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Model independent $H(z)$ reconstruction using the cosmic inverse distance ladder

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ABSTRACT

Recent distance ladder determinations of the Hubble constant H_0 disagree at about the 3.5σ level with the value determined from *Planck* measurements of the cosmic microwave background (CMB) assuming a ΛCDM[1] cosmology. This discrepancy has prompted speculation that new physics might be required beyond that assumed in the ΛCDM model. In this paper, we apply the inverse distance ladder to fit a parametric form of $H(z)$ to baryon acoustic oscillation (BAO) and Type Ia supernova (SNe) data together with priors on the sound horizon at the end of the radiation drag epoch, r_d. We apply priors on r_d, based on inferences from either *Planck* or the Wilkinson Microwave Anisotropy Probe (WMAP), and demonstrate that these values are consistent with CMB-independent determination of r_d derived from measurements of the primordial deuterium abundance, BAO, and supernova data assuming the ΛCDM cosmology. The $H(z)$ constraints that we derive are independent of detailed physics within the dark sector at low redshifts, relying only on the validity of the Friedmann–Robertson–Walker metric of General Relativity. For each assumed prior on r_d, we find consistency with the inferred value of H_0 and the *Planck* ΛCDM value and corresponding tension with the distance ladder estimate.

Key words: cosmology: observations – distance scale – cosmological parameters – large-scale structure of Universe.

1 INTRODUCTION

The *Planck* satellite has provided strong evidence in support of the ΛCDM cosmology and has measured the six parameters that define this model to high precision (Planck Collaboration et al. 2014, 2016, hereafter P14 and P16, respectively). In particular, P16[2] found a value of the Hubble constant of $H_0 = 67.27 \pm 0.66\,\mathrm{km\,s^{-1}Mpc^{-1}}$. As pointed out in P16, other data combinations give similar values of H_0, for example combining WMAP and BAO data gives $H_0 = 68.0 \pm 0.7\,\mathrm{km\ s^{-1}Mpc^{-1}}$. A 'low' value of H_0 is therefore not solely driven by high-multipole CMB anistoropies measured by *Planck* but is necessary if the ΛCDM cosmology is to fit a range of cosmological data.

In contrast, direct measurements of the cosmic distance scale have consistently found a higher value of H_0. The SH0ES[3] project uses Cepheid period–luminosity relations, together with local distance anchors, to calibrate distances to Type Ia SNe host galaxies. The SH0ES programme has reported measurements of H_0 of increasing precision over the last few years (Riess et al. 2009, 2011, 2016, 2018c). The latest value from the SH0ES collaboration[4] is $H_0 = 73.48 \pm 1.66\,\mathrm{km\ s^{-1}Mpc^{-1}}$ (Riess et al. 2018c, hereafter R18), which is consistent with but has a much smaller error than earlier determinations from the *Hubble Space Telescope* key project (Freedman et al. 2001).

The 3.5σ difference between the SH0ES determination of H_0 and the value inferred from *Planck* for the ΛCDM cosmology is one of the most intriguing problems in modern cosmology. Perhaps unsurprisingly, there have been many attempts to solve the problem by introducing new (and sometimes highly speculative) physics (e.g.

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[1] Here, ΛCDM refers to a spatially flat FRW cosmology dominated by cold dark matter and a cosmological constant at the present date with Gaussian initial adiabatic fluctuations characterized by a power-law spectrum.

[2] This value is for the full temperature and polarization analysis in P16. It is consistent with the value $H_0 = 67.36 \pm 0.54\,\mathrm{km\ s^{-1}Mpc^{-1}}$ from the latest *Planck* analysis (Planck Collaboration et al. 2018) derived for the TT,TE,EE+lowE + lensing likelihood combination.

[3] Supernovae and H_0 for the Equation of State.

[4] As this work was nearing completion, Riess et al. (2018a) reported new *Hubble Space Telescope* photometry of long period Milky Way Cepheids. Together with GAIA parallaxes (Gaia Collaboration et al. 2018) these measurements increase the tension between *Planck* and the distance ladder estimate of H_0 to 3.8σ.

Wyman et al. 2014; Solà, Gómez-Valent & de Cruz Pérez 2017; Zhao et al. 2017; Di Valentino et al. 2018a; Di Valentino, Linder & Melchiorri 2018b). There have also been several reanalyses of the SH0ES data (Efstathiou 2014; Cardona, Kunz & Pettorino 2017; Zhang et al. 2017; Follin & Knox 2018) which, apart from minor details, agree well with the analyses by the SH0ES collaboration, though Feeney, Mortlock & Dalmasso (2018b) conclude that the Gaussian likelihood assumption used in the SH0ES analysis may overestimate the statistical significance of the discrepancy.

In this paper, we apply the inverse ladder (Percival et al. 2010; Heavens, Jimenez & Verde 2014; Aubourg et al. 2015; Cuesta et al. 2015; Bernal, Verde & Riess 2016; DES Collaboration et al. 2017; Verde et al. 2017) to derive an estimate of H_0. In our application, we combine SNe data from the Pantheon sample (Scolnic et al. 2017) with BAO measurements from the 6dF Galaxy Survey (6dFGS) (Beutler et al. 2011), Baryon Oscillation Spectroscopic Survey (BOSS) (Alam et al. 2016) and Sloan Digital Sky Survey (SDSS) quasars (Bautista et al. 2017; du Mas des Bourboux et al. 2017; Zarrouk et al. 2018). To calibrate the inverse distance ladder, we impose priors on the sound horizon at the end of the radiation drag epoch, r_d. However, instead of assuming a particular cosmological model, we fit a flexible parametric model describing the evolution of the Hubble parameter $H(z)$. The FRW metric of General Relativity then fixes the luminosity distance $D_L(z)$ in terms of $H(z)$; the extrapolation of $H(z)$ to $z = 0$ is then independent of the low-redshift properties of dark matter and dark energy, as in the important analysis of Heavens et al. (2014).

The analysis presented here is similar to recent analyses by Feeney et al. (2018a), who parameterized $D_L(z)$ with a third-order Taylor expansion (characterized by the deceleration and jerk parameters q_0 and j_0), by Joudaki et al. (2017), who parameterized $H(z)$ on a discrete grid in z and by Bernal et al. (2016), who reconstruct $H(z)$ by interpolating piece-wise cubic splines specified by a small number of knots. In this paper, we parameterize $H(z)$ as a smooth function of redshift. Our analysis is closely related to that of Bernal et al. (2016), except that we use more recent (and more constraining) BAO and supernova data to extrapolate to a value of H_0 rather than fixing the sound horizon, and we demonstrate explicitly that the discrepancy with the direct measurement of H_0 is insensitive to whether the BAO scale is normalized using priors on the sound horizon derived from *Planck* or WMAP.

The layout of our paper is as follows: In Section 2, we introduce our parameterization of $H(z)$ and the priors on r_d that we use to calibrate the distance scale. The datasets used in this analysis are described in Section 3 and our results are presented in Section 4. Section 5 presents our conclusions.

2 INVERSE DISTANCE LADDER

2.1 $H(z)$ parameterizations

According to General Relativity, the Hubble parameter $H(z)$ fixes the luminosity distance $D_L(z)$ and comoving angular diameter distance $D_M(z)$ according to

$$D_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')}, \quad D_M(z) = \frac{D_L(z)}{(1+z)}, \tag{1}$$

where we have assumed that a spatially flat geometry is an accurate description of our Universe. We adopt the following parametric form for $H(z)$:

$$H^2(z) = H_{\rm fid}^2\left[A(1+z)^3 + B + Cz + D(1+z)^{\epsilon}\right], \tag{2}$$

with A, B, C, D, and ε as free parameters. We refer to this parameterization as the 'epsilon' model. The normalizing factor $H_{\rm fid}$ is fixed at $H_{\rm fid} = 67\,{\rm km\,s^{-1}Mpc^{-1}}$ and is introduced so that the free parameters A to D are dimensionless and of order unity. In the base ΛCDM cosmology,

$$H(z) = H_0\left[\Omega_m(1+z)^3 + (1-\Omega_m)\right]^{1/2}, \tag{3}$$

where Ω_m is the present day total matter density in units of the critical density. Equation (3) applies at low redshifts when contributions to the energy density from photons and neutrinos can be ignored. This equation is reproduced by the parametrization of equation (1) if

$$A = \left(\frac{H_0}{H_{\rm fid}}\right)^2\Omega_m, \quad B = \left(\frac{H_0}{H_{\rm fid}}\right)^2(1-\Omega_m),$$
$$C = D = 0, \epsilon \neq 0, \tag{4}$$

with a degeneracy between B and D for $\varepsilon = 0$.

The base ΛCDM model assumes that dark energy is a cosmological constant with equation of state $w = p/\rho = -1$. In models of evolving dark energy, the equation of state is often parameterized as

$$w(z) = w_0 + w_a\frac{z}{1+z}. \tag{5}$$

With this equation of state, and arbitrary curvature Ω_k,

$$H^2(z) = H_0^2[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{\rm DE}(1+z)^{-3(1+w_0+w_a)}e^{-3w_a z/(1+z)}], \tag{6}$$

where $\Omega_{\rm DE} = 1 - \Omega_m - \Omega_k$. In our application of the inverse distance ladder, the data that we use spans the redshift range 0.1–2.4 (see Section 3). Over this redshift range, the parameterization of equation (2) accurately reproduces equation (6) for extreme values of w_0, w_a and Ω_k (i.e. variations of order unity) that are strongly excluded by the data used in this paper. Provided $H(z)$ is a smoothly varying function of z, with no abrupt jumps, the epsilon model provides an accurate description of the evolution of $H(z)$ in a wide variety of theories involving dynamical dark energy and interactions between dark energy, dark matter, and baryons.

As we will see in Section 4, the parameters of the epsilon model are strongly degenerate. We have therefore also implemented a simpler parameterization, which we refer to as the 'log' model:

$$H^2(z) = H_{\rm fid}^2\left[A'(1+z)^3 + B' + C'z + D'\ln(1+z)\right]. \tag{7}$$

This is a less flexible parameterization than the epsilon model but the four free parameters in equation (7) are less degenerate. In fact, we will find that the data constrain $H(z)$ to be so close to the form expected in the base ΛCDM cosmology that the epsilon and log models give nearly identical results for H_0.

2.2 The sound horizon

The principal datasets used in this analysis are Type Ia supernovae, for which we require the luminosity distance $D_L(z)$, and BAO data which return joint estimates of $D_M(z)/r_d$ and $H(z)r_d/c$. Here, r_d is the sound horizon at the epoch z_d when baryons decouple from the photons:

$$r_d = \int_{z_d}^{\infty}\frac{c_s(z)}{H(z)}{\rm d}z, \tag{8}$$

where c_s is the sound speed in the photon-baryon fluid, given by

$$c_s^2(z) = \frac{c^2}{3}\left[1 + \frac{3}{4}\frac{\rho_b(z)}{\rho_\gamma(z)}\right]^{-1}, \tag{9}$$

Table 1. BAO measurements used in this paper. z_{eff} gives the effective redshift for each measurement. The BOSS DR12 and eBOSS DR14 QSO analyses adopt a fiducial sound horizon of $r_{d,\,\mathrm{fid}} = 147.78$ Mpc. Note that Beutler et al. (2011) use the Eisenstein & Hu (1998) formulae to calculate r_d.

Dataset	z_{eff}	Measurement	Constraint
6dFGS	0.106	$r_d/D_V(z_{\mathrm{eff}})$	0.336 ± 0.015
BOSS DR12	0.38	$D_M(z_{\mathrm{eff}})r_{d,\,\mathrm{fid}}/r_d$	1512 ± 25 Mpc
		$H(z_{\mathrm{eff}})r_d/r_{d,\,\mathrm{fid}}$	$81.2 \pm 2.4\ \mathrm{km\,s^{-1}Mpc^{-1}}$
	0.51	$D_M(z_{\mathrm{eff}})r_{d,\,\mathrm{fid}}/r_d$	1975 ± 30 Mpc
		$H(z_{\mathrm{eff}})r_d/r_{d,\,\mathrm{fid}}$	$90.9 \pm 2.3\ \mathrm{km\,s^{-1}Mpc^{-1}}$
	0.61	$D_M(z_{\mathrm{eff}})r_{d,\,\mathrm{fid}}/r_d$	2307 ± 37 Mpc
		$H(z_{\mathrm{eff}})r_d/r_{d,\,\mathrm{fid}}$	$99.0 \pm 2.5\ \mathrm{km\,s^{-1}Mpc^{-1}}$
eBOSS DR14 QSO	1.52	$D_A(z_{\mathrm{eff}})r_{d,\,\mathrm{fid}}/r_d$	1850^{+90}_{-115} Mpc
		$H(z_{\mathrm{eff}})r_d/r_{d,\,\mathrm{fid}}$	$159^{+12}_{-13}\ \mathrm{km\,s^{-1}Mpc^{-1}}$
BOSS DR12 Ly α	2.33	$D_M(z_{\mathrm{eff}})/r_d$	37.8 ± 2.1
		$c/(H(z_{\mathrm{eff}})r_d)$	9.07 ± 0.31
BOSS DR12 QSOxLy α	2.40	$D_M(z_{\mathrm{eff}})/r_d$	35.7 ± 1.7
		$c/(H(z_{\mathrm{eff}})r_d)$	9.01 ± 0.36

where ρ_b and ρ_γ are the energy densities of baryons and radiation, respectively. CMB experiments such as *Planck* and WMAP (Hinshaw et al. 2013) lead to precise determinations of r_d. From the 2015 Planck Legacy Archive (PLA) tables,[5] we have

$$r_d = 147.27 \pm 0.31\ \mathrm{Mpc}, \quad \text{Planck}, \tag{10a}$$

$$r_d = 148.5 \pm 1.2\ \mathrm{Mpc}, \quad \text{WMAP9}, \tag{10b}$$

where the *Planck* value is for the likelihood combination TE+TE+EE + lowTEB in the notation of P16.[6] We use the PLA value for the nine-year WMAP estimate, rather than the value quoted in Hinshaw et al. (2013), since (10b) is calculated consistently using the Boltzmann solver `CAMB` (Lewis, Challinor & Lasenby 2000).

The estimates of r_d in (10a) and (10b) are extremely insensitive to physics at low redshifts (Cuesta et al. 2015) (since the physical densities $\Omega_m h^2$ and $\Omega_c h^2$, which enter in equation (8) are fixed mainly by the relative heights of the CMB acoustic peaks) but assume the base ΛCDM cosmology at high redshifts. By using these values as priors in the inverse distance ladder, we are implicitly assuming that the base ΛCDM model is correct at high redshift though we allow deviations from the model at low redshifts via the parameterizations of equations (2) or (7). However, as discussed in P14 and P16, the parameters of the base ΛCDM found by *Planck* are consistent with Big Bang Nucleosynthesis (BBN) constraints on $\Omega_b h^2$ inferred from deuterium abundance measurements in low-metallicity systems at high redshift (Cooke et al. 2014, 2016; Cooke, Pettini & Steidel 2018). As emphasized by Addison et al. (2018), BBN constraints can be used together with BAO data to provide a consistency check of r_d and H_0 assuming the base ΛCDM model. We will revisit this constraint in Section 4.2.

3 DATA

The BAO measurements used in this paper are summarized in Table 1. We use the BAO measurements from the 6dF Galaxy Survey (6dFGS) (Beutler et al. 2011) which constrains r_d/D_V, where

$$D_V(z) = \left[D_M^2(z)\frac{cz}{H(z)}\right]^{1/3}. \tag{11}$$

Note that (Beutler et al. 2011) use the Eisenstein & Hu (1998) formulae to calculate r_d. The `CAMB` code gives values that are lower by a factor 1.027 and so the Beutler et al. (2011) numbers in Table 1 have been corrected to account for this difference (for a more detailed discussion see appendix B of Hamann et al. 2010). We use the BOSS DR12 consensus BAO measurements (Alam et al. 2016) on $D_M(z)$ and $H(z)$ in three redshift bands together with the associated 6×6 covariance matrix.[7] We also use the eBOSS BAO measurements from quasars in DR14 (Zarrouk et al. 2018), from BOSS DR12 analyses of Ly α absorption in quasar spectra (Bautista et al. 2017) and BAO constraints from a Ly α-quasar cross-correlation analysis with BOSS DR12 (du Mas des Bourboux et al. 2017). The high-redshift measurements are less accurate than the BOSS DR12 galaxy measurements, but serve to anchor the parametrizations (2) and (7) at redshifts greater than unity. Note also that since the likelihoods for these high-redshift measurements were not available to us, and these data are relatively unimportant for fixing H_0, we sampled over $D_M(z)$ and $H(z)$ assuming that they are Gaussian distributed and uncorrelated.

For the supernovae (SNe) data, we use the new Pantheon sample[8] (Scolnic et al. 2017). This dataset contains SNe spanning the redshift range $0.01 < z < 2.3$ drawn from a number of surveys: The Pan-Starrs1 survey (Rest et al. 2014; Scolnic et al. 2014), CfA1-CfA4 (Riess et al. 1998; Jha et al. 2006; Hicken et al. 2009, 2012), CSP (Contreras et al. 2010; Folatelli et al. 2010; Stritzinger et al. 2011), SNLS (Conley et al. 2011; Sullivan et al. 2011), SDSS (Frieman et al. 2008; Kessler et al. 2009), SCP survey (Suzuki et al. 2012), GOODS (Riess et al. 2007), and CANDELS/CLASH survey (Graur et al. 2014; Rodney et al. 2014; Riess et al. 2018b). We also used the Joint Light-Curve Analysis (JLA) sample (Betoule et al. 2014). The JLA compilation gives almost identical results for H_0 as the Pantheon sample, so we do not present those results here.

[5] http://www.cosmos.esa.int/web/planck/pla.

[6] This is consistent with the value $r_d = 147.09 \pm 0.26$ Mpc derived for the TT,TE,EE+lowE + lensing likelihood combination (Planck Collaboration et al. 2018).

[7] BAO_consensus_covtot_dM_Hz.txt downloaded from http://www.sdss3.org/science/BOSS_publications.php.

[8] The files are publicly available as a `CosmoMC` module at http://kicp.uchicago.edu/~dscolnic/Pantheon_Public.tar.

4 RESULTS

4.1 Constraints on the expansion history

We use the CosmoMC package[9] (Lewis & Bridle 2002; Lewis 2013) to sample the free parameters of the models. For r_d, we adopt Gaussian priors with dispersions as given in equations (10a) and (10b). For the epsilon model, the parameters A, B, and D are constrained to be positive and we impose the constraint that $\varepsilon > -5$. For the log model, we impose the conditions that $A^{'}$ and $B^{'}$ should be positive.

The constraints on the parameters of each model are illustrated in Fig. 1. The parameters in the epsilon model show complex degeneracies in comparison to the parameters of the log model. Nevertheless, the expansion histories $H(z)$ allowed by the two models are almost identical as shown in Fig. 2. The overall scaling of $H(z)$ is set by the r_d prior. The BAO and SNe data then strongly constrain the redshift dependence with the SNe and are particularly important in fixing the slope of $H(z)$ at low redshifts (as will be discussed in more detail below). The epsilon and log models give almost identical results, differing at redshifts $z > 2.4$, where the models become unconstrained by the BAO and SNe data. Fig. 2 also shows the effect of the mild tension between Ly α BAO measurements and *Planck* reported by Bautista et al. (2017) and discussed in Planck Collaboration et al. (2018). The Ly α BAO measurements serve to anchor the reconstruction at $z > 2$, but have little impact on the reconstruction at lower redshifts.

The main results of this paper are illustrated in Fig. 3 which shows posteriors on H_0 for the epsilon model. We find

$$H_0 = 68.42 \pm 0.88\ \text{km s}^{-1}\text{Mpc}^{-1}, \text{ Planck } r_d \text{ prior}, \tag{12a}$$

$$H_0 = 67.9 \pm 1.0\ \text{km s}^{-1}\text{Mpc}^{-1}, \quad \text{WMAP9 } r_d \text{ prior}. \tag{12b}$$

The estimate (12a) is about 1σ lower, and has a smaller error, than the similar analysis of Bernal et al. (2016) (which gives $H_0 = 69.4 \pm 1\,\text{km s}^{-1}\text{Mpc}^{-1}$) because of differences in methodology and improvements in the BAO and SNe data. Both estimates (12a) and (12b) are much closer to the *Planck* ΛCDM estimate of H_0 than the SHOES estimate of R18. The value inferred using the Planck and WMAP r_d priors are, respectively, 1σ and 0.5σ higher than *Planck* estimate and 2.7σ and 2.9σ lower than the R18 value. Evidently, provided General Relativity is valid, the discrepancy with the R18 estimate of H_0 is unlikely to be a consequence of new physics at redshifts $z \lesssim 1$.

These results are in excellent agreement with those of Feeney et al. (2018a), who used an $H(z)$ expansion in terms of the present day values of the deceleration and jerk parameters,

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2}, \qquad j \equiv \frac{\dddot{a}a^2}{\dot{a}^3}, \tag{13}$$

where a is the scale factor of the Friedman–Robinson–Walker metric and dots denote differentiation with respect to time. Expanding to second order in z:

$$H(z) = H_0[1 + (1 + q_0)z + (j_0 - q_0^2)\frac{z^2}{2}], \tag{14a}$$

$$D_L(z) = \frac{cz}{H_0}[1 + (1 + q_0)\frac{z}{2} + (1 - q_0 - 3q_0^2 + j_0)\frac{z^2}{6}]. \tag{14b}$$

[9] https://cosmologist.info/cosmomc.

For the base ΛCDM model with $\Omega_m = 0.31$, $q_0 = 1 - 3\Omega_m/2 = -0.535$ and $j_0 = 1$ and the expressions (14a) and (14b) agree well with the exact forms of $H(z)$ and $D_L(z)$ out to a redshift $z \approx 0.6$ (covering the redshift range of the BOSS DR12 galaxy measurements). Fig. 4 shows our constraints on q_0 and j_0, which are determined mainly by the Pantheon SNe sample and so are nearly independent of r_d. These distributions are consistent with the values expected in base ΛCDM. Although these distributions have extended tails, the gradient $dH(z)/dz$ at low redshifts is tightly constrained by the Pantheon SNe (Fig. 2), which is why it is not possible to match the BAO $H(z)$ measurements with the SH0ES estimate of H_0. It is also worth noting that the SH0ES methodology matches Cepheid-based distance measurements of SNe host galaxies to more distant supernovae assuming the relation (14b) with $q_0 = -0.55$ and $j_0 = 1$, based on fits to the SNe magnitude-redshift relation. It is inconsistent, therefore, to apply the R18 H_0 measurement as a fixed prior, independent of the underlying cosmological model, and to infer a cosmology that conflicts with the SNe magnitude–redshift relation since the SNe magnitude–redshift relation is a fundamental part of the H_0 determination. This inconsistency needs to be borne in mind when using the direct measurement of H_0 to set a scale for the sound horizon (e.g. Heavens et al. 2014; Bernal et al. 2016).

4.2 Consistency of r_d with high-redshift physics

The inverse distance ladder constraints on H_0 derived in this paper assume that there is no new physics at high redshift that can alter CMB estimates of r_d. BBN provides a strong test of new physics at high redshift and can, in principle, be used to test the consistency of CMB estimates of r_d. The most recent estimates (Cooke et al. 2018) of the deuterium to hydrogen ratio D/H, based on seven low-metallicity damped Ly α systems, give

$$10^5(\text{D/H}) = 2.527 \pm 0.030. \tag{15}$$

Assuming three (non-degenerate) neutrino families and BBN, the estimate (15) can be converted into a constraint on $\Omega_b h^2$. This conversion is, however, dependent on uncertainties in the $d(p,\gamma)^3$He reaction rate. Cooke et al. (2018) use the theoretical reaction rate from Marcucci et al. (2016) and the experimental value from Adelberger et al. (2011) to illustrate the sensitivity of $\Omega_b h^2$. They find:

$$100\Omega_b h^2 = 2.166 \pm 0.019, \quad \text{Marcucci et al.}, \tag{16a}$$

$$100\Omega_b h^2 = 2.235 \pm 0.037, \quad \text{Adelberger et al.}, \tag{16b}$$

where the error in (16b) is dominated by the error in the Adelberger et al. (2011) cross-section. The estimate (16a) is lower by 2.4σ compared to the P16 TT+TE+EE + lowP value of $100\Omega_b h^2 = 2.225 \pm 0.016$ for the base ΛCDM cosmology, whereas (16b) is consistent with the P16 value to within 0.25σ. We consider these two values and associated error estimates in the analysis below.

We then follow Addison et al. (2018) and DES Collaboration et al. (2017) in using these BBN estimates together with supplementary astrophysical data to infer r_d assuming the base ΛCDM cosmology. Here, we have combined the BBN constraints with the BAO measurements and the Pantheon SNe sample, as described in Section 3. The posteriors on r_d are shown in Fig. 5 and are consistent with the r_d constraints from WMAP and *Planck*. To the extent that BBN probes early Universe physics, we find no evidence for any inconsistency with the values of the sound horizon inferred from CMB measurements.

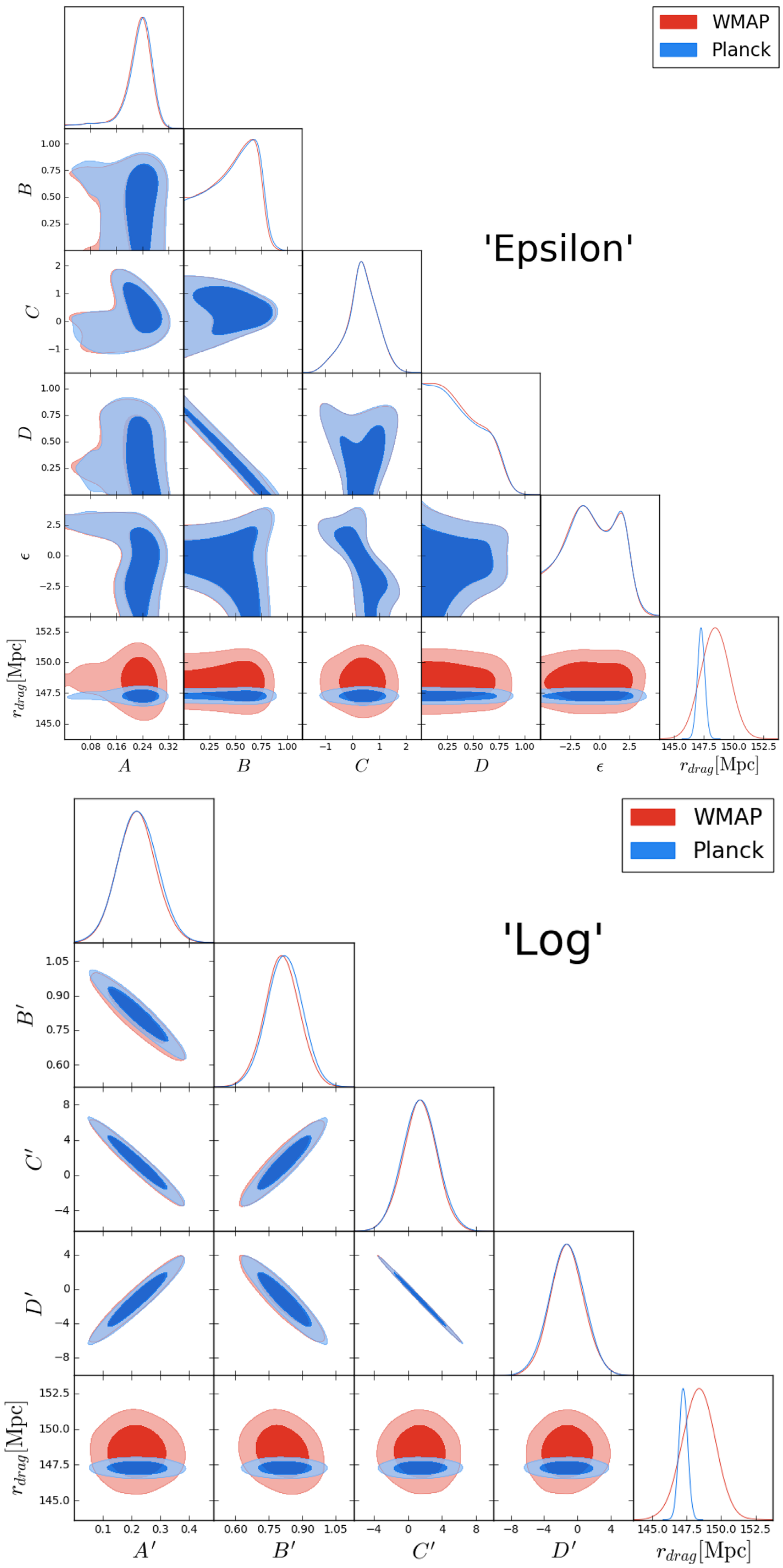

Figure 1. Posterior likelihoods for the 'epsilon' (above) and 'log' (below) parameterizations of $H(z)$. Blue contours show 68 per cent and 95 per cent constraints using the *Planck* prior on r_d. The red contours (largely hidden by the blue contours) show the constraints using the WMAP prior on r_d.

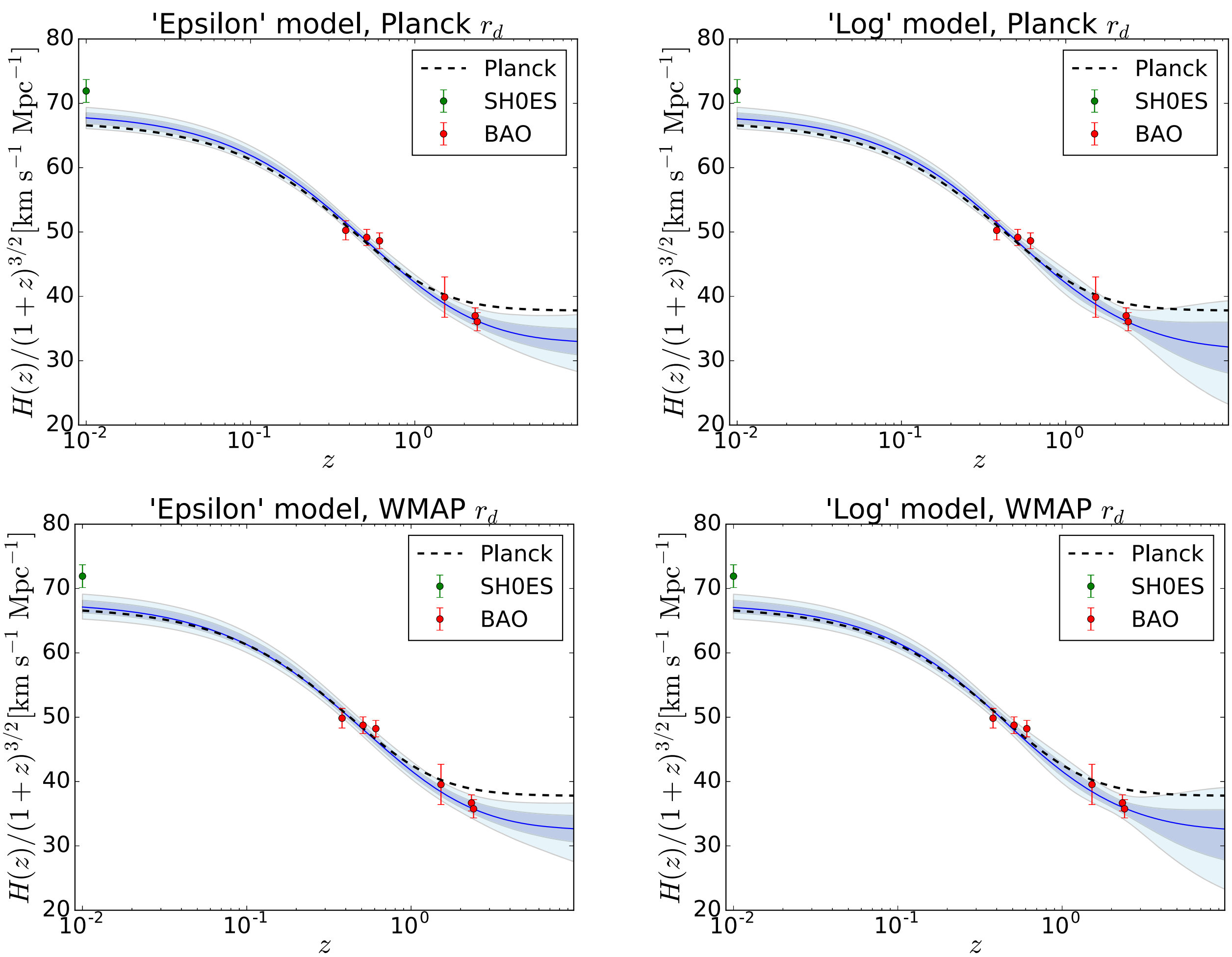

Figure 2. $H(z)$ reconstruction for the epsilon model (left-hand panels) and log model (right-hand panels) for the *Planck* and WMAP priors on r_d: The blue lines show the best fits, and the bands show the allowed one and two sigma ranges. The black dashed lines show $H(z)$ for the best-fitting ΛCDM cosmology obtained from *Planck*. The red points show the BAO estimates on $H(z)$ from Table 1 plotted assuming the central values of the priors on r_d. The R18 SH0ES forward distance ladder estimate of $H_0 = 73.48 \pm 1.66\,\mathrm{km\,s^{-1}Mpc^{-1}}$ is plotted as the green point in each panel.

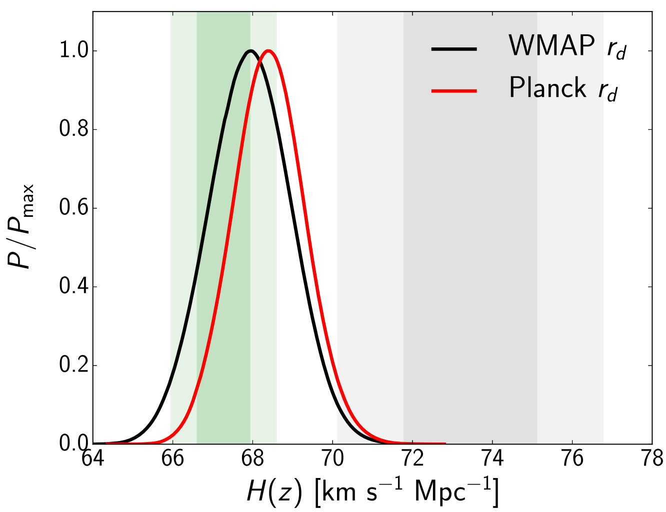

Figure 3. Posteriors for the Hubble constant H_0 derived from the epsilon model using the WMAP and *Planck* r_d priors. The grey bands show the 1σ and 2σ errors for the value obtained by R18, while the green bands show the *Planck* base ΛCDM value from P16.

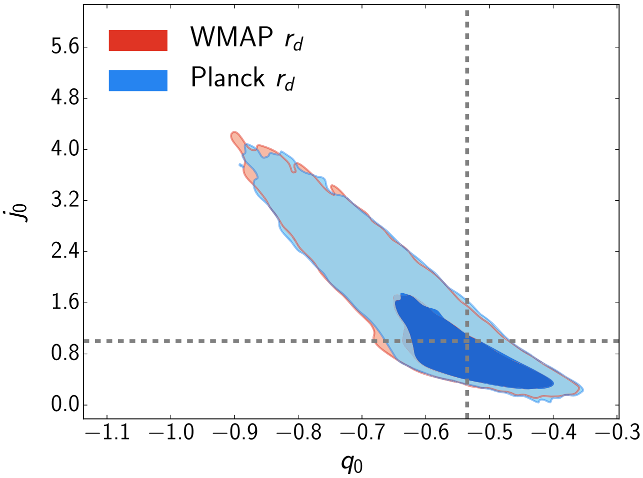

Figure 4. 68 per cent and 95 per cent constraints on the q_0 and j_0 parameters determined from the epsilon model. These constraints are set mainly by the Pantheon SNe sample and are almost independent of the prior on r_d. The lines give the values of j_0 and q_0 expected in the base ΛCDM model with $\Omega_m = 0.31$.

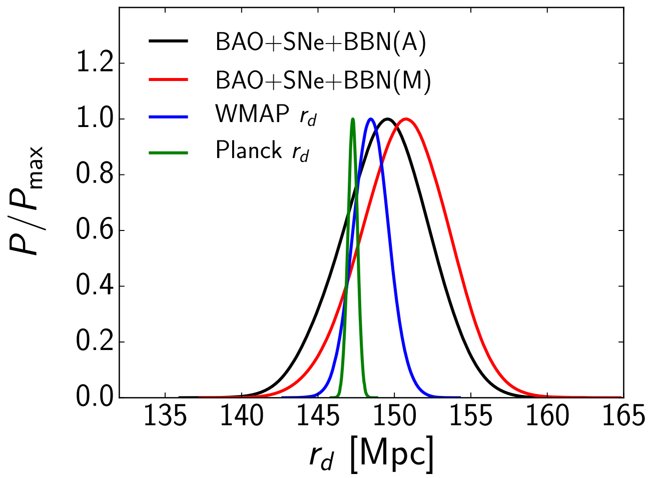

Figure 5. The CMB constraints on the sound horizon r_d from WMAP and *Planck* used in this paper. The black and red curves show the posteriors on r_d determined by fitting to the BAO and Pantheon SNe data assuming the base ΛCDM cosmology and BBN constraints on $\Omega_b h^2$. The curve labelled BBN(M) assumes the Marcucci et al. (2016) $d(p, \gamma)^3$He reaction rate. The curve labelled BBN(A) uses the experimental rate from Adelberger et al. (2011).

Bernal et al. (2016) suggested that the H_0 tension can be partially relieved by invoking extra relativistic degrees of freedom in addition to the $N_{\rm eff} = 3.046$ expected in the standard model. This solution is disfavoured by the latest *Planck* analysis. Allowing $N_{\rm eff}$ to vary as an extension to the base-ΛCDM cosmology, Planck Collaboration et al. (2018) find $N_{\rm eff} = 2.99 \pm 0.17$, $H_0 = 67.3 \pm 1.1\,{\rm km\,s^{-1}Mpc^{-1}}$ and $r_d = 147.9 \pm 1.8$ Mpc for the TT,TE,EE+lowE+BAO + lensing likelihood combination. Additional relativistic degrees of freedom are therefore tightly constrained by the latest data.

5 CONCLUSIONS

The precision and redshift reach of BAO measurements has improved substantially over the last few years. Together with SNe data, it is now possible to reconstruct the time evolution of $H(z)$ accurately without invoking any specific model of the physics of the late time Universe other than the validity of the FRW metric of General Relativity. If we assume that there is no new physics at early times, then CMB measurements constrain the sound horizon, r_d, and this in turn fixes the absolute scale of $H(z)$ allowing an extrapolation to $z = 0$ to infer H_0. Our results disagree with the direct measurement of H_0 from the SH0ES collaboration and are in much closer agreement with the H_0 value determined by *Planck* assuming the base ΛCDM cosmology. This conclusion holds irrespective of whether we use a prior on r_d from WMAP or from *Planck*.

Our results are consistent with previous work on the inverse distance ladder (e.g Percival et al. 2010; Aubourg et al. 2015; Feeney et al. 2018a, P16). In agreement with (Bernal et al. 2016), we reach this conclusion without having to assume any specific model for the time evolution of dark energy or its interaction with dark matter and baryons. As long as there is no new physics in the early Universe that can alter the CMB value of the sound horizon, the new BAO measurements from BOSS provide accurate absolute measurements of $H(z)$ in the redshift range 0.38–2.4. The SNe data then provide a strong constraint on the gradient of $H(z)$ at lower redshifts, which is compatible with the gradient expected in the base ΛCDM cosmology. The data therefore do not allow a rise in $H(z)$ at low redshift with which to match the SH0ES direct measurement of H_0. We conclude that it is not possible to reconcile CMB estimates of H_0 and the SH0ES direct measurements of H_0 by invoking new physics at low redshifts.

If the tension between the CMB estimates of H_0 and direct measurements is a signature of new physics, then we need to introduce new physics in the early Universe. This new physics must lower the sound horizon by about 9 per cent (i.e. to about 135 Mpc) compared to the values used in this paper while preserving the structure of the temperature and polarization power spectra measured by CMB experiments. This new physics also needs to preserve the consistency between BBN and observed abundances of light elements. These requirements pose interesting challenges for theorists.

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