Disturbance Observer Based Discrete Time Sliding Mode Control for a Continuous Stirred Tank Reactor

Luning Ma[1], Dongya Zhao[1], Sarah K. Spurgeon[2]

***Abstract*— The continuous stirred tank reactor (CSTR) is a typical element of equipment frequently found in the process industry. The control of a CSTR is very challenging due to its nonlinear dynamics and the presence of external disturbances. In this paper, a novel discrete time sliding mode control method for the CSTR is presented. Then a disturbance observer is designed to eliminate the effect of external disturbances on the CSTR system. Stability analysis is presented based on the Lyapunov method. Finally, the control performance of the proposed method is validated by using MATLAB simulation experiments.**

***Index Terms*— discrete time sliding mode control; disturbance observer; continuous stirred tank reactor**

I. INTRODUCTION

The strongly nonlinear and coupled nature of chemical processes provide particular challenges for control. Accurate and robust control of such processes is however essential to ensure the appropriate quality of chemical products. The CSTR is representative of a class of typical chemical process equipment. It is attractive from the point of view of costs and can ensure stable product quality, which makes it be widely used in the chemical industry [1]. However, due to the complexity and non-linearity of the CSTR system, traditional control methods such as PID [2], robust control [3], adaptive control [4] may not yield the required high control performance. Various advanced control methods have been proposed for the CSTR such as fuzzy control [5], predictive control [6] and sliding mode control [7].

Sliding mode control (SMC) is a special class of variable structure control and is known to exhibit excellent robustness properties. There are many successful applications studies which use SMC in the CSTR system [8]. However, most of the existing SMC algorithms applied to the CSTR are continuous in nature. In terms of implementation in industry which is likely to be computer based, it is of interest to explore discrete time SMC [9][10]. Direct discretisation of a continuous sliding mode control algorithms may induce large chattering, discretization errors, or even instability. In order to solve these issues, discrete time sliding mode control (DSMC) theory has been developed.

A large number of DSMC methods have been proposed in the literature [11][12]. One of the most commonly used methods is the reaching law based DSMC proposed by Gao et al [13]. The concept of DSMC was proposed in [14] and employed the reaching law method from [15]. The reaching law method has many advantages, such as simplifying the design process of DSMC and maintaining the overall robustness of the control system [16]. However, the property of total invariance to matched uncertainty which is exhibited in continuous time is lost, and chattering will occur due to the discontinuous control term. In practice, the system state trajectory in DSMC is restricted by the reaching law but it cannot move along the sliding surface. The area in which the system state moves near the sliding surface is called the quasi-sliding mode domain [17]. Therefore, the system states cannot reach to the equilibrium point and will perform a zigzag movement around the equilibrium point. The analysis in [18] shows that the chattering is caused by the control method, which is closely related to the sampling time and switching frequency. In theory, to eliminate chattering, the switching frequency must be infinite, which is impossible in practice. This motivated the original work by Utkin, who proposed a DSMC to achieve chatter free motion with a finite sample time. This early work developed DSMC for linear systems [18].

Practical systems frequently are subject to disturbances, and the existence of disturbances may affect the control performance. If the disturbance can be estimated, it is possible to eliminate or minimise the effects [19][20]. In recent decades, the notion of a disturbance observer has been extensively studied and applied within controller design frameworks [21][22]. In [23], disturbance observers for discrete-time nonlinear systems are designed, and the gains of the disturbance observers need to be obtained via solution to an LMI. In [24], an algorithm is developed to exactly decouple the disturbance estimation dynamics from the sliding mode dynamics. Such DSMC schemes incorporating a disturbance observer typically use the reaching law approach which may lead to chattering [25].

Inspired by the existing literature, this study proposes a novel discrete sliding mode control law to stabilize the nominal CSTR system. Then a disturbance observer is designed to eliminate the effects of an external disturbance. In comparison with the reaching law based DSMC[26], the proposed approach designs a novel DSMC using Uktin's method, which is known to reduce the chattering effectively; in comparison with Utkin's DSMC approach [18], the proposed approach is designed for a class of nonlinear systems and incorporates a disturbance observer. The method is shown to provide good control of a CSTR.

The remainder of this paper is organized as follows.

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Section II presents the CSTR dynamic model as well as some definitions that will be used in the subsequent sections. In Section III, a novel DSMC is proposed and applied to a nominal model of the CSTR. Section IV extends the DSMC concept to include a disturbance observer. Section V evaluates the performance of the designed controller using MATLAB simulations. Finally, section VI concludes this paper.

II. PROBLEM DESCRIPTION

The diagram of the CSTR is shown in Fig. 1 and a nondimensional dynamic model from [27] is given by

$$\begin{aligned} \dot{x}_1 &= -ax_1 + D_a(1-x_1)\exp(\frac{\gamma x_2}{\gamma + x_2}) \\ \dot{x}_2 &= -ax_2 - bD_a(1-x_1)\exp(\frac{\gamma x_2}{\gamma + x_2}) \\ &\quad + \beta(u - x_2 + d) \\ y &= x_2 \end{aligned} \tag{1}$$

where $x_1 \in R$ is the concentration, $x_2 \in R$ is the temperature, $u \in R$ is the control input, $y \in R$ is the system output and $d \in R$ is the external disturbance. The parameters of the dynamic model are given by: a, β, γ, b and D_a, which are all positive constants.

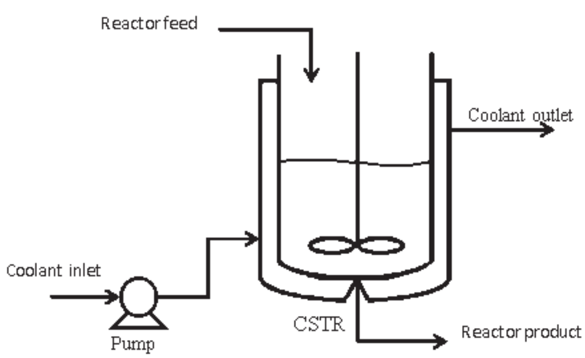

Fig. 1. The model of CSTR

From equation (1), a discrete time model of the nonlinear CSTR system can be obtained by using the Euler difference method as follows

$$\begin{aligned} x_1(k+1) &= \left[-ax_1(k) + Da(1-x_1(k))\,\mathrm{e}^{\left(\frac{\gamma x_2(k)}{\gamma + x_2(k)}\right)}\right]T \\ &\quad + x_1(k) \\ x_2(k+1) &= \left[-ax_2(k) + bDa(1-x_1(k))\,\mathrm{e}^{\left(\frac{\gamma x_2(k)}{\gamma + x_2(k)}\right)}\right. \\ &\quad \left. + \beta(u(k) - x_2(k) + d(k))\right]T + x_2(k) \\ y(k) &= x_2(k) \end{aligned} \tag{2}$$

For the sake of simplicity, this model can be rewritten as

$$\begin{aligned} x_1(k+1) &= f_1(x(k)) \\ x_2(k+1) &= f_2(x(k)) + g(u(k) + d(k)) \end{aligned} \tag{3}$$

where

$$\begin{aligned} f_1(x(k)) &= \left[-ax_1(k) + Da(1-x_1(k))\,\mathrm{e}^{\left(\frac{\gamma x_2(k)}{\gamma + x_2(k)}\right)}\right]T \\ &\quad + x_1(k) \\ f_2(x(k)) &= \left[-ax_2(k) + bDa(1-x_1(k))\,\mathrm{e}^{\left(\frac{\gamma x_2(k)}{\gamma + x_2(k)}\right)}\right. \\ &\quad \left. -\beta x_2(k)\right]T + x_2(k) \end{aligned}$$

$x(k) = [x_1(k), x_2(k)]^T$, $g = \beta T$, $d(k)$ is the disturbance existing in the system.

Define the tracking error of the controlled system:

$$\begin{aligned} e(k) &= y(k) - r \\ &= x_2(k) - r \end{aligned} \tag{4}$$

where $r \in R$ is the desired value of x_2 and it is a constant. The corresponding sliding variable is defined by

$$s(k) = ce(k) \tag{5}$$

where $c > 0$ is a constant.

III. SLIDING MODE CONTROLLER DESIGN

From equation (3), the nominal model of the CSTR system can be described as

$$\begin{aligned} x_1(k+1) &= f_1(x(k)) \\ x_2(k+1) &= f_2(x(k)) + gu(k) \end{aligned} \tag{6}$$

$s(k+1)$ can be obtained from (5):

$$\begin{aligned} s(k+1) &= ce(k+1) \\ &= c(x_2(k+1) - r) \\ &= c(f_2(x(k)) + gu(k) - r) \end{aligned} \tag{7}$$

To make the state of $x_2(k+1)$ converge to the desired value r in a finite number of steps, $s(k+1)$ should be zero, and the control input can be derived from (7) as

$$u(k) = -g^{-1}(f_2(x(k)) - r) \tag{8}$$

The control input in (8) is the equivalent control which can be rewritten as the sum of two functions:

$$u_{eq}(k) = -(cg)^{-1}(s(k) + c(f_2(x(k)) - x_2(k))) \tag{9}$$

and

$$s(k+1) = s(k) + c(f_2(x(k)) + gu(k) - x_2(k)) \tag{10}$$

To avoid $u_{eq}(k)$ exceeding actuator limitations, the bounds of the control must taken into account. Assume the practical input $u(k)$ is bounded by $\|u(k)\| \leq u_0$. The corresponding control law is defined by

$$u(k) = \begin{cases} u_{eq}(k) & \|u_{eq}(k)\| \leq u_0 \\ \frac{u_{eq}(k)}{\|u_{eq}(k)\|}u_0 & \|u_{eq}(k)\| > u_0 \end{cases} \tag{11}$$

When $u(k) = u_{eq}(k)$ for $u_{eq}\|(k)\| \leq u_0$, the system is already in the sliding mode $s = 0$, therefore only the case $\|u_{eq}(k)\| > u_0$ needs to be proved.

Assumption 1: The available control limits u_0 are satisfied with $u_0 > \left\|(cg)^{-1}\right\| \|c(f_2(x(k)) - x_2(k))\|$.

Theorem 1: If the control law is designed as (9) and (11) under Assumption 1, then the closed-loop control system will be asymptotically stable.

Proof:

Select a Lyapunov candidate function as

$$V(k) = \|s(k)\| \tag{12}$$

According to the form of the control law, there are two cases to be considered.

(1) For the case $\|u_{eq}(k)\| \leq u_0$.

It is obvious that $\|s(k+1)\| = 0 \leq \|s(k)\|$ when $u(k) = u_{eq}(k)$, hence the system is stable.

(2) For the case $\|u_{eq}(k)\| > u_0$.

From equation (7) and equation (11), it follows that

$$\begin{aligned} s(k+1) = (s(k) + c\,(f_2(x(k)) \\ -x_2(k)))\left(1 - \frac{u_0}{\|u_{eq}(k)\|}\right) \end{aligned} \tag{13}$$

so

$$\begin{aligned} \|s(k+1)\| &= \|(s(k) + c\,(f_2(x(k)) \\ &\quad -x_2(k)))\|\left(1 - \frac{u_0}{\|u_{eq}(k)\|}\right) \\ &\leq \|s(k)\| + \|c\,(f_2(x(k)) - x_2(k))\| \\ &\quad - \frac{u_0}{\left\|(cg)^{-1}\right\|} \\ &< \|s(k)\| \end{aligned} \tag{14}$$

Appealing to Assumption 1, $u_0 > \left\|(cg)^{-1}\right\| \|c\,(f_2(x(k)) - x_2(k))\|$. It follows that $\|s(k+1)\| < s(k)$ and the system is asymptotically stable.

IV. DISTURBANCE OBSERVER BASED SLIDING MODE CONTROLLER DESIGN

The dynamic model of the CSTR in the presence of disturbances is given by

$$\begin{aligned} x_1(k+1) &= f_1(x(k)) \\ x_2(k+1) &= f_2(x(k)) + g\,(u(k) + d(k)) \end{aligned} \tag{15}$$

In this case, $s(k+1)$ can be derived as

$$\begin{aligned} s(k+1) &= ce(k+1) \\ &= c\,(x_2(k+1) - r) \\ &= c\,(f_2(x(k)) + gu(k) + gd(k) - r) \end{aligned} \tag{16}$$

The control law is defined as

$$u(k) = u_1(k) + u_2(k) \tag{17}$$

where $u_1(k)$ is designed to stabilize the nominal portion of the dynamic model and $u_2(k)$ is designed to eliminate the effect of the disturbance on the closed-loop system. As in the nominal case, $u_1(k)$ (9) is given as

$$u_1(k) = -(cg)^{-1}\,(s(k) + c\,(f_2(x(k)) - x_2(k))) \tag{18}$$

To deal with the effect of the disturbance, $u_2(k)$ is designed as

$$u_2(k) = -\hat{d}(k) \tag{19}$$

where $\hat{d}$ is the output of a disturbance observer; this represents an estimate of the disturbance $d(k)$. The disturbance observer is defined by

$$\hat{d}(k) = \hat{d}(k-1) + \mu(cg)^{-1}s(k) \tag{20}$$

Define the estimation error by

$$\tilde{d}(k) = d(k) - \hat{d}(k) \tag{21}$$

Assumption 2: The rate of change of the external disturbance satisfies $|d(k+1) - d(k)| < m$ and is assumed small.

The estimation error dynamics are given by

$$\tilde{d}(k+1) = (1-\mu)\,\tilde{d}(k) + d(k+1) - d(k) \tag{22}$$

where $0 < \mu < 1$ is constant.

Lemma 1: If $|d(k+1) - d(k)| < m$ is satisfied, then the disturbance observer error will be bounded, that is, $\tilde{d}(k)$ will decrease to a small residual set m/μ [24].

After a finite number of steps, the disturbance observer tracking error $\tilde{d}(k)$ will satisfy

$$\tilde{d}(k) < m/\mu \tag{23}$$

and if m is small enough, $\tilde{d}(k)$ will converge to zero.

From the definition of the control input (17), substitute (18) and (19) into $s(k+1)$ to yield

$$\begin{aligned} s(k+1) &= c\,(f_2(x(k)) + g\,(u_1(k) + u_2(k)) \\ &\quad +gd(k) - r) \\ &= cg\tilde{d}(k) \end{aligned} \tag{24}$$

Then $\tilde{d}(k)$ is given by

$$\tilde{d}(k) = (cg)^{-1}s(k+1) \tag{25}$$

According to (22)

$$\hat{d}(k+1) = \hat{d}(k) + \mu(cg)^{-1}s(k+1) \tag{26}$$

As in the case of the nominal control law (11), the practical bounds for the control $u(k)$ should be taken into account to avoid exceeding the actuator limitations. Assume the input $u(k)$ is bounded by $\|u(k)\| \leq u_0$. Then the control law can be defined by

$$u(k) = \begin{cases} u(k) & \|u(k)\| \leq u_0 \\ \frac{u(k)}{\|u(k)\|}u_0 & \|u(k)\| > u_0 \end{cases} \tag{27}$$

Assumption 3: The available control limits u_0 satisfy $u_0 > (\|cf_2(x(k)) + cgd(k)\|)^{-1}\left\|(g)^{-1}f_2(x(k)) + \hat{d}(k)\right\|\left(\left\|cg\tilde{d}(k)\right\| + \|c\,(f_2(x(k)) - x_2(k)) + cgd(k)\|\right)$.

Theorem 2 : If the control law is designed as (27) and the disturbance observer is designed as (20) using Assumptions 2 and 3, then the closed-loop control system will be asymptotically stable.

Proof:

Select a Lyapunov candidate function as

$$V(k) = \|s(k)\| \tag{28}$$

According to the control law (27), there are two cases to consider.
(1) For the case $\|u(k)\| \le u_0$.

$$\begin{aligned} \|s(k+1)\| &= \|c(f_2(x(k)) + gu(k) + gd(k) - r)\| \\ &= \left\|cg\tilde{d}(k)\right\| \end{aligned} \tag{29}$$

According to Lemma 1 and (23), it can be shown that after a finite number of steps $s(k+1)$ satisfies

$$\begin{aligned} ||s(k+1)|| &= \left\|cg\tilde{d}(k)\right\| \\ &\le cgm/\mu \end{aligned} \tag{30}$$

When μ is small enough.

$$||s(k+1)|| \to 0$$

(2) For the case $\|u(k)\| > u_0$.

$$s(k+1) = s(k) + c(f_2(x(k)) + gu(k) + gd(k) - x_2(k)) \tag{31}$$

Then

$$\begin{aligned} &\|s(k+1)\| = \|s(k) + c(_2(x(k)) + gu(k) + gd(k) \\ &-x_2(k))\| \\ &= \left\|s(k) + c\left(f_2(x(k)) + g\left(\frac{u(k)}{\|u(k)\|}u_0\right) + gd(k)\right.\right. \\ &-x_2(k))\| \\ &\le \|s(k) + c(f_2(x(k)) - x_2(k)) \\ &+cgd(k)\| \left(1 - \frac{u_0}{\|u(k)\|}\right) + \left\|cg\tilde{d}(k)\right\| \\ &\le \|s(k)\| + \|c(f_2(x(k)) - x_2(k)) + cgd(k)\| \\ &- \|cf_2(x(k)) + cgd(k)\| \frac{u_0}{\|u(k)\|} + \left\|cg\tilde{d}(k)\right\| \\ &= \|s(k)\| + \|c(f_2(x(k)) - x_2(k)) + cgd(k)\| \\ &- \|cf_2(x(k)) + cgd(k)\| \frac{u_0}{\left\|(g)^{-1} f_2(x(k)) + \hat{d}(k)\right\|} \\ &+ \left\|cg\tilde{d}(k)\right\| \\ &\le \|s(k)\| \end{aligned} \tag{32}$$

According to Assumption 3, the closed loop system will be asymptotically stable.

V. SIMULATION ANALYSIS

The proposed control approach is now validated using MATLAB on a nonlinear CSTR model. The parameters of the dynamic equation of the CSTR are given as: $a = 1.0$, $\beta = 0.3$, $\gamma = 20.0$, $b = -8.0$, $D_a = 0.072$, $T = 0.05$, $u_0 = 50$. The initial values of the states are chosen as $x_1 = 0.5$, $x_2 = 3$ and the reference output is chosen as $r = 4$.

Case 1: DSMC in the absence of disturbance.

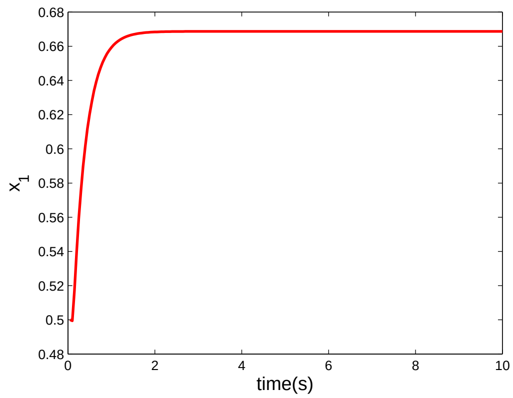

Fig. 2. x_1 performance of the DSMC controller for the nominal CSTR system without disturbance

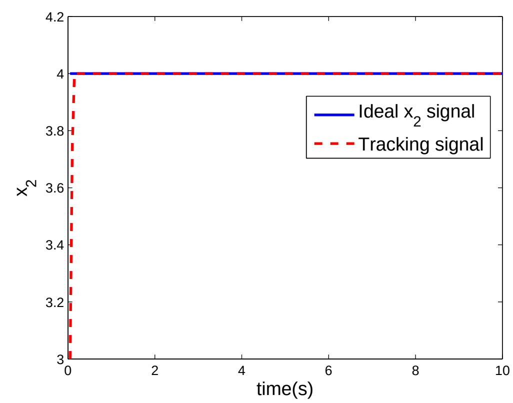

Fig. 3. x_2 performance of the DSMC controller for the nominal CSTR system without disturbance

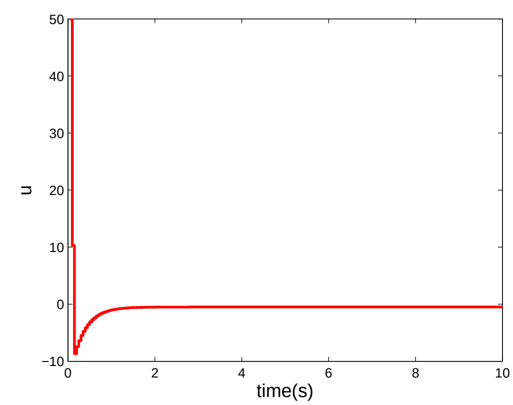

Fig. 4. The control input of the DSMC controller for the nominal CSTR system without disturbance

The control performance for the CSTR is shown in Fig. 2, Fig. 3 and Fig. 4. In the absence of the external disturbance, the closed-loop system performs very well and the control input is bounded. It can be seen from Fig. 2 and Fig. 3 that the proposed control has good convergence speed and there is no chattering.

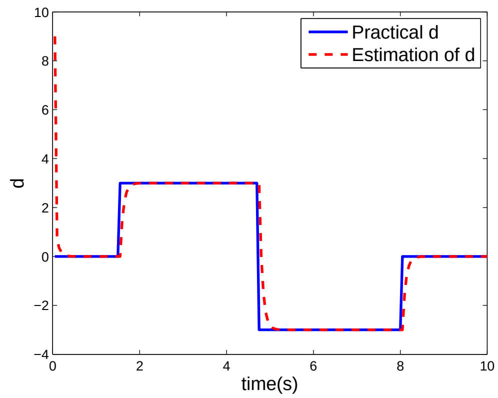

Fig. 5. The output of disturbance observer when disturbance is rectangular wave

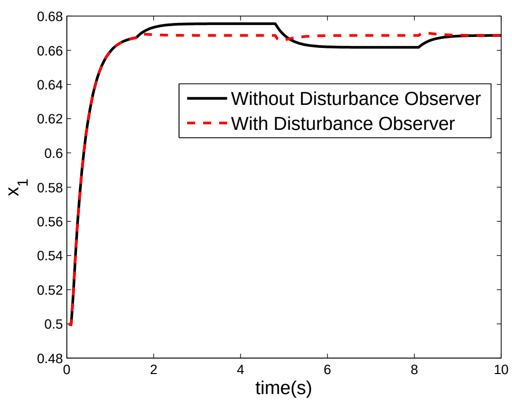

Fig. 6. Comparison of x_1 between DSMC with and without disturbance observer for rectangular wave disturbance

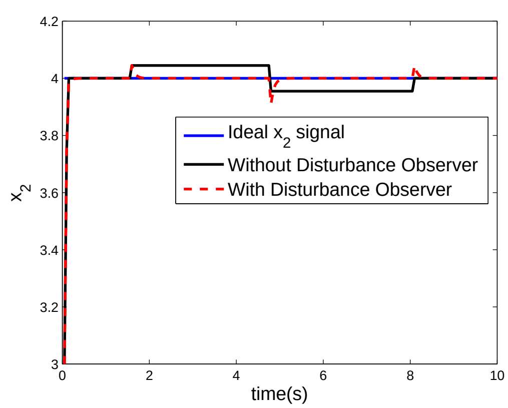

Fig. 7. Comparison of x_2 between DSMC with and without disturbance observer for rectangular wave disturbance

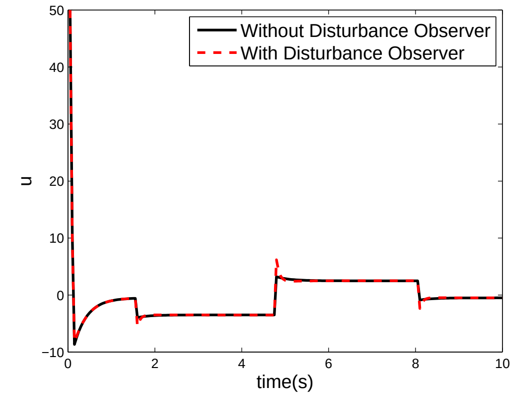

Fig. 8. Comparison of u between DSMC with and without disturbance observer for rectangular wave disturbance

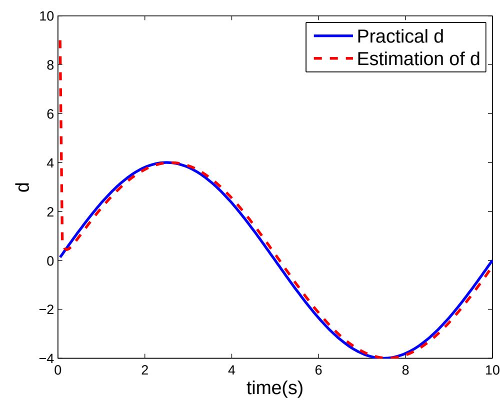

Fig. 9. The output of disturbance observer when disturbance is sine wave

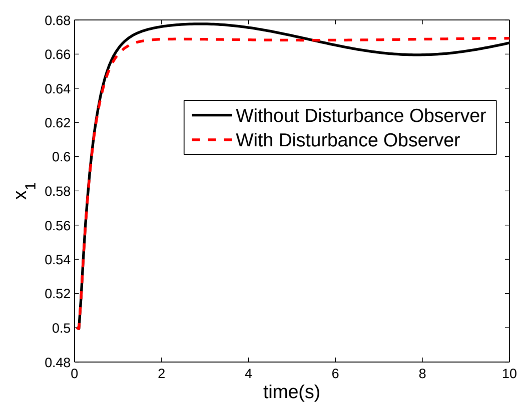

Fig. 10. Comparison of x_1 between DSMC with and without disturbance observer for sine wave disturbance $d = 4sin(0.2\pi t)$

Case 2: Disturbance observer based DSMC in the presence of a rectangular wave disturbance.

The output of the disturbance observer is shown in Fig. 5 when the disturbance is a rectangular wave. Fig. 6 and Fig. 7 show the tracking performance when the DSMC is augmented with and is without the disturbance observer. Fig. 8 shows the control input signals. It can be seen from Fig. 6

and Fig. 7 that the DSMC with disturbance observer exhibits stronger robustness in the presence of a rectangular wave disturbance.

Case 3: Disturbance observer based DSMC in the presence

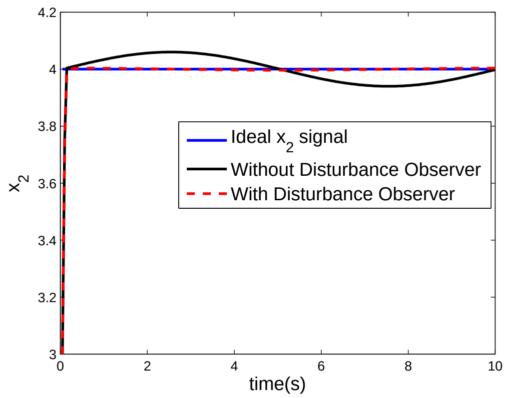

Fig. 11. Comparison of x_2 between DSMC with and without disturbance observer for sine wave disturbance $d = 4sin(0.2\pi t)$

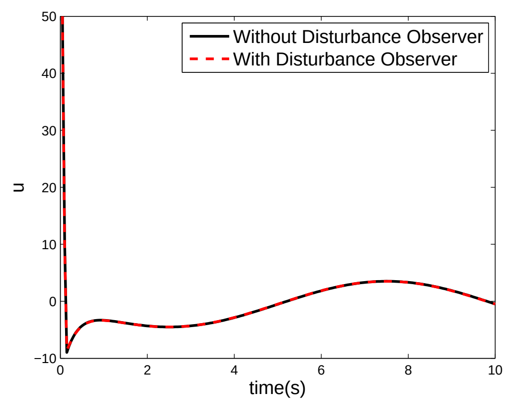

Fig. 12. Comparison of u between DSMC with and without disturbance observer for sine wave disturbance $d = 4sin(0.9\pi t)$

of a sine wave disturbance.

Fig. 9 shows the output of the disturbance observer when the external disturbance is the sine wave $d = 4\sin(0.2\pi t)$. Fig. 10 and Fig. 11 show the tracking performance when the DSMC is implemented with and without the disturbance observer, respectively. Fig. 12 shows the control inputs. It can be seen from Fig. 9 that the implementation with the disturbance observer has better performance when the disturbance is a sine wave. It can be seen from Fig. 10 and Fig. 11 that the DSMC with disturbance observer has stronger robustness to the sine wave disturbance. From Fig. 12, the control input in the presence of the sine wave disturbance is smoother than the control input in Fig. 8 as may be expected from the characteristics of the disturbance; no chattering is induced by the proposed scheme.

Case 4: Comparisons of the control performance of the disturbance observer based DSMC in the presence of high and low frequency sine wave disturbances.

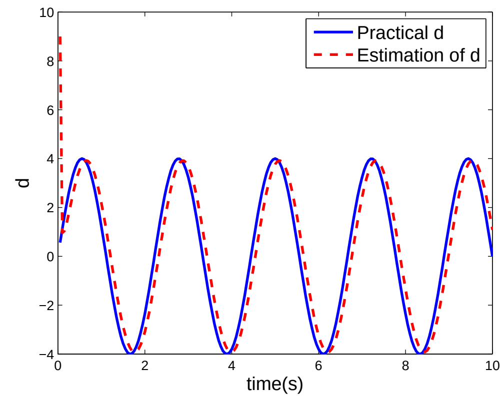

Fig. 13. The output of disturbance observer when disturbance is sine wave $d = 4sin(0.9\pi t)$

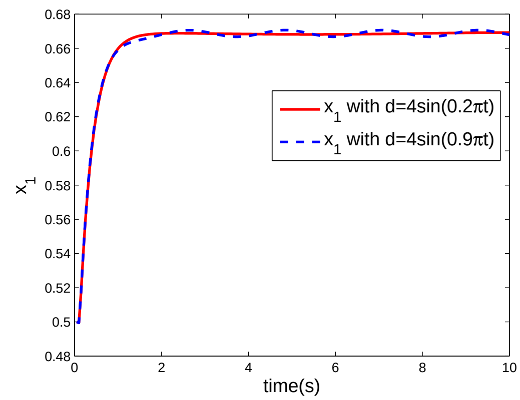

Fig. 14. Comparison of x_1 between disturbance $d = 4sin(0.9\pi t)$ and disturbance $d = 4sin(0.2\pi t)$

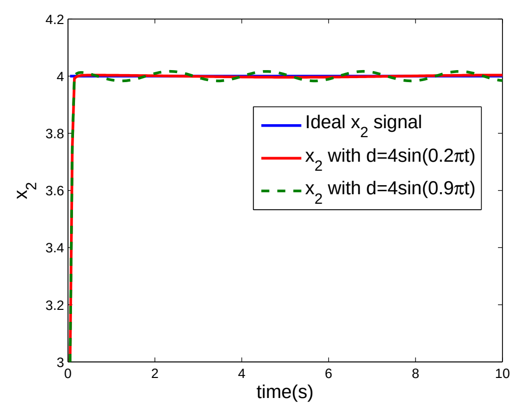

Fig. 15. Comparison of x_2 between disturbance $d = 4sin(0.9\pi t)$ and disturbance $d = 4sin(0.2\pi t)$

Fig. 13 presents the output of the disturbance observer when the external disturbance is a higher frequency sine wave $d = 4\sin(0.9\pi t)$. Fig. 14, Fig. 15 and Fig. 16 show the performances of x_1, x_2 and u in the presence of disturbances of different frequencies, respectively. It can be seen from the comparisons of Fig. 5, Fig. 9, and Fig. 13 that the observer

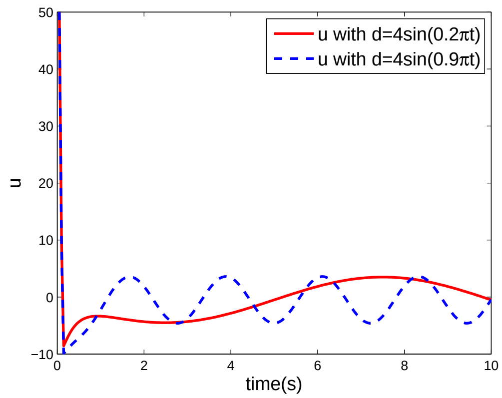

Fig. 16. Comparison of u between disturbance $d = 4sin(0.9\pi t)$ and disturbance $d = 4sin(0.2\pi t)$

has better performance with the low frequency disturbance as would be expected from Assumption 2. It can be seen from Fig. 14 and Fig. 15 that the controller has higher control precision for the low frequency sine wave.

VI. CONCLUSIONS

In this paper, a novel DSMC is proposed to achieve stability of a nonlinear CSTR. A key design requirement has been to reduce the chattering exhibited by the system. To eliminate the effect of external disturbances, a disturbance observer is combined with DSMC and the method is shown to be very effective when the external disturbance is slowly varying. The theoretical analysis and simulation results demonstrate that the proposed method is effective. Future work will focus on experiments with a CSTR rig.

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