The Role of Consumer Behaviour in Service Operations Management

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I, Hang Ren, confirm that the work presented in this thesis is my own. No part of this thesis has been presented before to any university or college for submission as part of a higher degree. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Signature: ____________

Date: April 16, 2018

Abstract

In this thesis, I study the impact of consumer behaviour on service providers' operations. In the first study, I consider service systems where customers do not know the distribution of uncertain service quality and cannot estimate it fully rationally. Instead, they form their beliefs by taking the average of several anecdotes, the size of which measures their level of bounded rationality. I characterise the customers' joining behaviour and the service provider's pricing, quality control, and information disclosure decisions. Bounded rationality induces customers to form different estimates of the service quality and leads the service provider to use pricing as a market segmentation tool, which is radically different from the full rationality setting. When the service provider also has control over quality, I find that it may reduce both quality and price as customers gather more anecdotes. In addition, a high-quality service provider may not disclose quality information if the sample size is small. In the second study, I analyse the performance of opaque selling in countering the negative revenue impact from consumers' strategic waiting behaviour in vertically differentiated markets. The advantage of opaque selling is to increase the firm's regular price, whereas the disadvantage lies in the inflexibility of segmenting different types of consumers. Both the advantage and the disadvantage are radically different from their counterparts in horizontally differentiated markets, and this contrast generates opposite policy recommendations across the two settings. In the third study, I investigate an online store's product return policy when competing with a physical store, in which consumers can try the product before purchase. I find that the online store should offer product return only if it is socially efficient. Moreover, it should allocate product return cost between the online store and the consumers to minimise the total return cost.

Impact Statement

Thanks to the development of modern technologies, especially mobile Internet and e-commerce, consumers can exhibit new behaviours and enhance their existing behaviours when engaging with the operations of service systems. The impact of these behaviours on service operations is addressed inadequately in the existing literature, and practitioners are relying mainly on intuition to make their operational decisions. This thesis aims to provide theoretical frameworks and policy recommendations about how service providers should adjust their operational and marketing decisions when faced with specific consumer behaviours.

From the academic perspective, this thesis lays the foundation for future research that considers consumer behaviour in operations management. For example, Chapter 2 proposes a modelling framework that incorporates customers' social learning from word-of-mouth information into service systems. Building on this framework, future modelling research can investigate the impact of customers' social learning on firms' operational and marketing decisions that have not been addressed in this thesis, e.g., capacity choice or pricing and quality decisions in a competitive market. Moreover, Chapter 4 establishes a model that considers online-offline competition on an experience good. This framework can serve as the basis for future research that studies the impact of various consumer behaviours (e.g., service free-riding) on competitive markets.

This thesis also contributes to the academia by improving its understanding regarding the impact of consumer behaviours on service systems. In Chapter 2, I study a service provider's pricing, quality information disclosure, and quality control decisions when its customers rely on word-of-mouth information to estimate service quality. In Chapter 3, I analyse the performance of opaque selling in helping travel service companies to cope with consumers' strategic waiting behaviour in vertically differentiated markets. In Chapter 4, I examine an online retailer's product return policy when consumers are ex ante unsure about the product fit and can learn about it from visiting an offline store. These studies not only address research questions that are novel to the modelling literature, they also provide testable hypotheses for future empirical research. For example, based on my modelling framework in Chapter 2, researchers can estimate the size of customers' word-of-mouth information for different types of service systems and how it evolves over time. Compounded with my analysis, researchers can provide solid policy recommendations about how a service provider should adjust its pricing, service quality, and information disclosure decisions to customers' word-of-mouth evolution.

From the practitioners' perspective, this thesis provides new insights regarding service providers' operational and marketing strategies. First, the thesis highlights the importance of gathering data to quantify the level of a particular consumer behaviour. Second, the modelling framework in this thesis can serve as the foundation to make concrete operational and marketing decisions based on consumer-level data. Finally, the analysis and discussions in this thesis provide policy recommendations for a service provider's operational and marketing strategies.

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Chapter 1

Introduction

With the advancement of modern technologies (e.g., mobile Internet, online social media, and e-commerce), consumers can exhibit new behaviours and enhance their existing behaviours when engaging with the operations of service systems. For example, the growing popularity of online social media and review websites enables consumers to acquire word-of-mouth information about the food quality of a particular restaurant before deciding to eat there. The availability of price-tracking websites makes consumers more strategic in the timing of purchase, i.e., they may wait for a future price cut instead of buying early. This strategic waiting behaviour is particularly salient in the travel service industry, where dynamic pricing is a common practice. In addition, the growth of online retailing underscores consumers' need to touch and feel a product before making the purchase decision.

These behavioural elements may fundamentally change a firm's operational and marketing strategies, thereby posing new challenges for managers in practice. For example, restaurant owners may need to change the menu (in terms of both prices and qualities) as consumers are better informed about the food from their social media. An online booking company should consider adjusting its dynamic pricing strategy to cope with consumers' strategic waiting behaviour. An online retailer may need to offer more lenient return policies to compensate consumers for the inability to try before purchase. This thesis addresses these challenges by modelling consumer behaviour in service operations to derive new insights into managers' operational and marketing strategies.

In Chapter 2, I consider service systems where customers do not know the distribution of uncertain service quality and cannot estimate it fully rationally. Instead, they form their beliefs by taking the average of several anecdotes, the size of which measures their level of bounded rationality. I characterise the customers' joining behaviour and the service provider's pricing, quality control, and information disclosure decisions. Bounded rationality induces customers to form different estimates of the service quality and leads the service provider to use pricing as a market segmentation tool, which is radically different from the full rationality setting. As customers gather more anecdotes, the service provider may first decrease and then increase price, and the revenue is U-shaped. Interestingly, a larger sample size may harm consumer surplus, although it always benefits social welfare. When the service provider also has control over quality, I find that it may reduce both quality and price as customers gather more anecdotes. In addition, a high-quality service provider may not disclose quality information if the sample size is small, while a low-quality service provider may disclose if the sample size is large. Furthermore, as the expected waiting cost increases, information non-disclosure is more attractive, thereby highlighting the importance of incorporating customer bounded rationality in congested settings.

In Chapter 3, I consider a new behavioural element in the setting of travel service operations. In this setting, service capacities (e.g., airline tickets and hotel rooms) are usually difficult to adjust in the short term and any leftover capacity after the expire date is worthless. As a result, it is common practice for online booking companies to mark down the price as the expire date approaches. However, this last-minute selling strategy may not improve the firms' aggregate revenue since it induces some consumers to wait for the sales instead of buying at the higher regular price. To counter the negative revenue impact from consumers' strategic waiting, Priceline and Hotwire have begun to adopt opaque/probabilistic selling strategies, i.e., mixing different types of leftover products to sell as a new product. The existing literature has analysed its performance in comparison to the classic last-minute selling strategy (i.e., selling the leftover products separately) in horizontally differentiated markets. However, it remains unclear how opaque selling performs in vertically differentiated markets. To address this question, I consider a dynamic pricing model in which a firm sells vertically differentiated products across two periods to strategic consumers with uncertain demand. I characterise the firm's optimal selling strategy and find that opaque selling may outperform last-minute selling because it increases the regular price by depriving consumers of a choice to buy the preferred type of product during the sales season. Its disadvantage, however, lies in the inflexibility of segmenting different types of consumers. Both the advantage and the disadvantage are radically different from their counterparts in horizontally differentiated markets, and this contrast generates opposite policy recommendations across the two settings. Specifically, under vertical differentiation, the firm may switch from opaque selling to last-minute selling as consumers become more differentiated or the probability of a low demand increases. However, it always switches from last-minute selling to opaque selling under horizontal differentiation.

Whereas Chapter 2-3 focus on monopoly settings, Chapter 4 considers the competition between an online store and a physical store in which the online store utilises money-back guarantees (MBGs) to compensate consumers for its disadvantage in instore services. Specifically, consumers who visit the physical store can try different variants of a product before purchase with instore assistance, whereas in an online store they buy a product without knowing whether it fits their need or not. To cope with this disadvantage, major online retailers (e.g., Amazon, Zappos, Newegg) offer consumers the chance to return products and get the full money back "no question asked." Using a game-theoretical model, I study how the online store should design its product return policy to compete with the physical store. I find that MBGs can effectively raise profits if it is more efficient to transfer an unfit product from the consumer side to the online store. Moreover, if the online store chooses to offer MBGs, it should allocate product return cost in the socially optimal way, i.e., to minimise the total return costs. I also study the impact of the stores' service quality on their optimal profits. Interestingly, the online store may lose profit from improving the service, whereas it may benefit from a better service from the physical store.

I am the first author of all chapters. Chapter 2 was undertaken as joint work with Tingliang Huang and Kenan Arifoglu. Chapter 3 draws on joint work with Tingliang Huang. Chapters 4 is based on conjoint work with Tingliang Huang, Christopher Tang, and Ying-Ju Chen. In all chapters, I performed all analyses and wrote all parts of the chapters myself.

Chapter 2

Managing Service Systems with Unknown Quality and Customer Anecdotal Reasoning

2.1 Introduction

Service systems such as call centres, hospitals, restaurants, night clubs, and amusement parks are prevalent in our everyday lives. In many situations, customers need to choose between waiting for service in a queue or balking based on their inference of the service quality. The existing literature (e.g., Debo et al. 2012, Guo et al. 2014) typically assumes that customers make this inference fully rationally, e.g., following the Bayesian rule. However, this is usually challenging for average customers in real-time decision-making. First, Bayesian updating requires customers to have prior knowledge about the service quality. In practice, they may lack such knowledge because of scarce learning opportunities (e.g., diners patronising a new restaurant) or because they do not have the relevant expertise (e.g., car owners incapable of evaluating all possible causes of a car breakdown). Second, the calculation involved in Bayesian updating is demanding. In fact, the psychology and economics literature has long argued that people usually do not follow the Bayesian rule in uncertain situations (e.g., Edwards 1968, Kahneman and Tversky 1979, 1984).

Because of the informational and computational challenges underlying Bayesian updating, customers naturally resort to simplified heuristics when estimating service quality. An example of such a heuristic is to ask acquaintances about their past service experiences (i.e., anecdotes) and then expect a service benefit equal to the sample average. Compared to Bayesian updating, this anecdotal reasoning is non-parametric in nature and thus allows customers to estimate service quality without knowing its distributional parameters. The computation involved in anecdotal reasoning is also usually much simpler than applying the Bayesian rule. Furthermore, the worldwide booming of online social media has greatly facilitated information sharing among customers, which makes anecdotal reasoning especially popular. According to surveys by Nielsen (2013, 2015), customers ranked word-of-mouth recommendations as the most trustworthy source of advertising from 2007 to 2015, with its percentage of trust nearly 20% higher than that of the runner-ups (i.e., consumer opinions posted online and editorial content) in 2015.

Although anecdotal reasoning greatly simplifies customers' inferences of service quality, it may lead them to hold incorrect estimates when service quality is intrinsically uncertain.[1] Specifically, anecdotal-reasoning customers attribute the server's performance inferred from anecdotes entirely to its capability rather than luck. Therefore they overestimate (underestimate) service quality if the performance happens to be good (bad) and thus exhibit a joining behaviour different from the full rationality setting. This presents new challenges for the management of service systems: (i) As customers gather more anecdotes, how should the service provider (e.g., a new restaurant with growing popularity) adjust price and service quality? (ii) Should the service provider disclose service quality information (e.g., restaurants inviting expert reviews or giving away free drinks/desserts to customers who post food photos on social media)?

To address these questions, I consider an M/M/1 unobservable queue with uncertain service quality. Customers do not know its expected value and estimate it as the average of several anecdotes. The sample size measures their level of bounded rationality: With more anecdotes, customers estimate service quality more accurately and thus their joining behaviour converges to the fully-rational benchmark where they know the expected service quality.

Using this anecdotal reasoning framework, I characterise customers' equilibrium joining rate and the service provider's pricing, service quality, and information disclosure decisions. Unlike the fully-rational benchmark, the service provider uses pricing as a tool to segment customers with different service quality estimates. In particular, a low-quality service provider prices higher than the fully-rational benchmark to target the niche customers who considerably overestimate service quality, and a high-quality service provider prices

[1] Uncertainty about service quality is prevalent because of the inherent variability in service providers' performance (e.g., a chef may be unable to guarantee perfect timing and seasoning for every dish) and unknown/unpredictable environmental factors (e.g., a pharmacist may be unable to predict all of a drug's side effects).

lower to target the mass customers who do not considerably underestimate service quality. As customers gather more anecdotes, the optimal revenue may first decrease and then increase, while social welfare always increases. Interestingly, consumer surplus may decrease due to intensified congestion. I also incorporate the service provider's quality control decision and find that counter-intuitively, it may target a lower quality level as customers gather more anecdotes (i.e., estimate service quality more accurately).

Apart from pricing, in practice service providers may also have the discretion to inform customers of the mean service quality. For example, restaurants can disclose the average food quality by inviting expert reviews and customers' social media posts. I characterise this information disclosure decision and find that high-quality service providers may not disclose information if customers are sufficiently boundedly rational, while low-quality service providers may disclose if customers are rational enough. In the former case, the high-quality service providers are better off by not disclosing the service quality and setting a high price to sell only to the niche customers. In the latter case, the low-quality service providers should price low to sell to the mass customers, and the service providers should disclose information since it increases the customer size. Moreover, a higher congestion cost makes the service providers switch from targeting the mass customers to targeting the niche customers. As a result, information non-disclosure is more attractive. This underscores the importance of considering customer anecdotal reasoning in congested settings.

The remainder of this chapter is organised as follows. I provide a literature review in §2.2, and present my model and preliminary analysis in §2.3. In §2.4, I study the service provider's pricing, quality, and information disclosure decisions. In §2.5, I extend the analysis by considering welfare-maximising service systems and customer heterogeneity in the sample size. I present concluding remarks in §2.6. The proofs of lemmas and propositions are relegated to Appendix A.1.

2.2 Related Literature

The traditional queueing economics literature incorporates customers' join-or-balk decision by assuming that they perfectly understand the queueing system (Hassin and Haviv 2003, and references therein). This assumption has been relaxed in several recent studies. For example, the behavioural operations management literature studies service systems where customers do not know the realised service quality and update their beliefs about it based on the queue length (Debo et al. 2012, Debo and Veeraraghavan 2014, Guo et al. 2014) or the expected waiting time (Kremer and Debo 2015). Another branch of literature investigates the operations of diagnostic services where customers rely on an expert service provider to identify the type of service they need (Alizamir et al. 2013, Wang et al. 2010). As with my model, the aforementioned research assumes that customers do not know the realised service quality. However, it focuses on the full rationality setting where customers infer service quality following the Bayesian rule, whereas I consider boundedly rational customers who estimate service quality based on anecdotes.

Recently, researchers have started to incorporate customer bounded rationality in service systems. Huang et al. (2013) capture customers' inability to accurately estimate the expected waiting time by introducing a random error term in their waiting time estimates. They derive the revenue and welfare implications of customer bounded rationality for a monopolistic service provider. Li et al. (2016) extend this analysis by considering market competition and the trade-off between service quality and service rate. Huang and Chen (2015) study a service provider's pricing and service rate decisions when faced with customers who estimate the expected waiting cost based on past experiences and anecdotal reasoning. Cui and Veeraraghavan (2016) consider customers who hold arbitrary beliefs about service rate and study the service provider's decision to disclose the true service rate. Although this chapter also focuses on customer bounded rationality in service systems, it is fundamentally different from this literature. First, the bounded rationality in Huang et al. (2013), Li et al. (2016), and Cui and Veeraraghavan (2016) stems from customers' inability to perfectly perceive the service rate or accurately calculate the expected waiting time. In this chapter, however, customer bounded rationality stems from their *cognitive* limitation of attributing service performance inferred from anecdotes entirely to capability rather than luck. Second, Huang and Chen (2015) focus on the extreme case where customers estimate the expected waiting time from only one anecdote. In contrast, I study customer anecdotal reasoning regarding service quality based on an *arbitrary* number of anecdotes. This allows me to derive insights into the service provider's information disclosure decision and the evolution of its pricing and quality strategies as customers acquire more anecdotes over time. Third, Cui and Veeraraghavan (2016) assume an exogenous price and service quality, whereas I endogenise both.

Researchers have also incorporated decision-makers' bounded rationality in inventory management. For example, Li et al. (2016) consider competing newsvendors' cognitive limitation in the form of inadequately perceiving demand uncertainty. In contrast, I study a different type of bounded rationality (anecdotal reasoning) in a different operational setting (monopolistic service systems).

My anecdotal reasoning framework is adapted from the $S(k)$-equilibrium proposed by Osborne and Rubinstein (1998). Unlike the classic Nash equilibrium where each player optimises based on a belief about other players' behaviours, in the $S(k)$-equilibrium she samples each action k times and chooses the one with the highest expected payoff. This equilibrium concept has been widely applied to model customer bounded rationality in several disciplines, including economics (Spiegler 2006a,b, Szech 2011), marketing (Huang and Yu 2014), and operations management (Huang and Chen 2015).

This chapter is also related to the extensive economics literature on quality information disclosure (see Dranove and Jin 2010, for a review). Grossman (1981) and Milgrom (1981) show that all firms should disclose their quality to fully-rational customers because they infer non-disclosing firms as having the lowest quality. This "unravelling" result is overturned if information disclosure is costly (Jovanovic 1982), customers ignore or cannot fully understand the disclosed information (Fishman and Hagerty 2003, Hirshleifer et al. 2004), or customers are unsure about the usefulness of a certain quality attribute (Stivers 2004). This chapter complements the literature by showing that customer bounded rationality alone can also overturn the unravelling result.

There is an emerging service operations literature on information disclosure. Hassin (1986, 2007) examines a service provider's decision to disclose its queue length, service quality, service rate, and unit waiting cost. Guo and Zipkin (2007, 2009) analyse the impact of different levels of waiting time information on the service system's performance and consumer surplus. Guo et al. (2011) study a service provider's decision to disclose delay information to customers who estimate waiting time by the entropy-maximisation principle. Note that this literature imposes the full rationality assumption on customers, whereas I focus on boundedly rational customers whose level of rationality (given by the sample size) is influenced by the service provider's information disclosure decision.

2.3 Model and Preliminaries

I consider an M/M/1 unobservable queue with homogeneous customers arriving according to a Poisson process at rate λ (arrival rate or market potential, hereafter). In what follows, I will refer to the service provider (server, manager, etc.) as "he" and each customer as "she."

Both the server and the customers are risk-neutral. Upon arrival, a customer chooses between joining the queue at price p or balking to obtain a constant payoff, which I normalise to zero without loss of generality. Joining customers are served on a first-come first-served basis, and the service time is exponentially distributed with rate μ. Waiting for service costs a customer c per unit of time, and completing the service entails a random benefit $\mathscr{R}$ (service quality or valuation, hereafter). For analytical simplicity, I assume that the benefit is normally distributed, i.e., $\mathscr{R} \sim N(R, \sigma^2)$, where R and σ^2 denote the expected benefit and its variance. The benefit is realised upon completion of the service, and *only* the service provider knows its distribution ex ante. In contrast, the existing queueing economics literature assumes that customers also know the service quality distribution (see, e.g., Guo et al. 2014). In my setting, this implies that their joining rate $\lambda_r(p)$ is given by (Hassin and Haviv 2003):

$$\lambda_r(p) = \begin{cases} 0, & \text{if} \quad \mu - \frac{c}{(R-p)^+} < 0, \\ \mu - \frac{c}{R-p}, & \text{if} \quad 0 \leqslant \mu - \frac{c}{(R-p)^+} \leqslant \lambda, \\ \lambda, & \text{if} \quad \mu - \frac{c}{(R-p)^+} > \lambda. \end{cases}$$

I will refer to this case as the fully-rational benchmark because customers have perfect knowledge of the expected service quality. In what follows, I will derive the joining rate of anecdotal-reasoning customers and show that it includes the fully-rational benchmark as a limiting case.

2.3.1 Customer Anecdotal Reasoning

Building on the established $S(k)$-equilibrium concept (see, e.g., Huang and Yu 2014, Spiegler 2006a,b), I develop customers' anecdotal reasoning framework as follows. Upon arrival, each customer gathers k service quality anecdotes/samples, which I denote as $\mathscr{R}_i$, $i = 1, \ldots, k$. Each anecdote is an independent draw from the service quality distribution, i.e., $\mathscr{R}_i \sim N(R, \sigma^2)$ for all i. The customer estimates service quality as the sample average of all anecdotes. As a result, her service quality estimate $\mathscr{R}$ is given by

$$\mathscr{R} = \left(\sum_{i=1}^{k} \mathscr{R}_i \right) / k \sim N(R, \sigma^2/k).$$

Note that customers' service quality estimates differ across each other and may not coincide with the mean service quality R (i.e., they are indeed boundedly rational). Both results are due to customer anecdotal reasoning: They attribute the service samples solely to the server's capability instead of luck, so some customers overestimate the expected service quality when the sample average happens to be high, while the others underestimate because the sample average happens to be low. Moreover, the sample size k measures customers' level of bounded rationality since their service quality estimates deviate less from the mean service quality as k increases (i.e., $\mathbf{Var}[\mathscr{R}]$ decreases in k). Intuitively, as customers acquire more anecdotes, the sample average is less influenced by luck and thus reflects the mean service quality more accurately.

Remark 1. *In practice, customers may also acquire anecdotes from online ratings/reviews, which are not independent across customers. I have incorporated this type of anecdote as an extension in Appendix A.2.4.*

2.3.2 Customer Joining Behaviour

Based on the service quality estimate $\mathscr{R}$, customers make the join-or-balk decision to maximise their estimated expected payoff. Therefore, unlike the fully-rational benchmark where customers play mixed strategies, in this setting they use a *pure* threshold strategy: All customers with $\mathscr{R} \geqslant p+W$ join the queue and the rest balk, where p is the price for service and W denotes their belief about the expected waiting cost. Since $\mathscr{R} \sim N(R, \sigma^2/k)$, the joining rate $\lambda P(\mathscr{R} > p+W)$ is equal to $\lambda\bar{\Phi}(\sqrt{k}(p+W-R)/\sigma)$, where $\bar{\Phi}$ denotes the complementary cumulative distribution function of the standard normal distribution.

Notably, this model is fundamentally different from an alternative model in which customers are fully rational and hold heterogeneous service valuations. First, the two models lead to different welfare implications. In my model, a customer's actual benefit from the service is independent of his service quality estimate, whereas they are equal in the alternative model. Second, the alternative model needs additional assumptions to replicate this model from the revenue perspective. For example, it requires the consumers to hold valuations for the service before they experience it, and the service provider needs to know the valuation distribution. Both assumptions may not hold in the settings of interest in this chapter. Lastly, the focus of this chapter is to provide policy recommendations and examine how they depend on the sample size k, whereas its counterpart in the alternative model is unexplored by the extant literature (see, e.g., Littlechild 1974, Larsen 1998).

As in the traditional queueing economics literature (Hassin and Haviv 2003), I characterise the equilibrium joining rate $\lambda_a^k(p)$ by the condition that customers' beliefs about the expected waiting time are correct. Therefore, using the PASTA property (Wolff 1982), I can derive the expected waiting time as $W = \frac{c}{[\mu-\lambda_a^k(p)]^+}$. Substituting this into the expression for the joining rate, I have

$$\lambda_a^k(p) = \lambda \bar{\Phi}\left(\sqrt{k}\left[p + \frac{c}{\mu - \lambda_a^k(p)} - R\right]/\sigma\right), \tag{2.1}$$

in which $[\mu - \lambda_a^k(p)]^+$ reduces to $\mu - \lambda_a^k(p)$ because $\lambda_a^k(p) > \mu$ cannot constitute an equilibrium—in this case, the expected waiting time goes to infinity and each joining customer can profitably deviate by balking. The next lemma shows that $\lambda_a^k(p)$ as defined by Equation (2.1) uniquely exists.

Lemma 1. *For any* $p \geqslant 0$*,*

(i) A unique $\lambda_a^k(p) \in (0, \min\{\lambda, \mu\})$ *exists and strictly decreases in* p*.*

(ii) The equilibrium joining rate $\lambda_a^k(p)$ *strictly decreases in* k *for* $R < p + \frac{c}{(\mu-0.5\lambda)^+}$*, strictly increases in* k *for* $R > p + \frac{c}{(\mu-0.5\lambda)^+}$*, and is invariant in* k *for* $R = p + \frac{c}{(\mu-0.5\lambda)^+}$*.*

(iii) $\lim_{k\to+\infty} \lambda_a^k(p) = \lambda_r(p)$*.*

Lemma 1 shows that the equilibrium joining rate is well-defined and converges to the fully-rational benchmark as k goes to infinity. Thus, my anecdotal reasoning framework includes the fully-rational benchmark as a limiting case. Intuitively, with more service samples, customers estimate the expected service quality more accurately because the sample average is influenced more by the server's capability than by luck.

Lemma 1 also suggests that bounded rationality leads to fewer customers joining high-quality service systems and more customers joining low-quality service systems. To see the intuition, first recall that the service quality estimates are normally distributed (i.e., Bell-shaped) among customers, who apply the threshold strategy of joining (balking) the queue if their service quality estimate is higher (lower) than the total cost (i.e., the price plus the expected waiting cost). In a low-quality (i.e., $R < p + \frac{c}{(\mu-0.5\lambda)^+}$) service system, the cost threshold is higher than the expected service quality, so only customers who considerably overestimate the service quality (i.e., the right tail of the Bell curve, dubbed as the *niche* customers) choose to join. In a high-quality (i.e., $R > p + \frac{c}{(\mu-0.5\lambda)^+}$) service system, the cost threshold is lower than the expected service quality, so all customers except those who considerably underestimate the service quality (i.e., the Bell curve without the left tail, dubbed as the *mass* customers) choose to join. Since bounded rationality lowers the accuracy of customers' service quality estimates (i.e., both tails of the Bell curves expand), it increases the number of niche customers and decreases the number of mass customers.

Therefore, the joining rate for low-quality service systems increases in k and the joining rate for high-quality service systems decreases in k.

2.4 Analysis

In this section, I study the service provider's revenue maximisation problem, as given by:

$$\max_{p \geqslant 0} \Pi(p,k) = p\lambda_a^k(p). \tag{2.2}$$

In §2.4.1, I characterise the optimal price and its revenue and welfare implications. Then I incorporate the server's quality control decision and examine the impact of the sample size on the optimal price and quality. In §2.4.2, I analyse the quality information disclosure decision and how it is influenced by the sample size and service system parameters.

2.4.1 Pricing and Quality Control

In this section, I analyse the service provider's pricing and quality control decisions and how they are influenced by the sample size k. This is of particular interest because it provides insights regarding a service provider's dynamic decisions in a setting where customers acquire more anecdotes over time.[2] As an example, consider a new restaurant with growing popularity (i.e., k is increasing). My analysis makes recommendations about how the restaurant owner should change the menu prices and the quality of the food and service accordingly.

I first examine the server's pricing decision, as given by (2.2). Let $p^*(k)$ and $\Pi^*(k)$ denote the optimal price and revenue for a given sample size k, and let p_r^* and Π_r^* denote the fully-rational counterparts. The next proposition characterises the impact of k on $p^*(k)$ and $\Pi^*(k)$.

Proposition 1. *(i) For any $k \geqslant 1$, a unique $p^*(k) > 0$ exists and $\lim_{k \to +\infty} p^*(k) = p_r^*$.*

(ii) The optimal revenue $\Pi^*(k)$ strictly decreases in k for $R < R_1(k)$, strictly increases in k for $R > R_1(k)$, and is invariant in k for $R = R_1(k)$, where $R_1(k) \equiv \frac{c\mu}{[(\mu-0.5\lambda)^+]^2} + \frac{\sigma}{2\sqrt{k}\Phi'(0)}$ is the mean service quality at which $\lambda_a^k(p^*(k)) = 0.5\lambda$.

(iii) For a sufficiently low λ, there exists a unique quality threshold $R_2(k) > R_1(k)$, which strictly decreases in k, such that $p^*(k)$ strictly decreases in k for $R < R_2(k)$, strictly increases in k for $R > R_2(k)$, and is invariant in k for $R = R_2(k)$.

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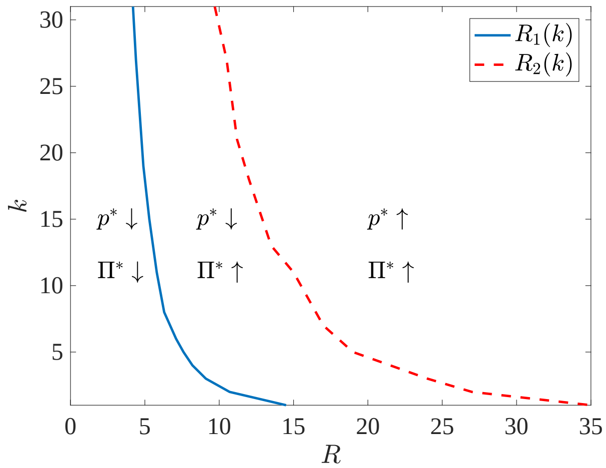
[2] See Appendix A.2.1 for a discussion of the dynamic decision process.

Proposition 1(i) shows that the revenue maximisation problem is well-defined and the optimal price converges to the fully-rational counterpart as the amount of quality information grows. Proposition 1(ii)-(iii) examine the impact of the sample size on the optimal price and revenue, which I illustrate in Figure 2.1.

Figure 2.1: The Impact of k on p^* and Π^* ($\mu = 2$, $\lambda = 2$, $c = 1$, $\sigma = 1$)



As shown by Proposition 1(ii) and Figure 2.1, a larger sample size decreases the optimal revenue of a low-quality (i.e., $R < \underline{R} \equiv \frac{c\mu}{[(\mu-0.5\lambda)^+]^2}$) service provider by reducing demand (i.e., the niche customers), and increases the optimal revenue of a high-quality (i.e., $R > R_1(1)$) service provider by increasing demand (i.e., the mass customers). Somewhat counter-intuitively, an intermediate-quality (i.e., $\underline{R} \leqslant R \leqslant R_1(1)$) service provider's optimal revenue first decreases and then increases in the sample size. This is because the service provider changes the pricing strategy as the number of anecdotes increases. Specifically, as the sample size increases, he switches from targeting exclusively the niche customers to targeting the mass customers. Since a larger sample size decreases demand in the former situation and increases demand in the latter situation, the optimal revenue is U-shaped in k.

The U-shaped revenue is similar in vein to Johnson and Myatt (2006) and Sun (2012). Johnson and Myatt (2006) study the optimal pricing strategy of a firm that sells to customers who hold heterogeneous product valuations. Sun (2012) extends the analysis by considering a 2-period model in which the period-2 customers infer product quality from ratings left by the period-1 customers. Assuming that customers are fully rational and hold heterogeneous product valuations, both papers show that the firm's optimal profit is U-shaped in the variance of customers' service quality estimates. My research contributes to this stream of literature by showing that when customers deviate from full rationality, the U-shaped revenue persists even when customers are ex ante homogeneous in their valuation of service.

Proposition 1(iii) characterises the impact of the sample size on the service provider's optimal pricing decision. As customers acquire more anecdotes, the number of niche customers decreases while the number of mass customers increases. In response, a low-quality (i.e., $R < R_1(k)$) service provider lowers price and a high-quality (i.e., $R > R_2(k)$) service provider raises price. Surprisingly, I find that an intermediate-quality (i.e., $R_1(k) < R < R_2(k)$) service provider who targets the mass customers lowers price despite the increase in demand. To fully understand this result, note that a larger sample size influences demand in both directions. First, it leads customers to estimate service quality more accurately and thus increases demand. Second, it intensifies congestion and thus decreases demand. When the service quality is not high enough, the second factor dominates and the service provider reduces price to incentivise customers to join. Notably, although I establish Proposition 1(iii) only for the low market potential case, I have numerically verified that it continues to hold when market potential is high (see Figure 2.1). Moreover, Proposition 1 continues to hold qualitatively for the no-congestion setting since my analysis includes $c = 0$ as a special case.

Managerially, Proposition 1 shows that customer anecdotal reasoning fundamentally changes the optimal pricing strategy in service systems: Managers should use pricing as a market segmentation tool to "milk" the most profitable consumer segment and should change it dynamically as customers acquire more word-of-mouth over time. For example, consider a new restaurant whose service quality is neither too high nor too low. Unlike the widely applied markup-on-cost pricing (Mealey 2016), the owner should start with high prices to target only customers who have received very positive word-of-mouth. As customers gather more anecdotes, he should lower prices to shift from this up-market strategy to the down-market strategy (i.e., even customers receiving occasional negative word-of-mouth patronise). With further growth in the restaurant's popularity, the owner should then switch to raising prices due to the substantial customer base.

2.4.1.1 Welfare implications.

Now I investigate the welfare implications of the service provider's optimal pricing strategy. Let $W(p,k)$ and $CS(p,k)$ denote the social welfare and consumer surplus for a given price

p and sample size k. By definition, I have

$$W(p,k) = \lambda_a^k(p)R - \frac{c\lambda_a^k(p)}{\mu - \lambda_a^k(p)},$$
$$CS(p,k) = \lambda_a^k(p)R - \frac{c\lambda_a^k(p)}{\mu - \lambda_a^k(p)} - p\lambda_a^k(p),$$

where the first terms on the right-hand side represent the expected social benefit of the service and the second terms are the expected waiting cost. I will denote social welfare by $W(\lambda_a^k(p))$ instead of $W(p,k)$ because it depends on p and k only indirectly through $\lambda_a^k(p)$. This observation suggests that examining the impact of the sample size on social welfare boils down to characterising its impact on $\lambda_a^k(p^*(k))$, as given by the lemma below.

Lemma 2. *The optimal joining rate $\lambda_a^k(p^*(k))$ strictly decreases in k for $R < \frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$, strictly increases in k for $R > \frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$, and is invariant in k for $R = \frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$, where $C \approx 0.7517$ is the unique solution of $\bar{\Phi}(C) = \Phi'(C)C$.*

Lemma 2 shows that a larger sample size decreases the demand for a low-quality service system (i.e., the niche customers) and increases the demand for a high-quality service system (i.e., the mass customers). Based on this, I examine the impact of k on social welfare and consumer surplus, as shown in the next proposition. For ease of exposition, I will abuse notation and write $W(\lambda_a^k(p^*(k)))$ as $W^*(k)$ and $CS(p^*(k),k)$ as $CS^*(k)$.

Proposition 2. *(i) Social welfare $W^*(k)$ strictly increases in k for $R \neq \frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$ and is invariant in k for $R = \frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$.*

(ii) Consumer surplus $CS^*(k)$ strictly increases in k for $R < R_1(k)$. If $c > 0$, $CS^*(k)$ strictly decreases in k for a sufficiently high R.

A larger sample size always benefits social welfare because it leads customers to make better decisions by inducing fewer (more) customers to join a low- (high-) quality service system. Somewhat unexpectedly, this improvement in their decision-making at the individual level may harm their aggregate benefit. In fact, Proposition 2(ii) shows that consumer surplus strictly decreases in k when service quality is sufficiently high. The key insight is that the improved decisions (more customers join a high-quality service system) present a negative externality (intensified congestion) for other joining customers. When service quality is high enough, the increased sample size significantly intensifies congestion such that the resulting consumer surplus loss outweighs the consumer surplus benefit due to better decision-making at the individual level. Overall, the consumer surplus decreases. Notably, this result is unique to the congested setting, although the rest of Proposition 2 continues to hold in the absence of congestion (see Appendix A.2.2). In fact, without congestion a larger sample size never harms consumer surplus since their decision of joining does not impact the other consumers' welfare.

From a managerial perspective, Proposition 2 implies that customers' quality information sharing through word-of-mouth (e.g., dining experiences in a new restaurant) may not create a win-win situation. In particular, information sharing leads fewer customers to visit a low-quality service system and thus harms the service provider's revenue, whereas it leads more customers to visit a high-quality service system, which intensifies congestion and thus may harm consumer surplus.

2.4.1.2 Quality control.

The preceding analysis focuses on the price as the only decision of the service provider. In practice, he may also have control over the average service quality through, e.g., staff training, facility upgrading, and service design. In this section, I characterise his joint pricing and quality decisions. As before, I focus on the impact of the sample size k on the optimal price and quality because this provides insights regarding a service provider's dynamic quality and pricing strategies in the setting where customers acquire more anecdotes over time (see Appendix A.2.1 for a detailed discussion).

I incorporate the quality decision by assuming that the service provider can choose R at cost aR^2 (the quality investment, hereafter), where $a > 0$ represents the rate of change of the marginal quality investment. This quadratic cost structure is standard in the product and service design literature (e.g., Anderson et al. 1997, Lahiri and Dey 2013). Moreover, all results are qualitatively preserved for convex cost functions that are independent of k and p.

The service provider's joint pricing and quality control problem is given by:

$$\max_{p,R\geqslant 0} \Pi_R(p,R) = p\lambda_a^k(p,R) - aR^2, \tag{2.3}$$

where I denote $\lambda_a^k(p)$ as $\lambda_a^k(p,R)$ to stress its dependence on R. The next proposition characterises the impact of the sample size k on the optimal price $\hat{p}$ and quality $\hat{R}$.

Proposition 3. *(i) A unique $\hat{p} > 0$ determined by $2a\hat{R} = \lambda_a^k(\hat{p},\hat{R})$ exists.*

(ii) When the market potential λ is sufficiently low, the optimal quality $\hat{R}$ strictly decreases in k. If, in addition, the standard deviation of service quality σ is sufficiently high or sufficiently low, the optimal price $\hat{p}$ strictly decreases in k.

(iii) When the market potential λ is sufficiently high and $c > 0$, the optimal price $\hat{p}$ and quality $\hat{R}$ strictly decrease in k.

(iv) When the market potential λ is sufficiently low, or sufficiently high and $c > 0$, the optimal revenue strictly decreases in k.

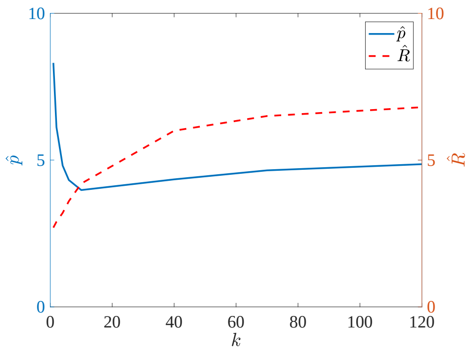
A larger sample size implies that customers estimate service quality more accurately. Therefore, conventional wisdom would probably recommend the service provider to improve quality and raise price. However, Proposition 3 points to just the opposite under certain conditions. To fully understand this result, first note that both quality improvement and price reduction may benefit the service provider: Quality improvement allows him to target a higher price, while price reduction allows him to lower quality and thus save the quality investment. The service provider chooses to reduce price because the quality improvement benefit is lower. Specifically, when market potential is sufficiently low, demand is low and raising price does not increase revenue much. When market potential is sufficiently high, quality improvement leads to significant congestion, so the server cannot raise price much. Notably, this result underscores the importance of incorporating customer bounded rationality in the *congested* setting: Without congestion, the optimal quality and price strictly increase in k under sufficiently high market potential.[3] Intuitively, in this case the server always sets a high quality and targets the mass customers. As they acquire more anecdotes, the joining rate increases and the server improves quality to better exploit the increase in demand, which further allows him to charge a higher price.

I would also like to note that the preceding analysis relies on the implicit assumption that quality control does not impact service quality uncertainty. This holds for certain types of quality improvements (e.g., interior refurbishment and tableware upgrading in a restaurant setting) but may fail for others. If quality improvement reduces service quality uncertainty (e.g., staff training), then my recommendation to reduce both quality and price is strengthened: Quality reduction not only lowers the quality investment, but also increases demand (i.e., the niche customers) by magnifying the variation of customers' service quality estimates. Nevertheless, if quality improvement increases service quality uncertainty (e.g., recipe innovation that requires more complicated cooking techniques), my recommendation may no longer hold.

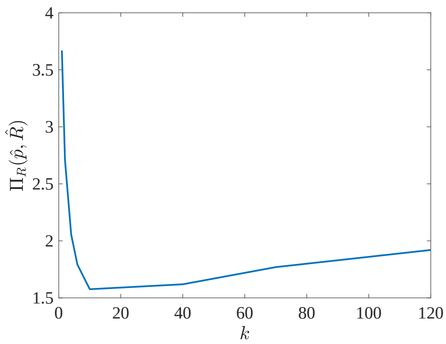
[3]The rest of Proposition 3 continues to hold. See Appendix A.2.2 for the proof and a related numerical study.

Proposition 3 characterises the quality control problem for sufficiently high and low market potential. The intermediate market potential case is analytically challenging, so I resort to a numerical study, as illustrated in Figure 2.2. Consistent with the exogenous quality scenario, the optimal price and revenue are both U-shaped in k. Perhaps more interestingly, I find that a larger sample size can induce the service provider to improve quality. This is because quality improvement is more profitable than price reduction under intermediate market potential: Quality improvement and the resulting price increase can increase revenue substantially due to the considerable market potential, while it does not significantly intensify congestion because market potential is not too high.

Figure 2.2: The Impact of k on $\hat{p}$, $\hat{R}$, and $\Pi_R(\hat{p},\hat{R})$ under Intermediate Market Potential ($\mu = 2, \lambda = 2, c = 1, \sigma = 10, a = 0.1$)



(a) $\hat{p}$ and $\hat{R}$



(b) $\Pi_R(\hat{p},\hat{R})$

2.4.2 Quality Information Disclosure

Apart from pricing and quality control, in practice service providers may also have the discretion to disclose service quality information. For example, a hospital can post key performance measures (e.g., patient satisfaction scale, readmission rate, and mortality rate) in its waiting rooms, and a restaurant can organise Yelp Elite events (Ayers 2011, Power 2011) and offer free drinks/desserts to customers who post food photos on social media such as Instagram (Dizik 2013), Facebook (Werner 2014), and WeChat (SAMPi 2015).

To incorporate the information disclosure decision, I assume that the service provider chooses to either inform customers of the mean service quality (i.e., $k \to +\infty$) or not (i.e., k is unchanged). Therefore, his joint pricing and information disclosure decisions are given by:

$$\max_{p \geqslant 0, i \in \{k,+\infty\}} \Pi(p,i) = p\lambda_a^i(p), \tag{2.4}$$

where I ignore the cost of the information disclosure to obtain sharper insights. As in Hassin (2007) and Guo and Zipkin (2007), I focus on truthful information disclosure. This fits my setting since the disclosed information (i.e., the expected service quality) is ex-post verifiable after collecting adequate service samples. Moreover, I do not explicitly consider partial information disclosure (i.e., increasing k to a finite number) because this is always sub-optimal (as will be shown in Proposition 4).

The joint optimisation problem in (2.4) can be treated as a 2-stage optimisation problem: The server first chooses k and then chooses p for any given k. By Proposition 1, the optimal price and revenue are $p^*(k)$ and $\Pi^*(k)$. As a result, the server discloses information if $\Pi_r^* > \Pi^*(k)$ and does not disclose if $\Pi_r^* < \Pi^*(k)$. The next proposition provides a full characterisation of the information disclosure decision, where $\tilde{k}$ is the sample size determined by $\Pi^*(\tilde{k}) = \Pi_r^*$ and $\tilde{R}$ is the mean service quality at which $\tilde{k} = 1$.[4]

Proposition 4. *(i) When $R \leqslant \underline{R}$, the service provider does not disclose information.*

(ii)When $\underline{R} < R \leqslant \tilde{R}$, the service provider does not disclose information for $k < \tilde{k}$, discloses information for $k > \tilde{k}$, and is indifferent between the two for $k = \tilde{k}$, where $\tilde{k}$ strictly decreases in R.

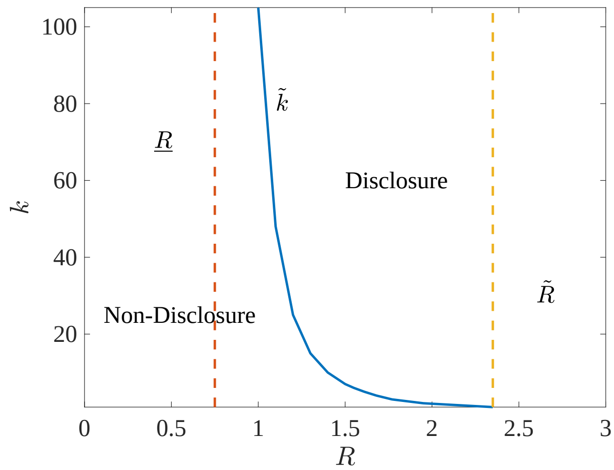
(iii) When $R > \tilde{R}$, the service provider discloses information.

I illustrate Proposition 4 by a numerical example in Figure 2.3. Intuitively, a low-quality (i.e., $R \leqslant \underline{R}$) service provider exploits customer bounded rationality by pricing high to target exclusively the niche customers. Therefore, he chooses not to inform them of the mean service quality. In contrast, a high-quality (i.e., $R > \tilde{R}$) service provider targets the mass customers. Since bounded rationality lowers revenue by leading some customers to balk due to occasional unfavourable anecdotes, the service provider chooses to inform customers of the mean service quality. Interestingly, I find that an intermediate-quality (i.e., $\underline{R} < R \leqslant \tilde{R}$) service provider informs customers of the mean service quality only when they are sufficiently rational (i.e., $k > \tilde{k}$). To see the intuition, first recall from Proposition 1 that the service provider switches from targeting exclusively the niche customers to targeting the mass customers as the sample size increases. Since information disclosure induces all

[4]As shown in the proof of Proposition 4, $\tilde{R}$ uniquely exists. Moreover, $\tilde{k} \in [1, +\infty)$ uniquely exists for $\underline{R} < R \leqslant \tilde{R}$ and does not exist otherwise.

customers to form an accurate service quality estimate (i.e., the niche customers vanish and the mass customers expand), the service provider should inform them of the mean service quality only when the sample size is sufficiently large.

Figure 2.3: The Impact of k and R on the Information Disclosure Decision ($\mu = 3$, $\lambda = 2$, $c = 1$, $\sigma = 5$, $\underline{R} = 0.75$, $\tilde{R} = 2.346$)



Proposition 4 contributes to the economics literature of quality disclosure (Dranove and Jin 2010, and references therein) by showing that customer bounded rationality alone can overturn the classic unravelling result that a *monopolistic* firm should disclose quality information whatever its quality is. This result is driven by the assumption of fully-rational customers: Even a low-quality firm should disclose information because customers rationally infer non-disclosing firms as having the lowest quality. In my setting, however, customers cannot make this rational inference. Therefore, the role of information disclosure is not to signal quality but to influence the market composition (i.e., the niche & mass customers). A low-quality service provider should not disclose quality information because it reduces demand (i.e., the niche customers).

Managerially, Proposition 4 underscores the importance of an up-to-date understanding of customers' service quality information. Specifically, contrary to the conventional wisdom that a high- (low-) quality service provider always (never) discloses quality information, I find that when service quality is not too high or too low, lower-quality service providers may disclose information if word-of-mouth abounds in the market, whereas higher-quality service providers may not disclose if word-of-mouth is scarce. In addition, a manager who has chosen not to disclose information may still need to keep track of customers' word-of-mouth: As they share more information over time, he may switch to disclosing quality information.

Comparative statics. The next corollary characterises the impact of service system parameters on the service provider's information disclosure decision; here I focus on the $\lambda < 2\mu$ case (since otherwise the server never discloses information).

Corollary 1. *When $\lambda < 2\mu$, $\tilde{k}$ strictly decreases in μ and strictly increases in c. When $\mu - \sqrt{c\mu/R} \leqslant \lambda < 2\mu$, $\tilde{k}$ strictly increases in λ.*

Corollary 1 again highlights the importance of incorporating customer bounded rationality in congested settings: Information non-disclosure (i.e., customers remain boundedly rational) is more attractive as the expected waiting cost increases (i.e., higher c, λ, or lower μ). In particular, compared to the no-congestion setting (i.e., $c = 0$), the server is incentivised to induce consumers to remain boundedly rational for a larger range of parameter values in congested settings. To see the intuition, first note that a higher expected waiting cost induces the service provider to reduce congestion by switching to targeting exclusively the niche customers. He does not disclose information because this reduces demand. Notably, I have numerically verified that $\tilde{k}$ strictly increases in λ even when $\lambda < \mu - \sqrt{c\mu/R}$. For completeness of the analysis, I have also examined the server's information disclosure decision under endogenous quality (see Appendix A.2.3). I find that all results in the present section continue to hold qualitatively.

2.5 Other Modeling Considerations

In this section I extend the analysis in §2.4 by considering two modelling variations, i.e., welfare-maximising service systems and customer heterogeneity in the sample size.

2.5.1 Welfare Maximisation

Consider a social planner that takes control of a service system to maximise social welfare. Typical examples include regulators levying tolls on public services, such as access to public roads/bridges/tunnels, and passport/driver's license applications. As in §2.4, I characterise the social planner's pricing, quality control, and information disclosure decisions.

2.5.1.1 Pricing.

The social planner's pricing decision is given by:

$$\max_{p \geqslant 0} W(p,k).$$

Consistent with Huang et al. (2013), I assume that customers are not compensated for receiving the service, i.e., $p \geqslant 0$. This prevents individuals with no need of the service from congesting the service system only for the compensation. Let $p_w^*(k)$ and $W_w^*(k)$ denote the optimal price and social welfare for a given sample size k, and let W_r^* denote the optimal social welfare in the fully-rational benchmark. The next proposition provides a full characterisation of $p_w^*(k)$ and $W_w^*(k)$.

Proposition 5. *(i) When $R > \underline{R}$ and $k < \hat{k}_w$, the optimal price $p_w^*(k) = 0$ and $W_w^*(k) < W_r^*$, where $\hat{k}_w \equiv \left[\sigma\Phi^{-1}\left(\left(1 - \frac{\mu - \sqrt{c\mu/R}}{\lambda}\right)^+\right)\right]^2 / \left(\sqrt{\frac{cR}{\mu}} - R\right)^2$.*

(ii) Otherwise, the optimal price $p_w^*(k) = R - \sqrt{\frac{cR}{\mu}} + \frac{\sigma\Phi^{-1}\left(1 - \frac{\mu - \sqrt{c\mu/R}}{\lambda}\right)}{\sqrt{k}}$ and $W_w^*(k) = W_r^*$. Moreover, the optimal price $p_w^*(k)$ strictly increases in k for $R > \frac{c\mu}{[(\mu-\lambda)^+]^2}$, strictly decreases in k for $R < \frac{c\mu}{[(\mu-\lambda)^+]^2}$, and is invariant in k for $R = \frac{c\mu}{[(\mu-\lambda)^+]^2}$.

Bounded rationality leads customers to deviate from the socially optimal joining rate, but does not influence joining customers' benefit from the service. Therefore, it never improves social welfare. To correct for the distorted joining rate, the social planner prices higher (lower) than the fully-rational benchmark if the mean service quality is low (high). This induces the socially optimal joining rate (i.e., welfare loss does not exist) unless the mean service quality is high and customers are sufficiently boundedly rational. In this case, fully correcting for the distortion requires the social planner to set a negative price, which is infeasible. Therefore, he prices at zero and bounded rationality leads to welfare loss. Notice that in the no-congestion setting, the social planner always prices at zero for $R > 0$ since $\hat{k}_w \to +\infty$. Intuitively, in the absence of congestion, joining is always socially beneficial as long as the service quality is positive. This result highlights the vital role of congestion in determining the impact of customer bounded rationality on managing service systems.

Compared to revenue maximisation, Proposition 5 shows that bounded rationality affects welfare-maximising service systems differently: It reduces the optimal revenue only when customers are sufficiently rational (i.e., $k > \tilde{k}$), whereas it reduces the optimal social welfare only when they are sufficiently boundedly rational (i.e., $k < \hat{k}_w$). The key insight is that the impact of bounded rationality on customers' joining behaviour may have contrasting revenue and welfare implications. For example, when $R > \underline{R}$ and $k < \min\{\tilde{k}, \hat{k}_w\}$, bounded rationality leads customers to join much more than they should and thus harms social welfare. However, it increases the optimal revenue by allowing the service provider to raise price.

2.5.1.2 Quality control.

Similar to §2.4.1.2, I incorporate the server's quality decision by assuming that he can choose the expected service quality R at cost aR^2. Therefore, his quality control problem is given by:

$$\max_{p,R\geqslant 0} W_R(p,R) = \lambda_a^k(p)R - \frac{c\lambda_a^k(p)}{\mu - \lambda_a^k(p)} - aR^2,$$

where the first two terms on the right-hand side represent social welfare and the last term is the quality investment. Let $\hat{p}_w$ and $\hat{R}_w$ denote the optimal price and quality. The following proposition characterises the impact of k on $\hat{p}_w$ and $\hat{R}_w$ for sufficiently high market potential.[5]

Proposition 6. *(i) A unique optimal price $\hat{p}_w$ exists.*

(ii) For sufficiently high market potential λ and $c > 0$, the optimal price $\hat{p}_w$ strictly decreases in k. The optimal quality $\hat{R}_w$ and social welfare are invariant in k when $a \leqslant \bar{a}$, and the social planner should not offer service when $a > \bar{a}$, where $\bar{a}$ uniquely exists.

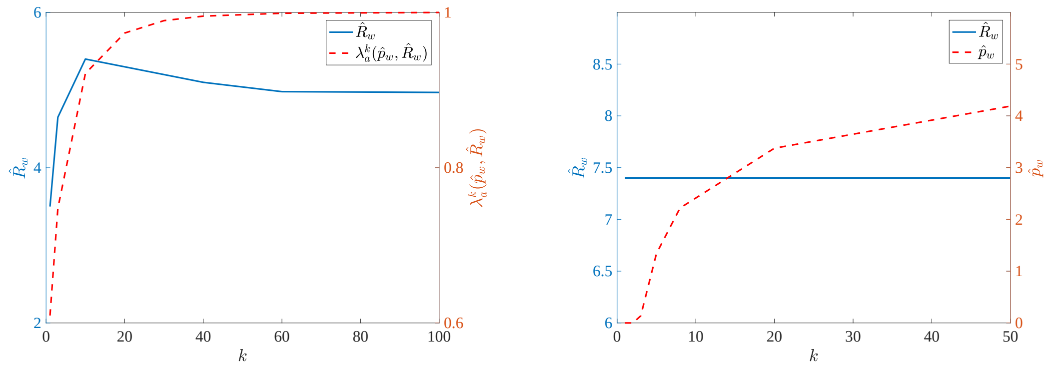
The social planner uses the pricing decision to maximise social welfare for a given quality level, and uses the quality decision to balance between the maximised social welfare and the corresponding quality investment. When market potential is sufficiently high and quality investment is not too costly (i.e., $a \leqslant \bar{a}$), the service system is sufficiently congested and bounded rationality leads customers to join more than socially desirable. In response, the social planner prices higher than the fully-rational benchmark and thus always achieves the first-best social welfare. As customers collect more anecdotes, they join less and the social planner prices lower to maintain the same joining rate and social welfare. This further implies that the trade-off between social welfare and quality investment is unchanged. As a result, the social planner maintains the same service quality. Notably, this result is unique to congested settings. Without congestion, a larger sample size leads more customers to join as long as the service quality is positive. Therefore, the social planner should improve quality (see Appendix A.2.2).

To complement Proposition 6, I have conducted a numerical study to examine the intermediate market potential case, as illustrated in Figure 2.4. In this situation, the service

[5] I have analytically shown that the social planner should not offer service for sufficiently low market potential (see the proof of Proposition 6). Here I omit the presentation for brevity.

system is not too congested and bounded rationality leads customers to join less than socially desirable. When λ is moderately low, the joining rate is very low such that inducing the socially optimal joining rate requires a negative price. Since this is not feasible, the social planner prices at zero. As customers acquire more anecdotes, they join more and this higher service utilisation drives the social planner to improve quality. However, when the service system is sufficiently congested, an even larger sample size leads the social planner to lower quality to alleviate congestion. When λ is moderately high, the joining rate is higher and the social planner can fully correct for customers' under-joining behaviour at a positive price. As customers acquire more anecdotes, they join more and the social planner prices higher to maintain the first-best social welfare. However, he maintains the same quality level since the trade-off between social welfare and quality investment is the same.

Figure 2.4: The Impact of k on $\hat{R}_w$, $\hat{p}_w$, and $\lambda_a^k(\hat{p}_w, \hat{R}_w)$ under Intermediate Market Potential ($a = 0.1, \mu = 2, c = 1, \sigma = 10$)



(a) Moderately Low Market Potential ($\lambda = 1$)

(b) Moderately High Market Potential ($\lambda = 1.8$)

2.5.1.3 Information disclosure.

Now I consider the social planner's decision to disclose service quality information. This has been widely practised in the healthcare sector. For example, the Centers for Medicare & Medicaid Services (CMS 2017) in the US and the National Health Service (NHS 2016) in the UK disclose hospitals' key performance measures to the public online.

As in revenue maximisation, I capture the information disclosure decision by assuming that the social planner chooses between informing customers of the mean service quality or not. Therefore, his joint pricing and information disclosure decisions are given by:

$$\max_{p \geqslant 0, i \in \{k, +\infty\}} W(p,i).$$

The next corollary, which follows immediately from Proposition 6, characterises the information disclosure decision.

Corollary 2. *The service provider discloses information if $R > \underline{R}$ and $k < \hat{k}_w$, and is indifferent between disclosure and non-disclosure otherwise.*

Corollary 2 suggests that a high-quality public service manager should inform customers of the mean service quality when they are sufficiently boundedly rational. This is because bounded rationality leads to welfare loss in this case: It induces customers to join significantly less than socially desirable such that the manager cannot fully correct for the distortion with a non-negative price.

2.5.2 Heterogeneous Sample Sizes

To obtain sharper insights, I assumed in §2.4 that all customers estimate service quality based on k anecdotes. In practice, however, some customers may have a larger sample size than others because of easier access to anecdotes or higher cognitive capabilities. Consistent with the cognitive hierarchy model (Nagel 1995, Stahl and Wilson 1994), I incorporate this heterogeneity into my model by assuming that k follows a zero-truncated Poisson distribution with rate n.[6] Therefore, the proportion of customers with sample size i ($i = 1, 2, \ldots$) is equal to $f_i = \frac{n^i e^{-n}}{i!(1-e^{-n})}$, where $n = \mathbf{E}[k]$ measures the average sample size across customers. Let λ_i denote the equilibrium joining rate of customers with i anecdotes, and let $\lambda_P^n \equiv \sum_{i=1}^{+\infty} f_i \lambda_i$ denote the total joining rate. I can show that

$$\lambda_i = \lambda \bar{\Phi}\left(\sqrt{i}\left(p + \frac{c}{\mu - \lambda_P^n} - R\right)/\sigma\right),$$

and then I have

$$\lambda_P^n = \lambda \sum_{i=1}^{+\infty} f_i \bar{\Phi}\left(\sqrt{i}\left(p + \frac{c}{\mu - \lambda_P^n} - R\right)/\sigma\right). \tag{2.5}$$

The next lemma shows that λ_P^n as defined by Equation (2.5) uniquely exists, where R_P is determined by $\sum_{i=1}^{+\infty} f_i \bar{\Phi}\left(\sqrt{i}\left[p + \frac{c}{(\mu - 0.5\lambda)^+} - R_P\right]/\sigma\right) = 0.5$.

[6] I have truncated $k = 0$ from the Poisson distribution because the anecdotal reasoning framework requires that customers have at least one anecdote.

Lemma 3. *(i) A unique $\lambda_P^n \in (0, \min\{\lambda, \mu\})$ exists and strictly decreases in p.*

(ii) The joining rate λ_P^n strictly decreases in n for $R < R_P$, strictly increases in n for $R > R_P$, and is invariant in n for $R = R_P$.

(iii) $\lim_{n \to +\infty} \lambda_P^n(p) = \lambda_r(p)$.

Consistent with Lemma 1, Lemma 3 shows that a larger average sample size leads to fewer customers joining a low-quality service system and more customers joining a high-quality service system. In other words, incorporating customer heterogeneity in k does not qualitatively change the impact of customer bounded rationality on their joining behaviour. Based on this, I have numerically verified that the impact of customer bounded rationality on the service provider's pricing, quality, and information disclosure decisions are also qualitatively preserved (see Appendix A.3.2).

2.6 Concluding Remarks

In this chapter, I studied the management of service systems with boundedly rational customers who infer service quality based on anecdotes. I characterised their equilibrium joining behaviour and the service provider's pricing, quality, and information disclosure decisions. Bounded rationality induces customers to form heterogeneous service quality estimates; thus the service provider adopts a pricing strategy different from the fully-rational benchmark. Specifically, a low-quality service provider targets the niche customers who considerably overestimate service quality, whereas a high-quality service provider targets the mass customers whose service quality estimates are not too low. With quality control, the service provider may reduce both price and quality as customers gather more anecdotes. I also characterised the service provider's quality information disclosure decision and found that it can be greatly influenced by customer bounded rationality: A lower-quality service provider may disclose information if customers are sufficiently rational, whereas a higher-quality service provider may not disclose if they are sufficiently boundedly rational.

This chapter has several implications for service system management in practice. First, estimating customers' level of bounded rationality (i.e., the number of service quality anecdotes) is critical because it greatly influences a manager's optimal pricing, quality, and information disclosure decisions. I conjecture that ignoring customer bounded rationality (by assuming that customers know the actual average service quality) can lead to significant profit loss. Second, even though conventional wisdom would probably suggest that managers should disclose quality information, I show that this may lower profit even for a high-quality service system. Third, when customers acquire more anecdotes over time, a service system manager may not improve service quality. In fact, when market potential is sufficiently high or low, he should reduce quality and compensate customers by lowering price. I hope that my theoretical model can stimulate future empirical studies in behavioural operations and help improve service management in practice.

For parsimony, I abstract away from several factors, which could serve as the basis for future modelling research. For example, I assumed that the service provider knows the service quality distribution. This is plausible for service systems with an established service content. However, it may fail if the service content is new, e.g., a restaurant experimenting with a new recipe. Future research can extend my work by considering service providers' decisions when they do not know the service quality distribution. Another research direction is to incorporate market competition in my model. It would be interesting to investigate whether customer anecdotal reasoning can soften service providers' competition on price and information disclosure. Finally, future research can consider customers' estimation of service quality based on both earlier customers' anecdotes and the service system itself (e.g., a prior belief, the queue length, or the expected waiting time).

Chapter 3

Opaque Selling and Last-Minute Selling: Revenue Management in Vertically Differentiated Markets

3.1 Introduction

Due to uncertain demand, firms across different industries (travel, electronics, fashion, etc.) may end up with leftover inventory/capacity (hereafter, inventory) after the regular (selling) season. To dispose of the leftovers, firms usually offer last-minute sales discounts. This strategy may generate additional revenue from the sales. However, it could harm the firms' overall profits by inducing consumers to strategically wait for the sales and be less willing to buy at the regular price. In fact, practitioners have long been sceptical about last-minute selling because it "gives away profits while simultaneously eroding the base of full-price business" (Schuster 1987).

As an alternative clearance strategy, opaque selling has recently been introduced by two online booking intermediaries: Priceline and Hotwire. Unlike last-minute selling, firms that adopt opaque selling offer a *new* product during the sales season that is equivalent to a lottery between different types of leftover products. Opaque selling has been widely applied since its introduction, most notably because it can"…generate incremental revenue by selling distressed inventory cheaply without disrupting existing distribution channels or retail pricing structures" (Smith et al. 2007, p. 75).

A salient feature of the application of opaque selling as a clearance strategy is that the leftover products within the opaque mix are usually vertically/quality differentiated, i.e., consumers *unanimously* prefer one product type to another. For example, Hotwire includes "special cars" in its rental car service, which are cars that have a compact size or larger, with the exact size revealed to consumers only after the transaction completes. Travellers typically prefer a larger size because there is more space for passengers and belongings. Hotwire and Priceline also offer opaque hotel rooms from brands that differ significantly in service quality as measured by the American Customer Satisfaction Index (ACSI 2016). For example, Priceline's 4-star opaque hotel brand list includes Hyatt Regency, Sheraton, and Holiday Inn (Priceline 2016), which are respectively ranked 4th, 18th, and 27th by ACSI. It seems likely that consumers who care more about hotel service than location (e.g., leisure travelers) will all be willing to pay a higher booking fee for Hyatt Regency than for Holiday Inn. As another example, consider the worldwide "Mystery Box" sale offered by Swatch during the winter sales season. Each box contains a watch randomly picked from the leftover inventory (Swatch 2015), and these may have considerably different regular prices, e.g., ranging between £32 and £105 in the 2014 UK sale (Swatch 2014). Since the price difference generally reflects differences in the quality of the design, construction, and materials, it seems reasonable to expect that consumers will prefer a higher-priced watch to a lower-priced one.

Despite the wide application of opaque selling as a clearance strategy in vertically differentiated markets, the existing literature has not yet examined its performance to cope with consumers' strategic waiting in this setting. Specifically, researchers have adopted single-period models to illustrate the advantage of opaque selling in terms of finer consumer segmentation (Anderson and Xie 2014, Rice et al. 2014) and higher inventory utilisation (Rice et al. 2014, Zhang et al. 2014), where the interaction between opaque selling and consumers' strategic waiting behaviour is absent. In contrast, Jerath et al. (2010) incorporate this interaction in horizontally differentiated markets. They find that consumers with heterogeneous preferences hold the same valuation for the opaque product (hereafter, the homogenizing effect). This allows the firm to charge a single price to extract full surplus from all purchasing consumers during the sales season (hereafter, the clearance advantage). Therefore, opaque selling may outperform last-minute selling. However, the homogenizing effect no longer exists under vertical differentiation: A consumer with a higher valuation is willing to pay more for *all* types of products, and thus more for their opaque mix. Consequently, it is unclear whether opaque selling can still outperform last-minute selling in vertically differentiated markets.

To address this question, I consider a stylised game-theoretical model in which a monopolistic firm sells vertically differentiated products across two periods to consumers who are heterogeneous in their product valuations. The inventory levels are exogenous, and the demand is uncertain. In period 1 (i.e., the regular season), the firm sells the products to the consumers separately, without knowing the realised state of demand. Then the demand realisation is revealed to all players. In period 2 (i.e., the sales season), the firm sells the leftover products, if any, either separately as transparent products (i.e., last-minute selling) or collectively as opaque products (i.e., opaque selling).

Using this model, I characterise the firm's optimal selling strategy and derive insights regarding the mechanism of opaque selling. The advantage of opaque selling is to allow the firm to price higher in the regular season because it prevents consumers from choosing their preferred product type if they delay their purchase until the sales season. However, opaque selling has the disadvantage of being less flexible in segmenting different consumer types, i.e., it lowers the sales-season revenue. Intriguingly, I find that both the advantage and the disadvantage are radically different from their counterparts in the horizontal differentiation setting. This contrast induces opposite policy recommendations across the two settings. Specifically, under vertical differentiation, the firm may switch from opaque selling to last-minute selling as consumers become more differentiated or the probability of the low-demand realisation increases. Under horizontal differentiation, however, the firm should always switch from last-minute selling to opaque selling.

I then extend the model by considering product damage. Consumers are usually willing to pay less if a product is obtained from the opaque mix because, e.g., opaque products are usually exempt from product returns/service cancellation. I find that such product damage lowers the firm's profit from the sales, but it makes consumers more willing to buy at the regular price. Therefore, the firm raises the regular price and its profit may go up.

The remainder of this chapter is organised as follows. §3.2 reviews the relevant literature. §3.3 sets up the base model, and §3.4 characterises the firm's optimal selling strategy and the mechanism of opaque selling. In §3.5, I extend the model by incorporating an additional consumer segment and product damage. I conclude in §3.6 by summarising the key findings and managerial insights. All proofs are relegated to Appendix B.1.

3.2 Literature Review

The investigation of firms' dynamic pricing decisions in the presence of strategic consumers can be traced back to the Coase conjecture: Coase (1972) shows that with an infinite number of selling seasons, consumers strategically delay their purchase until the price drops to the marginal cost. Since then, a growing body of operations management literature has examined the impact of strategic consumers on the pricing and inventory decisions of a firm that faces demand uncertainty and a limited inventory. In particular, Su (2007) characterises a firm's pricing strategy when consumers have heterogeneous product valuations and patience levels. Mersereau and Zhang (2012) study a firm's dynamic pricing decisions when some of the consumers are strategic but the firm does not know their proportion. Liu and van Ryzin (2008) consider a firm's capacity choice and find that it may deliberately understock to incentivise consumers to buy early at a higher price. Other papers in this literature investigate the impact of strategic consumers on the performance of different marketing and operations strategies, e.g., inventory replenishment (Cachon and Swinney 2009, Swinney 2011), enhanced design (Cachon and Swinney 2011), displaying all or one (Yin et al. 2009), posterior price matching (Lai et al. 2010), and single/dual rollover (Liang et al. 2014). My research complements this branch of literature by characterising the performance of opaque selling as a clearance strategy to counter the negative revenue impact of consumers' strategic waiting.

This chapter is also related to the emerging literature that studies the performance of opaque selling in horizontally differentiated markets. Opaque selling may emerge in equilibrium because of its advantages in facilitating price discrimination (Jiang 2007, Fay and Xie 2008), reducing capacity-demand mismatches (Fay and Xie 2008), and softening price competition (Fay 2008, Shapiro and Shi 2008). The literature has also studied the impact of opaque selling on a firm's other marketing and operations decisions, e.g., the timing of product allocation (Fay and Xie 2014) and product line design (Fay et al. 2015).

Recently, researchers have begun to examine the performance of opaque selling in vertically differentiated markets. Opaque selling may increase profits by inducing finer consumer segmentation (Anderson and Xie 2014, Rice et al. 2014) and higher inventory utilisation (Rice et al. 2014, Zhang et al. 2014). Moreover, when consumers are salient thinkers (Zheng et al. 2016) or cannot fully anticipate their post-purchase regret (Chao et al. 2016), opaque selling may emerge even when it cannot do so with rational consumers. In a more general setting in which the market can be either horizontally or vertically differentiated, Huang and Yu (2014) show that opaque selling can be optimal solely because of consumer anecdotal reasoning. In another paper, Fay and Xie (2010) compare the profitability of opaque selling with that of advance selling and show that the firm's optimal strategy depends on consumer heterogeneity in both horizontal and vertical dimensions. This stream of literature focuses on static settings and studies whether a firm should sell opaque products in addition to transparent products. In contrast, this chapter uses a dynamic model to identify the mechanism of opaque selling in helping a firm to cope with consumers' strategic waiting behaviour and characterise a firm's choice of offering opaque sales or transparent sales (i.e., last-minute selling).

From this perspective, this chapter is closely related to Jerath et al. (2010), who consider a two-period model in which firms offer either opaque sales or transparent sales to consumers who are horizontally differentiated in their product valuations. They find that opaque selling may outperform last-minute selling because of the previously mentioned homogenising effect. Moreover, opaque selling can be more attractive when there is a higher probability of low demand or a higher level of consumer differentiation. My research contributes to the literature by showing that the mechanism of opaque selling in vertically differentiated markets is radically different, and that it may induce policy recommendations that are exactly the opposite of those for the horizontal differentiation setting.

My work is also related to the extensive literature on substitute products (Bassok et al. 1999, Netessine et al. 2002, Shumsky and Zhang 2009, and references therein). Similar to opaque products, substitute products offer consumers the opportunity to receive either a high- or a low-quality product with a single purchase. However, the two selling strategies differ in several important respects. First, consumers of the substitute product do not anticipate being upgraded (i.e., receiving the high-quality product) and thus value the substitute the same as the low-quality product. In contrast, consumers of the opaque product take into account the possibility of receiving either product type and thus value the opaque product as a weighted average of their valuations for the high- and low-quality products. Second, substitute products help the firm meet otherwise unsatisfied demands and thus are usually offered when demand is high (Shumsky and Zhang 2009). In contrast, opaque products help the firm dispose of excess inventory and thus are usually offered when demand is low (Jerath et al. 2010).

3.3 Model

I consider a stylised game-theoretical model in which a monopolistic firm offers vertically differentiated products across two periods. The products can be physical goods or services, e.g., electronics, watches, or hotel rooms for a particular day. Moreover, they are perishable in the sense that products remaining after the second period are worthless. Consumers hold heterogeneous valuations for the products, and each consumer purchases at most one unit. I will refer to the firm as "she" and each consumer as "he."

Product valuations. The firm offers two types of products that differ in quality: A high-type product (denoted by subscript H) with inventory level M and a low-type product (denoted by subscript L) with inventory level N. All consumers value the high-type product more than the low-type product. However, they may hold different valuations for a given product type. As in Zhang et al. (2014), I capture this heterogeneity by considering two consumer segments, with one having higher valuations for both product types than the other. I will refer to the former segment as the high-type consumers (denoted by subscript h) and the latter segment as the low-type consumers (denoted by subscript l). Let V_{ij} denote a type-i consumer's valuation for a type-j product ($i \in \{h,l\}$, $j \in \{H,L\}$). By definition, $V_{iH} > V_{iL}$ for each i and $V_{hj} > V_{lj}$ for each j. In addition, I assume that $V_{hH} - V_{hL} > V_{lH} - V_{lL}$. This allows me to rule out the somewhat implausible equilibrium outcome where the firm sells the high-type products only to the low-type consumers and at the same time sells the low-type products only to the high-type consumers. All the above assumptions are standard in the classic vertical differentiation models (Moorthy 1988, Tirole 1988), which have been extensively adopted in the literature of strategic consumer behaviour (Parlaktürk 2012) and opaque selling (Chao et al. 2016, Zheng et al. 2016).

Demand uncertainty. Consistent with the opaque selling literature (see, e.g., Jerath et al. 2010, Zhang et al. 2014), I incorporate demand uncertainty by assuming that, with probability α, the demand turns out to be high in the sense that the numbers of the high- and low-type consumers (denoted by N_h and N_l) are respectively greater than the inventory levels of high- and low-type products (i.e., M and N).[7] Otherwise, the demand is low and the numbers of the high- and low-type consumers (denoted in this case by n_h and n_l) are less than M and N respectively.[8] In addition, I assume that $n_h + n_l > M$ and $N_h \leqslant M + N$.

[7] All insights continue to hold when demand is deterministic (see Appendix B.2).

[8] All insights continue to hold when the demands for two consumer types are negatively correlated (e.g., $n_h < M < N_h$ and $n_l > N > N_l$). I omit the formal presentation for brevity.

The first inequality, which is similar in spirit to Lemma 1 in Zhang et al. (2014), rules out the obvious case where opaque selling is not feasible because the high-type products are always out of stock in the sales season. Through the second inequality, I focus on the more plausible scenario where the firm cannot always meet the demand with only the high-type products. However, it can be verified that in other scenarios this chapter's major insights continue to hold.

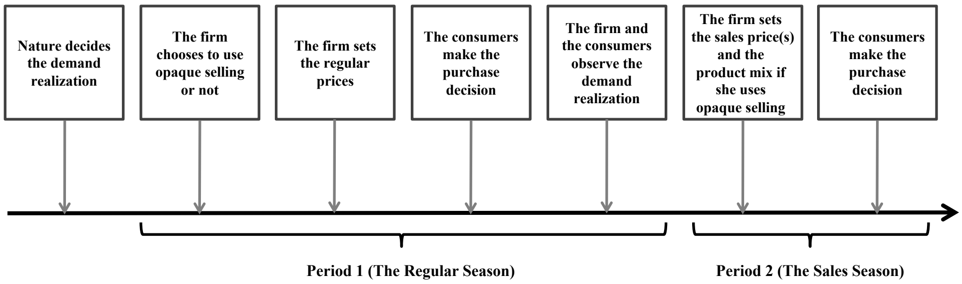
Below I summarise the set of assumptions that I adopt throughout this chapter, unless stated otherwise:

Assumption 1. *$V_{hj} > V_{lj}$ for each j, $V_{iH} > V_{iL}$ for each i, $V_{hH} - V_{hL} > V_{lH} - V_{lL}$, $n_h < M < N_h \leqslant M + N$, $n_l < N < N_l$, and $n_h + n_l > M$.*

For expositional convenience, I denote $K \equiv (1-\alpha)n_h + \alpha M$, $\Delta_i \equiv V_{iH} - V_{iL}$ for $i \in \{h, l\}$, and $\alpha' \equiv \alpha M / [(1-\alpha)N_h + \alpha M]$.

Timing. I consider two selling periods: In period 1 (the regular season), the firm sells the different types of products separately; in period 2 (the sales season), she sells them either separately as transparent products or collectively as opaque products (i.e., the different types of products are randomly allocated to the consumers). Following Jerath et al. (2010), I assume that (i) the firm cannot offer both opaque and transparent products in the sales season, and (ii) she chooses between opaque sales and last-minute sales (if applicable) before the regular season and commits to the choice in the sales season. However, the key mechanism of opaque selling is robust to these assumptions, and I omit the formal presentation for brevity. Within each selling season, the firm chooses the selling price(s) first, and then the consumers make their purchase decisions. Specifically, in the regular season, they decide whether to buy now or wait for the sales based on rational expectations about the future price and availability of each type of product. In the sales season, the consumers choose either to buy a product or to leave the market without a purchase. They make their purchase decisions to maximise their expected surplus. Anticipating the consumers' purchase behaviours, the firm determines the selling and pricing strategies to maximise her expected profit. Note that in both seasons she has the discretion to offer only one type of product by pricing the other type high enough to effectively prevent consumers from buying it. Both the firm and the consumers know the demand distribution, inventory levels, and consumer valuation structure from the beginning of the game. However, they do not know the demand realisation until the end of the regular season. Moreover, I assume that there is no

Figure 3.1: Sequence of Events



discounting between the two periods. All of these assumptions are consistent with Jerath et al. (2010).

I summarise the sequence of events as follows and illustrate it in Figure 3.1:

- *Stage 1* – Nature decides the demand realisation. The firm and the consumers do not observe it.
- *Stage 2* – The firm decides whether or not to use opaque selling in period 2, when applicable.
- *Stage 3* – The firm sets the period-1 (regular) prices for both types of products.
- *Stage 4* – The consumers choose to either buy a certain type of product in the regular season or wait for the sales season.
- *Stage 5* – The firm and the consumers observe the demand realisation.
- *Stage 6* – The firm sets the period-2 (sales) price(s). If she uses opaque selling, the firm also determines the product mix.
- *Stage 7* – The consumers choose to either buy a product or leave the market without a purchase.

Other assumptions. I assume that the firm cannot defer introducing the high-type product to the sales season. This may increase revenue by allowing the firm to condition its price on the demand realisation.[9] However, I rule out this strategy to reflect the general observation that firms usually introduce their high-quality products well before the start of

[9] I can show that this strategy never arises in equilibrium if $N_h < N$.

the sales season, possibly because (i) high-valuation consumers may not visit a firm in the sales season after discovering that no high-quality products are offered during the regular season, and (ii) firms cannot charge a high sales price for their high-quality products due to intense market competition. Notably, Parlaktürk (2012) has imposed a similar assumption.

I also adopt the following tie-breaking rules. First, I employ the efficient rationing scheme, i.e., a high-type consumer receives a particular type of product before a low-type consumer does when they are willing to pay the same price for it. This scheme has been widely adopted in the strategic consumer behaviour literature (Cachon and Swinney 2011, and references therein); the rationale is that the high-type consumers are more willing to compete for the product because they face a higher cost if it stocks out. However, I have verified that adopting the proportional rationing scheme does not qualitatively change the major insights of this chapter. Second, consumers who are indifferent between buying in either period will buy in period 1, and consumers who are indifferent between buying either type of product will buy the high-type product. Third, I assume that the firm strictly prefers last-minute selling when opaque selling yields the same expected profit. This break-even rule has also been adopted by Zhang et al. (2014) and can be justified by the fact that opaque selling usually involves additional transaction costs stemming from clarification and fulfilment. Finally, when different pricing strategies lead to the same expected profit, I assume that the firm adopts the strategy that entails the highest demand.

3.4 Analysis

In this section, I characterise the firm's optimal selling strategy and how it depends on demand uncertainty. In §3.4.1, I identify three candidate selling strategies and derive the corresponding equilibrium expected profits. In §3.4.2, I specify the firm's optimal selling strategy and how it depends on the probability of the high-demand realisation. Then I compare my results with Jerath et al. (2010) to highlight the difference in the mechanism of opaque selling across vertical and horizontal differentiation settings.

3.4.1 Candidate Selling Strategies

In this subsection, I describe all possible selling strategies and derive the corresponding profit expressions. Like Jerath et al. (2010), I assume that the consumers have rational expectations about the future price and availability of each type of product, and I solve for all pure strategy subgame perfect Nash equilibria using backward induction.

Before delving into the equilibrium analysis, I would first like to define the following selling strategies:

- *Traditional selling:* In period 1, the firm sells the high-type products to the high-type consumers and the low-type products to the low-type consumers. In period 2, she does not sell any products because there are either no leftover products (under high demand) or no remaining consumers (under low demand).

- *Last-minute selling:* In period 1, the firm sells the high-type products to the high-type consumers and does not sell the low-type products. In period 2, she sells the leftover products separately (i.e., as transparent products) to the remaining consumers.

- *Opaque selling:* In period 1, the firm sells the high-type products to the high-type consumers and does not sell the low-type products. In period 2, she mixes the leftover high- and low-type products (if applicable) and sells them collectively as opaque products.

For ease of exposition, I will denote traditional selling, last-minute selling, and opaque selling by subscripts T, L, and O respectively. Notably, this enumeration is not exhaustive. For example, in period 1 the firm may sell both types of products to the high-type consumers, or sell the high-type products to both types of consumers. The next lemma rules out these alternative selling strategies since they cannot improve revenue based on T, L, and O.

Lemma 4. *Under Assumption 1, selling strategies other than traditional selling, last-minute selling, and opaque selling cannot emerge in equilibrium.*

Next I will derive the profit expressions for the three selling strategies. This will serve as the basis for characterising the firm's optimal selling strategy in §3.4.2.

Traditional selling. In period 1, the firm prices the low-type products at V_{lL} to extract full surplus from the low-type consumers. She prices the high-type products at $V_{hH} - V_{hL} + V_{lL}$ so that the high-type consumers obtain the same surplus (i.e., $V_{hL} - V_{lL}$) from buying the high- and low-type products. The firm does not sell in period 2 because she has no leftover products if demand turns out to be high and no remaining consumers if it is low. Therefore, her expected profit is:

$$\Pi_T \equiv (1-\alpha)[n_h(V_{hH} - V_{hL} + V_{lL}) + n_l V_{lL}] + \alpha[M(V_{hH} - V_{hL} + V_{lL}) + N V_{lL}].$$

Last-minute selling. In period 1 the firm sells the high-type products exclusively to the high-type consumers and does not sell the low-type products. Therefore, if demand turns out to be low, in period 2 she has $M - n_h$ high-type products and N low-type products, and the market has n_l low-type consumers. By subgame perfection, she should price the high-type products at V_{lH} and the low-type products at V_{lL}. This will allow the firm to extract full consumer surplus, which is equal to $(M - n_h)V_{lH} + (n_l - M + n_h)V_{lL}$. If demand turns out to be high, in period 2 the firm has N low-type products and the market has $N_h - M$ high-type and N_l low-type consumers. Therefore, she sells the products either exclusively to the high-type consumers at price V_{hL}, or to both types of consumers at price V_{lL}. Since the corresponding profits are $(N_h - M)V_{hL}$ and NV_{lL} respectively, the firm should price at V_{hL} if $(N_h - M)V_{hL} > NV_{lL}$ and at V_{lL} otherwise.

Next I determine the period-1 price of the high-type products. Consider a high-type consumer who visits the firm in period 1 and finds the high-type products still in stock. To incentivise him to buy now, the firm should price the high-type products below V_{hH} so that the consumer is indifferent between buying in period 1 and in period 2. As a result, the period-1 price is equal to $V_{hH} - CS_L$, where CS_L denotes the consumer's expected surplus of buying in period 2. To derive the expression for CS_L, first note that if demand turns out to be low, in period 2 the consumer should buy a high-type product at price V_{lH} to obtain surplus $V_{hH} - V_{lH}$. If demand turns out to be high, he should buy a low-type product, and the surplus will be equal to $V_{hL} - V_{lL}$ if $(N_h - M)V_{hL} \leqslant NV_{lL}$ and 0 otherwise. Therefore $CS_L = (1-\alpha')(V_{hH} - V_{lH})$ if $(N_h - M)V_{hL} > NV_{lL}$ and $CS_L = (1-\alpha')(V_{hH} - V_{lH}) + \alpha'(V_{hL} - V_{lL})$ otherwise, where $\alpha' \equiv \alpha M/[(1-\alpha)N_h + \alpha M]$ denotes the consumer's updated belief about the probability of the high-demand realisation after discovering that the high-type products are still in stock when he visits the firm. The firm's expected profit is:

$$\Pi_L \equiv \begin{cases} (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(V_{hH} - V_{lH})] + (M - n_h)V_{lH} \\ +(n_l - M + n_h)V_{lL}\} + \alpha\{M[V_{hH} - (1-\alpha')(V_{hH} - V_{lH})] \\ +(N_h - M)V_{hL}\}, & \text{if} \quad (N_h - M)V_{hL} > NV_{lL}, \\ \\ (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(V_{hH} - V_{lH}) - \alpha'(V_{hL} - V_{lL})] \\ +(M - n_h)V_{lH} + (n_l - M + n_h)V_{lL}\} + \alpha\{M[V_{hH} \\ -(1-\alpha')(V_{hH} - V_{lH}) - \alpha'(V_{hL} - V_{lL})] + NV_{lL}\}, & \text{if} \quad (N_h - M)V_{hL} \leqslant NV_{lL}. \end{cases}$$

Opaque selling. This selling strategy is identical to last-minute selling except that under low demand, in period 2 the firm mixes the leftover high- and low-type products and sells them collectively as opaque products.[10] By subgame perfection, she should mix all $M-n_h$ high-type products with n_h+n_l-M low-type products and price the opaque products at $\frac{M-n_h}{n_l}V_{lH}+\frac{n_h+n_l-M}{n_l}V_{lL}$. This allows the firm to extract full consumer surplus, which is equal to $(M-n_h)V_{lH}+(n_h+n_l-M)V_{lL}$.

Next I derive the period-1 price of the high-type products. The firm should price them below V_{hH} so that a high-type consumer who finds the product in stock will be indifferent between buying in period 1 and waiting until period 2. Therefore, the period-1 price is equal to $V_{hH}-CS_O$, where CS_O denotes the consumer's expected surplus of waiting for period 2. Similar to the derivation of CS_L, I have

$$CS_O=\begin{cases}(1-\alpha')[\frac{M-n_h}{n_l}(V_{hH}-V_{lH})+\frac{n_h+n_l-M}{n_l}(V_{hL}-V_{lL})], & \text{if} \quad (N_h-M)V_{hL}>NV_{lL},\\ (1-\alpha')[\frac{M-n_h}{n_l}(V_{hH}-V_{lH})+\frac{n_h+n_l-M}{n_l}(V_{hL}-V_{lL})] & \\ +\alpha'(V_{hL}-V_{lL}), & \text{if} \quad (N_h-M)V_{hL}\leqslant NV_{lL}.\end{cases}$$

Based on this, I can derive the expected profit as:

$$\Pi_O\equiv\begin{cases}(1-\alpha)\{n_h[V_{hH}-(1-\alpha')(V_{hH}-V_{lH})]+(M-n_h)V_{lH} & \\ +(n_h+n_l-M)V_{lL}\}+\alpha\{M[V_{hH}-(1-\alpha')(V_{hH}-V_{lH})] & \\ +(N_h-M)V_{hL}\}+(1-\alpha')\frac{n_h+n_l-M}{n_l}K(V_{hH}-V_{lH} & \\ -V_{hL}+V_{lL}), & \text{if} \quad (N_h-M)V_{hL}>NV_{lL},\\ & \\ (1-\alpha)\{n_h[V_{hH}-(1-\alpha')(V_{hH}-V_{lH})-\alpha'(V_{hL}-V_{lL})] & \\ +(M-n_h)V_{lH}+(n_h+n_l-M)V_{lL}\}+\alpha\{M[V_{hH} & \\ -(1-\alpha')(V_{hH}-V_{lH})-\alpha'(V_{hL}-V_{lL})]+NV_{lL}\} & \\ +(1-\alpha')\frac{n_h+n_l-M}{n_l}K(V_{hH}-V_{lH}-V_{hL}+V_{lL}), & \text{if} \quad (N_h-M)V_{hL}\leqslant NV_{lL}.\end{cases}$$

I summarise the equilibrium prices and demands for the three selling strategies in Table 3.1. For ease of exposition, I denote $\psi\equiv(M-n_h)/n_l$, $\bar{\psi}\equiv 1-\psi$, and use p_i^j and D_i^j to denote the price and demand for product i in period j ($i\in\{H,L,o\}$, $j\in\{1,2h,2l\}$), where the subscript o denotes the opaque products and the superscripts $2h$ and $2l$ denote high and low demand in period 2.

[10]She does not offer opaque products under high demand because in this case the high-type products are out of stock.

Table 3.1: The Equilibrium Prices and Demands for Traditional Selling, Last-Minute Selling, and Opaque Selling

(a) Traditional Selling

Prices (p_H^1, p_L^1)	Demands for High-Type Products (D_H^{2l}, D_H^{2h})	Demands for Low-Type Products (D_L^{2l}, D_L^{2h})
$(V_{hH} - V_{hL} + V_{lL}, V_{lL})$	(n_h, M)	(n_l, N)

(b) Last-Minute Selling

$\frac{V_{lL}}{V_{hL}}$	The Regular Season		The Sales Season			
			Low Demand		High Demand	
	Price p_H^1	Demands (D_H^{2l}, D_H^{2h})	Prices (p_H^{2l}, p_L^{2l})	Demands (D_H^{2l}, D_L^{2l})	Price p_L^{2h}	Demand D_L^{2h}
$\frac{V_{lL}}{V_{hL}} < \frac{N_h - M}{N}$	$V_{hH} - (1-\alpha')(V_{hH} - V_{lH})$	(n_h, M)	(V_{lH}, V_{lL})	$(M - n_h, n_l - M + n_h)$	V_{hL}	$N_h - M$
$\frac{V_{lL}}{V_{hL}} > \frac{N_h - M}{N}$	$V_{hH} - (1-\alpha')(V_{hH} - V_{lH}) - \alpha'(V_{hL} - V_{lL})$				V_{lL}	N

(c) Opaque Selling

$\frac{V_{lL}}{V_{hL}}$	The Regular Season		The Sales Season			
			Low Demand		High Demand	
	Price p_H^1	Demands (D_H^{2l}, D_H^{2h})	Price p_o^{2l}	Demand D_o^{2l}	Price p_L^{2h}	Demand D_L^{2h}
$\frac{V_{lL}}{V_{hL}} < \frac{N_h - M}{N}$	$V_{hH} - (1-\alpha')[\psi(V_{hH} - V_{lH}) + \bar{\psi}(V_{hL} - V_{lL})]$	(n_h, M)	$\psi V_{lH} + \bar{\psi} V_{lL}$	n_l	V_{hL}	$N_h - M$
$\frac{V_{lL}}{V_{hL}} \geqslant \frac{N_h - M}{N}$	$V_{hH} - (1-\alpha')[\psi(V_{hH} - V_{lH}) + \bar{\psi}(V_{hL} - V_{lL})] - \alpha'(V_{hL} - V_{lL})$				V_{lL}	N

3.4.2 The Optimal Selling Strategy

In §3.4.1, I derived the profit expressions for all candidate selling strategies. The next proposition compares the expressions to characterise the firm's optimal selling strategy.

Proposition 7. *Under Assumption 1, opaque selling strictly dominates last-minute selling.*

(i) When $V_{hL} > \frac{N}{N_h - M} V_{lL}$, the firm should use opaque selling if $\Delta_h < [1 + \frac{(1-\alpha)n_l}{(1-\alpha')K}]\Delta_l + \frac{\alpha' n_l}{(1-\alpha')(M-n_h)}(V_{hL} - V_{lL}) + \frac{\alpha n_l}{(1-\alpha')(M-n_h)}[\frac{N_h - M}{K}V_{hL} - \frac{N}{K}V_{lL}]$ and traditional selling otherwise.

(ii) When $V_{hL} < \frac{N}{N_h - M} V_{lL}$, the firm should use opaque selling if $\Delta_h < [1 + \frac{(1-\alpha)n_l}{(1-\alpha')K}]\Delta_l$ and traditional selling otherwise.

Proposition 7 shows that opaque selling may still outperform last-minute selling in vertically differentiated markets. This is somewhat surprising, since the previously mentioned homogenising effect no longer exists in vertically differentiated markets. In fact, since the high-type consumers value all types of products higher than the low-type consumers, they also have a higher valuation for the opaque mix. To see this formally, consider a special case of my model in which $V_{hH}/V_{lH} = V_{hL}/V_{lL} \equiv \sigma$.[11] This ratio represents consumers' level of heterogeneity in terms of their valuations for transparent products. For the opaque mix with the proportion of the high-type products equal to ω, the high- and low-type consumers' valuations are $\omega V_{hH} + (1-\omega)V_{hL}$ and $\omega V_{lH} + (1-\omega)V_{lL}$ respectively. As a result, their level of heterogeneity regarding the opaque products is

$$\frac{\omega V_{hH} + (1-\omega)V_{hL}}{\omega V_{lH} + (1-\omega)V_{lL}} = \frac{\omega\sigma V_{lH} + (1-\omega)\sigma V_{lL}}{\omega V_{lH} + (1-\omega)V_{lL}} = \sigma.$$

In other words, opaque selling does not influence consumers' level of heterogeneity. Therefore, its clearance advantage no longer holds in vertically differentiated markets. In fact, opaque selling and last-minute selling lead to the same expected sales revenue (see the expressions of Π_L and Π_O).

Despite the absence of the homogenising effect, I find that opaque selling may still outperform last-minute selling in vertically differentiated markets. This is because opaque selling deprives high-type consumers of the opportunity to choose their preferred product type during the sales season (hereafter, the choice-deprivation advantage). Specifically, under low demand, a high-type consumer finds both types of products in stock in the sales season. If the firm sells them separately, the consumer should buy a high-type product since it entails a higher surplus. In contrast, if the firm sells them collectively as opaque products, the consumer may be allocated a low-type product that entails a lower surplus. Therefore, he has less incentive to wait for the sales, and the firm can charge a higher regular price on the high-type products than she can with last-minute selling.

Notably, opaque selling may also increase the regular prices in horizontally differentiated markets. However, this is because of the homogenising effect rather than the choice-deprivation advantage. In fact, this advantage does not exist under horizontal differentiation because consumers never face a choice between the two product types in the sales season: All remaining consumers would rather leave the market than buy the products at the other end of the Hotelling line (see Proposition 5.2 of Jerath et al. 2010). This difference leads to contrasting policy recommendations under vertical and horizontal differentiation (as will be shown in Proposition 10).

Another difference between the two settings is that opaque selling always outperforms last-minute selling in vertically differentiated markets, whereas it may not do so in horizontally differentiated markets. As shown by Jerath et al. (2010), opaque selling under

[11]This assumption is standard in the literature (see, e.g., Mussa and Rosen 1978, Parlaktürk 2012).

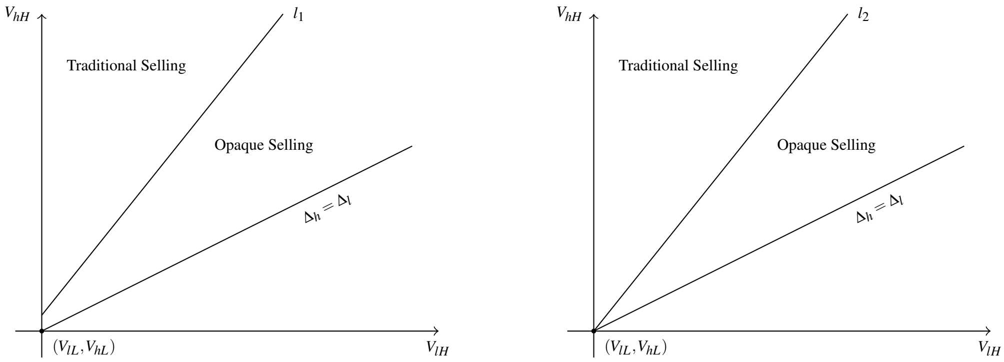
horizontal differentiation may allocate to a consumer the product type he dislikes (i.e., the net purchase payoff is negative). Therefore, it significantly reduces the product valuations of strong-preference consumers (i.e., those close to either endpoint of the Hotelling line), which further implies that opaque selling may be less effective in separating out these high-preference consumers and thus becomes suboptimal. This product deterioration effect no longer holds in vertically differentiated markets because all product types are valuable, i.e., there does not exist a product type that induces a negative net purchase payoff. In fact, since consumers value the opaque products as the weighted sum of their valuations for the transparent products, the firm obtains the same sales-season revenue from opaque selling and last-minute selling.

Since last-minute selling is always suboptimal, the firm effectively chooses between opaque selling and traditional selling. Proposition 7 provides a full characterisation of how this decision depends on consumer valuations, which I illustrate in Figure 3.2. Compared to traditional selling, opaque selling enables the firm to clean up the leftover high-type products by selling them to the low-type consumers in the sales season. However, this clean-up advantage comes at the cost of a lower regular price, which prevents the high-type consumers from waiting for the sales. Therefore, opaque selling outperforms traditional selling when the low-type consumers' valuation for the high-type products is close to the high-type consumers' valuation.

Interestingly, Figure 3.2 also shows that opaque selling dominates traditional selling for a larger range of parameter values when the high-type consumers value the low-type products much more than the low-type consumers do (i.e., $V_{hL} > \frac{N}{N_h - M} V_{lL}$). Intuitively, in this case with a high-demand realisation, it is more profitable to sell the leftover low-type products exclusively to the remaining high-type consumers than to both types of consumers. Opaque selling is more attractive than traditional selling since it provides an additional pricing opportunity for the firm to sell the low-type products to different consumer types across the two seasons.

Managerially, Proposition 7 indicates that selling quality-differentiated leftover products collectively (i.e., as an opaque mix) rather than separately can be optimal because it allows the firm to charge a higher regular price. Moreover, the firm should offer opaque sales rather than no sales when consumers' valuations for the high-quality products are close.

Figure 3.2: The Optimal Selling Strategy*



(a) $V_{hL} > \frac{N}{N_h-M} V_{lL}$ (b) $V_{hL} \leqslant \frac{N}{N_h-M} V_{lL}$

*$l_1 : \Delta_h = \{1+(1-\alpha)n_l/[(1-\alpha')K]\}\Delta_l + \alpha' n_l(V_{hL}-V_{lL})/[(1-\alpha')(M-n_h)] + \alpha n_l[(N_h-M)V_{hL}-NV_{lL}]/[(1-\alpha')K(M-n_h)]$
$l_2 : \Delta_h = \{1+(1-\alpha)n_l/[(1-\alpha')K]\}\Delta_l$

The impact of demand uncertainty. The previous discussion showed that opaque selling may outperform traditional selling by allowing the firm to clean up the leftover products under *both* high- and low-demand realisations. Therefore, it is not intuitively clear whether a higher probability of the high-demand realisation increases or decreases the attractiveness of opaque selling. I answer this question in the next proposition, where the expressions of thresholds for V_{lH} and V_{hL} are relegated to the proof for brevity.

Proposition 8. *Under Assumption 1,*

(i) when $V_{hL} \leqslant \frac{N}{N_h-M} V_{lL}$, opaque selling becomes less preferable than traditional selling as α increases;

(ii) when $V_{hL} > \frac{N}{N_h-M} V_{lL}$, opaque selling becomes less preferable than traditional selling as α increases if V_{lH} is high or V_{hL} is low, and becomes more preferable otherwise.

When the high-type consumers' valuation for the low-type products is close to the low-type consumers' valuation (i.e., $V_{hL} \leqslant \frac{N}{N_h-M} V_{lL}$), it is optimal to price the low-type products at the low-type consumers' valuations in both selling seasons. As a result, offering opaque sales cannot increase profits under the high-demand realisation. This further implies that the firm prefers traditional selling to opaque selling as the probability of high demand increases.

The situation is more complicated when the high-type consumers' valuation for the low-type products is much higher than the low-type consumers' valuation (i.e., $V_{hL} \geqslant \frac{N}{N_h-M} V_{lL}$). In this case, opaque selling generates higher profits from leftover products under both high- and low-demand realisations. Specifically, if demand turns out to be low, opaque selling allows the firm to clean up the leftover high-type products (at price V_{hL}); if demand turns out to be high, opaque selling increases the price of the leftover low-type products by allowing the firm to sell them exclusively to the remaining high-type consumers (at price V_{lH}). When V_{lH} is high or V_{hL} is low, the clean-up advantage under the low-demand realisation dominates. Since a higher α weakens the aggregate clean-up advantage, it makes opaque selling less preferable than traditional selling. Otherwise, opaque selling is more attractive as α increases.

3.5 Model Extensions

To derive richer insights into the mechanism of opaque selling, I extend the base model in this section by considering an additional consumer segment and damaged opaque products.

3.5.1 Three Consumer Segments

Based on the stylised model with two consumer segments, the analysis in §3.4.2 reveals several fundamental differences in opaque selling's mechanism for vertically and horizontally differentiated markets. In particular, its clearance advantage in horizontally differentiated markets no longer exists under vertical differentiation, and its choice-deprivation advantage in vertically differentiated markets does not exist under horizontal differentiation. In this section, I will consider an additional consumer segment and show that these differences continue to hold in the extended model.[12] This extension also allows me to derive richer insights into the firm's optimal selling strategy and how it depends on demand uncertainty.

I introduce the additional consumer segment by splitting the low-type consumers into two segments: Intermediate-type consumers (denoted by subscript m) and low-type consumers (denoted by subscript l). Let V_{ij} denote the valuation of a type-i consumer for a type-j product ($i \in \{h,m,l\}$, $j \in \{H,L\}$). Moreover, let $\Delta_i \equiv V_{iH} - V_{iL}$, and n_i (N_i) denote the number of type-i consumers under the low- (high-) demand realisation. I adopt the following set of assumptions throughout this section:

Assumption 2. *$V_{hj} > V_{mj} > V_{lj}$ for each j, $V_{iH} > V_{iL}$ for each i, $\Delta_h > \Delta_m > \Delta_l$, $n_h < M < N_h \leqslant M+N$, $n_h + n_m > M$, $n_h + n_l > M$, $N_h + N_m > M+N$, and $n_m + n_l \leqslant N < N_m + N_l$.*

Assumption 2 is a direct extension of Assumption 1 except $N_h + N_m > M+N$. I impose

[12] I do not incorporate more than three segments since the analysis would be prohibitively difficult. Presumably major insights in this section are robust to the three-segment assumption.

this assumption to rule out the somewhat implausible equilibrium outcome in which the firm sells the high-type products exclusively to the intermediate-type consumers and does not sell the low-type products in the regular season. However, it can be verified that major insights continue to hold when $N_h + N_m \leqslant M + N$.

Next I enumerate all candidate selling strategies and derive their profit expressions.

- *Traditional selling:* In period 1, the firm sells the high-type products to the high-type consumers at price $V_{hH} - V_{hL} + V_{lL}$ and the low-type products to the intermediate- and low-type consumers at price V_{lL}. As in the base model, the firm does not sell in period 2 because there are either no leftover products or no remaining consumers. I will denote this strategy by subscript Tm. The corresponding expected profit is $\Pi_{Tm} \equiv (1-\alpha)[(V_{hH} - V_{hL} + V_{lL})n_h + V_{lL}(n_m + n_l)] + \alpha[(V_{hH} - V_{hL} + V_{lL})M + V_{lL}N]$.
- *Last-minute selling:* The firm sells the high-type products to the high-type consumers in period 1 and sells the leftover products separately to the remaining consumers in period 2. Specifically, if demand turns out to be high, in period 2 she has N low-type products and the market has $N_h - M$ high-type consumers, n_m intermediate-type consumers, and n_l low-type consumers. Similar to the base model, the firm should sell the products exclusively to the remaining high-type consumers if $(N_h - M)V_{hL} > NV_{mL}$, and to both the high- and intermediate-type consumers otherwise. If demand turns out to be low, in period 2 the firm has $M - n_h$ high-type products and N low-type products, and the market has n_m intermediate-type consumers and n_l low-type consumers. She either sells both types of products exclusively to the intermediate-type consumers or sells the high-type products to the intermediate-type consumers and the low-type products to the low-type consumers. In the former case (denoted by subscript $L1m$), the firm prices the high- and low-type products at V_{mH} and V_{mL}, and her expected profit is: $\Pi_{L1m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(V_{hH} - V_{mH})] + (M - n_h)V_{mH} + (n_h + n_m - M)V_{mL}\} + \alpha\{M[V_{hH} - (1-\alpha')(V_{hH} - V_{mH})] + (N_h - M)V_{hL}\}$ if $(N_h - M)V_{hL} > NV_{mL}$, and $\Pi_{L1m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(V_{hH} - V_{mH}) - \alpha'(V_{hL} - V_{mL})] + (M - n_h)V_{mH} + (n_h + n_m - M)V_{mL}\} + \alpha\{M[V_{hH} - (1-\alpha')(V_{hH} - V_{mH}) - \alpha'(V_{hL} - V_{mL})] + NV_{mL}\}$ otherwise. In the latter case (denoted by subscript $L2m$), the firm prices the high-type products at $V_{mH} - V_{mL} + V_{lL}$ and the low-type products at V_{lL}. Therefore, the firm's expected profit is $\Pi_{L2m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(V_{hH} - V_{mH} + V_{mL} - V_{lL})] + (M - n_h)(V_{mH} - V_{mL} + V_{lL}) + (n_h + n_m + n_l - M)V_{lL}\} + \alpha\{M[V_{hH} - (1-\alpha')(V_{hH} - V_{mH} +$

$V_{mL} - V_{lL})] + (N_h - M)V_{hL}\}$ if $(N_h - M)V_{hL} > NV_{mL}$, and $\Pi_{L2m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(V_{hH} - V_{mH} + V_{mL} - V_{lL}) - \alpha'(V_{hL} - V_{mL})] + (M - n_h)(V_{mH} - V_{mL} + V_{lL}) + (n_h + n_m + n_l - M)V_{lL}\} + \alpha\{M[V_{hH} - (1-\alpha')(V_{hH} - V_{mH} + V_{mL} - V_{lL}) - \alpha'(V_{hL} - V_{mL})] + NV_{mL}\}$ otherwise.

- *Opaque selling:* This strategy is identical to last-minute selling except that under low demand, in period 2 the firm mixes the leftover high- and low-type products and sells them collectively either to the intermediate-type consumers only or to both the intermediate- and low-type consumers. In the former case (denoted by subscript $O1m$), the firm prices the opaque products at $\frac{M-n_h}{n_m}V_{mH} + \frac{n_h+n_m-M}{n_m}V_{mL}$, and her expected profit is $\Pi_{O1m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(\theta(V_{hH} - V_{mH}) + \bar{\theta}(V_{hL} - V_{mL}))] + (M - n_h)V_{mH} + (n_h + n_m - M)V_{mL}\} + \alpha\{M[V_{hH} - (1-\alpha')(\theta(V_{hH} - V_{mH}) + \bar{\theta}(V_{hL} - V_{mL}))] + (N_h - M)V_{hL}\}$ if $(N_h - M)V_{hL} > NV_{mL}$ and $\Pi_{O1m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(\theta(V_{hH} - V_{mH}) + \bar{\theta}(V_{hL} - V_{mL})) - \alpha'(V_{hL} - V_{mL})] + (M - n_h)V_{mH} + (n_h + n_m - M)V_{mL}\} + \alpha\{M[V_{hH} - (1-\alpha')(\theta(V_{hH} - V_{mH}) + \bar{\theta}(V_{hL} - V_{mL})) - \alpha'(V_{hL} - V_{mL})] + NV_{mL}\}$ otherwise. In the latter case (denoted by subscript $O2m$), the firm prices at $\frac{M-n_h}{n_m+n_l}V_{lH} + \frac{n_h+n_m+n_l-M}{n_m+n_l}V_{lL}$, and the expected profit is $\Pi_{O2m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(\phi(V_{hH} - V_{lH}) + \bar{\phi}(V_{hL} - V_{lL}))] + (M - n_h)V_{lH} + (n_h + n_m + n_l - M)V_{lL}\} + \alpha\{M[V_{hH} - (1-\alpha')(\phi(V_{hH} - V_{lH}) + \bar{\phi}(V_{hL} - V_{lL}))] + (N_h - M)V_{hL}\}$ if $(N_h - M)V_{hL} > NV_{mL}$, and $\Pi_{O2m} \equiv (1-\alpha)\{n_h[V_{hH} - (1-\alpha')(\phi(V_{hH} - V_{lH}) + \bar{\phi}(V_{hL} - V_{lL})) - \alpha'(V_{hL} - V_{mL})] + (M - n_h)V_{lH} + (n_h + n_m + n_l - M)V_{lL}\} + \alpha\{M[V_{hH} - (1-\alpha')(\phi(V_{hH} - V_{lH}) + \bar{\phi}(V_{hL} - V_{lL})) - \alpha'(V_{hL} - V_{mL})] + NV_{mL}\}$ otherwise, where $\phi \equiv (M - n_h)/(n_m + n_l)$ and $\bar{\phi} \equiv 1 - \phi$. The next lemma shows that I do not need to consider other selling strategies.

Lemma 5. *Under Assumption 2, selling strategies other than traditional selling, last-minute selling, and opaque selling cannot emerge in equilibrium.*

According to Lemma 5, characterising the firm's optimal selling strategy boils down to comparing Π_{Tm}, Π_{L1m}, Π_{L2m}, Π_{O1m}, and Π_{O2m}. The next proposition provides a full characterisation, where I relegate the expressions of thresholds for V_{hH} and V_{mH} to the proof for brevity.

Proposition 9. *Under Assumption 2,* *(i) when $(n_m + n_l)V_{lL} < n_m V_{mL}$, the firm should use traditional selling if V_{mH} is sufficiently low compared to V_{hH} and opaque selling otherwise;*

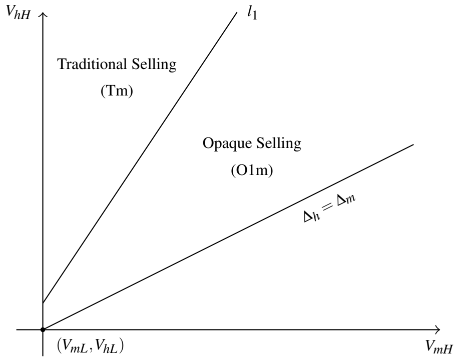
(ii) when $(n_m + n_l)V_{lL} \geqslant n_m V_{mL}$, the firm should use traditional selling if V_{mH} is sufficiently low compared to V_{hH}, last-minute selling if V_{mH} is sufficiently high compared to V_{hH}, and opaque selling otherwise.

I graphically illustrate Proposition 9 in Figure 3.3. When the intermediate-type consumers value the low-type products much more than the low-type consumers do (i.e., $(n_m + n_l)V_{lL} < n_m V_{mL}$), the firm should sell the leftover high- and low-type products exclusively to the intermediate-type consumers in the sales season under low demand. Therefore, opaque selling and last-minute selling generate the same sales-season revenue. Due to its choice-deprivation advantage, opaque selling allows the firm to raise the regular price of the high-type products and thus strictly dominates last-minute selling. Interestingly, I find that last-minute selling may outperform opaque selling when the intermediate- and low-type consumers' valuations for the low-type products are close (i.e., $(n_m + n_l)V_{lL} \geqslant n_m V_{mL}$). To see the intuition, first note that opaque selling is less flexible in segmenting different consumer types (hereafter, the segmentation inflexibility). Specifically, last-minute selling allows the firm to target the intermediate-type consumers with the leftover high-type products while at the same time targeting the low-type consumers with the low-type products. In contrast, she cannot separate them under opaque selling since she has only one product type (i.e., the opaque product) to offer.[13] Therefore, opaque selling may generate a lower revenue from the sales. This clearance disadvantage is strengthened when V_{mH} is high, and opaque selling's choice-deprivation advantage is weakened when V_{hH} is low. Therefore, the firm uses last-minute selling instead of opaque selling when V_{mH} and V_{hH} are close.

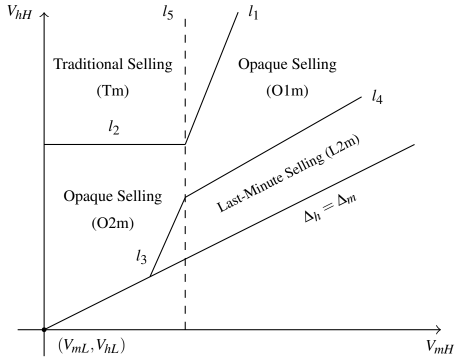
Perhaps unexpectedly, I find that the segmentation inflexibility *increases* the attractiveness of opaque selling when V_{mH} is high and $V_{mL} \leqslant \frac{n_m+n_l}{n_m} V_{lL}$ (i.e., to the right of l_5 in Figure 3.3(b)). Specifically, as V_{mH} increases and strengthens the segmentation inflexibility, the firm uses opaque selling for a larger range of parameter values. This is because the segmentation inflexibility helps maintain high sales price(s) for the leftover products, which further increases the regular price of the high-type products. In particular, with last-minute selling,

[13]The segmentation inflexibility is qualitatively preserved when the firm can offer both opaque and transparent sales. Here opaque selling makes the offered products less differentiated (i.e., opaque and low-type products v.s. high- and low-type products), so it is less efficient in separating different consumer types.

Figure 3.3: The Optimal Selling Strategy with Three Consumer Segments*



(a) $V_{mL} > \frac{n_m+n_l}{n_m} V_{lL}$



(b) $V_{mL} \leqslant \frac{n_m+n_l}{n_m} V_{lL}$

*See the proof of Proposition 9 for the expressions for l_1-l_5.

the firm prices the leftover high-type products below the intermediate-type consumers' valuation to prevent them from buying the low-type products at V_{lL}. In contrast, with opaque selling, the firm cannot separate consumers and thus chooses to sell the opaque products exclusively to the intermediate-type consumers at their expected valuation. Since the firm extracts full surplus from them with opaque selling and only partial surplus with last-minute selling, the high-type consumers who buy in the regular season are less willing to wait for the opaque sales. Therefore, the firm can raise the regular price with opaque selling, i.e., the segmentation inflexibility increases her profit.

The preceding discussion highlights several important differences in opaque selling's mechanism in vertically and horizontally differentiated markets. First, opaque selling is always less efficient in cleaning up leftover products in vertically differentiated markets. However, it may be more efficient under horizontal differentiation. Second, opaque selling is less preferable than last-minute selling as customers are more horizontally differentiated, whereas it can be more preferable as they are more vertically differentiated (e.g., higher V_{hH}/V_{mH} or V_{mL}/V_{lL}).

The impact of demand uncertainty. Next I examine how demand uncertainty influences the firm's choice between opaque selling and last-minute selling. In horizontally differentiated markets, a higher probability of a high-demand realisation makes opaque selling more preferable since its clearance advantage becomes more valuable (Jerath et al. 2010). Specifically, as the probability of high demand increases, the firm raises the regular price and more consumers remain active in the market under low demand in the sales season. Due to its clearance advantage, opaque selling yields higher sales-season revenue and thus is more attractive. However, this rationale no longer holds in vertically differentiated markets, since opaque selling is no more efficient in cleaning up the leftover products than last-minute selling. In fact, the next proposition shows that the opposite policy recommendation can be optimal. Notably, I focus on the $V_{mL} \leqslant (n_m + n_l)V_{lL}/n_m$ case (otherwise opaque selling always outperforms last-minute selling) and relegate the expressions of thresholds for α, V_{hH}, and V_{mH} to the proof for brevity.

Proposition 10. *Under Assumption 2, when $V_{mL} \leqslant \frac{n_m+n_l}{n_m} V_{lL}$, last-minute selling outperforms opaque selling for a larger range of parameter values as α increases if α and V_{hH} are sufficiently high and V_{mH} is sufficiently low.*

Unlike in the horizontally differentiated setting, in vertically differentiated markets the firm may switch from opaque selling to last-minute selling as high demand becomes more likely. To see the intuition, first recall that opaque selling's advantage here is that it increases the regular price by depriving consumers of the choice of buying the preferred product type during the sales season. Its disadvantage, however, lies in its inflexibility in segmenting different types of consumers. Since the choice deprivation and the segmentation inflexibility take place only when demand is low, both the advantage and the disadvantage are weakened as the probability of high demand α increases. When α and V_{hH} are high and V_{mH} is low, the advantage dominates the disadvantage. In particular, a higher α increases the expected demand and a higher V_{hH} increases the price in the regular season, both of which strengthen the choice-deprivation advantage. In contrast, a higher V_{mH} allows the firm to charge a higher sales price for the high-type products when selling them exclusively to the intermediate-type consumers. Therefore, this strengthens opaque selling's disadvantage of segmentation inflexibility. When α and V_{hH} are high and V_{mH} is low, the advantage dominates the disadvantage, so a higher α reduces the advantage more and the firm switches from opaque selling to last-minute selling.

3.5.2 Damaged Opaque Products

Throughout the preceding analysis, I assumed that opaque selling does not affect consumers' product valuations. In practice, however, it usually requires the firm to "damage" the products by, e.g., prohibiting product returns (Swatch 2017) or service cancellation (Priceline 2017). In this section, I study the impact of this product damage on the mechanism of opaque selling. As in Jerath et al. (2010), I incorporate product damage by assuming that opaque selling discounts consumers' product valuation by $1-\delta$. In other words, a type-j product from the opaque mix is worth δV_{ij} to a type-i consumer. Following the derivation of Π_O, I can show that the firm's expected profit from this new opaque selling strategy (denoted by subscript Od) is $\Pi_{Od} \equiv (1-\alpha)\{n_h[V_{hH}-(1-\alpha')\delta(V_{hH}-V_{lH})]+\delta(M-n_h)V_{lH}+\delta(n_l-M+n_h)V_{lL}\}+\alpha\{M[V_{hH}-(1-\alpha')\delta(V_{hH}-V_{lH})]+(N_h-M)V_{hL}\}+(1-\alpha')\delta\frac{n_l+n_h-M}{n_l}K(V_{hH}-V_{lH}-V_{hL}+V_{lL})$ if $(N_h-M)V_{hL} > NV_{lL}$, and $\Pi_{Od} \equiv (1-\alpha)\{n_h[V_{hH}-(1-\alpha')\delta(V_{hH}-V_{lH})-\alpha'(V_{hL}-V_{lL})]+\delta(M-n_h)V_{lH}+\delta(n_l-M+n_h)V_{lL}\}+\alpha\{M[V_{hH}-(1-\alpha')\delta(V_{hH}-V_{lH})-\alpha'(V_{hL}-V_{lL})]+NV_{lL}\}+(1-\alpha')\delta\frac{n_l+n_h-M}{n_l}K(V_{hH}-V_{lH}-V_{hL}+V_{lL})$ otherwise.

Based on this expression, I can characterise the impact of product damage on the profitability of opaque selling as follows.

Lemma 6. *Under Assumption 1,* $\Pi_{Od} > \Pi_O$ *if* $\Delta_h > [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_l - [KV_{hL} - (\frac{1-\alpha}{1-\alpha'}n_l + K)V_{lL}]/(\theta K)$, $\Pi_{Od} < \Pi_O$ *if* $\Delta_h < [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_l - [KV_{hL} - (\frac{1-\alpha}{1-\alpha'}n_l + K)V_{lL}]/(\theta K)$, *and* $\Pi_{Od} = \Pi_O$ *if* $\Delta_h = [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_l - [KV_{hL} - (\frac{1-\alpha}{1-\alpha'}n_l + K)V_{lL}]/(\theta K)$.

Perhaps unexpectedly, Lemma 6 shows that product damage may influence the expected profit from opaque selling in both directions. First, it reduces the sales-season revenue by lowering consumers' product valuations. Second, it increases the regular-season revenue since the high-type consumers have less incentive to wait for the sales. When V_{hH} is high and V_{lH} is low, the latter effect dominates and the expected profit increases. Notably, this is just the opposite of what happens in the horizontal differentiation setting, where product damage always reduces profit (Jerath et al. 2010). This difference is driven by the contrasting mechanisms of opaque selling across the two settings. Specifically, under horizontal differentiation, opaque selling increases the regular price since its homogenising effect allows the firm to extract full surplus from the consumers who wait for the sales. Since product damage cannot extract even more surplus, it does not increase the expected profit. Under vertical differentiation, however, opaque selling increases the regular price because of the choice-deprivation advantage. Since this advantage allows the firm to extract only partial surplus from the consumers who wait for the sales, it leaves room for product damage to extract even more surplus and thus raise the regular price. As a result, the expected profit from opaque selling may increase.

The next proposition characterises the firm's optimal selling strategy when product damage is considered, where I relegate the expressions of thresholds for V_{hH} and V_{lH} to the proof for brevity.

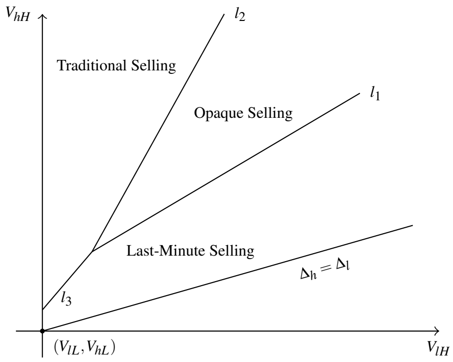
Proposition 11. *Under Assumption 1, the firm should use traditional selling when V_{hH} is high or V_{lH} is low, opaque selling when V_{hH} and V_{lH} are intermediate, and last-minute selling when V_{hH} is low or V_{lH} is high.*

I summarise Proposition 11 in Figure 3.4. With product damage, last-minute selling may still emerge in equilibrium since it induces a higher sales-season revenue than opaque selling. However, it is optimal only when the size of the sales-season revenue is considerable compared to the regular-season revenue (i.e., low V_{hH} or high V_{lH}). When the regular-season revenue dominates (i.e., high V_{hH} or low V_{lH}), the firm should use traditional selling to maintain the highest regular price for the high-type products. Moreover, the firm may still use opaque selling despite its damage to the products. Intuitively, opaque selling is a compromise between the profits from the regular season and those from the sales season. Therefore, it dominates the other selling strategies when these profits are comparable in size (i.e., intermediate V_{hH} and V_{lH}).

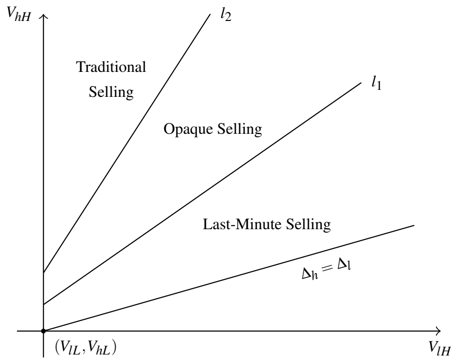
3.6 Concluding Remarks

In this chapter, I analysed the performance of opaque selling as a clearance strategy in vertically differentiated markets. I considered a two-period model with strategic consumers and a monopolistic firm selling two types of quality-differentiated products. I characterised the firm's optimal selling strategy and showed that opaque selling may outperform last-minute selling by virtue of preventing consumers from choosing their preferred product type if they delay their purchase to the sales season. The disadvantage of opaque selling is that it is less efficient in cleaning up the leftover products due to its inflexibility in segmenting different types of consumers. Both results are in sharp contrast to the horizontal differentiation setting. This difference further induces opposite policy recommendations for vertically and horizontally differentiated markets. Specifically, under vertical differentiation, the firm may switch from opaque selling to last-minute selling as consumers become more differentiated or the probability of the low-demand realisation increases. Under horizontal differentiation, however, she should always switch in the opposite direction. I have also shown that the advantage and disadvantage of opaque selling continue to hold in the presence of market competition and product damage. Surprisingly, I found that market competition adds a new

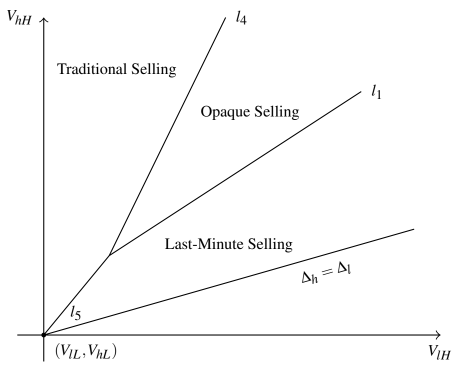
Figure 3.4: The Optimal Selling Strategy with Product Damage*



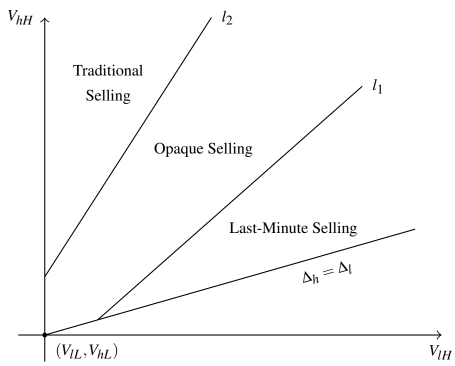
(a) $V_{hL} > \frac{N}{N_h-M}V_{lL}$, $n_l > \frac{(1-\delta+\alpha'\delta\bar{\theta})\Delta+\alpha(1-\delta\theta)R}{(1-\alpha)(1-\delta)V_{lL}}$



(b) $V_{hL} > \frac{N}{N_h-M}V_{lL}$, $n_l \leqslant \frac{(1-\delta+\alpha'\delta\bar{\theta})\Delta+\alpha(1-\delta\theta)R}{(1-\alpha)(1-\delta)V_{lL}}$



(c) $V_{hL} \leqslant \frac{N}{N_h-M}V_{lL}$, $n_l \geqslant \frac{(1-\alpha')K\Delta}{(1-\alpha)V_{lL}}$



(d) $V_{hL} \leqslant \frac{N}{N_h-M}V_{lL}$, $n_l < \frac{(1-\alpha')K\Delta}{(1-\alpha)V_{lL}}$

*See the proof of Proposition 11 for the expressions for l_1-l_5.

advantage to opaque selling: It makes better use of the leftover low-type products. Moreover, product damage may also make opaque selling more attractive since it allows for an even higher regular price.

This analytical framework provides the following insights for managers in practice. First, offering opaque sales is attractive only when high-valuation consumers (due, e.g., to brand loyalty or an urgent need for the product) are unwilling to pay a much higher price than the low-valuation consumers for the most valuable product type. Second, opaque sales are more attractive if demand is more likely to be low or consumers hold similar product valuations. Third, although consumers generally value opaque products lower than the corresponding transparent ones, product damage may in fact increase the attractiveness of opaque selling by making high-valuation consumers less willing to wait for the sales.

For the sake of parsimony, I have abstracted away from several factors, which could serve as the basis for future research. For example, to focus on the performance analysis of opaque selling purely as a clearance strategy, I followed the analytical framework of Jerath et al. (2010) by assuming that the firm can use opaque selling only during the sales season. Future research can extend my work by relaxing this assumption in order to examine the performance of opaque selling as both a market segmentation tool and a clearance strategy. Another research direction would be to endogenise the firm's quality choice. It would be interesting to investigate whether opaque selling induces more differentiated product types to better exploit its choice-deprivation advantage, or less differentiated product types to reduce its disadvantage of segmentation inflexibility. Finally, future research can incorporate behavioural elements and examine their impact on the performance of opaque selling. For example, consumers may use anecdotes to estimate the likelihood of each product in the opaque mix. Moreover, they may anticipate post-purchase regret when making their purchase decisions.

Chapter 4

Money Back Guarantees with Competing Physical and Online Stores

4.1 Introduction

As consumers have increasingly easier access to the Internet, online retailing has been growing steadily. For example, the global sales volume from online retailing has increased from 1,336 billion dollars in 2014 to 1,859 in 2016, and the figure is estimated to rise to 4,479 in 2021 (Statista 2018). Often-cited reasons for consumers to purchase online include saving time and money from travelling to a physical store and the convenience to purchase anytime and anywhere (Miller 2012, The Telegraph 2016, Domingo 2017).

Despite the convenience of online retailing, consumers may hesitate to buy online since physical stores usually provide better services in the sense that consumers can try different variants of a product with the help of instore assistance before making the purchase. In fact, recent surveys have shown that the ability to "touch and feel" products is the primary reason for consumers to buy in a physical store instead of an online store (Retail TouchPoints 2015, Retail Dive 2017).

To cope with the disadvantage in instore services, many online stores choose to offer money-back guarantees (MBGs). With MBGs, consumers can return a product to get the full money back "no question asked." For example, Zappos, an US-based online shoe and clothing retailer, allows customers to return products with a free return label for any reason (Zappos 2018). Other online retailers may offer a less lenient return policy by imposing a return fee on the consumers or a post-purchase time limit after which MBGs are not offered. For instance, Amazon requires customers to return products within 30 days after receiving the shipment to claim MBGs (Amazon 2018). In addition to this time limit, Newegg also charges customers for product returns if the returned item is non-defective (Newegg 2018).

Although MBGs may alleviate consumer concerns about product misfit before purchase, they can be costly for the online retailers in several aspects. First, handling product returns involve return repackaging, return shipment, and restocking, all of which are costly. Second, returned products are usually sold at a lower price as open-box items. The cost is especially prominent considering the fact that the online return rate are at least 30%, as compared to 8.89% in physical stores (Saleh 2018).

Since both the benefit and cost of MBGs are salient, it is critical for online retailers to understand how to design the product return policy in order to compete with brick-and-mortar retailers. This chapter aims to address this question using a stylised game-theoretical model. The model consists of a physical store and an online store competing for consumers on a single product. The consumers do not know whether the product fits or not before tying it, and they differ in their valuations for a fitting product. The physical store allows the consumers to try different variants of the product and thus they know whether it fits their need or not before purchase, whereas the online store cannot. However, the online store can choose to offer consumers MBGs, i.e., they can return a product after purchase to get the full money back. The online store also determines the allocation of product return cost between itself and the consumers.

Using this model, I characterise both stores' optimal pricing decisions and the online store's optimal product return policy. I find that the online store should offer MBGs only if it is more efficient to transfer an unfit product from the consumer side to the store. Moreover, if the online store chooses to offer MBGs, it should allocate product return cost in the socially optimal way, i.e., to minimise the total product return cost. I also study the impact of the stores' service quality on their optimal profit. I find that the online store may lose profit from improving the service, since it intensifies competition. Moreover, it may benefit from a better service from the physical store.

The remainder of this chapter is organised as follows. §4.2 reviews the relevant literature. §4.3 introduces model specifications, and §4.4 characterises both stores' pricing decisions an the online store's optimal product return policy. In §4.5, I conclude this chapter by summarising the key findings and managerial insights. All proofs are relegated to Appendix C.

4.2 Related Literature

This chapter is closely related to two streams of the extensive literature: MBGs and multi-channel competition.

MBGs. Product returns are widely provided for consumers who cannot fully evaluate a product before purchase. By promising them to refund the full purchase price "no question asked," retailers can mitigate consumers' uncertainty about their product valuations. Davis et al. (1995) examine a monopolistic firm's decision of offering MBGs or not to consumers who are ex ante unsure if the product fits or not. Che (1996) studies the welfare implications of MBGs for risk-averse consumers and shows that the firm adopts MBGs only if they are highly risk averse, whereas MBGs always improve social welfare. Moorthy and Srinivasan (1995) consider MBGs as a way to signal product quality and show that they can be more effective than uninformative advertising. Shulman et al. (2010) investigate the impact of reverse channel structure (i.e., returns are salvaged by the manufacturer v.s. by the retailer) on a firm's optimal return policy.

The aforementioned literature treats MBGs as a given policy in the sense that the firm and consumers incur a predetermined amount of cost associated with product returns. In practice, however, retailers typically can choose the amount of return cost incured by them and consumers. To analyse this MBG design problem, Davis et al. (1998) consider a monopolistic firm who decides on the return hassle imposed on consumers. They find that a firm should offer low-hassle MBGs if consumers do not benefit much from using the product and then returning it, or the returned item has a high salvage value. Hsiao and Chen (2014) extend Davis et al. (1998) by comparing the profitability of MBGs with a partial-refund policy.

Recently, researchers have started to examine the performance of MBGs in competitive markets. Shulman et al. (2011) and McWilliams (2012) study the product return policy of competing retailers with horizontal and vertical differentiation, respectively. Ofek et al. (2011) consider the impact of product returns on competing brick-and-mortar retailers' decisions to open an online channel. Although this chapter also considers MBGs in a competitive market, it is fundamentally different from this stream of literature. In fact, the competing firms in the literature operate in the same way (i.e., brick-and-mortar or brick-and-click), whereas they are different in my study: Physical-store customers can try the product before purchase, whereas online-store consumers cannot. The main contribution of this study is to consider how MBGs help the online store cope with this operational disadvantage.

Multi-channel competition. The existing literature in supply chain management, marketing, and information systems literature has studied competition between firms that operate on different channels, but the role of MBGs on market competition has not been examined. In supply chain management, the extant literature focuses on duel-channel coordination problems in which a manufacturer who sells to an independent retailer can choose to open its own direct-to-customer store to coordinate the supply chain (Tsay and Agrawal 2004, Gupta et al. 2009, and references therein). Introducing the online channel may benefit not only the manufacturer, but also the retailer, since it can mitigate double marginalisation (Chiang et al. 2003, Arya et al. 2007), improve market segmentation (Cattani et al. 2006), and induce customer free-riding on the manufacturer's store (Bernstein et al. 2009). In the marketing literature, researchers have studied competition between independent retailers from different channels. For example, Balasubramanian (1998) examines the impact of online sellers on the competition with and among brick-and-mortar sellers. Viswanathan (2005) extends Balasubramanian (1998) by considering network externality and switching cost among different channels. The information systems literature has also examined online-offline competition, yet it focuses on the role of consumers' instore service free riding (Wu et al. 2004, Balakrishnan et al. 2014, Mehra et al. 2017).

As in this stream of literature, this chapter also studies online-offline competition. However, my study differs from the literature fundamentally since it addresses the problem of how the online store should design its return policy in the face of competition from a physical store, whereas the literature typically abstracts away from consumer product returns.

4.3 Model

I consider heterogeneous-valuation consumers in need of at most one unit of an experience good (Nelson 1970, Shulman et al. 2010), i.e., a good that the consumer does not know if it fits her need until after trying it. Consistent with the MBG literature (e.g., Chu et al. 1998, McWilliams 2012), I incorporate this fitting uncertainty by assuming that each consumer values an unfit product at 0 and a fitting product at v, which is uniformly distributed between 0 and 1. Moreover, the consumers know whether the product fit their need or not only after trying it. Each store knows consumers' valuation distribution but not a specific consumer's valuation, which is her private knowledge.

In practice, the consumers may also be uncertain about their product valuation v before purchase and fully understand this valuation only after using it. This valuation uncertainty prevails in product categories such as books, beauty and health products, and has been incorporated by the MBG literature (Che 1996, Davis et al. 1998, Su 2009, Akçay et al. 2013). However, I abstract away from the valuation uncertainty to focus on the market where the fitting uncertainty is more prominent, e.g., clothes, shoes, and electronics. For example, the value of a pair of shoes can be almost fully revealed to the consumer after browsing the online product description, and the major uncertain that influences her purchase decision is whether the shoes fit her feet or not.

The product is offered by two competing retailers: A physical store (denoted by subscript 1) and an online store (denoted by subscript 2). We denote c_i as store i's per-unit procurement cost and α_i as the probability for consumers to find a fitting product in the store ($i \in \{1,2\}$). We can interpret α_i as service quality since it measures how effective a store helps consumers find the fitting product type. Because the physical store allows consumers to try different variations (colors, sizes, shapes, etc.) physically and offers professional advice about which one to choose, I assume that consumers have a higher chance to find a fitting product in the physical store,[14] i.e., $\alpha_1 > \alpha_2$. Notably, this assumption has also been adopted by Ofek et al. (2011).

Timing. The sequence of events is as follows.

In Stage 1, the physical store determines the price of the product P_1, and the online store determines the price P_2 and chooses to offer MBGs or not. If she offers MBGs, the online store allocates the total return hassle cost T_1 (due to, e.g., repacking, return shipment, restocking) between each consumer and the store. Each store's objective is to maximise her own expected profit.

In Stage 2, consumers arrive at the market and choose among three options: (i) Visiting the physical store and incurring the travelling hassle K; (ii) visiting the online store and buying there; (iii) leaving the market without a purchase to obtain null payoff. Notably,

[14] Admittedly, the online store may display more product variations because online display does not require the product to be in stock. However, consumers are still more likely to find a fitting product in the physical store, because they learn whether the product fits or not more by trying it than by comparing the specifications online. Moreover, brick-and-mortar retailers (e.g., J. C. Penney, Nordstrom, and Sears Holdings) have deployed in-store touchscreens and tablets to show more product variations (Townsend 2012).

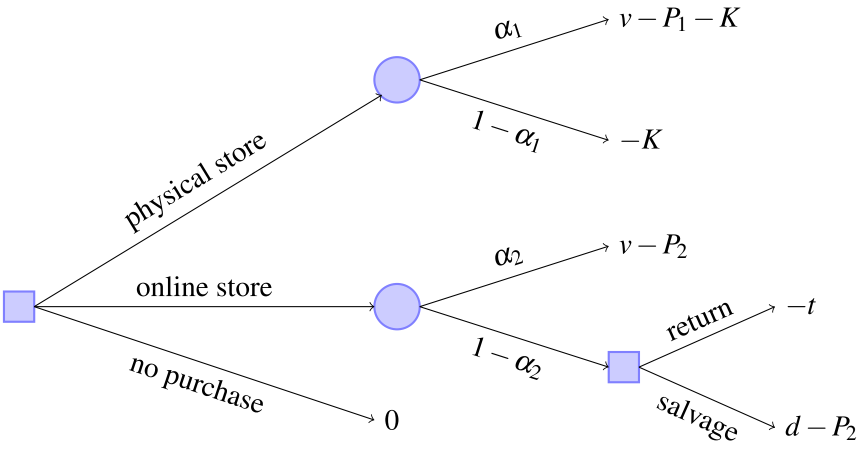
I do not consider consumer decision of visiting the online store first and then going to the physical store. In fact, if they do not buy online, then the former visit does not bring any benefit since product fit is revealed only after trying the product. Therefore, the decision is equivalent to going to the physical store directly. If the consumers indeed buy online after visiting the online store, then they should not go to the physical store after this purchase: 1. If the product fits, they do not need to buy offline. 2. If the product misfits, their surplus from visiting the physical store is equal to $-K$, so the consumers should leave the market instead.

In Stage 3, consumers who visit the physical store try the product and find it fit with probability α_1 and unfit otherwise. Consumers who visit the online store cannot try the product before purchase and thus buy it without resolving the fitting uncertainty (note that if they do not buy, they should not visit the online store in Stage 2). Upon receiving the product, consumers try the product and find it fit with probability α_2 and unfit otherwise.

In Stage 4, physical-store consumers, who have already known whether the product fits or not, choose to buy the product or leave the market without a purchase. Online-store consumers, who have bought the product and known whether it fits or not, choose among three options: (i) Using the product; (ii) salvaging the product to a secondary market at value d (the consumer salvage value, hereafter); (iii) returning the product to the online store for full refund (i.e., P_2) at hassle cost $t \in [0, T_1]$. The online store incurs the rest of the return hassle (i.e., $T_1 - t$) and salvages the returned product in the secondary market at price S (the store salvage value, hereafter).

Figure 4.1 illustrates consumers' decision tree. Note that I have simplified the decisions by removing branches that can never be reached. For example, although physical-store consumers have the option to buy an unfit product, they never choose it since the corresponding utility (i.e., $v - P_1 - K$) is lower than that of not buying (i.e., $-K$). Moreover, they prefer buying a fitting product than leaving the market without a purchase: If they do not buy, their expected utility is $-K$ so they should not visit the physical store in the first place. Third, online-store consumers never return a fitting product: Otherwise, they also return an unfit product, so their expected utility is $-t$ and the consumers should not buy from the online store in the first place. Lastly, online-store consumers never salvage a fitting product: Otherwise, their expected utility ($\alpha_2(d - P_2) + (1 - \alpha_2)\max\{-t, d - P_2\}$) is non-positive so they should not buy from the online store in the first place. Note that $d - P_2 \leqslant 0$ by

Figure 4.1: Consumers' Decision Tree



$d \leqslant S \leqslant c_2 \leqslant P_2$ (see Assumption 3 for details), where the last inequality holds because otherwise the online store obtains negative profit and thus should not participate in the market.

The MBG Framework. I incorporate the online store's MBGs by assuming that consumers can return a product back to the store at hassle t and get full money back. The online store chooses t to allocate the return hassle between the consumers and the store. This framework is consistent with the return policy of many online retailers (e.g., Amazon, RueLaLa, Zappos), and it is flexible enough to include many return policies studied in the literature as special cases. For example, the MBGs in Davis et al. (1998) and Hsiao and Chen (2014) correspond to the $t = T_1$ case, the restocking fee policy in Shulman et al. (2011) is a special case of my framework with $T_1 = 0$, and the hassle-free policy in Hsiao and Chen (2014) is equivalent to the $t = 0$ case in my model.

Consistent with the MBG literature (see, e.g., Moorthy and Srinivasan 1995, McWilliams 2012), I assume that consumers who receive the refund through MBGs do not buy the same product (maybe with a different color, size, shape, etc.) again from the online store. This is a good approximation for most experience goods I consider (e.g., clothes, shoes), where consumers usually buy the good elsewhere after returning it online. Also note that I purposely abstract away from the physical store's return policies: Physical-store consumers never return a product because they can fully resolve the fitting uncertainty by trying the product before purchase, i.e., consumers who return a product should not choose to buy it in the first place. This simplification allows me to draw sharper insight and I conjecture that incorporating the physical store's return policy does not change major insights qualitatively. Lastly, as a break-even rule, I assume that each store chooses to leave the market when staying in it leads to the same expected profit, and the online store offers MBGs when she is indifferent between offering and not offering it.

Apart from the above assumptions on the MBG framework, I also adopt the following set of assumptions throughout this chapter.

Assumption 3. $0 < \alpha_2 < \alpha_1 < 1$, $d \leqslant S \leqslant c_2$, $c_1 < \frac{\alpha_1 - K}{\alpha_1}$, $c_2 < \alpha_2 + (1-\alpha_2)\min\{d, S - T_1\}$.

I assume that the online store can salvage the unfit product in the secondary market with at least the same price as consumers. This is because the store is usually better at repackaging the product and has more resell channels than consumers. I also assume that the online store's marginal cost exceeds the store salvage value. This rules out uninteresting cases where the online store earns infinite profits by producing and salvaging as many products as possible. The last two inequalities in Assumption 3 guarantee that neither store is driven out of market because of inefficient operations. If either inequality fails, the corresponding store earns negative profits even without the competitor.

4.4 Analysis

In this section, I characterise all pure-strategy subgame-perfect Nash equilibria and investigate the online store's optimal return policy. To this end, I examine consumers' decisions in §4.4.1 and the stores' decisions in §4.4.2. Then I characterise the equilibrium outcome and derive insights into the online store's optimal return policy and the impact of service quality on each store's optimal profit, as shown in §4.4.3.

4.4.1 Consumers' Decisions

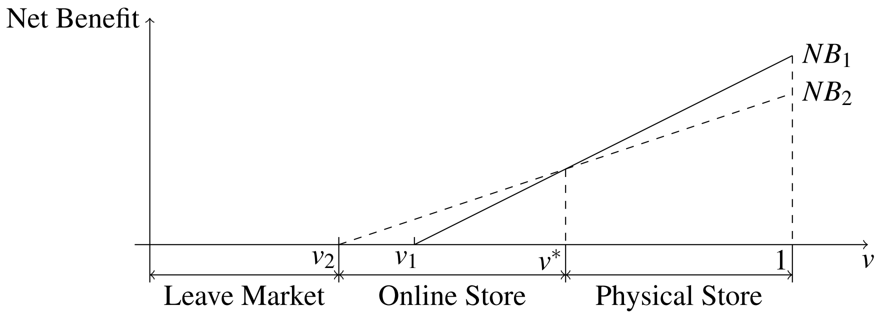
To characterise the pure strategy equilibrium, I first develop the consumers' purchase behaviour for any given pricing and return policy decisions. According to Figure 4.1, a physical-store consumer's expected net benefit is $NB_1 = \alpha_1 v - \alpha_1 P_1 - K$, while an online-store consumer's expected net benefit is

$$NB_2 = \begin{cases} \alpha_2 v + (1-\alpha_2)d - P_2, & \text{without MBG,} \\ \alpha_2(v - P_2) + (1-\alpha_2)t, & \text{with MBG.} \end{cases}$$

Denote $\tilde{P}_1 \equiv \alpha_1 P_1 + K$ and

$$\tilde{P}_2 \equiv \begin{cases} P_2 - (1-\alpha_2)d, & \text{without MBG,} \\ \alpha_2 P_2 + (1-\alpha_2)t, & \text{with MBG} \end{cases}$$

Figure 4.2: The Consumer's Store Visit Decision



as consumers' expected payment per visit (certainty equivalent price, hereafter) to the physical store and the online store respectively. Then $NB_1 = \alpha_1 v - \tilde{P}_1$ and $NB_2 = \alpha_2 v - \tilde{P}_2$. Consumers visit the physical store if $NB_1 > \max\{NB_2, 0\}$, visit the online store if $NB_2 > \max\{NB_1, 0\}$, and leave the market without a purchase if $\max\{NB_1, NB_2\} < 0$. Since $\alpha_1 > \alpha_2$, this suggests that high-valuation-consumers (i.e., $v > \max\{\frac{\tilde{P}_1}{\alpha_1}, \frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2}\}$) visit the physical store, intermediate-valuation (i.e., $\frac{\tilde{P}_2}{\alpha_2} < v < \frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2}$) consumers visit the online store, and low-valuation (i.e., $v < \min\{\frac{\tilde{P}_1}{\alpha_1}, \frac{\tilde{P}_2}{\alpha_2}\}$) consumers do not purchase. I illustrate the consumer's purchase decision in Figure 4.2, where I denote $v_1 \equiv \tilde{P}_1/\alpha_1$, $v_2 = \tilde{P}_2/\alpha_2$, and $v^* = (\tilde{P}_1 - \tilde{P}_2)/(\alpha_1 - \alpha_2)$.

4.4.2 The Stores' Decisions

Fully aware of consumers' decisions, each store maximises her profit by setting the certainty equivalent price as a function of the other store's certainty equivalent price. Note that the online store's MBG decisions (i.e., offering MBGs or not, and the return hassle allocation if she offers MBGs) have been internalised into her certainty equivalent price and thus will not be treated as explicit decisions.

First consider the online store. For any $\tilde{P}_1$, the online store's profit is:

$$\Pi_2(\tilde{P}_2) = \begin{cases} (1 - \frac{\tilde{P}_2}{\alpha_2})(\tilde{P}_2 - \tilde{c}_2), & \text{if } \frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2} \geqslant 1, \\ (\frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2} - \frac{\tilde{P}_2}{\alpha_2})(\tilde{P}_2 - \tilde{c}_2), & \text{if } \frac{\tilde{P}_1}{\alpha_1} < \frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2} < 1, \\ 0, & \text{if } \frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2} \leqslant \frac{\tilde{P}_1}{\alpha_1}, \end{cases} \tag{4.1}$$

where

$$\tilde{c}_2 \equiv \begin{cases} c_2 - (1 - \alpha_2)d, & \text{without MBG,} \\ c_2 - (1 - \alpha_2)(S - T_1), & \text{with MBG.} \end{cases}$$

The expression of $\Pi_2(\tilde{P}_2)$ suggests that the online store has two pricing strategies. First, she prices low (i.e., $\tilde{P}_2 \leqslant \tilde{P}_1 - (\alpha_1 - \alpha_2)$ or $\frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2} \geqslant 1$) to drive the physical store out of the market (the aggressive pricing strategy, hereafter). Here the best response is

$$\tilde{P}_2(\tilde{P}_1) = \min\{\frac{\alpha_2 + \tilde{c}_2}{2}, \tilde{P}_1 - (\alpha_1 - \alpha_2)\},$$

and the interior optimal profit is

$$\Pi_2(\frac{\alpha_2 + \tilde{c}_2}{2}) = \frac{(\alpha_2 - \tilde{c}_2)^2}{4\alpha_2}.$$

Second, she prices high (i.e., $\tilde{P}_1 - (\alpha_1 - \alpha_2) < \tilde{P}_2 < \frac{\alpha_2}{\alpha_1}\tilde{P}_1$ or $\frac{\tilde{P}_1}{\alpha_1} < \frac{\tilde{P}_1 - \tilde{P}_2}{\alpha_1 - \alpha_2} < 1$) so both stores obtain positive profit (the moderate pricing strategy, hereafter). Here the best response is

$$\tilde{P}_2(\tilde{P}_1) = \min\{\max\{\frac{\alpha_2\tilde{P}_1 + \alpha_1\tilde{c}_2}{2\alpha_1}, \tilde{P}_1 - (\alpha_1 - \alpha_2)\}, \frac{\alpha_2}{\alpha_1}\tilde{P}_1\},$$

and the interior optimal profit is

$$\Pi_2(\frac{\alpha_2\tilde{P}_1 + \alpha_1\tilde{c}_2}{2\alpha_1}) = \frac{(\alpha_2\tilde{P}_1 - \alpha_1\tilde{c}_2)^2}{4\alpha_1\alpha_2(\alpha_1 - \alpha_2)}.$$

By comparing the online store's profits under aggressive and moderate pricing for any given $\tilde{P}_1$ (see Appendix C.1 for the technical details), I derive the online store's best response as follows:

$$\tilde{P}_2(\tilde{P}_1) = \begin{cases} \text{N/A}, & \text{if } \tilde{P}_1 \leqslant \frac{\alpha_1}{\alpha_2}\tilde{c}_2, \\ \frac{\alpha_2\tilde{P}_1 + \alpha_1\tilde{c}_2}{2\alpha_1}, & \text{if } \frac{\alpha_1}{\alpha_2}\tilde{c}_2 < \tilde{P}_1 < \frac{2\alpha_1(\alpha_1 - \alpha_2) + \alpha_1\tilde{c}_2}{2\alpha_1 - \alpha_2}, \\ \tilde{P}_1 - \alpha_1 + \alpha_2, & \text{if } \frac{2\alpha_1(\alpha_1 - \alpha_2) + \alpha_1\tilde{c}_2}{2\alpha_1 - \alpha_2} \leqslant \tilde{P}_1 \leqslant \alpha_1 - \frac{\alpha_2 - \tilde{c}_2}{2}, \\ \frac{\alpha_2 + \tilde{c}_2}{2}, & \text{otherwise}, \end{cases} \tag{4.2}$$

where "N/A" represents leaving the market.

Following the same procedure (see Appendix C.1 for the technical details), I derive the physical store's best response as follows:

$$\tilde{P}_1(\tilde{P}_2) = \begin{cases} \text{N/A}, & \text{if } \tilde{P}_2 \leqslant \tilde{c}_1 - \alpha_1 + \alpha_2, \\ \frac{\alpha_1 - \alpha_2 + \tilde{c}_1 + \tilde{P}_2}{2}, & \text{if } \tilde{c}_1 - \alpha_1 + \alpha_2 < \tilde{P}_2 < \frac{\alpha_2(\alpha_1 - \alpha_2) + \alpha_2\tilde{c}_1}{2\alpha_1 - \alpha_2}, \\ \frac{\alpha_1}{\alpha_2}\tilde{P}_2, & \text{if } \frac{\alpha_2(\alpha_1 - \alpha_2) + \alpha_2\tilde{c}_1}{2\alpha_1 - \alpha_2} \leqslant \tilde{P}_2 \leqslant \frac{\alpha_2(\alpha_1 + \tilde{c}_1)}{2\alpha_1}, \\ \frac{\alpha_1 + \tilde{c}_1}{2}, & \text{otherwise}, \end{cases} \tag{4.3}$$

where $\tilde{c}_1 \equiv \alpha_1 c_1 + K$.

According to (4.2)-(4.3), each store prices more aggressively when the competitor prices higher: (i) When the competitor's price is low (i.e., $\tilde{P}_1 \leqslant \frac{\alpha_1}{\alpha_2}\tilde{c}_2$ or $\tilde{P}_2 \leqslant \tilde{c}_1 - \alpha_1 + \alpha_2$), the store does not participate in the market. (ii) When the competitor's price is moderate (i.e., $\frac{\alpha_1}{\alpha_2}\tilde{c}_2 < \tilde{P}_1 < \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$ or $\tilde{c}_1 - \alpha_1 + \alpha_2 < \tilde{P}_2 < \frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2}$), the store uses the moderate pricing strategy such that both stores obtain positive profits. (iii) When the competitor's price is high (i.e., $\tilde{P}_1 \geqslant \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$ or $\tilde{P}_2 \geqslant \frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2}$), the store uses the aggressive pricing strategy to drive the competitor out of the market.

4.4.3 Equilibria

Next we derive all pure-strategy Nash equilibria using the expressions of both stores' best-response functions, as given by (4.2)-(4.3). We find three types of equilibria. First, both stores adopt moderate pricing and thus earn positive profits (denoted by B). Second, the physical store adopts aggressive pricing to drive the online store out of market (denoted by P). Third, the online store adopts aggressive pricing to drive the physical store out of market (denoted by O). The next proposition characterises these equilibria.

Proposition 12. *(i) When $\tilde{c}_1 \leqslant \frac{-\alpha_1\alpha_2+2\alpha_1\tilde{c}_2}{\alpha_2}$, only Equilibrium P exists.*

(ii) When $\frac{-\alpha_2(\alpha_1-\alpha_2)+(2\alpha_1-\alpha_2)\tilde{c}_2}{\alpha_2} < \tilde{c}_1 < \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$, only Equilibrium B exists.

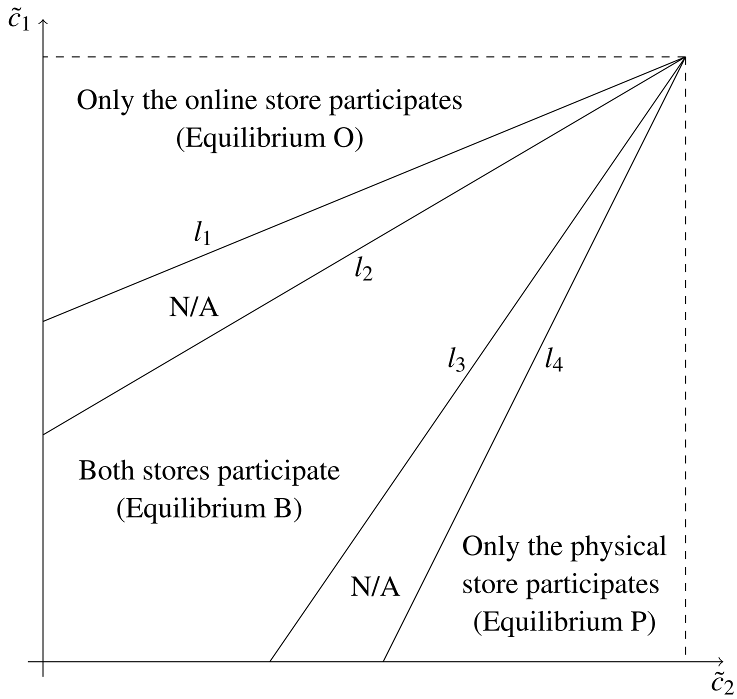
(iii) When $\tilde{c}_1 \geqslant \frac{2\alpha_1-\alpha_2+\tilde{c}_2}{2}$, only Equilibrium O exists.

Moreover, whenever the online store participates in the market, she offers MBGs and is indifferent between any return hassle allocation if $S - T_1 \geqslant d$, and does not offer MBGs otherwise.

I illustrate Proposition 12 in Figure 4.3. Intuitively, when one store is much more cost-efficient than the competitor (i.e., $\tilde{c}_1 \leqslant \frac{-\alpha_1\alpha_2+2\alpha_1\tilde{c}_2}{\alpha_2}$) or $\tilde{c}_1 \geqslant \frac{2\alpha_1-\alpha_2+\tilde{c}_2}{2}$), the competitor does not participate in the market and that store acts as the monopolist. However, when the stores' equivalent costs are close (i.e., $\frac{-\alpha_2(\alpha_1-\alpha_2)+(2\alpha_1-\alpha_2)\tilde{c}_2}{\alpha_2} < \tilde{c}_1 < \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$), both stores participate in the market and a duopoly equilibrium emerges.

The optimal return hassle allocation. Proposition 12 shows that when choosing to offer MBGs, the online store is indifferent between any return hassle allocation. This result is in sharp contrast to the MBG literature that shows the existence of a unique optimal return hassle (e.g., Davis et al. 1998, Hsiao and Chen 2014). To fully understand this difference, first recall that Davis et al. (1998) and Hsiao and Chen (2014) incorporate consumers' valuation uncertainty, i.e., they cannot perfectly estimate the benefit of a fitting product before purchase. Therefore, they may buy a product that turns out to be not valuable after purchase and thus is returned even if it fits. This suggests that the store can adjust the size of product returns by the return hassle allocation: When consumers face high (low) return hassle, they will return the fitting product only (even) if its benefit is low (high), so the return volume

Figure 4.3: Equilibrium Outcomes*



*$l_1 : \tilde{c}_1 = \frac{2\alpha_1-\alpha_2+\tilde{c}_2}{2}$
$l_2 : \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$
$l_3 : \frac{-\alpha_2(\alpha_1-\alpha_2)+(2\alpha_1-\alpha_2)\tilde{c}_2}{\alpha_2}$
$l_4 : \tilde{c}_1 = \frac{-\alpha_1\alpha_2+2\alpha_1\tilde{c}_2}{\alpha_2}$
N/A: No pure-strategy equilibrium exists

is low (high). As a result, the store sets the consumer return hassle to trade off between purchase volume (note that lower consumer return hassle leads to higher purchase volume and thus increases profits) and return volume (note that higher consumer return hassle leads to lower return volume and thus increases profits), so a unique optimal return hassle exists.

In my setting, however, the online store is indifferent between any return hassle allocation. This is because I focus on settings where valuation uncertainty is not salient: Since consumers can perfectly estimate the benefit of a fitting product before purchase, they never return a fitting product (otherwise, they will not buy it in the first place) and always return an unfit product, i.e., the return hassle allocation cannot adjust the return volume. Therefore, the aforementioned trade-off between purchase volume and return volume no longer holds. In fact, the consumer return hassle t has the same function as the price P_2: Both change only the money transfer between the consumer and the online store. In other words, the online store chooses t and P_2 to make the expected money transfer $\tilde{P}_2 = \alpha_2 P_2 + (1-\alpha_2)t$ the best response to the physical store's certainty equivalent price $\tilde{P}_1$, as given by Equation (4.2).

Note that this best response is unchanged by t because it does not influence consumers' benefit from visiting the physical store. Therefore, when t increases, the online store should maintain the same certainty equivalent price by decreasing P_2.

However, this result does not suggest the online store to allocate return hassle arbitrarily. In practice, the return hassle allocation usually influences the total return hassle, i.e., T_1 depends on t. Since the online store's optimal profit strictly decreases in T_1, she should allocate return hassle in the socially efficient way, i.e., assigning to each agent that is less costly for her than for the other agent. For example, the online store should take the responsibility of return shipment if she has extensive distribution network.

The optimal return policy. Proposition 12 also suggests that the online store should offer MBGs if the net social benefit of transferring an unfit product back to the online store (i.e., $S - T_1$) exceeds the benefit of leaving it to the consumer (i.e., d). This result is somewhat surprising because of its simplicity: The MBG decision is independent of the competitor and the online store's key operational parameters (i.e., the production cost and the fitting probability). The key insight is that offering MBGs is equivalent to changing the online store's operational cost of dealing with an unfit product. To illustrate this point with analytical precision, suppose that the online store switches to offering MBG with $t = -d$ and maintains the same price (recall that t does not influence her profit, so the result also holds for other t values). Online-store consumers do not change their purchase decision because they obtain the same benefit (i.e., $v - P_2$ when the product fit and $d - P_2$ when it does not). Therefore, the MBG policy does not influence the online store's profit except that she pays $c_2 + T_1 + d - S$ for an unfit product instead of c_2. To minimise this cost, the store offers MBGs when $S - T_1 - d \geqslant 0$ and does not offer MBGs otherwise. Note that the online store is always incentivised to reduce the operational cost regardless of the competitor. Moreover, the cost change due to the return policy is influenced only by the unfit product's salvage value and total return hassle. Therefore, the return policy is independent of the physical store and the online store's production cost and fitting probability.

The literature has shown a similar MBG decision rule in the monopoly setting with homogeneous consumers (Davis et al. 1995). This chapter extends the literature by showing that the decision rule also holds in the duopoly setting with physical and online stores competing on heterogeneous consumers. Note that the MBG decision in this chapter is consistent with the branch of literature (McWilliams 2012, Xu et al. 2015) where offering

MBGs dominates because consumers obtain no salvage value from the unfit product (i.e., $d = 0$), or obtain less value than the store and the total return hassle is zero (i.e., $S > d$ and $T_1 = 0$).

Comparative statics. Next I examine the impact of the stores' service quality on their equilibrium profits. When one store is much more cost-efficient than the competitor and thus drives it out of the market, the monopolistic store always benefits from improving her own service quality. Therefore, I will focus on the duopoly case. Conventional wisdom would suggest that improving service quality benefits the store and harms her competitor. However, Proposition 13 shows that this conjecture fails to hold for the online store, where I refer to the physical and online stores' profits under Equilibrium B as Π^*_{1B} and Π^*_{2B} respectively.

Proposition 13. *There exists $\bar{c}$, such that for $c_1 < \bar{c}$ and $c_2 < \bar{c}$,*

(i) Π^*_{1B} strictly increases in α_1 and strictly decreases in α_2.

(ii) Π^*_{2B} strictly decreases in α_2 for $\alpha_2 \in (\underline{\alpha}, \bar{\alpha})$ and strictly increases in α_2 for $\alpha_2 \in (\bar{\alpha}, \alpha_1)$, where a unique pair of $\underline{\alpha}, \bar{\alpha} \in (0, \alpha_1)$ exist and $\underline{\alpha} < \bar{\alpha}$.

(iii) When $(4 - 2\alpha_2 + \alpha_2^2)(1 - \alpha_2)\max\{d, S - T_1\} + (8 - 4\alpha_2 - \alpha_2^2)K < (2 + \alpha_2)(1 - \alpha_2)\alpha_2$, Π^*_{2B} strictly decreases in α_1 for $\alpha_1 \in (\alpha_2, \tilde{\alpha})$ and strictly increases in α_1 for $\alpha_1 \in (\tilde{\alpha}, 1)$, where a unique $\tilde{\alpha} < 1$ determined by $(4\tilde{\alpha}^2 - 2\tilde{\alpha}\alpha_2 + \alpha_2^2)(1 - \alpha_2)\max\{d, S - T_1\} + (8\tilde{\alpha}^2 - 4\tilde{\alpha}\alpha_2 - \alpha_2^2)K = (2\tilde{\alpha} + \alpha_2)(\tilde{\alpha} - \alpha_2)\alpha_2$ exists; otherwise Π^*_{2B} strictly decreases in α_1.

Perhaps unexpectedly, Proposition 13 shows that the online store may lose profits from improving her service quality. This is because quality improvement intensifies market competition (note that higher α_2 reduces the quality differentiation between the two stores) and thus lowers both stores' prices. When the online store is not sufficiently competitive ($\alpha_2 \in (\underline{\alpha}, \bar{\alpha})$), she has to lower the price significantly such that the resulting profit loss outweighs the profit increase due to higher service quality (i.e., higher demand of the online store). Managerially, this result shows that high product return rate due to misfit may benefit the online retailer by softening the competition with the brick-and-mortar retailer. Therefore, widely-applied misfit reduction strategies (e.g., stronger technical service (Lawton 2008) and displaying purchase history reminders (Kim 2013)) should be implemented only if the online retailer's return rate is sufficiently low.

Another interesting result from Proposition 13 is that improving the service quality of the physical store may increase the online store's profit. The intuition is that the quality improvement (i.e., higher α_1) softens market competition by increasing the quality differentiation. This softened competition allows both stores to raise the price. When the physical store is much more competitive than the online store (i.e., $\alpha_1 > \tilde{\alpha}$, K, d, and $S - T_1$ are small), the physical store raises price dramatically; it allows the online store to charge a much higher price. Therefore, the resulting profit increase of the online store outweighs the profit loss due to higher α_1 (i.e., lower demand of the online store), and the online store benefits on aggregate. From a managerial perspective, this result suggests that "reverse showrooming" or "webrooming" (i.e., searching online and buying in a brick-and-mortar store) benefits the online store if she competes with a sufficiently competitive physical store. As a result, the online store should embrace reverse showrooming by providing accurate and extensive product descriptions to divert high-valuation consumers to the physical store and benefit from a higher price in the physical store.

4.5 Conclusions

In this chapter, I studied how an online store should design the product return policy when competing with a physical store. I constructed a game-theoretical model in which a physical store and an online store sell one product to consumers who are ex ante unsure about whether the product fits their need or not. I characterised the stores' optimal pricing decisions and the online store's optimal product return policy. I found that she should offer MBGs only if they are socially efficient, and she should allocate product return cost to minimise the total return cost. In addition, I studied the impact of the stores' service quality on their optimal profits. I found that the online store may lose profit from improving her service, whereas she may benefit from a better service from the physical store.

The model proposed in this chapter can serve as the foundation for future modelling research that examines the impact of several important factors on an online store's product return policy when competing with a physical store. For example, physical-store consumers who have found that the product fits their need may choose to buy in the online store instead of in the physical store. In other words, some consumers may use the physical store as a showroom instead of a place to make the purchase. It would be interesting to consider the impact of consumers' showrooming behaviour on the online store's product return policy and online-offline competition. Another research direction is to consider consumers' free-riding behaviour on MBGs in the sense that they may buy a product only to return it after using it for some time. This extension introduces a new consumer segment (i.e., the free-riders) for the online store and thus may radically change her optimal product return policy. Finally, future research can extend MBGs by considering the policy of exchanging an unfit product for another type of the same product or merchandise credit.

Chapter 5

Conclusions

In this thesis, I use game-theoretical models to investigate the impact of three consumer behaviours for the operations of service systems. In Chapter 2, I consider service systems with boundedly rational customers who infer service quality based on anecdotes. I characterise their equilibrium joining behaviour and the service provider's pricing, quality, and information disclosure decisions. This work not only provides specific policy recommendations for operations managers in practice, it also builds the modelling framework for incorporating customer anecdotal reasoning in estimating service quality. This can serve as the foundation for future research that also considers this type of bounded rationality. In Chapter 3, I compare the performance of last-minute selling with that of opaque selling, which is widely used by travel service companies to cope with consumers' strategy waiting. This chapter contributes to the literature by showing that the mechanism of opaque selling under horizontally differentiated markets is radically from its mechanism under vertically differentiated markets, and this can lead to opposite policy recommendations across the two cases. Managerially, Chapter 3 provides insights for practitioners about whether to adopt opaque selling or not, and how this decision depends on their operational and marketing factors, including demand uncertainty, consumer valuations, and market competition. In Chapter 4, I examine the impact of consumers' ex ante uncertainty about product fit on the competition between an online store and a physical store. This research generates managerial implications about how an online store should design her return policy to compete with a physical store, and when the stores should not improve service quality as it intensifies competition.

I hope that this thesis can motivate future research to further investigate the impact of consumer behaviour in service operations. The specific research questions have been elaborated in the conclusion section in each chapter, and here I highlight several research themes that recur in all chapters. First, it is critical to empirically quantify the magnitude of consumers' specific behavioural elements in practice. As demonstrated in this thesis, consumer behaviour can have a significant impact on a firm's operations and marketing decisions. Therefore, a better understanding about the magnitude of these behaviour elements will enable firms to refine their decision-makings. Second, consumer behaviours may evolve over time as they have more interactions with the market. Therefore, it would be interesting to consider how this behavioural evolution interacts with the dynamics of service systems. For example, one can extend Chapter 2 by considering the interplay between customers' information acquisition with queueing dynamics. This may fit into observable queueing settings in which customers learn word-of-mouth information from other joining customers while they wait in line for service. Similarly, Chapter 3 can be extended to a multi-period game with customers becoming more strategic over time. This can provide policy recommendations for firms that serve regular customers. Finally, future research can apply the modelling framework in this thesis to address new research questions in service operations. The anecdotal reasoning framework in Chapter 2 can be utilised to study competing service providers' quality improvement and quality information disclosure decisions. Chapter 3 can serve as a benchmark for future research that incorporates customer bounded rationality that is relevant to opaque selling, e.g., loss aversion or post-purchase regret. The online-offline competition model in Chapter 5 allows researchers to consider the impact of consumers' showrooming behaviour on the firms' pricing competition and the online store's product return policy.

Appendix A

Appendix to "Managing Service Systems with Unknown Quality and Customer Anecdotal Reasoning"

This appendix includes three sections. §A.1 provides the technical proofs to the lemmas and propositions in Chapter 2. §A.2 includes supplemental discussions of the base model in Chapter 2.3 and additional analysis for Chapter 2.4. In §A.3, I show that major insights from the base model continue to hold for a more general set of service quality distributions and customer heterogeneity in the sample size.

A.1 Proofs

This section provides the proofs of all lemmas and propositions in the paper. Note that Equation (2.1) is well-defined as long as k is a positive real number (not only an integer). For technical convenience, I will adopt this generalisation throughout the proof. I will also abuse notations by writing $\lambda_a^k(p,R)$ as λ_a, $\Phi\left(\frac{\sqrt{k}}{\sigma}(p+\frac{c}{\mu-\lambda_a}-R)\right)$ as Φ, $\Phi'\left(\frac{\sqrt{k}}{\sigma}(p+\frac{c}{\mu-\lambda_a}-R)\right)$ as Φ', Π_r^* as $\Pi^*(+\infty)$, and will denote $\frac{\sqrt{k}}{\sigma}\left[p+\frac{c}{\mu-\lambda_a}-R\right]$ as y, $\frac{\partial y}{\partial p}$ as y', and $\frac{\sqrt{k}}{\sigma}$ as s when there is no confusion.

Proof of Lemma 1. Define $F(x) \equiv \lambda\left[1-\Phi\left(\sqrt{k}\frac{p+\frac{c}{(\mu-x)^+}-R}{\sigma}\right)\right]-x$. $F(0)>0, F(\lambda)<0, F(\mu)<0$. Moreover, $F(x)$ is continuous and strictly decreasing in x. Therefore, there exists a unique solution to $F(x)=0$ for $x\in(0,\min\{\lambda,\mu\})$, i.e., $\lambda_a^k(p)\in(0,\min\{\lambda,\mu\})$ exists and is unique.

Differentiating both sides of Equation (2.1) with respect to p and rearranging the terms, I have

$$\frac{\partial\lambda_a^k(p)}{\partial p}=-\frac{\lambda s\Phi'}{1+\lambda s\Phi' c/[\mu-\lambda_a^k(p)]^2}. \tag{A-1}$$

Since $\frac{\partial \lambda_a^k(p)}{\partial p} < 0$, $\lambda_a^k(p)$ strictly decreases in p.

To see how $\lambda_a^k(p)$ is influenced by k, I differentiate both sides of Equation (2.1) with respect to k, which leads to

$$\frac{\partial \lambda_a^k(p)}{\partial k} = -\frac{\lambda y \Phi'/2k}{1+\lambda s \Phi' c/[\mu - \lambda_a^k(p)]^2}. \tag{A-2}$$

By (A-2), $\frac{\partial \lambda_a^k(p)}{\partial k} > 0$ iff $R > p + \frac{c}{\mu - \lambda_a^k(p)}$, which is equivalent to $\lambda_a^k(p) > 0.5\lambda$ by Equation (2.1). Since $F(x)$ strictly decreases in x, $\lambda_a^k(p) > 0.5\lambda$ iff $F(0.5\lambda) > F(\lambda_a^k(p)) = 0$. By the definition of $F(x)$, I can simplify $F(0.5\lambda) > 0$ into $R > p + \frac{c}{(\mu-0.5\lambda)^+}$. Similarly, $\frac{\partial \lambda_a^k(p)}{\partial k} < 0$ iff $R < p + \frac{c}{(\mu-0.5\lambda)^+}$ and $\frac{\partial \lambda_a^k(p)}{\partial k} = 0$ iff $R = p + \frac{c}{(\mu-0.5\lambda)^+}$.

Next I will show $\lim_{k\to+\infty} \lambda_a^k(p) = \lambda_r(p)$ by enumerating all possible cases.

Case 1: $0 \leqslant \mu - \frac{c}{R-p} \leqslant \lambda$ and $R > p$

First, $p + \frac{c}{[\mu - \lim_{k\to+\infty} \lambda_a^k(p)]^+} - R \leqslant 0$ because otherwise $\lim_{k\to+\infty} \Phi\left(\sqrt{k}\frac{p+\frac{c}{[\mu-\lambda_a^k(p)]^+}-R}{\sigma}\right) = 1$ and $\lim_{k\to+\infty} \lambda_a^k(p) = 0$. This suggests that $p + \frac{c}{\mu} > R$, which violates the assumption that $\mu - \frac{c}{R-p} \geqslant 0$.

Second, $p + \frac{c}{[\mu - \lim_{k\to+\infty} \lambda_a^k(p)]^+} - R \geqslant 0$, because otherwise $\lim_{k\to+\infty} \Phi\left(\sqrt{k}\frac{p+\frac{c}{[\mu-\bar{\lambda}_a^k(p)]^+}-R}{\sigma}\right) = 0$, and $\lim_{k\to+\infty} \lambda_a^k(p) = \lambda$. This suggests that $p + \frac{c}{(\mu-\lambda)^+} < R$, which violates the assumption that $\mu - \frac{c}{R-p} \leqslant \lambda$.

Consequently, $p + \frac{c}{[\mu - \lim_{k\to+\infty} \lambda_a^k(p)]^+} - R = 0$, i.e., $\lim_{k\to+\infty} \lambda_a^k(p) = \mu - \frac{c}{R-p} = \lambda_r(p)$

Case 2: $\lambda < \mu - \frac{c}{R-p}$ and $R > p$

In this case, $p + \frac{c}{\mu-\lambda} - R < 0$. By $\lambda_a^k(p) \leqslant \lambda$, $p + \frac{c}{\mu - \lambda_a^k(p)} - R \leqslant p + \frac{c}{\mu-\lambda} - R < 0$, so $\lim_{k\to+\infty} \Phi\left(\sqrt{k}\frac{p+\frac{c}{\mu-\lambda_a^k(p)}-R}{\sigma}\right) = 0$, thus $\lim_{k\to+\infty} \lambda_a^k(p) = \lambda = \lambda_r(p)$.

Case 3: $\mu - \frac{c}{R-p} < 0$ and $R > p$

In this case, $p + \frac{c}{\mu - \lambda_a^k(p)} - R \geqslant p + \frac{c}{\mu-\lambda} - R > 0$, so $\lim_{k\to+\infty} \Phi\left(\sqrt{k}\frac{p+\frac{c}{\mu-\lambda_a^k(p)}-R}{\sigma}\right) = 1$ and $\lim_{k\to+\infty} \lambda_a^k(p) = 0 = \lambda_r(p)$.

Case 4: $R \leqslant p$

In this case, $p + \frac{c}{\mu - \lambda_a^k(p)} - R > 0$, so $\lim_{k\to+\infty} \Phi\left(\sqrt{k}\frac{p+\frac{c}{\mu-\lambda_a^k(p)}-R}{\sigma}\right) = 1$ and $\lim_{k\to+\infty} \lambda_a^k(p) = 0 = \lambda_r(p)$. $\square$

Proof of Proposition 1. *Proof of Proposition 1(i).* The first-order condition (FOC) of $\Pi(p,k)$ with respect to p is

$$\lambda_a^k(p^*(k)) + p^* \frac{\partial \lambda_a^k(p^*(k))}{\partial p} = 0.$$

Denoting $\Phi'\left(\sqrt{k}\frac{p+\frac{c}{\mu-\lambda_a^k(p^*(k))}-R}{\sigma}\right)$ as Φ'^*, $\Phi\left(\sqrt{k}\frac{p+\frac{c}{\mu-\lambda_a^k(p^*(k))}-R}{\sigma}\right)$ as Φ^*, and substituting (A-1) into the FOC, I have:

$$
\begin{aligned}
p^*(k) &= \left\{\frac{\sigma}{\Phi'^*\lambda\sqrt{k}} + \frac{c}{[\mu-\lambda_a^k(p^*(k)]^2}\right\}\lambda_a^k(p^*(k)) \\
&= \left\{\frac{\sigma}{\Phi'^*\sqrt{k}} + \frac{c\lambda}{[\mu-\lambda_a^k(p^*(k)]^2}\right\}(1-\Phi^*).
\end{aligned}
$$

Define

$$
H(x) \equiv \left\{\frac{\sigma}{\sqrt{k}\Phi'(\sqrt{k}\frac{x+\frac{c}{\mu-\lambda_a^k(x)}-R}{\sigma})} + \frac{c\lambda}{[\mu-\lambda_a^k(x)]^2}\right\}\left[1-\Phi(\sqrt{k}\frac{x+\frac{c}{\mu-\lambda_a^k(x)}-R}{\sigma})\right] - x.
$$

Therefore $H(p^*(k)) = 0$. To show that $p^*(k)$ uniquely exists, it suffices to show that $H(0) > 0$, $H(+\infty) = -\infty$, $H(x)$ is continuous, and $H'(x) < 0$. The first three conditions can be verified easily, and I will focus on the last one. When $k = 0$, $H'(x) < 0$ immediately holds. Therefore, I will focus on the $k > 0$ case. By (A-1) and Equation (2.1), $\lambda_a^k(x)$ strictly decreases in x and $x + \frac{c}{\mu-\lambda_a^k(x)} - R$ strictly increases in x. As a result, in order to prove $H(x)$ strictly decreases in x, it suffices to prove $\frac{1-\Phi\left(\frac{\sqrt{k}}{\sigma}(x+\frac{c}{\mu-\lambda_a^k(x)}-R)\right)}{\Phi'\left(\frac{\sqrt{k}}{\sigma}(x+\frac{c}{\mu-\lambda_a^k(x)}-R)\right)}$ decreases in x. Since y strictly increases in x, it suffices to prove that $\frac{1-\Phi(y)}{\Phi'(y)}$ strictly decreases in y.

$$
\left[\frac{1-\Phi(y)}{\Phi'(y)}\right]' = \frac{-\Phi'(y)^2 - \Phi''(y)[1-\Phi(y)]}{\Phi'(y)^2}.
$$

Therefore $\left[\frac{1-\Phi(y)}{\Phi'(y)}\right]' < 0$ iff $-\Phi'(y)^2 - \Phi''(y)[1-\Phi(y)] < 0$. Since $\Phi(y)$ is the cumulative distribution function of the standard normal distribution, $\Phi''(y) = -y\Phi'(y)$, so it suffices to prove that $\Phi'(y) - y[1-\Phi(y)] > 0$. The inequality holds because the left-hand side (LHS) strictly decreases in y and tends to zero from above as $y \to +\infty$. $\lim_{k\to+\infty} p^*(k) = p_r^*$ follows from Lemma 1(ii).

Proof of Proposition 1(ii). I apply the Envelope Theorem by extending the definitions of $\lambda_a^k(p)$ and $p^*(k)$ such that k can be non-integer. I replace k by $l \geqslant 0$ and $p^*(k)$ by $\dot{p}(l)$. The extension preserves all previous results.

$$
\begin{aligned}
\Pi^{*'}(l) &= \frac{\partial\Pi(\dot{p}(l),l)}{\partial p}\dot{p}'(l) + \frac{\partial\Pi(\dot{p}(l),l)}{\partial l} = \frac{\partial\Pi(\dot{p}(l),l)}{\partial l} = p\frac{\partial\lambda_a^l(p)}{\partial l}\bigg|_{p=\dot{p}(l)} \\
&= -p\frac{\Phi'\left(\sqrt{l}\frac{p+\frac{c}{\mu-\lambda_a^l}-R}{\sigma}\right)\frac{p+\frac{c}{\mu-\lambda_a^l}-R}{2\sqrt{l}\sigma}\lambda}{1+\Phi'\left(\sqrt{l}\frac{p+\frac{c}{\mu-\lambda_a^l}-R}{\sigma}\right)\frac{c\sqrt{l}}{\sigma(\mu-\lambda_a^l)^2}\lambda}\Bigg|_{p=\dot{p}(l)}
\end{aligned}
$$

$\Pi^{*'}(l) > 0$ if $\dot{p}(l) + \frac{c}{\mu - \lambda_a^l(\dot{p}(l))} - R < 0$, i.e., $\dot{p}(l) < R - \frac{c}{(\mu - 0.5\lambda)^+}$ by Lemma 1. The last inequality is equivalent to $H\left(R - \frac{c}{(\mu-0.5\lambda)^+}\right) < 0$, which yields $R > R_1(k)$ after rearranging the terms. Similarly, $\Pi^{*'}(l) < 0$ iff $R < R_1(k)$, and $\Pi^{*'}(l) = 0$ iff $R = R_1(k)$.

Proof of Proposition 1(iii). I will show $\frac{\partial^2 \Pi}{\partial p \partial k} < 0$ when $y > y_1$ and $\frac{\partial^2 \Pi}{\partial p \partial k} > 0$ when $y < y_1$, where $y_1 < 0$ determines $R_2(k)$ by $H\left(\frac{\sigma}{\sqrt{k}} y_1 + R - \frac{c}{\mu - \lambda_a}\right) < 0$. After rearranging the terms, I have $R_2(k) > R_1(k)$. Denote $\lambda_a' \equiv \frac{\partial \lambda_a}{\partial p}$ and $\lambda_a'' \equiv \frac{\partial^2 \lambda_a}{\partial^2 p}$.

$$\frac{\partial \Pi(p,k)}{\partial k} = \frac{p \partial \lambda_a^k(p)}{\partial k} = -\frac{p \lambda y \Phi'/2k}{1 + \lambda s \Phi' c/[\mu - \lambda_a^k(p)]^2} = \frac{\sigma p y}{2k^{1.5}} \lambda_a'$$

When $\lambda \to 0$, $\lambda_a \to 0$, $\lambda_a' \to -\lambda s \Phi'$, $y' \to s$, $\lambda_a'' \to \lambda s \Phi' y y' = \lambda s^3 \Phi' \left(p + \frac{c}{\mu} - R\right)$. Define $V(p) \equiv \frac{\partial (p y \lambda_a')}{\partial p} = y \lambda_a' + p y' \lambda_a' + p y \lambda_a''$. When $\lambda \to 0$, $V(p) \approx -\lambda s^2 \Phi' (p + \frac{c}{\mu} - R) - \lambda s^2 p \Phi' + \lambda s^4 p \Phi' \left(p + \frac{c}{\mu} - R\right)^2 = -\lambda s^2 \Phi' \left[2p + \frac{c}{\mu} - R - s^2 p (p + \frac{c}{\mu} - R)^2\right]$.

$$\begin{aligned}
V'(p)\big|_{V(p)=0} &= -\lambda s^2 \Phi' \left[2 - s^2 \left(p + \frac{c}{\mu} - R\right)^2 - 2s^2 p \left(p + \frac{c}{\mu} - R\right)\right]\Bigg|_{s^2(p+\frac{c}{\mu}-R)^2 = 2 - (R - \frac{c}{\mu})/p} \\
&= -\lambda s^2 \Phi' \left[\frac{R - \frac{c}{\mu}}{p} + 2s^2 p \left(R - p - \frac{c}{\mu}\right)\right]\Bigg|_{s^2(p+\frac{c}{\mu}-R)^2 = 2 - (R - \frac{c}{\mu})/p} \\
&= -\lambda s^2 \Phi' \left[\frac{R - \frac{c}{\mu}}{p} + \frac{4p - 2\left(R - \frac{c}{\mu}\right)}{R - p - \frac{c}{\mu}}\right]\Bigg|_{s^2(p+\frac{c}{\mu}-R)^2 = 2 - (R - \frac{c}{\mu})/p} \\
&= -\lambda s^2 \Phi' \left[\frac{\frac{\left(R - \frac{c}{\mu}\right)^2}{p} - \left(R - \frac{c}{\mu}\right) + 4p - 2\left(R - \frac{c}{\mu}\right)}{R - p - \frac{c}{\mu}}\right]\Bigg|_{s^2(p+\frac{c}{\mu}-R)^2 = 2 - (R - \frac{c}{\mu})/p} \\
&= -\lambda s^2 \Phi' \left[\frac{\frac{\left(R - \frac{c}{\mu}\right)^2}{p} - 3\left(R - \frac{c}{\mu}\right) + 4p}{R - p - \frac{c}{\mu}}\right]\Bigg|_{s^2(p+\frac{c}{\mu}-R)^2 = 2 - (R - \frac{c}{\mu})/p}
\end{aligned}$$

First suppose $R - \frac{c}{\mu} > 0$. Since $\frac{\left(R - \frac{c}{\mu}\right)^2}{p} - 3\left(R - \frac{c}{\mu}\right) + 4p > 0$, $V'(p)|_{V(p)=0} < 0$ for $p < R - \frac{c}{\mu}$ and $V'(p)|_{V(p)=0} > 0$ for $p > R - \frac{c}{\mu}$. By $V(0) > 0$ and $V\left(R - \frac{c}{\mu}\right) < 0$, $V(p) = 0$ has a unique solution for $p \in \left(0, R - \frac{c}{\mu}\right)$; by $V(+\infty) < 0$[15] and $V(R - \frac{c}{\mu}) < 0$, $V(p) < 0$ for $p \geqslant R - \frac{c}{\mu}$. As a result, $V(p) = 0$ has a unique solution for $p \in \left(0, R - \frac{c}{\mu}\right)$. Denote the solution as p_1 and set $y_1 = s\left[p_1 + \frac{c}{\mu - \lambda_a^k(p_1)} - R\right]$. $y_1 \to s\left[p_1 + \frac{c}{\mu} - R\right] < 0$ as $\lambda \to 0$. Therefore, I have shown that $\frac{\partial^2 \Pi}{\partial p \partial k} < 0$ when $y > y_1$ and $\frac{\partial^2 \Pi}{\partial p \partial k} > 0$ when $y < y_1$.

[15] Since $p \approx \frac{1 - \Phi(y)}{s \Phi'(y)}$ and $\frac{1 - \Phi(y)}{\Phi'(y)}$ strictly decreases in y, $sp \leqslant \frac{0.5}{\Phi'(0)} \approx 1.2533$. Therefore, When $p \to +\infty$, $2p + \frac{c}{\mu} - R - s^2 p \left(p + \frac{c}{\mu} - R\right)^2 \to 2p + \frac{c}{\mu} - R - 1.2533p \approx +\infty$ and thus $V(+\infty) < 0$.

Now suppose $R-\frac{c}{\mu}\leqslant 0$. Since $\frac{\left(R-\frac{c}{\mu}\right)^2}{p}-3\left(R-\frac{c}{\mu}\right)+4p>0$ and $R-p-\frac{c}{\mu}<0$, $V'(p)|_{V(p)=0}>0$. By $V(0)\leqslant 0$ and $V(+\infty)<0$, $V(p)<0$ for $p>0$, thus $\frac{\partial^2\Pi}{\partial p\partial k}<0$. Since $\frac{c}{\mu}<R_2(k)$, I can combine this case into the case where $R>\frac{c}{\mu}$.

In order to prove that $R_2(k)$ strictly decreases in k, it suffices to prove that p_1 strictly increases in k. Since $V\left(\frac{1}{2}\left(R-\frac{c}{\mu}\right)\right)<0$, $p_1<\frac{1}{2}\left(R-\frac{c}{\mu}\right)$. By definition, $2p_1+\frac{c}{\mu}-R-s^2p_1\left(p_1+\frac{c}{\mu}-R\right)=\left(p_1+\frac{c}{\mu}-R\right)+p_1-\frac{k}{\sigma^2}p_1\left(p_1+\frac{c}{\mu}-R\right)^2=0$, which yields $k=\sigma^2\left[\frac{1}{p_1\left(p_1+\frac{c}{\mu}-R\right)}+\frac{1}{\left(p_1+\frac{c}{\mu}-R\right)^2}\right]$. When k increases, p_1 increases because the RHS strictly increases in p_1 for $p_1\in\left(0,\frac{1}{2}\left(R-\frac{c}{\mu}\right)\right)$. $\square$

Proof of Lemma 2. I will apply the supermodularity argument to show that $\frac{\partial^2\Pi}{\partial\lambda_a^k\partial s}<(>,=)0$ when $R<(>,=)\frac{c\mu}{[\mu-\bar{\Phi}(C)\lambda]^2}$, where $\Pi=p\lambda_a^k(p)$ is the server's revenue. To this end, I treat λ_a^k instead of p as the service provider's sole decision variable and denote it by x. I also denote $p(x)$ as the inverse function of $\lambda_a^k(p)$. Therefore, Equation (2.1) can be rewritten into

$$x=\lambda\bar{\Phi}\left(s\left[p(x)+\frac{c}{\mu-x}-R\right]\right).$$

Differentiating both sides with respect to x, I have

$$1=-\lambda s\left[p'(x)+\frac{c}{(\mu-x)^2}\right]\Phi',$$

which further implies

$$p'(x)=-\frac{1}{\lambda s\Phi'}-\frac{c}{(\mu-x)^2}.$$

Substituting the above expression of $p'(x)$ into the FOC of the server's revenue function (i.e., $p'(x)x+p(x)=0$), I have

$$p(x)=\frac{x}{\lambda s\Phi'}+\frac{cx}{(\mu-x)^2}.$$

Denote $\tilde{H}(x)\equiv\frac{x}{\lambda s\Phi'}+\frac{cx}{(\mu-x)^2}-p(x)$. Therefore, $\lambda_a^k(p^*(k))$ satisfies $\tilde{H}(\lambda_a^k(p^*(k)))=0$. The existence and uniqueness of $\lambda_a^k(p^*(k))$ follow from $\tilde{H}'(x)>0$, $\tilde{H}'(0)<0$, $\tilde{H}'(\min\{\mu,\lambda\})>0$.

Next, I will show that $\frac{\partial^2\Pi}{\partial\lambda_a^k\partial s}<(>,=)0$ when $R<(>,=)\frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$.

$$\frac{\partial\Pi}{\partial s}=\frac{\partial p}{\partial s}x=-xy/s,$$

where $\frac{\partial p}{\partial s}=-y/s$ follows by definition of $p(x)$. As a result,

$$\frac{\partial^2\Pi}{\partial\lambda_a^k\partial s}=-\left\{y/s+x\left[p'(x)+\frac{c}{(\mu-x)^2}\right]\right\}=-\left[y-\frac{x}{\lambda\Phi'}\right]/s=\left[\frac{\bar{\Phi}(y)}{\Phi'(y)}-y\right]/s,$$

where the second equality follows from the FOC of Π and the last equality follows from $x = \lambda\bar{\Phi}(y)$.

Since Φ and Φ' are the cumulative and density functions of the standard normal distribution, $y > (<,=)\frac{\bar{\Phi}(y)}{\Phi'(y)}$ and thus $\frac{\partial^2\Pi}{\partial\lambda_a^k\partial s} < (>,=)0$ if $y > (<,=)C$. Therefore, to complete the proof, it suffices to show that $R < (>,=)\frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$ when $y > (<,=)C$. To show this, note that $y > (<,=)C$ iff $x < (>,=)\bar{\Phi}(C)\lambda$. By monotonicity of $\tilde{H}(x)$, $x < (>,=)\bar{\Phi}(C)\lambda$ iff $\tilde{H}(\bar{\Phi}(C)\lambda) > (<,=)0$. Since

$$\begin{aligned}\tilde{H}(\bar{\Phi}(C)\lambda) &= \frac{\bar{\Phi}(C)}{s\Phi'(C)} + \frac{c\bar{\Phi}(C)\lambda}{[\mu-\bar{\Phi}(C)\lambda]^2} - \frac{C}{s} - R + \frac{c}{\mu-\bar{\Phi}(C)\lambda}\\ &= \frac{\bar{\Phi}(C)}{s\Phi'(C)} + \frac{c\mu}{[\mu-\bar{\Phi}(C)\lambda]^2} - \frac{C}{s} - R\\ &= \frac{c\mu}{[\mu-\bar{\Phi}(C)\lambda]^2} - R,\end{aligned}$$

$\tilde{H}(\bar{\Phi}(C)\lambda) > (<,=)0$ iff $R < (>,=)\frac{c\mu}{\{[\mu-\bar{\Phi}(C)\lambda]^+\}^2}$. □

Proof of Proposition 2. Part (i) is immediate from Lemma 2 and the equivalence between revenue and welfare maximisations in the fully-rational benchmark (see Hassin and Haviv 2003). Therefore, I will focus on part (ii). Since the $R < R_1(k)$ case follows from Proposition 1(i) and Lemma 2, I will only consider the sufficiently high R case. Denote $z \equiv p^* + \frac{c}{\mu-\lambda_a^k(p^*)} - R$. According to the proof of Lemma 2, z is determined by

$$\frac{\bar{\Phi}(sz)}{s\Phi'(sz)} + \frac{c\mu}{[\mu-\bar{\Phi}(sz)\lambda]^2} - z - R = 0.$$

Since the LHS strictly decreases in z and R, $\lim_{R\to+\infty} z = -\infty$.

The consumer surplus under the revenue-maximising price p^* is given by:

$$CS = -\lambda\bar{\Phi}(sz)z.$$

Therefore,

$$\frac{\partial CS}{\partial s} = -\lambda[-(z+sz')z\Phi' + z'\bar{\Phi}] = -\lambda[-z^2\Phi' + z'(\bar{\Phi} - sz\Phi')].$$

When $z \to -\infty$,

$$\frac{\partial CS}{\partial s} \to -\lambda z'(\bar{\Phi} - sz\Phi').$$

Therefore, to show $\frac{\partial CS}{\partial s} < 0$, it suffices to show that $z' > 0$. By definition,

$$z' = \frac{\partial p^*}{\partial s} + \frac{c}{[\mu-\lambda_a^k(p^*(k))]^2}\frac{\partial\lambda_a^k(p^*(k))}{\partial s}.$$

By Lemma 2, $\frac{\partial \lambda_a^k(p^*(k))}{\partial s} > 0$. Therefore, to complete the proof, it suffices to show that $\frac{\partial p^*}{\partial s} > 0$ when R is sufficiently large. Similar to the proof of Lemma 2, I treat z instead of p as the service provider's sole decision variable. Then Equation (2.1) can be rewritten as

$$\mu - \frac{c}{R+z-p(z)} = \lambda \bar{\Phi}(sz).$$

Therefore,

$$\frac{\partial p(z)}{\partial z} = 1 + \frac{s\lambda(R+z-p)^2\Phi'}{c},$$
$$\frac{\partial p(z)}{\partial s} = \frac{\lambda z(R+z-p)^2\Phi'}{c}.$$

To prove $\frac{\partial p^*}{\partial s} > 0$, I will invoke the supermodularity argument by showing that $\frac{\partial^2 \Pi}{\partial z \partial s} > 0$. Since $\Pi = p\lambda_a^k(p) = p\left(\mu - \frac{c}{R+z-p}\right)$,

$$\begin{aligned}
\frac{\partial \Pi}{\partial s} &= \frac{\partial p}{\partial s}\left(\mu - \frac{c}{R+z-p}\right) - \frac{pc}{(R+z-p)^2}\frac{\partial p}{\partial s} \\
&= \frac{\partial p}{\partial s}\left[\mu - \frac{c(R+z)}{(R+z-p)^2}\right] \\
&= \frac{\lambda z \Phi'}{c}[\mu(R+z-p)^2 - c(R+z)].
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial^2 \Pi}{\partial z \partial s} =& \frac{\lambda}{c}(1-s^2z^2)\Phi'[\mu(R+z-p)^2 - c(R+z)] \\
&+ \frac{\lambda}{c} z\Phi'\left[2\mu\,(R+z-p)\left(1 - \frac{\partial p}{\partial z}\right) - c\right].
\end{aligned}$$

By the expression of $\frac{\partial p}{\partial z}$, $1 - \frac{\partial p}{\partial z} < 0$. Since $R+z-p = \frac{c}{\mu - \lambda_a^k(p)}$ and $z \to -\infty$ as $R \to +\infty$, $z\Phi'\left[2\mu(R+z-p)\left(1 - \frac{\partial p}{\partial z}\right) - c\right] > 0$. Therefore, to prove $\frac{\partial^2 \Pi}{\partial z \partial s} > 0$, it suffices to show that $\mu(R+z-p)^2 < c(R+z)$, i.e., $\frac{c\mu}{[\mu - \lambda_a^k(p)]^2} < p + \frac{c}{\mu - \lambda_a^k(p)}$ or $p > \frac{c\lambda_a^k(p)}{[\mu - \lambda_a^k(p)]^2}$. The last inequality holds by the proof of Proposition 1(i). □

Proof of Proposition 3. *Proof of Proposition 3(i).* We can think of the revenue maximisation problem as a 2-stage optimisation problem: In Stage 1, the server chooses the optimal price $\hat{p}$ for any given quality R. In Stage 2, the server optimises over R for the optimal price obtained in Stage 1, i.e., $R(p)$ is a function of the optimal price.

First consider the server's pricing decision. The optimal price satisfies the FOC:

$$\lambda_a^k(p,R) + p\frac{\partial \lambda_a^k(p,R)}{\partial p} + p\frac{\partial \lambda_a^k(p,R)}{\partial R}\frac{\partial R}{\partial p} = 2aR\frac{\partial R}{\partial p}. \tag{A-3}$$

Now return to the quality decision. The optimal quality satisfies the FOC:

$$\hat{p}\frac{\partial \lambda_a^k(\hat{p},R)}{\partial R} = 2aR. \tag{A-4}$$

The solution pair $(\hat{R}, \hat{p})$ satisfies Equation (A-3)-(A-4), which yields

$$\lambda_a^k(\hat{p}, \hat{R}) = 2a\hat{R}.$$

By Proposition 1, a unique positive optimal price $\hat{p}$ exists for any positive quality.

Proof of Proposition 3(ii). By Equation (A-4), $\Pi_R(p,R) = p\lambda_a^k(p,R) - a\left[p\frac{\partial\lambda_a^k(p,R)}{\partial R}/2a\right]^2 = p\lambda_a^k(p,R) - \frac{p^2\{[\lambda_a^k(p,R)]'\}^2}{4a}$, where $[\lambda_a^k(p,R)]' \equiv \frac{\partial\lambda_a^k(p,R)}{\partial p}$ and $\frac{\partial\lambda_a^k(p,R)}{\partial R} = -\frac{\partial\lambda_a^k(p,R)}{\partial p}$ by the following equation.

$$\frac{\partial\lambda_a}{\partial R} = \frac{\lambda s\Phi'}{1 + \lambda s\Phi'\frac{c}{(\mu-\lambda_a)^2}}$$

I will prove $\hat{p}' < 0$ using the submodularity argument.

$$\frac{\partial\Pi_R(p,R)}{\partial k} = p\frac{\partial\lambda_a}{\partial k} - \frac{p^2\lambda_a'}{2a}\frac{\partial^2\lambda_a}{\partial k\partial p}$$

Since $\frac{\partial\lambda_a}{\partial k} = \frac{\sigma y}{2k^{1.5}}\lambda_a'$ and $\frac{\partial^2\lambda_a}{\partial k\partial p} = \frac{\partial^2\lambda_a}{\partial p\partial k} = \frac{\sigma}{2k^{1.5}}(y'\lambda_a + y\lambda_a')$,

$$\frac{\partial\Pi_R(p,R)}{\partial k} = \frac{\sigma py}{2k^{1.5}}\lambda_a' - \frac{p^2\lambda_a'}{2a}\frac{\sigma}{2k^{1.5}}(y'\lambda_a + y\lambda_a')$$

When $\lambda \to 0$, $\lambda_a \to 0$, $\lambda_a' \to -\lambda s\Phi'$, $y \to s\left(p + \frac{c}{\mu}\right)$, $y' \to s$, $\lambda_a'' \to \lambda s\Phi' yy' = \lambda s^3\Phi'\left(p + \frac{c}{\mu}\right)$. As a result, $\frac{\partial\Pi(p,R)}{\partial k} = \frac{\sigma py}{2k^{1.5}}\lambda_a'$ and

$$\begin{aligned}
\frac{\partial^2\Pi_R(p,R)}{\partial p\partial k} &\approx \frac{\sigma}{2k^{1.5}}[y\lambda_a' + py'\lambda_a' + py\lambda_a''] \\
&\approx \frac{\sigma}{2k^{1.5}}\left[-\lambda s^2\Phi'\left(p + \frac{c}{\mu}\right) - \lambda s^2 p\Phi' + \lambda s^4 p\Phi'\left(p + \frac{c}{\mu}\right)^2\right] \\
&= -\frac{\sigma\lambda s^2\Phi'}{2k^{1.5}}\left[2p + \frac{c}{\mu} - s^2 p\left(p + \frac{c}{\mu}\right)^2\right].
\end{aligned}$$

When σ is sufficiently high, $s \to 0$ and $p \to \frac{\lambda_a}{\Phi'\lambda s}$, so $\frac{\partial^2\Pi_R(p,R)}{\partial p\partial k} \approx -\frac{\sigma\lambda s^2\Phi'}{2k^{1.5}}\left(2p + \frac{c}{\mu}\right) < 0$. When σ is sufficiently low, $p \approx \frac{\sigma(1-\Phi)}{\Phi'\sqrt{k}} \to 0$, thus $\frac{\partial^2\Pi_R(p,R)}{\partial p\partial k} \approx -\frac{\sigma\lambda s^2\Phi'}{2k^{1.5}}\frac{c}{\mu} < 0$. As a result, $\hat{p}$ strictly decreases in k when σ is sufficiently high or sufficiently low.

Next I will characterises how k influences $\hat{R}$. By Equation (A-4), $\frac{\partial\lambda_a}{\partial k} = -\frac{\sigma y}{2k^{1.5}}\frac{\partial\lambda_a}{\partial R} = -\frac{a\sigma yR}{pk^{1.5}}$. As a result, $\frac{\partial\Pi_R(p,R)}{\partial k} = p\frac{\partial\lambda_a}{\partial k} = -\frac{a\sigma yR}{k^{1.5}}$. $\frac{\partial^2\Pi_R(p,R)}{\partial R\partial k} = -\frac{a\sigma}{k^{1.5}}\left[\frac{\partial y}{\partial R}R + y\right] < 0$ because $y \to s\left(p + \frac{c}{\mu}\right) > 0$, $R \to 0$, and $\frac{\partial y}{\partial R} \to -s$ when $\lambda \to 0$.

Proof of Proposition 3(iii). When $\lambda \to +\infty$, $y \to +\infty$, $\frac{1-\Phi}{\Phi'} = \frac{\lambda_a}{\lambda\Phi'} \to 0$, so $\lambda\Phi' \to +\infty$. As a result, $\lambda_a' \to -\frac{(\mu-\lambda_a)^2}{c}$, $\lambda_a'' \to -\frac{2(\mu-\lambda_a)^3}{c^2}$, and $y' \to 0$. By the expression of $\frac{\partial\Pi_R(p,R)}{\partial k}$, $\frac{\partial\Pi_R(p,R)}{\partial k} \approx \frac{\sigma py}{2k^{1.5}}\lambda_a'$ and

$$\begin{aligned}
\frac{\partial^2\Pi_R(p,R)}{\partial p\partial k} &\approx \frac{\sigma}{2k^{1.5}}[y\lambda_a' + py'\lambda_a' + py\lambda_a''] \\
&\approx -\frac{\sigma y}{2k^{1.5}}\left[\frac{(\mu-\lambda_a)^2}{c} + p\frac{(\mu-\lambda_a)^3}{c^2}\right] < 0.
\end{aligned}$$

Since $\frac{\partial^2 \Pi_R(p,R)}{\partial p \partial k} < 0$, $\hat{p}$ strictly decreases in k.

Next I characterise how k influences $\hat{R}$. $\frac{\partial \Pi_R(p,R)}{\partial k} = p\frac{\partial \lambda_a}{\partial k} = -\frac{a\sigma y R}{k^{1.5}}$. Thus $\frac{\partial^2 \Pi_R(p,R)}{\partial R \partial k} = -\frac{a\sigma}{k^{1.5}}\left(\frac{\partial y}{\partial R}R + y\right) = -\frac{a\sigma}{k^{1.5}}(Ry' + y) \to -\frac{a\sigma y}{k^{1.5}} < 0$. Note that R is finite when $\lambda \to +\infty$ by $\lambda_a^k(\hat{p},\hat{R}) = 2a\hat{R}$. As a result, $\hat{R}$ strictly strictly decreases in k.

Proof of Proposition 3(iv).

$$\frac{\partial \Pi_R(\hat{p},\hat{R})}{\partial k} = \left.\frac{\partial \Pi_R(p,R)}{\partial k}\right|_{p=\hat{p},R=\hat{R}} = p\left.\frac{\partial \lambda_a^k(p,R)}{\partial k}\right|_{p=\hat{p},R=\hat{R}} < 0.$$

The last inequality follows by $\hat{p} + \frac{c}{\mu - \lambda_a^k(\hat{p},\hat{R})} - \hat{R} > 0$ when λ is sufficiently high or low: (i) When $\lambda \to +\infty$, $\hat{p} + \frac{c}{\mu - \lambda_a^k(\hat{p},\hat{R})} - \hat{R} \to +\infty$ by Equation (2.1). (ii) When $\lambda \to 0$, $\hat{R} = 2a\lambda_a^k(\hat{p},\hat{R}) \to 0$, thus $\hat{p} + \frac{c}{\mu - \lambda_a^k(\hat{p},\hat{R})} - \hat{R} \to \hat{p} + \frac{c}{\mu} > 0$. $\square$

Proof of Proposition 4. By Proposition 1(ii), $\Pi^*(k) > \Pi^*(+\infty)$ for all k if $R \leqslant \underline{R}$, and $\Pi^*(k) < \Pi^*(+\infty)$ for all k if $R \geqslant R_1(1)$. Therefore, I will focus on the $\underline{R} < R < R_1(1)$ case. When $R_1(k) \leqslant R < R_1(1)$, $\Pi^*(k) < \Pi^*(+\infty)$ always holds because $\Pi^*(k)$ is U-shaped, so I will further restrict the attention to the $\underline{R} < R < R_1(k)$ case. Denote $Q(k,R) \equiv \Pi^*(k,R) - \Pi^*(+\infty,R)$ as the optimal revenue benefit compared to the fully-rational benchmark, where

$$\Pi^*(+\infty,R) = \begin{cases} (\sqrt{R\mu} - \sqrt{c})^2, & \text{if} \quad \lambda \geqslant \mu - \sqrt{\frac{c\mu}{R}}, \\ R\lambda - \frac{c\lambda}{\mu - \lambda}, & \text{if} \quad \lambda < \mu - \sqrt{\frac{c\mu}{R}}. \end{cases}$$

Next I will show that $Q(k,R)$ strictly decreases in k for $\underline{R} < R < R_1(k)$. To this end, I first denote $\lambda_a^*(k) \equiv \lambda_a^k(p^*(k))$ as the optimal arrival rate. According to the FOC of the revenue maximisation problem, $\lambda_a^*(k) = p^*(k)\frac{\partial \lambda_a^*(k)}{\partial R}$. Moreover, $\lambda_a^*(k) < 0.5$ for $\underline{R} < R < R_1(k)$ by the proof of Proposition 1(ii).

$$\begin{aligned}\frac{\partial Q(k,R)}{\partial R} &= \frac{\partial \Pi^*(k,R)}{\partial R} - \frac{\partial \Pi^*(+\infty,R)}{\partial R} \\ &= p^*(k)\frac{\partial \lambda_a^*(k)}{\partial R} - \frac{\partial \Pi^*(+\infty,R)}{\partial R} \\ &= \lambda_a^*(k) - \frac{\partial \Pi^*(+\infty,R)}{\partial R} < 0.5\lambda - \frac{\partial \Pi^*(+\infty,R)}{\partial R}.\end{aligned}$$

When $\lambda < \mu - \sqrt{\frac{c\mu}{R}}$, $\Pi^*(+\infty,R) = R\lambda - \frac{c\lambda}{\mu-\lambda}$ so $\frac{\partial \Pi^*(+\infty,R)}{\partial R} = \lambda$. $\frac{\partial Q(k,R)}{\partial R} < 0.5\lambda - \frac{\partial \Pi^*(+\infty,R)}{\partial R} = -0.5\lambda < 0$.

When $\lambda \geqslant \mu - \sqrt{\frac{c\mu}{R}}$, $\Pi^*(+\infty,R) = (\sqrt{R\mu} - \sqrt{c})^2$ so $\frac{\partial \Pi^*(+\infty,R)}{\partial R} = \mu - \sqrt{\frac{c\mu}{R}} \leqslant \mu -$

$\sqrt{\frac{c\mu}{\bar{R}}}$.

$$\frac{\partial Q(k,R)}{\partial R} < 0.5\lambda - \frac{\partial \Pi^*(+\infty,R)}{\partial R} \leqslant 0.5\lambda - \mu + \sqrt{\frac{c\mu}{\frac{c\mu}{[(\mu-0.5\lambda)^+]^2} + \frac{\sigma}{2\Phi'(0)}}}$$
$$< 0.5\lambda - \mu + \sqrt{\frac{c\mu}{\frac{c\mu}{[(\mu-0.5\lambda)^+]^2}}} \leqslant 0$$

Therefore, $Q(k,R)$ strictly decreases in R. Based on this, I will prove Proposition 4 for the $\underline{R} < R < R_1(k)$ case. When $\tilde{R} < R < R_1(k)$, $\Pi^*(1) < \Pi^*(+\infty)$, so the server discloses information. When $\underline{R} < R \leqslant \tilde{R}$, $\tilde{k} = \{k \geqslant 1 | \Pi^*(k) = \Pi^*(+\infty)\}$ uniquely exists because $Q(1,R) > 0$, $Q(+\infty,R) < 0$, and $Q(k,R)$ is U-shaped in k. Moreover, $\tilde{k}$ strictly decreases in R because $Q(k,R)$ strictly decreases in R and strictly increases in k for $\underline{R} < R \leqslant \tilde{R}$. □

Proof of Corollary 1. By the proof of Proposition 4, in order to show that $\tilde{k}$ strictly decreases in μ, it suffices to prove that $Q(k,R)$ strictly decreases in μ for $\underline{R} < R < R_1(k)$.

$$\frac{\partial \lambda_a^k}{\partial \mu} = \frac{\lambda s \Phi' \frac{c}{(\mu-\lambda_a^k)^2}}{1+\lambda s \Phi' \frac{c}{(\mu-\lambda_a^k)^2}} = -\frac{c}{(\mu-\lambda_a)^2}\frac{\partial \lambda_a^k}{\partial p}.$$

By the Envelope Theorem,

$$\begin{aligned}
\frac{\partial Q(k,R)}{\partial \mu} &= p^*(k)\frac{\partial \lambda_a^k(p^*(k))}{\partial \mu} - \frac{\partial \Pi_r^*}{\partial \mu} \\
&= -\frac{c}{[\mu-\lambda_a^k(p^*(k))]^2} p^*(k) \frac{\partial \lambda_a^k(p^*(k))}{\partial p} - \frac{\partial \Pi_r^*}{\partial \mu} \\
&= \frac{c\lambda_a^k(p^*(k))}{[\mu-\lambda_a^k(p^*(k))]^2} - \frac{\partial \Pi_r^*}{\partial \mu} < \frac{0.5c\lambda}{(\mu-0.5\lambda)^2} - \frac{\partial \Pi_r^*}{\partial \mu},
\end{aligned}$$

where $\lambda_a^k(p^*(k)) < 0.5\lambda$ by $R < R_1(k)$. According to the traditional queueing economics (e.g., Hassin and Haviv 2003),

$$\Pi_r^* = \begin{cases} (\sqrt{R\mu}-\sqrt{c})^2, & \text{if} \quad \lambda \geqslant \mu - \sqrt{\frac{c\mu}{R}}, \\ R\lambda - \frac{c\lambda}{\mu-\lambda}, & \text{if} \quad \lambda < \mu - \sqrt{\frac{c\mu}{R}}. \end{cases}$$

Therefore,

$$\frac{\partial \Pi_r^*}{\partial \mu} = \begin{cases} R - \sqrt{\frac{cR}{\mu}}, & \text{if} \quad \lambda \geqslant \mu - \sqrt{\frac{c\mu}{R}}, \\ \frac{c\lambda}{(\mu-\lambda)^2}, & \text{if} \quad \lambda < \mu - \sqrt{\frac{c\mu}{R}}. \end{cases}$$

When $\lambda \geqslant \mu - \sqrt{\frac{c\mu}{R}}$,

$$\frac{\partial Q(k,R)}{\partial \mu} < \frac{0.5c\lambda}{(\mu-0.5\lambda)^2} - R + \sqrt{\frac{cR}{\mu}}.$$

Since $R-\sqrt{\frac{cR}{\mu}}$ strictly decreases in R for $R>\underline{R}$,

$$\frac{\partial Q(k,R)}{\partial \mu}<\frac{0.5c\lambda}{(\mu-0.5\lambda)^2}-\frac{c\mu}{(\mu-0.5\lambda)^2}+\sqrt{\frac{c}{\mu}\frac{c\mu}{(\mu-0.5\lambda)^2}}=-\frac{c}{\mu-0.5\lambda}+\frac{c}{\mu-0.5\lambda}=0.$$

When $\lambda<\mu-\sqrt{\frac{c\mu}{R}}$,

$$\frac{\partial Q(k,R)}{\partial \mu}=\frac{c\lambda_a^k(p^*(k))}{[\mu-\lambda_a^k(p^*(k))]^2}-\frac{\partial \Pi_r^*}{\partial \mu}<\frac{c\lambda}{(\mu-\lambda)^2}-\frac{c\lambda}{(\mu-\lambda)^2}=0.$$

In order to prove that $\tilde{k}$ strictly increases in c, it suffices to prove that $Q(k,R)$ strictly increases in c for $\underline{R}<R<R_1(k)$. By the Envelope Theorem,

$$\begin{aligned}\frac{\partial Q(k,R)}{\partial c}&=p^*(k)\frac{\partial \lambda_a^k(p^*(k))}{\partial c}-\frac{\partial \Pi_r^*}{\partial c}\\&=\frac{p^*(k)\frac{\partial \lambda_a^k(p^*(k))}{\partial p}}{\mu-\lambda_a(p^*(k))}-\frac{\partial \Pi_r^*}{\partial c}\\&=-\frac{\lambda_a(p^*(k))}{\mu-\lambda_a(p^*(k))}-\frac{\partial \Pi_r^*}{\partial c}>-\frac{0.5\lambda}{\mu-0.5\lambda}-\frac{\partial \Pi_r^*}{\partial c},\end{aligned}$$

where

$$\frac{\partial \Pi_r^*}{\partial c}=\begin{cases}1-\sqrt{\frac{R\mu}{c}}, & \text{if} \quad \lambda\geqslant\mu-\sqrt{\frac{c\mu}{R}},\\ -\frac{\lambda}{\mu-\lambda}, & \text{if} \quad \lambda<\mu-\sqrt{\frac{c\mu}{R}}.\end{cases}$$

When $\lambda\geqslant\mu-\sqrt{\frac{c\mu}{R}}$,

$$\frac{\partial Q(k,R)}{\partial \lambda}>-\frac{0.5\lambda}{\mu-0.5\lambda}+\frac{\lambda}{\mu-\lambda}>0.$$

When $\lambda<\mu-\sqrt{\frac{c\mu}{R}}$,

$$\frac{\partial Q(k,R)}{\partial \lambda}>-\frac{0.5\lambda}{\mu-0.5\lambda}-1+\sqrt{\frac{R\mu}{c}}=\sqrt{\frac{R\mu}{c}}-\frac{\mu}{\mu-0.5\lambda}>0.$$

The last inequality follows from $R>\underline{R}$.

In order to prove that $\tilde{k}$ strictly increases in λ, it suffices to prove that $Q(k,R)$ strictly increases in λ for $\lambda\geqslant\mu-\sqrt{\frac{c\mu}{R}}$ and $\underline{R}<R<R_1(k)$. Note that

$$\frac{\partial \lambda_a^k}{\partial \lambda}=\frac{1-\Phi}{1+\lambda s\Phi'\frac{c}{(\mu-\lambda_a^k)^2}}=-\frac{1-\Phi}{\Phi' s\lambda}\frac{\partial \lambda_a^k}{\partial p}.$$

By the Envelope Theorem,

$$\begin{aligned}\frac{\partial Q(k,R)}{\partial \lambda}&=p^*(k)\frac{\partial \lambda_a^k(p^*(k))}{\partial \lambda}-\frac{\partial \Pi_r^*}{\partial \lambda}\\&=-\frac{(1-\Phi)}{\Phi' s\lambda}p^*(k)\frac{\partial \lambda_a^k(p^*(k))}{\partial p}-\frac{\partial \Pi_r^*}{\partial \lambda}\\&=\frac{(1-\Phi)\lambda_a^k(p^*(k))}{\Phi' s\lambda}-\frac{\partial \Pi_r^*}{\partial \lambda},\end{aligned}$$

where

$$\frac{\partial \Pi_r^*}{\partial \lambda} = \begin{cases} 0, & \text{if} \quad \lambda \geqslant \mu - \sqrt{\frac{c\mu}{R}}, \\ R - \frac{c\mu}{(\mu-\lambda)^2}, & \text{if} \quad \lambda < \mu - \sqrt{\frac{c\mu}{R}}, \end{cases}$$

Therefore when $\lambda \geqslant \mu - \sqrt{\frac{c\mu}{R}}$,

$$\frac{\partial Q(k,R)}{\partial \lambda} = \frac{(1-\Phi)\lambda_a^k(p^*(k))}{\Phi' s \lambda} > 0. \quad \square$$

Proof of Proposition 5. According to the FOC of $W(\lambda_a^k(p))$ with respect to $\lambda_a^k(p)$, I can obtain the socially optimal joining rate λ_w^* as:

$$\lambda_w^* = \begin{cases} 0, & \text{if} \quad R \leqslant \frac{c}{\mu}, \\ \mu - \sqrt{\frac{c\mu}{R}}, & \text{if} \quad \frac{c}{\mu} < R < \frac{c\mu}{[(\mu-\lambda)^+]^2}, \\ \lambda, & \text{if} \quad R \geqslant \frac{c\mu}{[(\mu-\lambda)^+]^2}. \end{cases}$$

Let $\check{p}_w(k)$ denote the price such that $\lambda_a^k(\check{p}_w(k)) = \mu - \sqrt{\frac{c\mu}{R}}$. By Equation (2.1), I have

$$\check{p}_w(k) = R - \sqrt{\frac{cR}{\mu}} + \frac{\sigma \Phi^{-1}\left(1 - \frac{\mu - \sqrt{\frac{c\mu}{R}}}{\lambda}\right)}{\sqrt{k}}.$$

When $\underline{R} < R < \frac{c\mu}{[(\mu-\lambda)^+]^2}$ and $k < \hat{k}_w$, the socially optimal joining rate is $\mu - \sqrt{\frac{c\mu}{R}}$ and $\check{p}_w(k) < 0$. Therefore, the social planner should price at 0. When $R \geqslant \frac{c\mu}{[(\mu-\lambda)^+]^2}$, the optimal joining rate is λ. Since $\lambda_a^k(p)$ strictly decreases in p, the social planner should price at 0. Note that $\hat{k}_w = +\infty$ when $R \geqslant \frac{c\mu}{[(\mu-\lambda)^+]^2}$, thus I can merge the above two cases, i.e., when $R > \underline{R}$ and $k < \hat{k}_w$, $p_w^* = 0$. Otherwise the server should price at $\check{p}_w(k)$ and no welfare loss exists because $\lambda_w^* = \mu - \sqrt{\frac{c\mu}{R}}$ can be induced. $\square$

Proof of Proposition 6. *Proof of Proposition 6(i).* Similar to the proof of Proposition 3, we can think of the social welfare maximisation problem as a 2-stage optimisation problem: In Stage 1, the social planner chooses the optimal price $\hat{p}_w$ for any given quality R. In Stage 2, the server optimises over R given he prices at $\hat{p}_w$, i.e., $R_w(p)$ is a function of the optimal price.

First consider the social planner's pricing decision. The optimal price satisfies the FOC:

$$\lambda_a \frac{\partial R}{\partial p} + \left[R - \frac{c\mu}{(\mu-\lambda_a)^2}\right]\left(\frac{\partial \lambda_a}{\partial p} + \frac{\partial \lambda_a}{\partial R}\frac{\partial R}{\partial p}\right) = 2aR\frac{\partial R}{\partial p}. \tag{A-5}$$

Now return to the quality decision. The optimal quality satisfies the FOC:

$$\lambda_a + R\frac{\partial \lambda_a}{\partial R} - \frac{c\mu}{(\mu-\lambda_a)^2}\frac{\partial \lambda_a}{\partial R} = 2aR. \tag{A-6}$$

The solution pair $(\hat{R}_w, \hat{p}_w)$ satisfies Equation (A-5) - (A-6), which yields

$$\hat{R}_w = \frac{c\mu}{[\mu - \lambda_a(\hat{p}_w, \hat{R}_w)]^2}.$$

When $R \leqslant \frac{c}{\mu}$, Equation (A-5)-(A-6) suggest that the optimal social welfare strictly decreases in p, so $\hat{p}_w = +\infty$. When $R \geqslant \frac{c\mu}{[(\mu-\lambda)^+]^2}$, Equation (A-5)-(A-6) suggest that the optimal social welfare strictly increases in p, so $\hat{p}_w = 0$. When $\frac{c}{\mu} < R < \frac{c\mu}{[(\mu-\lambda)^+]^2}$, the optimal price $\hat{p}_w$ is determined by $\lambda_a(\hat{p}_w, R) = \mu - \sqrt{\frac{c\mu}{R}}$. The optimal price exists and is unique for any positive quality by the proof of Proposition 5.

Proof of Proposition 6(ii). By Proposition 5, the social planner chooses price to induce joining rate closest to the optimal value $\mu - \sqrt{\frac{c\mu}{R}}$. Suppose there exists p such that $\lambda_a(p) = \mu - \sqrt{\frac{c\mu}{R}}$. After pricing at p, the social planner chooses quality to maximise social welfare:

$$\max_{R \geqslant 0} \ (\sqrt{\mu R} - \sqrt{c})^2 - aR^2.$$

The FOC is:

$$2aR + \sqrt{\frac{c\mu}{R}} = \mu. \tag{A-7}$$

Since the optimal quality is independent of λ, $\lambda < \mu - \sqrt{\frac{c\mu}{R}}$ when $\lambda \to 0$, thus $\lambda_a(p) < \mu - \sqrt{\frac{c\mu}{R}}$. The social planner should price at $\hat{p}_w = 0$ to induce joining rate closest to $\mu - \sqrt{\frac{c\mu}{R}}$. I abuse notation and write $\lambda_a(0)$ as λ_a. After pricing at 0, the social planner optimises over R:

$$\max_{R \geqslant 0} \ W_R(0,R) = R\lambda_a - \frac{c\lambda_a}{\mu - \lambda_a} - aR^2.$$

$\frac{\partial W_R(0,R)}{\partial R} = \lambda_a + \left[R - \frac{c\mu}{(\mu-\lambda_a)^2}\right] \frac{\partial \lambda_a}{\partial R} - 2aR \approx \lambda_a + \left(R - \frac{c}{\mu}\right) \lambda s \Phi' - 2aR < 0$ because $\lambda_a \to 0$ and $\frac{\partial \lambda_a}{\partial R} \to \lambda s \Phi'$ when $\lambda \to 0$. Therefore, $\frac{\partial^2 W_R(0,R)}{\partial R^2} \to -2a < 0$. I will show $\hat{R}_w < \frac{c}{\mu}$ and thus operating the queue leads to negative social welfare. Since $\frac{\partial^2 W_R(0,R)}{\partial R^2} < 0$, it suffices to show $\frac{\partial W_R(0,R)}{\partial R} < 0$ when $R = \frac{c}{\mu}$. The last inequality holds because $\frac{\partial W_R(0,R)}{\partial R} \to -\frac{2ac}{\mu} < 0$.

When λ is sufficiently high, $\frac{\lambda_a}{\lambda} \to 0$ so $p + \frac{c}{\mu - \lambda_a} - R \to +\infty$. When $p = 0$, this implies $\lambda_a(0) \to \mu$. As a result, $\lambda_a(0) > \mu - \sqrt{\frac{c\mu}{R}}$ when λ is sufficiently large. Since $\lambda_a(p) < \mu - \sqrt{\frac{c\mu}{R}}$ for sufficiently large p and $\lambda_a(p)$ is continuous and strictly decreases in p, there exists a positive price such that $\lambda_a(p) = \mu - \sqrt{\frac{c\mu}{R}}$, i.e., the price induces the socially optimal joining rate by Proposition 5. Since the cost for improving service quality is independent of price, the price (attainable when λ is sufficiently high, as argued above) remains optimal when the social planner can control quality, i.e., $\hat{p}_w$ is determined by:

$$\lambda_a(\hat{p}_w) = \mu - \sqrt{\frac{c\mu}{R}}.$$

$\hat{p}_w$ strictly decreases in k because when $\lambda > 2\mu$, $\lambda_a < \mu < 0.5\lambda$ and thus λ_a strictly decreases in k.

After pricing at $\hat{p}_w$, the social planner chooses quality to maximise social welfare, and the FOC is given by Equation (A-7). Define $m(R) \equiv 2aR + \sqrt{\frac{c\mu}{R}}$. By its FOC and second-order condition, the unique minimiser $R_m^* = \frac{(c\mu)^{1/3}}{(4a)^{2/3}}$ and $m(R_m^*) = \left(\frac{27ac\mu}{2}\right)^{1/3}$.

When $a \geqslant \frac{2\mu^2}{27c}$, $m(R) > \mu$ for $R \neq R_m^*$ and $m(R_m^*) \geqslant \mu$. Thus $\hat{R}_w = 0$ and the social planner should not offer service.

When $a < \frac{2\mu^2}{27c}$, Equation (A-7) has two positive solutions, and $\hat{R}_w$ is the larger one because the smaller solution is less than $\frac{c}{\mu}$. According to Equation (A-7), $\hat{R}_w$ is independent of k. To complete the proof, I will show that the optimal social welfare is positive when a is sufficiently small, and the optimal social welfare strictly decreases in a. The first claim holds because the optimal social welfare function $\left[\sqrt{\mu\hat{R}_w(a)} - \sqrt{c}\right]^2 - a\hat{R}_w^2(a)$ is continuous in a and is positive when $a \to 0^+$ (note that by Equation (A-7), $\hat{R}_w(a) > \frac{c}{\mu}$ when $a \to 0^+$). Next, I will prove that the second claim also holds. Define $n(a) \equiv \left[\sqrt{\mu\hat{R}_w(a)} - \sqrt{c}\right]^2 - a\hat{R}_w^2(a)$ where $\hat{R}_w$ depends on a by Equation (A-7). Denote $\frac{\partial \hat{R}_w}{\partial a}$ as $(\hat{R}_w)'$.

$$n'(a) = \mu(\hat{R}_w)' - \sqrt{\frac{c\mu}{\hat{R}_w}}(\hat{R}_w)' - (\hat{R}_w)^2 - 2a\hat{R}_w(\hat{R}_w)' = -(\hat{R}_w)^2 < 0,$$

where the last equality holds by Equation (A-7). □

Proof of Lemma 3. Define $D(x) \equiv \lambda \sum_{i=1}^{+\infty} f_i \bar{\Phi}(\sqrt{i}(p + \frac{c}{\mu - x} - R)/\sigma) - x$. $D(0) > 0$, $D(\min\{\lambda, \mu\}) < 0$. Since $D(x)$ is continuous and $D(x)$ strictly decreasing in x, $D(x) = 0$ has a unique solution $\lambda_P^n \in (0, \min\{\lambda, \mu\})$.

To show that $\frac{\partial \lambda_P^n}{\partial p} < 0$, I differentiating both sides of Equation (2.5) with respect to p, which yields

$$\frac{\partial \lambda_P^n}{\partial p} = -\lambda \sum_{i=1}^{+\infty} f_i \Phi' \sqrt{i} \left[1 + \frac{c}{(\mu - \lambda_P^n)^2} \frac{\partial \lambda_P^n}{\partial p}\right] / \sigma.$$

Reorganising the terms leads to

$$\frac{\partial \lambda_P^n}{\partial p} = -\frac{\lambda \sum_{i=1}^{+\infty} f_i \Phi' \sqrt{i}/\sigma}{1 + \lambda \sum_{i=1}^{+\infty} f_i \Phi' \sqrt{i} \frac{c}{(\mu - \lambda_P^n)^2}/\sigma} < 0.$$

To show the comparative statics with respect to n, first note that Equation (2.5) can be rewritten as

$$\lambda_P^n = \lambda \mathbf{E}\left[\bar{\Phi}\left(\sqrt{K}(p + \frac{c}{\mu - \lambda_P^n} - R)/\sigma\right)\right] \equiv d(n, \lambda_P^n),$$

where $\mathbf{E}$ denotes the expectation on the random variable $K \sim \text{Poisson}(n)$. For any $n_1 > 0$ and $n_2 > n_1$, denote $K_i \sim \text{Poisson}(n_i)$. Therefore K_2 has strictly first-order stochastic dominance over K_1, which I denote by $K_2 \succ_{FSD} K_1$. When $p + \frac{c}{\mu-\lambda_P^n} - R < 0$, d strictly increases in K and thus the RHS of Equation (2.5) is higher for $n = n_2$ than $n = n_1$ for a fixed λ_P^n. This further suggests that $\lambda_P^{n_2} > \lambda_P^{n_1}$: If $\lambda_P^{n_2} \leqslant \lambda_P^{n_1}$, $\lambda_P^{n_2} = \lambda d(n_2, \lambda_P^{n_2}) > \lambda d(n_1, \lambda_P^{n_2}) \geqslant \lambda d(n_1, \lambda_P^{n_1}) = \lambda_P^{n_1}$, which contradicts $\lambda_P^{n_2} \leqslant \lambda_P^{n_1}$. Therefore, λ_P^n strictly increases in n if $p + \frac{c}{\mu-\lambda_P^n} - R < 0$. Moreover, $p + \frac{c}{\mu-\lambda_P^n} - R < 0$ is equivalent to $\lambda_P^n > 0.5\lambda$ by Equation (2.5), which is further equivalent to $D(0.5\lambda) > 0$ or $\sum_{i=1}^{+\infty} f_i \bar{\Phi}\left(\sqrt{i}\left(p + \frac{c}{(\mu-0.5\lambda)^+} - R\right)/\sigma\right) > 0.5$. Since the LHS strictly increases in R, this inequality is equivalent to $R > R_P$, where R_P strictly increases in p, c and strictly decreases in μ because $\sum_{i=1}^{+\infty} f_i \bar{\Phi}\left(\sqrt{i}\left(p + \frac{c}{(\mu-0.5\lambda)^+} - R\right)/\sigma\right)$ strictly increases in p, c and strictly decreases in μ. By mimicking this proof, I can show that λ_P^n strictly decreases in n when $R < R_P$. When $R = R_P$, $\lambda_P^n = \lambda \mathbf{E}[\bar{\Phi}(0)] = 0.5\lambda$, which is invariant in n.

To prove $\lim_{n\to+\infty} \lambda_P^n(p) = \lambda_r(p)$, first note that as $n \to +\infty$, K converges in distribution to infinity. Therefore, the RHS of Equation (2.5) is equal to $\lim_{k\to+\infty} \lambda \bar{\Phi}\left(\sqrt{k}\left(p + \frac{c}{\mu-\lambda_P^n} - R\right)/\sigma\right)$. According to Lemma 1, the unique solution that satisfies Equation (2.5) is $\lambda_r(p)$. Therefore, $\lim_{n\to+\infty} \lambda_P^n(p) = \lambda_r(p)$. $\square$

A.2 Supplemental Discussions and Analysis

In this section, I propose three repeated-generation models to incorporate customers' anecdotes acquisition process in the base model and provide supplemental analysis that focuses on the no-congestion setting, the server's information disclosure with quality control, and his joint information disclosure decision on both public and private anecdotes.

A.2.1 Repeated-Generation Models

Here I introduce three repeated-generation models (dubbed as M1, M2, M3) to endogenise customers' anecdotes acquisition process and the server's dynamic pricing and quality decisions in Chapter 2.4.1. These models not only provide a rigorous formulation of service systems with growing popularity over time, they also illustrate the generality of my anecdotal reasoning framework in capturing different types of anecdotes acquisition processes.

First, consider the following repeated-generation model (i.e., M1). In period 0, generation-0 customers arrive and join at any positive rate. In period i ($i \geqslant 1$), generation-i customers arrive and make the join-or-balk decision based on k anecdotes. Each anecdote is a service quality realisation experienced by a customer from any of the previous generations. In addition, I assume that: (i) Each generation is long enough for the system to reach steady state. (ii) All anecdotes acquired by customers of the same generation are independent draws from the service quality distribution. The former assumption has been widely adopted in the existing multi-period queueing models (Huang and Chen 2015, Cui and Veeraraghavan 2016, and references therein), and the latter one is a standard assumption in the anecdotal reasoning literature (Spiegler 2006b, Szech 2011, Huang and Yu 2014). Notably, Huang and Chen (2015) adopt both assumptions in a similar repeated-generation model. Consistent with Huang and Chen (2015), I will implicitly adopt both assumptions for all repeated-generation models in this section.

It can be verified that in each period, customers apply the anecdotal reasoning framework described in Chapter 2.3.1. In other words, M1 generalises the base model by incorporating an anecdotes acquisition process, which is general in the following aspects. First, customers across different generations can be different customers or the same customers that visit the service system repeatedly. Second, a generation-i customer's anecdotes may come from earlier customers or her own past service experiences, and I do not impose any assumption regarding the distribution of anecdotes from different generations (e.g., recent generations are sampled more often, as assumed by Huang and Chen 2015).

Building on M1, the next repeated-generation model (i.e., M2) incorporates the server's dynamic pricing decision in Chapter 2.4.1. In period 0, generation-0 customers arrive and join at any positive rate. Then in period i ($i = 1, \ldots, \bar{k}$), the service provider sets price p_i and generation-i customers arrive and make the join-or-balk decision based on k_i anecdotes, where $k_i > 0$ and increases in i. Each anecdote is a service quality realisation experienced by a customer from any of the first $i+1$ period.

M2 inherits the general anecdotes acquisition process from M1. In addition, M2 endogenises the growing popularity of the service system (i.e., k_i increases in i) and the service provider's dynamic pricing decision (i.e., p_i may depend on i). Since customers' join-or-balk decision is independent of prices in earlier generations, the multi-period optimisation problem underlying M2 can be decoupled into $\bar{k}$ single-period revenue maximisation problems defined by (2.2), where $k = k_1, \ldots, k_{\bar{k}}$. As a result, studying the evolution of the optimal price, revenue, social welfare, and consumer surplus boils down to investigating the impact of k on $p^*(k)$, $\Pi^*(k)$, $W^*(k)$, and $CS^*(k)$, as given by Proposition 1-2.

Next I introduce a new repeated-generation model (i.e., M3) based on M2 to capture the server's dynamic joint pricing and quality decisions in Chapter 2.4.1.2. In period 0, generation-0 customers arrive and join at any positive rate. In period i ($i = 1,\dots,\bar{k}$), the service provider sets price p_i and quality R_i at cost aR_i^2. Then generation-i customers arrive and make the join-or-balk decision based on k_i anecdotes, where $k_i > 0$ and increases in i. Generation-i customers' anecdotes are service quality realisations experienced by customers from an earlier generation A_i, where $A_i \leqslant i$ and increases in i. Note that M3 captures a more restrictive set of anecdotes acquisition processes than M2: Customers within a generation cannot acquire anecdotes from different earlier generations. This assumption is for analytical tractability, and it has also been adopted by the literature (Huang and Chen 2015, and references therein).

Similar to M2, it can be verified that the multi-period pricing and quality optimisation problem underlying M3 can be decoupled into $\bar{k}$ single-period revenue maximisation problems defined by (2.3), where $k = k_1,\dots,k_{\bar{k}}$. As a result, studying the evolution of the optimal price and quality boils down to investigating the impact of k on $\hat{p}$ and $\hat{R}$, as given by Proposition 3.

A.2.2 The No-Congestion Setting

In this section, I characterise customers' joining rate and the server's pricing, quality, and information disclosure decisions in the no-congestion setting, i.e., $c = 0$. For expositional convenience, I focus on the underloaded case in which $\mu \geqslant \lambda$. Notably, all insights continue to hold for the overloaded case (i.e., $\mu < \lambda$).

Following the formulation of Equation (2.1), I can derive the equilibrium joining rate as

$$\lambda_a^k(p) = \lambda \bar{\Phi}\left(\frac{\sqrt{k}}{\sigma}(p-R)\right).$$

Consistent with the congestion setting (see Lemma 1), the above expression shows that a larger sample size increases the demand of a high-quality (i.e., $R > p$) service system and decreases the demand of a low-quality (i.e., $R < p$) service system. Based on this, I characterise the service provider's pricing, quality, and information disclosure decisions under revenue and welfare maximisations.

Revenue maximisation. Similar to the congestion setting, the revenue maximisation problem is given by:

$$\max_{p\geqslant 0}\ \lambda p\bar{\Phi}\left(\frac{\sqrt{k}}{\sigma}(p-R)\right).$$

The next proposition characterises the optimal price $p^*(k)$ and revenue $\Pi^*(k)$.

Proposition A-1. *(i) A unique $p^*(k)>0$ exists and $\lim\limits_{k\to+\infty} p^*(k)=p_r^*$.*

(ii) The optimal revenue $\Pi^*(k)$ strictly decreases (increases) in k for $R<(>)\frac{0.5\sigma}{\sqrt{k}\Phi'(0)}$, and is invariant in k for $R=\frac{0.5\sigma}{\sqrt{k}\Phi'(0)}$.

(iii) There exists a unique quality threshold $\bar{R}_2(k)$, which strictly decreases in k, such that $p^*(k)$ strictly decreases (increases) in k for $R<(>)\bar{R}_2(k)$, and is invariant in k for $R=\bar{R}_2(k)$.

Proof. Proof of Proposition A-1(i). The FOC of revenue is

$$\bar{\Phi}(s(p-R))=sp\Phi'(s(p-R)),$$

where I denote $s\equiv\frac{\sqrt{k}}{\sigma}$. Therefore,

$$p^*=\frac{\bar{\Phi}(s(p^*-R))}{s\Phi'(s(p^*-R))}.$$

The existence and uniqueness of $p^*(k)$ can be established following the proof of Proposition 1(i).

Next, I will prove $\lim\limits_{s\to+\infty} p^*(k)=p_r^*=R$ by contradiction. First, suppose $\lim\limits_{k\to+\infty} p^*(k)<R$.

$$p^*=\frac{\bar{\Phi}(s(p^*-R))}{s\Phi'(s(p^*-R))}=\frac{1}{s\Phi'(s(p^*-R))}.$$

Since $\lim\limits_{s\to+\infty}\Phi'(s(p^*-R))=0$, $\lim\limits_{k\to+\infty} p^*(k)=+\infty>R$.

Second, suppose $\lim\limits_{k\to+\infty} p^*(k)>R$.

$$p^*=\frac{(p^*-R)\bar{\Phi}(s(p^*-R))}{s(p^*-R)\Phi'(s(p^*-R))}.$$

Since

$$\lim_{s\to+\infty}\frac{\bar{\Phi}(s(p^*-R))}{s(p^*-R)\Phi'(s(p^*-R))}=0,$$

$\lim\limits_{k\to+\infty} p^*(k)=0<R$.

Proof of Proposition A-1(ii). By the Envelope Theorem,

$$\frac{\partial\Pi^*}{\partial s}=-p(p-R)\Phi'(s(p-R))\big|_{p=p^*}.$$

Therefore, $\frac{\partial \Pi^*}{\partial s} > 0$ iff $p^* < R$, which holds iff

$$\frac{\bar{\Phi}(s(x-R))}{s\Phi'(s(x-R))} - x\bigg|_{x=R} = \frac{0.5}{s\Phi'(0)} - R < 0.$$

Proof of Proposition A-1(iii). The proof is identical to the proof of Proposition 1(iii), except that here $\lambda_a' = -\lambda s\Phi'$, $y' = s$, $\lambda_a'' = \lambda s\Phi' yy' = \lambda s^3\Phi'(p-R)$ always hold. □

Proposition A-1 shows that without congestion, targeting the mass customers instead of exclusively the niche customers dominates for a larger range of parameter values. This is an intuitive result since the advantage of the former strategy is to increase demand and it is strengthened when there is no congestion: In this case, a higher demand no longer poses the negative externality of congestion on other joining customers.

Next I study the welfare implications of the service provider's optimal pricing strategy. As in Chapter 2.4.1.1, I derive the expressions of social welfare and consumer surplus as follows:

$$W(p,k) = R\lambda_a^k(p),$$
$$CS(p,k) = (R-p)\lambda_a^k(p).$$

I denote the social welfare and consumer surplus under the optimal price $p^*(k)$ as $W^*(k)$ and $CS^*(k)$ respectively. By definition, I have $W^*(k) = W(p^*(k),k)$ and $CS^*(k) = CS(p^*(k),k)$. The next proposition characterises the impact of k on $W^*(k)$ and $CS^*(k)$.

Proposition A-2. *Social welfare $W^*(k)$ and consumer surplus $CS^*(k)$ strictly increase in k for $R \neq 0$ and are invariant of k for $R = 0$.*

Proof. The comparative statics of $W^*(k)$ is a special case of Proposition 2(i) with $c = 0$. Therefore, I will only prove the comparative statics of $CS^*(k)$. According to Lemma 2 and the proof of Proposition 1(ii), I have:

1. If $R < 0$, $p^*(k) > R$ and $\lambda_a^k(p^*(k))$ strictly decreases in k.
2. If $R = 0$, $p^*(k) = R$ and $\lambda_a^k(p^*(k))$ is invariant of k.
3. If $R > 0$, $p^*(k) < R$ and $\lambda_a^k(p^*(k))$ strictly increases in k.

The proposition follows by noting that $CS^*(k) = [R - p^*(k)]\lambda_a^k(p^*(k))$. □

Proposition A-2 shows that a large sample size never harms social welfare or consumer surplus. This is in sharp contrast to the congestion setting, in which a large sample size decreases consumer surplus when R is large enough (see Proposition 2(ii)). This contrast highlights the role of congestion in the welfare impact of customer bounded rationality. Specifically, as customers become more rational, they join a high-quality service system more. Therefore, consumer surplus increases in the absence of congestion. The opposite is true with congestion since the increase in demand also implies longer waits for all joining customers.

Now I investigate the service provider's decision of disclosing quality information or not. The next corollary, which follows immediately from Proposition A-1, fully characterises this information disclosure decision in the no-congestion setting.

Corollary A-1. *(i) When $R \leqslant \underline{R}$, the service provider does not disclose information.*

(ii)When $\underline{R} < R < \tilde{R}$, the service provider does not disclose information for $k < \tilde{k}$, discloses information for $k > \tilde{k}$, and is indifferent between the two for $k = \tilde{k}$.

(iii) When $R \geqslant \tilde{R}$, the service provider discloses information.

Corollary A-1 shows that the server's information disclosure decision in the congestion setting continues to hold in the no-congestion setting. Moreover, incorporating congestion leads the server not to disclose information for a larger range of parameter values (see Corollary 1). This highlights the importance of incorporating customer bounded rationality in the congested setting.

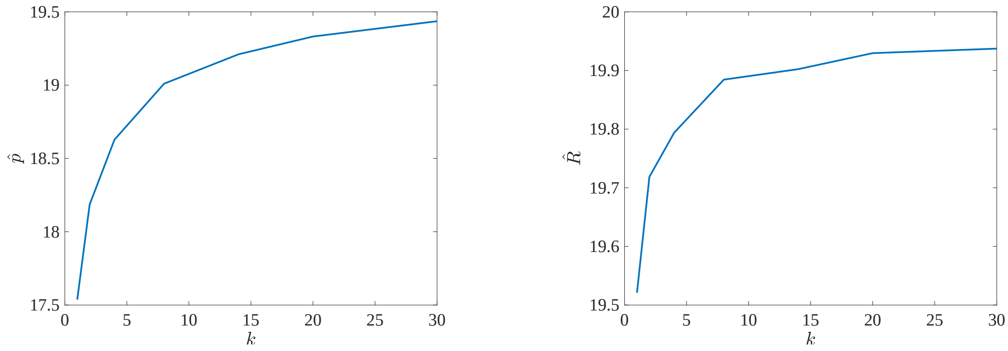
Next I examine the server's quality control problem as defined in (2.3).

Proposition A-3. *A unique $\hat{p} > 0$ determined by $2a\hat{R} = \lambda_a(\hat{p}, \hat{R})$ exists. Moreover, when market potential λ is sufficiently low, the optimal quality $\hat{R}$ strictly decreases in k and the optimal revenue strictly decreases in k. If, in addition, the standard deviation of service quality σ is sufficiently high, the optimal price $\hat{p}$ strictly decreases in k.*

I omit the proof because it follows immediately from the proof of Proposition 3. Similar to the congestion setting, a larger sample size leads the server to lower price and quality under low market potential. Under high market potential, however, the congestion and no-congestion settings may lead to opposite pricing and quality recommendations. In the congestion setting, the server sets a low quality to target exclusively the niche customers. With more anecdotes, customers join less and the server reduces both price and quality. In the no-congestion setting, however, I numerically find that the server increases both price and quality, as illustrated in Figure A-1.

Welfare maximisation. Now I characterise a social planner's pricing, quality, and

Figure A-1: The Impact of k on $\hat{p}$ and $\hat{R}$ with No Congestion and High Market Potential ($\mu = 2$, $\lambda = 100, a = 1, \sigma = 1$)



(a) $\hat{p}$ (b) $\hat{R}$

information disclosure decisions, as given by:

$$W = R\lambda\bar{\Phi}\left(\frac{\sqrt{k}}{\sigma}(p-R)\right).$$

Since W strictly decreases in p, the optimal price p_w^* and social welfare W_w^* are given by:

$$p_w^* = 0,$$
$$W_w^* = R\lambda\bar{\Phi}\left(-\frac{\sqrt{k}}{\sigma}R\right).$$

Intuitively, the social planner prices at zero because joining always improves social welfare in the absence of congestion. Since bounded rationality leads a portion of customers to considerably underestimate service quality and thus choose to balk, it always reduces social welfare. Therefore, the social planner should disclose quality information for all positive quality levels (i.e., $R > 0$).

Next, I consider the quality control problem, as given by:

$$\max_{R \geqslant 0} \; R\lambda\bar{\Phi}\left(-\frac{\sqrt{k}}{\sigma}R\right) - aR^2.$$

The following proposition characterises the optimal quality $\hat{R}_w^*$ and how it depends on the sample size k.

Proposition A-4. *A unique $\hat{R}_w$ exists and strictly increases in k.*

Proof.

$$W' = \lambda[\bar{\Phi}(-sR) + sR\Phi'(-sR) - 2aR/\lambda].$$

Define $Z(x) \equiv x + x\Phi'(-x) - \bar{\Phi}(-x)$. It can be shown that $Z(x)$ strictly increases in x and $Z(x) = 0$ has a unique solution. Therefore, $\hat{R}_w$ exists and is unique. I will show $\frac{\partial \hat{R}_w}{\partial s} > 0$ using the supermodularity argument.

$$\begin{aligned}\frac{\partial^2 W}{\partial s \partial R} &= \lambda[2R\Phi'(-sR) - sR^2\Phi''(-sR)] \\ &= \lambda R[2 + s^2R^2]\Phi(-sR) > 0 \quad \square\end{aligned}$$

Proposition A-4 shows that as customers share more quality information over time, the social planner always improve service quality. Intuitively, as sample size increases, more customers join the free service system. This strengthens the social benefit of service and thus induces the social planner to improve quality.

A.2.3 Information Disclosure with Quality Control

To complement Chapter 2.4.2, I incorporate the service provider's quality decision and characterise his information disclosure decision in this setting, as shown below.

Corollary A-2. *When market potential λ is sufficiently high or sufficiently low, the service provider does not disclose quality information.*

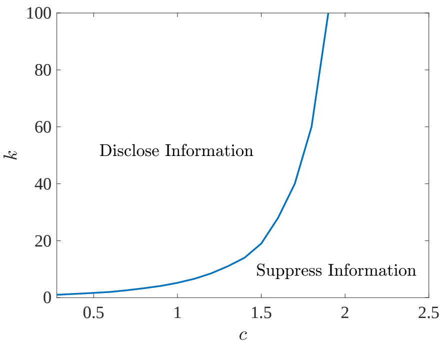
Corollary A-2 follows immediately from Proposition 3. Intuitively, the service provider chooses not to disclose information because he targets exclusively the niche customers. Targeting the mass customers leads to a lower revenue since: (i) It cannot increase demand considerably when market potential is sufficiently low. (ii) It significantly intensifies congestion and thus lowers demand when market potential is sufficiently high.

I complement Corollary A-2 by a numerical study to examine intermediate market potential case, as illustrated in Figure A-2. In line with Proposition 4 and Corollary 1, the server switches from not disclosing to disclosing quality information at a larger sample size or a lower unit waiting cost. Interestingly, I also find that a higher a makes information non-disclosure more attractive. This is because as quality improvement becomes more costly, the server sets a lower quality and targets exclusively the niche customers.

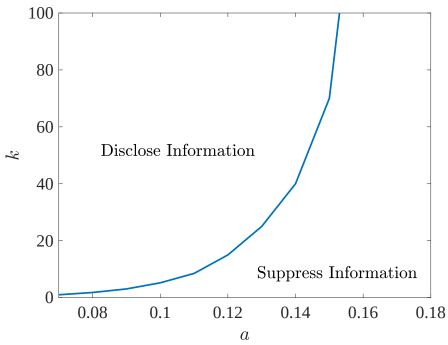
A.2.4 Ratings/Reviews from Third-Party Review Websites

Throughout the paper, I focus on word of mouth from acquaintances as customers' only source of service quality information. In practice, however, they may acquire information

Figure A-2: The Impact of c and a on the Information Disclosure Decision under Intermediate Market Potential with Quality Control ($\mu = 2, \lambda = 2, \sigma = 10$)



(a) c ($a = 0.1$)



(b) a ($c = 1$)

also from ratings/reviews on third-party review websites. I will refer to this type of anecdotes as public anecdotes since they are publicly available to all customers. Moreover, I will refer to word of mouth from acquaintances as private anecdotes because they are private information to each customer. The terminology highlights the key difference between the two types of anecdotes: Public anecdotes can target a much larger audience and thus are more correlated among customers than private anecdotes. This observation implies that the service provider may adopt different information disclosure strategies in public and private channels. As a result, instead of choosing between informing customers of the mean service quality or not, the service provider now has four information disclosure strategies, i.e., informing customers in the private/public channel only, in both channels, or in neither one. In practice, restaurants usually give away free desserts/drinks to customers who upload food photos on social media, and/or remind them to write Yelp ratings/reviews by posting a "Review Us on Yelp" sign on the door front (Marrs 2013). In the hotel industry, Marriott rewards 25 membership points to guests who share content about Marriott hotels on their own social media, and 250 points to guests who "like" a Marriott hotel's Facebook page or follow a Marriott property on Twitter (Nayer 2014).

In what follows, I will incorporate public anecdotes in customers' service quality estimation and examine its impact on the server's pricing, quality control, and information disclosure decisions. I will first characterise customers' equilibrium joining rate and show that the server's decisions in the base model are qualitatively preserved. Then I will focus on the server's joint information disclosure decision on both public and private anecdotes.

Model and preliminaries. To introduce public anecdotes in customers' join-or-balk decision, I assume that they estimate service quality as the weighted average of k_1 public anecdotes and k_2 private anecdotes. Let θ denote the mental weight on public anecdotes and $\bar{\theta} \equiv 1-\theta$. Then a customer's service quality estimate $\mathscr{R} = \theta r + \bar{\theta}\left(\sum_{i=1}^{k_2} \mathscr{R}_i\right)/k_2$, where r is the average of k_1 public anecdotes (e.g., the overall star rating of a restaurant on Yelp), and $\mathscr{R}_i$ ($i = 1,\ldots,k_2$) are private anecdotes. To capture the aforementioned difference between private and public anecdotes, I assume that r is the same for all customers and $\mathscr{R}_i \sim N(R,\sigma^2)$ is independent across customers for all i.

Although public anecdotes are the same for all customers, they may still exhibit different joining behaviours because private anecdotes are different across them. As in Chapter 2.3.2, customers adopt the pure threshold strategy of joining the queue iff the service quality estimate exceeds price and the expected waiting cost. Therefore, the joining rate for any given r, which I denote by $\lambda_p(r)$, is given by:

$$\begin{aligned}\lambda_p(r) &= \lambda P\left(\mathscr{R} > p + \frac{c}{\mu - \lambda_p(r)}\right)\\ &= \lambda P\left(\theta r + \bar{\theta}\left(\sum_{i=1}^{k_2} \mathscr{R}_i\right)/k_2 > p + \frac{c}{\mu - \lambda_p(r)}\right).\end{aligned}$$

Since each private anecdote is an independent draw from the service quality distribution, I have $\left(\sum_{i=1}^{k_2} \mathscr{R}_i\right)/k_2 \sim N(R,\sigma^2/k_2)$. Therefore, I can simplify the expression of $\lambda_p(r)$ into

$$\lambda_p(r) = \lambda\bar{\Phi}\left(\frac{\sqrt{k_2}}{\bar{\theta}\sigma}\left[p + \frac{c}{\mu - \lambda_p(r)} - \theta r - \bar{\theta}R\right]\right).$$

Note that this equation can be derived by replacing σ and R in Equation (2.1) by $\bar{\theta}\sigma$ and $\theta r + \bar{\theta}R$ respectively. Intuitively, relying on both private and public anecdotes reduces the dispersion of customers' service quality estimates by θ (recall that public anecdotes are the same for all customers) and changes the average service quality estimate from R to $\theta r + \bar{\theta}R$. This observation suggests that a unique positive joining rate exists (see Lemma 1) and its impact by k_2 is qualitatively the same as the impact of k in the base model. Specifically, a larger sample size decreases the joining rate of a low-quality service system and increases the joining rate of a high-quality service system.

Next I study the server's pricing decision. There are two cases to consider depending on whether he makes the decision contingent on public anecdotes realisations. If he does, the server maximises revenue $p\lambda_p(r)$ for a given r; otherwise, he maximises the *expected* revenue $p\lambda_a(p,k_1,k_2)$, where $\lambda_a(p,k_1,k_2) \equiv \int_{-\infty}^{+\infty} \lambda_p(r) f(r)\, dr$ is the expected joining rate and $f(x)$ denotes the probability density function of r. I will focus on the latter case because: (i) All results in the main body of the paper continue to hold for the former case because $\lambda_p(r)$ can be derived directly from Equation (2.1). (ii) The latter case reflects the common practice in many service systems. For example, restaurants usually cannot change price frequently because it requires redesigning and reproducing the menu, brochures, and websites, all of which are costly and take time.

To investigate the server's pricing decision in the latter case, I first characterise $\lambda_a(p,k_1,k_2)$ and then examine how it depends on k_1 and k_2. Since public anecdotes are independent draws (i.e., independent from other public anecdotes but are the same across customers) from the service quality distribution, I have $r \sim N(R, \sigma^2/k_1)$. Therefore,

$$\lambda_a(p,k_1,k_2) = \frac{\sqrt{k_1}\lambda}{\sigma} \int_{-\infty}^{+\infty} \lambda_p(s) \phi\left(\frac{\sqrt{k_1}(s-R)}{\sigma}\right) ds. \tag{A-8}$$

The next lemma shows that $\lambda_a(p,k_1,k_2)$ uniquely exists.

Lemma A-1. *For any $p \geqslant 0$ and $k_1, k_2 \geqslant 1$, a unique $\lambda_a(p,k_1,k_2) \in (0, \min\{\lambda, \mu\})$ exists and strictly decreases in p. Moreover, $\lim_{k_1,k_2 \to +\infty} \lambda_a(p,k_1,k_2) = \lambda_r(p)$.*

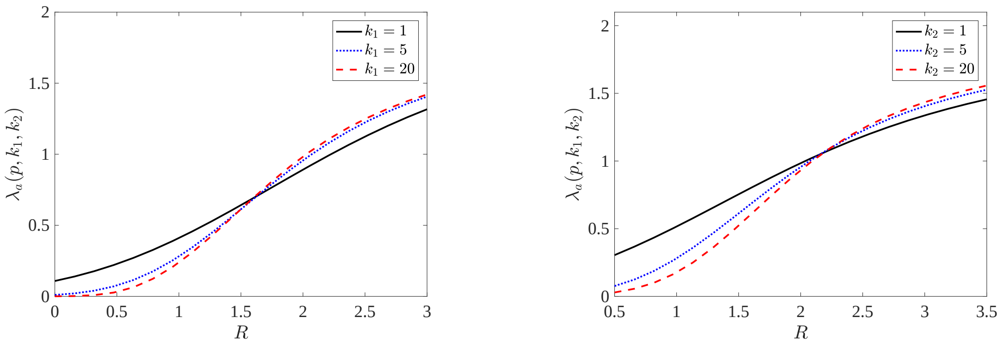
The proof of Lemma A-1 is similar to that of Lemma 1 and thus is omitted.

Next I study the impact of sample sizes (i.e., k_1 and k_2) on the expected joining rate. Since an analytical characterisation is prohibitively difficult, I examine it numerically, as illustrated in Figure A-3. As in the base model, a smaller sample size in either channel leads customers to join a low-quality service system more and a high-quality service system less. However, the intuitions for the two channels are strikingly different. Specifically, fewer private anecdotes lead customers' service quality estimates to be more dispersed, whereas fewer public anecdotes do not because they are the same for all customers. Instead, fewer public anecdotes increase the likelihood for the sample average in the public channel to be considerably inaccurate, thereby shifting all customers' service quality estimates (either up or down) by the same magnitude. As a result, the ex ante (before public anecdotes are realised) expected size of the niche customers increases and the ex ante expected size of the mass customers decreases. This induces a higher joining rate in a low-quality service system and a lower joining rate in a high-quality service system.

Now I will examine the server's pricing decision, as given by:

$$\max_{p \geqslant 0}\ p\lambda_a(p,k_1,k_2).$$

Figure A-3: The Impact of k_1 and k_2 on $\lambda_a(p,k_1,k_2)$ $(\mu=2,\lambda=2,c=1,p=1,\sigma=2,\theta=0.5)$



(a) k_1 $(k_2=5)$ (b) k_2 $(k_1=5)$

Denote the optimal price by $p^*(k_1,k_2)$ and the optimal revenue by $\Pi^*(k_1,k_2)$. The next lemma shows that this revenue maximisation problem is well-defined.

Lemma A-2. *For any $k_1,k_2 \geqslant 1$, the optimal price $p^*(k_1,k_2)>0$ exists and is unique.*

Proof. Denote $\mathbf{E}[f(x)] \equiv \frac{\sqrt{k_1}\lambda}{\sigma}\int_{-\infty}^{+\infty} f(x)\phi\left(\frac{\sqrt{k_1}(x-R)}{\sigma}\right)dx$ as the expectation of function $f(x)$ with respect to x with $x\sim N(R,\sigma^2/k_1)$. I will abuse the notation by writing $p^*(k_1,k_2)$ as p^*. The FOC of $\Pi(p,k_1,k_2)$ with respect to p is

$$\mathbf{E}[\lambda_{p^*}(r)]+p^*\mathbf{E}\left[\left.\frac{\partial\lambda_p(s)}{\partial p}\right|_{p=p^*}\right]=0.$$

By the definition of λ_p^r,

$$\frac{\partial\lambda_p(r)}{\partial p}=-\frac{\lambda\frac{\sqrt{k_2}}{\bar{\theta}\sigma}\Phi'\left(\sqrt{k_2}\frac{p+\frac{c}{\mu-\lambda_p(r)}-\theta x-\bar{\theta}R}{\bar{\theta}\sigma}\right)}{1+\lambda\frac{\sqrt{k_2}}{\bar{\theta}\sigma}\Phi'\left(\sqrt{k_2}\frac{p+\frac{c}{\mu-\lambda_p(r)}-\theta x-\bar{\theta}R}{\bar{\theta}\sigma}\right)\frac{c}{[\mu-\lambda_p(r)]^2}}.$$

Denoting $\Phi'\left(\sqrt{k_2}\frac{p+\frac{c}{\mu-\lambda_p(r)}-\theta x-\bar{\theta}R}{\bar{\theta}\sigma}\right)$ and $\Phi\left(\sqrt{k_2}\frac{p+\frac{c}{\mu-\lambda_p(r)}-\theta x-\bar{\theta}R}{\bar{\theta}\sigma}\right)$ as Φ' and Φ, $\Phi'\left(\sqrt{k_2}\frac{p^*+\frac{c}{\mu-\lambda_{p^*}(r)}-\theta x-\bar{\theta}R}{\bar{\theta}\sigma}\right)$ and $\Phi\left(\sqrt{k_2}\frac{p^*+\frac{c}{\mu-\lambda_{p^*}(r)}-\theta x-\bar{\theta}R}{\bar{\theta}\sigma}\right)$ as Φ'^* and Φ^*, and substituting the above equation into the FOC, I have:

$$p^*=\mathbf{E}\left[\left\{\frac{\bar{\theta}\sigma}{\Phi'^*\lambda\sqrt{k_2}}+\frac{c}{[\mu-\lambda_{p^*}(r)]^2}\right\}\lambda_{p^*}(r)\right].$$

Similar to the proof of Proposition 1, I can show that for any given r,

$$\left\{\frac{\bar{\theta}\sigma}{\Phi'\lambda\sqrt{k_2}}+\frac{c}{[\mu-\lambda_p(r)]^2}\right\}\lambda_p(r)-p$$

strictly decreases in p and is positive (negative) when $p \to 0(+\infty)$. Therefore,

$$p < (>)\mathbf{E}\left[\left\{\frac{\bar{\theta}\sigma}{\Phi'\lambda\sqrt{k_2}} + \frac{c}{[\mu - \lambda_p(r)]^2}\right\}\lambda_p(r)\right]$$

for $p < (>)p^*$, and thus the optimal price p^* exists and is unique. $\square$

As in the base model, Lemma A-2 implies that the server's pricing, quality, and information disclosure decisions are well-defined. Next I will derive insights regarding the impact of k_1 and k_2 on these decisions. To this end, first recall from Figure A-3 that k_1 and k_2 influence customers' joining behaviour in the same way as k in the base model. Based on this, I have numerically verified that the impact of the sample size on the server's decisions are also qualitatively preserved. Specifically, for a given k_1 (k_2), the impact of k_2 (k_1) on the server's optimal price, quality, and information disclosure decision in the private (public) channel are qualitatively the same as the impact of k on the corresponding decisions in the base model. For brevity I have omitted the presentation of the numerical study.

The joint information disclosure decision. The preceding discussion shows that for a fixed sample size in the private/public channel, the server's information disclosure decision in the other channel is qualitatively the same as the base model. In this section, I will examine his *joint* information disclosure decision, i.e., informing customers of the mean service quality in one channel only, in both channels, or in neither one. Notably, I focus on the $k_1 \gg k_2$ case because it reflects general observations where the size of online ratings/reviews usually far exceeds the size of word of mouth from acquaintances.

Due to analytical difficulties, I examine the joint information disclosure through an extensive numerical study, as illustrated in Figure A-4. Intuitively, a high-quality (i.e., $R > \tilde{R}_2$) service provider informs customers of the mean service quality in both channels, while a low-quality (i.e., $R < \tilde{R}_1$) service provider does not inform in either channel. Somewhat surprisingly, an intermediate-quality (i.e., $\tilde{R}_1 < R < \tilde{R}_2$) service provider informs customers in the public channel only. According to the interpretation of Proposition 4, choosing not to inform them in the private channel implies that the server sets a high price to target exclusively the niche customers. Since information disclosure in the public channel lowers their size, I would expect the service provider not to do so. However, my numerical study points to just the opposite. The key insight is that information disclosure in the public channel improves the efficiency of the pricing strategy (the pricing efficiency advantage, hereafter): More public anecdotes lead customers' average service quality estimate to deviate less from the actual mean service quality, so the server is more likely to target the most profitable customer segment by the price which he sets without observing the anecdotes realisations. Managerially, this result suggests that compared to customers' social media posts, their online ratings/reviews are the more effective medium in disclosing service quality information: Even if the service quality is low such that the service provider prices high to target exclusively the niche customers, he may still inform them of the mean service quality in the public channel due to the pricing efficiency advantage.

Figure A-4: The Impact of R and θ on the Joint Information Disclosure Decision ($k_1 = 5, k_2 = 1, \mu = 2, \lambda = 2, c = 1, \sigma = 3$)

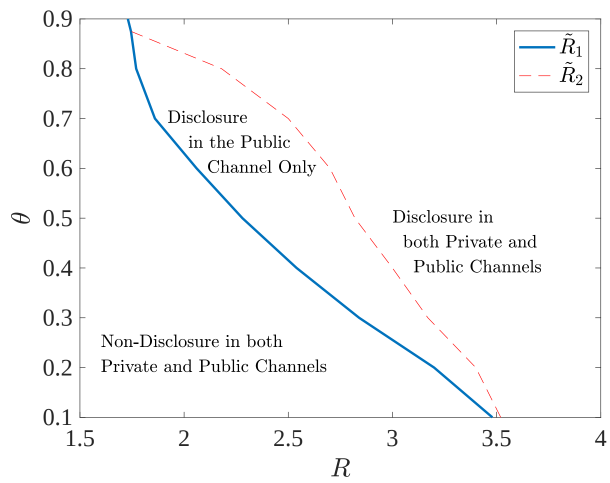


Figure A-4 also illustrates the impact of θ on the service provider's information disclosure decision. Intuitively, as customers rely more on public anecdotes (i.e., θ increases), their quality estimates become less dispersed. Therefore, the size of the niche (mass) customers decreases (increases) and the server switches from targeting exclusively the niche customers to targeting the mass customers. This implies that the server discloses information in both channels for sufficiently high θ and discloses no information in both channels for sufficiently low θ.

Interestingly, when θ is intermediate, the server discloses information in the public channel only. To fully understand this result, note that this strategy is less efficient in increasing demand compared with the others: A high (low)-quality service provider targets the mass (niche) customers and can increase demand more effectively by disclosing (not disclosing) information in both channels. When customers rely predominantly on one channel, this disadvantage is amplified and information disclosure only in the public channel becomes less attractive. In contrast, under intermediate service quality, the pricing efficiency advantage dominates. Therefore, information disclosure exclusively in the public channel becomes more attractive.

This result leads to the following managerial implications. If customers come from centralised communities (e.g., residents in close neighborhood) and thus estimate service quality mainly by word of mouth from acquaintances, the service provider should encourage neither public nor private platform posts. However, if they come from dispersed locations (e.g., travelers) and thus estimate service quality mainly by online ratings/reviews, the service provider should encourage both public and private platforms posts. Moreover, if customers are loosely connected with each other (e.g., frequent visitors of a shopping mall) and thus attach similar mental weights to online ratings/reviews and word of mouth from acquaintances, the service provider should encourage only public platform posts.

A.3 Robustness Check

In this section, I will relax several assumptions in Chapter 2.3 and show that major insights of the paper continue to hold.

A.3.1 Uncertain Service Quality with a Logconcave Density Function

In the base model, I assumed that each anecdote is an independent and identical draw from a normal distribution. In this section, I will generalise the model by considering all continuous distributions with a symmetric and logconcave density function, where logconcavity is defined as follows.

Definition A-1 (Boyd and Vandenberghe 2004)**.** *A function $f(x):\mathbb{R}\rightarrow\mathbb{R}$ is logconcave if $f(x)>0$ for all $x\in dom\ f$ and $log(f(x))$ is concave, i.e.,*

$$f(\theta x_1+\bar{\theta}x_2)\geqslant f(x_1)^{\theta}f(x_2)^{\bar{\theta}}$$

for any $x_1,x_2\in dom\ f$, $\theta\in[0,1]$, and $\bar{\theta}\equiv 1-\theta$.

Note that this generalisation not only extends the set of service quality distributions, it also allows me to consider anecdotes that are *biased* draws from the service quality distribution. For example, suppose that the service quality is normally distributed, i.e., $\mathscr{R}\sim N(R,\sigma^2)$. Moreover, each anecdote $\mathscr{A}_i$ ($i=1,\ldots,k$) is an independent logistic perturbation of service quality, i.e., $\mathscr{A}_i=\mathscr{L}_i\mathscr{R}$, where $\mathscr{L}_i$ ($i=1,\ldots,k$) are independent and identically draws from a logistic distribution with mean R. My generalised model can capture this situation because the distribution of each anecdote, i.e., a logistic perturbation of normal distribution, is continuous and has a symmetric and log-concave density function.

Below I summarise several important properties of logconcavity, which I will invoke later.

Lemma A-3 (Boyd and Vandenberghe 2004). *(i) Logconcavity is closed under convolution.*

(ii) Random variables with logconcave density functions have an increasing failure rate.

Next I characterise customers' equilibrium joining rate $\lambda_L^k(p)$. Let $B_k(x)$, $\bar{B}_k(x)$, and $b_k(x)$ denote the cumulative, inverse cumulative, and density functions of customers' service quality estimate $\mathscr{R}_k = \left(\sum_{i=1}^{k} \mathscr{R}_i\right)/k$. According to the analysis in Chapter 2.3.2, $\lambda_L^k(p)$ is given by:

$$\lambda_L^k(p) = \lambda \bar{B}_k\left(p + \frac{c}{[\mu - \lambda_L^k(p)]^+}\right).$$

The next lemma, which can be proven by mimicking the proof of Lemma 1, shows that $\lambda_L^k(p)$ is well-defined.

Lemma A-4. *For any $p \geqslant 0$, a unique $\lambda_L^k(p) \in [0, \min\{\lambda, \mu\}]$ exists and strictly decreases in p. Moreover, $\lim_{k \to +\infty} \lambda_L^k(p) = \lambda_r(p)$.*

Note that when the service quality distribution has a finite support, customers may all join or all balk: Since each anecdote cannot be higher (lower) than the upper (lower) bound of service quality, all customers will balk (join) the queue if price is sufficiently high (low).

As in Lemma 1, I will examine the impact of k on $\lambda_L^k(p)$. To this end, I apply the stochastic comparison technique by comparing $\mathscr{R}_k$ and $\mathscr{R}_{k-1}$ in the peakness order, which is defined as follows.

Definition A-2 (Shaked and Shanthikumar 2007). *Let $\mathscr{X}$ and $\mathscr{Y}$ be two random variables with the same mean and different distribution functions. Suppose that the distribution functions F and G, of $\mathscr{X}$ and $\mathscr{Y}$ respectively, are symmetric about the common mean. Then $\mathscr{X}$ is smaller than $\mathscr{Y}$ in the peakness order (denoted by $\mathscr{X} \leqslant_{peak} \mathscr{Y}$) iff G crosses F only once from above.*

The lemma below presents an important property of the peakness order.

Lemma A-5 (Shaked and Shanthikumar 2007). *If $\mathscr{X}_1, \mathscr{X}_2, \ldots$ are independently and identically distributed random variables, having a common logconcave density function that is* *symmetric about a common value, then for each $n \geqslant 2$, one has $\bar{\mathscr{X}_n} \leqslant_{peak} \bar{\mathscr{X}}_{n-1}$, where $\bar{\mathscr{X}_i} \equiv (\mathscr{X}_1 + \cdots + \mathscr{X}_i)/i$.*

Based on Lemma A-5, the next proposition characterises the impact of sample size k on customers' equilibrium joining behaviour.

Proposition A-5. *The equilibrium joining rate $\lambda_L^k(p)$ strictly decreases in k if $R < p + \frac{c}{(\mu-0.5\lambda)^+}$, strictly increases in k if $R > p + \frac{c}{(\mu-0.5\lambda)^+}$, and is invariant in k if $R = p + \frac{c}{(\mu-0.5\lambda)^+}$.*

Proof. To prove this proposition, it suffices to show that for any integer $k > 1$, $\lambda_L^k(p) < \lambda_L^{k-1}(p)$ when $R < p + \frac{c}{(\mu-0.5\lambda)^+}$, $\lambda_L^k(p) > \lambda_L^{k-1}(p)$ when $R > p + \frac{c}{(\mu-0.5\lambda)^+}$, and $\lambda_L^k(p) = \lambda_L^{k-1}(p)$ when $R = p + \frac{c}{(\mu-0.5\lambda)^+}$. I will prove the $R < p + \frac{c}{(\mu-0.5\lambda)^+}$ case, and the other two cases can be proven in the same way.

Since $\mathscr{R}_1, \mathscr{R}_2, \ldots$ are i.i.d. random variables with a common logconcave density function that is symmetric about $x = R$, $\bar{\mathscr{R}}_k \leqslant_{peak} \bar{\mathscr{R}}_{k-1}$ for $k \geqslant 2$ by Lemma A-5. This suggests that $B_k(x)$ crosses $B_{k-1}(x)$ only once from below at $x = R$ (see Definition A-2). By mimicking the proof of Lemma 1, I can show that both $\lambda_L^k(p)$ and $\lambda_L^{k-1}(p)$ are below 0.5λ when $R < p + \frac{c}{(\mu-0.5\lambda)^+}$. I will show $\lambda_L^k(p) < \lambda_L^{k-1}(p)$ by proof of contradiction. Suppose $\lambda_L^k(p) \geqslant \lambda_L^{k-1}(p)$. Then by definition

$$\begin{aligned}\lambda_L^k(p) = \lambda \bar{B}_k\left(p + \frac{c}{\mu - \lambda_L^k(p)}\right) &< \lambda \bar{B}_{k-1}\left(p + \frac{c}{\mu - \lambda_L^k(p)}\right)\\ &\leqslant \lambda \bar{B}_{k-1}\left(p + \frac{c}{\mu - \lambda_L^{k-1}(p)}\right) = \lambda_L^{k-1}(p),\end{aligned}$$

which contradicts the assumption that $\lambda_L^k(p) \geqslant \lambda_L^{k-1}(p)$. $\square$

Proposition A-5 shows that the impact of customer bounded rationality on their joining behaviour, as characterised in Lemma 1, continues to hold in the generalised setting. Based on this, I have verified that the impact of customer bounded rationality on the server's pricing, quality control, and information disclosure decisions are also qualitatively preserved, as presented for the rest of this section.

Pricing and quality control. As in the base model, I will examine how the sample size influences the service provider's pricing and quality decisions, as given respectively by:

$$\max_{p \geqslant 0} \; p\lambda_L^k(p),$$

$$\max_{p,R \geqslant 0} \; p\lambda_L^k(p,R) - aR^2.$$

The next proposition characterises the server's revenue maximisation problem without quality control, where $p_L^*(k)$ and $\Pi_L^*(k)$ denote the optimal price and revenue.

Proposition A-6. *(i) For any $k \geqslant 0$, a unique $p_L^*(k) > 0$ exists and $\lim_{k\to+\infty} p_L^*(k) = p_r^*$.*

(ii) The optimal revenue $\Pi_L^*(k)$ strictly decreases in k for $R < R_{1L}(k)$ and increases in k for $R > R_{1L}(k)$, where $R_{1L}(k) \equiv \frac{c\mu}{[(\mu-0.5\lambda)^+]^2} + \frac{1}{2b_k(R)}$ is the mean service quality at which $\lambda_L^k(p_L^*(k)) = 0.5\lambda$. $R_{1L}(k)$ strictly decreases in k.

Proof. I will abuse the notation by writing $p_L^*(k)$ as p_L^* and $\Pi_L^*(k)$ as Π_L^*. Moreover, I extend the definition of λ_L^k to incorporate non-integer rationality levels. In particular, for any non-negative real number k, define

$$\tilde{b}_k(x) \equiv [b_{\lfloor k\rfloor}(x)]^{(k-\lfloor k\rfloor)}[b_{\lceil k\rceil}(x)]^{(\lceil k\rceil-k)}$$

as the probability density function of $\mathscr{R}_k$, where $\lfloor x\rfloor$ and $\lceil x\rceil$ denote the floor and ceiling functions respectively. Let $\tilde{B}_k(x)$ denote the corresponding probability cumulative function and $\bar{\tilde{B}}_k(x)$ the inverse cumulative function. Therefore, $\lambda_L^k(p)$ is given by:

$$\lambda_L^k(p) = \lambda\bar{\tilde{B}}_k\left(p + \frac{c}{\mu-\lambda_L^k(p)}\right).$$

I will adopt this extension throughout this appendix.

Proof of Proposition A-6(i). The FOC of $p\lambda_L^k(p)$ with respect to p is

$$\lambda_L^k(p_L^*) + p^*\frac{\partial\lambda_L^k(p_L^*)}{\partial p} = 0.$$

Differentiating both sides of the defining equation of $\lambda_L^k(p)$ with respect to p, I have

$$\frac{\partial\lambda_L^k(p)}{\partial p} = -\frac{\lambda\tilde{b}_k\left(p+\frac{c}{\mu-\lambda_L^k(p)}\right)}{1+\tilde{b}_k\left(p+\frac{c}{\mu-\lambda_L^k(p)}\right)\frac{c\lambda}{\left[\mu-\lambda_L^k(p)\right]^2}}.$$

Therefore,

$$p_L^* = \left\{\frac{1}{\lambda\tilde{b}_k\left(p+\frac{c}{\mu-\lambda_L^k(p_L^*)}\right)} + \frac{c}{[\mu-\lambda_L^k(p_L^*)]^2}\right\}\lambda_L^k(p_L^*).$$

Denote

$$H_L(x) \equiv \left\{\frac{1}{\tilde{b}_k\left(x+\frac{c}{\mu-\lambda_L^k(x)}\right)} + \frac{c\lambda}{[\mu-\lambda_L^k(x)]^2}\right\}\bar{\tilde{B}}_k\left(x+\frac{c}{\mu-\lambda_L^k(x)}\right) - x.$$

Therefore, $H_L(p_L^*) = 0$. To show that p_L^* uniquely exists, it suffices to show that $H_L(0) > 0$, $H_L(+\infty) = -\infty$, $H_L(x)$ is continuous, and $H_L'(x) < 0$. The first three conditions can be verified easily, and I will focus on the last one. Similar to the proof of Proposition 1(i), to complete the proof, it suffices to show that $\frac{\bar{\tilde{B}}_k(x)}{\tilde{b}_k(x)}$ strictly decreases in x, i.e., $\tilde{b}_k(x)$ has an increasing failure rate. According to Lemma A-3, $b_{\lfloor k \rfloor}(x)$ and $b_{\lceil k \rceil}(x)$ are logconcave and thus have an increasing failure rate. Therefore, $\tilde{b}_k(x)$ is also logconcave and has an increasing failure rate. $\lim_{k\to+\infty} p^*(k) = p_r^*$ follows from Lemma 3.

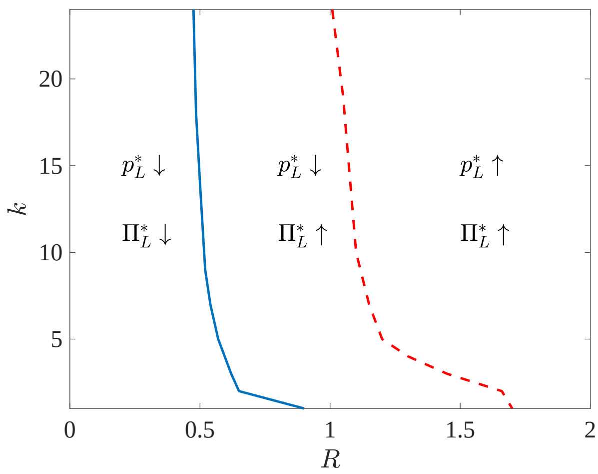
Proof of Proposition A-6(ii). According to the envelop theorem,

$$\Pi_L^{*'}(k) = p\frac{\partial \lambda_L^k(p)}{\partial k}\bigg|_{p=p_L^*(k)}$$

By the proof of Proposition A-5, $\frac{\partial \lambda_L^k(p)}{\partial k} > 0$ if $p_L^* + \frac{c}{\mu-\lambda_L^k(p_L^*)} - R < 0$, i.e., $p_L^* < R - \frac{c}{(\mu-0.5\lambda)^+}$. The last inequality is equivalent to $H_L(R - \frac{c}{(\mu-0.5\lambda)^+}) < 0$, which yields $R > R_{1L}(k)$ after rearranging the terms. The $R < R_{1L}(k)$ and $R = R_{1L}(k)$ cases follow by mimicking the above proof. $R_{1L}'(k) < 0$ holds because by Lemma A-5, $B_{\lceil k \rceil}(x)$ crosses $B_{\lfloor k \rfloor}(x)$ only once from below at $x = R$, i.e., $b_{\lceil k \rceil}(R) > b_{\lfloor k \rfloor}(R)$. □

Proposition A-6 extends Proposition 1 by showing that the impact of sample size on the server's optimal revenue continues to hold with general service quality distributions. Numerically, I find that the server's optimal pricing strategy is also qualitatively preserved, as illustrated in Figure A-5.

Figure A-5: The Impact of k on p_L^* and Π_L^* ($\mu = 1$, $\lambda = 1$, $c = 0.1$, the service quality is uniformly distributed with support $[R-0.5, R+0.5]$)



Next, I examine the server's quality control decision. Denote $\hat{p}_L$, $\hat{R}_L$, and $\hat{\Pi}_L$ as the optimal price, quality, and revenue respectively. Due to analytical difficulties, I characterise the impact of sample size on $\hat{p}_L$, $\hat{R}_L$, and $\hat{\Pi}_L$ numerically, as illustrated in Figure A-6. Notably, all results in the base model (see Proposition 3 and the following numerical study) continue to hold. Specifically, when market potential is sufficiently low or high, a larger sample size leads the service provider to lower price and quality, and his optimal revenue decreases. When market potential is intermediate, a larger sample size leads to a higher quality, and both the optimal price and revenue are U-shaped in k.

Quality information disclosure. In this section I consider the service provider's decision to inform customers of the mean service quality. The next proposition, which follows by mimicking the proof of Proposition 4 and Corollary 1, characterises this information disclosure decision, where $\tilde{k}_L \equiv \{k|\Pi_L^*(k) = \Pi_L^*(+\infty)\}$ and $\tilde{R}_L$ is the service quality level at which $\tilde{k}_L = 1$.[16]

Proposition A-7. *(i) When $R \leqslant \underline{R}$, the service provider does not disclose information.*

(ii)When $\underline{R} < R < \tilde{R}_L$, the service provider does not disclose information for $k < \tilde{k}_L$, discloses information for $k > \tilde{k}_L$, and is indifferent between the two for $k = \tilde{k}_L$, where $\tilde{k}_L$ strictly decreases in R.

(iii) When $R \geqslant \tilde{R}_L$, the service provider discloses information.

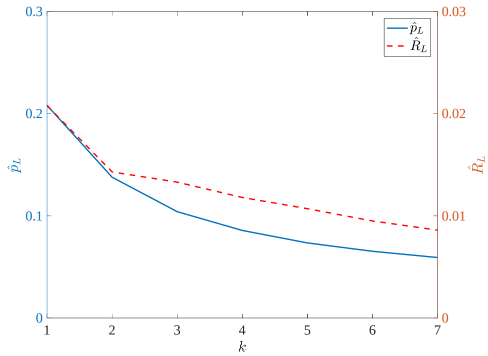
(iv) When $\lambda < 2\mu$, $\tilde{k}_L$ strictly decreases in μ and strictly increases in c. When $\mu - \sqrt{\frac{c\mu}{R}} \leqslant \lambda < 2\mu$, $\tilde{k}_L$ strictly increases in λ.

Proposition A-7 shows that the server's information disclosure decision is robust to the assumption that each anecdote is normally distributed. In particular, I find that an intermediate-quality service provider chooses to inform customers of the mean service quality when they are sufficiently rational, and chooses not to inform when they are sufficiently boundedly rational. Moreover, information non-disclosure is optimal for a larger range of parameter values when the expected waiting cost increases (i.e., higher c, λ, or lower μ).

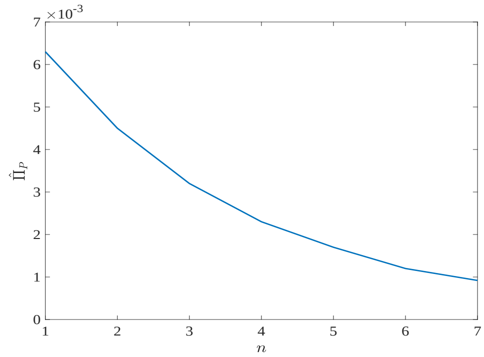
Next, I numerically investigate the information disclosure decision with quality control. As shown in Figure A-6, the server chooses not to inform customers when market potential is sufficiently high or low. Under intermediate market potential, the service provider chooses to inform iff the unit waiting cost is low, quality investment is not too costly, and customers are sufficiently rational. A numerical example is presented in Figure A-7. Notably, all results are consistent with the base model (see A.2.3).

[16] As in Proposition 4, I can show that $\tilde{R}_L$ uniquely exists, and $\tilde{k}_L \in [1, +\infty)$ uniquely exists for $\underline{R} < R \leqslant \tilde{R}_L$ and does not exist otherwise.

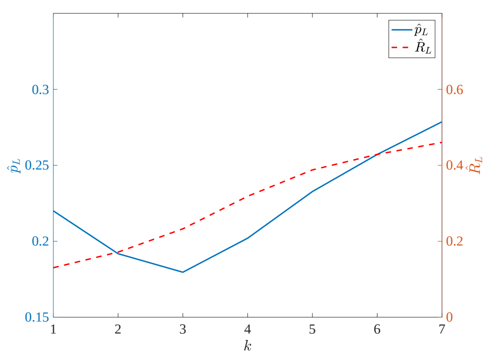
Figure A-6: The Impact of k on $\hat{p}_L$, $\hat{R}_L$, and $\hat{\Pi}_L$ ($\mu = 1$, $c = 0.1$, the service quality is uniformly distributed with support $[R-0.5, R+0.5]$)



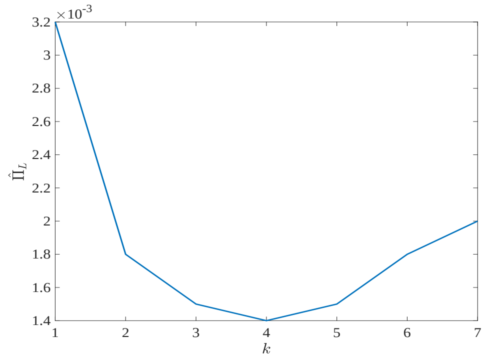
(a) $\hat{p}_L$ and $\hat{R}_L$ under Low Market Potential ($\lambda = 0.01$, $a = 0.05$)



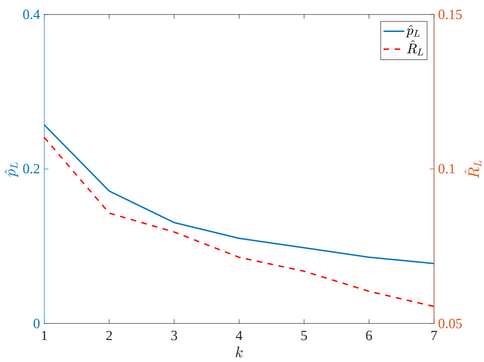
(b) $\hat{\Pi}_L$ under Low Market Potential ($\lambda = 0.01$, $a = 0.05$)



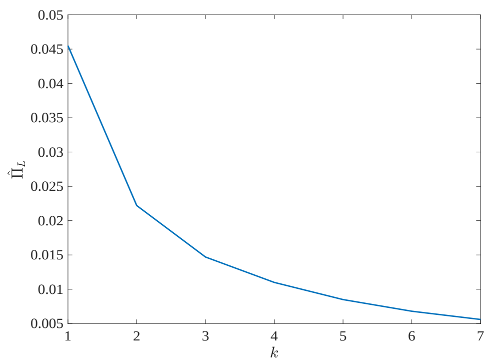
(c) $\hat{p}_L$ and $\hat{R}_L$ under Intermediate Market Potential ($\lambda = 0.06$, $a = 0.05$)



(d) $\hat{\Pi}_L$ under Intermediate Market Potential ($\lambda = 0.06$, $a = 0.05$)

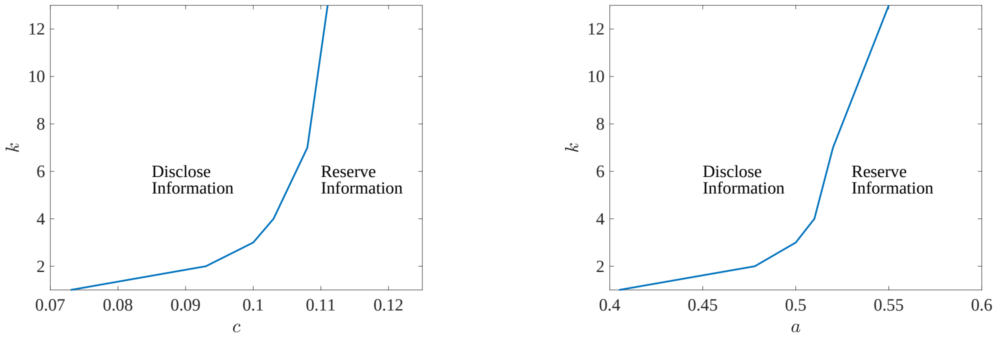


(e) $\hat{p}_L$ and $\hat{R}_L$ under High Market Potential ($\lambda = 100$, $a = 1$)



(f) $\hat{\Pi}_L$ under High Market Potential ($\lambda = 100$, $a = 1$)

Figure A-7: The Impact of c and a on the Information Disclosure Decision under Intermediate Market Potential with Quality Control ($\mu = 1$, $\lambda = 1$, the service quality is uniformly distributed with support $[R-0.5, R+0.5]$)



(a) c ($a = 0.5$)   (b) a ($c = 0.1$)

A.3.2 Customer Heterogeneity in the Sample Size

This section complements Chapter 2.5.2 by numerically verifying that the impact of sample size on the server's pricing, quality control, and information disclosure decisions, as characterised in the main body of the paper, continues to hold when customers are heterogeneous in the sample size.

Pricing and quality control. In this section, I examine the impact of the average sample size n on the service provider's pricing and quality control decisions, as given respectively by:

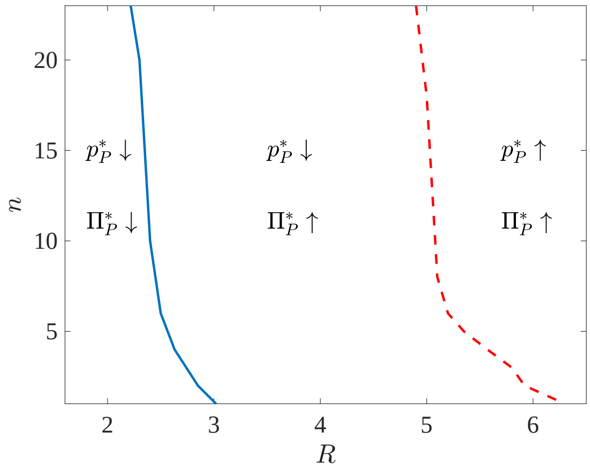
$$\max_{p \geqslant 0} \; p\lambda_P^n(p), \tag{A-9}$$

$$\max_{p,R \geqslant 0} \; p\lambda_P^n(p,R) - aR^2. \tag{A-10}$$

First, I consider the pricing decision characterised by Equation (A-9). Let $p_P^*(n)$ and $\Pi_P^*(n)$ denote the optimal price and revenue for a given n. Numerically, I find that the impact of the average sample size n on the optimal price and revenue are qualitatively the same as the impact of sample size n in the base model (see Figure A-8 for an example).

Next I consider the quality control problem given by Equation (A-10). Let $\hat{p}_P(n)$, $\hat{R}_P(n)$, and $\hat{\Pi}_P(n)$ denote respectively the optimal price, quality, and revenue for a given n. Recall that in the homogeneous-k setting, a larger sample size lowers the optimal price, quality, and revenue when market potential is sufficiently low or high, and induces a higher quality and U-shaped price and revenue when market potential is intermediate. Numerically,

Figure A-8: The Impact of n on p_P^* and Π_P^* ($\mu = 2$, $\lambda = 2$, $c = 1$, $\sigma = 1$)



I have verified that these results continue to hold qualitatively in the heterogeneous-k setting, and Figure A-9 presents an example.

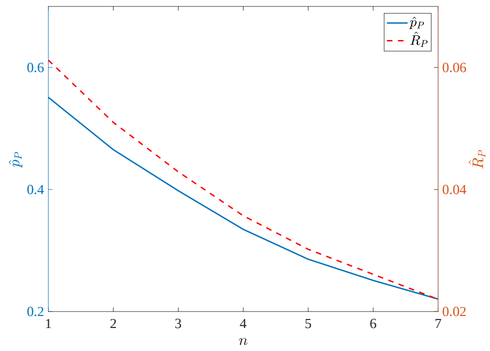
Information disclosure. Next I investigate the service provider's decision to inform customers of the mean service quality, i.e., $n \to +\infty$. In the homogeneous-k setting, Proposition 1 shows that a high (low)-quality service provider discloses (does not disclose) information, and an intermediate-quality service provider discloses information iff customers are sufficiently rational. Through an extensive numerical study (see Figure A-10 for an illustrative example), I have verified that these results continue to hold in the heterogeneous-k setting.

As in §A.2.3, I also numerically investigate the server's information disclosure decision with quality control, as illustrated in Figure A-11. In line with the homogeneous-k model, the server discloses (does not disclose) information when market potential is sufficiently high (low); when market potential is intermediate, disclosing information is optimal when c, a are low and n is high.

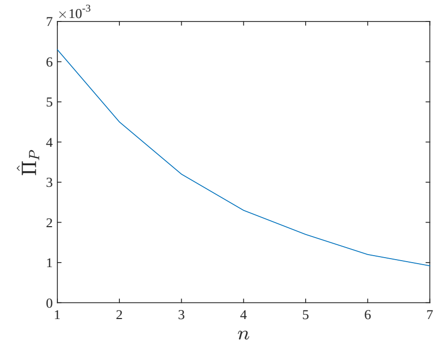
Joint information disclosure. Now I consider the service provider's joint information disclosure decision on both public and private anecdotes, the sizes of which (i.e., k_1 and k_2 respectively) are heterogeneous among customers. Consistent with Chapter 2.5.2, I assume that the joint distribution of k_1 and k_2 has zero-truncated Poisson marginal distributions, i.e.,

$$\sum_{k_2=1}^{+\infty} f_{k_1 k_2} = \frac{n_1^{k_1} e^{-n_1}}{k_1!(1-e^{-n_1})},$$

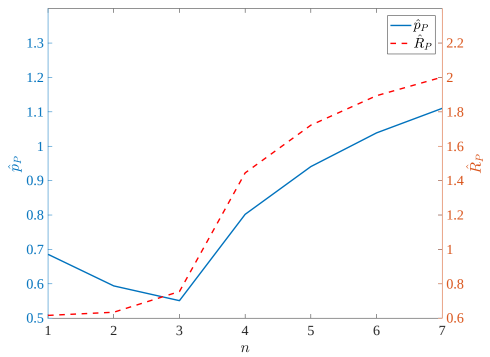
Figure A-9: The Impact of n on $\hat{p}_P$, $\hat{R}_P$, and $\hat{\Pi}_P$ ($\mu = 2$, $c = 1$, $\sigma = 1$)



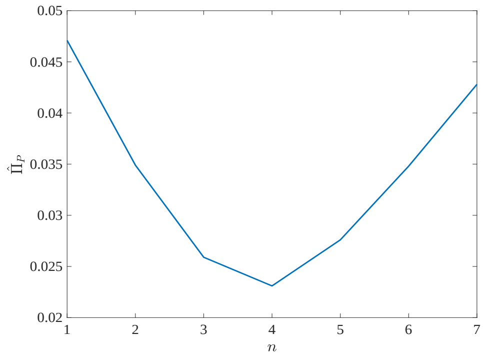
(a) $\hat{p}_P$ and $\hat{R}_P$ under Low Market Potential ($\lambda = 0.1$, $a = 0.1$)



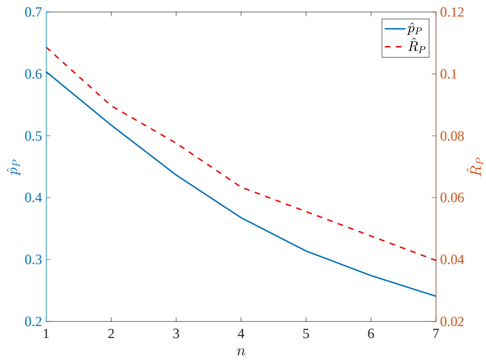
(b) $\hat{\Pi}_P$ under Low Market Potential ($\lambda = 0.1$, $a = 0.1$)



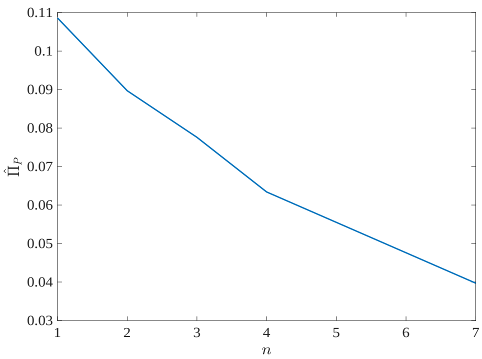
(c) $\hat{p}_P$ and $\hat{R}_P$ under Intermediate Market Potential ($\lambda = 0.53$, $a = 0.1$)



(d) $\hat{\Pi}_P$ under Intermediate Market Potential ($\lambda = 0.53$, $a = 0.1$)



(e) $\hat{p}_P$ and $\hat{R}_P$ under High Market Potential ($\lambda = 100$, $a = 1$)



(f) $\hat{\Pi}_P$ under High Market Potential ($\lambda = 100$, $a = 1$)

Figure A-10: The Impact of n and R on the Quality Information Disclosure Decision ($\mu = 2$, $\lambda = 2$, $c = 1$, $\sigma = 1$)

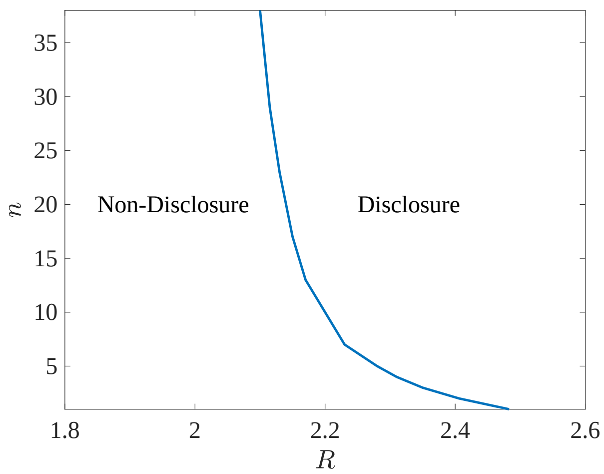
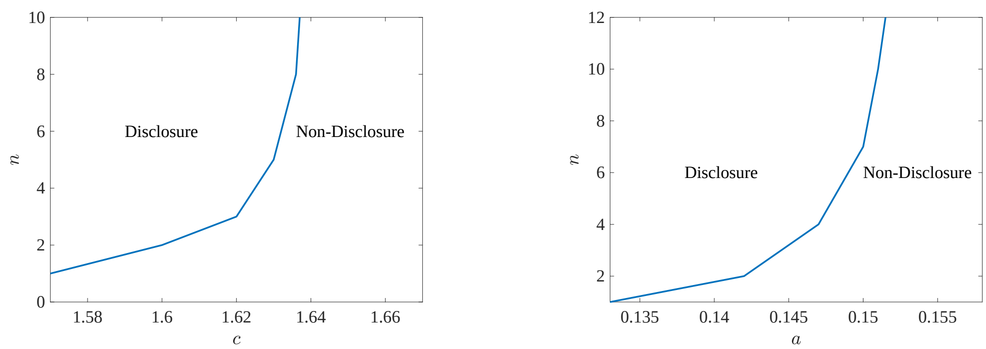


Figure A-11: The Impact of c and a on the Information Disclosure Decision under Intermediate Market Potential with Quality Control ($\mu = 2, \lambda = 0.53, \sigma = 1$)



(a) The Impact of c ($a = 0.1$)

(b) The Impact of a ($c = 1$)

$$\sum_{k_1=1}^{+\infty} f_{k_1k_2} = \frac{n_2^{k_2}e^{-n_2}}{k_2!(1-e^{-n_2})},$$

where $f_{k_1k_2}$ is the probability mass function of k_1 and k_2, n_1 is the Poisson rate of the marginal distributions of k_1, and n_2 is the corresponding Poisson rate for k_2. By definition, n_1 and n_2 measure customers' overall rationality levels regarding public and private anecdotes respectively.

Similar to the derivation of $\lambda_p(r)$, I can show that for a given set of public anecdotes realisations $\mathscr{R}_l = x_l$ $(l = 1,\ldots)$, the joining rate of customers with $k_1 = i$ and $k_2 = j$, which I denote as λ_P^{ij}, is given by:

$$\lambda_P^{ij} = \lambda\bar{\Phi}\left(\frac{\sqrt{j}}{\bar{\theta}\sigma}\left[p+\frac{c}{\mu-\lambda_P^l}-\theta\left(\sum_{h=1}^{i}x_h\right)/i-\bar{\theta}R\right]\right),$$

where $\lambda_P^l \equiv \sum_i\sum_j f_{ij}\lambda_P^{ij}$ is the total joining rate for all customers. Summing up the expression of λ_P^{ij} with respect to i and j, I have

$$\lambda_P^l = \sum_i\sum_j f_{ij}\lambda\bar{\Phi}\left(\frac{\sqrt{j}}{\bar{\theta}\sigma}\left[p+\frac{c}{\mu-\lambda_P^l}-\theta(\sum_{h=1}^{i}x_h)/i-\bar{\theta}R\right]\right).$$

Denote the joint distribution of x_l $(l = 1,\ldots)$ by $g(x_1,x_2,x_3,\ldots)$, I can derive the expression of the expected joining rate λ_P^{het} as follows:

$$\lambda_P^{het} = \int_{x_1,x_2,\ldots}\cdots\int \lambda_P^l(x_1,x_2,\ldots)g(x_1,x_2,\ldots)dx_1dx_2\ldots$$

The next lemma shows that λ_P^{het} is well-defined.

Lemma A-6. *For any $p \geqslant 0$, a unique $\lambda_P^{het} \in (0,\min\{\lambda,\mu\})$ exists and strictly decreases in p. Moreover, $\lim_{n_1,n_2\to+\infty}\lambda_P^{het} = \lambda_r(p)$.*

Proof. Denote $F_{het}(x) \equiv \sum_i\sum_j f_{ij}\lambda\bar{\Phi}\left(\frac{\sqrt{j}}{\bar{\theta}\sigma}\left[p+\frac{c}{\mu-x}-\theta(\sum_{h=1}^{i}x_h)/i-\bar{\theta}R\right]\right)-x$. Therefore, λ_P^l is defined by $F_{het}(\lambda_P^l) = 0$. To prove the existence and uniqueness of $\lambda_P^{het} \in (0,\min\{\lambda,\mu\})$, it suffices to show that: (i) $F_{het}(x)$ strictly decreases in x; (ii) $F_{het}(0) > 0$; (iii) $\lim_{x\to\min\{\lambda,\mu\}^-} F_{het}(x) \leqslant 0$, where the superscript "$-$" denotes x approaching $\min\{\lambda,\mu\}$ from the left/below. Part (i) holds because $\bar{\Phi}\left(\frac{\sqrt{j}}{\bar{\theta}\sigma}\left[p+\frac{c}{\mu-x}-\theta(\sum_{h=1}^{i}x_h)/i-\bar{\theta}R\right]\right)$ strictly decreases in x and Part (ii) follows by $\bar{\Phi}(x) > 0$ for all $x < +\infty$. To show Part (iii), I consider the following cases. First, when $\mu > \lambda$,

$$\begin{aligned}F_{het}(\min\{\lambda,\mu\}) &= F_{het}(\lambda)\\ &= \lambda\left\{\sum_i\sum_j f_{ij}\bar{\Phi}\left(\frac{\sqrt{j}}{\bar{\theta}\sigma}\left[p+\frac{c}{\mu-\lambda}-\theta(\sum_{h=1}^{i}x_h)/i-\bar{\theta}R\right]\right)-1\right\} < 0.\end{aligned}$$

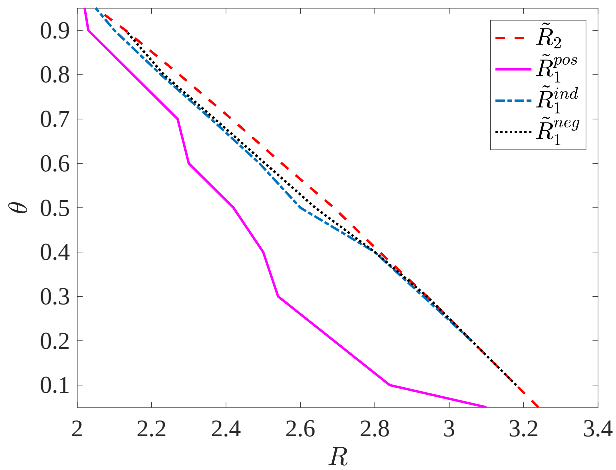
since $p+\frac{c}{\mu-\lambda}-\theta(\sum_{h=1}^{i} x_h)/i-\bar{\theta}R>-\infty$. Second, when $\mu\leqslant\lambda$,

$$\lim_{x\to\min\{\lambda,\mu\}^-} F_{het}(x)=0-\mu<0.$$

To show that λ_p^{het} strictly decreases in p, it suffices to show that λ_p^l strictly decreases in p. This holds because $F_{het}(x)$ strictly decreases in p for any x. $\lim_{n_1,n_2\to+\infty}\lambda_p^{het}=\lambda_r(p)$ follows by mimicking the proof of Lemma 1. $\square$

Next, I examine the server's joint information disclosure decision. Since an analytical characterisation is prohibitively difficult, I turn to a numerical study, as illustrated in Figure A-12. I construct the joint distribution of k_1 and k_2 using the Frank's copula. This approach has been widely adopted in the economics, finance, and statistics literature (see, e.g., Genest et al. 2003, Cameron et al. 2004, Chavez-Demoulin et al. 2006, McHale and Scarf 2007). I find that the joint information disclosure decision in the homogeneous-k_1&k_2 setting continue to hold qualitatively. In particular, the service provider discloses (does not disclose) information in both channels when R and θ are high (low), and discloses information only in the public channel when R and θ are intermediate. Interestingly, the correlation between k_1 and k_2 may significantly influence the server's joint information disclosure decision: As k_1 and k_2 become more positively correlated, the server chooses to disclose information in the public channel instead of information non-disclosure for a larger range of parameter values. To fully understand this result, first recall that compared to information non-disclosure, disclosing information exclusively in the public channel benefits the server by increasing the efficiency of the pricing strategy at the cost of lower demand (i.e., the niche customers). When k_1 and k_2 are more positively correlated, the expected demand increases, so the disadvantage of information disclosure (i.e., reducing demand) leads to a lower profit loss. As a result, information disclosure exclusively in the public channel becomes more attractive.

Figure A-12: The Impact of R and θ on the Joint Information Disclosure Decision with Customers Heterogeneity in k_1 and k_2 ($n_1 = 5, n_2 = 1, \mu = 2, \lambda = 2, c = 1, \sigma = 3$, $\tilde{R}_1^{pos}, \tilde{R}_1^{ind}, \tilde{R}_1^{neg}$ correspond to $\tilde{R}_1$ for $\rho = 0.5, 0, -0.5$ respectively)



Appendix B

Appendix to "Opaque Selling and Last-Minute Selling: Revenue Management in Vertically Differentiated Markets"

B.1 Proofs

In this section I prove all lemmas and propositions in Chapter 3. For expositional convenience, we denote p_i^j as the price of the type-i product in period j, where $i \in \{H, L\}$ and $j \in \{1, 2\}$.

Proof of Lemma 4. Other selling strategies are:

1. In period 1, the firm targets the low-type consumers by the high-type product and do not sell the low-type product. The high-type consumers do not buy in period 1.

2. In period 1, the firm targets the high-type consumers by the low-type product and the low-type consumers by the high-type product.

3. In period 1, the firm targets both types of consumers by the high-type product and does not sell the low-type product.

4. In period 1, the firm targets the high-type consumers by the high-type product and the low-type consumers by both types of products.

5. In period 1, the firm targets the high-type consumers by both types of products and does not sells to the low-type consumers.

For the rest of this proof, I will show that the above selling strategies cannot emerge in equilibrium.

First, I will rule out Case 1. Since the low-type consumers buy the high-type products in period 1, I have

$$V_{lH} - p_H^1 \geqslant 0.$$

To incentivise the high-type consumers to buy in period 2 instead of period 1, I need

$$V_{hH} - p_H^1 < \theta_1(V_{hH} - p_H^2) + \theta_2(V_{hL} - p_L^2),$$

where θ_1 (θ_2) is the probability of receiving a high (low)-type product in period 2, and p_H^2 (p_L^2) is the expected price of the high (low)-type product in period 2. By subgame perfection, $p_H^2 \geqslant V_{lH}$ and $p_L^2 \geqslant V_{lL}$. Therefore, I need

$$V_{hH} - p_H^1 < \theta_1(V_{hH} - V_{lH}) + \theta_2(V_{hL} - V_{lL}).$$

This inequality does not hold because $V_{lH} - p_H^1 \geqslant 0 = \theta_1(V_{lH} - V_{lH}) + \theta_2(V_{lL} - V_{lL})$, $V_{hH} - V_{hL} > V_{lH} - V_{lL}$, and $\theta_1 + \theta_2 \leqslant 1$.

Next, I will rule out Case 2. By incentive compatibility, I have

$$\begin{cases} V_{hL} - p_L^1 \geqslant V_{hH} - p_H^1, \\ V_{lH} - p_H^1 \geqslant V_{lL} - p_L^1 \end{cases}$$

The above equations cannot hold simultaneously because $V_{hH} - V_{hL} > V_{lH} - V_{lL}$.

Now, I will rule out Case 3 by showing that the firm's equilibrium expected profit is lower than Π_O. In this case with a low demand realisation in period 2, the firm has N low-type products and the market has $n_h + n_l - M$ low-type consumers. By subgame perfection, she should price at V_{lL} and all remaining consumers will buy. If demand turns out to be high, the firm has N low-type products and the market has $N_h - M$ high- and N_l low-type consumers. The firm prices either at V_{hL} to target the high-type consumers only, or at V_{lL} to target both types of consumers. Therefore the optimal period-2 profit under high-demand realisation is $\Pi_2 = \max\{(N_h - M)V_{hL}, NV_{lL}\}$.

Next I calculate the period-1 profit. To incentivise both types of consumers to buy in period 1, the firm should price the high-type product at most at V_{hL}. Therefore, the period-1 profit is no larger than MV_{lH} and the total expected profit is no larger than Π_3, as given by: $\Pi_3 \equiv (1-\alpha)[MV_{lH} + (n_h + n_l - M)V_{lL}] + \alpha[MV_{lH} + (N_h - M)V_{hL}]\}$ if $(N_h - M)V_{hL} > NV_{lL}$, and $\Pi_3 \equiv (1-\alpha)[MV_{lH} + (n_h + n_l - M)V_{lL}] + \alpha(MV_{lH} + NV_{lL})$ otherwise.

By the expressions of Π_L and and Π_O,

$$\begin{aligned} \Pi_O - \Pi_3 > \Pi_L - \Pi_3 &= K[\alpha' V_{hH} + (1-\alpha')V_{lH}] + (1-\alpha)(M - n_h)V_{lH} - MV_{lH} \\ &= K[\alpha' V_{hH} + (1-\alpha')V_{lH}] - KV_{lH} > 0. \end{aligned}$$

Next, I will show that the firm's equilibrium expected profit under Case 4 (denoted by Π_4) is weakly lower than Π_3. In Case 4, the high-type consumers buy the high-type product and the low-type consumers buy both types of products in period 1. Therefore, if demand turns out to be low, the market has no consumers in period 2. If demand turns out to be high, the firm has no remaining products. Consequently, I have:

$$\Pi_4 = (1-\alpha)[MV_{lH} + (n_h + n_l - M)V_{lL}] + \alpha(MV_{lH} + NV_{lL}) \leqslant \Pi_3.$$

Last, I will show that Case 5 cannot be an equilibrium. When demand turns out to be high in period 2, the firm has $M + N - N_h$ low-type products and N low-type consumers. Therefore, the firm prices at V_{lL} and sells all of them to the consumers. When a high-type consumer finds only low-type products in stock in period 1, he rationally infers that the demand is high. Therefore, by delaying his purchase to period 2, he can obtain surplus $V_{hL} - V_{lL}$. To incentivise him not to delay the purchase, the firm should price the low-type product at $V_{hL} - V_{hL} + V_{lL} = V_{lL}$ in period 1. This indicates that the low-type consumers should buy in period 1 and thus Case 5 cannot be an equilibrium. $\square$

Proof of Proposition 7. $\Pi_O > \Pi_L$ by definition.

When $(N_h - M)V_{hL} > NV_{lL}$, $\Pi_O - \Pi_T = K[\alpha'(V_{hL} - V_{lL}) - (1-\alpha')(\Delta_h - \Delta_l)] + (1-\alpha)(M - n_h)\Delta_l + \alpha[(N_h - M)V_{hL} - NV_{lL}] + (1-\alpha')\frac{n_l + n_h - M}{n_l}K(\Delta_h - \Delta_l)$. Therefore, $\Pi_O > \Pi_T$ if and only if:

$$\Delta_h < \left[1 + \frac{(1-\alpha)n_l}{(1-\alpha')K}\right]\Delta_l + \frac{\alpha' n_l}{(1-\alpha')(M-n_h)}(V_{hL} - V_{lL}) + \frac{\alpha n_l}{(1-\alpha')(M-n_h)}\left[\frac{N_h - M}{K}V_{hL} - \frac{N}{K}V_{lL}\right].$$

When $(N_h - M)V_{hL} \leqslant NV_{lL}$, $\Pi_O - \Pi_T = -(1-\alpha')K(\Delta_h - \Delta_l) + (1-\alpha)(M - n_h)(V_{lH} - V_{lL}) + (1-\alpha')\frac{n_l + n_h - M}{n_l}K(\Delta_h - \Delta_l)$. Therefore, $\Pi_O > \Pi_T$ if and only if

$$\Delta_h < \left[1 + \frac{(1-\alpha)n_l}{(1-\alpha')K}\right]\Delta_l. \quad \square$$

Proof of Proposition 8. When $V_{hL} \leqslant \frac{N}{N_h - M}V_{lL}$, the firm should use opaque selling when $\Delta_h < [1 + \frac{(1-\alpha)n_l}{(1-\alpha')K}]\Delta_l$ and traditional selling otherwise. Part (i) holds because the right-hand side (RHS) strictly decreases in α due to

$$\frac{(1-\alpha)n_l}{(1-\alpha')K} = \frac{(1-\alpha+\alpha M/N_h)n_l}{(1-\alpha)n_h + \alpha M} = \frac{n_l}{N_h}\frac{(1-\alpha)N_h + \alpha M}{(1-\alpha)n_h + \alpha M} = \frac{n_l}{n_h N_h}\left[N_h - \frac{M(N_h - n_h)}{\frac{(1-\alpha)n_h}{\alpha} + M}\right].$$

When $V_{hL} > \frac{N}{N_h - M} V_{lL}$, the firm should use opaque selling when $\Delta_h < [1 + \frac{(1-\alpha)n_l}{(1-\alpha')K}]\Delta_l + \frac{\alpha' n_l}{(1-\alpha')(M-n_h)}(V_{hL} - V_{lL}) + \frac{\alpha n_l}{(1-\alpha')(M-n_h)}[\frac{N_h - M}{K} V_{hL} - \frac{N}{K} V_{lL}]$. Let $F(\alpha)$ denote the RHS, $\Delta \equiv V_{hL} - V_{lL}$, $R \equiv (N_h - M)V_{hL} - NV_{lL}$, and $T(\alpha) \equiv \frac{\alpha}{(1-\alpha')K}$. By definition, I have

$$F'(\alpha) = -\frac{n_l M(N_h - n_h)}{N_h K^2}\Delta_l + \frac{n_l M}{(M-n_h)N_h(1-\alpha)^2}\Delta + \frac{n_l R T'}{M - n_h}.$$

Since

$$T(\alpha) = \frac{\alpha(1-\alpha+\alpha M/N_h)}{(1-\alpha)[(1-\alpha)n_h + \alpha M]} = \frac{1}{n_h}(\frac{1}{1-\alpha} - 1)\left[1 - \frac{M(\frac{1}{n_h} - \frac{1}{N_h})}{(1-\alpha)/\alpha + M/n_h}\right],$$

I have

$$\begin{aligned} n_h T' &= \frac{1}{(1-\alpha)^2}\frac{\frac{1-\alpha}{\alpha} + M/N_h}{\frac{1-\alpha}{\alpha} + M/n_h} - \frac{1}{(1-\alpha)\alpha}\frac{M(\frac{1}{n_h} - \frac{1}{N_h})}{[(1-\alpha)/\alpha + M/n_h]^2} \\ &= \frac{1}{(1-\alpha)^2\alpha}\frac{\alpha(\frac{1-\alpha}{\alpha} + \frac{M}{N_h})(\frac{1-\alpha}{\alpha} + \frac{M}{n_h}) - (1-\alpha)(\frac{M}{n_h} - \frac{M}{N_h})}{[(1-\alpha)/\alpha + M/n_h]^2} \\ &= \frac{1}{(1-\alpha)^2\alpha}\frac{\frac{(1-\alpha)^2}{\alpha} + (1-\alpha)(\frac{M}{n_h} + \frac{M}{N_h}) + \frac{\alpha M^2}{n_h N_h} - (1-\alpha)(\frac{M}{n_h} - \frac{M}{N_h})}{[(1-\alpha)/\alpha + M/n_h]^2} > 0. \end{aligned}$$

Therefore, when $\frac{n_l M(N_h - n_h)}{N_h K^2} V_{lH} \leqslant \frac{n_l M(N_h - n_h)}{N_h K^2} V_{lL} + \frac{n_l M}{(M-n_h)N_h(1-\alpha)^2}\Delta + \frac{n_l R T'}{M - n_h}$, $F'(\alpha) \geqslant 0$ and thus opaque selling becomes more preferable as α increases. Otherwise, $F'(\alpha) \leqslant 0$ and thus opaque selling becomes less preferable as α increases. $\square$

Proof of Lemma 5. I will enumerate all other possible selling strategies and show that they cannot emerge in equilibrium.

Case 1: Suppose that the firm sells the high-type products to the high- and intermediate-type consumers in period 1. When demand turns out to be high in period 2, the firm has N low-type products, and the market has $N_h - M$ high-type consumers, N_m intermediate-type consumers and N_l low-type consumers. When demand turns out to be low, the firm has N low-type products, and the market has $n_h + n_m - M$ intermediate-type consumers and n_l low-type consumers. There are two subcases depending on the firm's selling strategy under low demand in period 2.

1. When $(n_h + n_m - M)V_{mL} > (n_h + n_m + n_l - M)V_{lL}$, the firm sells the low-type products exclusively to the intermediate-type consumers. I denote this equilibrium by L3a and derive the firm's expected profit as:

$$\Pi_{L3a} \equiv \begin{cases} MV_{mH} + (1-\alpha)(n_h + n_m - M)V_{mL} + \alpha(N_h - M)V_{hL}, & \text{if} \quad (N_h - M)V_{hL} > NV_{mL}, \\ MV_{mH} + (1-\alpha)(n_h + n_m - M)V_{mL} + \alpha NV_{mL}, & \text{if} \quad (N_h - M)V_{hL} \leqslant NV_{mL}. \end{cases}$$

L3a cannot emerge in equilibrium since $\Pi_{L3a} < \Pi_{O1m}$ and O1m exists whenever L3a does.

2. When $(n_h + n_m - M)V_{mL} \leqslant (n_h + n_m + n_l - M)V_{lL}$, the firm sells the low-type products to the intermediate- and low-type consumers. I denote this equilibrium by L3b and derive the firm's expected profit as:

$$\Pi_{L3b} \equiv \begin{cases} M(V_{mH} - V_{mL} + V_{lL}) + (1-\alpha)(n_h + n_m + n_l - M)V_{lL} & \\ +\alpha(N_h - M)V_{hL}, & \text{if} \quad (N_h - M)V_{hL} > NV_{mL}, \\ & \\ M(V_{mH} - V_{mL} + V_{lL}) + (1-\alpha)(n_h + n_m + n_l - M)V_{lL} & \\ +\alpha NV_{mL}, & \text{if} \quad (N_h - M)V_{hL} \leqslant NV_{mL}. \end{cases}$$

Note that the period-1 price is $V_{mH} - V_{mL} + V_{lL}$ because intermediate-type consumers who find the high-type products in stock in period 1 rationally infer that the demand realisation is low.

When $n_m V_{mL} > (n_m + n_l)V_{lL}$, O1m exists and $\Pi_{O1m} > \Pi_{L3b}$ since

$$\begin{aligned} \Pi_{O1m} - \Pi_{L3b} >& [(1-\alpha)n_h + \alpha M][V_{mH} + \bar{\theta}(\Delta_h - \Delta_m)] + (1-\alpha)(M - n_h)V_{mH} \\ & - (1-\alpha)[(n_h + n_m + n_l - M)V_{lL} - (n_h + n_m - M)V_{mL}] - M(V_{mH} - V_{mL} + V_{lL}) \\ =& [(1-\alpha)n_h + \alpha M]\bar{\theta}(\Delta_h - \Delta_m) + M(V_{mL} - V_{lL}) \\ & - (1-\alpha)[(n_h + n_m + n_l - M)V_{lL} - (n_h + n_m - M)V_{mL}] > 0 \end{aligned}$$

for $(n_h + n_m)V_{mL} > (n_h + n_m + n_l)V_{lL}$.

When $n_m V_{mL} \leqslant (n_m + n_l)V_{lL}$, it can be verified that $\Pi_{L1m} > \Pi_{L3b}$. Since L1m is strictly dominated for all parameter values (see the proof of Proposition 9), L3b cannot emerge in equilibrium.

Case 2: Suppose that the firm sells the high-type products to all three types of consumers in period 1. When demand turns out to be high in period 2, the firm has N low-type products, and the market has $N_h - M$ high-type consumers, N_m intermediate-type consumers, and N_l low-type consumers. When demand turns out to be low, the firm has N low-type products, and the market has $n_h + n_m - M$ intermediate-type consumers and n_l low-type consumers. Therefore, Case 2 is identical to Case 1 expect that the period-1 price in Case 2 is lower. Case 2 cannot emerge in equilibrium since Case 1 cannot.

Case 3: Now consider all other selling strategies in which the firm sells only the high-type product in period 1. Similar to the proof of Lemma 4, I can show that whenever the low-type consumers are willing to buy in period 1, both the high- and intermediate-type consumers are also willing to buy. Therefore, the only remaining strategy is that the firm sells the high-type product exclusively to the intermediate-type consumers in period 1. I will rule out this strategy by enumerating all possible subcases.

First, suppose that $M \leqslant n_m < N_m$. When demand turns out to be high in period 2, the firm has N low-type products, and the market has N_h high-type consumers, $N_m - M$ intermediate-type consumers, and N_l low-type consumers. When demand turns out to be low, the firm has N low-type products, and the market has n_h high-type consumers, $n_m - M$ intermediate-type consumers, and n_l low-type consumers. Note that under both demand realisations, when an intermediate-type consumer delays his purchase to period 2, he is assured to receive the low-type products. I will show that the strategy does not emerge in equilibrium because the high-type consumers are incentivised to buy in period 1. When the firm prices the low-type product above V_{lL} under both demand realisations in period 2, it can be verified that the high-type consumers benefit more by buying in period 1. Therefore, I only need to consider the strategies in which the firm prices the low-type products at V_{lL} under either demand realisation. Notice that she cannot price at V_{lL} due to $N_h + N_m > M + N$. Consequently, the only remaining case is that the firm prices at V_{lL} under the low demand realisation.

First, suppose that the firm prices the low-type product at V_{hL} under the high demand realisation in period 2. Here a high-type consumer's expected surplus of buying in period 2 is $(1-\alpha)(V_{hL} - V_{lL})$, whereas his surplus of buying in period 1 is $V_{hH} - V_{mH} + (1-\alpha)(V_{mL} - V_{lL})$. Since $V_{hH} - V_{mH} + (1-\alpha)(V_{mL} - V_{lL}) > (1-\alpha)(V_{hL} - V_{lL})$, the consumer should not buy in period 2 and the strategy does not emerge in equilibrium.

Second, suppose that the firm prices the low-type product at V_{mL} under the high demand realisation in period 2. Here a high-type consumer's expected surplus of buying in period 2 is $(1-\alpha)(V_{hL} - V_{lL}) + \alpha(V_{hL} - V_{mL})$, whereas his surplus of buying in period 1 is $V_{hH} - V_{mH} + (1-\alpha)(V_{mL} - V_{lL})$. Since $V_{hH} - V_{mH} + (1-\alpha)(V_{mL} - V_{lL}) > (1-\alpha)(V_{hL} - V_{lL}) + \alpha(V_{hL} - V_{mL})$, the consumer should not buy in period 2 and the strategy does not emerge in equilibrium.

Next suppose that $n_m < M < N_m$. When demand turns out to be high in period 2, the firm has N low-type products, and the market has N_h high-type consumers, $N_m - M$ intermediate-type consumers, and N_l low-type consumers. When demand turns out to be low, the firm has $M - n_m$ high-type products and N low-type products, and the market has

n_h high-type consumers and n_l low-type consumers. Note that under the high demand realisation, the firm targets either exclusively the high-type consumers or both the high- and intermediate-type consumers. Therefore, the intermediate-type consumers who delay their purchase to period 2 obtain null surplus under the high demand realisation.

As the first subcase, suppose that the firm targets exclusively the high-type consumers under high demand in period 2. Therefore, the highest period-1 price is V_{mH}. First, suppose that the firm targets exclusively the high-type consumers under low demand in period 2, i.e., $(M-n_m)V_{hH}+(n_h+n_m-M)V_{hL} > (M-n_m)(V_{hH}-V_{hL}+V_{lL})+(n_h+n_m+n_l-M)V_{lL}$. Since the high-type consumers' expected surplus of buying in period 2 is zero, they should switch to buying in period 1. Second, suppose that the firm targets both the high- and low-type consumers under low demand in period 2, i.e., $(M-n_m)V_{hH}+(n_h+n_m-M)V_{hL} \leqslant (M-n_m)(V_{hH}-V_{hL}+V_{lL})+(n_h+n_m+n_l-M)V_{lL}$. Here a high-type consumer's expected surplus of buying in period 2 is $(1-\alpha)(V_{hL}-V_{lL})$ and his surplus of buying in period 1 is $V_{hH}-V_{mH}+(1-\alpha)(V_{mL}-V_{lL})$. As a result, they should buy in period 1 and the strategy cannot exist in equilibrium.

As the second subcase, suppose that the firm targets both the high- and intermediate-type consumers under high demand in period 2. First, suppose that the firm targets exclusively the high-type consumers under low demand in period 2, i.e., $(M-n_m)V_{hH}+(n_h+n_m-M)V_{hL} > (M-n_m)(V_{hH}-V_{hL}+V_{lL})+(n_h+n_m+n_l-M)V_{lL}$. Therefore, a high-type consumer's expected surplus of buying in period 2 is $\alpha(V_{hL}-V_{mL})$ and his surplus of buying in period 1 is $V_{hH}-V_{mH}$. As a result, they should buy in period 1 and the strategy cannot exist in equilibrium. Second, suppose that the firm targets both the high- and low-type consumers under low demand in period 2, i.e., $(M-n_m)V_{hH}+(n_h+n_m-M)V_{hL} \leqslant (M-n_m)(V_{hH}-V_{hL}+V_{lL})+(n_h+n_m+n_l-M)V_{lL}$. Here a high-type consumer's expected surplus of buying in period 2 is $(1-\alpha)(V_{hL}-V_{lL})+\alpha(V_{hL}-V_{mL})$ and his surplus of buying in period 1 is $V_{hH}-V_{mH}+(1-\alpha)(V_{mL}-V_{lL})$. As a result, the high-type consumers should buy in period 1 and the strategy cannot exist in equilibrium.

Last suppose that $M \geqslant N_m$. When demand turns out to be high in period 2, the firm has $M-N_m$ high-type products and N low-type products, and the market has N_h high-type consumers and N_l low-type consumers. Since $N_h+N_m > M+N$, the firm should target only the high-type consumers. When demand turns out to be low, the firm has $M-n_m$ high-type products and N low-type products, and the market has n_h high-type consumers and n_l low-type consumers. Following the same line of reasoning as the $n_m < M < N_m$ case, I can show that the high-type consumers obtain higher surplus by buying in period 1, so the strategy does not exist in equilibrium.

Case 4: Suppose that the firm sells both types of products exclusively to the high-type consumers in period 1. When demand turns out to be high in period 2, the firm has $M + N - N_h$ low-type products and the market has N_m intermediate-type consumers and N_l low-type consumers. Suppose that the firm targets only the intermediate-type consumers in this scenario. Since the high-type consumers who find the high-type product out of stock in period 1 rationally infer that the demand is high, the firm has to lower the regular price of the low-type product to $V_{hL} - V_{hL} + V_{mL} = V_{mL}$ to incentivise the high-type consumers to buy the low-type product in period 1. However, this implies that the intermediate-type consumers should deviate to buying in period 1. Therefore, the strategy cannot exist in equilibrium. By the same reasoning, the firm cannot target both the intermediate- and low-type consumers under high demand in period 2. Consequently, Case 4 cannot exist in equilibrium.

The above reasoning also indicates that the strategy in which the firm sells both types of products exclusively to the intermediate-type consumers in period 1 cannot exist in equilibrium. Moreover, the strategy in which the firm sells both types of products exclusively to the low-type consumers in period 1 cannot exist in equilibrium by the proof of Lemma 4. Therefore, I only need to consider strategies in which the firm sells different types of products to different types of consumers in period 1. By the proof of Lemma 4, the type(s) of consumers that buy the high-type product in period 1 is(are) higher than the type(s) of consumers that buy the low-type product in period 1. As a result, we can further restrict our attention to the following strategies.

Case 5: Suppose that the firm sells the high-type product to the high-type consumers and the low-type product to the intermediate-type consumers (and also the high-type consumers if the high-type product stocks out) in period 1. As the first subcase, suppose that $N_h + N_m > M + N$. When demand turns out to be high in period 1, M high-type consumers buy the high-type product, the remaining high-type consumers buy the low-type product,[17] and the intermediate-type consumers buy the leftover low-type products. Therefore, under high demand in period 2, the firm has no remaining products and the market has

[17] They prefer buying the low-type product in period 1 to delaying the purchase to period 2 because they rationally infer from the stockout of the high-type products that the demand is high, in which case their expected surplus of waiting is zero.

$N_h + N_m - M - N$ intermediate-type consumers and N_l low-type consumers. Under low demand in period 2, the firm has $M - n_h$ high-type products and $N - n_m$ low-type products, and the market has n_l low-type consumers. In this scenario, the firm should mix all high-type products and $n_h + n_l - M$ low-type products and sell them to the low-type consumers as the opaque product at price $\beta_1 V_{lH} + \bar{\beta}_1 V_{lL}$, where $\beta_1 \equiv (M - n_h)/n_l$ and $\bar{\beta}_1 \equiv 1 - \beta_1$.

Now I characterise the period-1 prices of both types of products. Since the intermediate-type consumers who find the high-type product out to stock in period 1 rationally infer that the demand is low, they anticipate surplus $\beta_1(V_{mH} - V_{lH}) + \bar{\beta}_1(V_{mL} - V_{lL})$ when delaying their purchase to period 2. To incentivise them to buy in period 1, the firm should lower the regular price of low-type products to $V_{mL} - \beta_1(V_{mH} - V_{lH}) + \bar{\beta}_1(V_{mL} - V_{lL}) < V_{lL}$. Therefore, the low-type consumers are also incentivised to buy in period 1, and the strategy fails to exist in equilibrium.

As the second subcase, suppose that $N_h + N_m \leqslant M + N$. The analysis is the same as the previous case under low demand in period 2. Under high demand in period 2, there are two cases. First, the high-type consumers who find the high-type product out of stock in period 1 buy the low-type product in period 1 instead of waiting for period 2. Therefore, there are $M + N - N_h - N_m$ low-type products and N_l low-type consumers under high demand in period 2. By subgame perfection, the firm should price these products at V_{lL}. Similar to the analysis of the previous case, it can be easily verified that the regular price of the low-type products is below V_{lL} to prevent the intermediate-type consumers from waiting for the sales. Therefore, this strategy cannot exist in equilibrium. Second, the high-type consumers who find the high-type products out of stock in period 1 wait for period 2 instead of buying the low-type product in period 1. This choice indicates that the sales price of the low-type products is V_{lL} instead of V_{hH} (recall that only high- and low-type consumers remain in the market under high demand in period 2). Similar to the first case, I can show that the regular price of the low-type products is below V_{lL}. Therefore, this strategy equilibrium cannot exist in equilibrium.

Case 6: Suppose that the firm sells the high-type product to the high- and intermediate-type consumers and sells the low-type product to the low-type consumers in period 1. Under high demand in period 1, M high-type consumers buy the high-type product, the remaining high-type consumers buy the low-type product, and the remaining low-type products are purchased by the intermediate-type consumers.[18] Therefore, the firm has no leftover products in period 2. Under low demand in period 1, all high-type consumers buy the high-type product, $M - n_m$ intermediate-type consumers buy the remaining high-type product, the remaining intermediate-type consumers buy the low-type product, and all low-type consumers buy the low-type product. Therefore, the firm has no remaining consumer in period 2 and the firm's expected profit is:

$$\Pi_{T2m} = (1-\alpha)[(V_{mH} - V_{mL} + V_{lL})M + V_{lL}(n_h + n_m + n_l - M)] + \alpha[(V_{mH} - V_{mL} + V_{lL})M + V_{lL}N],$$

where the subscript T2m denotes this type of strategy. Next I will show that $\Pi_{T2m} < \Pi_{O1m}$ whenever O1m exists and $\Pi_{T2m} < \Pi_{L2m}$ whenever L1m exists.

First, suppose $n_m V_{mL} > (n_m + n_l)V_{lL}$ and O1m exists.

$$\begin{aligned}\Pi_{O1m} - \Pi_{T2m} > &MV_{mH} + (1-\alpha)(n_h + n_m - M)V_{mL} + \alpha N V_{mL} \\ &- M(V_{mH} - V_{mL} + V_{lL}) - (1-\alpha)(n_h + n_m + n_l - M)V_{lL} - \alpha N V_{lL} \\ = &[M + (1-\alpha)(n_h + n_m - M) + \alpha N](V_{mL} - V_{lL}) - (1-\alpha)n_l V_{lL}\end{aligned}$$

Therefore, $\Pi_{O1m} > \Pi_{T2m}$ if

$$V_{mL} > \frac{(1-\alpha)(n_m + n_l) + M - (1-\alpha)(M - n_h) + \alpha N}{(1-\alpha)n_m + M - (1-\alpha)(M - n_h) + \alpha N} V_{lL}.$$

This inequality holds because $n_m V_{mL} > (n_m + n_l)V_{lL}$ and $M - (1-\alpha)(M - n_h) + \alpha N > 0$.

Second, suppose $n_m V_{mL} \leqslant (n_m + n_l)V_{lL}$ and L2m exists.

$$\begin{aligned}\Pi_{L2m} - \Pi_{T2m} > &M(V_{mH} - V_{mL} + V_{lL}) + (1-\alpha)(n_h + n_m + n_l - M)V_{lL} + \alpha N V_{mL} \\ &- M(V_{mH} - V_{mL} + V_{lL}) - (1-\alpha)(n_h + n_m + n_l - M)V_{lL} - \alpha N V_{lL} > 0\end{aligned}$$

Therefore, Case 6 cannot exist in equilibrium. $\square$

Proof of Proposition 9. By Lemma 5, I only need to consider Tm, L1m, L2m, O1m, and O2m. Moreover, L1m can be ruled out because it is strictly dominated by O1m (note that $\Pi_{L1m} < \Pi_{O1m}$ and O1m exists whenever L1m exists). Since

$$\begin{aligned}\Pi_{O2m} - \Pi_{L2m} = &[(1-\alpha)n_h + \alpha M](1-\alpha')[\Delta_h - \Delta_m - \phi(\Delta_h - \Delta_l)] \\ &- (1-\alpha)(M - n_h)(\Delta_m - \Delta_l),\end{aligned}$$

$\Pi_{O2m} > \Pi_{L2m}$ if and only if:

$$\Delta_h > \frac{1}{\bar{\phi}}[1 + \frac{(1-\alpha)(M - n_h)}{(1-\alpha')K}]\Delta_m - \frac{1}{\bar{\phi}}[\phi + \frac{(1-\alpha)(M - n_h)}{(1-\alpha')K}]\Delta_l.$$

[18] As in Case 5, I can show that the high-type consumers prefer buying the low-type product in period 1 to buying in period 2.

If $(N_h - M)V_{hL} > NV_{mL}$,

$$\begin{aligned}\Pi_{O1m} - \Pi_{Tm} =& [(1-\alpha)n_h + \alpha M]\{V_{hL} - V_{lL} - (1-\alpha')[\theta(\Delta_h - \Delta_m) + V_{hL} - V_{mL}]\} \\ &+ (1-\alpha)(M - n_h)\Delta_m + (1-\alpha)[n_m V_{mL} - (n_m + n_l)V_{lL}] \\ &+ \alpha[(N_h - M)V_{hL} - NV_{lL}].\end{aligned}$$

Therefore, $\Pi_{O1m} > \Pi_{Tm}$ if and only if:

$$\begin{aligned}\Delta_h <& [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_m + \frac{V_{hL} - V_{lL} - (1-\alpha')(V_{hL} - V_{mL})}{(1-\alpha')\theta} \\ &+ \frac{(1-\alpha)[n_m V_{mL} - (n_m + n_l)V_{lL}] + \alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')\theta K}.\end{aligned}$$

Since

$$\begin{aligned}\Pi_{L2m} - \Pi_{Tm} =& [(1-\alpha)n_h + \alpha M][\alpha'(V_{hL} - V_{lL}) - (1-\alpha')(\Delta_h - \Delta_m)] \\ &+ (1-\alpha)(M - n_h)\Delta_m + \alpha[(N_h - M)V_{hL} - NV_{lL}],\end{aligned}$$

$\Pi_{L2m} > \Pi_{Tm}$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha'}{1-\alpha'}(V_{hL} - V_{lL}) + \frac{\alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')K}.$$

Since

$$\begin{aligned}\Pi_{O2m} - \Pi_{Tm} =& [(1-\alpha)n_h + \alpha M][\alpha'(V_{hL} - V_{lL}) - (1-\alpha')\phi(\Delta_h - \Delta_l)] \\ &+ (1-\alpha)(M - n_h)\Delta_l + \alpha[(N_h - M)V_{hL} - NV_{lL}],\end{aligned}$$

$\Pi_{O2m} > \Pi_{Tm}$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l + \frac{\alpha'}{(1-\alpha')\phi}(V_{hL} - V_{lL}) + \frac{\alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')\phi K}.$$

Since $\Pi_{O1m} - \Pi_{L2m} = (1-\alpha')[(1-\alpha)n_h + \alpha M][\bar{\theta}(\Delta_h - \Delta_m) + V_{mL} - V_{lL}] - (1-\alpha)[(n_m + n_l)V_{lL} - n_m V_{mL}]$, $\Pi_{O1m} > \Pi_{L2m}$ if and only if

$$\Delta_h > \Delta_m + \frac{(1-\alpha)[(n_m + n_l)V_{lL} - n_m V_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h + \alpha M]} - \frac{V_{mL} - V_{lL}}{\bar{\theta}}.$$

If $(N_h - M)V_{hL} \leqslant NV_{mL}$,

$$\begin{aligned}\Pi_{O1m} - \Pi_{Tm} =& -[(1-\alpha)n_h + \alpha M](1-\alpha')\theta(\Delta_h - \Delta_m) + (1-\alpha)(M - n_h)\Delta_m \\ &+ (1-\alpha)[n_m V_{mL} - (n_m + n_l)V_{lL}] + \alpha N(V_{mL} - V_{lL}).\end{aligned}$$

Therefore, $\Pi_{O1m} > \Pi_{Tm}$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_m + \frac{(1-\alpha)[n_m V_{mL} - (n_m + n_l)V_{lL}] + \alpha N(V_{mL} - V_{lL})}{(1-\alpha')\theta K}.$$

Since

$$\Pi_{L2m} - \Pi_{Tm} = -[(1-\alpha)n_h + \alpha M](1-\alpha')(\Delta_h - \Delta_m) + (1-\alpha)(M-n_h)\Delta_m + \alpha N(V_{hL} - V_{lL}),$$

$\Pi_{L2m} > \Pi_{Tm}$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha N(V_{hL} - V_{lL})}{(1-\alpha')K}.$$

Since

$$\Pi_{O2m} - \Pi_{Tm} = -[(1-\alpha)n_h + \alpha M](1-\alpha')\phi(\Delta_h - \Delta_m) + (1-\alpha)(M-n_h)\Delta_l + \alpha N(V_{hL} - V_{lL}),$$

$\Pi_{O2m} > \Pi_{Tm}$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l + \frac{\alpha N(V_{hL} - V_{lL})}{(1-\alpha')\phi K}.$$

Since

$$\Pi_{O1m} - \Pi_{L2m} = (1-\alpha')[(1-\alpha)n_h + \alpha M]\bar{\theta}(\Delta_h - \Delta_m) - (1-\alpha)[(n_m+n_l)V_{lL} - n_m V_{mL}],$$

$\Pi_{O1m} > \Pi_{L2m}$ if and only if

$$\Delta_h > \Delta_m + \frac{(1-\alpha)[(n_m+n_l)V_{lL} - n_m V_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h + \alpha M]}.$$

Organising the above inequalities for all possible cases (i.e., $(n_m+n_l)V_{lL} - n_m V_{mL} < 0$, $0 \leqslant (n_m+n_l)V_{lL} - n_m V_{mL} < (M-n_h)(\Delta_m - \Delta_l)$, and $(n_m+n_l)V_{lL} - n_m V_{mL} \geqslant (M-n_h)(\Delta_m - \Delta_l)$) leads to the proposition. Specifically, when $(n_m+n_l)V_{lL} - n_m V_{mL} < 0$ and $(N_h - M)V_{hL} > NV_{mL}$, the firm should use traditional selling if

$$\begin{aligned}\Delta_h >& [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_m + \frac{V_{hL} - V_{lL} - (1-\alpha')(V_{hL} - V_{mL})}{(1-\alpha')\theta} \\ &+ \frac{(1-\alpha)[n_m V_{mL} - (n_m+n_l)V_{lL}] + \alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')\theta K}\end{aligned}$$

and use opaque selling otherwise.

When $(n_m+n_l)V_{lL} - n_m V_{mL} < 0$ and $(N_h - M)V_{hL} \leqslant NV_{mL}$, the firm should use traditional selling if

$$\Delta_h > [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_m + \frac{(1-\alpha)[n_m V_{mL} - (n_m+n_l)V_{lL}] + \alpha N(V_{mL} - V_{lL})}{(1-\alpha')\theta K}$$

and use opaque selling otherwise.

When $0 \leqslant (n_m+n_l)V_{lL} - n_mV_{mL} < (M-n_h)(\Delta_m - \Delta_l)$ and $(N_h - M)V_{hL} > NV_{mL}$, the firm should use traditional selling if

$$\Delta_h > [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha'}{1-\alpha'}(V_{hL}-V_{lL}) + \frac{\alpha[(N_h-M)V_{hL}-NV_{lL}]}{(1-\alpha')K}$$

and

$$\Delta_h > [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l + \frac{\alpha'}{(1-\alpha')\phi}(V_{hL}-V_{lL}) + \frac{\alpha[(N_h-M)V_{hL}-NV_{lL}]}{(1-\alpha')\phi K},$$

use opaque selling if

$$\Delta_h > \Delta_m + \frac{(1-\alpha)[(n_m+n_l)V_{lL}-n_mV_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h+\alpha M]} - \frac{V_{mL}-V_{lL}}{\bar{\theta}}$$

and

$$\begin{aligned}\Delta_h <&[1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_m + \frac{V_{hL}-V_{lL}-(1-\alpha')(V_{hL}-V_{mL})}{(1-\alpha')\theta}\\ &+\frac{(1-\alpha)[n_mV_{mL}-(n_m+n_l)V_{lL}]+\alpha[(N_h-M)V_{hL}-NV_{lL}]}{(1-\alpha')\theta K},\end{aligned}$$

and use last-minute selling if

$$\Delta_h < [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha'}{1-\alpha'}(V_{hL}-V_{lL}) + \frac{\alpha[(N_h-M)V_{hL}-NV_{lL}]}{(1-\alpha')K}$$

and

$$\Delta_h < \Delta_m + \frac{(1-\alpha)[(n_m+n_l)V_{lL}-n_mV_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h+\alpha M]} - \frac{V_{mL}-V_{lL}}{\bar{\theta}}.$$

When $0 \leqslant (n_m+n_l)V_{lL} - n_mV_{mL} < (M-n_h)(\Delta_m - \Delta_l)$ and $(N_h - M)V_{hL} \leqslant NV_{mL}$, the firm should use traditional selling if

$$\Delta_h >[1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_m + \frac{(1-\alpha)[n_mV_{mL}-(n_m+n_l)V_{lL}]+\alpha N(V_{mL}-V_{lL})}{(1-\alpha')\theta K}$$

and

$$\Delta_h > [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha N(V_{hL}-V_{lL})}{(1-\alpha')K},$$

opaque selling if

$$\Delta_h <[1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_m + \frac{(1-\alpha)[n_mV_{mL}-(n_m+n_l)V_{lL}]+\alpha N(V_{mL}-V_{lL})}{(1-\alpha')\theta K}$$

and

$$\Delta_h > \Delta_m + \frac{(1-\alpha)[(n_m+n_l)V_{lL} - n_m V_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h + \alpha M]},$$

and last-minute selling if

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha N(V_{hL} - V_{lL})}{(1-\alpha')K}$$

and

$$\Delta_h < \Delta_m + \frac{(1-\alpha)[(n_m+n_l)V_{lL} - n_m V_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h + \alpha M]}.$$

When $(n_m+n_l)V_{lL} - n_m V_{mL} \geqslant (M-n_h)(\Delta_m - \Delta_l)$ and $(N_h - M)V_{hL} > NV_{mL}$, the firm should use traditional selling if

$$\Delta_h > [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha'}{1-\alpha'}(V_{hL} - V_{lL}) + \frac{\alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')K}$$

and

$$\Delta_h > [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l + \frac{\alpha'}{(1-\alpha')\phi}(V_{hL} - V_{lL}) + \frac{\alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')\phi K},$$

opaque selling if

$$\Delta_h > \frac{1}{\bar{\phi}}[1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m - \frac{1}{\bar{\phi}}[\phi + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l$$

and

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l + \frac{\alpha'}{(1-\alpha')\phi}(V_{hL} - V_{lL}) + \frac{\alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')\phi K},$$

and last-minute selling if

$$\Delta_h < \frac{1}{\bar{\phi}}[1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m - \frac{1}{\bar{\phi}}[\phi + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l$$

and

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha'}{1-\alpha'}(V_{hL} - V_{lL}) + \frac{\alpha[(N_h - M)V_{hL} - NV_{lL}]}{(1-\alpha')K}.$$

When $(n_m+n_l)V_{lL} - n_m V_{mL} \geqslant (M-n_h)(\Delta_m - \Delta_l)$ and $(N_h - M)V_{hL} \leqslant NV_{mL}$, the firm should use traditional selling if

$$\Delta_h > [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m + \frac{\alpha N(V_{hL} - V_{lL})}{(1-\alpha')K}$$

and

$$\Delta_h > [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l+\frac{\alpha N(V_{hL}-V_{lL})}{(1-\alpha')\phi K},$$

opaque selling if

$$\Delta_h > \frac{1}{\bar{\phi}}[1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m-\frac{1}{\bar{\phi}}[\phi+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l$$

and

$$\Delta_h < [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l+\frac{\alpha N(V_{hL}-V_{lL})}{(1-\alpha')\phi K},$$

and last-minute selling if

$$\Delta_h < \frac{1}{\bar{\phi}}[1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m-\frac{1}{\bar{\phi}}[\phi+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l$$

and

$$\Delta_h < [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m+\frac{\alpha N(V_{hL}-V_{lL})}{(1-\alpha')K}.$$

The above analysis yields Proposition 9, which I illustrate in Figure 3.3 and provide the expressions of l_1-l_5 are given as follows.

l_1 : $\Delta_h = \{1+(1-\alpha)(M-n_h)/[(1-\alpha')\theta K]\}\Delta_m + [V_{hL}-V_{lL}-(1-\alpha')(V_{hL}-V_{mL})]/[(1-\alpha')\theta] + \{(1-\alpha)[n_m V_{mL}-(n_m+n_l)V_{lL}]+\alpha[(N_h-M)V_{hL}-NV_{lL}]\}/[(1-\alpha')\theta K]$ when $(N_h-M)V_{hL} > NV_{mL}$ and $\Delta_h = \{1+(1-\alpha)(M-n_h)/[(1-\alpha')\theta K]\}\Delta_m + \{(1-\alpha)[n_m V_{mL}-(n_m+n_l)V_{lL}]+\alpha N(V_{mL}-V_{lL})\}/[(1-\alpha')\theta K$ when $(N_h-M)V_{hL} \leqslant NV_{mL}$.

l_2 : $\Delta_h = [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l + \frac{\alpha'}{(1-\alpha')\phi}(V_{hL}-V_{lL}) + \frac{\alpha[(N_h-M)V_{hL}-NV_{lL}]}{(1-\alpha')\phi K}$ when $(N_h-M)V_{hL} > NV_{mL}$ and $\Delta_h = [1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')\phi K}]\Delta_l + \frac{\alpha N(V_{hL}-V_{lL})}{(1-\alpha')\phi K}$ when $(N_h-M)V_{hL} \leqslant NV_{mL}$.

$l_3 : \Delta_h = \frac{1}{\bar{\phi}}[1+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_m - \frac{1}{\bar{\phi}}[\phi+\frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l$.

$l_4 : \Delta_h = \Delta_m + \frac{(1-\alpha)[(n_m+n_l)V_{lL}-n_m V_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h+\alpha M]} - \frac{V_{mL}-V_{lL}}{\bar{\theta}}$ when $(N_h-M)V_{hL} > NV_{mL}$ and $\Delta_h = \Delta_m + \frac{(1-\alpha)[(n_m+n_l)V_{lL}-n_m V_{mL}]}{(1-\alpha')\bar{\theta}[(1-\alpha)n_h+\alpha M]}$ when $(N_h-M)V_{hL} \leqslant NV_{mL}$.

$l_5 : \Delta_m = \Delta_l + \frac{(n_m+n_l)V_{lL}-n_m V_{mL}}{M-n_h}$. □

Proof of Proposition 10. First, consider the case where $(n_m+n_l)V_{lL}-n_m V_{mL} < (M-n_h)(\Delta_m-\Delta_l)$. By Proposition 9, only Tm, O1m, and L2m may exist in equilibrium. As the first subcase, suppose $(N_h-M)V_{hL} > NV_{mL}$. Denote $g_1(\alpha) \equiv \Pi_{O1m}-\Pi_{L2m} = (1-\alpha')[(1-$

$\alpha)n_h + \alpha M][\bar{\theta}(\Delta_h - \Delta_m) + V_{mL} - V_{lL}] - (1-\alpha)[(n_m + n_l)V_{lL} - n_m V_{mL}]$.

$$
\begin{aligned}
g_1'(\alpha) =& \left\{(M-n_h)(1-\alpha') - \frac{MN_h[(1-\alpha)n_h + \alpha M]}{[\alpha M + (1-\alpha)N_h]^2}\right\}[\bar{\theta}(\Delta_h - \Delta_m) \\
&+ V_{mL} - V_{lL}] + (n_m + n_l)V_{lL} - n_m V_{mL} \\
=& \frac{(M-n_h)(1-\alpha)N_h[\alpha M + (1-\alpha)N_h] - MN_h[(1-\alpha)n_h + \alpha M]}{[\alpha M + (1-\alpha)N_h]^2}[\bar{\theta}(\Delta_h - \Delta_m) \\
&+ V_{mL} - V_{lL}] + (n_m + n_l)V_{lL} - n_m V_{mL} \\
=& \frac{(M-n_h)(1-\alpha)^2 N_h^2 - MN_h[(1-\alpha^2)n_h + \alpha^2 M]}{[\alpha M + (1-\alpha)N_h]^2}[\bar{\theta}(\Delta_h - \Delta_m) + V_{mL} - V_{lL}] \\
&+ (n_m + n_l)V_{lL} - n_m V_{mL}.
\end{aligned}
$$

Since

$$
\begin{aligned}
&(M-n_h)(1-\alpha)N_h[\alpha M + (1-\alpha)N_h] - MN_h[(1-\alpha)n_h + \alpha M] \\
=& -(M-n_h)(M+N_h)N_h\alpha^2 - 2(M-n_h)N_h^2\alpha + N_h(MN_h - n_h N_h - Mn_h),
\end{aligned}
$$

this expression is negative if $\alpha > \bar{\alpha}$, where $\bar{\alpha} \equiv -\infty$ if $N_h^2 + (M+N_h)(MN_h - n_h N_h - Mn_h)/(M-n_h) < 0$, and $\bar{\alpha} \equiv \frac{\sqrt{N_h^2 + (M+N_h)(MN_h - n_h N_h - Mn_h)/(M-n_h)} - 2N_h}{M+N_h}$ otherwise.

Assume that $\alpha > \bar{\alpha}$, then $g_1'(\alpha) < 0$ iff

$$
\begin{aligned}
&\frac{(M-n_h)(1-\alpha)^2 N_h^2 - MN_h[(1-\alpha^2)n_h + \alpha^2 M]}{[\alpha M + (1-\alpha)N_h]^2}[\bar{\theta}(\Delta_h - \Delta_m) + V_{mL} - V_{lL}] \\
&+ (n_m + n_l)V_{lL} - n_m V_{mL} < 0,
\end{aligned}
$$

or

$$
\begin{aligned}
V_{hH} - V_{mH} >& V_{hL} - V_{mL} \\
&+ \frac{1}{\bar{\theta}}\left\{-\frac{[(n_m + n_l)V_{lL} - n_m V_{mL}][\alpha M + (1-\alpha)N_h]^2}{(M-n_h)(1-\alpha)^2 N_h^2 - MN_h[(1-\alpha^2)n_h + \alpha^2 M]} - V_{mL} + V_{lL}\right\}.
\end{aligned}
$$

As the second subcase, suppose $(N_h - M)V_{hL} \leqslant NV_{mL}$. Denote $g_2(\alpha) \equiv \Pi_{O1m} - \Pi_{L2m} = (1-\alpha')[(1-\alpha)n_h + \alpha M]\bar{\theta}(\Delta_h - \Delta_m) - (1-\alpha)[(n_m + n_l)V_{lL} - n_m V_{mL}]$.

$$
g_2'(\alpha) = \frac{(M-n_h)(1-\alpha)^2 N_h^2 - MN_h[(1-\alpha^2)n_h + \alpha^2 M]}{[\alpha M + (1-\alpha)N_h]^2}\bar{\theta}(\Delta_h - \Delta_m) + (n_m + n_l)V_{lL} - n_m V_{mL}.
$$

Similar to the first subcase, it can be verified that $g_2'(\alpha) < 0$ when $\alpha > \bar{\alpha}$ and

$$
V_{hH} - V_{mH} > V_{hL} - V_{mL} - \frac{[(n_m + n_l)V_{lL} - n_m V_{mL}][\alpha M + (1-\alpha)N_h]^2}{\bar{\theta}\{(M-n_h)(1-\alpha)^2 N_h^2 - MN_h[(1-\alpha^2)n_h + \alpha^2 M]\}}.
$$

Second, consider the case where $(n_m + n_l)V_{lL} - n_m V_{mL} \geqslant (M-n_h)(\Delta_m - \Delta_l)$. Denote $g_3(\alpha) \equiv \Pi_{O2m} - \Pi_{L2m} = [(1-\alpha)n_h + \alpha M](1-\alpha')[\Delta_h - \Delta_m - \phi(\Delta_h - \Delta_l)] - (1-\alpha)(M-$

$n_h)(\Delta_m - \Delta_l)$.

$$g_3'(\alpha) = \frac{(M-n_h)(1-\alpha)^2 N_h^2 - MN_h[(1-\alpha^2)n_h + \alpha^2 M]}{[\alpha M + (1-\alpha)N_h]^2}[\Delta_h - \Delta_m - \phi(\Delta_h - \Delta_l)] + (M-n_h)(\Delta_m - \Delta_l).$$

Similar to the first case, I can show that $g_3'(\alpha) < 0$ when $\alpha > \bar{\alpha}$ and

$$V_{hH} - \frac{M-n_h-A}{-\bar{\phi}A}V_{mH} > \frac{M-n_h+\phi A}{A\bar{\phi}}\Delta_l + V_{hL} - \frac{M-n_h-A}{-\bar{\phi}A}V_{mL},$$

where $A \equiv \frac{(M-n_h)(1-\alpha)^2 N_h^2 - MN_h[(1-\alpha^2)n_h+\alpha^2 M]}{[\alpha M+(1-\alpha)N_h]^2} < 0$. □

Proof of Lemma 6. $\Pi_{Od} - \Pi_O = (1-\alpha')(1-\delta)[K(\Delta_h - \Delta_l + V_{hL} - V_{lL}) - \bar{\theta}K(\Delta_h - \Delta_l) - \frac{1-\alpha}{1-\alpha'}(M-n_h)\Delta_l - \frac{1-\alpha}{1-\alpha'}n_l V_{lL}]$. Therefore product damage increases profits if

$$\Delta_h > \frac{[\frac{1-\alpha}{1-\alpha'}(M-n_h) + \theta K]\Delta_l - KV_{hL} + (\frac{1-\alpha}{1-\alpha'}n_l + K)V_{lL}}{\theta K}$$

and decreases profits otherwise. □

Proof of Proposition 11. Similar to the proof of Lemma 4, I can show that selling strategies other than traditional selling, last-minute selling, and opaque selling cannot exist in equilibrium. Therefore, I will focus on T, L, and Od for the rest of the proof.

$\Pi_{Od} - \Pi_L = (1-\alpha')(1-\delta)K(\Delta_h - \Delta_l + V_{hL} - V_{lL}) + (1-\alpha')\delta\bar{\theta}K(\Delta_h - \Delta_l) - (1-\alpha)(1-\delta)[(M-n_h)\Delta_l + n_l V_{lL}]$. Thus $\Pi_{Od} > \Pi_L$ if and only if

$$\Delta_h > [1 + \frac{(1-\alpha)(1-\delta)(M-n_h)}{(1-\alpha')(1-\delta\theta)K}]\Delta_l + \frac{(1-\delta)[(K+\frac{1-\alpha}{1-\alpha'}n_l)V_{lL} - KV_{hL}]}{(1-\delta\theta)K}.$$

I will denote the RHS by x_1.

When $(N_h - M)V_{hL} > NV_{lL}$, $\Pi_{Od} - \Pi_T = K[(V_{hL} - V_{lL}) - (1-\alpha')\delta(V_{hH} - V_{lH})] + (1-\alpha)[\delta(M-n_h)\Delta_l - (1-\delta)n_l V_{lL}] + \alpha[(N_h - M)V_{hL} - NV_{lL}] + (1-\alpha')\delta\bar{\theta}K(\Delta_h - \Delta_l)$. Thus $\Pi_{Od} > \Pi_T$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_l + \frac{[1-(1-\alpha')\delta]K(V_{hL} - V_{lL}) + \alpha[(N_h - M)V_{hL} - NV_{lL}] - (1-\alpha)(1-\delta)n_l V_{lL}}{(1-\alpha')\delta\theta K}.$$

I will denote the RHS by x_2.

By the proof of Proposition 7, $\Pi_L > \Pi_T$ if and only if

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l + \frac{\alpha'}{1-\alpha'}(V_{hL} - V_{lL}) + \frac{\alpha}{1-\alpha'}[\frac{N_h - M}{K}V_{hL} - \frac{N}{K}V_{lL}].$$

I will denote the RHS by x_3. To summarise, the firm should use last-minute selling when $\Delta_h < \min\{x_1, x_3\}$, opaque selling when $x_1 < \Delta_h < x_2$, and traditional selling when $\Delta_h > \max\{x_2, x_3\}$.

When $(N_h - M)V_{hL} \leqslant NV_{lL}$, $\Pi_{Od} - \Pi_T = (1-\alpha')K[(V_{hL} - V_{lL}) - \delta(V_{hH} - V_{lH})] + (1-\alpha')\delta\bar{\theta}K(\Delta_h - \Delta_l) + (1-\alpha)[\delta(M-n_h)\Delta_l - (1-\delta)n_l V_{lL}]$. Thus $\Pi_{Od} > \Pi_T$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_l + \frac{(1-\delta)[K(V_{hL} - V_{lL}) - \frac{1-\alpha}{1-\alpha'}n_l V_{lL}]}{\delta\theta K}.$$

I will denote the RHS by x_4.

By Proposition 7, $\Pi_L > \Pi_T$ if and only if:

$$\Delta_h < [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l.$$

I will denote the RHS by x_5.

To summarise, the firm should use last-minute selling when $\Delta_h < \min\{x_1, x_5\}$, opaque selling when $x_1 < \Delta_h < x_4$, and traditional selling when $\Delta_h > \max\{x_4, x_5\}$.

The above analysis yields Proposition 11, which I illustrate in Figure 3.4. The expressions corresponding to l_1-l_5 are given as follows, where $\theta \equiv \frac{M-n_h}{n_l}$, $\Delta \equiv V_{hL} - V_{lL}$, and $R \equiv (N_h - M)V_{hL} - NV_{lL}$.

$l_1 : \Delta_h = [1 + \frac{(1-\alpha)(1-\delta)(M-n_h)}{(1-\alpha')(1-\delta\theta)K}]\Delta_l + \frac{(1-\delta)[(K+\frac{1-\alpha}{1-\alpha'}n_l)V_{lL} - KV_{hL}]}{(1-\delta\theta)K}$.

$l_2 : \Delta_h = [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_l + \{[1-(1-\alpha')\delta]K(V_{hL} - V_{lL}) + \alpha[(N_h - M)V_{hL} - NV_{lL}] - (1-\alpha)(1-\delta)n_l V_{lL}\}/[(1-\alpha')\delta\theta K]$.

$l_3 : \Delta_h = [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l + \frac{\alpha'}{1-\alpha'}(V_{hL} - V_{lL}) + \frac{\alpha}{1-\alpha'}[\frac{N_h - M}{K}V_{hL} - \frac{N}{K}V_{lL}]$.

$l_4 : \Delta_h = [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')\theta K}]\Delta_l + \frac{(1-\delta)[K(V_{hL} - V_{lL}) - \frac{1-\alpha}{1-\alpha'}n_l V_{lL}]}{\delta\theta K}$.

$l_5 : \Delta_h = [1 + \frac{(1-\alpha)(M-n_h)}{(1-\alpha')K}]\Delta_l$. $\square$

B.2 Deterministic Demand

In this section, I consider the deterministic demand setting in which I denote the numbers of the high- and low-type consumers by D_h and D_l respectively. I adopt all assumptions in the base model except those on the demand distribution. In addition, I assume that $D_h < M < D_h + D_l$ to rule out the obvious cases where opaque selling is unfeasible because of insufficient high-type leftover products ($D_h \geqslant M$) or insufficient remaining consumers ($D_h + D_l \leqslant M$).

First, consider the strategies in which the firm sells the high-type products to the high-type consumers and does not sell the low-type products in period 1. At the beginning of period 2, the firm has $M - D_h$ high-type products and N low-type products, and the market has D_l low-type consumers. When the firm uses last-minute selling (denoted by subscript $L1$), she targets them by the high and low-type products. To prevent the high-type consumers from delaying their purchase to period 2, the firm should price the high-type product at V_{lH} in period 1. Therefore, her total profit is:

$$\Pi_{L1} \equiv MV_{lH} + \min\{D_h + D_l - M, N\}V_{lL}.$$

If the firm uses opaque selling (denoted by subscript $O1$), the high-type consumers' delaying surplus becomes $\gamma(V_{hH} - V_{lH}) + \bar{\gamma}(V_{hL} - V_{lL})$, where $\gamma \equiv \frac{M-D_h}{\min\{D_l, M-D_h+N\}}$ and $\bar{\gamma} \equiv 1 - \gamma$. Therefore, the regular price of the high-type products increases to $V_{lH} + \frac{D_h+D_l-M}{D_l}(\Delta_h - \Delta_l)$. The firm's total profit under O1 is:

$$\Pi_{O1} \equiv D_h[V_{lH} + \bar{\gamma}(\Delta_h - \Delta_l)] + (M - D_h)V_{lH} + \min\{D_h + D_l - M, N\}V_{lL}.$$

Second, consider the strategies in which the firm sells the high-type products to the high-type consumers and the low-type products to the low-type consumers in period 1. When $D_l \leqslant N$, the market has no remaining consumers in period 2, so the firm does not sell. I denote this strategy by subscript $T1$. The firm's total profit under T1 is:

$$\Pi_{T1} \equiv D_h(V_{hH} - V_{hL} + V_{lL}) + D_l V_{lL}.$$

When $D_l > N$, the firm has $M - D_h$ high-type products and the market has $D_l - N$ low-type consumers at the beginning of period 2. Therefore, the firm sells all products at price V_{lH}. I denote this strategy by subscript $L2$. The firm's total profit under L2 is:

$$\Pi_{L2} \equiv D_h V_{lH} + NV_{lL} + \min\{M - D_h, D_l - N\}V_{lH}.$$

Third, consider the strategies in which the firm sells the high-type product to both types of consumers in period 1. At the beginning of period 2, she has N low-type products and the market has $D_h + D_l - M$ low-type consumers. . Therefore, the firm sells all products at price V_{lL}. I denote this strategy by subscript $L3$. The firm's total profit under L3 is:

$$\Pi_{L3} \equiv MV_{lH} + \min\{D_h + D_l - M, N\}V_{lL}.$$

It can be verified that no other strategy exists in equilibrium. Therefore, characterising the optimal strategy boils down to comparing the profit expressions of L1, O1, T1, L2, and L3, as given by the proposition below.

Proposition A-8. *Opaque selling strictly dominates last-minute selling, and*

(i) when $D_l > N$, the firm should use opaque selling;

(ii) when $D_l \leqslant N$, the firm should use opaque selling if $\Delta_h < (1+\frac{M-D_h}{\gamma D_h})\Delta_l$, and use traditional selling otherwise.

Proof. Opaque selling strictly dominates last-minute selling since $\Pi_{O1} > \Pi_{L1} = \Pi_{L3} \geqslant \Pi_{L2}$. Therefore, the firm should use O1 if $\Pi_{O1} > \Pi_{T1}$ and use T1 otherwise. Simplifying this inequality yields the proposition. □

Proposition A-8 shows that opaque selling strictly dominates last-minute selling due to its choice-deprivation advantage. Compared to traditional selling, opaque selling leads to a higher profit if the low-type consumers can generate sufficiently high sales profit due to a large volume or a high valuation for the high-type product. These results are the consistent with those of the base model, which indicates that the mechanism of opaque selling is preserved without demand uncertainty.

Appendix C

Appendix to "Money Back Guarantee with Competing Physical and On-line Stores"

C.1 Best Responses

First I characterise the online store's best response for any given $\tilde{P}_1$. As the first case, suppose $\tilde{P}_1 \leqslant \frac{\alpha_1}{\alpha_2}\tilde{c}_2$ (i.e., $\frac{\alpha_2}{\alpha_1}\tilde{P}_1 \leqslant \frac{\alpha_2\tilde{P}_1+\alpha_1\tilde{c}_2}{2\alpha_1}$) so the online store's profit is 0 under moderate pricing. Under aggressive pricing, she sets $\tilde{P}_2 = \tilde{P}_1 - \alpha_1 + \alpha_2$ (note that $\frac{\alpha_2+\tilde{c}_2}{2} \geqslant \tilde{P}_1 - (\alpha_1 - \alpha_2)$ when $\tilde{P}_1 \leqslant \frac{\alpha_1}{\alpha_2}\tilde{c}_2$ because $\tilde{c}_2 \leqslant -\alpha_2$). Since $\Pi_2(\tilde{P}_1 - (\alpha_1 - \alpha_2)) < \Pi_2(\frac{\alpha_2}{\alpha_1}\tilde{P}_1) = 0$, the online store does not use the aggressive pricing strategy either.

Second, suppose $\tilde{P}_1 \geqslant \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$ (i.e., $\frac{\alpha_2\tilde{P}_1+\alpha_1\tilde{c}_2}{2\alpha_1} \leqslant \tilde{P}_1 - \alpha_1 + \alpha_2$) so the online store sets $\tilde{P}_2 = \tilde{P}_1 - \alpha_1 + \alpha_2$ under the moderate pricing strategy. Since this is the boundary price of the aggressive pricing strategy, I will only need to consider the aggressive pricing strategy. Since $\frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2} \leqslant \alpha_1 - \alpha_2 + \frac{\alpha_2+\tilde{c}_2}{2}$, there are two subcases: (i) when $\frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2} \leqslant \tilde{P}_1 \leqslant \alpha_1 - \frac{\alpha_2-\tilde{c}_2}{2}$, the online store sets $\tilde{P}_2 = \tilde{P}_1 - \alpha_1 + \alpha_2$ and her profit is

$$\Pi_2(\tilde{P}_1 - \alpha_1 + \alpha_2) = \frac{(\alpha_1 - \tilde{P}_1)(\tilde{P}_1 - \alpha_1 + \alpha_2 - \tilde{c}_2)}{\alpha_2} \geqslant 0.$$

The last inequality follows from $\tilde{P}_1 \leqslant \alpha_1$ (otherwise no consumer visits the physical store) and $\alpha_1 - \alpha_2 + \tilde{c}_2 \leqslant \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$ (by $\tilde{c}_2 \leqslant \alpha_2$). (ii) When $\tilde{P}_1 > \alpha_1 - \frac{\alpha_2-\tilde{c}_2}{2}$, the online store sets $\tilde{P}_2 = \frac{\alpha_2+\tilde{c}_2}{2}$ and her profit is

$$\Pi_2(\frac{\alpha_2+\tilde{c}_2}{2}) = \frac{(\alpha_2-\tilde{c}_2)^2}{4\alpha_2}.$$

Lastly, suppose $\frac{\alpha_1}{\alpha_2}\tilde{c}_2 < \tilde{P}_1 < \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2}$ (i.e., $\tilde{P}_1 - \alpha_1 + \alpha_2 < \frac{\alpha_2\tilde{P}_1+\alpha_1\tilde{c}_2}{2\alpha_1} < \frac{\alpha_2}{\alpha_1}\tilde{P}_1$) so the online store sets $\tilde{P}_2 = \frac{\alpha_2\tilde{P}_1+\alpha_1\tilde{c}_2}{2\alpha_1}$ under the moderate pricing strategy and her profit is

$$\Pi_2(\frac{\alpha_2\tilde{P}_1+\alpha_1\tilde{c}_2}{2\alpha_1}) = \frac{(\alpha_2\tilde{P}_1-\alpha_1\tilde{c}_2)^2}{4\alpha_1\alpha_2(\alpha_1-\alpha_2)}.$$

Now consider the aggressive pricing strategy. Since $\tilde{P}_1 < \frac{2\alpha_1(\alpha_1-\alpha_2)+\alpha_1\tilde{c}_2}{2\alpha_1-\alpha_2} \leqslant \alpha_1-\alpha_2+\frac{\alpha_2+\tilde{c}_2}{2}$, the online store sets $\tilde{P}_2 = \tilde{P}_1 - \alpha_1 + \alpha_2$, which coincides with her boundary price under the moderate pricing strategy. Since the optimal moderate price is interior, the online store should not use the aggressive pricing strategy.

Summarising the above results lead to the online store's best response, as given by (4.2).

Next consider the physical store. For any $\tilde{P}_2$, the physical store's profit is:

$$\Pi_1(\tilde{P}_2) = \begin{cases} 0, & \text{if } \frac{\tilde{P}_1-\tilde{P}_2}{\alpha_1-\alpha_2} \geqslant 1, \\ (1-\frac{\tilde{P}_1-\tilde{P}_2}{\alpha_1-\alpha_2})(\tilde{P}_1-\tilde{c}_1), & \text{if } \frac{\tilde{P}_1}{\alpha_1} < \frac{\tilde{P}_1-\tilde{P}_2}{\alpha_1-\alpha_2} < 1, \\ (1-\frac{\tilde{P}_1}{\alpha_1})(\tilde{P}_1-\tilde{c}_1), & \text{if } \frac{\tilde{P}_1-\tilde{P}_2}{\alpha_1-\alpha_2} \leqslant \frac{\tilde{P}_1}{\alpha_1}. \end{cases}$$

Similar to the online store, the physical store also has two pricing strategies. First, she prices low (i.e., $\frac{\tilde{P}_1-\tilde{P}_2}{\alpha_1-\alpha_2} \leqslant \frac{\tilde{P}_1}{\alpha_1}$ or $\tilde{P}_1 \leqslant \frac{\alpha_1}{\alpha_2}\tilde{P}_2$) to drive the online store out of the market (the aggressive pricing strategy, hereafter). Here the best-response is

$$\tilde{P}_1(\tilde{P}_2) = \min\{\frac{\alpha_1+\tilde{c}_1}{2}, \frac{\alpha_1}{\alpha_2}\tilde{P}_2\}$$

and the interior optimal profit is

$$\Pi_1(\frac{\alpha_1+\tilde{c}_1}{2}) = \frac{(\alpha_1-\tilde{c}_1)^2}{4\alpha_1}.$$

Second, she prices high (i.e., $\frac{\tilde{P}_1}{\alpha_1} < \frac{\tilde{P}_1-\tilde{P}_2}{\alpha_1-\alpha_2} < 1$ or $\frac{\alpha_1}{\alpha_2}\tilde{P}_2 < \tilde{P}_1 < \tilde{P}_2+\alpha_1-\alpha_2$) so both stores obtain positive profit (the moderate pricing strategy, hereafter). Here the best-response is

$$\tilde{P}_1(\tilde{P}_2) = \min\{\max\{\frac{\alpha_1-\alpha_2+\tilde{c}_1+\tilde{P}_2}{2}, \frac{\alpha_1}{\alpha_2}\tilde{P}_2\}, \tilde{P}_2+\alpha_1-\alpha_2\}$$

and the interior optimal profit is

$$\Pi_1(\frac{\alpha_1-\alpha_2+\tilde{c}_1+\tilde{P}_2}{2}) = \frac{(\alpha_1-\alpha_2-\tilde{c}_1+\tilde{P}_2)^2}{4(\alpha_1-\alpha_2)}.$$

Now I characterise the physical store's best response for any given $\tilde{P}_2$. As the first case, suppose $\tilde{P}_2 \leqslant \tilde{c}_1-\alpha_1+\alpha_2$ (i.e., $\frac{\alpha_1-\alpha_2+\tilde{c}_1+\tilde{P}_2}{2} \geqslant \tilde{P}_2+\alpha_1-\alpha_2$) so the physical store's profit is 0 under moderate pricing, Under aggressive pricing, she sets $\tilde{P}_1 = \frac{\alpha_1}{\alpha_2}\tilde{P}_2$ (note that $\frac{\alpha_1}{\alpha_2}\tilde{P}_2 \leqslant \frac{\alpha_1+\tilde{c}_1}{2}$ when $\tilde{P}_2 \leqslant \tilde{c}_1-\alpha_1+\alpha_2$ by $\tilde{c}_1 \leqslant \alpha_1$). Since this is the boundary price of moderate pricing, the physical store cannot obtain positive profit under aggressive pricing.

Second, suppose $\tilde{P}_2 \geqslant \frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2}$ (i.e., $\frac{\alpha_1-\alpha_2+\tilde{c}_1+\tilde{P}_2}{2} \leqslant \frac{\alpha_1}{\alpha_2}\tilde{P}_2$) so the physical store sets $\tilde{P}_1 = \frac{\alpha_1}{\alpha_2}\tilde{P}_2$ under moderating pricing. Since this is the boundary price of aggressive pricing, we will only need to consider the aggressive pricing strategy. Since $\frac{\alpha_2(\alpha_1+\tilde{c}_1)}{2\alpha_1} \geqslant \frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2}$ by $\tilde{c}_1 \leqslant \alpha_1$, there are two subcases: (i) when $\frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2} \leqslant \tilde{P}_2 \leqslant \frac{\alpha_2(\alpha_1+\tilde{c}_1)}{2\alpha_1}$, the physical store sets $\tilde{P}_1 = \frac{\alpha_1}{\alpha_2}\tilde{P}_2$ and her profit is

$$\Pi_1(\frac{\alpha_1}{\alpha_2}\tilde{P}_2) = \frac{(\alpha_2-\tilde{P}_2)(\alpha_1\tilde{P}_2-\alpha_2\tilde{c}_1)}{\alpha_2^2} \geqslant 0.$$

The last inequality follows from $\tilde{P}_2 \leqslant \alpha_2$ (otherwise no consumer visits the online store) and $\frac{\alpha_2}{\alpha_1}\tilde{c}_1 \leqslant \frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2}$. (ii) When $\tilde{P}_2 > \frac{\alpha_2(\alpha_1+\tilde{c}_1)}{2\alpha_1}$, the physical store sets $\tilde{P}_1 = \frac{\alpha_1+\tilde{c}_1}{2}$ and her profit is

$$\Pi_1(\frac{\alpha_1+\tilde{c}_1}{2}) = \frac{(\alpha_1-\tilde{c}_1)^2}{4\alpha_1}.$$

Lastly, suppose $\tilde{c}_1-\alpha_1+\alpha_2 < \tilde{P}_2 < \frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2}$ (i.e., $\frac{\alpha_1}{\alpha_2}\tilde{P}_2 < \frac{\alpha_1-\alpha_2+\tilde{c}_1+\tilde{P}_2}{2} < \tilde{P}_2 + \alpha_1-\alpha_2$) so the physical store sets $\tilde{P}_1 = \frac{\alpha_1-\alpha_2+\tilde{c}_1+\tilde{P}_2}{2}$ under moderate pricing and her profit is

$$\Pi_1(\frac{\alpha_1-\alpha_2+\tilde{c}_1+\tilde{P}_2}{2}) = \frac{(\alpha_1-\alpha_2-\tilde{c}_1+\tilde{P}_2)^2}{4(\alpha_1-\alpha_2)}.$$

Now consider the aggressive pricing strategy. Since $\tilde{P}_2 < \frac{\alpha_2(\alpha_1-\alpha_2)+\alpha_2\tilde{c}_1}{2\alpha_1-\alpha_2} \leqslant \frac{\alpha_2(\alpha_1+\tilde{c}_1)}{2\alpha_1}$, the physical store sets $\tilde{P}_1 = \frac{\alpha_1}{\alpha_2}\tilde{P}_2$, which coincides with her boundary price under the moderate pricing strategy. Since the optimal moderate price is interior, the physical store should not use the aggressive pricing strategy.

Summarising the above results lead to the online store's best response, as given by (4.3).

C.2 Proofs

Proof of Proposition 12. First I characterise the online store's optimal return policy. Since Π_2 is invariant of t (I will verify that consumers are willing to return unfit product instead of salving them even at the highest return hassle T_1) for any given $\tilde{P}_2$, the online store is indifferent between any return hassle allocation under MBG. Moreover, $\frac{\partial \Pi_2}{\partial \tilde{c}_2} > 0$ when $\frac{\tilde{P}_1-\tilde{P}_2}{\alpha_1-\alpha_2} \leqslant \frac{\tilde{P}_1}{\alpha_1}$. Therefore, according to the expression of $\tilde{c}_2$, the online store offers MBG when $S-T_1 \geqslant d$ and does not offer MBG otherwise. Note that when she offers MBG, consumers choose to return unfit products at even the highest return hassle T_1: consumers' benefit of returning the product (i.e., $-T_1$) is higher than the benefit of salvaging the product (i.e., $d-P_2$) because $P_2 \geqslant c_1 > S \geqslant d+T_1$.

Next I will characterise all equilibria. First consider Equilibrium B where both stores obtain positive profit in equilibrium. According to each store's best response in (4.2)-

(4.3), this equilibrium is supported only when both stores use the moderate pricing strategy. Therefore,

$$\begin{cases} \tilde{P}_2 &= \frac{\alpha_2 \tilde{P}_1 + \alpha_1 \tilde{c}_2}{2\alpha_1}, \\ \tilde{P}_1 &= \frac{\alpha_1 - \alpha_2 + \tilde{c}_1 + \tilde{P}_2}{2}. \end{cases}$$

Solving the simultaneous equations leads to the interior optimal prices, as given by:

$$\tilde{P}_{1B}^* = \frac{2\alpha_1(\alpha_1 - \alpha_2) + 2\alpha_1 \tilde{c}_1 + \alpha_1 \tilde{c}_2}{4\alpha_1 - \alpha_2},$$
$$\tilde{P}_{2B}^* = \frac{\alpha_2(\alpha_1 - \alpha_2) + \alpha_2 \tilde{c}_1 + 2\alpha_1 \tilde{c}_2}{4\alpha_1 - \alpha_2}.$$

The expressions of equilibrium demand are:

$$D_{1B}^* = \frac{2\alpha_1(\alpha_1 - \alpha_2) - (2\alpha_1 - \alpha_2)\tilde{c}_1 + \alpha_1 \tilde{c}_2}{(\alpha_1 - \alpha_2)(4\alpha_1 - \alpha_2)},$$
$$D_{2B}^* = \frac{\alpha_1[\alpha_2(\alpha_1 - \alpha_2) + \alpha_2 \tilde{c}_1 - (2\alpha_1 - \alpha_2)\tilde{c}_2]}{\alpha_2(\alpha_1 - \alpha_2)(4\alpha_1 - \alpha_2)}.$$

By $\Pi_{1B}^* = (\tilde{P}_{1B}^* - \tilde{c}_1)D_{1B}^*$ and $\Pi_{2B}^* = (\tilde{P}_{2B}^* - \tilde{c}_2)D_{2B}^*$, I can derive the equilibrium profits, as given by:

$$\Pi_{1B}^* = \frac{[2\alpha_1(\alpha_1 - \alpha_2) - (2\alpha_1 - \alpha_2)\tilde{c}_1 + \alpha_1 \tilde{c}_2]^2}{(\alpha_1 - \alpha_2)(4\alpha_1 - \alpha_2)^2},$$
$$\Pi_{2B}^* = \frac{\alpha_1[\alpha_2(\alpha_1 - \alpha_2) + \alpha_2 \tilde{c}_1 - (2\alpha_1 - \alpha_2)\tilde{c}_2]^2}{\alpha_2(\alpha_1 - \alpha_2)(4\alpha_1 - \alpha_2)^2}.$$

Note that Equilibrium B exists when $D_{1B}^* > 0$ and $D_{2B}^* > 0$, i.e.,

$$\frac{-\alpha_2(\alpha_1 - \alpha_2) + (2\alpha_1 - \alpha_2)\tilde{c}_2}{\alpha_2} < \tilde{c}_1 < \frac{2\alpha_1(\alpha_1 - \alpha_2) + \alpha_1 \tilde{c}_2}{2\alpha_1 - \alpha_2}.$$

Moreover, under this condition, no store has any profitable deviation (e.g., aggressive pricing or leaving the market) from $\tilde{P}_{1B}^*$ and $\tilde{P}_{2B}^*$ because

$$\frac{\alpha_1}{\alpha_2}\tilde{c}_2 < \tilde{P}_{1B}^* < \frac{2\alpha_1(\alpha_1 - \alpha_2) + \alpha_1 \tilde{c}_2}{2\alpha_1 - \alpha_2},$$
$$\tilde{c}_1 - \alpha_1 + \alpha_2 < \tilde{P}_{2B}^* < \frac{\alpha_2(\alpha_1 - \alpha_2) + \alpha_2 \tilde{c}_1}{2\alpha_1 - \alpha_2}.$$

Second, consider Equilibrium P where only the physical store participates in the market. According to Equation (4.3), there are two subcases. First, $\frac{\alpha_2(\alpha_1 - \alpha_2) + \alpha_2 \tilde{c}_1}{2\alpha_1 - \alpha_2} \leqslant \tilde{P}_2 \leqslant \frac{\alpha_2(\alpha_1 + \tilde{c}_1)}{2\alpha_1}$ and the physical store sets $\tilde{P}_2 = \frac{\alpha_1}{\alpha_2}\tilde{P}_2$. This cannot constitute an equilibrium because by Equation (4.2), the online store leaves the market instead of setting $\tilde{P}_2 \geqslant \frac{\alpha_2(\alpha_1 - \alpha_2) + \alpha_2 \tilde{c}_1}{2\alpha_1 - \alpha_2}$. Second, the physical store sets

$$\tilde{P}_{1P}^* = \frac{\alpha_1 + \tilde{c}_1}{2}$$

and the online store leaves the market. Therefore, the demand and expected profit of the physical store are

$$D_{1P}^{*}=1-\frac{\tilde{P}_{1P}^{*}}{\alpha_1}=\frac{\alpha_1-\tilde{c}_1}{2\alpha_1},$$
$$\Pi_{1P}^{*}=(\tilde{P}_{1P}^{*}-\tilde{c}_1)D_{1P}^{*}=\frac{(\alpha_1-\tilde{c}_1)^2}{4\alpha_1}.$$

Note that $D_{1P}^{*}>0$ (i.e., $\tilde{c}_1<\alpha_1$) according to Assumption 3. Next I verify that no store has any profitable deviation when $\tilde{c}_1\leqslant\frac{-\alpha_1\alpha_2+2\alpha_1\tilde{c}_2}{\alpha_2}$. The physical store has no profitable deviation because by (4.3), $\tilde{P}_1^{*}=\frac{\alpha_1+\tilde{c}_1}{2}$ is her best response when the online store is out of market. The online store has no profitable deviation because when $\tilde{c}_1\leqslant\frac{-\alpha_1\alpha_2+2\alpha_1\tilde{c}_2}{\alpha_2}$, $\tilde{P}_{1P}^{*}=\frac{\alpha_1+\tilde{c}_1}{2}\leqslant\frac{\alpha_1}{\alpha_2}\tilde{c}_2$ so the online store should leave the market according to (4.2).

Finally, consider Equilibrium O where only the online store participates in the market. As in the analysis of Equilibrium P, I can derive the equilibrium price, demand, and profit of the online store as follows:

$$\tilde{P}_{2O}^{*}=\frac{\alpha_2+\tilde{c}_2}{2},$$
$$D_{2O}^{*}=1-\frac{\tilde{P}_{2O}^{*}}{\alpha_2}=\frac{\alpha_2-\tilde{c}_2}{2\alpha_2},$$
$$\Pi_{2O}^{*}=(\tilde{P}_{2O}^{*}-\tilde{c}_2)D_{2O}^{*}=\frac{(\alpha_2-\tilde{c}_2)^2}{4\alpha_2}.$$

It can be verified that both stores have no profitable deviation when $\tilde{c}_1\geqslant\frac{2\alpha_1-\alpha_2+\tilde{c}_2}{2}$. □

Proof of Proposition 13. Since Π_{1B}^{*} and Π_{2B}^{*} are continuous in c_1 and c_2, I will proceed by treating $c_1=c_2=0$. Note that the comparative statics continue to hold if c_1 and c_2 are positive yet sufficiently low.

Denote $W\equiv\max\{d,S-T_1\}$. Naturally $W>0$.

$$\begin{aligned}\frac{\partial\Pi_{1B}^{*}}{\partial\alpha_1}=&[2\alpha_1(\alpha_1-\alpha_2)-\alpha_1(1-\alpha_2)W-(2\alpha_1-\alpha_2)K][2(\alpha_1-\alpha_2)(4\alpha_1^2-3\alpha_1\alpha_2+2\alpha_2^2)\\&+(4\alpha_1^2+\alpha_1\alpha_2-2\alpha_2^2)(1-\alpha_2)W+(8\alpha_1^2-10\alpha_1\alpha_2+5\alpha_2^2)K]\\&/[(\alpha_1-\alpha_2)^2(4\alpha_1-\alpha_2)^3].\end{aligned}$$

Since the denominator is positive, in order to prove Proposition 13(i) it suffices to prove that the nominator is positive, which holds since $2\alpha_1(\alpha_1-\alpha_2)-\alpha_1(1-\alpha_2)W-(2\alpha_1-\alpha_2)K>0$ by $D_{1B}^{*}>0$ and $2(\alpha_1-\alpha_2)(4\alpha_1^2-3\alpha_1\alpha_2+2\alpha_2^2)+(4\alpha_1^2+\alpha_1\alpha_2-2\alpha_2^2)(1-\alpha_2)W+(8\alpha_1^2-10\alpha_1\alpha_2+5\alpha_2^2)K>0$ by $\alpha_1>\alpha_2$.

$$\frac{\partial \Pi_{1B}^*}{\partial \alpha_2} = -[2\alpha_1(\alpha_1-\alpha_2)-\alpha_1(1-\alpha_2)W-(2\alpha_1-\alpha_2)K][2\alpha_1(\alpha_1-\alpha_2)(2\alpha_1+\alpha_2)- \\ \alpha_1(8\alpha_1^2-4\alpha_1\alpha_2-6\alpha_1-\alpha_2^2+3\alpha_2)W+(4\alpha_1^2-2\alpha_1\alpha_2+\alpha_2^2)K] \\ /[(\alpha_1-\alpha_2)^2(4\alpha_1-\alpha_2)^3].$$

In order to prove that Π_{1B}^* strictly decreases in α_2, it suffices to show that $2\alpha_1(\alpha_1-\alpha_2)(2\alpha_1+\alpha_2)-\alpha_1(8\alpha_1^2-4\alpha_1\alpha_2-6\alpha_1-\alpha_2^2+3\alpha_2)W>0$. This inequality holds by $2\alpha_1(\alpha_1-\alpha_2)(2\alpha_1+\alpha_2)-\alpha_1(8\alpha_1^2-4\alpha_1\alpha_2-6\alpha_1-\alpha_2^2+3\alpha_2)W>\alpha_1(1-\alpha_2)(2\alpha_1+\alpha_2)W-\alpha_1(8\alpha_1^2-4\alpha_1\alpha_2-6\alpha_1-\alpha_2^2+3\alpha_2)W=2\alpha_1(1-\alpha_1)(4\alpha_1-\alpha_2)W>0$.

$$\frac{\partial \Pi_{2B}^*}{\partial \alpha_2} = \frac{\alpha_1\{[\alpha_2(\alpha_1-\alpha_2)+(2\alpha_1-\alpha_2)(1-\alpha_2)W+\alpha_2K][E-FW+GK]\}}{\alpha_2^2(\alpha_1-\alpha_2)^2(4\alpha_1-\alpha_2)^3},$$

where $E\equiv\alpha_1\alpha_2(4\alpha_1-7\alpha_2)(\alpha_1-\alpha_2)$, $F\equiv 8\alpha_1^3-2\alpha_2^3-10\alpha_1^2\alpha_2^2+9\alpha_1\alpha_2^2-18\alpha_1^2\alpha_2+5\alpha_1\alpha_2^3+8\alpha_1^3\alpha_2$ and $G\equiv\alpha_2(4\alpha_1^2+\alpha_1\alpha_2-2\alpha_2^2)$. Since $\alpha_2(\alpha_1-\alpha_2)+(2\alpha_1-\alpha_2)(1-\alpha_2)W+\alpha_2K$ by $D_{1B}^*>0$, the sign of $\frac{\partial \Pi_{2B}^*}{\partial \alpha_2}$ is the same as $E-FW+GK$. Define $f(\alpha_2)\equiv E-FW+GK=[7\alpha_1-5W\alpha_1+2W-2K]\alpha_2^3-[11\alpha_1^2-K\alpha_1-10W\alpha_1^2+9W\alpha_1]\alpha_2^2+[4\alpha_1^3+4K\alpha_1^2+18W\alpha_1^2-8W\alpha_1^3]\alpha_2-8W\alpha_1^3$. $f(\alpha_2)$ is in a cubic form and $7\alpha_1-5W\alpha_1+2W-2K>0$. As a result, in order to prove that there exists $0<\underline{\alpha}<\bar{\alpha}<\alpha_1$ such that Π_{2B}^* strictly decreases in α_2 when $\underline{\alpha}<\alpha_2<\bar{\alpha}$ and strictly increases in α_2 when $\bar{\alpha}<\alpha_2<\alpha_1$, it suffices to prove that $f(0)<0$ and $f(\alpha_1)>0$. $f(0)=-8W\alpha_1^3<0$ and $f(\alpha_1)=3\alpha_1^3[K+(1-\alpha_1)W]>0$, so both inequalities hold.

$$\frac{\partial \Pi_{2B}^*}{\partial \alpha_1} = [\alpha_2(\alpha_1-\alpha_2)+(2\alpha_1-\alpha_2)(1-\alpha_2)W+\alpha_2K][\alpha_2(2\alpha_1+\alpha_2)(\alpha_1-\alpha_2)- \\ (4\alpha_1^2-2\alpha_1\alpha_2+\alpha_2^2)(1-\alpha_2)W-(8\alpha_1^2-4\alpha_1\alpha_2-\alpha_2^2)K]/[(\alpha_1-\alpha_2)^2(4\alpha_1-\alpha_2)^3].$$

Therefore, $\frac{\partial \Pi_{2B}^*}{\partial \alpha_1}>0$ if and only if $(4\alpha_1^2-2\alpha_1\alpha_2+\alpha_2^2)(1-\alpha_2)W+(8\alpha_1^2-4\alpha_1\alpha_2-\alpha_2^2)K<\alpha_2(2\alpha_1+\alpha_2)(\alpha_1-\alpha_2)$. Define $G(\alpha_1)=\alpha_2(2\alpha_1+\alpha_2)(\alpha_1-\alpha_2)-(4\alpha_1^2-2\alpha_1\alpha_2+\alpha_2^2)(1-\alpha_2)W-(8\alpha_1^2-4\alpha_1\alpha_2-\alpha_2^2)K$. $G'(\alpha_1)=(4\alpha_1-\alpha_2)[\alpha_2-2(1-\alpha_2)W-4K]$. When $(4-2\alpha_2+\alpha_2^2)(1-\alpha_2)W+(8-4\alpha_2-\alpha_2^2)K<(2+\alpha_2)(1-\alpha_2)\alpha_2$, $G(1)>0$, $G(\alpha_2)<0$, and $\alpha_2-2(1-\alpha_2)W-4K>0$ so $G'(\alpha_1)>0$. As a result, the first part of Proposition 13(iii) holds. When $(4-2\alpha_2+\alpha_2^2)(1-\alpha_2)W+(8-4\alpha_2-\alpha_2^2)K>(2+\alpha_2)(1-\alpha_2)\alpha_2$ and $\alpha_2-2(1-\alpha_2)W-4K>0$, $G(\alpha_1)$ strictly increases in α_1 and $G(1)<0$, so Π_{2B}^* strictly decreases in α_1. When $(4-2\alpha_2+\alpha_2^2)(1-\alpha_2)W+(8-4\alpha_2-\alpha_2^2)K>(2+\alpha_2)(1-\alpha_2)\alpha_2$ and α_2-

$2(1-\alpha_2)W-4K \leqslant 0$, $G'(\alpha_1) \leqslant 0$ and $G(\alpha_2) < 0$, so Π^*_{2B} strictly decreases in α_1. Therefore, the second part of Proposition 13(iii) holds. $\square$

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