Sum Rate and Fairness Analysis for the MU-MIMO Downlink under PSK Signalling: Interference Suppression vs Exploitation

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***Abstract*—In this paper, we analyze the sum rate performance of multi-user multiple-input multiple-output (MU-MIMO) systems, with a finite constellation phase-shift keying (PSK) input alphabet. We analytically calculate and compare the achievable sum rate in three downlink transmission scenarios: 1) without precoding, 2) with zero forcing (ZF) precoding 3) with closed form constructive interference (CI) precoding technique. In light of this, new analytical expressions for the average sum rate are derived in the three cases, and Monte Carlo simulations are provided throughout to validate the analysis. Furthermore, based on the derived expressions, a power allocation scheme that can ensure fairness among the users is also proposed. The results in this work demonstrate that, the CI strictly outperforms the other two schemes, and the performance gap between the considered schemes increases with increase in the MIMO size. In addition, the CI provides higher fairness and the power allocation algorithm proposed in this paper can achieve maximum fairness index.**

***Index Terms*—Finite constellation signaling, zero forcing, constructive interference, phase-shift keying signaling, multiple-input multiple-output.**

I. Introduction

The recent decades have witnessed the widespread application of multi-user multiple-input multiple-output (MU-MIMO) communication systems, due to their high spectral efficiency and reliability [1]–[3]. However, these potential advantages of MU-MIMO systems are often undermined by strong interference in practical wireless systems [1]–[3]. Consequently, considerable amount of researches have focused on reducing/ mitigating the interference in MU-MIMO channels [3]–[5].

A number of information theoretic works have studied the sum rate performance of MU-MIMO systems by assuming Gaussian input signals. However, in practical communication systems, signals are generated from finite discrete constellation sets. In light of this, several works have considered MU-MIMO systems for finite alphabet input signals. In [3] optimal linear precoder for MU-MIMO interference channels with finite alphabet inputs was designed. In [5] the design of linear precoders in multi-cell MIMO systems for finite alphabet signals was studied. The authors in [6] considered the capacity of a MIMO fading channel with PSK input alphabet; in this work a downlink transmission without precoding was studied. In [7] the design of optimal precoders which maximize the mutual information of MIMO channels were investigated, by assuming non-Gaussian inputs of finite alphabet. The design of linear transmit precoder for MIMO broadcast channels with finite alphabet input signals was investigated in [8], where an explicit expression for the achievable rate region was derived. The work in [9] proposed low complexity precoding scheme that aimed to maximize the mutual information for MIMO systems with finite alphabet inputs. Linear precoder design that maximizes the average mutual information of MIMO fading channels with finite-alphabet inputs was proposed in [10], in which the statistical channel state information (CSI) was assumed to be known at the transmitter side. In [11], the authors studied a linear precoding for MIMO channels with finite discrete inputs, in which the capacity region for the MU-MIMO has been derived. In [12] a linear precoder design for MU-MIMO transceivers under finite alphabet inputs was proposed, where the optimal transmission strategies in both low and high signal-to-noise ratio (SNR) regions were studied. In addition, the authors in [13] have formulated MIMO transmission game, in which based on the users' sum rate, the base station (BS) obtained the transmit power. Although the aforementioned algorithms produced optimal performances, the fact that they have no closed form solutions and their resulting high computational complexity make them inapplicable in practical scenarios.

Furthermore, when the BS has full CSI the capacity achieving dirty-paper coding (DPC) scheme has been proposed by pre-canceling the interference at the BS. However, due to the impractical assumption of infinite codewords length and its very high computational cost, DPC is difficult to implement in practical communication networks. In order to perform a compromise between complexity and performance, its non-linear counterparts, Tomlinson-Harashima precoding (THP) and vector perturbation (VP) have been proposed. Nevertheless, THP and VP are still complicated and hard to implement in practical communication systems, due to the inclusion of the sophisticated sphere-search algorithm [14]–[16]. Consequently, linear precoding schemes with low complexity such as zero forcing (ZF) precoding, have received increasing research attention [14], [17]. However, the conventional linear precoding schemes such as ZF have ignored the fact that the interference can be beneficial for the communication networks and further exploited [14], [17].

Recently, constructive interference (CI) precoding technique has been proposed to enhance the performance of downlink

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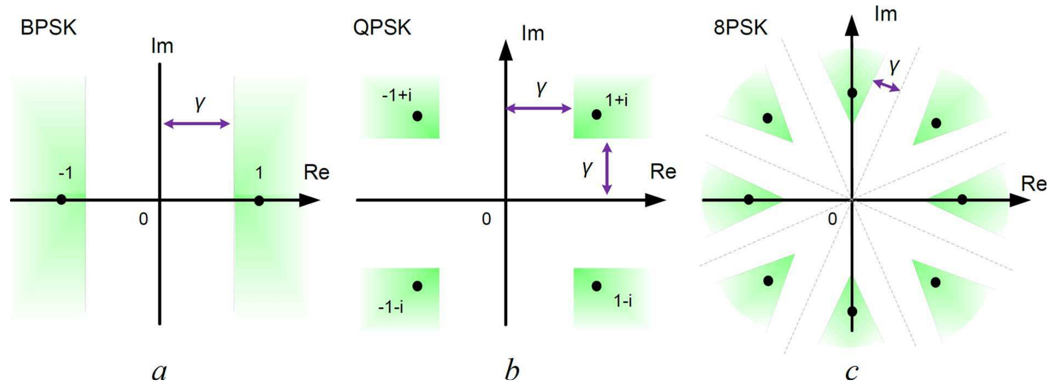


Figure 1: Concept of Constructive interference in a) BPSK, b) QPSK and c) 8PSK, where γ is the threshold distance [21].

MU-MIMO systems [18]–[22]. In contrast to the conventional techniques where the knowledge of the interference is used to cancel it, the main idea of the CI is to use the interference to improve the system performance. Specifically, the CI precoding technique exploits interference that can be known to the transmitter to increase the useful signal received power [18], [20]–[22]. That is, with the knowledge of the CSI and users' data symbols, the interference can be classified as constructive and destructive. The interference signal is considered to be constructive to the transmitted signal if it moves/ pushes the received symbols away from the decision thresholds of the constellation towards the direction of the desired symbol. Accordingly, the transmit precoding can be designed such that the resulting interference is constructive to the desired symbol. For clarity, the basic idea of CI is summarized in Fig. 1 for PSK constellations. Briefly, CI is the interference that pushes the received symbols away from the decision thresholds of the constellation, which represents the green areas in the figure, and thus improves the detection [21].

The concept of the CI has been extensively studied in literature. This line of work has been introduced in [18], where the CI precoding scheme for the downlink of PSK-based MIMO systems has been proposed. In this work it was shown that the system performance can be enhanced by exploiting the interference signals. As a result, the effective signal to interference-plus-noise ratio (SINR) can be enhanced without the need to increase the transmitted signal power at the BS. In [19] instead of eleminating interference, as in conventional schemes, the authors proposed adaptive code allocation to exploit the constructive interference in order to enhance the SINR at the receiver. In [20] the concept of CI was used to design an optimization based precoder in the form of pre-scaling for the first time. Thereof, [21] proposed transmit beamforming schemes for the MU-MIMO down-link that minimize the transmit power for generic PSK signals. In [22], [23] CI precoding scheme was applied in wireless power transfer scenario in order to minimize the transmit power while guaranteeing the energy harvesting and the quality of service (QoS) constraints for PSK messages. Further work in [24] applied the CI concept to massive multi-input multi-output (M-MIMO) systems. The authors in [25], [26] studied general category of CI regions, namely distance preserving CI region, where full characterization for a generic constellation was provided. Very recently, the authors in [14] derived closed-form precoding expression for CI exploitation in the MU-MIMO down-link. The closed-form precoder in this work has for the first time made the application of CI exploitation practical, and has further paved the way for the development of communication theoretic analyses of the benefits of CI, which is the focus of this work.

Accordingly this paper investigates the sum rate and fairness of downlink transmission with finite constellation PSK signalling for interference suppression and interference exploitation techniques. Therefore, interference suppression zero forcing (ZF) precoding technique, and CI precoding technique have been considered and investigated. We would like to mention here that, this work is the first ever analytical performance evaluation of CI precoding technique. In addition and in order to provide clear explanation/comparison between the two techniques a scenario when the CSI is unknown at the BS is also considered, in which we study downlink transmission without precoding. The un-precorded scheme can be considered as a lower-bound and can give clear explanation about the gain attained by using the other precoding techniques [1]. In this regard, new closed-form explicit expressions for the average sum rate are derived for each transmission scheme. The derived rate expressions provide practical design insights into the impact of different system parameters on the achievable sum-rate. Then, based on the derived sum-rate expressions we extend our investigation on the performance of the interference suppression and exploitation techniques to include the fairness among the users. The fairness of communication systems can be supported through appropriate power allocation of the transmitted signals. Therefore, power allocation scheme is studied for the sake of improving the system performance and ensuring the fairness among users for the considered techniques.

For clarity we summarize the major contributions of this work as:

1) New closed-form explicit analytical expressions for the average achievable sum rate of MU-MIMO with PSK inputs are derived for, a) un-precoded transmission, b) ZF precoded transmission and c) CI precoded transmission, considering the channels to be of Rayleigh fading. The analytical expressions

[1] Additionally, in Section numerical results some simulated results for maximum ratio transmission (MRT) are also included.

for the average sum rate provided in this paper are new and simple expressions, from these expressions the impact of different system parameters on the achievable sum-rate can be clearly observed and the optimal system design, e.g., the number of BS antennas, for enhancing the system performance can be chosen. We would like to emphasize that, this paper is the first work on the theoretical performance analysis of the achievable sum rate for CI precoders. In addition, while traditional analysis focus on Gaussian signaling, the study of finite constellation signaling is of particular importance, since finite constellations are applied in all practical applications of the explored precoders. The performance analysis of finite alphabet systems is difficult and challenging, since the signals are non-Gaussian and as a result the analytical expression to calculate the rate is too complicated, which makes the analysis of this model hard and challenging. Due to the complexity of deriving the average sum rate, in this work we use Jensen inequality to derive closed-form bounds as alternatives, which greatly reduce the computational complexity as will be explained in Section VII. It is shown that the derived expressions actually are very tight to the average rate. To the best of our knowledge, the derived closed form sum-rate bound expressions in literature on finite alphabet scenarios are approximated based on very low/high SNR, and/or large number of antennas, and/or conditioning on the precoding matrix [5], [6], [9], [10], [12], [27].

2) Based on the derived sum rate expressions, we analyze and compare the fairness among the users of the considered transmission techniques. In this regard a power allocation scheme that can provide fairness of the considered system is proposed. In this power allocation scheme, first we calculate the transmit power required to achieve a target data rate, and then a simple algorithm that can be used to provide fairness in MU-MIMO systems with finite alphabet signals is introduced. In the proposed method, the BS allocates power to meet a target rate, thereby the system fairness can be enhanced by reducing the excessive power of the users who have good channel condition. Simulation results explain clearly that the proposed algorithm can achieve higher system performance. We would like to mention again that, there is no other sum-rate fairness technique for CI in the literature and this is only made possible through our new analysis. Furthermore, due to the essential difference between Gaussian and finite-alphabet inputs, a significant performance gap exists between the power allocation techniques based on Gaussian-input and finite-alphabet input. Thus, power allocation schemes based on finite alphabet constraints becomes a substantial issue [28], [29].

3) Monte-Carlo simulations are provided to validate the accuracy of our analysis, and the impact of different system parameters on the system performance of the considered schemes are examined and investigated.

Results provided in this paper show that CI scheme outperforms the other schemes, for the same system parameters. Also, it is shown that increasing the SNR and the number of the BS antennas enhances the system performance, whereas the gap between the minimum transmission power required for ZF and CI to achieve same target rate is almost fixed with increasing the distance between the BS and the users. Furthermore, the CI provides higher fairness than ZF and the power allocation algorithm proposed in this paper achieves high fairness index.

$\mathbf{H}$ and $\mathbf{h}$	Channel matrix and vector
$(\cdot)^H$ and $(\cdot)^T$	Conjugate transposition, and transposition
$\mathcal{E}\,[.]$ and diag $(.)$	Average operation and diagonal of a matrix
$[\mathbf{h}]_k$ and h	Element k in vector $\mathbf{h}$and a scalar
$\lvert.\rvert$and $\lVert.\rVert^2$	Absolute value and Second norm
$\mathbb{C}^{K\times N}$ and $\mathbf{I}$	K ×N matrix, and the identity matrix
$\mathbf{W}$	Precoding matrix
R_k	Rate for user k
N	Number of BS antennas
K	Number of users
d_k	Distance between the BS and the k^{th} user
m	Path loss exponent
p	Transmission power

Table I: Summary of Symbols and Notations.

Next, Section II describes the system model under consideration. Sections III, IV, and V derive the analytical expressions for the average sum rate in conventional transmission, ZF and CI precoding techniques, respectively. Section VI, considers the users fairness in the three transmission schemes. Numerical and simulation results are presented and discussed in Section VII. Finally, the main conclusions of this work are stated in Section VIII.

II. System Model

Consider a downlink MU-MIMO system, consisting of a BS equipped with N antennas communicating with K single antenna users, where $N \geq K$. All the channels are modeled as independent identically distributed (i.i.d) Rayleigh fading channels. The channel matrix between the BS and the K users is denoted by $\mathbf{H} \in \mathbb{C}^{K\times N}$, which can be represented as $\mathbf{H} = \mathbf{D}^{1/2}\mathbf{H}_1$where $\mathbf{H}_1 \in \mathbb{C}^{K\times N}$ contains i.i.d $\mathcal{CN}\,(0{,}1)$ entries which represent small scale fading coefficients and $\mathbf{D} \in \mathbb{C}^{K\times K}$ is a diagonal matrix with $[\mathbf{D}]_{kk} = \varpi_k$ which represent the path-loss attenuation $\varpi_k = d_k^{-m}$. For clarity, Table I summarizes the commonly used symbols and notations. It is also assumed that the signal is equiprobably drawn from an M-PSK constellation and denoted as $\mathbf{s} \in \mathbb{C}^{K\times 1}$ [14]. The received signal at the k^{th} user can be expressed as,

$$y_k = \mathbf{h}_k\mathbf{W}\mathbf{s} + n_k \tag{1}$$

where $\mathbf{h}_k$ is the channel vector from the BS to user k, $\mathbf{W}$ is the precoding matrix, n_k is the additive wight Gaussian noise (AWGN) at the k^{th} user, $n_k \sim \mathcal{CN}\left(0,\, \sigma_k^2\right)$.

It was shown in [6], [8], [12] that, the achievable rate for the k-th user in general MU-MIMO system with finite constellation signaling, is given by,

$$R_k = N\log_2 M - \frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{h},\mathbf{n}}\left\{\log_2\sum_{i=1}^{M^N} e^{\frac{-\|\mathbf{h}_k\mathbf{W}\boldsymbol{s}_{m,i}+n_k\|^2}{\sigma_k^2}}\right\}$$
$$+\frac{1}{M^{N-1}}\sum_{c=1}^{M^{N-1}}\mathcal{E}_{\mathbf{h},\mathbf{n}}\left\{\log_2\sum_{t=1}^{M^{N-1}} e^{\frac{-\|\mathbf{h}_k\mathbf{W}\boldsymbol{s}_{c,t}+n_k\|^2}{\sigma_k^2}}\right\}, \tag{2}$$

where $\mathbf{s}_{m,i}=\mathbf{s}_m-\mathbf{s}_i$, $\mathbf{s}_m$ and $\mathbf{s}_i$ contain symbols taken from the M signal constellation.

In the following, the average sum rate is derived for three cases, without precoding, with ZF precoding and with CI precoding technique.

III. Downlink Transmission Without Precoding

In this case the BS transmits the users' signals without any precoding technique, such scenario occurs when the CSI of the users is unknown at the BS. Let the k^{th}signal be equiprobably drawn from an M-PSK constellation, the average rate at a user k can be written as [5], [8], [12],

$$\bar{R}_k=\log_2 M^N-\underbrace{\frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{h},n_k}\log_2\sum_{i=1}^{M^N}e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2}{\sigma_k^2}}}_{T_1}$$

$$+\underbrace{\frac{1}{M^{N-1}}\sum_{\substack{c=1\\c\neq k}}^{M^{N-1}}\mathcal{E}_{\mathbf{h},n_k}\log_2\sum_{\substack{t=1\\t\neq k}}^{M^{N-1}}e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}+n_k\right|^2}{\sigma_k^2}}}_{T_2}, \quad (3)$$

where $p_N=\frac{p}{N}$, is the power transmitted by each antenna. The second and third terms in (3), $\{T_1,T_2\}$, can be simplified to more familiar formulas. The second term, T_1, can be simplified by taking the j^{th} term, $\left(e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}\right)$, out as follows

$$T_1=\frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{h},n_k}\Bigg\{\log_2 e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}$$

$$\times\left(1+\sum_{i=1,i\neq j}^{M^N}e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}\right)\Bigg\} \quad (4)$$

$$=\frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{h},n_k}\Bigg\{\left(\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}\log_2 e\right)$$

$$+\log_2\left(1+\sum_{i=1,i\neq j}^{M^N}e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}\right)\Bigg\}. \quad (5)$$

where $j\in\left\{1:M^N\right\}$. Please note that, in case $j=m$, $e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}=e^{\frac{-|n_k|^2}{\sigma_k^2}}$. Finally, with the use of some auxiliary notation the second term can be expressed as

$$T_1=\frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{h},n_k}\left\{\left(\Upsilon_1+\log_2\left(1+\Xi_1\right)\right)\right\}, \quad (6)$$

where $\Upsilon_1=\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2\log_2 e}{\sigma_k^2}$ and $\Xi_1=\sum_{i=1,i\neq j}^{M^N}e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}$. Likewise, by following similar steps for T_2 we can get

$$T_2=\frac{1}{M^{N-1}}\sum_{\substack{c=1\\c\neq k}}^{M^{N-1}}\mathcal{E}_{\mathbf{h},n_k}\left\{\left(\Upsilon_2+\log_2\left(1+\Xi_2\right)\right)\right\}, \quad (7)$$

where $\Upsilon_2=\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}+n_k\right|^2\log_2 e}{\sigma_k^2}$ and $\Xi_2=\sum_{t=1,t\neq k,j}^{M^{N-1}}e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}+n_k\right|^2}{\sigma_k^2}}$.

Theorem 1. *Accurate approximation of the average sum rate of the un-precoded downlink transmission scheme in MU-MIMO systems under PSK signaling can be calculated by*

$$R=\sum_{k=1}^{K}\bar{R}_k, \quad (8)$$

where $\bar{R}_k$ is given by (9), shown at the top of next page.

Proof: The proof is provided in Appendix A. ■

It is worth mentioning that, Jensen's inequality has been implemented in the two terms in (3), which are subtracted in (3). Accordingly, the resulting expression cannot lead to a strict bound of the resulting rate. Nevertheless, since the involved rate is based on a finite constellation, the resulting low-SNR and high SNR approximation correspond the exact rate. In the intermediate SNR regions it can be observed that the bounding errors of the two terms have similar values which results in an accurate overall approximation, as already verified in relevant analysis in [12]. We note that the bottom line rate approximations show a very close match to our Monte Carlo simulations in our results Section VII.

The average sum-rate with respect to each user location can be obtained by averaging the derived sum-rate over all possible user locations.

IV. Zero Forcing Precoding

In this case the BS has perfect CSI and ZF precoding technique is implemented. Therefore, the precoding matrix can be written as [14], [30],

$$\mathbf{W}=\frac{1}{\beta}\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}, \quad (10)$$

where β is the scaling factor to meet the transmit power constraint. Therefore, the received signal at the k^{th} user can be expressed as,

$$\begin{aligned}y_k&=\beta\mathbf{h}_k\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\mathbf{s}+n_k,\\&=\beta\left[\mathbf{s}\right]_k+n_k.\end{aligned} \quad (11)$$

Consequently, the rate in this scenario is given by [6], [8]

$$\bar{R}_k^{ZF}=N\log_2 M-\log_2 e$$

$$-\frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{H},n_k}\left\{\log_2\sum_{i=1}^{M^N}e^{\frac{-\left|\beta\left[\mathbf{s}_{m,i}\right]_k+n_k\right|^2}{\sigma_k^2}}\right\}. \quad (12)$$

$$\bar{R_k} = N \quad \log_2 M - \frac{1}{M^N}\sum_{m=1}^{M^N}\left(-\log_2 e + \left\{\log_2\left(1+\sum_{i=1, i\neq j}^{M^N}\frac{2\sigma_k^2}{2\sigma_k^2+p_N d_k^{-m}\lambda_{mi}}\right)\right\}\right) + \frac{1}{M^{N-1}}\sum_{\substack{c=1\\ c\neq k}}^{M^{N-1}}\left(-\log_2 e + \left\{\log_2\left(1+\sum_{t=1, t\neq k,j}^{M^{N-1}}\frac{2\sigma_k^2}{2\sigma_k^2+p_N d_k^{-m}\lambda_{ct}}\right)\right\}\right). \tag{9}$$

$$\bar{R_k}^{ZF} = N\log_2 M - \log_2 e - \frac{1}{M^N}\sum_{m=1}^{M^N}\left\{\left(-\left(\left|\beta\left[\mathbf{s}_{m,j}\right]_k\right|^2+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right) + \log_2\left(1+\sum_{i=1, i\neq j}^{M^N} e^{-\frac{\left|\beta[\mathbf{s}_{m,i}]_k\right|^2-\left|\beta[\mathbf{s}_{m,j}]_k\right|^2}{2\sigma_k^2}}\right)\right\} \tag{15}$$

By taking the j^{th} term $\left(e^{\frac{-\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2}{\sigma_k^2}}\right)$ out, (12) can be expressed as

$$\bar{R_k}^{ZF} = N\log_2 M - \log_2 e - \frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{H},n_k}\left\{\left(\Upsilon+\log_2\left(1+\Xi\right)\right)\right\}, \tag{13}$$

where $j \in \left[1, M^N\right]$, $\Upsilon = \frac{-\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2}{\sigma_k^2}\log_2 e$ and $\Xi = \sum_{i=1, i\neq j}^{M^N} e^{\frac{-\left|\beta[\mathbf{s}_{m,i}]_k+n_k\right|^2+\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2}{\sigma_k^2}}$.

Theorem 2. *The average sum rate of the ZF transmission scheme in MU-MIMO systems under PSK signaling can be lower bounded by*

$$R^{ZF} = \sum_{k=1}^{K}\bar{R_k}^{ZF}, \tag{14}$$

where $\bar{R_k}^{ZF}$ is given by (15), shown at the top of this page.

Proof: The proof is provided in Appendix B. ∎

V. Constructive Interference Precoding

The concept and characterization for the CI have been extensively investigated in MIMO systems [18], [31], [32]. In order to avoid repetition and for more details, we refer the reader to the aforementioned works in this paper. Here in this section, we analyze the performance of CI precoding technique in MU-MIMO systems, for the first time. We focus on the recent closed form CI precoding scheme, where the precoding matrix is given by [14],

$$\mathbf{W}\boldsymbol{s} = \frac{1}{K}\beta\,\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\operatorname{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\mathbf{s}, \tag{16}$$

where β is the scaling factor to meet the transmit power constraint, which can be expressed as [14] $\beta = \sqrt{\frac{p}{\mathbf{u}^H\mathbf{V}^{-1}\mathbf{u}}}$, while $\mathbf{1}^H\mathbf{u} = \mathbf{1}$ and $\mathbf{V} = \operatorname{diag}\left(\mathbf{s}^H\right)\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\operatorname{diag}\left(\mathbf{s}\right)$. For the sake of comparison, the normalization factor β is designed to ensure that the long-term total transmit power at the source is constrained, and it is given by [4] $\beta = \sqrt{\frac{p}{\mathbf{u}^H\mathcal{E}[\mathbf{V}^{-1}]\mathbf{u}}}$, where $\mathcal{E}\left[\mathbf{V}^{-1}\right] = \operatorname{diag}\left(\mathbf{s}^H\right)^{-1}\mathcal{E}\left[\left(\mathbf{H}\mathbf{H}^H\right)\right]\left(\operatorname{diag}\left(\mathbf{s}\right)\right)^{-1} = \operatorname{diag}\left(\mathbf{s}^H\right)^{-1}N\boldsymbol{\Sigma}\left(\operatorname{diag}\left(\mathbf{s}\right)\right)^{-1}$ [33]. The received signal at the k^{th} user now can be written as,

$$y_k = \frac{\beta}{K}\mathbf{h}_k\,\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\operatorname{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\mathbf{s}+n_k,$$

$$= \frac{\beta}{K}\mathbf{a}_k\left(\operatorname{diag}\left(\mathbf{s}^H\right)\right)^{-1}\mathbf{H}\mathbf{H}^H\left(\operatorname{diag}\left(\mathbf{s}\right)\right)^{-1}\mathbf{u}\left[\boldsymbol{s}\right]_k+n_k \tag{17}$$

where $\mathbf{a}_k$ is a $1\times K$ vector all the elements of this vector are zeros except the k^{th} element is one. Following the principles of CI, and the symmetric properties of the PSK constellation, the rate at the user k can be written as [6], [8],

$$\bar{R_k}^{CI} = N\log_2 M - \log_2 e - \frac{1}{M^N}\sum_{m=1}^{M^N}\mathcal{E}_{\mathbf{H},n_k}\left\{\left(\Upsilon+\log_2\left(1+\Xi\right)\right)\right\}, \tag{18}$$

wherewhere $\Upsilon = \frac{-|\mathbf{h}_k\mathbf{W}\boldsymbol{s}_{m,j}+n_k|^2\log_2 e}{\sigma_k^2}$ and $\Xi = \sum_{i=1, i\neq j}^{M^N} e^{\frac{-|\mathbf{h}_k\mathbf{W}\boldsymbol{s}_{m,i}+n_k|^2+|\mathbf{h}_k\mathbf{W}\boldsymbol{s}_{m,j}+n_k|^2}{\sigma_k^2}}$.

$$\Lambda_{m,i} = \left(\frac{2^{\left(\frac{1}{2}(N-K-1)\right)} K^{(N-K+1)} \left|[\boldsymbol{s}_{m,i}]_k\right|^{-2+K-N}}{(N-K)!} \right) \left(\left(\frac{c_k^2}{\sigma_k^2} \right)^{\frac{1}{2}(K-N-1)} \right)$$
$$\times \left(\left(c_k^2 \left|[\boldsymbol{s}_{m,i}]_k\right| \right) \Gamma\left(\frac{1}{2}(N-K+1) \right) {}_1\mathrm{F}_1\left(\frac{1}{2}(N-K+1), \frac{1}{2}, \frac{K^2\sigma_k^2}{2c_k^2 \left|[\boldsymbol{s}_{m,i}]_k\right|^2} \right) \right.$$
$$\left. - \sqrt{2}K\, c_k\, \sigma_k \Gamma\left(\frac{1}{2}(N-K+2) \right) {}_1\mathrm{F}_1\left(\frac{1}{2}(N-K+2), \frac{3}{2}, \frac{K^2\sigma_k^2}{2c_k^2 \left|[\boldsymbol{s}_{m,i}]_k\right|^2} \right) \right). \tag{21}$$

Theorem 3. *The average sum rate of the CI transmission scheme in MU-MIMO systems under PSK signaling can be lower bounded by*

$$R^{CI} = \sum_{k=1}^{K} \bar{R_k}^{CI}, \tag{19}$$

where

$$\bar{R_k}^{CI} = N \log_2 M - \log_2 e - \frac{1}{M^N} \sum_{m=1}^{M^N} \left(-\log_2 e + \left\{ \log_2 \left(1 + \sum_{i=1, i\neq j}^{M^N} \Lambda_{m,i} \right) \right\} \right). \tag{20}$$

and $\Lambda_{m,i}$ is given by (21), shown at the top of this page.

Proof: The proof is provided in Appendix C. ■

VI. Fairness for MU-MIMO Systems with PSK Signaling

In this section we consider the fairness among the users for the system model under consideration. Firstly, based on the derived expressions in the previous sections, we calculate the minimum power required to achieve a target data rate, and then we propose a fairness algorithm that can be used to provide fairness in MU-MIMO systems with finite alphabet signals.

A. Minimum Power Transmission

Consider that the BS transmits the messages at a target data rate. Let the target data rate be denoted by R_T, so that for perfect transmission the achievable rate at the k^{th} user, $\bar{R}_k$, has to satisfy the condition, $\bar{R}_k \geq R_T$. In order to determine the minimum power transmission required to achieve R_T, we simplify the derived expressions in the previous sections as follows.

1) un-precoded downlink Transmission: To find the minimum power transmission in this case we simplify (9) into,

$$R_T = \bar{R}_k$$
$$= X - \left\{ \frac{1}{M^N} \sum_{m=1}^{M^N} \log_2 \left(1 + \sum_{i=1, i\neq j}^{M^N} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{mi}} \right) \right\}$$
$$+ \left\{ \frac{1}{M^{N-1}} \sum_{\substack{c=1 \\ c\neq k}}^{M^{N-1}} \log_2 \left(1 + \sum_{t=1, t\neq k}^{M^{N-1}} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{ct}} \right) \right\}, \tag{22}$$

where $X = N \log_2 M$. The minimum p_k can be obtained by solving,

$$X - \left\{ \frac{1}{M^N} \sum_{m=1}^{M^N} \log_2 \left(1 + \sum_{i=1, i\neq j}^{M^N} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{mi}} \right) \right\} +$$
$$\frac{1}{M^{N-1}} \sum_{\substack{c=1 \\ c\neq k}}^{M^{N-1}} \log_2 \left(1 + \sum_{\substack{t=1 \\ t\neq k}}^{M^{N-1}} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{ct}} \right) - R_T = 0 \tag{23}$$

Therefore, minimum value of p_k is the value that satisfies (23). By exploiting the symmetric properties of the constellation [6, Eq(5)], (23) can be reduced to

$$X - \left\{ \log_2 \left(1 + \sum_{i=1, i\neq j}^{M^N} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{\bar{m}i}} \right) \right\}$$
$$+ \left\{ \log_2 \left(1 + \sum_{t=1, t\neq k}^{M^{N-1}} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{\bar{c}t}} \right) \right\} - R_T = 0, \tag{24}$$

where $\bar{m}$ and $\bar{c} = [1, 1, ...1]^T$ [6]. Hence, (24) can be written as

$$\frac{1 + \sum_{t=1, t\neq k}^{M^{N-1}} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{\bar{c}t}}}{1 + \sum_{i=1, i\neq j}^{M^N} \frac{2\sigma_k^2}{2\sigma_k^2 + p_k d_k^{-m} \lambda_{\bar{m}i}}} = Y, \tag{25}$$

and

$$\sum_{t=1, t\neq k}^{M^{N-1}} \frac{2}{2 + \frac{p_k}{\sigma_k^2} d_k^{-m} \lambda_{\bar{c}t}}$$
$$-Y \sum_{i=1, i\neq j}^{M^N} \frac{2}{2 + \frac{p_k}{\sigma_k^2} d_k^{-m} \lambda_{\bar{m}i}} = Y - 1, \tag{26}$$

where $Y = 2^{R_T - X}$, therefore at high SNR, the minimum value of the transmission power p_k can be expressed as

$$p_k = \frac{\sum\limits_{t=1,t\neq k}^{M^{N-1}} \frac{2\sigma_k^2}{d_k^{-m}\lambda_{\bar{c}t}} - Y \sum\limits_{i=1,i\neq j}^{M^N} \frac{2\sigma_k^2}{d_k^{-m}\lambda_{\bar{m}i}}}{Y-1}. \quad (27)$$

2) Zero Forcing Precoding: To find the minimum power transmission in ZF precoding case, we simplify (15) into,

$$R_T = \bar{R_k}^{ZF}$$

$$= X - \frac{1}{M^N}\sum_{m=1}^{M^N}\left\{\left(-\left(\left|\beta\left[\mathbf{s}_{m,j}\right]_k\right|^2 + \sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right)\right.$$
$$\left. + \log_2\left(1 + \sum_{i=1,i\neq j}^{M^N} e^{-\frac{\left|\beta\left[\mathbf{s}_{m,i}\right]_k\right|^2 - \left|\beta\left[\mathbf{s}_{m,j}\right]_k\right|^2}{2\sigma_k^2}}\right)\right\} \quad (28)$$

where $X = N\log_2 M - \log_2 e$. The minimum p_k can be obtained by solving,

$$X - \frac{1}{M^N}\sum_{m=1}^{M^N}\left\{\left(-\left(\left|\beta\left[\mathbf{s}_{m,j}\right]_k\right|^2 + \sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right)\right.$$
$$\left. + \log_2\left(1 + \sum_{i=1,i\neq j}^{M^N} e^{-\frac{\left|\beta\left[\mathbf{s}_{m,i}\right]_k\right|^2 - \left|\beta\left[\mathbf{s}_{m,j}\right]_k\right|^2}{2\sigma_k^2}}\right)\right\} - R_T = 0 \quad (29)$$

The minimum p_k is the value that satisfies (29). Again using the simplification in [6, Eq(5)], (29) reduces to

$$X - \left\{\left(-\left(\left|\beta\left[\mathbf{s}_{\bar{m},j}\right]_k\right|^2 + \sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right)\right.$$
$$\left. + \log_2\left(1 + \sum_{i=1,i\neq j}^{M^N} e^{-\frac{\left|\beta\left[\mathbf{s}_{\bar{m},i}\right]_k\right|^2 - \left|\beta\left[\mathbf{s}_{\bar{m},j}\right]_k\right|^2}{2\sigma_k^2}}\right)\right\} - R_T = 0 \quad (30)$$

which can be written as

$$\left\{\left(-\left(\left|\beta\left[\mathbf{s}_{\bar{m},j}\right]_k\right|^2 + \sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right)\right.$$
$$\left. + \log_2\left(1 + \sum_{i=1,i\neq j}^{M^N} e^{-\frac{\left|\beta\left[\mathbf{s}_{\bar{m},i}\right]_k\right|^2 - \left|\beta\left[\mathbf{s}_{\bar{m},j}\right]_k\right|^2}{2\sigma_k^2}}\right)\right\} = X - R_T \quad (31)$$

Therefore, in high SNR the minimum transmission power can be obtained by

$$p_k = \frac{\sigma_k^2\left(R_T - N\log_2 M\right)}{\log_2 e\,\zeta^2\varpi_k^2\left|\left[\mathbf{s}_{\bar{m},j}\right]_k\right|^2} \quad (32)$$

where $\zeta = \frac{\Gamma\left(\frac{3}{2} - K + N\right)}{\sqrt{(\mathbf{s}^H\Sigma^{-1}\mathbf{s})}K\sqrt{K}(N-K)!}$.

3) Constructive Interference Precoding: To find the minimum transmission power in CI precoding scheme we simplify (20) into,

$$\begin{aligned} R_T &= \bar{R_k}^{CI} \\ &= X - \left\{\frac{1}{M^N}\sum_{m=1}^{M^N}\log_2\left(1 + \sum_{i=1,i\neq j}^{M^N}\Lambda_{m,i}\right)\right\} \end{aligned} \quad (33)$$

where $X = N\log_2 M$. The minimum p_k can be obtained by solving,

$$X - \left\{\frac{1}{M^N}\sum_{m=1}^{M^N}\log_2\left(1 + \sum_{i=1,i\neq j}^{M^N}\Lambda_{m,i}\right)\right\} - R_T = 0. \quad (34)$$

Therefore, the minimum value of p_k is the value that satisfies (34). Using the simplification in [6, Eq(5)], (34) becomes,

$$X - \left\{\log_2\left(1 + \sum_{i=1,i\neq j}^{M^N}\Lambda_{\bar{m},i}\right)\right\} - R_T = 0, \quad (35)$$

which can be written as,

$$\sum_{i=1,i\neq j}^{M^N}\Lambda_{\bar{m},i} = 2^{X - R_T} - 1. \quad (36)$$

The Hypergeometric function is defined as,

$${}_1\mathrm{F}_1\left(a,b,z\right) = \sum_{\upsilon=0}^{\infty}\frac{(a)_\upsilon}{(b)_\upsilon}\frac{z^\upsilon}{\upsilon!}, \quad (37)$$

where $(a)_\upsilon$ and $(b)_\upsilon$ are Pochhammer symbols. By substituting (37) into (21) we can notice that, at high SNR or when number of the users is much smaller than number of the antennas, only the first term in (37) has great impact on the value of Hypergeometric function, and therefore the other terms can be ignored with high accuracy. Hence, at high SNR we can get

$$\sum_{i=1,i\neq j}^{M^N}\left(AC_1C_2\right) = 2^{X-R_T} - 1, \quad (38)$$

where $A = \left(\frac{2^{\left(\frac{1}{2}(N-K-1)\right)}K^{(N-K+1)}\left|\left[\boldsymbol{s}_{\bar{m},i}\right]_k\right|^{-2+K-N}}{(N-K)!}\right)$, $C_1 = \left(\left(\frac{c_k^2}{\sigma_k^2}\right)^{\frac{1}{2}(K-N-1)}\right)$,
$C_2 = \left(\left(c_k^2\left|\left[\boldsymbol{s}_{\bar{m},i}\right]_k\right|\right)\Gamma\left(\frac{1}{2}\left(N-K+1\right)\right)\right)$. From (38) we can find,

$$a_{1k}\sum_{i=1,i\neq j}^{M^N}\left|\left[\boldsymbol{s}_{\bar{m},i}\right]_k\right|^{-1+K-N}\left(\left(c_k^2\right)^{\frac{1}{2}(K-N+1)}\right) = \left(2^{X-R_T} - 1\right), \quad (39)$$

Algorithm 1 Optimal Algorithm for R^*.

1) Initialize $R_{TLB}=0,$ and $R_{TUB}=\log_2 M$.
2) While $(R_{TUB}-R_{TLB}\geq\epsilon)$do
3) Set $R_T=\frac{R_{TLB}+R_{TUB}}{2}$.
4) Obtain $p_1,...,p_K$ from (27), (32) and (41).
5) If $\left(\sum\limits_{k=1}^{K} p_k\leq P_t\right)$then,
6) Set $R_{TLB}=R_T$; $R^*=R_T$
7) Else
8) Set $R_{TUB}=R_T$

$$\sum_{i=1,i\neq j}^{M^N}\left|\left[\boldsymbol{s}_{\bar{m},i}\right]_k\right|^{-1+K-N}\left(\frac{\beta}{a_{2k}}\right)^{(K-N+1)}=\frac{\left(2^{X-R_T}-1\right)}{a_{1k}}, \tag{40}$$

where $a_{1k}=\frac{\sigma_k^{(N-K+1)}2^{\left(\frac{1}{2}(N-K-1)\right)}K^{(N-K+1)}\Gamma\left(\frac{1}{2}(N-K+1)\right)}{(N-K)!}$ and $a_{2k}=\frac{K}{\left(\mathbf{a}_k(\mathrm{diag}(\mathbf{s}^H))^{-1}\Sigma(\mathrm{diag}(\mathbf{s}))^{-1}\mathbf{u}\right)}$. Finally, the minimum transmission power in CI scenario can be obtained by

$$p_k=\sqrt[\left(\frac{K-N+1}{2}\right)]{\frac{\left(2^{X-R_T}-1\right)}{a_{1k}\sum\limits_{i=1,i\neq j}^{M^N}\left(a_{2k}a_3\right)^{N-K-1}\left|\left[\boldsymbol{s}_{\bar{m},i}\right]_k\right|^{-1+K-N}}}. \tag{41}$$

where $a_3=\sqrt{\mathbf{u}^H\mathrm{diag}\left(\mathbf{s}^H\right)^{-1}N\boldsymbol{\Sigma}\left(\mathrm{diag}\left(\mathbf{s}\right)\right)^{-1}\mathbf{u}}$.

B. $Max-Min$ Fairness Algorithm

In this section, based on the achievable data rate, the fairness problem is formulated. Specifically, we propose a power allocation scheme which maximizes the minimum user rate, whilst satisfying the total power constraint as in the following expression,

$$\begin{aligned}&\max_{p_k}\quad\min_{k=1,\dots K}\quad R_k\\ &\text{s.t.}\sum_{k=1}^{K}p_k\leq P_t\end{aligned} \tag{42}$$

where P_t is the total power. As we can see from the previous sections that, the achievable data rate expression, R_k, is complex and this complexity makes the optimization problem in (42) hard to solve using standard optimization solvers. However, some iterative algorithms can be used to solve a power allocation problem. Consequently, for the target data rate R_T, we can consider the following problem,

$$\begin{aligned}&\max_{p_k} R_T\\ &\text{s.t. } R_k\geqslant R_T,\ k=1,..,K\\ &\sum_{k=1}^{K}p_k\leq P_t,\ p_k\neq 0\end{aligned} \tag{43}$$

According to the last formula in (43), the optimal objective function value of (42) (R^*)is larger than or equal to R_T. It has been presented in literature that, in finite alphabet systems the rate for the k^{th} user, R_k, is an increasing function with the power p_k and there is minimum power, p_m, in which the rate R_k achieves the maximum vlaue $\log_2 M$. In other words, the rate R_k is increasing when $p_k<p_m$, and constant when $p_k\geq p_m$ [6], [8], [34]. Based on this fact, the required power for each user, $p_1,...,p_K$, in each transmission scheme can be calculated using the derived equations (27), (32) and (41), and the optimal R_T can be obtained using Bisection method as explained in Algorithm 1, shown at the top of this page, where R_{TLB} and R_{TUB} are the lower and upper values of the achievable data rate in finite alphabet scenarios.

For a user and given error ϵ, the number of bisections is bounded by $\log_2\left(\frac{\Omega}{\epsilon}\right)$, where in finite alphabet scenarios $\Omega=\log_2 M$. Thus, the total complexity of Algorithm 1 is logarithmic dependence on ϵ and can be expressed by $O\left(K\,\log_2\left(\frac{1}{\epsilon}\right)\right)$, where $O(\cdot)$ is the big-O notation [35], [36].

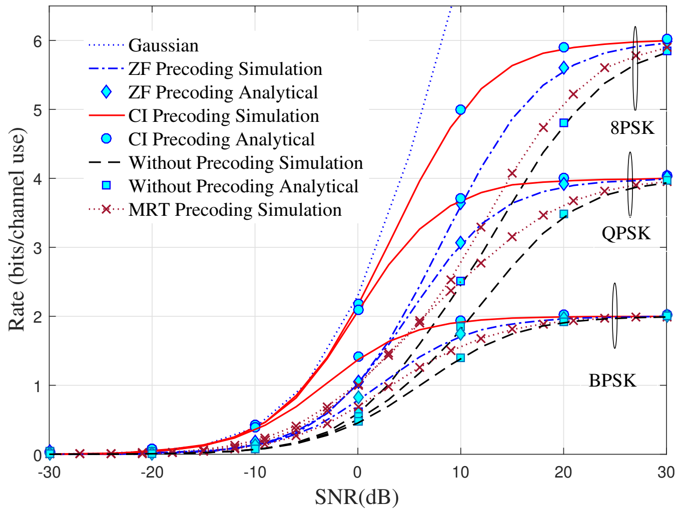
VII. Numerical Results

In this section some numerical results of the considered transmission techniques are presented. Monte-Carlo simulations are conducted with 10^6 independent trials, where the channel coefficients and the noise are randomly generated in each simulation run. Assuming the BS transmission power is p, and the users have same noise power σ^2, the SNR ratio can be defined as SNR = $\frac{p}{\sigma^2}$, when the channels are normalized and the path loss exponent is chosen to be $m=2.7$. Additionally, in this section we add some results of maximum ratio transmission, MRT, precoder in which $\mathbf{W}=\beta\mathbf{H}^H$, i.e., the precoder is the conjugate transposition of the channel matrix, and β is a normalization constant that ensures the power constraint is satisfied [2].

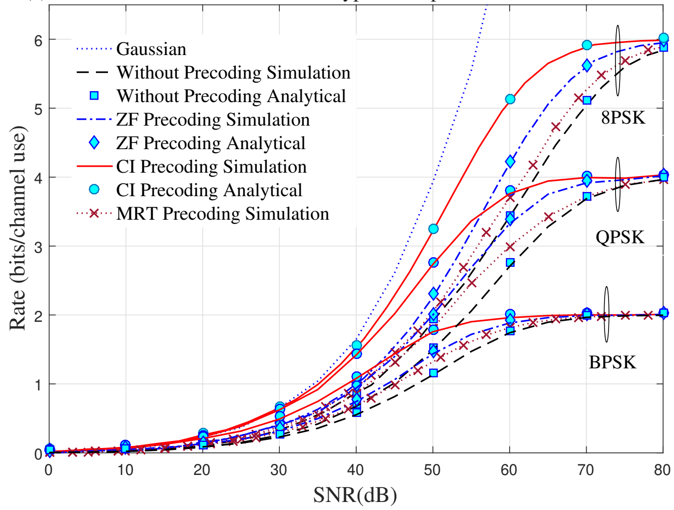
First note in this section is that, the computational complexities of evaluating average sum rate using Monte-Carlo simulation and the derived expressions are $O\left(\kappa^2\,M^N\right)$, and $O\left(M^N\right)$, respectively, where κ is number of points used for calculating expectations over the noise and the channels [37].

Fig. 2 illustrates the sum-rate for the three transmission schemes, subject to different types of input, BPSK, QPSK and 8PSK, when $N=2$, and $K=2$. Fig. 2a, presents the sum-rate when the distances between the BS and the users are

[2]The analytical derivation of MRT scheme will be investigated in future work.



(a) Rate versus SNR with different types of input, when $d_1 = d_2 = 1m$



(b) Rate versus SNR with different types of input, when the users are randomly distributed.

Figure 2: Rate versus SNR with different types of input, when $N = 2, K = 2$.

normalized to unit value, .i.e, without the impact of the path-loss. Fig. 2b shows the sum-rate when the users are uniformly distributed inside a circle area with a radius of 80m, and no user is closer to the BS than 10m where the BS is located at the center of this area. The good agreement between the analytical and simulated results confirms the validity of our analysis in the previous sections. From this figure, we have several observations. Firstly, it is evident that the sum rate saturates to the value of, $K \log_2 M$, past a certain SNR, owing to the finite constellation; the sum rates saturate at 2 bits/s/Hz in BPSK, at 4 bits/s/Hz in QPSK and at 6 bits/s/Hz in 8PSK. In addition, the CI technique always outperforms the ZF, MRT and non-precoding techniques for a wide SNR range with an up to 5dB gain in the SNR for a given sum rate. Furthermore, comparing Fig. 2a and Fig. 2b, one can notice that, in general, for a given SNR value, increasing the distance always degrades the achievable sum rates, due to larger path-loss. For instance when SNR=30 dB, the sum rates reach its maximum values in case the distances are normalized to unit value as in Fig. 2a, while the sum rates are ≤ 1 bits/ channel use in case the distance between the BS and the users are increased as in Fig. 2b. In addition, when the distance between the BS and the users increases the rate saturation occurs at high SNR values. It is worth noting that, the gain attained by CI over the ZF does not depend on the users' locations. Finally, from the figures we can also notice that at low SNR values, MRT and ZF almost have similar performance with some superiority of MRT. However, CI always has better performance than MRT. This is because, CI exploits all the well known interference to increase the useful signal received power [21].

To capture the influence of number of BS antennas and number of users on the system performance, we present in Fig. 3 the sum-rate for the considered transmission schemes for BPSK, QPSK and 8PSK, when $N = 3$, and $K = 3$. Comparing the results in this figure with the ones in Fig. 2, it is clear that increasing N and/or K leads to enhance the system performance. In addition, the CI technique always outperforms the ZF technique with an up to 7dB gain in the SNR for a given sum rate. Furthermore, comparing the sum rate achieved in Fig. 3a and Fig. 3b, we can see similar observations as in the case when $N = K = 2$.

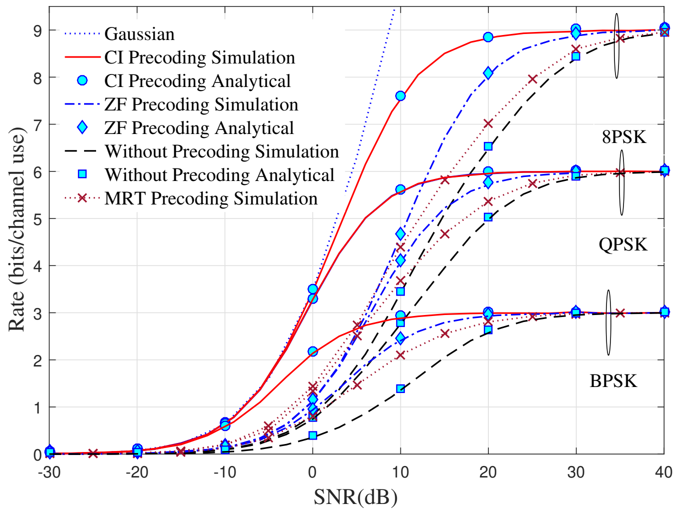
In Fig. 4 we plot the minimum power transmission versus a user distance, when $N = 2$, $K = 2$ and the target data rate, $R_T = 0.5$ bits/s/Hz in BPSK scenario. It should be pointed out that the results for the conventional, ZF and CI techniques in this figure are obtained from Section VI. Generally and as anticipated, the CI technique consumes much smaller power transmission than the other two schemes to achieve the same target data rate, and this superiority is almost fixed with the distance.

Moreover, the max-min rate of the considered system versus the total power is shown in Fig. 5. From this figure, it can be clearly noticed that, the rate can be enhanced significantly by using the proposed power allocation algorithm. Furthermore, the CI always has higher sum-rate than ZF and un-precoded techniques.

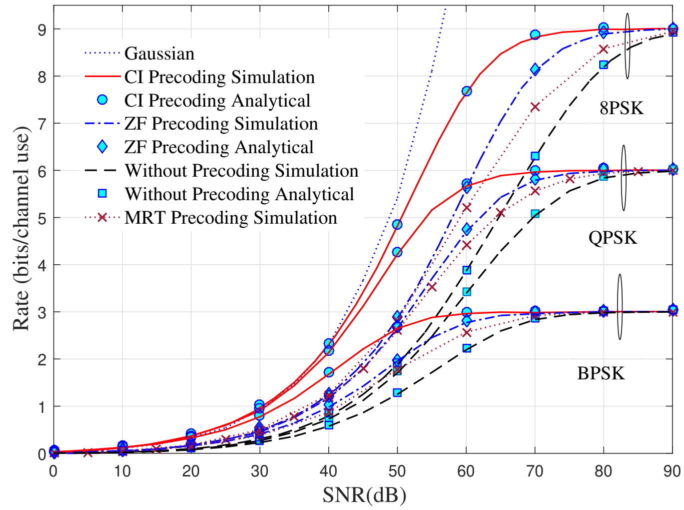
Finally, Fig. 6 illustrates the Jain's fairness index versus the SNR. The fairness index is defined as [38], $\frac{\left(\sum_{k=1}^{K} R_k\right)^2}{K \sum_{k=1}^{K} R_k^2}$, the range of Jain's fairness index is between 0 and 1, where the maximum achieved when users' rates are equal. It can be observed that, in case equal power allocation transmission, the fairness index increases as the SNR increases, and the CI achieves higher fairness than the other transmission techniques. In addition and as anticipated, the proposed power allocation algorithm performs higher fairness index than equal power allocation transmission scheme.

VIII. Conclusions

In this paper we analyzed for the first time the performance of CI precoding technique in MU-MIMO systems with a PSK input alphabet. In light of this, new explicit analytical expressions for the average sum rate are derived for three downlink transmission schemes: 1) without precoding, 2) ZF precoding technique 3) CI precoding technique. In addition, based on



(a) Sum rate versus SNR with different types of input, when $d_1 = d_2 = d_3 = 1m$.



(b) Sum rate versus SNR with different types of input, when the users are randomly distributed.

Figure 3: Rate versus SNR with different types of input, when $N = 3, K = 3$.

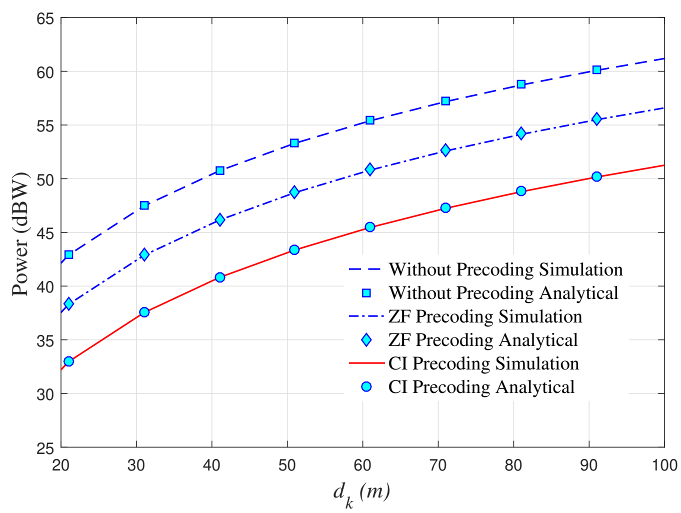


Figure 4: Minimum power transmission versus d_k with BPSK input, when $R_T = 0.5$ (bits/s/Hz) , $N = 2$, and$K = 2$.

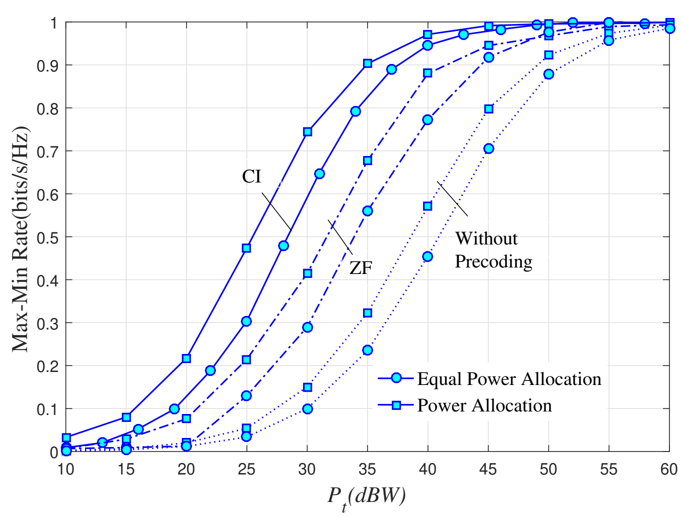


Figure 5: Total max-min rate versus power with BPSK input, when $N = 2$, $K = 2$, $d_k = 15$m..

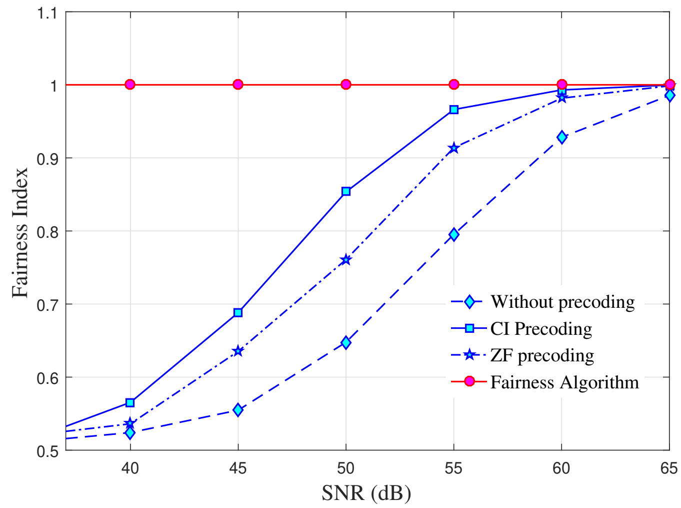


Figure 6: Fairness index versus SNR with BPSK, when $N = K = 2$, $d_1 = 10$m, $d_2 = 90$m.

the derived sum-rate expressions, the minimum transmission power that performs a target data rate was obtained for each transmission scheme, and then power allocation algorithm has been proposed to provide fairness among the users. The results in this work demonstrated that no matter what the values of the system parameters are, the CI scheme outperforms the other two schemes, and the performance gap between the considered schemes depends essentially on the system parameters. Furthermore, increasing the SNR enhances the sum rate to a certain level, and increasing the distance between the BS and the users has no impact on the gap between the minimum transmission power required for ZF and CI to achieve same target data rate. Finally, it was shown that, the CI can provide higher fairness than ZF technique, and the power allocation algorithm proposed can perform high fairness index.

Appendix A
Proof of Theorem 1

In this appendix, we provide the proof of Theorem 1. In order to derive the average rate in this scenario, we need to find the averages of T_1 and T_2 over the noise and the channel states. To start with, the average of the first term in T_1, $\mathcal{E}_{\mathbf{h},n_k}\{\Upsilon_1\}$, can be obtained as

$$\mathcal{E}_{\mathbf{h},n_k}\left\{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}\log_2 e\right\}$$

$$\begin{aligned} = \quad & \mathcal{E}_{n_k}\left\{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}\right|^2+\left|n_k\right|^2+2\left(\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}\right)^H n_k\right\} \\ & \times\frac{-\log_2 e}{\sigma_k^2} \\ & =\mathcal{E}_{\mathbf{h}}\left\{-\left(\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}\right|^2+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right\}, \\ & =-\left(p\varpi_k\left\|\mathbf{s}_{m,j}\right\|^2+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2} \end{aligned} \tag{44}$$

which can be reduced to $-\log_2 e$, by choosing $j = m$. In order to calculate the average of the second term in T_1, since log is a concave function, we can apply Jensen's inequality. Therefore, using Jensen inequality the upper bound can be obtained as

$$\begin{aligned} & \mathcal{E}_{\mathbf{h},n_k}\left\{\log_2\left(1+\sum_{\substack{i=1\\i\neq j}}^{M^N}e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}\right)\right\} \\ & \triangleq\log_2\left(1+\sum_{\substack{i=1\\i\neq j}}^{M^N}\mathcal{E}_{\mathbf{h},n_k}\left\{e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}\right\}\right) \end{aligned} \tag{45}$$

Since n_k has Gaussian distribution, the average over the noise can be derived as

$$\begin{aligned} & \mathcal{E}_{n_k}\left\{e^{\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2}{\sigma_k^2}}\right\}= \\ & \frac{1}{\pi\sigma^2}\int_{n_k}e^{-\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2+\left|n_k\right|^2}{\sigma_k^2}}dn_k. \end{aligned} \tag{46}$$

Using the integrals of exponential function in [39], we can find

$$\begin{aligned} & \mathcal{E}_{n_k}\left\{e^{\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2}{\sigma_k^2}}\right\}= \\ & e^{-\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}\right|^2}{2\sigma_k^2}}. \end{aligned} \tag{47}$$

Therefore, the average over $\mathbf{h}$ can be written as,

$$\begin{aligned} & \mathcal{E}_{\mathbf{h}}\left\{\mathcal{E}_{n_k}\left\{e^{\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}+n_k\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}+n_k\right|^2}{\sigma_k^2}}\right\}\right\}= \\ & \mathcal{E}_{\mathbf{h}}\left\{e^{-\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}\right|^2}{2\sigma_k^2}}\right\}. \end{aligned} \tag{48}$$

Now, to derive the average over $\mathbf{h}$, it is more convenient to use the Quadratic form as follows

$$\begin{aligned}\Phi_i &= \left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,i}\right|^2 - \left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{m,j}\right|^2 \\ &= p_N\mathbf{h}_k\underbrace{\left(\mathbf{s}_{m,i}\mathbf{s}_{m,i}^H - \mathbf{s}_{m,j}\mathbf{s}_{m,j}^H\right)}_{\mathbf{S}_{mi}}\mathbf{h}_k^H, \\ &= p_N\mathbf{h}_k\mathbf{S}_{mi}\mathbf{h}_k^H \\ &=\sum_{n=1}^{l} p_N d_k^{-m}\lambda_{mi,n}\left|\mathbf{q}_{in}^T\mathbf{h}_{1k}\right|^2, \end{aligned} \tag{49}$$

where $\lambda_{mi,n}$ is the n^{th} eigenvalue of matrix $\mathbf{S}_{mi}$ and $\mathbf{q}_{in}^T$ is the corresponding eigenvector. The distribution of Φ_i depends on the number of the eigenvalues (l) and the values of the eigenvalues. In case $l=1$, Φ_i has exponential distribution, in case, $l>1$ Φ_i has sum of exponential distributions, and in case all of the eigenvalues are ones and zeros, Φ_i has Gamma distribution. By choosing $j=m$ the matrix $\mathbf{S}_{mi}$ will have one eigenvalue λ_{mi}. Therefore, Φ_i will follow exponential distribution, and we can get,

$$\begin{aligned}\mathcal{E}_{\mathbf{h}}\left\{e^{-\frac{\Phi_i}{2\sigma_k^2}}\right\} &= \int_0^\infty e^{-\frac{\Phi}{2\sigma_k^2}}\,e^{-\Phi}d\Phi, \\ &= \frac{2\sigma_k^2}{2\sigma_k^2+d_k^{-m}p_N\lambda_{mi}}.\end{aligned} \tag{50}$$

Similarly, the average of the first term in T_2, $\mathcal{E}_{\mathbf{h},n_k}\left\{\Upsilon_2\right\}$, can be obtained as

$$\begin{aligned}&\mathcal{E}_{\mathbf{h},n_k}\left\{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}+n_k\right|^2}{\sigma_k^2}\log_2 e\right\} \\ &= -\left(p\varpi_k\left\|\mathbf{s}_{c,j}\right\|^2+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}.\end{aligned} \tag{51}$$

which can be reduced to $-\log_2 e$, by choosing $c=j$. In order to derive the average of the second term in T_2, using Jensen's inequality, we can calculate the upper bound as

$$\begin{aligned}&\mathcal{E}_{\mathbf{h},n_k}\left\{\log_2\left(1+\sum_{\substack{t=1\\t\neq k,j}}^{M^{N-1}} e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}+n_k\right|^2}{\sigma_k^2}}\right)\right\} \\ &\triangleq \log_2\left(1+\sum_{\substack{t=1\\t\neq k,j}}^{M^{N-1}}\mathcal{E}_{\mathbf{h},n_k}\left\{e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}+n_k\right|^2}{\sigma_k^2}}\right\}\right)\end{aligned} \tag{52}$$

Since n_k has Gaussian distribution, following similar steps as in (46) and (47), the average over the noise can be obtained as

$$\begin{aligned}&\mathcal{E}_{n_k}\left\{e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}+n_k\right|^2}{\sigma_k^2}}\right\} = \\ &e^{-\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}\right|^2}{2\sigma_k^2}}.\end{aligned} \tag{53}$$

Now, the average over $\mathbf{h}$ can be written as,

$$\begin{aligned}&\mathcal{E}_{\mathbf{h}}\left\{\mathcal{E}_{n_k}\left\{e^{\frac{-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}+n_k\right|^2+\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}+n_k\right|^2}{\sigma_k^2}}\right\}\right\} = \\ &\mathcal{E}_{\mathbf{h}}\left\{e^{-\frac{\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}\right|^2-\left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}\right|^2}{2\sigma_k^2}}\right\}.\end{aligned} \tag{54}$$

In order to derive the average over $\mathbf{h}$, the Quadratic form can be used as follows

$$\begin{aligned}\Phi_t &= \left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,t}\right|^2 - \left|\sqrt{p_N}\mathbf{h}_k\mathbf{s}_{c,j}\right|^2 \\ &= p_N\mathbf{h}_k\underbrace{\left(\mathbf{s}_{c,t}\mathbf{s}_{c,t}^H - \mathbf{s}_{c,j}\mathbf{s}_{c,j}^H\right)}_{\mathbf{S}_{ct}}\mathbf{h}_k^H, \\ &= p_N\mathbf{h}_k\mathbf{S}_{ct}\mathbf{h}_k^H \\ &=\sum_{n=1}^{l} p_N d_k^{-m}\lambda_{ct,n}\left|\mathbf{q}_{tn}^T\mathbf{h}_{1k}\right|^2. \end{aligned} \tag{55}$$

where $\lambda_{ct,n}$ is the n^{th} eigenvalue of matrix $\mathbf{S}_{ct}$and $\mathbf{q}_{tn}^T$ is the corresponding eigenvector. The distribution of Φ_t depends on the number of the eigenvalues (l) and the values of the eigenvalues. By choosing $j=c$ the matrix $\mathbf{S}_{ct}$ will have one eigenvalue λ_{ct}, and then Φ_t will follow exponential distribution. Therefore we can get,

$$\mathcal{E}_{\mathbf{h}}\left\{e^{-\frac{\Phi_t}{2\sigma_k^2}}\right\} = \frac{2\sigma_k^2}{2\sigma_k^2+p_N d_k^{-m}\lambda_{ct}}. \tag{56}$$

APPENDIX B
PROOF OF THEOREM 2

In this appendix, we provide the proof of Theorem 2. To derive the average sum rate in this case, firstly we need to derive the average of the Υ term in (13). The scaling factor in this scenario is given by, $\beta=\sqrt{\frac{p}{\mathbf{s}^H(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{s}}}$ [14], [30], where p is the power transmission. For simplicity but without loss of generality, and in order to provide fair comparison between the considered schemes, in this paper we consider constant power scaling factors. It was shown that, the term, $X=\frac{\mathbf{s}^H\Sigma^{-1}\mathbf{s}}{\mathbf{s}^H(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{s}}$, follows Gamma distribution [18], [40], where $\Sigma=\mathbf{D}$, so the average of the scaling factor can be obtained as $\beta=\frac{\sqrt{\frac{p}{\mathbf{s}^H\Sigma^{-1}\mathbf{s}}}\Gamma\left(\frac{3}{2}-K+N\right)}{K\sqrt{K}(N-K)!}$. Therefore the average of term Υ in (13), $\mathcal{E}_{\mathbf{H},n_k}\left\{\Upsilon\right\}$, can be obtained as

$$\begin{aligned}&\mathcal{E}_{\mathbf{H},n_k}\left\{\frac{-\left|\beta\left[\mathbf{s}_{m,j}\right]_k+n_k\right|^2}{\sigma_k^2}\log_2 e\right\} = \\ &\left\{-\left(\left|\beta\left[\mathbf{s}_{m,j}\right]_k\right|^2+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right\}.\end{aligned} \tag{57}$$

Now, in order to calculate the average of the last term in (13), $\mathcal{E}_{\mathbf{H},n_k}\left\{\log_2\left(1+\Xi\right)\right\}$, using Jensen inequality, the upper bound can be written as

$$\mathcal{E}_{\mathbf{H},n_k}\left\{\log_2\left(1+\sum_{\substack{i=1\\ i\neq j}}^{M^N} e^{\frac{-\left|\beta[\mathbf{s}_{m,i}]_k+n_k\right|^2+\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2}{\sigma_k^2}}\right)\right\}$$

$$\triangleq \log_2\left(1+\sum_{\substack{i=1\\ i\neq j}}^{M^N}\mathcal{E}_{\mathbf{H},n_k}\left\{e^{\frac{-\left|\beta[\mathbf{s}_{m,i}]_k+n_k\right|^2+\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2}{\sigma_k^2}}\right\}\right) \tag{58}$$

Since n_k has Gaussian distribution, the average over n_k can be derived as

$$\mathcal{E}_{n_k}\left\{e^{\frac{-\left|\beta[\mathbf{s}_{m,i}]_k+n_k\right|^2+\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2}{\sigma_k^2}}\right\}=$$

$$\frac{1}{\pi\sigma_k^2}\int_{n_k} e^{-\frac{\left|\beta[\mathbf{s}_{m,i}]_k+n_k\right|^2-\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2+|n_k|^2}{\sigma_k^2}}dn_k. \tag{59}$$

Using the integrals of exponential function in [39], we can find

$$\mathcal{E}_{n_k}\left\{e^{\frac{-\left|\beta[\mathbf{s}_{m,i}]_k+n_k\right|^2+\left|\beta[\mathbf{s}_{m,j}]_k+n_k\right|^2}{\sigma_k^2}}\right\}=$$

$$e^{-\frac{\left|\beta[\mathbf{s}_{m,i}]_k\right|^2-\left|\beta[\mathbf{s}_{m,j}]_k\right|^2}{2\sigma^2}} \tag{60}$$

Appendix C
Proof of Theorem 3

In this appendix, we provide the proof of Theorem 3. To start with, the average of the Υ term in (18), $\mathcal{E}_{\mathbf{H},n_k}\left\{\Upsilon\right\}$, can be obtained as

$$\mathcal{E}_{\mathbf{H},n_k}\left\{\frac{-\left|\frac{\beta}{K}\left[\operatorname{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\boldsymbol{s}_{m,j}\right]_k+n_k\right|^2}{\sigma_k^2}\log_2 e\right\}=$$

$$\mathcal{E}_{\mathbf{H}}\left\{-\left(\left|\frac{\beta}{K}\mathbf{a}_k\operatorname{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\left[\boldsymbol{s}_{m,j}\right]_k\right|^2+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right\}. \tag{61}$$

In [33] it was shown that, the term $Z=\frac{\mathbf{a}_k\left(\operatorname{diag}\left(\mathbf{s}^H\right)\right)^{-1}\left(\mathbf{H}\mathbf{H}^H\right)(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}[\boldsymbol{s}_{m,j}]_k}{\mathbf{a}_k(\operatorname{diag}(\mathbf{s}^H))^{-1}(\boldsymbol{\Sigma})(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}[\boldsymbol{s}_{m,j}]_k}$ has Gamma distribution with N degrees of freedom. Hence the average in (61) can be derived as in (62), shown at the top of next page.

In order to calculate the average of the last term in (18), $\mathcal{E}_{\mathbf{H},n_k}\left\{\log_2\left(1+\Xi\right)\right\}$, using Jensen inequality, the upper bound can be calculated as follows. Since n_k has Gaussian distribution, the average over the noise can be obtained as

$$\mathcal{E}_{n_k}\left\{e^{\frac{-\left|\mathbf{h}_k\mathbf{W}\boldsymbol{s}_{m,i}+n_k\right|^2+\left|\mathbf{h}_k\mathbf{W}\boldsymbol{s}_{m,j}+n_k\right|^2}{\sigma_k^2}}\right\}$$

$$=e^{-\frac{\left|\frac{\beta}{K}\mathbf{a}_k\operatorname{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\boldsymbol{s}_{m,i}\right|^2-\left|\frac{\beta}{K}\mathbf{a}_k\operatorname{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\boldsymbol{s}_{m,j}\right|^2}{2\sigma_k^2}}. \tag{63}$$

Now, the average of (63) over $\mathbf{H}$ can be written as in (64), shown at the top of next page, where $\chi_{i,j}=\left|\frac{\beta\left(\mathbf{a}_k\left(\operatorname{diag}\left(\mathbf{s}^H\right)\right)^{-1}\Sigma(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}\right)}{K}X\left[\boldsymbol{s}_{m,i}\right]_k\right|^2-\left|\frac{\beta\left(\mathbf{a}_k\left(\operatorname{diag}\left(\mathbf{s}^H\right)\right)^{-1}\Sigma(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}\right)}{K}X\left[\boldsymbol{s}_{m,j}\right]_k\right|^2$, $X=\frac{\mathbf{a}_k\left(\operatorname{diag}\left(\mathbf{s}^H\right)\right)^{-1}\left(\mathbf{H}\mathbf{H}^H\right)(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}}{\mathbf{a}_k(\operatorname{diag}(\mathbf{s}^H))^{-1}\Sigma(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}}$.
Since $\chi_{i,j}$ has Gamma distribution [18], [40]; we can find the average as in (65), where $\xi=\left(\left|[\boldsymbol{s}_{m,i}]_k\right|^2-\left|[\boldsymbol{s}_{m,j}]_k\right|^2\right)$, $c_k=\frac{\beta\left(\mathbf{a}_k\left(\operatorname{diag}\left(\mathbf{s}^H\right)\right)^{-1}\Sigma(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}\right)}{K}$ and ${}_1\mathrm{F}_1$ is the Hypergeometric function. By choosing $j=m$, $\chi_{i,j}$ reduces to, $\chi_i=\left|\frac{\beta\left(\mathbf{a}_k\left(\operatorname{diag}\left(\mathbf{s}^H\right)\right)^{-1}\Sigma(\operatorname{diag}(\mathbf{s}))^{-1}\mathbf{u}\right)}{K}X\left[\boldsymbol{s}_{m,i}\right]_k\right|^2$ and the expression can be further simplified as in (21).

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$$\mathcal{E}_{\mathbf{H}}\left\{-\left(\left|\frac{\beta}{K}\mathbf{a}_k\text{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\left[\boldsymbol{s}_{m,j}\right]_k\right|^2+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}\right\}$$
$$=-\left(\mathcal{E}_{\mathbf{H}}\left\{\left|\frac{\beta\left(\mathbf{a}_k\left(\text{diag}\left(\mathbf{s}^H\right)\right)^{-1}\left(\boldsymbol{\Sigma}\right)\left(\text{diag}\left(\mathbf{s}\right)\right)^{-1}\mathbf{u}\left[\boldsymbol{s}_{m,j}\right]_k\right)}{K}Z\right|^2\right\}+\sigma_k^2\right)\frac{\log_2 e}{\sigma_k^2}$$
$$=\left(\frac{-\Gamma\left(2+N\right)}{\sigma_k^2\Gamma\left(N\right)}\left|\frac{\beta\left(\mathbf{a}_k\left(\text{diag}\left(\mathbf{s}^H\right)\right)^{-1}\left(\boldsymbol{\Sigma}\right)\left(\text{diag}\left(\mathbf{s}\right)\right)^{-1}\mathbf{u}\left[\boldsymbol{s}_{m,j}\right]_k\right)}{K}\right|^2-1\right)\log_2 e. \tag{62}$$

$$\mathcal{E}_{\mathbf{H}}\left\{e^{-\frac{\left|\frac{\beta}{K}\mathbf{a}_k\text{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\boldsymbol{s}_{m,i}\right|^2-\left|\frac{\beta}{K}\mathbf{a}_k\text{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\}\boldsymbol{s}_{m,j}\right|^2}{2\sigma_k^2}}\right\}$$
$$=\mathcal{E}_{\mathbf{H}}\left\{e^{-\frac{\left|\frac{\beta}{K}\mathbf{a}_k\left(\text{diag}\left(\mathbf{s}^H\right)\right)^{-1}\left(\mathbf{H}\mathbf{H}^H\right)\left(\text{diag}(\mathbf{s})\right)^{-1}\mathbf{u}\left[\boldsymbol{s}_{m,i}\right]_k\right|^2-\left|\frac{\beta}{K}\mathbf{a}_k\left(\text{diag}\left(\mathbf{s}^H\right)\right)^{-1}\left(\mathbf{H}\mathbf{H}^H\right)\left(\text{diag}(\mathbf{s})\right)^{-1}\mathbf{u}\left[\boldsymbol{s}_{m,j}\right]_k\right|^2}{2\sigma_k^2}}\right\},$$
$$=\mathcal{E}_{\mathbf{H}}\left\{e^{-\frac{\left|\frac{\beta\left(\mathbf{a}_k\left(\text{diag}\left(\mathbf{s}^H\right)\right)^{-1}\Sigma\left(\text{diag}(\mathbf{s})\right)^{-1}\mathbf{u}\right)}{K}X\left[\boldsymbol{s}_{m,i}\right]_k\right|^2-\left|\frac{\beta\left(\mathbf{a}_k\left(\text{diag}\left(\mathbf{s}^H\right)\right)^{-1}\Sigma\left(\text{diag}(\mathbf{s})\right)^{-1}\mathbf{u}\right)}{K}X\left[\boldsymbol{s}_{m,j}\right]_k\right|^2}{2\sigma_k^2}}\right\},$$
$$=\mathcal{E}_{\mathbf{H}}\left\{e^{-\frac{\chi_{i,j}}{2\sigma_k^2}}\right\}. \tag{64}$$

$$\mathcal{E}_{\mathbf{H}}\left\{e^{-\frac{\chi_{i,j}}{2\sigma^2}}\right\}=\Lambda_{i,j}=\left(\frac{2^{\left(\frac{1}{2}(N-K-1)\right)}K^{(N-K+1)}}{(N-K)!}\right)\left(\left(\frac{c_k^2\xi}{\sigma_k^2}\right)^{\frac{1}{2}(K-N-1)}\right)$$
$$\times\left(\left(c_k^2\left(\left|\left[\boldsymbol{s}_{m,i}\right]_k\right|^2\right)\right)\Gamma\left(\frac{1}{2}\left(N-K+1\right)\right){}_1\mathrm{F}_1\left(\frac{1}{2}\left(N-K+1\right),\frac{1}{2},\frac{K^2\sigma_k^2}{2c_k^2\xi}\right)\right.$$
$$-\left(c_k^2\left(\left|\left[\boldsymbol{s}_{m,j}\right]_k\right|^2\right)\right)\Gamma\left(\frac{1}{2}\left(N-K+1\right)\right){}_1\mathrm{F}_1\left(\frac{1}{2}\left(N-K+1\right),\frac{1}{2},\frac{K^2\sigma_k^2}{2c_k^2\xi}\right)$$
$$\left.-\sqrt{2}K\,\sigma_k^2\sqrt{\frac{\xi}{\sigma_k^2}}\Gamma\left(\frac{1}{2}\left(N-K+2\right)\right){}_1\mathrm{F}_1\left(\frac{1}{2}\left(N-K+2\right),\frac{3}{2},\frac{K^2\sigma_k^2}{2c_k^2\xi}\right)\right). \tag{65}$$

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