Dynamics of Charged Particles Trapped in a Gas Giant Magnetodisc

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Abstract

The Earth's internal magnetic field is to a good approximation dipolar, and charged particles from the magnetosphere, depending on their kinetic energy, pitch angle and distance can remain trapped in this field. The motion of such trapped particles is characterised by their time scales —cyclotron (gyration), bounce and drift periods— and the position of the mirror point. High-energy electron and proton populations in the two radiation (van Allen) belts are such examples. At the gas giants, Jupiter and Saturn, the total magnetic field deviates from a dipolar configuration due to internal sources of plasma provided by the moons Io and Enceladus respectively. In addition, the rapid rotation of these planets (period of order $\sim 10\,\mathrm{h}$) plays a role in the development of a disk-like field structure near the equator where centrifugal force is dominant —a configuration referred to as a magnetodisc.

We present results of numerical simulations of charged particle motion in such a magnetodisc field structure using particle tracing and the UCL Magnetodisc Model, and we use these simulations to characterise the time scales and mirror point, and quantify the differences between particle motion in magnetodisc and dipole fields.

Introduction

We use the relativistic formulation of the motion of a charged particle of mass m and charge q in a magnetic field $\boldsymbol{B}$ described by the Newton-Lorentz equation (*Öztürk*, 2012)

$$\frac{\mathrm{d}(\gamma m\boldsymbol{v})}{\mathrm{d}t} = q\boldsymbol{v}\times\boldsymbol{B}, \tag{1}$$

where $\gamma = 1/\sqrt{1-v^2/c^2}$ is the relativistic factor and $\boldsymbol{v}$ is the particle speed, to study *numerically* the motion of trapped charged particles in a planet's magnetic field with our particle tracing code.

Conservation of the first adiabatic invariant μ, defined as the ratio of the kinetic energy associated with the gyratory motion perpendicular to the magnetic field (with velocity $v_\perp$) to the intensity of the field B, $\mu = mv_\perp^2/(2B)$ implies that the quantity $\sin^2\alpha/B$, where α is the pitch angle of the particle with respect to the magnetic field, remains constant. Thus the pitch angle becomes larger for more intense magnetic field.

In a planetary dipole-like magnetic field, the loss cone α_0 defined by:

$$\sin^2\alpha_0 = \frac{B_{\mathrm{eq}}}{B_{\mathrm{m}}}, \tag{2}$$

is the smallest pitch angle where particles will be bounced back when travelling from the equator with magnetic field B_{eq} along the field line to the 'mirror point' of reflection with magnetic field B_{m}.

In the assumption of a dipole magnetic field $\boldsymbol{B}_{\mathrm{dip}}$ with magnetic moment $\boldsymbol{M}$, or equivalently B_{P}, the intensity of the purely tangential field at the surface magnetic equator of the planet ($L = 1$), the bounce motion period τ_b related to the second invariant and the drift motion period τ_d related to the third adiabatic invariant are given by the following approximate expressions:

$$\tau_b \sim LR_{\mathrm{P}}\frac{m^{1/2}}{W^{1/2}}(3.7 - 1.6\sin\alpha_{\mathrm{eq}}), \tag{3}$$

$$\tau_d \sim \frac{\pi q B_{\mathrm{eq}} R_{\mathrm{P}}^2}{3LW}\frac{1}{0.35 + 0.15\sin\alpha_{\mathrm{eq}}}, \tag{4}$$

where LR_{P}, α_{eq} and W are respectively the radial distance of the particle at the equator, its pitch angle, and its kinetic energy.

The bounce period depends linearly on the distance of the particle at the equator (dipole L parameter), and is also a function of the kinetic energy and, more weakly, of the pitch angle. Bounce period is not dependent on mass nor charge.

The drift period depends inversely on the parameters L and W, varies linearly with charge, relies less sensitively on pitch angle, and is independent of mass. Thus electrons and protons with same kinetic energy will have the same drift period but will drift in opposite directions.

Also the mirror point magnetic latitude λ_{m} for a dipole magnetic field is the solution of the following equation:

$$\sin^2\alpha_{\mathrm{eq}} = \frac{\cos^6\lambda_{\mathrm{m}}}{\sqrt{1+3\sin^2\lambda_{\mathrm{m}}}}. \tag{5}$$

Thus the mirror point latitude depends only upon the pitch angle of the particle on the equatorial plane. It does not depend on its charge, mass, and kinetic energy; nor does it depends on L but the associated radial position r_{m} does $r_{\mathrm{m}} = LR_{\mathrm{P}}\cos^2\lambda_{\mathrm{m}}$.

Code validation for dipole field: Earth

We developed a MATLAB® code to *numerically* solve the Newton-Lorentz equation for any charged particle in a prescribed magnetic field, and derive the characteristic properties of the particle motion such as the bounce and drift periods as well as the mirror point latitude.

For the Earth the magnetic field is to a good approximation a dipole field and we validated our numerical code against the analytic expressions for the bounce and drift periods, and the mirror point latitude of a $1\,\mathrm{MeV}$ proton with varying pitch angle and initial equatorial distance. Such proton energy corresponds to an average proton in the Earth's van Allen belt (*Mauk*, 2014).

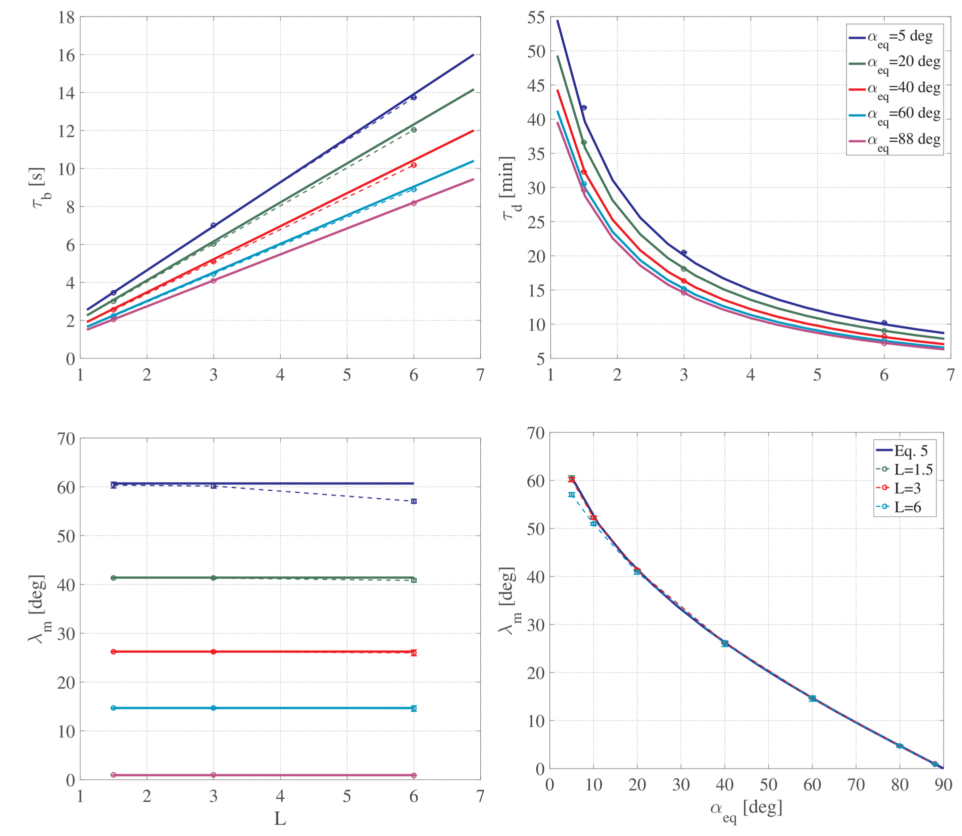

Fig. 1: Comparison of bounce and drift periods, and mirror point latitudes for a $1\,\mathrm{MeV}$ proton given by our numerical code and the expressions in Eqs. (3–4), as function of initial L-shell values and initial pitch angle α_{eq}. As expected the mirror point latitude is not dependent on L.

Magnetodisc field structure: Jupiter

The UCL Magnetodisc model (*Achilleos et al.*, 2010) uses the formalism developed in *Caudal* (1986) to compute axisymmetric models of the rotating Jovian (or Kronian) plasmadisc in which magnetic, centrifugal and plasma pressure forces are in equilibrium.

We use the output of the model for a standard Jovian disc configuration where the magnetopause is located at $90\,\mathrm{R_J}$.

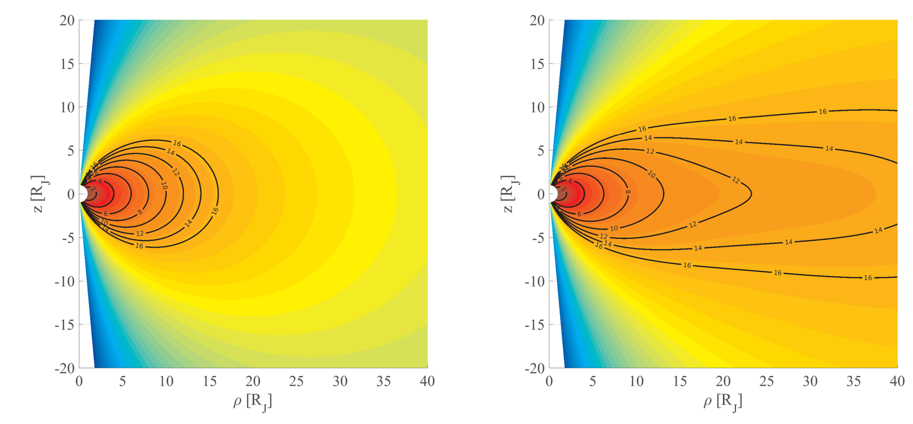

Fig. 2: Comparison of dipole and magnetodisc field lines. Field lines are labelled with an 'equivalent dipole L' parameter. For the pure dipole field, this parameter is equal to the equatorial distance of the field line in units of planetary radii. For the magnetodisc field, this parameter is equal to the equatorial distance to which a pure dipole field line, emanating from the *same ionospheric foot point* as the labelled ma

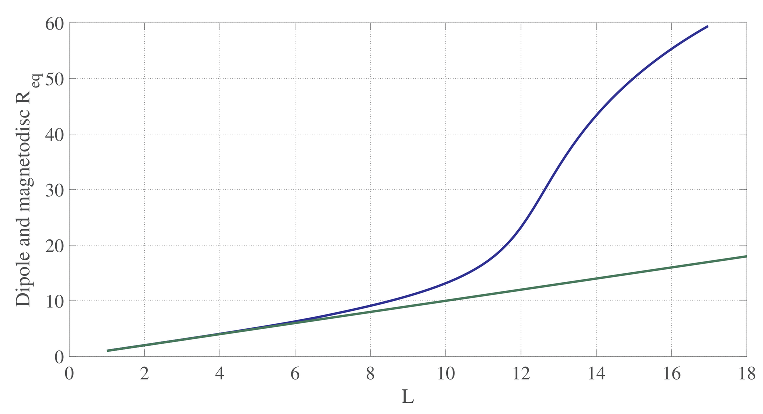

Fig. 3: The equatorial distance R_{eq} (in units of planet radii) for dipole (green) and magnetodisc (blue) field lines having the same foot point on the planet surface, as specified by the equivalent dipole L (see Fig. 2). The magnetodisc field is apparently dipolar to a good approximation for equatorial distances corresponding to $L \lesssim 6$.

Trapped motion properties for Jovian magnetodisc

We use again a $1\,\mathrm{MeV}$ proton which is an average energy for a proton in the radiation belt of Jupiter (*Mauk*, 2014).

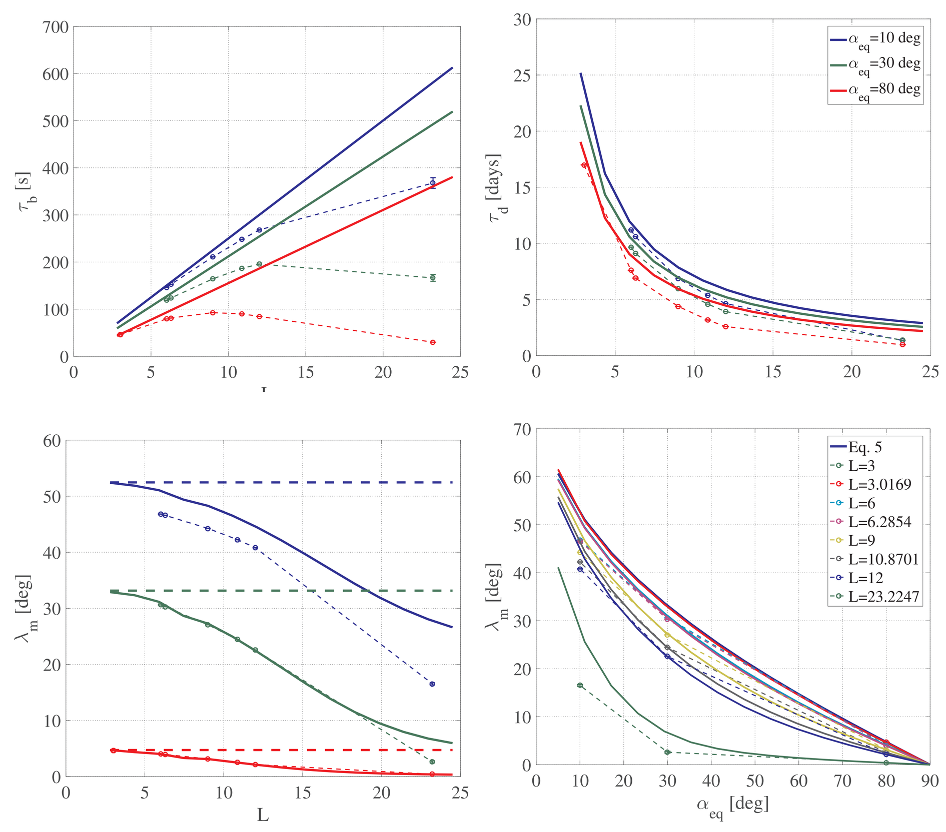

Fig. 4: Same as Fig. 1 but for the magnetodisc as in Fig. 2. Note how the bouncing period drops for large L due to the strong decrease of λ_{m} with increasing L, reflecting the equatorial confinement of the plasma. The drift period as a function of L is marginally less than the dipole value, which is the signature of the magnetic flux invariance through the drift path (dipole and magnetodisc drift shells of the same equivalent L enclose similar magnetic flux).

Conclusion

We have presented some preliminary results on how a magnetodisc structure modifies the characteristic geometry and time scales of trapped-particle motion.

We plan to explore how to get semi-analytic expressions for the bounce and drift periods, as well as the mirror latitude within a magnetodisc structure analogous to the dipole field case.

Further studies could include the effect of centrifugal force that would confine further the motion of the trapped particle towards the equator.

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