QUASI EX-ANTE INFLATION FORECAST UNCERTAINTY

WOJCIECH CHAREMZA*, CARLOS DÍAZ † AND SVETLANA MAKAROVA+

+: University College London, UK, s.makarova@ucl.ac.uk (corresponding author), 16 Taviton Street, London WC1H 0BW, UK

*: Vistula University, Poland,

†: University of Leicester, UK

February 2018

Please do not quote without permission

Forthcoming in the

International Journal of Forecasting

KEYWORDS: macroeconomic forecasting, monetary policy, non-normality, inflation uncertainty, statistical distributions, central banking

JEL codes: C54, E37, E52

ABSTRACT

We propose a measure of the effects of monetary policy based on the distribution of *ex-post* inflation forecast uncertainty. We argue that the difference between the distributions of the *ex-ante* and *ex-post* uncertainties reflects the impact of monetary policy decisions. Using the New Keynesian model with imperfect information and a monetary policy rule, we derive a proxy for *ex-ante* inflation uncertainty called *quasi ex-ante* forecast uncertainty, which is to an extent free of the effects of monetary policy decisions. Further, we introduce the *compound strength* measure of monetary policy and the *uncertainty ratio*, which approximates the impact that monetary policy has on reducing inflation forecast uncertainty. Empirical results show that the compound strength is non-linearly related to measures of bank independence and the greatest policy effect in reducing inflation forecast uncertainty occurs for countries which conduct either strict or clandestine inflation targeting.

1. INTRODUCTION

The concept of forecast uncertainty is usually understood in either an *ex-ante* or an *ex-post* sense (see Clements, 2014; Clements and Galvão, 2014). Ex-ante uncertainty is the variance of an unpredictable error in a forecast formulated at a given time, say $t-h$ for time *t*, or alternatively it is an explicitly formulated statement by expert forecasters about such uncertainty. The ex-post uncertainty is measurable at time *t* by the variance of forecast errors, where the forecast was made at time $t-h$. Under conditions of (a) stationarity, (b) ergodicity of the forecast errors, (c) perfect model specification, (d) the absence of structural breaks in the period of the forecast, and (e) independence, unbiasedness and confidence neutrality in the experts' forecasts, the ex-post and ex-ante uncertainties should be identical. However, it has often been observed that they are not the same in practice, even when conditions (a)-(d) are likely to hold (see e.g. Dowd, 2007). Condition (e) is particularly easy to question. The criticism concentrates mainly on the outcomes of various surveys of forecasters, like the Surveys of Professional Forecasters in Europe and the US, various consensus forecasts, and others. Indeed it is often noted that the experts participating in such pools can be inattentive and fail to update their forecasts, may disagree when updating, and do not always learn from past experience (Andrade and Le Bihan, 2013). Furthermore, forecasts in a panel may be highly correlated (Makarova, 2014), and the constitution of panels of forecasters can frequently change, depending on the phase of the business cycle (López-Pérez, 2016). Additionally, psychological bias, overconfidence and underconfidence may play a role when probabilistic and interval forecasts are being formulated (Soll and Klayman, 2004; Hansson, Juslin and Winman, 2008; Clements, 2014). In a cross-country comparison, an additional important drawback seems to be that panels of forecasts across countries are often incomplete, unavailable or not comparable because the definitions of the aggregates are different, the timing of forecasts may differ, and so on (for a critique and development see Lahiri and Sheng, 2010; Lahiri, Peng and Sheng, 2014; and Ozturk and Sheng, forthcoming).

It is evidently easier and less expensive to compute ex-post uncertainty rather than ex-ante uncertainty. Inference in ex-post uncertainty does not require access to a panel of unbiased, independently formulated forecasts. Similar forecasting models can be applied to different countries, making international comparison feasible.

Under conditions (a)-(d), ex-post uncertainty could be a reasonable proxy for ex-ante uncertainty, but only if it is not affected by monetary policy. We assume here that monetary decisions follow a policy rule. If the application of a monetary policy rule in an imperfect information economy is to result in expectational stability, the agents must know this rule (see Orphanides and Williams, 2005; and Preston, 2006). However, they might not necessarily know the values of the forecasts used as instruments by the central bankers, who have access to some additional information. Consequently, if such a monetary policy is successful in minimising the observed volatility of inflation, it might well be successful in reducing forecast uncertainty. In this case, the distributions of the uncertainty originally expected by the agents and the evaluated ex-post uncertainty will be different, with the ex-post uncertainty having a smaller dispersion.

In fact the empirical distribution of the ex-post uncertainty might be affected by numerous other factors which are not related to monetary policy. In order to clean the distribution of the influence of such factors to make part of the uncertainty predictable from the past, we introduce the concept of *policy-prone ex-post* forecast uncertainty of inflation. This is based on ex-post forecast errors (similar to those in Clements, 2014) where the predictable element of variability has been removed.

The effects of monetary policy can be identified by fitting the weighted skew-normal distribution (WSN) introduced in Section 3, to observations on the policy-prone uncertainty. The parameters of this distribution can be interpreted in terms of an outcome of monetary policy. This can then be used to formulate a measure of the *compound strength* of a policy resulting in reducing inflation uncertainty. Next, we derive an approximation of ex-ante uncertainty by partially removing the expected average effects of the monetary policy on the distribution of forecast errors. We call this the *quasi ex-ante* uncertainty.

The further structure of the paper is as follows. In Section 2 we outline the New Keynesian model with imperfect information and a monetary policy rule based on the forecasts of central bankers (CB). Section 3 explains why we treat CB forecasts as unobservable and described by a random variable, which, in turn, constitutes a core part of the skew-normal distribution described later in this section. Section 4 discusses the settings of the estimation. Section 5 introduces the concept of *quasi ex-ante* forecast uncertainty and applies it to construct a measure of the monetary policy effects named the *uncertainty ratio*. Details and derivations are given in the Supplementary Material, Part 1. In Section 6 the main empirical results are given. It is shown that for 38 countries the compound strength relates to the independence and transparency of the central bank in a nonlinear way, as it relates positively if the compound strength is smaller than it is for the maximum level of the uncertainty ratio, and negatively otherwise. The concept of quasi ex-ante uncertainty is used to evaluate the monetary policy effects for the BRICS countries (Brazil, Russia, India, China and South Africa), the UK and the US. Section 7 gives the summary of the stability and robustness analysis, based on the detailed results given in the Supplementary Material, Part 2, and Section 8 concludes. The paper contains Appendix with the description of data.

2. THEORETICAL BACKGROUND

We have grounded our approach within the settings of the New Keynesian supply and demand model with imperfect information. In the description below, we closely follow the discussion by Preston (2006) of the efficiency of a monetary policy based on information available to central bankers and not the agents (see also Eusepi and Preston, 2016). The aggregate structural supply/demand equations which constitute the monetary transmission mechanism, are:

$$g_t = \bar{E}_t \sum_{T=t}^{\infty} b^{T-t} \left[(1-b) g_{T+1} - s(i_T - \pi_{T+1}) + \xi_{1,T} \right] \quad ; \tag{1}$$

$$\pi_t = \kappa g_t + \bar{E}_t \sum_{T=t}^{\infty} (ab)^{T-t} \left[\kappa ab g_{T+1} + (1-a) b \pi_{T+1} + \xi_{2,T} \right] \quad , \tag{2}$$

where g_t denotes log-deviations from the steady state, $0 < b < 1$ is a discount factor, $s > 0$ is the intertemporal elasticity of substitution, i_T is nominal interest rate, π_{T+1} is the inflation rate at time $T+1$, $\kappa > 0$ is the slope of a generalised New-Keynesian Phillips curve, $(1-a)$ is the Calvo probability, which is the probability that a firm will be able to revise its prices at time *t*, and $\xi_{1,T}$ and $\xi_{2,T}$ are exogenous disturbances. The notation $\bar{E}_t$ stands for the averaged expectations of agents based on possibly imperfect information, taken at time *t*. The model results from the aggregation of the approximation of the optimal decision rules for households and firms described in Woodford (2003) among others; for the derivation see Preston (2005).

The intertemporal loss function minimised by the central bank is given by:

$$W = E_{t_0} \sum_{t=t_0}^{\infty} b^{t-t_0} L_t \quad ,$$

where E_{t_0} is rational expectations at time t_0, and the period loss function L_t is defined as:

$$L_t = \pi_t^2 + \lambda g_t^2 , \quad \lambda > 0 \quad .$$

We assume that the following main assumptions hold:

[1] the central bank aims to stabilise the variation of inflation in the time-invariant policy (see Woodford, 2003, chapter 7);

[2] following Preston (2006), the policy rule is:

$$i_t - i_t^* = \psi_1 (E_t^{CB} \pi_{t+1} - E_t \pi_{t+1}^*) + \psi_2 (E_t^{CB} g_{t+1} - E_t g_{t+1}^*) \quad , \tag{3}$$

where E_t^{CB} is a forecast the central bank responds to, and i_t^*, π_t^* and g_t^* denote the optimal time-invariant paths of i_t, π_t and g_t;

[3] agents know the policy rule (3);

[4] agents conduct adaptive linear least-squares learning, as in Evans and Honkapohja (1994);

[5] the Taylor principle: $\kappa(\psi_1 - 1) > (1-b)\psi_2$ holds.

Preston (2006) shows in Proposition 3 that under assumptions [1]-[4], [5] constitutes the necessary and sufficient condition for expectational stability (E-Stability). For a similar approach and result see Orphanides and Williams (2005).

In the stochastic model derived further in Section 3 we are assuming that the economy is described by (1)-(2) and the assumptions [1]-[5] hold. In particular, our settings will be directly related to the implementation of the policy rule (3) as a fundamental for the statistical distribution explaining the development of the policy-prone uncertainty.

3. EX-POST INFLATION FORECAST UNCERTAINTY AND ITS DISTRIBUTION

In this section we consider how monetary policy conducted using policy rule (3) affects the distribution of inflation forecast uncertainty. As we limit our interest to a univariate problem of inflation uncertainty, we ignore the effect of the second component of the right-hand side of (3), which is related to output. We call the forecast uncertainty for a particular horizon policy-prone if it is unforecastable in the first and second moments using public imperfect knowledge. It might, however, be partly forecastable by the CB forecasters. For each forecast horizon, the policy-prone uncertainty is a random variable denoted by U.

Let us initially consider the case where monetary policy is either absent or is fully ineffective. In such case, U coincides with a random variable X. Assuming normality, we have

$$U = X \sim N(\mu_X, \sigma_X^2) \quad . \tag{4}$$

However, if the conditions [1]-[5] hold, monetary policy is conducted in accordance with rule (3), and might be effective, at least to some extent. There is some evidence that this rule is often implemented in practice by central banks. Hubert (2015) uses data for the US to show that the Federal Open Market Committee (FOMC) bases its decisions on different information from that used for forming the baseline forecast. He also argues that the forecast information used by FOMC might be orthogonal to that used by other, independent, forecasters. Supporting theory can be found in Baeriswyl and Cornand (2010) and an anthropological discussion in Holmes (2014). Similar conclusions can be drawn from Nunes (2013). However, it does not appear that the official forecasts supplied by the forecasting divisions of central banks to the monetary policy committees constitute a good approximation of $E_t^{CB}\pi_{t+1}$. In fact, the decision makers consider various additional quantitative and qualitative factors or forecasts of various sources and strengths, possibly including some anecdotal and random evidence. Moreover, Charemza and Ladley (2016) show empirically that the official CB forecasts are often significantly biased towards the target and, as such, do not constitute valid warning signals, particularly for longer forecast periods.

As the CB official forecasts are not useful here, we treat the first component in (3), $E_t^{CB}\pi_{t+1} - E_t\pi_{t+1}^*$, as not observable. To incorporate the effects of the monetary policy into the distribution of U, we assume that $E_t^{CB}\pi_{t+1} - E_t\pi_{t+1}^*$ is a realisation of a normally distributed random variable Y with mean μ_Y and variance σ_Y^2:

$$Y \sim N(\mu_Y, \sigma_Y^2) \quad . \tag{5}$$

If the CB forecasters are not completely ignorant, they might have some relevant knowledge of X in (4). In this case, there should be a positive correlation between Y and X. The higher this correlation is, the more competent the CB forecasters are, as it means they can explain more of the variability of X.

Next, we assume that monetary policy is costly, so that it pays to issue a monetary policy signal only if the forecast signal to the central bank from (3) is clear as it is sufficiently large; otherwise the policy may be too expensive, ineffective or even counter-efficient (see e.g. Morris and Shin, 2002; Charemza and Ladley, 2016). In our settings, it is not necessary to define monetary policy signals solely as changes in the CB interest rate. Following Hubert (2015) we accept that the policy effects are transmitted to the economy not only through changes in the nominal interest rate, but also by various formal and informal communication and signalling of the intentions and expectations of the policy decision makers. Hence we assume that a policy action is undertaken if the value of such a forecast signal is above or below certain thresholds at which the marginal cost of issuing a signal equals its expected benefit. As (3) makes these thresholds binding for $E_t^{CB}\pi_{t+1} - E_t\pi_{t+1}^*$, they should not be confused with inflation targeting bands or other published quantitative indicators for the level of inflation.

Finally, using (4) and (5) and the policy rule with clear signals we formulate the following process for explaining the distribution of the policy-prone uncertainty U under the assumptions [1]-[5]:

$$U = X + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \ , \tag{6}$$

$$(X,Y) \sim N\left(\begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix}, \begin{bmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{bmatrix} \right) , \tag{7}$$

where components X and Y are introduced in (4) and (5), $I_{\{\bullet\}}$ is the indicator function of a set $\{\bullet\}$, $\alpha, \beta, \mu_X, \mu_Y, \bar{k} < \bar{m} \in \mathbb{R}$, $\sigma_X^2, \sigma_Y^2 \in \mathbb{R}^+$, and $0 < \rho < 1$. We call the distribution of U the *weighted skew-normal* and denote it as $U \sim \mathrm{WSN}_{\sigma_X,\sigma_Y}^{(\mu_X,\mu_Y)}(\alpha, \beta, \bar{m}, \bar{k}, \rho)$.

The parameter ρ is the correlation coefficient between X and Y and represents the ability of the CB forecasters in explaining X. The variance of Y cannot practically be greater than the variance of X, as the overall expertise of CB forecasters cannot exceed the overall uncertainty. So in order to simplify the model it is reasonable to analyse (6)-(7) for the marginal case assuming that the variances of X and Y are identical, meaning $\sigma_X^2 = \sigma_Y^2 = \sigma^2$. Additionally, we can set $\mu_X = \mu_Y = 0$, which results from the assumption that the forecasts $E_t \pi_{t+1}^*$ are ex-ante unbiased.

According to (6) and (7) and under the additional assumptions about the equality of variances of X and Y, $U = X \sim N(0, \sigma^2)$ if $\alpha = \beta = 0$. This suggests the interpretation of α and β in the light of actions and outcomes of some policy that affects inflation. If the policy is to be effective in reducing inflation uncertainty, the parameters α and β should be negative.

The parameters $\bar{m}$ and $\bar{k}$ denote the upper and lower signal thresholds. A breach of these thresholds means that a CB forecast signal is clear and that a monetary policy decision needs to be taken: an anti-inflationary decision if $\bar{m}$ is breached from below or a pro-inflationary one if $\bar{k}$ is breached from above.

For $\mu_X = \mu_Y = 0$ and $\sigma_X = \sigma_Y = \sigma$, we have six unknown parameters in the WSN: $U \sim \mathrm{WSN}_{\sigma,\sigma}^{(0,0)}(\alpha, \beta, \bar{m}, \bar{k}, \rho)$. The probability density function (*pdf*) of a random variable that follows this distribution with $\sigma = 1$ is given by:

$$f_{\mathrm{WSN}_{1,1}^{(0,0)}(\alpha,\beta,\bar{m},\bar{k},\rho)}(t) = \frac{1}{\sqrt{A_\alpha}} \varphi\left(\frac{t}{\sqrt{A_\alpha}}\right) \Phi\left(\frac{B_\alpha t - \bar{m} A_\alpha}{\sqrt{A_\alpha (1-\rho^2)}}\right) + \frac{1}{\sqrt{A_\beta}} \varphi\left(\frac{t}{\sqrt{A_\beta}}\right) \Phi\left(\frac{-B_\beta t + \bar{k} A_\beta}{\sqrt{A_\beta (1-\rho^2)}}\right)$$
$$+ \varphi(t) \cdot \left[\Phi\left(\frac{\bar{m} - \rho t}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{\bar{k} - \rho t}{\sqrt{1-\rho^2}}\right)\right] \quad , \tag{8}$$

where φ and Φ denote the density and cumulative distribution functions of the standard normal distribution respectively, $A_\tau = 1 + 2\tau\rho + \tau^2$, and $B_\tau = \tau + \rho$. The derivation of the *pdf* in (8) is shown in the Supplementary Material, Part 1.1, alongside the derivation of the moment generating function and other properties.

It can be shown that if $\alpha = -2\rho$ and $\beta = \bar{m} = 0$ in (6)-(7), the distribution of U coincides with the Azzalini (1985, 1986) skew-normal $\mathrm{SN}(\xi)$ distribution with a *pdf* of $f_{\mathrm{SN}}(t;\xi) = 2\varphi(t)\Phi(\xi t)$, where $\xi = \frac{-\rho}{\sqrt{1-\rho^2}}$. It is shown in the Supplementary Material, Part 1.1, that the *pdf* of the weighted skew-normal distribution $\mathrm{WSN}_{1,1}^{(0,0)}(\alpha, \beta, \bar{m}, \bar{k}, \rho)$ can be interpreted as a weighted sum of the *pdf*'s of two Azzalini-type skew-normal densities with different ξ's and a *pdf* of the conditional distribution $\left.\frac{X}{\sigma}\right| k \le \frac{Y}{\sigma} \le m$; hence the name for the distribution. It immediately follows from (6) and (7) that the WSN distribution is symmetric only if $\alpha = \beta = 0$ or if $\bar{k} = -\bar{m}$ and $\alpha = \beta$; otherwise, it is asymmetric.

4. SETTINGS FOR ESTIMATION

In order to obtain observations on U it is necessary to estimate the expected optimal time-invariant path of inflation $E_t\pi^*_{t+1}$, compute ex-post forecast errors and then adjust them using forecasts of past-dependent second moments. The experience of inflation forecasting in the US suggests single-equation models have some predictive advantage over multivariate models, with evidence usually favouring autoregressive models (Aron and Muellbauer, 2013; Clark and Ravazzolo, 2015) and unobserved components models (Stock and Watson, 2007, 2010). The advantage in using simple univariate models rather than more complex models in forecasting for sample sizes smaller than 500 has been confirmed by Constantini and Kunst (2011) and Mitchell, Robertson, and Wright (2015). The additional complication here is in deciding about the type of volatility. Results obtained for the US by Groen, Paap and Ravazzolo (2013), Clark (2011) and Eisenstat and Strachan (2015) show that stochastic volatility models are slightly superior to the more conventional autoregressive integrated moving average models with generalised autoregressive conditional heteroscedasticity (ARIMA-GARCH). However, there is some evidence for European countries, though it is weak, that ARIMA-GARCH models are superior to other models (see e.g. Bjørnland et al., 2012; Buelens, 2012). The forecasts of all these models can most probably be beaten by the forecasts derived through model averaging (Koop and Korobilis, 2012). Given such vastly different conclusions, we have decided for the sake of simplicity and transparency to resort to simple ARIMA-GARCH predictions.

Consequently, we recover observations on U by estimating the ARIMA-GARCH model in the *pseudo out-of-sample* way (see Stock and Watson, 2007), that is by computing predictions recursively within the observed sample, adding one observation at a time (see Appendix for details), re-estimating the model, and then computing forecast errors and forecast conditional standard deviations:

$$u_{t|t-h} = (\pi_t - \hat{\pi}_{t|t-h}) \cdot (\hat{\sigma}_h / \hat{\sigma}_{t|t-h}) \ , \ t = t_0,\ t_0+1, \dots, T-h,\ h = h_{\min}, h_{\min}+1 \dots, H \ , \quad (9)$$

where h is the forecast horizon, $\hat{\pi}_{t|t-h}$ is the ARIMA-GARCH baseline point forecast of inflation recorded at time t, obtained by using information available at time $t-h$, and $\hat{\sigma}_{t|t-h}$ and $\hat{\sigma}_h$ are respectively the conditional and unconditional standard deviations of the h-step ahead forecast error $\pi_t - \hat{\pi}_{t|t-h}$. In first recursion the sample is from 1 to t_0 ; T is the total sample size, so that the last sample for which the estimation is performed for a particular horizon h is from 1 to $T-h$; $h_{\min}$ is the minimal forecast horizon for identification of the policy-prone uncertainty and H is the maximal forecast horizon. In order to retrieve observations on the policy-prone uncertainty, we need to remove the uncertainty which is forecastable using agent's historical knowledge. This is the reason for scaling forecast errors in (9) by $\hat{\sigma}_h / \hat{\sigma}_{t|t-h}$.

After observations on the policy-prone uncertainty $u_{t|t-h}$ are recovered as in (9), they are used for estimating the WSN parameters. It is known that the maximum likelihood estimation (MLE) of the parameters of various types of skew-normal distribution is often numerically awkward, even though the closed form expression of the density functions makes it formally straightforward since there are possible bias and convergence problems (see e.g. Pewsey, 2000; Monti, 2003). As we also encountered such problems while trying to use the MLE for estimating WSN, we have decided to apply minimum distance estimators (MDEs) rather than the MLE. Under some general conditions, the MDE is asymptotically efficient and asymptotically equivalent to the maximum likelihood estimators (see Basu, Shioya and Park, 2011).

The minimum distance criteria can be defined in different ways. To estimate the WSN parameters $\theta = \{\alpha, \beta, \bar{m}, \bar{k}, \rho, \sigma\} \in \Theta \subset \mathbb{R}^6$ (some of which are later fixed and restricted in order to reduce dimensionality; see Section 6.1) we have used the Hellinger twice squared distance criterion (see Basu, Shioya and Park, 2011):

$$HD(d_n\ ,\ f_\theta) = 2\sum_{i=1}^{m}[d_n(i)^{1/2} - f_\theta(i)^{1/2}]^2 \quad , \qquad (10)$$

where n is the sample size, m is the number of disjoint intervals, $d_n(i)$ is the empirical frequency of data falling into the i^{th} interval and $f_\theta(i)$ is the corresponding theoretical probability for this interval. The properties of estimators based on Hellinger distances have been well researched in the context of other skew-normal distributions (see Greco, 2011), and it is known that the estimates are reasonably robust to the presence of outliers.

We approximate $f_\theta(i)$ by simulation. Details of this procedure, called the *simulated minimum distance estimator*, *SMDE,* are given in Charemza et al. (2012); a similar approach is used by Dominicy and Veredas (2013). The *SMDE* estimator of θ applied here can be defined as:

$$\hat{\theta}_n^{SMDE} = \arg\min_{\theta\in\Theta} \left\{ \bar{M} \left\{ HD(d_n\ ,\ f_{\theta,N,r} \right\}_{r=1}^{Nnepl} \right\} \quad ,$$

where:

- d_n — the empirical density of a sample of size n, as in (10);
- $f_{\theta,N,r}$ — an approximation of theoretical *pdf,* f_θ, obtained by generating a large random sample of size *N* (*N* was set to 10,001 in the empirical analysis described in Section 6). In order to avoid the problem of the 'noisy' criterion function, empirical approximation of the theoretical *pdf* is replicated *Nrepl* times, where $r = 1, 2, ..., Nrepl$;
- $\bar{M}$ — the aggregation operator, which is the median from $Nrepl$ runs.

The entire estimation process is numerically quite expensive. As the computations have to be made recursively, for a large number of countries and for different specifications of the model in the stability and robustness analysis (see Section 7) the computational burden can be cumbersome. Because of that we have decided to fix some of the parameters (discussed in Section 6.1).

5. MEASURES OF MONETARY POLICY EFFECTS

In order to formulate a measure which reflects the impact of the monetary policy on inflation forecast uncertainty, we introduce the concept of *quasi ex-ante* uncertainty as an approximation of the ex-ante uncertainty. It is defined as:

$$V = U - E(X \mid Y) = U - \rho Y \quad .$$

The quasi ex-ante uncertainty, V, approximates the ex-post uncertainty; it does not contain the elements known to CB forecasters which might be the cause of a monetary policy action. In the absence of such elements, it is equal to U.

If $U \sim \mathrm{WSN}_{(\sigma,\sigma)}^{(0,0)}\left(\alpha,\beta,\bar{m},\bar{k},\rho\right)$, the distribution of *V* also belongs to the WSN family as

$$\frac{1}{\sigma\sqrt{1-\rho^2}}V \sim \mathrm{WSN}_{(1,1)}^{(0,0)}\left(\frac{\alpha}{\sqrt{1-\rho^2}},\frac{\beta}{\sqrt{1-\rho^2}},\frac{\bar{m}}{\sigma},\frac{\bar{k}}{\sigma},0\right) \quad .$$

The standard deviation of *V*, denoted further as σ_V, can be interpreted as a proxy measure for the uncertainty which is fully unpredictable from the past and by the CB forecasters. Comparison between the root mean square error of *U*, $RMSE_U = \sqrt{\mathrm{var}(U) + E^2(U)}$, and σ_V could provide an idea of the influence that policy decisions might have on the distribution of inflation forecast errors.

Although *V* is not observable, it is possible to compute the ratio of σ_V^2 to the squares of $RMSE_U$ by evaluating the *uncertainty ratio* UR using the estimated parameters of WSN, as:

$$\mathrm{UR} \overset{def}{=} \frac{\sigma_V^2}{RMSE_U^2} = 1 - 2\rho\frac{\alpha D_m + \beta D_k + \rho/2 + \left[E(U^*)\right]^2}{RMSE_{U^*}^2} \quad ,$$

where $m = \bar{m}/\sigma$, $k = \bar{k}/\sigma$ and $U^* = \frac{1}{\sigma}U$ so that $U^* \sim \mathrm{WSN}_{1,1}^{(0,0)}(\alpha,\beta,\bar{m}/\sigma,\bar{k}/\sigma,\rho)$, with $E(U^*) = \alpha\cdot\varphi(m) - \beta\cdot\varphi(k)$, and

$$D_a = \int_{|a|}^{+\infty} t^2\varphi(t)dt = 1 - \Phi(|a|) + |a|\varphi(a) \quad . \tag{11}$$

In an unbiased case, which is where $\alpha = \beta$ and $\bar{k} = -\bar{m}$, UR is equal to unity if $\rho = 0$ or $\rho = -2\left[(\alpha D_m + \beta D_k)\right]$. For the derivation see the Supplementary Material, Part 1.2. Note that UR does not depend on σ, but rather on the ratios $m = \bar{m}/\sigma$ and $k = \bar{k}/\sigma$.

Deviations of UR from unity can be interpreted as the effects of CB forecasts (through ρ) and the strength of monetary policy (through $\alpha D_m + \beta D_k$), where α and β reflect the marginal intensity of monetary policy actions, and D_m and D_k defined in (11) reflect the frequency of such actions. Let us define the compound strength of monetary policy as:

$$S = |\alpha|\, D_m + |\beta|\, D_k \,. \tag{12}$$

Figure 1 plots UR for the fully symmetric case, where $\alpha = \beta < 0$, $\sigma^2 = 1$, $m = -k = 1$, and there are different values of ρ, against $|\alpha| + |\beta| = S / D_1$, representing the normalised strength of the policy. When the monetary policy has very low strength, the UR is smaller than one (the yellow area on the plot) and it decreases as ρ increases. In this case, the variance of *U* increases relative to the variance of *V*, meaning the policy action actually increases uncertainty. Alternatively, if the strength of the monetary policy increases to the point where UR peaks, inflation uncertainty represented by $RMSE_U$ decreases relative to the variance of *V*. In this case, the monetary policy is effective in the sense that it results in reduced inflation uncertainty.

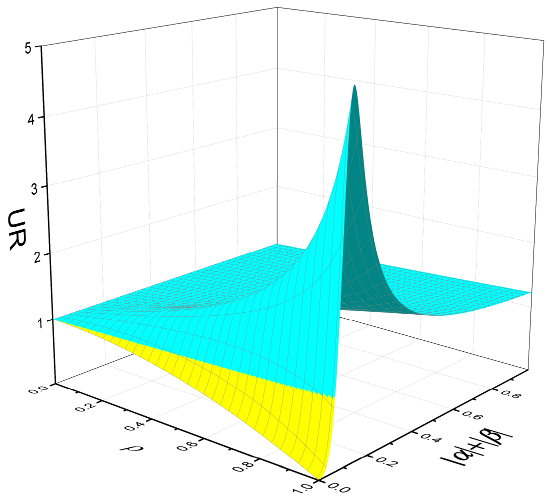
It is shown in the Supplementary Material, Part 1.2, that the maximum of UR for a given ρ and $m = -k$ is

$$\mathrm{UR}_{\max}(\rho)=1+\frac{4\rho}{\rho(1-4D)/D+2\sqrt{2(1-\rho^2)/D+\rho^2/(4D^2)}} \quad ,$$

where $D=D_m=D_k$, achieved when $\alpha=\beta=-\left(\rho+\sqrt{8D(1-\rho^2)+\rho^2}\right)/(4D)$. Therefore, the compound strength S defined by (12) that maximises the ratio of the ex-post to ex-ante uncertainty is

$$S_{\max}(\rho)=\left(\rho+\sqrt{8D(1-\rho^2)+\rho^2}\right)/2 \quad .$$

Figure 1: UR for the case where $\sigma^2=1$, $\alpha=\beta$, $m=-k=1$ and for different values of ρ. Values of UR smaller than one are in a lighter shade (yellow).



The ratio of UR to $\mathrm{UR}_{\max}$, called the *normalised uncertainty ratio*, NUR, can be interpreted as a compound, albeit symptomatic, measure of the effects of monetary policy. If NUR=1, then $\mathrm{UR}=\mathrm{UR}_{\max}$ and the parameters α and β are set at such levels that the ex-post uncertainty expressed by $RMSE_U^2$ cannot be reduced any further by changing α and β. The smaller NUR is, the greater the potential room for improvement might be in using the forecastable elements in X to reduce the uncertainty.

6. EMPIRICAL RESULTS

To assess the practical relevance of UR and confirm the rationale of the assumptions imposed, we use data on inflation forecast errors for 38 countries, which are 32 OECD countries, 5 BRICS countries (Brazil, Russia, India, China and South Africa), and Indonesia. The series for CPI inflation are of different lengths for the different countries and all end in March 2017. The longest series, starting in January 1949, is for Canada with 807 observations, and the two shortest are for Estonia with 231 observations, and for China with

255 observations.[1] It is conjectured that if these countries conducted an effective monetary policy of some sort, it might affect the distribution of their policy-prone uncertainties. Although members of the European Monetary Union have not had autonomous monetary policies since the creation of the Eurozone, the decisions of the European Central Bank still influence inflation uncertainty in individual countries. In our approach, it is not relevant how the monetary policy decisions are made, as their effect on uncertainties is what matters.

Since 2013, the long-lasting downward trend in inflation in most of the countries in the group analysed here has also been accompanied by a decline in the dispersion in inflation rates. This might affect the statistical properties of the estimates. Therefore, to account for the effects of the recent stabilisation of global inflation, the computations have also been performed on a shorter sample, ending for all the countries in February 2013.

The aims of the empirical part are to:

(i) compute and interpret URs and NURs;

(ii) test the rationale of URs and the compound strength by showing their relationships with measures of central bank independence and transparency;

(iii) evaluate and discuss the aggregated forecast uncertainty measures.

6.1. Data on policy-prone uncertainty

Observations on U have been recovered by computing the uncertainties defined by (9) recursively using as $\hat{\pi}_{t|t-h}$ the ARIMA-GARCH(1,1) h-steps ahead point forecasts of the mean of inflation, and forecasts of the conditional and unconditional standard deviations of the ex-post forecast errors.[2]

The orders of integration of the series have initially been identified using a battery of 28 tests, which are the traditional GLS-detrended and optimal point unit root tests (see Ng and Perron, 2001, and Perron and Qu, 2007), and tests allowing for the presence of structural breaks under the null and alternative (see Carrion-i-Silvestre, Kim and Perron, 2009). These tests gave results that are overwhelmingly consistent with the automated differencing and the selection of lag polynomials, which is based on minimisation of the Bayesian Information Criteria (BIC). Models have been estimated by the quasi-maximum likelihood (QML) method.[3] Forecasts have been made for up to 24 periods ahead, meaning 24 months. For each country, we have started the recursions using the first 20% of observations if this totals more than 80 observations. Otherwise, we have used the first 80 observations. These forecasts have not been adjusted or manipulated. As a result, we have obtained a reasonably long series of forecasts for different forecast horizons, and then forecast errors, with the maximum number of sample observations for Canada at 663, and the smallest for Estonia at 231.

Next, we have used $u_{t|t-h}$ to estimate the parameters of the WSN distribution defined by equations (6) and (7) for each forecast horizon. To reduce the computational burden we have

[1] The raw CPI data can be downloaded from http://stats.oecd.org/. A full list of countries together with the details of data spans for individual countries is given in Appendix.

[2] For the sake of comparison, we have also repeated the computations for the forecast errors not scaled by the conditional standard errors. The results are similar to those reported here.

[3] For consistency and robustness of the QML method with asymmetric and non-normally distributed errors see e.g. Francq and Zakoïan (2012). The computations were made in GAUSS, mainly using the ALICE High Performance Computing Facility at the University of Leicester. The FANPAC package, by Ronald Schoenberg, has been adopted for estimation and forecasting of the GARCH model.

assumed that the decision thresholds are fixed relative to σ and are identical for all countries so that $m = \bar{m} / \sigma = -k = -\bar{k} / \sigma = 1$, meaning the thresholds are equal to one standard deviation of X and Y, and that the correlation coefficient ρ = 0.75, so that the level of knowledge of the CB forecasters is reasonably high. This leaves us with three parameters to be estimated: α, β and σ. The computations have also been repeated for different thresholds and correlation coefficients and the results seem to be relatively robust to changes in these parameters.

6.2. Uncertainty ratios and central bank efficiency

The rationale of the concepts of UR and compound strength defined in Section 5 can be confirmed if there is indeed a nonlinear relationship between them and some externally developed measures of the efficiency of central banks. If the findings illustrated in Figure 1 are to be supported by data, there should be a positive relationship between strength and the efficiency of the central banks as expressed by the indicators for independence and transparency, but only up to the optimal point given by $S_{\max}(\rho)$. Beyond this point, the compound strength is too big, and has an adverse influence on the policy effects (for a theoretical background see Amato, Morris and Shin, 2002; for empirical observations see Blinder et al., 2008).

We have used four measures of central bank independence introduced by Dincer and Eichengreen (2014) and have tested for possible relationships between them and URs and the compound strength of policy. These measures are LVAU, LVAW, CBIU and CBIW, and are based on the methodology by Cukierman, Webb and Neyapti (1992)[4]. They are the weighted and unweighted averages of a range of independence indicators computed from disaggregated data up to 2010 (see also Bodea and Hicks, 2015).

Additionally, we have used the Dincer and Eichengreen measure of the transparency of central banks, denoted here as TRM, which is not correlated with the measures of independence. To maintain approximate time correspondence, we have decided to use the URs and compound strengths computed with a shorter data series finishing in February 2013 rather than with the data up to March 2017. The Spearman's rank correlation coefficient between the measures of independence for the banks and the URs for the 23 countries with independent central banks, taking Germany as the main euro country, computed separately for each forecast horizon, are predominantly negative and insignificant.[5] However, we have found some mild evidence of a nonlinear relationship between the URs and the measures of central bank independence, illustrated by Figure 2.

Figure 2 plots the Dincer and Eichengreen (2014) LVAW against the compound strengths on the right vertical axis, and against the theoretical URs (that is, a slice of Figure 1 at $\rho = 0.75$) on the left vertical axis.

In the settings used, $\mathrm{UR}_{\max}$ corresponds to $S_{\max}(0.75) = 1.08$, as in Figure 1. There are 13 empirical URs with a compound strength greater than 1.08, and only 10 URs with smaller strength. Below the point of 1.08, the URs increase with the increase in the compound

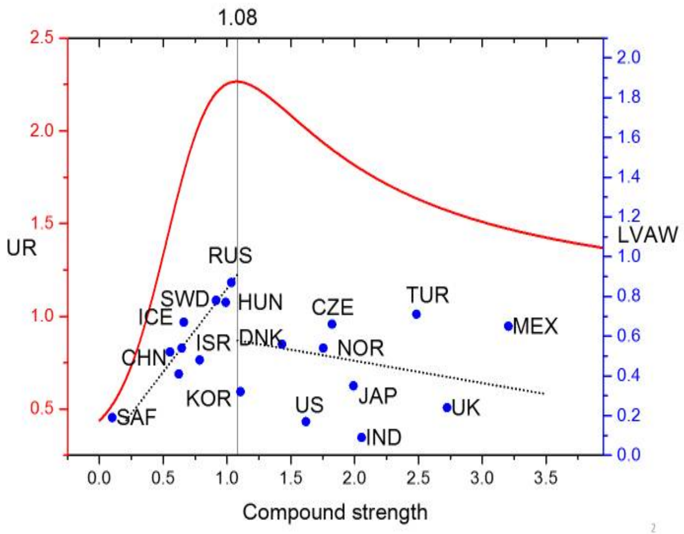
[4] LVAU and LVAW are central bank independence measures computed by Dincer and Eichengreen (2014), using the original Cukierman, Webb and Neyapti (1992) methodology. They are, respectively, unweighted and weighted, averages over a number of aggregates describing bank independence. CBIU and CBIV are analogous measures, also obtained by Dincer and Eichengreen (2014), using a modified methodology. For a critique and analysis of drawbacks of measures of the independence of central banks see Cargill (2013).

[5] In Appendix, countries with independent central banks are in boldface.

strength, and they decrease after that point, in line with the theoretical findings depicted in Figure 1. As expected, there is a clear positive correlation between the measures of central bank independence and the compound strength for the points below the threshold of 1.08 and a negative correlation for the points above this threshold.

The findings above can be summarised in Table 2 for all forecast horizons in the form of a simple split test. In this test, we claim that we have a weak confirmation of the split relationship between the compound strength and the measures of central bank independence if, for a given forecast horizon, the coefficient of regression of a particular measure of independence or transparency on the compound strength is positive for compound strength below and negative otherwise. We claim to have a semi-strong confirmation if one of these coefficients is significant at the 10% significance level, and a strong confirmation if both regression coefficients are significant. We have applied it for the LVAU, LVAW, CBIU, CBIV and TRM measures.

Figure 2 URs, compound strength and central bank efficiency



Legend: Continuous line represent UR's for $\rho = 0.75$ (see Figure 1), measured at the left-hand side vertical axis. Dots represent values of LVAW index, measured on the right-hand side vertical axis, against the compound strength. The dotted lines represent least-squares regression lines, fitted separately to points to the left and to the right of the compound strength which corresponds to the maximal UR.

Table 2 shows the number of forecast horizons, out of 24, where there is a positive result for the weak, semi-strong and strong tests. The tests have been computed in three versions: for the full sample of 38 countries; for 23 countries with independent central banks, so excluding the euro countries; and for 22 countries, where South Africa has been additionally removed as an outlier, as it is possible that some data for South Africa might have been misrecorded. The results confirm to an extent the existence of the relationship, as the signs of the regression coefficients are consistent with those expected in 17 forecast horizons out of 24 for all the measures of independence and transparency if the sample of all 38 countries is used, for 19-21 cases for LVAU, LVAW CBIU and CBIW, and for 15 cases for TRM for samples of 23 and 22 countries. As most of the regression coefficients are statistically insignificant and the data sample is small, this confirmation is, however, quite weak. Nevertheless, it provides some evidence that the independence of the central bank contributes positively towards a reduction in forecast uncertainty if the strength of the monetary policy is not more than the optimal; otherwise, the policy is too strong, and the contribution is negative. This is in line with current findings that a high degree of independence for central banks can sometimes be sub-optimal (see e.g. Hefeker and Zimmer, 2012; Hielscher and Markwardt, 2012).

Table 2: Split test results for the compound strength and measures of central bank independence and transparency for 24 forecast horizons.

	Independence measures				Transparency measure
	LVAU	LVAW	CBIU	CBIW	TRM
Weak test, 38 countries	17	17	17	17	17
Weak test, 23 countries	19	19	19	19	15
Weak test, 22 countries	21	21	21	21	15
Semi-strong test, 38 countries	2	2	2	2	4
Semi-strong test 23 countries	2	3	2	2	5
Semi-strong test 22 countries	2	3	2	2	6
Strong test, 38 countries	0	0	0	0	0
Strong test 22 & 23 countries	0	0	0	0	1

Note: Tests have been computed separately for each forecast horizon from 1 to 24, using samples of 38, 23 and 22 countries. The numbers in the main body of the table indicate the number of forecast horizons out of a maximum of 24 for which a particular test result is positive.

The relationship with the transparency of banks, if discovered, is more strongly confirmed than that with the independence of banks. One of two split regressions is significant for six forecast horizons, and the sign is consistent with expectations if the sample of 22 countries is used. For the relationship with measures of bank independence (TRM), this is the case for only two or three horizons. It can also be noted that including the euro countries, which do not have their own central banks, in the sample increases the significance and frequency of the relationship between compound strength and transparency very substantially, but it acted adversely for the relationship with the independence of the banks. This was also an attempt to relate strength and the UR to the measures of the conservatism of the central banks in a linear and a nonlinear way (see Levieuge, Lucotte, and Pradines-Jobert, 2017). However, no such relationship has been found.

6.3. Aggregated forecast uncertainty measures

To summarise the empirical estimates of policy effects, we present the uncertainty characteristics introduced in Section 5, in aggregated form for the UK, the US and the BRICS countries. These countries conduct some sorts of monetary policy systematically, but apart from that there is not much in common between them for how policy is implemented in practice. The UK, Brazil and South Africa were among the first countries to adopt transparent inflation targeting, with the UK doing so in 1992, Brazil in 1999, and South Africa in 2000, and though the US explicitly adopted inflation targeting only in 2012, in practice it had been implemented much earlier in a clandestine way (see e.g. Goodfriend, 2003). Inflation targeting in Brazil has been contaminated to some extent by systematic efforts to stabilise exchange rate fluctuations and occasional administrative decisions to freeze wages and prices (see e.g. Garcia, Gullén, and Kehoe, 2015). India adopted targeting much later, in 2016, while China and Russia, although officially targeting inflation, were effectively trying to control the exchange rate for most of the time under study. Moreover, the BRICS countries have not coordinated their monetary policies, as their aims have mainly been political and strategic. What they have in common is their desire to conduct monetary policy without involving the International Monetary Fund.

Although there is a relatively large body of empirical evidence on the dynamic effects of monetary policy on inflation, we were not able to find convincing results for such effects on inflation forecast uncertainty. There must surely be a time delay between a monetary policy action and its eventual effect on inflation forecast uncertainty, but the length of any such delay has not so far been investigated. Consequently, it is difficult to identify values of the minimum and maximum forecast horizons for which the monetary policy effects might be visible in the policy-prone uncertainty on an empirical basis. To minimise the arbitrariness of choice and, at the same time, present the results in a more compact way, we have aggregated the uncertainty characteristics across forecast horizons using Samuelson time weights (for the behavioural interpretation of the Samuelson concept of temporal aggregation, properties and alternatives, see al-Nowaihi and Dhami, 2014). We also use the reversed weights. More precisely, we use weights which decay exponentially with the increase in forecast horizon then reverse their order, that is we assign the highest weight to the longest forecast horizon. We call the former *short aggregation* and the latter *long aggregation*. The reason for using two different ways of weighting is a consequence of the fact that we are unsure of the time lag between the policy action and its effect on uncertainty. The comparison of short- and long-aggregated results shows whether the response of the uncertainty to a policy shock is stronger in shorter, or longer periods. If, for a given country, the short-aggregated is greater than the long-aggregated one, it can be seen that the policy effects for short forecast horizons are, on average, greater, than for the longer.

Tables 3a and 3b give the characteristics of the quasi ex-ante uncertainties for the same group of countries aggregated over the forecast horizons. Table 3a shows that for the shorter span of data ending in February 2013, China, as represented by its $RMSE_U$, has the second smallest short-aggregated inflation forecast errors after the US. However, China's short-aggregated UR is smaller than those of other countries. This shows that such low uncertainty was due to external factors rather than to successful policy interventions. China's short-aggregated NUR is also small, indicating potential room for improvement as an increase in the policy strength would, in this case, be likely to lead to a decrease in uncertainty. Indeed, for the longer span of data until March 2017 (Table 3b), China's short-aggregated UR increased. A similar phenomenon can be observed for Russia, where low values of short-aggregated UR for the data up to February 2013 are accompanied by low NUR. The low NUR demonstrates that a subsequent improvement is possible in the effects of monetary policy shown by the increased short-aggregated UR for the data up to March 2017. Such an improvement resulted in a significant decrease in inflation forecast uncertainty, as expressed by a reduction in $RMSE_U$.

Table 3a: Forecast uncertainty measures for the BRICS countries, the UK and the US, aggregated across forecast horizons (data until Feb 2013)

	Short aggregation	Long aggregation

Country	$RMSE_U$	σ_V	UR	NUR	$\hat{\sigma}$	$RMSE_U$	σ_V	UR	NUR	$\hat{\sigma}$
BRA	4.14	5.05	1.45	0.86	0.82	21.7	24.4	1.02	0.82	3.29
CHN	1.69	2.02	1.24	0.76	1.53	8.88	10.2	1.05	0.84	3.98
IND	5.28	7.13	1.49	0.91	1.40	23.4	27.4	1.17	0.94	3.78
RUS	3.70	4.62	1.30	0.79	1.92	22.4	25.0	0.98	0.79	3.99
SAF	3.69	5.11	1.41	0.87	1.10	5.51	6.71	1.20	0.96	3.76
UK	2.79	3.49	1.41	0.88	0.68	14.6	16.8	1.15	0.92	1.84
US	1.32	1.76	1.45	0.87	0.60	7.26	8.67	1.19	0.95	1.62

Table 3b: Forecast uncertainty measures for the BRICS countries, the UK and the US, aggregated across forecast horizons (data until March 2017)

	Short aggregation					Long aggregation				
Country	$RMSE_U$	σ_V	UR	NUR	$\hat{\sigma}$	$RMSE_U$	σ_V	UR	NUR	$\hat{\sigma}$
BRA	0.45	0.60	1.34	0.84	0.27	1.26	1.45	1.07	0.85	0.69
CHN	0.81	1.00	1.33	0.82	0.66	8.34	9.95	1.19	0.95	1.94
IND	1.19	1.46	1.28	0.78	0.78	7.06	8.14	1.06	0.85	3.81
RUS	0.82	1.03	1.42	0.88	0.66	5.12	6.17	1.20	0.96	1.61
SAF	1.06	1.33	1.32	0.82	0.82	5.57	6.37	1.21	0.87	1.79
UK	1.06	1.29	1.37	0.84	0.56	13.4	15.4	1.15	0.92	1.69
US	0.52	0.63	1.38	0.85	0.43	6.56	7.84	1.18	0.94	1.78

Legend: $\hat{\sigma}$ is the estimated σ parameter in the WSN distribution (6)-(7).

It can also be noted that for the period up to February 2013, the long-run aggregated URs and NURs are the highest for India, South Africa, the US and the UK. India did not introduce inflation targeting until 2016, and the other three countries have long-established and reasonably strong inflation targeting policies, official or clandestine. This is less evident for the longer data period, as URs and NURs do not differ much between countries. Consequently, it indicates that once monetary policy targets inflation, the monetary authorities should not allow the policies of inflation targeting and exchange rate stabilisation to be mixed, as they were in China and Russia before 2013.

7. FURTHER RESULTS ON ESTIMATION, ROBUSTNESS AND STABILITY

In this section we outline more detailed results related to testing the rationale of the estimation approach we applied, the robustness to changing data definition and estimation periods and the stability of the results over time. To provide a rationale for the choice of distribution, we compare fit of the WSN to that of other distributions used for modelling inflation forecast errors and uncertainty. To check, to what extent the type of autoregressive volatility used for scaling forecast errors in (9) affects the outcomes, we compare the results obtained using symmetric and asymmetric GARCH respectively. To check whether our results are robust to the way inflation is measured, and to the choice of estimation period, we use different types of inflation data and estimate the model using windows of different length. We also comment on the development in time and stability of the estimated URs. Below we provide a summary of this investigation, with the supporting data given in the Supplementary Material, Part 2.

7.1 Model estimation, validation and comparison

We provide an ad-hoc validation of our estimation approach by counting the frequency of positive signs of the estimated α's and β's in (6), that is, the cases where there is an inconsistency with the policy rule described in Section 2. In order for the monetary policy to be consistent with the goal of minimisation the intertemporal policy rule, signs of α's and β's should be negative, that is, an anti-inflationary policy should result in reducing inflation, and pro-inflationary policy in its increase. Tables S1.1-S1.2 in the Supplementary Material, Part 2, give the frequencies of wrong, that is, positive, signs of α's and β's, aggregated for forecast horizons from 1 to 24. The results are reported for countries carrying out, at some stage, active inflation targeting (that is, excluding China and Russian Federation, as these countries conducted, in reality, a mixed targeting policy). The estimates were made in rolling windows of the length of 60, with forecast errors in (9) respectively scaled by the predicted conditional standard deviations of the symmetric and asymmetric GARCH processes. The results have been obtained for windows covering the entire span of data, and also, separately, for periods prior and after the introduction of inflation targeting, prior and during the period of geopolitical instability defined by the beginning of the Libyan crisis (February 2011) and for the period of Great Moderation (June 1985-June 2007) and after.

The results given in Tables S1.1-S1.2 in the Supplementary Material, Part 2, show that, for most countries, the frequency of wrong (positive) signs of the estimated α's and β's is negligible, that is, smaller than 0.1. The exceptions are India, Israel and, to a lesser extent, Korea. Disaggregated results (available on request) reveal a tendency of an increase in the frequency of wrong signs with the increase in forecast horizon. For all countries, there are no wrong signed estimates for the forecast horizons below 6. The results are remarkably robust to a change of geopolitical (Libyan crisis) and economic (Great Moderation) regimes. Also, they do not seem to be strongly affected by an introduction of inflation targeting. For most countries, the results for the case where asymmetric GARCH was used for scaling do not differ much from that of the symmetric GARCH. The exceptions here are Korea, for which numerical problems appeared in computing the asymmetric GARCH results, and Indonesia, where the frequency of wrong signs is markedly higher in the asymmetric case.

We have also compared the fit of WSN to data with that of two other distributions are often used for modelling inflation forecast errors, these being the two-piece skew-normal distribution, TPN, and the generalised three-parameters beta distribution, GB. TPN has often been used for representing uncertainty in fan charts of inflation (for the statistical properties of TPN see John, 1982, and Kimber, 1985; for wider discussion and its use in the context of fan-chart modelling see e.g. Tay and Wallis, 2000). Three-parameters GB has been used for the US by Engelberg, Manski and Williams (2009) to approximate the empirical distribution of a panel of forecasts, and by Clements (2014) to interpolate the histograms representing probabilistic forecasts, or ex-ante uncertainty; for other economic applications and generalisations see McDonald and Xu (1995).

In order to fare WSN against these two competing distributions, we analyse the relative (that is, divided by that of WSN) Hellinger distance measures averaged over 60-months windows, presented in Tables S1.3-S1.4 in the Supplementary Material, Part 2. The TPN or GB distribution has a better (worse) fit that that of WSN if the relative Hellinger distance is smaller (greater) than one. Overall, the fit of GB is worse than that of WSN and TPN in most cases, though it gains some advantage for longer forecast horizons. For data ending in February 2013 and shorter forecast horizons, WSN fits the data best more often than the other distributions do. The fit of WSN is also the best for the UK and the US, except with the forecast horizon of one. It loses its advantage for data ending in March 2017, but the differences in fit between WSN and TPN are not substantial.

For the samples ending in March 2017, TPN has a clear advantage over WSN, though it is small in absolute terms (see Tables S1.3-S1.4 in the Supplementary Material, Part 2). It is most likely that the stabilisation of the inflation dispersion after 2013 resulted in some fuzziness in the identification of policy effects in the uncertainty. It should be noted that the fit of all distributions to data for such samples is much better than it is for the shorter samples. Similar results have been obtained for other countries.

7.2 Use of different inflation indicators

We have repeated the computations for the US and the UK using data for core and HICP inflation rather than CPI. For the US, we also investigated the robustness of our findings by using the Personal Consumption Expenditures price index (PCE). The PCE is the price index for which the Federal Reserve sets its inflation target. The results are given in Table S2.4, with data described in Tables S2.1-S2.3 in the Supplementary Material, Part 2.

Core inflation results for both countries and both short and long aggregations exhibit markedly smaller ex-post and pseudo ex-ante uncertainties than that for CPI, HICP and, in case of the US, PCE. It is understandable as, in core inflation, unlike in other inflation measures, the transitory elements of inflation are removed, which makes it easier to predict. There is not much difference between the aggregated URs for CPI and core inflation in the UK. For the US, PCE has the smallest aggregated uncertainty among all compared, but only for relatively long forecast horizons. However, its UR and NUR are below that for CPI and core inflation. This indicates that there is some room for a further reduction in PCE uncertainty. It is quite difficult to interpret differences between uncertainties based on HICP and those from other inflation measures. This might be because the theoretical foundations of HICP are not usually regarded as sound (e.g. Wynne, 2008).

7.3 Time stability

Time stability of the parameters’ estimates has been already discussed in Section 7.1 above, where the estimates for different geopolitical and economic regimes have been discussed. For further checks, the computations of URs have been repeated for the UK, US and the BRICS countries in time-moving windows. The windows are of two types: rolling windows with a width of 48 observations, so moving by one observation at a time, and windows of expanding size, with subsequent observations added one by one. Time paths of these measures are given in the Supplementary Material, Part 2, at Figures S3.1-S3.2.

The series of URs presented at Figures S3.1-S3.2 generally show a relative stability over time. There is a clear upward shift of the short-aggregated URs for the US in the second half of 1996, after which the short-run policy effects constantly prevail over the long-run effects, as the short-aggregated UR is greater than the long-aggregated UR. This could be attributed to the relaxation of monetary policy in 1996 (see e.g. Goodfriend, 2015). The period of the Great Moderation of 1985, -2007 does not leave a mark on the time series of the URs in the UK and the US. Relaxing exchange rate controls in 2014-15 gave China and Russia a marked increase in the short-aggregated UR, indicating monetary policy was more effective in the short run than in the long run.,

7. CONCLUSIONS

The ex-post forecast uncertainty measured simply by the variance of past forecast errors is usually of interest to economic agents who do not have any influence on economic policy and who do not really care about what is known to the central bankers and what is not. However, inflation forecast errors can tell us more. In particular, if the weighted skew-normal distribution is fitted to data on policy-prone uncertainty, that is, to inflation forecast errors adjusted for the forecast of the historical volatility, the footprints of this policy can, at least partly, be revealed. Consequently, we suggest a measure that uses a comparison of the dispersion characteristics of the quasi ex-ante and ex-post distributions of the uncertainty to reveal such footprints. It can be applied as an alternative or as a substitute for a purely ex-ante uncertainty obtained from surveys of forecasters, particularly given that this is often either not available or not reliable. Our proposed measure is really a *back to the future* one, as it requires knowledge of ex-post forecast errors and the parameters of the weighted skew-normal distribution fitted to them. This measure is, to an extent, free of the elements known to central bank forecasters and consequently of the effects of the monetary policy decisions. Because of that, it might be of interest to policy makers who do not want the picture of uncertainty to be blurred by the outcomes of their own decisions. Instead, they may be more interested in learning what the uncertainty would have been if they had not carried out the policy. Empirical results obtained for the BRICS countries, the US and the UK, suggest that better effects in reducing inflation forecast uncertainty may be achieved if there is no inflation targeting at all rather than if an inflation targeting policy is mixed with exchange rate stabilisation.

We also explained that indicators for the independence and transparency of central banks do not seem to be correlated with the policy outcomes. We showed that such a relationship is, in fact, nonlinear, and that policy strength is positively related to the independence and transparency of the central bank only if that strength is not too big; otherwise, the relation is negative. However, our evidence is statistically not very strong, and so it calls for further investigation.

ACKNOWLEDGEMENT

We thank the Referees of the earlier draft of the paper and the Editors of this issue for their constructive comments and advice. We are grateful to the participants at the 21st IIF workshop in Washington, D.C. *Workshop on Forecasting Issues in Developing Economies* (IMF, Washington, April 2017) and, in particular, Sangyup (Sam) Choi, Neil Ericsson and Gloria Gonzalez-Rivera for their comments and discussion. Financial support of the ESRC/ORA project RES-360-25-0003 *Probabilistic Approach to Assessing Macroeconomic Uncertainties* and the Opus 8 project 2014/15/B/HS4/04263 *Modelling macroeconomic uncertainty* of the National Science Centre in Poland is gratefully acknowledged. This research used the ALICE High Performance Computing Facility at the University of Leicester. We also thank Cristina Bodea for sharing her data with us. We are solely responsible for all remaining deficiencies.

Appendix: Country codes and data description

Data and estimation periods, CPI. Countries with independent central banks are in boldface.

Legend: *h*- forecast horizon

Country	Code	First recursion		Total sample size	Size first estimation window	No. of observations for WSN for h=1	No. of observations for WSN for h=24
		Start	End				
Austria	AUT	Jan-59	Aug-70	699	140	559	536
Belgium	BEL	Jan-56	Mar-68	735	147	588	565
Brazil	BRA	Jan-00	Aug-06	207	80	127	104
Canada	CAN	Jan-50	Jun-63	807	162	645	622
Chile	CHL	Jan-71	Mar-80	555	111	444	421
China	CHN	Jan-96	Aug-02	255	80	175	152
Czech Republic	CZE	Jan-92	Aug-98	303	80	223	200
Denmark	DNK	Jan-67	Jan-77	603	121	482	459
Estonia	EST	Jan-98	Aug-04	231	80	151	128
Finland	FIN	Jan-56	Mar-68	735	147	588	565
France	FRA	Jan-56	Mar-68	735	147	588	565
Germany	GER	Jan-56	Mar-68	735	147	588	565
Greece	GRC	Jan-56	Mar-68	735	147	588	565
Hungary	HUN	Jan-99	Aug-05	219	80	139	116
Iceland	ICE	Jan-87	Aug-93	363	80	283	260
India	IND	Jan-99	Aug-05	219	80	139	116
Indonesia	IDS	Jan-00	Aug-06	207	80	127	104
Ireland	IRE	Nov-75	Feb-84	497	100	397	374
Israel	ISR	Jan-87	Aug-93	363	80	283	260
Italy	ITA	Jan-56	Mar-68	735	147	588	565
Japan	JAP	Jan-71	Mar-80	555	111	444	421
Korea	KOR	Jan-56	Mar-68	735	147	588	565
Luxembourg	LUX	Jan-56	Mar-68	735	147	588	565
Mexico	MEX	Jan-00	Aug-06	207	80	127	104
Netherlands	NLD	Apr-60	Aug-71	684	137	547	524
Norway	NOR	Jan-56	Mar-68	735	147	588	565
Poland	POL	Jan-99	Aug-05	219	80	139	116
Portugal	PRT	Jan-56	Mar-68	735	147	588	565
Russian Federation	RUS	Jan-97	Aug-03	243	80	163	140
Slovak Republic	SLK	Jan-92	Aug-98	303	80	223	200
Slovenia	SLV	Jan-95	Aug-01	267	80	187	164
South Africa	SAF	Jan-58	Nov-69	711	143	568	545
Spain	SPA	Mar-55	Jul-67	745	149	596	573
Sweden	SWE	Jan-56	Mar-68	735	147	588	565
Switzerland	SWZ	Jan-56	Mar-68	735	147	588	565
Turkey	TUR	Jan-04	Aug-10	159	80	79	56
United Kingdom	UK	Jan-56	Mar-68	735	147	588	565
United States	US	Jan-56	Mar-68	735	147	588	565

References

Al-Nowaihi, A., Dhami, S. (2014). Foundations and properties of time discount functions', Department of Economics, University of Leicester, Working Paper No. 14/11.

Amato, J. D., Morris S., Shin, H. (2002). Communication and monetary policy. *Oxford Review of Economic Policy* **18**: 495-503.

Andrade, P., Le Bihan, H. (2013). Inattentive professional forecasters. *Journal of Monetary Economics* **60**: 967-982.

Aron, J., Muellbauer J. (2013). New method for forecasting inflation, applied to the US. *Oxford Bulletin of Economics and Statistics* **75**: 637-661.

Azzalini, A. (1985). A class of distribution which includes the normal ones. *Scandinavian Journal of Statistics* **12**: 171-178.

Azzalini, A. (1986). Further results on a class of distributions which includes the normal ones. *Statistica* **46**: 199-208.

Baeriswyl, R., Cornand C. (2010). 'The signaling role of policy action', *Journal of Monetary Economics*, 57: 682–695.

Basu. A., Shioya H., Park, C. (2011). *Statistical inference: the minimum distance approach*. CRC Press.

Bjørnland, H. C., Gerdrup, K., Jore, A. S., Smith, C., Thorsrud, L. A. (2012). Does forecast combination improve Norges Bank inflation forecasts*? Oxford Bulletin of Economics and Statistics* **74**: 163-179.

Blinder, A. S., Ehrmann, M., Fratzscher, M., De Haan, J., Jansen, D.-J. (2008). Central bank communication and monetary policy: a survey of theory and evidence, *Journal of Economic Literature* **46**: 910-945.

Bodea, C., Hicks, R. (2015) Price stability and central bank independence: discipline, credibility, and democratic institutions. *International Organization* **69**, 35-61.

Buelens, C. 2012. Inflation forecasting and the crisis: assessing the impact of the performance of different forecasting models and methods. European Economy, European Commission, Economic Papers 451.

Cargill, T. F. (2013). A critical assessment of measures of central bank independence. *Economic Inquiry* **51**: 260-272.

Carrion-i-Silvestre, J. L., Kim, D., Perron, P. (2009). GLS-based unit root tests with multiple structural breaks both under the null and the alternative hypotheses. *Econometric Theory* **25**: 1754-1792.

Charemza, W., Fan, Z., Makarova, S., Wang, Y. (2012). Simulated minimum distance estimators in macroeconomics. Paper presented at the conference *Computational and Financial Econometrics,* Oviedo.

Charemza, W., Ladley, D. (2016). Central banks' forecasts and their bias: evidence, effects and explanation. *International Journal of Forecasting* **32**, 804-817,

Clark, T. E. (2011). Real-time density forecasts from Bayesian vector autoregressions with stochastic volatility. *Journal of Business and Economic Statistics* **29**: 327-341.

Clark, T. E., Ravazzolo, F. (2015), Macroeconomic forecasting performance under alternative specification of time-varying volatility, *Journal of Applied Econometrics* **30**, 551-575.

Clements, M. P. (2014). Forecast uncertainty – ex ante and ex post: US inflation and output growth. *Journal of Business and Economic Statistics* **32**: 206-216.

Clements, M. P., Galvão, A. B. (2014). Measuring macroeconomic uncertainty: US inflation and output growth. ICMA Centre Discussion Papers in Finance icma-dp2014-04, Henley Business School, Reading University.

Constantini, M., Kunst, R. M. (2011). Combining forecasts based on multiple encompassing tests in a macroeconomic core system. *Journal of Forecasting* **30**: 579-596.

Cukierman, A., Webb, S., Neyapti, B. (1992). Measuring the independence of central banks and its effect on policy outcomes. *World Bank Economic Review* **6**: 353–98.

Dincer, N. N., Eichengreen, B. (2014). Central bank transparency and independence: updates and new measures. *International Journal of Central Banking* **10**: 189-253.

Dominicy, Y., Veredas, D. (2013). The method of simulated quantiles. *Journal of Econometrics* **172**: 235-247.

Dowd, K. (2007). Too good to be true? The (In)credibility of the UK inflation fan chart. *Journal of Macroeconomics* **29**: 91-102.

Engelberg, J., Manski, C. F., Williams, J. (2009). Comparing the point predictions and subjective probability distributions of professional forecasters. *Journal of Business and Economic Statistics* **27**: 30-41.

Eisenstat, E., Strachan, R. W. (2015). Modelling inflation volatility. *Journal of Applied Econometrics*, published online.

Evans, G. W., Honkapohja, S. (1994). Learning, convergence and stability with multiple expectations equilibria, *European Economic Review* **38**, 1071-1098.

Eusepi, S., Preston, B. (2016). The science of monetary policy: an imperfect knowledge perspective', Staff report No. 782, Federal Reserve Board of New York.

Francq, C., Zakoïan, J.-M. (2012). QML estimation of a class of multivariate asymmetric GARCH models. *Econometric Theory* **28**: 179-206.

Garcia, M., Gullén, D., Kehoe, P. (2015). The monetary and fiscal history of Latin America: Brazil (mimeo).

Goodfriend, M. (2003). Inflation targeting in the United States? NBER Working Paper 9981.

Goodfriend, M. (2015). Monetary policy lessons from the 1990's for today, SOMC, New York,

Greco, L. (2011). Minimum Hellinger distance based inference for scalar skew-normal and skew-t distributions. *Test* **20**: 120-137.

Groen, J. J. J., Paap, R., Ravazzolo, F. (2013). Real-time inflation forecasting in a changing world. *Journal of Business and Economic Statistics* **31**: 29-44.

Hansson, P., Juslin, P., Winman, A. (2008). The role of short-term memory capacity and task experience for overconfidence in judgment under uncertainty. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, **34**, 1027–1042.

Hefeker, C., Zimmer, B. (2011). The optimal choice of central bank independence and conservatism under uncertainty. *Journal of Macroeconomics* **33**: 595-606.

Hielscher, K., Markwardt, G. (2012). The role of political institutions for the effectiveness of central bank independence. *European Journal of Political Economy* **28**: 286-301.

Holmes, D. R. (2014) '*Communicative Imperatives in Central Banks, Cornell International Law Journal*: **47** , 15-61,

Hubert, P. (2015). 'The influence and policy signalling role of FOMC forecasts'. *Oxford Bulletin of Economics and Statistics* **77**: 655-680.

John, S. (1982). The three parameter two-piece normal family of distributions and its fitting. *Communications in Statistics* **-** *Theory and Methods* **11**: 879-885.

Kimber, A. C. (1985). Methods for the two-piece normal distribution. *Communications in Statistics* **-** *Theory and Methods* **14**: 235-245.

Koop, G., Korobilis, D. (2012). Forecasting inflation using dynamic model averaging. *International Economic Review* **53**: 867-886.

Lahiri, K., Peng, H., Sheng, X. (2014). Measuring historical uncertainty and some tests for forecaster heterogeneity. Presented at the workshop 'Uncertainty and Economic Forecasting', UCL, London.

Lahiri, K., Sheng, X. (2010). Measuring forecast uncertainty by disagreement: the missing link. *Journal of Applied Econometrics* **25**: 514-538.

Levieuge, G., Lucotte, Y., Pradines-Jobert, F. (2017). Central banks' preferences and banking sector vulnerability. Eesti Pank, Working Paper 3/2017

López-Pérez, V. (2016). Does uncertainty affect participation in the European Central Bank's Survey of Professional Forecasters? Presented at the 10th International Conference on Computational and Financial Econometrics (CFE 2016), Seville.

Makarova, S. (2014). Risk and uncertainty: macroeconomic perspective, UCL SSEES Economic and Business Working Paper No. 129.

McDonald, J. B., Xu, Y. J. (1995). A generalization of the beta distribution with applications. *Journal of Econometrics* **66**: 133-152.

Mitchell, J., Robertson, D., Wright, S. (2015), 'What univariate models can tell us about multivariate macroeconomic models?', presented at WBS/Monash Workshop, the University of Warwick.

Monti, A. C. (2003). A note on estimation of the skew-normal and the skew exponential power distributions. *METRON – International Journal of Statistics* **61**: 205-219. (Accessed May 2016).

Morris, S., Shin, H.-S. (2002). Social value of public information. *The American Economic Review* **92**, 1521-1534.

Ng, S., Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica* **69**: 1519-1554.

Nunes, R. (2013), 'Do central banks' forecasts take into account public opinion and views?', Board of Governors of the Federal Reserve System,

Orphanides, A., Williams, J. C. (2005). 'Inflation scares and forecast-based monetary policy, *Review of Economic Dynamics* **8**: 498-527.

Ozturk, E., Sheng, X. (forthcoming), Measuring global and country-specific uncertainty, *Journal of International Money and Finance*.

Perron, P., Qu, Z. (2007). A simple modification to improve the finite sample properties of Ng and Perron's unit root tests. *Economics Letters* **94**: 12-19.

Pewsey, A. (2000). Problems of inference for Azzalini's skew-normal distribution. *Journal of Applied Statistics* **27**: 859-870.

Preston, B. (2005). 'Learning about monetary policy rules when long-horizon expectations matter', *International Journal of Central Banking* **1**: 81–126.

Preston, B. (2006). 'Adaptive learning, forecast-based instrument rules and monetary policy', Journal of Monetary Economics **53**: 507-535.

Soll, J. B., Klayman, J. (2004). Overconfidence in interval estimates. *Journal of Experimental Psychology: Learning, Memory, and Cognition* **30**, 299–314.

Stock, J. H., Watson, M. W. (2007). Why has U.S. inflation become harder to forecast? *Journal of Money, Credit and Banking* **39** (Supplement): 3-33.

Stock, J. H, Watson, M. W. (2010). Modelling inflation after the crisis. NBER, Working Paper No. 16488.

Tay, A. S., Wallis, K. F. (2000). Density forecasting: a survey. *Journal of Forecasting* **19**: 235-254.

Woodford, M. (2003). *Interest and prices: foundations of a theory of monetary policy,* Princeton University Press.

Wynne, M. W. (2008). Core inflation: a review of some conceptual issues. Review, Federal Reserve Bank of St. Louis 90, 205-228,

QUASI EX-ANTE INFLATION FORECAST UNCERTAINTY

Supplementary Material

February 2018

WOJCIECH CHAREMZA*, CARLOS DÍAZ[†] AND SVETLANA MAKAROVA[+]

[+]: University College London, UK, s.makarova@ucl.ac.uk (corresponding author), 16 Taviton Street, London WC1H 0BW, UK

*: Vistula University, Poland

[†]: University of Leicester, UK

The acknowledgements and disclaimers given in the paper apply also to this Supplementary Material.

CONTENTS

Part 1: WEIGHTED SKEW-NORMAL DISTRIBUTION AND THE UNCERTAINTY RATIO: PROPERTIES AND DERIVATIONS

1.1. WEIGHTED SKEW-NORMAL DISTRIBUTION

For a random variable Y and a real number a, notation $I_{Y>a}$ (or $I_{Y<a}$) denotes an indicator of the event $\{Y>a\}$ (or $\{Y<a\}$), which is equal to unity if $Y>a$ and zero otherwise.

Definition A1.1. Let X and Y constitute a bivariate normal random variable such as:

$$(X,Y) \sim N\left(\begin{bmatrix}\mu_X \\ \mu_Y\end{bmatrix},\begin{bmatrix}\sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2\end{bmatrix}\right), \text{ with } |\rho|<1. \tag{A1.1}$$

Let random variable U be defined as

$$U = X + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \text{ , where } \alpha, \beta, \bar{k} < \bar{m} \in \mathbb{R} . \tag{A1.2}$$

We call the distribution of U defined by (A1.1)-(A1.2) *weighted skew-normal* and denote it as $U \sim \mathrm{WSN}_{\sigma_X,\sigma_Y}^{(\mu_X,\mu_Y)}(\alpha,\beta,\bar{m},\bar{k},\rho)$.

Definition A1.2. A weighted skew-normal variable U^* with $\mu_X=\mu_Y=0$ and $\sigma_X=\sigma_Y=1$ is called *standard weighted skew-normal*. To simplify notation we use $U^* \sim \mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$, instead of $U^* \sim \mathrm{WSN}_{1,1}^{(0,0)}(\alpha,\beta,m,k,\rho)$.

Proposition A1.1. The probability density function (*pdf*) of the standard weighted skew-normal distribution $U^* \sim \mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$ is given by:

$$\begin{aligned} f_{\mathrm{WSN}_1}(t) = {} & \frac{1}{\sqrt{A_\alpha}}\varphi\left(\frac{t}{\sqrt{A_\alpha}}\right)\Phi\left(\frac{B_\alpha t - mA_\alpha}{\sqrt{A_\alpha(1-\rho^2)}}\right) + \frac{1}{\sqrt{A_\beta}}\varphi\left(\frac{t}{\sqrt{A_\beta}}\right)\Phi\left(\frac{-B_\beta t + kA_\beta}{\sqrt{A_\beta(1-\rho^2)}}\right) \\ & + \varphi(t)\cdot\left[\Phi\left(\frac{m-\rho t}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{k-\rho t}{\sqrt{1-\rho^2}}\right)\right] \quad , \end{aligned} \tag{A1.3}$$

where φ and Φ denote respectively the density and cumulative distribution functions of the standard normal distribution, $A_\tau = 1 + 2\tau\rho + \tau^2$, and $B_\tau = \tau + \rho$.

Proof. The cumulative distribution function (*cdf*) F_{WSN_1} of U^* can be obtained by integrating a normal bivariate *pdf* with zero means, unit variances, and the correlation coefficient of ρ over three disjoint areas as follows: $F_{WSN_1}(t) = \int_{-\infty}^{t-\alpha m} dx \int_{m}^{(t-x)/\alpha} + \int_{-\infty}^{t-\beta k} dx \int_{(t-x)/\beta}^{k} + \int_{-\infty}^{t} dx \int_{k}^{m}$. Taking the first derivative $dF_{WSN_1}(t)/dt$ completes the proof.

It immediately follows from Proposition A1.1 that f_{WSN_1} can be interpreted as a weighted sum of three *pdf*'s as $f_{\mathrm{WSN}_1}(t) = \alpha_1 f_1(t) + \alpha_2 f_2(t) + \alpha_3 f_3(t)$, where $\alpha_1 = \Phi(-m)$, $\alpha_2 = \Phi(k)$, and $\alpha_3 = \Phi(m) - \Phi(k)$, f_i $(i=1,2,3)$ are the three corresponding consecutive components of *pdf* (A1.3). The *pdf* f_3 is a *pdf* of the conditional variable $\left(X \mid k \le Y \le m\right)$. The relations between

f_1, f_2 and skew-normal distribution are as follows. A simple Azzalini (1985, 1986) skew-normal distribution $SN(\lambda,\omega)$ can be defined by its *pdf* as $f_{SN}(t;\lambda,\omega)=\frac{2}{\omega}\varphi(t/\omega)\Phi(\lambda x/\omega)$. Hence, for $m=k=0$ and $\alpha=-2\rho$ the functions f_1 and f_2 reduce to *pdf*'s of $SN(\lambda_1,\omega_\alpha)$ and $SN(\lambda_2,\omega_\beta)$ with $\lambda_{1,2}=\mp\rho/\sqrt{1-\rho^2}$ and $\omega_\tau=\sqrt{A_\tau}$ ($\tau=\alpha,\beta$). This representation allows for another interpretation of the Azzalini distribution, as $U^{SN}\sim\mathrm{WSN}_1(-2\rho,0,0,0,\rho)$, or $U^{SN}=X-2\rho Y\cdot I_{Y>0}$, where $X,Y\sim N(0,1)$ and $corr(X,Y)=\rho$.

The representation $f_{\mathrm{WSN}_1}(t)=\alpha_1 f_1(t)+\alpha_2 f_2(t)+\alpha_3 f_3(t)$ can now be interpreted as a weighted sum of the conditional *pdf* of $\left(X\,|k\le Y\le m\right)$ and the two *pdf*'s that, under some restrictions on parameters, coincide with that of the Azzalini skew-normal (hence the name of the WSN distribution).

Proposition A1.2. The moment generating function (MGF) of $U^*\sim\mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$ is given by:

$$R_{\mathrm{WSN}_1}(u)=e^{\frac{u^2}{2}A_\beta}\Phi(k-B_\beta u)+e^{\frac{u^2}{2}}\cdot\left[\Phi(m-\rho u)-\Phi(k-\rho u)\right]+e^{\frac{u^2}{2}A_\alpha}\Phi(B_\alpha u-m) \qquad \text{(A1.4)}$$

Proof. By definition,

$$R_{WSN_1}(u)=E\left(e^{u\cdot U^*}\right)=\frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{+\infty}dx\left[\int_{-\infty}^{k}e^{u[x+\beta\cdot y]}+\int_{k}^{m}e^{ux}+\int_{m}^{+\infty}e^{u[x+\alpha\cdot y]}\right]\cdot e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}}dy\,.$$

Changing the order of integration in each of the integrals above and noting that the MGF of the standard normal distribution is $e^{u^2/2}$ completes the proof.

Corollary. The moment generating function R_{WSN} of $U\sim\mathrm{WSN}_{\sigma_X,\sigma_Y}^{(\mu_X,\mu_Y)}(\alpha,\beta,\overline{m},\overline{k},\rho)$ is given by:

$$R_{\mathrm{WSN}}(u)=e^{u\mu_X}\left\{e^{\alpha\mu_Y u+\frac{(u\sigma_X)^2}{2}A_{\alpha\frac{\sigma_Y}{\sigma_X}}}\Phi\left(B_{\alpha\frac{\sigma_Y}{\sigma_X}}u\sigma_X-\frac{\overline{m}-\mu_Y}{\sigma_Y}\right)+e^{\beta\mu_Y u+\frac{(u\sigma_X)^2}{2}A_{\beta\frac{\sigma_Y}{\sigma_X}}}\Phi\left(\frac{\overline{k}-\mu_Y}{\sigma_Y}-B_{\beta\frac{\sigma_Y}{\sigma_X}}u\sigma_X\right)\right.$$
$$\left.+e^{\frac{(u\sigma_X)^2}{2}}\cdot\left[\Phi\left(\frac{\overline{m}-\mu_Y}{\sigma_Y}-\rho u\sigma_X\right)-\Phi\left(\frac{\overline{k}-\mu_Y}{\sigma_Y}-\rho u\sigma_X\right)\right]\right\}$$

Proof. It follows from the representation of $U\sim\mathrm{WSN}_{\sigma_X,\sigma_Y}^{(\mu_X,\mu_Y)}(\alpha,\beta,\overline{m},\overline{k},\rho)$ via $U^*\sim\mathrm{WSN}_1\left(\alpha\frac{\sigma_Y}{\sigma_X},\beta\frac{\sigma_Y}{\sigma_X},\frac{\overline{m}-\mu_Y}{\sigma_Y},\frac{\overline{k}-\mu_Y}{\sigma_Y},\rho\right)$ as $U=\sigma_X U^*+\mu_X+\mu_Y\cdot\left(\alpha I_{Y_0>m}+\beta I_{Y_0<k}\right)$, where Y_0 is standard normal. ■

Proposition A1.3. Let R_{WSN_1} be an MGF given by (A1.4), then

$R'_{\mathrm{WSN}_1}(0)=\alpha\cdot\varphi(m)-\beta\cdot\varphi(k)$;

$$R''_{\mathrm{WSN}_1}(0) = A_\alpha + \left[1 - A_\alpha\right]\Phi(m) + \left[B_\alpha^2 - \rho^2\right]m\varphi(m) + \left[A_\beta - 1\right]\Phi(k) - \left[B_\beta^2 - \rho^2\right]k\varphi(k)\,;$$

$$R^{(3)}_{\mathrm{WSN}_1}(0) = \varphi(m)\cdot\left\{B_\alpha\cdot\left[3A_\alpha + B_\alpha^2\left(m^2-1\right)\right] - \rho\cdot\left[3+\rho^2\left(m^2-1\right)\right]\right\} +$$
$$\varphi(k)\cdot\left\{-B_\beta\cdot\left[3A_\beta + B_\beta^2\left(k^2-1\right)\right] + \rho\cdot\left[3+\rho^2\left(k^2-1\right)\right]\right\} \quad ;$$

$$R^{(4)}_{\mathrm{WSN}_1} = 3\cdot\left\{A_\alpha^2 + \Phi(m)\cdot\left[1 - A_\alpha^2\right]\right\} + m\cdot\varphi(m)\cdot\left\{\left(3-m^2\right)\cdot\left(\rho^4 - B_\alpha^4\right) + 6\cdot\left(A_\alpha B_\alpha^2 - \rho^2\right)\right\}$$
$$+3\cdot\Phi(k)\cdot\left[A_\beta^2 - 1\right] - k\cdot\varphi(k)\left\{\left(3-k^2\right)\cdot\left(\rho^4 - B_\beta^4\right) + 6\cdot\left(A_\beta B_\beta^2 - \rho^2\right)\right\} \quad .$$

Proof. Substituting Taylor expansions of $e^{\frac{au^2}{2}}$ and $\Phi(bu+c)$ into $g_{a,b,c}(u) = e^{\frac{au^2}{2}}\Phi(bu+c)$ yields:

$$g_{a,b,c}(u) = \Phi(c) + b\varphi(c)\cdot u + \frac{1}{2}\left[a\cdot\Phi(c) - b^2c\cdot\varphi(c)\right]\cdot u^2 + \frac{1}{3!}\left[3a + b^2(c^2-1)\right]b\varphi(c)\cdot u^3$$
$$+\frac{1}{4!}\left[3a^2\Phi(c) - 6ab^2c\varphi(c) + b^4c(3-c^2)\varphi(c)\right]\cdot u^4 + \ldots \quad .$$

Bearing in mind that the MGF in (A1.4) can be expressed via $g_{a,b,c}$ as:

$$R_{\mathrm{WSN}_1}(u) = g_{A_\alpha, B_\alpha, (-m)}(u) + g_{A_\beta, (-B_\beta), k}(u) + g_{1,(-\rho),m}(u) - g_{1,(-\rho),k}(u)\,, \qquad \text{(A1.5)}$$

taking the derivative of both sides of (A1.5) and substituting the corresponding derivatives of $g_{a,b,c}$ at zero completes the proof.

Note. For $m>0$ and it is convenient to simplify the expression for $R''_{\mathrm{WSN}_1}(0)$ as:

$$R''_{\mathrm{WSN}_1}(0) = 1 + C_\alpha D_m + C_\beta D_k = 1 - 2\rho S + \alpha^2 D_m + \beta^2 D_k \;, \qquad \text{(A1.6)}$$

where

$$C_\tau = \tau(\tau + 2\rho)\,,\; D_a = 1 - \Phi(|a|) + |a|\varphi(a) = \int_{|a|}^{+\infty} t^2\varphi(t)dt \text{ and } S = -\alpha D_m - \beta D_k\,. \qquad \text{(A1.7)}$$

It immediately follows from the properties of MGF, Proposition A1.3 and (A1.6)-(A1.7) that the first and second moments of $U^* \sim \mathrm{WSN}_1(\alpha, \beta, m, k, \rho)$ can be given as

$$EU^* = \alpha\cdot\varphi(m) - \beta\cdot\varphi(k) \text{ and } E(U^*)^2 = 1 - 2\rho S + \alpha^2 D_m + \beta^2 D_k\,.$$

1.2. UNCERTAINTY RATIO

Definition A2.1. Let $U \sim \mathrm{WSN}_\sigma(\alpha, \beta, \overline{m}, \overline{k}, \rho)$ be defined by (A1.1)-(A1.2) and $V = U - E\left(X\middle|Y\right) = U - \rho Y$. Let also $RMSE_U = \sqrt{\mathrm{var}(U) + E^2(U)}$ and $\sigma_V = \sqrt{\mathrm{var}(V)}$.

We will define the *uncertainty ratio* of *U* and *V* as

$$\mathrm{UR} = \frac{\sigma_V^2}{RMSE_U^2} \quad .$$

The properties of the uncertainty ratio UR and its useful re-parameterisations are summarised in the following proposition.

Proposition A2.1. Properties of UR.

1) Noting that $EU = \sigma EU^* = \sigma\left[\alpha\varphi(m) - \beta\varphi(k)\right]$ and

$$\begin{aligned}\mathrm{var}(V) = \mathrm{var}(U) + \rho^2\sigma^2 - 2\rho E\left(X + \alpha\cdot Y\cdot I_{Y>\bar{m}} + \beta\cdot Y\cdot I_{Y<\bar{k}}\right)Y = \sigma^2\,\mathrm{var}(U^*) - \rho^2\sigma^2 \\ -2\rho\sigma^2\left(\alpha D_m + \beta D_k\right)\end{aligned},$$

where $U^* = \dfrac{1}{\sigma}U$, $m = \bar{m}/\sigma$, $k = \bar{k}/\sigma$.

This yields the following representation:

$$\mathrm{UR} = 1 - 2\rho\frac{\alpha D_m + \beta D_k + \rho/2 + \left[EU^*\right]^2}{\mathrm{var}(U^*) + \left[EU^*\right]^2} \quad ,$$

where $D_a = 1 - \Phi(|a|) + |a|\varphi(a) = \int_{|a|}^{+\infty} t^2\varphi(t)dt$.

2) For $m > 0$ and $k < 0$, by applying Proposition A1.1 we get another convenient expression for UR:

$$\mathrm{UR} = 1 + 2\rho\frac{S - \rho/2 + \left[\alpha\varphi(m) - \beta\varphi(k)\right]^2}{1 - 2\rho S + \alpha^2 D_m + \beta^2 D_k} \quad . \qquad \text{(A2.1)}$$

3) Derivation of the maximum UR (for fixed ρ) in a symmetric case of $m = -k$.

Denote: $D = D_m = D_k$, $\varphi = \varphi(m) = \varphi(k)$, $t = \left(S - \rho/2\right)/D = -(\alpha+\beta) - \dfrac{\rho}{2D}$. Hence (A2.1) reduces to

$$\mathrm{UR} = 1 + \frac{2\rho Dt + \left(\alpha - \beta\right)^2\varphi^2}{1 - 2\rho(Dt + \rho/2) + (\alpha^2 + \beta^2)D}. \qquad \text{(A2.2)}$$

Let $\tau = \alpha - \beta$, then

$$\alpha^2 + \beta^2 = \frac{1}{2}\left[(\alpha+\beta)^2 + (\alpha-\beta)^2\right] = \frac{t^2}{2} + \frac{t\rho}{2D} + \frac{\rho^2}{8D^2} + \frac{\tau^2}{2}. \qquad \text{(A2.3)}$$

Substituting (A2.3) into (A2.2) we get

$$\mathrm{UR} = 1 + \frac{4\rho}{F(t)} - \frac{\tau^2\varphi^2}{F(t)} \qquad \text{(A2.4)}$$

where

$$F(t) = t + \frac{(1-\rho^2)\dfrac{2}{D} + \dfrac{\rho^2}{4D^2} + \tau^2}{t} + \frac{\rho}{D}(1 - 4D). \qquad \text{(A2.5)}$$

Note, that the maximum UR, $\mathrm{UR}_{\max}$, is achieved for minimum F when t>0. Consider the function $G(t) = t + \frac{A}{t}$, where t, A>0. Hence: $\arg\min G(t) = t_0 = \sqrt{A}$ and $G_{\min} = G(t_0) = 2\sqrt{A}$. Therefore, for F defined by (A2.5), $A = (1-\rho^2)\frac{2}{D} + \frac{\rho^2}{4D^2} + \tau^2$ and for fixed $\tau > 0$ we have:

$$F_{\min,\tau} = 2\sqrt{(1-\rho^2)\frac{2}{D} + \frac{\rho^2}{4D^2} + \tau^2} + \frac{\rho}{D}(1-4D).$$

Obviously, $F_{\min,\tau} \geq F_{\min,\tau=0}$, and

$$F_{\min,\tau=0} = 2\sqrt{(1-\rho^2)\frac{2}{D} + \frac{\rho^2}{4D^2}} + \frac{\rho}{D}(1-4D),$$

which means that F achieves its minimum when $\tau = 0$, that is for $\alpha = \beta = \alpha_0$, and

$$t_0 = \sqrt{(1-\rho^2)\frac{2}{D} + \frac{\rho^2}{4D^2}}.$$

Noting that $t = \left(S - \rho/2\right)/D = -(\alpha+\beta) - \frac{\rho}{2D}$ gives $\alpha_0 = \frac{\rho}{4D} + \frac{1}{2}\sqrt{(1-\rho^2)\frac{2}{D} + \frac{\rho^2}{4D^2}}$.

Note also that $\min\left\{\frac{\tau^2\varphi^2}{F(t)}\right\} = 0$ and is achieved at $\tau = 0$, which, in combination with (A2.4), gives

$$\mathrm{UR}_{\max}(\rho) = 1 + \frac{4\rho}{F_{\min}} = 1 + \frac{4\rho}{2\sqrt{(1-\rho^2)\frac{2}{D} + \frac{\rho^2}{4D^2}} + \frac{\rho}{D}(1-4D)}$$

and is achieved at $\alpha_0 = \beta_0 = -\left(\rho + \sqrt{8D(1-\rho^2) + \rho^2}\right)/(4D)$.

Part 2: DATA DESCRIPTION AND ADDITIONAL EMPIRICAL RESULTS

2.1. MODELS ESTIMATION, VALIDATION AND COMPARISON

Table S1.1: Frequency of positive (that is, wrong) signs of $\hat{\alpha}$ and $\hat{\beta}$ for where $m = -k = \hat{\sigma}$ and $\rho = 0.75$; estimates made in rolling windows of the length 60, $h = 1,...,24$. Symmetric case: data for uncertainty is obtained from symmetric ARIMA-GARCH models.

Note: for forecast horizons h from 1 to 5 all frequencies are zeros for all countries for the full sample and for all splits.

	Dates of IT[1)]	Full sample	Split by IT[1)] date		Split by Libyan crisis February 2011		Split by Great. Moderation. June 1985 – June 2007	
			before	after	before	After	during	after
BRA	Jun-99	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CAN	Feb-91	0.003	0.007	0.000	0.004	0.000	0.005	0.000
CHL	Sep-99	0.008	0.015	0.003	0.008	0.008	0.010	0.005
CZE	Dec-97	0.001	0.001	0.001	0.001	0.002	0.000	0.002
IND	Jul-99	0.204	0.209	0.167	0.299	0.206	0.205	0.204
IDS	Jun-05	0.095	0.092	0.095	0.092	0.095	0.092	0.095
ISR	Jul-97	0.251	0.252	0.251	0.266	0.224	0.262	0.242
JAP	Jan-13	0.000	0.000	0.000	0.000	0.000	0.000	0.000
KOR	Jan-99	0.128	0.179	0.059	0.149	0.001	0.142	0.001
MEX	Jan-01	0.012	0.012	0.012	0.012	0.012	0.012	0.012
NOR	Mar-01	0.004	0.007	0.000	0.005	0.000	0.004	0.000
POL	Sep-98	0.031	0.031	0.031	0.046	0.031	0.031	0.031
SWE	Jan-93	0.015	0.009	0.021	0.018	0.000	0.024	0.000
UK	Oct-92	0.041	0.081	0.011	0.047	0.006	0.039	0.004
US	Jan-12	0.022	0.025	0.000	0.025	0.000	0.018	0.000

[1)] IT stands for Inflation Targeting

Table S1.2: Frequency of positive (that is, wrong) signs of $\hat{\alpha}$ and $\hat{\beta}$ for where $m = -k = \hat{\sigma}$ and $\rho = 0.75$; estimates made in rolling windows of the length 60, $h = 1,...,24$. Asymmetric case: data for uncertainty is obtained from asymmetric ARIMA-GARCH models.

Note: for forecast horizons h from 1 to 5 all frequencies are zeros for all countries for the full sample and for all splits.

Country Code	Dates of IT[1]	Full sample	Split by IT[1] date		Split by Libyan crisis February 2011		Split by Great. Moderation. June 1985 – June 2007	
			before	after	before	after	during	after
BRA	Jun-99	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CAN	Feb-91	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CHL	Sep-99	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CZE	Dec-97	0.020	0.020	0.020	0.046	0.000	0.012	0.021
IND	Jul-99	0.004	0.003	0.005	0.005	0.004	0.004	0.004
IDS	Jun-05	0.209	0.206	0.209	0.206	0.209	0.206	0.209
ISR	Jul-97	0.150	0.150	0.150	0.199	0.057	0.199	0.111
JAP	Jan-13	0.000	0.000	0.000	0.000	0.000	0.000	0.000
KOR[2]	Jan-99	NA	NA	NA	NA	NA	NA	NA
MEX	Jan-01	0.000	0.000	0.000	0.000	0.000	0.000	0.000
NOR	Mar-01	0.000	0.000	0.000	0.000	0.000	0.000	0.000
POL	Sep-98	0.002	0.002	0.002	0.002	0.002	0.002	0.002
SWE	Jan-93	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UK	Oct-92	0.013	0.029	0.000	0.015	0.000	0.001	0.000
US	Jan-12	0.000	0.000	0.000	0.000	0.000	0.000	0.000

[1] IT stands for Inflation Targeting
[2] Results not reported due to convergence problems

Table S1.3: Relative minimum distance characteristics of distributions fitted to *U*-uncertainties for 38 countries. Data until February 2013. Full sample estimates for selected forecast horizons. Countries with independent central banks are in boldface.

Legend: *h*- forecast horizon; MD_{WSN}^{TPN} - ratio of Hellinger Distance of TPN distribution to Hellinger Distance of WSN distribution fitted to *U*-uncertainties; MD_{WSN}^{GB} - ratio of Hellinger Distance of GB distribution to Hellinger Distance of WSN distribution fitted to *U*-uncertainties. In the body of the Table, DIV denotes the cases where there is a possible divergence in one of the estimates.

Country Code	*h* = 1		*h* = 3		*h* = 6		*h* = 12		*h* = 24	
	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}
AUT	2.23	25.05	1.14	9.98	1.90	3.86	0.95	1.93	0.82	1.25
BEL	2.43	23.02	2.63	5.30	1.15	2.98	1.25	2.26	1.04	12.35
BRA	1.60	1.76	9.76	9.99	4.11	1.78	1.39	0.32	1.27	0.25
CAN	0.90	0.90	1.40	1.42	1.83	2.58	2.75	5.21	0.70	0.89
CHL	1.62	1.03	0.41	0.88	0.22	1.36	0.27	0.25	0.02	0.05
CHN	3.33	6.69	0.63	1.15	0.28	0.65	0.20	0.64	0.27	0.24
CZE	1.35	7.65	0.96	1.58	0.66	1.06	1.76	2.57	0.65	1.65
DNK	2.39	4.51	0.83	1.63	1.82	4.01	0.71	1.32	0.34	0.73
EST	0.81	0.75	0.76	2.28	3.21	1.73	3.73	1.22	5.25	0.57
FIN	1.01	1.49	0.70	1.82	0.89	1.45	0.48	1.52	0.76	4.36
FRA	1.26	12.25	0.88	1.27	1.39	2.96	0.70	1.87	1.04	1.71
GER	4.76	5.90	2.79	4.41	6.86	12.06	0.52	0.78	0.27	0.49
GRC	1.36	0.67	1.56	1.52	1.52	1.97	1.15	1.83	0.36	3.72
HUN	1.07	2.21	1.33	3.09	0.75	5.08	3.25	1.12	DIV	0.97
ICE	3.22	6.35	0.76	2.48	0.20	1.07	1.36	0.51	DIV	0.06
IND	1.17	2.18	1.06	6.17	1.51	5.95	3.72	1.77	DIV	1.09
IDS	0.35	0.80	0.64	1.20	DIV	1.01	1.00	DIV	1.00	DIV
IRE	0.53	1.30	0.35	0.68	0.77	0.89	0.85	0.87	0.33	0.51
ISR	1.72	1.93	1.10	1.10	0.81	1.31	0.15	0.36	0.09	0.07
ITA	0.47	0.86	1.53	2.19	2.65	2.44	1.76	1.41	1.05	2.83
JAP	1.20	2.26	1.03	1.70	2.43	1.58	1.87	1.39	2.32	1.83
KOR	0.56	1.13	0.49	1.58	1.24	1.32	0.84	2.76	0.79	2.16
LUX	3.34	5.03	0.87	1.49	4.39	41.24	2.26	3.96	3.72	4.97
MEX	1.52	1.03	2.05	2.10	1.00	0.00	1.00	0.00	1.00	0.00
NLD	4.17	30.69	1.57	18.70	0.76	1.32	0.52	0.92	0.44	0.62
NOR	0.74	1.51	1.86	3.58	2.82	1.21	0.68	3.98	1.66	6.86
POL	0.63	1.34	0.17	0.48	0.15	0.16	0.07	0.11	0.06	0.08
PRT	2.09	1.14	2.09	0.95	0.85	0.94	0.73	2.81	0.75	1.07
RUS	0.56	1.26	1.23	0.93	0.14	0.08	0.12	0.19	0.02	0.03

Country Code	h = 1		h = 3		h = 6		h = 12		h = 24	
	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}
SLK	1.09	4.26	1.71	1.49	0.56	1.47	0.70	1.54	0.69	0.50
SLV	1.34	4.78	0.08	0.47	0.04	0.25	0.05	0.07	0.07	0.06
SAF	0.81	1.65	2.14	4.27	3.05	5.03	1.25	6.61	1.52	2.53
SPA	0.32	0.46	1.55	3.84	1.15	1.35	1.92	4.42	0.50	1.00
SWE	0.71	1.30	2.52	6.88	1.57	2.52	1.81	23.06	0.42	12.63
SWZ	2.96	2.96	1.36	2.06	0.69	1.58	1.93	3.03	0.85	0.76
TUR	0.32	0.85	0.73	1.22	1.27	0.72	DIV	0.15	1.00	DIV
UK	0.44	0.47	1.51	3.77	3.68	2.21	2.61	2.17	1.68	3.05
USA	2.58	2.68	2.58	4.05	3.56	9.45	2.65	4.98	2.01	5.43

Table S1.4: Relative minimum distance characteristics of distributions fitted to *U*-uncertainties for 38 countries. Data until March 2017. Full sample estimates for selected forecast horizons. Countries with independent central banks are in boldface.

Legend: *h*- forecast horizon; MD_{WSN}^{TPN} - ratio of Hellinger Distance of TPN distribution to Hellinger Distance of WSN distribution fitted to *U*-uncertainties; MD_{WSN}^{GB} - ratio of Hellinger Distance of GB distribution to Hellinger Distance of WSN distribution fitted to *U*-uncertainties.

Country Code	*h* = 1		*h* = 3		*h* = 6		*h* = 12		*h* = 24	
	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}
AUT	2.20	2.46	0.79	2.49	1.90	2.02	0.68	1.10	1.75	2.95
BEL	1.17	2.87	1.15	2.60	0.72	1.48	0.92	1.63	1.64	15.01
BRA	0.22	0.30	0.63	1.66	0.34	0.61	2.30	2.76	1.12	0.51
CAN	1.76	1.84	1.06	1.53	1.20	1.22	1.82	3.03	1.53	4.51
CHL	6.13	8.42	2.19	5.45	1.36	2.07	0.55	7.33	0.41	1.22
CHN	0.33	0.38	0.80	1.25	0.65	1.52	0.87	1.26	1.39	2.35
CZE	0.91	2.00	1.52	3.78	1.10	2.25	1.10	1.82	0.04	0.60
DNK	0.66	1.15	0.40	0.68	0.38	0.61	0.34	0.75	0.61	0.97
EST	1.12	0.90	0.85	2.98	0.66	0.45	0.80	0.36	4.49	0.90
FIN	0.35	1.03	0.52	0.53	0.72	0.72	0.73	6.65	1.08	8.40
FRA	0.95	11.26	1.29	1.76	0.42	1.74	0.29	0.78	0.24	0.74
GER	0.97	14.76	0.87	1.75	1.00	1.47	0.97	1.25	0.75	1.10
GRC	1.36	2.39	0.95	4.50	1.33	4.74	2.01	3.35	1.05	1.58
HUN	0.75	0.85	0.67	1.63	0.60	1.45	0.24	0.55	0.53	0.29
ICE	1.22	5.52	0.77	1.22	0.79	3.12	0.83	2.48	0.30	0.80
IND	1.94	1.64	3.98	2.26	0.84	1.08	0.31	0.57	0.08	0.13
IDS	0.63	2.86	0.54	1.01	0.36	0.66	0.93	0.74	0.97	1.89
IRE	1.23	2.44	0.81	2.01	0.90	1.84	1.18	1.55	1.00	2.71
ISR	1.71	2.25	1.01	2.03	0.81	1.80	0.27	1.16	0.10	0.47
ITA	1.00	1.26	0.32	0.87	1.78	1.07	2.48	5.62	1.82	2.82
JAP	0.96	2.01	0.45	0.51	0.58	1.08	0.41	0.80	0.43	0.37
KOR	0.84	1.40	0.60	1.10	1.66	4.49	1.33	1.94	2.01	2.26
LUX	1.00	1.72	0.95	1.50	0.62	1.65	1.14	2.06	1.50	10.95
MEX	0.53	1.59	0.46	0.90	0.52	0.43	0.37	0.49	0.28	0.20
NLD	1.19	12.86	1.82	23.92	1.06	1.72	0.74	0.95	0.71	1.33
NOR	1.16	2.03	0.60	1.66	0.47	1.49	1.04	0.35	1.84	6.60
POL	0.60	0.63	0.77	1.97	0.71	0.96	0.16	0.88	0.51	0.23
PRT	1.06	1.87	1.65	3.75	2.14	6.07	2.08	2.48	0.79	1.21
RUS	0.50	0.58	2.38	4.59	0.98	21.64	1.00	1.14	1.00	0.93

Country Code	*h* = 1		*h* = 3		*h* = 6		*h* = 12		*h* = 24	
	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}	MD_{WSN}^{TPN}	MD_{WSN}^{GB}
SLK	1.34	9.86	0.23	0.51	0.71	0.54	0.25	0.99	0.22	0.24
SLV	1.56	4.11	0.62	2.20	0.39	0.47	0.17	0.47	0.07	0.22
SAF	0.69	1.50	1.70	1.92	0.93	12.29	0.70	21.80	2.54	35.38
SPA	0.38	0.58	0.51	1.47	0.68	0.79	0.96	3.53	1.19	3.14
SWE	0.69	1.34	0.56	1.31	0.81	1.58	1.72	18.92	0.99	14.99
SWZ	1.18	1.71	0.77	1.00	0.50	1.22	1.61	2.48	0.98	6.68
TUR	1.78	1.44	3.52	3.14	0.44	1.26	0.74	4.69	0.31	28.98
UK	0.87	1.55	0.40	0.66	0.80	0.56	1.89	2.90	3.02	3.84
USA	1.60	3.32	0.91	2.10	0.99	3.84	1.37	10.72	1.26	2.82

2.2. USE OF DIFFERENT INFLATION INDICATORS

Table S2.1: Data and estimation periods, core inflation.
Countries with independent central banks are in boldface.

Legend: *h*- forecast horizon.

Country	Code	First recursion		Total sample size	Size first estimation window	No. of observations for WSN for h=1	No. of observations for WSN for h=24
		Start	Start				
Austria	AUT	Jan-67	Jan-77	603	121	482	459
Belgium	BEL	Jun-76	Jul-84	490	98	392	369
Canada	CAN	Jan-62	Jan-73	663	133	530	507
Chile	CHL	Jan-99	Aug-05	219	80	139	116
Czech Republic	CZE	Jan-96	Aug-02	255	80	175	152
Denmark	DNK	Jan-71	Mar-80	555	111	444	421
Estonia	EST	Jan-98	Aug-04	231	80	151	128
Finland	FIN	Jan-56	Mar-68	735	147	588	565
France	FRA	Jan-71	Mar-80	555	111	444	421
Germany	GER	Jan-63	Nov-73	651	131	520	497
Greece	GRC	Jan-71	Mar-80	555	111	444	421
Hungary	HUN	Jan-99	Aug-05	219	80	139	116
Iceland	ICE	Nov-92	Jun-99	293	80	213	190
Ireland	IRE	Nov-75	Feb-84	497	100	397	374
Israel	ISR	Jan-87	Aug-93	363	80	283	260
Italy	ITA	Jan-61	Mar-72	675	135	540	517
Japan	JAP	Jan-56	Mar-68	735	147	588	565
Korea	KOR	Jan-90	Aug-96	327	80	247	224
Luxembourg	LUX	Jan-68	Nov-77	591	119	472	449
Mexico	MEX	Feb-00	Aug-06	207	80	127	104
Netherlands	NLD	Jan-61	Mar-72	675	135	540	517
Norway	NOR	Jan-79	Aug-86	459	92	367	344
Poland	POL	Jan-99	Aug-05	219	80	139	116
Portugal	PRT	Jan-71	Mar-80	555	111	444	421
Slovak Republic	SLK	Jan-96	Aug-02	255	80	175	152
Slovenia	SLV	Jan-00	Aug-06	207	80	127	104
South Africa	SAF	Jan-03	Aug-09	171	80	91	68
Spain	SPA	Jan-76	Mar-84	495	99	396	373
Sweden	SWE	Jan-71	Mar-80	555	111	444	421
Switzerland	SWZ	Jan-56	Mar-68	735	147	588	565
Turkey	TUR	Jan-04	Aug-10	159	80	79	56
United Kingdom	UK	Jan-71	Mar-80	555	111	444	421
United States	US	Jan-58	Nov-69	711	143	568	545

Table S2.2: Data and estimation periods, HICP inflation.
Countries with independent central banks are in boldface.

Legend: *h*- forecast horizon

Country	Code	First recursion		First recursion	Size first estimation window	No. of observations for WSN for *h*=1	No. of observations for WSN for *h*=24
		Start	Start				
Austria	AUT	Jan-97	Aug-03	243	80	163	140
Belgium	BEL	Jan-97	Sep-03	243	80	163	140
Czech Republic	CZE	Jan-97	Oct-03	243	80	163	140
Denmark	DNK	Jan-97	Nov-03	243	80	163	140
Estonia	EST	Jan-97	Dec-03	243	80	163	140
Finland	FIN	Jan-97	Jan-04	243	80	163	140
France	FRA	Jan-97	Feb-04	243	80	163	140
Germany	GER	Jan-97	Mar-04	243	80	163	140
Greece	GRC	Jan-97	Apr-04	243	80	163	140
Hungary	HUN	Jan-97	May-04	243	80	163	140
Iceland	ICE	Jan-97	Jun-04	243	80	163	140
Ireland	IRE	Jan-97	Jul-04	243	80	163	140
Italy	ITA	Jan-97	Aug-04	243	80	163	140
Luxembourg	LUX	Jan-97	Oct-04	243	80	163	140
Netherlands	NLD	Jan-97	Nov-04	243	80	163	140
Norway	NOR	Jan-97	Dec-04	243	80	163	140
Poland	POL	Jan-99	Jan-05	243	80	163	140
Portugal	PRT	Jan-97	Feb-05	243	80	163	140
Slovak Republic	SLK	Jan-97	Mar-05	243	80	163	140
Slovenia	SLV	Jan-97	Apr-05	243	80	163	140
Spain	SPA	Jan-97	May-05	243	80	163	140
Sweden	SWE	Jan-97	Jun-05	243	80	163	140
Switzerland	SWZ	Dec-05	Jul-12	136	80	56	33
Turkey	TUR	Jan-97	Aug-05	243	80	163	140
United Kingdom	UK	Jan-97	Sep-05	243	80	163	140
United States	US	Dec-98	Jul-05	220	80	140	117

Table S2.3: Data and estimation periods, PCE inflation.

Legend: *h*- forecast horizon

Country	Code	First recursion		First recursion	Size first estimation window	No. of observations for WSN for *h*=1	No. observations for WSN for *h*=24
		Start	Start				
United States	US	Jan-60	Jun-71	687	138	549	538

Table S2.4: Forecast uncertainty characteristics for the UK and the US aggregated across forecast horizons computed for different of measures of inflation (data until March 2017).

Legend: $\hat{\sigma}$ is the estimated σ parameter in the WSN distribution (6)-(7). For explanations of other symbols see the main body of the paper.

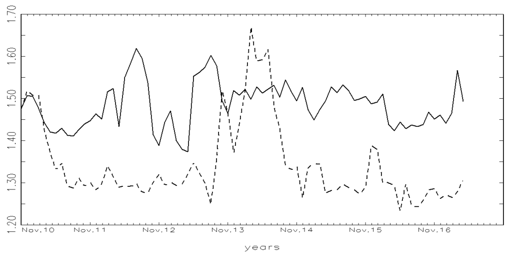
	Short aggregation					Long aggregation				
Infl. measure	$RMSE_U$	σ_V	UR	NUR	$\hat{\sigma}$	$RMSE_U$	σ_V	UR	NUR	$\hat{\sigma}$
UK										
CPI	1.06	1.29	1.37	0.84	0.56	13.4	15.4	1.15	0.92	1.69
CORE	0.56	0.71	1.41	0.87	0.45	5.90	6.90	1.14	0.91	1.18
HICP	0.28	0.35	1.35	0.84	0.27	1.18	1.39	1.15	0.92	0.93
US										
CPI	0.52	0.65	1.38	0.85	0.43	6.56	7.84	1.18	0.94	1.78
CORE	0.25	0.31	1.41	0.87	0.22	3.80	4.55	1.17	0.93	1.19
HICP	1.03	1.21	1.07	0.71	0.71	4.11	4.92	1.19	0.95	1.88
PCE	0.36	0.45	1.37	0.85	0.27	2.74	3.23	1.12	0.90	1.44

2.3. TIME STABILITY

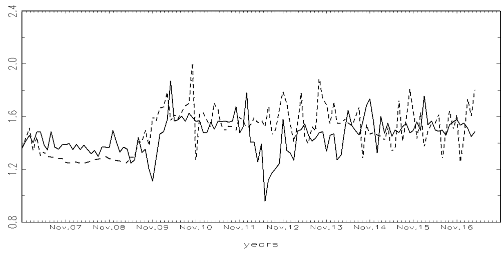
Figure S3.1: Aggregated URs computed in fixed size windows.

Legend: solid lines indicate the long aggregation and the dashed line the short aggregation.

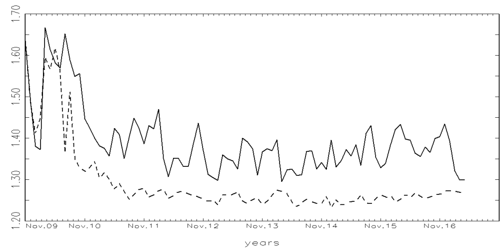
Brazil



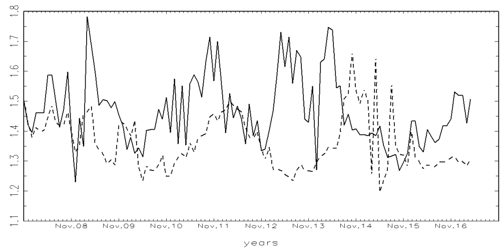
China



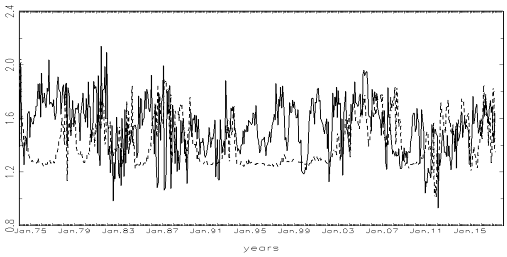
India



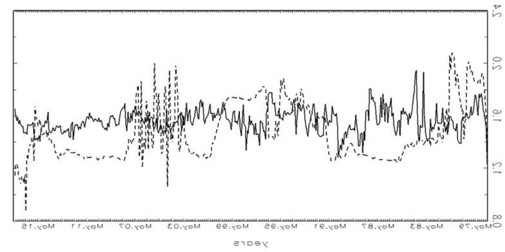
Russia



South Africa



UK



US

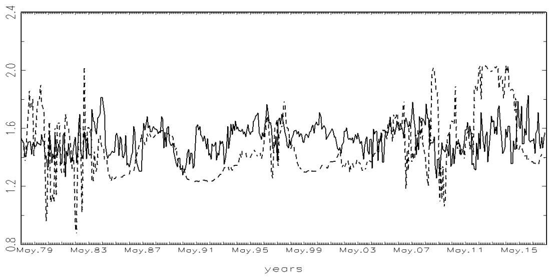
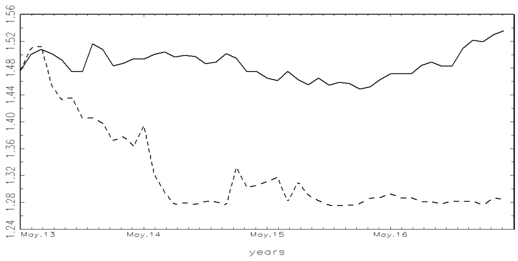


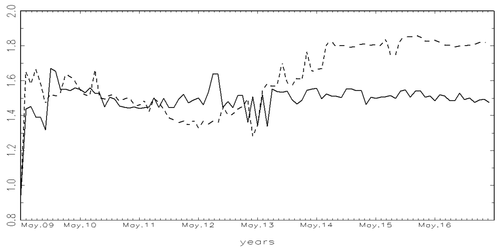
Figure S3.2: Aggregated URs computed in expanding size windows.

Legend: solid lines indicate the long aggregation and the dashed line the short aggregation.

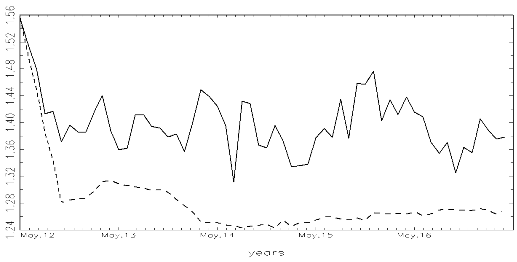
Brazil



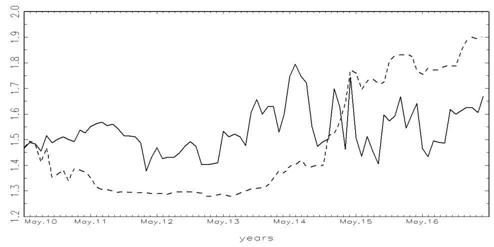
China



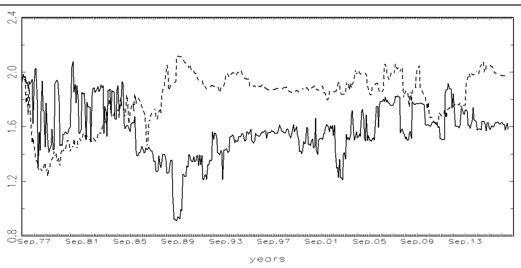
India



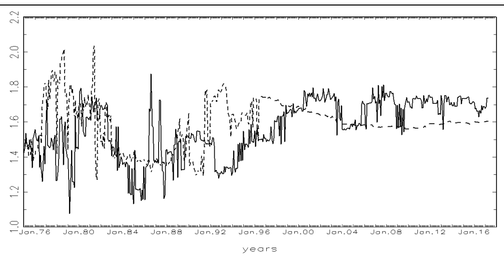
Russia



South Africa



UK



US