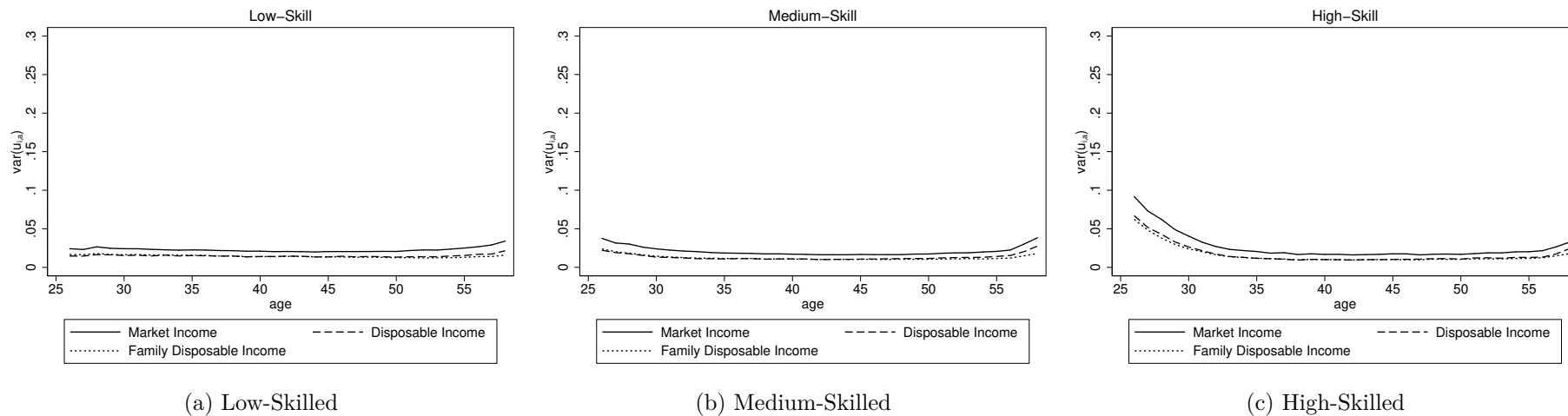


Figure B.8: Age profiles in the variance of permanent shocks to individual income when excluding individuals with low market income.

**Notes:** This figure graphs the age profiles in the variances of permanent shocks. The age profiles are based on the model of income dynamics described in Section 2.3.1. We refine the baseline sample: In each year, we exclude individuals with market income less than one basic amount. We estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. The age profiles are adjusted for education-specific calendar time effects.

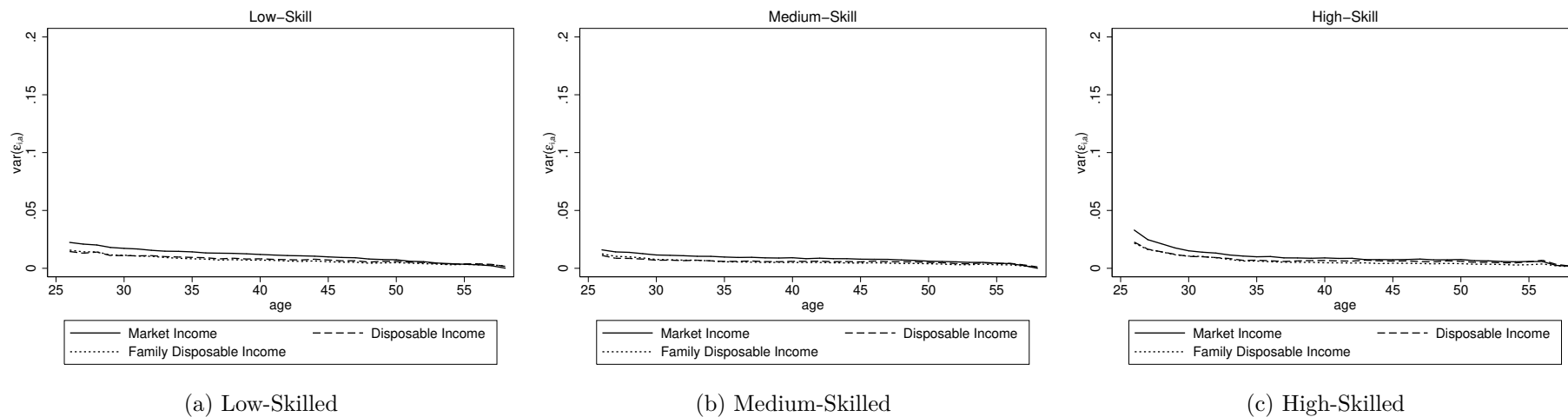


Figure B.9: Age profiles in the variance of transitory shocks to individual income when excluding individuals with low market income.

**Notes:** This figure graphs the age profiles in the variances of transitory shocks. The age profiles are based on the model of income dynamics described in Section 2.3.1. We refine the baseline sample: In each year, we exclude individuals with market income less than one basic amount. We estimate the model separately by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. The age profiles are adjusted for education-specific calendar time effects.



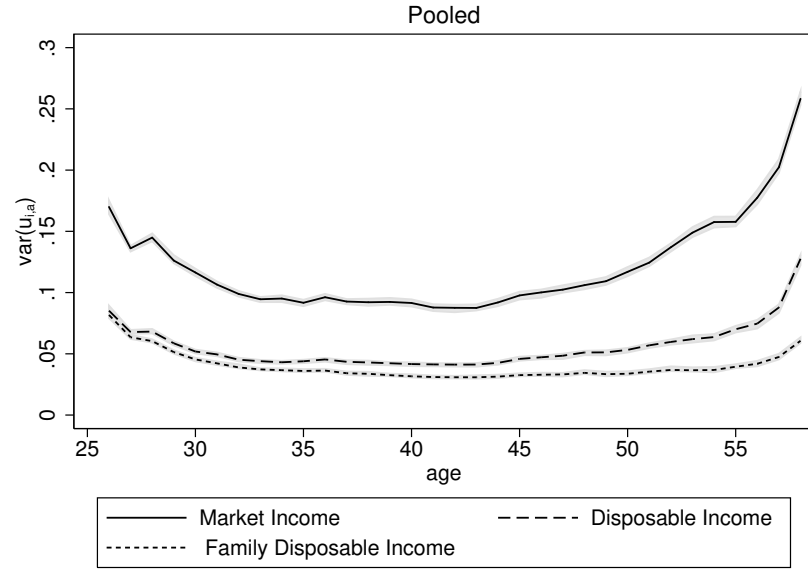


Figure B.10: Age profiles in the variances of permanent shocks in the pooled sample

**Notes:** This figure graphs the age profiles in the variances of permanent shocks. The age profiles are based on the model of income dynamics described in Section 3.1. We use the baseline sample, but do *not* estimate the model separately by educational levels. The age profiles are adjusted for calendar time effects. The 95 percent confidence interval is based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.

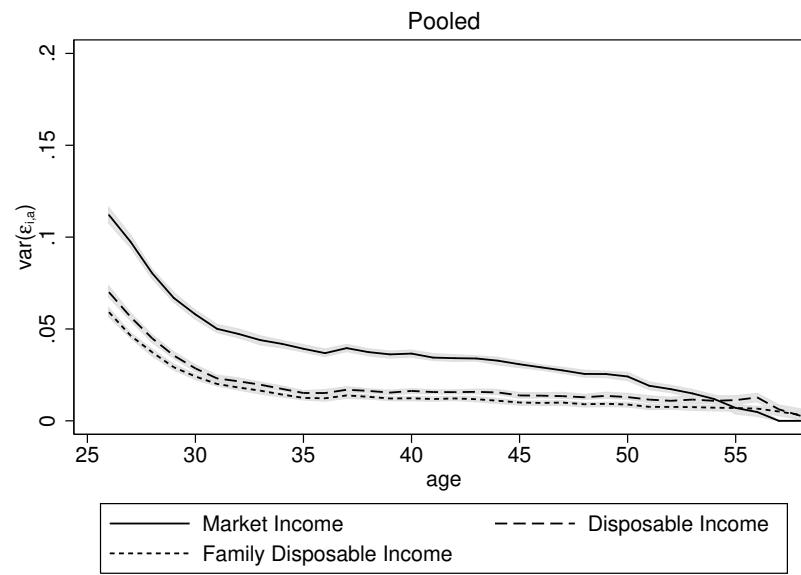


Figure B.11: Age profiles in the variances of transitory shocks in the pooled sample

**Notes:** This figure graphs the age profiles in the variances of transitory shocks. The age profiles are based on the model of income dynamics described in Section 3.1. We use the baseline sample, but do *not* estimate the model separately by educational levels. The age profiles are adjusted for calendar time effects. The 95 percent confidence interval is based on nonparametric bootstrap (of both estimation stages) with 70 bootstrap replications.

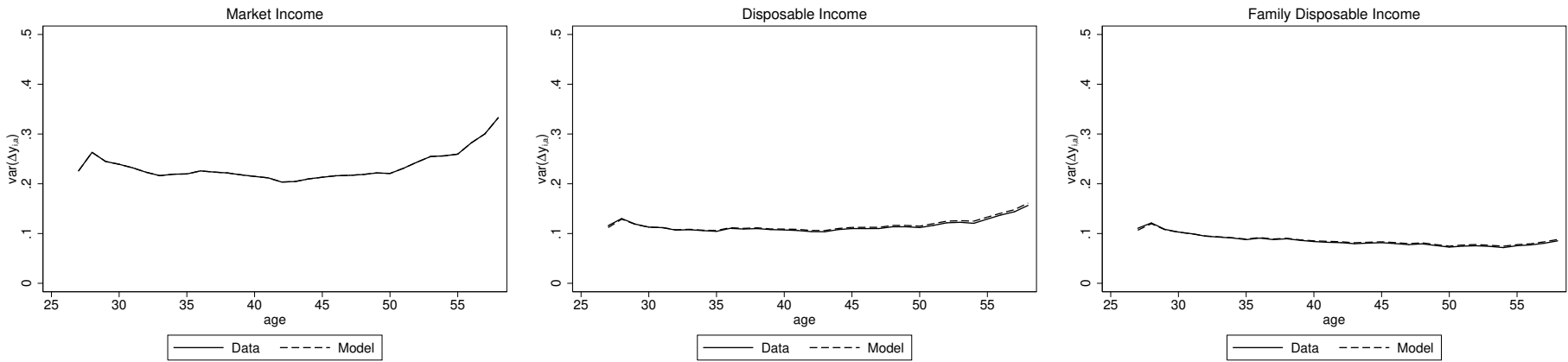


Figure B.12: Variance profiles of residual income growth rates (low-skilled)

**Notes:** This figure graphs the age profiles in the variance of the residual income growth rate for the low-skilled. The age profiles are based on the model of income dynamics described in Section 2.3.1 and the empirical counterpart is calculated using the baseline sample. Low skilled is defined as not having completed high school.

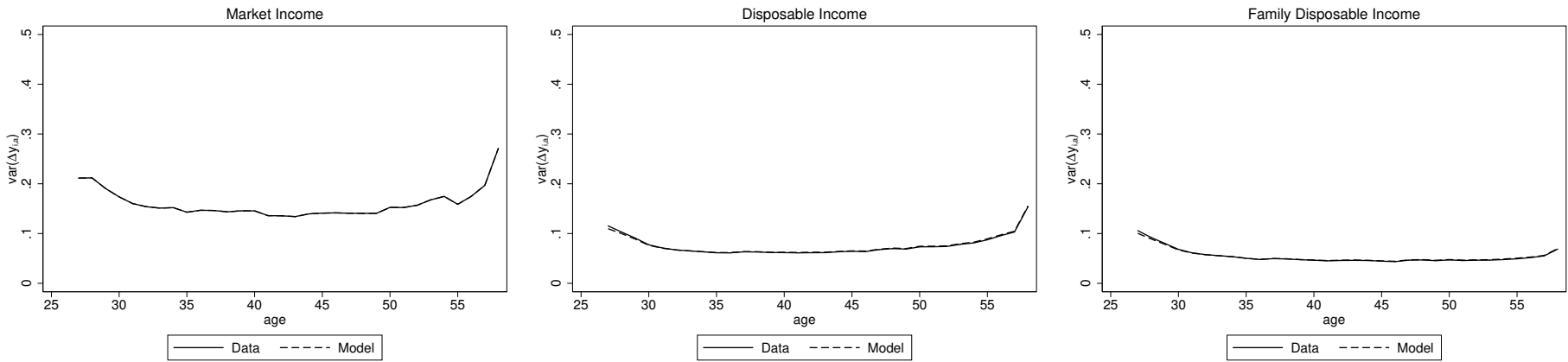


Figure B.13: Variance profiles of residual income growth rates (medium-skilled)

**Notes:** This figure graphs the age profiles in the variance of the residual income growth rate. The age profiles are based on the model of income dynamics described in Section 2.3.1 and the empirical counterpart is calculated using the baseline sample. Medium skilled includes individuals with a high school degree.

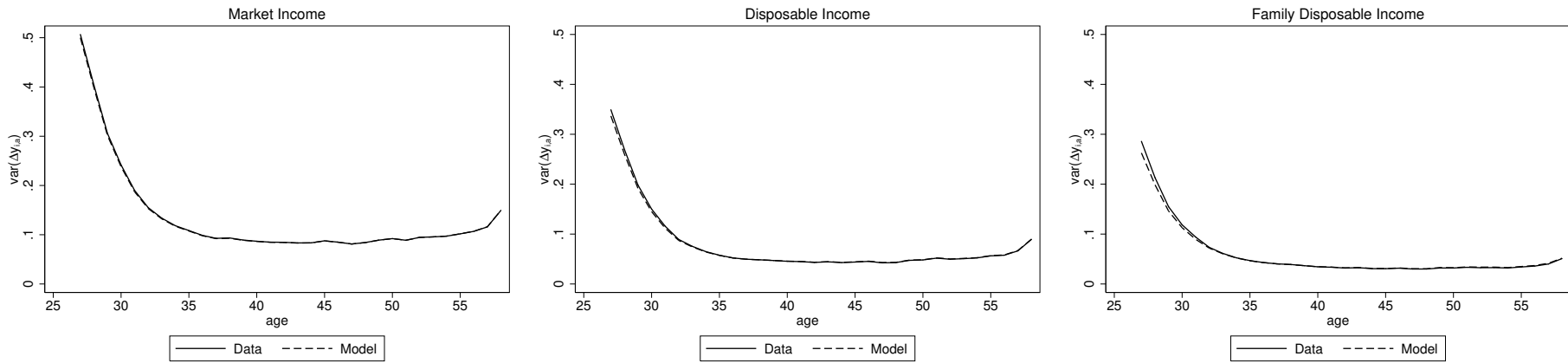


Figure B.14: Variance profiles of residual income growth rates (high-skilled)

**Notes:** This figure graphs the age profiles in the variance of the residual income growth rate. The age profiles are based on the model of income dynamics described in Section 2.3.1 and the empirical counterpart is calculated using the baseline sample. Medium skilled includes individuals with a high school degree.

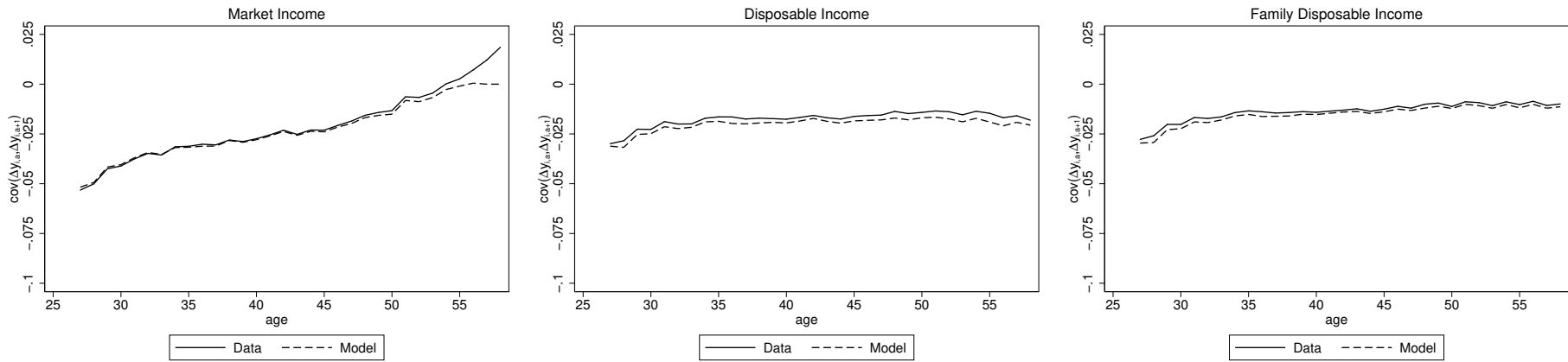


Figure B.15: Covariance profiles of residual income growth rates (low-skilled)

**Notes:** This figure graphs the age profiles in the covariance of the residual income growth rate at one lag. The age profiles are based on the model of income dynamics described in Section 2.3.1 and the empirical counterpart is calculated using the baseline sample. Low skilled is defined as not having completed high school. The age profiles are adjusted for education-specific calendar time effects.

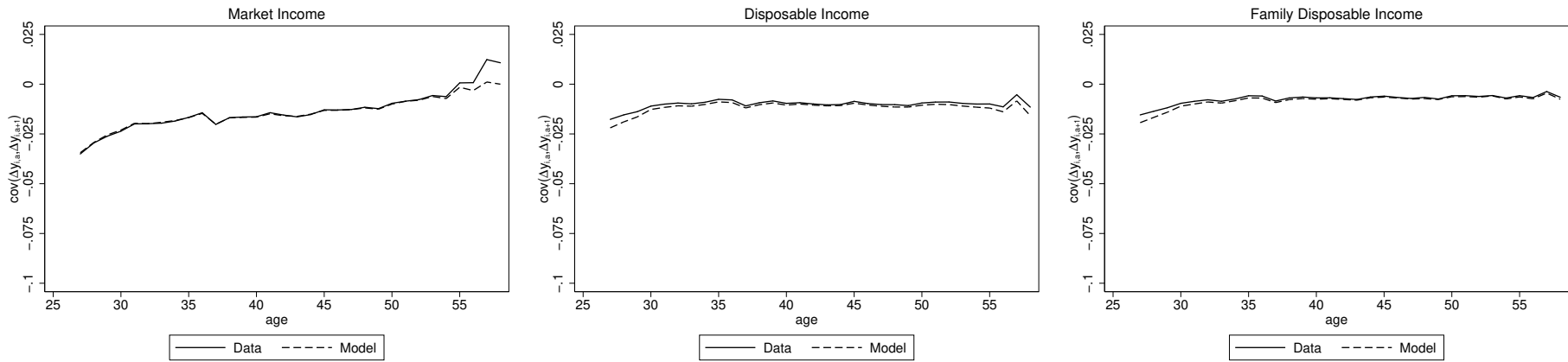


Figure B.16: Covariance profiles of residual income growth rates (medium-skilled)

**Notes:** This figure graphs the age profiles in the covariance of the residual income growth rate at one lag. The age profiles are based on the model of income dynamics described in Section 2.3.1 and the empirical counterpart is calculated using the baseline sample. Medium skilled includes individuals with a high school degree. The age profiles are adjusted for education-specific calendar time effects.

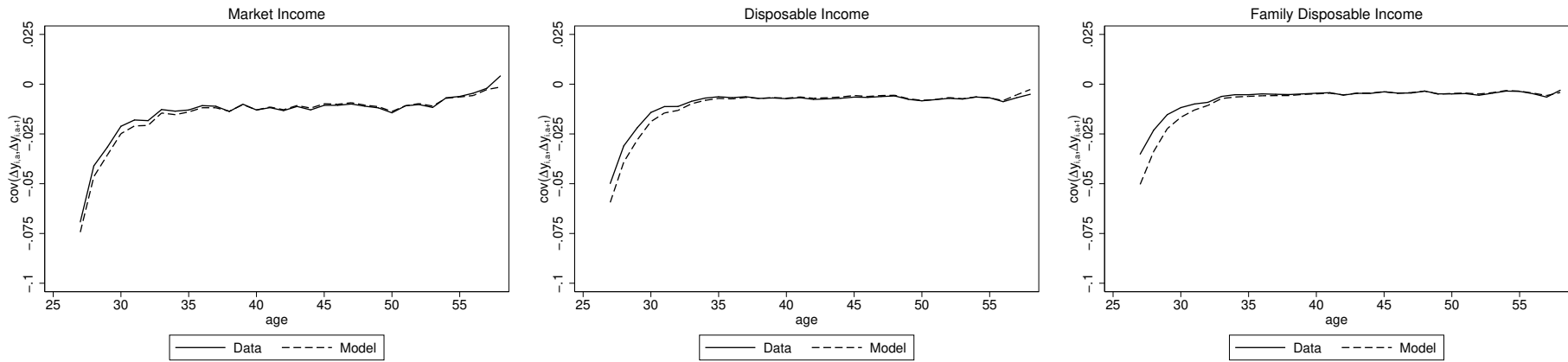


Figure B.17: Covariance profiles of residual income growth rates (high-skilled)

**Notes:** This figure graphs the age profiles in the covariance of the residual income growth rate at one lag. The age profiles are based on the model of income dynamics described in Section 2.3.1 and the empirical counterpart is calculated using the baseline sample. High skilled consists of individuals who have attended college. The age profiles are adjusted for education-specific calendar time effects.



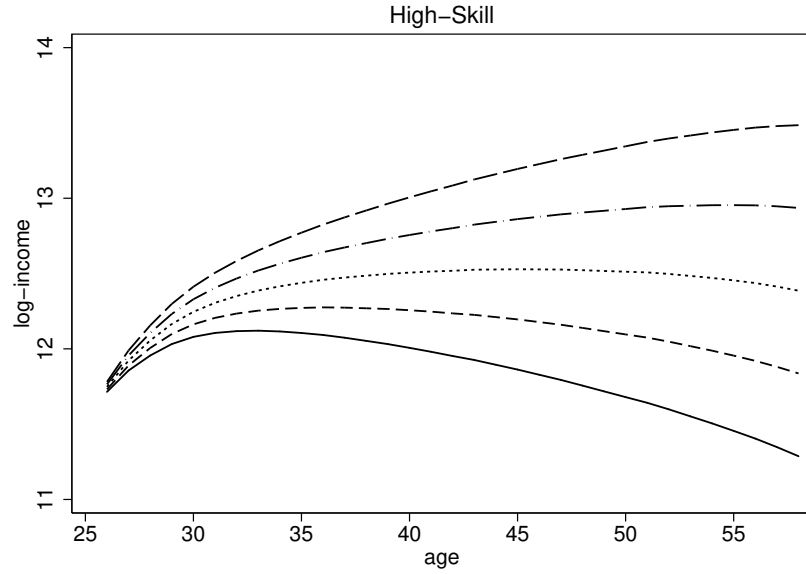


Figure B.18: Heterogenous income profiles in market income among the high-skilled

**Notes:** This figure graphs the heterogenous profiles in individual market income for the high skilled. High skilled consists of individuals who have attended college. The dotted line in the middle shows the age profile for the average growth rate ( $\beta = 0$ ). The lines above and below show age profiles for  $\beta = \pm\sqrt{\text{var}(\beta_i)}$  and  $\beta = \pm 2 \times \sqrt{\text{var}(\beta_i)}$ . The profiles are based on the model of income dynamics described in Section 3.1 with  $p_a = a - 25$ .

# Appendix C

## C.1 Additional Tables and Figures

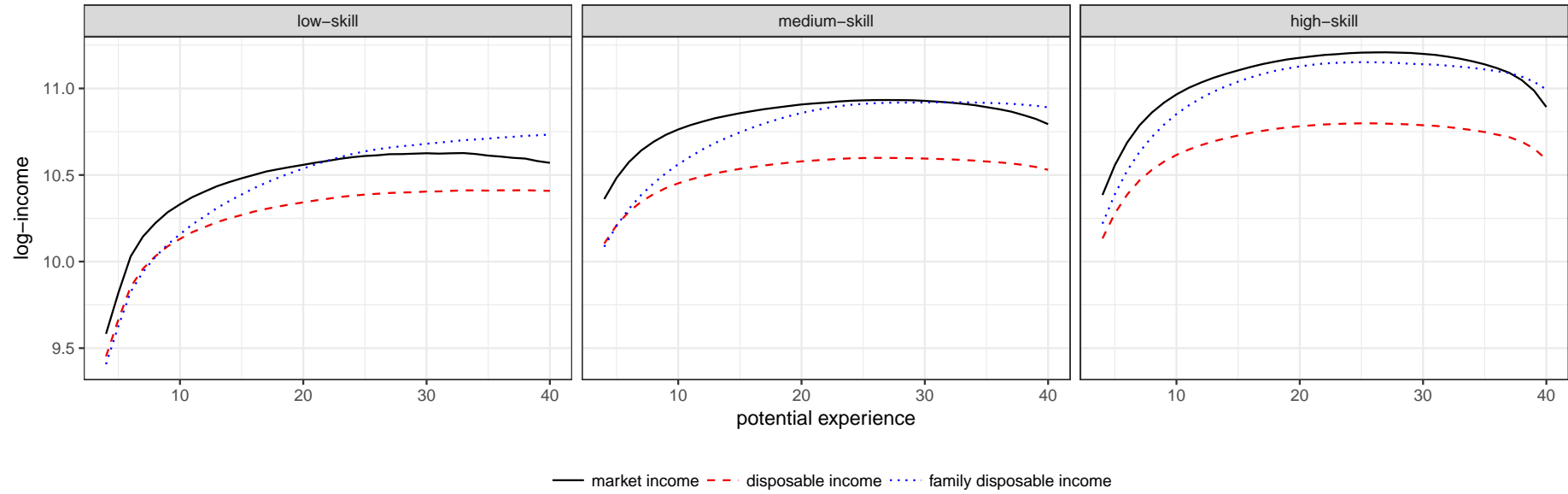


Figure C.1: Age profiles in the log of income

**Notes:** This figure shows the age profiles in the log of income by educational levels. The age profiles are adjusted for education-specific calendar time effects. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

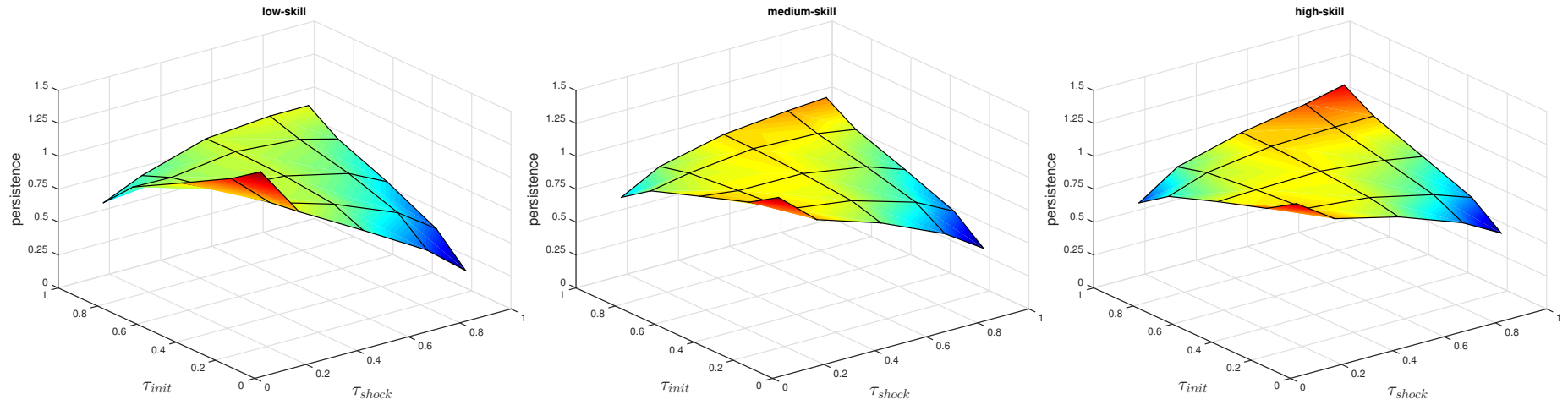


Figure C.2: Persistence in family disposable income

**Notes:** This figure shows the average persistence in family disposable income for households with 20 years of experience. Persistence is defined in (3.5) as the derivative of the conditional quantile function of  $y_{it}$  with respect to  $y_{i,t-1}$ . For each cohort and time period we evaluate the derivative at the rank of the income shock ( $\tau_{shock}$ ) and at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{init}$  percentile of the time and cohort specific distribution of  $y_{i,t-1}$ . Cohort and business cycle effects are taken out based on model (3.12). Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

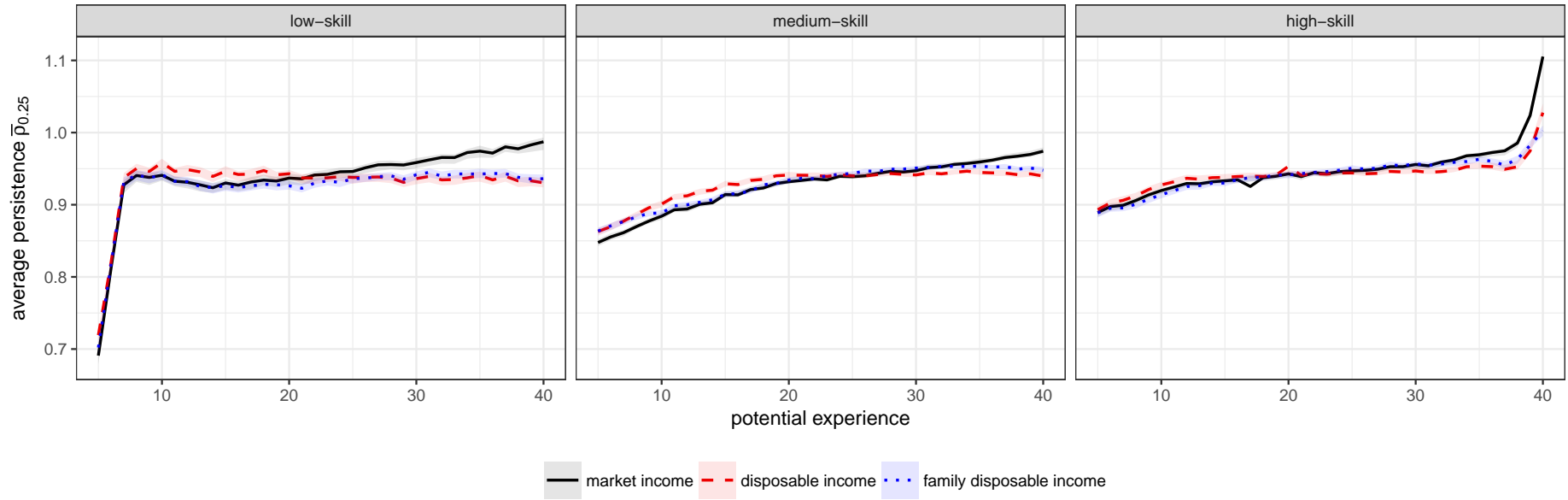


Figure C.3: Average persistence over the life cycle:  $\bar{\rho}(0.25)$

**Notes:** This figure shows the average persistence of a shock of rank 0.25, given by  $\bar{\rho}_t(0.25) = \mathbb{E} \frac{\partial Q_t(y_{i,t-1}, 0.25)}{\partial y_{i,t-1}}$ . Cohort and business cycle effects are taken out based on model (3.12). Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

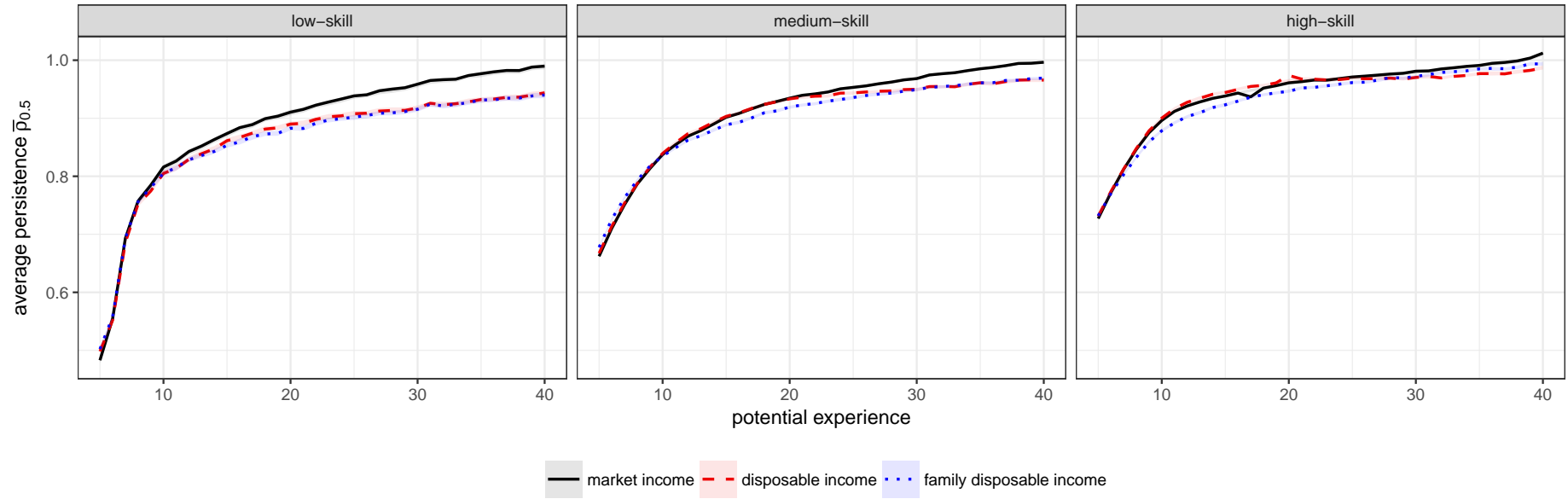


Figure C.4: Average persistence over the life cycle

**Notes:** This figure shows the average persistence of a shock of rank 0.5, given by  $\bar{\rho}_t(0.5) = \mathbb{E} \frac{\partial Q_t(y_{i,t-1}, 0.5)}{\partial y_{i,t-1}}$ . Cohort and business cycle effects are taken out based on model (3.12). Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

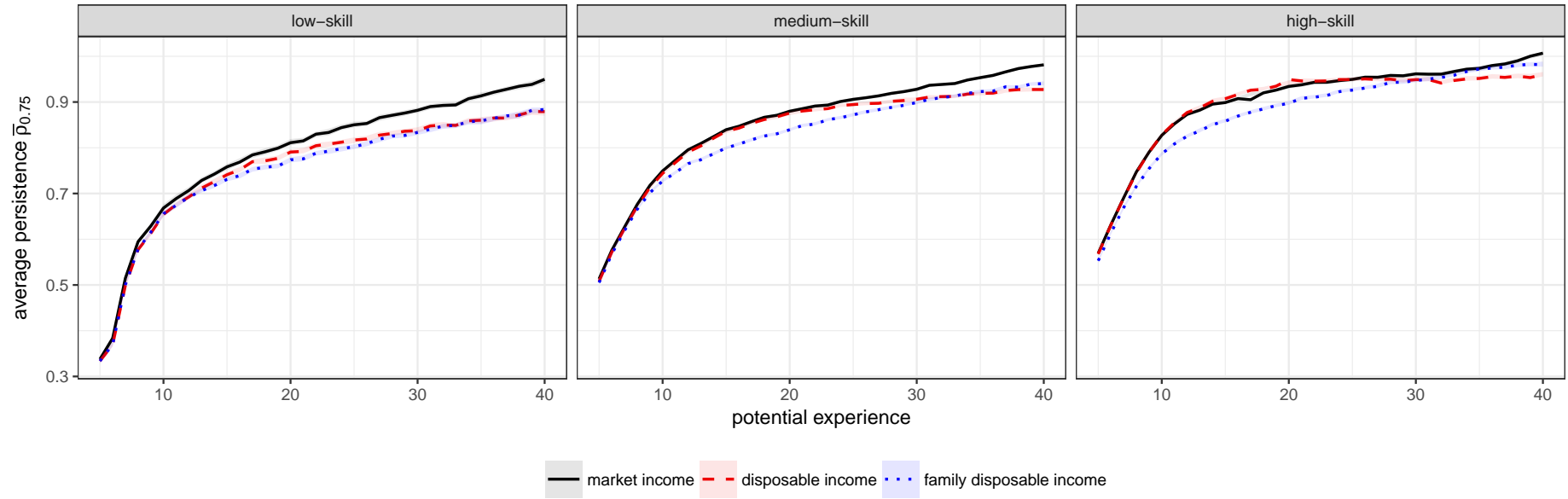


Figure C.5: Average persistence over the life cycle

**Notes:** This figure shows the average persistence of a shock of rank 0.5, given by  $\bar{\rho}_t(0.75) = \mathbb{E} \frac{\partial Q_t(y_{i,t-1}, 0.75)}{\partial y_{i,t-1}}$ . Cohort and business cycle effects are taken out based on model (3.12). Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

## Appendix C

	market income		disposable income		family disposable income	
unemp. rate, de-meaned	−0.046*** (0.000)		−0.041*** (0.000)		−0.034*** (0.000)	
$\mathbb{1}\{\text{exp.}=5\} \times \text{unemp. rate}$	−0.104*** (0.002)		−0.099*** (0.002)		−0.101*** (0.002)	
$\mathbb{1}\{\text{exp.}=10\} \times \text{unemp. rate}$	−0.052*** (0.001)		−0.043*** (0.001)		−0.036*** (0.001)	
$\mathbb{1}\{\text{exp.}=20\} \times \text{unemp. rate}$	−0.044*** (0.001)		−0.041*** (0.001)		−0.029*** (0.001)	
$\mathbb{1}\{\text{exp.}=30\} \times \text{unemp. rate}$	−0.023*** (0.001)		−0.023*** (0.001)		−0.017*** (0.001)	
$\mathbb{1}\{\text{exp.}=40\} \times \text{unemp. rate}$	−0.034*** (0.002)		−0.031*** (0.001)		−0.028*** (0.001)	
$R^2$	0.16	0.16	0.26	0.27	0.49	0.50
Num. Obs.	5,377,227	5,377,227	5,377,227	5,377,227	5,377,227	5,377,227

(a) low-skilled

	market income		disposable income		family disposable income	
unemp. rate, de-meaned	−0.036*** (0.000)		−0.026*** (0.000)		−0.019*** (0.000)	
$\mathbb{1}\{\text{exp.}=5\} \times \text{unemp. rate}$	−0.068*** (0.001)		−0.058*** (0.001)		−0.057*** (0.001)	
$\mathbb{1}\{\text{exp.}=10\} \times \text{unemp. rate}$	−0.039*** (0.001)		−0.028*** (0.000)		−0.016*** (0.000)	
$\mathbb{1}\{\text{exp.}=20\} \times \text{unemp. rate}$	−0.035*** (0.001)		−0.024*** (0.000)		−0.016*** (0.000)	
$\mathbb{1}\{\text{exp.}=30\} \times \text{unemp. rate}$	−0.022*** (0.001)		−0.014*** (0.000)		−0.010*** (0.000)	
$\mathbb{1}\{\text{exp.}=40\} \times \text{unemp. rate}$	−0.039*** (0.001)		−0.025*** (0.001)		−0.025*** (0.001)	
$R^2$	0.17	0.17	0.28	0.28	0.54	0.54
Num. Obs.	14,106,136	14,106,136	14,106,136	14,106,136	14,106,136	14,106,136

(b) medium-skilled

	market income		disposable income		family disposable income	
unemp. rate, de-meaned	−0.034*** (0.000)		−0.015*** (0.000)		−0.010*** (0.000)	
$\mathbb{1}\{\text{exp.}=5\} \times \text{unemp. rate}$	−0.046*** (0.001)		−0.037*** (0.001)		−0.034*** (0.001)	
$\mathbb{1}\{\text{exp.}=10\} \times \text{unemp. rate}$	−0.035*** (0.001)		−0.019*** (0.001)		−0.010*** (0.001)	
$\mathbb{1}\{\text{exp.}=20\} \times \text{unemp. rate}$	−0.034*** (0.001)		−0.012*** (0.001)		−0.006*** (0.000)	
$\mathbb{1}\{\text{exp.}=30\} \times \text{unemp. rate}$	−0.026*** (0.001)		−0.006*** (0.001)		−0.003*** (0.001)	
$\mathbb{1}\{\text{exp.}=40\} \times \text{unemp. rate}$	−0.012*** (0.003)		−0.002 (0.002)		−0.002 (0.002)	
$R^2$	0.22	0.22	0.30	0.30	0.55	0.55
Num. Obs.	7,789,349	7,789,349	7,789,349	7,789,349	7,789,349	7,789,349

(c) high-skilled

Table C.1: Aggregate cyclical income risk: Unemployment rate

**Notes:** This table presents coefficients of pooled OLS regressions. The results from specifications (3.2) and (3.3) are presented in column (1) and (2) respectively. The business cycle indicator is the demeaned unemployment rate. Other controls include potential experience fixed effects, cohort fixed effects, region fixed effects and marital status and dummies for the number of children in the household. Standard errors are presented in parentheses, and are clustered at the individual-level. \*\*\*, \*\*, and \*, represent statistical significance at 0.1%, 1%, and 5% levels, respectively.



## Appendix C

	market income		disposable income		family disposable income	
ln(real GDP, det.)	1.570*** (0.018)		1.228*** (0.014)		1.155*** (0.013)	
1{exp.=5}×ln(real GDP, det.)	3.854*** (0.143)		3.141*** (0.118)		3.153*** (0.118)	
1{exp.=10}×ln(real GDP, det.)	2.005*** (0.102)		1.419*** (0.081)		1.289*** (0.079)	
1{exp.=20}×ln(real GDP, det.)	1.335*** (0.101)		1.011*** (0.077)		1.024*** (0.072)	
1{exp.=30}×ln(real GDP, det.)	1.133*** (0.089)		0.844*** (0.062)		0.757*** (0.055)	
1{exp.=40}×ln(real GDP, det.)	1.208*** (0.184)		1.158*** (0.098)		1.014*** (0.087)	
$R^2$	0.58	0.58	0.56	0.57	0.71	0.71
Household FE?	yes	yes	yes	yes	yes	yes
Num. Obs.	5,377,227	5,377,227	5,377,227	5,377,227	5,377,227	5,377,227

(a) low-skilled

	market income		disposable income		family disposable income	
ln(real GDP, det.)	1.179*** (0.009)		0.919*** (0.007)		0.839*** (0.006)	
1{exp.=5}×ln(real GDP, det.)	2.421*** (0.073)		1.821*** (0.058)		1.794*** (0.057)	
1{exp.=10}×ln(real GDP, det.)	1.282*** (0.053)		0.921*** (0.039)		0.837*** (0.038)	
1{exp.=20}×ln(real GDP, det.)	1.285*** (0.045)		0.921*** (0.033)		0.818*** (0.030)	
1{exp.=30}×ln(real GDP, det.)	0.783*** (0.053)		0.662*** (0.038)		0.602*** (0.033)	
1{exp.=40}×ln(real GDP, det.)	0.914*** (0.099)		0.865*** (0.060)		0.776*** (0.049)	
$R^2$	0.55	0.55	0.60	0.60	0.75	0.75
Household FE?	yes	yes	yes	yes	yes	yes
Num. Obs.	14,106,136	14,106,136	14,106,136	14,106,136	14,106,136	14,106,136

(b) medium-skilled

	market income		disposable income		family disposable income	
ln(real GDP, det.)	0.832*** (0.012)		0.556*** (0.010)		0.516*** (0.009)	
1{exp.=5}×ln(real GDP, det.)	1.984*** (0.096)		1.506*** (0.079)		1.383*** (0.076)	
1{exp.=10}×ln(real GDP, det.)	0.881*** (0.059)		0.551*** (0.047)		0.451*** (0.044)	
1{exp.=20}×ln(real GDP, det.)	0.863*** (0.057)		0.489*** (0.047)		0.442*** (0.041)	
1{exp.=30}×ln(real GDP, det.)	0.415*** (0.069)		0.346*** (0.055)		0.295*** (0.047)	
1{exp.=40}×ln(real GDP, det.)	-0.292 (0.223)		-0.408* (0.180)		-0.091 (0.132)	
$R^2$	0.61	0.61	0.64	0.64	0.77	0.77
Household FE?	yes	yes	yes	yes	yes	yes
Num. Obs.	7,789,349	7,789,349	7,789,349	7,789,349	7,789,349	7,789,349

(c) high-skilled

Table C.2: Aggregate cyclical income risk: Household fixed effects

**Notes:** This table presents coefficients of household fixed effects regressions. The business cycle indicator is the HP-filtered natural logarithm of real GDP of mainland Norway. Other controls include potential experience fixed effects, region fixed effects, marital status and dummies for the number of children in the household. Standard errors are presented in parentheses, and are clustered at the individual-level. \*\*\*, \*\*, and \*, represent statistical significance at 0.1%, 1%, and 5% levels, respectively.

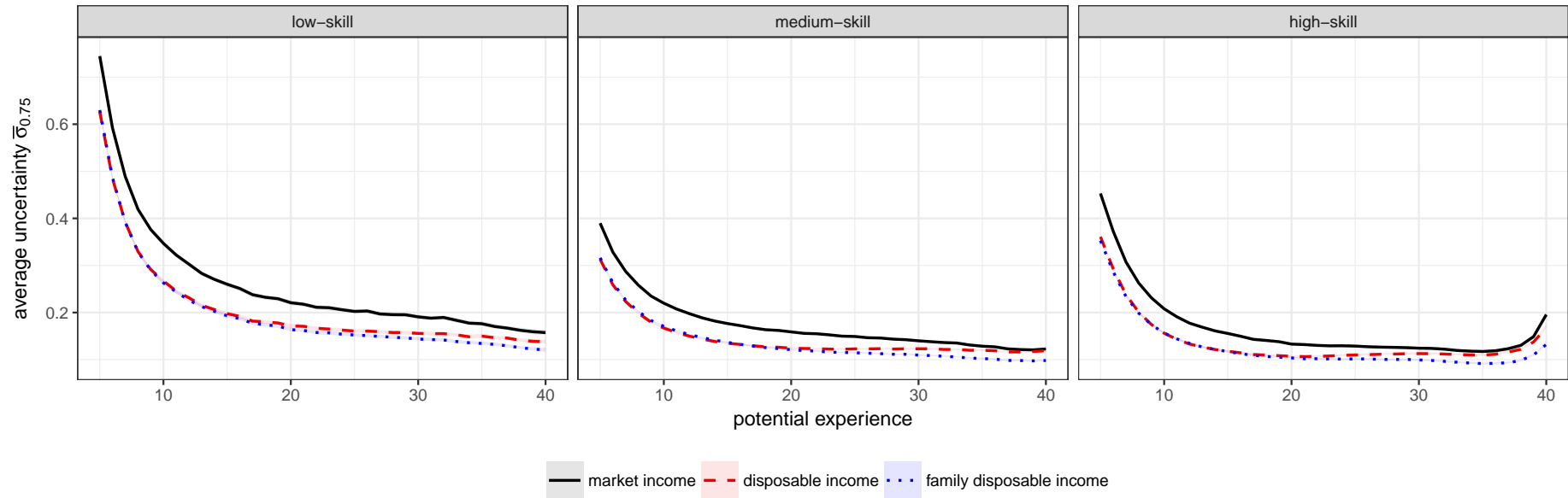


Figure C.6: Average dispersion over the life cycle

**Notes:** This figure shows the life cycle profile of the average dispersion defined in (3.7) with  $\tau = 0.75$ . The life cycle profiles are based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

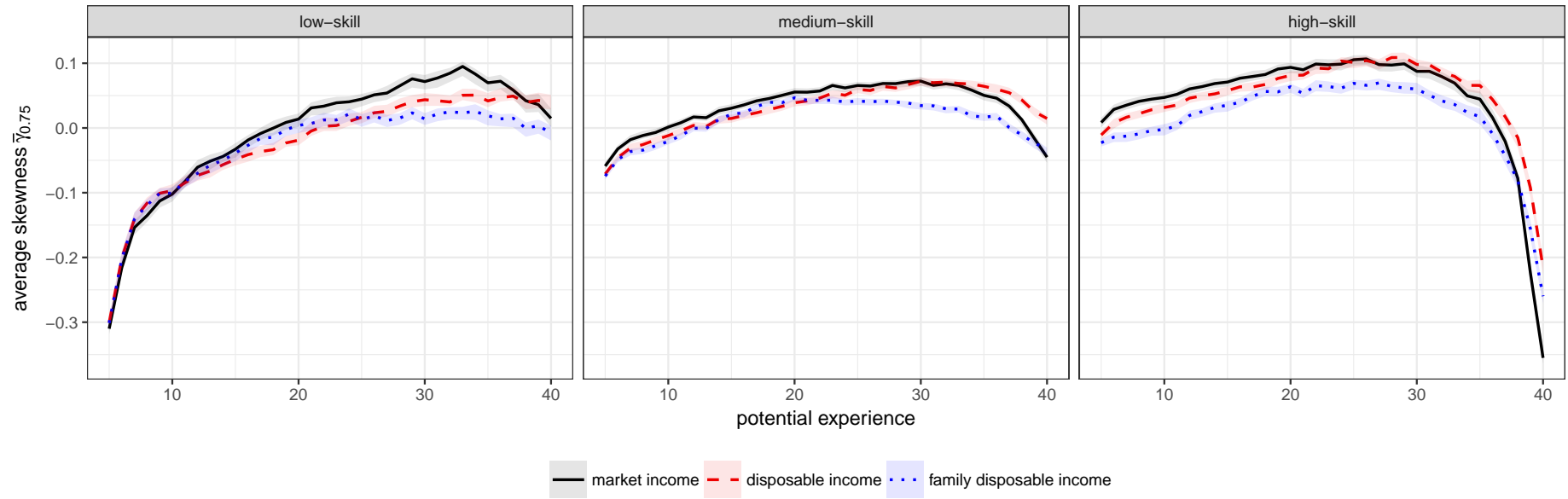
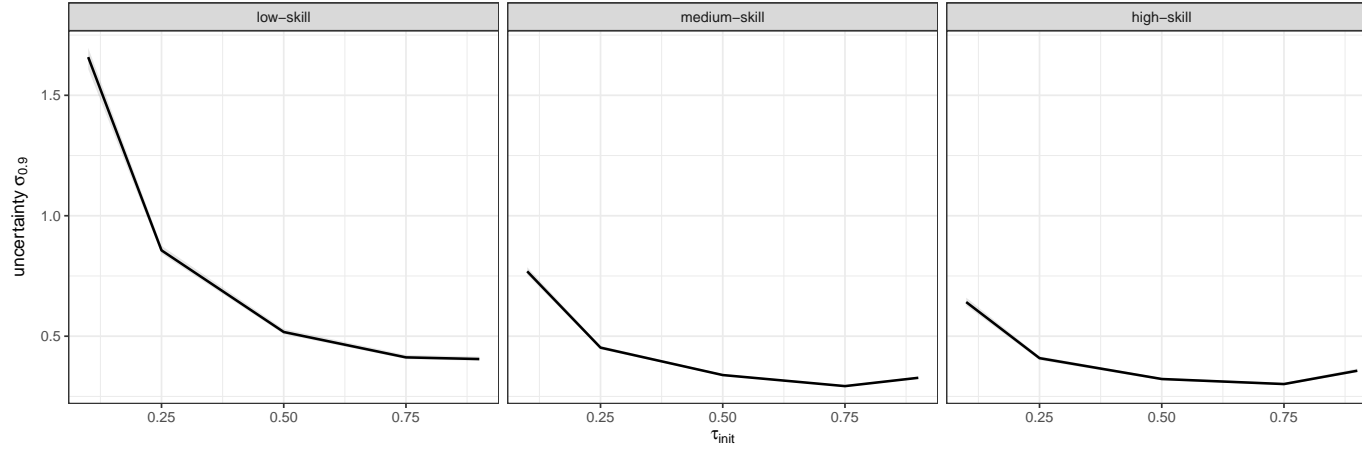
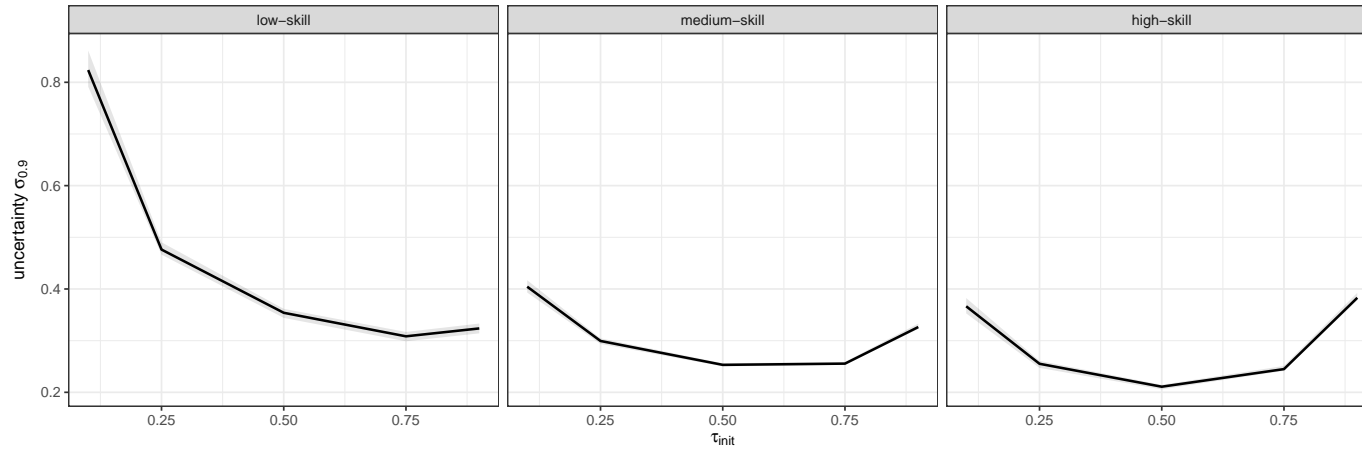


Figure C.7: Average skewness over the life cycle

**Notes:** This figure shows the life cycle profile of the average Bowley skewness defined in (3.9) with  $\tau = 0.75$ . The life cycle profiles are based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



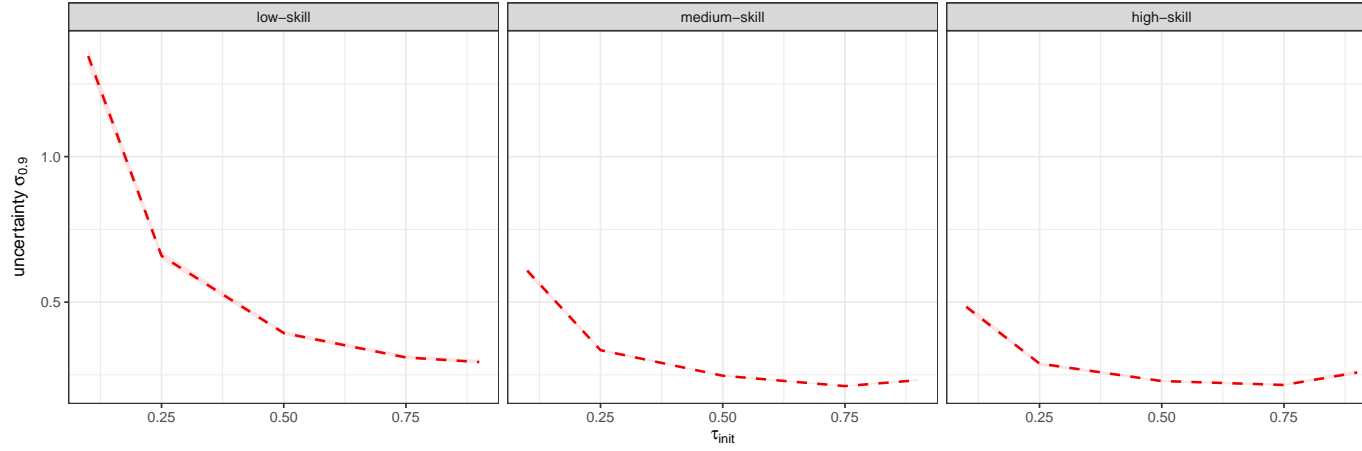
(a) 10 years of potential experience



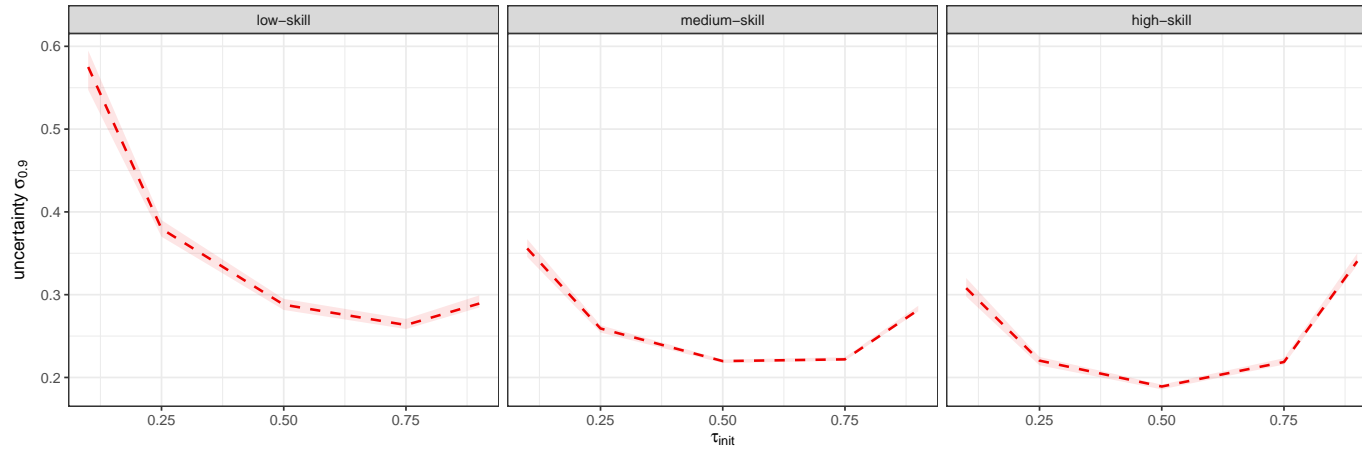
(b) 30 years of potential experience

Figure C.8: Conditional dispersion in market income

**Notes:** This figure shows the conditional dispersion in market income, defined in (3.6) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{\text{init}}$  percentile of the distribution of  $y_{i,t-1}$ ; 10 and 30 years of potential experience. The estimates are net of cohort and business cycle effects based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



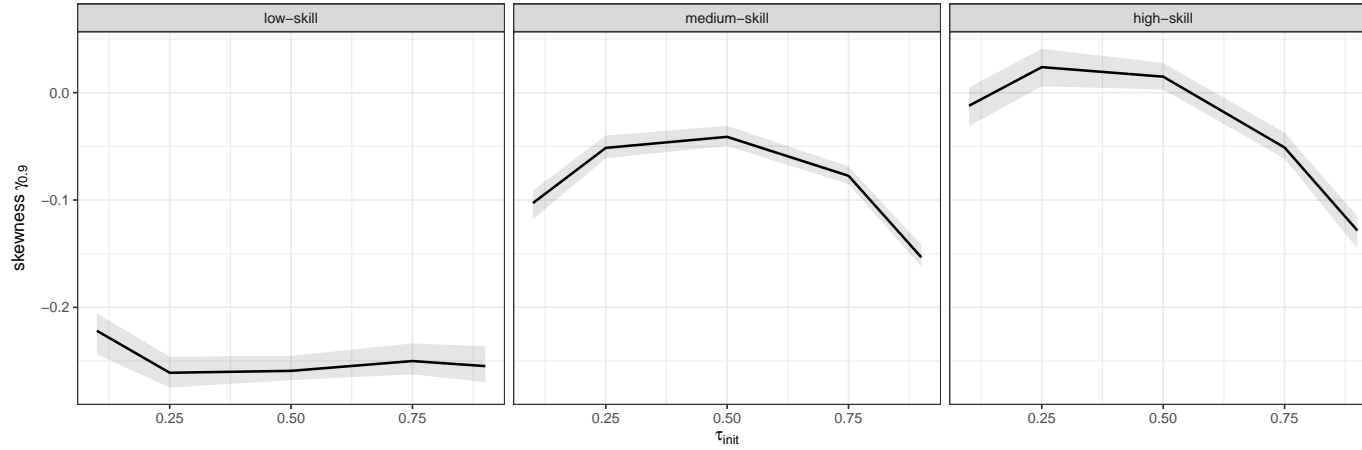
(a) 10 years of potential experience



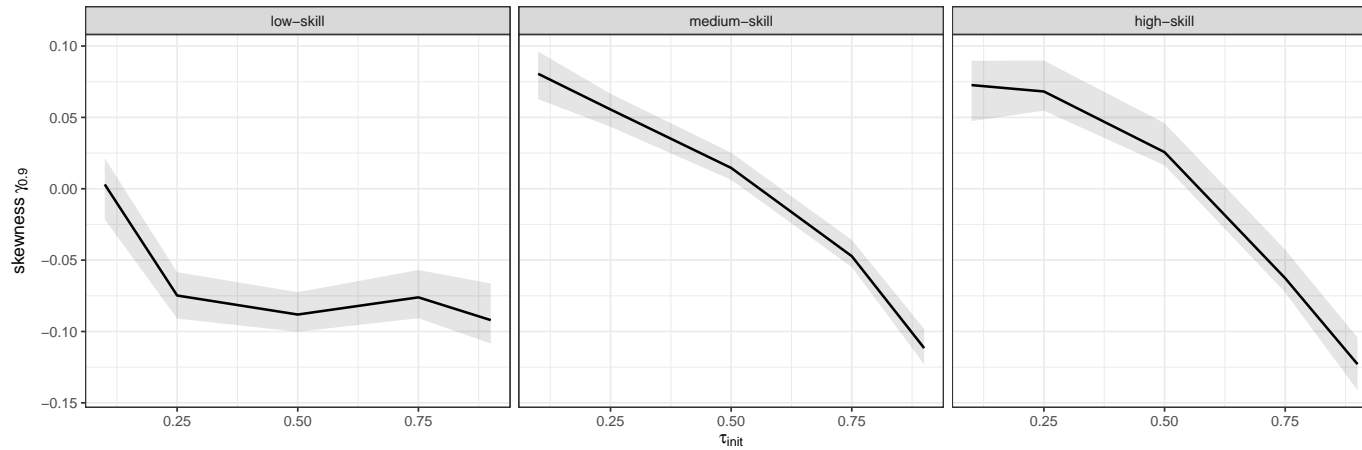
(b) 30 years of potential experience

Figure C.9: Conditional dispersion in disposable income

**Notes:** This figure shows the conditional dispersion in disposable income, defined in (3.6) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{\text{init}}$  percentile of the distribution of  $y_{i,t-1}$ ; 10 and 30 years of potential experience. The estimates are net of cohort and business cycle effects based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



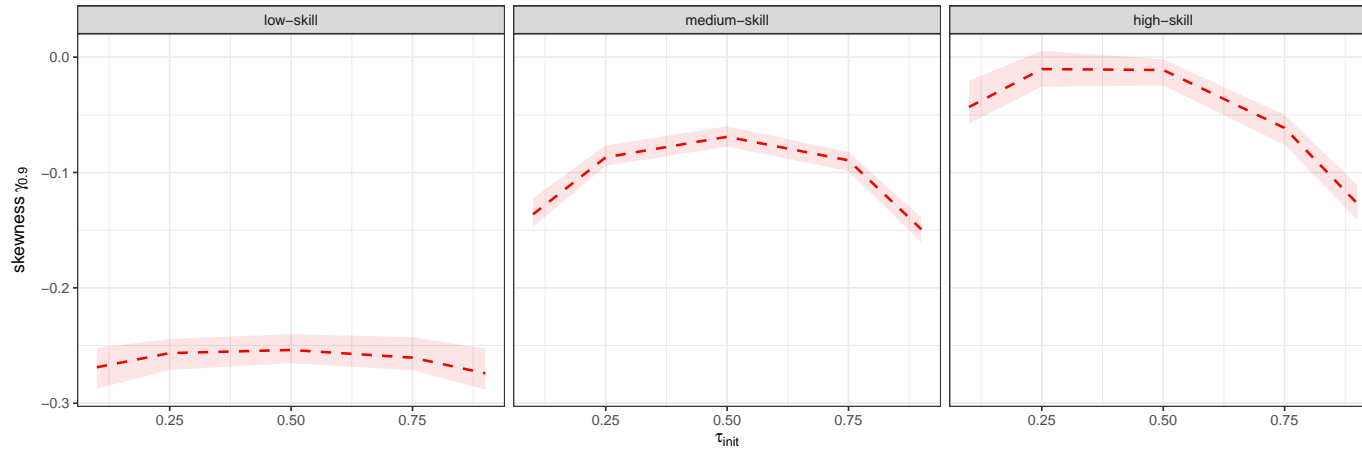
(a) 10 years of potential experience



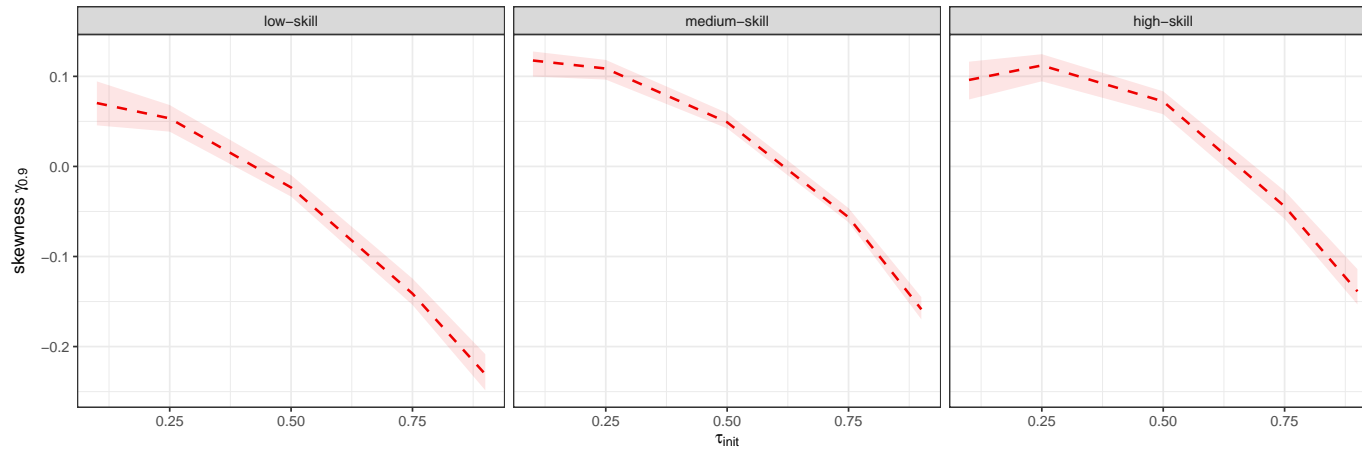
(b) 30 years of potential experience

Figure C.10: Conditional skewness in market income

**Notes:** This figure shows the conditional skewness in market income, defined in (3.8) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{\text{init}}$  percentile of the distribution of  $y_{i,t-1}$ ; 10 and 30 years of potential experience. The estimates are net of cohort and business cycle effects based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



(a) 10 years of potential experience



(b) 30 years of potential experience

Figure C.11: Conditional skewness in disposable income

**Notes:** This figure shows the conditional skewness in disposable income, defined in (3.8) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{init}$  percentile of the distribution of  $y_{i,t-1}$ ; 10 and 30 years of potential experience. The estimates are net of cohort and business cycle effects based on model (3.12). Shaded areas represent 95% point-wise confidence intervals based on non-parametric bootstrap of the quantile auto-regressions (3.11), clustered at the household level, with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

## Appendix C

$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \text{unemp. rate}$	-0.0258*	0.0020*	0.0105*	-0.0227*	0.0011*	0.0071*	-0.0233*	0.0004	0.0071*
standardised	(0.0020)	(0.0006)	(0.0006)	(0.0014)	(0.0004)	(0.0005)	(0.0013)	(0.0005)	(0.0005)

(a) low-skilled

$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \text{unemp. rate}$	-0.0035*	0.0023*	0.0037*	-0.0050*	0.0018	0.0017	-0.0078	0.0008	0.0024
standardised	(0.0007)	(0.0002)	(0.0003)	(0.0006)	(0.0001)	(0.0002)	(0.0005)	(0.0001)	(0.0002)

(b) medium-skilled

$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \text{unemp. rate}$	-0.0049*	-0.0007*	-0.0021*	-0.0060*	-0.0002*	-0.0010*	-0.0081*	-0.0009*	-0.0002
standardised	(0.0005)	(0.0001)	(0.0002)	(0.0008)	(0.0002)	(0.0004)	(0.0007)	(0.0002)	(0.0004)

(c) high-skilled

Table C.3: Cyclicalitity of conditional dispersion

**Notes:** This table presents the results of model (3.12). Cyclicalitity is captured by the coefficient on the change in the unemployment rate, normalised to have mean zero and unit standard deviation. The dependent variables are the cohort and time specific measures of conditional dispersion, defined in (3.6) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{\text{init}}$  percentile of the distribution of  $y_{i,t-1}$ ; Bootstrap standard errors are presented in parentheses. \* represents statistical significance at 5% level.

$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \text{unemp. rate}$	-0.0213*	-0.0387*	-0.0346*	-0.0132*	-0.0416*	-0.0282*	-0.0146*	-0.0393*	-0.0278*
standardised	(0.0014)	(0.0010)	(0.0014)	(0.0013)	(0.0009)	(0.0014)	(0.0013)	(0.0010)	(0.0012)

(a) low-skilled

$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \text{unemp. rate}$	-0.0312*	-0.0441*	-0.0251*	-0.0254*	-0.0397*	-0.0184*	-0.0186*	-0.0322*	-0.0172*
standardised	(0.0010)	(0.0007)	(0.0009)	(0.0009)	(0.0006)	(0.0009)	(0.0009)	(0.0006)	(0.0009)

(b) medium-skilled

$\tau_{\text{init}}$	market income			disposable income			family disposable income		
	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$\Delta \text{unemp. rate}$	-0.0217*	-0.0284*	-0.0164*	-0.0176*	-0.0256*	-0.0141*	-0.0131*	-0.0186*	-0.0138*
standardised	(0.0018)	(0.0010)	(0.0014)	(0.0018)	(0.0011)	(0.0013)	(0.0015)	(0.0009)	(0.0014)

(c) high-skilled

Table C.4: Cyclicalitity of conditional skewness

**Notes:** This table presents the results of model (3.12). Cyclicalitity is captured by the coefficient on the change in HP-filtered log real GDP of mainland Norway, normalised to have mean zero and unit standard deviation. The dependent variables are the cohort and time specific measures of conditional skewness, defined in (3.6) with  $\tau = 0.9$  and evaluated at a value of  $y_{i,t-1}$  that corresponds to the  $\tau_{\text{init}}$  percentile of the distribution of  $y_{i,t-1}$ ; Bootstrap standard errors are presented in parentheses. \* represents statistical significance at 5% level.



# Appendix D

## D.1 Derivation of the Hamilton-Jacobi-Bellman Equation

We first derive the Bellman equation in discrete time for a small time interval  $\Delta t$  and then take the limit  $\lim_{\Delta t \rightarrow 0}$  to derive the continuous time HJB equation. Alternatively, the continuous time Hamilton-Jacobi-Bellman equation can be directly derived applying the methods presented in Wälde (2011).

**Discrete time representation of (4.1).** First consider the random walk representation of the human capital accumulation equation (4.1):

$$\Delta h_{1t} = \begin{cases} \Delta x & \text{with probability } p \\ -\Delta x & \text{with probability } 1 - p \end{cases}.$$

with

$$\mathbb{E}(\Delta h_{1t}) = \mu(\mathbf{h}, y)\Delta t; \quad \text{var}(\Delta h_{1t}) = \sigma(\mathbf{h})^2\Delta t.$$

It follows that

$$\begin{aligned} (2p - 1)\Delta x &= \mu(\mathbf{h}, y)\Delta t \\ 2p(1 - p)\Delta x &= \sigma(\mathbf{h})^2\Delta t. \end{aligned}$$

Taking into account that  $\Delta x\Delta t = \Delta t^2 = 0$  for small  $\Delta t$ , we therefore get

$$\Delta h_{1t} = \begin{cases} \sigma(\mathbf{h})\sqrt{\Delta t} & \text{with probability } \frac{1}{2} \left[ 1 + \frac{\mu(\mathbf{h}, y)}{\sigma(\mathbf{h})} \sqrt{\Delta t} \right] \\ -\sigma(\mathbf{h})\sqrt{\Delta t} & \text{with probability } \frac{1}{2} \left[ 1 - \frac{\mu(\mathbf{h}, y)}{\sigma(\mathbf{h})} \sqrt{\Delta t} \right] \end{cases}. \quad (\text{D.1})$$

**HJB equation.** Consider the Bellman equation in discrete time,

$$W(a, \mathbf{h}, y, \theta) = \max_{a - \underline{a} \geq c \geq 0} \left\{ u(c)\Delta t + \frac{1}{1 + \rho\Delta t} \mathbb{E}W(a', \mathbf{h}', y', \theta') \right\}, \quad (\text{D.2})$$

Using a Taylor expansion, the expected continuation value can be written as:

$$\begin{aligned} \mathbb{E}W(a', \mathbf{h}', y', \theta') &= \frac{\partial}{\partial a}W(a, \mathbf{h}, y, \theta)[ra + \theta f(\mathbf{h}, y) - c]\Delta t + \mu(\mathbf{h}, y)\Delta t \frac{\partial}{\partial h_1}W(a, \mathbf{h}, y, \theta) \\ &+ \frac{\sigma(\mathbf{h})^2}{2}\Delta t \frac{\partial^2}{\partial h_1^2}W(a, \mathbf{h}, y, \theta) + [1 - \lambda(h_0)\Delta t - \delta(h_0)\Delta t - \xi\Delta t]W(a, \mathbf{h}, y, \theta) + \delta(h_0)\Delta t W(a, \mathbf{h}, \underline{y}, 1) \\ &+ \xi\Delta t R(a, \mathbf{h}) + \lambda(h_0)\Delta t \int \max\{W(a, \mathbf{h}, y', \frac{y}{y'}), W(a, \mathbf{h}, y, \max\{\theta, \frac{y'}{y}\})\}dF(y') + o(\Delta t). \end{aligned}$$

Substituting into the Bellman equation and multiplying with  $[1 + \rho\Delta t]$  gives after rearranging,

$$\begin{aligned} [\rho + \lambda(h_0) + \delta(h_0) + \xi]W(a, \mathbf{h}, y, \theta)\Delta t &= \max_{a - \underline{a} \geq c \geq 0} \left\{ [1 + \rho\Delta t]u(c)\Delta t \right. \\ &+ \frac{\partial}{\partial a}W(a, \mathbf{h}, y, \theta)[ra + \theta f(\mathbf{h}, y) - c]\Delta t + \mu(\mathbf{h}, y)h_1\Delta t \frac{\partial}{\partial h_1}W(a, \mathbf{h}, y, \theta) \\ &+ \frac{\sigma(\mathbf{h})^2}{2}\Delta t \frac{\partial^2}{\partial h_1^2}W(a, \mathbf{h}, y, \theta) + \delta(h_0)\Delta t W(a, \mathbf{h}, \underline{y}, 1) + \xi\Delta t R(a) \\ &\left. + \lambda(h_0)\Delta t \int \max\{W(a, \mathbf{h}, y', \frac{y}{y'}), W(a, \mathbf{h}, y, \max\{\theta, \frac{y'}{y}\})\}dF(y') + o(\Delta t) \right\}. \end{aligned}$$

Dividing by  $\Delta t$  and taking the limit  $\lim_{\Delta t \rightarrow 0}$  yields the continuous time HJB (4.2).

**Reflecting barriers.** When the process for human capital  $h_1$  reaches the upper (lower) barrier, then the process moves down (up) with probability 1 by assumption. Thus, when the process is at the upper barrier  $\bar{h}_1$ , the expected continuation value becomes

$$\begin{aligned} \mathbb{E}W(a', \mathbf{h}', y', \theta') &= \frac{\partial}{\partial a}W(a, \bar{\mathbf{h}}, y, \theta)[ra + \theta f(\bar{\mathbf{h}}, y) - c]\Delta t - \sigma(\bar{\mathbf{h}})\sqrt{\Delta t} \frac{\partial}{\partial h_1}W(a, \bar{\mathbf{h}}, y, \theta) \\ &+ \frac{\sigma(\bar{\mathbf{h}})^2}{2}\Delta t \frac{\partial^2}{\partial h_1^2}W(a, \bar{\mathbf{h}}, y, \theta) + [1 - \lambda(h_0)\Delta t - \delta(h_0)\Delta t - \xi\Delta t]W(a, \bar{\mathbf{h}}, y, \theta) + \delta(h_0)\Delta t W(a, \bar{\mathbf{h}}, \underline{y}, 1) \\ &+ \xi\Delta t R(a, \bar{\mathbf{h}}) + \lambda(h_0)\Delta t \int \max\{W(a, \bar{\mathbf{h}}, y', \frac{y}{y'}), W(a, \bar{\mathbf{h}}, y, \max\{\theta, \frac{y'}{y}\})\}dF(y') + o(\Delta t). \end{aligned}$$

Substitute this expression into the Bellman equation (D.2) and multiply with  $\frac{[1 + \rho\Delta t]}{\sqrt{\Delta t}}$ . Taking the limit  $\lim_{\Delta t \rightarrow 0}$  yields the boundary condition

$$\frac{\partial}{\partial h_1}W(a, \bar{\mathbf{h}}, y, \theta) = 0.$$

Similarly, the lower barrier  $\underline{h}_1$  implies the boundary condition

$$\frac{\partial}{\partial h_1}W(a, \underline{\mathbf{h}}, y, \theta) = 0.$$

## D.2 Parameters for the Simulation

Parameter	Value	Parameter	Value
interest rate $r$ , annual	0.03	job contact rate $\lambda_0$	-1.8
time-preference rate $\rho$ , annual	0.05	job contact rate $\lambda_1$	0.05
coefficient of relative risk aversion $\gamma$	2	job destruction rate $\delta_0$	-3.9
retirement rate $\xi$	0.0021	job destruction rate $\delta_1$	-0.05
lower bound firm productivity $\underline{y}$	1	human capital accumulation $\mu_0$	0.001
upper bound firm productivity $\bar{y}$	2	human capital accumulation $\mu_1$	0.0001
shape parameter $\alpha$	2	human capital accumulation $\mu_2$	0.0001
		diffusion parameter $\sigma$	0.0125

Table D.1: Parameters for the Simulation

# Bibliography

- AASTVEIT, K. A., A. S. JORE, AND F. RAVAZZOLO (2016): “Identification and real-time forecasting of Norwegian business cycles,” *International Journal of Forecasting*, 32(2), 283–292.
- ABOWD, J. M., AND D. CARD (1989): “On the Covariance Structure of Earnings and Hours Changes,” *Econometrica*, 57(2), 411–45.
- ABRAHAM, K. G., AND H. S. FARBER (1987): “Job Duration, Seniority, and Earnings,” *The American Economic Review*, 77(3), 278–297.
- ACHDOU, Y., J. HAN, J.-M. LASRY, AND B. MOLL (2015): “Heterogeneous Agent Models in Continuous Time,” unpublished manuscript.
- ALTONJI, J. G., AND R. A. SHAKOTKO (1987): “Do Wages Rise with Job Seniority?,” *Review of Economic Studies*, 54(3), 437–59.
- ALTONJI, J. G., A. SMITH, AND I. VIDANGOS (2013): “Modeling Earnings Dynamics,” *Econometrica*, 81, 1395–1454.
- ANDOLFATTO, D. (1996): “Business Cycles and Labor-Market Search,” *American Economic Review*, pp. 112–132.
- ARELLANO, M., R. BLUNDELL, AND S. BONHOMME (2016): “Earnings and consumption dynamics: a nonlinear panel data framework,” *forthcoming: Econometrica*.
- ATKINSON, A., L. RAINWATER, AND T. SMEEDING (1995): “Income Distribution in OECD Countries: Evidence from the Luxembourg Income Study,” Report, OECD.
- BAGGER, J., F. FONTAINE, F. POSTEL-VINAY, AND J.-M. ROBIN (2014): “Tenure, Experience, Human Capital, and Wages: A Tractable Equilibrium Search Model of Wage Dynamics,” *American Economic Review*, 104(6), 1551–96.
- BAGGER, J., AND R. LENTZ (2014): “An empirical model of wage dispersion with sorting,” Discussion paper, National Bureau of Economic Research WP 20031.
- BAKER, M., AND G. SOLON (2003): “Earnings Dynamics and Inequality among Canadian Men, 1976-1992: Evidence from Longitudinal Income Tax Records,” *Journal of Labor Economics*, 21, 267–288.

- BARLEVY, G. (2008): “Identification of Search Models using Record Statistics,” *Review of Economic Studies*, 75(1), 29–64.
- BAYER, C., R. LUETTICKE, L. PHAM-DAO, AND V. TJADEN (2017): “Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk,” *Working Paper*.
- BENMELECH, E., N. K. BERGMAN, AND A. SERU (2011): “Financing Labor,” Discussion paper, National Bureau of Economic Research.
- BIKKER, J. A., AND K. HAAF (2002): “Competition, concentration and their relationship: An empirical analysis of the banking industry,” *Journal of Banking & Finance*, 26(11), 2191–2214.
- BLANCHARD, O. J. (1985): “Debt, deficits, and finite horizons,” *Journal of Political Economy*, 93(2), 223–247.
- BLANCHARD, O. J., P. DIAMOND, R. E. HALL, AND J. YELLEN (1989): “The beveridge curve,” *Brookings papers on economic activity*, 1989(1), 1–76.
- BLUNDELL, R., L. PISTAFERRI, AND I. PRESTON (2008): “Consumption inequality and Partial Insurance,” *American Economic Review*, 98, 1887–1921.
- BLUNDELL, R., L. PISTAFERRI, AND I. SAPORTA-EKSTEN (2012): “Consumption Inequality and Family Labor Supply,” Working paper, NBER.
- BOWLUS, A. J., AND H. LIU (2013): “The contributions of search and human capital to earnings growth over the life cycle,” *European Economic Review*, 64, 305 – 331.
- BROWNING, M., M. EJRNAES, AND J. ALVAREZ (2010): “Modelling Income Processes with Lots of Heterogeneity,” *Review of Economic Studies*, 77(4), 1353–1381.
- BUCHINSKY, M., D. FOUGÈRE, F. KRAMARZ, AND R. TCHERNIS (2010): “Interfirm Mobility, Wages and the Returns to Seniority and Experience in the United States,” *The Review of Economic Studies*, 77(3), 972–1001.
- BUERA, F. J., R. N. F. JAEF, AND Y. SHIN (2015): “Anatomy of a Credit Crunch: From Capital to Labor Markets,” *Review of Economic Dynamics*, 18(1), 101–117.
- BUNZEL, H., B. CHRISTENSEN, N. KIEFER, AND L. KORSHOLM (1999): “Equilibrium Search with Human Capital Accumulation,” unpublished manuscript.
- BURDETT, K., C. CARRILLO-TUDELA, AND M. G. COLES (2011): “Human Capital Accumulation and Labor Market Equilibrium,” *International Economic Review*, 52(3), 657–677.
- BURDETT, K., AND M. COLES (2003): “Equilibrium Wage-Tenure Contracts,” *Econometrica*, 71(5), 1377–1404.

- BURDETT, K., AND D. T. MORTENSEN (1998): “Wage Differentials, Employer Size, and Unemployment,” *International Economic Review*, 39(2), 257–273.
- BUSCH, C., D. DOMEIJ, F. GUVENEN, AND R. MADERA (2016): “Asymmetric Business Cycle Risk and Government Policy,” Discussion paper, Working Paper.
- BUSCH, C., AND A. LUDWIG (2016): “Labor Income Risk in Germany over the Business Cycle,” Discussion paper, Working Paper, University of Cologne.
- CABALLERO, R. J., AND P. D. KURLAT (2009): “The ‘surprising’ origin and nature of financial crises: a macroeconomic policy proposal,” *Financial Stability and Macroeconomic Policy*, Federal Reserve Bank of Kansas City.
- CARLSTROM, C. T., AND T. S. FUERST (1997): “Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis,” *American Economic Review*, pp. 893–910.
- CHUGH, S. K. (2013): “Costly External Finance and Labor Market Dynamics,” *Journal of Economic Dynamics and Control*, 37(12), 2882–2912.
- DE SANTIS, M. (2007): “Individual Consumption Risk and the Welfare Cost of Business Cycles,” *American Economic Review*, 97(4), 1488–1506.
- DEATON, A. (1997): *The analysis of household surveys: a microeconomic approach to development policy*. World Bank Publications.
- DEBACKER, J., B. HEIM, V. PANOUSI, S. RAMNATH, AND I. VIDANGOS (2013): “Rising Inequality: Transitory or Persistent? New Evidence from a Panel of U.S. Tax Returns,” *Brookings Papers on Economic Activity*, pp. 67–142.
- ELSBY, M. W., B. HOBIJN, AND A. ŞAHİN (2013): “Unemployment dynamics in the OECD,” *Review of Economics and Statistics*, 95(2), 530–548.
- ELSBY, M. W., AND J. C. SMITH (2010): “The great recession in the UK labour market: a transatlantic perspective,” *National Institute Economic Review*, 214(1), R26–R37.
- GARIN, J. (2015): “Borrowing Constraints, Collateral Fluctuations, and the Labor Market,” *Journal of Economic Dynamics and Control*, 57, 112–130.
- GERALI, A., S. NERI, L. SESSA, AND F. M. SIGNORETTI (2010): “Credit and Banking in a DSGE Model of the Euro Area,” *Journal of Money, Credit and Banking*, 42(s1), 107–141.
- GERTLER, M., AND A. TRIGARI (2009): “Unemployment Fluctuations with Staggered Nash Wage Bargaining,” *Journal of Political Economy*, 117(1), 38–86.
- GOURINCHAS, P.-O., AND J. A. PARKER (2002): “Consumption Over the Life Cycle,” *Econometrica*, 70, 47–89.
- GUVENEN, F. (2007): “Learning Your Earning: Are Labor Income Shocks Really Very Persistent?,” *American Economic Review*, 97(3), 687–712.

- (2009a): “An Empirical Investigation of Labor Income Processes,” *Review of Economic Dynamics*, 12, 58–79.
- GUVENEN, F., F. KARAHAN, S. OZKAN, AND J. SONG (2016): “What Do Data on Millions of U.S. Workers Reveal about Life-Cycle Earnings Dynamics?,” Discussion paper, Working Paper.
- GUVENEN, F., S. OZKAN, AND J. SONG (2014): “The nature of countercyclical income risk,” *Journal of Political Economy*, 122(3), 621–660.
- HAIDER, S. J. (2001): “Earnings Instability and Earnings Inequality of Males in the United States: 1967-1991,” *Journal of Labor Economics*, 19, 799–836.
- HALL, R. E. (2005): “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review*, pp. 50–65.
- HALL, R. E., AND P. R. MILGROM (2008): “The Limited Influence of Unemployment on the Wage Bargain,” *American Economic Review*, 98(4), 1653–1674.
- HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2005): “Two Views of Inequality Over the Life Cycle,” *Journal of the European Economic Association*, 3, 765–775.
- HECKMAN, J., AND R. ROBB (1985): “Using longitudinal data to estimate age, period and cohort effects in earnings equations,” in *Cohort Analysis in Social Research*, pp. 137–150. Springer.
- HECKMAN, J. J., L. LOCHNER, AND P. E. TODD (2003): “Fifty Years of Mincer Earnings Regressions,” Discussion papers, IZA.
- HUGGETT, M., G. VENTURA, AND A. YARON (2011): “Sources of Lifetime Inequality,” *American Economic Review*, 101, 2923–2954.
- JERMANN, U., AND V. QUADRINI (2012): “Macroeconomic Effects of Financial Shocks,” *American Economic Review*, 102(1), 238–271.
- JUDD, K. L. (1998): *Numerical Methods in Economics*. MIT press.
- KADIR, H., M. S. HABIBULLAH, L. S. HOOK, AND A. MOHAMED (2015): “Market Structure and Bank Competition in Malaysia,” *Mimeo, University Putra Malaysia, Serdang*.
- KALAI, E., AND M. SMORODINSKY (1975): “Other solutions to Nash’s bargaining problem,” *Econometrica*, pp. 513–518.
- KAPLAN, G. (2012a): “Inequality and the life cycle,” *Quantitative Economics*, 3, 471–525.
- KARAHAN, F., AND S. OZKAN (2013): “On the Persistence of Income Shocks over the Life Cycle: Evidence, Theory, and Implications,” *Review of Economic Dynamics*, 16, 452–476.

- KOPCZUK, W., E. SAEZ, AND J. SONG (2010): “Earnings Inequality and Mobility in the United States: Evidence from Social Security Data Since 1937,” *The Quarterly Journal of Economics*, 125, 91–128.
- KREBS, T. (2007): “Job Displacement Risk and the Cost of Business Cycles,” *American Economic Review*, 97(3), 664–686.
- KRUEGER, D., K. MITMAN, AND F. PERRI (2016): “On the distribution of the welfare losses of large recessions,” Discussion paper, National Bureau of Economic Research.
- LA CROCE, C., AND L. ROSSI (2015): “Firms Endogenous Entry and Monopolistic Banking in a DSGE Model,” Discussion paper, DEM Working Paper, University of Pavia, Italy.
- LAMADON, T. (2014): “Productivity shocks, Optimal Contracts and Income Dynamics,” unpublished working paper.
- LENTZ, R., AND N. ROYS (2015): “Training and Search On the Job,” unpublished working paper.
- LILLARD, L. A., AND Y. WEISS (1979): “Components of Variation in Panel Earnings Data: American scientists, 1960-1970,” *Econometrica*, 47, 437–454.
- LISE, J. (2013): “On-the-Job Search and Precautionary Savings,” *Review of Economic Studies*, 80(3), 1086–1113.
- LUCAS, R. E. (1987): *Models of Business Cycles*. Basil Blackwell.
- LUNDBERG, S. (1985): “The Added Worker Effect,” *Journal of Labor Economics*, 3, 11–37.
- MACURDY, T. E. (1982): “The Use of Time Series Processes to Model the Error Structure of Earnings in Longitudinal Data Analysis,” *Journal of Econometrics*, 18, 82–114.
- MARCUSSE, J. (2016): “Wage Bargaining and the Beveridge Curve,” *Mimeo, University of Essex, Department of Economics, UK*.
- MCKAY, A. (2017): “Time-varying idiosyncratic risk and aggregate consumption dynamics,” *Journal of Monetary Economics*, 88, 1–14.
- MEGHIR, C., AND L. PISTAFERRI (2004): “Income Variance Dynamics and Heterogeneity,” *Econometrica*, 72(1), 1–32.
- (2011): “Earnings, Consumption and Lifecycle Choices,” *Handbook of Labor Economics*, 4, 773–854.
- MINCER, J. (1974): “Schooling, Experience, and Earnings,” *NBER Press*.
- (2012): “Trends in the Transitory Variance of Male Earnings: Methods and Evidence,” *Journal of Human Resources*, 47(1), 204–236.



- MONACELLI, T., V. QUADRINI, AND A. TRIGARI (2011): “Financial markets and unemployment,” Discussion paper, National Bureau of Economic Research.
- MORTENSEN, D. T., AND E. NAGYPAL (2007): “More on Unemployment and Vacancy Fluctuations,” *Review of Economic Dynamics*, 10(3), 327–347.
- MOSCARINI, G., AND F. POSTEL-VINAY (2013): “Stochastic Search Equilibrium,” *Review of Economic Studies*, 80(4), 1545–1581.
- OSTROVSKY, Y. (2010): “Long-Run Earnings Inequality and Earnings Instability among Canadian Men Revisited, 1985–2005,” *The B.E. Journal of Economic Analysis & Policy*, 10, Article 20.
- (2012): “The correlation of spouses’ permanent and transitory earnings and family earnings inequality in Canada,” *Labour Economics*, 19, 756–768.
- PETROSKY-NADEAU, N. (2014): “Credit, Vacancies and Unemployment Fluctuations,” *Review of Economic Dynamics*, 17(2), 191–205.
- PETROSKY-NADEAU, N., AND E. WASMER (2013): “The Cyclical Volatility of Labor Markets under Frictional Financial Markets,” *American Economic Journal: Macroeconomics*, 5(1), 193–221.
- PISSARIDES, C. (2000): *Equilibrium Unemployment Theory*. MIT Press, 2 edn.
- POSTEL-VINAY, F., AND J.-M. ROBIN (2002): “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity,” *Econometrica*, 70(6), pp. 2295–2350.
- RAVN, M. O., AND H. UHLIG (2002): “On adjusting the Hodrick-Prescott filter for the frequency of observations,” *The Review of Economics and Statistics*, 84(2), 371–376.
- RUBINSTEIN, Y., AND Y. WEISS (2006): “Chapter 1 Post Schooling Wage Growth: Investment, Search and Learning,” vol. 1 of *Handbook of the Economics of Education*, pp. 1 – 67. Elsevier.
- SCHULHOFER-WOHL, S. (2013): “The Age-Time-Cohort Problem and the Identification of Structural Parameters in Life-Cycle Models,” *Unpublished paper*.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, pp. 25–49.
- SILVA, J. I., AND M. TOLEDO (2009): “Labor turnover costs and the cyclical behavior of vacancies and unemployment,” *Macroeconomic Dynamics*, 13(S1), 76–96.
- SNIEKERS, F. (2016): “Persistence and Volatility of Beveridge Cycles,” *forthcoming: International Economic Review*.
- STORESLETTEN, K., C. I. TELMER, AND A. YARON (2001): “The Welfare Cost of Business Cycles Revisited: Finite lives and Cyclical Variation in Idiosyncratic risk,” *European Economic Review*, 45, 1311–1339.

- (2004b): “Cyclical Dynamics in Idiosyncratic Labor Market Risk,” *Journal of Political Economy*, 112, 695–717.
- TOPEL, R. H. (1991): “Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority,” *Journal of Political Economy*, 99(1), 145–76.
- TOPEL, R. H., AND M. P. WARD (1992): “Job Mobility and the Careers of Young Men,” *The Quarterly Journal of Economics*, 107(2), 439–79.
- VERAMENDI, G. (2011): “Labor Market Dynamics: A Model of Search and Human Capital Accumulation,” unpublished manuscript.
- WÄLDE, K. (2011): *Applied Intertemporal Optimization, Edition 1.1*. Mainz University Gutenberg Press, Available at [www.waelde.com/aio](http://www.waelde.com/aio).
- WASMER, E., AND P. WEIL (2004): “The Macroeconomics of Labor and Credit Market Imperfections,” *American Economic Review*, 94(4), 944–963.
- YAMAGUCHI, S. (2010): “Job Search, Bargaining, and Wage Dynamics,” *Journal of Labor Economics*, 28(3), pp. 595–631.

# Note on co-authored work

Note on co-authored work contained in Michael Graber's thesis "Essays in Labour Economics":

- Chapter 1: "Unemployment and Vacancy Dynamics with Imperfect Financial Markets" is joint work with Carlos Carrillo-Tudela and Klaus Wälde, and each author contributed equally.
- Chapter 2: "Labour Income Dynamics and the Insurance from Taxes, Transfers, and the Family" is joint work with Richard Blundell and Magne Mogstad, and each author contributed equally.
- Chapter 3: "Labour Income Dynamics over the Business Cycle" is single-authored by Michael Graber
- Chapter 4: "Labour Market Frictions, Human Capital Accumulation, and Consumption Inequality" is joint work with Jeremy Lise, and each author contributed equally.