MA in Music Education

A Computer Program for Investigating Transformation Geometry Concepts in Musical and Visual Form: From Theory to Realisation

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Abstract

This study begins with the hypothesis that music, like the Logo Turtle, can function as an *object-to-think-with*. It asserts that an extensible, ‘microworld’ computer environment is best placed to facilitate and develop a transitional relationship between music and mathematics.

Transformation geometry is proposed as an area of mathematics that exhibits conceptual parallels with musical composition techniques. It is a subject rich in creative potential.

A survey of research into the development of transformation geometry concepts in childhood is compared with existing models of musical development. It is suggested that exposure to symmetrical phenomena, both in the world around them and through education, may engender a tendency for children to ‘symmetrize’ melodies and drawings.

Since transformation geometry and music have so much in common, it is hypothesised that children may be able to use their understanding of one domain to inform their understanding of the other. A computer program – GeoMusE – is developed to explore these issues.

GeoMusE allows learners to enter melodies or shapes (collectively known as *phrases*) on a co-ordinate grid. The horizontal axis represents time and the vertical axis represents pitch. Phrases can be reflected and translated to develop musical and graphical material. Since it is written entirely in Logo, GeoMusE can be extended by the user to perform further transformations.

The results of a school trial of GeoMusE are reported. Six Year 8 students attempted tasks such as composing palindromic melodies and canon, and creating music using shapes and patterns. The students moved fluidly between the musical and mathematical domains to produce some interesting compositions. Analysis of questionnaire data shows that the students were able to transfer their knowledge from one domain to another. It also shows that through the provision of feed back, audio-visual representational modes, and the facility to edit work, the computer environment cultivated this transfer.

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Chapter 1

Introduction

Can music be an *object-to-think-with*?

"So what is the rhythm of that gear?"…
"I'll play it"[1]

Seymour Papert, creator of the Logo programming language, tells a now famous story about his early childhood fascination with gears:

> I loved rotating circular objects against one another in gearlike motions and, naturally, my first "erector set" project was a crude gear system. I became adept at turning wheels in my head and at making chains of cause and effect: "This one turns this way so that must turn that way so..." I found particular pleasure in such systems as the differential gear, which does not follow a simple linear chain of causality... I believe that working with differentials did more for my mathematical development than anything I was taught in elementary school. Gears, serving as models, carried many otherwise abstract ideas into my head. (Papert, 1980: *vi*)

Reflecting on this intense relationship, Papert (1980) identifies four qualities that made the gears such effective tools in his mathematical development. Firstly, gears were embedded in the surrounding culture - an easily accessible feature of his technological landscape. Secondly, as the concern of adults, gears and mechanics provided a means of relating to the adult world. Thirdly, they exhibited what Papert terms *body syntonicity*: 'I could feel how gears turn by imagining my body turning' (1980: 11). Finally, as a young child, Papert had yet to encounter the formal theory of mechanics but the physical presence of the gears provided a window onto the mathematics beyond. They were a *transitional object*, an 'object-to-think-with' (Papert, 1980: 11).

Taking his lead from Suzanne Langer, Swanwick argues that humans understand their environment through symbolic systems. These include language and mathematics, but also the arts:

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[1] Exchange between researcher and student in Bamberger (1999: 238)

> This symbol-making facility enables us to become aware of and articulate dimensions of our personal history, elements of our culture, the perceived feeling and actions of other people, the movement of the planets, the natural world around us. It also allows us to speculate, to predict, to make attempts to shape the future.
> (Swanwick, 1994, p. 37)

The claims made by Swanwick for the symbolising properties of the arts appear consistent with those identified in Papert's gears. Malcolm Ross has argued that education should encourage children to explore the world around them through the art forms found in everyday life, a concept which he terms the *vernacular spirit* (Ross, 1984).

Rena Upitis (1997), a researcher active in both music and mathematics education, describes how children naturally discriminate between 'kids' stuff' and 'real stuff'. As Papert did, they recognise the potential of the latter to project them into the adult world. During her work with *Project Headlight* at the Hennigan School in Boston, Upitis organised weekly concerts featuring her students' compositions alongside those of more established musicians: 'If their songs could be played with Mozart's songs, then they must be, in some way, "real music"' (Upitis, 1990b: 42).

Papert recalls drawing on his 'body knowledge' when thinking about gear systems (1980: 11). He writes of a similar relationship between children and the Logo Turtle:

> The Turtle circle is *body syntonic* in that the circle is firmly related to children's sense and knowledge of their own bodies.
> (Papert, 1980: 63)

Music too shares an intense relationship with physical movement. As Chernoff (1979) points out, the main beat of much African music is only externalised in the associated dance. Without the physical accompaniment, the music is incomplete and thus cannot be fully understood. Similarly, music therapists highlight the congruence between musical structures and the organisation of the human body. Ansdell (1995) describes how certain kinds of music, which he terms *kinetic melodies*, are able to temporarily restore a sense of control, direction and naturalness to clients with debilitating neuromotor conditions such as Parkinson's disease. The pioneering work of the mathematician Zoltan Dienes (1973) demonstrates how this relationship can be employed in education. In *Mathematics*

Through the Senses, Dienes presents music and movement activities which explore the nature of repeating patterns and fractions. Just as Upitis' weekly concerts 'served to legitimise the children's compositions' (1990b: 41), Henle (1996) praises Dienes' book for enhancing mathematics education by introducing a sense of *performance*.

The gears of Papert's childhood remained an object-to-think-with throughout his development as a professional mathematician. As an educational researcher, his goal, 'has been the design of other objects that children can make theirs for themselves and in their own ways' (1980: 11). The pre-eminent example is the Logo Turtle. Yet, in order for children to form such intense relationships, I would argue the transitional objects must reflect their personal and cultural surroundings. After all, Papert recalls falling in love with the gears after discovering them *for himself* in the world around him. As Noss and Hoyles say of these objects:

> They should evoke intuitions, current understandings and personal images – even preferences and pleasures. (1996: 68)

Music is particularly effective at evoking (and influencing) the personal images and preferences of young people (Zillmann and Gan, 1997). Can music, like Papert's gears and the Logo Turtle, function as an object-to-think-with?

This question does not place music in a subservient role to the learning of mathematics. As Noss and Hoyles caution, to be effective, transitional objects 'must evoke something worthwhile in the learner, some rationale for wanting to explore with them, play with them, learn with them' (1996: 68). The objects must be valuable in themselves – as Upitis reminds us, they must be *real*. Upitis describes a project where students composed music to accompany an animated film they had made using geometric principles. During the premiere of the finished product, Upitis recalls:

> We watched their faces change from nervous anticipation to proud satisfaction as their work unfolded – a quiet drama spun without words or sound, but eloquent in its message; I wrote that, I made that, I am proud of that – I am an artist, a mathematician, an animator, a composer. (1997: 122)

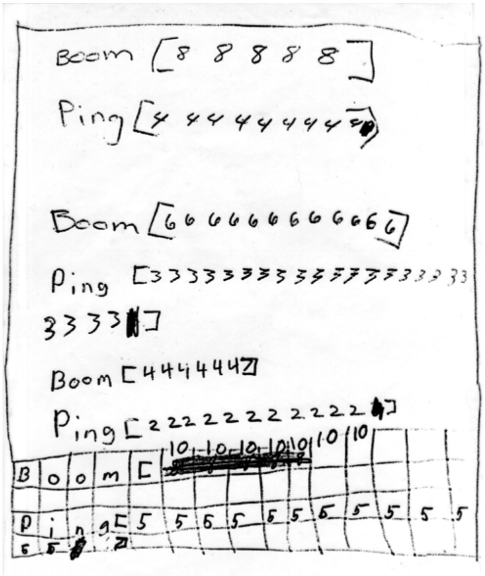
Bamberger's (1999) *Laboratory for Making Things* project was motivated by the observation that many children who excel in practical, everyday skills such as building and fixing things (including music) have trouble learning formally in school. Bamberger argues that these children learn by extracting principles from their constructed objects rather than through standard 'school symbol systems' such as graphing, calculations and written language.

Figure 1 Children playing with large cardboard gears (Bamberger, 1999: 48)

As a long-time collaborator of Papert, Bamberger was interested in exploring his theory of gears as transitional objects. In one activity, children compared the rotation speeds of two large, cardboard gears (figure 1). They were then asked how this relationship might be illustrated by clapping the 'rhythm' of the gears. The children responded as follows: One clap for each rotation of the large cog was subdivided into four claps for the smaller cog. Bamberger recalls Papert's assertion that the power of gears to objectify mathematics is due to their 'double relationship' between the abstract and the sensory (Papert, 1980: *xx*). She believed that 'moving between clapping the rhythms and playing with the gears could be a particularly lively playground for making this "double relationship" manifest' (Bamberger, 1999: 239).

In a subsequent activity, the children were asked to clap a simpler 2:1 gear ratio and then recreate it on a computer using a version of Logo developed by Bamberger called *MusicLogo*. This tool uses two sound commands, a low 'boom' and a high 'ping'. Lists of numbers between 1 and 8, supplied as arguments to the commands, control the quantity and duration of each sound. For instance, `BOOM [8 8 8 8 8 8 8]` would produce seven long low sounds, while `PING [2 2 2 2 2]` would produce five high sounds whose durations were a quarter of the length of the 'booms'. In order to model the motion of the gears, the children had to deduce that the ratio between them was 2:1. One student, Leon, appears to have grasped this effectively enough to *generalise* the relationship. Figure 2 shows his experiments with different pairs of numbers, each of which retain the 2:1 ratio.

Figure 2 Leon's 2:1 ratios (Bamberger, 1999: 259)



Area of Enquiry

Bamberger's use of MusicLogo in the gears project was motivated by the following question:

> Could the children use the computer as a vehicle for effectively moving between their own body actions in clapping/drumming, the actions of the gears, and numeric-symbolic descriptions of the shared, embodied principles? (1999: 240)

This study proceeds from a similar position. I assert the hypothesis that music *can* function as an object-to-think-with. Moreover, I consider how a computer environment might facilitate the exploration and development of a musical *transitional relationship*.

In the first four chapters, I argue that transformation geometry is a particularly suitable area of mathematics on which to base integrative projects with music. I explore transformation geometry from educational, musical and developmental perspectives. In Chapter 5, I describe GeoMusE – a computer environment designed to enable children to explore the relationship between transformation geometry and music. In the last four chapters, I report the results of a school-based trial of the software. I conclude with a re-examination of my initial hypotheses in the light of empirical evidence.

The integration of music and mathematics education

> 'Our maths teacher had musical notes on his tie once.
> That was about as close as it got.'[2]

A review of the professional literature of music and mathematics education reveals a great deal of enthusiasm for combining ideas from the two subjects. Unfortunately, the majority of this work concentrates on superficial connections.

In the primary age group, much is made of the mathematical features of western musical notation. For instance, adding up series of note durations or considering time signatures as fractions (Roy, 1988; Harding, 1990; Moses and Proudfit, 1992). Authors appear to have assumed that *music* is interchangeable with *music notation*. An article by Berurla (1989) is intriguingly entitled *The Use of Music in the Teaching of Logo*. Upon reading, it becomes clear that students used Logo neither to compose nor to play any music. Rather, they programmed the turtle to *draw* musical notation.

In older age groups, suggested activities focus on the physics of sound. Moses and Proudfit (1992) calculate the length of rubber bands required to play notes of the Pythagorean scale whilst Fernandez (1999) uses graphing calculators connected to microphones to measure frequency and plot waveforms. Norman (1989) uses a BASIC program to generate the frequencies necessary to play musical scales. Such activities perhaps help students understand acoustics yet I would question their potential to engender genuine musical understanding.

In an insightful article, Henle (1996) establishes historical parallels between the conceptual development of western mathematics and classical music. Discussing the nineteenth century, he links the move towards *naturalism* in music to mathematicians' preoccupation with practical, everyday problems. Similarly, he equates the highly theoretical and abstract music of the twentieth-century with

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[2] Comment made by student in GeoMusE school trial – see chapter 9

formalism in contemporary mathematics. Henle argues that the true affinity between music and mathematics is *cultural* rather than abstract. His views have educational implications:

> Children are taught from an early age that music is exact, unforgiving and arbitrary. Their relationship to it is complex and troubled. They see it as a trap, a contest in which they must guess what the teacher is teaching... I would argue that mathematics should be taught as music is taught. Students should make mathematics *together*. Creative ideas should be stressed over the "right way to do it". Mathematics should be a treat, not a chore (Henle, 1996: 28)

This argument is attractive, yet research shows that the real picture is more complex. Bahna-James (1991) asked secondary school students for comments on the links between work in their mathematics and music lessons. The majority denied any connection existed. Some students reacted with anger to the suggestion; one wrote 'MUSIC IS NOT MATHEMATICS; IT IS EMOTIONS! WE ARE NOT COMPUTERS!' (Bahna-James, 1991: 484). For Bahna-James, these dismissive reactions stem from the contradiction between the students' negative image of mathematics (what Papert (1980) calls *mathophobia*) and their image of music as a 'human', creative subject.

This contradiction might be alleviated if we follow Henle and search for conceptual foundations on which to base interdisciplinary projects. Wiggins and Wiggins (1997) consider the problems of integrating music with other subjects. They argue that through conceptual parallels, connections are made not between subject matter, but between the ways in which we *understand* the subject matter:

> When these connections are made explicit to students, they will come to use the understanding gained through one intellectual perspective to help them gain a greater understanding of others. The connections need to be natural human connections, not forced subject-area connections. (Wiggins and Wiggins, 1997: 41)

Transformation geometry is an excellent subject with which to investigate the potential of an integrated music and mathematics project. The use of symmetry and tessellation is widespread in many artistic traditions. Furthermore, there are distinct conceptual similarities between these graphical operations and compositional devices in music.

Chapter 2

Transformation Geometry in Education

Why include Transformation Geometry in the curriculum?

The study of transformation geometry and the associated subject of tessellation involve the application of a great deal of mathematics[3]. From the elementary calculation of interior angles to the use of matrix arithmetic, the subjects provide a challenge to students at all levels. However, as Edwards points out, this area is attractive not only for its mathematical richness but also because it 'connects with children's everyday experience with motions, shapes, and actions' (1992: 130). In this sense, it resembles Papert's gears and the Logo Turtle as another *body syntonic*, transitional object. The mathematical concepts underpinning transformation geometry are encountered *implicitly* as the student applies this everyday experience. Since examples of transformation geometry and tessellation abound in the natural and cultural worlds, the range of experiences on which students can draw is extensive (Upitis, 1997). The ability to identify the geometric properties of our environment forms part of the concept of *visual literacy* promoted by Knupfer and Clark:

> Through this process of visual thinking we are able to evaluate our visual experiences and make visual statements regarding our perception. (Knupfer and Clark, 1992: 62)

Regarding tessellation, Mason makes a similar observation:

> Two mathematical results, two generalities which are, at first sight, quite surprising – that any triangle and any quadrilateral… can be used to tessellate the plane – emerge from this exploration. There is a certain power, a certain sense of control and knowledge which such a generality offers. (1990: 69)

Knupfer, Clark and Mason believe that the study of transformation geometry and tessellation can enhance students' abilities to generalise and form conjectures. After all, it is not enough to prove a particular shape tessellates over a small area.

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[3] The reader is advised to consult Maxwell (1975), and Lockwood and Macmillan (1978) for a comprehensive introduction to transformation geometry.

As one of Upitis' students confidently asserted: "tessellation is where there is *no* floor showing" (1997: 18, emphasis added).

Experience accumulated through the study of transformation geometry is clearly of great benefit to the learning of other areas of mathematics (Law, 1993).

The cross-curricular potential of transformation geometry

In common with the 'best mathematics', Gailiunas writes, tessellation 'involves showing how one thing can be many things at once… The richness in the pattern might be reflected in the mathematics' (1993: 8). The aesthetic appeal of transformation and tessellation activities is clearly a significant part of their popularity. Perhaps more than any other areas of the mathematics curriculum, transformation geometry and tessellation provide rich opportunities for cross-curricular work[4]. Whist there are obvious possibilities in the art, craft and design areas, the humanities subjects offer forums for exploring the artistic cultures of Ancient Rome, Islam and Africa.

An area that combines artistic and mathematical processes particularly effectively is the study of Escher's paintings and drawings. Many of the Dutch artist's images are based on elaborate geometric transformations. Ranucci and Teeters (1977) present a detailed approach to reproducing the Escher's methods in the classroom. In particular, they demonstrate the important *put and take* technique (see also NCET, 1993), where one side of a shape is altered and the result transformed to the other sides.

Projects such as those offered by Ranucci and Teeters, where children produce artwork which they are proud to share with peers and family, are perhaps the best examples of Upitis' concept of *real mathematics*:

> The children were interested in tessellations as a concept and learned a great deal from using the manipulatives; they became even more eager to understand tessellations more fully when the possibility of creating something that was uniquely their own was presented. Suddenly, understandings good enough for 'maths purposes' were pushed further. (Upitis, 1997: 41)

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[4] The literature contains many examples of these activities. *Micromath*, Vol. 9, No. 2 (1993), Upitis *et al* (1997), and Ranucci and Teeters (1977) provide a selection.

By situating the mathematics within a cultural context, transformation geometry provides an example of the purposeful activities called for by Noss and Hoyles (Chapter 1).

Chapter 3

Transformation Geometry Concepts in Music

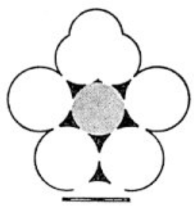
Transformations and Analogous Musical Devices

‘You could just move it.’
‘Yes, that’s called a translation.’…
‘You could also spin it.’
‘Oh yes, that’s called a rotation.’
‘But that would be kinda hard for music, right?’
‘Yup’[5]

As Henle correctly points out, mathematical studies of music are a ‘staple of mathematicians, musicians and philosophers’ (1996: 19). The relationship between geometric concepts, in particular symmetry, and music has been investigated thoroughly and from various perspectives.

Some work has investigated composers’ use of symmetry on the musical ‘macro’ level. For instance, Lendvai (1990) examines how axes of symmetry in the cycle of fifths affected Bartok’s use of key modulation. Apagyi (1989) and Kempf (1996) discuss symmetrical tonal schemes. In addition, they both highlight the potential for symmetrical musical forms. From the simple A-B-A form of Beethoven’s *Écossaise in G major*, in which the final A is an almost exact recapitulation of the first, to the complexity of Stravinsky’s *Canticum Sacrum*. This symphony received its premiere in St. Marks Cathedral, Venice. Critics maintain that its five-movement structure represents the five domes of the cathedral. The final movement is an almost exact retrograde version of the first, suggesting the roof’s circular design (figure 3) (Apagyi, 1989).

Figure 3 A Graphical representation of the structure of Stravinsky’s *Canticum Sacrum* showing the similarities with St. Marks, Venice (White, 1966: 443).



[5] Exchanges between students and Teacher during a project to compose music using geometric transformations (Upitis, 1997: 115).

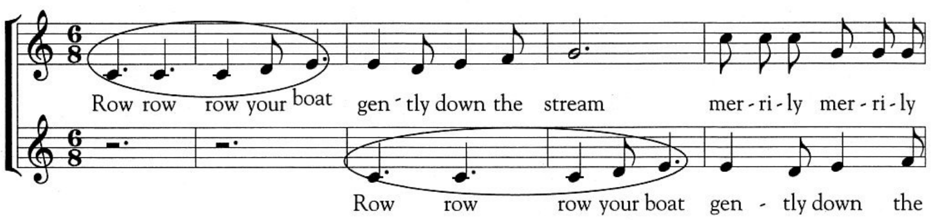
Other studies deal with musicians' use of transformation geometry concepts on the 'micro' level; in particular, the generation and development of basic melodic and rhythmic material.

Translation

Translation is the foundation for several common musical devices. Most simply, repetition is obtained through the horizontal translation of the musical motive. An example cited by Wilson (1986) is the first phrase of Mozart's *Symphony No. 40 in G minor* (example 1).

Example 1 Opening melody of Mozart's *Symphony No. 40 in G minor* (adapted from Wilson, 1986: 102)

Repeated horizontal translation of a phrase results in *ostinato* (Wilson, 1986). A *canon* (round) is derived by locating the translation far enough forward in time to coincide with the successive motive in the original; see example 2 (Garland and Kahn, 1995).



Example 2 *Row Row Row Your Boat* (Garland and Kahn, 1995: 70)

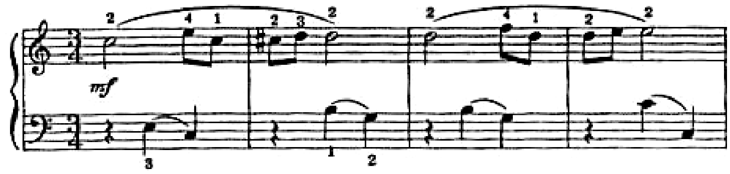
In a *sequence*, an iterative motive gradually ascends or descends stepwise. This is achieved through two-dimensional translation (Apagyi, 1989). The second phrase of example 1 provides an example. A related operation is *transposition*, the wholesale repetition of a motive at a given pitch interval. The transposed motive may appear at the same horizontal position as the original, producing a harmony part. Perhaps the most sophisticated example of translation is the *fugue* where a motive is displaced both horizontally and vertically; see example 3 (Garland and Kahn, 1995).

Example 3 Opening of the *Fugue II* from Bach's *48 Preludes and Fugues* (Garland and Kahn, 1995: 79)

Reflection

This transformation is an important technique in the *twelve-tone*, or *serial*, method of composition developed by Schoenberg and the Second Viennese School. Yet, it is often found in other music. Reflection of a melody by a vertical axis of symmetry results in the musical operation *retrogression* (sometimes called *retrogradation*). Often, a composer will vertically reflect only the pitch information, articulating the retrogression with a different rhythm. This is the case in example 4, taken from Bartók's *Microcosmos*. On other occasions, the rhythm is reflected but the pitch is new; see the top line of example 5, for instance (Apagyi, 1989).

Example 4 Extract from Bartók's *Six Unisono Melodies*, *Microcosmos*, Vol. 1 (Apagyi, 1989: 675)



Example 5 Opening of Mozart's *Minuet in C major* (Apagyi, 1989: 688)

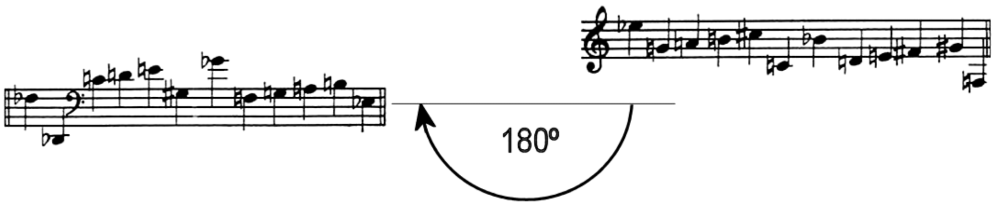
Reflection of a melody through a horizontal mirror line produces *inversion*. When performed exactly, the inversion appears directly beneath the original and the two motives exhibit contrary motion; see example 6 (Apagyi, 1989). Clearly, this introduces a harmonic dimension, where previously there may have been none. As is noted by Albrecht (1995), these harmonies will rarely remain diatonic. More frequently, horizontal reflection is performed concurrently with a translation, resulting in the geometric transformation *slide reflection* (Garland and Kahn, 1995). For instance, see the inverted phrase in example 3.

Example 6 Extract from *Leggiero* by Lajos Papp (Apagyi, 1989: 677)

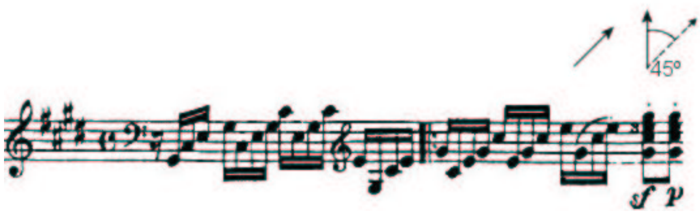
Some composers appear to have regarded mirrored compositions as 'intellectual show pieces'. Machaut named a rondeau *My End is My Beginning* with obvious pride (Abraham, 1979). Haydn informed performers of his *Sonata for Piano and Violin*, No. 4, 2nd movement: 'the minuet will be played backwards at the da capo' (Albrect, 1995).

Rotation

Use of a linearly oriented stave system restricts the potential of rotation as a compositional device. However, the serialist operation of retrograde-inversion does resemble a 180º rotation; see example 7 (Garland and Kahn, 1995). Wilson (1986) suggests that under certain circumstances, the *arpeggiation* of chords can be considered as a 45º rotation (example 8).



Example 7 Tone row from Schoenberg's *Wind Quintet, Opus 26* in original (right) and retrograde-inversion (left) forms (adapted from Machlis, 1961: 341)



Example 8 Extract from Beethoven's *"Moonlight" Sonata*, Op. 27, No. 2, 3rd Movement (adapted from Wilson, 1986: 105)

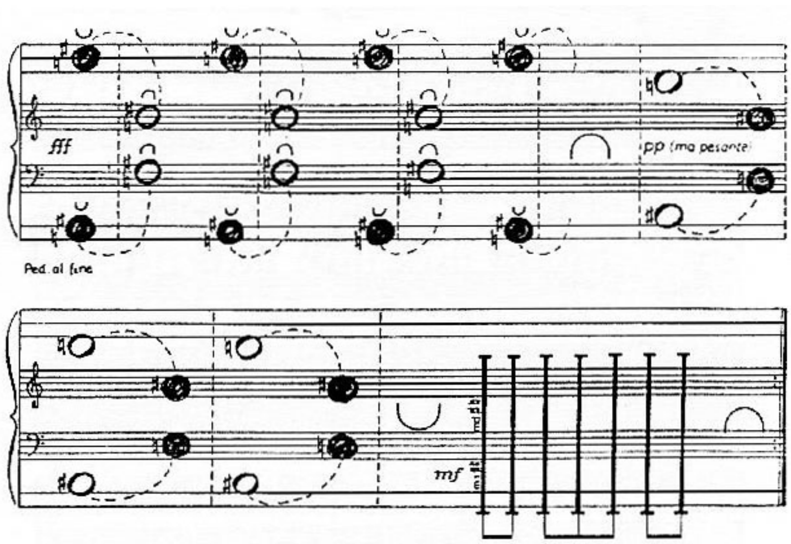
Symmetry and listener expectancy

Geometric transformations provide the composer with a useful array of devices for musical development. Yet some writers have suggested their role is beyond simply the functional. Wilson (1986) writes that when listening to a previously unknown piece of music, the hearer constructs understanding by relating the latest musical events to those that have already past, whilst also making predictions about future directions. Often such expectations are of symmetrical resolution – as the saying goes 'what goes up must come down'. Wilson argues that the job of the composer is to either satisfy or deny this desire for symmetry in order to achieve an expressive outcome. The transformation itself can remain unheard by the listener: 'reflection is more often "felt"… and provides an "arch" form' (Wilson, 1986: 104). Wilson cites the beginning of Mozart's Symphony No. 40 (quoted in example 1) as an illustration of the arch form. Here, the expectation engendered by the sudden upward leap at the end of the first phrase is resolved by the subsequent, gentler descending sequence. Wilson identifies three common methods for the composer to deny this sense of balance: denial through phrase extension, denial through phrase truncation and an overall lack of unity between phrases.

Liebermann and Liebermann (1990) have developed Wilson's argument further. They cite examples of question and answer melodies by Correlli and Handel that rely on reflection devices for resolution. The writers conclude that symmetry appears to 'manifest itself consistently in a question and answer sequence' (Liebermann and Liebermann, 1990: 66).

Parallels between musical symmetry and instrument ergonomics

In addition to theories of expectation and resolution, there is another argument for the occurrence of symmetry in some music. Returning to the above discussion of reflection through a horizontal mirror line (example 6), it can be seen that the resultant contrary motion is best suited to keyboard instruments. Each hand can reach one half of the keyboard with greater ease. This, in turn, influences the parts played by each hand. Kurtág's *Palm Exercise* (example 9) provides a clear illustration of this.



Example 9 Extract from Kurtág's *Palm Exercise*. Black circles indicate clusters of black notes, white circles indicate clusters of white notes (Apagyi, 1989: 678)

It appears that the physical design of an instrument may *encourage* certain symmetrical devices. Sutton (1978) for instance, argues that the style of ornamentation found in Javanese Gamelan music reflects the ergonomics of the individual instruments. For example, the notes of the large metallophone known as the *gender* are struck with two soft mallets and dampened by the side of the hand. Sutton suggests this continuous contrary rocking motion of the hands has led to a performance practice based upon melodic inversion.

Parallels between musical and graphical transformations

Many cultures of the world decorate their environment with geometric patterns. The mosaics of Islamic communities are among the best known. Al Faruqi (1978) makes connections between these artefacts and the highly ornamented Arabic traditional music.

As was noted in Chapter 2, the work of Escher has done a great deal to popularise transformation geometry amongst professional mathematicians and educators alike. Interestingly, the artist himself appears to have been fascinated by the possibility of composing music using the same principles. He explored these ideas during a series of letters to his nephew, the composer Rudolf Escher (Schönberger, 1988). One letter in particular is worth quoting at length:

> Despite the dissimilarity and the contrast between their natures, I nonetheless hold that the image sequence of a divided surface and the sonic progression of a piece of music can be reconstructed as different steps of the same ladder… Regular elements such as rhythm and recurrence, which have an important function expressed auditively [sic] in music and visually with reference to geometrical configuration, similarly suggest a degree of relationship between the two.
>
> Going into particulars, it emerges that terms used in the canon like augmentation, diminution, inversion and even retrogradation, are visually indicated in the score in a manner which is directly analogous with and similar to figures on a regularly divided place.
>
> Let me conclude this essay with a testimony to the influence which Bach’s music has had on my work. His reason, his mathematical order, the firmness of his rules probably play an important yet direct role in this… When hearing his music, the influence it exerts is of an emotional kind… both in the sense of evoking specific notions or

images, and more generally by arousing an irresistible urge to produce.

(Escher in Schönberger, 1988: 13)

As was observed in the case of rotation, the conventional musical stave can restrict what transformations are possible. These limitations can be partially overcome if the conventional system is extended in ways that increase the opportunities for the kind of ‘visual indication’ identified by Escher. Two musicians who have attempted this are the Argentinean-German composer Mauricio Kagel and the American theorist Larry Solomon.

Kagel has suggested superimposing shapes onto the stave, the corners of which indicate musical pitches (Cole, 1974). By revolving these shapes around a chosen pivot or ‘screwing’ point, Kagel achieves a useful kind of melodic rotation (figure 4). It is also possible to reflect shapes in the same fashion.

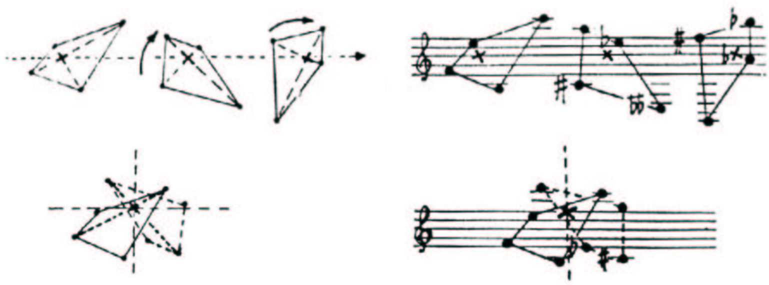


Figure 4 Part of Kagel’s theory of notation-manipulation. The top row indicates how a four-note melody might be rotated. The bottom indicates how it might be reflected (Adapted from Cole, 1974: 126).

Solomon (1973) proposes an extension to serial theory in which the first and last, highest and lowest notes in a four-note row are used as points to define a quadrilateral. By transforming this quadrilateral, the composer can reorder the notes of the row. This *quadrate-variation* can produce up to eight different combinations of notes (figure 5).

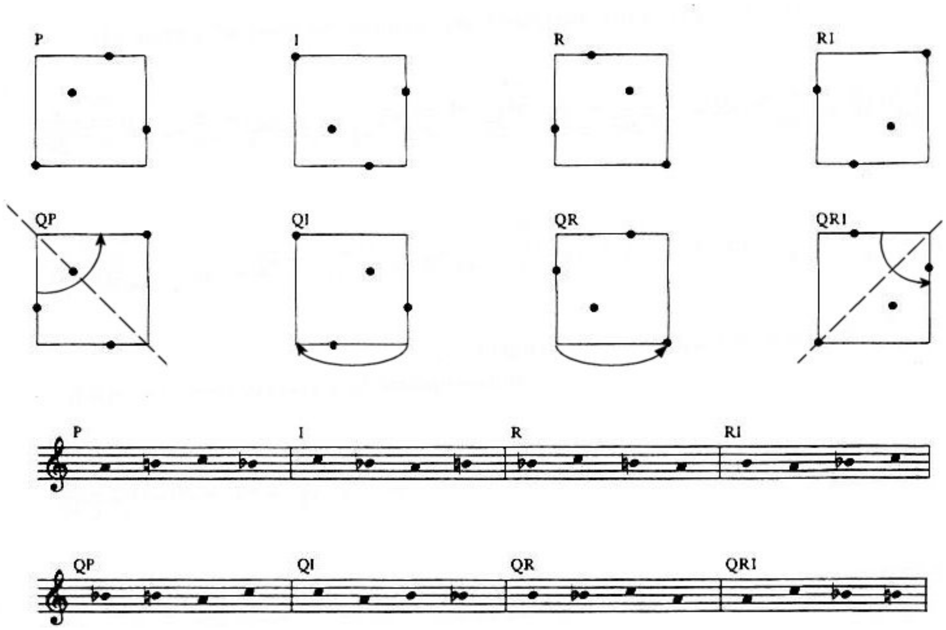


Figure 5 Solomon's theory of *quadrate variation*: *P* is the original row, *I* is the inversion, *R* is the retrograde and *RI* is the retrograde-inversion. Solomon's extends these by adding *QP*, derived by reflecting the original square through a line from top left to bottom right. *QI* is derived by rotating the original clockwise through 90º. *QR* is derived by rotating the original anticlockwise by 90º and *QRI* is derived by reflecting the original through a line from bottom left to top right (adapted from Solomon, 1973: 260).

Chapter 4

Transformation Geometry and Conceptual Development

Research into children's development of transformation geometry concepts has typically proceeded from one of two perspectives. Some work has concentrated on children's ability to classify symmetrical and asymmetrical figures. Other work has analysed children's artwork and identified the implicit use of transformations. A common theme is the acknowledgement of the role played by natural and cultural environments in the acquisition of these concepts. With this in mind, I offer an exploration of the use of 'transformation-like' operations in children's musical compositions.

The ability of children to identify symmetry

Genkins (1975) investigated the ability of children between the ages of 5 years and 8½ years to differentiate between figures exhibiting bilateral symmetry, point symmetry and no symmetry at all. She also compared different methods for testing the symmetrical properties of the figures. The control group used simple visual estimation. A second group folded the paper on the axis of symmetry and a third group used small mirrors. The latter groups received a demonstration of the different types of symmetry. The control group received only a minimal introduction. The three groups were then asked to identify the type of symmetry exhibited by 48 test figures.

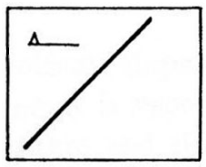
Genkins' results suggest a hierarchy of symmetry types based on the difficulty children had identifying them. Across the age range, those children without access to mirrors or paper folding found the identification of vertical lines of symmetry easier than horizontal lines. They also found it hard to discriminate point symmetry. Children at lower ages found paper folding a useful tool, particularly in the discrimination of point symmetry. However, they were unable to use the mirrors to identify any symmetry type properly. Genkins suggests that children at the kindergarten level may not have developed the necessary perceptual and comparison skills to compare the image in the mirror with that on the paper. The performance of older children was enhanced significantly by the use of both mirrors and paper folding. Across all three groups, the older children were much better at identifying bilateral symmetry. However, there were less pronounced differences in the ability to distinguish point symmetry.

Overall, Genkins' work suggests children's understanding of symmetrical concepts develops significantly between the ages of 5 and 8½ years. The ability to distinguish bilateral symmetry improves but vertical lines of reflection remain the easiest to recognise. One reason suggested by Genkins is that children are used to visually scanning the page from left to right when they read. Another is the profusion of objects exhibiting vertical bilateral symmetry in the natural and man-made worlds. Children across this age range find point symmetry much harder to identify. However, learning aids such as mirrors and tracing paper can improve this situation. Again, there are fewer examples of point symmetry readily available in the surrounding environment[6].

Küchemann (1981) investigated the abilities of 13, 14 and 15 year-old students to perform reflections correctly. As might have been expected from the conclusions of Genkins, nearly all the children in the study showed some understanding of reflection. However, certain features of the reflected figures mitigated this understanding.

Küchemann's subjects found it much easier to locate a reflected figure correctly when the line of symmetry was either vertical or horizontal. When the line was sloping, the children would often ignore it and place the transformation in parallel to the original. When tackling figure 6, for instance, only 25% of 13 year olds and 40% of 15 year olds answered correctly.

Figure 6 Reflection through a sloping mirror line (Kürchemann, 1981: 141)

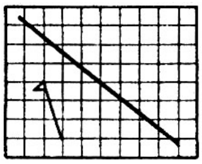


[6] Gallou-Dumiel (1989) suggests beams of light converging on a point before subsequent diffusion as an example of point symmetry. However, this seems unlikely to be experienced by most children out of the school science context.

Küchemann suggests one reason for these difficulties is that students can no longer simply assume that a reflected figure shares the same latitude to the original, as they can with vertical lines of reflection. Instead, they must develop an *analytical* approach, deconstructing the figure into its constituent endpoints and reflecting them individually. Only then are they able to see the orientation of the subsequent transformation. It would appear that some in this age group have yet to develop the necessary memory facilities and abilities to abstract individual elements from figures.

A second problematic factor was the complexity of the figures to be transformed. Students capable of reflecting single points were often unable to replicate this action for more advanced shapes such as a flag design. In the case of figure 7, 15% of 14 year olds submitted completely disoriented transformations. These did not match a single endpoint of the original.

Figure 7 Reflection of sloping flag figure through sloping mirror line (Kürchemann, 1981: 142)



Küchemann concludes that reflection involving vertical mirror lines utilises concrete operational thought developed through other activities – in this case, paper folding. However, correct interpretation of sloped mirror lines requires at least some formal, analytic operations.

Transformation geometry concepts in children's artwork

Booth (1980; 1989) researched the emergence of transformation geometry concepts in children's pattern painting. She defines this kind of art activity as the 'child's non-representational or non-figurative art-form' and notes that it 'arises completely spontaneously if young children are not influenced by peers or adults to paint figurative pictures' (Booth, 1980: 71). After a survey of over two thousand of these paintings, Booth (1980) identified three developmental stages: the *scribble* stage, the *topology* stage and the *geometric pattern* stage.

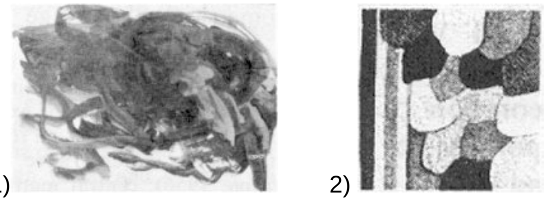


Figure 8 Examples of spontaneous pattern painting: 1) Scribble stage (Booth, 1980: 124) 2) Topology stage (Booth, 1989: 72)

1) 2)

During the scribble stage, there is little spatial awareness and the brush is moved in oscillating 'scribbles' (figure 8.1). Booth relates this stage of development to the Piagetian notions of assimilation and accommodation. The child's experience of holding the brush is assimilated into previous knowledge of holding pencils and crayons. However, since the brush differs from these other implements, the child must accommodate their own style and technique to suit the new tool. During the topology stage, marks are restricted to randomly placed short strokes, dots or irregular patches (figure 8.2). Children are able to accommodate their brush strokes to both imagined images and the physical size of the paper. There is also a pronounced interest in notions of area and boundary, which Booth identifies as the precursor to the final geometric pattern stage.

The primary characteristic of the geometric pattern stage is the imposition of regular order. This is established through either the repetition of elements or the regular division of the plane. This desire for order engenders strategies for organising designs based on the concepts of translation, reflection and rotation.

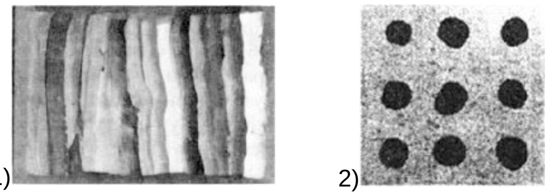


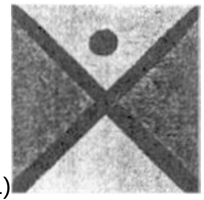
Figure 9 Examples of spontaneous pattern painting: 1) Vertical translation (Booth, 1980: 124) 2) Vertical and horizontal translation (Booth, 1989: 72)

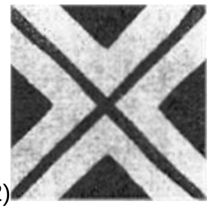
1) 2)

In general, one-dimensional translation is the first of these to appear in the guise of a single element repeated regularly across the paper. Booth believes the high level of repetition is due to the child's desire to practise a skill for its own sake – Piaget's concept of *functional assimilation*. The most likely pattern to occur first in this stage is a repetition of vertical lines (figure 9.1). This preoccupation with the vertical axis is consistent with the findings of Genkins and Küchemann. Vertical lines are often followed by paintings exhibiting translation in two dimensions.

Typically, these feature repeated shapes such as circles and squares (figure 9.2). Booth notes that when constructing these two-dimensional translations, children generally work in horizontal rows rather than vertical columns. This supports Genkins' view that children perceive graphical space in the same way as they do text; i.e. from left to right.

Figure 10 Examples of spontaneous pattern painting: 1) One-fold reflection (Booth, 1989: 72) 2) Four-fold reflection (Booth, 1989: 72)

1) 

2) 

Booth found reflection was generally the next transformation to appear in the children's artwork. Initially, this is manifested as simple designs with single lines of reflection (figure 10.1) but later, as a child's control of colour becomes advanced, pictures might feature two or four lines of reflection (figure 10.2). Significantly, the examples of one-fold reflection in both Booth (1980) and Booth (1989) are all examples of vertical bilateral symmetry again.

Figure 11 Example of spontaneous pattern painting: rotation (Booth, 1989: 72)

The concept of rotation is the last to develop in this style of children's painting (figure 11). Booth notes its rarity in comparison with translation and reflection. When it does occur, it is often at the expense of colour symmetry suggesting the child is unable to manage transformation of the shapes and colour matching simultaneously. However, toward the end of the primary age range, rotation designs become more frequent.

Booth is careful to point out that the three stages are not bound to biological age since development is affected by the amount of painting experience a child has. However, with regular painting sessions, simple translations may be achieved between 4 and 5 years (Booth, 1980) whilst rotations will be rare before the age of 8 or 9 (Booth, 1989).

Comparisons with early musical development

Booth’s theory bears interesting comparison with the lower sections of the model of music development proposed by Swanwick and Tillman (1986). Whilst there is not quite a one-to-one correlation between Booth’s three phases and the modes on the Swanwick-Tillman spiral, there is a sense that the two models develop together, at least up until the *vernacular* mode.

For Swanwick, the *sensory* mode is characterised by a fascination of music’s primary material, sound itself: ‘There is much ‘strumming’ – experimentation with instruments and other sound sources… Unpredictable and fitful sound exploration is characteristic of these early years’ (1988: 77). Similarly, for Booth, the scribble stage is concerned primarily with ‘making marks’ using paint (1980: 122). Just as the child struggles to control the paintbrush, ‘the physical aspects of the instruments themselves determine the organisation of the music’ (Swanwick, 1988: 64).

As children master the materials in both domains, they become more interested in exploring technical devices. During the *manipulative* phase of music making, Swanwick reports a preoccupation with devices ‘such as glissandi, scalic and intervallic patterns, trills and tremolo’ (1988: 77). In common with Booth’s topology stage where shapes are bound by the physical features of the paper (typically, one or more edges), musical composition is inspired by ‘the physical structure and layout of the instruments’ (Swanwick, 1988: 77). When familiarity with technical devices in both domains increases, repetition becomes common as children enjoy their sense of mastery.

Swanwick proceeds with the *personal expressiveness* mode, characterised by the ‘exploitation of changes in speed and loudness levels, often… in a fairly shapeless way’ (1988: 78). Perhaps we may draw a parallel here with those topological paintings which are ‘covered with irregular shapes placed in irregular order’ (Booth, 1989: 72) (figure 8.2).

With the onset of the *vernacular* mode, melodic and rhythm patterns become common. The music has a regular metre and phrases are subdivided into the standard two, four or eight bar units. Devices such as syncopation, ostinato and sequence are used to develop ideas. The adoption of such conventions show that the children have ‘absorbed musical ideas from elsewhere while singing, playing and listening to others’ (Swanwick, 1988: 78). In Booth’s theory too, the arrival of the geometric pattern stage is accompanied with a *regularisation* of material. The implicit usage of symmetrical operations shows that children also absorb the conventions of art and design. Booth describes geometric decoration as a ‘universal human activity’ (1980: 71).

Children’s use of symmetry operations in musical compositions

The parallels I have drawn above require a great deal of further investigation. In particular, whilst Booth’s model covers the ages up to about 9 years with children reaching the vernacular mode at roughly the same time, it is far from clear that the inner stages are chronologically comparative. Clearly, as both Booth (1980) and Swanwick (1988) point out, schooling and other external factors can influence development in these domains. Nevertheless, the similarities led me to consider the extent to which children may use ‘transformation geometry-like’ operations in their musical compositions.

Werner (1948) established that children as young as 3¼ exhibit a preference for singing glissandi which ascend several tones before descending back to the starting pitch. This impulse to ‘balance’ the antecedent part of the melody with the consequent was also observed in a second experiment. A series of asymmetric melodies were played to children of all school ages who sang back what they believed they had heard. Often, the subjects would unintentionally amend one half of the melody to mirror the other. Werner calls this tendency ‘symmetrization’ and notes that a similar phenomenon occurs when children are asked to redraw an asymmetric graphical figure.

Hargreaves (1986) quotes the melody in figure 12 as an example of a ’pop song’ improvised by a child of between four and five years. Consistent with Werner’s observations, the melody ascends the pentatonic scale from Eb to Bb where it reiterates the Bb before descending back to Eb. This extract features not only melodic symmetry: with the exception of the last note, the ascending and descending sections are rhythmically symmetrical too.

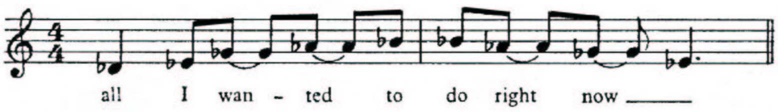


Figure 12 Improvised ‘pop song’ by a child of 4-5 years (Hargreaves, 1986: 80)

Figure 13 Composition by girl of 5 years (Swanwick and Tillman, 1986: 330)

Figure 14 Composition by girl of 7.8 years (Swanwick and Tillman, 1986: 332)

In figure 13, a girl of 5 years has repeatedly played the notes of the pentatonic scale from end to end of an Orff xylophone. This results in two examples of melodic symmetry along a vertical axis and illustrates the extent to which the physical layout of an instrument can affect the melody produced (Swanwick and Tillman, 1986). This reinforces the argument put forward by Sutton (1978) that the physical features of an instrument can *encourage* symmetrical ornamentation. In figure 14, a girl of nearly 8 years creates a melody that exhibits horizontal symmetry about the note D.

Educational Applications

Gromko (1996) investigated children's ability to apply their understanding of visual transformations in a musical context. She played short melodies to five children aged 6 to 9 and asked them to notate what they heard using small magnets on a metal sheet. The children were then asked to play the melodies in retrograde, inverted and retrograde-inverted formation. Rather than attempt the reversal immediately in their heads, the majority of the children realised they could perform the retrograde by reading the magnet notation in reverse. Similarly, they accomplished the inversion by first inverting the contour of the magnets. One boy commented, "It's the opposite. The ones that are high will be low and the ones that were low will be high" (in Gromko, 1996: 44). Most interestingly, all five children discovered that they could produce the retrograde-inversion by rotating the metal sheet by 180º . As she rotated the magnets, one girl commented, "You can turn those (change the highs and lows), but if you turn the sheet over, you can also get it backwards" (p. 44). Gromko observes that in accomplishing the tasks, all the subjects moved freely between graphical, musical and lingual modes of representation.

Upitis (1997) carried out a classroom research project with primary school children that investigated how transformation geometry concepts could be used to develop musical compositions. An initial session introduced the children to the visual characteristics of the different transformations and went on to identify the similarities between these and musical processes. They composed short melodies and recorded them using standard western notation drawn onto sheets of transparent acetate. By turning the acetate back-to-front and upside-down, the children were able to visually reflect and invert the melodies. The transformed versions were notated and arranged to form longer compositions. The final step was to record the compositions using a computer sequencer, allowing an accompaniment part to be added. The computer also facilitated detailed editing and final printing. Figure 15 shows one student's original melody, their subsequent transformations and the order in which the versions form a longer composition.

Figure 15 Child's melody and transformations. Composed during classroom research project by Upitis (1997: 118)

Implications

Children are surrounded by symmetrical patterns in the world around them. They find it easier to identify those that they encounter frequently, e.g. horizontal and vertical symmetry. This exposure can sometimes result in a tendency to 'symmetrize' their graphical and musical products. Composers, too, use symmetrical devices to achieve a sense of resolution. Moreover, Gromko and Upitis' work suggest that children are able to move freely between graphic and visual domains. In Chapter 1, I referred to the position of Wiggins and Wiggins (1997) who believe that the integration of music with other subjects should arise from conceptual foundations. I now proceed to assess whether transformation geometry can meet the criteria laid down by Wiggins and Wiggins; that the subjects to be integrated should 'share as their common root… the human mind' (1997: 40).

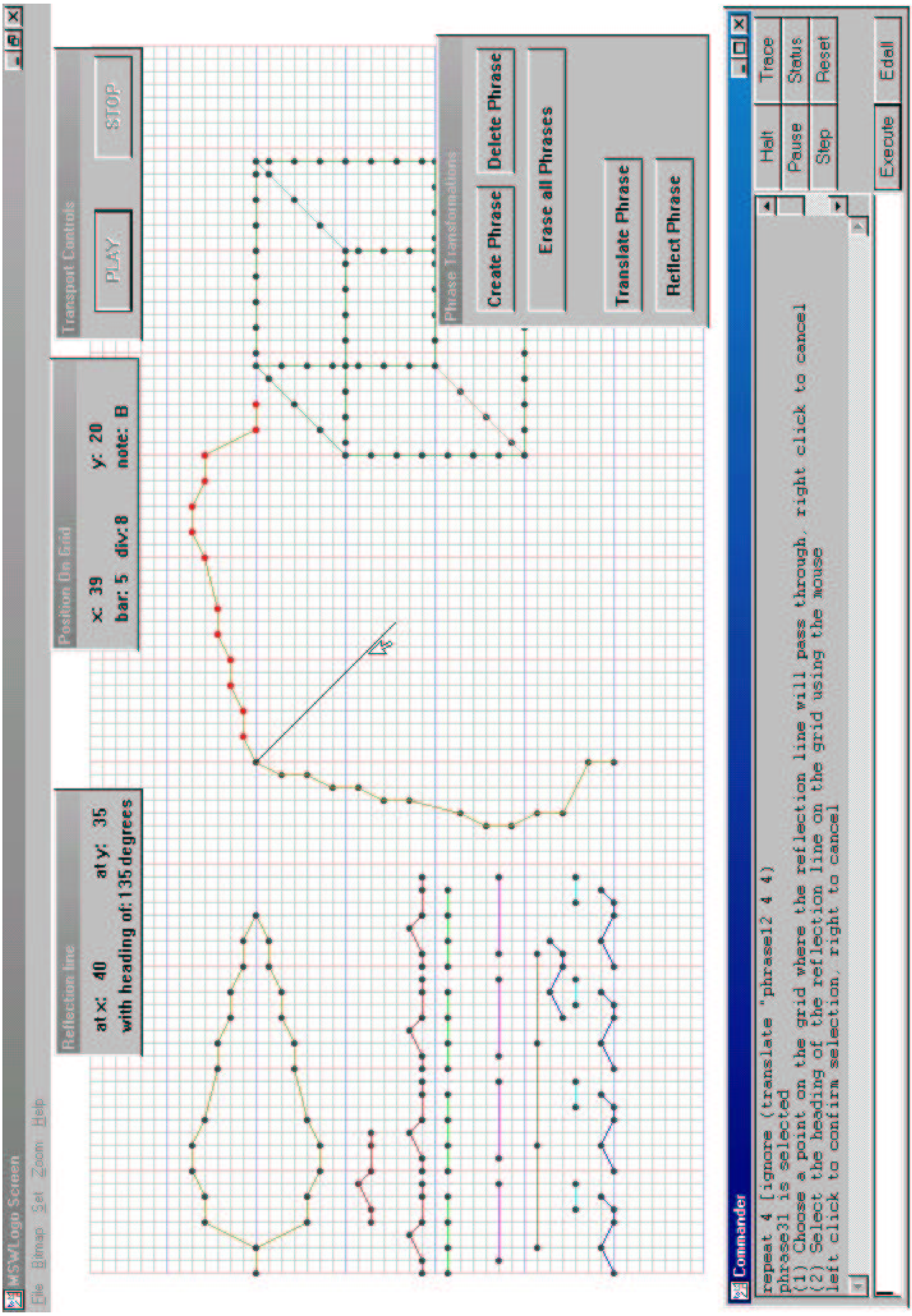


Figure 16 Screenshot from GeoMusE

Chapter 5
The GeoMusE microworld

I developed the *GEOmetric MUsical Environment* to explore the three hypotheses of this investigation: Firstly, that music could function as an *object-to-think-with*. Secondly, the computer provides an effective medium to develop this transitional relationship. Thirdly, transformation geometry operations have conceptual parallels with devices used in musical composition. In Chapters 6 to 9, I describe a series of case studies in which GeoMusE was tested with school children and adults.

Due to space limitations, I have limited my account of developing the GeoMusE program to the following discussion of important design considerations. The accompanying CDROM contains the GeoMusE software along with a copy of *MSWLogo*, the Logo environment in which it runs. These programs are written for computers running Windows 95 or higher. Full instructions on installing them are included on the CD. The CD also contains a comprehensive multimedia presentation of GeoMusE in Microsoft PowerPoint format.

It is essential that the written part of this dissertation be considered alongside the GeoMusE software itself. Reference should be made, at the very least, to the multimedia presentation since this demonstrates the complete functionality of GeoMusE.

Appendix 1 contains a printout of the MSWLogo source code for GeoMusE.

The microworld methodology

The microworld is a computer-based learning environment in which students can explore a particular knowledge domain using open-ended tools (Papert, 1980). Extensibility is a particularly important feature of such environments and is defined by Hoyles and Noss as ‘the extent to which the elements of the microworld can be combined, recombined and extended to form new elements’ (1996: 65). By situating GeoMusE within Logo, this extensibility is built in. Program development in Logo is carried out in the same environment as program execution, ensuring that end users have access to the individual procedures that constitute the software. By combining or adding to these procedures, users are able to extend and adapt the functionality of GeoMusE.

In order to take advantage of this extensibility, some knowledge of the Logo language is required. For a user base now firmly wedded to the cut-and-paste, click-and-drag world of modern graphical user interfaces (GUIs), being prompted to actually *type* something is rare. Noss and Hoyles (1996) describe the debate between those who value the sense of engagement with screen objects such as windows and mouse pointers and those who stress the descriptive, communicative qualities of programming. They point to Eisenberg's concept of *programmable applications* as a possible solution to the conflict:

> Programmable applications… are software systems that integrate the best features of two important paradigms of software design – namely, direct manipulation interfaces and interactive programming environments. The former paradigm – popularly associated with menus, palettes, icon-based interaction techniques and so forth – stresses the value of learnability, explorability, and aesthetic appeal; the latter, by providing a rich linguistic medium in which users can develop their own domain-oriented 'vocabularies', stresses the value of extensibility and expressive range.
> (Eisenberg in Noss and Hoyles, 1996: 56)

GeoMusE was developed as a programmable application. As a cross-curricular enterprise, I anticipated many potential users of the program would have no previous experience of either Logo or making music with a computer. It was vital that this group would find GeoMusE's interface comfortable and familiar. This meant employing windows and pointers. As this group of users advanced, I predicted their expectations might outgrow the potential of the GUI, hastening a move toward the programming interface. Like Edwards (1991; 1992), I envisaged a situation where users would want to extend the small selection of pre-programmed transformations, combining these to form more powerful *symbolic sentences*, capable of simultaneously performing several transformations.

Design influences

Two pieces of experimental computer software influenced the design of GeoMusE. These were *Streamer*, developed by Balzano (1987), and *TEMPER*, developed by Haus and Morini (1993).

Balzano's *Streamer* software allows students to manipulate three 'streams' of musical data, representing pitch, note duration and timbre.

Underlying this program are the mathematical concepts of phase and super-periodicity. When all three data streams have equal numbers of elements, the composite musical cycle repeats with each note playing for the same duration and with the same timbre. Yet, when one or more of the streams contains a different number of elements they do not stay in phase but instead flow through the different combinations until becoming unified again only on the *least common multiple*. For instance, if the three streams had 3, 4 and 6 elements respectively, the least common multiple would be 12. The pitch pattern would repeat twice for every cycle of the timbre pattern and the duration pattern would repeat thrice for every two cycles of the timbre pattern. Streamer's graphical interface, in which the three streams are superimposed on to a grid, provides a visual representation of the shifting phase patterns. (Figure 17).

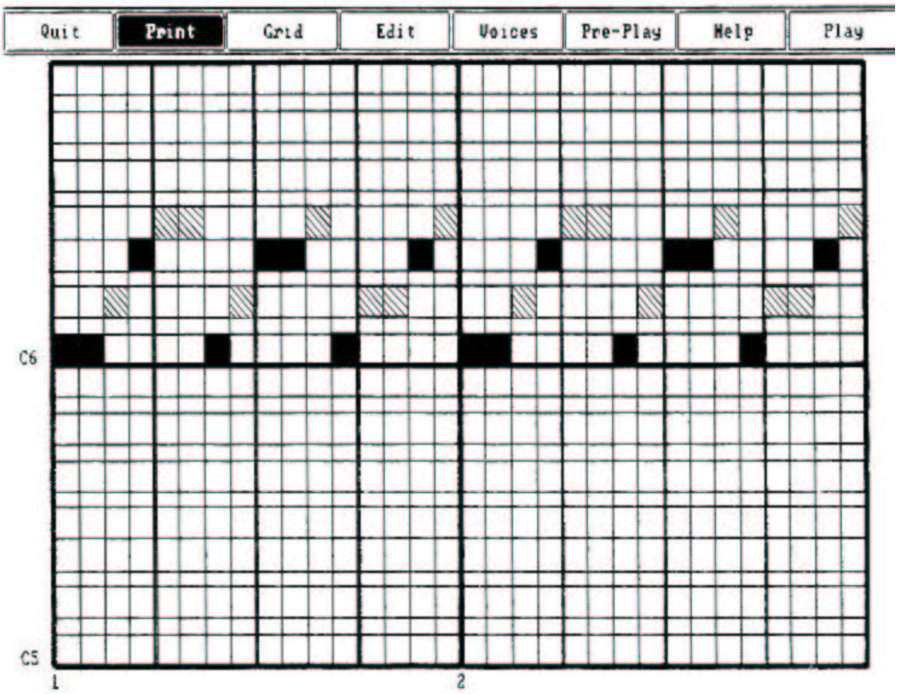


Figure 17 The *Streamer* grid. Pitch is represented by vertical axis, duration by horizontal extent and timbre by shading pattern. The pitch pattern is of length four: [C D E F], the timbre pattern is of length two and the rhythm pattern is of length three: [2 1 1] (adapted from Balzano, 1987: 96).

While not a true example of tessellation, the idea of producing a musical piece from the composite of several streaming patterns was appealing. In particular, I saw the potential of such a technique to produce complex rhythms by taking monotonic rhythmic patterns of different lengths and combining them using translation and reflection. The result would 'tessellate' (along the time axis) only when the periods of all the individual rhythms came together, i.e. the point of super-periodicity.

Haus and Morini's (1993) TEMPER (TEssellating Music PERformer) program generates slowly evolving tessellations, which the computer converts to music. In a tessellation, the borders of one figure also form the borders of adjacent figures. It is thus possible to regard the two shapes as being divided by only one line. These lines are joined at vertices with at least three meeting at any one vertex. TEMPER associates each vertex with a musical note. The screen is scanned from left to right with each vertex being 'played' as the scanning line passes over it. To increase musical interest, TEMPER gradually evolves one tessellation into another using a special transformation algorithm. An example is shown in Figure 18. Here, an Escher-like tessellation of birds in flight is transformed into another of winged horses through two intermediary stages.

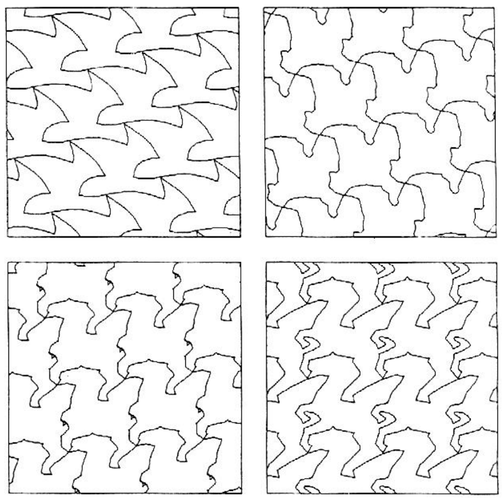


Figure 18 Evolving tessellation produced by *TEMPER* (Haus and Morini, 1993: 173)

Since TEMPER uses the position of vertices to dictate the properties of the musical notes, it is possible to perceive the result in either domain. A user can create a melody from the vertices of a tessellating shape. Alternatively, they can compose a melody and use the note positions as an outline for a shape. TEMPER allows the user to specify different timbres for different components of the tessellation. GeoMusE adopts this idea. In addition, each shape in GeoMusE can be displayed with a different colour.

Graphical Representation of Musical Information

Transformation geometry is, by definition, a visually oriented branch of mathematics so it was vital to ensure that the chosen system of musical representation was flexible enough to accommodate the fullest range of transformative procedures. In Upitis' classroom research project (Chapter 4), melodies written in conventional stave notation were transformed using transparent acetate. Such an approach limits the range of possible transformations. For instance, it is not possible to reflect a melody through a sloping mirror line. The horizontal lines of the stave would appear vertical and the curious result would be as though reading a musical score on its side (figure 19). The use of stave notation in GeoMusE was therefore discounted.

Figure 19 Reflecting standard stave notation through a sloping mirror line (musical extract adapted from Rosenberg, 1991: 476).

Bamberger (1991) has identified a class of children's invented musical notations that she terms *metric drawings*. In these, an attempt has been made to record accurately the properties of the music heard. Often metric drawings are drawn with reference to pitch and time, reminiscent of a co-ordinate system. Upitis (1990a) played *Twinkle, Twinkle, Little Star* to 50 children between the ages of six and nine years and asked them to notate what they heard. She found that the majority used discrete marks to indicate pitch (figure 20). Nisbet (1993) played a series of short melodies to ten and eleven year-olds, asking them to choose the corresponding representation from a selection of melodic contour graphs. Some children explained that they were able to look at the graph and hear the tune in their heads. Others said they were able to hear the tune and imagine the contour. Children with a musical background were more likely to identify the correct graph than those with who had shown a good ability to read mathematical graphs. In addition, Kürchemann (1981) found that the superimposition of a grid improved children's ability to locate reflections accurately. With these findings in mind, I decided to design GeoMusE around a co-ordinate grid system of musical representation.

Figure 20 Invented notation of *Twinkle Twinkle Little Star* by child of eight years. Note the use of discrete marks indicating pitch, duration and phrasing (Upitis, 1990a: 96).

The use of a co-ordinate grid to display pitch and timing information has become widespread in computer music. Usually implemented with pitch represented by the vertical, *y*-axis and time represented by the horizontal, *x*-axis, this system retains a conceptual similarity with standard stave notation. The *pitch-time grid*, as this form of representation is often known, is the format traditionally used in player-pianos and fairground organs. More recently, it has become a feature of computer sequencing software where the ability to trace melodic contours makes it particularly useful for those unfamiliar with stave notation. Figure 21 shows the 'piano-roll' edit screen from the *Cubase* sequencing program.

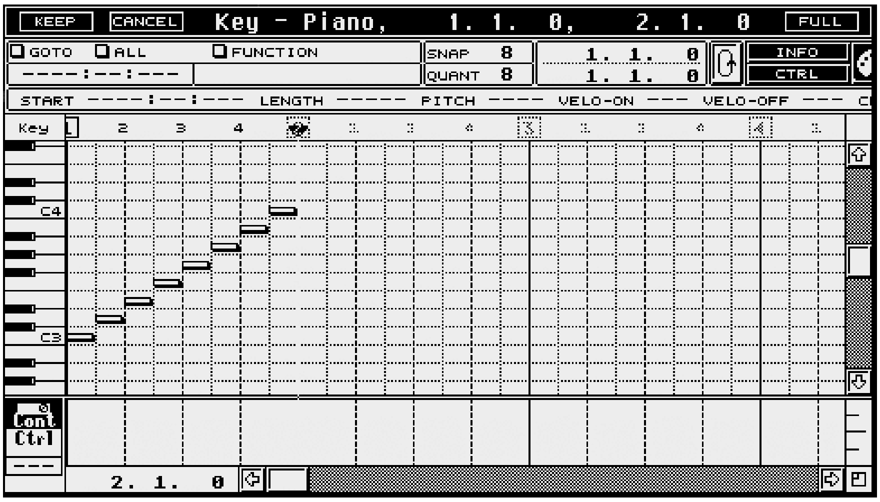
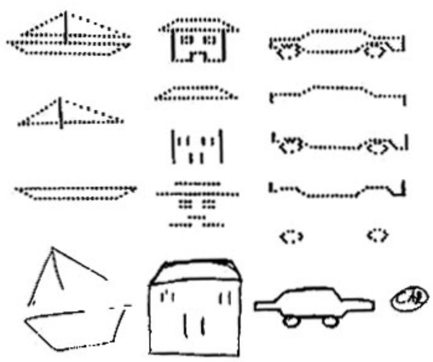


Figure 21 The pitch-time grid (key edit) screen from the *Cubase* sequencer for the Atari ST computer.

Cronly-Dillon *et al* (2000) have developed *SmartSight*, a computer program capable of converting images into musical information. The aim has been to enable blind people to perceive their surroundings through hearing rather than sight. Initial research (Cronly-Dillon *et al*, 1999) demonstrated that subjects (some of whom were blind, some blindfolded), were able to draw shapes after hearing musical analogues. Simplified images, including those of a yacht, a house and a car, were plotted on a pitch-time grid (the top row of figure 22). The musical results were played back with the option of 'masking' certain regions of the images (the middle two rows of figure 22).

Figure 22 Drawings made by non-sighted subjects after hearing musical representations encrypted using the *SmartSight* pitch-time grid. The top row shows the figures as drawn on the grid. The middle rows show certain areas of the figures that could be heard independently from the rest. The bottom row shows the subject's consequent drawings (Cronly-Dillon *et al*, 1999: 2431).



After time for consideration lasting from three to six minutes, the subjects drew the images they perceived (the bottom row of Figure 22). They included an impressive degree of detail in their drawings, leading Cronly-Dillon to conclude that it was possible for people deprived of sight to perceive the structure of complex visual forms using musical representation.

Cronly-Dillon’s results suggested that it might be possible for certain individuals to distinguish symmetrical properties audibly. Given that children often have difficulty reflecting shapes using sloping mirror lines (Chapter 4), I was keen to assess whether *hearing* the results of this transformation might make it any easier to comprehend.

Chapter 6

Beta Testing GeoMusE

A thorough trial of the GeoMusE software was essential for several reasons. Most importantly, it was necessary to establish the validity of my three hypotheses. Yet, before assessing these, it was also important to evaluate the performance of the program itself. As Vixie (1999) points out, problems often emerge when a program is first tried by others simply because they use it in a different way from the developer. As with any new software product, GeoMusE first needed *beta* testing. This process, where selected users explore the program in depth, is intended to detect problems and highlight areas requiring improvement.

Based on suggestions offered by Harrison (1991), the testing process was expanded into three distinct sections: the *expert review*, the *one-to-one evaluation* and the *small group pilot*. Due to time and resource limitations, the small group pilot was additionally designed to test the hypotheses. The assumption was that by this point in the testing process, the majority of the bugs and other problems would have been located and fixed.

The expert review

When designing educational software, it is important to ensure the educational efficacy of the content. Whilst it is students who ultimately judge a program, the expert, or subject-matter review can be used to 'gain insight into the completeness, basic assumptions, and acceptability of the design' (Harrison, 1991: 232). With this in mind, it is important to engage experienced teachers to conduct the expert review. GeoMusE was first demonstrated to the heads of the music and mathematics departments at a comprehensive secondary school. This school later became the venue for the small group pilots. Several important issues emerged from these sessions:

- Before piloting the software with Key Stage 3 students, it would be necessary to revise reflection by the horizontal and vertical axes as some students would have not have studied this since leaving primary school.

- It was unlikely that Key Stage 3 students would have yet encountered reflection by a sloping mirror line. They would also be unfamiliar with translation - a topic not covered until the following academic year. It would therefore be necessary to introduce these concepts before the pilot.
- If GeoMusE were to be made publicly available, detailed documentation would be needed explaining how teachers could get the most from the software.
- The head of music was impressed by GeoMusE's grid representation and has since incorporated this system of graphic notation into his own teaching.

The results of the expert review were particularly useful in designing the subsequent small group trials.

One-to-one evaluation

These informal sessions were used to identify potential problems with the software. During one-to-one evaluation, the developer is concerned both with the reaction of the subjects to the material under evaluation and the behaviour of the program (Harrison, 1991). Subjects are prompted for feed back as they work. They are later 'debriefed' in a semi-structured interview. A copy of the activity sheet and interview questions can be found in Appendix 2. Two subject experts were used in this phase of testing.

Evaluator A

Evaluator A was a highly experienced music education researcher who had previously taught in secondary schools. At the time of Evaluator A's session, the GeoMusE graphical user interface was still in its infancy and it was necessary to call each procedure from the command line (e.g. *create*, *reflect*, *delete* etc). This caused problems because Evaluator A often mistyped commands and was confused when the intended action did not result. At first, Evaluator A found the grid system difficult – particularly the time axis - but soon mastered it, entering some well-known melodies. He was then able to translate and reflect these with ease. The session and subsequent interview raised several significant issues, summarised below:

- If an experienced QWERTY keyboard user such Evaluator A found typing commands initially difficult, the problem would be even worse for school students. It was vital that a more complete graphical user interface was in place by the time of the school trials.
- Evaluator A’s problems entering melodies on the grid were exasperated by the GeoMusE’s inability to correct notes. During the interview he requested a ‘backspace’ function which, like its word-processing namesake, could proceed backwards through a phrase, deleting erroneous notes. This was duly implemented in time for Evaluator B’s session.

Evaluator B

Evaluator B was a primary school teacher who was completing an MA in music education at the time of the test. She had studied mathematics to A level. Evaluator B had the benefit of a fully functioning graphical interface, along with the ‘backspace’ function. She appeared to take to GeoMusE quickly and soon developed idiosyncratic techniques for its use, several of which revealed bugs in the software. As a serving teacher, Evaluator B was well placed to advise on special needs and differentiation issues. In particular, she observed that colour blind children would find distinguishing the differently coloured lines making up the grid difficult. She also suggested reducing the number of divisions in a bar from eight (quaver resolutions) to four (crochet resolution) at first, since children might find it difficult to place notes in the correct position. Following Evaluator B’s session, several important changes were made to the software. These included:

- The phrase creation process was amended so that any words included in a phrase name after a space character are automatically truncated.
- The variables controlling the colour of the grid lines were made available for users to alter in the ‘config’ procedure. Anyone finding the default colours difficult to distinguish could now change them.
- The small group pilot sessions were designed to use a grid that divided the bar into 4 rather than 8.

The one-to-one sessions were invaluable in detecting bugs and shaping the design of the school trials. A comparison of the evaluators’ experiences revealed a common problem: Neither subject naturally used the grid position window to ascertain the current pitch value, preferring to count the lines manually. In order to avoid similar misunderstandings during the school trials, I realised it would be necessary to explicitly introduce the students to the position window at the outset.

Chapter 7

Designing the School Trial

With most serious bugs isolated and fixed, the small group pilot was used to assess the pedagogical and psychological implications of the software. In the case of GeoMusE, the small group pilot was used to establish the following:

1. The efficacy of the hypothesis that music could function as an *object-to-think-with*.
2. If, through using the software, students could identify conceptual parallels between transformation geometry and musical composition.
3. What else the students might learn from the work.
4. The role of preparation work covered before the pilot trial.
5. How the software performed under the direction of target students.
6. Possible future directions for the software.

Choice of students

The students used in a small group pilot must be representative of the target user base of the software (Harison, 1991). In the case of GeoMusE, however, there was to be no specific user base. It was hoped that students of varying ages and abilities would be able to find their own level of comfortable interaction with the software and progress accordingly. With only a short amount of time available for the small group pilot, the trial was restricted to Year 8 students (12-13 years old). At this age, students are approximately halfway through their studies of transformation geometry according to the English National Curriculum (DfEE, 1999). By this point, students should have a firm conceptual grasp of the subject but there remain areas (such as sloping mirror lines and translation) with which they are substantially unfamiliar. I was anxious to evaluate the contribution of GeoMusE in introducing these unknown areas.

As a multidisciplinary project, I was keen for students to encounter GeoMusE from different curricular perspectives. To this end, I designed the trial around three groups. Each group contained a pair of students. This was intended to encourage subjects to voice their feelings about the work as they discussed their progress with one another (Harrison, 1991). The first two pairs - Louise and

Hassan, and Elizabeth and Susannah[7] - worked with the software during their mathematics lessons. The third pair, Mark and Helen, worked during their music lessons. It was anticipated all groups would receive the same preparatory work with the program but that the final project would be oriented towards the associated subject on the timetable.

The venue for the pilot was a large secondary comprehensive school in Buckinghamshire. In the case of mathematics, students were selected by the head of department in consultation with her staff and form tutors. In the case of music, responsibility for selection lay with the subject teacher. My only stipulation was that students should be enthusiastic about learning to use an unfamiliar computer tool. In the short amount of time available for the trial, it was important that the students could quickly get to grips with the program if possible, allowing me to concentrate on evaluating other, non-technical factors. The trials took place over two consecutive weeks in June and July 2001.

Methods of data collection

As Blease (1986) reminds us, data triangulation is particularly important when evaluating educational software using qualitative methods. In addition to summary interviews, several other data collection methods were employed.

All sessions were video recorded, facilitating later transcription. Video also retains important non-verbal information such as facial expression and body movement (Segall, 1991).

The students were encouraged to make notes and sketch out possible phrase shapes on paper. This provided a record of their initial ideas and intentions and offered a comparison with the subsequent, actual results from using the software. The worksheets completed during the initial preparation stages of the pilot also permitted comparison with previously published research (Chapter 4).

A facility for the production of *dribble* files was added to GeoMusE for the pilot test. A dribble file contains a record of all operations, including erroneous ones, performed by the user during the course of a session. Such files offer a rich source of data for analysis. In particular, they offer an insight into the exploratory

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[7] Names have been changed to preserve the anonymity of the subjects.

heuristics employed by students when tackling a Logo problem (for instance, see Hoyles and Noss, 1992). An example of a GeoMusE dribble file can be found in Appendix 3. A further source of data was the students' completed musical and graphical products. Audio copies can be found on the accompanying audio CD.

Pedagogical methodology for the pilot sessions

I have been impressed by the methodologies adopted by researchers working on Project Headlight at the Hennigan School, Boston (Upitis, 1990; Harel and Papert, 1991). This programme introduced a rich array of computers and other, non-traditional learning resources into the curriculum of an inner-city elementary school. Researchers typically adopted the role of class teacher or project leader, allowing them to act as not only facilitators and observers, but also participants in the learning process:

> More of my time was spent as an activity leader…Sometimes, if children were having a problem with a particular concept… I would pull a small group of students together aside and work with them for two or three minutes… Most of my time, however, was spent as a player – a learner. I took great delight in playing and learning with the children. But it was not a hollow form of learning of the "Let's learn together" variety… I learned with the children, but on my level, interpreting and directing my own explorations based on what I had previously explored. This in turn modelled for them "real" learning as opposed to "school" learning. (Upitis, 1990: 16)

This convergence of teacher and researcher is, of course, reminiscent of the ideas of Stenhouse (1975). Given the small numbers of students per group and the project-oriented nature of the pilot, I felt that his *teacher as researcher* methodology was particularly appropriate for this phase of evaluation. As Lawton puts it, 'it is a probe through which to explore and test hypotheses, not a recommendation to be adopted' (1983: 104). By combining the roles of project facilitator and researcher, I could perform the essential task of guiding the students' first steps with an unfamiliar, potentially confusing computer program whilst subsequently orienting the project work to best illuminate their evolving expertise.

Structure of the pilot sessions

The pilot sessions were organised in three sections:

- Preparation
- Practical activities and project work
- Evaluation

Preparation

The expert review had identified the need for preparation work to refresh the students' knowledge of transformation geometry. This was vital to ensure all subjects were commencing work on GeoMusE with similar levels of experience. Edwards (1992) similarly set aside time for initial preparation sessions before introducing students to her *TGEO Microworld*. With no formal pre-test planned, this preparation also provided a simple means of assessing each student's understanding of the subject at the project's outset.

The preparation material was adapted from Cox and Bell (1989a; 1989b). Copies of the worksheets used can be found in Appendix 4. The first two sheets dealt with reflection. Small mirrors were available for students to check their solutions if they wished (Genkins, 1975; Gromko, 1996). Exercises (a) through (d) of sheet 1 each featured a shape and either a horizontal or vertical reflection line (mirror line). Students were asked to draw in the corresponding reflected figure (for example, figure 23.1). Exercises (e) through (h) each showed a shape with its reflection. Students had to draw in the corresponding line of reflection (figure 23.2). Exercises (i) to (n) followed the same two conventions, but featured diagonal lines of reflection (figure 23.3).

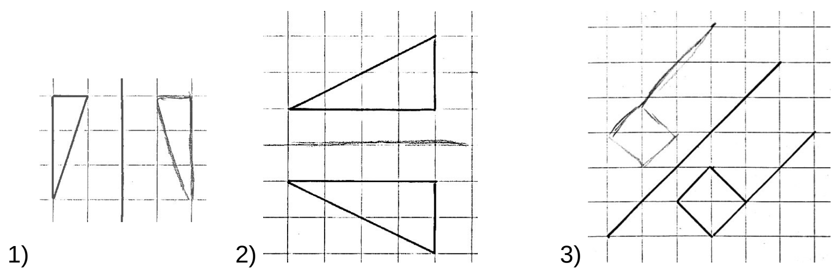

Figure 23 Examples of reflection preparation exercises: 1) Drawing in the reflected figure 2) Drawing in the mirror line 3) Drawing in the reflected figure (sloping mirror line)

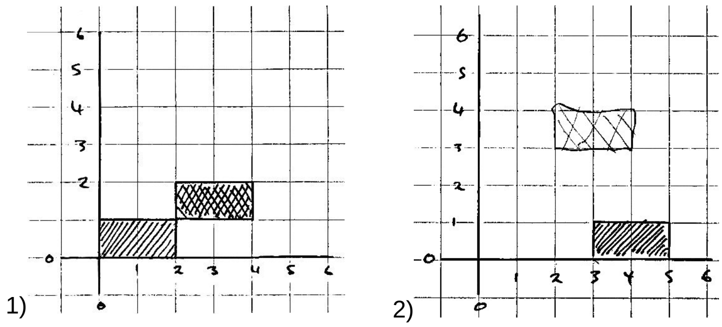

Figure 24 Examples of translation preparation exercises: 1) Identifying shift vector: [2 1] 2) Drawing in shift vector: [-1 3]

The third worksheet dealt with translation. In exercises (a) to (c), students were asked to determine the *shift vector* which linked the two shaded rectangles (figure 24.1). In exercises (d) through (f), they were given one shaded rectangle and a vector and asked to draw in the corresponding translation (figure 24.2).

The members of each pair took it in turns to complete an exercise but discussion about possible solutions was encouraged. I would often intervene if students appeared to be unsure of the solution they had offered, instigating further discussion. On completion of these worksheets, I asked each group if they could describe the musical equivalents of these transformations. These discussions followed a similar format to those in Upitis' (1997) case study. First, we established the musical features of each transformation. For instance, in the case of a reflection by vertical mirror line, a melody beginning low and ending high would begin high and end low. Later, the correct musical names for each transformation were introduced. The discussion of musical translation naturally introduced the notion that the *x*-axis could represent time and the *y*-axis pitch.

Practical activities and project work

These activities were designed as not only exercises to familiarise the subjects with GeoMusE but also promoted significant educational outcomes in their own right.

Before commencing work on the activities, I introduced the pitch-time grid concept more formally. I demonstrated how one could create a phrase on the grid, playing in the first phrase of *Twinkle Twinkle Little Star*. I also asked the subjects to place the mouse in random locations on the grid and read off the information displayed in the position window. I hoped such an explicit introduction to this feature would encourage reference to it from the beginning. This had not been the case in the one-to-one trials (Chapter 6).

The first two activities involved reflection and were preceded with a discussion concerning the representation of angles as *headings* (Abelson and diSessa, 1980; Edwards, 1992). The subjects were asked to reflect the first phrase of *Twinkle Twinkle Little Star* so that the notes played backwards. They were then asked to reflect the first phrase of *Twinkle Twinkle Little Star* so that the tune was inverted. After each activity, we discussed how the musical properties of the transformed phrases had changed. For instance, was the rhythm the same? What about the pitch? What happened to the intervals between consecutive notes? Did the transformation retain the visual shape of the original? We also discussed how altering the horizontal and vertical position of the mirror line would affect the pitch, or timing of the transformation.

The next two activities were concerned with translation. The subjects were asked to create an *ostinato* from the first phrase of *Twinkle Twinkle Little Star*. Later, they created a *round* from the same phrase. These tasks were an effective way of introducing the students to GeoMusE's representation of the time domain since it was necessary to carefully monitor the position of each translated phrase on the *x*-axis.

Two further activities were to create *non-retrogradable* and *non-invertible* phrases[8]. I demonstrated palindromic words and sentences and explained that some composers had written music which was the same when played both forwards and backwards (Garland and Kahn, 1995; Kempf, 1996). For the work on non-invertible phrases, I revised the concept of harmony since it is only possible to create such a phrase by having more than one note sounding at a time (discounting the possibility of a monotonic melody).

The last activity explored the musical effects of reflecting a phrase (*Twinkle Twinkle Little Star* again) through a mirror line with a heading of 45º.

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[8] These activities were inspired by aspects of Messiaen's theory: He used *Non-retrogradable rhythms*, which sound the same both forward and backward (and thus cannot be reversed). *Modes of limited transposition* are scales that can only be modulated once or twice before the original version is obtained again. An example is the whole-tone scale (Taylor, 1991).

Following the activities, the subjects engaged in longer, more involved project work. For those piloting GeoMusE during their mathematics classes, I prepared a main project and a smaller secondary project.

In the main mathematics project, I asked the students to think of either a short musical phrase or a shape (or pattern) that could be repeatedly transformed to create a longer, more complex composition. Following Upitis' (1997) experiences, I explained that they could commence work in either the musical or the visual domain. A regular shape was likely to sound pleasing to the ear whilst an interesting tune would often generate an attractive shape. Once the initial phrases had been entered, the students took turns to transform them in some way. After each transformation, we paused to discuss both the musical and mathematical consequences of their actions. Occasionally, when it seemed that a student had exhausted their particular battery of transformations, I would suggest an additional technique. For instance, I pointed out that a mirror line could go *through* a phrase instead of being adjacent. I made these suggestions as subtly as possible since I was aware that when introduced like this, children often adopt them *verbatim*, immediately abandoning their own intentions (Upitis, 1990b; Gromko, 1996). I also explained that GeoMusE could display phrases in different colours. Whilst not affecting the musical result, this was a simple and effective way of enhancing the visual product.

The follow-up project planned for the mathematics students involved composing rhythms. The computer running GeoMusE featured a *General MIDI* compatible sound card[9]. This was configured to map a percussion sound to each line on the *y*-axis. Since there was no longer a relationship between pitch and vertical placement, attention moved from creating melodic shapes to rhythmic patterns, each located on a single horizontal line. Initially, the students improvised rhythms on the grid, exploring the range of sounds available. I was prepared with a book of pictures of percussion instruments, since many found in the General MIDI set were unfamiliar to them. Finally, I asked the students to compose a rhythm in the style of modern electronic dance music.

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[9] General MIDI is an extension to MIDI which specifies 127 standard tuned voices and another 52 percussion sounds.

In the music project, students were asked to build up a composition featuring rhythm, bass, harmony and melody parts. They were free to use the transformations as they wished and could choose different instrument sounds for each part. An important motivation for this project was to assess the students' ability to bring what they had so far learned about musical transformations to bear on conventional musical forms.

Evaluation

The final session for each group concluded with a revision of the material covered and a semi-structured interview. This probed their thoughts on both the preparatory materials and the range of computer-based activities. It also established their attitude to using the software and solicited their suggestions for improvement. The interview concluded with a discussion about what they believed they had learned from the sessions. Each session took place at the computer so that the students could refer to it during the discussion. They were also encouraged to make notes and draw diagrams. The questions relating to the program itself were adapted from Burkhardt and Fraser (1983), Blease (1986) and Harrison (1991). A copy of the questionnaire can be found in Appendix 5.

Chapter 8

Results of the School Trial

Results of the preparation work

The students had no difficulties in correctly answering the reflection exercises that featured vertical and horizontal axes of symmetry. Consistent with Küchemann's (1981) findings, however, they found it harder to complete exercises with sloping lines of reflection. Hassan initially misplaced the reflection in figure 25.1, later correcting it. He also misplaced the reflection in figure 25.2 but was unable to correct it when prompted. Susannah also found this exercise difficult and needed to check her answer with the mirror card. Mark found it difficult to draw in a mirror line when the figure itself was sloped (figure 25.3). This, again, is in keeping with Küchemann's results.

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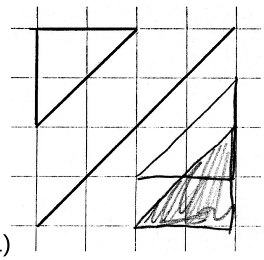
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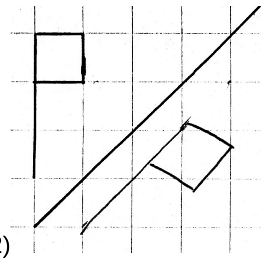
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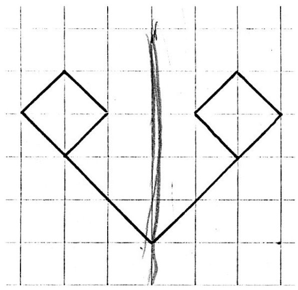
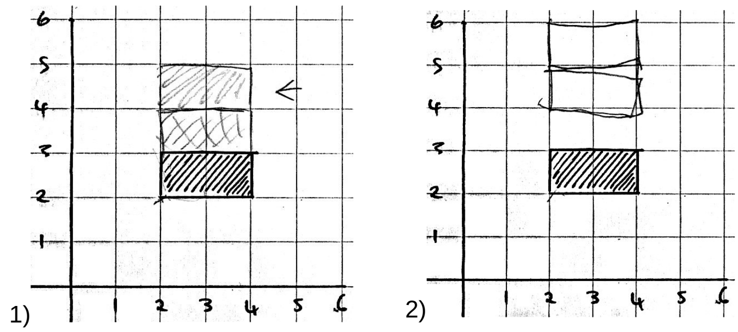


Figure 25 Difficulties encountered during the reflection preparation

Since none of the six students had any previous experience of the transformation, all found the translation exercises more challenging. The most common problem was to miscount the number of grid lines by which to translate a figure. Hassan tended to place the transformations one increment too few up or across because he began counting at one rather than zero (figure 26.1). Mark occasionally placed them one increment too far because he counted the whole squares rather than the lines (figure 26.2).

Figure 26 Mistakes made during translation preparation



Whilst all the subjects appeared to accept the idea that reflection and translation could be applied to melodies, initial attempts at explaining the consequent musical effect varied in accuracy. The easiest concept to grasp was that a melody reflected through a vertical mirror line would be reversed. Louise suggested that the transformation would “go the other way” whilst Mark said it would be the “opposite”. Susannah stated that it would go “backwards”.

Melodic inversion due to a horizontal reflection proved harder to verbalise. Hassan correctly said the transformed version would be “lower” than the original and qualified this by added that if the original phrase started low and got higher, the reflection would “start high and go lower”. Similarly, Helen explained that a melody with the contour low-high-low would reflect to become high-low-high. Conversely, Susannah believed the reflection would sound the same as the original. None of the students used words such as ‘upside down’ or ‘inversion’ to describe the transformed melody.

When asked what the result of horizontally translating a melody to an adjacent position would be, several students replied that either the individual notes or the whole melody would be longer. This suggests they had realised that when heard together, the original and transformed melodies would sound for a greater amount of time than the original alone. However, it is unclear that they understood that the transformation would be a repetition of the original. Conversely, Elizabeth and Susannah immediately identified the translation as a repetition. After clearing up the initial confusion regarding horizontal translation, all the students quickly came to understand vertical translation. When asked what the musical effect would be, Louise replied, “it would be a higher pitch but the same shape”

These results are largely consistent with those of Gromko (1996) (Chapter 4), even down to the similar descriptions of the various transformations. Like

Gromko, I would suggest that all the students in the present study were initially able to conceptualise the musical results of the transformations to a greater or lesser degree. This is despite their frequent inability to provide complete and accurate verbal descriptions of the operations.

Results of the Activities

Reflecting a melody through horizontal and vertical mirror lines

Overall, the subjects found it easy to describe the orientation of a mirror line by its heading. They knew, for instance, that the mirror line needed to reverse the melody should have a heading of 0º. The majority also realised that the mirror lines could equally be described using the opposite heading; e.g., a 0º heading produced the same mirror line as a 180º heading. Since all the subjects had used Logo at some point in their school careers, I would echo Edwards' (1992) argument that they had applied the knowledge of headings gained through this experience to the new situation. A comment made by Elizabeth during the evaluation reinforces this view:

Ross: Did the program's layout confuse you in any way?
Susannah: No.
Elizabeth: I reckon that's because we've been on something like it before.
Ross: What was that?
Elizabeth: That Logo thing. We've seen it before.

Louise, Hassan and Mark soon realised that the initial point needed to 'pin down' a reflection line (Edwards, 1992: 137) could be placed at any position on the reflecting axis. Furthermore, Hassan correctly stated that the position of the point on the perpendicular axis determined the distance between the original and its reflection.

The students found it easier to describe the musical consequences of vertical reflection than horizontal reflection. When the computer had plotted the reversed melody on the screen, Susannah exclaimed, "It's turned it the other way now" and attempted to sing the tune backwards. As had been the case during the preparation exercises, there was a tendency to describe the results of horizontal reflection either incorrectly or incompletely. Initially, Helen and Mark described the inverted and original melodies as "playing at the same time". Susannah found a verbal explanation of inversion difficult and resorted to hand movements to demonstrate how she thought the original had been "flipped". Hassan was more accurate:

Sussannah: You can hear the same tune but with two different pitches playing at the same time.
Ross: Is it the same tune?
Hassan: No, it's upside down.

Whilst the students found it difficult to describe inversion, they all appeared to realise that the reflected melody had retain the same melodic contour or shape to the original. When asked what the relationship between the notes in the original and reflected melodies was, Susannah answered, "they've been put in the same place but on the other side [of the mirror line]". Similarly, Louise and Hassan correctly stated that if the interval between two notes in the original would be four tones, the interval between the corresponding notes in the reflection would also be four tones. Susannah, Louise and Hassan appeared to be applying Piagetian *conservation* to relate the two melodies. In his comparison between the Piagetian developmental stages and theories of musical development, Warrener notes the appearance of melodic conservation during the *concrete operations* period:

> As he now recognises that a cup of milk retains its quantity whether poured in a narrower or wider container, the individual should also recognise that a tune is the same in spite of changes brought about be melodic augmentation or diminution, a different harmonisation, a new modality, an inversion of certain notes, rhythmic alteration, or other variations. (Warrener, 1985: 25)

Using translation to create ostinati *and* canon

All the students had learnt about ostinati in their music lessons. After a brief revision, they all appeared comfortable with the concept of canon. Except in one case, the students understood that these musical devices worked through time displacement only and that the transformed melodies would retain the pitch of the original.

As has been noted, Hassan and Mark initially had problems relating a shift vector to the grid co-ordinates during the preparation exercises. This confusion was not repeated here. In the ostinato activity, both confidently positioned the translated melodies so that they would begin not immediately following the original but at the beginning of the next bar. They were aware that this would be necessary to maintain a regular rhythm. Repetition is a prominent feature of Swanwick's (1988) *vernacular* mode and I would argue that it is through their exposure to conventional musical devices during everyday musical experience that Hassan and Mark intuitively knew where to locate the melody.

After the initial work on ostinati, the students quickly mastered the translations necessary to create a canon. All realised the repeated melodies had to *overlap* the preceding one. The richly contrapuntal and harmonious result was often met with excitement (figure 27, track 1[10]).

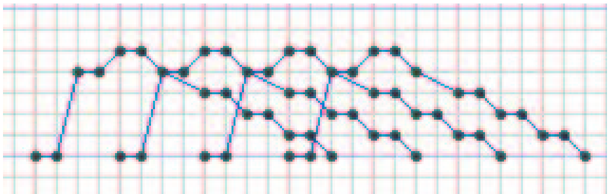


Figure 27 *Twinkle Twinkle Little Star* horizontally translated three times to form a canon

Creating palindromic and non-invertible melodies

Since the students were not familiar with the term *palindrome*, literal examples were offered including 'MADAM IM ADAM' and 'ANNA'. From their comments, it appeared that the students could then generalise the 'rule' for such constructions:

Mark: If its first it will be last and if its second it will be second from last.

Susannah was already thinking in musical terms:

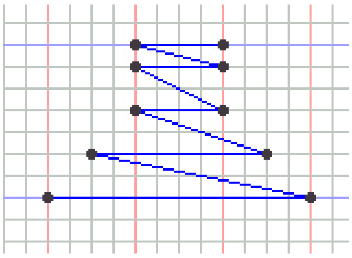
Susannah: You'd use the same note to start and finish and use the same notes as you go in. The first five will be the same as the last five but the other way.

Interestingly, whilst all three groups arrived at the same rule they differed in the way they applied it. Louise and Hassan constructed the palindromic melody by initially entering the first and last notes and then working inwards in both directions. This meant pairs of notes could be matched around an imagined axis of symmetry.

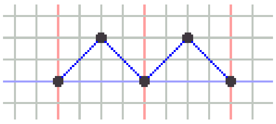
[10] Track numbers refer to the accompanying audio CD

Their strategy is evident from the transverse lines joining the notes (figure 28, track 2).

Figure 28 Palindromic melody composed by Louise and Hassan. Note traverse construction technique



The other two groups worked linearly from the first to last note. This strategy appears to have required greater concentration as it necessitated continual reference to what had gone before. Perhaps because of this additional mental overhead, they produced simpler melodies featuring only two pitches and fewer notes (figures. 29.1 and 29.2, tracks 3 and 4).

1) 

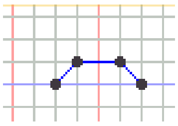
2) 

Figure 29 Palindromic melodies composed by:
1) Susannah and Elizabeth 2) Helen and Mark

When Gromko (1996) asked her subjects to play the given melody in retrograde, she also identified two distinct strategies. As in the present study, some students tackled the problem linearly, playing through the tune repeatedly until they had memorised it accurately. They then performed it in reverse by referencing the internalised sequence. Another boy divided the melody into 4-beat patterns, commenting: “I’m breaking it up so I can play some parts of it backward so I can memorise it” (in Gromko, 1996: 44). Like Louise and Hassan, who had constructed the palindrome from note pairs, this boy appears to have realised the potential of deconstructing larger problems into smaller ones. The use of procedures to break down complex operations is naturally promoted in Logo. Through their use, Papert writes:

> It is possible to build a large intellectual system without ever making a step that cannot be comprehended. And building with a hierarchical structure makes it possible to grasp the system as a whole, that is to say, to see the system as “viewed from the top.” (1980: 103)

Papert recounts the testimony of one young covert to procedural programming: “See, all my procedures are mind-sized bites…I used to get mixed up by my programs. Now I don’t bite off more than I can chew” (in Papert, 1980: 103).

Whilst the students had managed to compose palindromic melodies, it was unclear that they had all realised these were necessarily vertically symmetrical. Before creating non-invertible melodies, I instigated discussions about the features of the palindromes, hoping that this fact would emerge. Gradually, the students understood that this kind of melody would require more than one note to play at a time. At first, Susannah returned to the misconception that inversion was simply “writing the same underneath”. She finally understood, as her comments demonstrate:

Susannah: You could do a straight line.
Ross: Yes, and that’s a clue actually.
Susannah: Oh? [pauses to consider] Or a cross, or a star, or something that has exact lines of symmetry.

Once they realised the horizontally symmetrical nature of non-invertible melodies, two groups of students quickly produced their own (figures 30.1 and 30.2, tracks 5 and 6). Hassan encountered problems with a melody formed from an outlined rectangle. He had correctly reasoned that the symmetrical properties of this shape would ensure his phrase was non-invertible, yet the notes forming its sides were themselves asymmetric (figure 30.3, track 7). Like some of the children observed by Edwards (1991), it might be that Hassan had *overgeneralised* his understanding of the symmetrical properties of rectangles, simply assuming all quadrilaterals could be reflected in the same way. His mistake only became clear to him when he listened back to the musical result. Since the two vertical sides of the square sounded with different harmonies and the two horizontal sides sounded with different rhythms, the phrase could be inverted. Hassan’s corrected version featured a greatly simplified square (figure 30.4, track 8).

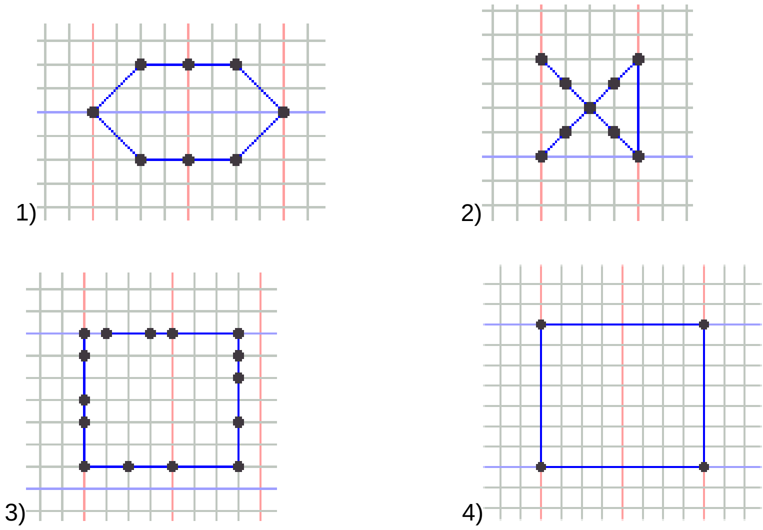


Figure 30 Non-invertible melodies by: 1) Mark and Helen 2) Susannah and Elizabeth 3) Louise and Hassan (erroneous) 4) Louise and Hassan (correct)

Reflecting a melody through a sloping mirror line

I began this activity by asking the students what they thought *Twinkle Twinkle Little Star* would sound like if it were reflected through a mirror line with a heading of 315º. Figure 31 and track 9 on the CD demonstrate the form this transformation might take. Most realised that the result would bear little musical resemblance to the original. As Susannah put it, “It would come out kind of weird”. Given that Hassan had found the preparation exercises on sloping mirror lines particularly difficult (figure 25.1 and 25.2), it is significant that he now offered the most complete and accurate prediction of the musical effect of this transformation:

Hassan: Some notes would be going at the same time. It would be twisted around.
Ross: Which notes would play at the same time?
Hassan: The first one and the last one.
Ross: Any others?
Hassan: I’m not sure.

After hearing the reflection back, he refined his prediction, correctly identifying all the notes that played together. Subsequently, he went further:

Ross: Thinking back to the original, what was special about the notes that now play at the same time?
Louise: Were they on the same note?

Hassan: Same pitch there [points at original], same time here [points at reflection].

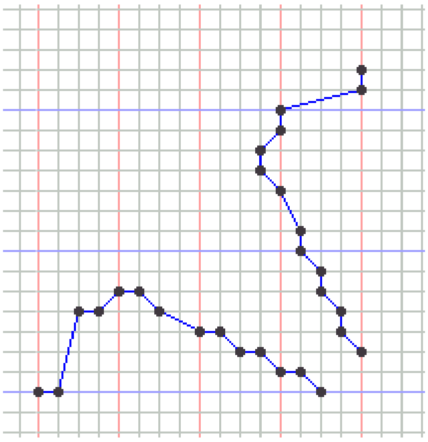


Figure 31 *Twinkle Twinkle Little Star* reflected through a mirror line with 315º heading

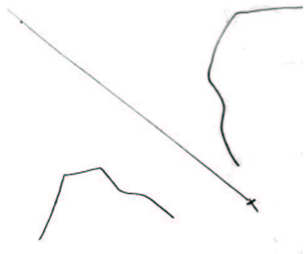


Figure 32 Susannah's prediction of figure 31

Whilst Susannah was unable to find the words to describe the visual effect of the sloping reflection line, she was able to draw what might happen (figure 32). Again, hearing the result appeared to help clarify her position:

Ross: What's happened?
Susannah: The notes have joined up because they were on the same lines.
Ross: Lines on which axis?
Susannah: 'y' [pitch]… The notes are now in lines on the 'x' axis.

Results of the project work

Using shapes as the basis for musical compositions

All of the students undertaking this project decided to begin working with shapes rather than melodies. However, as Susannah insightfully pointed out, "if you draw a shape it will end up as a tune". After their initial ideas had been sketched out on paper, it became clear that some students were already thinking both of the musical and geometric properties of their figures. Hassan and Elizabeth included note positions on their drawings (figures 33.1 and 33.2), whilst Hassan also indicated a simple strategy for reflection. He later slanted his 'H' shape so that the notes would play at different times (figure 34.1, track 10). In Hassan, Susannah and Elizabeth's designs, the notes are placed at the corners of the shape. Louise wanted to use notes more often, outlining the perimeter of her 'L' shape. She decided not to place the notes contiguously since they would sound like "a bit of a blur" and instead included every other note. Louise had previously explained that harmony meant doing "one note, then miss a note, then another note". Her final version (figure 34.2, track 11) uses alternate notes on the two vertical sides to create a musically very satisfying result.

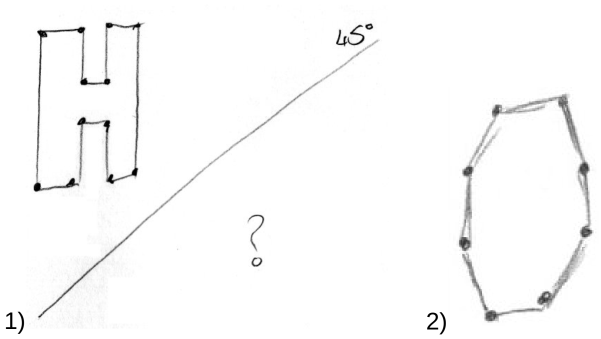
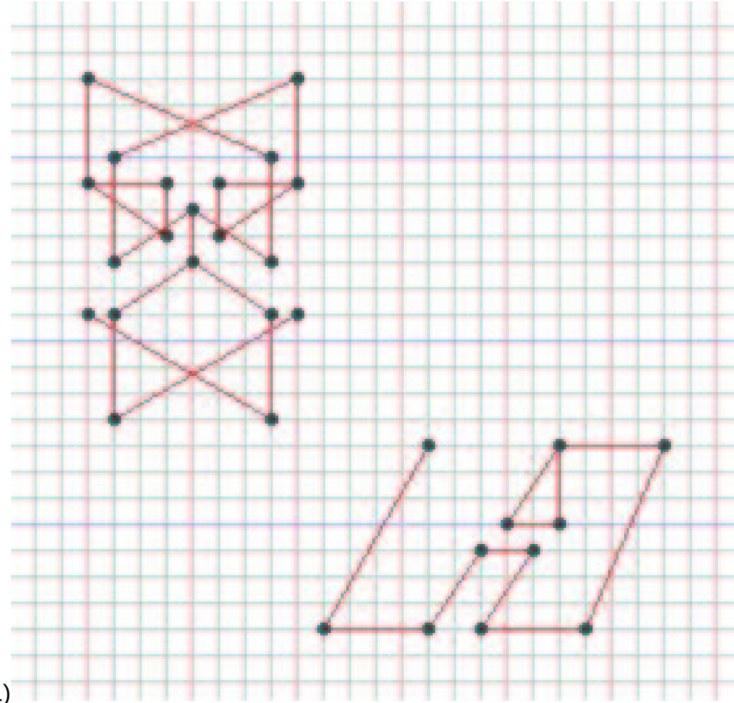


Figure 33 Plans for shapes showing note positions and transformation ideas: 1) Hassan 2) Elizabeth


1)

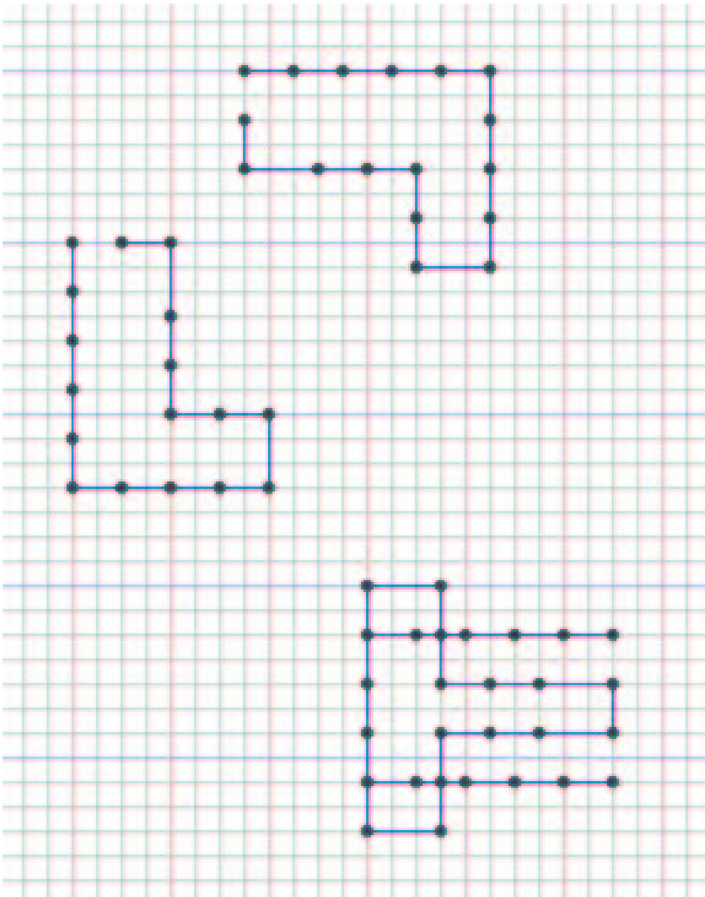
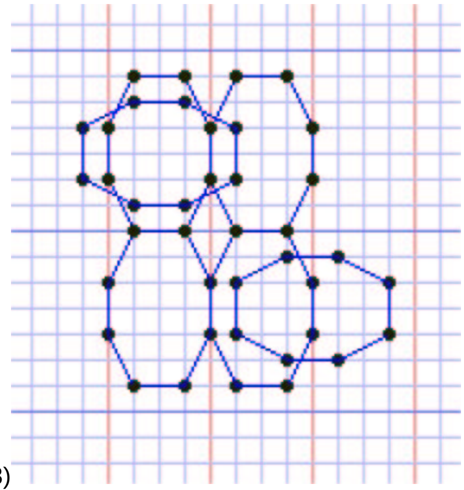

2)

Figure 34



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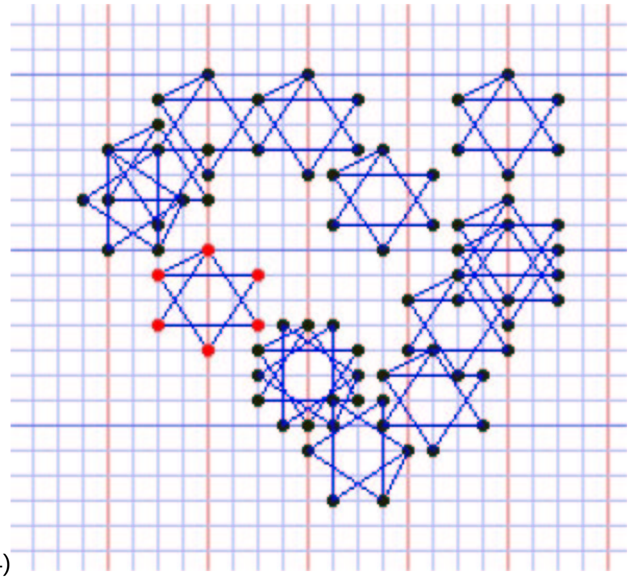

4)

Figure 34 (cont.) Shape compositions: 1) Hassan 2) Louise 3) Elizabeth 4) Susannah

Reflection through a sloping mirror line proved to be the favourite transformation. The students seemed fascinated by the results. When Louise reflected her ‘L’ by 315º, she discovered that the notes from the vertical legs now produced a rhythmic, hocket effect. Conversely, Hassan’s slanted ‘H’ now became a series of widely spaced chords. Placing a mirror line inside a shape was also a popular technique. Elizabeth and Susannah’s shapes each had two lines of symmetry. By internally reflecting them through 45º, the pair discovered it was possible to rotate their shapes about their centre points by 90º. This led Susannah to remark that the two designs produced musical harmonies that began close, moved apart and came back together. I explained that melodies are often divided into *question* and *answer* sections that together sound ‘complete’.

Intrigued by the symmetrical aspect of their shapes, Elizabeth and Susannah continued this theme on a larger scale. Elizabeth reflected her original shape vertically and again horizontally to produce a tiling effect. Her final composition, whilst not a precise example of rotational symmetry, certainly conveys a strong sense of intended balance (figure 34.3, track 12). Susannah’s various transformations formed an interesting curve (the left side of figure 34.4). Seizing its potential, she carefully translated more stars to replicate a reflection of the curve, thus approximating a heart shape (track 13). These designs are consistent with Werner’s (1948) argument that some children exhibit a natural desire to ‘symmetrize’ melodies and drawings (Chapter 4).

Using translation to compose rhythms

Due to time constraints, only Louise and Hassan undertook this project. The pair had some good ideas about how one rhythm should relate to another:

Hassan: It should flow into the other
Louise: As long as it isn’t completely different from the rest it will join OK.

Since I wanted the students to create a rhythm in the style of modern electronic dance music, we discussed the importance of a regular beat:

Ross: What happens if you are dancing to something and the rhythm keeps changing?
Louise: You can’t dance to it. You’d have to change the moves completely.

	If it goes slow but then really quick then it will look odd in the dance.
Hassan:	It won't really fit into the direction of things.
Louise:	With the stuff I listen to, the words might be different but it does the same thing all the way through. It's a kind of ostinato.

Louise began by creating a bass drum part that played on every beat (the bottom pattern in figure 35). Rather than repeat the same pattern throughout, she translated the first three bars twice commenting: "it's the kind of rhythm you can tap you feet to". As we listened back to the bass drum part, I asked the pair to clap where they thought the snare drum should go. They clapped on the second and fourth beats of each bar and Hassan subsequently entered the pattern on the grid (the middle line in figure 35).

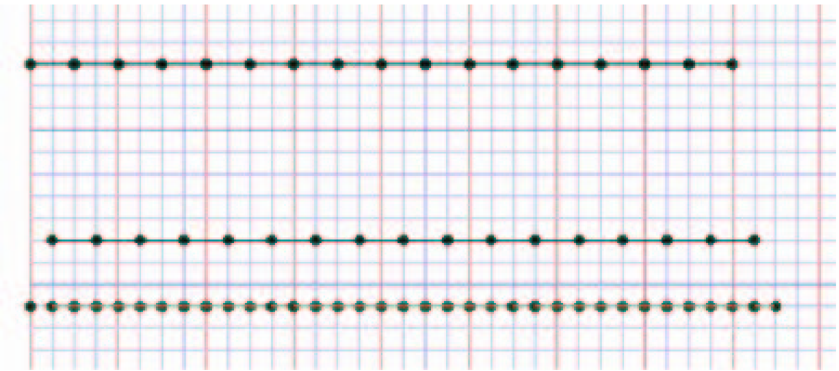

Figure 35 Rhythm composition by Louise and Hassan. Top line: hi-hat, middle line: snare drum, bottom line: bass drum

I was keen to discover whether Louise and Hassan were able to apply what they had learnt about translation in a new context:

Ross:	Where would you put the high hat [cymbals] in the bar?
Louise:	The other one to the snare drum?
Ross:	The snare is on two and four.
Louise:	So have it on one and three.
Ross:	So do we need to create a new phrase?
Louise:	No. We can translate that [the snare pattern] and put it up. Move it to a different place.

This exchange led to a discussion about rhythms in which some instruments played the same part but began in different places. The completed rhythm can be heard on track 14 of the CD.

Using transformations to create a layered musical composition

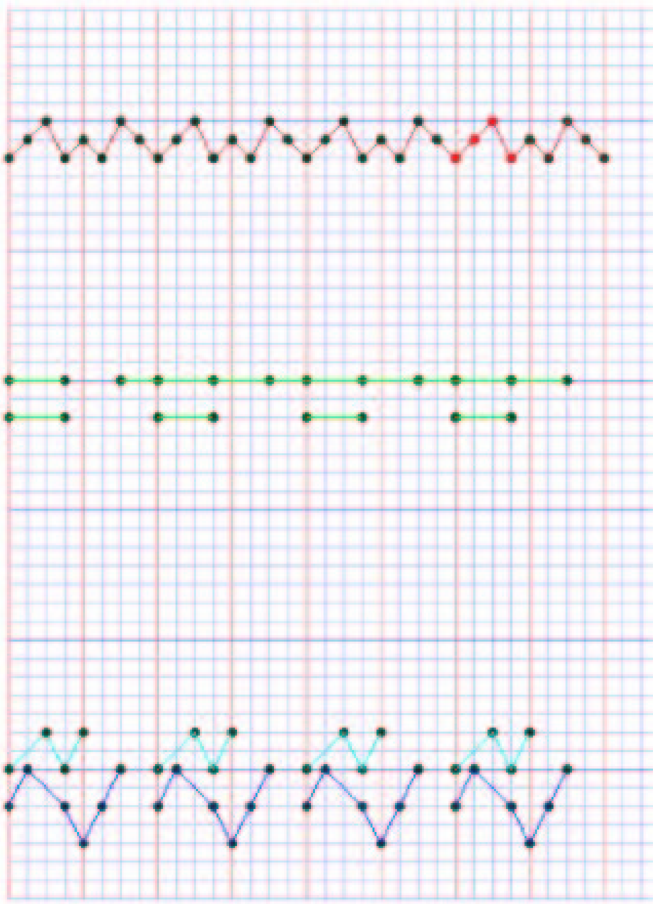

Figure 36 Layered musical composition by Helen and Mark. Dark blue: bass line, light blue: bass and snare drums, green: harmony (guitar), red: tune

For this project, Helen and Mark endeavoured to produce a more ‘conventional’ musical composition using transformations. The need to consider the stylistic appropriateness of their actions meant the pair found the task particularly challenging. Helen began by entering a two-bar bass line (coloured dark blue in figure 36). Mark repeated this phrase three times by translating it along the horizontal axis. They derived the lower harmony part (coloured green) by positioning its notes in line with the first and third notes of the bass line. Helen believed this would ensure the part remained consonant and in time. Her desire for concordance was evident again when the pair translated the first harmony part to create a second. They considered placing it either one or two notes higher:

Ross:	Where do you want to translate it?
Mark:	Anywhere. Does it matter?
Helen:	It does. You want it on the second one otherwise the notes won’t sound right together.
Ross:	Why not?

Helen: If you play the notes next to each other on a piano they don't sound right.

With the two harmony parts in place, Helen observed that there was a "big gap" in which they did not play and where the bass part was isolated. She remedied this by overlaying an additional harmony part whose notes played in time with the last note of each bass phrase.

Whilst I demonstrated the range of drum sounds available, Mark began vocalising a bass drum and snare rhythm which he thought would fit the piece. Helen pointed out that this rhythm would require a 'gap' to remain in time. She calculated the length of the rest by counting along the increments on the time axis. The consequent rhythm part is shown in light blue in figure 36.

When contemplating adding a melody part, Mark explained he was unsure which notes would 'work'. From her experiences with the harmony parts, Helen had formed a conjecture that she now shared with her partner:

Helen: As long as the notes are somewhere near to the rest – so it doesn't sound really out of tune – then it should sound alright.

I continued by explaining that melodies often contained *suspended* notes outside the key but that these were usually followed by stepwise *resolutions*. The pair elected to reflect the opening phrase vertically. Mark's comment that the complete melody resembled a spider suggested that he was still concerned with the visual aspect of the composition. They continued with the reflection process, transforming each phrase in turn until they had filled eight bars. The completed composition can be heard on track 15 on the CD.

Chapter 9

Analysis and Conclusions

Well I guess every time I think about
geometry I'll think of music [11]

By way of conclusion, I wish to revisit some of the issues raised in this study in the light of the school trial projects and evaluations.

Music as an object-to-think-with

I began this report by recalling Seymour Papert's childhood fascination with gears. Papert gives several reasons why gears were such effective models for exploring mathematical ideas. I suggested that the first three of these pertained equally to music and considered whether the fourth might also apply: Could music, like the gears and the Logo Turtle, function as an *object-to-think-with*? The school trial of GeoMusE went some way in proving this hypothesis.

Just as the gears were part of Papert's cultural landscape, I argued that music is a ubiquitous feature of the modern child's environment. The students in the trial invariably brought their experience of composing and listening to bear on their geometric constructions. During the composition project, Helen and Mark mastered the difficult task of mediating transformations through their understanding of harmonic and rhythmic conventions. There were indications that the others were also considering the musical implications of their actions. For instance, the melody created by Hassan's slanted 'H' and Louise's use of two different chords in her 'L'. Both Helen and Louise exploited knowledge of harmony they may not have known they had. Indirectly, Susannah appeared to discover the common antecedent-consequent melody writing formula. The children who took part in Upitis' (1997) transformation composition project also discovered traditional musical 'rules'. In just a few hours, Upitis reports, the children had were articulating rules which, as a harmony student, she had only learnt through memorisation.

Children associate meaningful, *real* objects and activities with the Adult world. For Papert, the objects were gears. For other children, as has Upitis

[11] Comment made by Elizabeth during the school trial evaluation

reminded us, they could be musical performances and compositions. That Susannah regarded her work as *real* was demonstrated when she stayed past the end of school to finish her ‘star heart’, later asking for a printed copy. In Louise’s case, the work with GeoMusE may have served to open up aspects of music from which she had previously felt excluded:

Louise: Personally, I don’t really like music that much at school because I don’t really get the keyboardy [sic] stuff. If I write it out, like A-B-C then it’s all right but now we’ve had the letters rubbed off our keyboards and its just all confusing.

Her sense that there is a body of knowledge (the ‘keyboardy stuff’) which she cannot understand is compounded by an image of composition as a cerebral activity where you just ‘sit there’. The realisation that there are alternative strategies for writing music appears to have come as a relief:

Louise: I thought people had to just sit there, decide what sound you want and compose a piece of music. But you could come up with a bit you like and then put it backwards, change it around, translate it to do a low bit and a high bit… I’d like to try making different shapes… At home it would be fine to do it on your own because you could muck about with what you want.

Louise may have been exposed to the myth that musicians are in some way unreachable, ‘special people’. As Walker laments:

> It is a fact that adults in western culture either regard musical activity as something that has to be taught or consider that to make up his or her own music, a child would have to be a genius (1996: 78).

As a teacher trainer, Welch (2000) would ask each new group of PGCE students how many had played instruments as children. Most hands would be raised. Yet, when he asked how many saw themselves *as musicians*, the number dropped dramatically.

After he had located the translated melody during the ostinato activity, Mark assuredly nodded his head to each note whilst the resulting tune played back. Just as Papert (1980) found it necessary to move *like* gears in order to understand them, Mark evidently needed *kinaesthetic proof* of the translation. As Bunt reminds us: ‘Anticipation of the ensuing event is crucial in synchronising a movement with a sound' (1994: 63). Mark was not the only one. Nearly all of the students exhibited rhythmic movement at some point during the sessions. Often this took the form of nodding or tapping the table but Louise would also wave her hand as though 'conducting' the computer. Bamberger writes of a similar episode during her *Laboratory for Making Things* project (Chapter 1). Leon had typed in a string of numbers, which the computer converted into a rhythm:

> Listening was not enough; to really *grasp* the meaning of the numbers, Leon needed to echo their actions in his own actions. As the numbers on the screen played, Leon moved along with them, keeping time. In this way he was literally co-ordinating symbol with sound and action. (Bamberger, 1999: 258)

The most effective way of testing the musical object-to-think-with hypothesis was deemed to be the extent to which the students made geometric observations and decisions based on what they could *hear*. As is known from the work of Cronly-Dillon *et al* (1999; 2000), humans are capable of reconstructing non-trivial geometric figures from musical representation (figure 22). Hearing the musical result of a 315º reflection certainly helped some students explain what had happened. Hassan, who had previously struggled with sloping mirror lines during the preparation exercises (figures 25.1 and 25.2) now proposed a concise and accurate rule: "same pitch there, same time here". Susannah, who could not explain this transformation before hearing it, was also able to relate the values on one axis with the other. Comments made by some of the students indicated that they viewed the music as a form of 'auditory triangulation':

Ross:	When you were thinking about the shapes and comparing them, did the fact you could hear music help when comparing?
Hassan:	Hearing it helped because you could see if you had made an error
Ross:	You could see or you could hear?
Hassan:	Hear.
Louise:	Especially on the drum kit. You might not notice whether the line was 24 or 25 or whatever, but you could hear the Chinese cymbal isn't what you wanted and you wanted the tambourine or whatever, so you know that you've got to rethink it.

Ross:	Thinking about when we translated or reflected shapes, did the music help…
Susannah:	…Yeah, you know when I did that star shape? I found it helped because [points at screen] those two notes, those two notes and those two notes were together and the ones in the middle were

separate. That was really good because you know what the shape is because when they play together you know whether you're going in, up or down again.

Ross: The melodies which were the same both forwards and backwards, before we played them could you look at the screen and see if they…

Susannah: …were like they were played? When I did a heart shape, you could tell what line was being played and how many notes there were in it. Some of them had shorter [fewer?] notes than others did so you could tell what shape you were on.

The computer as mediator between abstract and concrete

For Hassan, Louise and Susannah, it appears music did function as an object-to-think-with. Like Bamberger (1999), I was interested in the role played by the computer in mediating this transitional relationship.

During the preparation exercises, students found the outcome of horizontal and vertical reflection much easier to predict than diagonal reflection. For Kürchemann (1981), this is because the former transformations bear a resemblance to the concrete act of folding paper. Their early hesitation was in contrast to the confidence shown after handling sloping mirror lines on the computer:

Susannah: I had problems doing the reflections on the 45º angle. I couldn't grasp that.

Ross: Would you say now that you've done it, it makes sense?

Susannah: Yeah, it makes sense.

Elizabeth: Yeah.

In a call for a 'revaluation' of concrete thinking, Turkle and Papert argue:

> The computer stands betwixt and between the world of formal systems and physical things; it has the ability to make the abstract concrete. (1991: 162)

In the present case, I would suggest that the computer was more than an electronic paper folding 'equivalent' for understanding sloped mirror lines. Through its simultaneous graphical and musical representations, students could approach the formal mathematics from different perspectives. Louise and Hassan seemed comfortable switching between these representational modes to suit their level of understanding:

Louise: I'm not that good at music but I'm all right on the maths side of it. So if you can understand one bit of it then it makes it much easier to understand the other side of it. So someone really good at music but not so good at maths, they can interpret the music side of it into the maths and others, the maths into music.
Ross: OK, so there's a kind of cross-over?
Louise: Yeah, I mean its fine if you not very good at either cause it will still help you. But if you know one side of it, it will make it a lot easier to be able to do the other side. Like some people will find music really fun but maths boring so to have it in both, they can do what they like rather than something they need.
Hassan: I think in both at the same time because I enjoy it more.

The computer can support the learner by providing instant feed back on their progress. The students found the position window particularly useful when they constructed their melodies and shapes on the grid:

Loren: That bit at the top [the position window] helped quite a lot.
Hassan: Cause you don't know what note you're on.
Loren: Yeah and it's a lot easier than looking at a whole sheet of music notes and trying to play it. You look there [the position window] and you know what note it's on and then you've got the maths stuff, with the 'y' on 44 and the line on 90.

Susannah: I think it helped by saying what position on the grid, what note and what bar you are on. That box was quite helpful.
Ross: Have you come across the term 'feedback'?
Susannah: Yes.
Ross: Would you say the computer is giving you feedback on what you are doing?
Susannah: Yes.

Hoyles and Noss (1992) describe the *Target Game* in which children use Logo to repeatedly multiply numbers to get a result as close as possible to 100. At each stage, the computer feeds back the result of the previous multiplication on the screen, allowing the learners to reassess their tactics. Hoyles and Noss contend that the computer, 'opens a range of alternatives, *strategic apertures*, through which children can gain access to approaches and solutions which are simply unavailable with pencil-and-paper' (Hoyles and Noss, 1992: 43). The potential for non-destructive experimentation offered by GeoMusE was valued by the students, as the following comment demonstrates:

Hassan: You could experiment with it. Its not like if you do it on a piece of paper and then you do it wrong so you have to rub it all out or start again, if you've done too much. You just go back and erase it if you want to or take bits out or translate it if you want it again.

Student attitudes to music and mathematics integration

Like the respondents in Bahna-James (1991) study, several students admitted to being sceptical of the connections between music and mathematics before the project commenced, fearing the work would be "weird" or "boring". Others said they had been confused or that they had not known what to expect. Elizabeth explained that when she told her brother about the trial, he had asked, "what, is it smarter you are in maths, the smarter you are in music or something?"

After the trial, all the students reported having enjoyed using GeoMusE. More significantly, all had altered their opinions on the links between music and geometry. Most mentioned the joint emphasis on shape and patterns and the consequential importance of transformations. Elizabeth and Susannah also pointed out the similarities between graphs in mathematics and stave notation. Some saw the education potential in making these connections explicit. Susannah's comment is strongly reminiscent of Kagel's and Solomon's composition techniques reviewed in Chapter 3:

Susannah: Instead of a 'making up tune' task, you could use shapes. Say if you used a five-line stave, you could draw shapes on it where you want the notes to be You could do little patterns or whatever you wanted and find the notes there.

Elizabeth: I think it's a good help with their maths or their music certainly. If they were studying for music GCSE and they did this before, it could help them.

Susannah: That would be quite good because it would help them with both maths and music even though it's only one program. They might get better marks in maths and music because they're using the same program for both pieces of work.

Several of the students said their opinions of the individual subjects had also changed. I have already referred to Louise's realisation that musicians might use strategies for composing rather than simply 'sitting there'. Susannah admitted that previously for her, “geometry was just about messing around with a load of shapes” whilst Helen now knew that “maths isn’t just about numbers”.

Implications and opportunities for further work

The results of this study support the hypothesis that music *can* function as an object-to-think-with. In particular, it highlights the educational efficacy of designing activities that integrate music with mathematics on a *conceptual* level. I would argue that a computer environment offers significant potential for students to explore and develop their understanding of these conceptual parallels, not least because the results of transformations can be evaluated in several different perceptual modes simultaneously.

During the trials described in Chapters 6 to 9, it was only possible to begin to investigate the possibilities offered by GeoMusE. In particular, there was no opportunity to introduce students to the ‘programming mode’. Future work could proceed from this point, with students designing advanced transformations based on those provided. Ultimately, this could lead to the production of complex tessellations.

Future versions of the GeoMusE software could enhance the potential for geometric modelling. Two possible developments are anticipated. Firstly, a move from two-dimensional to three-dimensional representation. This would allow pitch and time to be joined by a third musical property, most probably dynamic intensity. Phrases entered further back on the grid would be heard more quietly than those nearer the front. This would correspond to our perception of the world around us. A second avenue for development would be to adapt Logo’s Turtle commands (e.g. `forward`, `right` or `pendown`) to draw directly on the grid. This would allow phrases to be automatically generated from Logo procedures.

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Appendix 1

MSWLogo Source Code for GeoMusE

```
to .info
;	GeoMusE v1.0
;	(C) Ross Purves 2001
;	Distributed under the terms of the GNU Public License (see www.gnu.org)
;	ross@ductape.net
end

to bresenham :xP :yP :xQ :yQ
	; this is not used as it stands but forms the basis of other procedures below
	; converted and extended from the C in Ammeraal (1992: 90)

	localmake "x :xP
	localmake "y :yP
	localmake "D 0
	localmake "dx :xQ - :xP
	localmake "dy :yQ - :yP
	localmake "xinc 1
	localmake "yinc 1
	(local "c "M)

	localmake "point_list []

	if (:dx < 0) [make "xinc -1 make "dx minus :dx]
	if (:dy < 0) [make "yinc -1 make "dy minus :dy]
	ifelse (:dy < :dx) [
		while [NOT :x = :xQ] [
			make "point_list lput (list :x :y) :point_list
			make "x :x + :xinc
			make "D :D + :M
			if (:D > :dx) [make "y :y + :yinc make "D :D - :C]
		]
	] [
		make "c 2 * :dy
		make "M 2 * :dx
		while [NOT :y = :yQ] [
			make "point_list lput (list :x :y) :point_list
			make "y :y + :yinc
			make "D :D + :M
			if (:D > :dy) [make "x :x + :xinc make "D :D - :C]
		]
	]
	output :point_list
end
```

```
to calculate_grid_coords
	make "time_line_list []
	make "pitch_line_list []

	; Derive these two values now since we use them a lot
	localmake "total_time_lines ((:x_max-(2*:x_margin))/(:beat_division*10))*:beat_division
	localmake "total_pitch_lines ((:y_max-(2*:y_margin))/(:scale*10))*:scale

	; Time (x axis)
	repeat :total_time_lines+1 [
		ifelse (modulo repcount :beat_division) = 1 [
			queue "time_line_list (list 2 (g2s (list repcount-1 0))~
			  (g2s (list repcount-1 :total_pitch_lines)))
		] [
			queue "time_line_list (list 1 (g2s (list repcount-1 0))~
			  (g2s (list repcount-1 :total_pitch_lines)))
		]
	]

	; Pitch (y axis)
	repeat :total_pitch_lines+1 [
		ifelse (modulo repcount :scale) = 1 [
			queue "pitch_line_list (list 3 (g2s (list 0 repcount-1))~
			  (g2s (list :total_time_lines repcount-1)))
		] [
			queue "pitch_line_list (list 1 (g2s (list 0 repcount-1))~
			  (g2s (list :total_time_lines repcount-1)))
		]

	]
end

to cartesian2heading :cartesian
	pprop "conversion_table "90 0
	pprop "conversion_table "45 45
	pprop "conversion_table "0 90
	pprop "conversion_table "315 135
	pprop "conversion_table "270 180
	pprop "conversion_table "225 225
	pprop "conversion_table "180 270
	pprop "conversion_table "135 315
	pprop "conversion_table "360 90
```

```
        ; need the following to clean up after property list
        localmake "temp_value (gprop "conversion_table :cartesian)
        erpl [conversion_table]
        output :temp_value
end

to closetrace
        if (NOT :tracing_to = "null) [
                close :tracing_to
                print se [Closing] :tracing_to
                make "tracing_to "null
        ]
end

to config
        ; Edit these global variables to alter the GeoMusE environment:
        make "x_max 1020                        ; graphics screen width
        make "y_max 540                         ; graphics screen height
        make "x_margin 30                       ; margin between left and right of screen and grid
        make "y_margin 30                       ; margin between top and bottom of screen and grid
        make "beat_division 8                   ; number of divisions per bar (4 = 4 crochets ber bar, 8 = 8 quavers)
        make "scale 7                   ; number of notes in octave (7 for major, 12 for chromatic)
        ;this is for midi procedures
        make "offset_note 24                    ; Lowest note playable on the GeoMusE Grid
        make "bpm 120                   ; Tempo for playback
        make "scale_id 1                        ; Selects scale type (0 for chromatic, 1 for major - or add your own)

        ; The following matrix specifies the colours of the lines in the grid
        ; 1st line: general increment
        ; 2nd line: time, large increment
        ; 3rd line: pitch, large increment
        make "colour_matrix [[200 200 200]~
                                [255 160 160]~
                                [160 160 255]]  ; Pitch, large increment
        make "pcl [60 60 60]                    ; controls colour of unselected vertices points
end

to create [:name "****dummy_name****] [:temp_ch "0] [:temp_colour [100 200 100]]
        gcont
        ifelse (:name = "****dummy_name****) [
                localmake "window_output window_createdialog_create gen_phrase_name
                if (:window_output = [cancel]) [
```

```
			print [Create operation cancelled]
			rcont stop
		]
		make "name item 1 :window_output
		make "ch item 2 :window_output
		make "colour item 3 :window_output
] [
		make "ch :temp_ch ; this makes channel available inside the mouse callback
		make "colour :temp_colour ; this makes colour available inside the mouse callback
]

; Check for previous creation of identically named phrase
repeat (:phrase_index) [
		if (item 0 item repcount-1 :phrase_list) = :name [
			print se :name [is already created]
			totrace (se :name [already created. Stop])
			rcont stop
		]
]

totrace (se [creating] :name)
pu

; Since MSWLogo doesn't pass local variables to mouse call-backs
; we have to make them global and destroy them afterwards with ern

make "global_name :name
make "switch_me 0 ; put something in switch_me  so midi_monitor won't complain
make "st "false
make "v_list []
make "last []
make "hist []
make "vel 127
; velocity is manually set at maximum for now. In the future
; it may be possible to vary velocity for each event (3D?)

print (se [Use the mouse to create ] :name [on the grid - right click to finish])

; the following provides 'backspace' functionality
(keyboardon [
		if (AND (keyboardvalue = 8) (NOT emptyp :v_list)) [
			redraw_line last :hist
			make "switch_me midi_monitor last :last "on :ch
```

```
                        make "hist butlast :hist
                        make "v_list butlast :v_list
                        ifelse (NOT emptyp :v_list) [make "last first last :v_list] [make "st "false]
                ]
        ] [
                        if (keyboardvalue = 8) [
                                midi_monitor :switch_me "off :ch
                        ]
                        totrace (se [deleting event:] :last [from] :global_name)
        ])

        mouseon [
                localmake "mpos rmousepos
                if AND (NOT :mpos = :last) (NOT (memberp "null :mpos)) [
                        queue "v_list (list :mpos :vel :ch)
                        make "switch_me (midi_monitor (last :mpos) "on :ch)
                        queue "hist (plot_event :colour :mpos :last :pcl :st)
                        make "st "true
                        make "last :mpos

                ]
        ] [
                        midi_monitor :switch_me "off :ch
                        totrace (se [adding event:] :last [to] :global_name)
        ] [
                mouseoff
                ifelse (NOT emptyp :v_list) [
                        phrase_add :global_name :colour :v_list
                        print (se [You created ] :global_name)
                ] [print se :global_name [not created]]
                keyboardoff
                ern [v_list global_name switch_me ch vel last colour st hist]
                rcont stop
        ] [] [window_position_update]

end

to dec :variable [:by 1]
        make :variable (thing :variable) - :by
end

to delete [:name "****dummy_name****]
        gcont
```

```
	if (:name = "****dummy_name****) [
		if (:highlighted_phrase = "null) [
			print [No phrase is highlighted]
			rcont stop
		]
		localmake "name :highlighted_phrase
	]
	if (remove_phrase :name) = "false [
		print se :name [does not exist]
		rcont stop
	]

	print se [Deleting] :name
	totrace se [Deleting] :name
	make "highlighted_phrase "null

	cs
	plot_grid_coords
	refresh_phrases
	rcont
end

to draw_line :P :Q :colour
	; converted and extended from the C in Ammeraal (1992: 90)

	pu ; pen remains up throughout this procedure

	localmake "x (first :P)
	localmake "y (last :P)
	localmake "D 0
	localmake "dx (first :Q) - (first :P)
	localmake "dy (last :Q) - (last :P)
	localmake "xinc 1
	localmake "yinc 1
	(local "c "M)

	localmake "pixel_list []

	if (:dx < 0) [make "xinc -1 make "dx minus :dx]
	if (:dy < 0) [make "yinc -1 make "dy minus :dy]
	ifelse (:dy < :dx) [
		make "c 2 * :dx
		make "M 2 * :dy
```

```
		while [NOT :x = (first :Q)] [
			setxy :x :y
			make "pixel_list lput (list :x :y pixel) :pixel_list
			setpixel :colour
			make "x :x + :xinc
			make "D :D + :M
			if (:D > :dx) [make "y :y + :yinc make "D :D - :c]
		]
	] [
		make "c 2 * :dy
		make "M 2 * :dx
		while [NOT :y = (last :Q)] [
			setxy :x :y
			make "pixel_list lput (list :x :y pixel) :pixel_list
			setpixel :colour
			make "y :y + :yinc
			make "D :D + :M
			if (:D > :dy) [make "x :x + :xinc make "D :D - :c]
		]
	]
	output :pixel_list
end

to find_last_matching_event :point
	localmake  "result "null
	if (:phrase_index > 0) [
		for [i :phrase_index-1 0] [
			if (memberp :point (firsts item 2 item :i :phrase_list)) [
				make "result item 0 item :i :phrase_list
			]
		]
	]
	output :result
end

to g2s :grid_coords
	output list ((minus (:x_max/2)-:x_margin)+((first :grid_coords)*10)) ((minus (:y_max/2)-:y_margin)+((last
:grid_coords)*10))
end

to gcont
	windowenable "control "false
	buttonenable "play "false
```

```
end

to gen_phrase_name
	inc "gen_phrase_index
	output ( word "phrase :gen_phrase_index)
end

to get_reflection_line
	pu

	window_reflectat_create

	print [(1) Choose a point on the grid where the reflection line will pass through, right click to cancel]

	make "pivot []
	while [:pivot = []] [
		mouseon [
			if (NOT (memberp "null rmousepos)) [
				make "pivot rmousepos
				mouseoff
			]
		] [] [
			mouseoff
			windowdelete "reflect_at
			print [Operation Cancelled]
			make "pivot [cancel]
		] [] [
			window_reflectat_update rmousepos "null]
	]

	if (:pivot = [cancel]) [
		ern [pivot]
		output [cancel]
	]

	print [(2) Select the heading of the reflection line on the grid using the mouse]
	print [    left click to confirm selection, right to cancel]

	make "last_pos 999
	make "angle "
	make "line []
	setpos g2s :pivot
	while [emptyp :angle] [
```

```
		mouseon [
			if (NOT (memberp "null rmousepos)) [
				make "angle r45 towards g2s rmousepos
				mouseoff
				windowdelete "reflect_at
			]
		] [] [
			mouseoff
			windowdelete "reflect_at
			ern [pivot angle last_pos]
			make "angle [cancel]
		] [] [
			if (NOT (memberp "null rmousepos)) [
				update_reflection_line rmousepos
			]
		]
	]
	redraw_line :line
	if (:angle = [cancel]) [
		ern [line]
		output [cancel]

	]
	localmake "final_pivot :pivot    ; this is so we can get rid of the globals
	localmake "final_angle :angle
	ern [angle pivot line last_pos]

	output (list :final_pivot :final_angle)
end

to heading2cartesian :heading
	pprop "conversion_table "0 90
	pprop "conversion_table "45 45
	pprop "conversion_table "90 0
	pprop "conversion_table "135 315
	pprop "conversion_table "180 270
	pprop "conversion_table "225 225
	pprop "conversion_table "270 180
	pprop "conversion_table "315 135
	pprop "conversion_table "360 90

	; need the following to clean up after property list
	localmake "temp_value (gprop "conversion_table :heading)
```

```
	erpl [conversion_table]
	output :temp_value
end

to highlight_phrase :name :switch
	pu
	localmake "dummy []

	ifelse (:switch = "true) [
		localmake "xcolour [255 0 0]
	] [
		localmake "xcolour :pcl
	]

	foreach (last vertices.out :name) [
		ignore plot_event :xcolour (list (first ?) (last ?)) :dummy :xcolour "false
	]
end

to inc :variable [:by 1]
	make :variable (thing :variable) + :by
end

to merge

	; we need a procedure which merges phrases together so they can be treated as one
	; alternatively, we need a 'grouping' procedure which allows us to transform more than
	; phrase at a time

end

to midi2note_name :note_number
	localmake "note_name_list [C C# D Eb E F F# G G# A Bb B C C# D Eb E F F# G G# A Bb B ~
	                           C C# D Eb E F F# G G# A Bb B C C# D Eb E F F# G G# A Bb B ~
	                           C C# D Eb E F F# G G# A Bb B C C# D Eb E F F# G G# A Bb B ~
	                           C C# D Eb E F F# G G# A Bb B C C# D Eb E F F# G G# A Bb B ~
	                           C C# D Eb E F F# G G# A Bb B C C# D Eb E F F# G G# A Bb B ~
	                           C C# D Eb E F F# G]

	output item :note_number+1 :note_name_list
end

to midi_monitor :x_value :switch :channel
```

```
	ifelse (:switch = "on) [
		midimessage (list 144+:channel ((note_mapper :x_value :scale_id) + :offset_note) 127)
		output ((note_mapper :x_value :scale_id) + :offset_note)
	] [
		midimessage (list 128+:channel :x_value 1 0)
	]
end

to midi_setup
	ignore midiopen
	midimessage [255 0 0]
	repeat 16 [
		midimessage (list 191+repcount 0 0)
	]
end

to note_mapper :grid_number :scale_id

	; scale_id 0 = chromatic
	; scale_id 1 = major

	localmake "scale_matrix [ [0 2 4 5 7 9 11 12 14 16 17 19 21 23 24 26 28 29 31 33 35 36 38 40 41 43 45 47 48 50 52 53
55 57 59 60 62 64 65 67 69 71 72 74 76 77 79 81 83 84 86 88 89 91 93 95 96] ]

	ifelse (:scale_id = 0) [
		output :grid_number
	] [
		output item (:grid_number+1) item :scale_id :scale_matrix
	]
end

to opentrace :name
	openwrite :name
	make "tracing_to :name
	print se [Tracing to] :name
end

to  phrase_add :name :colour :vertices
	; construct a temporary, internal array first
	localmake "internal (array 3 0)
	setitem 0 :internal :name
	setitem 1 :internal :colour
	setitem 2 :internal :vertices
```

```
        ; now add it to the phrase_list
        setitem :phrase_index :phrase_list :internal
        inc "phrase_index
        totrace (se [Adding phrase] :name [with vertices:] :vertices)
end

to play
        gcont
        ; first indicate we are in 'play mode'
        make "play_flag "true
        totrace [now playing back]
        buttonenable "stop "true

        ; initialise array of lists that will hold event information
        ; the +2 at the end is to accommodate the last note-off message
        ; it is +1 to compensate for the first x_event possible being 0, not 1 and
        ; + another 1 because the last x_event possible is actually at the
        ; beginning of the next bar

        localmake "event_array (array ((:x_max-(2*:x_margin))/10)+2 0)

        ; now fill each element with an empty list, otherwise lput complains

        repeat ((:x_max-(2*:x_margin))/10)+2 [
                setitem repcount-1 :event_array []
        ]

        print [Now Playing...]
        print [Click on [STOP] to finish]

        repeat :phrase_index  [
                foreach (item 2 item repcount-1 :phrase_list) [
                localmake "temp_vertices (item (first first ?) :event_array)
                localmake "temp_vertices (lput (list (note_mapper (last first ?) :scale_id)+:offset_note (item 2 ?) (item 3
?)+144) :temp_vertices)
                setitem (first first ?) :event_array :temp_vertices
                localmake "temp_vertices (item (first first ?)+1 :event_array)
                localmake "temp_vertices (lput (list (note_mapper (last first ?) :scale_id)+:offset_note 0 (item 3 ?)+128)
:temp_vertices)
                setitem (first first ?)+1 :event_array remdup :temp_vertices
                ]
        ]
```

```
        ; do calculation for tempo once, not each time around the repeat loop
        localmake "wait_time (round (4/:beat_division) * (3600/:bpm))

        scrollx -200

        repeat ((:x_max-(2*:x_margin))/10)+2 [
                if NOT (emptyp (item repcount-1 :event_array)) [
                        foreach (item repcount-1 :event_array) [
                                midimessage (LIST (last ?) (first ?) (item 2 ?))
                        ]
                ]

                if :stop_flag [
                        repeat 16 [midimessage (list 175+repcount 123)]
                        print [Now Stopping...]
                        make "stop_flag "false
                        make "play_flag "false
                        setfocus [Commander]
                        rcont stop
                ]
                wait :wait_time
        ]
        make "play_flag "false
        ; reset the transport controls
        buttonenable "stop "false
        buttonenable "play "true
        rcont
end

to plot_event :colour :coordinates :last_coords :pcolour :switch

        localmake "list (plot_point first g2s :coordinates last g2s :coordinates :pcolour)

        if (:switch = "true) [
                localmake "list se (draw_line g2s :last_coords g2s :coordinates :colour) :list
        ]

        output :list
end

to plot_grid_coords
        pu
        foreach :time_line_list [
```

```
		setpencolor item (first ?) :colour_matrix
		setpos item 2 ?
		pd
		setpos last ?
		pu
	]
	foreach :pitch_line_list [
		setpencolor item (first ?) :colour_matrix
		setpos item 2 ?
		pd
		setpos last ?
		pu
	]
end

to plot_point :screen_x :screen_y :colour
	; The following applies offset coordinates to the current pos
	; to generate a 'point' shape. It is hard coded for speed

	localmake "pixel_list []

	repeat 3 [
		setxy :screen_x-1+(repcount-1) :screen_y+2
		make "pixel_list lput (list (first pos) (last pos) pixel) :pixel_list
		setpixel :colour
	]
	repeat 5 [
		setxy :screen_x-2+(repcount-1) :screen_y+1
		make "pixel_list lput (list (first pos) (last pos) pixel) :pixel_list
		setpixel :colour
	]
	repeat 5 [
		setxy :screen_x-2+(repcount-1) :screen_y
		make "pixel_list lput (list (first pos) (last pos) pixel) :pixel_list
		setpixel :colour
	]
	repeat 5 [
		setxy :screen_x-2+(repcount-1) :screen_y-1
		make "pixel_list lput (list (first pos) (last pos) pixel) :pixel_list
		setpixel :colour
	]
	repeat 3 [
		setxy :screen_x-1+(repcount-1) :screen_y-2
```

```
		make "pixel_list lput (list (first pos) (last pos) pixel) :pixel_list
		setpixel :colour
	]
	output :pixel_list
end

to r45 :input_angle

	localmake "input_angle round :input_angle
	localmake "result modulo :input_angle 45

	ifelse (:result = 0) [
		output :input_angle
	] [
		ifelse (OR (:result = 23) (:result > 23)) [
			output :input_angle + (45 - :result)
		] [
			output :input_angle - :result
		]
	]
end

to rcont
	mouseoff
	mouseon [select] [] [] [] [window_position_update]
	windowenable "control "true
	buttonenable "play "true
end

to redraw_line :pixel_list

	pu ; pen remains up through out this procedure

		repeat count :pixel_list [
			setxy (item 1 item repcount :pixel_list) (item 2 item repcount :pixel_list)
			setpixel item 3 item repcount :pixel_list

		]
end

to reflect [:in_name "****dummy_name****] [:x_origin 20] [:y_origin 20] [:angle 0] [:out_name "****dummy_name****]

	gcont
```

```
ifelse (:in_name = "****dummy_name****) [
	localmake "window_output window_reflectdialog_create
	if (:window_output = [cancel]) [
		print [Create operation cancelled]
		rcont stop
	]
	localmake "in_name first :window_output
	localmake "out_name item 2 :window_output

	if (vertices.out :in_name) = "null [rcont stop]


	localmake "reflection_vector get_reflection_line
	if (:reflection_vector = [cancel]) [
		print [Create operation cancelled]
		rcont stop
	]
	localmake "x_origin (first first :reflection_vector)
	localmake "y_origin (last first :reflection_vector)
	localmake "angle (last :reflection_vector)
	localmake "output_flag "false
] [
	if (:out_name = "****dummy_name****) [localmake "out_name gen_phrase_name]
	localmake "output_flag "true
]

localmake "temp_list []
localmake "diff_list []
localmake "temp_x 0
localmake "temp_y 0

make "angle heading2cartesian r45 :angle

totrace (se [reflecting] :in_name [by line of] :angle [through] :x_origin :y_origin [to form] :out_name)

foreach (last vertices.out :in_name) [
	queue "diff_list (list ((first ?)-:x_origin) ((last ?)-:y_origin))
]

foreach (:diff_list) [
	make "temp_x (round (cos (2*:angle)) * ((first ?))) + (round (sin (2*:angle)) * ((last ?)))
	make "temp_y (round (sin (2*:angle)) * ((first ?))) + (minus (round (cos (2*:angle))) * ((last ?)))
	queue "temp_list (list (:x_origin+:temp_x) (:y_origin+:temp_y))
```

```
	]
	vertices.in :out_name (first vertices.out :in_name) (item 2 vertices.out :in_name) :temp_list

	if (:output_flag) [output :out_name]

	rcont
end

to refresh_phrases

	pu

	localmake "status "false
	localmake "last []

	repeat :phrase_index [
		localmake "colour item 1 item repcount-1 :phrase_list
		foreach (firsts item 2 item repcount-1 :phrase_list) [
			ignore plot_event :colour ? :last :pcl :status
			make "last ?
			make "status "true
		]
		make "status "false
	]
end

to remove_phrase :name
	repeat :phrase_index [
		if ((item 0 item repcount-1 :phrase_list) = :name) [
			for [count repcount :phrase_index] [
				setitem :count-1 :phrase_list (item :count :phrase_list)
			]
			dec "phrase_index
			output "true
		]
	]
	output "false
end

to reset
	cs ht pu
	totrace [system reset]
	config
```

```
	; these are for phrase creation
	make "phrase_list (array 100 0)
	make "phrase_index 0

	calculate_grid_coords
	plot_grid_coords
	scrollx -200
	make "stop_flag "false
	make "play_flag "false

	; this is for automatic naming of phrases
	make "gen_phrase_index 0

	; this records which phrase is currently highlighted
	make "highlighted_phrase "null
	rcont
end

to rmousepos
	localmake "acuracy 5
	localmake "derived_x "null
	localmake "derived_y "null

	; first check that the mouse is on the grid
	if (OR ((first mousepos) < (minus (:x_max/2)-:x_margin))
	  ((first mousepos) > ((:x_max/2)-:x_margin))
	  ((last mousepos) < (minus (:y_max/2)-:y_margin))
	  ((last mousepos) > ((:y_max/2)-:y_margin))) [
		output (list :derived_x :derived_y)
	]
	; now do rounding
	ifelse (modulo (first mousepos) 10) > 0 [
		if (modulo (first mousepos) 10) < :acuracy [
			localmake "derived_x (((:x_max/2)-:x_margin) + (first mousepos) - (modulo (first mousepos) 10))/10

		]
		if (modulo (first mousepos) 10) > (10-:acuracy) [
			localmake "derived_x (((:x_max/2)-:x_margin) + (first mousepos) + (10-(modulo (first mousepos) 10)))/10

		]
	] [
		localmake "derived_x (((:x_max/2)-:x_margin) + (first mousepos))/10
```

```
	]
	ifelse (modulo (last mousepos) 10) > 0 [
		if (modulo (last mousepos) 10) < :acuracy [
			localmake "derived_y (((:y_max/2)-:y_margin) + (last mousepos) - (modulo (last mousepos) 10))/10

		]
		if (modulo (last mousepos) 10) > (10-:acuracy) [
			localmake "derived_y (((:y_max/2)-:y_margin) + (last mousepos) + (10-(modulo (last mousepos) 10)))/10

		]
	] [
		localmake "derived_y (((:y_max/2)-:y_margin) + (last mousepos))/10
	]

	output (list :derived_x :derived_y)
end

to select
	gcont
	localmake "phrase find_last_matching_event rmousepos
	if :phrase = :highlighted_phrase [rcont stop]
	ifelse (NOT :phrase = "null) [
		if (NOT :highlighted_phrase = "null) [
			highlight_phrase :highlighted_phrase "false
		]
		make "highlighted_phrase :phrase
		highlight_phrase :phrase "true
		rcont
		print se :phrase [is selected]
		totrace se :phrase [is selected]
	] [
		rcont stop
	]
end

to start
	ht
	windowset [MSWLogo Screen] 3
	window_control_create
	window_position_create
	window_transport_create
	midi_setup
	make "tracing_to "null
```

```
	reset
end

to totrace :data
	if (NOT :tracing_to = "null) [
		setwrite :tracing_to
		print :data
		setwrite []
	]
end

to translate [:in_name "****dummy_name****] [:by_x 0] [:by_y 0] [:out_name "****dummy_name****]
	gcont
	ifelse (:in_name = "****dummy_name****) [
		localmake "window_output window_translatedialog_create
		if (:window_output = [cancel]) [
			print [Create operation cancelled]
			rcont stop
		]

		localmake "in_name first :window_output
		localmake "out_name item 2 :window_output

		if (vertices.out :in_name) = "null [stop]

		print [Click on the grid where you want to place the translation - right click to cancel]
		localmake "translation_vector translate_mouse_pos :in_name
		if (:translation_vector = [cancel]) [
			print [Create operation cancelled]
			rcont stop
		]
		localmake "by_x (first :translation_vector) - (first first last vertices.out :in_name)
		localmake "by_y (last :translation_vector) - (last first last vertices.out :in_name)
		localmake "output_flag "false
	] [
		if (:out_name = "****dummy_name****) [localmake "out_name gen_phrase_name]
		localmake "output_flag "true
	]

	totrace (se [translating] :in_name [by] :by_x :by_y [to form] :out_name)

	localmake "temp_list []
	localmake "colour first vertices.out :in_name
```

```
        localmake "channel item 2 vertices.out :in_name
        foreach (last vertices.out :in_name) [
                queue "temp_list (list (first ?)+:by_x (last ?)+:by_y)
        ]
        vertices.in :out_name :colour :channel :temp_list
        if :output_flag [output :out_name]
        rcont
end

to translate_mouse_pos :name
        window_translateby_create
        make "global_name :name
        make "global_position []

        while [:global_position = []] [
                mouseon [
                        if (NOT memberp "null rmousepos) [
                                make "global_position rmousepos
                                mouseoff
                                windowdelete "translate_by
                        ]
                ] [] [
                        make "global_position [cancel]
                        windowdelete "translate_by
                        mouseoff
                        print [Operation Cancelled]
                ] [] [window_translateby_update (first last vertices.out :global_name)]
        ]

        ; have to do the following to ensure this procedure leaves no global variables behind
        localmake "position :global_position
        ern [global_position global_name]
        output :position
end

to update_reflection_line :coords

        ; this controls the length of the reflection line (measured in grid units)
        localmake "line_length 15

        pu
        setpos g2s :pivot
        localmake "pos (heading2cartesian r45 towards g2s :coords)
```

```
	if (AND (NOT emptyp :line) (NOT :last_pos = :pos)) [redraw_line :line]

	if (NOT :last_pos = :pos) [
		make "line draw_line g2s :pivot g2s (list (first :pivot)+(round :line_length * cos :pos) (last :pivot)+(round
:line_length * sin :pos)) [0 0 0]
		make "last_pos :pos
		window_reflectat_update :pivot cartesian2heading :pos
	]
end

to vertices.in :name :colour :channel :vertices_list
	; Check for previous creation of identically named phrase
	repeat (:phrase_index) [
		if (item 0 item repcount-1 :phrase_list) = :name [
			print se :name [is already created]
			rcont stop
		]
	]

	localmake "velocity 127
	localmake "temp_list []
	localmake "create_flag "empty
	localmake "switch "false
	localmake "last_coords []

	pu ; make a clean break between each phrase graphic

	foreach :vertices_list [
		ifelse (OR ((first ?) < 0) ((first ?) > ((:x_max-(:x_margin*2))/10))
		   ((last ?) < 0) ((last ?) > ((:y_max-(:y_margin*2))/10))) [
			print se ? [is not on the grid]
			totrace (se ? [is not on the grid])
		] [
			queue "temp_list (list ? :velocity :channel)
			ignore plot_event :colour ? :last_coords :pcl :switch
			make "create_flag "filled
			make "switch "true
			make "last_coords ?
		]
	]
	ifelse (:create_flag = "filled) [
		phrase_add :name :colour :temp_list
```

```
	] [
		print se :name [not created]
	]
end

to vertices.out :name
	localmake "result []

	repeat :phrase_index [
		if ((item 0 item repcount-1 :phrase_list) = :name) [
			make "result (list (item 1 item repcount-1 :phrase_list) (last first item 2 item repcount-1 :phrase_list)
firsts (item 2 item repcount-1 :phrase_list))
		]
	]

	ifelse (NOT emptyp :result) [
		output :result
	] [
		print se :name [has no value]
		output "null
	]
end

to voice :channel :number
	midimessage (list 192+:channel :number 0)
end

to window_control_create
	windowcreate "main "control [Phrase Transformations] 380 190 115 107 [window_control_setup]
;	mouseon [select] [] [] [] []
end

to window_control_setup
	buttoncreate "control "create_control [Create Phrase]		05 05 49  15 [create]
	buttoncreate "control "delete_control [Delete Phrase]		59 05 49  15 [delete]
	buttoncreate "control "reset_control [Erase all Phrases]	05 25 103 15 [reset]
	buttoncreate "control "translate_control [Translate Phrase]	05 55 60  15 [translate]
	buttoncreate "control "reflect_control [Reflect Phrase]		05 75 60  15 [reflect]
end

to window_createdialog_create :current_name
	(local "status "my.channel "my.colour)
	localmake  "my.new_name []
```

```
	dialogcreate "main "create_dialog [Create phrase] 100 100 82 108 [
		window_createdialog_setup
		comboboxsettext "name :current_name
	]

	ifelse (OR (:status = [cancel]) (emptyp :my.new_name)) [
		output [cancel]
	] [
			output (se first :my.new_name :my.channel :my.colour)
	]
end

to window_createdialog_setup
	staticcreate "create_dialog "name_text [Name for new phrase:] 5 2 80 8
	comboboxcreate "create_dialog "name 5 13 70 10

	staticcreate "create_dialog "channel_text [MIDI channel:] 5 31 40 20
	listboxcreate "create_dialog "channel 40 30 20 20
	repeat 16 [
		listboxaddstring "channel repcount
	]

	staticcreate "create_dialog "colour_text [Colour:] 5 58 40 20
	listboxcreate "create_dialog "colour 40 55 20 20
	repeat 16 [
		listboxaddstring "colour repcount
	]

	buttoncreate "create_dialog "cancel "cancel 5 80 30 10 [
		dialogdelete "create_dialog
		make "status [cancel]
	]

	buttoncreate "create_dialog "ok "ok 45 80 30 10 [
		make "status [ok]
		make "my.new_name comboboxgettext "name
		make "my.channel listboxgetselect "channel
		make "my.colour listboxgetselect "colour
		dialogdelete "create_dialog
	]
end
```

```
to window_position_create
	windowcreate "main "mouse [Position On Grid] 260 20 115 35 []

	staticcreate "mouse "x_name [x:]	10  3 10 10
	staticcreate "mouse "x []		20  3 10 10
	staticcreate "mouse "y_name [y:]	70  3 10 10
	staticcreate "mouse "y []			80  3 10 10
	staticcreate "mouse "bar_name [bar:]	10 13 15 10
	staticcreate "mouse "bar []			25 13 10 10
	staticcreate "mouse "div_name [div:]	35 13 18 10
	staticcreate "mouse "div []			48 13 10 10
	staticcreate "mouse "note_name [note:]		70 13 17 10
	staticcreate "mouse "note []			90 13 10 10
end

to window_position_update
	if (NOT (memberp "null rmousepos)) [
		localmake "div (modulo (first rmousepos) :beat_division)+1
		localmake "bar (((first rmousepos)-(modulo (first rmousepos) :beat_division))/:beat_division)+1

		localmake "name midi2note_name ((note_mapper (last rmousepos) :scale_id) + :offset_note)

		staticupdate "x first rmousepos
		staticupdate "y last rmousepos
		staticupdate "bar :bar
		staticupdate "div :div
		staticupdate "note :name
	]
end

to window_reflectat_create
	windowcreate "main "reflect_at [Reflection line] 100 20 105 35 []

	staticcreate "reflect_at "ref_x_name [at x:]		8  3 20 10
	staticcreate "reflect_at "ref_x []				28  3 10 10
	staticcreate "reflect_at "ref_y_name [at y:]			68  3 20 10
	staticcreate "reflect_at "ref_y []				88  3 10 10
	staticcreate "reflect_at "ref_heading_name [with heading of:]	08 13 60 10
	staticcreate "reflect_at "ref_heading []			60 13 13 10
	staticcreate "reflect_at "ref_degree_name [degrees]	73 13 30 10
end

to window_reflectat_update :origin :heading
```

```
	if (NOT (memberp "null rmousepos)) [
		staticupdate "ref_x (first :origin)
		staticupdate "ref_y (last :origin)
		if (NOT :heading = "null) [
			staticupdate "ref_heading :heading
		]
	]
end

to window_reflectdialog_create
	local "status
	localmake "my.new_name []
	localmake "my.reflect []
;	(local "my.by_x local "my.by_y)

	dialogcreate "main "reflect_dialog [Reflect phrase] 100 100 114 108 [
		window_reflectdialog_setup
	]

	if (OR (:status = [cancel]) (emptyp :my.new_name) (emptyp :my.reflect)) [
		output [cancel]
	]

	output (se :my.reflect :my.new_name)
end

to window_reflectdialog_setup
	staticcreate "reflect_dialog "reflect_text [reflect:] 5 5 60 20
	comboboxcreate "reflect_dialog "reflect 40 5 65 30
	repeat :phrase_index [
		comboboxaddstring "reflect (item 0 item repcount-1 :phrase_list)
	]
	if (NOT :highlighted_phrase = "null) [
		comboboxsettext "reflect :highlighted_phrase
	]

	staticcreate "reflect_dialog "ref_new_phrase_text [Name for reflected phrase:] 5 44 40 30
	comboboxcreate "reflect_dialog "new_name 40 50 65 10
	comboboxsettext "new_name gen_phrase_name

	buttoncreate "reflect_dialog "cancel "cancel 5 80 30 10 [
		dialogdelete "reflect_dialog
		make "status [cancel]
```

```
	]

	buttoncreate "reflect_dialog "ok "ok 45 80 30 10 [
		make "status [ok]
		make "my.reflect comboboxgettext "reflect
		make "my.new_name comboboxgettext "new_name
		dialogdelete "reflect_dialog
	]
end

to window_translateby_create
	windowcreate "main "translate_by [Translate by] 100 20 105 35 []

	staticcreate "translate_by "trans_x_name [by x:]      8  3 20 10
	staticcreate "translate_by "trans_x []               28  3 10 10
	staticcreate "translate_by "trans_y_name [by y:]     68  3 20 10
	staticcreate "translate_by "trans_y []                       88  3 10 10
	staticcreate "translate_by "trans_bar_name [bar:]    08 13 15 10
	staticcreate "translate_by "trans_bar []                     23 13 10 10
	staticcreate "translate_by "trans_div_name [div:]    33 13 18 10
	staticcreate "translate_by "trans_div []                     46 13 10 10
	staticcreate "translate_by "trans_note_name [note:]  68 13 17 10
	staticcreate "translate_by "trans_note []            88 13 10 10
end

to window_translateby_update :origin
	if (NOT (memberp "null rmousepos)) [
		localmake "div (modulo (first rmousepos) :beat_division)+1
		localmake "bar (((first rmousepos)-(modulo (first rmousepos) :beat_division))/:beat_division)+1

		localmake "name midi2note_name ((note_mapper (last rmousepos) :scale_id) + :offset_note)

		staticupdate "trans_x (first rmousepos)-(first :origin)
		staticupdate "trans_y (last rmousepos)-(last :origin)
		staticupdate "trans_bar :bar
		staticupdate "trans_div :div
		staticupdate "trans_note :name
	]
end

to window_translatedialog_create
	local "status
	localmake "my.new_name []
```

```
        localmake "my.translate []
;       (local "my.by_x "my.by_y)

        dialogcreate "main "translate_dialog [Translate phrase] 100 100 114 108 [
              window_translatedialog_setup
        ]

        if (OR (:status = [cancel]) (emptyp :my.new_name) (emptyp :my.translate)) [
              output [cancel]
        ]

        output (se :my.translate :my.new_name)
end

to window_translatedialog_setup
        staticcreate "translate_dialog "translate_text [Translate:] 5 5 60 20
        comboboxcreate "translate_dialog "translate 40 5 65 30
        repeat :phrase_index [
              comboboxaddstring "translate (item 0 item repcount-1 :phrase_list)
        ]
        if (NOT :highlighted_phrase = "null) [
              comboboxsettext "translate :highlighted_phrase
        ]

        staticcreate "translate_dialog "trans_new_phrase_text [Name for translated phrase:] 5 45 40 30
        comboboxcreate "translate_dialog "new_name 40 50 65 10
        comboboxsettext "new_name gen_phrase_name

        buttoncreate "translate_dialog "cancel "cancel 5 80 30 10 [
              dialogdelete "translate_dialog
              make "status [cancel]
        ]

;       staticcreate "translate_dialog "x_text [Translate by (x):] 100 70 40 20
;       comboboxcreate "translate_dialog "by_x 100 90 40 10
;       comboboxsettext "by_x "grid

;       staticcreate "translate_dialog "y_text [Translate by (y):] 150 70 40 20
;       comboboxcreate "translate_dialog "by_y 150 90 40 10
;       comboboxsettext "by_y "grid

        buttoncreate "translate_dialog "ok "ok 45 80 30 10 [
              make "status [ok]
```

```
            make "my.translate comboboxgettext "translate
            make "my.new_name comboboxgettext "new_name
            dialogdelete "translate_dialog
      ]

; to reinstate :by_x and :by_y, put the following two lines back in the above block of code:
;           make "my.by_x comboboxgettext "by_x
;           make "my.by_y comboboxgettext "by_y
end

to window_transport_create
      windowcreate "main "transport [Transport Controls] 380 20 115 35 []

      buttoncreate "transport "play "PLAY 10 5 40 15 [
            if NOT :play_flag [
                  buttonenable "stop "true
                  buttonenable "play "false
                  play
            ]
      ]

      buttoncreate "transport "stop "STOP 70 5 40 15 [
            if :play_flag [
                  buttonenable "stop "false
                  buttonenable "play "true
                  make "stop_flag "true
            ]
      ]
end
```

Appendix 2

Activity Sheet and Questionnaire for One-to-One Evaluation

GeoMusE One-to-One Evaluation

Some Important Points:

- This is new, experimental software. It is not yet at a publishable standard. Today's session is aimed at discovering how users respond to the basic interface, and to help locate any errors ('bugs'). This kind of session is known in software development as *beta testing*.
- Please feel free to mention *any* thoughts you have about the software. If you do not understand something, it is the software that is wrong, not you. If necessary, we can change it.

Starting Points for GeoMusE tasks:

1. Try to create a favourite melody on the grid

2. Create a short phrase and repeat it 8 times

3. Can you create turn a short phrase into a round? What about the beginning of a fugue?

4. Reflect a phrase through a line with a 90º heading. Compare the original and reflected phrases. How do they sound together?

5. What happens if you reflect a phrase by 45º?

6. Can you create a phrase that has 180º rotational symmetry? What about 90º rotational symmetry?

Appendix 3

Example GeoMusE Dribble File

Extracts from Session 3 with Louise and Hassan:

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creating phrase1
adding event: 10 25 to phrase1
adding event: 13 28 to phrase1
adding event: 14 26 to phrase1
adding event: 23 30 to phrase1
deleting event: 14 26 from phrase1
deleting event: 13 28 from phrase1
deleting event: 10 25 from phrase1
deleting event: 10 25 from phrase1
creating phrase2
adding event: 0 21 to phrase2
adding event: 16 21 to phrase2
adding event: 2 23 to phrase2
Adding phrase phrase2 with vertices: [[0 21] 127 1] [[16 21] 127 1] [[2
23] 127 1]
phrase2 is selected
Deleting phrase2
creating phrase3
adding event: to phrase3
adding event: 0 21 to phrase3
adding event: 12 21 to phrase3
adding event: 2 23 to phrase3
adding event: 10 23 to phrase3
adding event: 4 25 to phrase3
adding event: 8 25 to phrase3
adding event: 4 27 to phrase3
adding event: 8 27 to phrase3
adding event: 4 28 to phrase3
adding event: 8 28 to phrase3
Adding phrase phrase3 with vertices: [[0 21] 127 1] [[12 21] 127 1] [[2
23] 127 1] [[10 23] 127 1] [[4 25] 127 1] [[8 25] 127 1] [[4 27] 127 1]
[[8 27] 127 1] [[4 28] 127 1] [[8 28] 127 1]
now playing back
phrase3 is selected
reflecting phrase3 by line of 90 through 12 18 to form phrase4
Adding phrase phrase4 with vertices: [[24 21] 127 1] [[12 21] 127 1] [[22
23] 127 1] [[14 23] 127 1] [[20 25] 127 1] [[16 25] 127 1] [[20 27] 127
1] [[16 27] 127 1] [[20 28] 127 1] [[16 28] 127 1]
now playing back
system reset
creating phrase1
Adding phrase phrase1 with vertices: [[8 21] 127 1]
system reset
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<CUT>

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phrase2 is selected
translating phrase2 by 37 10 to form phrase4
Adding phrase phrase4 with vertices: [[53 31] 127 1] [[49 24] 127 1] [[53
24] 127 1] [[55 27] 127 1] [[57 27] 127 1] [[55 24] 127 1] [[59 24] 127
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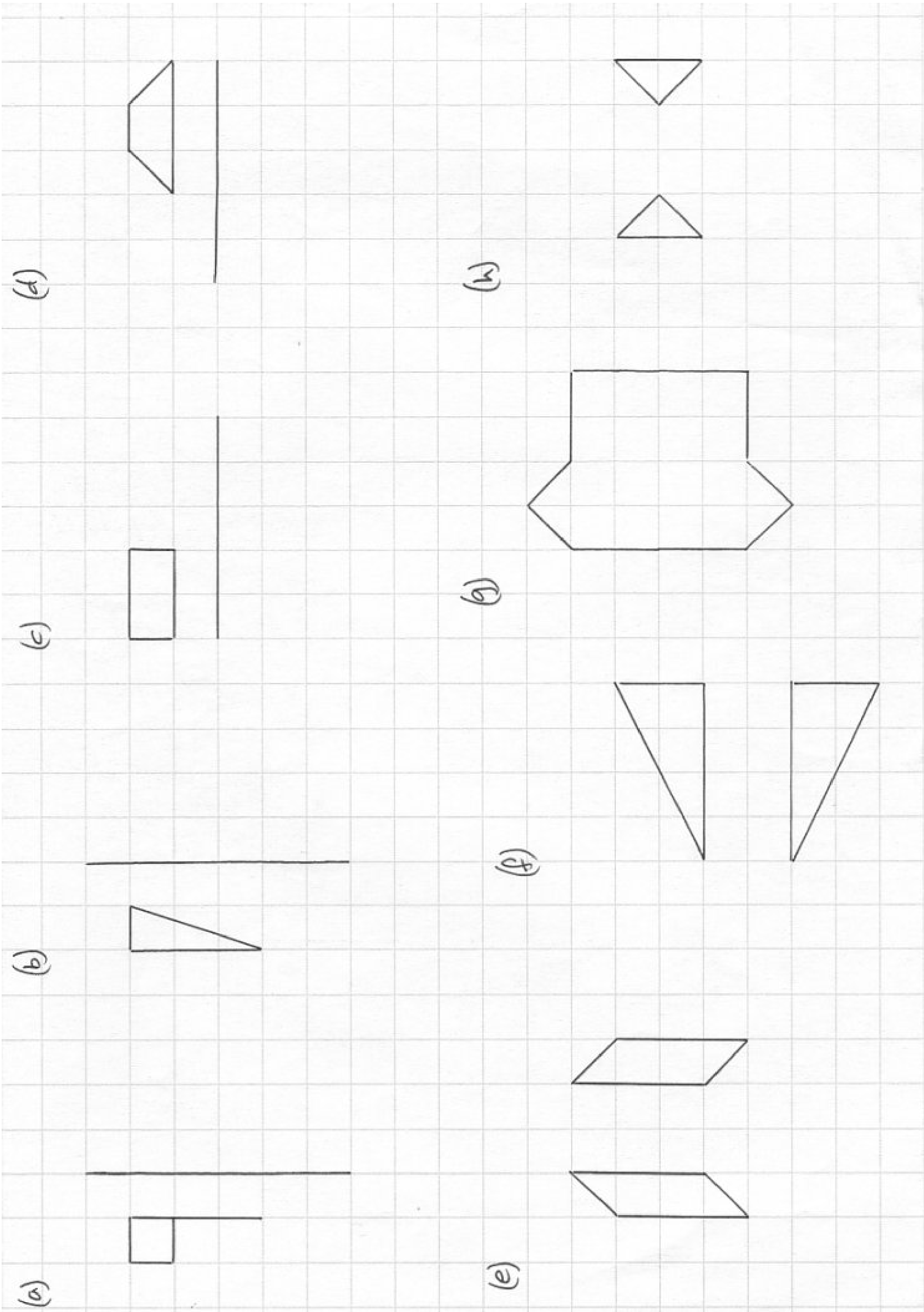
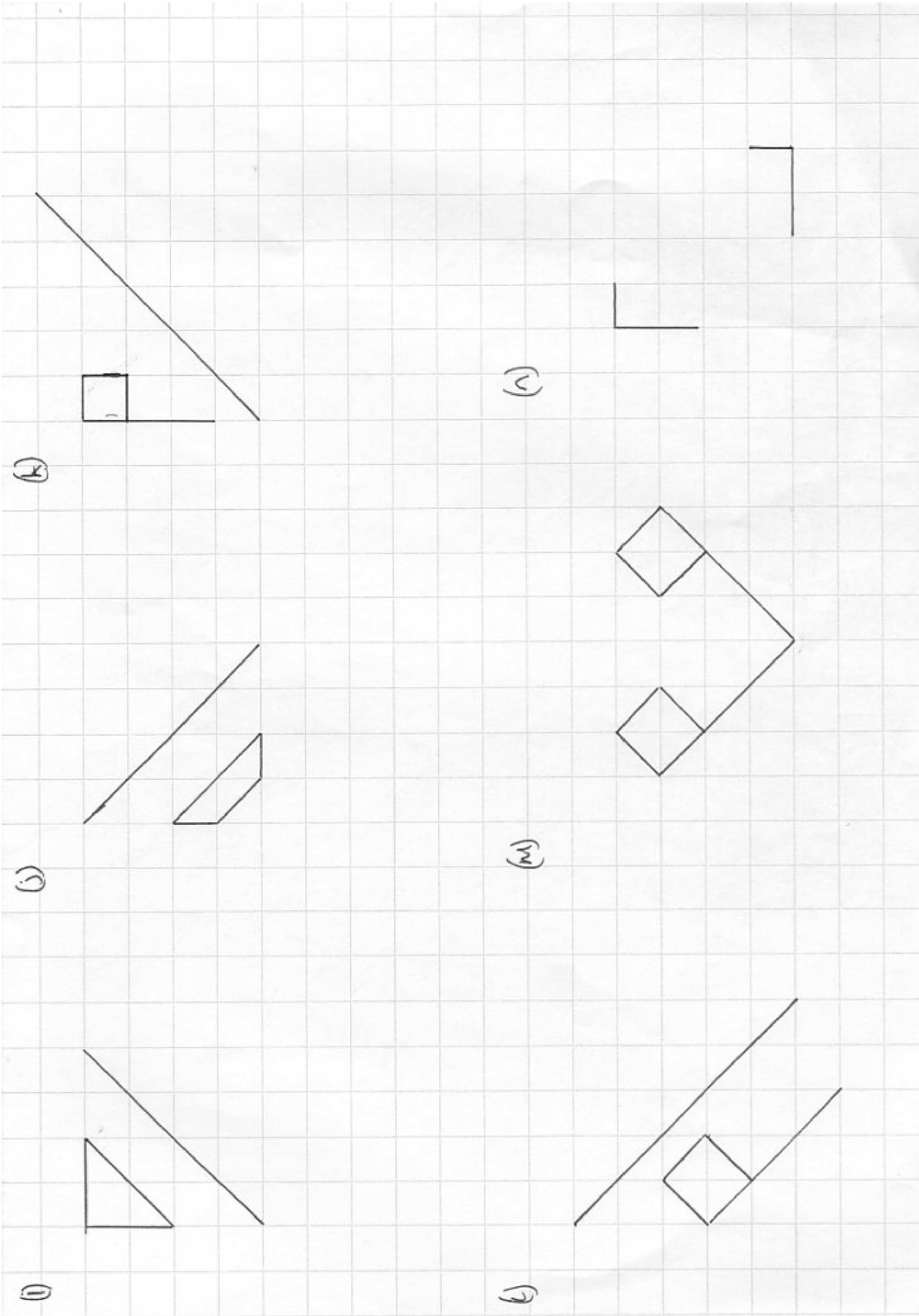
1] [[62 31] 127 1] [[58 31] 127 1] [[56 28] 127 1] [[58 28] 127 1] [[58
31] 127 1]
Deleting phrase2
now playing back
phrase4 is selected
reflecting phrase4 by line of 45 through 44 27 to form phrase5
Adding phrase phrase5 with vertices: [[48 36] 127 1] [[41 32] 127 1] [[41
36] 127 1] [[44 38] 127 1] [[44 40] 127 1] [[41 38] 127 1] [[41 42] 127
1] [[48 45] 127 1] [[48 41] 127 1] [[45 39] 127 1] [[45 41] 127 1] [[48
41] 127 1]
now playing back
phrase1 is selected
reflecting phrase1 by line of 315 through 0 24 to form phrase6
-11 20 is not on the grid
-9 20 is not on the grid
-7 20 is not on the grid
-5 20 is not on the grid
-3 20 is not on the grid
-1 20 is not on the grid
-1 18 is not on the grid
-1 16 is not on the grid
-1 14 is not on the grid
-1 12 is not on the grid
-4 12 is not on the grid
-4 14 is not on the grid
-4 16 is not on the grid
-6 16 is not on the grid
-8 16 is not on the grid
-11 16 is not on the grid
-11 18 is not on the grid
creating phrase7
adding event: 16 25 to phrase7
deleting event: 16 25 from phrase7
reflecting phrase1 by line of 225 through 16 25 to form phrase8
Adding phrase phrase8 with vertices: [[26 13] 127 1] [[24 13] 127 1] [[22
13] 127 1] [[20 13] 127 1] [[18 13] 127 1] [[16 13] 127 1] [[16 15] 127
1] [[16 17] 127 1] [[16 19] 127 1] [[16 21] 127 1] [[19 21] 127 1] [[19
19] 127 1] [[19 17] 127 1] [[21 17] 127 1] [[23 17] 127 1] [[26 17] 127
1] [[26 15] 127 1]
now playing back
phrase5 is selected
reflecting phrase5 by line of 90 through 44 31 to form phrase9
Adding phrase phrase9 with vertices: [[40 36] 127 1] [[47 32] 127 1] [[47
36] 127 1] [[44 38] 127 1] [[44 40] 127 1] [[47 38] 127 1] [[47 42] 127
1] [[40 45] 127 1] [[40 41] 127 1] [[43 39] 127 1] [[43 41] 127 1] [[40
41] 127 1]
now playing back
phrase8 is selected
reflecting phrase8 by line of 0 through 12 16 to form phrase10
Adding phrase phrase10 with vertices: [[26 19] 127 1] [[24 19] 127 1]
[[22 19] 127 1] [[20 19] 127 1] [[18 19] 127 1] [[16 19] 127 1] [[16 17]
127 1] [[16 15] 127 1] [[16 13] 127 1] [[16 11] 127 1] [[19 11] 127 1]
[[19 13] 127 1] [[19 15] 127 1] [[21 15] 127 1] [[23 15] 127 1] [[26 15]
127 1] [[26 17] 127 1]
now playing back
```

Appendix 4

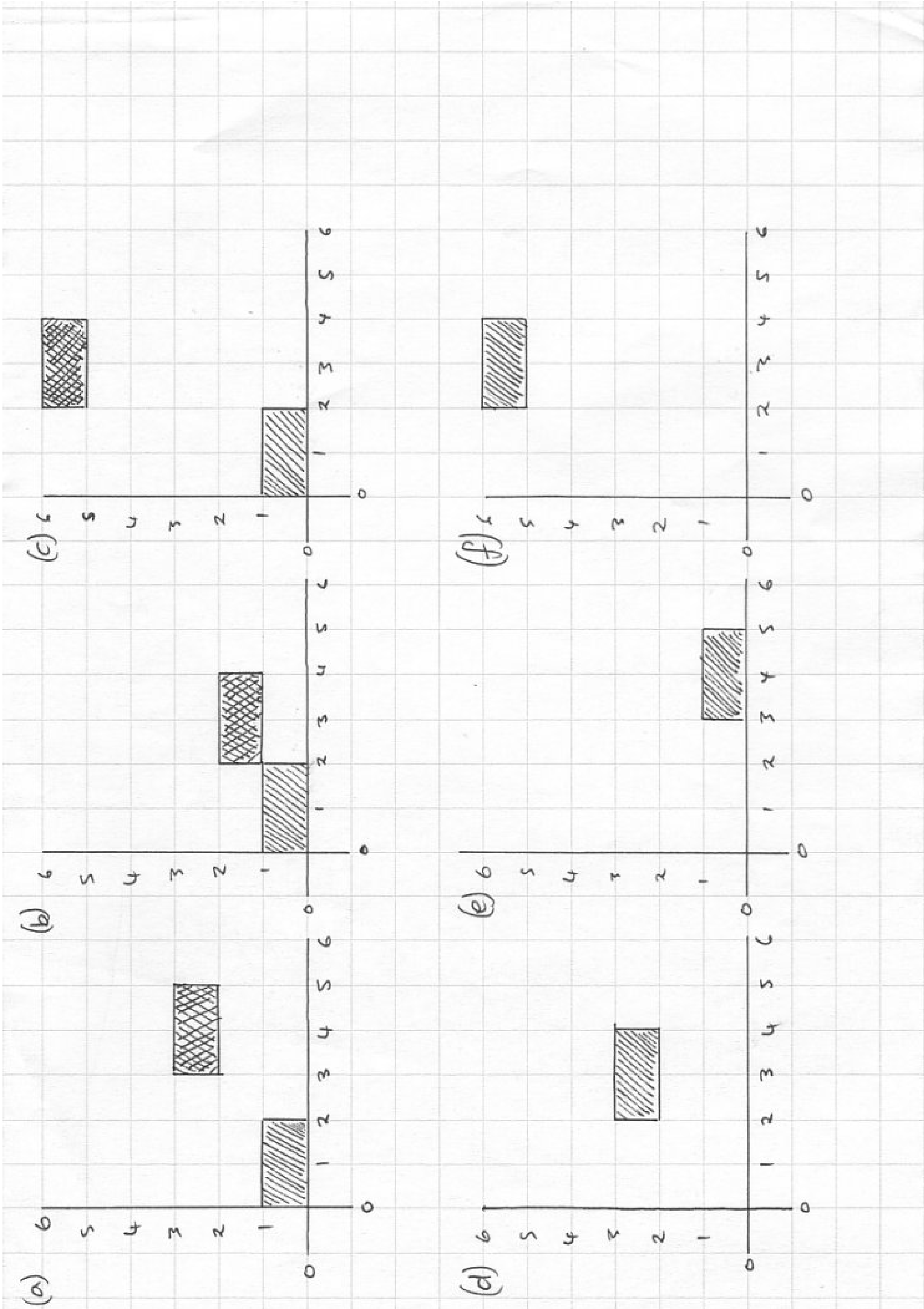
Preparation Worksheets



1. Reflection



2. Reflection



3. Translation

Appendix 5

Pilot Study Evaluation Questions

Environmental factors:
- Could you see the screen properly?
- Were you able to use the mouse and keyboard comfortably?

Preparation:
- Did the preparation work help with the activities on the computer?
- How did it help?
- Could any of the preparation have been clearer?
- Was there enough information to be able to use the program properly?
- Could the information have been clearer?

Activities:
- Where you able to do the activities?
- Which activities were the hardest?
- Was anything difficult to follow? How could it have been clearer?
- Were the activities interesting, motivating, and involving? Which ones?
- Which were not as interesting?
- How did you find working in pairs on the computer? Might it have been better another way?

Program:
- How did you find using the software, what did you think of the way it looked?
- Did the way it was set out help, or not help, your understanding of the mathematical and musical ideas?
- Did the software work as you might have expected? If not, was this because it was confusing, worked in a strange way, or because it did not work?
- How might the software be made better? Please mention *anything*.
- What could be added or changed?

Learning:
- Which activities helped you learn the most about mathematics and about music?
- What did you think about the connections between mathematics and music before doing this work?
- Has your opinion changed?
- Based on the work we have done, what do you think the connections might be?
- Has this work changed the way you think about geometry?
- Has it changed the way you think about composing music?
- Did the music help when comparing different shapes on the grid? How?

Finally:
- Given the chance would you like to do more of this kind of work in the future?
- How does this work compare with your regular mathematics and music lessons?
- Have you enjoyed it?
- Is there anything else you would like to say?