SUPPLEMENTAL MATERIAL: Neutrinoless Double Beta Decay with Non-standard Majoron Emission

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A. CALCULATION OF THE EXOTIC MAJORON PROCESS

We here detail the computation of the amplitude and differential decay rate of the $0\nu\beta\beta\phi$ process. We follow the calculation of the standard long-range contributions presented in [1] and start from the effective Lagrangian

$$\mathcal{L}_{0\nu\beta\beta\phi} = \mathcal{L}_{\rm SM} + \mathcal{L}_{R\phi}, \tag{SM.1}$$

with the SM charged current

$$\mathcal{L}_{\rm SM} = \frac{G_F \cos\theta_C}{\sqrt{2}} j_L^\mu J_{L\mu} + \text{h.c.}, \tag{SM.2}$$

and the exotic 7-dimensional operators incorporating right-handed lepton currents and the Majoron ϕ,

$$\mathcal{L}_{R\phi} = \frac{G_F \cos\theta_C}{\sqrt{2} m_p} \left(\epsilon_{RL}^\phi j_R^\mu J_{L\mu}\phi + \epsilon_{RR}^\phi j_R^\mu J_{R\mu}\phi \right) + \text{h.c.}. \tag{SM.3}$$

Here, G_F is the Fermi constant, θ_C is the Cabbibo angle and the leptonic and hadronic currents are defined as

$$j_{L,R}^\mu = \bar{e}\gamma^\mu(1 \mp \gamma_5)\nu, \quad J_{L,R}^\mu = \bar{u}\gamma^\mu(1 \mp \gamma_5)d, \tag{SM.4}$$

respectively.

To lowest order of perturbation, the amplitude for the process of $0_I^+ \to 0_F^+\ 0\nu\beta\beta\phi$ decay depicted in Fig. 1 (center) of the main text is

$$\mathcal{M} = -\int d^4x d^4y \langle F|\mathcal{T}\left\{\mathcal{L}_{\rm SM}(x)\mathcal{L}_{R\phi}(y)\right\}|I\rangle. \tag{SM.5}$$

The time-ordered product is expanded as

$$\begin{aligned}\mathcal{T}\left\{\mathcal{L}_{\rm SM}(x)\mathcal{L}_{R\phi}(y)\right\} = 2\,\epsilon_{RX}\frac{(G_F\cos\theta_C)^2}{m_p} \\ \times \mathcal{T}\{J_L^\mu(x)J_X^\nu(y)\underbrace{\bar{e}(x)\gamma_\mu P_L\nu(x)\bar{\nu}(y)\gamma_\nu P_L e^c(y)}_{\Xi^L_{\mu\nu}(x,y)}\phi(y)\},\end{aligned} \tag{SM.6}$$

with the chiral projectors defined as $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. Using the neutrino propagator with momentum q and mass m_ν, the highlighted term $\Xi^L_{\mu\nu}(x,y)$ can be expressed as

$$\begin{aligned}\Xi^L_{\mu\nu}(x,y) &= \int \frac{d^4q}{(2\pi)^4}\frac{e^{-iq(x-y)}}{q^2 - m_\nu^2 + i\varepsilon}\bar{e}(x)\gamma_\mu P_L(\not{q} + m_\nu)\gamma_\nu P_L e^c(y) \\ &= \int \frac{d^4q}{(2\pi)^4} q^\alpha \frac{e^{-iq(x-y)}}{q^2 - m_\nu^2 + i\varepsilon}\bar{e}(x)\gamma_\mu\gamma_\alpha\gamma_\nu P_L e^c(y).\end{aligned} \tag{SM.7}$$

The amplitude needs to be antisymmetric under the exchange of the electrons e_1 and e_2, and thus we generalize

$$\Xi^{L/R}_{\mu\nu}(x,y) = \frac{1}{\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq(x-y)}}{q^2 - m_\nu^2 + i\varepsilon} \left(u^{L/R}_{\mu\nu}(E_1 x, E_2 y) - u^{L/R}_{\mu\nu}(E_2 x, E_1 y) \right), \quad \text{(SM.8)}$$

with $u^{L/R}_{\mu\nu}(E_1 x, E_2 y) = q^\alpha \bar{e}(E_1, x)\gamma_\mu \gamma_\alpha \gamma_\nu P_{L/R}\, e^c(E_2, y)$ and E_i is the energy of each electron.

We now perform the integral over the temporal variables. The integration over q_0 is straightforward by means of the residue theorem,

$$\int \frac{dq_0}{2\pi} \frac{1}{q_0^2 - \omega^2} f(q_0) = \frac{i}{2\omega} f(\omega), \quad \text{(SM.9)}$$

with $\omega^2 = \mathbf{q}^2 + m_\nu^2$. On the other hand, expanding the time-ordered product as

$$\mathcal{T}\left\{\mathcal{L}_{\rm SM}(x)\mathcal{L}_{R\phi}(y)\right\} = \Theta(x^0 - y^0)\mathcal{L}_{\rm SM}(x)\mathcal{L}_{R\phi}(y) + \Theta(y^0 - x^0)\mathcal{L}_{R\phi}(y)\mathcal{L}_{\rm SM}(x), \quad \text{(SM.10)}$$

and using the operator e^{iHt} to extract the temporal dependence from the different wave functions, for example $\phi(y) = e^{iE_\phi y_0}\phi(\mathbf{y})$, one can directly integrate over x_0 and y_0 obtaining the analogous expression to Eq. (C.2.19) in [1],

$$\begin{aligned} \mathcal{M} = \epsilon^\phi_{RX} \frac{(G_F \cos\theta_C)^2}{\sqrt{2} m_p} \sum_N \int d^3x d^3y \int \frac{d^3q}{2\pi^2\omega} J^{\rho\sigma}_{LX}(\mathbf{x},\mathbf{y})\phi(\mathbf{y}) \\ \times \Bigg\{ \quad e^{i\mathbf{q}(\mathbf{x}-\mathbf{y})} \left[\frac{u^L_{\rho\sigma}(E_1\mathbf{x}, E_2\mathbf{y})}{\omega + A_2 + \frac{1}{2}E_\phi} - \frac{u^R_{\sigma\rho}(E_1\mathbf{y}, E_2\mathbf{x})}{\omega + A_1 + \frac{1}{2}E_\phi} \right] \\ - e^{i\mathbf{q}(\mathbf{y}-\mathbf{x})} \left[\frac{u^L_{\rho\sigma}(E_1\mathbf{x}, E_2\mathbf{y})}{\omega + A_1 - \frac{1}{2}E_\phi} - \frac{u^R_{\sigma\rho}(E_1\mathbf{y}, E_2\mathbf{x})}{\omega + A_2 - \frac{1}{2}E_\phi} \right] \Bigg\}, \end{aligned} \quad \text{(SM.11)}$$

where $A_{1/2} = E_N - E_I + \frac{1}{2}Q_{\beta\beta} + m_e \pm \frac{1}{2}(E_1 - E_2)$. We anticipate the closure approximation and define the matrix element of the hadronic currents as

$$J^{\rho\sigma}_{LX}(\mathbf{x},\mathbf{y}) = \frac{1}{2}\left[\langle F| J^\rho_L(\mathbf{x}) |N\rangle \langle N| J^\sigma_X(\mathbf{y}) |I\rangle + \langle F| J^\sigma_X(\mathbf{y}) |N\rangle \langle N| J^\rho_L(\mathbf{x}) |I\rangle \right]. \quad \text{(SM.12)}$$

In addition, the following properties under the exchange of position and electron energies were used in Eq. (SM.11),

$$u^{L/R}_{\rho\sigma}(E_1\mathbf{x}, E_2\mathbf{y}) = u^{R/L}_{\sigma\rho}(E_1\mathbf{x}, E_2\mathbf{y}), \qquad J^{\rho\sigma}_{LX}(\mathbf{x},\mathbf{y}) = J^{\sigma\rho}_{XL}(\mathbf{y},\mathbf{x}). \quad \text{(SM.13)}$$

The integration over x_0 and y_0 in Eq. (SM.11) also provides the overall energy conservation condition $\delta(Q_{\beta\beta}+2m_e-E_1-E_2-E_\phi)$ with $Q_{\beta\beta} = E_I-E_F-2m_e$. It is included in the phase space, Eq. (SM.31) below, by requiring $E_\phi = Q_{\beta\beta}+2m_e-E_1-E_2$. We additionally assume that the Majoron ϕ is emitted predominantly in an S-wave configuration, $\phi(\mathbf{y}) \approx 1$.

Considering the term between braces in Eq. (SM.11), one can write everything under the same exponential by interchanging $\mathbf{x}$ and $\mathbf{y}$,

$$e^{i\mathbf{q}(\mathbf{x}-\mathbf{y})}\Bigg\{ \left[\frac{J^{\rho\sigma}_{LX}(\mathbf{x},\mathbf{y})u^L_{\rho\sigma}(E_1\mathbf{x},E_2\mathbf{y})}{\omega+A_2+\frac{1}{2}E_\phi} + \frac{J^{\rho\sigma}_{XL}(\mathbf{x},\mathbf{y})u^R_{\rho\sigma}(E_1\mathbf{x},E_2\mathbf{y})}{\omega+A_2-\frac{1}{2}E_\phi}\right] - \left[\frac{J^{\rho\sigma}_{LX}(\mathbf{x},\mathbf{y})u^L_{\rho\sigma}(E_2\mathbf{x},E_1\mathbf{y})}{\omega+A_1+\frac{1}{2}E_\phi} + \frac{J^{\rho\sigma}_{XL}(\mathbf{x},\mathbf{y})\,u^R_{\rho\sigma}(E_2\mathbf{x},E_1\mathbf{y})}{\omega+A_1-\frac{1}{2}E_\phi}\right]\Bigg\}. \tag{SM.14}$$

It is furthermore useful to split the leptonic $u^{L,R}_{\rho\sigma}$ functions by separating out the part containing γ_5 as $u^{L/R}_{\rho\sigma} = \frac{1}{2}\left[u_{\rho\sigma} \mp u^5_{\rho\sigma}\right]$. We then define

$$F^{\pm}_{\rho\sigma} = u_{\rho\sigma}(E_1\mathbf{x},E_2\mathbf{y}) \pm u_{\sigma\rho}(E_1\mathbf{y},E_2\mathbf{x}), \tag{SM.15}$$

$$F^{5\pm}_{\rho\sigma} = u^5_{\rho\sigma}(E_1\mathbf{x},E_2\mathbf{y}) \pm u^5_{\sigma\rho}(E_1\mathbf{y},E_2\mathbf{x}), \tag{SM.16}$$

$$J^{\pm}_{\rho\sigma} = J^{LX}_{\rho\sigma}(E_1\mathbf{y},E_2\mathbf{x}) \pm J^{XL}_{\rho\sigma}(E_1\mathbf{y},E_2\mathbf{x}). \tag{SM.17}$$

These definitions become useful if one recalls that in the non-relativistic impulse approximation, the J^L part of $J^{LX}_{\rho\sigma}$ acts on the n-th nucleon whereas the J^X part acts on the m-th when performing the sum over all neutrons in the initial nucleus. The superscript $\pm$ in $J^{\pm}_{\rho\sigma}$ thus indicates if the combination of currents is symmetric or antisymmetric under the interchange of $m \leftrightarrow n$. The same applies to $F^{\pm}_{\rho\sigma}$ and $F^{5\pm}_{\rho\sigma}$.

The closure approximation implies that the sum over all possible intermediate states is performed analytically using the completeness of all intermediate states and by replacing the intermediate state energies E_N with a common average $\langle E_N\rangle$. This means that the antisymmetric combinations under the interchange of the nucleons m and n will vanish, as the sum is performed over all possible configurations. From Eqs. (SM.11) and (SM.14), the non-vanishing terms are

$$\mathcal{M} = \epsilon_{RX}\frac{(G_F\cos\theta_C)^2}{2\sqrt{2}m_p} \times \sum_N (H_{\omega 2}-H_{\omega 1})\left\{J^+_{\mu\nu}F^{+,\mu\nu} - J^-_{\mu\nu}F^{5-,\mu\nu} + \frac{E_\phi}{E_{12}}\left(J^+_{\mu\nu}F^{5+,\mu\nu} - J^-_{\mu\nu}F^{-,\mu\nu}\right)\right\}, \tag{SM.18}$$

where $H_{\omega i}$ are neutrino potentials defined as

$$H_{\omega i} = \int \frac{d^3q}{2\pi^2\omega} \frac{\omega}{\omega + A_i} e^{i\mathbf{q}(\mathbf{x}-\mathbf{y})}. \tag{SM.19}$$

Now, the connection with the results of [1] can be done by contracting the leptonic and nuclear currents within the impulse approximation. The only change in our case is in the ω term,

$$\begin{aligned}\mathcal{M}_\omega \propto (H_{\omega 2} - H_{\omega 1}) \Big\{ &(X_3 + X_{5R}) \left[F_+^0 + \frac{E_\phi}{E_{12}} F_{5+}^0 \right] + Y_{3R} \left[F_{5-}^0 + \frac{E_\phi}{E_{12}} F_-^0 \right] \\ &+ (X_{4R}^l + X_5^l) \left[F_+^l + \frac{E_\phi}{E_{12}} F_{5+}^l \right] + (Y_4^l - Y_{5R}^l) \left[F_{5-}^l + \frac{E_\phi}{E_{12}} F_-^l \right] \Big\},\end{aligned} \tag{SM.20}$$

where the X and Y terms are functions of nuclear parameters and operators defined in Appendix C of [1]. The $F^\alpha_{(5)\pm}$-terms are generated by the contraction of the hadronic and leptonic parts in Eqs. (SM.15)-(SM.17) factorizing out the dependence with the momentum q_α from the leptonic part (see Eq. (C.2.25) in [1]). One trivially recovers the ω term in the expression (C.2.23) of [1] for $E_\phi \to 0$.

Comparing Eq. (SM.20) with the results from [1], one can track the dependence with E_ϕ in the decay rate down to Eq. (C.3.9) of [1]. The main change for $0_I^+ \to 0_F^+$ transitions is in the terms N_3 and N_4 where a contribution proportional to E_ϕ appears explicitly,

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \alpha^*_{-1-1} \\ \alpha^*_{11} \end{pmatrix} \left[\frac{4}{3} Z_6 \mp \frac{4}{m_e R} \left(Z_{4R} - \frac{1}{6} \zeta Z_6 \right) \right], \tag{SM.21}$$

$$\begin{pmatrix} N_3 \\ N_4 \end{pmatrix} = \begin{pmatrix} \alpha^*_{1-1} \\ \alpha^*_{-11} \end{pmatrix} \left[-\frac{2}{3} Z_5 \mp \frac{E_{12}}{m_e} \left(Z_3 + \frac{1}{3} Z_5 \right) + \frac{E_\phi}{m_e} Z_3 \right]. \tag{SM.22}$$

Here, $\alpha_{jk} = \tilde{A}_j(E_1)\tilde{A}_k(E_2)$ describe the Coulomb-corrected relativistic electron wave functions and $\zeta = 3\alpha Z + (Q_{\beta\beta} + 2m_e)R$ the correction of the electron P wave, with the fine structure constant α and the radius R and charge Z of the final state nucleus. The information about the electron wave functions is encoded in

$$\tilde{A}_{\pm k}(E) = \sqrt{\frac{E \mp m_e}{2E} F_{k-1}(Z, E)}, \tag{SM.23}$$

with the Fermi factor

$$F_{k-1}(Z, E) = \left[\frac{\Gamma(2k+1)}{\Gamma(k)\Gamma(2\gamma_k + 1)} \right]^2 (2pR)^{2(\gamma_k - k)} |\Gamma(\gamma_k + iy)|^2 e^{\pi y}, \tag{SM.24}$$

Isotope	$Q_{\beta\beta}$ [MeV]	M_{GT}	χ_F	$\chi_{GT\omega}$	$\chi_{F\omega}$	χ'_{GT}	χ'_F	χ_T	χ_R	χ_P
^{82}Se	2.99	2.993	−0.134	0.947	−0.131	1.003	−0.103	0.004	1.086	0.430
^{136}Xe	2.46	1.770	−0.158	0.908	−0.149	1.092	−0.167	−0.031	0.955	0.256

TABLE I. Energy release $Q_{\beta\beta}$ and relevant nuclear matrix elements for ^{82}Se and ^{136}Xe used in the calculation of the $0\nu\beta\beta\phi$ decay rate and distributions. The nuclear matrix elements were taken from the shell model calculations [2] (^{82}Se) and [3] (^{136}Xe), except for M_{GT} in ^{136}Xe where we use an updated value from the same group [4].

where $\gamma_k = \sqrt{k^2 + (\alpha Z)^2}$, $y = \alpha Z E/p$ and $p = \sqrt{E^2 - m_e^2}$.

In order to arrive at Eq. (SM.20) one should neglect the higher order terms E_{12}^2, $E_{12}E_\phi$ and E_ϕ^2 as they are suppressed with an extra denominator $(\omega + A_i)$ compared to Eq. (SM.19).

The Z_i terms are given in Eqs. (SM.25)-(SM.28) below and they contain the nuclear matrix elements and effective particle physics couplings. The Z_i terms are the same as in [1], with the relevant couplings $\lambda \to \epsilon^\phi_{RR}$ and $\eta \to \epsilon^\phi_{RL}$ substituted. Note that the term with Z_1 in Eq. (C.3.9) from [1] related to the standard $0\nu\beta\beta$ decay disappears from Eq. (SM.21), as we are not considering the interaction $\mathcal{L}_{SM}(x)\mathcal{L}_{SM}(y)$.

$$Z_3 = \left[-\epsilon^\phi_{RR}(\chi_{GT\omega} - \chi_{F\omega}) + \epsilon^\phi_{RL}(\chi_{GT\omega} + \chi_{F\omega})\right] M_{GT}, \tag{SM.25}$$

$$Z_{4R} = \epsilon^\phi_{RL}\chi_R M_{GT}, \tag{SM.26}$$

$$Z_5 = \frac{1}{3}\left[\epsilon^\phi_{RR}(\chi'_{GT} - 6\chi_T + 3\chi'_F) - \epsilon^\phi_{RL}(\chi'_{GT} - 6\chi_T - 3\chi'_F)\right] M_{GT}, \tag{SM.27}$$

$$Z_6 = \epsilon^\phi_{RL}\chi_P M_{GT}. \tag{SM.28}$$

The above equations are valid when both ϵ^ϕ_{RL} and ϵ^ϕ_{RR} are present. For our numerical calculations, we use the $Q_{\beta\beta}$ values and nuclear matrix elements M_{GT}, χ_F, etc. presented in Table I for ^{82}Se and ^{136}Xe. We use the following values for the remaining parameters: $G_F = 1.2 \times 10^{-5}$ GeV^{-2}, $\alpha = 1/137$, $g_A = 1.27$, $R = 1.2A^{1/3}$ fm with the mass number A of the isotope in question. The factors N_1, N_2, N_3 and N_4 in Eqs. (SM.21) and (SM.22) are then fully described and the energy-dependent coefficients are

$$a(E_1, E_2, E_\phi) = |N_1|^2 + |N_2|^2 + |N_3|^2 + |N_4|^2, \tag{SM.29}$$

$$b(E_1, E_2, E_\phi) = -2\,\mathrm{Re}\left(N_1^* N_2 + N_3^* N_4\right). \tag{SM.30}$$

The differential decay rate for the $0^+ \to 0^+$ $0\nu\beta\beta\phi$ decay can then be written as [1]

$$d\Gamma = C \left[a(E_1, E_2, E_\phi) + b(E_1, E_2, E_\phi)\cos\theta\right] w(E_1, E_2, E_\phi)\, dE_1\, dE_2\, d\cos\theta, \qquad \text{(SM.31)}$$

with

$$C = \frac{(G_F \cos\theta_C g_A)^4 m_e^9}{256\pi^7 (m_p R)^2}, \qquad \text{(SM.32)}$$

$$w(E_1, E_2, E_\phi) = m_e^{-7} p_1 p_2 E_1 E_2 E_\phi. \qquad \text{(SM.33)}$$

Here, g_A is the axial coupling of the nucleon and R is the radius of the nucleus. The magnitudes of the electron momenta are given by $p_i = \sqrt{E_i^2 - m_e^2}$ and $0 \le \theta \le \pi$ is the angle between the emitted electrons. Throughout the above expressions, the Majoron energy is implicitly fixed by the electron energies as $E_\phi = Q_{\beta\beta} + 2m_e - E_1 - E_2$ due to overall energy conservation.

The total decay rate Γ and the half life $T_{1/2}$ are then calculated as

$$\Gamma = \frac{\ln 2}{T_{1/2}} = 2C \int_{m_e}^{Q_{\beta\beta}+m_e} dE_1 \int_{m_e}^{Q_{\beta\beta}+2m_e-E_1} dE_2\, a(E_1, E_2, E_\phi) w(E_1, E_2, E_\phi). \qquad \text{(SM.34)}$$

The fully differential energy information is encoded in the normalized double energy distribution

$$\Gamma^{-1} \frac{d\Gamma}{dE_1 dE_2} = \frac{2C}{\Gamma} a(E_1, E_2, E_\phi) w(E_1, E_2, E_\phi). \qquad \text{(SM.35)}$$

This function, in terms of the kinetic energies normalized to the Q value, $(E_i - m_e)/Q_{\beta\beta}$, is plotted in the top row of Fig. 1 for the case of $0\nu\beta\beta\phi$ Majoron emission through ϵ^ϕ_{RL} (left) and ϵ^ϕ_{RR} (center) as well as for the SM $2\nu\beta\beta$ decay (right). The plots are for the isotope ^{82}Se but would be qualitatively similar for ^{136}Xe. As can be seen, the shapes depicted as contours are different between all three modes. Especially the ϵ^ϕ_{RR} exhibits an asymmetry in that one of the electrons takes the majority of the visible energy. If the individual electron energies can be measured, as e.g. in the NEMO-3 or SuperNEMO experiments, this can be exploited to enhance the signal over the $2\nu\beta\beta$ background. As an illustrating example, requiring that any one of the electrons in a signal event has a kinetic energy $E_i - m_e > Q_{\beta\beta}/2$ would reduce the $0\nu\beta\beta\phi$-ϵ^ϕ_{RR} rate only by a factor of 2 but would suppress the $2\nu\beta\beta$ rate by a factor of 20. The distributions in Fig. 2 of the main text can be easily determined by appropriately integrating over $\frac{d\Gamma}{dE_1 dE_2}$.

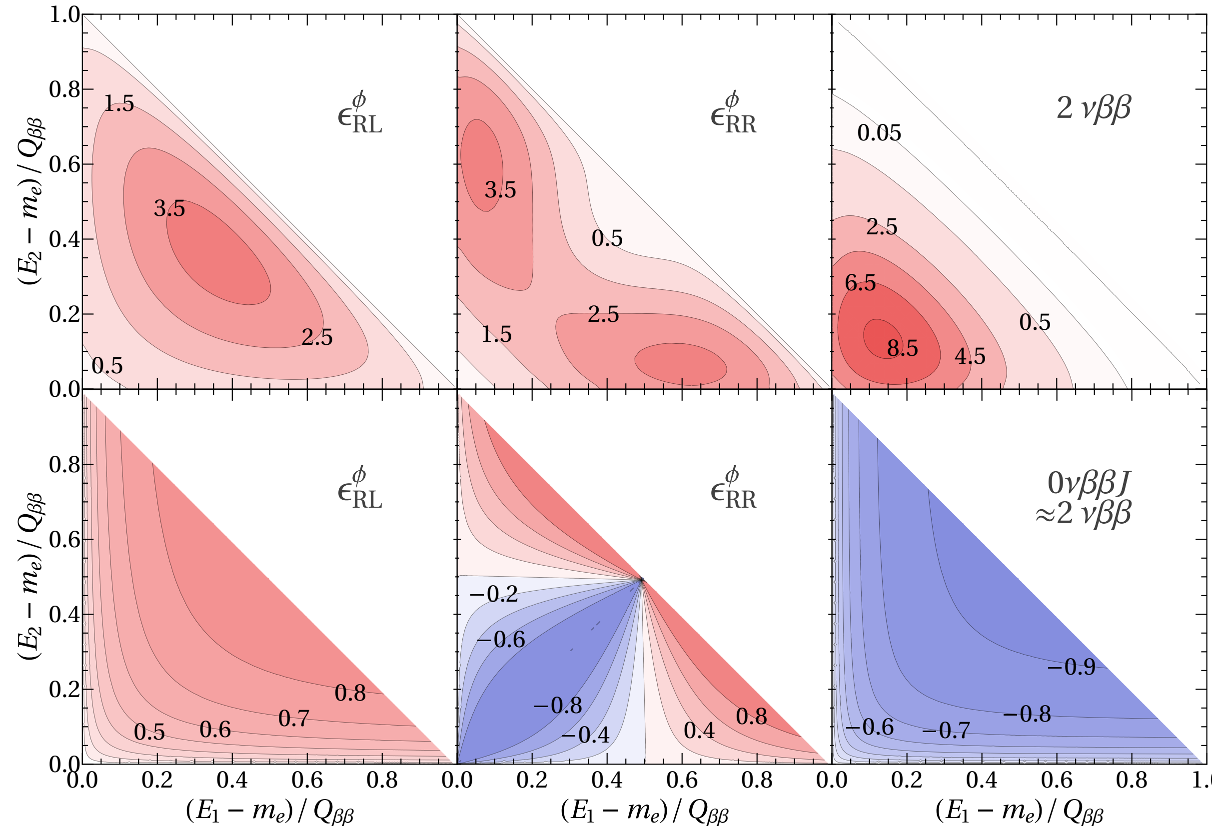

FIG. 1. Double electron energy distribution $\frac{d\Gamma}{dE_1 dE_2}$ (top row) and electron angular correlation α (bottom row) as function of the individual electron kinetic energies for ^{82}Se. Each column is for a specific scenario: $0\nu\beta\beta\phi$ Majoron emission through ϵ^{ϕ}_{RL} (left) and ϵ^{ϕ}_{RR} (center); SM $2\nu\beta\beta$ decay (right). The angular correlation of the latter is approximately identical to ordinary Majoron emission $0\nu\beta\beta J$.

In addition to the energies, the angle between the electron momenta also contains useful information. The so-called angular correlation defined by

$$\alpha(E_1, E_2) = \frac{b(E_1, E_2, E_\phi)}{a(E_1, E_2, E_\phi)}, \tag{SM.36}$$

is a function of the individual electron energies which can take values between -1 (the two electrons are dominantly emitted back-to-back) and $+1$ (the two electrons are dominantly emitted collinearly). For ^{82}Se it is plotted in the bottom row of Fig. 1 in the three modes of interest. As expected from angular momentum considerations, the electrons are dominantly emitted back-to-back in the SM $2\nu\beta\beta$ decay with $(V-A)$ lepton currents, $\alpha < 0$ for all energies. For ϵ^{ϕ}_{RL}, they are dominantly emitted collinearly, $\alpha > 0$ for all energies. In the case of ϵ_{RR}^{ϕ}, the behaviour is complex due to the asymmetry of the amplitude under the exchange of electrons and nuclear recoil effects. The correlation α changes sign, with $\alpha > 0$ when any one electron has a kinetic energy $E_i - m_e > Q_{\beta\beta}/2$ but $\alpha < 0$ when both electrons each have a kinetic energy $E_i - m_e < Q_{\beta\beta}/2$. Note that Fig. 1 provides the full kinematical information in each mode; all measurable quantities can be constructed from these distributions.

As discussed in the main text, averaged over all energies, the electrons are actually emitted almost isotropically for ϵ_{RR}^{ϕ}. This is quantified by integrating Eq. (SM.31) over all energies analogous to Eq. (SM.34) to yield the angular distribution

$$\frac{d\Gamma}{d\cos\theta} = \frac{\Gamma}{2}(1 + k\,\cos\theta), \tag{SM.37}$$

with the average angular correlation factor k.

B. ULTRAVIOLET COMPLETE SCENARIOS

Left-right symmetry

Here we briefly discuss the Left-Right symmetric scenario mentioned in the main text as a possible ultraviolet completion generating the effective operators in Eq. (1) in the main text. At some energy scale above the electroweak scale we assume a left-right symmetric gauge symmetry

$$G_{\mathrm{LR}} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X. \tag{SM.38}$$

It breaks down to the SM gauge group $G_{\mathrm{SM}} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ at a scale M_R. The SM electric charge is related to the generators of the gauge groups by the relation

$$Q = T_{3L} + T_{3R} + \frac{X}{2} = T_{3L} + Y. \tag{SM.39}$$

In the minimal Left-Right symmetric model, the quantum number X is identified with $B - L$, i.e. $B - L$ is a gauge symmetry in the model. Consequently, left-right symmetry breaking is usually assumed to induce several $B - L$ violating interactions, including generation of Majorana neutrino masses via a seesaw mechanism.

In general, though, one can define a new quantum number ζ such that

$$X = (B - L) + \zeta. \tag{SM.40}$$

Field	$SU(2)_L$	$SU(2)_R$	$B-L$	ζ	X	$SU(3)_C$
q_L	2	1	1/3	0	1/3	3
q_R	1	2	1/3	0	1/3	3
ℓ_L	2	1	-1	0	-1	1
ℓ_R	1	2	-1	0	-1	1
$U_{L,R}$	1	1	1/3	+1	4/3	3
$D_{L,R}$	1	1	1/3	-1	-2/3	3
$E_{L,R}$	1	1	-1	-1	-2	1
$N_{L,R}$	1	1	-1	+1	0	1
χ_L	2	1	0	+1	1	1
χ_R	1	2	0	+1	1	1
ϕ	1	1	2	-2	0	1

TABLE II. Field content and quantum numbers under $G_{\rm LR}$ in the Left-Right symmetric scenario proposed.

If $\zeta \neq 0$ then $B-L$ can remain a global symmetry, independent of the left-right gauge symmetry. We here propose such a scenario where the field content and their quantum numbers for such a realisation of a Left-Right symmetric model are summarised in Table II. Apart from the SM fields q_L, ℓ_L and the SM quark singlets which transform as a doublet q_R under $G_{\rm LR}$, left-right symmetry naturally includes right-handed neutrino fields ν_R as part of the right handed lepton doublet ℓ_R.

As noted, the assumed symmetry breaking pattern is given by

$$G_{\rm LR} \overset{M_R}{\to} G_{\rm SM} \overset{m_W}{\to} SU(3)_c \times U(1)_Q. \tag{SM.41}$$

For the left-right symmetry breaking, we use a doublet Higgs scalars χ_R, whose vacuum expectation value (VEV) breaks the left-right symmetry [5–10]. This field does not have any exclusive interaction with the SM fermions and hence the $B-L$ quantum number is no longer uniquely determined. Thus for χ_R, we can choose $B-L=0$, and hence, $\zeta = 1$ in Eq. (SM.40). Note that our model differs from earlier models in this choice of the $B-L$ quantum number [1]. The left-right symmetry ensures that we have a second

[1] Nevertheless, the model remains anomaly free under the charge X as the new vector-like fermions do

doublet Higgs scalar χ_L with the same assignment of $B-L=0$ and $\zeta=1$. Interestingly, these assignments do not require any additional global symmetries, but will allow $B-L$ to remain as a global symmetry after the electroweak symmetry breaking.

A priori, we have two choices of Higgs scalar for breaking the electroweak symmetry. The first choice is that we retain the Higgs bi-doublet from the conventional model, which, after electroweak symmetry breaking, will generate Dirac masses for all the fermions. One particularly interesting scenario arises if we assume that only quarks acquire their masses through the VEV of the bi-doublet and the Yukawa couplings giving rise to such masses of leptons are forbidden by some symmetry[2]. Both the charged and the neutral leptons would then acquire Dirac seesaw masses in this scenario [11–17]. An alternative is that there is no Higgs bi-doublet and the left-handed Higgs doublet χ_L breaks the electroweak symmetry. In such a scenario the quark masses and the charged lepton masses are generated through a seesaw mechanism introducing new vector-like states. This scheme is often called the universal seesaw mechanism. We will mainly focus on this second scenario, however all the discussion presented are applicable for both the scenarios.

In the leptonic sector, we introduce four singlet vector-like fermions, which are the charged and neutral heavy leptons N_L, N_R, E_L, E_R in Table II, all carrying $B-L=1$, and hence $\zeta=-1$ for the neutral fermions $N_{L,R}$ and $\zeta=1$ for the charged fermions $E_{L,R}$. The left-right symmetry breaking will allow mixing of these fermions with the light leptons and the assignment of lepton number is somewhat more natural than in conventional left-right symmetric models where similar new singlets carry vanishing lepton numbers. The VEVs $u_{L,R}$ of the fields $\chi_{L,R}$ introduce mixing of the new neutral leptons $\sigma_{L,R}$ with the neutrinos and the new charged leptons $E_{L,R}$ with the charged leptons.

In the absence of the Higgs bi-doublet, χ_L breaks the electroweak symmetry. In this case we need to introduce vector-like states for all fermions in order to generate their masses. Consequently, all the masses of quarks, leptons and neutrinos are generated by a Dirac seesaw mechanism.

For the charged and neutral leptons there are no bare Dirac mass terms. The Yukawa

not contribute to the anomalies and the assignments of X for the chiral fermions are the same as that of $B-L$ in the conventional Left-Right symmetric model.

[2] For example one may introduce an additional discrete Z_2 symmetry such that ℓ_R, N_R and E_R are odd under this discrete symmetry. Note that in such a case the vector-like mass term for N and E (see. Eq. (SM.42)) will break this Z_2 symmetry softly.

interactions that give masses to the leptons are given by

$$\mathcal{L}_Y = \ell_L^T \cdot f_L \cdot C^{-1} N_L \, \chi_L + \ell_R^T \cdot f_R \cdot C^{-1} N_R \, \chi_R + \bar{N}_L \cdot m_N \cdot N_R \\ + \bar{\ell}_L \cdot h_L \cdot E_R \, \chi_L + \bar{\ell}_R \cdot h_R \cdot E_L \, \chi_R + \bar{E}_L \cdot m_E \cdot E_R + \text{h.c.}. \tag{SM.42}$$

Here, we suppress generation indices and thus $f_{L,R}$, m_N, $h_{L,R}$ and m_E are 3×3 matrices in the space of fermion generations. The charged lepton masses are generated through a Dirac seesaw mechanism and the mass matrix is given by

$$m_\ell = u_L u_R \left(h_L \cdot m_E^{-1} \cdot h_R^\dagger \right). \tag{SM.43}$$

In discussing the fields associated with neutrinos, we now work with the CP conjugates of the right-handed fields for convenience,

$$\nu_R \overset{CP}{\to} (\nu_R)^c = (\nu^c)_L = \sigma_L, \quad N_R \overset{CP}{\to} (N^c)_L = \Sigma_L, \tag{SM.44}$$

so that the mass matrix of the fields $(\nu_L, \sigma_L, N_L, \Sigma_L)^T$ can be written as

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & u_L f_L & 0 \\ 0 & 0 & 0 & u_R f_R \\ u_L f_L & 0 & 0 & m_N \\ 0 & u_R f_R & m_N & 0 \end{pmatrix}. \tag{SM.45}$$

This results in a spectrum of six Dirac neutrinos, three very heavy with masses $\approx m_N$ and three light with masses $\approx u_L u_R f_L f_R / m_N$. The heavy Dirac neutrinos are composed of N_L and Σ_L while the light Dirac neutrinos represent the active SM neutrinos in a combination of ν_L and σ_L or ν_R. Unlike the usual models of light Dirac neutrinos [18–25], where the light neutrino masses are proportional to a small induced VEV, in this scenario the smallness of neutrino masses is due to a seesaw mechanism where the heavy seesaw scale corresponds to the masses of the vector-like neutrino states.

Finally, the last ingredient of this model is a light charge-neutral scalar particle ϕ that can potentially be a Dark Matter candidate [26–28]. Of main interest to us, the presence of such a particle with a Yukawa coupling to N of the form $g_\phi N N \phi$, can lead to $0\nu\beta\beta\phi$ decay with emission of a light neutral scalar ϕ from a single effective dimension-7 operator of the form $\Lambda_{\text{NP}}^{-3} (\bar{u}\mathcal{O}d)(\bar{e}\mathcal{O}\nu)\phi$ as discussed in the main text. This model provides a working example of a scenario where purely Dirac neutrinos can mimic conventional

$0\nu\beta\beta$ decay through emission of extra particle that carries lepton number. This illustrates the necessity of searches for extra particles in double beta decay to understand the nature of neutrinos.

Leptoquarks and R-parity violating supersymmetry

Here we briefly discuss an alternative scenario for generating the effective Majoron current in Eq. (1) of the main text. This setup is based on a simple extension of the SM, introducing two heavy scalar leptoquarks $S_{3,2,1/6}$, $S_{3^*,1,1/3}$ and a scalar singlet ϕ. Here, the quantum numbers of the leptoquarks under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ are as indicated. The interesting part of the Lagrangian is given by

$$\mathcal{L}^{\rm LQ} = Y^1_{\alpha\beta} L_\alpha \bar{d}_{R\beta} S_{3,2,1/6} + Y^2_{\alpha\beta} e_{R\alpha} u_{R\beta} S_{3^*,1,1/3} + Y^S \phi H S^\dagger_{3,2,1/6} S^\dagger_{3^*,1,1/3}. \tag{SM.46}$$

Assigning lepton numbers as $L(S_{3,2,1/6}) = -1$, $L(S_{3^*,1,1/3}) = -1$ and $L(\phi) = -2$ this Lagrangian conserves lepton number. In Eq. (SM.46), we have written out explicitly generation indices of the lepton and quark fields, $\alpha, \beta = 1, 2, 3$. Thus, Y^1 and Y^2 are in general 3×3 matrices. However, double beta decay will be sensitive only to first generation.

Integrating out the heavy leptoquark states, this Lagrangian leads to an effective coupling $\epsilon_{\alpha\beta}(\bar{d}_R L^\alpha)(\bar{e}^c_R u_R) H^\beta \phi$. After electroweak symmetry breaking and Fierz rearrangement of the fields at low energies, the effective current $\frac{\epsilon^\phi_{RR}}{m_p} j^\mu_R J_{R\mu} \phi$ is generated as shown in Fig. 2 (right).

The phenomenology of this setup depends on whether the scalar ϕ develops a VEV,

$$Y^S \Phi H S^\dagger_{3,2,1/6} S^\dagger_{3^*,1,1/3} \Rightarrow Y^S \langle\phi\rangle H S^\dagger_{3,2,1/6} S^\dagger_{3^*,1,1/3}. \tag{SM.47}$$

In this case, lepton number is spontaneously broken and a massless (exotic) Majoron appears automatically and Majorana neutrino masses are generated. In Fig. 2 (right) we show the 2-loop neutrino mass diagram, which will result unavoidably for $\langle\phi\rangle \neq 0$. A rough estimate of the neutrino mass generated by this diagram is

$$m_\nu \approx \frac{Y^1 Y^2 m_u m_d m_\ell}{(16\pi^2)^2} \frac{Y^S \langle\phi\rangle v_{SM}}{\Lambda^4}, \tag{SM.48}$$

where Λ is of the order of the leptoquark masses and m_u, m_d, m_ℓ indicate the SM quark and lepton masses. For couplings of order one, 3rd generation SM fermion masses and

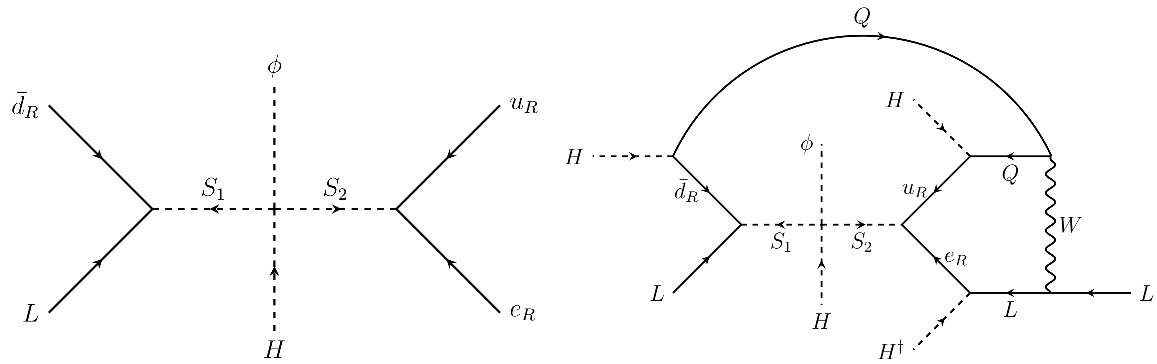

FIG. 2. Possible leptoquark contribution to the decay $0\nu\beta\beta\phi$ (left). If ϕ develops a VEV, the 2-loop diagram (right) is unavoidable in this model. Note that the presence of this singlet VEV signals lepton number violation.

$\Lambda = \mathcal{O}(1)$ TeV, neutrino masses of the order of the atmospheric scale can be generated for $\langle\phi\rangle \approx 10$ GeV. However, due to the smallness of the first generation fermion masses, no constraint on their couplings to leptoquarks can be derived from neutrino masses.

As in all such Majoron models, whether the constraints from non-observation of $0\nu\beta\beta\phi$ are more important than neutrino mass constraints or from the non-observation of ordinary $0\nu\beta\beta$ depends on the unknown value of $\langle\phi\rangle$. For $\langle\phi\rangle$ approaching zero lepton number is effectively restored and at low energies $0\nu\beta\beta\phi$ will provide the only constraint.

Finally, we would like to remark that the two leptoquarks in this model have the same quantum numbers as the scalar quark doublet and down-type scalar quark singlet fields, $S_{3,2,1/6} \equiv \tilde{Q}$ and $S_{3^*,1,1/3} \equiv \tilde{d}^c$ in supersymmetric models. This opens up to possibility to speculate about Majoron model variants in R-parity violating supersymmetry. However, different from the model discussed above, for R-parity violating supersymmetry the Lagrangian would contain terms $L_\alpha Q_\beta S_{3^*,1,1/3}$ instead of $e_{R\alpha} u_{R\beta} S_{3^*,1,1/3}$. This affects the discussion of the phenomenology, since (i) neutrino masses become 1-loop effects and (ii) at low energies different currents from the ones considered in the main text would be generated.

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