Trapped Particle Motion In Magnetodisc Fields

SM33C-2668

P. Guio[1,2], N. Staniland[1,2], N. Achilleos[1,2], C. Arridge[3]

[1] Department of Physics and Astronomy, University College London (UCL), UK; [2] Centre for Planetary Science, UCL/Birkbeck; [3] Lancaster University

Email contacts: p.guio@ucl.ac.uk, n.achilleos@ucl.ac.uk

Download poster from `http://www.ucl.ac.uk/~ucappgu`

Abstract

The spatial and time characterisation of trapped charged particle trajectories in magnetospheres has been extensively studied using dipole magnetic field structures. Such studies have allowed the calculation of spatial quantities such as equatorial loss cone size as a function of radial distance, the location of the mirror points along particular field lines ('L shells') as a function of the particle's equatorial pitch angle, and time quantities such as the bounce period and drift period as a function of the radial distance and the particle's pitch angle at the equator.
In this study, we present analogous calculations for the 'disc-like' field structure associated with the giant rotation-dominated magnetosphere of Jupiter as described by the UCL/Achilleos-Guio-Arridge (UCL/AGA) magnetodisc model. We discuss the effect of the magnetodisc field on various particle parameters, and make a comparison with the analogous motion in a dipole field.

Introduction

Conservation of the first adiabatic invariant μ, defined as the ratio of the kinetic energy associated with the gyratory motion perpendicular to the magnetic field (with velocity $v_\perp$) to the intensity of the field B, $\mu = mv_\perp^2/(2B)$ implies that the quantity $\sin^2\alpha/B$, where α is the pitch angle of the particle with respect to the magnetic field, remains constant. Thus the pitch angle becomes larger for more intense magnetic field.
In the *guiding centre* approximation, where particles are assumed to travel along the field line, the mirror point magnetic latitude λ_m is defined implicitly in B_m by:

$$\sin^2\alpha_\mathrm{eq} = \frac{B_\mathrm{eq}}{B_\mathrm{m}}, \tag{1}$$

where α_eq is the pitch angle of the particle at the equator with magnetic field $B_\mathrm{eq} = B(R_\mathrm{eq}, 0)$, and $B_\mathrm{m} = B(r_\mathrm{m}, \lambda_\mathrm{m})$ the magnetic field at the 'mirror point' where the particle bounces back.
The bounce period τ_b and the azimuthal drift period τ_d related to the second and third adiabatic invariants are then given by the following integrals (*Baumjohann and Treumann*, 1996):

$$\tau_b = 4\int_0^{\lambda_\mathrm{m}} \frac{ds}{d\lambda}\frac{d\lambda}{v_\parallel}, \tag{2}$$

$$\tau_d = \frac{2\pi}{\Delta\phi}\tau_b, \quad \text{where} \quad \Delta\phi = 4\int_0^{\lambda_\mathrm{m}} \frac{v_D}{r\cos\lambda}\frac{ds}{d\lambda}\frac{d\lambda}{v_\parallel}, \tag{3}$$

where ds is an arc element of the particle's path along its field line, $\Delta\phi$ the rate of change of longitude during one bounce period τ_b. The magnetic drift velocity v_D is the sum of curvature drift v_c and gradient drift v_g. A *first order* approximation of Taylor-expanding $\boldsymbol{B}$ about the guiding centre gives the following expressions:

$$v_D = v_c + v_g = \frac{mv_\parallel^2}{q}\frac{\boldsymbol{R}_c\times\boldsymbol{B}}{R_c^2B^2} + \frac{mv_\perp^2}{2q}\frac{\boldsymbol{B}\times\boldsymbol{\nabla}B}{B^3}, \tag{4}$$

$$\Delta\phi = \Delta\phi_c + \Delta\phi_g, \tag{5}$$

where q and $\boldsymbol{R}_c$ are the particle charge and the radius of curvature vector.
For a field line parameterised in polar coordinates $r(\lambda)$, the element of arc length along a general magnetic field is given by $ds^2 = dr^2 + r^2d\lambda^2$, and by definition $dr/d\lambda = -rB_\mathrm{r}/B_\lambda$, thus $ds/d\lambda = r(\lambda)(1 + B_\mathrm{r}^2/B_\lambda)^{1/2}$. In addition since $v^2 = v_\parallel^2 + v_\perp^2$ is a constant of motion and the adiabatic invariant μ is conserved, we can write $v_\parallel = v(1 - B/B_\mathrm{m})^{1/2}$ and $v_\perp = v(B/B_\mathrm{m})^{1/2}$. Thus the bouncing period τ_b can be expressed as:

$$\tau_b = LR_\mathrm{P}\frac{2(2m)^{1/2}}{W^{1/2}}\Phi(R_\mathrm{eq}, \alpha_\mathrm{eq}), \quad \text{with} \quad \Phi(R_\mathrm{eq}, \alpha_\mathrm{eq}) = \frac{1}{L}\int_0^{\lambda_\mathrm{m}} \left(\frac{1 + B_\mathrm{r}^2/B_\lambda^2}{1 - B/B_\mathrm{m}}\right)^{\frac{1}{2}} \hat{r}(\lambda)d\lambda, \tag{6}$$

where $\hat{r} = r/R_\mathrm{P}$ and $L = R_\mathrm{eq}/R_\mathrm{P}$ are *normalised* distance to the planetary radius R_P, and W the particle kinetic energy.
In polar coordinates $r(\lambda)$, the radius of curvature vector $\boldsymbol{R}_c$ and curvature κ are given by the expressions:

$$\boldsymbol{R}_c = \frac{(r^2 + (dr/d\lambda)^2)^{\frac{3}{2}}}{\left|r^2 + 2(dr/d\lambda)^2 - rd^2r/d\lambda^2\right|}\boldsymbol{n}, \quad \text{and} \quad \kappa = 1/R_c, \tag{7}$$

where $\boldsymbol{n}$ is the unit normal vector, and $d^2r/d\lambda^2$ can be expressed as function of B_r, B_λ and their first derivative with respect to λ. The drift period τ_d can be expressed as:

$$\tau_d = \frac{\pi qB_\mathrm{P}R_\mathrm{P}^2}{3LW}\frac{\Phi(R_\mathrm{eq}, \alpha_\mathrm{eq})}{\Omega(R_\mathrm{eq}, \alpha_\mathrm{eq})}, \quad \text{where} \quad \Omega = \Omega_c + \Omega_g \quad \text{and,} \tag{8}$$

$$\Omega_c(R_\mathrm{eq}, \alpha_\mathrm{eq}) = \frac{1}{L^2}\int_0^{\lambda_\mathrm{m}} \left(1 + \frac{B_\mathrm{r}^2}{B_\lambda^2}\right)^{\frac{1}{2}} \frac{\kappa}{\hat{B}} \left(1 - \frac{B}{B_\mathrm{m}}\right)^{\frac{1}{2}} \frac{d\lambda}{3\cos\lambda}, \tag{9}$$

$$\Omega_g(R_\mathrm{eq}, \alpha_\mathrm{eq}) = \frac{1}{L^2}\int_0^{\lambda_\mathrm{m}} \frac{B_\mathrm{r}\nabla_\lambda B - B_\lambda\nabla_r B}{B^2\hat{B}_\mathrm{m}} \left(\frac{1 + B_\mathrm{r}^2/B_\lambda^2}{1 - B/B_\mathrm{m}}\right)^{\frac{1}{2}} \frac{d\lambda}{6\cos\lambda}, \tag{10}$$

where $\hat{B} = B/B_\mathrm{P}$ and $\hat{B}_\mathrm{m} = B_\mathrm{m}/B_\mathrm{P}$ are *normalised* field strength to the field at the surface equator B_P, and ∇_r and ∇_λ are gradient components in polar coordinates.
Both periods can be approximated in the case of a dipole field by the following analytic expressions (*Öztürk*, 2012):

$$\tau_b^d \sim LR_\mathrm{P}\frac{2(2m)^{1/2}}{W^{1/2}}(1.31 - 0.57\sin\alpha_\mathrm{eq}), \tag{11}$$

$$\tau_d^d \sim \frac{\pi qB_\mathrm{P}R_\mathrm{P}^2}{3LW}\frac{1}{0.35 + 0.15\sin\alpha_\mathrm{eq}}, \tag{12}$$

We developed a MATLAB® code to *numerically* solve the integrals $\Phi(R_\mathrm{eq}, \alpha_\mathrm{eq})$ and $\Omega(R_\mathrm{eq}, \alpha_\mathrm{eq})$ for a prescribed magnetic field. The code was validated for a dipole field: (i) we estimated the integrals Φ and Ω/Φ for a range of R_eq and α_eq, (ii) we then computed the best fit to the following function linear in $\sin\alpha_\mathrm{eq}$:

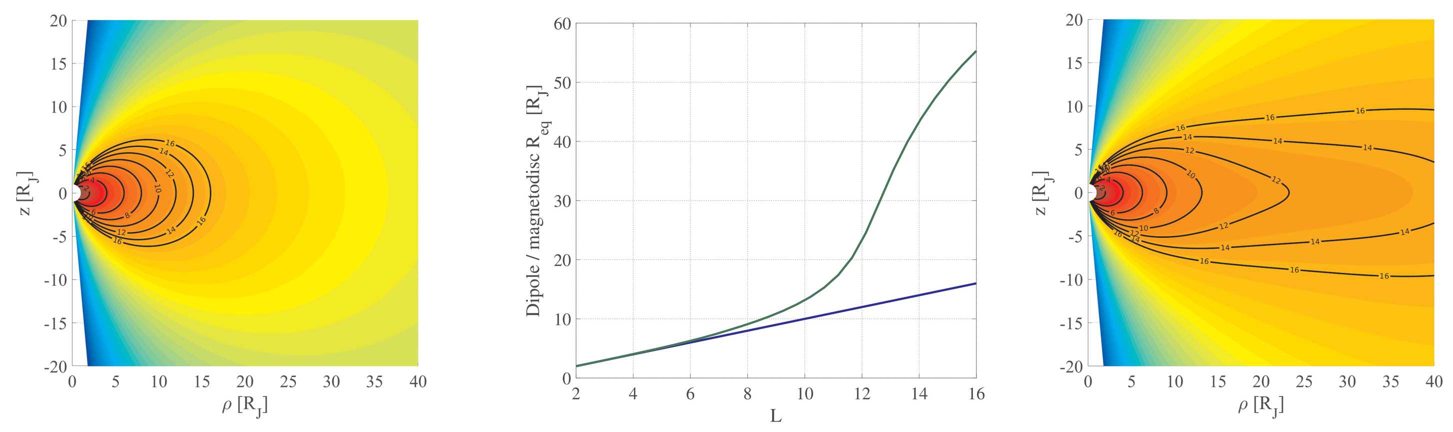
$$f(\alpha_\mathrm{eq}) = a + b\sin\alpha_\mathrm{eq}. \tag{13}$$

The fitted coefficients a and b are in very good agreement with the ones given by Eqs. (11–12) and are summarised in the following table:

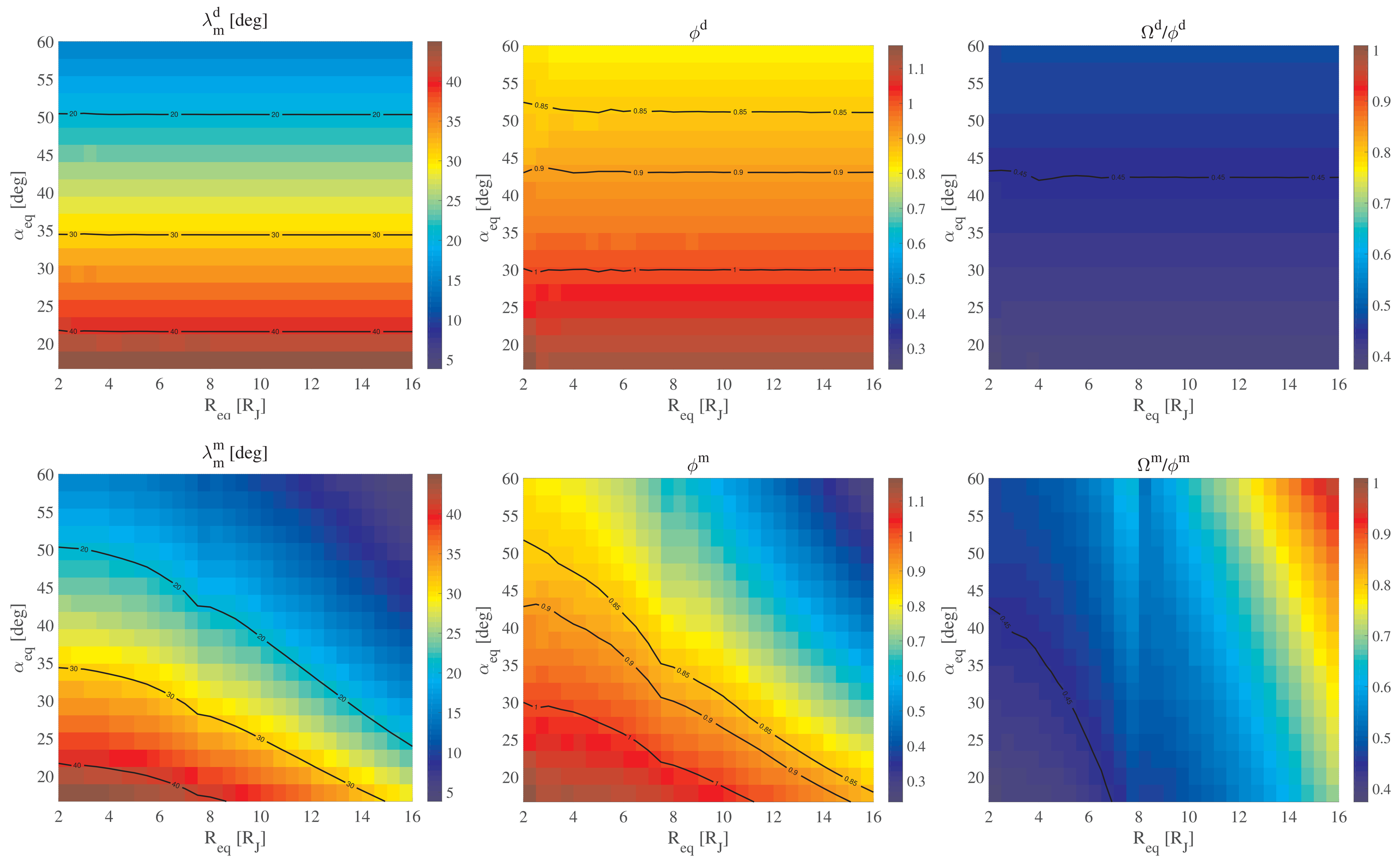
	a	b
Φ^d	1.28	−0.55
Ω^d/Φ^d	0.35	0.14

Trapped Motion Properties for Jovian Magnetodisc

The UCL Magnetodisc model (*Achilleos et al.*, 2010) uses the formalism developed in *Caudal* (1986) to compute axisymmetric models of the rotating Jovian (or Kronian) plasmadisc in which magnetic, centrifugal and plasma pressure forces are in equilibrium.
We use the output of the model for a *standard* Jovian disc configuration where the magnetopause is located at $90\,\mathrm{R_J}$.



Here we compare the dipole and magnetodisc field. Field lines are labelled with an 'equivalent dipole L' parameter. For the dipole field, this parameter is equal to the equatorial distance of the field line in R_J. For the magnetodisc field, this parameter is equal to the equatorial distance to which the dipole field line, emanating from the *same ionospheric foot point* as the labelled magnetodisc field line, would extend. The middle panel shows the equatorial distance R_eq in R_J for dipole (green) and magnetodisc (blue) field lines having the same foot point on the planet surface, as specified by the equivalent dipole L. The magnetodisc field is dipolar to a good approximation for R_eq corresponding to $L \lesssim 6$.



From left to right, the latitude for mirror point λ_m, and the two integrals Φ and Ω/Φ characterising the bouncing and drift periods. The upper panels are for the dipole field and the lower ones for the magnetodisc as the figure below. For the dipole, there is no dependency on R_eq for any of the integral quantities due to the property of the field. For the magnetodisc, note how Φ^d, thus the bouncing period drops for both large R_eq and α_eq due to the strong decrease of λ_m with increasing R_eq, reflecting the equatorial confinement of the plasma. From the Ω/Φ integral, inversely proportional to the drift period, it can be seen that the drift period for large R_eq and α_eq is marginally less than the dipole value, which is the signature of the magnetic flux invariance through the drift path (dipole and magnetodisc drift shells of the same equivalent L enclose similar magnetic flux).
We then computed the fits to our results, using the following function with a *correction* term to account for the magnetodisc structure (cross term bi-linear in L and $\sin\alpha_\mathrm{eq}$):

$$f(L, \alpha_\mathrm{eq}) = a + b\sin\alpha_\mathrm{eq} + cL\sin\alpha_\mathrm{eq}, \tag{14}$$

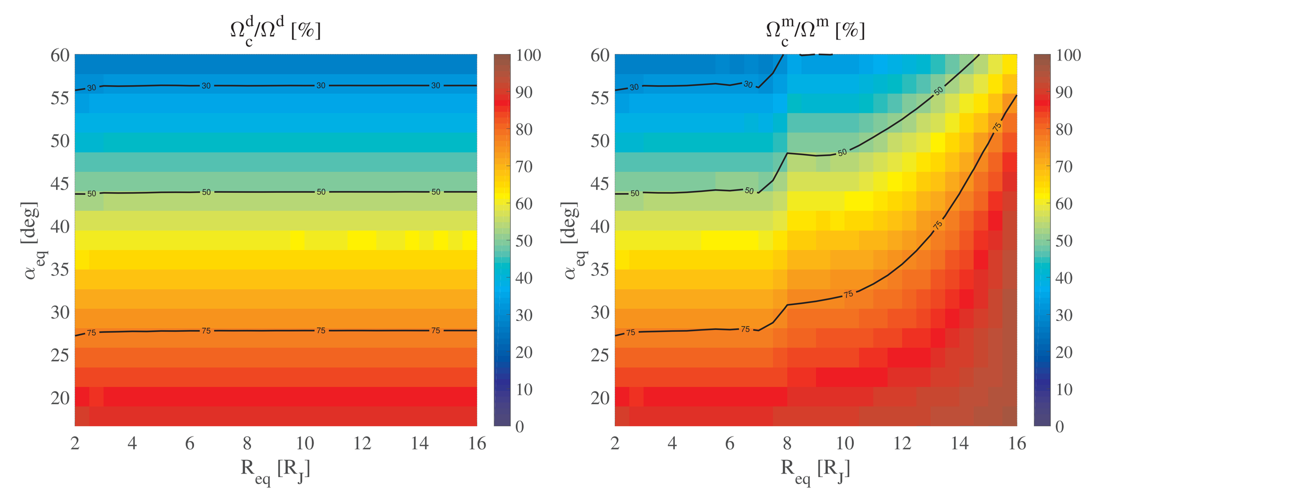
to find analytic approximation formulae similar to Eqs. (11–12) for the bounce and drift periods of the Jovian magnetodisc studied here:

$$\tau_b^m \sim LR_\mathrm{J}\frac{2(2m)^{1/2}}{W^{1/2}}(1.27 - 0.37\sin\alpha_\mathrm{eq} - 0.05L\sin\alpha_\mathrm{eq}), \tag{15}$$

$$\tau_d^m \sim \frac{\pi qB_\mathrm{J}R_\mathrm{J}^2}{3LW}\frac{1}{0.40 - 0.06\sin\alpha_\mathrm{eq} + 0.04L\sin\alpha_\mathrm{eq}}. \tag{16}$$

Curvature Versus Gradient Drift

Finally we examine the respective contribution of curvature drift rate $\Delta\phi_c \propto \Omega_c$ and gradient drift rate $\Delta\phi_g \propto \Omega_g$ to the total azimuthal drift rate $\Delta\phi \propto \Omega$.



Here we compare the percentage of drift due to curvature in the total azimuthal drift, as function of R_eq and α_eq, for the dipole case (left) and the magnetodisc (right). For the dipole field, the drift contribution is not a function of R_eq, and for $\alpha_\mathrm{eq} \ll 45\,\mathrm{deg}$ the curvature drift dominates as λ_m becomes larger, while for $\alpha_\mathrm{eq} \gg 45\,\mathrm{deg}$ the gradient drift dominates as the motion is confined around the equator. The magnetodisc exhibits the same behaviour as the dipole for $R_\mathrm{eq} \le 6R_\mathrm{J}$ as expected, but for $R_\mathrm{eq} \ge 6R_\mathrm{J}$ the curvature drift largely dominates, even at large pitch angle due to the equatorial confinement in the disc-like field structure.

Conclusion

We have presented a formalism to calculate the bounce and drift periods in the guiding centre approximation for any prescribed magnetic field and applied it to nominal Jupiter's magnetodisc.
We have derived analytic expressions for the bounce and drift periods for a magnetodisc structure, analogous to expressions for the dipole field.
Further studies are needed to check the validity range for these approximations, and how the solar wind and supra-thermal population influence the bounce and drift periods (compressed and expanded magnetosphere).

References

Achilleos, N., P. Guio, C. S. Arridge, N. Sergis, R. J. Wilson, M. F. Thomsen, and A. J. Coates, Influence of hot plasma pressure on the global structure of Saturn's magnetodisk, *Geophys. Res. Lett.*, *37*, L20,201, doi: 10.1029/2010GL045159, 2010.

Baumjohann, W., and R. A. Treumann, *Basic Space Plasma Physics*, Imperial College Press, London, ISBN 1-86094-079-X, 1996.

Caudal, G., A self-consistent model of Jupiter's magnetodisc including the effects of centrifugal force and pressure, *J. Geophys. Res.*, *91*, 4201–4221, 1986.

Öztürk, M. K., Trajectories of charged particles trapped in Earth's magnetic field, *Am. J. Phys.*, *80*, 420–428, doi:10.1119/1.3684537, 2012.