Federal Reserve Bank of Minneapolis
Research Department Staff Report 316

April 2003

Financial Crises as Herds: Overturning the Critiques

V. V. Chari*

Federal Reserve Bank of Minneapolis
and University of Minnesota

Patrick J. Kehoe*

Federal Reserve Bank of Minneapolis,
University of Minnesota,
and National Bureau of Economic Research

ABSTRACT

Financial crises are widely argued to be due to herd behavior. Yet recently developed models of herd behavior have been subjected to two critiques which seem to make them inapplicable to financial crises. Herds disappear from these models if two of their unappealing assumptions are modified: if their zero-one investment decisions are made continuous and if their investors are allowed to trade assets with market-determined prices. However, both critiques are overturned—herds reappear in these models—once another of their unappealing assumptions is modified: if, instead of moving in a prespecified order, investors can move whenever they choose.

*We thank Susan Athey, Abhijit Banerjee, Sushil Bikhchandani, Harold Cole, David Levine, George Mailath, Enrique Mendoza, Stephen Morris, Andrew Postlewaite, Hyun Shin, and an anonymous referee for useful comments. We thank Kathy Rolfe for editorial assistance. We thank the National Science Foundation for financial support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Capital flows to emerging markets are notoriously volatile. Many researchers have argued that a substantial fraction of this volatility is due to herd behavior. In discussing financial crises in developing countries, for example, Calvo and Mendoza [5] say that " 'the fall from grace' in world capital markets . . . may be driven by herding behavior not necessarily linked to fundamentals." Similar views can be found in other recent work, including that of Chari and Kehoe [8], Cole and Kehoe [9], and Sachs, Tornell, and Velasco [16]. More generally, the belief that herd behavior is widespread in financial markets is held by both market participants and economists (Devenow and Welch [10, p. 603]).

Recently, models of herd behavior in which agents rationally mimic the behavior of other agents have been developed (by, for example, Banerjee [3] and Bikhchandani, Hirshleifer, and Welch [4]). In these models, herds occur because information becomes trapped; agents' actions do not reveal their underlying signals. While these models, at first glance, seem appealing for understanding financial crises in emerging markets, a closer look reveals a problem: In order to generate herds, the models include two stark simplifying assumptions which make applying the models to financial crises difficult.

The two stark assumptions are that investment decisions are discrete, zero-one decisions and that there are no traded assets with market-determined prices. Researchers have shown that if the assumptions are relaxed, then the models no longer generate herds. Lee [14] shows that if investment decisions are continuous instead of discrete, then herds disappear from the models. Avery and Zemsky [2] and Glosten and Milgrom [11] show that once the models allow for trade in financial markets, prices reveal information, and herds disappear. With the more natural assumptions, these models do not generate herds because investors can use the continuous variable, either investment or prices, to infer private signals; hence, no information gets trapped. We label these critiques the *continuous investment critique* and the *price critique*.

In modeling financial crises it seems undesirable to abstract from continuous investment or prices. The scale of investments in financial markets can often be easily changed so that discrete investment assumptions seem unappealing. Moreover, prices are central to the operation of financial markets so that abstracting from prices seems particularly unappealing. Taken together, the critiques suggest that the early models of herd behavior, at least as they stand, are not applicable to financial crises.

Yet, as we will show, both critiques can be overturned by replacing another stark simplifying assumption of the early herd models with an assumption that is more natural for applied situations. The stark assumption is that of *exogenous timing,* namely, that investors move in a prespecified order. We show that when this assumption is replaced by the assumption of *endogenous timing*—when investors can move whenever they choose—the two critiques are overturned; herds reappear in both the model with continuous investment and the model with prices.

In our continuous investment model, investors must choose how to divide their assets between a risky project and a safe one. The returns in the risky project are determined by whether the economy's underlying state is high or low. Information about that state arrives slowly in the economy, in the form of a signal in each period. This signal is received privately by one of the investors. Other investors attempt to infer this signal from observed levels of investment. In each period, investors face a trade-off between investing and waiting to invest: waiting is potentially beneficial because it lets investors gain information, but it is costly because of discounting. In this model, a small number of high signals leads all investors to invest in the risky project while a small number of low signals leads all investors to not invest in the risky project. The model generates herd-like behavior because, at some point, the benefits from waiting for more information are outweighed by the costs from waiting due to discounting, and investors choose not to wait for future signals. The information contained in future signals is never revealed to the market, and in this sense, information becomes trapped.

We call these outcomes *herds* because they satisfy two criteria: investors make the same decisions regardless of their private signals, so that the outcomes are a *cascade*, and the outcomes are inefficient relative to those that emerge from solving a mechanism design problem. The outcomes in our continuous investment model are inefficient because of an information externality that leads the private return to waiting to be lower than the social return. If uninformed investors would wait to receive signals before acting, the market would receive more information in the future, and all uninformed investors would benefit. Each uninformed investor ignores this social benefit to waiting when making decisions, thus rendering equilibrium outcomes inefficient. Our model with prices works similarly.

Thinking about policy toward financial crises requires distinguishing between situations in which outcomes are efficient and those in which they are inefficient. Suppose a model generated a cascade in which investors fled a country, leading to a financial crisis, but this cascade was efficient. While this outcome may be striking, there is no role for policy in trying to prevent it. If, however, the cascade were inefficient, and therefore a herd, there may be a role for policy.

Our two models share the key feature with existing herd models that investors are informationally large: the decisions of just a few investors can set off herds. This feature is desirable in the context of capital flows to emerging markets because such capital flows are dominated by a relatively small number of portfolio managers. While these flows are large from the perspective of the emerging markets, they are small from the perspective of markets in developed countries. Not surprisingly, developed countries have a considerable amount of specialization in acquiring information about emerging markets, so that investing is dominated by a relatively few portfolio managers.

Our work here is related to an extensive literature on herds. Several studies are closely related to ours. Caplin and Leahy [6], Chamley and Gale [7], and Gul and Lundholm [13] allow for private signals and endogenous timing of decisions; however, none of that work is directed toward the critiques of the early herd literature. See also the work by Vives [18] on social learning.

Two studies have attempted to overturn the critiques of the herd literature. Lee [15] shows that, with exogenous timing, fixed costs of trading can lead informed investors to stop trading after some point, so that information becomes trapped. If trading costs are small, however, almost all of the information is revealed through prices. In international financial markets, the volume of trades is enormous, and fixed costs of trading seem small. Avery and

Zemsky [2] offer a definition of herd behavior quite different from that in the literature. They show that with exogenous timing, herds in their sense can occur, but cascades cannot. We follow the rest of the literature in requiring that for an outcome to be a herd, it must also be a cascade. Hence, Avery and Zemsky's model does not generate herds in our sense.

Finally, our work is also related to the large literature about prices revealing information which has followed Grossman and Stiglitz [12] and to an extensive literature on bubbles in asset prices (for example, Allen, Morris, and Postlewaite [1]).

1. Herds with Continuous Investment

We develop an economy with continuous investment along with endogenous timing and slow arrival of information. We show that this economy has herds and that endogenous timing is critical in generating herds of investment. We then describe Lee's [14] continuous investment critique of the early herd models in terms of our setup.

1.1. *The Economy*

Consider an infinite horizon economy with periods denoted $t = 0, 1, \ldots$ with an infinite number of risk-neutral investors. Each investor starts with one unit invested in a safe project. Investors must make a one-time decision to invest in a risky project. If they have not invested in the risky project before period t, then in period t they can either wait until period $t+1$ or invest some amount $x_t \in [0, 1]$ in the risky project and the rest in the safe project. Investors discount the future at rate $1/(1+r)$.

The returns on the risky project depend on the state of the economy y, which is either *high,* H, or *low,* L, but is initially unknown to investors. The present discounted value of investing x units in the risky project is $f(x)$ if the state is high and 0 if the state is low. (If the state is high, we can think of the project as yielding a per period return of $\tilde{f}(x)$, so that $f(x) = \tilde{f}(x)/r$ is the perpetuity value of $\tilde{f}(x)$.) The safe project yields a per period dividend of r per unit invested in the safe project, so that the perpetuity value of one unit of the safe project is one. This dividend is paid at the beginning of the period.[1]

We assume that f is strictly concave, $f'(0)$ is finite, and $f(0) = 0$. The assumption that $f'(0)$ is finite is natural in any applied situation with either a fixed cost or a minimum scale of production. If in any period t an investor decides to invest and assigns probability p that the state is high, then this investor faces a standard static portfolio problem given by

$$V(p) = \max_{x \in [0,1]} pf(x) + (1 - x) \tag{1}$$

where $V(p)$ is the value of investing when the probability of a high state is p. Let $x(p)$ denote the solution to this problem. Notice that $V(p) \geq 1$ since it is feasible to set $x = 0$ and that $V(p) > 1$ if and only if $x(p) > 0$. The first-order condition at an interior point is $f'(x) = 1/p$. Let $\underline{p} = 1/f'(0)$. Clearly, the optimal investment of an investor in period t given beliefs p is

$$x(p) = \left\{ (f')^{-1} \left(\frac{1}{p} \right) \text{ if } p \geq \underline{p} \right\} \tag{2}$$

and 0 otherwise, so that $\underline{p}$ is the *cutoff level* for investment.

Information arrives slowly in our economy. In each period t, the economy receives a signal which can take on one of two possible values $s \in \{H, L\}$: that the state of the economy is high or low. The signals are informative and symmetric in the sense that

$$\Pr(s = H \mid y = H) = \Pr(s = L \mid y = L) = q > 1/2 \tag{3}$$

as well as conditionally independent over time. Each period the signal is randomly distributed to one and only one agent among the set of investors who have not already received a signal and is privately observed by that agent.[2]

The timing in each period is that first an investor receives a signal and then investment decisions are made. The only publicly observable event in any period t is the aggregate quantity of investment, denoted X_t. The *public history* $h_t = (X_0, X_1, \ldots, X_{t-1})$ records the aggregate quantity of investment in each period up through the beginning of period t. Investors also privately record the signal they receive, if any, and the period in which they receive it. Thus, the history of an investor i in period t who has received a signal in r is $h_{it} = (h_t, s_r, r)$, and the history of an investor who has not received a signal is simply the public history.

In each period t, given their histories, investors can be described as belonging to one of four groups. Any investor who has already invested is *inactive*. The active investors in period t consist of a *newly informed* investor who receives the signal at the beginning of period t, *previously informed* investors who received a signal in some period r before t, and *uninformed* investors who have not yet received a signal.

An investor's strategy and beliefs are sequences of functions $x_t(h_{it})$ and $p_t(h_{it})$ that map the investor's histories into actions and into the probability that the state is high. Notice that we have imposed symmetry by supposing that all investors who have the same histories take the same actions and have the same beliefs. Let $p_t(h_t)$ be the *public beliefs,* that is, the probability that the state is high, conditional on the public history $h_{t.}$

The payoffs are defined as follows. The payoff to an investor who makes an investment decision in period t with history h_{it} is

$$V_t(h_{it}) = \max_{x \in [0,1]} p_t(h_{it}) f(x) + (1 - x). \tag{4}$$

Note that $V_t(h_{it})$ does not include current dividends from the safe project and, hence, is the post-dividend payoff to investing.

The post-dividend payoff to an investor who waits in period t is

$$W_t(h_{it}) = \Big[r + \sum_{h_{it+1}} \mu_t(h_{it+1}|h_{it}) \max\{V_{t+1}(h_{it+1}), W_{t+1}(h_{it+1})\} \Big] \Big/ (1 + r) \tag{5}$$

where $\mu_t(h_{it+1}|h_{it})$ is the conditional distribution over histories in $t+1$ given the history in t. Clearly, an investor invests in period t if $V_t(h_{it}) \geq W_t(h_{it})$ and waits otherwise. Notice that the conditional distribution $\mu_t(h_{it+1}|h_{it})$ is induced from the strategies and the structure of exogenous uncertainty of the game in the obvious way.

We refer to this game as the *private signal game.* A *perfect Bayesian equilibrium* of this game is a set of strategies $x_t(h_{it})$, a set of conditional distributions $\mu_t(h_{it+1}|h_{it})$, and a set of beliefs $p(h_{it})$ such that (i) for every history h_{it}, the investment and waiting decisions are optimal and (ii) the conditional distributions $\mu_{it}(h_{it+1}|h_{it})$ and the beliefs $p_t(h_{it})$ are consistent with Bayes' rule wherever possible and arbitrary otherwise.

In the game, investors get information from two sources: they receive private signals, and they see the investment decisions of others and try to infer their underlying signals. In a region where uninformed investors can accurately infer the underlying signals of informed investors, it is as if these uninformed investors see the signal (with a lag) and update their beliefs using Bayes' rule. For arbitrary beliefs p, we use Bayes' rule to define $P_H(p)$ and $P_L(p)$ as the updated beliefs that the state is high, given that signals H and L were either directly received or indirectly inferred:

$$P_H(p) = \frac{pq}{pq + (1-p)(1-q)} \tag{6}$$

$$P_L(p) = \frac{p(1-q)}{p(1-q) + (1-p)q} \tag{7}$$

where q is defined in (3). Let $p(0) = p_0$, $p(1) = P_H\big(p(0)\big)$, $p(2) = P_H\big(p(1)\big)$, and so on, and let $p(-1) = P_L\big(p(0)\big)$, $p(-2) = P_L\big(p(-1)\big)$, and so on. Thus, $p(k)$ for $k > 0$ is the prior probability that the state is high if k high signals have been received, and $p(k)$ for $k < 0$ is the prior probability that the state is high if k low signals have been received. Notice from the symmetry in (3) that

$$P_H\big(P_L(p)\big) = P_L\big(P_H(p)\big) = p \tag{8}$$

so that the effect on the prior of a given set of signals is summarized by the number of high signals minus the number of low signals in the set. Thus, for example, receiving two high signals and one low signal yields the same prior as receiving one high signal.

Now focus on the region of the parameter space that satisfies these two assumptions:

$$V\big(p(0)\big) > 1 \tag{9}$$

$$V\big(p(-1)\big) = 1. \tag{10}$$

To interpret these assumptions, recall that $V(p) > 1$ if and only if $x(p) > 0$. Assumption (9) implies that if an investor is forced to invest at the initial prior, this investor will invest a strictly positive amount in the risky project. Assumption (10) implies that if an investor is forced to invest at beliefs $p(-1)$, this investor will invest nothing in the risky project. These assumptions imply that the cutoff level for investment $\underline{p}$ defined in (2) is between $p(0)$ and $p(-1)$. For an alternative interpretation, suppose $f(x) = Ax$, for some constant A, so that f is linear. Then the assumptions reduce to $p(0)A > 1$ and $p(-1)A \leq 1$.

The key decision in the model is to invest in the current period or to wait. Waiting is costly because of discounting, but waiting is valuable for two information-related reasons. First, waiting to receive information is beneficial because investors have the option of not investing if the information makes investors sufficiently pessimistic. We call this value the *no investment option value.* Second, even if investors are sufficiently optimistic that they know they will eventually invest, information allows the investor to fine tune the scale of their investment. We call this value the *fine-tuning value.*

In addition to (9) and (10), we make an assumption that ensures that the no investment option value is large relative to discounting, and one that ensures that the fine-tuning value is small relative to discounting. To ensure that the no investment option value is large, we assume that

$$V\big(p(0)\big) < \Big[r + v_H\big(p(0)\big)V\big(p(1)\big) + v_L\big(p(0)\big)\Big]\Big/(1+r) \tag{11}$$

where $v_H(p) = pq + (1-p)(1-q)$ is the probability that a high signal is received when the prior is p and $v_L(p) = p(1-q) + (1-p)q$ is the probability that a low signal is received when the prior is p.

Assumption (11) essentially says that discounting is small relative to the value of information if that information could lead the investor to invest nothing in the risky project. The left side of (11) is the value of investing at $p(0)$. Now suppose that an uninformed investor knows that waiting (rather than investing) will allow the investor to draw the following inferences from the actions of informed investors: with probability $v_H\big(p(0)\big)$, a high signal has occurred, so that the prior rises to $p(1)$ and, with probability $v_L\big(p(0)\big)$, a low signal has occurred, so that the prior falls to $p(-1)$. The right side of (11) is the payoff to the (possibly) suboptimal strategy of investing if the prior rises to $p(1)$ and never investing if the prior falls to $p(-1)$, where the payoff to never investing is clearly 1. Then (11) says that the investor is better off waiting to receive this type of information rather than investing immediately. In this sense, assumption (11) says that the value of the no investment option is large relative to discounting. This assumption is satisfied if r is sufficiently small.

To ensure that the fine-tuning value is relatively small, we assume that

$$V(p) > \Big[r + v_H(p)V\big(P_H(p)\big) + v_L(p)V\big(P_L(p)\big)\Big]\Big/(1+r) \tag{12}$$

for all $p \geq p(1)$. This assumption requires that investing at a prior of, say, p dominates waiting one period, indirectly inferring a signal, and then investing the optimal larger amount if the inferred signal is high, so that the prior rises to $P_H(p)$, and the optimal smaller amount if the inferred signal is low, and the prior falls to $P_L(p)$. Assumption (12) thus says that the fine-tuning value is small relative to discounting.

To understand the intuition for assumption (12), suppose that $f(x) = Ax$ for $x \leq 1/2$ and $A/2$ for $x > 1/2$. Then, if (9) holds, (12) automatically holds, since the value of fine-tuning is zero: it is optimal to run the project at rate $1/2$, regardless of whether the inferred signal about the state in the next period turns out to be high or low. That is, the optimal size of the project does not vary at all with marginal changes in information. To see this, note that since p, $P_H(p)$, and $P_L(p)$ are all greater than or equal to $p(0)$ in each of the corresponding portfolio problems, it is optimal to set $x = 1/2$, so that $V(p') = p'A/2$ for p' equal to p, $P_H(p)$, or $P_L(p)$. Since the prior p is the mean of the posterior distribution, it follows that $p = v_H(p)P_H(p) + v_L(p)P_L(p)$. With some manipulation, we can rewrite (12) as $pA > 1$, which is implied by (9). More generally, when f is sufficiently concave at $x(p)$ for $p \geq p(1)$, (12) is likely to be satisfied because the optimal size of the project varies little with marginal changes in information.

We now informally describe investor strategies. The strategy of uninformed and previously informed investors is to invest $x(p)$ if and only if the prior is at least $p(1)$. The strategy of newly informed investors is to invest $x(p)$ if and only if the prior p is at least $p(0)$. From these strategies, it is easy to construct how beliefs evolve.

These strategies lead to the following equilibrium outcomes. The newly informed investor in period 0 invests if the signal is high and waits if the signal is low. All uninformed investors wait.

The decisions in period 1 depend on the history from period 0. If there was positive investment in period 0, then the uninformed investors infer that the signal in period 0 was high, their priors rise to $p(1)$, and they all invest, while the newly informed investor in period 1 invests regardless of the signal. We say that this history starts a *cascade of investment,* in that all investors invest, and future signals are never revealed to the market. If there was zero investment in period 0, then the uninformed investors infer that the signal in period 0 was low, their priors fall to $p(-1)$, and they wait. The newly informed investor in period 1 invests if the signal is high, but otherwise waits.

At the beginning of period 2, if there has been no investment in both periods 0 and 1, then uninformed investors' priors fall to $p(-2)$, and no investor invests in period 2 or any subsequent period. We will say that this history starts a *cascade of no investment,* in that no investors invest, and future signals are never revealed to the market. If there has been no investment in period 0 but an investment in period 1, then both the uninformed investors and the previously informed investor have a prior of $p(0)$, they wait, and the newly informed investor invests if and only if the signal is high.

Let $X(k) = x\big(p(k)\big)$ denote the investment level associated with prior $p(k)$. Then, more generally, histories of the form $\big(0, X(0), 0, X(0), \ldots, 0, X(0), X(1)\big)$ start cascades of investment while histories of the form $\big(0, X(0), 0, X(0), \ldots, 0, X(0), 0, 0\big)$ start cascades of no investment.

In the appendix, we formally define the strategies and beliefs and prove the following proposition:

PROPOSITION 1. *Under assumptions* (9)–(10) *and* (11)–(12), *the strategies and beliefs described above constitute a perfect Bayesian equilibrium.*

Our model generates cascades because, as soon as the public beliefs reach $p(1)$, the benefits from waiting for more information are outweighed by the costs from waiting due to discounting, and investors choose to invest rather than to wait for future signals. Once public beliefs reach $p(-2)$, no newly informed investor invests, and all uninformed investors keep their assets in the safe project forever. Thus, in both cases, the information contained in future signals is never revealed to the market, and in this sense, information becomes trapped.

1.2. *Herds*

So far we have shown that our equilibrium generates *cascades,* that is, outcomes in which information becomes trapped. This trapping of information need not be inefficient. An interesting feature of our model is that the equilibrium cascades are also inefficient relative to some benchmark. We refer to such inefficient cascades as *herds*. We make the distinction between efficient and inefficient cascades in order to facilitate future policy analysis of cascade-like behavior.

Our benchmark captures some of the physical restrictions inherent in the environment. In the private signal game, the uninformed agents can react to the revealed information only with a one-period lag. Our benchmark public signal game which captures this lag is as follows: uninformed agents learn the realization of the period t signal after they have made their period t investment decisions.

We find the public signal game a useful benchmark for efficiency because its outcomes are the efficient outcomes of a certain mechanism design problem in the private signal game. In the mechanism design problem, privately informed investors can report their signals to the mechanism designer in each period t, and the mechanism designer can communicate these signals to other investors after period t investment decisions are made. Clearly, truth-telling is an equilibrium of this mechanism, and the truth-telling outcomes correspond to the outcomes of the public signal game.

In our public signal game, the signals are observed by all investors. In this game, the public history at the beginning of period t is $s^{t-1} = (s_0, s_1, \ldots, s_{t-1})$, and there is no need to record the history of investments. Let $z_{Ut}(s^{t-1})$ denote the investment decision in period t of the uninformed and the previously informed investors with the signal history s^{t-1}. The relevant history for the newly informed agent in period t is s^t, and the corresponding investment decision is $z_{It}(s^t)$. Let $Z_t(s^t)$ denote the aggregate investment in period t. Beliefs follow from Bayes' rule. An equilibrium is defined as before.

We can make our original private signal game parallel to this public signal game as follows. In the private signal game, the uninformed investors' strategies are defined over histories of investment while the informed investors' strategies include their private signals as well as histories of investment. Given any realization of a history of signals s^t, we can use these strategies recursively to determine aggregate investment as a function of s^t. For example, given the initial signal s_0 in some history s^t, the strategies determine the amount that the informed investor invests and the amount the uninformed investors invest in period 0 and, hence, determine the aggregate investment in period 0, written as $X_0(s_0)$. Given this investment and the signal in period 1, the strategies determine the resulting investment in period 1, written as $X_1(s^1)$. Continuing this process, we can recursively determine $X_t(s^t)$, which denotes the aggregate investment outcome of the original private signal game for a history of signals s^t.

Definition. The private signal game has a *herd at* s^t if (i) for all future histories s^r containing $s^t, X_r(s^r)$ does not vary with $(s_{t+1}, \ldots, s_r)$ and (ii) for some future history s^r containing s^t, $Z_r(s^r) \neq X_r(s^r)$, so that the outcomes in the public and private signal games do not coincide. A herd at s^t is a *herd of investment* if $X_r(s^r) > 0$ for all future histories s^r containing s^t. And a herd at s^t is a *herd of no investment* if $X_r(s^r) = 0$ for all future histories s^r containing s^t.

The first clause in this definition of a herd requires that aggregate investments are the same regardless of the signals, or that the outcome is a cascade. The second clause requires that aggregate investments are inefficient relative to the public signal game, so that a herd is an inefficient cascade. Several researchers (including Banerjee [3]) have defined notions of herd behavior which require only the first clause, our notion of a cascade. In common usage, however, the term *herd* entails some form of mistaken behavior. The second clause of the definition attempts to capture this usage by ensuring that if everyone is doing the same thing, then relative to the public signal game, they are doing the wrong thing.

We can show that the model has herds under the following assumption:

$$V\big(p(1)\big) < \big[r + v_H\big(p(1)\big)V\big(p(2)\big) + v_L\big(p(1)\big)\bar{W}\big(p(0)\big)\big]\big/(1+r) \tag{13}$$

where $\bar{W}\big(p(0)\big) = \big[r + v_H\big(p(0)\big)V\big(p(1)\big) + v_L\big(p(0)\big)\big]/(1+r)$. Assumption (13) is a strengthened version of (11) since, given (10), it is easy to see that (13) implies (11). Assumption (13) implies that in the public signal game, in any period t, investing at $p(1)$ is dominated by the following strategy: Wait until $t+1$; if the signal is high, invest, and if the signal is low, wait until $t+2$ and invest if and only if the signal is high. The following proposition is immediate:

PROPOSITION 2. (*Herds with Continuous Investment*) *Under* (9)–(10), (11)–(12), *and* (13), *the model with endogenous timing has herds of both investment and no investment.*

1.3. *The Continuous Investment Critique*

Our model differs in three key respects from the early models in the herd literature. First, and most importantly, we have endogenous rather than exogenous timing of investment decisions. Second, investments are continuous rather than discrete, zero-one decisions. Third, information arrives slowly over time rather than arriving all at once at the beginning.

Our results overturn one critique of the early herd models, that their ability to generate herds depends critically on the discreteness of the investment decisions, that is, on the action space being coarse relative to the signal space (Lee [14]). In particular, Lee shows that if investment decisions are continuous and the timing of investment decisions is exogenous, then the early herd models cannot generate herds.

To understand this critique, consider a version of our benchmark model in which the timing of investment decisions is exogenously specified. Specifically, suppose that in each period t, only the newly informed investor can invest. We define both a perfect Bayesian equilibrium and herds for this game in analogous ways to those in the game with endogenous timing. The proof of the following proposition is a straightforward adaptation of the arguments of Lee [14] and is available in the working paper version of our paper.

Proposition 3. (*The Continuous Investment Critique*) *Under assumptions* (9) *and* (10), *the model with exogenous timing has no herds of investment.*

The idea behind the proof is that as long as the prior is above the cutoff level $\underline{p}$, the investor in period t invests some positive amount, say, x_t, given by (2). From the investor's first-order condition, the belief of the investor can be inferred to be $p_t = 1/f'(x_t)$. The investor's signal can then be uncovered using the public belief p_{t-1}.

The basic idea in the exogenous timing model is that investors are forced to wait for their signals, and given the continuous nature of investment, investors' actions reveal their signals. Hence, information does not become trapped, and there can be no inefficient cascades of investment. In contrast, in the endogenous timing model, investors are not forced to wait for their signals, and investors find it privately optimal to invest before they receive their private signals. Since investors do not internalize the benefits to others of waiting to receive signals and transmitting this information to the market, the outcomes are inefficient.

The assumption that, in each period, only the newly informed investor can invest can be interpreted in at least two ways. The interpretation in the literature on herds with exogenous timing is that all investors receive signals at the beginning of the game, but each investor is assigned a specific period in which to invest. An alternative interpretation is that both investment opportunities and information arrive slowly and at the same time.

2. Herds with Prices

The other critique of the early herd models is that if investors can trade investment projects in the models, then the prices of the projects reveal information, and herds disappear from the models (as in Avery and Zemsky [2] and Glosten and Milgrom [11]). Here we show that with endogenous timing as well as prices, herds reappear. We also discuss the significant differences between Avery and Zemsky's [2] work and ours.

2.1. *The Economy*

We consider a variant of the continuous investment model in which investors trade investment projects. Here we replace one continuous variable, investment, with another continuous variable, prices, and show that the model can generate herds. We add an infinite number of risk-neutral *market makers,* who set prices in a competitive fashion, together with some *idiosyncratic traders*. Otherwise, the basic setup is quite similar.

Including market makers is a convenient way of modeling competitive trade between informed and uninformed investors. In our model, all trades occur between investors and market makers. We could dispense with the market makers and have direct trades between investors, but doing so would complicate the notation of the investors' decision problems without altering the results.

Introducing idiosyncratic traders guarantees that the equilibrium of our model has trade. Moreover, in models without idiosyncratic traders, adverse selection leads market makers to price assets at the extremes of the distribution of values: market makers will sell a project only at its highest possible value and will buy a project only at its lowest possible value. As the number of signals increases, so does the adverse selection problem, and trade eventually disappears. While the adverse selection problem in our model with two signals is not severe, having idiosyncratic traders ensures that our results immediately generalize to models with many signals.

The signals about the underlying state arrive slowly to the economy, as in the continuous investment model, and are drawn from a distribution given by (3). In addition to the market makers, the model has an infinite number of risk-neutral agents. A fraction $1 - \alpha$ of the agents are idiosyncratic traders, and the rest are (standard) investors.

Idiosyncratic traders are imagined to have some personal reason that makes them want to buy or sell with probability $1/2$ each, in any period, regardless of the price and any other information. Since these agents trade for idiosyncratic reasons, we do not need to model their investment decisions or payoffs.

Briefly, the model works as follows. The market maker loses money to the informed investors, makes money with the idiosyncratic traders, and makes zero expected profits on trades. The zero expected profit condition is supposed to capture the idea that market makers are competitive.

Uninformed investors observe the trades in the market and infer some information about the informed traders' signals. This inference works as follows. In the relevant region of the equilibrium, when an investor sells a project to the market maker, the uninformed investor infers that the seller was either an informed investor with a low signal or an idiosyncratic trader. Likewise, when an investor buys a project from the market maker, the uninformed investor infers that the buyer was either an informed investor with a high signal or an idiosyncratic trader. The uninformed investors then update their beliefs according to Bayes' rule. The prices the market maker charges for buying and selling reflect the same set of inferences.

The uninformed investors face the same tension as in the continuous investment model, between investing immediately and waiting for more information, which is costly because of discounting. At some point, the gains from more information are outweighed by the costs of discounting, and the uninformed investors invest. Since these investors ignore any social benefits their actions may have in providing more information to the market, their actions are privately optimal but socially suboptimal. Thus, the model can generate inefficient cascades that we call *herds*.

More specifically, the rest of the model is as follows. Each investor is endowed with a risky project. Each project requires an investment of one unit of effort to become viable. Here the cost of effort generates an opportunity cost to investing in the risky project similar to that of the safe project in the continuous investment model. This investment pays off A units if the state is high and zero if the state is low. Thus, if an investor has beliefs p that the state is high and already owns a project, then the payoff from investing, net of the effort cost, is $pA - 1$.

In terms of market trades, the investors have three options: sell the project to a market maker, buy a second project from the market maker, or not trade. In terms of investments, we assume that if an investor buys a second project, then the investor must invest in both. This assumption is innocuous since in equilibrium an investor who wants to buy will always also want to invest as much as possible. The choices of investors in any period are, then, four: to sell a project, to buy a second project and invest in both, to invest in their own project without trading, or to do nothing and wait. We denote these choices by S, B, I, N, respectively.

These choices are affected by market prices. In general, there may be a vector of selling prices and a vector of buying prices. Clearly, only the highest selling price and the lowest buying price are relevant to agents' decisions. We let Q_{St} denote the price at which projects are sold to market makers, namely, the highest selling price, and Q_{Bt} the price at which projects are bought from market makers, namely, the lowest buying price.

Let B_t denote the number of agents who buy and S_t the number who sell in period t. These agents include both the idiosyncratic traders and the investors. The publicly observable events are $z_t = (B_t, S_t, Q_{Bt}, Q_{St})$. The public history is $h_t = (z_0, z_1, \ldots, z_{t-1})$. Here, as before, h_{it} denotes the history of investor i in period t. This history records the signal the investors have received, if any, and the period in which they received it, in addition to the public history. The model's active agents include idiosyncratic traders as well as the newly informed, the previously informed, and the uninformed investors. Strategies do not need to be defined for either idiosyncratic traders or inactive investors.

An investor's strategy and beliefs are sequences of functions $x_t(h_{it})$ and $p_t(h_{it})$ that map the investor's histories into actions $\{S, B, I, N\}$ and into the probability that the state is high. The payoff to an investor who buys in period t with history h_{it} and current prices Q_{Bt} and Q_{St} is

$$V_{Bt}(h_{it}) = [p_t(h_{it})A - 1] + [p_t(h_{it})A - 1 - Q_{Bt}]$$

while the payoff to an investor who sells is Q_{St} and the payoff to an investor who chooses I is $V_{It}(h_{it}) = p_t(h_{it})A - 1$. The payoff to an investor who waits in period t is

$$W_t(h_{it}) =$$

$$\sum_{h_{it+1}} \mu_t(h_{it+1}|h_{it}) \max\{V_{Bt+1}(h_{it+1}), Q_{St+1}(h_{t+1}), V_{It+1}(h_{it+1}), W_{t+1}(h_{it+1})\}/(1+r)$$

where $\mu_t(h_{it+1}|h_{it})$ is the conditional distribution over history in $t+1$ given the history in t. The future histories and the conditional distributions are induced from the strategies and the structure of exogenous uncertainty of the game in the obvious way.

Consider now the market makers. They do not receive signals. In each period t, each market maker posts prices at which an agent can buy or sell one risky project. The market maker understands that investors' signals determine whether investors want to buy or sell. Thus, when an agent wants to buy a risky project, the market maker has a different posterior than when an agent wants to sell. Hence, the market maker charges different prices for buying and selling. In equilibrium, competition among market makers ensures that the buying and selling prices each yield zero expected profits.

Let $Q^{'}_{Bt}$ and $Q^{'}_{St}$ denote the prices of a particular market maker. If an agent buys a project from this market maker, then the market maker receives a payoff of

$$Q'_{Bt} - \max\{p_B(h_t, Q'_{Bt})A - 1, 0\} \tag{14}$$

where $p_B(h_t, Q'_{Bt})$ is the posterior of this market maker. This posterior depends on the market maker's posted price because the mix of agents attracted to the market maker depends on the posted price. (Of course, the mix also depends on prices posted by other market makers, $Q_{Bt}(h_t)$ and $Q_{St}(h_t)$, but these prices are known functions of the history h_t and, hence, are suppressed.) If an agent sells a project, then the market maker receives a payoff of

$$\max\{p_S(h_t, Q'_{St})A - 1, 0\} - Q'_{St} \tag{15}$$

where $p_S(h_t, Q'_{St})$ is the posterior of the market maker given Q'_{St}.

One interpretation of these payoffs is that each market maker is endowed with a project which the market maker can run, and each market maker can buy a second project and run it too. The opportunity cost of selling the project to an agent is $\max\{p_S(h_t, Q'_{Bt})A-1, 0\}$. Thus, if an agent buys a project, the profits of the market maker are given by (14). Clearly, if an agent sells a project, the profits of the market maker are given by (15). Another interpretation of the payoffs is that market makers are intermediaries between the agents in this market and uninformed agents outside of it.

2.2. *Equilibrium*

An equilibrium in this model is a collection of strategies $x_t(h_{it}, Q_{Bt}, Q_{St})$, prices $Q_{Bt}(h_t)$ and $Q_{St}(h_t)$, and beliefs $p_t(h_t)$, $p_B(h_t, Q_{Bt})$, $p_S(h_t, Q_{St})$, and $p_t(h_{it})$ such that (i) the strategies $x_t(\cdot)$ are optimal for the investors, (ii) the prices set by market makers maximize profits given in (14) and (15), (iii) a market maker's profits evaluated at $Q_{Bt}(h_t)$ and $Q_{St}(h_t)$ are equal to zero, and (iv) where possible, the beliefs satisfy Bayes' rule.

Consider first the beliefs and strategies of the market makers. Competition among market makers drives their expected profits to zero on both purchases and sales. In equilibrium, the market maker loses by trading with informed investors and gains by trading with idiosyncratic investors. Before trading, the market makers' beliefs are the public beliefs. The information revealed in trading leads market makers to value individual projects between the value implied by public beliefs and the value assigned by the investor given the investor's private information.

In terms of characterizing the equilibrium, it is easier to write the strategies as functions of the public beliefs $p = p_t(h_t)$ rather than the histories h_t directly. The equilibrium prices charged by market makers fall into one of two regions; which one depends on the level of public beliefs p. First, consider a *discriminating* region in which the newly informed investor buys when the signal is high and sells when the signal is low. This is the only interesting discriminating action by the informed investor because selling when the signal is high and buying when the signal is low cannot be part of an equilibrium. We claim that in such a region the equilibrium prices are

$$Q_{Bt}(p) = \max\{P_U(p)A - 1, 0\} \tag{16}$$

$$Q_{St}(p) = \max\{P_D(p)A - 1, 0\} \tag{17}$$

where

$$P_U(p) = \frac{p[(1-\alpha)/2 + \alpha q]}{(1-\alpha)/2 + \alpha[pq + (1-p)(1-q)]} \tag{18}$$

$$P_D(p) = \frac{p[(1-\alpha)/2 + \alpha(1-q)]}{(1-\alpha)/2 + \alpha[p(1-q) + (1-p)q]}. \tag{19}$$

In this region, $P_U(p)$ turns out to be the posterior beliefs of the market maker conditional on receiving a buy order, and $P_D(p)$ turns out to be the posterior beliefs conditional on receiving a sell order. (The subscripts U and D indicate that the beliefs move up and down, respectively.)

We show that in the discriminating region, profits are zero at the equilibrium prices, and no market maker can gain by deviating. To evaluate the expected profits of the market maker, we need to form the posteriors of the market maker. Consider a buy decision by an agent. Using Bayes' rule, we see that the posterior of the market maker is $P_U(p)$, given in (18). The probability that the agent is an idiosyncratic trader is $(1-\alpha)/2$. The probability that the agent is an investor and receives a high signal is αq. The prior probability that the state is H is p. Thus, the probability that the state is high and an agent buys from the market maker is given by the numerator of (18). The denominator of (18) is simply the probability that an agent buys from the market maker. Similar reasoning establishes that the posterior of the market maker to whom agents sell is given by $P_D(p)$, or (19). Given these posteriors, it follows from (14) and (15) that the expected profits of the market maker are zero at the prices given by (16) and (17).

In this region, no market maker can gain by deviating. The interesting deviations are those that lower the price at which investors can buy and raise the price at which investors can sell. Clearly, in either case, expected profits are negative.

Consider next the region of beliefs in which the newly informed investor takes a *nondiscriminating action,* that is, the same action regardless of the signal. We claim that in such a region, the equilibrium prices are $Q_{Bt}(p) = \max\{pA-1, 0\}$ and $Q_{St}(p) = \max\{pA-1, 0\}$. Clearly, now a buy or sell decision does not convey any information to the market maker, and the market maker's posterior stays at the public belief p. Substituting these posteriors and the equilibrium prices into (14) and (15) gives that the expected profits of the market maker are zero at these prices. Clearly, any deviation by a market maker leads to negative profits.

In equilibrium, public beliefs in period $t+1$ are the same as the market maker's posteriors after trading in period t. These public beliefs will always be equal to $p(k)$ for some integer k, where $p(0) = p_0$, $p(1) = P_D(p_0)$, $p(-1) = P_U(p_0)$, and so on. This follows since P_U and P_D are symmetric, in the sense that $P_U\big(P_D(p)\big) = P_D\big(P_U(p)\big) = p$.

To characterize the equilibrium strategies of investors, we assume the analogs of (9), (10), and (11), that

$$p(0)A > 1 \tag{20}$$

$$p(-1)A < 1 \tag{21}$$

$$p(0)A - 1 < v_B\big(p(0)\big)[p(1)A - 1]/(1 + r) \tag{22}$$

where $v_B(p) = [(1 - \alpha)/2] + \alpha[pq + (1 - p)(1 - q)]$ is the probability that investors will see a buy when the public beliefs are p.

Under these assumptions, if the fraction of idiosyncratic traders, $1 - \alpha$, is sufficiently small, then the equilibrium outcome path is very similar to that in the continuous investment model. For such a fraction, the strategy for newly informed investors is to take a discriminating action if public beliefs are at $p(-1)$ or higher and to wait otherwise. The discriminating action is to buy if their signal is high and sell if their signal is low. The strategy of the uninformed and previously informed investors is to invest in the risky project if their beliefs are at least $p(1)$ and to wait if their beliefs are at or below $p(0)$. These investors never trade. From these strategies, it is easy to construct how beliefs evolve.

These strategies lead to the following equilibrium outcomes. Whenever the newly informed agent is an idiosyncratic trader, the equilibrium outcome is obvious. We focus on outcomes in which the newly informed agent is an investor. The newly informed investor in period 0 takes a discriminating action, and all uninformed investors wait.

If the agent in period 0 buys, then the public beliefs in period 1 rise to $p(1)$, and all uninformed investors invest, setting off a *cascade of investment*. If the agent in period 0 sells, these public beliefs fall to $p(-1)$, and uninformed investors wait. The newly informed investor in period 1 buys if the signal is high and sells if the signal is low.

If the agents in both periods 0 and 1 sell, then public beliefs fall to $p(-2)$, and no investor invests in period 2 or any subsequent period. This history starts a *cascade of no investment*. If there has been no investment in period 0, but an investment in period 1, then both the uninformed investors and the previously informed investor have a prior of $p(0)$, they wait, and the newly informed investor invests if and only if the signal is high.

Consider now the equilibrium outcomes for arbitrary α. For any given α, let k be defined as the smallest integer that satisfies

$$P_L\big(p(k)\big)A < 1 < P_H\big(p(k)\big)A.$$

The newly informed investors' strategies are to take a discriminating action if public beliefs are at $p(k)$ or greater and to not trade otherwise. Note that under (20) and (21) when $\alpha = 0$, $k = -1$. Thus, for α sufficiently small, $k = -1$ as well.

2.3. *Herds*

As before, we now define a *herd* relative to a public signal game. The public signal game which captures some of the information constraints of the private signal game is the following. In each period t, with probability α, a signal H or L is drawn from the distribution in (3); this signal is received by the agent at the beginning of the period and becomes public at the end of the period. With probability $(1-\alpha)/2$, an idiosyncratic trader buys, and at the end of the period, the public sees the signal H. With probability $(1-\alpha)/2$, an idiosyncratic trader sells, and at the end of the period, the public sees the signal L. The outcomes of the public signal game again correspond to the efficient outcomes of a suitably defined mechanism design problem.

The following assumption, the analog of (13), implies that the equilibrium cascades are inefficient:

$$p(1)A - 1 < \Big\{v_B\big(p(1)\big)[p(2)A-1] + v_S\big(p(1)\big)v_B\big(p(0)\big)[p(1)A-1]\Big\}\Big/(1+r) \qquad (23)$$

where $v_S(p) = [(1-\alpha)/2] + \alpha[p(1-q)+(1-p)q]$ is the probability that investors will see a sell when the public beliefs are p. In the appendix, we prove the following proposition:

Proposition 4. (*Herds with Prices*) *Under* (20)–(23), *the model has herds of both investment and no investment.*

In the model, we have investors making investment decisions as well as trading decisions. To better understand the role the investment decisions play, consider a version of the model with no investment decisions. In this version, imagine that all investment projects have been undertaken before the model starts. In the exogenous timing version of this model, in each period of the active phase, the private signal of the informed investor is revealed through market trades, and herd behavior cannot occur. (This version of the model and the results are very similar to those of Avery and Zemsky [2] and Glosten and Milgrom [11].)

The endogenous timing version of the model without investment decisions also has no herds. Here the only possibilities are to buy, to sell, and to wait. Clearly, the only way to get a herd in this version of the model is to have all the uninformed investors buy or sell at some public prior. Suppose that in some period there is a herd in which all uninformed investors buy. In this period, with positive probability, one of the buyers is informed. The market maker sets the buy price so as to make zero profits across the different types of buyers. At this price, the market maker loses by trading with informed investors and therefore must gain by trading with uninformed investors. Since uninformed investors would lose if they traded with the market maker, they would optimally choose not to trade. Therefore, there cannot be such a herd. A similar argument applies to herds of sales.

Our notion of a herd is quite different from that of Avery and Zemsky [2]. We follow the rest of the literature in assuming that a herd must be a cascade. In a cascade with investment, the uninformed investors choose not to wait for their private signals and invest in the market. In a cascade without investment, the uninformed investors choose not to wait for their private signals and invest outside of the market. In either case, the notion of cascade captures the idea that the uninformed investors are in some sense "following the crowd." In our definition of a *herd,* we add to this notion of cascade that the crowd is taking a socially inefficient action relative to some benchmark. Avery and Zemsky [2] modify the model of Glosten and Milgrom [11] in an attempt to revive herds. In their Proposition 2, however, they show that their model cannot generate cascades. Thus, if they had used any version of the standard definition of herds in the literature, their Proposition 2 also would prove that there are no herds in their model. Indeed, in their model, the public beliefs converge to a degenerate distribution on the true state, so there is no sense in which information becomes trapped forever.

Avery and Zemsky [2] pursue a different tack. They propose a definition of a herd quite different from that in the literature and then show that their model can produce this kind of behavior. Briefly, Avery and Zemsky say that an informed investor who buys in period t engages in herd behavior if three conditions are met. First, this investor's private information about the state is negative. Second, along the equilibrium path, the pattern of trading leads to an increase in the market maker's prior about the mean value of the asset. Third, after such a path of trading, the investor buys. Avery and Zemsky show that this seemingly odd behavior can occur when, in addition to private signals, all investors have a common piece of information that market markers do not have. In this setup, the market makers infer a noisier version of the underlying signal from the equilibrium trades than investors infer. This noisier inference leads market makers to update their beliefs more slowly than investors. Hence, even though an investor's private signal is negative, a string of buys can lead this investor to become more optimistic about the value of an asset than the market maker and hence lead the investor to buy.

Our view is that while the behavior generated by Avery and Zemsky's [2] model is both interesting and perverse, their notion of herds is quite different from that in the rest of the literature. In relation to our definition, it might be more precise to label the behavior they uncover "waves of optimism and pessimism" rather than herds.

3. Conclusion

We have here taken a step toward developing models of herd behavior that can be used in applied work. We have demonstrated that recent critiques of early herd models can be overturned when the exogenous timing of investment decisions in the models is replaced by a more natural endogenous timing. We think, therefore, that models of herd behavior have the potential to help us understand financial crises in emerging markets and elsewhere.

Here we have considered a variant of the early herd models with homogenous individuals and a simple structure for signals. Smith and Sorensen [17] extend the herd literature to heterogeneous individuals and a more general structure for signals to allow for situations in which, in equilibrium, agents' actions settle down to some limit distribution over a nontrivial set of actions, but learning is never complete. To capture this situation, the notion of a *herd* must be appropriately modified. With that modification, our results might fruitfully be extended to some of the more general setups considered by Smith and Sorensen.

Appendix
Definitions and Proofs

Defining Strategies and Beliefs

Here we formally define strategies and beliefs for our continuous investment model. At $p = p_t(h_{it})$, the strategy for all uninformed and previously informed investors is

$$x_t(h_{it}) = \{x(p) \text{ if } p \geq p(1)\} \tag{24}$$

and $x_t(h_{it}) = 0$ otherwise, while the strategy for newly informed investors is

$$x_t(h_{it}) = \{x(p) \text{ if } p \geq p(0)\} \tag{25}$$

and 0 otherwise. Notice that in order to invest, the uninformed and previously informed investors must be more optimistic than the newly informed investors. We refer to $p(1)$ and $p(0)$ as the *cutoff levels*—$p(1)$ for the uninformed and previously informed investors and $p(0)$ for the informed investors.

The beliefs of uninformed investors clearly coincide with the public beliefs. We say that public beliefs are in the *discriminating* region if they are equal to either $p(-1)$ or $p(0)$ and are in the *nondiscriminating* region otherwise. Given $p_{t-1}(h_{t-1})$ and a total investment of X_{t-1} in period $t-1$, the public beliefs in t are as follows. For $p_{t-1}(h_{t-1})$ in the discriminating region,

$$p_t(h_t) = \left\{ \begin{array}{l} P_L\big(p_{t-1}(h_{t-1})\big) \text{ if } X_{t-1} = 0 \\ P_H\big(p_{t-1}(h_{t-1})\big) \text{ if } X_{t-1} > 0 \end{array} \right\} \tag{26}$$

where $p_0(h_0) = p(0)$. In the nondiscriminating region, $p_t(h_t) = p_{t-1}(h_{t-1})$.

The beliefs of the newly informed investors at history $h_{it} = (h_t, s, t)$ are simply the public beliefs, updated by the newly informed investor's signal: $p_t(h_t, s, t) = P_s\big(p_t(h_t)\big)$ for $s = H, L$.

The beliefs of the previously informed investor in t who received a signal in $t-1$ with history $h_{it} = (h_t, s, t-1)$ are defined as follows. If no other investor invested in $t-1$, this investor's beliefs are the same in t as they were in $t-1$: $p_t(h_t, s, t) = P_s\big(p_{t-1}(h_{t-1})\big)$ for $s = H, L$. If some other investor invested in $t-1$, then $p_t(h_t, s, t) = p(2)$. The beliefs of previously informed investors who received their signals before period $t-1$ are recursively defined using (26) except that the recursion starts in period v, with the beliefs of the newly informed investor in v: $p_v(h_{iv}, s, v)$. These strategies and beliefs induce the conditional distributions $\mu_t(h_{it+1}|h_{it})$ in the obvious way.

Built into these beliefs is the idea that investors look at previous investors' actions and try to infer their signals. On the equilibrium path and for deviations that they cannot detect, investors infer the following. Consider the uninformed investors with public beliefs p in the discriminating region. If these investors see $X_t = x\big(P_H(p)\big)$, then they infer that the newly informed investor received a high signal. If they see $X_t = x\big(P_L(p)\big) = 0$, then they infer that the newly informed investor received a low signal. If public beliefs are equal to $p(-2)$, then uninformed investors expect to see no investment regardless of the newly informed investor's signal. Both the newly informed and the previously informed investors simply update the public beliefs with their private signal. We have also filled in beliefs off the equilibrium path in an intuitive way. Our results are unaffected by these choices.

Proving Propositions

PROPOSITION 1. *Under assumptions* (9)–(10) *and* (11)–(12), *the strategies and beliefs described above constitute a perfect Bayesian equilibrium.*

Proof of Proposition 1. By construction, the beliefs in (26) satisfy Bayes' rule. We repeatedly use the observation that by construction, for any history h_{it}, $p_t(h_{it}) = p(k)$ for some integer k.

Consider first optimality for histories with no detectable deviations. Start with the strategies of the uninformed investors. With public beliefs $p(0)$, (11) ensures that waiting is optimal, while with beliefs $p(-1)$ and below, (10) ensures this. With public beliefs $p(1)$ and above, all uninformed investors are supposed to invest. Suppose that an uninformed investor instead deviates and waits. Recall that for such a history, all other active investors have already invested. Thus, by waiting, the uninformed investor receives no new information from others. The only reason to wait, therefore, is that the investor might receive a signal. An upper bound on the payoffs from waiting is given by the case in which the investor is certain to receive a signal in the following period. Assumption (12) ensures that this deviation is not profitable.

Now turn to the strategies of the informed investors at some history h_{it}. The interesting histories are those in which public beliefs are $p(0)$ or $p(-1)$ and the newly informed investor has just received a high signal. Suppose, first, that the public beliefs are at $p(-1)$. A deviation by the informed investor causes public beliefs to drop to $p(-2)$ and triggers a cascade with no investment. This deviation brings the investor no new information, so by discounting, it is better to invest immediately.

Suppose next that the public beliefs are at $p(0)$, and the newly informed investor receives a high signal, so now has beliefs $p(1)$. The strategy for the newly informed investor specifies invest, but suppose this investor instead deviates and waits, presumably to garner information about the signals of subsequent informed investors. Note first that, in all subsequent periods, the beliefs of this deviating investor are 2 higher than those of the uninformed investors, in the sense that if public beliefs are at $p(k)$, the deviating investor's beliefs are at $p(k+2)$. The reason is that the newly informed investor's private signal raised the investors' beliefs by 1 and the deviation by the newly informed investor lowered the uninformed investors' beliefs by 1 without affecting the investor's own beliefs.

Next note that a herd of investment starts when uninformed investors' beliefs are at $p(1)$, and a herd of no investment starts when they are at $p(-2)$. Hence, in any period after the deviation, the deviating investor's beliefs can never be below $p(0)$ or above $p(3)$. Moreover, if this investor's beliefs reach $p(0)$, they stay there because the uninformed investors' beliefs are at $p(-2)$, and there is a cascade without investment. Consequently, it is optimal for the deviating investor to invest immediately at beliefs $p(0)$. If this investor's beliefs reach $p(3)$, they stay there because the uninformed investors' beliefs are at $p(1)$, and there is a cascade with investment. Consequently, it is optimal for the deviating investor to invest immediately at beliefs $p(3)$.

Since it is optimal for the deviating investor to invest at $p(0)$ and $p(3)$, a recursive application of (12) implies that the original deviation is not profitable. To see this formally, suppose that the optimal continuation strategy for the deviating investor is to invest at $p(2)$. Then (12) implies that the original deviation cannot be profitable. The reason is that then the right side of (12) is the payoff to the deviation of waiting and the left side is the payoff to investing immediately, where both sides are evaluated at $p = p(1)$. Similar reasoning establishes that if the optimal continuation strategy for the deviating investor is to invest at $p(1)$, then the original deviation cannot be profitable. The only possibility that remains is that the optimal continuation strategy for the deviating investor is to wait at both $p(1)$ and $p(2)$. We will rule out this possibility by contradiction. Suppose that the deviating investor waits at $p(1)$ and $p(2)$. Let $W\big(p(1)\big)$ and $W\big(p(2)\big)$ denote the payoffs at these beliefs, respectively. These payoffs are given by

$$W\big(p(1)\big) = \big[r + v_H\big(p(1)\big)W\big(p(2)\big) + v_L\big(p(1)\big)V\big(p(0)\big)\big]\big/(1+r) \tag{27}$$

$$W\big(p(2)\big) = \big[r + v_H\big(p(2)\big)V\big(p(3)\big) + v_L\big(p(2)\big)W\big(p(1)\big)\big]\big/(1+r). \tag{28}$$

Using (12), we can see that Eqs. (27) and (28) imply that

$$W\big(p(1)\big) < V\big(p(1)\big) + v_H\big(p(1)\big)\big[W\big(p(2)\big) - V\big(p(2)\big)\big]\big/(1+r) \tag{29}$$

$$W\big(p(2)\big) < V\big(p(2)\big) + v_L\big(p(2)\big)\big[W\big(p(1)\big) - V\big(p(1)\big)\big]\big/(1+r). \tag{30}$$

Substituting (30) into (29) gives an immediate contradiction. Thus, the optimal continuation strategy for a deviating investor must be to invest at either $p(1)$ or $p(2)$. It follows that the original deviation cannot be profitable.

For histories in which the newly informed investor's beliefs are at or below $p(-1)$, (10) implies that deviating to investing is not optimal.

Finally, it is easy to show that for histories off the equilibrium path, the strategies for all investors are optimal. *Q.E.D.*

PROPOSITION 4. (*Herds with Prices*) *Under* (20)–(23), *the model has herds of both investment and no investment.*

Proof of Proposition 4. The proof that these strategies of the investors are optimal and that the beliefs satisfy Bayes' rule is essentially identical to the proof of Proposition 1. The proof that the prices are optimal is in the text. The proof that there are herds of no investment is immediate.

To see that there are herds of investment, consider a period t in either the private or the public signal game in which public beliefs are at $p(1)$. In the private signal game, all investors invest. In the public signal game, (23) implies that the uninformed investors wait. Waiting dominates investing. To see that, consider the following (possibly suboptimal) strategy: Wait in t. If the end-of-period t public signal is H, invest in $t+1$. If the end-of-period t public signal is L, wait in $t+1$. If the end-of-period $t+1$ signal is H, invest in $t+2$; otherwise, never invest. The payoff to this strategy is the right side of (23), which by assumption is larger than the payoff to investing in t, $p(1)A-1$. *Q.E.D.*

Notes

[1]One interpretation of this environment is that the risky project consists of investments in an emerging economy while the safe project consists of investments in the United States. Returns from the risky project depend on whether the emerging market's government expropriates the investments. Herds in this context consist of sudden reversals of capital flows. For a model of herd behavior and expropriation with this interpretation, see Chari and Kehoe [8].

[2]We can imagine that agents are randomly drawn without replacement from the pool of agents and assigned a number designating the period in which each will receive a signal. Neither the names of the agents who will receive the signals nor the periods in which these agents will receive signals are observed, but the process for assigning names and periods is common knowledge.

Also note that conceptually, it is easy to instead allow signals to arrive intermittently, say, according to a Poisson process. The results are similar; however, the resulting algebra is more complicated.

References

[1] F. Allen, S. Morris, and A. Postlewaite, Finite bubbles with short sale constraints and asymmetric information, *J. Econ. Theory* **61** (1993), 206–229.

[2] C. Avery and P. Zemsky, Multidimensional uncertainty and herd behavior in financial markets, *Amer. Econ. Rev.* **88** (1998), 724–748.

[3] A. V. Banerjee, A simple model of herd behavior, *Quart. J. Econ.* **107** (1992), 797–817.

[4] S. Bikhchandani, D. Hirshleifer, and I. Welch, A theory of fads, fashion, custom, and cultural change in informational cascades, *J. Polit. Economy* **100** (1992), 992–1026.

[5] G. A. Calvo and E. G. Mendoza, "Mexico's Balance-of-Payments Crisis: A Chronicle of a Death Foretold," manuscript, University of Maryland, 1995.

[6] A. Caplin and J. Leahy, Business as usual, market crashes, and wisdom after the fact, *Amer. Econ. Rev.* **84** (1994), 548–565.

[7] C. Chamley and D. Gale, Information revelation and strategic delay in a model of investment, *Econometrica* **62** (1994), 1065–1085.

[8] V. V. Chari and P. J. Kehoe, "Hot Money," Research Department Staff Report 228, Federal Reserve Bank of Minneapolis, 2003.

[9] H. L. Cole and T. J. Kehoe, Self-fulfilling debt crises, *Rev. Econ. Stud.* **67** (2000), 91–116.

[10] A. Devenow and I. Welch, Rational herding in financial economics, *Europ. Econ. Rev.* **40** (1996), 603–615.

[11] L. R. Glosten and P. R. Milgrom, Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders, *J. Finan. Econ.* **14** (1985), 71–100.

[12] S. Grossman and J. E. Stiglitz, On the impossibility of informationally efficient markets, *Amer. Econ. Rev.* **70** (1980), 393–408.

[13] F. Gul and R. Lundholm, Endogenous timing and the clustering of agents' decisions, *J. Polit. Economy* **103** (1995), 1039–1066.

[14] I. H. Lee, On the convergence of informational cascades, *J. Econ. Theory* **61** (1993), 395–411.

[15] I. H. Lee, Market crashes and informational avalanches, *Rev. Econ. Stud.* **65** (1998), 741–759.

[16] J. D. Sachs, A. Tornell, and A. Velasco, The Mexican peso crisis: Sudden death or death foretold? *J. Int. Econ.* **41** (1996), 265–283.

[17] L. Smith and P. Sorensen, Pathological outcomes of observational learning, *Econometrica* **68** (2000), 371–398.

[18] X. Vives, Learning from others: A welfare analysis, *Games Econ. Behav.* **20** (1997), 177–200.