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On the Need for Fiscal Constraints in a Monetary Union*

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ABSTRACT

We show that the desirability of fiscal constraints in monetary unions depends critically on the extent of commitment of the monetary authority. If the monetary authority can commit to its policies, fiscal constraints can only impose costs. If the monetary authority cannot commit, there is a free-rider problem in fiscal policy, and fiscal constraints may be desirable.

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Over the last 10 years, interest in the design of monetary unions has grown. Whether to impose constraints on the fiscal policies of the member states is a central issue in designing such unions. Our purpose is to address this issue using standard economic models with benevolent policy makers. We find that the desirability of imposing constraints depends critically on the extent to which the monetary authority can commit to its future policies. If the monetary authority can commit, then the formation of the monetary union, by itself, implies that there are no gains from imposing constraints. If the monetary authority cannot commit, then there are gains from imposing fiscal constraints.

The intuition for our results is as follows. Without commitment, a benevolent monetary authority finds it optimal to set high inflation rates when the inherited debt levels of the member states are large. The costs of this higher inflation are borne by the residents of all the member states. When a fiscal authority in a member state decides how much debt to issue, it recognizes the incentives of the monetary authority in the future to partially monetize its debt, but it ignores the costs this induced inflation imposes on other member states. This *free-rider problem* leads to inefficient outcomes in the sense that each fiscal authority issues too much debt. All the member states can be made better off if constraints are imposed on the amount of debt that each fiscal authority can issue. With commitment by the monetary authority, this free-rider problem disappears, and there are no gains from imposing constraints on the debt.

Interest in the rules governing monetary unions has been spurred, in part, by the rules laid out in the Maastricht treaty governing the formation of the European Monetary Union. From our perspective, we see two readings of the portions of the Maastricht treaty dealing with the European Central Bank. One reading is that the primacy of the goal of price stability and the independence of the Central Bank emphasized in the treaty ensure complete commitment to future monetary policy. In this reading, debt constraints can only be harmful. (See Buiter et al. 1993 for a forceful argument that debt constraints are harmful.) Our analysis with commitment supports this view. An alternative reading is that, notwithstanding the solemn expressions of intent in the treaty, as a practical matter, monetary policy is set sequentially by majority rule. As such, nothing in the treaty ensures effective commitment to monetary policy. In such a reading, our analysis shows that debt constraints are desirable. Our analysis is consistent with the view that the framers of the treaty thought that it is extremely difficult to commit to monetary policy and therefore wisely included debt constraints as an integral part of the treaty.

An extensive literature has discussed the gains from international cooperation in setting fiscal policy. This literature shows that cooperation is desirable if a country's fiscal policy affects world prices and real interest rates. (See Chari and Kehoe 1990 and Canzoneri and Diba 1991 for details.) The kind of desirable cooperation that this literature points to applies as well to Germany and Canada as it does to Germany and Italy in that it is not especially related to countries being in a monetary union. Because the issues raised in this literature are well understood, we abstract from them here. We do so by considering models in which the fiscal policies of the cooperating countries taken as a group do not affect world prices and real interest rates. In such models, there can be no gains from cooperation of this sort.

There is also an extensive literature on the role of monetary unions as devices to achieve cooperative monetary policy. We focus on the rules in a monetary union which lead noncooperative fiscal authorities to achieve good outcomes in terms of setting their fiscal policies. In order to maintain this focus, we have simply assumed that monetary policy is set in a cooperative fashion. We have also abstracted from a variety of issues associated with forming monetary unions, such as the reduction in transaction costs involved in having a single currency.

We begin with a simple reduced-form two-period model. In the first period, fiscal authorities issue nominal debt to outside risk-neutral lenders. In the second period, the monetary authority decides the common inflation rate. We assume that second-period output is a decreasing function of the inflation rate, so the monetary authority balances the benefits of devaluing nominal debt against the costs of lowering output. The larger the debt the monetary authority inherits, the higher it sets the inflation rate. This effect implies that when a fiscal authority issues more debt, the outside lenders demand a higher nominal interest rate. Thus, when one of the fiscal authorities issues more debt, it makes the other fiscal authority worse off. This free-rider problem implies that there is too much debt in a noncooperative equilibrium relative to debt in a cooperative equilibrium. We go on to show that if the monetary authority commits to its policies at the start of the first period, there is no free-rider problem.

A union can adopt a number of institutional arrangements to solve the free-rider problem. One arrangement is to impose constraints on the amount of debt any country can issue as a condition for joining the monetary union. Another is to impose a tax on new debt issues, payable to the union. A third is to impose various penalties, such as the removal of subsidies, for failing to behave cooperatively. In practice, the main difficulty in sustaining these arrangements is that they require some degree of commitment by the union. For example, if one of the countries happens to violate its debt constraints, the union may find it optimal ex post to forgive this transgression. Anticipation of such forgiveness would make the constraints vacuous.

We then consider a general equilibrium cash-in-advance model with the timing assumptions as in Svensson (1985). Under these timing assumptions, households are constrained to using cash held over from the previous period for current consumption. An increase in the money supply leads to an increase in the price level and reduces current consumption, because of the cash-in-advance constraint. An increase in the price level also decreases the value of the outstanding nominal government debt. Devaluing this debt makes households better off in our model because we assume that at the margin, government expenditures are financed in a distorting fashion. The monetary authority trades off the benefits of devaluing the debt against the costs of lowering current consumption. Thus, as in the reduced-form model, a higher inherited debt leads to a rise in inflation. When a fiscal authority issues more debt, it leads to higher future inflation and, thus, higher current nominal interest rates which, in turn, imposes costs on the other fiscal authority. Hence, the free-rider problem in the general equilibrium model is similar to that in the reduced-form model, and it can be solved by imposing appropriate debt constraints. As in the reduced-form model, if the monetary authority can commit to its policies, there is no free-rider problem.

An issue worth raising in this context is whether our analysis is applicable to federal systems like the United States. In our view, it is not applicable because we analyze situations in which fiscal policy is decentralized. In the United States, a large federal government has a variety of means to discipline rogue states and achieve good outcomes. In a monetary union without a large federal government, some means need to be devised to discourage member states from deviant behavior. Debt constraints are one such means.

1. A simple example

We use a simple reduced-form example that is useful for building intuition. Consider a two-period model with two identical countries that are small in the world economy. In period 0, the countries start with an identical price level p_0, which is given. Each country issues nominal debt in period 0 to lenders who live outside these two countries. These lenders are risk neutral and have discount factors β. In period 1, the countries form a monetary union in which a common monetary authority determines monetary policy. We model monetary policy as the choice of the price level in period 1, p_1. Period 0 output in both countries is a constant given by ω, while period 1 output y is a decreasing and convex function of the common inflation rate from period 0 to period 1, denoted by $\pi = p_1/p_0$.

We begin by setting up the budget constraints and objective functions. We denote country 1 allocations without an asterisk and country 2 allocations with an asterisk. The budget constraints of the government in country 1 are

$$p_0 c_0 = \omega + qb$$

and

$$p_1 c_1 = p_1 y - b$$

where b is nominal debt sold to foreign lenders at price q and c_0 and c_1 denote consumption of the citizens of country 1 in the two periods. The objective function of the country 1 government is

$$U(c_0) + \beta U(c_1).$$

The model starts with p_0 given, so it is convenient to set $p_0 = 1$. It is also convenient to let the repayment rate $r = 1/\pi$ denote the inverse of the inflation rate. Output is then an increasing and concave function of r, denoted $y(r)$. The period 1 budget constraint is then

$$c_1 = y(r) - rb.$$

Notice that r is the fraction of nominal debt that is repaid. We will assume that ω is sufficiently smaller than $y(1)$ so that the governments have an incentive to borrow. The government of country 2 has similar budget constraints and objective functions. The monetary authority's objective function is the sum of the objective functions of the two governments.

The timing of the model is as follows. In period 0, the two governments choose their debt levels (b, b^*). Then the price of debt q is determined. In period 1, the monetary authority chooses the common repayment rate r. We consider two regimes: a noncooperative regime in which the governments simultaneously choose their debt levels to maximize their own objective functions and a cooperative regime in which the governments choose their debt levels to maximize the sum the objective functions. This timing reflects the idea that two countries recognize that they will form the monetary union but that they cannot commit to the policies the union will follow. Specifically, the monetary authority takes (b, b^*) as given and then chooses the repayment rate optimally. When choosing their debt levels, the two governments recognize the effect of their choices on future inflation by influencing the actions of the monetary authority.

In both regimes, we solve the model by starting at the end. The problems of the monetary authority and the lenders are the same in both regimes. Taking b and b^* as given, the monetary authority chooses r to solve

$$(1) \qquad \max_r U(y(r) - rb) + U(y(r) - rb^*).$$

Let $r(b, b^*)$ denote the resulting repayment function.

Consider next the foreign lenders. Since they are risk neutral and have discount factors β, the debt price function is given by

$$(2) \qquad q(b, b^*) = \beta r(b, b^*).$$

In the noncooperative regime, the government of country 1, taking b^* as given, solves the following problem:

$$(3) \qquad \max_b U(\omega + q(b, b^*)b) + \beta U(y(r(b, b^*)) - r(b, b^*)b).$$

The government of country 2 solves an analogous problem. In the cooperative regime, b and b^* are chosen to solve the following problem:

$$(4) \qquad \max_{b,b^*} \left[U(\omega + q(b, b^*)b) + \beta U(y(r(b, b^*)) - r(b, b^*)b)\right] +$$
$$\left[U(\omega + q(b, b^*)b^*) + \beta U(y(r(b, b^*)) - r(b, b^*)b^*)\right].$$

A *noncooperative equilibrium* is a repayment function r that solves (1), a debt price function q that solves (2), and a pair of debt levels (b_N, b_N^*) that solves (3) and its analog.

A *cooperative equilibrium* is a repayment function r that solves (1), a debt price function q that solves (2), and a pair of debt levels (b_C, b_C^*) that solves (4).

When we compare the two regimes, we find it convenient to assume that

$$(5) \qquad y = \bar{y} - r^{-\sigma}/\sigma \text{ with } \sigma > 0.$$

We restrict consideration to symmetric equilibrium in which the debt levels are the same in the two countries. We then have the following proposition:

Proposition 1. Under assumption (5), the symmetric noncooperative debt level b_N is greater than the symmetric cooperative debt level b_C. Moreover, inflation is higher and welfare is lower in the noncooperative regime than in the cooperative regime.

Proof: The first-order condition for the monetary authority is

$$U'(c_1)(y_r - b) + U'(y^* - rb^*)(y_r - b^*) = 0.$$

Differentiating this first-order condition with respect to b gives

$$(6) \qquad (\partial r/\partial b) = \frac{U'(c_1) + rU''(c_1)(y_r - b)}{U'(c_1)y_{rr} + U''(c_1)(y_r - b)^2 + U'(c_1^*)y_{rr} + U''(c_1^*)(y_r - b^*)^2}.$$

In a symmetric equilibrium, the monetary authority's first-order condition implies that

$$(7) \qquad y_r = b.$$

Hence, in a symmetric equilibrium, (6) reduces to

$$(8) \qquad (\partial r/\partial b) = \frac{1}{2y_{rr}}.$$

Differentiating the equilibrium condition for the lenders, $q = \beta r$, gives

$$(9) \qquad \frac{\partial q}{\partial b} = \beta \frac{\partial r}{\partial b}.$$

The first-order condition for debt in the noncooperative equilibrium is

$$(10) \qquad [U'(c_0)q - \beta r U'(c_1)] + U'(c_0)b\frac{\partial q}{\partial b} + \beta U'(c_2)(y_r - b)\frac{\partial r}{\partial b} = 0.$$

Substituting $q = \beta r$, together with (7)–(9) and using symmetry, we can reduce (10) to

$$r(U'(c_0) - U'(c_1)) + U'(c_0)\frac{y_r}{2y_{rr}} = 0$$

which, when we use (5), we can rewrite as

$$\text{(11)} \quad \left[1 - \frac{1}{2}\frac{1}{1+\sigma}\right] = \frac{U'(y - ry_r)}{U'(\omega + \beta r y_r)}.$$

The first-order condition for debt in the cooperative equilibrium is

$$\text{(12)} \quad \begin{aligned} &[U'(c_0)q - \beta r U'(c_1)] \; + \; [U'(c_0)b + U'(c_0^*)b^*]\,\frac{\partial q}{\partial b} \\ &+ \beta\,[U'(c_2)(y_r - b) + U'(c_2^*)(y_r - b^*)]\,\frac{\partial r}{\partial b} = 0. \end{aligned}$$

Substituting $q = \beta r$, together with (7)–(9), and using symmetry, we can reduce (12) to

$$r(U'(c_0) - U'(c_1)) + U'(c_0)\frac{y_r}{y_{rr}} = 0$$

which we can rewrite as

$$\text{(13)} \quad \left[1 - \frac{1}{1+\sigma}\right] = \frac{U'(y - ry_r)}{U'(\omega + \beta r y_r)}.$$

From (5), it follows that $(y - ry_r)$ is increasing in r and $(\omega + \beta r y_r)$ is decreasing in r, so the right sides of (11) and (13) are decreasing in r. Since the left side of (11) is greater than the left side of (13), it follows that $r_N < r_C$. Since $y_r = b$ and y is concave, it follows that $b_N > b_C$. Q.E.D.

The following corollary is immediate.

Corollary: If the two countries in the noncooperative regime face the constraints $b \leq b_C$ and $b^* \leq b_C$, they will achieve the cooperative outcome.

Thus far, we have assumed that it is not possible to commit in period 0 to the common inflation rate between period 0 and period 1. This lack of commitment is crucial for our result that debt constraints improve welfare. To see this, consider a change in the timing of our model in which the repayment rate is chosen first, then the two governments choose debt levels, and finally, the price of debt is determined. Clearly, in this scenario, the repayment rate does not depend on the debt levels. Optimal behavior by lenders implies that

$$q = \beta r$$

and, thus, implies that the price of debt also does not depend on the debt levels. In the noncooperative regime, the government of country 1 solves

$$\max_{b} U(\omega + qb) + \beta U(y(r) - rb). \tag{14}$$

The government of country 2 solves an analogous problem. In the cooperative regime, b and b^* are chosen to solve the following problem:

$$\max \left[U(\omega + qb) + \beta U(y(r) - rb)\right] + \left[U(\omega + qb^*) + \beta U(y(r) - rb^*)\right]. \tag{15}$$

Since b^* does not affect the utility level of country 1 and b does not affect the utility level of country 2, we have the following proposition:

Proposition 2. With commitment by the monetary authority, the noncooperative and cooperative equilibria coincide.

Propositions 1 and 2, together, imply that the issue of whether debt constraints are desirable is intimately connected to the extent to which the monetary authority can commit to future monetary policy. Proposition 2 says that once a monetary authority commits to a monetary policy, binding constraints on future debt issues can only reduce welfare. Proposition 1 implies that as long as such commitment is not possible, appropriately chosen debt constraints improve welfare.

The economy with commitment is broadly similar to the economies in an extensive literature that discusses the gains from international cooperation in setting fiscal policy. (See Chari and Kehoe 1990 and Canzoneri and Diba 1991.) As we noted in the introduction, this literature shows that cooperation is desirable if a country's fiscal policy affects world prices and real interest rates. In our model, there are no gains from cooperation under commitment because we assume that the monetary union is small in the world in the sense that the world interest factor β is independent of the fiscal policy decisions of the union. Consider, instead, a general equilibrium model with no outside lenders so that countries 1 and 2 constitute the entire world. Specifically, suppose that each country chooses its spending level on a public good that benefits its own residents and finances the spending with debt and distorting taxes. In such a formulation, even with commitment by the monetary authority, the noncooperative and cooperative equilibria do not coincide, because any country's spending decision affects the world interest rate and, hence, the other country's welfare. Since these types of gains from cooperation are not related to the formation of a monetary union, we have chosen a formulation in which these effects don't appear.

2. A general equilibrium version

In this section, we develop a general equilibrium model of a fiscally decentralized monetary union. The underlying model is a two-country version of the cash-in-advance model in Svensson (1985). In each period t, $t = 0, 1, ...$, labor is used to produce a nonstorable good which can be used for consumption or government spending. We denote country 1 allocations without an asterisk and country 2 allocations with an asterisk. The resource constraint is

$$c_t + c_t^* + g_t + g_t^* = n_t + n_t^*$$

where c_t, g_t, and n_t denote consumption, government spending, and labor input in country 1 and c_t^*, g_t^*, and n_t^* denote the analogous allocations for country 2. The consumer in country 1 solves the problem

$$\max \sum_{t=0}^{\infty} \beta^t \left[U(c_t, n_t) + W(g_t) \right]$$

subject to the sequence of budget constraints

$$m_{t+1} + Q_t b_{t+1} = p_t n_t + m_t - p_t c_t + b_t$$

and the cash-in-advance constraints

$$p_t c_t \leq m_t$$

where m_t and b_t denote holdings of money and nominal debt at the start of period t, p_t denotes the price level, and Q_t denotes the price of nominal debt. The consumer in country 2 solves the problem,

$$\max \sum_{t=0}^{\infty} \beta^t \left[U(c_t^*, n_t^*) + W(g_t^*) \right]$$

subject to the sequence of budget constraints

$$m_{t+1}^* + Q_t b_{t+1}^* = p_t n_t^* + m_t^* - p_t c_t^* + b_t^*$$

and the cash-in-advance constraints

$$p_t c_t^* \leq m_t^*.$$

We assume that consumers in the two countries have the same initial money and debt holdings.

The cash-in-advance constraint reflects the timing assumption that consumers enter each period with money holdings m_t, purchase goods and supply labor, and then trade money for bonds in a securities market. This timing assumption, made in Svensson (1985), has the useful feature that it is costly to raise the inflation rate from the preceding period to the current period because such a raise in inflation reduces current consumption. (In contrast, Lucas and Stokey 1987 assume that securities market trading occurs at the start of the period. With this timing, it is costless to raise the inflation rate from the preceding period to the current period. See Nicolini 1997 for details.)

The budget constraint for the government of country 1 is

$$Q_t B_{t+1} = B_t + p_t g_t - (M_t - M_{t-1})/2$$

where B_t denotes the nominal debt of the government of country 1 at the start of period t and M_t denotes the aggregate stock of money for the two countries. We assume that seignorage revenues are equally distributed to the two governments. The government of country 2 has an analogous constraint.

For given sequences of money M_t and nominal debt B_t and B_t^*, a *competitive equilibrium* is a set of allocations for consumers in the two countries and a set of prices such that (*i*) consumers solve their problems, (*ii*) the government's budget constraints are satisfied, (*iii*) the resource constraint is satisfied, and (*iv*) the money and debt markets clear:

$$m_t + m_t^* = M_t$$

$$b_t + b_t^* = B_t + B_t^*.$$

The timing of events in each period is as follows. Consumers enter the period with money and debt, the monetary authority chooses the amount of new money creation, consumers and governments choose how many goods to purchase, consumers choose how much labor to supply, and finally, consumers choose new money and bond holdings. The amount of new government debt is then determined by the government's budget constraint.

In this paper, we assume that in period 1, a single authority sets both monetary and fiscal policy for the infinite future and can commit once and for all in period 1. From a strategic perspective, this assumption effectively reduces the strategic interaction to that in a two-period model. (With the restrictions on utility functions that we adopt below, we can show that having a single authority is equivalent to having the monetary authority commit to infinite sequences of prices and then having the governments noncooperatively choose their fiscal policy.)

Before we set up the governments' problem in period 0 and the single authority's problem from period 1 onward, we characterize the competitive equilibrium. The first-order conditions of consumers in country 1 are

$$c_t : \beta^t U_{ct} = (\lambda_t + \delta_t) p_t$$

$$n_t : -\beta^t U_{nt} = \lambda_t p_t$$

$$m_{t+1} : \lambda_t = \lambda_{t+1} + \delta_{t+1}$$

$$b_{t+1} : Q_t \lambda_t = \lambda_{t+1}$$

where λ_t and δ_t are the Lagrange multipliers on the budget constraint and the cash-in-advance constraints. We can reduce these conditions to

$$-\frac{U_{nt}}{\beta U_{ct+1}} = \frac{p_t}{p_{t+1}}$$

$$Q_t = \frac{\beta U_{nt+1}}{U_{nt}} \frac{p_t}{p_{t+1}}.$$

Since the utility function is separable in government spending, the consumers in the two countries choose the same allocations. We then have the following proposition:

Proposition 3. Given B_1, B_1^*, and M_1, the allocations in a competitive equilibrium for $t = 1, ...,$ satisfy

$$(16) \quad c_t = c_t^*, \; n_t = n_t^*$$

$$(17) \quad 2c_t + g_t + g_t^* = 2n_t$$

and

$$(18) \quad \beta U_{n1} n_1 + \sum_{t=2}^{\infty} \beta^t \left[U_{ct} c_t + U_{nt} n_t \right] + \beta U_{n1} \frac{(B_1 + B_1^*)}{2p_1} = 0$$

where U_{ct} and U_{nt} denote marginal utilities in period t and $p_1 = M_1/2c_1$. Furthermore, any allocations that satisfy these conditions can be decentralized as a competitive equilibrium.

The proof is standard. (See, for example, Chari and Kehoe 1998.)

The preferences of the single authority in period 1 are given by

$$(19) \quad \sum_{t=1}^{\infty} \beta^t \left[U(c_t, n_t) + W(g_t) + U(c_t^*, n_t^*) + W(g_t^*) \right].$$

This authority's problem is to maximize this objective over the set of competitive equilibria and, thus, to maximize (19) subject to (16)–(18). Let $V(B_1 + B_1^*)$ denote the maximized value of this objective function, let $c_t(B_1 + B_1^*), n_t(B_1 + B_1^*)$, and $g_t(B_1 + B_1^*)$ denote the decision rules for the two countries, and let $p_1(B_1 + B_1^*)$ denote the price level in period 1.

For given levels of g_0 and g_0^*, we can summarize the competitive equilibrium at period 0 as allocation functions $c_0(g_0, g_0^*), n_0(g_0, g_0^*), B_1(g_0, g_0^*)$, and $B_1^*(g_0, g_0^*)$ and price functions $p_0(g_0, g_0^*)$ and $Q_0(g_0, g_0^*)$. These functions are implicitly defined by the resource constraint

$$(20) \quad 2c_0 + g_0 + g_0^* = 2n_0$$

the cash-in-advance constraint

$$(21) \quad p_0 c_0 = M_0/2$$

the government budget constraints

$$(22) \quad Q_0 B_1 = B_0 + p_0 g_0 - (M_1 - M_0)/2$$

$$(23) \quad Q_0 B_1^* = B_0 + p_0 g_0^* - (M_1 - M_0)/2$$

and the consumer first-order conditions

$$(24) \quad -\frac{U_n(c_0, n_0)}{\beta U_c(c_1, n_1)} = \frac{p_0}{p_1}$$

$$(25) \quad Q_0 = \frac{\beta U_n(c_1, n_1)}{U_n(c_0, n_0)} \frac{p_0}{p_1}$$

where $c_1 = c_1(B_1 + B_1^*)$, $n_1 = n_1(B_1 + B_1^*)$, and $p_1 = p_1(B_1 + B_1^*)$ are from the single authority's problem.

The preferences of the governments of the two countries coincide with those of the consumers. In the noncooperative regime, the government of country 1 takes g_0^* as given and chooses g_0 to solve the following problem:

$$(26) \quad \max_{g_0} U(c_0(g_0, g_0^*), n_0(g_0, g_0^*)) + W(g_0) + \beta V(B_1(g_0, g_0^*) + B_1^*(g_0, g_0^*)).$$

In the cooperative regime, the spending levels are chosen to maximize the sum of the objective functions of these two governments.

A *noncooperative equilibrium outcome* is a pair of spending levels (g_0, g_0^*) that solves the noncooperative problem (26), together with allocations and prices $c_0, n_0, B_1, B_1^*, p_0$, and Q_0 that satisfy (20)–(25). The allocations and prices from then onward are obtained from the single authority's problem from period 1 onward. A *cooperative equilibrium outcome* is defined in a similar fashion. Recall that any competitive equilibrium (even with $g_0 \neq g_0^*$) has identical allocations for private agents in the two countries. Thus, the spending levels (g_0, g_0^*) solve the following problem:

$$\max_{g_0, g_0^*} 2U(c_0(g_0, g_0^*), n_0(g_0, g_0^*)) + W(g_0) + W(g_0^*) + 2\beta V(B_1(g_0, g_0^*) + B_1^*(g_0, g_0^*)). \tag{27}$$

In what follows, we assume that the first-order conditions for the cooperative equilibrium are necessary and sufficient.

Proposition 4. The spending levels in period 0 in a symmetric noncooperative equilibrium are higher than those in a symmetric cooperative equilibrium.

Proof: The first-order condition for g_0 in the noncooperative equilibrium is

$$U_{c0}\frac{\partial c_0}{\partial g_0} + U_{n0}\frac{\partial n_0}{\partial g_0} + W'(g_0) + \beta\left(\frac{\partial B_1}{\partial g_0} + \frac{\partial B_1^*}{\partial g_0}\right)V' = 0. \tag{28}$$

The first-order condition for g_0 in the cooperative equilibrium is

$$2U_{c0}\frac{\partial c_0}{\partial g_0} + 2U_{n0}\frac{\partial n_0}{\partial g_0} + W'(g_0) + 2\beta\left(\frac{\partial B_1}{\partial g_0} + \frac{\partial B_1^*}{\partial g_0}\right)V' = 0. \tag{29}$$

Let $F_N(g_0)$ denote the left side of (28), and let $F_C(g_0)$ denote the left side of (29) divided by 2, where both are evaluated at $g_0 = g_0^*$. We then have

$$F_N(g_0) - F_C(g_0) = \frac{W'(g_0)}{2} > 0$$

so that $F_N > F_C$ for all g_0. Since the first-order condition for the cooperative equilibrium is necessary and sufficient, a unique value of g_0, say, g_C satisfies $F_C(g_0) = 0$. Since F_N is always greater than F_C, it follows that F_N can be equal to zero only for values of g_0, say, g_N which are greater than g_C. Q.E.D.

In general, it difficult to show that debt is higher in the noncooperative equilibrium. Under the assumption that

$$(30) \quad U(c,n) = \log c - \alpha n$$

we have the following proposition.

Proposition 5. Under (30), end of period 0 debt B_1 is higher in the noncooperative equilibrium than in the cooperative equilibrium.

Proof: Equations (20)–(25) and the problem of the single authority in period 1 determine the allocations in the economy as functions of the levels of government spending (g_0, g_0^*). We are interested in symmetric spending levels that satisfy $g_0 = g_0^* = g$. We will show that the end of period 0 debt is increasing in g. We begin by simplifying (20)–(25). Using (30), (21), and $p_1 c_1 = M_1/2$ in (24) gives

$$\frac{\alpha}{\beta} = \frac{p_0}{p_1 c_1} = \frac{M_0}{M_1 c_0}$$

so $c_0 = \beta M_0 / \alpha M_1$. Hence, c_0 and p_0 are both constants. Using (30), (21), and $p_1 c_1 = M_1/2$ in (25), together with symmetry of the equilibrium, gives

$$Q_0 = \alpha c_1$$

where $c_1 = c_1(2B_1)$ comes from the problem of the single authority. With symmetric spending levels g, the budget constraints of the governments can be summarized by

$$(31) \quad g = \frac{\alpha c_1(2B_1) B_1}{p_0} - e$$

where $e = (M_1 - M_0)c_0/M_0 - 2B_0 c_0/M_0$ is a constant. This equation implicitly defines the equilibrium end of period 0 debt as a function of g, namely, $B_1(g)$. Clearly, if $c_1(2B_1)B_1$ is increasing in B_1, it follows that $B_1(g)$ is increasing in g. The problem of the single authority is

$$\max \sum_{t=1}^{\infty} \beta^t \left[\log c_t - \alpha(c_t + g_t) + W(g_t)\right]$$

subject to

$$(32) \quad -\beta\alpha(c_1 + g_1) + \sum_{t=2}^{\infty} \beta^t \left[1 - \alpha(c_t + g_t)\right] - \beta\alpha \frac{B_1 c_1}{M_1} = 0.$$

The first-order conditions for this problem are

$$(33) \quad c_1 : \frac{1}{c_1} - \alpha = \lambda\alpha\left(1 + \frac{B_1}{M_1}\right)$$

$$(34) \quad c_t : \frac{1}{c_t} - \alpha = \lambda\alpha \text{ for } t \geq 2$$

$$(35) \quad g_t : W'(g_t) - \alpha = \lambda\alpha \text{ for } t \geq 1$$

where λ is the Lagrange multiplier on the implementability constraint. We first show that if λ is increasing in B_1, then $B_1 c_1$ is increasing in B_1. We then show that λ is, indeed, increasing. If λ is increasing in B_1, it follows from (33) that c_1 is decreasing in B_1, while it follows from (34) and (35) that c_t and g_t are decreasing in B_1. When we use these results in (32) it follows that $B_1 c_1$ is increasing in B_1.

To see that λ is increasing in B_1, substitute consumption and government spending from (33)–(35) into the implementability constraint (32) to obtain

$$-\frac{1}{1 + \lambda(1 + \frac{B_1}{M_1})} - \alpha W'^{-1}(\alpha(1+\lambda)) +$$
$$\sum_{t=2}^{\infty} \beta^{t-1} \left[1 - \frac{1}{(1+\lambda)} - \alpha W'^{-1}(\alpha(1+\lambda))\right] - \frac{B_1}{M_1} \frac{1}{1 + \lambda(1 + \frac{B_1}{M_1})} = 0.$$

We can rewrite this as

$$H(\lambda, B_1) = \frac{(1+\frac{B_1}{M_1})}{1+\lambda(1+\frac{B_1}{M_1})} + \frac{\alpha W'^{-1}(\alpha(1+\lambda))}{1-\beta} + \frac{\beta}{1-\beta}\left[\frac{1}{(1+\lambda)} - 1\right] = 0 \tag{36}$$

which implicitly determines λ as a function of B_1. It is clear that H is decreasing in λ, and it is easy to show that it H increasing in B_1. Thus, λ is increasing in B_1. Q.E.D.

As in the reduced-form economy, the extent of the commitment of the monetary authority plays a central role in our results. To see this, consider a scenario in which the monetary authority moves first and sets monetary policy forever. Specifically, suppose that in period 0, the monetary authority chooses a sequence of price levels, $p_0, p_1, ...$, and then each fiscal authority chooses a sequence of spending levels. We first develop the constraints facing the governments. Recall the consumer first-order conditions

$$-\frac{U_{nt}}{\beta U_{ct+1}} = \frac{p_t}{p_{t+1}}$$

$$Q_t = \frac{\beta U_{nt+1}}{U_{nt}} \frac{p_t}{p_{t+1}}.$$

With our utility function $U(c, n) = \log c - \alpha n$, these conditions reduce to

$$\frac{\alpha c_{t+1}}{\beta} = \frac{p_t}{p_{t+1}}$$

$$Q_t = \beta \frac{p_t}{p_{t+1}}.$$

Thus, once the sequence of price levels is fixed, then so are the levels of consumption c_t and the debt prices Q_t. From the cash-in-advance constraint, it follows that the levels of the money stock are pinned down. We can iterate on the government budget constraint to obtain

$$\sum_{t=1}^{\infty} [Q_0 Q_1 ... Q_{t-1}] \left(p_t g_t - \frac{(M_{t+1} - M_t)}{2}\right) + p_0 g_0 + B_0 - \frac{(M_1 - M_0)}{2} = 0. \tag{37}$$

Using the first-order conditions, we can rewrite (37) as

$$(38) \quad \sum_{t=0}^{\infty} \beta^t g_t = A/p_0$$

where A is a constant equal to the present value of the seignorage less the initial debt. The objective function of the fiscal authority in country 1 can be written as

$$\max \sum_{t=0}^{\infty} \beta^t \left[\log c_t - \alpha(c_t + (g_t + g_t^*)/2) + W(g_t)\right]$$

where we use the resource constraint to substitute for n_t. As we have shown, since prices are fixed, so are consumption levels c_t. From (38) and its analog for country 2, it follows that

$$(39) \quad -\alpha \sum_{t=0}^{\infty} \beta^t (g_t + g_t^*)/2$$

is also fixed. Thus, the problem of the fiscal authority in country 1 reduces to

$$\max \sum_{t=0}^{\infty} \beta^t W(g_t)$$

subject to (38).

In a cooperative equilibrium, the problem of the single fiscal authority reduces to

$$\max \sum_{t=0}^{\infty} \beta^t (W(g_t) + W(g_t^*))$$

subject to (38) and its analog for country 2. It should be clear that the solutions to both problems are identical. We summarize this discussion with the following proposition:

Proposition 6. Under (30), when the monetary authority commits to its policy in period 0, the noncooperative and cooperative equilibria coincide.

Propositions 5 and 6 again illustrate that the desirability of debt constraints in monetary unions depends critically on the extent of commitment available to the monetary authority.

In our formulation, there are no world real interest rate effects from policy choices that give rise to the gains from cooperation discussed by Chari and Kehoe (1990) and Canzoneri and Diba (1991). The quasi-linear utility functions that we have assumed imply that the real interest rate is independent of policy choices. The gains from cooperation implied by Proposition 5 thus emerge from the lack of commitment by the monetary authority.

3. Conclusion

In this paper, we have argued that the desirability of debt constraints in monetary unions depends critically on the extent of commitment of the monetary authority. If the monetary authority can commit to its policies, debt constraints can only impose costs. If the monetary authority cannot commit, there is a free-rider problem in fiscal policy, and debt constraints may be desirable.

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