Managing Intrinsic Motivation in a Long-Run Relationship*

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Abstract

We study a repeated principal-agent interaction, in which the principal offers a "spot" wage contract at every period, and the agent's outside option follows a Markov process with *i.i.d* shocks. If the agent rejects an offer, the two parties are permanently separated. At any period during the relationship, the agent is productive as long as his wage does not fall below a "reference point", which is defined as his lagged-expected wage in that period. We characterize the game's unique Markov perfect equilibrium. The equilibrium path exhibits an aspect of wage rigidity. The agent's total discounted rent is equal to the maximal shock value.

1 Introduction

The standard principal-agent model is built on the premise that the agent needs to be incentivized in order to exert effort on a task. This requires the principal to condition the agent's wage on a verifiable signal of his effort. However, in many environments such information is either unavailable or very imprecise, which forces the principal to rely on the agent's "intrinsic motivation". For instance, think of a parent hiring a nanny, or a hospital employing a surgeon.

Intrinsic motivation is a *dynamic* property - an agent who is initially motivated may temporarily *lose* his motivation in the course of his relationship with the principal. In addition, numerous studies in the literature - notably, Akerlof (1982), Akerlof and Yellen (1990), Bewley (1999), Fehr et al. (2009) - have argued that intrinsic motivation is *reference-dependent*. An agent may become demotivated when his compensation falls

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below his expectations. This means that temporal variations in the agent's compensation that reflect changes in the external environment can adversely affect the agent's motivation. Hence, in situations with limited contractual instruments, the principal is faced with the problem of optimally managing the agent's motivation: trading-off the cost and benefit of keeping the agent motivated.

This paper studies a simple dynamic principal-agent model that explores this trade-off. The principal makes a "spot" wage offer at every period, and the agent decides whether to accept it. Once the agent rejects an offer, the two parties are permanently separated and the agent receives an outside payment θ_t at every t, where θ_t evolves according to some Markov process. The agent's output is reference-dependent, dropping from its normal level to zero whenever his wage drops below his reference wage e_t by more than $\lambda > 0$.

Inspired by Kőszegi and Rabin (2006), we assume that the agent's reference wage is equal to the "rational" expectation of his wage at period t (conditional on continued employment), calculated at the end of period $t-1$ according to the parties' continuation strategies. The expectational aspect of the reference point captures the idea that a wage is treated as a disappointment or as a pleasant surprise, depending on how it compares with the agent's former expectations. The lagged-expectations aspect captures the idea that like habits, reference points are sluggish in adapting to new circumstances.[1]

Our task is to characterize Markov perfect equilibria in this game, where the state at period t is (θ_t, θ_{t-1}). To illustrate the possible effects of reference dependence, consider first the case of perfectly myopic parties. The agent's participation wage at period t is θ_t. Assume that θ_t can take two values, $\underline{\theta}$ and $\overline{\theta}$, with equal probability (independently of the history), where $\underline{\theta} < \overline{\theta} < 1$. Suppose that in equilibrium the parties' relationship is not severed at t for any realization of θ_t. Let $w(\theta)$ denote the equilibrium wage when $\theta_t = \theta$. Then, $e_t = \frac{1}{2}w(\underline{\theta}) + \frac{1}{2}w(\overline{\theta})$. If the principal paid the agent his reference wage in equilibrium, we would have $e = \frac{1}{2}\underline{\theta} + \frac{1}{2}\overline{\theta} > \underline{\theta}$. If λ is small, the agent will produce zero output when $\theta_t = \underline{\theta}$. Therefore, it would be profitable for the principal to deviate to $w_t = e$ in the state $\underline{\theta}$. In fact, the only wage strategy that is consistent with equilibrium in the $\lambda \to 0$ limit is $w(\underline{\theta}) = w(\overline{\theta}) = \overline{\theta}$.

The equilibrium strategy in this example has two noteworthy features: (*i*) *wage rigidity* - the wage is invariant to the fluctuations in the agent's outside option; (*ii*) *efficiency wages* - the principal pays the agent a wage above the reservation level in order to ensure high output. The example thus naturally links the two phenomena

[1] For earlier models in which an agent's productivity depends directly on his beliefs, see Compte and Postlewaite (2004) and Fang and Moscarini (2005).

together.

When parties are not myopic, the efficiency-wage effect means that the agent expects to earn rents in the future, and this lowers his current reservation point. Since this wage in turn determines the equilibrium reference wage, finding the equilibrium wage strategy requires us to find a *fixed point of a coupled pair of functional equations*: the dynamic reservation-wage equation after every history, and the equation that defines the reference wage after every history. From a technical point of view, this novel fixed-point problem constitutes the paper's core. The unique solution to this problem extends the wage-rigidity effect of the myopic example: the equilibrium wage at any period is not responsive to the current shock, and the agent's discounted rent is the same as in a one-period model.

This note follows up Eliaz and Spiegler (2013), which essentially embedded an elaborate version of the myopic case in a search-matching model of the labor market.[2] The technical challenge in Eliaz and Spiegler (2013) arose from the possibility of rematching. Here we abstract from this complication and focus on the pure principal-agent relationship and the new considerations that arise from its infinite horizon. Re-incorporating it in a larger model of the labor market is a challenge for future research.

2 A Model

Two players, referred to as a principal and an agent, play a discrete time, infinite-horizon game with perfect information. At the beginning of every period $t = 1, 2, ...$, the principal makes a wage offer $w_t \in \mathbb{R}$. If the agent rejects the offer, the relationship is terminated, and the agent (principal) collects a payoff of θ_s (0) at every period $s \geq t$. We assume that $\theta_t = \Psi(\theta_{t-1}) + \varepsilon_t$, where Ψ is a deterministic function and ε_t is *i.i.d* according to a *cdf* F with mean zero. Let $\bar{\varepsilon}$ denote the highest value that ε_t can take. We assume that Ψ and F are such that θ_t always takes values in $(0, 1)$.[3]

If the agent accepts the offer at period t, he collects a payoff w_t, and the principal's payoff is $y_t = \mathbf{1}(w_t \geq e_t - \lambda) - w_t$, where $\lambda > 0$ and e_t is the agent's *reference point* at period t. We refer to $\mathbf{1}(w_t \geq e_t - \lambda)$ as the agent's *output* in period t. The parameter λ captures the tolerance of the agent's intrinsic motivation to frustrated wage expectations.[4] However, our analysis will focus on the $\lambda \to 0$ limit. Both parties

[2] Effective myopia arose from a short horizon of the employment relation, rather than from a zero discount factor.

[3] E.g., $\Psi(\theta) = \alpha\theta + (1-\alpha)\frac{1}{2}$ and $F \sim U[-\bar{\varepsilon}, \bar{\varepsilon}]$, where $\bar{\varepsilon} \in (0, \frac{1}{2}(1-\alpha))$.

[4] Eliaz and Spiegler (2013) assume a stochastic, multiplicative version of reference-dependent output.

maximize discounted expected payoffs, with a discount factor $\delta \in [0, 1)$.

For every period t in which the agent is employed, let h_t denote the history of realized wages, the principal's payoff and the outside option up to and including period t, i.e. $h_t = (w_s, y_s, \theta_s)_{s=1}^{t}$. The history is commonly observed by both players. However, the agent's output is unverifiable, which is why the principal cannot condition the agent's wage on his output. A strategy for the principal is a function w that specifies a wage offer for every history h_{t-1} and realized outside option θ_t. A strategy for the agent is a function a that specifies for every (h_{t-1}, θ_t) and wage offer w_t a binary decision: "accept" $(a = 1)$ or "reject" $(a = 0)$.

To complete the description of the game, we need to specify how e_t is formed. Inspired by Kőszegi and Rabin (2006), we assume that it is equal to the agent's lagged-expected wage at period t. More precisely, consider a history at the end of period $t-1$ (i.e., before θ_t is realized), and fix the parties' continuation strategies from period t onwards. Then, e_t is the expectation of w_t, calculated according to these continuation strategies at the end of the period-$(t-1)$ history, conditional on the event that the agent accepts the principal's offer at period t (if this is a null event, we set $e_t = 0$). Thus, e_t - and therefore the principal's payoff at period t - is a function of the expectations that players hold at the end of period $t - 1$. In equilibrium, these expectations will be correct. Given a strategy pair (w, a), we let e denote the function that assigns for every history h_{t-1} a reference wage for period t.

Since the principal's payoff is defined in terms of the players' beliefs, this is not strictly speaking a conventional extensive game, but an extensive *psychological game* in the sense of Geanakopolos, Pearce and Stachetti (1989). However, since the belief-dependence is straightforward, we can work with the usual and familiar Subgame Perfect Equilibrium concept, which can be defined in terms of the usual single-deviation property: in equilibrium, each player's action at every history maximizes his discounted expected payoffs, given the continuation strategies of both players.

For simplicity, we restrict attention to SPE that are Markovian, where the state in period t is (θ_{t-1}, θ_t). Thus, a Markov Perfect Equilibrium (MPE) is a triple (w, a, e) that satisfies the following properties for every (θ_{t-1}, θ_t). First, given (w, a, e), the wage $w(\theta_{t-1}, \theta_t)$ maximizes the principal's discounted sum of expected payoffs. Second, for every wage offer w_t, the decision $a(\theta_{t-1}, \theta_t, w_t)$ maximizes the agent's discounted sum of expected payoffs. Third, given the principal's strategy w and the agent's strategy a, the reference function e satisfies

$$e(\theta_{t-1}) = \mathbb{E}[w(\theta_{t-1}, \theta_t) \mid \theta_{t-1} \ ; \ a(\theta_{t-1}, \theta_t, w(\theta_{t-1}, \theta_t)) = 1]$$

and $e(\theta_{t-1}) = 0$ if the event $\{\theta_{t-1}, \theta_t \mid a(\theta_{t-1}, \theta_t, w(\theta_{t-1}, \theta_t)) = 1\}$ is null for the given θ_{t-1}.

3 Analysis

Let us first consider a reference-independent benchmark model, in which the agent's output is always 1, independently of the history. (In other words, set $\lambda = \infty$.)

Claim 1 *Let $\lambda = \infty$. Then, there is a unique MPE: the agent's accepts any $w_t \geq \theta_t$ at every period t, and the principal offers $w_t = \theta_t$ at every t, independently of the history.*

This is a standard result due to the principal having all the bargaining power. Therefore, the proof is omitted. The equilibrium wage is entirely flexible and the agent earns no rent in equilibrium.

We now provide a characterization of MPE in the $\lambda \to 0$ limit, where the agent becomes unproductive whenever the actual wage falls below his reference point, however slightly.

Theorem 1 *There exists a unique MPE in the $\lambda \to 0$ limit. At every period t:*

(i) The principal offers $w_t = \Psi(\theta_{t-1}) + (1 - \delta)\bar{\varepsilon}$. This wage is equal to the agent's reference wage e_t.

(ii) The agent accepts any $w_t \geq \theta_t - \delta\bar{\varepsilon}$.

Proof. Let us begin with a few preliminary definitions and observations. Throughout the proof, we use h_{t-1} to denote a history $(\theta_s, w_s)_{s=1,\ldots,t-1}$, where θ_s is the realized outside option in period s and w_s is the wage offer that the principal made in period s, such that the agent accepted all wage offers up to period $t - 1$. Let (h_{t-1}, θ_t) denote the immediate concatenation of h_{t-1}, right after θ_t is realized. With slight abuse of notation, we use $F(\theta_{t+1} \mid \theta_t)$ to denote the *cdf* over θ_{t+1} conditional on θ_t. Denote the agent's reference point at period t following the history h_{t-1} by $e(h_{t-1})$.

Fix some MPE. The agent necessarily follows a cutoff strategy: If after some history he accepts some wage w, then he would also accept any higher wage because this has no effect on the future behavior of the players in an MPE. Hence, for every (θ_{t-1}, θ_t), we can define the lowest accepted wage $\bar{w}(\theta_{t-1}, \theta_t)$. If the agent rejects an offer at t, his continuation payoff is $B(\theta_t) = \mathbb{E}\left[\sum_{s \geq t} \delta^{s-t}\theta_s \mid \theta_t\right]$. Recall that by assumption, $\theta_t < 1$. Therefore, the agent will strictly prefer to accept every $w_t \in (\theta_t, 1)$, because accepting w_t and rejecting the principal's offer at $t+1$ will give him a higher payoff. Thus, $\bar{w}(\theta_{t-1}, \theta_t) \leq \theta_t$ for every θ_t.

Note that in MPE, the principal's payoff at (h_{t-1}, θ_t) is purely a function of (θ_{t-1}, θ_t). Our first step is to show that for every (θ_{t-1}, θ_t),

$$w_t(\theta_{t-1}, \theta_t) = \max\{\bar{w}(\theta_{t-1}, \theta_t), e(\theta_{t-1}) - \lambda\}$$

To show this, suppose that $w_t(\theta_{t-1}, \theta_t) > \bar{w}(\theta_{t-1}, \theta_t)$ and $w_t(\theta_{t-1}, \theta_t) \neq e(\theta_{t-1}) - \lambda$ after some history (θ_{t-1}, θ_t). The principal's continuation payoff at period $t+1$ is independent of w_t, conditional on the event that the agent accepts it. If $w_t(\theta_{t-1}, \theta_t) > e(\theta_{t-1}) - \lambda$, then by the definition of $\bar{w}$, there exists a wage $\max\{\bar{w}(\theta_{t-1}, \theta_t), e(\theta_{t-1}) - \lambda\} < w < w_t(\theta_{t-1}, \theta_t)$, such that if the principal deviates to w, the agent will accept this offer and his output at t will not be affected. If $w_t(\theta_{t-1}, \theta_t) < e(\theta_{t-1}) - \lambda$, then if the principal deviated to a wage $w \in (\bar{w}(\theta_{t-1}, \theta_t), w_t(\theta_{t-1}, \theta_t))$, the agent would accept this offer and his output at t would not be affected. In both of these cases the principal's deviation will have no implication for the principal's continuation payoff. Therefore, the deviation in both cases is profitable. By the same reasoning, it must be the case that the worker would accept $\bar{w}(\theta_{t-1}, \theta_t)$. It follows that if $w_t(\theta_{t-1}, \theta_t) \geq \bar{w}(\theta_{t-1}, \theta_t)$, then $w_t(\theta_{t-1}, \theta_t) \in \{\bar{w}(\theta_{t-1}, \theta_t), e(\theta_{t-1}) - \lambda\}$. By the definition of $e(\theta_{t-1})$ and the result that $\bar{w}(\theta_{t-1}, \theta_t) \leq \theta_t < 1$, it follows that $e(\theta_{t-1}) < 1$. Therefore, the principal will always choose $w_t(\theta_{t-1}, \theta_t) = \max\{\bar{w}(\theta_{t-1}, \theta_t), e(\theta_{t-1}) - \lambda\}$ after every (θ_{t-1}, θ_t), because this maximizes his period t payoff, without affecting his continuation payoff.

Let us now derive a formula for e in the $\lambda \to 0$ limit. By the previous paragraph and the definition of the reference wage:

$$e(\theta_{t-1}) = \int_{\theta_t} \max\{\bar{w}(\theta_{t-1}, \theta_t), e(\theta_{t-1}) - \lambda\} dF(\theta_t \mid \theta_{t-1})$$

In the $\lambda \to 0$ limit, the solution to this equation is

$$e(\theta_{t-1}) = \max_{\theta_t \mid \theta_{t-1}} \bar{w}(\theta_{t-1}, \theta_t)$$

Thus, the principal pays $w_t = e(\theta_{t-1})$ after every (θ_{t-1}, θ_t), and by the definition of $\bar{w}(\theta_{t-1}, \theta_t)$, the agent always accepts this offer. The agent's participation wage

$\bar{w}(\theta_{t-1},\theta_t)$ is the wage that makes him indifferent between accepting and rejecting an offer following θ_t:

$$\bar{w}(\theta_{t-1},\theta_t)+\mathbb{E}\left[\left(\sum_{s>t}\delta^{s-t}\max_{\theta_s|\theta_{s-1}}\bar{w}(\theta_{s-1},\theta_s)\right)\mid\theta_t\right]=B(\theta_t)$$

This can be rewritten as

$$\begin{aligned}\bar{w}(\theta_{t-1},\theta_t) &= B(\theta_t)-\delta\max_{\theta_{t+1}|\theta_t}\bar{w}(\theta_t,\theta_{t+1})-\delta\mathbb{E}\left[\left(\sum_{s>t+1}\delta^{s-t}\max_{\theta_s|\theta_{s-1}}\bar{w}(\theta_{s-1},\theta_s)\right)\mid\theta_t\right]\\ &= B(\theta_t)-\delta\max_{\theta_{t+1}|\theta_t}\bar{w}(\theta_t,\theta_{t+1})-\delta\left[\mathbb{E}B(\theta_{t+1})-\int_{\theta_{t+1}}\bar{w}(\theta_t,\theta_{t+1})dF(\theta_{t+1}\mid\theta_t)\right]\end{aligned}$$

which is simplified into the recursive functional equation

$$\bar{w}(\theta_{t-1},\theta_t)=\theta_t-\delta\left[\max_{\theta_{t+1}|\theta_t}\bar{w}(\theta_t,\theta_{t+1})-\int_{\theta_{t+1}}\bar{w}(\theta_t,\theta_{t+1})dF(\theta_{t+1}\mid\theta_t)\right]$$

or

$$\bar{w}(\theta_{t-1},\theta_t)=\theta_t+\delta\mathbb{E}\left[\bar{w}(\theta_t,\theta_{t+1})\mid\theta_t\right]-\delta\max_{\theta_{t+1}|\theta_t}\bar{w}(\theta_t,\theta_{t+1})$$

We claim that this functional equation has a unique solution. To show this, let W be the set of all possible MPE functions $\bar{w}$. These are functions that associate a real number with every (θ_{t-1},θ_t). The reservation wage is equal to the outside option plus the discounted sum of future rents. Therefore, its value at every history is bounded by some finite number (as the maximal rent that the principal would pay at any period is less than 1).

For every function $\bar{w}\in W$, define

$$q(\bar{w})\equiv\max_{\theta_t}[\max_{\theta_{t+1}|\theta_t}(\bar{w}(\theta_t,\theta_{t+1}))-\mathbb{E}(\bar{w}(\theta_t,\theta_{t+1})\mid\theta_t)]$$

This is the maximal gap between the maximal and expected participation wage at any period $t+1$ given θ_t, according to the agent's strategy. For any pair $\bar{w},\bar{v}\in W$, define

$$d(\bar{w},\bar{v})\equiv|q(\bar{w})-q(\bar{v})|+\max_{\theta_t,\theta_{t+1}}|\bar{w}(\theta_t,\theta_{t+1})-\bar{v}(\theta_t,\theta_{t+1})|$$

It is straightforward to verify that d is a metric. Hence, (W,d) is a complete metric space.[5]

[5]Note that we are using a non-standard metric. Standard techniques that rely on the sup metric would not establish that $\bar{w}$ is a contraction for $\delta>\frac{1}{2}$.

Let $H(w)$ be a self-map on W defined by the R.H.S. of the final expression for $\bar{w}$. This self-map is a contraction in (W, d). To see this, note that for any pair $\bar{w}, \bar{v} \in W$,

$$q(H(\bar{w})) = q(H(\bar{v})) = \max_{\theta_t}[\max(\theta_{t+1} \mid \theta_t) - \mathbb{E}(\theta_{t+1} \mid \theta_t)]$$

and

$$\max_{\theta_t, \theta_{t+1}} |H(\bar{w}) - H(\bar{v})| = \delta |q(\bar{w}) - q(\bar{v})|$$

It follows that

$$\begin{aligned} d(H(\bar{w}), H(\bar{v})) &= |q(H(\bar{w})) - q(H(\bar{v}))| + \delta |q(\bar{w}) - q(\bar{v})| \\ &= \delta |q(\bar{w}) - q(\bar{v})| \\ &< \delta \left[|q(\bar{w}) - q(\bar{v})| + \max_{\theta_t, \theta_t + 1} |\bar{w}(\theta_t, \theta_{t+1}) - \bar{v}(\theta_t, \theta_{t+1})| \right] \\ &= \delta d(\bar{w}, \bar{v}) \end{aligned}$$

Thus, for any $\delta \in (0,1)$, $d(H(\bar{w}), H(\bar{v})) < \delta d(\bar{w}, \bar{v})$, implying that H is a contraction. Therefore, by the Banach Fixed Point Theorem, there exists a unique fixed point $\bar{w} = H(\bar{w})$, which means that the functional equation for $\bar{w}$ has a unique solution. It is easy to verify that the definitions of e and w as in the statement of the theorem constitute a solution. Therefore, this must be the unique solution. ∎

The unique MPE has several noteworthy properties. First, the agent's acceptance rule is Markovian w.r.t θ_t, whereas the principal's behavioral rule is Markovian w.r.t θ_{t-1}. Second, the agent is always paid his reference wage in equilibrium and therefore he always produces an output of 1 along the equilibrium path. Third, the equilibrium wage is sluggish, in the sense that it is totally unresponsive to the current shock ε_t. The wage at t is a weighted average of the expected and maximal values of θ_t conditional on θ_{t-1}, where the weight on the latter is $1 - \delta$. Fourth, observe that if $\bar{\varepsilon}$ is sufficiently large and δ is sufficiently close to one, the agent's participation wage can take negative values. However, his actual equilibrium wage is of course strictly positive.

Finally, the agent earns an expected discounted rent of $\bar{\varepsilon}$, namely the difference between the maximal and expected values of ε. As F is subjected to a mean preserving spread, $\bar{\varepsilon}$ weakly increases, and thus the agent's rent goes up. The rent is independent of the discount factor: a higher δ simply means greater smoothing of the rent over time. Our model thus establishes a link between two phenomena: wage rigidity and efficiency wages, and it links them to the fundamentals $\delta, \bar{\varepsilon}$.

Comment: The role of the assumption that $\lambda \to 0$

The assumption that $\lambda > 0$ is crucial for equilibrium uniqueness. If $\lambda = 0$, it is possible to sustain equilibria in which the principal pays $w_t = e_t$, where e_t can take any value below 1 and above the highest participation wage that is possible given h_{t-1}. In this case, the agent's wage (lagged) expectations are self-sustaining: the principal does not wish to cut the wage below e_t because this would result in loss of output.

If λ were bounded away from 0, the equilibrium wage path would change as follows. First, the reference wage e_t would be strictly below the maximal participation wage that is possible given h_{t-1}. As a result, the wage at t would cease to be purely a function of θ_{t-1}: it would coincide with e_t at relatively low realizations of ε_t but it would coincide with the (higher) participation wage at relatively high realizations of ε_t. Second, the agent's equilibrium rent would be lower than in the $\lambda \to 0$ limit. Since our main objective in this note is to characterize the maximal rent that a reference-dependent agent can get in his long-run relationship with the principal, we do not provide a detailed characterization of this more general case.

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