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SYSTEMS INNOVATION, INERTIA AND PLIABILITY: A MATHEMATICAL EXPLORATION WITH IMPLICATIONS FOR CLIMATE CHANGE ABATEMENT

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12 March 2018

This paper develops a stylised mathematical interpretation of innovation and inertia in economic systems, characteristics which feature in economics literature traceable back at least to Schumpeter and other economic theorists of innovation, as well as economic historians. Such characteristics are particularly important in energy systems and their potential response to climate change, where it is important to distinguish operational/fuel substitution from investment because the latter necessarily embodies both inertia and innovation, in systems as well as technologies.

We argue that integrated assessments of climate abatement need to focus on investment, including the associated characteristics of both learning and inertia, and derive in detail the mathematical basis for incorporating these factors through marginal investment cost curves. From this we also introduce the concept of 'pliability' as an expression of the ratio between costs which are significant but transitional (including learning investments, infrastructure and overcoming inertia), as compared to the enduring costs implied by purely exogenous technology assumptions.

We then incorporate these features in a global model of optimal climate mitigation and show that they can generate a very different profile and pattern of results from traditional 'integrated assessment' models, pinpointing the key sensitivities. We conclude that alongside all the attention devoted to evaluating climate change impacts and technology scenarios, far more effort should be devoted to understanding the structural characteristics of how the global energy system may respond to climate change mitigation.

EPRG WORKING PAPER

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1. Introduction: the historical intellectual context

Economic theory has long struggled with the phenomena of innovation and inertia. Solow (et al., 1953; 1957) acknowledged innovation as a crucial component of the 'residual' in his basic growth model, and more than half a century of subsequent research has underlined its importance to economic development. Preceding Solow, Schumpeter (1947) - best known for clarifying the distinction between invention and innovation, and for his concept of 'creative destruction' - emphasised that a crucial function of the entrepreneur is 'the doing of new things, or the doing of things that are already being done in a new way'.[6] Arrow (1962) was the first to formalise analysis of another crucial factor, namely 'learning by doing', that captures effects such as scale economies. Theories of evolutionary economics, often traced back to Nelson & Winter (1982), in effect build upon such concepts. Modern growth theory now habitually emphasises the importance of innovation as an engine of growth (Romer, 1986; Aghion & Howitt, 1992; Acemoglu, 1998, 2002; Acemoglu et al. 2012).

Some of these approaches have also discussed inertia and potential costs of change. Thus, Schumpeter (1947, p. 152) refers to 'the resistances and difficulties which action always meets with outside of the ruts of established practice'; and notes (p.157) that 'the teaching [...] according to which capital "migrates" from declining to rising industries, is obviously unrealistic: the capital "invested" in railroads does not migrate into trucking and airport transportation but will perish in and with the railroads'. Subsequent theories and empirics have similarly emphasised the importance of inertia (see e.g. Freeman & Perez, 1988; Grubler et al., 1999; Geels, 2002, 2005). The combination of these forces form some of the intellectual foundations for theories of evolutionary economics, path dependence and lock-in (Unruh, 2000) in economic systems, and particularly in complex socio-technological systems when viewed over long timescales (e.g. Beinhocker, 2006).

In contrast to the growing complexity of much of this literature, our inquiry in this paper can be considered as attempting to develop a simple, stylised mathematics of some core implication of innovation (including creative responses, learning and other adaptive responses), along with inertia and path dependence, in economic systems.

Application: energy systems response to climate change mitigation

We then apply this to a problem concerning some of the largest and most complex capital-intensive systems on the planet, over very long timescales: namely, the challenge that climate change poses to the development of the global energy system.

[6] From this, Schumpeter drew another distinction, less acknowledged: between what he called 'adaptive response' (using more or less hands, brains and capital to adapt the mix of established tasks – in effect, resource substitution) as compared to 'creative response' – when 'the economy or an industry or firms do something else, something that is outside the range of existing practice.'(Schumpeter, 1947, p. 150). In this paper we use innovation as the broadest term, but emphasize the aspects relating to investment, which can encompass both aspects and also structural and system-level responses.

Here, if anywhere, we might expect to see substantial implications arising from the underlying phenomena.

Developments over the past few years in this field suggest a serious need to try and take account of the factors explored here (learning and inertia). For example, most economic 'integrated assessment' studies of the problem embodied exogenous assumptions about the cost of clean energy that have been significantly overtaken by major cost reductions in key renewable sources, partly due to extensive learning and scale effects. The most extensive empirical study of this phenomenon (Bettencourt et al., 2013) finds that related innovation has been strongly correlated with deployment even as measured through patents, neglecting pure scale economy or related factors[7]. These cost reductions, however, required large (and relatively expensive) initial investments, and their growing scale has also led to major and costly disruptions in the incumbent industry (particularly electric utility) sectors.

Our analysis thus explores a new way to take account of learning and the sources and characteristics of inertia in stylised economic models, in particular to understand better the economics and dynamics of private (consumer and business) sector responses to a carbon price or equivalent emissions constraint. We go on to demonstrate that, indeed, optimal climate change responses may be profoundly affected by such a focus in economic perspectives.

We structure this paper in three main parts. Part I reviews relevant literature and clarifies our terminology and focus, including emphasising the distinction between operational substitution and investment, and distinctions between traditional 'abatement cost' curves and the cost of *investment* in abatement technologies: we argue the need to focus on this because it is capital investment which is most obviously associated both with long-lived assets and technology learning.

In Part II we develop a micro-economic framework to represent capital investment in abatement technologies, and both the learning and lifetimes associated with it. First, we derive the functional form of the cost of scrapping capital earlier than its expected lifetime. This shows a first relationship of the costs of emissions abatement to its rate: the faster emissions are reduced beyond a certain rate, the more stranded assets accumulate. Second, we explore investment for intermediate production and supply chains, and show that this has an inertial term, related to the non-linearity of the abatement cost curve and learning rates. Third, we consider processes of technology adoption and diffusion and again suggest a functional form relating cost to the rate of emissions abatement.

[7] Bettencourt et al (2013) document 'A sharp increase in rates of patenting [during 2000-2009], particularly in renewable technologies, despite continued low levels of R&D funding. To solve the puzzle of fast innovation despite modest R&D increases, we develop a model that explains the nonlinear response observed in the empirical data of technological innovation to various types of investment. The model reveals a regular relationship between patents, R&D funding, and growing markets across technologies, and accurately predicts patenting rates at different stages of technological maturity and market development. We show quantitatively how growing markets have formed a vital complement to public R&D in driving innovative activity. These two forms of investment have each leveraged the effect of the other in driving patenting trends over long periods of time.'

In Part III, we take these concepts to the systems/macro-level. We argue that the essential dynamic features can be represented in terms of two main factors. The *adaptability* of the techno-economic system (e.g. to pressures such as emissions pricing or constraints) determines the extent to which costs of meeting a constrain will endure or diminish due to learning and other adjustments. The *transitional costs* involved in moving from one state to another embody the various sources of inertia we identified at the microeconomic level, plus additional aspects of transitional costs including the investments in learning that drive reductions in enduring costs. We introduce and define the term *pliability* to express the ratio of these transitional costs, compared to the ongoing costs associated with meeting a given level of emissions constraint.

The microeconomic analysis thereby informs the structure of a simple two-term model, which we then apply to an optimising cost-benefit appraisal of global climate change. The emphasis is not upon prediction, but rather to show in a transparent manner how (often implicit) assumptions around learning and inertia actually drive results, and how the ratio between these forces – the 'pliability' of the system - is a determining factor in optimal policy.

Part I: Literature and concepts

2. Existing modelling approaches

Efforts to conduct global economic assessment of climate change responses have spawned a range of 'Integrated Assessment Models' (IAMs), which seek to balance the damages of climate change against the costs of emission cutbacks, or otherwise seek an optimal trajectory to meet an emissions constraint. The vast majority of such models represent mitigation costs as a function of the degree of mitigation, relative to an assumed reference trajectory (e.g. as reviewed in Clarke et al., 2014). In such models, if there is no endogenous learning, timing tends to enter in quite indirect ways, for example as a product of discount rates, or as affected by the economics of waiting for improved technologies.

Whilst various energy sector models have developed capacity to model endogenous technical change, few IAMs (with a climate damage module) have done so, one example being the PAGE model which explicitly incorporated learning-by-doing (Alberth and Hope, 2006). Still, the rate of change tends not to enter as a *direct* factor determining abatement costs in any of these IAMs.

The most widespread efforts to represent inertia in abatement have focused on capital stock lifetimes in energy system models. These date back to at least the early 1990s (e.g. Manne & Richels, 1992). Early forays looking at implications of inertia in its own right included those of Ha-Duong et al., (1997). But little attention was paid in these efforts to understanding or formally characterising the various sources of inertia (or of learning) and, consequently, quite how these might be more fully represented. By far the most extensive effort is by Vogt-Schilb et al. (2017), who focus on the implications of capital stock lifetimes for optimal strategy with a fixed concentration limit, but do not incorporate learning, or optimal cost-benefit analysis.

Modelling studies have repeatedly underlined the importance of technology. But efforts to represent technology, innovation, and inertia have followed mostly different tracks. As noted, most studies continue to use exogenous assumptions about future technology costs, with ever increasing degrees of technological detail (as seen in the IPCC Fifth Assessment).

2.1. Representing learning and diffusion processes in global energy-economy models: efforts and challenges

In reaction to purely exogenous cost assumptions, early studies using Learning-by-Doing, building on the early forays of BCG (1972), emphasised the potential for non-linearities and bifurcations in pathway costs (e.g. Gritsevskyi & Nakićenovi, 2000).

Meanwhile, the closely connected concept of diffusion of innovations is well established, but has traditionally been studied separately (Rogers, 2010). Diffusion and learning have however a close relationship as they re-inforce each other, where learning leads to further adoption, which induces further learning and so on.

A comparative study over a decade ago (the Innovation Modeling Comparison (IMCP) project) identified over a dozen global models incorporating endogenous innovation in various ways (Grubb et al., 2006; Edenhofer et al., 2006) and the literature has continued to expand. Yet no consensus has emerged on how best to incorporate systems innovation, and it remains neglected in many of the most widely used global energy models.

We suggest that this is due to at least three factors: complexity, uncertainty, and the centrality of inertia.

Incorporating innovation is *complex*. The technology learning literature itself distinguishes 'learning by searching' from 'learning by doing' (Grubler, 2003), which have spawned sometimes divergent modelling approaches. Much of the wider literature is explicitly concerned with 'complex systems' and has earned the term 'complexity theory' (Anderson et al., 1988; Arthur, 1999). The individual processes and possibilities involved themselves are complex; together it seems they make for a degree of complexity which is hard for analysts to manage, and even harder for others to understand.

The non-linearity and potential for increasing returns to scale makes the resulting models frequently very sensitive. The Innovation Modeling Comparison (IMCP) project (Grubb et al., 2006; Edenhofer et al., 2006) underlined that models with endogenous technical change can be sensitive to assumptions (the 'butterfly effect' of chaos theory), in which minor changes may radically change outcomes. In turn, this creates a modelling environment within which optimisation is difficult, sometimes to the point of impossibility, in sharp contrast to the tractability of traditional linear programming or convexity assumptions. Tractability, however, is not generally robust scientific grounds onto which to support a choice of modelling approach (Mercure et al., 2016), where a clear danger exists to oversimplify reality.

The second major issue is *uncertainty*. Almost by definition, the outcome of 'learning by searching' (typically equated with R&D) is hard to predict. This approach is central to seminal work by Acemoglu et al. (2012), which projected that R&D combined with a small carbon tax could lead to radical emission reductions at very little economic cost. However, the assumptions were subject to a strong critique, which concluded that in fact their model, populated with equally or more plausible assumptions, would lead to the opposite conclusion (Pottier et al., 2014). Large differences between results of the three models of the RECIPE project (Edenhofer et al., 2006; Luderer et al., 2011), for example, are largely about different parametrisations of learning processes.

At first sight, learning-by-doing might seem subject to less uncertainty, as correlation between deployed volume and cost reduction has now been measured for hundreds of technologies (e.g. Weiss et al., 2010). However, the estimates span a wide range and the value may vary with stage of development (e.g. see review in Grubb et al., 2014, Chapter 9). And more fundamentally, correlation is not causality, as emphasised by Nordhaus (2014): scale and cost reduction could be expected to reinforce each other, and it is hard to disentangle induced learning from other forms of learning. Hence there is risk that empirical estimates of technology learning rates (i.e. cost reductions driven by scale of deployment) are exaggerated. Based on such considerations, Nordhaus (2014) warned against 'the perils of the learning model'.

However, complexity and uncertainty are not valid scientific reasons for ignoring important phenomena. Indeed, doing so, just because they are complex and hard to quantify, risks 'throwing out the baby with the bathwater'. Assuming *no* capacity for induced learning, scale economies, or 'creative responses' to energy or environmental policy is clearly inconsistent with multiple lines of evidence (see eg. note 7). In almost all cases, increasing deployment of a given technology has led to cost reductions, businesses frequently plan on the basis of assessed scale economies, and there is very strong evidence that the *ex-post* costs of meeting environmental constraints have generally proved to be far lower than *ex-ante* assumptions or expectations (Hammitt, 2000; Harrington et al., 2000). Assuming zero scope for learning or creative response – the implication of purely exogenous technology assumptions - is a poor and biased approximation to something we know to be positive. Assuming something to be zero just because it is hard to measure or model, may be the most misleading approach of all – particularly when diverse modelling approaches so far attempted suggest that these processes can *radically* change results.

Central to charting a path through this maze, we suggest that a third factor needs to be embedded with any theory of learning, namely a time dimension most simply conceived as *inertia*. We show below that without any time dimension, endogenous learning on its own either changes little. It is also notable that in Acemoglu et al. (2012), time enters *indirectly*, through an assumption on substitution elasticity between dirty and clean fuels, and it was precisely this parametrisation which formed a central focus of the critique by Pottier et al. (2014). They argued that the assumed substitution elasticity of 10 was without foundation and entirely implausible, and yet largely determined the results, because it meant that a modest gain from R&D could rapidly and radically change the energy system at little cost.

2.2. On operational substitution versus investment

We consider how the limited attention given to issues of systems innovation and inertia[8] may be partly due to the fact that much of theoretical literature does not distinguish between *operational substitution* and *investment in abatement.*

[8] In the rest of this paper we use the terms 'systems innovation and inertia' to encompass the multiple processes of (a) endogenous learning, scale economies and creative responses, and (b) the various factors which make the

Operational substitution – like switching between running a clean gas plant instead of a dirtier coal plant – has almost no inertia, and involves little or no learning. The cost of such abatement can in most respects be considered as entirely exogenous, and indeed the action is reversible. Time is largely irrelevant.

But tackling climate change is largely about investment, both in end-use (vehicles, buildings) and supply. An almost universal conclusion from the modelling literature is that most of the existing capital stock ultimately needs to be either retrofitted with additional stock (e.g. building insulation or carbon capture and storage), or replaced, much of it with new or improved low carbon technologies.

Capital stock can have extended lifetimes, which in the energy sector typically span decades (even longer in urban and transport infrastructure). With changes of capital stock, intermediate production and supply chains also need to be transformed. Investment with associated learning is thus crucial, and it all takes time (for a review see Hanna et al., 2015).

In this paper, we argue that even the most stylised economic models need to recognise that large scale investment involves both innovation and inertia, and that these need to be represented in integrated and consistent ways. We have a particular focus on clarifying some sources of inertia and their implications for the functional forms of emissions abatement, and then illustrate this with application to optimised emission trajectories, thus making an important extension to develop a stylised, but theoretically-grounded, integrated representation of systems innovation and inertia: a formalised approach to investment in cleaner technologies

If abatement were only about switching fuels (e.g. replacing coal with gas), actions in one period would have little influence on later periods. In standard cost-benefit 'integrated assessment' studies (e.g. Nordhaus, 2010; Stern, 2007), the apparently smooth profiles are basically determined by the discount rate and exogenous unit cost assumptions.

Such a framework may be appropriate to operational substitution. In reality, however, only a modest part of long-run mitigation is likely to be achieved by fuel substitution, at least based on existing assets (e.g. switching existing coal and gas power generations).

The dominant driver will be investment in new technologies with significantly lower greenhouse gas emissions. By technologies, we mean technological systems that produce societal services that the economy consumes (e.g. electricity, transport, chemicals, steel, cement, etc.), for which there exist alternatives. Typical examples of alternatives include: renewables or nuclear in place of fossil fuel power generation; more efficient vehicles and new types of vehicles able to run on different energy

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cost of energy provision (in one period directly dependent upon that in preceding periods; and hence, the cost of an absolute emissions level dependent on the emissions level in previous periods.

sources supplied through different infrastructure; and investment in ultra-efficient building stock.

Such investment in new technologies has two generic characteristics that distinguish it from purely operational substitution: learning, and inertia. In this section we begin the development of a mathematical framework that can be used to consider more directly issues of capital stock investment (including inertia) and associated innovation (including scale economies and endogenous learning).

2.3. Investment paths and marginal abatement cost curves

Providing energy for the twenty-first century will cost trillions of dollars every year (IEA, 2015). Under most 'business-as-usual' scenarios, investment allocation follows its historic trend, preserving the dominance of the fossil fuel industry. Energy investments are predominantly allocated towards technology and processes that support exploration, extraction, processing, transport and sale of fossil fuels. This is not consistent with climate science or international commitments, as adopted in the Paris Agreement (UN, 2015).

Following a different and low carbon trajectory is generally assessed to require higher investment, given that most new low carbon equipment (e.g. solar panels, buildings insulation) is more capital-intensive, per unit energy delivered (or saved), than equivalent alternatives (e.g. a coal plant).[9]

In most of the relevant literature it is common to display technology options in order of increasing net cost to define a 'marginal abatement cost curve' (MACC). This necessarily involves assumptions (whether explicit or implicit) about both capital and fuel costs, along with discount rates, which are generally assumed to be applicable across all global abatement, and the assumption that measures are implemented strictly in order of least cost assessed at this universal discount rate.

Whilst this approach is common, the use of technology-based cost curves has also generated considerable controversy (see eg. Murphy and Jaccard 2011); and review of MACC methodologies by Huang, Kuo and Chou 2016). In particular, attempts to apply empirical data in this form have generated considerable controversy between engineering and economic perspectives – controversy which, we argue, are partly explicable by reliance on data which bundles investment and fuel savings into a unified net abatement cost curve.

Instead, our approach starts with an explicit and distinct focus on investment. Capital investment often involves newer technologies with potential for learning, and

[9] The most extensive study of global investment implications of a low carbon scenario concludes: 'A fundamental reorientation of energy supply investments and a rapid escalation in low carbon demand-side investments would be necessary to achieve the 66% 2°C Scenario. Around USD 3.5 trillion in energy sector investments would be required on average each year between 2016 and 2050, compared to USD 1.8 trillion in 2015. Fossil fuel investment would decline, but would be largely offset by a 150% increase in renewable energy supply investment between 2015 and 2050. Total demand-side investment into low carbon technologies would need to surge by a factor of ten over the same period.' (IEA / IRENA, 2017)

tends to embody capital stock, and hence inertia, in ways that other forms of abatement – such as operating gas instead of coal plants – do not. These characteristics are our main focus, and we return to consider net abatement costs after developing methodologies to analyse these dimensions of investment in abatement technologies.

Part II: Micro-economic representation of investment, learning and inertia

3. Integrating across technology investment cost curves

3.1. On abatement investment cost curves

Whilst most of the methodology we develop in this paper could be applied to MACCs, our focus on investment as the source of both learning and inertia suggests instead attention to a 'marginal *investment* cost curve' (MICC), which focuses on the capital intensity of abatement options; Vogt-Schilb et al (2017) similarly emphasise the need to focus on *investment* when considering the dynamics of abatement. An example curve is illustrated in Figure 1.[10]

By construction, abatement technologies represented in the positive side of the MICC curve tend to be more investment-intensive than the carbon technologies they displace. This reflects our initial focus on investment – which as noted is key to low carbon scenarios and is the source of both inertia and learning. By focusing on investment in the end-use emitting systems we can set aside debate about whether some low carbon technologies might be (or become) more cost-effective than fossil fuels when the stream of fuel savings are taken into account at universal discount rate.

For analytic tractability, the traditional economic assumption that low carbon option must be more expensive than the fossil fuel alternative, in terms of NPV, is thus also amended to the assumption that it is more capital intensive. This is less restrictive and more consistent with the evidence and nature of the abatement problem: the up-front capital investment requirement is a major factor deterring the adoption of clean technologies, even when on an NPV basis they may result in net savings (as in many consumer efficiency options, and more recently some renewable energies); it is also striking that macro-assessments increasingly emphasise the capital investment

[10] Figure 1 is drawn from one of the biggest global assessments. The 'net cost' analysis of this has been controversial in part due to a strong element of apparently 'negative costs'. We suggest that in part, what appears as 'negative cost' arises because of the asset nature of the associated investments: many of the options identified by McKinsey as negative cost at a standardised (and low) discount rate either involve long term returns (such as buildings insulation) or involve the adoption of relatively new technologies (such as LED lighting, much more efficient vehicles, etc.), in sectors where decisions are taken by entities with generally high discount rates and/or behavioural characteristics including risk aversion to new technologies.

requirements of low carbon scenarios, rather than absolute cost differences.[11] A focus on investment correspondingly avoids the complexities of widely different discount rates in many end-use investments[12]. We briefly consider the net cost, including fuel savings, subsequently.

We represent abatement by invoking a certain number N of independent sectors where abatement measures, involving changes of technologies, could reduce greenhouse gas emissions (in carbon equivalent). We use the subscript k to denote the order of successive sectors, and assume that all abatement options on the curve involve either positive net cost (MACC) or higher investment (MICC) relative to a more carbon-intensive 'reference' pathway. Whereas the MACC curve would order abatement technology sectors according to net present value at a universal discount rate relative to this reference, as noted the MICC curve orders options in order of increasing capital intensity. With our main focus on the latter, we return to consider fuel savings later in the paper.

This construction of a MICC generates an increasing function of investment cost $I_k = f(\mu_k)$, which can be most simply represented as an explicit power function with exponent γ:

$$I_k(\mu_k) = \alpha_I \mu_k^{\gamma}, \quad and \quad \mu_k(I_k) = \int_0^{I_k} \rho_k(I'_k) dI'_k, \tag{1}$$

where α_I is a scaling factor obtained from the MICC data, I_k is the investment on abatement in sector *k*, ρ_k is abatement per unit of investment and μ_k is the cumulative abatement (in tCO_2/year), following the MICC curve from sector 1 until sector *k*. Note that the investment cost function is obtained by inverting the result of the integral for $\mu_k(I_k)$, and abatement measures are ordered according to their capital intensity, following the MICC curve.

[11] See note 5. Also, global scenarios by the International Energy Agency (including the most recent (IEA/IRENA 2017) increasingly conclude that low carbon scenarios may not be more costly in aggregate, because higher investment costs are offset by the fuel savings and the impact in globally reducing fossil fuel prices.

[12] Most notably, there are in particular many end-use efficiency investments for which the fuel savings appear to outweigh additional investment costs assessed at any reasonable global discount rate, but many do not happen because the investment faces numerous barriers including high consumer discount rates. Also, most renewable energy generation is capital intensive but then delivers energy at negligible variable operating costs. Overall, energy system modeling by the International Energy Agency estimates that low carbon scenarios are systematically more *capital intensive*, but may not involve overall higher costs (IEA/IRENA, 2017).

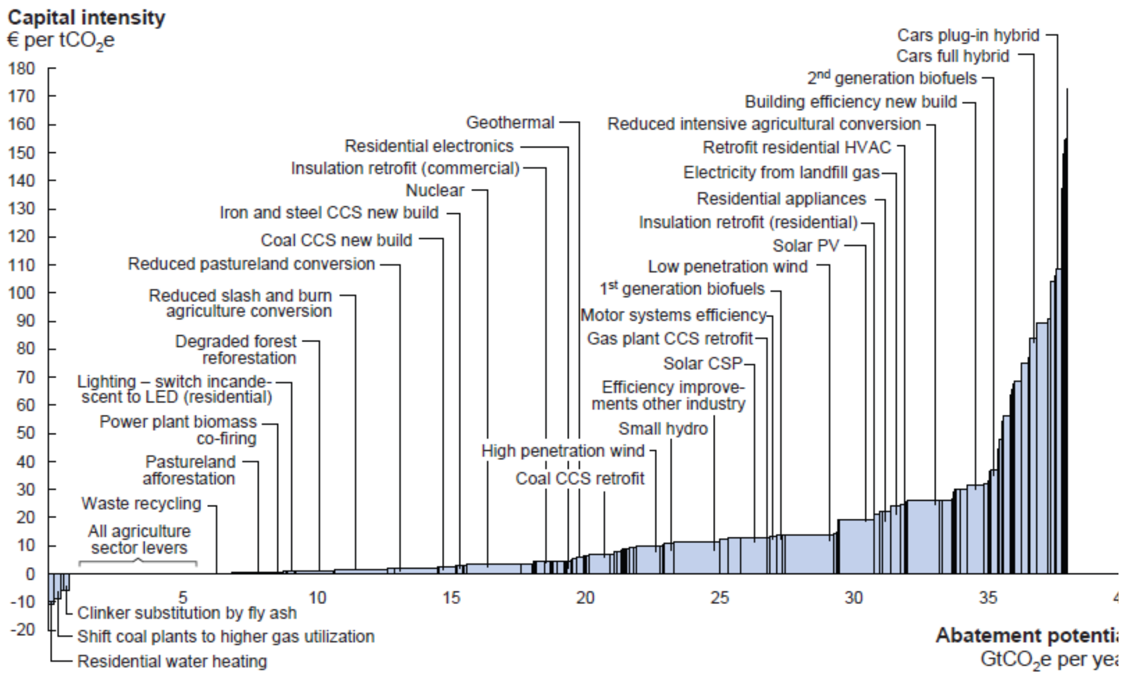

Figure 1 Curve of marginal investment (capital) intensity – a *marginal investment cost curve* (MICC). Source: "Pathways to a low-carbon economy: Version 2 of the global greenhouse gas abatement cost curve" (Exhibit 8), 2009, McKinsey & Company, www.mckinsey.com. Copyright (c) 2017 McKinsey & Company. All rights reserved. Reprinted by permission.

We see here that when dealing with investment, rather than operational substitution, we discuss segments of deployed abatement technologies which displace emitting stock, rather than displacing emissions. We thus have a different relation to time than most studies in the literature to date. Once a capital emission source is displaced, its contribution to growing emissions stops.

Most global assessments of potential abatement focus on net abatement costs and imply a somewhat non-linear abatement cost curve, at least without technology learning. This applies to all approaches: engineering cost curves (e.g. McKinsey 2009); global energy system models – both partial and general equilibrium (e.g. Kriegler et al. 2015) [13], and the stylised representations in IAM models (e.g. The DICE model of Nordhaus, 2008 and the PAGE09 model of Hope, 2013[14]). DICE – which to an extent has set a standard in IAM literature, assumes total abatement costs rise quadratically, which is equivalent to linearity of the marginal cost curve. This appears a plausible form, though uncertainties increase at higher abatement levels, where there is tension in assumptions between the rising cost of known identified technologies, versus the

[13] Kriegler et al (2015) summarise, as part of a Special Issue on global abatement scenarios, the results of the AMPERE studies comparing a wide range of energy & industrial system models; Figure 9 displays the clear non-linearity of abatement costs in both general and partial equilibrium models.

[14] In the PAGE09 model, the abatement cost curve is specified by three points, and by two parameters describing the curvature of the MACC curve below and above zero cost respectively (Hope, 2013).

possibility of ‘backstop’ technologies available at large volumes if the price gets high enough.

This also means that the supply of emissions reductions measures per unit cost decreases with increasing abatement. Clearly the MACC curve is likely to some degree to reflect the investment MICC curve, and the MICC curve illustrated in Figure 1 also suggest that investment intensity increases faster than linearly but slower than quadratically i.e. with an exponent γ between 1 and 2, at least until the most investment-intensive options are reached (the costs of which already appear to have come down more recently).[15]

3.2. Integrating the investment cost curve

Abatement measures in sector *k* take place by gradually replacing existing polluting units of technology (e.g. power plants, cars, industrial machines, etc) by less polluting units. There may be several options within the range $I_k + \Delta I_k$; however we skip over the details of individual technologies and assume that they have an average abatement factor β_k, in units of tonnes of avoided CO_2/y per unit of capacity (e.g. GW).

We denote the *ex-ante* assumed MICC as $I_k^0(\mu_k)$, and represent the cost of implementing abatement measures in sector *k* an *overnight* investment (capital cost) per unit capacity (e.g. \$/vehicle, \$/MW). Thus the cost before learning of all abatement measures in sector *k* with respect to baseline is $I_k^0 U_k$,[16] where U_k is the additional capacity of low carbon technology in sector *k* (e.g. in MW). This capacity has a capacity factor and an emissions factor, the latter lower than that of the incumbent technology, and their combination generates the factor β_k, where $d\mu_k = \beta_k dU_k$.

With learning, however, the values of I_k may change with cumulative constructions, each becoming less capital intensive than the previous. Clearly, the total cost is path (i.e. investment history) dependent. This requires the use of a *path integral* as a calculation method. Thus the total investment cost c_A over all sectors *k* is determined by integration of the marginal cost over a *pathway of technology development* in each sector, and summed over all sectors:

$$c_A = \sum_k \int_0^{U_k} I_k(U_k') dU_k', \tag{2}$$

If the approach is applied to *marginal abatement cost (MACC),* I_k has to be understood as the net cost by sector, taking account of fuel savings relative to baseline, and c_A is the additional net cost of the abatement pathway. Where the focus is on investment (*MICC*), I_k is the incremental investment cost and c_A is to be interpreted as

[15] Most notably, since estimation of the MICC curve in Figure 1, the cost of hybrid and electric vehicles, which populate the very high end of the investment curve, has come down dramatically.

[16] In the baseline, equipment has to be gradually replaced with a gradual scrapping rate, or turnover rate, and thus some investments are made. Mitigation costs in this context correspond to investments that are more expensive than what would have been paid for in a non-mitigation scenario. For example, wind turbines are more expensive than coal power plants when measured on the same capacity basis.

the additional *investment* requirement of the abatement pathway. In what follows we focus on the latter interpretation and consider fuel savings separately later (though most of the mathematical analysis would be applicable to either approach).

Time is not (yet) included in this because we are only calculating the annualised investment cost associated with a given level of abatement: the learning in this form is 'instantaneous'. In assuming that $I_k(U_k^{'})$ is levelised, all operation & maintenance (O&M) costs are included, but fuel costs can be separated as discussed later (since fuel costs would not be affected by learning). In the case without learning, the marginal cost of measures within a given sector would be constant and the total cost across sectors is:

$$\sum_k \int_0^{U_k} I_k(U_k^{'})dU_k^{'} = \sum_k \frac{1}{\beta_k}\int_{\mu_{k-1}}^{\mu_k} \alpha_I {\mu_k'}^{\gamma} d\mu_k' = \sum_k \frac{\alpha_I}{(\gamma+1)\beta_k} {\mu_k'}^{\gamma+1}\Big|_{\mu_{k-1}}^{\mu_k} \quad (3)$$

Where as in equation (1), α_I is the scaling factor and γ is the exponent for the marginal cost curve, $I_k(\mu_k) = \alpha_I \mu_k^{\gamma}$, and μ_k is the cumulative abatement up to sector *k*, assuming that all abatement measures included in the MICC curve up to k have been implemented.[17] Assuming an average emissions factor $\beta \approx \beta_k$ in the sum every term in *k* cancels with every term in *k-1*, except for the end terms, obtaining:

$$c_A \approx \frac{\alpha_I \mu^{\gamma+1}}{(\gamma+1)\beta} \quad (4)$$

Where μ is the cumulative abatement from sectors 1 until *k,* given that $\mu_0 \approx 0$, and c_A is the cost difference with respect to the baseline scenario (so that if μ = 0 then c_A = 0). Thus without learning, our approach as expected yields the basic cost curve in the pre-defined form, the only point being that we have structured this explicitly by integrating deployment across the abatement sectors.

3.3. Abatement cost curves with ('instantaneous') learning

The economics literature that seeks to represent and quantify learning tends to divide the processes into knowledge accumulation through direct *learning-by-searching*, and indirect *learning-by-doing*, to which the innovation systems literature also adds consumer-based *learning-by-using.* In the earlier stages of innovation, the first of these may dominate. Still, as a technology moves to market, for entrepreneurs to improve their products, they critically need sales, of which part of the profits are re-invested into the expansion of production capacity and (private) R&D to improve production processes and products. These reduce production costs and also generate economies of scale. The total amount of investment in expansions and R&D depends

[17] The relation between capacity and abatement is: $\mu_k = \beta_k U_k$. In order to keep tractability, abatement measures are assumed to be implemented in order of initial investment cost. Therefore, abatement in sector *k* only happens after abatement measures in sectors 1 ... *k*-1 are already implemented. In that context, it is valid to assume that $d\mu_k = \beta_k dU_k$,

ultimately on sales, this process may stop if sales stop, and proceeds if sales proceed, almost irrespective of time.

In its purest conceptual form therefore such learning in the private sector – *including learning-by-searching as well as learning-by-doing and learning-by-using* - occurs as function of sales. Empirical studies emphasise not only an association of deployment with cost reduction (which as Nordhaus (2013) notes, can also reflect cost reductions increasing deployment), but also patenting (Bettencourt et al, 2013); thus, costs change with deployment levels of technologies through multiple channels.[18] This means that the actual building costs may turn out to be lower than the expected (*a priori*) costs, as represented in the assumed MICC*.* While the MICC represents costs estimated *a priori*, costs *post fact* turn out lower than initially assumed to the extent there is learning. For simplicity we use the term learning-by-doing but emphasise it actually encompasses all the different learning processes associated with growing investment and sales.

Analysing this requires an explicit approach to integrating costs across the curve as abatement proceeds. The cost of the first additional investment in each sector taken corresponds to that assumed in the initial cost curve, which we denote as $I_k^0(\mu_k)$, while the actual cost curve incurred, which continuously changes with increasing abatement, as $I_k(\mu_k)$. This we term *instantaneous learning.*

Thus where instantaneous learning occurs, the cost of measures I_k changes with the cumulative production of similar units W_k, following learning curves. A large literature now estimates learning rates (e.g. Kohler et al., 2006; Weiss et al., 2010)) – cost reductions associated with a doubling of capacity – which can be converted to learning exponents $-b_k$, which we use here, of positive values between 0 and 1. In particular the studies of Weiss (2010) covered a wide range of both supply and demand-side technologies and estimated learning rates for most technologies between 5 and 30%, with a median at around 15%. The learning exponent *b* relates to the learning rate *LR* through $b = \frac{\ln(LR-1)}{\ln(2)}$ which (when using as a negative exponent) suggests values typically between 0 and 0.5.

Costs including instantaneous learning can be expressed in terms of cumulative capacity additions and emissions reductions. Note that the very concept of learning rates assumes some pre-existing deployment. This again emphasises that our framework concerns not radically new and untested technologies, but rather is appropriate to the fact that many of the low carbon technologies represented in detailed abatement scenarios are now well known and deployed at significant scales (consider high efficiency gas, wind, solar, batteries, deep building insulation, electric vehicles, etc.), but require radical scaling-up to achieve deep emission reductions.

[18] We use the terms 'learning' and 'learning-by-doing' in this section because structuring the analysis around abatement cost curves implies a mathematical focus on industrial investment in specific technology- or sector-areas. Such learning is subset of overall systems innovation, which can for example include structural shifts and the impact of technological and institutional innovation in facilitating the adoption of new technologies.

Our focus is on how to analyse the potential pathway and cost implications of learning and inertia given this.

Learning is relative to an initial cost, which in the form of marginal cost curves is either the additional abatement cost (MACC) or additional investment cost (MICC) of the lower carbon option. In common with almost all the economics literature, the construction above assumes that the cost curves reflect *additional cost* (so that any 'negative costs' are already included in the high carbon reference). With learning they decline, but cannot reach zero or go negative; such analysis thus assumes by construction that, relative to the baseline, the low carbon options are and remain either more costly overall (MACC) or more capital intensive (MICC). The latter, on which we focus, is clearly less restrictive since the great majority of abatement options involve little (e.g. renewables) or no (enhanced energy efficiency) running costs.

Cumulative production corresponds to all units produced in history to date. We define here historical cumulative production W_k^0 (since $t' = -\infty$ up to the present $t' = 0$),[19] and future additional cumulative production W_k (from the present $t' = 0$ to the end of the projection period $t' = t$):

$$W_k(\mathrm{t}) = \int_0^t \big(\dot{U}_k(t') + \delta_k U_k(t')\big) dt',$$
$$W_k^0 = \int_{-\infty}^0 \big(\dot{U}_k(t') + \delta_k U_k(t')\big)\, dt', \quad (5)$$

where δ_k corresponds to a depreciation rate, meaning that new cumulative capacity at time *t* includes all additions as well as replacements for decommissioned units. We also assume that for abatement technologies, $\dot{U}_k > 0$. Costs including instantaneous learning can then be expressed in terms of cumulative capacity additions, emissions reductions and the sector learning rate b_k:

$$I_k(\mathrm{t}) = I_k^0 \left(\frac{W_k^0 + W_k(\mathrm{t})}{W_k^0}\right)^{-b_k} = I_k^0 \left(1 + \frac{\int_0^t \big(\dot{U}_k(t') + \delta_k U_k(t')\big) dt'}{W_k^0}\right)^{-b_k}$$
$$= I_k^0 \left(1 + \frac{\int_0^t (\dot{\mu}_k(t') + \delta_k \mu_k(t')) dt'}{\beta_k W_k^0}\right)^{-b_k} \quad (6)$$

Since the numerator is always larger than the denominator, and W_k cannot decrease, costs can only go down with capacity additions, meaning that the system does not 'un-learn'. Note that the denominator here $\beta_k W_k^0$ is effectively the cumulative emission saving attributable to existing deployment of technology *k*, which may for example have been promoted previously by technology-specific government policies.

We calculate the two terms in the integral. The first term is simply μ_k (assuming $\mu_k(0) = 0$), while the second,

[19] Technology capacity growth is typically exponential, and therefore missing capacity additions very far in the past does not generate significant uncertainty.

$$\int_0^t \delta_k \mu_k(t')dt' \quad (7)$$

produces a fixed value, the total abatement $\bar{E}$ at the end of the projection period, a constant, times the depreciation rate δ_k. We then have

$$I_k(\mathrm{t}) = I_k^0 \left(1 + \frac{\mu_k(\mathrm{t}) + \delta_k \bar{E}}{\beta_k W_k^0}\right)^{-b_k} \quad (8)$$

where $I_k^0 = \alpha_I \mu_{\mathrm{k}}^{\gamma}$.[20]

Knowing the investment cost of the *k*th option as function of abatement does not tell us the total cost yet until we know how many were carried out at which cost. This requires evaluating the path integral eq. (2) under the new expression for $I_k(U_k^{'})$:

$$c_A = \sum_k \int_{\mu_{k-1}}^{\mu_k} \alpha_I \mu^{\gamma} \left(1 + \frac{\mu + \delta_k \bar{E}}{\beta_k W_k^0}\right)^{-b_k} \frac{d\mu}{\beta_k}. \quad (9)$$

The term in $\delta_k \bar{E}$ relates to amount of depreciated low carbon technology units in a scenario in comparison to the total number of units. For long-lived new low carbon technologies, with rising investment as abatement proceeds, $\delta_k \bar{E} \ll \mu$.[21] It is also the case that their current cumulative capacities are small relative to the scale of deployment implied in any deep emission reduction pathway, i.e. $\mu \gg \beta_k W_k^0$.[22] We thus obtain for the total investment cost of long-lived abatement options with learning (see Appendix 2 for details):

$$c_A = \sum_k \int_{\mu_{k-1}}^{\mu_k} \alpha_I \mu^{\gamma} \left(1 + \frac{\mu + \delta_k \bar{E}}{\beta_k W_k^0}\right)^{-b_k} \frac{d\mu}{\beta_k}$$
$$= \sum_k \left[{\mu_k}^{\gamma+1-\mathrm{b_k}} - {\mu_{k-1}}^{\gamma+1-\mathrm{b}_k}\right] \frac{\alpha_{\mathrm{I}} (\beta_k W_k^0)^{\mathrm{b_k}}}{(\gamma + 1 - \mathrm{b_k})\beta_k}$$

We impose the assumption that in the chosen ordering of the cost curve, all abatement sectors up to k are fully implemented (most simply interpreted as being implemented in order of increasing *ex-ante* cost). The critical term in this, $\frac{(\beta_k W_k^0)^{\mathrm{b_k}}}{(\gamma+1-\mathrm{b_k})}$, represents the % cost reduction due to learning in sector after it is fully deployed, with

[20] The superindex 0 in I_k^0 does not indicate time (otherwise investment would be zero, because $\mu_k(0) = 0$). Instead, it indicates the *a priori* investment cost (without learning).

[21] Strictly speaking, $\delta_k E$ is always smaller than $\mu\mu$. In many cases, it is much smaller, which simplifies the analysis. Cases in which the order is similar are straightforward to calculate, although tedious, and not shown here, since they do not add to the argument.

[22] In cases where lifetimes are extremely short, $\delta_k E \gg \mu\mu$, or where learning is also insignificant, such costs are taken care of by a classical component without learning.

$\beta_k W_k^0$ being the emission reduction associated with pre-existing capacity relative to the overall scale of the sector (when fully deployed). With typical learning rates $\mathrm{b_k}$ of around 0.1– 0.15, the denominator is quite constant. If the % cost reduction in each sector after fully implementing its capacity is similar, the ratio $\frac{(\beta_k W_k^0)^{\mathrm{b_k}}}{(\gamma+1-\mathrm{b_k})}$ does not vary much among sector, and the total investment cost can be calculated as a telescopic sum. With $\mu_0 \approx 0$, and taking average cost reduction in each sector $\frac{(\beta_k W_k^0)^{\mathrm{b_k}}}{(\gamma+1-\mathrm{b_k})\beta_k} \approx \frac{(\beta W^0)^{\mathrm{b}}}{(\gamma+1-\mathrm{b})\beta}$ we obtain (see the full calculation in Appendix 2):

$$c_A \approx \frac{\alpha_{\mathrm{I}}(\beta W^0)^b}{(\gamma+1-b)\beta}\mu^{\gamma+1-b}. \tag{10}$$

with μ being the cumulative abatement. Here βW^0 represents the emission reductions associated with the *existing* capacities of the abatement technologies – the annual emission savings (relative to the total emissions of the system) associated with cumulative production to date of the technologies represented in the curve.

The exponent $\gamma+1-b$ of the power law cost curve turns out lower than $\gamma+1$ as measured at $t=0$, by the learning exponent *b*, meaning a less steep rise in costs. Thus if the marginal *ex-ante* curve is slightly convex ($\gamma > 1$), making the total curve slightly steeper than quadratic, then learning will tend to pull the *ex-post* curve back towards DICE-like assumptions of linear marginal and quadratic total costs.

The pre-factor also makes the curve lower in absolute value, in comparison to the classical case without learning $\frac{\alpha_I \mu^{\gamma+1}}{(\gamma+1)\beta}$, to a degree dependent on the typical degree of learning once a sector is fully deployed.

The assumption of similar learning across the different sectors required to derive this overall cost curve is formally a strong one, but the basic form and insights of the result are far more generic and evident from the form of the telescopic sum itself (eq.9). For example, if a sector has already been mostly deployed, leaving little room for further learning, it is also only a very small component of the projected overall cost curve. Also if a technology sector is higher up the cost curve but displays little learning as deployment proceeds, it might be abandoned – in effect, simply ejected from the cost curve in favour of those displaying higher learning potential. Such variations are thus unlikely to radically change the general insight about the potential impact of learning on both the shape and scale of the *ex-ante* cost curve.

4. The cost of early scrapping of equipment

In a process of rapid decarbonisation, additional costs arise, which have not been explored here yet, due to the inertia inherent to energy systems. These include potential 'stranded assets'. Technologies and capital are fixed, and have a lifetime during which they are expected to operate. A rapid transition may involve scrapping equipment before its expiry date, in other words, forgone planned income: physical capital is *not* liquid and firms cannot recover their money if these become stranded.

Abatement may thus become more costly when *units of capital are decommissioned or scrapped much earlier than the date up to which they were expected to operate when their business plan was made, and more importantly, whether planned income is forgone*. Of course, after investment these costs are sunk, but not utilising them may still imply opportunity costs, so in general the inertia associated with capital lifetimes implies an additional cost when abatement is rapid. Formally, this can be described by considering the case where a loan is taken to finance a unit of technology, repaid over its lifetime (e.g. a power plant, a car), during which the user (the firm, the consumer) uses the income generated by the operation of the technology (e.g. selling electricity, transport services) to repay the loan. If the consumer/investor is required by climate policy to scrap this unit while still paying, and being still required to provide the same service with a unit of cleaner technology, he will most likely have to take out a *second loan* before he has finished paying back the first, and thus have to repay *two loans* simultaneously instead of one, with the service income of only a single unit of producing technology. The outstanding amount to pay for the first unit corresponds to the early scrapping loss, which is thus easy to define.[23] Further losses incurred after the end of the repayment schedule due to loss of income in the operation of the capital are not considered here; such values are subjective and not possible to define clearly, since it is not possible to know at what date exactly the capital is to be considered obsolete and due for replacement.[24]

A unit of polluting technology (e.g. fossil fuelled) has a lifetime τ_k, defined by the length of the repayment schedule of the initial loan, in other words, the *expected lifetime at the time of purchase*.[25] We denote $\tau_k^{'} < \tau_k$ the time at which it is scrapped. At the system level, the loss is therefore

$$c_k^{ES} = -\beta_k \dot{U}_k \frac{I_k^{ES}}{\beta_k} \int_{\tau_k^{'}}^{\tau_k} e^{-r\tau} d\tau \approx -\beta_k \dot{U}_k \frac{I_k^{ES}}{\beta_k} \left(\frac{e^{-r\tau_k^{'}} - e^{-r\tau_k}}{r} \right) \approx -\beta_k \dot{U}_k \frac{I_k^{ES}}{\beta_k} \left(\tau_k - \tau_k^{'} \right) \tag{11}$$

for a small $\Delta\tau_k = \tau_k - \tau_k^{'}$, where I_k^{ES} is the forgone income, a constant with no learning or relationship with abatement μ_k, $\dot{U}_k$ is the polluting capacity scrapped and replaced

[23] Note that an early scrapping loss can only be defined with respect to a difference between income and income planned at the time when the business plan was made and a loan may have been taken. However, it can lead to default and bankruptcy.

[24] The research area called ‘vintage capital’, initially explored by Solow et al. (1966), and more recently developed by Boucekkine et al. (2011), attempt to construct models of optimal capital use and decommission. Such models involve optimisations with time delays that are not very tractable and challenging to solve. The gain that would be obtained in involving such problems here is significant enough to justify the work, as we already capture the principal component of the loss.

[25] At the end of this, the investor may keep using the unit of technology or scrap and change it at any time he wants, he is not bound financially anymore.

(a negative value), and β_k is the emissions factor per unit of capacity of the polluting technology (in contrast to the abatement factors used above in units of avoided carbon per unit of capacity, but the function is the same). In fact, the cost should be written as

$$c_k^{ES} \approx \begin{cases} -\beta_k \dot{U}_k \frac{I_k^{ES}}{\beta_k}\left(\tau_k - \tau_k^{'}\right) & \text{if} \quad \tau_k^{'} < \tau_k \\ 0 & \text{if} \quad \tau_k^{'} \geq \tau_k \end{cases}, \tag{12}$$

since no loss occurs if the operating lifetime $\tau_k^{'}$ is equal or longer than the expected rated lifetime τ_k. The capacity change can be expressed in terms of abatement (since it is replaced by low carbon equivalent technologies), $\dot{U}_k = \dot{\mu}_k / \beta_k$, and the early scrapping cost per unit of abatement. Since additional avoided emissions and abatement rates are

$$\mu_k = U_k \beta_k \frac{\tau_k - \tau_k^{'}}{\tau_k}, \qquad \dot{\mu}_{\mathrm{k}} = \dot{U}_k \beta_k \frac{\tau_k - \tau_k^{'}}{\tau_k}, \tag{13}$$

The total early scrapping cost, calculated using the path integral eq. (4), scales with

$$c^{ES} \approx -\sum_k \int_{\dot{\mu}_{k-1}}^{\dot{\mu}_k} \frac{I_k^{ES}}{{\beta_k}^2} \tau_k \dot{\mu}_{\mathrm{k}}^{'} d\dot{\mu}_{\mathrm{k}}^{'} = -\sum_k \left(\frac{I_k^{ES}}{{\beta_k}^2} \dot{\mu}_k^2 \tau_k - \frac{I_{k-1}^{ES}}{{\beta_{k-1}}^2} \dot{\mu}_{k-1}^2 \tau_{k-1} \right) \tag{14}$$

when $\dot{\mu}_{\mathrm{k}} < 0$, otherwise if emissions are not declining, there is no early scrapping and $c^{ES} = 0$. We do not know the *k* dependence of I_k^{ES}, β_k or τ_k. Were they constant, in this telescopic sum over *k*, each term would cancel out with the previous, leaving the first and the last. They are not constant, however, where for instance the emissions, capital intensity and lifetimes of gas and coal plants are different. Being unrelated to *k*, and having relatively similar parameters (lifetimes, emission factors, etc.) the sum will nevertheless yield a result that is proportional to $\dot{\mu}^2$ (roughly speaking, an average),

$$c^{ES} \approx \begin{cases} \zeta_{ES} \dot{\mu}^2 - c_0^{ES}(\mu_0^{ES}) & \text{if} \quad \tau^{'} < \tau \; and \; \alpha_{ES} \dot{\mu}^2 > c_0^{ES}(\mu_0^{ES}) \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

where $\tau^{'}$ and τ are mean lifetimes and expected lifetimes and $\zeta\alpha_{ES}$ is just a proportionality factor rising from the average approximation. The term μ_0^{ES} is a threshold value, equivalent to the minimum rate of abatement above which premature retirement is required. For abatement below this threshold level, no early scrapping is required, and therefore the early scrapping cost is zero ($\zeta_{ES}\dot{\mu}^2 < c_0^{ES}(\mu_0^{ES})$). For abatement above the threshold level, the cost of early scrapping increases quadratically, proportional to the rate of abatement.

The faster abatement is made, proportionally more units are scrapped each year. Thus, inertia related to the fossil fuel capital stock results in a cost component proportional to the rate of abatement. However, as abatement proceeds, there are less and less early scrapping losses since there are less and less existing polluting technologies, and therefore this cost vanishes at high levels of abatement unless the rate is very fast. Overall an optimal trajectory would tend to pathways which reduce the need for premature scrapping of equipment.

5. The cost of developing new production capacity for abatement

Abatement takes place by replacing polluting units of technology by clean ones. The associated investment scales with replacements multiplied by their price, which decreases with learning following cumulative sales. This cost calculation is incomplete since new production capacity, factories, supply chains and new infrastructure were required to produce these units on a possibly large scale, which didn't exist before. Furthermore, some old production capacity for polluting equipment could also become stranded assets. In other words, inertia characterises not only individual units of technology, but also the technologies and infrastructures required to produce these units. The associated costs do not scale simply with emissions reductions. Instead, they scale with the rate of transformation, which can readily be understood in terms of a transitional cost.

A number of units of new clean technology (e.g. electric cars) for abatement can be produced following all kinds of possible profiles in time. Each unit of abatement takes a certain amount of time t_k to produce, and each unit of production capacity can produce $1/t_k$ units per unit time. If the demand for units is exactly $1/t_k$ per unit time, and one single unit of production capacity exists, then no additional production capacity is required. However, if the rate of production demanded becomes higher, for example, for an identical final number of units produced that we require to be produced in a shorter time span, then several units of production capacity must work in parallel, and more factories must be built. Thus for the same amount of abatement, depending on the profile of abatement in time, different amounts of investment in production capacity may be required. Critically, we assume these are long-lived, and may thus also become stranded assets in a scenario where the production rate is very high but stops abruptly when decarbonisation is achieved, and only the factories replacing the depreciation of existing units are needed. We thus define the production capacity for technologies as $N_k = \dot{U}_k$.

Effectively, while emissions reductions μ scale with the number of units of polluting technologies replaced by clean units, the rate of change of emissions abatement $\dot{\mu}$ scales with the production capacity for clean technology. The cost of transformation, or transitional cost, corresponds to the expansion of production capacity for clean technology, where learning is also present in this development. We assume that the cost of production capacity for abatement technologies follows the same order as the technologies themselves (i.e. the more expensive technologies have more expensive production lines as well). Therefore we use another MICC, but for the production capacity of abatement $\dot{\mu}$, rather than abatement μ itself. In this case, we have a similar set-up as to eq. (3):

$$I_k^N = \alpha_B \dot{\mu}_k^\gamma, \qquad \text{where } \dot{\mu}_k = \int_0^{C_k^B} \rho_k^N(C_k')dC_k' \tag{16}$$

Learning also exists in creating production capacity. We denote as b_N the learning exponents related to production capacity, and follow the same path integral method as in section 3.3 (eq. 6), where the cumulative cost is integrated along a path of abatement:

$$c_B = \sum_k \int_{\mu_{k-1}}^{\mu_k} \frac{I_k^N}{\beta_k}\left(1 + \frac{\int_0^t \dot{N}_k + \delta_k^N N_k dt}{W_k^{N0}}\right)^{-b_N} d\mu \tag{17}$$

where N_k is the production capacity of technology for sector *k* which is being expanded, I_k^N its unit cost and δ_k^N its depreciation rate. This production capacity must cover both the building of new units of technology for abatement U_k and replace those coming to the end of their lives,

$$\frac{N_k}{t_k} = \dot{U}_k + \delta_k U_k, \qquad \frac{\dot{N}_k}{t_k} = \ddot{U}_k + \delta_k \dot{U}_k. \tag{18}$$

Where as noted, t_k is the average time to produce the abatement product *k* from the production process. This can be transformed into abatement and abatement rates:

$$\frac{N_k}{t_k} = \frac{\dot{\mu} + \delta_k \mu}{\beta_k} \frac{\dot{N}_k}{t_k} = \frac{\ddot{\mu} + \delta_k \dot{\mu}}{\beta_k}. \tag{19}$$

We now calculate the two terms in the (internal) integral:

$$\int_0^t \frac{\ddot{\mu} + (\delta_k^N + \delta_k)\dot{\mu} + \delta_k^N \delta_k \mu}{\beta_k} dt = \frac{\dot{\mu} + (\delta_k^N + \delta_k)\mu + \delta_k^N \delta_k E}{\beta_k.} \tag{20}$$

In comparison to section 0, we obtain a result of similar form, but with an additional time derivative. The third term in $\delta_k^N \delta_k \bar{E}$ is a cumulative impact of the product of the combined depreciation (of manufacturing capacity, and of the abatement technology itself) in tonnes of CO_2, a term very small if both capital and factory lifetimes are large. The second term in μ can be small depending on lifetimes; or otherwise can be added to the cost term of section 0. We are interested in the first term in $\dot{\mu}$, which yields a rate-dependent term. By the same procedure as in section 0, equation 19 follows a similar form as in equation 12, namely:

$$c_B \approx \frac{\alpha_B (\beta W^{N0})^{b_N}}{(\gamma + 1 - b_N)\beta} \dot{\mu}^{\gamma + 1 - b_N} \tag{21}$$

where $\dot{\mu}(0) = 0$ and one may decide whether or not to include learning-by-doing on building factories.[26] In the latter case, $b_N = 0$. In equation (21), W^{N0} is the equivalent to W^0 in equation (10).

Here we have a cost component that depends again on the rate of abatement. This cost arises due to the inertia associated with the existing production capacity. This implies that *all* scenarios of abatement incur an investment cost element that increases with the rate of transformation, however slow. An optimal pathway will therefore balance costs of transformation against damages, and will require balancing the benefits of technological transformation against the costs of rapid transition. This would suggest starting abatement earlier in order to achieve the maximum abatement whilst minimising rates of change.

6. Adoption and diffusion capabilities

New abatement technologies do not only require new productive capacity. The ability to diffuse new technologies also requires investment of varied types. Economic history is littered with examples where seemingly attractive technologies took many years or even decades to be adopted. Often new technologies must not only overcome barriers of trust, but their widespread diffusion requires development of markets, delivery networks, and appropriate regulations, institutions and infrastructure.

The history of 'technology transfer' to developing countries – and indeed, the general pattern of global technology diffusion - has testified to the significant inertia these factors involve. Processes of diffusion thus also involve investment in 'capabilities'. A poor country with limited education, grid or marketing capability will only be able to slowly adopt abatement technologies. Investment in "capacity building" for developing countries has been a major feature of climate negotiations aimed at enhancing their abatement efforts. All this reasonably reflects a direct demand-side analogue to the need to develop productive capacity.

Innovations diffuse following S-shaped patterns ('logistic curves'; Rogers, 2010). Following the diffusion literature, it is well known that the pace of diffusion (the width of the S-shaped curve) depends on both capital stock turnover rates (the rate at which people replace equipment such as cars, electricity generators, etc), and the degree to which people are attracted to these technologies (Mercure, 2015; Rogers, 2010; Young, 2009). The empirical literature provides many examples of this (Grubler et al., 1999; Marchetti & Nakicenovic, 1978; Mcshane et al., 2012). In a classical network problem, in order to gain information, agents typically wait to see innovations used by peers before adopting them (Rogers, 2010).

When that is the case, one way to accelerate the rate of adoption beyond what is feasible through peer influence is through information campaigns (Knobloch & Mercure, 2016). If we seek to isolate the costs of diffusion occurring faster than the

[26] Learning-by-doing scales with cumulative production of units, and factories are not typically built in large numbers. Cost reductions for factories are complex to identify. This, however, does not change our argument.

'natural' rate of diffusion, we argue that one is primarily looking at the costs of information campaigns. Taking the example of a society that undertakes a program of social marketing targeting the adoption of low carbon technology, this marketing cost will scale in a particular way with abatement. Knowledge grows with the adoption of new technology; this means that social marketing is primarily needed at the early stages of diffusion in order to accelerate kick-starting the diffusion process. The cost scales with the number of people targeted, and will yield a rate of adoption, i.e. each marketing investment will lead to a certain number of adoptions of new technology, and thus, marketing investment scales with the rate of abatement:

$$I_k^M = \alpha_M \dot{\mu}_k \tag{22}$$

which, needs to be integrated along a path of investment history, using once more the path integral of eq. (4). This yields

$$c_M = \sum_k \int_0^{U_k} I_k^M d\dot{U}_k = \sum_k \int_{\mu_k}^{\mu_{k+1}} \frac{\alpha_M \dot{\mu}_k}{\beta_k} d\dot{\mu}_k \approx \frac{\alpha_M}{2\beta} \dot{\mu}^2 \tag{23}$$

This yields again a cost term in the square of the *rate* of abatement.

7. Interpretation and simplified forms of cost components

7.1. Investment and inertia cost components

We summarise the cost components developed in this paper in Table 1. So far, it appears as if our analysis has simply increased complexity, by separating investment from fuel-related costs, and introducing learning and rate-dependant terms in the former. We now turn to show how, to the contrary, identifying these components can suggest a much simplified version of stylised analysis, incorporating learning and inertia, with a simple analytic way to cut through many of the complexities.

Table 1 shows the cost components in both more general and simplified forms. The simplified form takes the general case and illustrates, for the investment cost, the result if *either* the marginal cost is rising linearly (quadratic in total abatement) without learning, *or* if the learning rate b is of a magnitude which roughly offsets any assumed non-linearity γ in the *ex-ante* MICC, such that $b = \gamma - 1$. (recalling that $\gamma = 1$ is a linearly increasing marginal cost, i.e. quadratically increasing total cost). In this case, the resulting functional form of investment cost is same as in the MACC curve implied by the DICE model, though the level of cost is lowered by any positive learning component, and we account separately for the fuel savings (discussed in the next section).

In addition, however we have the rate-dependent terms – which also feature terms which are generally quadratic, but in the *rate* of abatement, $\dot{\mu}^2$. We note that in Vogt-Schilb et al (2017), because investment results in a stream of subsequent abatement, their representation of investment is mathematically similar to our $\dot{\mu}^2$ term;

they assume quadratically rising investment costs for their modelling. In effect, our analysis above (as summarised in Table 1), offers a more solid theoretical basis for this assumption.

Cost component	Expression	Simplification if no learning and linearly rising marginal abatement cost, *or* if learning rate compensates for non-linearity of marginal abatement cost ($b = \gamma - 1$)	Dependence	Units
New technologies following a marginal abatement cost curve	$\frac{\alpha_I(\beta W^0)^b}{(\gamma+1-b)\beta}\mu^{\gamma+1-b}$	$\frac{\alpha_I(\beta W^0)^b}{2\beta}\mu^2$	Emission sources avoided in a given year	$/tCO$_2$/y
Early scrapping of capital stock	$\alpha_{ES}\dot{\mu}^2 - c_0^{ES}(\mu_0^{ES})$		Rate of abatement, non-linear	$/tCO$_2$/y^2
Transforming production capital	$\frac{\alpha_B(\beta W^{N0})^{b_N}}{(\gamma+1-b_N)\beta}\dot{\mu}^{\gamma+1-b_N}$	$\frac{\alpha_B(\beta W^{N0})^{b_N}}{2\beta}\dot{\mu}^2$	Rate of abatement, non-linear	$/tCO$_2$/y^2
Diffusion / utilisation capability	$c_M = \frac{\alpha_M}{2\beta_k}\dot{\mu}^2$		Rate of abatement, non-linear	$/tCO$_2$/y^2

Table 1 Different cost components and their scaling with respect to time derivatives of the abatement variable. E refers to a quantity of CO_2 emissions (in tCO_2), $\boldsymbol{\mu}$ to a quantity of abatement (in tCO_2/y), $\dot{\boldsymbol{\mu}}$ is a rate of abatement (in tCO_2/y^2). See section 7 for discussion of fuel cost savings.

Note that only the first term (the adoption of technologies according to a static 'abatement curve') and fuel savings (below) involve the absolute degree of abatement μ relative to an assumed reference projection. The other investment-related terms all concern the transitional costs of change. When referring to costs that relate to *transformation* processes, we are referring to the *transitional component* of the economic process. For example, if costs were only of the early scrapping nature, then as long as early scrapping is avoided, any transformation could take place and the economy could adapt to any circumstances cost-less. This is of course never the case; however, transformation will hurt if capital in good working order must be decommissioned. Meanwhile, the production of any new technology necessarily involves investments into fixed productive capital that will remain for a certain amount of time, producing a chosen number of units each year, and remain available even when costs are sunk, enabling low cost production. Once all factories producing internal combustion engine cars have been transformed into factories producing electric cars, no further transitional costs will be required in the sector.

7.2. Value of fuel savings

As noted earlier, most of the above analysis could be applied either to *marginal abatement cost curves (MACC)*, or *marginal investment cost curves (MICC)*. We have chosen to focus on the latter, partly because it is investment that is most directly associated with both the learning and the inertia effects on which we have focused. Abatement costs may be a poor proxy for these effects. This means, however that an overall cost assessment needs to consider fuel savings separately.

In the general case, we can assume an economy facing a choice between a dirty fossil fuel and a cleaner fuel (e.g. gas instead of coal), each with its own price and an elasticity of substitution. Appendix 1 details this more formal calculation, which yields - as one would expect - that in equilibrium, the cost per year of reducing emissions through fuel switching in sector k c_k^{FS} is proportional to the price difference between the fuels

$$c_k^{FS} \sim (\mu_{\mathrm{k}} - \mu_{\mathrm{k}}{}^0).\frac{(p_C - p_F)}{\beta_F - \beta_C} \tag{24}$$

where β_F, β_C are the carbon content of the fossil and cleaner fuel, respectively, and $(\mu_{\mathrm{k}} - \mu_{\mathrm{k}}{}^0)$ is the emission reductions associated with the corresponding sector k.

Cost is linearly proportional *to* the abatement *as long as the prices are independent of abatement*. Locally, this is plausible for a given sector displacing internationally traded fuels; however as a generalisation across the cost curve and globally it is more problematic.[27]

First, the fuel costs of abatement options vary widely. Coal or gas with carbon capture and storage intrinsically increase operating costs (since they reduce the operational efficiency, as well as requiring additional investment). Gas displacing coal will reduce emissions, but is often (though not always) more expensive to run.

Nuclear power generally has lower operating costs than fossil fuel plants; for renewables, the variable operating cost is negligible, and for most end-use efficiency options there is no additional operating cost, only fuel savings. For these options, therefore, both p_C and β_C are effectively zero. These options in fact dominate most abatement cost curves, particularly as emission reductions deepen (and are likely to do so even more in the light of the large cost reductions already observed in many renewable technologies and energy efficiency options).

Moreover, the value of end-use savings is affected by the overall system – thus, saving electricity is more valuable per unit of emissions than displacing generating fuels alone. There are other costs that vary by sector and region: if we are considering the market response, we need to take account of different end-use prices, which are

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[27] Note also that reducing use of fossil fuels is likely to reduce global fossil fuel prices (as emphasised by IEA, 2015), thereby increasing *local* abatement costs (such possibilities have also generated literature on the 'green paradox', see Sinn, 2008).

not constant across sectors, but display wide differences. This is illustrated in Table 2.

Process	End-use Value at current energy prices ($/tCO2)	Notes
Carbon capture and storage	Negative	Same fuel, lower efficiency
Coal to gas switching	Minus $0-40	Newbery (2016) based on range of gas and coal prices
Zero carbon displacing coal generation	$30	2015 UK steam coal price (IEA) for electricity generation & tCO_2 conversion factor (DECC)
Zero carbon displacing gas generation	$110	2015 UK (IEA) natural gas price for electricity generation & tCO_2 conversion factor (DECC)
End-use electricity saving – coal based system	$230	2015 Polish (IEA) household electricity price (inc. taxes) & approximate 2015 grid CO_2 intensity
Transport fuel savings – US/Canada	$280	2015 USA (IEA) unleaded gasoline price (inc. taxes)
End-use electricity saving – gas based system	$500	2015 UK (IEA) household electricity price (inc. taxes) & 2015 grid CO_2 intensity
Transport fuel savings – EU/Japan	$520	2015 Japanese (IEA) unleaded gasoline price (inc. taxes)

Table 2: **Value of operational fuel savings associated with emission reductions ($/tCO2)**

If we compare Table 2 against the kind of technologies in the investment cost curve (Figure 1), there is some tendency for the options that involve higher investment cost per unit emission savings to also have a higher value of fuel savings. This is not surprising: options requiring little investment but yielding big fuel savings would tend to be already implemented without a carbon price, and higher investment costs would tend to go along with higher fuel savings.[28] The range is considerable, but the broad pattern suggested is again a non-linear increase, at least initially, this time in the value of fuel savings, as abatement proceeds across different sectors with higher investment costs.

7.3. Dynamics of Learning

Against this backdrop, we can also return to the dynamics of technology learning. In section 3 we observed that the traditional mathematical representation of 'learning-

[28] As noted, the fact that many end-use improvements (like building insulation, and high efficiency or alternate fuelled vehicles) combine high investment needs with valuable fuel savings helps to explain why some appear as negative in classical engineering cost curves at low discount rates, and yet are not implemented.

by-doing' through learning curves does not involve any time component, and the concept itself remains somewhat controversial.

Obviously, learning-by-doing does in reality take time and it is clear that this is, partly, because of the three inertial components we have traced: each new round of improved technology may displace earlier investment, requires new factories to produce the equipment, and the development of supply chains and markets to diffuse the newer, improved products.

In the energy sector the importance of investment-based learning – and the associated transitional costs - has been illustrated most dramatically by the evolution of renewable energy over the past fifteen years. Many regions – but with Germany dominant in Europe – have sought to drive an energy transformation to renewable energies by huge investments driven by public subsidies to renewable generation.

This expansion has indeed been accompanied by remarkable cost reductions. Even offshore wind energy, long assumed to be very expensive, has seen costs tumbling to the point where contracts in 2017 in many European countries were awarded at levels close to wholesale electricity prices.[29]

These gains were not cheap – the cost of the contracts struck over these fifteen years under Germany's *Energiewende* now amounts to about €20bn/yr. At times, the pace of expanding production and supply chains 'overheated' the effort, keeping costs high, illustrating the importance of dynamics in learning-by-doing processes. But as the capacity-to-produce-the-product became established, and with the associated learning across all elements (private R&D, learning-by-doing and learning-by-using, as documented *i.a.* by Betterncourt et al 2013), dramatic cost reductions ensued which have led to wind and solar investments being cheaper than new fossil power generation in some regions particularly where the resources are good: in effect, to this extent, learning combined with fuel savings have reduced the first component of Table 1 to very small levels.

As noted, however, this progress was not cheap. Essentially, such learning can be expressed as a transitional investment associated with the expansion of emerging technology classes. *Ex-ante* cost estimates, which typically inform models, may turn out to be widely wrong as a guide to the enduring costs of an abatement technology, but this does not mean the costs were not real. Rather, *ex-post* they appear as having been a transitional investment which has lowered the ongoing costs for subsequent deployment.

The faster the growth, the faster the learning, but the bigger the learning investment in a given period. Hence learning investments can be considered as a function of $\dot{\mu}$. Moreover, it is clear that very rapid expansion can drive up the overall costs (though it also accelerates the benefits), implying non-linearity with respect to

[29] Perhaps the most dramatic example being the results of the second UK renewable energy auction: see https://www.gov.uk/government/publications/contracts-for-difference-cfd-second-allocation-round-results

pace of abatement. In a generalised context of representing abatement costs, in other words, learning-by-doing can be interpreted as reducing costs associated with but adding another rate-dependent component. This combination of learning investment and the need for the new technologies to overcome the sources of inertia characterised in sections 4-6 above, there, place further emphasis upon the transitional component of abatement costs we have associated with $\dot{\mu}^2$. This can thus be understood as encompassing both a learning investment, as well as the varied forms of inertial cost, required to secure the enduring gains of building up the production capacity and supply chains of whole new industries.

Part III: From micro to systems analysis

8. From technology innovation to system-level adaptability

The analysis we have presented thus far has sought to build up an understanding of potential cost structures from micro foundations. We now turn to consider this in a wider context of how technology-specific innovation may translate into a wider adaptability of systems, such as energy systems, and the analytic and policy implications of this shift in the level of analysis. This is relevant because "integrated assessment" of global climate change economics – as with IAM discussion in section 2 - needs to be conducted at the level of overall systems, not individual technologies.

8.1. Empirical basis

We noted above the recent experience of renewable electricity sources, notably wind and solar, which have seen dramatic cost reductions to the point where, following the huge investments made and resulting cost reductions, they can be cost-competitive with fossil fuel generation in a growing range of circumstances.

We note also that the industrial economics of this 'renewables revolution' has much in common with that in the energy sector more generally. After the oil shocks of the 1970s, for example, the UK offshore oil industry enjoyed around £10bn/yr investment for well over a decade – initially stimulated by government, then with increased private funding given the global oil price shock and expectations thereof. Cost projections were initially around US$80-100/bbl, but they subsequently fell dramatically as the pressure increased after the oil price collapse. Technology learning studies have charted the decline of technology costs with the scale of investment and markets across dozens of different technology areas (Weiss et al, 2010).

The experience of wind and solar technologies is thus emblematic, but not unique. Moreover, energy systems themselves are adapting to accommodate the very different characteristics of renewables – again, involving transitional costs, such as in enhanced transmission capacity. Echoing the wider institutional literature on innovation, all this points to the wider transformations that effective innovation can involve, including Schumpeter's ideas of 'creative destruction' as old capital is stranded by better technologies and systems.

Similarly, examining the longer-term history of the response to the oil shocks, it is possible to identify the broader systemic nature of transformations at the level of energy systems. The response saw extensive innovation and structural adjustment through the energy system, including dramatic improvements in energy efficiency which persisted; nowhere did these gains reverse after the energy price collapse, and particularly in countries which maintained higher energy prices (partly through taxation) the efficiency improvements continued.

A recent study (Grubb et al 2018), reinforces the findings of Bashmakov (2007) and Newbery (2003) that countries with higher end-use energy prices have adjusted so that in general they spend no more on energy (as a proportion on GDP) than those with lower prices – indeed many spend less. In other words, at a system level, the features of technology-specific learning appear to be matched by a significant capacity of energy systems overall to adapt to conditions, like structurally higher prices, through a variety of mechanisms including specific areas of higher efficiency, innovation, infrastructure better matched to high energy costs, and changes in industry structure.

Thus, as we move from technology-specific innovation to systems, we are led to recognise the overall *adaptability of systems* to constraints and incentives as a closely-related, but distinct and broader, concept: it embodies not only technology innovation but innovation and structural change in systems and associated infrastructures. The cross-country data assembled in Grubb et al (2018) points to these wide-ranging processes, but as also illustrated by the time-series data in their study, such adaptive response of the system is slow, to be measured in decades.

8.2. On public versus private investment & innovation

Though innovation is increasingly recognised as essential in tackling climate change, most of the existing modelling literature either (a) does not make any distinction (implicit or not) between public and private investment, subsuming all in some kind of implicit globally socially optimal response to emission constraints; or (b) in effect ascribes innovation to government investment (e.g. public R&D) in knowledge accumulation.

In the latter class of models (which includes for example Acemoglu et al., 2012), public R&D can compensate for example for spillovers and other long-recognised market failures around innovation. Whether or not models allow for such overt knowledge investment, most of them by implication treat the private sector response to emission constraints or carbon price as one that does not (in the modelling) involve any innovation, and is defined entirely either exogenously, or by public investment in knowledge.

This is somewhat paradoxical, given that innovation is supposed to be one of the key benefits of private sector competition and incentives. Indeed, in the energy area, one of the key observations of Bettencourt et al. (2013) is that the huge increase in renewable energy patents and cost reductions have occurred despite large declines in public energy R&D; most of the innovation has been private. We indicate below some of the reasons for this, but note here the dilemma that much of this has hinged upon technology-specific interventions (like renewable energy mandates) to drive industrial development.

Continuing technology and sector-specific supports has obvious drawbacks as such industries mature. Whilst unit costs decline, the total costs may rise due to the sheer scale (as noted, global low carbon scenarios involve investments running over US$trn/yr). There is a growing risk of enduring 'industry capture', as it becomes increasingly difficult to disentangle supports, initially justified in terms of learning spillovers, from simple subsidies to locally less efficient producers. In addition, such policy approaches do not fit well with IAMs, which tend to focus on national and global trajectories. Hence, we turn now to examine the investment and innovation dimensions of responses to technology-neutral emission policies – whether emission constraints or pricing. This is the approach (frequently recommended by economists) that one might hope to do the bulk of emission reductions, separated from the risks commonly associated with governments' technology-specific supports.

Consequently, we raise the level of our analysis from the micro-economics of learning and inertia to considering the implications of adaptive responses at systems level. Correspondingly, we are turning the lens from consideration of technology-specific innovation which might be fostered by government intervention across the innovation chain, to a generalised analysis of optimal technology-neutral policy (such as a common emission constraint or carbon price) with respect to systems which have some capacity to adapt, but with considerable inertia.

From this we offer a new approach to representing investment responses to CO2 constraints and carbon pricing in technology sectors, including innovation and inertia, and explore how that in turn might affect optimal timing and policy on the related economic instruments.

8.3. Stylised characteristics of systems with innovation and inertia: pliability and the representation of adaptability and transitional costs

The above discussion sets our mathematical formulation in a wider context concerning the extent to which emitting systems are *adaptive* in the sense that technologies and systems over time adapt to external conditions and constraints, for example through induced technological and systems innovation, different infrastructure choices, and structural changes. This might also imply that some costs typically assumed to be enduring are actually manifestations of transitional costs (as with renewable energy costs, the decade of industry-building in the North Sea oil and gas industries, and the initial costs of more efficient vehicles and appliances which subsequently have become dominant).

Many lines of evidence point to the apparent capacity of energy systems to adapt to circumstances. Consistent with the 'Bashmakov-Newbery constant' of energy expenditure, described above, there are sustained differences in structures and levels of energy consumption between different industrialised countries, with no sign of convergence (Grubb, et al., 2014, Chapter 1). Technology studies underline the huge array of energy technologies available or under development, and the extent of both fossil fuel and renewable energy resources, as well as continuing wide potentials to improve energy efficiency. Consequently, there is every reason to assume that further improvements are possible – and yet, face many transitional constraints.

Indeed, history offers innumerable examples of how complex systems evolve in path dependent ways, with consequent phenomena of lock-in (e.g. Unruh, 2002). Such characteristics are clear in the history – and projections – of key sectors like transport, electricity and urban systems, all of which have the potential to be transformed over the next few decades, but embody very long lived capital stock and infrastructure.

As examined more extensively in Grubb, et al. (2014), one factor in such behaviour reflects the extent to which apparently cost-effective measures (particularly relating to energy-efficiency) are impeded by other factors – overcoming these obstacles so as to install insulation or become aware of more efficient equipment leads to gains that are unlikely to be reversed, for example. Innovation and infrastructure investments are, likewise, to an important degree long-lived public goods whose influence is again enduring. In the terminology of Grubb, et al. (2014), these are First and Third Domain processes, and are unlikely to reverse.

Such processes contribute to the capacity of energy systems to adapt to external forces, through innovation and structural change. Indeed, more classical economic processes do *not* exclude that possibility, particularly in relation to investment, though they have generally been interpreted otherwise in modelling.

The term *adaptive* is here meant to capture the evidence that technologies and systems can respond to investment, adjustments or external pressures which combine to lower the cost of sequent action in enduring ways, and should not be confused with the term 'adaptation' – generally used in the climate change literature to adapting to the *impacts* of climate change. Changes however also involve the cost of the investment and have to overcome the many sources of *inertia* as detailed in this paper.

The basic implication is we can define emissions abatement as a cost which relates not only to the *degree* of reduction (abatement), but also to the *rate* of deviation from the baseline. In essence, we group the cost components identified in Table 1 into an overall cost of abatement:

$$C = \alpha_I \mu^2 + \alpha_B \dot{\mu}^2 - \alpha^{FS} f(\mu) \quad (25)$$

where $\alpha^{FS} f(\mu)$ represents the value of fossil fuel savings. Compared to the classical formulation, we are separating investment and fuel costs, and *adding* a generalised *rate-dependent* term derived from our analysis. The *enduring* cost term – the overall degree of abatement at time t – may be moderated by learning on the investment cost, and offset to some degree against the value of fuel savings.

In this formulation, note that as compared with exogenous ex-ante modelling, learning-by-doing investments, and indeed wider system adjustments (such as long-lived infrastructure associated with low carbon systems) have the effect of reducing the enduring component α_I, as the costs take the form of transitional investment associated with α_B.

The tension between these two forces we refer to as the *pliability* of the system – reflecting the potential for changes in ways that endure, but inevitably with a trade-off between this and the effort or pressure exerted – the scale of investment or other costs incurred in response to incentives or constraints. A system with no pliability simply returns to the 'status quo ex ante' once the incentive or constraint is alleviated. A fully pliable system will retain all the advances, of technologies, infrastructure, etc., and hence stay in the new state unless some new pressure is exerted.

9. Some implications for timing and effort: a model exploration

9.1. Interpreting and parametrising the stylised abatement costs

Consequently, we turn to show the implications of this for studying how optimal abatement paths might depend on the presence of learning and inertia. As noted, the form of fuel savings is somewhat indeterminate, but section 7 suggests the value would rise non-linearly as abatement proceeds at least initially. The simplest treatment is to approximate fuel cost savings as offsetting some (positive but uncertain) portion of the higher investment costs assumed for low carbon abatement, at least initially; fuel cost savings at high abatement would be capped, but may be eclipsed by the other cost components of rising investment, transitional costs and the internalised costs of carbon, the last of which we consider separately in the following model.

Our energy system equation then approximates to two basic functional dependencies, respectively quadratic on the degree and rate of abatement:

$$C(t) = C_A(t) + C_B(t) = \alpha_A \mu(\mathrm{t})^2 + \alpha_B \left(\frac{d\mu(t)}{dt}\right)^2$$

The first element now expresses the 'enduring' component of *enduring abatement* costs – investment less fuel savings – and thus mimics the classic DICE formulation in functional form, but we have in additional a rate-dependent term, also quadratic in form based on the enquiries in this paper. Recalling that learning reduces the coefficient α_I, note that innovation is now subsumed in a wider interpretation of the coefficient α_A, in which a lower value reflects not only the estimated scope for learning to reduce investment costs (the $(\beta W^0)^b$ of equation 10), but also the wider potential of the system to adjust over time, including the potential for specific investment costs in energy efficiency, zero carbon sources and related infrastructure, to be offset against fuel savings.

The degree and rate of abatement refer to the deviation at any time *t* from the 'default' or 'business-as-usual' trajectory. The constants α_A and α_B can now be seen as representing more broadly the magnitudes of ongoing vs transitional abatement costs respectively. The first term represents the ongoing cost element associated with a given *degree* of abatement relative to the high carbon baseline; the second term reflects inertia, i.e. the cost of *changing* the level of abatement. A lower ongoing component α_A and higher transitional component α_B means that abatement cost (or effort) is increasingly dominated by the transitional (learning and infrastructure investments and adjustment costs) cost of moving from one state to another, relative to the recurring costs of staying at any given distance from the 'reference' path. The ratio of the transitional to enduring component thus represents the pliability of the system. Lower α_A and higher α_B increases the relative influence of transitional costs, ie. efforts to overcome inertia, which have an enduring impact in lowering the cost of subsequent cutbacks or altering the underlying pathway.

Exploring the implications of this 'pliability' requires a way to trade-off between the two components, which we do with reference to a total abatement cost over time horizon $\hat{t}$. Thus, for a time horizon $\hat{t}$ years after 2015, we define the relationship between the enduring (C_A) and transitional (C_B) cost components by assuming the cost of linear emission reductions, at rate $\bar{\varepsilon}$%/yr (slope), over a time horizon $\hat{t}$ is invariant. (Specifically, in the analysis, below, we parametrise C_A and C_B in terms of the costs of halving global emissions from current levels by mid Century).

So the abatement - the difference between rising baseline and declining actual emissions is[30]

$$\mu = (\varepsilon_1 - \bar{\varepsilon})t$$
$$\dot{\mu} = (\varepsilon_1 - \bar{\varepsilon})$$

Integrating (undiscounted) abatement cost to time $\hat{t}$, we thus define:

[30] Note that a linear abatement schedule implies a quadratically rising cost in the conventional (enduring only cost) case, but a constant rate of expenditure if the system is fully pliable – we find in the model that this can be the result of optimisation over the first few decades in this case.

$$\bar{C} = \int_0^{\hat{t}} (C_A + C_B)dt = \int_0^{\hat{t}} \alpha_A \mu^2 dt + \int_0^{\hat{t}} \alpha_B \dot{\mu}^2 dt = (\varepsilon_1 - \bar{\varepsilon})^2 \cdot \left(\alpha_A \frac{\hat{t}^3}{3} + \alpha_B \hat{t}\right)$$

Denote $\hat{\alpha}_A$ as the value of α_A when all the abatement costs are enduring ($C_B = 0$), thus defining the abatement cost curve when there are no transitional costs, and so the cost is

$$\bar{C} = (\varepsilon_1 - \bar{\varepsilon})^2 \hat{\alpha}_A \frac{\hat{t}^3}{3}$$

Similarly, denote $\hat{\alpha}_B$ as the value of α_B when there are *only* transitional costs ($C_A = 0$), in which case:

$$\bar{C} = (\varepsilon_1 - \bar{\varepsilon})^2 \hat{\alpha}_B \hat{t}$$

The trade-off of course does not depend on the rate (the slope drops out when we are comparing C_A and C_B). This form is intuitively reasonable since the adjustment costs rise as the square of the rate, which is inversely proportional to the timescales over which adjustment costs are defined. These define the extreme cases of no ($C_A=\bar{C}$, $C_B=0$) or complete ($C_A=0$, $C_B=\bar{C}$) pliability of the system, for a characteristic adjustment time $\hat{t}$.

We write the general case in terms of a parameter σ which defines the relative pliability of the system, in terms of the ratio of the two components:

Abatement cost = $\hat{\alpha}_A (1-\sigma)\mu^2 + \hat{\alpha}_B \sigma \dot{\mu}^2$

Substituting for $\hat{\alpha}_B$, we can write the overall cost for a system of pliability σ as:

$$C(t) = \hat{\alpha}_A[(1-\sigma).\mu(t)^2 + \sigma.\frac{\hat{t}^2}{3}\dot{\mu}(t)^2]$$

In which $\hat{\alpha}_A$ serves as the overall cost scaling factor, and $\hat{t}$ is the characteristic timescale of major system adjustments (see note 30 below). Starting with the classical notion of a reference high carbon projection assumed *ex ante* to be least cost, we take the above equation as the first order approximation of the abatement cost, and now apply it in a DICE-like integrated assessment context.

9.2. Representation of climate damages

The task of applying this to studies of optimal global responses is greatly eased by the finding from the scientific community that global temperature change at a given time is closely related to cumulative emissions to that point. This enables a simple representation of climate impacts linked to temperature change (the vast majority of optimising 'integrated assessment' studies, which try to compare the cost of cutbacks with the cost of avoided damages, express climate damages in terms of global average temperature increase; see e.g. IPCC 2014).

Our general model allows for climate damage with both linear and quadratic dependence on the global temperature change respectively, but in the illustration here and again in common with much of the 'integrated assessment' literature, we assume that global damage increases in proportion to the square of temperature change.[31] A central estimate is that 500GtC cumulative emissions increases global temperature by about 1 deg.C. (IPCC 2013; Figure SPM-10) so that:

Annual damage from climate change at time t,

$$D(t \mp \tau) \propto \ (temperature\ change)^2 = \left(\frac{E(t)}{500\ GtCO_2}\right)^2, \tag{26}$$

where τ is a time lag between emissions and impacts, and E(t) is the cumulative CO_2 emissions in billion tonnes (gigatonnes) of carbon (GtC) at time t:

$$E(T) = \int_0^T \varepsilon(t)dt,$$

Contrary to common assumption, the time lag between emissions and resulting temperature is small – a recent scientific study (Ricke and Caldeira 2014)) estimates mean lag to be about one decade, and most of the temperature rise occurs in the first few years.[32]

The cumulative damage from climate change is then:

$$\int_0^T e^{-r\cdot t} \cdot D(t)dt$$

[31] A much earlier study by the author (Grubb et al., 1995) presented the basic idea of the mitigation analysis, and showed results that emerged if the damages from climate change were assumed to be proportional to the atmospheric concentration of CO_2. At the time this seemed the analytically tractable approach and a useful approximation to illustrate the underlying themes. However, maths does what is specified and this treatment had the serious drawback that especially in cases with a highly adaptive energy system, results could be driven by the long-run benefits of negative emissions which reduce concentrations, without limit. Particularly at low discount rates (or high damage coefficients) this could go to implausible extremes. The treatment here, in which damage is related directly to the square of temperature change since pre-industrial levels, avoids this problem since the benefits of reducing concentrations decline non-linearly, and turn negative if global temperature drops below pre-industrial levels.

[32] Although there is significant thermal time lag particularly in ocean response, this inbuilt thermal inertia is largely offset by the CO2 absorption so that in fact temperatures occur at a time quite close to the point of cumulative emissions. Ricke and Caldeira (2014) examine the impact of a CO2 pulse with a variety of representations and conclude: "The median estimate of the time until maximum warming occurs is 10.1 years after the CO2 emission"; most relevant most of the warming occurs well before this. ".. while the temperature consequences of CO2 emission materialize more quickly than commonly assumed, they are long lasting. The fraction of maximum warming still remaining one century after an emission has a median value of 0.82, with a very likely range of 0.65–0.97." The relationship observed in the IPCC (2013, Figure SPM-10) does not include a time lag, and is stated to hold for most scenarios excepting extreme rates of emissions change, suggesting that representing temperature by E(t) is a reasonable approximation; our central case here thus also does not include an explicit time lag between cumulative emissions and temperature. In the PAGE model, if emissions cease, about 70% of the maximum committed warming occurs in the first decade (Chris Hope, personal communication). The DICE model however (and particularly DICE 2016) appears to have a far slower temperature response.

Where $e^{-r \cdot t}$ is the time discount (at rate r) applied to climate damages, and $D(t)$ are the annual damages at time *t*, which are costed as above.

The optimisation problem is then to minimise the total discount costs out to time T:

Min. Function $\mathrm{F(T)} = D(T) + C_A(T) + C_B(T)$

Where

$$\int_0^T F(t)dt = \int_0^T e^{-r \cdot t} \left\{ \begin{array}{c} d_1 \cdot E(t) + \frac{d_2}{2} \cdot E(t)^2 \\ +\alpha_A \cdot \left(\varepsilon_{ref}(t) - \dot{E}(t)\right)^2 \\ +\alpha_B \cdot \left(\varepsilon_1 - \ddot{E}(t)\right)^2 \end{array} \right\} dt$$

Where d_1 and d_2 are the coefficients of damage costs related to linear and quadratic dependence on the global temperature change respectively;[33] $\varepsilon_{ref}(t)$ is the reference emissions path, and α_A and α_B are the coefficients associated with the ongoing and transitional elements of abatement cost, respectively.

The optimality conditions associated to the equation above generate a second-order differential equation. In Appendix 3 we give details and nomenclature and show it can be solved analytically using the Euler Lagrange Method. Thus for any given level of assumed damage associated with accumulating CO_2 in the atmosphere, we can explore how the path of theoretically optimal responses depends on the balance between ongoing abatement costs, and the transitional costs associated with induced learning investments and overcoming the various sources of inertia, all set against the long-run benefits associated with reduced atmospheric change.

9.3. Illustrative Numerical assumptions – climate and baseline

Emissions since the late nineteenth century to 2016 amount to around 565GtC, and annual emissions in 2016 were around 9.9GtC/yr (Le Quéré et al., 2016). Whilst the main interest is in the influence of shifting costs between enduring and transitional components, to illustrate the impact of adaptability in the energy system, a number of other assumptions are necessary and chosen in part to facilitate comparison of the conventional cost case (with non-adaptive abatement costs) with other modelling works in the field.

- ***Climate change damage $3trn/yr for one degree additional temperature increase (an additional 500GtC emission).*** The cost associated with any given degree of atmospheric change is extremely uncertain and needs to factor in corresponding issues of risk. If an additional 500GtC (from present) emissions increases global average temperatures to 2 deg.C above pre-industrial levels, $3trn/yr damages amounts to around 2% of projected global GDP in the corresponding decades (around or soon after 2050); this is towards the high

[33] For simplicity we have omitted the temperature lag as being not material to the analysis, as indicated in note 32; it is equivalent to a reduction in d, which is a highly uncertain parameter, by just a few years discounting).

end of 'classical economic' estimates, but modest compared to many other estimates which place more weight on risks, non-linear responses and the welfare of future generations, for example.[34]

- ***Real discount rate 2.5%/yr.*** This is a compromise between the 'prescriptive' and the 'descriptive' rates, though leaning more towards the latter. This leads to significant discounting of costs after a few decades. Note that when the emitting system is 'pliable', the results suggest that the economic case for substantial action is not so dependent on the valuation of damages – either the absolute damage assessment, or weighting thereof through eg. very low (Stern-type) discounting assumptions or related distributional assumptions.
- ***Reference emissions growth 120MtC/yr.*** Global emissions growth has tended to be approximately linear over extended periods, whilst fluctuating significantly. There are various reasons why emissions growth is not exponential and is even less likely to be so in the future. We thus use linear projections for the reference case, and emissions are reduced relative to this:

$$\varepsilon_{ref}(t) = \varepsilon_0 + \varepsilon_1 \cdot t$$

  where ε_1 is the linear growth rate. Over the past few decades the average increase of fossil fuels CO2 has been about 1.5% of 2010 emissions, were substantially higher in the early 2000s (with the Asian boom, before the energy price rises and the credit crunch) but have since paused (due to structural changes and growing abatement efforts), whilst the dramatic cost reductions in renewable sources are likely to contain future growth. Also historical data already include the impact of extensive energy efficiency measures. We take as a reference (no action) case a view in which global emissions rise at approximately 120MtC/yr: = 1.2 % of annual CO2 emissions over 2014-16, which is slower than historical trend but well within the range of IEA projections.

9.4. Illustrative Abatement costs parameters

We are left with the question of how to parametise the abatement cost parameters. There is a huge literature on abatement costs, but hardly any of it examines *transitional* costs – and, as noted, a core part of our argument is that the traditional measures actually comprise both enduring and transitional costs.

To address these, we estimate abatement cost parameters with reference to a 50% cut in global emissions *from recent (2014-16) levels* as follows. We reference abatement costs to the existing economics literature, integrated over a specific time

[34] A review by Tol (2015, Table 1) summarises standard estimates of climate damage by classical economists, but curiously omits many other views including those of Stern (2013), Weitzman (2012), Ackerman and Munitz (2012), Hope (2013) and Pindyck (2013) which place more emphasis upon risk aversion, ethically-grounded discount rates, and numerous other factors; and which are now complemented by Pezzey (2017) who argues that the value is truly and deeply unknowable, particularly at the higher ends.

period, but estimate the coefficients from two possible extremes of assumptions about the actual source of these costs:

- Purely ongoing costs (no transitional component: C_B =0): 50% cut in global CO2 emissions (from recent levels) by 2050 costs 1.5% of projected GDP, around at $2trn/yr) This corresponds approximately to central estimates by a number of the more detailed energy system models in recent comparative studies (eg. the AMPERE studies reported in Kriegler et al 2015) [35].
- Purely transitional costs (no ongoing component: C_A =0): the same cutback, on a linear trajectory of abatement, results in the same total integrated cost over the period 2015-2050; however, these are attributed as transitional costs of reorienting the energy system over these decades.

The analysis in Grubb et al (2018) estimates the adjustment timescale of energy systems to price shocks has been at least 25 years, but notes that this is still incomplete adjustment: they suggest a characteristic timescale of fuller adjustment at 30-40 years. Here, we fix the characteristic timescale of adjustment used for the above comparison at $\hat{t}$ = 35-year time horizon, which is an empirically reasonable estimate of the characteristic time constant of major transformations in energy systems, and corresponds to a depreciation rate of 2%/yr. [36] This matches the timescale from 2015 to 2050, on which many published mitigation cost estimates are reported.

Thus for the cost parametrisation, we incur the cost indicated over the specified 35-year abatement schedule, to 2050. As indicated above, the overall abatement cost is $\hat{\alpha}_A(1-\sigma)\mu^2 + \hat{\alpha}_B\sigma\dot{\mu}^2$ and $\hat{\alpha}_B = \frac{\hat{\alpha}_A\hat{t}^2}{3} = 408\,\hat{\alpha}_A$, so the general cost form is: $\hat{\alpha}_A[(1-\sigma)\mu^2 + 408\sigma\dot{\mu}^2]$. For a system with no capacity to adapt to emission constraints ($\sigma = 0$) we recover the classical form of abatement cost curve $\hat{\alpha}_A\mu^2$. For a fully pliable system ($\sigma = 1$; C_A = 0), the costs incurred from such an emission reduction to mid-century are the costs of learning and related investments, inertia and pathway adjustments, so that by the end of this transition, there are no enduring costs associated with remaining at that level after 2050: the system has responded that far.

[35] Kriegler et al (2015), Figure 9, indicates that a *cumulative* emission reduction of 60% below reference costs has an *NPV policy cost* of about 1.7% GDP in the MESSAGE and MERGE-ETL models, and 1.3 – 1.5% GDP in the IMAGE and DNE21 models. Several other models do give higher numbers. However, cumulative emission reduction of 60% clearly implies much greater reductions by 2050; also note that renewable energy costs in particular have declined significantly even since these studies were carried out.

[36] The data collected in Grubb et al (2017) indicates that the US took 1-2 "cycles" to adjust fully to the oil price shocks (with one cycle being about 25 years), and that adjustment processes in eastern Europe have been of similar timescales. The parameters in Vogt-Schilb et al (2017) include depreciation rates for forestry and buildings well below 2%/yr, and energy at 2.5%/yr, but the latter is based mostly on power plants and does not take full account of e.g. transmission systems and other system / infrastructure dependences.

Beyond simply deriving the cost coefficients in this way, a great deal of insight can be gained simply by comparing the results of these two opposing conceptual assumptions – contrasting the two extremes of zero pliability, or total pliability.

These are the cases shown in bold lines (dashed and solid respective) in the results below. The latter appears as a radical assumption, but it reflects many of the observations in the previous section about the apparent capacity of energy systems to adjust; recall, moreover, the projections of the International Energy Agency, which estimate that the additional investment costs of such an emission reduction scenario are almost exactly offset by the cost savings in the fossil fuel industries. We emphasise however that we are not *assuming* this but offering it to illustrate the implications of such a fully pliable energy system.

It is of course quite likely that reality lies somewhere between, and hence a particular interest might be in our mid-scenario, in which the costs are weighted 50:50 between the ongoing and transitional components, i.e. pliability σ = 0.5.[37]

9.5. Illustrative Results

Figure 2 shows the optimal trajectories of emissions and cumulative emissions. For the classical, non-adaptive/low inertia case (zero pliability), there is a substantial jump of initial abatement which then increases slowly as climate damages accumulate. The effort is defined by distance from the default trajectory, and the abatement cost directly reflects the assumed 'social cost of carbon damages' as discounted.

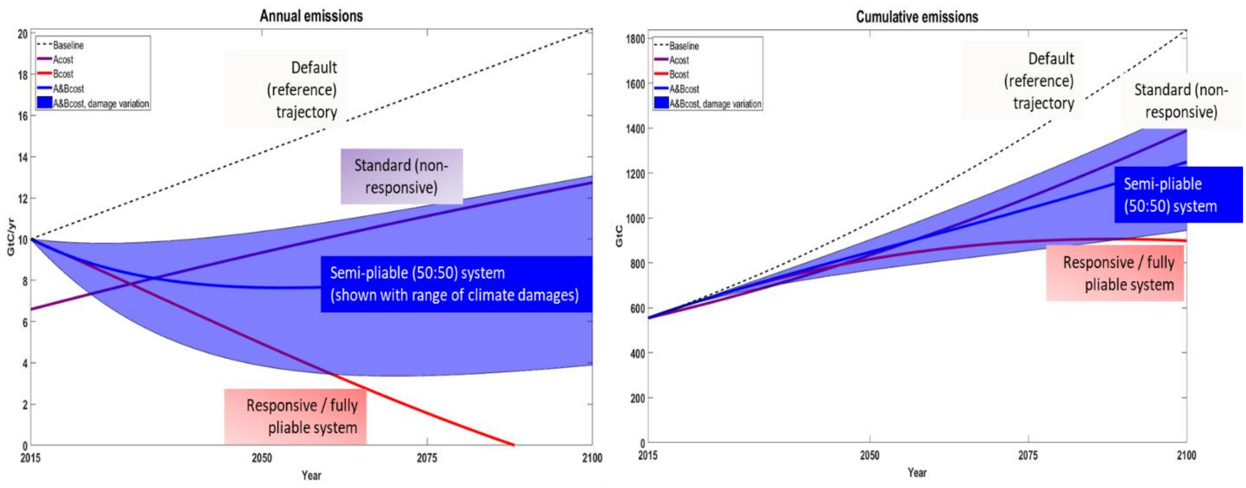

Figure 2 Implications of pliability (adaptability and inertia) for abatement trajectories. Notes: The Figures show the least-cost global response given different assumptions about the structure of energy systems costs. The panels show trajectories of annual (left) and cumulative (right) CO2 emissions (in GtC). The violet lines reflect classical assumptions in which abatement costs relate purely to the degree of abatement, relative to baseline global emissions that are steeply rising (the top line in the emission figures). The lower red lines reflect a fully pliable system, in which abatement costs (the degree of effort) relate to the rate of abatement, but shift the trajectory thereafter. The blue line in between

[37] Going further requires some estimate of how much of the apparent costs of any given path may actually be ongoing, and how much are transitional (with enduring benefits). Further discussion is given in the authors' book *Planetary Economics* (Grubb, Hourcade, & Neuhoff, 2014).

reflects a mixed case. The blue range corresponds to a variation of the marginal cost of damages, using a minimum and maximum factors of 0.5 (upper trajectory – less abatement) and 2 (bottom trajectory – more abatement), respectively. For assumptions see text. The fully pliable case results in steadily declining global emissions, reaching zero in the second half of the Century (left panel) and limiting the additional cumulative emissions to less than 1000GtC (right panel).

The combination of such assumptions means that after the initial prompt cutback, global emissions continue to rise, as shown in the top 'classical case' (violet) lines; the abatement cannot keep pace with the rising emissions of the reference case. This is broadly the result that emerged from many of the modelling studies embodying classical assumptions (particularly prior to the debates provoked by the Stern Review, which have tended to increase the damage estimate). Cumulative emissions by the end of the century reach around 1400GtC.

In contrast, if the energy system itself is highly adaptive in the long run (offset by high inertia as discussed), the pattern is quite different ('fully pliable system', red lines as indicated in Figure 2). The deviation from the default trajectory rises to exceed the 'steady state' level of the classical case after 10-15 years, and it carries on diverging at a rate which does not slacken. The optimal response in this case involves global emissions halving before 2050 and continuing to decline, reaching zero before the end of the century. The corresponding cumulative emissions reach around 900GtC, after which atmospheric concentrations slowly start to decline.

Note that *the assumed marginal damage associated with a given degree of climate change in the two cases is identical by 2050*. It is the dynamics of response that differs. At first glance, this appears to be somewhat paradoxical – one might suppose that the effort would be less when inertial/transitional costs increase. Yet this is not the case, because abatement in the pliable case is associated with an enduring change in trajectories. The benefits are not only those of the immediate emission reduction, but they extend over time - initial efforts carry through to a pattern of more extensive abatement spanning over decades.

The intermediate cases allow for the possibility that we continue to develop fossil fuels in ways intrinsically cheaper than the alternatives. This contrasts with the fully pliable assumption that systems have huge capacity to innovate, evolve and respond over time, and that there is no inherent reason why a system based on fossil fuels should ultimately be cheaper than one that is based on more intensive energy efficiency and zero carbon energy resources.

Figure 3 then shows the corresponding abatement investment and climate damage over the century.

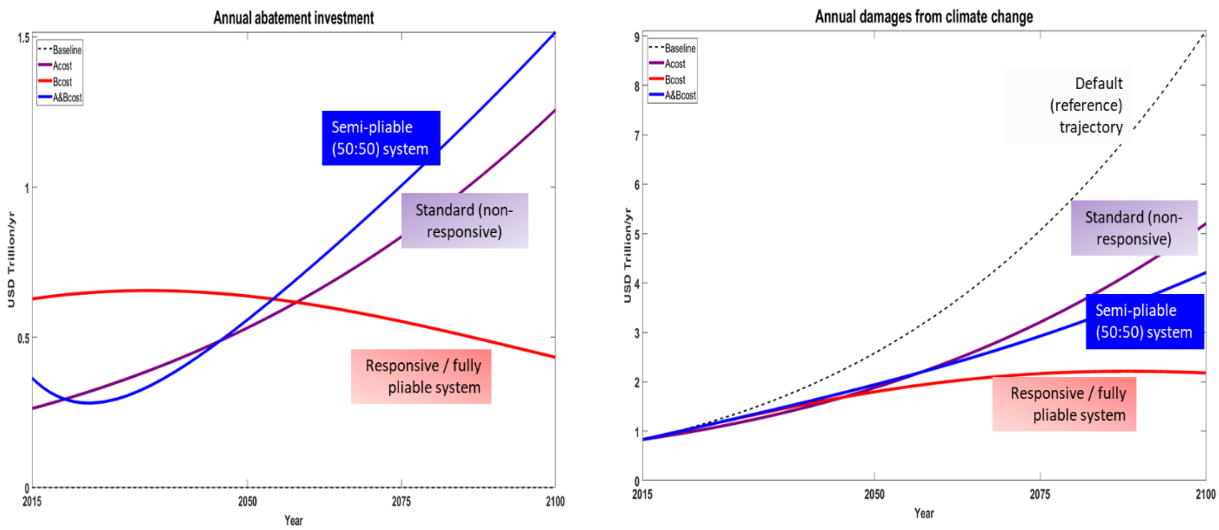

Figure 3 Implications of adaptability and inertia for abatement trajectories and effort (see notes to Figure 2.)

In the classical (non-adaptive, no inertia: i.e. zero pliability) case, for the given assumptions, optimal expenditure on abatement starts at just $250bn/yr and then rises steadily over the century.

The optimal investment of initial action – to be more precise, the effort worth exerting – is more than twice as big if the system is fully pliable, because in addition to the direct value of emission reductions, there are benefits from both the cost-reduction response of the emitting system (i.e. abatement costs coming down in response to learning and infrastructure investment) and the avoided costs of overcoming future inertia. However, after a few decades the effort needed starts to decline, as the system moves closer towards atmospheric stability. For the given assumptions, the initial effort is around $700bn/yr and declines towards $400bn/yr by the end of the century. This shows that responses which help to adjust the long-run trajectory may be more valuable than the value of cutting emissions alone. In this example, they are worth more than twice as much.

The corresponding costs of climate damages are illustrated in the right panel of Figure 3. In the classical, non-adaptive case, the level of damages arising from the 'optimal' response increases to around $6trillion/yr by the end of the century, since emissions carry on rising. In the fully pliable case, the damages stabilise around $2trn/yr. For optimal responses, the total equivalent-cost by the end of the century is thus less than $3trn/yr for a pliable energy system, and over $8trn in the opposite case.

The importance of this is not so much the absolute numbers, but rather the more generic headline insights (the absolute numbers are interesting, but completely dependent on assumptions).[38]

[38] Technically-minded readers may wish to email the authors for a copy of the model (which is implemented in MatLab) and experiment with different assumptions, including those on damage-equivalence and discount rates which can have a strong bearing on the absolute results, and to compare energy-related assumptions with other

10. Conclusion

Energy systems are characterised by both learning and inertia: energy technology costs have generally declined with investment, and there is wider evidence of the capacity of systems to respond, but this requires substantial investment and involves a range of other transitional costs. Whilst most operational changes can happen rapidly and are irreversible, long-term energy futures are dominated by investment, and for example low carbon scenarios are characterised by radically different investment structures. Most new supply investments in particular embody the development of whole new industries and long-lived assets (like the systems they displace), and also considerable technological change and scale economies.

To date, most such developments have been driven by technology-specific supports or regulatory programmes. These have helped to radically reduce the cost of key low carbon technologies, but have also required large investments and government direction. As the costs have come down and scale grows, there is a case to consider again the economics of technology neutral policies, based on direct emission constraints or carbon pricing. This however is only credible if analysis encompasses the demonstrated potential for investment-related learning, and takes account of transitional / investment costs at a whole systems level.

Most of the models applied to global energy-environment studies, however, have little or no direct representation of costs related to the *rate* of change, and often make exogenous assumptions about technology costs, implying no investment-related learning. Temporal effects in such models arise indirectly, either as a result of optimisation with discount rates; imposed constraints on rates of change (such as exogenous and often arbitrary constraints on the growth of new sources); or attention to just one dimension, in terms of capital stock lifetime.

We have explored in depth the characterisation of both learning associated with investment, and the various sources of inertia. We have argued that the conjoined effects of inertia and learning deserve far more direct attention in energy systems modelling, in particular for example, concerning some of the more ambitious climate change goals. We have thus attempted a systematic study of potential sources of inertia and the mathematical forms they might take, consistent also with learning, in optimisation frameworks.

We show that when learning is instantaneous, it has no direct effect on optimal timing. However, time appears through effects of capital displacement; the investment in productive capital to manufacture the energy-producing equipment; and the business and network investments required to support diffusion of technologies.

studies with strong empirical content such as those by the International Energy Agency. A good analysis of how results in such modelling depend on key assumptions – and the consequent implications of uncertainty for estimates of the 'social cost of carbon' – is given in Hope (2012).

These terms appear as costs related to the time derivative of the pathway of abatement, i.e. rate of change.

We refer to the inherent tension between adaptability and inertia as the *pliability* of the system. One essential insight from the resulting modelling is that an adaptive emitting system, in which costs decline in response to learning and infrastructure investments, can greatly lower the overall costs associated with climate change – both impacts and responses - but only if requisite effort is put up-front into changing course. Given this, it also serves to narrow the gulf between the "cost/benefit" and the "security" approach to the problem: a pliable emitting system implies that the costs of remaining 'secure' – or of avoiding many $trns of climate damages - will end up much lower than classical approaches suggests. With an energy system that is largely adaptive, either approach to the costs and risks of climate change then yield similar conclusions about the benefits of strong action.

As the results here show, the extent to which energy systems are indeed adaptive is a very important economic question. Unquestionably, the degree of adaptability and the determinants (and parametrisation) of inertia deserve more attention. The idea that energy and other emitting systems have no capacity to learn and respond to constraints and incentives is clearly inconsistent with the evidence; this paper shows how much this matters.

Ultimately, attention to learning and inertia result in smoother time profiles of abatement, but also substantially higher investment efforts early on to build up new capabilities and change the course of the energy and other emitting systems, as early and smoothly as possible.

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Appendix 1. Maximising output with fuel switching

To evaluate the general case of fuel switching we can define a Constant Elasticity of Substitution (CES) function in which substitution occurs between fuels used to produce output,

$$\mathrm{Y} = \varphi_0 (J_C{}^{\eta} + J_F{}^{\eta})^{\frac{1}{\eta}}, \tag{27}$$

where Y is output, J_C is clean fuel use, and J_F is the more carbon intensive fossil fuel, both for production, 1/(1-η) is the substitution elasticity ($-\infty < \eta < 1$) , and φ_0 is a constant productivity factor.

For simplicity, we maximise Y with a budget constraint, $X = p_C J_C + p_F J_F$, with p_C, p_F fuel prices.

The Lagrangian is $\mathrm{L} = \mathrm{Y} - \lambda \mathrm{X}$, with λ an arbitrary Lagrange multiplier. We consider that total fuel use $\mathrm{J} = \mathrm{J_C} + \mathrm{J_F}$ does not depend on the scenario considered. In equilibrium, we obtain:

$$\lambda \mathrm{p_C} = \varphi_0 (\mathrm{J_C}{}^{\eta} + \mathrm{J_F}{}^{\eta})^{\frac{1}{\eta}-1} \mathrm{J_C}{}^{\eta-1} \text{ and } \lambda \mathrm{p_F} = \varphi_0 (\mathrm{J_C}{}^{\eta} + \mathrm{J_F}{}^{\eta})^{\frac{1}{\eta}-1} \mathrm{J_F}{}^{\eta-1} \tag{28}$$

We want to know what is the cost to the economy of switching fuels. Denoting fuel combustion emissions reductions μ in tonnes of CO2/y, this is

$$\frac{\mathrm{dY}}{\mathrm{d}\mu} = \frac{\partial \mathrm{Y}}{\partial \mathrm{J_C}} \frac{\mathrm{dJ_C}}{\mathrm{d}\mu} + \frac{\partial \mathrm{Y}}{\partial \mathrm{J_F}} \frac{\mathrm{dJ_F}}{\mathrm{d}\mu} \tag{29}$$

We assume that the economy uses a total amount of fuel $\mathrm{J} = \mathrm{J_C} + \mathrm{J_F}$ (in e.g. GJ), and that these fuels have carbon contents β_C and β_F (in MtCO2/GJ). In a baseline scenario, clean fuels are not used, and $\mathrm{J} = \mathrm{J_F}$. When changes in the use of fossil fuels is compensated by clean fuels, we can write emissions as (assuming constant total fuel use J):

$$\mu = \mathrm{J}\beta_F - (\mathrm{J_C}\beta_C + \mathrm{J_F}\beta_F) = \mathrm{J_C}(\beta_F - \beta_C) = (\mathrm{J} - \mathrm{J_F})(\beta_F - \beta_C) \tag{30}$$

The terms of (33) are:

$$\frac{\partial \mathrm{Y}}{\partial \mathrm{J_C}} = \lambda \mathrm{p_C}, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{J_F}} = \lambda \mathrm{p_F}, \quad \left.\frac{\mathrm{dJ_C}}{\mathrm{d}\mu}\right|_{\mathrm{J}} = \frac{1}{(\beta_F - \beta_C)}, \quad \left.\frac{\mathrm{dJ_F}}{\mathrm{d}\mu}\right|_{\mathrm{J}} = \frac{-1}{(\beta_F - \beta_C)}, \tag{31}$$

And therefore, in equilibrium,

$$\frac{\mathrm{dY}}{\mathrm{d}\mu} = \frac{\lambda(\mathrm{p_C} - \mathrm{p_F})}{\beta_F - \beta_C}, \tag{32}$$

Which thus requires that $\lambda \sim J_C$ in order for the cost to scale correctly in terms of fuel expenditure. We see that in equilibrium, the cost of reducing emissions through fuel switching, per ton of carbon, is proportional to the price difference between the fuels (assuming that the clean fuels are more expensive). The total cost per year is linear in both the emissions reductions, in a very intuitive form,

$$c^{FS} = \mu \frac{dY}{d\varepsilon} \propto \mu \frac{(p_C - p_F)}{\beta_F - \beta_C}. \tag{33}$$

Appendix 2. Mathematics of abatement investment cost curves with instantaneous learning

We give here the mathematical details of the calculations used to arrive at the first cost expression including learning-by-doing. The infrastructure cost calculation follows the exact same scheme however with a time derivative.

ρ_k = abatement density: marginal abatement in sector k per unit of marginal cost

$\rho_k = \frac{d\mu}{dI_k}$, $\mu = \int_0^{I_k} \rho_k dI_k{}'$

U_k = installed capacity in sector k

W_k = cumulative production in sector k

I_k = Investment cost on installed capacity in sector k

β_k= emissions intensity savings per unit of installed capacity

We assume that abatement in sector k is proportional to installed capacity in the same sector:

$$\beta_k dU_k = \rho_k dc_k = d\mu_k$$

Following the nomenclature of section 4, the total cost c_A over all sectors *k* is determined by integration of the marginal cost over a *pathway of technology development* in each sector, and summed over all sectors:

$$c_A = \sum_k \int_0^{U_k} I_k(U_k^{'}) dU_k^{'}, \qquad (34)$$

The cost in sector *k* is, therefore, an integral along a technology pathway:

$$C_{Ak} = \int_0^{U_k} I_k dU_k^{'} = \int_{\mu_{k-1}}^{\mu_k} \frac{I_k}{\beta_k} d\mu$$

Equations (3) and (4) in section 3.2 show the total costs in the case without learning. If learning is taken into consideration, then the costs associated with sector *k* change as measures are applied. Denoting the initial MACC as I_k^0, while the actual cost curve incurred, which continuously changes with increasing abatement, as I_k:

$$I_k = I_k^0 \left(\frac{W_k^0 + W_k}{W_k^0} \right)^{-b_k} = I_k^0 \left(1 + \frac{\int_0^t \left(\dot{U}_k + \delta_k U_k \right) dt'}{W_k^0} \right)^{-b_k} = I_k^0 \left(1 + \frac{\int_0^t (\dot{\mu}_k + \delta_k \mu_k) dt'}{\beta_k W_k^0} \right)^{-b_k}$$

where δ_k corresponds to a depreciation rate, meaning that new cumulative capacity at time *t* includes all additions as well as replacements for decommissioned units. The first term is simply $\mu_k - \mu_k(0) \approx \mu_k$ ($\mu_k \gg \mu_k(0)$), while the second,

$$\int_0^t \delta_k \mu_k(t')dt',$$

produces a fixed value, the total abatement $\bar{E}$ at the end of the projection period, a constant, times the depreciation rate δ_k. We then have

$$I_k = I_k^0 \left(1 + \frac{\mu_k + \delta_k \bar{E}}{\beta_k W_k^0}\right)^{-b_k}$$

In cases where lifetimes are long in comparison to the time span of the analysis, such as in the electricity sector, $\delta_k \bar{E} \ll \mu_k$ and we can safely neglect this term (see section 3.3 for more details). W_k^0 corresponds to the cumulative production in sector k before the abatement measures are implemented. Therefore, $\beta_k W_k^0$ is effectively the cumulative emission saving attributable to existing deployment of technology k. Replacing in C_{Ak}

$$C_{Ak} = \int_{\mu_{k-1}}^{\mu_k} \frac{I_k^0}{\beta_k} \left(1 + \frac{\mu}{\beta_k W_k^0}\right)^{-b_k} d\mu$$

Following the assumptions presented in section 4, equation (3), we assume $I_k^0 = \alpha_A \mu^\gamma$, with α_A being a scaling factor obtained from the MICC data. Setting $\theta_k = \beta_k W_k^0$:

$$C_{Ak} = \alpha_A \int_{\mu_{k-1}}^{\mu_k} \mu^\gamma \left(\frac{\mu}{\theta_k} + 1\right)^{-b_k} d\mu$$

This integral is solved by parts:

$$\int u dv = uv - \int v du$$

$$u = \mu^\gamma \xrightarrow{\text{yields}} du = \gamma \mu^{\gamma-1} d\mu$$

$$dv = \left(\frac{\mu}{\theta_k} + 1\right)^{-b_k} d\mu \xrightarrow{\text{yields}} v = \frac{\theta_k}{1 - b_k} \left(\frac{\mu}{\theta_k} + 1\right)^{1-b_k}$$

Therefore:

$$\alpha_A \int_{\mu_{k-1}}^{\mu_k} \mu^\gamma \left(\frac{\mu}{\theta_k} + 1\right)^{-b_k} d\mu$$
$$= \frac{\alpha_A \theta_k \mu^\gamma}{1 - b_k} \left(\frac{\mu}{\theta_k} + 1\right)^{1-b_k} \Bigg|_{\mu_{k-1}}^{\mu_k} - \left(\frac{\gamma \alpha_A \theta_k}{1 - b_k}\right) \int_{\mu_{k-1}}^{\mu_k} \mu^{\gamma-1} \left(\frac{\mu}{\theta_k} + 1\right)^{1-b_k} d\mu$$

The first term would be:

$$\frac{\alpha_A \theta_k \mu^\gamma}{1 - b_k} \left(\frac{\mu}{\theta_k} + 1\right)^{1-b_k} \Bigg|_{\mu_{k-1}}^{\mu_k} = \frac{\alpha_A {\theta_k}^{b_k}}{1 - b_k} \left[{\mu_k}^\gamma (\mu_k + \theta_k)^{1-b_k} - {\mu_{k-1}}^\gamma (\mu_{k-1} + \theta_k)^{1-b_k}\right]$$

While the right hand side integral can also be solved by parts:

$$\int_{\mu_{k-1}}^{\mu_k} \mu^{\gamma-1}\left(\frac{\mu}{\theta_k}+1\right)^{1-b_k} d\mu = \int udv = uv - \int vdu$$

$$u = \mu^{\gamma-1} \xrightarrow{\text{yields}} du = (\gamma-1)\mu^{\gamma-2}d\mu$$

$$dv = \left(\frac{\mu}{\theta_k}+1\right)^{1-b_k} d\mu \xrightarrow{\text{yields}} v = \frac{\theta_k}{2-b_k}\left(\frac{\mu}{\theta_k}+1\right)^{2-b_k}$$

Therefore:

$$\left(\frac{\gamma\alpha_A\theta_k}{1-b_k}\right)\int_{\mu_{k-1}}^{\mu_k} \mu^{\gamma-1}\left(\frac{\mu}{\theta_k}+1\right)^{-b_k} d\mu$$
$$= \left(\frac{\gamma\alpha_A\theta_k}{1-b_k}\right)\left(\frac{\theta_k\mu^{\gamma-1}}{2-b_k}\right)\left(\frac{\mu}{\theta_k}+1\right)^{2-b_k}\Bigg|_{\mu_{k-1}}^{\mu_k}$$
$$- \left(\frac{\gamma\alpha_A\theta_k}{1-b_k}\right)\left(\frac{(\gamma-1)\theta_k}{2-b_k}\right)\int_{\mu_{k-1}}^{\mu_k} \mu^{\gamma-2}\left(\frac{\mu}{\theta_k}+1\right)^{2-b_k} d\mu$$
$$= \frac{\gamma\alpha_A{\theta_k}^2\mu^{\gamma-1}}{(1-b_k)(2-b_k)}\left(\frac{\mu}{\theta_k}+1\right)^{2-b_k}\Bigg|_{\mu_{k-1}}^{\mu_k}$$
$$- \frac{\gamma\alpha_A{\theta_k}^2(\gamma-1)}{(1-b_k)(2-b_k)}\int_{\mu_{k-1}}^{\mu_k} \mu^{\gamma-2}\left(\frac{\mu}{\theta_k}+1\right)^{2-b_k} d\mu$$

Combining all the terms of C_{Ak}:

$$C_{Ak} = \frac{\alpha_A{\theta_k}^{b_k}}{1-b_k}\left[{\mu_k}^{\gamma}(\mu_k+\theta_k)^{1-b_k} - {\mu_{k-1}}^{\gamma}(\mu_{k-1}+\theta_k)^{1-b_k}\right]$$
$$- \frac{\gamma\alpha_A{\theta_k}^2\mu^{\gamma-1}}{(1-b_k)(2-b_k)}\left(\frac{\mu}{\theta_k}+1\right)^{2-b_k}\Bigg|_{\mu_{k-1}}^{\mu_k}$$
$$+ \frac{\gamma\alpha_A{\theta_k}^2(\gamma-1)}{(1-b_k)(2-b_k)}\int_{\mu_{k-1}}^{\mu_k} \mu^{\gamma-2}\left(\frac{\mu}{\theta_k}+1\right)^{2-b_k} d\mu$$

Solving the integral above requires to define the value of γ. In the classical approach of DICE model (Nordhaus, 2008), *total* abatement costs rise quadratically, which is equivalent to linearity of the marginal cost curve, i.e. $\gamma = 1$. In that case:

$$C_{Ak} = \frac{\alpha_A{\theta_k}^{b_k}}{1-b_k}\left[\mu_k(\mu_k+\theta_k)^{1-b_k} - \mu_{k-1}(\mu_{k-1}+\theta_k)^{1-b_k}\right]$$
$$- \frac{\alpha_A{\theta_k}^{b_k}\left[(\mu_k+\theta_k)^{2-b_k} - (\mu_{k-1}+\theta_k)^{2-b_k}\right]}{(1-b_k)(2-b_k)}$$
$$= \left(\frac{\alpha_A{\theta_k}^{b_k}}{1-b_k}\right)\Bigg[\mu_k(\mu_k+\theta_k)^{1-b_k} - \mu_{k-1}(\varepsilon_{k-1}+\theta_k)^{1-b_k}$$
$$- \left(\frac{1}{(2-b_k)}\right)\left[(\mu_k+\theta_k)^{2-b_k} - (\mu_{k-1}+\theta_k)^{2-b_k}\right]\Bigg]$$

Assuming $\mu_{k,k-1} \gg \theta_k$, we obtain:

$$C_{Ak} = \frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left[\left(\mu_k{}^{2-b_k} - \mu_{k-1}{}^{2-b_k}\right)\left(1 - \frac{1}{(2-b_k)}\right)\right]$$

$$= \frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left[\left(\mu_k{}^{2-b_k} - \mu_{k-1}{}^{2-b_k}\right)\left(\frac{1-b_k}{2-b_k}\right)\right] = \frac{\alpha_A \theta_k{}^{b_k}}{2-b_k}\left[\left(\mu_k{}^{2-b_k} - \mu_{k-1}{}^{2-b_k}\right)\right]$$

If $\gamma = 2$, the integral becomes:

$$C_{Ak} = \frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left[\mu_k{}^2(\mu_k + \theta_k)^{1-b_k} - \mu_{k-1}{}^2(\mu_{k-1} + \theta_k)^{1-b_k}\right]$$

$$- \left(\frac{2\alpha_A \theta_k}{1-b_k}\right)\int_{\mu_{k-1}}^{\mu_k} \mu\left(\frac{\mu}{\theta_k} + 1\right)^{-b_k} d\mu$$

The right hand side integral can also be solved by parts:

$$\int_{\mu_{k-1}}^{\mu_k} \mu\left(\frac{\mu}{\theta_k} + 1\right)^{1-b_k} d\mu = \int u dv = uv - \int v du$$

$$u = \mu \xrightarrow{\text{yields}} du = d\mu$$

$$dv = \left(\frac{\mu}{\theta_k} + 1\right)^{1-b_k} d\mu \xrightarrow{\text{yields}} v = \frac{\theta_k}{2-b_k}\left(\frac{\mu}{\theta_k} + 1\right)^{2-b_k}$$

Therefore:

$$\int_{\mu_{k-1}}^{\mu_k} \mu\left(\frac{\mu}{\theta_k} + 1\right)^{1-b_k} d\mu = \frac{\theta_k \mu}{2-b_k}\left(\frac{\mu}{\theta_k} + 1\right)^{2-b_k}\Bigg|_{\mu_{k-1}}^{\mu_k} - \left(\frac{\theta_k}{2-b_k}\right)\int_{\mu_{k-1}}^{\mu_k}\left(\frac{\mu}{\theta_k} + 1\right)^{2-b_k} d\mu$$

Again, the left side would be:

$$\frac{\theta_k \mu}{2-b_k}\left(\frac{\mu}{\theta_k} + 1\right)^{2-b_k}\Bigg|_{\mu_{k-1}}^{\mu_k} = \frac{\theta_k{}^{b_k - 1}}{2-b_k}\left[\mu_k(\mu_k + \theta_k)^{2-b_k} - \mu_{k-1}(\mu_{k-1} + \theta_k)^{2-b_k}\right]$$

The last integral can be solved directly:

$$\int_{\mu_{k-1}}^{\mu_k}\left(\frac{\mu}{\theta_k} + 1\right)^{2-b_k} d\mu = \left(\frac{\theta_k}{3-b_k}\right)\left(\frac{\mu}{\theta_k} + 1\right)^{3-b_k}\Bigg|_{\mu_{k-1}}^{\mu_k}$$

$$= \left(\frac{\theta_k{}^{b_k - 2}}{3-b_k}\right)\left[(\mu_k + \theta_k)^{3-b_k} - (\mu_{k-1} + \theta_k)^{3-b_k}\right]$$

Putting all together:

$$C_{Ak} = \frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left[\mu_k{}^2(\mu_k+\theta_k)^{1-b_k} - \mu_{k-1}{}^2(\mu_{k-1}+\theta_k)^{1-b_k}\right]$$
$$- \left(\frac{2\alpha_A\theta_k}{1-b_k}\right)\left[\frac{\theta_k{}^{b_k-1}}{2-b_k}\left[\mu_k(\mu_k+\theta_k)^{2-b_k} - \mu_{k-1}(\mu_{k-1}+\theta_k)^{2-b_k}\right]\right.$$
$$\left. - \left(\frac{\theta_k}{2-b_k}\right)\left(\left(\frac{\theta_k{}^{b_k-2}}{3-b_k}\right)\left[(\mu_k+\theta_k)^{3-b_k} - (\mu_{k-1}+\theta_k)^{3-b_k}\right]\right)\right]$$

$$C_{Ak} = \frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left\{\left[\mu_k{}^2(\mu_k+\theta_k)^{1-b_k} - \mu_{k-1}{}^2(\mu_{k-1}+\theta_k)^{1-b_k}\right]\right.$$
$$- \frac{2}{2-b_k}\left[\mu_k(\mu_k+\theta_k)^{2-b_k} - \mu_{k-1}(\mu_{k-1}+\theta_k)^{2-b_k}\right]$$
$$\left. + \left(\frac{2(1-b_k)}{(2-b_k)(3-b_k)}\right)\left[(\mu_k+\theta_k)^{3-b_k} - (\mu_{k-1}+\theta_k)^{3-b_k}\right]\right\}$$

Doing the same approximation $\mu_{k,k-1} \gg \theta_k$, we obtain

$$C_{Ak} \approx \frac{\alpha_A \theta_k{}^{b_k}}{1-b}\left\{\left[\mu_k{}^{3-b_k} - \mu_{k-1}{}^{3-b_k}\right] - \frac{2}{2-b}\left[\mu_k{}^{3-b_k} - \mu_{k-1}{}^{3-b_k}\right]\right.$$
$$\left. + \left(\frac{2}{(2-b_k)(3-b_k)}\right)\left[\mu_k{}^{3-b_k} - \mu_{k-1}{}^{3-b_k}\right]\right\}$$
$$= \left[\mu_k{}^{3-b_k} - \mu_{k-1}{}^{3-b_k}\right]\frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left\{1 - \frac{2}{2-b_k} + \frac{2}{(2-b_k)(3-b_k)}\right\}$$
$$= \left[\mu_k{}^{3-b_k} - \mu_{k-1}{}^{3-b_k}\right]\frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left\{\frac{6-5b_k+b_k{}^2-6+2b_k+2}{(2-b_k)(3-b_k)}\right\}$$
$$= \left[\mu_k{}^{3-b} - \mu_{k-1}{}^{3-b}\right]\frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left\{\frac{2-3b_k+b_k{}^2}{(2-b_k)(3-b_k)}\right\}$$
$$= \left[\mu_k{}^{3-b_k} - \mu_{k-1}{}^{3-b_k}\right]\frac{\alpha_A \theta_k{}^{b_k}}{1-b_k}\left\{\frac{(2-b_k)(1-b_k)}{(2-b_k)(3-b_k)}\right\}$$
$$= \left[\mu_k{}^{3-b_k} - \mu_{k-1}{}^{3-b_k}\right]\frac{\alpha_A \theta_k{}^{b_k}}{3-b_k}$$

By induction, it is possible to demonstrate that the general expression for the investment cost on sector k as a function of γ (being γ a natural number) is:

$$C_{Ak} \approx \left[\mu_k{}^{\gamma+1-b_k} - \mu_{k-1}{}^{\gamma+1-b_k}\right]\frac{\alpha_A \theta_k{}^{b_k}}{\gamma+1-b_k}$$

for low values of $\theta_k{}^0$ (which is typically the case of low carbon technologies) the ratio $\frac{\alpha_A \theta_k{}^{b_k}}{\gamma+1-b_k}$ does not suffer significant variations among sectors. Under those conditions, the total investment cost (calculated as the sum over k of the sectoral investment cost C_{Ak}) becomes a telescopic sum, where only the first and the last term survive. Assuming $\mu_k = \mu$ and $\mu_0 \approx 0$, we obtain the expression presented in equation (10) of section 3 (using $\gamma + 1 - b = 3 - b$ and $\beta W^0 = \theta$).

For the intermediate case where γ is between 1 and 2, this calculation is more complex and requires a numerical integral unless the simplification given above is made, but in which case the result is not very different from where γ equals 1 or 2.

Appendix 3 Mathematical formulation of the integrated model and assumptions

The mathematical specification of the model outlined in this Appendix is as follows.

- **Emissions at time *t*** $\varepsilon(t)$
- **Cumulative Emissions** $E(T) = \int_0^T \varepsilon(t)dt$
- **Reference Emissions** $\varepsilon_{ref} = \varepsilon_0 + \varepsilon_1 \cdot t$
- **Marginal Damage (X=temp)**[39] $D(t) = d_1 \cdot X(t) + \frac{d_2}{2} \cdot X(t)^2$
- **Cumulative Damage (r=real discount rate)**

$$\int_0^T e^{-r \cdot t} \cdot D(t)dt$$

- **Cost Abatement Type A:**

$$C_A(t) = \alpha_A \cdot \left(\varepsilon_{ref}(t) - \varepsilon(t)\right)^{\gamma+1-\mathrm{b}} \approx \alpha_A \cdot \left(\varepsilon_{ref}(t) - \varepsilon(t)\right)^2$$

- **Cumulative A. Cost Type A** $\int_0^T e^{-r \cdot t} \cdot C_A(t)\, dt$
- **Cost Abatement Type B:**

$$C_B(t) = \alpha_B \cdot (\varepsilon_1 - \dot{\varepsilon}(t))^{\gamma+1-b_N} \approx \alpha_B \cdot (\varepsilon_1 - \dot{\varepsilon}(t))^2$$

- **Cumulative A. Cost Type B** $\int_0^T e^{-r \cdot t} \cdot C_B(t)\, dt$
- **Min. Function** $\mathrm{F(t)} = (D(t) + C_A(t) + C_B(t))e^{-r \cdot t}$
- $$\int_0^T F(t)dt = \int_0^T e^{-r \cdot t} \left\{ \begin{matrix} d_1 \cdot E(t) + \frac{d_2}{2} \cdot E(t)^2 \\ +\alpha_A \cdot \left(\varepsilon_{ref}(t) - \dot{E}(t)\right)^2 \\ +\alpha_B \cdot \left(\varepsilon_1 - \ddot{E}(t)\right)^2 \end{matrix} \right\} dt$$
- **Euler Lagrange Method to find the optimal trajectory**

$$\frac{\partial F}{\partial E} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{E}}\right) + \frac{d^2}{dt^2}\left(\frac{\partial F}{\partial \ddot{E}}\right) = 0$$

[39] To avoid confusion with the time horizon T in the model, X(t) is here used to denote temperature change; as explained in the text this is approximately proportional to cumulative emissions: X(t) = E(t) * 500. In all the modelling work presented here we set $d_1 = 0$, so that the focus is simply upon the quadratic damage function.

Quadratic Damage and type A abatement

$$\int_0^T F(t)dt = \int_0^T e^{-r\cdot t}\left\{d_1 \cdot E(t) + \frac{d_2}{2}\cdot E(t)^2 + \alpha_A \cdot \left(\varepsilon_{ref}(t) - \dot{E}(t)\right)^2\right\}dt$$

$$\frac{\partial F}{\partial E} = \left(d_1 + d_2 \cdot E(t)\right)\cdot e^{-r\cdot t}$$

$$\frac{\partial F}{\partial \dot{E}} = -2.\alpha_A \cdot \left(\varepsilon_{ref}(t) - \dot{E}(t)\right)\cdot e^{-r\cdot t}$$

$$\frac{\partial F}{\partial \ddot{E}} = 0$$

Quadratic Damage and type B abatement

$$\int_0^T F(t)dt = \int_0^T e^{-r\cdot t}\left\{d_1 \cdot E(t) + \frac{d_2}{2}\cdot E(t)^2 + \alpha_B \cdot \left(\dot{\varepsilon}_{ref} - \dot{\varepsilon}(t)\right)^2\right\}dt$$

$$\int_0^T F(t)dt = \int_0^T e^{-r\cdot t}\left\{d_1 \cdot E(t) + \frac{d_2}{2}\cdot E(t)^2 + \alpha_B \cdot \left(\varepsilon_1 - \ddot{E}(t)\right)^2\right\}dt$$

$$\frac{\partial F}{\partial E} = \left(d_1 + d_2 \cdot E(t)\right)\cdot e^{-r\cdot t}$$

$$\frac{\partial F}{\partial \dot{E}} = 0$$

$$\frac{\partial F}{\partial \ddot{E}} = -2.\alpha_B \cdot \left(\varepsilon_1 - \ddot{E}(t)\right)\cdot e^{-r\cdot t}$$

Quadratic Damage and types A and B abatement

$$\int_0^T F(t)dt = \int_0^T e^{-r\cdot t}\left\{d_1 \cdot E(t) + \frac{d_2}{2}\cdot E(t)^2 + \cdot\, \alpha_A \cdot \left(\varepsilon_{ref}(t) - \dot{E}(t)\right)^2 + \alpha_B \cdot \left(\dot{\varepsilon}_{ref} - \dot{\varepsilon}(t)\right)^2\right\}dt$$

$$\int_0^T F(t)dt = \int_0^T e^{-r\cdot t}\left\{d_1 \cdot E(t) + \frac{d_2}{2}\cdot E(t)^2 + \alpha_A \cdot \left(\varepsilon_{ref}(t) - \dot{E}(t)\right)^2 + \alpha_B \cdot \left(\varepsilon_1 - \ddot{E}(t)\right)^2\right\}dt$$

$$\frac{\partial F}{\partial E} = \left(d_1 + d_2 \cdot E(t)\right)\cdot e^{-r\cdot t}$$

$$\frac{\partial F}{\partial \dot{E}} = -2.\alpha_A \cdot \left(\varepsilon_{ref}(t) - \dot{E}(t)\right)\cdot e^{-r\cdot t}$$

$$\frac{\partial F}{\partial \ddot{E}} = -2.\alpha_B \cdot \left(\varepsilon_1 - \ddot{E}(t)\right)\cdot e^{-r\cdot t}$$