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Stretching Hollow Jets in Potential Flow

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Abstract. Why model hollow stretching jets? As well as offering some interesting new mathematics, the dynamics of such jets are of potential interest in the consideration of methods of creating circular penetration cuts into targets, in understanding better the coherency of solid shaped charge jets, and possibly as a means of engendering cracking in targets over a wider target area than can be accomplished with a solid shaped charge jet. This paper presents and solves numerically the boundary-value problem of a stretching hollow jet in potential flow. The behavior of the hollow cylindrical jet is analyzed as a function of the initial inner and outer radii, the rate of stretching, and the initial radial velocity component associated with the inner radius. It is shown that under different initial conditions the hollow can close up or expand. The evolution of the associated pressure field is determined.

INTRODUCTION

How do the characteristics of elongating hollow jets differ from solid jets? This question is of importance in the study of the stretching jet created by a shaped charge. Shaped charges are widely used for military applications and for oil recovery. They usually produce solid jets which stretch and become thinner. Much work on modelling such solid jets has been done [1] — far less on hollow jets. This is because, in general, hollowness is not a desirable characteristic for a shaped charge jet. For one example, links between hollow jets and incoherency (radial dispersion of jet) have been postulated [2]. Hollowness is likely in general to degrade the penetrative capability [3]. But understanding hollowness effects may nonetheless offer other benefits, *e.g.* by making greater damaged zones in some targets or by making circular cuts in targets.

To gain such understanding of the dynamics of a stretching hollow jet, it was logical to consider first the simplest constitutive model, namely potential flow as in earlier work by the authors on instability [4] and bi-material jets [5]. The adoption of potential flow implicitly implies incompressibility. Following again the philosophy of working initially with the simplest possible scenario, it was decided to model a stretching uniform tubular jet, making the assumptions that the uniformity is maintained as the jet stretches and that the axial velocity component increases linearly with distance along the jet. These assumptions are entirely in accord with previous work on the stability of shaped charge jets [4, 6-10].

In the next section the mathematical model for a stretching hollow jet in potential flow is derived, assuming that the pressure is so small as to be effectively zero on the curved inner and outer surfaces. The analysis results in three ordinary differential equations in time to be solved simultaneously. The numerical solution scheme used to solve these equations is described and applied for input parameters of values typical in the shaped charge regime. Predictions are made for a variety of initial conditions. A correspondingly varied range of responses of the jet is seen. The results are discussed, conclusions are drawn and finally a range of potential future research is outlined.

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MATHEMATICAL MODEL

The boundary-value problem for the hollow jet in potential flow is posed in and on the surfaces of the liquid part of density ρ occupying

$$a(t) \le r \le b(t); \quad 0 \le z \le L(t) \tag{1}$$

Here $a(t)$, $b(t)$,and $L(t)$ are respectively the inner and outer radii and the length of the stretching tubular jet, which are all time-dependent and r and z are radial and axial cylindrical polar co-ordinates. In this axisymmetric region the potential $\phi = \phi(r,z,t)$ satisfies Laplace's equation expressed in the cylindrical polars:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{\partial^2 \phi}{\partial z^2} = 0\,. \tag{2}$$

The radial and axial velocity components in cylindrical polars are respectively:

$$u = \frac{\partial \phi}{\partial r}, \quad w = \frac{\partial \phi}{\partial z}\,. \tag{3}$$

Equation (2) in fact expresses the incompressibility of the fluid. The boundary conditions we consider are as follows:

$$u = \dot{a}(t) \qquad \text{on } r = a(t), \tag{4}$$

$$u = \dot{b}(t) \qquad \text{on } r = b(t), \tag{5}$$

$$w = 0 \qquad \text{on } z = 0, \tag{6}$$

$$w = V(t) \qquad \text{on } z = L(t), \tag{7}$$

$$p = 0 \text{ on } r = a(t), \text{ and on } r = b(t). \tag{8}$$

Here and later a superscript dot denotes differentiation with respect to time. The pressure boundary conditions (8) are believed to be a good approximation in the shaped charge regime, where pressures due to inertia greatly exceed atmospheric pressure, at least soon after the formation of the jet. The boundary conditions (6) and (7) imply in general

$$\dot{L}(t) = V(t), \tag{9}$$

and in particular for constant $V(t) = V$, that

$$L(t) = L_0 + Vt, \tag{10}$$

where $L_0 = L(0)$.

One further assumption is made, specifically that the axial velocity component is linear in z, *i.e.*

$$w = \frac{Vz}{L(t)}. \tag{11}$$

This is an assumption that has widely been made in undertaking analytical studies of shaped charge jet instability and is known to be a good approximation for shaped charges based on radiographic evidence. Henceforth the explicit time dependence of *L(t)* will not be shown. Note boundary conditions (6) and (7) are automatically satisfied by Eq. (11).

On the basis of assumption (11) it may be shown that the general solution for the potential is

$$\phi = \frac{V}{4L}\left(2z^2 - r^2\right) + q(t)\ln r + f(t). \tag{12}$$

Here the functions *q(t)* and *f(t)* are as yet arbitrary functions of time. From Eq. (3) the radial velocity component is

$$u = -\frac{Vr}{2L} + \frac{q}{r}. \tag{13}$$

Application of conditions (4) and (5) yields

$$\dot{a} = -\frac{Va}{2L} + \frac{q}{a} \tag{14}$$

and

$$\dot{b} = -\frac{Vb}{2L} + \frac{q}{b}. \tag{15}$$

The momentum equations are

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial r} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial r}, \tag{16}$$

$$\rho\frac{\partial w}{\partial t} + \rho u\frac{\partial w}{\partial r} + \rho w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z}. \tag{17}$$

Equations (11), (13) and the boundary conditions (4), (5) and (8) are applied with Eqs. (16) and (17) to obtain, after some manipulation, the following two equations for p and q:

$$\dot{q} = \frac{\left(b^2 - a^2\right)}{2\ln\left(\frac{b}{a}\right)}\left(\frac{q^2}{a^2b^2} - \frac{3V^2}{4L^2}\right), \tag{18}$$

$$p = p(r,t) = \frac{\rho\left(r^2 - a^2\right)}{2}\left(\frac{q^2}{a^2r^2} - \frac{3V^2}{4L^2}\right) + \frac{\rho\left(b^2 - a^2\right)\ln\left(\frac{a}{r}\right)}{2\ln\left(\frac{b}{a}\right)}\left(\frac{q^2}{a^2b^2} - \frac{3V^2}{4L^2}\right). \tag{19}$$

Equations (14), (15) and (18) provide three ordinary differential equations in the three variables a, b, q. However, it is sufficient to solve just the two Eqs. (14) and (18), eliminating b by means of the condition that the volume of the jet remains constant because of the incompressibility:

$$\pi L\left(b^2 - a^2\right) = \pi L_0\left(b_0^2 - a_0^2\right), \tag{20}$$

where subscript 0 denotes the initial value at $t=0$. The solution of (14) and (18) may be done by use of a fourth-order Runge-Kutta method. The initial value for q, namely $q(0)=q_0$, is most easily and understandably set by prescribing the initial value of $\dot{a}$, *i.e.* $\dot{a}(0)$ and using Eq. (14) to calculate q_0.

SIMULATION OF A HOLLOW SHAPED CHARGE JET

Parameter values which are representative of a copper shaped charge jet have been used as presented in Table 1. Tip speeds of 8-10 km/s and tail speeds of 1-2 km/s are typical for such jets. The tip and tail speeds remain constant and the initial values of the inner and outer radii are constant along the jet. Only the initial radial velocity component is varied in the simulations; the values investigated to date are shown at the foot of Table 1.

TABLE 1. Typical parameter values for a stretching hollow shaped charge jet.

Parameter and Units	Symbol	Values
Initial Length (m)	L_0	0.05
Initial Inner Radius (m)	a_0	0.005
Initial Outer Radius (m)	b_0	0.01
Density of Jet Material (kg/m^3)	ρ	8900.0
Tip Speed of Jet (m/s)	V	6000.0
Initial Inner Surface Radial Speed (m/s)	$\dot{a}(0)$	0; 300; 400

Figure 1 shows the predictions of the change with time of the inner and outer radii for some selected values of $\dot{a}(0)$ studied. Other values have been run with consistent monotonic behavior – the results for only three values are shown in the figure for clarity. For values of approximately 400 m/s and above the radii both increase radially. For values between 300 and 400 m/s there is limited expansion or contraction of the hollow – the lower the value the more contraction, the higher the more expansion. For values below 300 m/s the inner radius reduces until the simulation fails as $a(t)\mapsto 0$, owing to the reciprocal and logarithmic terms evident in Eqs. (14) and (18). The value 0 m/s is a prime example as shown in Fig. 1.

For cases where the radii increase and the hollow grows, the maximum value of the pressure decreases rapidly with time. For cases where the radius *a* decreases to zero, the maximum pressure is seen initially to decrease with time before rising again to reach values of the same order as at the commencement of the simulation. The example of this behavior for $\dot{a}(0)=0$ is shown in Figure 2, which depicts the evolution of the radial pressure distribution across the jet as given by Eq. (19).

DISCUSSION

In answer to the question about how hollow jet behavior differs from solid jet behavior, the following features have emerged. Hollowness brings the need to prescribe one of the initial normal velocity components at the free surfaces, with an associated diversity of the subsequent jet dynamic response. It has been shown that, in the shaped charge regime, hollow jets either expand or contract radially depending on the value of the initial radial velocity at the inner surface. Higher values lead to radial outward expansion and diminishing pressure. Lower values lead to closing of the hollow, with initial decrease in pressure with time followed by a later rise as hollow closes. The simulations fail as complete closure approaches because of the singular nature of the solution on the axis of symmetry, the model being predicated on the existence of the hollow.

The model is of potential relevance to considerations of jet coherency, but additional considerations of strength and compressibility effects would be needed to address that topic properly. It is intended to investigate these extensions.

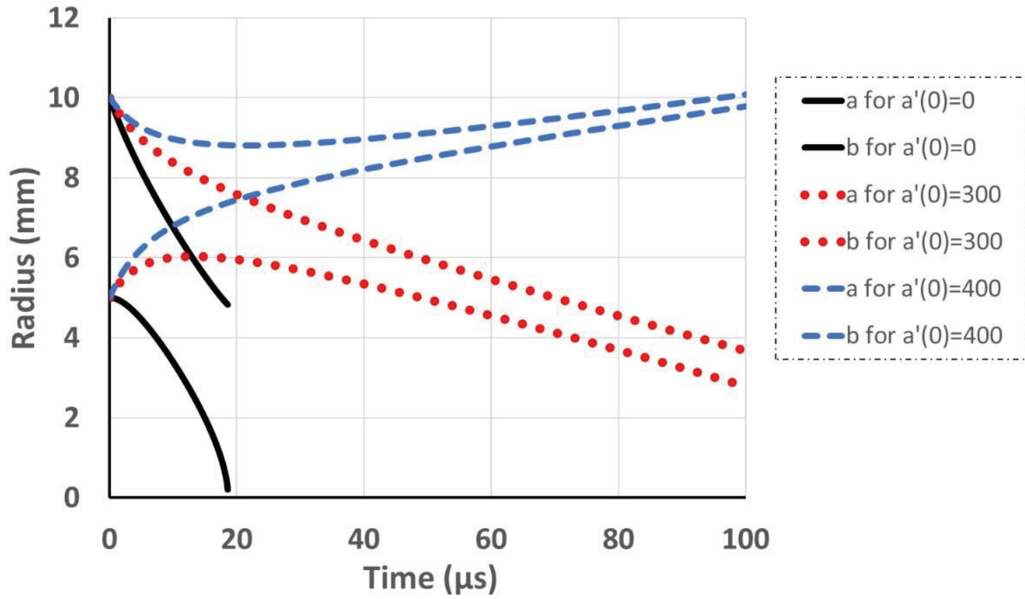

FIGURE 1. Evolution of inner and outer radii with time for different values of $\dot{a}(0)$.

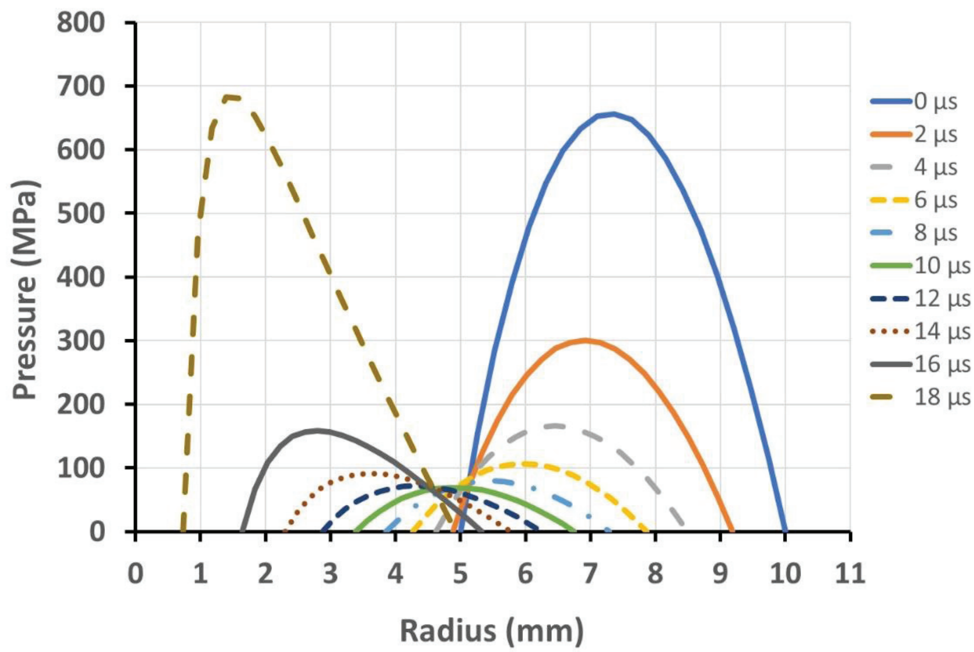

FIGURE 2. Evolution of pressure distribution across jet with time for $\dot{a}(0) = 0$.

The model is as yet limited to the treatment of ideal hollow cylindrical jets with uniform radii and initial conditions – real ones would have varying internal and external radii and initial radial surface speeds. There is likely to be scope to consider such variations using asymptotic methods and again it is planned to explore the modelling of these more general situations. Other promising avenues for future research include the treatments of viscosity, compressibility, and associated stability studies.

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