Essays on frictional labour markets in the presence of capital skill complementarity

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I, David Zentler-Munro, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.

Abstract

The first paper of this thesis considers the impact of minimum wages when search frictions are present and firms can substitute away from low skilled workers to both higher skilled workers and to capital. This represents a contribution to the search literature, which typically assumes labour is the only input of production and perfect substitution between labour inputs. I examine whether the model I develop features significant nonlinearities in the impact of the minimum wage on unemployment. I find that the theoretical contribution of this paper, i.e. allowing for search frictions and imperfect substitutability of factor inputs, is quantitatively significant. Specifically, the nonlinear unemployment response in my model does not occur if I use the typical assumptions of the search literature.

In my second paper, I develop a structural model that can help to quantify the relative importance of institutions, labour market frictions and technology in explaining wage inequality. This contribution is a complement to the empirical literature on wage inequality, which tends to emphasise either technological explanations or institutional ones but rarely considers the two jointly. I take my model to the data to test whether estimates of capital skill complementarity in Krusell et al. (2000) are robust to the inclusion of search frictions. I find this is indeed the case: parameter estimates change very little when allowing for search frictions.

My final paper returns to the minimum wage model of my first paper and considers how allowing for asset accumulation by workers changes the model's predictions regarding the relationship between the minimum wage and consumption inequality. I find that allowing for asset accumulation by workers suggests the minimum wage is more successful at reducing consumption inequality than models without asset accumulation would indicate.

Impact Statement

The issues analysed and findings in this thesis are relevant both to academia and policy makers.

Two of the papers in my thesis look at the impact of minimum wages, a topic highly relevant to policy makers' efforts to boost the living standards of low paid workers and reduce inequality. It is a policy that many countries around the world have turned to recently - Germany introduced a minimum wage in 2015, Spain increased its minimum wage by 20% in January 2019, and the UK is in the process of increasing its minimum wage from values that were below many peers' (as a ratio of the median wage) to one of the highest levels of advanced economies.

My first paper suggests there is a risk that further minimum wage increases in the UK could lead to significantly reduced vacancy creation, and so to higher unemployment. The relevance of this analysis to policy makers is clear, however this paper also makes a contribution to the academic literature. Specifically, it develops a tractable model featuring labour market imperfections, due to search costs in the labour market, and firms that can substitute away from low skilled labour to capital or to higher skilled labour. This model is likely to be useful for analysing a range of policy issues beyond the minimum wage i.e. unemployment insurance or labour and capital taxes.

My second paper considers whether allowing for labour market imperfections, again from search costs, changes our understanding of increasing wage inequality and the role of technology in this. Rising wage inequality is an important social concern, and understanding what drives this is both of significant academic interest and is important for determining the most appropriate policy response.

My third paper, also on the minimum wage, makes an important contribution to both the academic and policy debate by showing that the relationship between the minimum wage and consumption inequality - a key welfare criterion for judging the impact of policies - crucially depends on whether workers can insure themselves against any extra unemployment risk from the minimum wage. My model looks at one such insurance mechanism - asset accumulation - and finds this mechanism significantly changes the model's predictions concerning the impact of the minimum wage on consumption inequality. This represents a novel finding in the academic literature.

In all cases, I hope to maximise the potential impact of my research by further development of the papers, continuing to present the research at relevant workshops and seminars, and eventually through publication.

Acknowledgements

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My friends and family have been an enormous source of strength and support throughout this process. I would not have been able to finish without the love and support of my partner, Sarah. Nor would any of my studies been possible without the constant encouragement provided by my mother to "get my head down and get on with it". I would like to dedicate my thesis to my late father, who saw me start this journey but not complete it, and to my beautiful daughter Rosa, who has given me the inspiration and impetus to finish this thesis.

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Chapter 1

Introduction

This thesis comprises three chapters that examine the economics of frictional labour markets in the presence of capital skill complementarity.

Chapter 2 considers the impact of minimum wages in this setting, and considers whether there are likely to be significant nonlinearities in the relationship between the minimum wage and unemployment. I consider this question in context of the UK economy, where the empirical evidence suggests that the introduction of the minimum wage and subsequent increases to its real terms value have had no significant employment effects but have boosted the wages of low paid workers (Stewart (2004), Draca et al. (2011)). Such empirical findings are not unusual in the wider literature; for example when considering the aggregate labour market impacts of variation in state level minimum wages in the U.S, Cengiz et al. (2018) find no evidence of significant negative employment impacts. However, there is evidence suggesting larger labour demand impacts at relatively high minimum wage values. Jardim et al. (2017) consdier the case of Seattle, where the city authorities raised the local minimum wage from \$9.47 to \$11 in 2015 and to \$13 in 2016. The authors report that they find *"evidence of nonlinear effects, as the rise to \$11 per hour had an insignificant effect on employment, whereas the rise to \$13 per hour resulted in a large drop in employment"*.

Assessing whether further minimum wage increases in the UK might produce nonlinear unemployment impacts requires a structural model of the labour market to make out-of-sample predictions. In this chapter, I develop a model featuring frictional labour markets and capital skill complementarity. I consider the former necessary for the model to have a chance of replicating findings of low employment impacts combined with positive wage impacts when the minimum wage was first introduced in the UK (1999). However, while there are many structural models of the minimum wage incorporating frictional labour markets (e.g. Flinn (2006) and van den Berg and Ridder (1998)) these models typically have production technology that has labour as its only input with constant returns to scale. This rules out some dimensions of interest when it comes to the minimum wage, such as whether firms are likely to substitute low skilled labour with capital or with higher skilled labour in response to a minimum wage increase. I therefore develop a model featuring both search frictions, with a similar bargaining set-up to Cahuc et al. (2006), and a production technology with capital skill complementarity, as in Krusell et al. (2000).

I compare the model's predictions to the experience of introducing the minimum wage in the UK, and examine whether the model features significant nonlinearities in the impact of the minimum wage on unemployment. I find that the model is able to match the wage and profits impact of the introduction of the minimum wage in the UK, but suggests a counter-factually large employment impact. When I consider model predictions for higher values of the minimum wage, I find there are indeed nonlinearities in the unemployment impact and that these lie close to the current level of the minimum wage in the UK, which has increased significantly over the last five years.

Chapter 3 proposes a model that can help to quantify the relative importance of institutions, labour market frictions and technology in explaining inequality trends. It makes a contribution to the empirical literature on wage inequality, which has tended to focus either on technological explanations for wage inequality, as in Krusell et al. (2000) and Katz and Murphy (1992), or institutional factors such as the minimum wage, as in Card and DiNardo (2002), but hasn't generally assessed both factors jointly. Equally the empirical literature has paid relatively little attention to the impacts of changes in labour market frictions on wage inequality. I develop a model that again features search frictions, as in Cahuc et al. (2006), and a production technology with capital skill complementarity, as in Krusell et al. (2000). I take this model to the data to test whether estimates of capital skill complementarity in Krusell et al. (2000) are robust to the inclusion of labour market frictions. I find that estimates that allow for frictions are not substantially different from estimates that assume perfect competition. In particular, both models produce similar estimates of the strength of capital skill complementarity and are reliant on this channel to match the observed increase in the graduate wage premium.

Chapter 4 returns to the minimum wage model of Chapter 2 and considers the impact of introducing asset accumulation by risk averse workers. I focus in particular on how allowing for asset accumulation changes the model's predictions regarding the relationship between consumption inequality and the minimum wage. The model proposed is close in spirit to the growing literature that combines search frictions with macroeconomic models of asset accumulation, e.g. Andolfatto (1996), Lise (2011) and Krusell et al. (2010), though none of these papers consider the impacts of a minimum wage. Introducing asset accumulation into the model of Chapter 2 allows us to consider the impact of the minimum wage on savings decisions by workers, which represents the key contribution of this chapter to the structural minimum wage literature, and facilitates analysis of impacts on consumption inequality. The scope for such analysis is limited in typical search models of the minimum wage where workers are risk neutral and hence consumption is not well defined.

I find that the workers' ability to self-insure via asset accumulation has an important role in determining the impact of minimum wages on consumption inequality. In my model, workers increase their savings to self-insure against the increased unemployment risk of higher minimum wage levels. This means minimum wages achieve reductions in consumption inequality even when set at relatively high levels that cause unemployment to rise. In a model without savings, increasing the minimum wage level to such levels would increase consumption inequality because increased unemployment risk has a more significant pass-through to consumption inequality.

Chapter 2

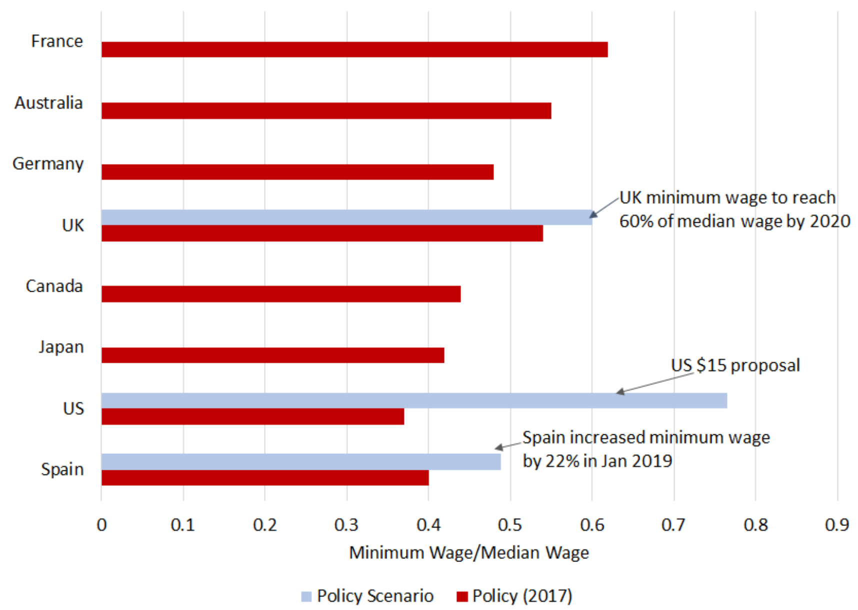
Minimum Wages in the UK: Searching for Nonlinearities

2.1 Introduction

Minimum wages are an increasingly popular policy response to concerns about low pay growth and wage inequality. In the 2015 Budget the former UK Chancellor announced a significant increase in the minimum wage, taking it from around 45% of the median wage in 2015 to a planned level of 60% of the median wage in 2020. UK policy makers are not the only ones turning towards higher minimum wages, as shown in Figure 2.1. In the US, there is an active campaign to increase the minimum wage to $15, which has had considerable success at a state/municipality level if not at the federal level. The German government introduced a $11.75 minimum wage in 2015, where previously trade unions had been the sole form of protection against low pay. More recently, the Spanish Government increased the minimum wage by 22% as of January 2019.

Much of the academic literature has focused on econometric evaluation of past increases to the minimum wage, and particularly on estimating impacts on employment rates. In the UK at least, the consensus of this empirical literature seems to be that the introduction of the minimum wage in 1998, and subsequent increases in the 2000s, had relatively benign effects: increasing

Figure 2.1: Minimum Wages on the Rise



Source: OECD, own calculations

wages for low paid workers without a significant decrease in employment (Draca et al. (2011), Stewart (2004)). This has led many to call for further increases; for example the Labour party proposed a £10 minimum wage (current rate is £8.21 for employees aged 25 and over) in their 2017 election manifesto. This represents something of a risk as past performance may not be a reliable guide to future impacts.

This chapter explores just how risky this logic could be by examining, in the context of the UK economy: (i) whether there are likely to be significant nonlinearities in the impact of the minimum wage on unemployment; and (ii) what channels drive the location and severity of any such nonlinearities.

Given this involves considering minimum wage levels outside of those already observed in the UK, any answer will require a structural model of the economy. I consider a reasonable requirement of any such model is that it replicates the impacts of the introduction of the minimum wage described above. Equally, it should also be able to asses the risk of less favourable impacts as the minimum wage is increased to significantly higher levels. I therefore propose a model that combines a production process featuring several margins of substitution between factor inputs with frictional labour markets.

Frictional labour markets, and the monopsony power they imply, are likely to be necessary to replicate the results of empirical studies in the UK concerning the introduction of the minimum wage. In particular, findings of a significant increase in wages, a fall in corporate profits, without an increase in firm exits (Draca et al. (2011)), are suggestive of some degree of monopsony power, as is the absence of significant unemployment impacts (Leonard et al. (2014)). This consideration also points towards a wage bargaining model with random search, rather than a directed search or wage posting model, which tend to replicate many of the features of competitive labour market models.

I model the labour market using a heterogeneous agent on-the-job (OTJ) search model, with a similar wage bargaining mechanism to Cahuc et al. (2006). I consider heterogeneity a necessary ingredient as the biting point of the minimum wage on employment is likely to depend on the ability distribution of workers. OTJ search provides an endogenous source of worker bargaining power and employer competition that can, to some extent, be disciplined by the data.

To this labour market structure, I add two features that are potentially helpful in analysing the employment reaction to minimum wages and the latter of which represents a key contribution of my approach: (i) endogenous vacancy creation; and (ii) firms that can respond to minimum wage increases by substituting both capital and high skilled labour for low skilled labour using the production function developed and estimated in Krusell et al. (2000).

In this context, nonlinearities are driven by: (i) endogenous nonlinearities in labour demand from using a multi-input production function and from endogenous vacancy creation; and (ii) exogenous non-linearities in the distribution of workers across ability types.

When calibrated to match the UK labour market, the model is able to replicate, qualitatively and quantitatively, empirical estimates of the profit and wage response to the introduction of the minimum wage in the UK. However, the model predicts a counter-factually large unemployment increase in response to the minimum wage's introduction. This illustrates that the inclusion of search frictions is certainly not a sufficient condition for matching empirical evidence of small/non-existent employment impacts. When considering minimum wage levels above those experienced already, the model suggests a nonlinear unemployment reaction that starts well before the planned level of the minimum wage in 2020.

Quantitatively, I find that imperfect substitution between inputs is the most significant endogenous source of nonlinearities in the model. If we instead use a constant returns to scale production function with labour as the only input, the model predicts that unemployment increases with the minimum wage in a much more linear fashion. This is significant as the search literature on minimum wages generally assumes constant returns to scale production with labour as the only factor of production, and that different worker types are perfectly substitutable (Flinn (2006), van den Berg and Ridder (1998)).

The assumption of constant returns to scale in labour input typically made in the search literature ensures the common restriction that firms employ a maximum of one worker can be made without loss of generality (in the context of the model at least). This assumption of one worker firms avoids the complexity of firm owners bargaining with multiple workers, as described in Stole and Zwiebel (1996). The theoretical contribution of this chapter is to develop an internally consistent model of production and the labour market that effectively incorporates both imperfect substitution between labour inputs and wage bargaining, without the complexities of Stole and Zwiebel (1996). I achieve this by confining search frictions to intermediate goods firms, where there is constant returns to scale production using labour inputs only. These intermediate goods firms sell their output to a final good producer that has production technology featuring imperfect substitution between all inputs and capital skill complementarity as per Krusell et al. (2000). [1] The quantitative contribution of this chapter is to show that allowing for imperfect substitution between inputs has a significant impact on the nonlinear relationship between the minimum wage and unemployment in my model.

The model I develop captures only the 'disemployment' impacts of the minimum wage, which can only ever have a negative impact on employment rates in the model for workers for whom the minimum wage binds. I do not consider gains in participation from minimum wage increases as discussed in Flinn (2006). There are also other margins of response for firms than the employment margin that I focus on, like changing hours worked or decreasing fringe benefits. In that sense, predictions from the model outlined here could be viewed as a somewhat cautious lower bound estimate of where any unemployment nonlinearities might lie.

The rest of the chapter is organised as follows. Section 2.2 reviews both the search literature featuring minimum wages and the 'reduced form' empirical literature, with a focus on the UK experience. Section 2.3 sets out the model, and considers the factors determining the employment impacts of the minimum wage and how this differs from more standard competitive and frictional models. Section 2.4 describes my calibration strategy and assesses whether the calibrated model can match empirical findings concerning the impact of the introduction of the minimum wage in the UK. Section 2.5 examines the quantitative implications of the model to asses whether there is indeed a nonlinear relationship between unemployment and the minimum wage, and investigates what determines the location and strength of any such nonlinearity. Section 2.6 concludes.

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[1] Acemoglu (2001) uses a similar production structure to consider the impact of the minimum wage, though his focus is on the composition of jobs and he uses his model for qualitative purposes only.

2.2 Related Literature

2.2.1 Search Literature on Minimum Wages

The nature of wage setting in frictional models is crucial in determining the impact of minimum wages. Two forms of wage setting are commonly used in the search literature: wage posting, where firms offer a take-it-or-leave it wage, or wage bargaining.

Wage posting models of the type pioneered by Burdett and Mortensen (1998) typically feature pure wage dispersion, i.e. dispersion that is not generated by worker or firm heterogeneity but is the result of a mixed strategy played by rival firms. The presence of pure wage dispersion means minimum wage increases will raise workers' wages with no employment impact as long as the minimum wage remains below the level of match productivity. However, any increases in the minimum wage beyond this point will destroy all such matches due to the common assumption of constant returns to scale in production. Engbom and Moser (2017) find minimum wages have sizable, and realistic, impacts on the wage distribution in a wage posting model that is estimated using data from a large minimum wage increase in Brazil. Although it's not the focus of their paper, their model also predicts a large rise in unemployment in response to the minimum wage increase.

Wage bargaining models with exogenous contact rates, e.g. Flinn (2006), also feature stark unemployment impacts whereby the minimum wage has no impact on employment until it hits the level of match ability, whereupon all matches of this ability level are destroyed. Wage bargaining models that have endogenous vacancy creation - again Flinn (2006) looks at this case - have a more gradual increase in unemployment until the minimum wage reaches the level of match ability at which point again matches are destroyed. The reduction in labour demand before this point occurs because the minimum wage decreases the share of profits going to firms, which disincentivises vacancy creation.

A common assumption running through this literature is that labour has a constant marginal product. This produces the stark "cliff-edge" results discussed above i.e. once the minimum wage exceeds this fixed marginal product, the match is destroyed. A key contribution of the model I present is that it combines search frictions with a production structure exhibiting diminishing marginal product in labour inputs (strictly speaking, the intermediate good produced by labour has a diminishing product in my model, rather than labour itself). This means that even if the current minimum wage exactly equals the marginal product of a match, an increase in the minimum wage need not destroy all such matches as at zero employment labour has an infinite marginal product (i.e. I assume Inada conditions hold).

Haanwinckel (2018) presents a model of the minimum wage featuring imperfect substitution between different worker types, who perform tasks of varying routine skill intensity, and imperfectly competitive labour markets. While his model has rich implications for minimum wage spillovers on the wage distribution, which is the focus of the paper, it captures imperfections in the labour market in a relatively reduced-form way, i.e. through a inelastic labour supply function to firms, and so has less scope to explore unemployment impacts.

2.2.2 Empirical Evaluation of Minimum Wage Changes

This section focuses on studies that evaluate changes to the minimum wage in the UK, as I will calibrate my model to UK data, however I start with a brief discussion of the sizable US evidence base.

A large fraction of US studies focus on the employment response of teenagers to the minimum wage or on particular sectors like fast-food restaurants (Neumark and Wascher (1995), Card and Krueger (1994)). While there is a clear interest in looking at areas where the minimum wage bites hardest, such studies offer little guidance regarding the macroeconomic impacts of the minimum wage, which is the focus of this chapter. However, more recent studies such as Cengiz et al. (2018) consider aggregate employment responses to state level minimum wage changes. The employment change induced by an increase in the minimum wage is calculated by comparing the increase in the density of workers paid at or just above the newly increased minimum wage to the decrease in density below the minimum wage. Looking at 138 state level minimum wage changes, they find no evidence of significant employment impacts.

Of course my concern is finding nonlinearities in the impact of the minimum wage on employment. In the U.S context, it is therefore instructive to consider evidence on aggregate employment impacts for states/regions that have introduced particularly high minimum wage levels. Jardim et al. (2017) consider the case of Seattle, where the city authorities raised the local minimum wage from \$9.47 to \$11 in 2015 and to \$13 in 2016. The authors report that they find *"evidence of nonlinear effects, as the rise to \$11 per hour had an insignificant effect on employment, whereas the rise to \$13 per hour resulted in a large drop in employment"*.

One critique of U.S studies is that federal and municipal minimum wage increases tend to be done in nominal terms and are soon eroded by inflation, so that the findings above are more relevant for short term impacts and may not be indicative of long term effects. This is less of a concern in the U.K where, when not in recession, the minimum wage tends to keep track with, or exceed, earnings and prices. Leonard et al. (2014) perform a meta-analysis of studies looking at the employment response to the introduction, and subsequent increases, of the UK minimum wage. They find the mean estimate of the employment elasticity is not significantly different from zero.

There are of course many margins of adjustment available to firms other than employment, e.g. hours worked, non-wage benefits, prices or profits. Taking hours first, there appears to be more evidence of effects through this channel in the UK than with employment, although the estimated reductions in hours have generally not been sufficient to reduce weekly earnings (Stewart and Swaffield (2008), Dickens et al. (2012).and Connolly and Gregory (2002).

Firms facing increases in their labour costs due to the minimum wage may also raise their prices. In their 2014 annual report, the UK Low Pay Commission (henceforth the LPC), who are responsible for recommending the level of the minimum wage to central government, note that prices have risen considerably faster in those sectors where minimum wage workers are concentrated. While this evidence is suggestive of price pass-through it is far from conclusive. Wadsworth (2009) tests this hypothesis in a slightly more formal regression framework and finds evidence of mild, but statistically significant, price pass through. In her survey of the impact of minimum wages on prices, Lemos (2008) comes to similar conclusions. This is consistent with international evidence of price pass through i.e. Harasztosi and Lindner (2015).

The other major avenue for employers to avoid the incidence of increased wages is to reduce non-wage benefits (e.g. pension contributions or bonus payments). The LPC commissioned research on this which concluded that firms did indeed reduce labour costs by reducing pay premia for overtime and unsocial hours; and by restricting non-wage benefits such as subsidised meals and transport, annual leave, pensions, and staff discounts (Grimshaw and Carroll (2002), Cronin and Thewlis (2004) and Denvir and Loukas (2006)). However, the introduction of default employee enrollment into pension schemes ('auto-enrollment'), with a mandatory contribution from the employer, will limit the extent to which employers can lower pension contributions.

This is of course a piecemeal approach to examining who bears the incidence of minimum wage increases. Arguably a more direct test of this is to examine the impact on firm profitability. This is exactly the approach of Draca et al. (2011) who look at firm profitability for extended periods before and after the introduction of the minimum wage in the UK. They find firms employing relatively large numbers of minimum wage workers have lower profit growth than those employing higher wage workers. The authors also note that the size of profit reduction is consistent with a static model where employers do not change their behaviour in response to the minimum wage change. A final finding is that there is not a statistically significant change in firm exit rates or employment.

In summary, the UK evidence points to muted impacts of previous minimum wage changes on employment which, combined with findings that firms absorbed a substantial amount of the wage increase through reduced profits, is suggestive of monopsonistic labour markets. However, muted short term employment impacts can be reconciled with competitive models of the labour market, as in the putty clay model of Aaronson et al. (2013). Competitive models are also consistent with findings of price pass-through and reductions in employee benefits and hours. This guides my choice of modeling assumptions in that, while I allow for some monopsony power by assuming search frictions and wage bargaining, I also allow for employer competition by assuming workers can search OTJ and that employers can respond to poaching by rivals as in Cahuc et al. (2006).

2.3 The Model

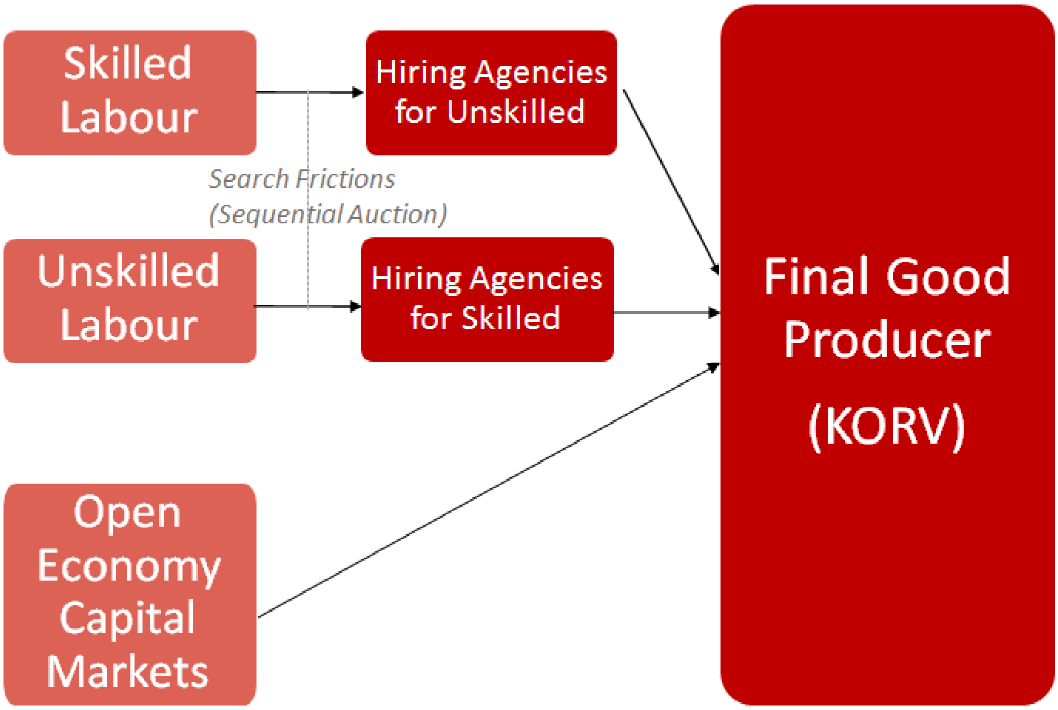
2.3.1 Model Environment

Model Environment: Workers

There are two skill types of workers, unskilled and skilled, and within each skill type there is a distribution of worker ability, with skill indexed by h and ability indexed by i. A worker of skill type h and ability type i has an efficiency level, which we will define precisely later, denoted by $x_{h,i}$. We assume a worker's skill type is observable to the researcher and firms, but their ability is observable to firms only. Workers' ability levels are distributed within a skill type according to the cdf $L_h(x)$, and pdf $\ell_h(x)$. For quantitative purposes later I will use a discrete approximation to a log normal ability distribution, with a total of M ability types for each skill group (I set $M = 400$ in my baseline calibration). All workers and firm owners have a common discount factor, β, and are risk neutral.

For notational convenience, the subscript j - which I refer to as a worker type - will be a vector valued index containing both the skill index ($h \in \{u, s\}$) and ability index ($i \in \{1..M\}$) of a worker.

Figure 2.2: Model Economy Overview



Model Environment: Production Structure

The production structure in my model has two layers. First there is an intermediate goods sector with search frictions, where I maintain the typical assumptions of the search literature (no capital and constant returns to scale production in labour inputs). Second, I include a final good sector with a production function that combines intermediate goods with capital, and features imperfect substitutability of all factors and capital skill complementarity as per Krusell et al. (2000) (henceforth referred to as the "KORV" production function).

There will be a segmented intermediate goods sector for each of the $2M$ possible pairings of skill type ($h \in \{u, s\}$) and ability level ($i \in \{1..M\}$). Firms in these intermediate sectors can be thought of as hiring agencies for the final goods firm, that face search frictions and wage bargaining. This economy is represented in stylised form in Figure 2.2.

I do not introduce search frictions and wage bargaining directly into the final good production stage as that would involve firms bargaining with many workers, e.g. a multi-player game, as described in Stole and Zwiebel (1996). In this environment, each worker would consider the impact of their negotiation on the negotiations of all other workers. Such considerations do not feel particularly relevant for investigating the macroeconomic impacts of minimum wages, so I choose to abstract from these effects using the production structure described above.

Model Environment: Final Good Firms

I use the same production structure as in Krusell et al. (2000), which is shown in equation (2.1). Final goods are produced using capital structures, K_{st}, capital equipment, K_{eq}, and aggregates of the intermediate goods produced by unskilled and skilled workers; these aggregate inputs are denoted U and S respectively.[2] U and S are aggregates of the output of the M types of intermediate goods firm in each skill sector, which correspond to the M heterogeneous ability levels of unskilled and skill workers. The output of the intermediate good sector employing unskilled (skilled) labour of ability type i is denoted $y_{u,i}$ $(y_{s,i})$.

$$
\begin{aligned}
Y &= AG(K_{st}, K_{eq}, U, S) \\
&= AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \qquad (2.1) \\
U &= \left(\sum_{i=1}^{M}(x_{u,i}y_{u,i})^{\frac{\Psi_u-1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u-1}}, \; S = \left(\sum_{i=1}^{M}(x_{s,i}y_{s,i})^{\frac{\Psi_s-1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s-1}} \qquad (2.2)
\end{aligned}
$$

with $\sigma, \rho < 1$, $\alpha, \lambda, \mu \in (0,1)$ and $\Psi_u, \Psi_s > 1$. The elasticity of substitution between the aggregate unskilled intermediate input and capital equipment, denoted by $\varepsilon_{u,k_{eq}}$, equals $1/(1-\sigma)$. The elasticity of substitution between the unskilled intermediate input and the skilled intermediate input, denoted $\varepsilon_{u,s}$, is also given by $1/1-\sigma$. The elasticity of substitution between the skilled intermediate input and capital equipment, denoted by $\varepsilon_{s,k_{eq}}$, is given by $1/(1-$

[2]Krusell et al. (2000) assume a perfectly competitive labour market, and U and S are simply the total hours worked by each skill group.

ρ). The parameter α, together with λ, determine the capital share of output, and μ impacts the output share of unskilled intermediate good sectors. The production function will exhibit capital skill complementarity, i.e $\varepsilon_{u,k_{eq}} > \varepsilon_{s,k_{eq}}$, whenever $\sigma > \rho$. This is exactly what Krusell et al. (2000) find to be the case and I will use their parameter estimates (I discuss this further in Section **??**).

Equation (2.2) states that the efficiency level of a worker of skill type h and ability type i, $x_{h,i}$, corresponds to the efficiency of the intermediate good they produce in final good production.

Model Environment: Intermediate Goods Sectors

There is a separate intermediate goods sector for each worker type j (recall j is a vector index of skill and ability: $j = (h, i)$) and one intermediate firm for each worker in the economy, so the density of intermediate goods firms in sector j equals the population density of workers ℓ_j. I assume all intermediate firms sell competitively to the final good firm. Intermediate goods sectors are completely segmented in the sense that a type j firm can only ever employ a type j workers and vice versa.

The assumption of constant returns to scale in intermediate good sectors means the output of sector j (y_j) will simply be the population density of type j workers multiplied by their employment rate and hours worked, $\bar{H}$ i.e. ($y_j = \ell_j(1 - e^{ue}_j)\bar{H}$), where e^{ue}_j denotes the unemployment rate of type j workers. I include hours worked as the KORV production function was originally specified with labour input measured in terms of total hours, however, I assume both worker types are full-time, i.e. work a fixed 40 hour week, and do not model the intensive margin of labour supply.

Model Environment: Search Frictions and Wage Bargaining in the Intermediate Goods Sectors

I assume that both unemployed and employed workers randomly search for jobs. The homogeneity of intermediate goods firms means workers exist in one of three employment states: unemployed; employed but not yet poached by another employer ('not-poached'); or employed and poached ('poached'). The employment state for a worker of skill type j is denoted as $\Upsilon_j \in \{ue, np, p\}$, where the indices $\{ue, np, p\}$ represent the unemployed, not-poached and poached employment states respectively.

The number of newly formed job matches is given by matching function $M(S_j, V_j)$, where S_j is the effective number of type j job searchers (unemployed and not-poached workers) and V_j is the number of type j vacancies. I assume that unemployed workers search more intensely than non-poached workers so that $S_j = N_j^{ue} + \chi_j N_j^{np}$, where N_j^{ue} is the number of unemployed type j workers, N_j^{np} is the number of not-poached workers, and χ_j is the search intensity rate for employees relative to the unemployed ($\chi > 0$). Once a worker is poached they stop searching as all firms are the same.

Defining $\theta_j \equiv V_j/S_j$ as labour market tightness, the contact rate is $q(\theta_j) \equiv M(S_j, V_j)/V_j$ for type j firms, and $(\theta_j q(\theta_j),\ \chi_j \theta_j q(\theta_j))$ for type j unemployed and not-poached workers respectively. The fraction of type j workers who are poached is denoted by e_j^p and the fraction who are not-poached by e_j^{np} (with the residual fraction unemployed denoted by e_j^{ue}). The share of effective job searching workers that are not-poached is denoted as $s_j^{np} \equiv \frac{\chi_j e_j^{np}}{\chi_j e_j^{np} + e_j^{ue}}$, and the share that are unemployed as $s_j^{ue} \equiv 1 - s_j^{np}$. Finally matches are destroyed with exogenous probability, δ_j.

I follow the approach of Cahuc et al. (2006) where all firms and workers engage in Nash bargaining. For unemployed workers matched with a firm, who then become 'not-poached' workers in my terminology, standard Nash bargaining takes place. This bargaining is subject to the constraint that the bargained wage must be at least as large as the legally binding minimum wage, m_w.

When a not-poached worker makes contact with another employer, becoming a poached worker, they also engage in Nash bargaining but this time the bargain is between the incumbent and poaching employer and the worker, as in Cahuc et al. (2006). The rival employers bid-up the wage until the value of employing a poached worker to the firm equals the value of carrying a vacancy.

Free entry will drive the latter to zero, due to the existence of a fixed vacancy cost κ_j. As type j firms are a priori identical, the poaching firm will offer the same wage as the incumbent (which we will see is the price of the intermediate good) leaving the worker indifferent between the two rival firms.

I arbitrarily assume the worker moves with probability one to a poaching firm conditional on making contact with them. This assumption means job contact rates, which are unobservable in the data, are equal to job mobility rates, which are observable. If in reality job contact rates for employees were significantly greater than job mobility rates, then this effectively moves the model closer to a competitive labour market or equivalently one with higher worker bargaining power. In that sense bargaining power and the probability of moving to a poaching firm conditional on contact with them are not separately identifiable. I therefore estimate the former and set the latter equal to one as a normalisation.

2.3.2 Behaviour in the Model Economy

Behaviour: workers

A worker of a given type j exists in one of three employment states: unemployed and receiving flow income b, employed but not poached and receiving the higher of the Nash bargained wage w_j^b and the minimum wage m_w, or employed and poached and receiving wage w_j^p. The expected lifetime utility of being in each of these states will be denoted by V_j^{ue}, V_j^{np} , and V_j^p respectively.

Workers face only one trivial decision: whether to participate in the labour market which they do as long as they are paid more than their reservation wage. Given vacancies are costly, rational firms will always offer at least the reservation wage so this decision is trivial. The Bellman equations for a unemployed, not poached and poached worker are therefore as follows:

$$V_j^{ue} = b + \beta[\theta_j q(\theta_j) V_j^{np} + (1 - \theta_j q(\theta_j)) V_j^{ue}] \tag{2.3}$$

$$\begin{aligned} V_j^{np} = {} & \max(w_j^b, m_w) + \\ & \beta\big[\delta_j V_j^{ue} + (1 - \delta_j)[\chi_j \theta_j q(\theta_j) V_j^p + (1 - \chi_j \theta_j q(\theta_j)) V_j^{np}]\big] \end{aligned} \tag{2.4}$$

$$V_j^p = w_j^p + \beta[\delta_j V_j^{ue} + (1-\delta_j)V_j^p] \tag{2.5}$$

Equation (2.3) states that an unemployed worker of skill level j receives benefits, b, in the current period and in the next period either gets a job offer with probability $\theta_j q(\theta_j)$, which they will always accept and so become a not poached worker, or remains unemployed with probability $1-\theta_j q(\theta_j)$. Equation (2.4) states that a not poached worker gets the higher of the Nash bargained wage and the minimum wage in the current period and in the following period loses their job with probability δ_j , gets poached with probability $(1-\delta_j)\chi\theta_j q(\theta_j)$ or remains not poached with probability $(1-\delta_j)(1-\chi\theta_j q(\theta_j))$. Finally equation (2.5) states that a poached worker gets a wage w_j^p in the current period and the next period either loses their job with probability δ_j or remains employed as a poached worker (since they have already reached the top of the job ladder) with probability $1-\delta_j$.[3]

Behaviour: Final Good Producers

The final good producer's profit maximisation problem is as follows, where we normalise the price of the final good to one:

$$\begin{aligned}\max_{K_{st},K_{eq},y_{1,u}\cdots y_{M,u},y_{1,s}\cdots y_{M,s}} \quad \Pi &= AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \\ &\quad - \sum_{i=1}^{M} p_{i,u}y_{u,i} - \sum_{i=1}^{M} p_{i,s}y_{s,i} - r_{st}K_{st} - r_{eq}K_{eq} \qquad (2.6) \\ U &= \left(\sum_{i=1}^{M}(x_{u,i}y_{u,i})^{\frac{\Psi_u-1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u-1}}, S = \left(\sum_{i=1}^{M}(x_{s,i}y_{s,i})^{\frac{\Psi_s-1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s-1}}\end{aligned}$$

As in Krusell et al. (2000), I impose a no arbitrage condition between capital equipment and capital structures. This implies that the net of depreciation rental rates for capital equipment and structures must be equal to some com-

[3]We will see later that poached workers are paid a wage equal to the price of the intermediate good they produce. This is equal to the marginal product of the intermediate good, which will always exceed the minimum wage, $w_j^p = p^j >= m_w$: if this were not the case intermediate firms would be loss making and leave the market, until the price of the intermediate good is bid up by the final good producer to the level of the minimum wage (Inada conditions guarantee this point will be reached)

mon interest rate, r, which implies their gross rental rates, r_{eq} and r_{st}, are related as follows: $r_{eq} - \delta_{eq} = r_{st} - \delta_{st} = r$, where δ_{eq} and δ_{st} are the depreciation rates for capital equipment and structures respectively.[4] I assume the final goods sector is competitive, and that intermediate goods sectors sell their output competitively, meaning factors of production are paid their marginal products as shown in equations (2.7) through to (2.10).

$$
\begin{aligned}
p_{i,u} &= A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (2.7)\\
&\times \mu U^{\sigma-1}\left(\sum_{i=1}^{M}(x_{u,i}y_{u,i})^{\frac{\Psi_u-1}{\Psi_u}}\right)^{\frac{1}{\Psi_u-1}}(x_{u,i}y_{u,i})^{\frac{-1}{\Psi_u}}x_{u,i}\\
p_{i,s} &= A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} \times && (2.8)\\
&(1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma-\rho}{\rho}}(1-\lambda)S^{\rho-1}\times\\
&\left(\sum_{i=1}^{M}(x_{s,i}y_{s,i})^{\frac{\Psi_s-1}{\Psi_s}}\right)^{\frac{1}{\Psi_s-1}}(x_{s,i}y_{s,i})^{\frac{-1}{\Psi_s}}x_{s,i}\\
r_{eq} &= A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} \times && (2.9)\\
&(1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma-\rho}{\rho}}K_{eq}^{\rho-1}\\
r_{st} &= \alpha A K_{st}^{\alpha-1}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} && (2.10)
\end{aligned}
$$

Behaviour: Intermediate Goods Producers

Intermediate firms are either inactive, generating zero expected liftetime utility for their owners (I refer to the expected lifetime utility of firm ownership as the firm's value), or exist in one of three active states: (i) carrying a vacancy, with a firm value denoted by J_j^v; (ii) employing a not-poached worker, with a firm value denoted by J_j^{np}; or (iii) employing a poached worker at a wage w_j^p, with a firm value denoted by J_j^p. The corresponding bellman equations are as follows:

$$
\begin{aligned}
J_j^v &= -\kappa_j + \beta[q(\theta_j)\{s_j^{ue}J_j^{np} + (1-s_j^{ue})J_j^p\} + (1-q(\theta_j))J_j^v] && (2.11)\\
J_j^{np} &= p_j - \max(w_j^b, m_w) +
\end{aligned}
$$

[4] When it comes to calibrating the model I will assume that both net of depreciation rates equal the natural rate of interest $r = \frac{1}{\beta} - 1$.

$$\beta\left[(1-\delta_j)\{\chi_j\theta_j q(\theta_j)J_j^p+(1-\chi_j\theta_j q(\theta_j))J_j^{np}\}+\delta_j J_j^v\right] \quad (2.12)$$

$$J_j^p \;=p_j-w_j^p+\beta[(1-\delta_j)J_j^p+\delta_j J_j^v] \quad (2.13)$$

Equation (2.11) states that a firm in intermediate good sector j carrying a vacancy pays a vacancy cost, κ_j, in the current period and in the next period makes contact with an unemployed worker with probability $q(\theta_j)s_j^{ue}$, makes contact with an employed worker with probability $q(\theta_j)(1-s^{ue})$, or remains carrying a vacancy with probability $1-q(\theta_j)$. Equation (2.12) states that a firm employing a not poached worker gets profits $p_j-\max(w_j^b,m_w)$ in the current period and in the next period remains employing that worker with the probability $(1-\delta_j)(1-\chi\theta_j q(\theta_j))$, loses the worker to a rival firm with probability $(1-\delta_j)\chi\theta_j q(\theta_j)$, or the job is destroyed with probability δ_j. Finally equation (2.13) states that a firm employing a poached worker gets profit $p_j-w_j^p$ in the current period and in the next period the job is either destroyed with probability δ_j or they remain employing the poached worker with probability $1-\delta_j$.

Free entry into markets by inactive firms will drive the value of holding a vacant job, J_j^v, to zero, and competition between employers drives the value of employing a poached worker to the value of holding vacancy e.g. $J_j^p=0$ too. The free entry condition ($J_j^v=0$) and poaching condition ($J_j^p=0$) imply the poached wage, w_j^p equals the price of the intermediate good p_j.

Equations (2.11) and (2.12), combined with the free entry condition, imply:

$$\kappa_j=\beta q(\theta_j)s_j^{ue}\frac{p_j-\max(w_j^b,m_w))}{1-\beta(1-\delta_j)(1-\chi_j\theta_j q(\theta_j))} \quad (2.14)$$

Inactive intermediate firms enter the market, by posting a new vacancy, until the discounted expected profits from hiring a worker equal the vacancy cost. This discounting reflects both the discount factor and the risk that the worker will be exogenously seperated from the firm (with probability δ_j) or be poached by another firm (with probability $\chi_j\theta_j q(\theta_j)$).

The Nash bargained wage is determined in the standard maximisation problem, shown in equation (2.15).

$$\begin{aligned} w_j^b &= \underset{w_j^b}{\operatorname{argmax}}(V_j^{np} - V_j^u)^{\phi_j}(J_j^{np})^{1-\phi_j} \\ &= \Phi_j p_j + (1-\Phi_j)\big(V_j^u(1-\beta) - \beta(1-\delta_j)\chi_j\theta_j q(\theta_j)(V_j^p - V_j^u)\big) \end{aligned} \tag{2.15}$$

The fact that the minimum wage acts as a side constraint on the Nash bargained wage implies that, in the absence of equilibrium impacts on prices of intermediate goods or contact rates, there are no "spillover" impacts of the minimum wage. Only workers with initial wages lower than the minimum wage benefit from its imposition, and will see their wages bid up to the value of the minimum wage and no higher.[5,6] However, once I allow for equilibrium effects, such as changes to the prices of intermediate goods or contact rates, the absence of minimum wage spillovers is no longer a given. While this chapter focuses on the unemployment impact of minimum wages, Appendix B discusses their impact on the shape of the wage distribution.

2.3.3 Equilibrium

One condition for a steady state equilibrium in the model, which I will formally define later, is that the labour market is in steady state. This requires the following equations to hold:

$$\delta_j(1-e_j^{ue}) = \theta_j q(\theta_j)e_j^{ue} \tag{2.16}$$

$$\theta_j q(\theta_j)e_j^{ue} = (\delta_j + (1-\delta_j)\chi_j\theta_j q(\theta_j))e_j^{np} \tag{2.17}$$

[5] I need that the solution to the Nash maximisation both exists and is the unique global maximum to justify my assertion that the wage outcome as the higher of the Nash wage and minimum wage. However, this is given from the linearity of all value functions and compactness of the feasible set.

[6] This would not necessarily be the case if I had used alternative bargaining solutions such as the Kalai-Smoridinsky solution concept (see e.g. Dittrich and Knabe (2013)). It is also will generally not be true when there is match heterogeneity, and the minimum wage may influence the reservation match quality accepted by workers (see Flinn (2003)).

Equation (2.16) equates inflows into unemployment (LHS of the equation) to outflows (RHS), where the inflow consists of employees losing their jobs, with probability δ_j, and the outflow is unemployed workers gaining jobs, with probability $\theta_j q(\theta_j)$. Similarly equation (2.17) equates the inflow in of workers into the not-poached state (LHS) with the outflow (RHS), where the inflow consists of unemployed workers gaining employment with probability $\theta_j q(\theta_j)$, and the outflow (RHS) is not-poached workers either losing their job, with probability δ_j, or becoming poached, with probability $(1-\delta_j)\chi_j\theta_j q(\theta_j)$.

The steady-state Equations (2.16) and (2.17) combined with a specification of the matching function can provide an expression for the steady-state level of labour market tightness as a function of the unemployment rate, which I denote as $\theta^{ss}(e_j^{ue})$ respectively. I derive an inverse supply function for intermediate goods, shown in equation (2.18), from these steady state conditions and the no entry condition in the intermediate good sector. The demand price equation comes from the first order conditions of the final good producer's first order conditions, as shown in equation (2.19).

$$p_j^s = \max(w_j^b, m_w) + \frac{\kappa_j\Big(1-(\beta(1-\delta_j)(1-\chi_j\theta^{ss}(e_j^{ue})q(\theta^{ss}(e_j^{ue}))))\Big)}{\beta q(\theta^{ss}(e_j^{ue}))s_j^{ue}} \tag{2.18}$$

$$p_j^d = \frac{\partial Y}{\partial y_j(e_j^{ue})} \tag{2.19}$$

The supply price of intermediate goods is the sum of the wage payment to a type j not-poached worker and discounted expected vacancy costs. The demand price is simply the marginal product of the intermediate good. The prices of intermediate goods follow from equating supply (from intermediate good producers) and demand (from the final good producer).

Definition 1. *The recursive stationary equilibrium consists of, $\forall j \in \{(u,1)..(u,M),(s,1)..(s,M)\}$ and for a fixed interest rate, r, and minimum wage, m_w:*

(i) a set of worker value functions $\{V_j^{ue}, V_j^{np}, V_j^{p}\}$,

(ii) a set of firm value functions $\{J_j^v, J_j^p, J_j^{np}\}$*,and vacancies,* v_j*,*

(iii) a set of employment states $\{e_j^{ue}, e_j^{np}, e_j^p\}$*,*

(iv) a choice of capital equipment, capital structures, and intermediate goods (K_{eq}, K_{st}, y_j) *by the final good producer*

(v) prices $\{p_j,\ w_j^b,\ w_j^p\}$ *; which satisfy:*

1. *Worker Optimisation:*

   The worker value functions satisfy equations (2.3), (2.4) *and* (2.5).

2. *Final Good Producer Optimisation:*

   The final good producer's choice of capital equipment and structures, K_{eq} *and* K_{st} *and intermediate goods* y_j *satisfy the FOCs* (2.7) *through to* (2.10) .

3. *Steady State in the Intermediate Good Sector:*

   The no-entry condition, 2.14, and steady state conditions 2.16 and 2.17 are met.

4. *Intermediate Goods Market Clearing:*

   Demand and supply for each intermediate good must be equal, implying conditions 2.18 and 2.19 hold simultaneously.

5. *Wage Determination:*

   Not poached workers are paid the higher of the Nash bargained wage wage w_j^b*, as specified in equation* (2.15)*, and the minimum wage,* m_w*. Poached workers are paid the competitive wage,* $w_j^p = p_j$

6. *Consistency:*

   Given employment and vacancy rates, the job contact rates determined by the matching function are consistent with those used in the worker and firm optimisation problems.

2.3.4 Minimum Wage impacts on Unemployment

Equations (2.18) and (2.19), imply that the equilibrium wage paid to a not-poached worker equals the marginal productivity of the intermediate good they produce minus expected recruitment costs at the equilibrium unemployment rate, e_j^{ue*}, and corresponding level of labour market tightness, $\theta(e_j^{ue*})$:

$$\max(w_j^b, m_w) = \frac{\partial Y}{\partial y_j(e_j^{ue*})} - \frac{\kappa_j\Big(1-(\beta(1-\delta_j)(1-\chi_j\theta(e_j^{ue*})q(\theta(e_j^{ue*}))))\Big)}{\beta q(\theta(e_j^{ue*}))s_j^{ue}} \tag{2.20}$$

The employment impacts of a minimum wage increase are unambiguously negative due to two features of the model. First, the marginal product of an intermediate good, and hence its price, are decreasing in the amount of intermediate good used. Second, recruitment costs are increasing in the steady state employment rate. This holds because extra vacancy creation is needed to sustain a higher employment rate, which results in a reduced vacancy filling rate $q(\theta_j)$ and higher recruitment costs. However, an important implication of equation (2.20) is that the employment impact of the minimum wage in my model will be more muted than in a comparable model with perfect competition and no labour market frictions, and compared to a model with with a more typical frictional labour market structure (i.e. CRS production with labour as the only factor input).

In a comparable competitive model, an increase in the marginal product via reduced employment levels is the only force that can restore equilibrium in the labour market following a minimum wage increase. Adding frictional labour markets to this set-up means the fall in employment necessary to restore equilibrium is less as recruitment costs fall when employment is reduced.

The same logic applies when I compare my model to a more typical frictional benchmark. When labour effectively has a constant marginal product, the fall in recruitment costs is the only force that can restore equilibrium following a minimum wage increase. Adding a production function where labour produces an intermediate good with a decreasing marginal product means the fall in employment necessary to restore equilibrium is again less as the marginal product of the intermediate good now increases when employment falls.

2.3.5 Solution Algorithm

For a fixed world interest rate, r, I:

1. Guess the unemployment rate e^{ue}_{j0}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$.

2. Use this guess to construct the aggregate output of intermediate goods produced in the unskilled and skilled intermediate sectors (these aggregate outputs, U and S, are defined in equation (2.2)).

3. Solve the final good firm's FOCs (equations (2.9) and (2.10)) to get their optimal choice of capital equipment and structures, K_{eq} and K_{st}, that is consistent with the implied levels of U and S from above and firm optimisation given the interest rate r. Then derive the price of each intermediate good p_j that is consistent with firm optimisation at the unemployment guess e^{ue}_{j0} using the FOCs in equations (2.7) and (2.8).

4. Use the conditions (2.16) and (2.17) to derive vacancy levels necessary for the unemployment guess $e^{ue}_{j,0}$ to be consistent with steady state in the labour market. This then implies employment transition probabilities for the unemployed and employed via the matching function: $\theta_j q(\theta_j)$ and $\chi_j \theta_j q(\theta_j)$ respectively.

5. Use employment transition probabilities from above and condition that poached worker is paid $w^p_j = \max(p_j, m_w)$ to solve worker value functions and Nash bargained wage using equations (2.3) to (2.5) and (2.15) respectively. Wage of not-poached worker is whatever is highest of this bargained wage and minimum wage. [7]

[7] As argued previously, we will always have $p_j(= w^p_j) \geqslant m_w$ in equilibrium however this does not necessarily hold outside of equilibrium so I must impose that $w^p_j == \max(p_j, m_w)$ when solving the model.

6. Use wage levels from above step to give an updated unemployment guess, e^{ue}_{j1}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$ that simultaneously solves free entry condition (2.14) for the intermediate firm and the final good firm's FOC i.e. equations (2.18) and (2.19).

7. Repeat iteration until convergence of unemployment guess.

2.4 Estimation

This section first describes my estimation strategy, before presenting parameter estimates and examining the model's fit to targeted and non-targeted empirical moments. In particular, I will consider two types of non-targeted empirical moments:(i) macro moments i.e. labour and profit share of output, the capital-output ratio, average firm mark-ups and fit of the wage distribution; and (ii) micro moments i.e. reduced form evidence from the introduction of the minimum wage in the UK (principally Draca et al. (2011)).

As the model moments are largely intractable, and therefore simulated numerically, I do not provide formal identification arguments but instead examine the relationship between the parameters I estimate and the model moments used in their estimation: this is done in Appendix A. I also discuss the logic of choosing the empirical targets I use in the section below.

2.4.1 Estimation Strategy

I will take all but one of the parameters of the final good production function from Krusell et al. (2000). This means applying parameter estimates from a model with a competitive labour market to my model that assume labour market frictions. However, results from Chapter 3 of this thesis suggest the parameter estimates obtained by Krusell et al. (2000) are robust to allowing for labour market frictions. This provides some reassurance that applying their parameter estimates to the model developed here is not unreasonable. There is a seperate issue that the estimates that Krusell et al. (2000) provide are based on calibration to the US economy, and I will be calibrating my model to the UK. However, this again seems reasonable as a calibration approach given the UK has exhibited similar, if not identical, trends in wage inequality and in the labour share to the U.S, particularly in the 1980s and 1990s.

I use the matching function specification, and the associated parameter estimate, from Hagedorn and Manovskii (2008b) - $M(u, v) = uv/(u^{\gamma} + v^{\gamma})^{1/\gamma}$, which ensures job contact rates are bounded between zero and one.

I focus on estimating: (i) the parameters in the exogenous distributions of worker ability, with separate distributions for unskilled and skilled labour (which I interpret as non-graduate and graduates respectively); (ii) the elasticities of substitution between workers within these two skill types, ψ_u, ψ_s; (iii) recruitment costs, κ_u, κ_s, which I assume are fixed within each skill type of worker; (iv) bargaining parameters, Φ_u, Φ_s, which I also assume are fixed within each skill type of worker; and (v) the share parameter, μ, in the KORV production function.

I assume a log normal distribution for worker ability within each skill type, meaning in principle there are two distributional parameters to estimate for each skill type i.e the mean and variance parameters, ζ_h and η_h respectively for $h \in \{u, s\}$. I normalise the mean of the ability distribution to one for skilled and unskilled workers but allow differing scale parameters η_u, η_s. This normalisation is justified on the basis that I will instead estimate the share parameter, μ, in the final good production function and TFP.[8] I denote the parameters to be estimated as $\Phi = (\psi_u, \psi_s, \kappa_u, \kappa_s, A, \phi_u, \phi_s, \eta_u, \eta_s, \mu)$.

The remaining parameters, denoted by Ω, are taken from the literature, directly from the data or are set at their legislative levels, as detailed in Table 2.1. I calibrate the model to data from 2013-14, as this precedes the significant increases in the minimum wage that started in 2014-15 and are planned to end when the minimum wage reaches 60% of the median wage in 2020-21. I assume job destruction rates are fixed within a given skill type but vary between skill

[8] μ plays an important role in determine the skill premium in the model (see Appendix A) and hence is not separately identifiable from the relative level of the mean ability parameters, ζ_s/ζ_u. Similarly TFP determines average wages in the model, and so is not separately identifiable from the absolute values of the parameters ζ_s, ζ_u. I therefore normalise the mean parameters of the ability distributions in absolute and relative terms.

Table 2.1: Calibrated Parameters

Parameter	Description	Source	Value
δ_u	Job destruction rate: unskilled	LFS 2013q4-2014q3	0.011
δ_s	Job destruction rate: skilled	LFS 2013q4-2014q3	0.007
χ_u	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.112
χ_s	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.075
b	Monthly Unemployment benefits (job seekers allowance)	Legislative level 2013-14	313.492
m_w	Hourly minimum wage	Legislative level 2013-14	6.31
σ	Elasticity of substitution between unskilled and skilled workers	Krusell et al. (2000)	0.401
ρ	Elasticity of substitution between skilled workers and capital equipment	Krusell et al. (2000)	-0.495
α	Capital Structures Parameter	Krusell et al. (2000)	0.117
λ	Input share parameter for capital equipment and skilled labour	Krusell et al. (2000)	0.3
γ	Matching Parameter	Hagedorn and Manovskii (2008a)	0.407
β	Monthly discount factor for workers and firms	By assumption	0.996

types, whereas I assume unemployment income is paid at a fixed rate that is common for all workers.[9]

I estimate the parameters in Φ by simulated method of moments, targeting the following empirical moments for non-graduates and graduates: median wages, variance of log wages, p90/10 and p50/10 ratios and the proportion of unskilled and skilled workers being paid at or less than the minimum wage (I refer to this moment as the minimum wages coverage).[10] The absolute magnitudes of median wages help to discipline the TFP parameter, A, and their relative magnitudes will discipline the output share parameter, μ. Unemployment rates for the unskilled and skilled are informative of both vacancy costs

[9] Unlike in many other jurisdictions, the main form of unemployment benefits in the UK is paid at a flat rate, as under my baseline calibration, rather than as a fixed percentage of previous earnings. Of course, workers may have access to other forms of insurance: Chapter 4 of this thesis considers minimum wage impacts when workers can self-insure themselves through asset accumulation.

[10] I allow for measurement error in minimum wage coverage in two ways. First I count anyone earning less than the minimum wage in my coverage statistic, and include anyone earning within 20 pence over the minimum wage in the data and model as being covered.

(κ_u, κ_s) and the elasticities of substitution between workers of heterogeneous ability within each skill group (ψ_u, ψ_s). This follows because the vacancy costs influence the unemployment rate of all workers within a given skill type, and the elasticity of substitution influences the unemployment impact of a given minimum wage on low ability workers within a given skill type. I use several measures of wage dispersion i.e. log wage variance, p90/10 and p50/10 wage ratios, to help pin down the variance parameters η_u, η_s and because they are also informative of vacancy costs (which determine the proportion of not-poached and poached workers). Finally, I use the minimum wage coverage rates for the unskilled and skilled as they help to discipline the bargaining parameters (ϕ_u, ϕ_s).

Equation (2.21) summarises the estimation method, where $\hat{M}$ denotes a vector of the empirical moments given above, $M(\Phi, \Omega)$ denotes the model predictions of these moments for given choice of estimated and calibrated parameters, and W is the weighting matrix.[11]

$$\Phi^* = \underset{\Phi}{\operatorname{argmin}}(M(\Phi, \Omega) - \hat{M})' W (M(\Phi, \Omega) - \hat{M}) \tag{2.21}$$

2.4.2 Estimation Results

Table 2.2 summarises the ability of the model to match its empirical targets. Given I have over identification (10 parameters vs 12 moments), the fact that the maximum absolute deviation is just above 8% is reassuring. The estimated parameters are shown in Table 2.3. It is perhaps counter-intuitive that my estimation delivers lower elasticities of substitution and higher bargaining parameters for unskilled workers compared to skilled workers. Both results are explained by the fact that the minimum wage bites further into the wage distribution of unskilled workers than skilled workers; in the model the minimum wage is 63% of median wages for unskilled workers, but just 40%

[11] The weighting matrix W, is chosen so I effectively minimise the percentage deviation of model moments from their empirical moments, which avoids the scale of absolute moment deviations biasing estimates i.e. $W = I.\frac{1}{\hat{M}}$.

Table 2.2: Estimation Results

Moment	Model Moment	Empirical Moment	% Deviation (Model - Data)
Median Hourly Wage: Unskilled	9.85	9.5	3.61
Median Hourly Wage: Skilled	16.08	15.71	2.41
Var Log Wages: Unskilled	0.45	0.49	-7.97
Var Log Wages: Skilled	0.54	0.57	-5.33
p90/50 Wages: Unskilled	2.02	1.92	4.99
p90/50 Wages: Skilled	2.03	1.96	3.35
p50/10 Wages: Unskilled	1.56	1.57	-0.54
p50/10 Wages: Skilled	2.08	2.07	0.7
Min Wage Coverage: Unskilled	0.16	0.16	-0.77
Min Wage Coverage: Skilled	0.06	0.06	-0.56
Unemployment: Unskilled	0.07	0.07	0.95
Unemployment: Skilled	0.03	0.03	0.92

Table 2.3: Estimated Parameters

Parameter	Description	Source	Value
Ψ_u	Elasticity of substitution between unskilled workers	SMM Estimation	7.218
Ψ_s	Elasticity of substitution between skilled workers	SMM Estimation	28.875
μ	Share parameter determining skill premium in KORV production function	SMM Estimation	0.336
A	Total Factor Productivity	SMM Estimation	11.885
η_u	Variance parameter of worker ability distribution: unskilled workers	SMM Estimation	0.452
η_s	Variance parameter of worker ability distribution: skilled workers	SMM Estimation	0.494
ϕ_u	Nash Bargaining Parameter for unskilled workers	SMM Estimation	0.235
ϕ_s	Nash Bargaining Parameter for skilled workers	SMM Estimation	0.143
κ_u	Hiring cost: unskilled workers	SMM Estimation	308.889
κ_s	Hiring cost: skilled workers	SMM Estimation	1228.192

for skilled workers. Without a lower elasticity of substitution for unskilled workers than skilled, the unemployment gap between the two groups would be counter-facutally large.[12] Similarly, without a higher bargaining parameter for unskilled workers than skilled the gap between the minimum wage coverage for the two groups would be counter-factually high.[13]

While studies such as Cahuc et al. (2006) find bargaining power decreases

[12]If I raise the elasticity of substitution for unskilled workers to the level for skilled workers, their respective unemployment rates increase from 7.1% and 3.1% respectively in the model (which matches the data) to 17.8% and 3.6%.

[13]If I lower the bargaining parameter for unskilled workers to the level for skilled workers, their respective minimum wage coverage rates increase from 15.8% and 5.7% respectively in the model (and data) to 18.6% and 5.7%.

with skill, the parameter plays a very different role in their estimation strategy than in mine. In Cahuc et al. (2006) the bargaining parameter is informative in matching model predictions regarding the wage distribution to the data, using their prior estimates of employer and employee fixed effects and job transition rates. In my estimation, its primary impact, as discussed above, is to match model predictions of minimum wage coverage to the data, and makes use of employee data only.

The estimates of vacancy costs are perhaps more intuitive and suggest it is approximately 4.5 times more costly to post a vacancy for skilled workers than unskilled workers. Given vacancy posting costs reflect the flow value of all recruitment costs in the model, this differential appears qualitatively reasonable on the grounds that skilled workers are likely to require greater screening and on-the-job training.

2.4.3 Non-targeted Empirical Moments: Macro Moments

Table 2.4 compares the model's predictions regarding a range of macroeconomic moments to the data. I hit the labour share precisely, which is perhaps surprising given the parameters of the KORV production function were originally estimated in the context of a competitive labour market model, and in the U.S where the labour share has tended to be lower than the UK. Though far from conclusive, this suggests the model, at a macro level, features relatively strong levels of employer competition despite the presence of frictions. This impression is reinforced when I compare markups in the model to empirical estimates, as is done in the second row of Table 2.4. The mark-up measure I use comes from De Loecker and Eeckhout (2018), and is the ratio of output price to estimated marginal costs (so a perfectly competitive economy would have a mark-up ratio of 1).[14] My model gives a mark-up measure that is significantly below the De Loecker and Eeckhout (2018) estimate for the UK (1.06 in the

[14]De Loecker and Eeckhout (2018)'s estimator for markups at a given firm, υ_f is:

$$\upsilon_f = \varepsilon_f^I \frac{P_f^Q Q_f}{P_f^I I_f}$$

model vs an estimate of 1.5). This is likely to reflect two features of my model: (i) free entry in vacancy creation, which drives the expected profits from issuing a vacancy to zero; and (ii) when a worker is poached in the model, they receive a wage equal to their marginal product (i.e. the competitive wage). The model's mark-up prediction is not out of the range of the estimates given by De Loecker and Eeckhout (2018) however (e.g. it's consistent with mark-up estimates for the UK in the 1980s).

The third and final moment considered in Table 2.4 is the net capital to gross value added ratio, where the model suggests less capital intensity than observed in the data (1.77 in model vs 2.66 in data). Differences could partly reflect methodological differences in capital stock measurement in the data used to estimate the KORV production function and in the empirical moment so I do not attach too much importance to this discrepancy.

Table 2.4: Non-targeted Macro Moments

Moment	**Model Moment**	**Empirical Moment**
Labour Share of GVA[1]	0.76	0.76
Mark-Up Ratio[2]	1.06	1.5
Net Capital Stock/GVA[3]	1.77	2.6

[1] Bank of England, includes self-employed labour income (imputing it as compensation per employee multiplied by number of self-employed). GVA=Gross Value Added

[2] Empirical moment taken from De Loecker and Eeckhout (2018), model moment is calculated analagously (as described in text).

[3] UK National accounts, ONS.

Although I do target some moments of the wage distribution, it is nevertheless instructive to consider the fit of the entire model wage distribution to that

---

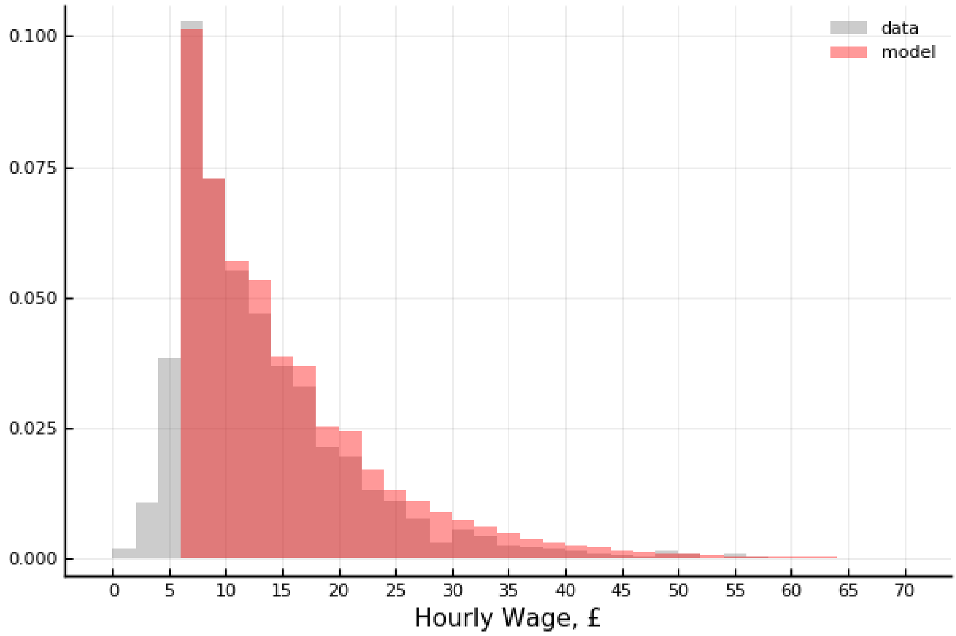
where I_f is the firms use of variable input I (with price denoted P_f^I), Q_f is their output (with price denoted P_f^Q) and ε_f^I is their output elasticity with respect to I_f. In my model, only intermediate firms have mark-ups, and have output elasticity of one due to constant returns to scale in intermediate good production. Constant returns makes the definition of an intermediate firm in principle ambiguous so I choose to define it as a collection of all firms employing a worker in sector j (where j again indexes the skill and ability of the workers employed in that sector). The model counterpart to De Loecker and Eeckhout (2018)'s mark up measure is therefore:

$$\upsilon_j = \frac{p_j(1 - e_j^{ue})\ell_j}{(\max(m_w, w_j^b)e_j^{np} + p_j e_j^p)\ell_j}$$

In both De Loecker and Eeckhout (2018) and my model counterpart, an average mark-up measure is calculated by taking the sales-weighted means of the firm mark-up measures shown above.

in the data: see Figure 2.3. The model closely fits the empirical wage distribution except where the empirical distribution lies below the legal minimum wage, which likely reflects a mixture of measurement error and non-compliance.

Figure 2.3: Model vs Empirical Wage Distribution



Notes: Data from LFS 2013-14, for workers who are paid by hour.

2.4.4 Non-targeted Empirical Moments: Matching Reduced Form Evidence

An important test of the model is whether it matches the reduced form evidence in the UK on minimum wage impacts. In this section, I examine whether the model can replicate the findings of Draca et al. (2011). They use a difference-in-difference methodology to estimate the impact of the introduction of the minimum wage in 1999 on firm profitability and average wages. Their treatment group is firms with average wages less than £12,000 in 1999. The average wage of this group is close to the level of the minimum wage. Their control group is firms with average wages between £12,000 and £20,000.

The only firms that earn profits in my model are intermediate goods firms, who sell competitively to final good firms but have some monopsony power over workers due to labour market frictions. The definition of a firm is in principle ambiguous due to constant returns to scale within the intermediate goods sector. I define the firm as a collection of all vacancies and active jobs in intermediate goods market j. The profit level, π_j, profit margin, π_j^m and average wage, $\bar{w}_j$ for a firm of type j are therefore as follows:

$$\pi_j = [(p_j - \max(w_j^b, m_w))e_j^{np} - \upsilon_j \kappa_j]\ell_j \tag{2.22}$$

$$\pi_j^m = \pi_j/(p_j(1 - e_j^{ue})\ell_j) \tag{2.23}$$

$$\bar{w}_j = (p_j e_j^p + \max(w_j^b, m_w)e_j^{np})/(1 - e_j^u) \tag{2.24}$$

Note that in the model the only variance in wages within a given firm comes from the proportion of workers that are poached or not, which can loosely be thought of as wage heterogeneity due to tenure.

As in Draca et al. (2011), I run the regressions in equations (2.25) through (2.28), where the subscript zero denotes the level of a variable before the minimum wage was introduced. I replicate the introduction of the minimum wage by first reestimating the set of parameters shown in Table 2.3 so that they match the models' predictions to the same empirical targets specified in section **??** but for 1998-99, i.e. before the minimum wage was introduced. I then simulate the steady-state impact of introducing the minimum wage at the level it was set at in April 1999, and running the regressions shown in equations 2.25 through to 2.28. I assume the economy is in steady state before and after the introduction of the minimum wage in my analysis, so the Δ in the regression equations represents the change in the dependent variable between steady states in the model. This is broadly consistent with the empirical exercise in Draca et al. (2011) which considers average profit and wage rates for the three years before and after the minimum wage introduction and thus also attempts to estimate a 'long-run' impact. [15]

$$\Delta\pi_j^m = const + \hat{\beta}_1\ Treatment_j + \epsilon_j \tag{2.25}$$

[15] The minimum wage did not significantly change in real terms in the three years after its introduction, either as a % of the median wage or in terms of consumer prices.

Table 2.5: Replicating Reduced Form Evidence

	Dependent Variable		
	Change in ln(average wage)	Abs Change in Profit Margin	% Change in Profit Margin
Results from Model:			
Dummy: Low Wage Firm	0.081	-0.0032	-22.81
	(0.0252)	(0.0017)	
-ln(initial average wage)	0.1939	-0.0074	
	(0.0266)	(0.0022)	
Results from Draca et al. (2011):			
Dummy: Low Wage Firm	0.09	-0.029	-22.66
	(0.026)	(0.012)	
-ln(initial average wage)	0.188	-0.032	
	(0.033)	(-0.015)	

$$\Delta\pi_j^m = const - \hat{\beta}_2 \ \log \bar{w}_{j0} + \epsilon_j \tag{2.26}$$

$$\Delta\bar{w}_j = const + \hat{\beta}_3 \ Treatment_j + \epsilon_j \tag{2.27}$$

$$\Delta\bar{w}_j = const - \hat{\beta}_4 \ \log \bar{w}_{j0} + \epsilon_j \tag{2.28}$$

The four regression coefficients $(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4)$ and their standard errors are shown in Table 2.5, with their standard errors, both for simulations from the model and the original findings in Draca et al. (2011). The model comes close to replicating the average wage impact of the minimum wage, a result that is not mechanical given the degree of minimum wage spillover is endogenous to the model. While the model does not manage to replicate the absolute fall in profit margins, it almost exactly matches the % fall in profit margins (see third column of Table 2.5).

While it is not the focus of their chapter, Draca et al. (2011) also estimate the employment impact of the minimum wage introduction and “do not find any significant negative effects on employment”, which is consistent with results elsewhere (e.g Leonard et al. (2014)). In contrast, the model predicts a significant increase in unemployment for low skilled workers (from 6.7% to 13.7%). This reflects the fact that the model captures only the disemployment impact of minimum wages and does not include other labour related margins of adjustment for firms like hours worked, or employee benefits (pensions, training etc). It also reinforces the earlier caveat that the predicted unemployment nonlinearities in the model should be viewed as a cautious/lower bound estimate of where any nonlinearities may be located in reality.

2.5 Results

Figure 2.4 shows the simulated relationship between steady state unemployment in my model to the level of the minimum wage. All simulations shown in this section are steady state equilibrium outcomes, conforming to the equilibrium definition provided in Section 2.3.3, and so do not account for transition dynamics. The results suggest a significant risk of increased unemployment in the range of minimum wage values planned in the UK.

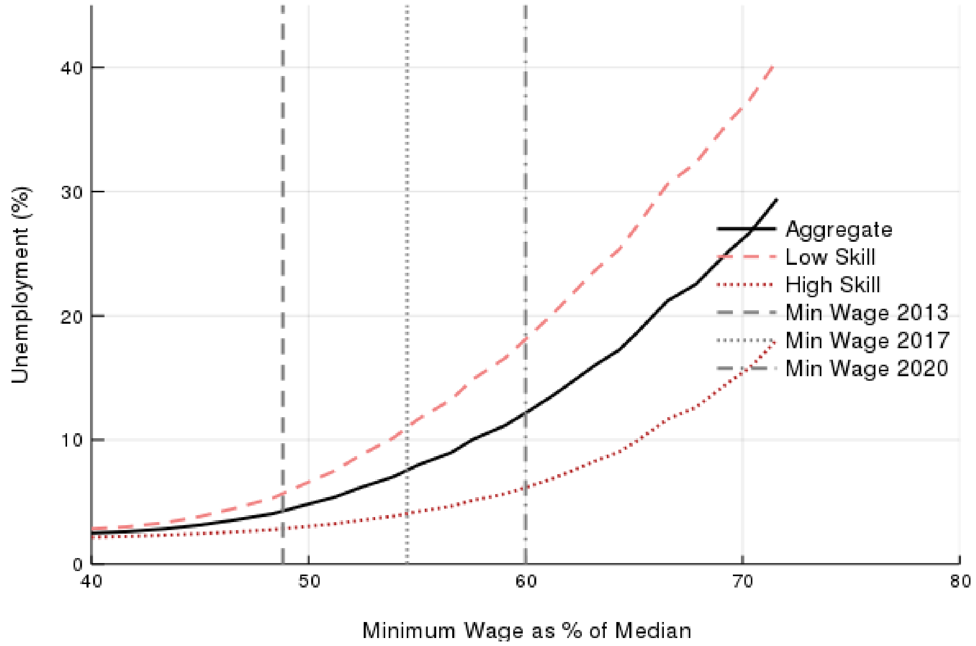
At first sight the unemployment response appears counter-factual; the model predicts that unemployment should have increased due to the minimum wage increases introduced from 2013 to the present date, but emprically this has not been the case.[16] However, inspecting headline movements in the unemployment data is not a substitute for econometric evaluation as there are likely to have been contemporaneous changes in the UK economy that might explain the fall in unemployment e.g. a cyclical improvement following the global financial crisis, and a structural decrease in unemployment due to a tightening of the welfare regime for unemployed workers. As far as I am aware, the only econometric research that considers both employment entry and exit impacts of the post 2013 increases in the minimum wage - Dickens and Lind (2018) - finds a small but significant decrease in employment, using variation in minimum wage bite by geographic area.

I now proceed to investigate what generates this nonlinear response in the model. I look at this question in two parts, first investigating the mechanisms that account for the existence of the nonlinearity, and then examining which parameters determine its location and strength.

The drivers of the nonlinear unemployment response in my model are: (i) endogenous nonlinearities in labour demand that arise both from using a multi-input production function with imperfect substitution between all inputs and

[16]the unemployment rate for those aged between 16 and 64 decreased from 7.8% in 2013 Q1 to 4.1% in 2018Q4 (source: Labour Force Survey)

Figure 2.4: Nonlinear Unemployment Response



from endogenous vacancy creation; and (ii) exogenous nonlinearities in the distribution of ability across workers, within a given skill type.

I investigate the quantitative importance of each of these factors by altering my baseline model in three ways. First I move to a final good production with labour as it's only input, and with perfect substitution between all skill and ability types.[17] This shuts off the endogenous nonlinearity in labour demand driven by imperfect substitution between input types, but keeps the other sources of nonlinearities (endogenous vacancy creation and a non-uniform distribution of ability types). Second I shut off endogenous vacancy creation as a driver of a nonlinear unemployment response by no longer imposing the free entry condition 2.14.[18] The third change I make is to impose a uniform distribution of abilities and skills, rather than my baseline assumption of a log

[17] The final good production function becomes $Y = \sum_{i=1}^{M} x_{u,i} y_{u,i} + \sum_{i=1}^{M} x_{s,i} y_{s,i}$, where as before $y_j = (1 - e_j^{ue})\ell_j$ but the ability level, x_j, of each worker type is set equal to the equilibrium marginal product of the intermediate good produced by that worker type, $\frac{\partial Y}{\partial y_j(e_j^{uess})}$, in my baseline model when the minimum wage is set to zero.

[18] I impose that job contact rates for the unemployed, $\lambda_{0,j}$, are initially set a fixed level equal to the equilibrium job contact rates in my baseline model, $\theta_j^{ss} q(\theta_j^{ss})$, when there is no minimum wage

normal distribution of ability.[19] These channels are eliminated one at a time, rather than sequentially. Appendix C describes in more detail how I implement these alterations to my baseline model.

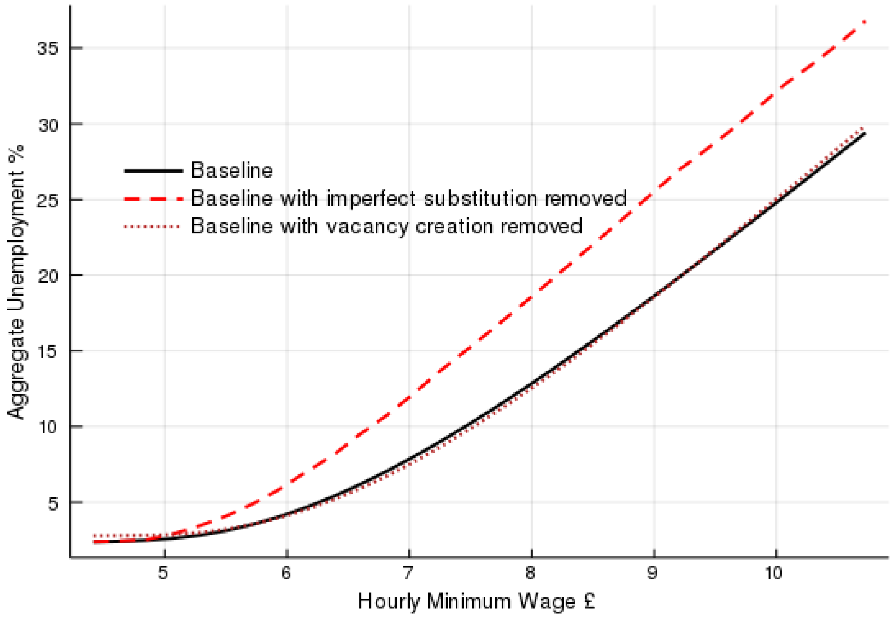
Figure 2.5 starts by shutting down the two endogenous sources of nonlinearities: vacancy creation and imperfect substitution between inputs. It suggests that, quantitatively, the impact of imperfect substitution is a much more significant driver of the nonlinear unemployment response than endogenous vacancy creation. Removing endogenous vacancy creation from the model yields almost exactly the same nonlinear relationship between the minimum wage and unemployment, whereas removing imperfect substitution between factor inputs yields a much more linear relationship. This is a significant implication of the model since existing search models of the minimum wage in the literature typically assume perfect substitution between inputs of production, and so are not able to capture this source of nonlinearity. When I look at the impact of a uniform distribution of skill types, as in Figure 2.6, I see that this change alone is enough to drive a largely linear response in unemployment, even in the presence of both endogenous nonlinearities, however the wage distribution in Figure 2.3 strongly suggests a non-uniform distribution of ability.

The picture that emerges from figures 2.5 and 2.6 is that, over the range of minimum wage values I consider, imperfect substitution between inputs is the most significant endogenous mechanism driving the nonlinear unemployment response. However, switching to a uniform distribution of ability types dominates the combined effect of both of the endogenous sources of nonlinearities. While this exercise is helpful in identifying which factors account for the existence of the nonlinearity, switching to completely uniform distribution of worker ability is clearly an extreme and likely unrealistic scenario: for example, the wage distribution shown in Figure 2.3 is suggestive of a non-uniform distribution of abilities that is approximately log normal.

I now turn to the question of which parameters in my baseline model de-

[19] That is I assume $x \sim U(x_{min}, x_{max})$ where the boundaries of this interval are the same as under my baseline calibration.

Figure 2.5: Sources of Nonlinearities: Endogenous

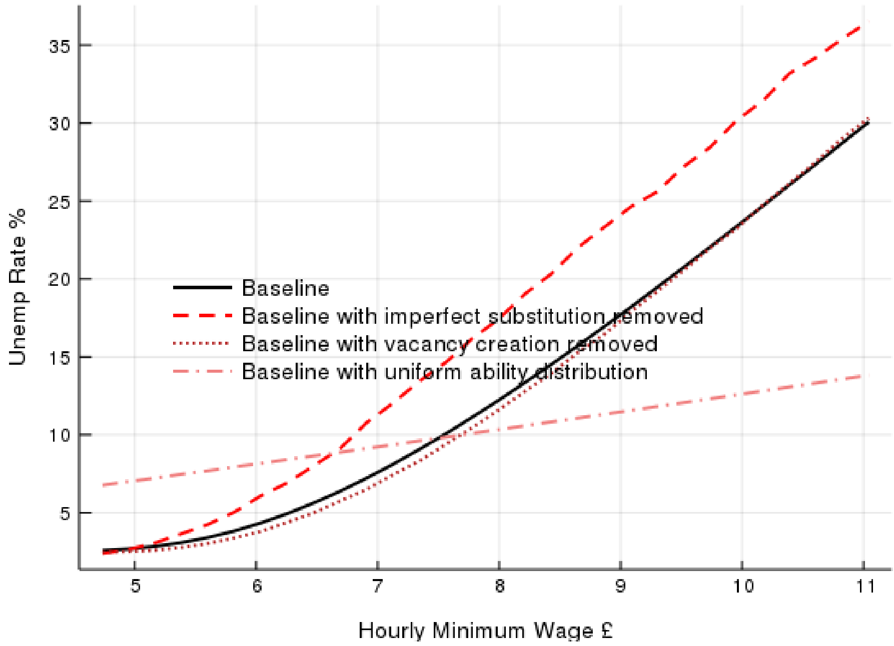


termine the strength of the nonlinear unemployment response. I do this by altering the parameter values used in my baseline calibration by plus and minus 25%. In each case I only alter one of the parameter values and leave the others unchanged.

The results are shown in Figure 2.7. The quantitative importance of the worker ability distribution again emerges; the first row of Figure 2.7 shows that the biting point of the nonlinearity occurs significantly later (i.e. at a higher minimum wage level) as I either decrease the dispersion of the unskilled workers' ability distribution, moving closer to a representative agent model, or by increasing the output share of the aggregate unskilled labour input (broadly equivalent to a rightward shift of the entire distribution of unskilled workers' ability).

I concluded above that the imperfect substitution of inputs in final good production was a more significant endogenous driver of the nonlinear unemployment response than vacancy creation. Figure 2.7 gives us the more specific conclusion that varying the elasticity of substitution between unskilled workers of differing abilities has a more significant impact on the location and strength

Figure 2.6: Sources of Nonlinearities: All

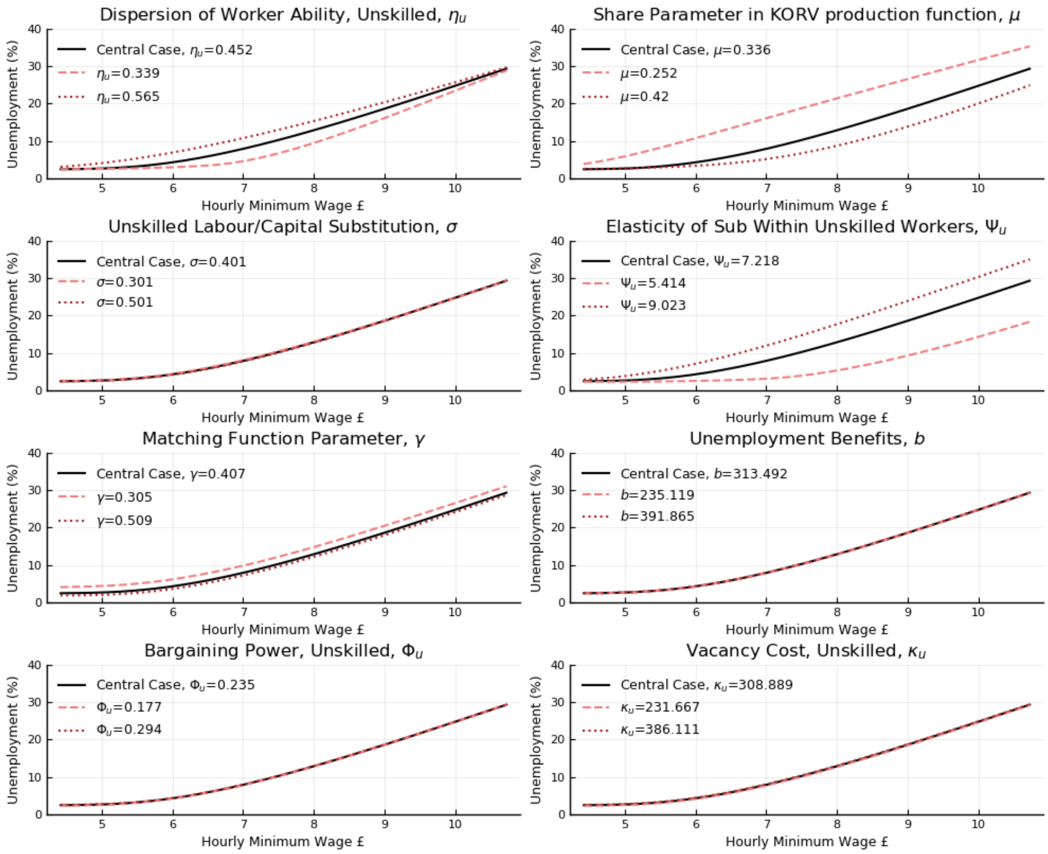


of the nonlinearity than varying the elasticity of substitution between the aggregate unskilled input, U, and capital equipment, K_{eq}.[20]

The remaining plots in Figure 2.7 reflect the quantitative lack of importance of endogenous vacancy creation in the model. The matching function parameter has a level impact on unemployment, but does not significantly change the unemployment response to the minimum wage. Varying the level of unemployment benefits, bargaining power and the cost of vacancy posting by plus and minus 25% all have negligible impacts on the strength and location of the nonlinear unemployment response. This is not particularly surprising given that all of these parameters have a direct impact on the vacancy creation channel only, which I have previously found to be a relatively unimportant source of the unemployment nonlinearity in the model.

[20]The specification of the KORV production function implies the elasticity of substitution between the aggregate unskilled input and capital equipment always equals the elasticity of substitution between the aggregate unskilled and skilled intermediate inputs, U and S.

Figure 2.7: Sensitivity Analysis



2.6 Conclusion

This chapter has examined whether there are likely to be significant nonlinearities in the impact of the minimum wage on unemployment. I explored this question using a model that combines search frictions with a production process featuring several margins of substitution between factor inputs. In this context, nonlinearities are driven by: (i) endogenous nonlinearities in labour demand that arise both from using a multi-input production function and from endogenous vacancy creation; and (ii) exogenous nonlinearities in the distribution of skill across workers. When calibrated to match the UK economy, the model suggests a nonlinear unemployment reaction that bites well within the range of minimum wage levels planned in the UK over the next two years.

Quantitative results from the model suggest that the most significant endogenous driver of a nonlinear relationship between the minimum wage and unemployment is the imperfect substitution between different worker ability types in the production function. If we instead assume constant returns to scale production using labour as the only input of production, as is commonly done in the search literature, the predicted relationship becomes significantly more linear. This highlights the importance of allowing for imperfect substitution between factor inputs when considering the unemployment impacts of the minimum wage.

I view the predictions of this model as a somewhat cautious lower bound estimate of where any nonlinearities might lie since the minimum wage can only ever have a negative impact on the employment rates of workers for whom the minimum wage binds in the model. There are plausible mechanisms that could break this result. The minimum wage could increase worker search effort or labour market participation, as in Flinn (2006), or could screen low productivity firms out of the market and so allow higher productivity firms to expand (Mayneris et al. (2014)). Including these mechanisms in the model developed in this chapter is a worthwhile goal for further research, particularly as it could address the counterfactually large unemployment response in the model.

Chapter 3

Wage Inequality, Technological Change and Search Frictions

3.1 Introduction

This paper seeks to make a contribution to the large and varied literature that examines the rise in wage inequality since the later 1970s. This literature, though broad in scope, has generally been based on a competitive labour market framework. This is true of both relatively early studies such as Katz and Murphy (1992) and later contributions such as Krusell et al. (2000) and Acemoglu and Autor (2011).

A competitive labour market framework, while restrictive in some dimensions, does not preclude a rich variety of explanations for the rise in wage inequality; ranging from the fall in capital prices documented in Krusell et al. (2000) to cohort specific supply changes emphasized in Card and Lemieux (2001). However, in a competitive framework these explanations are naturally restricted to two broad categories: those based on changes to the technology governing production of output; and those based on changes to the skill distribution of workers.

This precludes some explanations that have received some empirical support but so far have not been grounded in theory. For example, the importance of firm heterogeneity in explaining growing wage inequality found in Song et al. (2015) would be difficult to incorporate in a perfectly competitive model where workers (and consumers) would instantly relocate to firms that are more productive.

Similarly changes to institutions such as the minimum wage or unions could also play an important role is explaining the rise in wage inequality, particularly in the 1980s, as discussed in Card and DiNardo (2002) and Lee (1999). Such institutions are certainly not irrelevant in perfectly competitive models but it can be difficult to generate quantitatively large impacts from institutional factors when assuming perfect competition. As an example of this difficulty, various papers have argued that competitive models of the labour market featuring minimum wages struggle to generate the sort of wage spillovers or spikes in the wage distribution observed in the data, see Flinn (2006), Manning (2003) and Teulings (2000).

A key contribution of this paper is to develop a structural model that can incorporate both technological based explanations for rising wage inequality and explanations reflecting labour market frictions and institutional change as outlined above. Specifically, I do this by combining the production framework specified in Krusell et al. (2000) with the sequential auction wage bargaining model developed in Postel-Vinay and Robin (2002) and Cahuc et al. (2006).

Krusell et al. (2000) relate the rise in the graduate wage premium to the fall in the price of capital equipment. They show that a production specification where skilled and unskilled labour combine with capital to produce output can provide an explanation for the rise in the graduate wage premium when capital is more complementary with skilled than unskilled labour. Quantitatively, the authors show that this capital skill complementarity channel, when combined with the large falls in the price of capital equipment observed in the data, is able to explain almost all of the rise in the graduate wage premium seen over the 1980s and early 1990s.

In the sequential auction model of the labour market in Cahuc et al. (2006), average wages of a given skill type of worker depend on the worker's marginal productivity at a given firm - as in the competitive framework - but also on job to job transition rates, bargaining strength, the distribution of firm heterogeneity and outside options in unemployment. The eventual goal of this project is to evaluate the contribution of changes to each of these factors to the rise in wage inequality, and see whether the estimates of capital skill complementarity from Krusell et al. (2000) are materially different once these factors are accounted for. This could be the case if, for example, job market frictions have significantly worsened for unskilled workers relative to skilled. This would mean my model, because it allows for these change of frictions, would be less reliant on the technology channel emphasised in Krusell et al. (2000) to explain the growth in the graduate wage premium, and would deliver parameter estimates suggesting a smaller degree of capital skill complementarity.

This has important consequences for policy; the technological explanation for rising wage inequality implies governments face a relatively acute trade-off if they wish to boost living standards of low skill workers through policies such as the minimum wage or increasing unionization rates; on the hand such policies can improve the incomes of those in work but, if low skilled labour is indeed significantly substitutable with capital, these policies risk pushing more workers into unemployment. Any findings suggesting a lower level of substitutability between unskilled labour and capital will therefore have an important bearing on how acute this trade-off is.

Indeed a closely related motivation for this project is to test the validity of the approach of Chapter 2 of this thesis, where I used the production technology and parameter estimates from Krusell et al. (2000) in a frictional labour market environment to predict minimum wage impacts. The empirical exercise performed here will test whether the parameter estimates in Krusell et al. (2000) are robust to moving from a perfectly competitive labour market environment (as assumed in the original study) to one with frictions; I will see that the estimates are robust to this change.

My strategy for taking the model to the data has been to maintain consistency with Krusell et al. (2000) by focusing on the graduate wage premium as my measure of labour market inequality, and by using national accounts and the Current Population Survey (CPS) data. As suggested above, I find that estimates of the strength of the capital-skill complementarity channel are not materially changed by allowing for search frictions in the labour market.

This finding is driven by the fact that, contrary to my expectations, the empirical measures of labour market frictions I use, such as job-to-job mobility and job destruction rates, do not show any trends favouring skilled workers (graduates) relative to unskilled, indeed if anything the reverse is true, and so do not provide an alternative explanation for the rise in the graduate wage premium.

The rest of this chapter is organised as follows. Section 3.2 will present the model, starting first with an overview of both the production technology in Krusell et al. (2000) (henceforth KORV) and the sequential model of Cahuc et al. (2006) before examining how I combine them in my model. Section 3.3 discusses the data I use to estimate the combined model, before Section 3.4 presents my econometric approach. Section 3.5 presents findings and Section 3.6 concludes.

3.2 The Model

Introducing search frictions, with wage bargaining, into the production technology in KORV comes up against a key theoretical challenge, which is that doing so directly would mean firms bargaining with many workers i.e. a multi-player game as per Stole and Zwiebel (1996). These multi-player games seem unlikely to be relevant for considering aggregate dynamics in the labour market. I therefore abstract from such effects by specifying a competitive final good firm, where production is as in KORV, and an intermediate good sector with a labour market structure identical to the sequential auction model of Cahuc et al. (2006) i.e. with random search by unemployed and employed workers, firm heterogeneity, and where incumbent employers can respond to job offers made to their employees by rivals. There are segmented intermediate goods sectors for unskilled and skilled labour, and firms within each intermediate goods sector have heterogeneous quality.

I will first present an overview of the KORV production environment in its original form, before explaining how I incorporate intermediate goods sectors with search frictions into the KORV production environment. Finally, I explain how search frictions and wage bargaining operate within the intermediate goods sector.

3.2.1 KORV Production Function: No Frictions or Intermediate Goods

In the original formulation of KORV, final good in period t, Y_t is produced using capital structures, $K_{st,t}$, capital equipment, $K_{eq,t}$, and skilled and unskilled labour, S_t & U_t, as inputs, as shown in equation (3.1).

$$
\begin{aligned}
Y_t &= A_t G(K_{st,t}, K_{eq,t}, U_t, S_t) && (3.1)\\
&= A_t K_{st,t}^{\alpha}[\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} && (3.2)
\end{aligned}
$$

with $\sigma, \rho < 1$ and $\alpha, \lambda, \mu \in (0,1)$. The elasticity of substitution between unskilled labour input and capital equipment, denoted by $\varepsilon_{u,k_{eq}}$, equals $1/(1-\sigma)$. The elasticity of substitution between unskilled and skilled labour inputs, denoted $\varepsilon_{u,s}$, is also given by $1/(1-\sigma)$. Finally, the elasticity of substitution between the skilled labour input and capital equipment, denoted by $\varepsilon_{s,k_{eq}}$, is given by $1/(1-\rho)$. The parameter, α, together with λ, determine the capital share of output, and μ determines the output share of unskilled workers.

Unskilled and skilled labour input are hours worked by non-graduates and graduates in efficiency units e.g $U_t \equiv \Psi_{u,t}h_{u,t}$, $S_t \equiv \Psi_{s,t}h_{s,t}$, where $\Psi_{i,t}$ is the efficiency of labour input of a given skill level, where skill is indexed by $i \in \{u, s\}$, and $h_{i,t}$ is the total amount of hours worked. Krusell et al. (2000), in their baseline model, impose that $\Psi_{u,t}$ and $\Psi_{s,t}$ both follow stationary stochastic processes (iid) as allowing for any time trend would introduce an unexplained source of skills-biased technical change, contrary to the aim of their paper which is to examine the extent to which increased capital use can explain the rise in the graduate wage premium.

The final good is used for consumption c_t, investment in capital equipment $x_{eq,t}$ and investment in capital structures $x_{st,t}$, as shown in equation (3.3), where q_t is the relative efficiency of producing capital equipment from the final good (or equivalently $1/q_t$ is the relative price of capital equipment).

$$Y_t = c_t + x_{st,t} + \frac{x_{eq,t}}{q_t} \tag{3.3}$$

The final good producer has the following profit maximisation problem, where $(w_{u,t}, w_{s,t})$ denote the wages for unskilled and skilled workers respectively, and $(r_{st,t}, r_{eq,t})$ denote the rental rates for capital structures and equipment respectively:

$$\begin{aligned}\max_{K_{st,t},K_{eq,t},h_{u,t},h_{s,t}} \Pi = {} & A_t K_{st,t}^{\alpha}[\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} \\ & - w_{u,t}h_{u,t} - w_{s,t}h_{s,t} - r_{st,t}K_{st,t} - r_{eq,t}K_{eq,t} \end{aligned} \tag{3.4}$$

In both KORV's original model and in my adaptation the final good producer is assumed to be competitive, so the first order conditions (FOCs) for its profit maximisation problem are as shown in Equations (3.5) through (3.8).

$$\begin{aligned} w_{u,t} &= A_t(1-\alpha)K_{st,t}^{\alpha}[\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (3.5)\\ &\quad \times \mu U^{\sigma-1}\Psi_{u,t} \\ w_{s,t} &= A_t(1-\alpha)K_{st,t}^{\alpha}[\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (3.6)\\ &\quad \times (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma-\gamma}{\gamma}}(1-\lambda)S_t^{\gamma-1}\Psi_{s,t} \\ r_{eq,t} &= A_t(1-\alpha)K_{st,t}^{\alpha}[\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (3.7)\\ &\quad \times (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma-\gamma}{\gamma}} K_{eq,t}^{\gamma-1} \\ r_{st,t} &= \alpha A_t K_{st,t}^{\alpha-1}[\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda)S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} && (3.8) \end{aligned}$$

In the absence of frictions, growth in the graduate wage premium (denoted by $\pi_t = w_{s,t}/w_{u,t}$) is given in equation (3.9), where g_z denotes the growth rate in variable z.[1]

$$\begin{aligned} g_{\pi_t} \simeq (1-\sigma)(g_{h_{u,t}} - g_{h_{s,t}}) + \sigma(g_{\Psi_{s,t}} - g_{\Psi_{u,t}}) + \\ (\sigma-\gamma)\lambda\Big(\frac{K_{eq,t}}{S_t}\Big)(g_{K_{eq,t}} - g_{\Psi_{s,t}} - g_{h_{s,t}}) \end{aligned} \tag{3.9}$$

3.2.2 KORV production function: Incorporating Intermediate Goods

I now interpret U_t and S_t as the effective amount of intermediate goods produced in the unskilled and skilled intermediate goods sectors respectively. Specifically I define $U_t \equiv \Psi_{u,t} y_{u_t}$ and $S_t \equiv \Psi_{s,t} y_{s_t}$ where $y_{i,t}$ is the volume of intermediate goods produced in skill sector $i \in \{u, s\}$ and $\Psi_{i,t}$ is, analogously to the KORV environment, the efficiency level of that intermediate good.

In each segmented intermediate goods market, unemployed workers are randomly matched to intermediate firms of quality ν (I refer to this as match quality), and with a sampling distribution $F_{i,t}(\nu)$ and pdf, $f_{i,t}(\nu)$. I denote the cdf and pdf of the cross-section distribution of match quality across all workers as $L_{i,t}(\nu)$ and $l_{i,t}(\nu)$, which differs from the offer distribution as workers can search for higher quality matches on the job.

A worker in a match of quality ν produces exactly ν units of intermediate good for every hour they work, though hours worked are assumed to be fixed for each skill type of worker.[2] The effective input of intermediate goods from the unskilled and skilled intermediate sectors are therefore as shown in equation (3.10), where $h_{i,t}$ is again the raw total amount of hours worked by workers of skill type i.

$$U_t \equiv \Psi_{u,t} y_{u,t} = \Psi_{u,t} h_{u,t} \int_{\nu_{inf}}^{\nu_{max}} \nu \ell_{t,u}(\nu) d\nu \,,\; S_t \equiv \Psi_{s,t} y_{s,t} = \Psi_{s,t} h_{s,t} \int_{\nu_{inf}}^{\nu_{max}} \nu \ell_{t,s}(\nu) d\nu \tag{3.10}$$

[1] This by derived by taking logs of the graduate wage premium - given by the final goods firm's FOCs - and then differentiating with respect to time to give equation (3.9).

[2] I make this assumption to maintain consistency with the original formulation of the KORV production function where labour inputs are measured in efficiency units of total hours worked.

Final good producers are again assumed to be competitive and so pay a price, p_i, for a unit of type i intermediate good given by $p_i = \frac{\partial Y}{\partial y_i} \Psi_i$. An intermediate good firm of match quality ν in intermediate sector i receives revenue equal to $p_i \nu$.

3.2.3 Intermediate Goods Sector

All intermediate firms and workers have common discount rate, ρ, and are risk neutral. As is standard in the search literature, I assume firms can employ a maximum of one worker so intermediate firms become synonymous to matches or jobs. Job destruction rates are exogenously given, but allowed to vary by skill sectors and are denoted by $\delta_{i,t}$. Workers receive flow income in unemployment equal to $b_{i,t} * p_{i,t}$, where $b_{i,t}$ is their replacement rate and $p_{i,t}$ is the price of the intermediate good they produce as defined above.[3]

The job offer arrival rates in unemployment and employment are denoted $\lambda_{0,i,t}$,$\lambda_{1,i,t}$ and will be assumed to be exogenously given.

Intermediate Goods Sector: Wage Bargaining with Unemployed Workers

I start by stating the Bellman equation for an unemployed worker of skill type i in equation (3.11), where $V_{0,i}(p_i)$ is the expected lifetime utility of an unemployed worker, $\phi_0(p_i, \nu)$ is the wage paid to a previously unemployed worker now in a match of quality ν and $V\left(p_i, \phi_0(p_i, \nu), \nu\right)$ is the expected lifetime utility of that worker.

$$(\rho + \lambda_{0,i})\, V_{0,i}(p_i) = p_i b_i + \lambda_{0,i} \int_{\nu_{inf_i}}^{\nu_{\max}} V\left(p_i, \phi_0(p_i, x), x\right) dF_i(x) \qquad (3.11)$$

Equation (3.11) indicates that the unemployed worker receives flow income $b_i p_i$ in the current period and in the next period, which is discounted at rate ρ, they encounter a match with probability $\lambda_{0,i}$, where the match quality is drawn from the distribution $F_i(\nu)$ and lies in the interval $[\nu_{inf_i}, \nu_{max}]$.

As in Cahuc et al. (2006), I assume that there is a latent vacancy posting

[3] This implies that unemployment income is independent of the match quality that the worker had in their previous employment. Reemployment wages are therefore not path dependent, which aids tractability in the model - see Cahuc et al. (2006) for further discussion.

cost, which ensures that intermediate firms won't post a match unless it will be accepted by a worker, so the lower bound of the match quality distribution is the workers reservation match quality ν_{inf_i}.

Cahuc et al. (2006) propose a generalized form of Nash bargaining both for unemployed and employed workers. For unemployed workers, this takes the standard form whereby previously unemployed workers (I henceforth refer to these as 'entrant' workers) are paid a wage, $\phi_0(p_i, \nu)$, that equalizes the expected lifetime utility of working at a match of quality ν with the expected lifetime utility of being unemployed plus a share, β (the bargaining parameter), of match surplus $V(p_i, p_i\nu, \nu) - V_{0,i}(p_i)$, as expressed in equation (3.12).[4]

$$V\left(p_i, \phi_0(p_i, \nu), \nu\right) = V_{0,i}(p_i) + \beta\left[V(p_i, p_i\nu, \nu) - V_{0,i}(p_i)\right] \tag{3.12}$$

From equations (3.12) and (3.11), Cahuc et al. (2006) derive the closed form solution for entrant wages shown in equation (3.13), where $\bar{F} \equiv 1 - F$.

$$\phi_0(p_i, \nu) = p_i \cdot \left(\nu_{inf_i} - (1-\beta)\int_{\nu_{inf_i}}^{\nu} \frac{\rho + \delta + \lambda_1 \bar{F}_i(x)}{\rho + \delta + \lambda_1 \beta \bar{F}_i(x)} dx\right) \tag{3.13}$$

Note that Cahuc et al. (2006) also use equation (3.12) and 3.11 to derive the expression for the reservation match quality, ν_{inf_i}, shown in equation (3.14).

$$\nu_{inf_i} = b_i + \int_{\nu_{inf_i}}^{\nu_{max}} \frac{\beta(\lambda_{0,i} - \lambda_{1,i})\bar{F}_i(x)}{\rho + \delta_i + \beta_i \lambda_{1,i} \bar{F}_i(x)} dx \tag{3.14}$$

Intermediate Goods Sector: Wage Bargaining with Employed Workers

A key novelty in the sequential auction model of Cahuc et al. (2006) is that incumbent employers can respond to rival job offers made to their employees, in contrast to wage posting models such as Burdett and Mortensen (1998). In this environment, the wage paid to an employee will depend on (i) the

[4] I have assumed there is zero value to a firm from having a vacancy i.e. a free entry condition holds, which when combined with the assumption of a common discount rate for firms and workers and risk neutrality of all agents means the match suprlus can be expressed as $V(p_i, p_i\nu, \nu) - V_{0,i}(p_i)$ i.e. the match suprlus equals the worker surplus when they are paid a wage equal to their marginal product.

match quality of the highest ranked match they have encountered in their employment spell, ν^+ , which will be at their current employer, (ii) the match quality at their outside option, ν^-, which is the second highest match they have encountered in their employment spell, and (iii) the price of the intermediate good they produce, p_i, which will be the same for all workers in a given skill group i . I denote this wage $\phi(p_i, \nu^-, \nu^+)$.

In order to state the Bellman equation for the employed worker, I must first specify what happens to a worker employed at a match of quality ν encounters a match of quality ν', and is currently paid a wage w. First, if $\nu' > \nu$, then the employee moves to higher quality match and gets wage $\phi(p_i, \nu, \nu')$. Encountering a match of quality $\nu' < \nu$ will trigger a renegotiation of the employees wage contract at their current employer if ν' exceeds a threshold, denoted $\chi(p_i, w, \nu)$, where $\chi(p_i, w, \nu)$ is defined by the equality $\phi(p_i, \chi(p_i, w, \nu), \nu) = w$.

The Bellman equation for a worker employed at a match of quality ν and paid a wage, w, is therefore as shown in equation (3.15).

$$\begin{aligned}
&\left[\rho + \delta_i + \lambda_{1,i}\bar{F}_i(\chi(p_i, w, \nu))\right] V_i(p_i, w, \nu) \\
&= w + \delta_i V_{0,i}(p_i) + \lambda_{1,i} \int_{\chi(p_i,w,\nu)}^{\nu} V_i(p_i, \phi(p_i, x, \nu), x) dF_i(x) \\
&+ \lambda_{1,i} \int_{\nu}^{\nu_{\max}} V_i(p_i, \phi(p_i, \nu, x), x) dF_i(x)
\end{aligned} \tag{3.15}$$

Equation (3.15) indicates that the worker, after receiving wage w in the current period, will either lose their job with probability δ_i or, failing that, make contact with a match that triggers a renegotiation of their wage in the next period with probability $\lambda_{1,i}\bar{F}_i(\chi(p_i, w, \nu))$. If the match quality at the alternative match, x, lies in the region $(\chi(p_i, w, \nu), \nu]$ the worker stays at their current employer and receives a pay rise $\phi(p_i, x, \nu) - w$. If $x > \nu$ the worker moves to the alternative match and gets a wage $\phi(p_i, \nu, x)$. Note that this wage need not be greater than their previous wage as workers may be willing to take a pay cut if the possibility of future wage increases at the higher quality match employer is sufficiently greater than at their incumbent employer.

All this is left to do is specify the result of wage bargaining that occurs between an employee who encounters a match of sufficient quality to trigger a wage renegotiation. I denote the higher of the incumbent and rival employer's match quality as ν^+ , and the lower match quality as ν^-. The worker will supply their labour to the higher quality match, and the lower quality match becomes their outside option. Cahuc et al. (2006) adapt the Nash bargaining game of Osborne and Rubinstein (1990) to an environment with rival bidders, and show the bargained wage must satisfy equation (3.16).

$$\begin{aligned} V(p_i, \phi(p_i, \nu^-, \nu^+), \nu^+) =& V(p_i, p_i\nu^-, \nu^-) + \\ & \beta[V(p_i, p_i\nu^+, \nu^+) - V(p_i, p_i\nu^-, \nu^-)] \end{aligned} \tag{3.16}$$

Equation (3.16) indicates that a worker receives their outside option - the value of working at the firm with productivity ν^- at a wage equal to their marginal product, $p_i\nu^-$, plus a share, β, of the match surplus from working at the higher productivity firm.

Cahuc et al. (2006) prove that the wage, $\phi(p_i, \nu^-, \nu^+)$, satisfying equation (3.16) has the form shown in equation (3.17) when value functions are as defined as in equation (3.15).

$$\phi(p_i, \nu^-, \nu^+) = p_i\left(\nu^+ - (1-\beta)\int_{\nu^-}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx\right) \tag{3.17}$$

Intermediate Goods Sector: Wage and Employment Distributions

The key objects of interest in the model are the wage distributions for each skill type of worker. The analysis of the preceding sections indicates that a worker's wage depends on two stochastic variables: their current match quality, ν, and that of their outside option, χ. As in Cahuc et al. (2006), I impose that the labour market is in steady state in order to derive expressions for the cross section distributions of ν, $L_i(\nu)$, and of χ conditional on ν, $L_i(\chi|\nu)$.

Steady state in the labour market requires equations (3.18) through to (3.20) to hold (where e_i^{ue} denotes the unemployment rate of skill type i, and I have suppressed that there is a total population, N_i, of type i workers that would multiply both sides of each equation).

$$\delta_i(1-e_i^{ue}) = \lambda_{0,i}e_i^{ue} \tag{3.18}$$

$$\lambda_{0,i}F_i(\nu)e_i^{ue} = [(\lambda_{1,i}\bar{F}_i(\nu)+\delta_i)](1-e_i^{ue})L_i(\nu) \tag{3.19}$$

$$\lambda_{0,i}e_i^{ue}f(\nu)+\lambda_{1,i}L_i(\chi)f(\nu)(1-e_i^{ue}) = [(\lambda_{1,i}\bar{F}_i(\chi)+\delta_i)]\times (1-e_i^{ue})L_i(\chi|\nu)\ell_i(\nu) \tag{3.20}$$

Equation (3.18) requires that the inflows of workers into unemployment - the left hand side (LHS) of the equation - equals the outflow from unemployment. Equation (3.19) requires the inflow into the measure of workers employed at a match of quality less than ν equals the outflow: the inflow consists of unemployed workers who make contact with a match of quality less than ν (LHS of equation (3.19)) and the outflow is employed workers with match quality below μ who either lose their job, with probability δ_i or make contact with a higher quality match, with probability $\lambda_{1,i}\bar{F}_i(\nu)$. Finally equation (3.20) requires that the inflow into the measure of workers employed a match quality equal to ν and with an outside option of quality less than χ equals the outflow: the inflow again consists of unemployed workers meeting a match of quality ν (by definition their outside option, i.e. unemployment, has a match quality less than all feasible values of χ), plus workers employed at a match of quality less than χ who make contact with a match of quality ν; the outflow is employed workers with a match quality equal to ν and with an outside option of quality less than χ who either lose their job or receive an offer of a match of quality exceeding χ.

The expressions for the steady state cross sectional distribution of workers across matches and outside options derived from the steady state requirements above are shown in equations (3.21) and (3.22), where $\kappa_{1,i} \equiv \frac{\lambda_{1,i}}{\delta_i}$.

$$L_i(\nu) = \frac{F_i(\nu)}{1 + \kappa_{1,i}\bar{F}_i(\nu)} \tag{3.21}$$

$$L_i(\chi|\nu) = \left[\frac{1 + \kappa_{1,i}L_i(\chi)}{1 + \kappa_{1,i}L_i(\nu)}\right]^2 \tag{3.22}$$

The expected wage for a worker of type i is given by:

$$E(w_i) = p_i \int_{\underline{\nu}}^{\nu_{max}} \Bigg[\nu - [1 + \kappa_{1,i}\bar{F}_i(\nu)]^2 \times \tag{3.23}$$

$$\int_{\nu_{inf}}^{\nu} \frac{(1-\beta)[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\bar{F}_i(x)]}{[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\beta\bar{F}_i(x)][1 + \kappa_{1,i}\bar{F}_i(x)]^2} dx\Bigg]\ell_i(\nu)d\nu$$

The graduate wage premium will therefore depend on the same variables as in Krusell et al. (2000), which influence the price of the intermediate good produced by skill type i, p_i , but also on relative job mobility rates, outside options in unemployment, distributions of match quality, and bargaining strength. In this paper I use the same data as in Krusell et al. (2000) which limits my ability to identify the impact of all these potential channels on the graduate wage premium: I can however consider the impact of changes to relative job mobility rates, outside options in unemployment and to the distribution of match quality.

3.3 Data

3.3.1 Data: Krusell et al. (2000)

In keeping with KORV's original study, I use labour market data from the Current Population Survey (CPS) and data on capital inputs and the labour share of income from U.S national accounts. Skilled labour is defined as total hours worked by graduates and unskilled labour input is the total hours worked by non-graduates. The authors split each skill type down further into education, gender and race cells to impute hours for those with missing data.I follow their exact approach for comparability.[5]

The authors differentiate between capital equipment, such as machinery, hardware and software, and capital structures e.g. buildings. The theoretical basis for doing so is presumably that capital skill complementarity is much more likely to occur with equipment than with structures. An important element of KORV's approach is their use of a relative price deflator for capital equipment that is based on the approach of Gordon (1990), which they use to calculate the real value of the stock of capital equipment (all other variables are deflated using a GDP deflator). This relative price of equipment falls significantly over KORV's sample period, which in turn implies that the real value of capital equipment used by firms increases appreciably faster than capital structures. Polgreen and Silos (2008) show that use of alternative price series suggest significantly less capital skill complementarity.

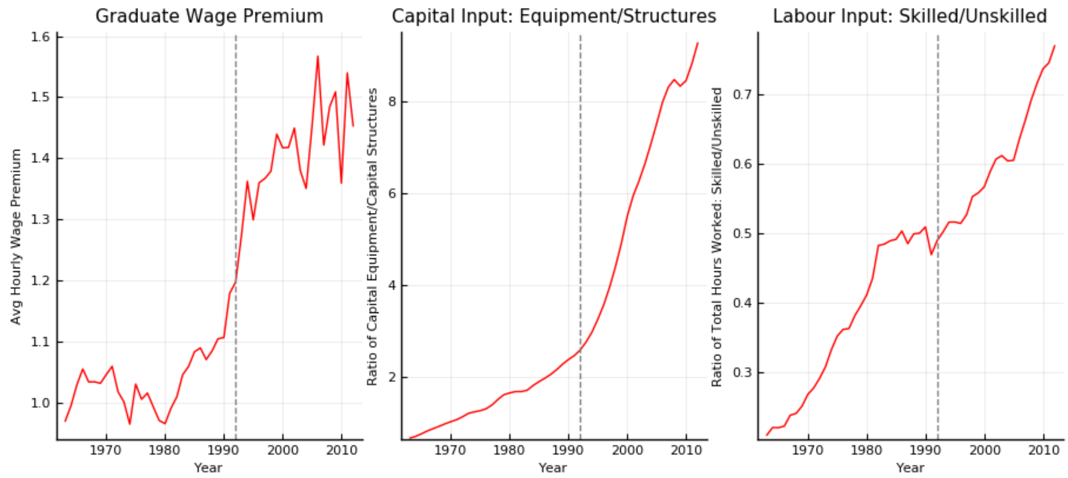
The key trends driving results in KORV are summarised in Figure 3.1. The rise in the graduate wage premium happens despite an increase in relative supply of skilled labour: given the authors assume constant relative labour efficiency in their baseline specification, the only possible driver of the rise in the graduate wage premium can be the growing use of capital equipment combined with some degree of capital skill complementarity, which is indeed what their results suggest. The authors estimate an elasticity of substitution between capital equipment and unskilled labour of 1.67 vs an equivalent elasticity of 0.67 for skilled labour.

3.3.2 Data: Labour Market Frictions

I supplement the core KORV data with data on labour market frictions. With each measure of labour market friction, the key dimension of interest will be the trend in frictions for skilled workers relative to unskilled. In the absence of distinct trends in relative frictions it is unlikely that incorporating labour market frictions into KORV will offer a different explanation for the rise in the graduate wage premium than the original KORV specification.

[5] See Appendix 1 of Krusell et al. (2000) for details of this approach

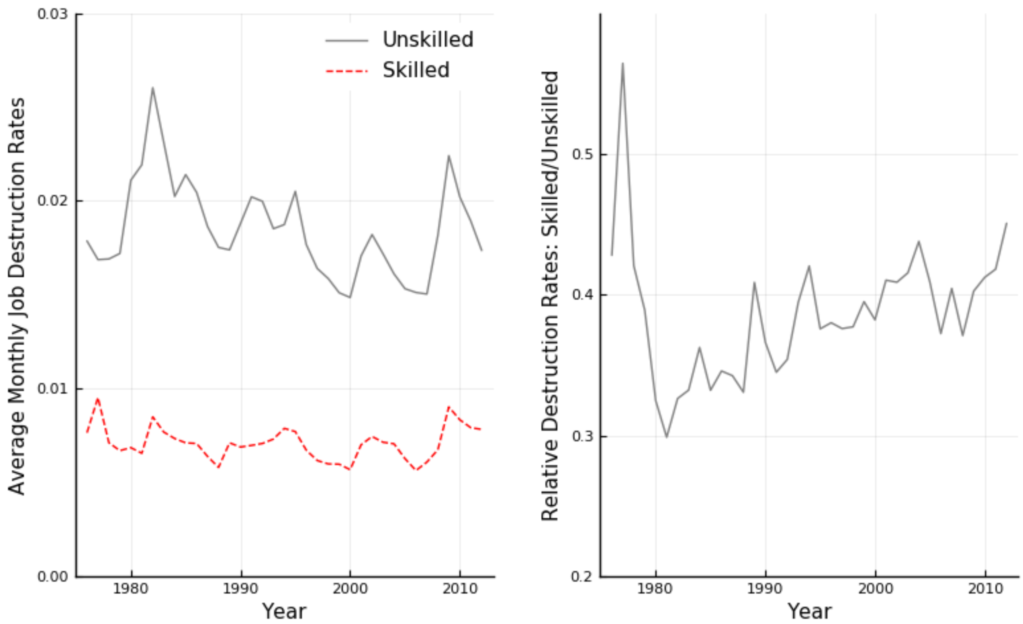
Figure 3.1: Key Data Trends in KORV



Notes: Dashed line represents end of sample period used in Krusell et al. (2000).

A key friction will be the degree of competitive intensity, $\kappa_{1,i}$ the rate of job to job contact rates relative to job destruction rates, which determines how quickly workers proceed up the job ladder. Job destruction rates can easily be taken from the panel element of the CPS, with the results given in Figure 3.2, which shows that, if anything, this measure of job market frictions shifted in favour of unskilled workers relative to skilled.

Figure 3.2: Job destruction rates



Job contact rates are not readily observable in the CPS and there has only been a question on change of employers since 1994, which hampers comparison with KORV since their original sample period finished in 1992. Since in any case job-to-job transitions would be used to infer job contact rates (not all contacts result in a transition), I use an alternative measure of job mobility which is the proportion of continuously employed individuals in a year that report having at least two employers (not concurrently) in that year, which is shown in Figure 3.3 and is referred to henceforth as the multiple employer rate. Figure 3.4 shows that movements in the multiple employer rate track movements to job-to-job mobility very closely. [6]

In relative terms, the multiple job rate rises falls for skilled workers: in Appendix E I show that the multiple employer rate is an increasing monotonic function of the job contact rate for employees, $\lambda_{1,i}$, so the downward trend in multiple employer rates for skilled workers relative to unskilled workers, taken together with the increase in relative job destruction rates, suggests a decrease in relative competitive intensity for skilled workers.

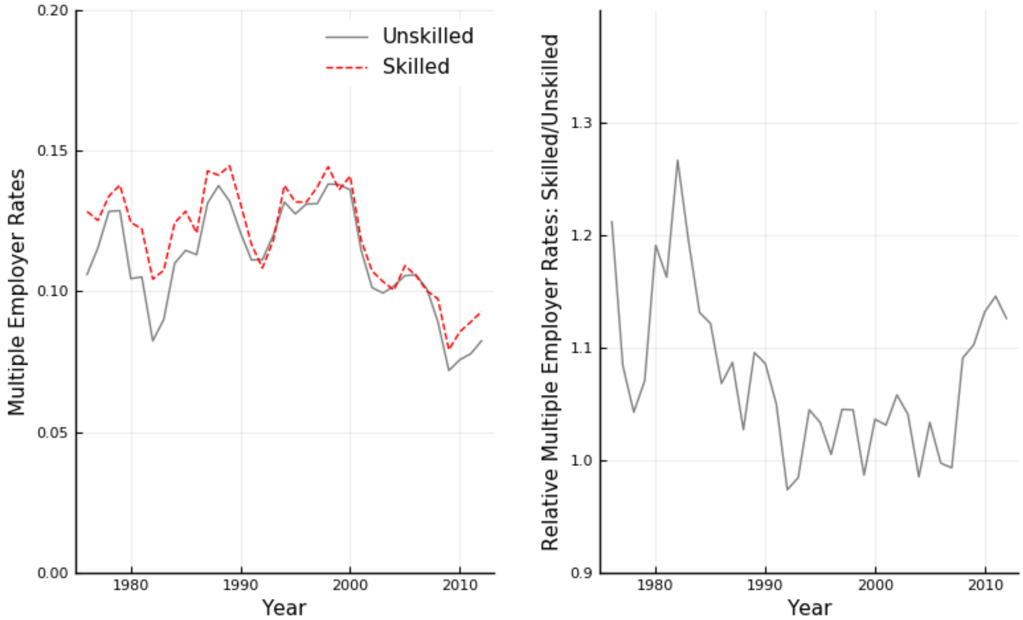
Of course the importance of movements up and down the job ladder for changes to the graduate wage premium depends on the dispersion of match quality: for example if there is little dispersion then even big changes in frequency of movements are unlikely to make much of a difference to changes in relative wage levels.

I use the standard deviation of log residual wages as my measure of dispersion - that is wages controlling for education, race, sex and year. I purposefully do not control for age since there are some endogenous returns to job tenure and total employment duration in my model, both of which are correlated with age (and are not directly measured in the CPS over the duration of my sample period).[7] This measure of dispersion increases in relative terms for skilled

[6]The CPS question on number of employers in the last year started in 1976, so I am forced to start my sample 13 years after Krusell et al. (2000) start theirs.

[7]I acknowledge this risks attributing some variance in wages driven by human capital accumulation over a worker's career to job ladder effects. However, I find that my estimates of the parameters of the KORV production function do not change if I control for age when calculating residual wage variance - see Appendix F for this and other robustness checks.

Figure 3.3: Multiple Employer Rates



workers as shown in Figure 3.5.[8]

Controlling for ex-ante differences in agents' human capital levels has the advantage of maintaining consistency with the datasets used in Krusell et al. (2000), and so facilitating comparison with their results. However, it risks attributing some unobservable differences in human capital to job ladder impacts, so a longer term research goal is to estimate this model in the context of matched employer-employee data so I can better separate out individual and firm fixed effects.

Finally the environment workers face in unemployment, both in terms of unemployment flow income and job contact rates, has an impact on the average quality of matches and wages for employed workers. This impact is less in models with on-the-job search than in models without, but it nonetheless must be accounted for.

Specifically I will need to estimate or infer the lower bound of the match quality distribution for each skill type, $\nu_{inf,i}$. In principle, this could be done by

[8]Note that using other measures of wage dispersion, such as the interquartile range, does not change my estimates of the KORV production function parameters - see Appendix F.

Figure 3.4: Multiple Employer and Job-to-Job Mobility Rates

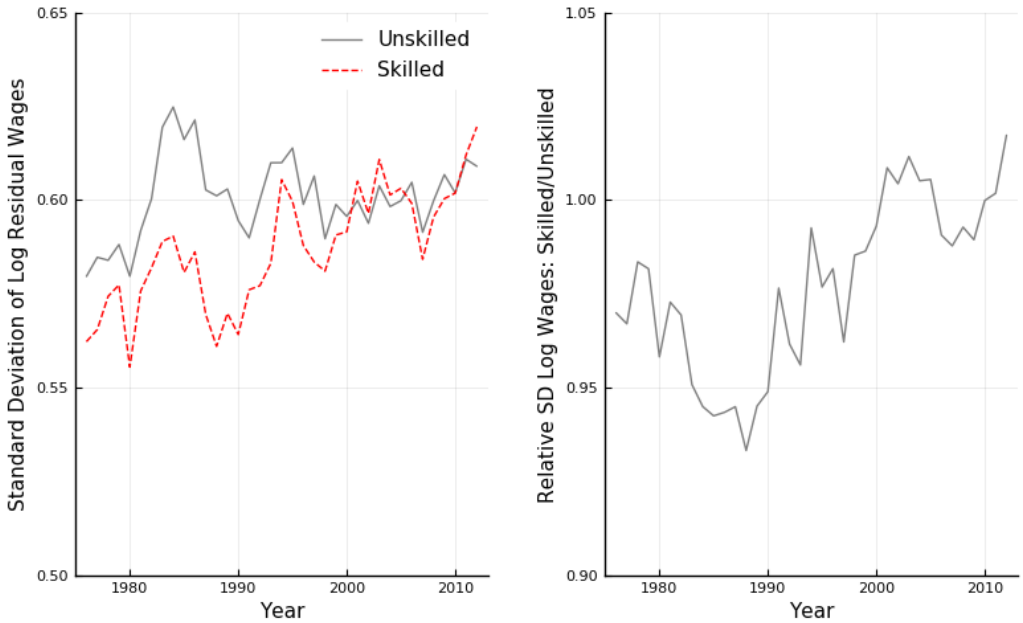
exploiting the tractable relationship between the lower bound of the sampling distribution of match quality and unemployment replacement rates and job contact rates shown previously in equation (3.14).

However, replacement rates are determined not only by legislative framework but also by the degree of insurance provided by asset accumulation, family/social relationships and many other factors beyond this making it difficult to observe in practice. I therefore directly estimate $\nu_{inf,i}$ by targeting the ratio of average wages of workers in the first two percentiles of the wage distribution to the median wage: the next section on my estimation approach and Appendix E discuss identification in more detail. The empirical moment is shown in Figure 3.6, where the trends shown suggest changes to outside options have compressed the tail of the unskilled wage distribution more than for skilled workers.[9]

Note that when I simply replicate KORV's estimation approach I use exactly the same treatment of the data as they do, however when it comes to

[9]The estimated parameters of the KORV production function are not sensitive to using different measures of the lower bound of the wage distribution i.e. the actual minimum, or different wage percentiles - see Appendix F.

Figure 3.5: Wage Dispersion



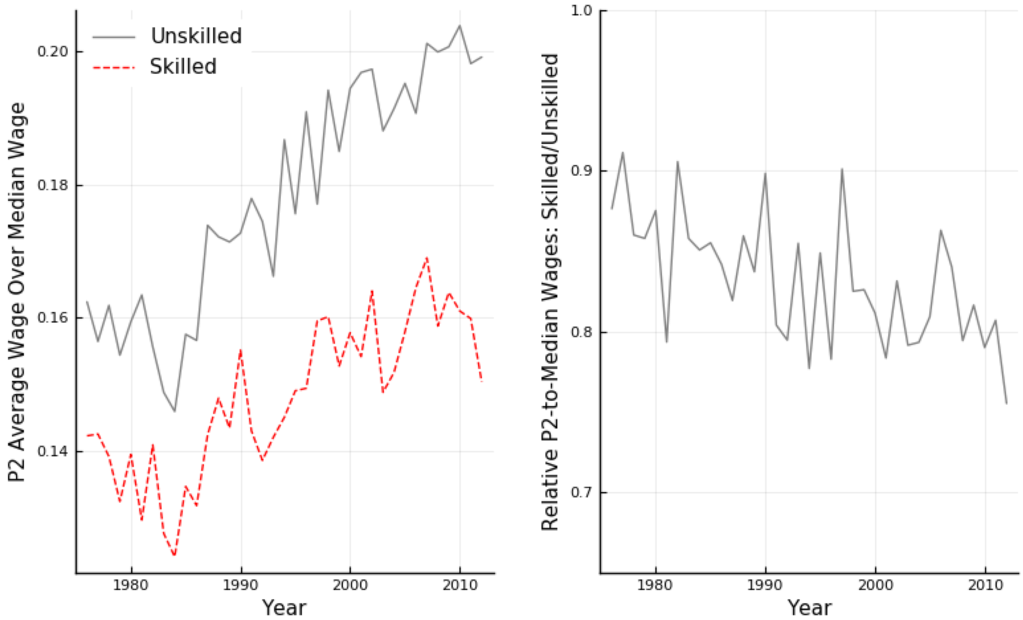
incorporating frictions I will trim the bottom and top percentile of the wage distribution out of the sample to minimise measurement error.

Perhaps surprisingly, the balance of the data on labour market frictions considered here has, if anything, moved in favour of unskilled workers relative to skilled workers, when skill is equated with having a degree. Skilled workers have seen their relative job destruction rates increase, and their relative job to job mobility rates (using the proxy measure discussed above) decease.

The one potential mitigating factor that may has moved in the favour of skilled workers, at least in the context of a job ladder model, is an increase in their residual wage variance, both in absolute terms and relative to unskilled workers. I will see that this means my estimates of the variance parameter of the match quality sampling distribution increases in relative terms for skilled workers, which leads to a relative increase in the cross sectional average of their match quality.[10]

[10]This is a function of job to job contact rates that significantly exceed job destruction rates in all years, which means that $L_i(\nu)$ will generally first order stochastically dominate $F_i(\nu)$. This in turn means a mean preserving increase in the spread of the $F_i(\nu)$, which is implied by my estimation strategy of matching the increasing dispersion of skilled workers wages while keeping the mean of the sampling distribution constant, will lead to a increase

Figure 3.6: Lower Bound of Wages Relative To Median



3.4 Estimation Approach

As with my exposition of the model, I will first present the original estimation approach used by Krusell et al. (2000), i.e. under perfect competition and with no intermediate goods sectors. I then set-out a two stage strategy for estimating the KORV parameters in the context of my model. The first stage is to estimate the parameters of the sequential auction model in the intermediate goods markets. The second stage is to incorporate results from the previous step to estimate the parameters of the KORV production function in the final good sector.

3.4.1 KORV Estimation: Without Frictions

Krusell et al. (2000) estimate their model by simulated pseudo maximum likelihood (SPML), targeting the model's predictions for the labour share of output and the wage bill ratio of skilled workers relative to unskilled workers, denoted lsh_t and wbr_t respectively, to their empirical counterparts.[11] Both of

in the cross section mean of match quality.

[11]SPML is generally attributed to Laroque and Salanie (1993) and is used when a closed form solution for the exact likelihood or quasi likelihood are both unavailable. Just as MLE can be viewed as a specific form of GMM (where the expectation of the score is the

these model moments come from the first order conditions of the final good firm's profit maximisation condition. In addition, Krusell et al. (2000) impose a no arbitrage condition between capital structures and equipment, i.e. their empirical strategy aims to minimise the difference between the model's predictions for the rate-of-return (RoR) on capital structures and the predicted RoR for capital equipment, alongside the other empirical targets mentioned above.[12] This estimation strategy is summarised in equations (3.24), (3.25) and (3.26) respectively, where X_t is the set of factor inputs $(K_{st,t}, K_{eq,t}, U_t, S_t)$, $(\kappa_{eq}, \kappa_{st})$ are the depreciation rates for capital equipment and structures respectively, and ϕ is the vector of all parameters to be estimated.

$$\frac{w_{u,t}h_{u,t} + w_{s,t}h_{s,t}}{Y_t} = lsh_t(X_t, \psi_t; \phi) \tag{3.24}$$

$$\frac{w_{s,t}h_{s,t}}{w_{u,t}h_{u,t}} = wbr_t(X_t, \psi_t; \phi) \tag{3.25}$$

$$0 = (1 - \kappa_{st}) + A_{t+1}G_{K_{s,t}}((X_t, \psi_t; \phi) - E_t(\frac{q_t}{q_{t+1}})(1 - \kappa_{eq}) -$$
$$q_t A_{t+1} G_{k_{eq,t}}((X_t, \psi_t; \phi) \tag{3.26}$$

Equations (3.24), (3.25) and (3.26) can be represented in vector form as $Z_t = f(X_t, \psi_t, \epsilon_t; \phi)$, where Z_t is a vector of the empirical moments on the left hand side of equations (3.24), (3.25) and (3.26) and $f(X_t, \psi_t, \epsilon_t; \phi)$ is a vector of the model moments on the right hand side of these equations.

Note that there are two stochastic elements in this system of estimation equations. First ψ_t is a (2×1) vector of the log of the efficiency levels of unskilled and skilled labour respectively, and follows a stationary process in KORV's benchmark estimation as set out in equation (3.27).

$$\psi_t = ln(\Psi_t), \psi_t = \psi_0 + \omega_t \text{ and } \Psi_t \equiv (\Psi_{u,t}, \Psi_{s,t}) \tag{3.27}$$

relevant moment), so SPML can be viewed as specific form of SMM where I am taking the expectation of a set of moments across both simulations and across time.

[12]This is done as neither RoR is directly observable in the data.

ω_t is a vector shock process to the log of labour efficiency that is assumed to be multivariate normal and *iid* with covariance matrix Ω i.e. $\omega \overset{i.i.d}{\sim} N(0, \Omega)$, and ψ_0 is a vector of the log of intial values of unskilled and skilled labour efficiency $(\psi_{0,u}, \psi_{0,s})$. In the benchmark estimation, the authors impose that there is no covariance between the two labour efficiency shocks and that they have a common variance so Ω can be rewritten as $\Omega = \eta_\omega^2 I$.[13]

The other stochastic process in this estimation procedure is in the no arbitrage condition, equation (3.26), where the third term on the right hand side of this equation $E_t(\frac{q_t}{q_{t+1}})(1 - \kappa_{eq})$ is the undepreciated capital equipment multiplied by the expected rate of change in the relative price of equipment. Krusell et al. (2000) make the simplifying assumption that this term can be replaced with $\frac{q_t}{q_{t+1}}(1 - \kappa_{eq}) + \epsilon_t$, where $\epsilon_t \sim N(0, \eta_{\epsilon_t})$.

In principle, the vector of parameters to be estimated, ϕ, contains 11 elements :$\{\kappa_{st}, \kappa_{eq}, \alpha, \mu, \lambda, \sigma, \gamma, \eta_\varepsilon, \eta_\omega, \psi_{0,u}, \psi_{0,s}\}$. However the authors calibrate $(\kappa_{st}, \kappa_{eq})$ using estimates from the literature, estimate η_ε separately , and normalize $\psi_{0,s} = 0$.[14] [15] This leaves $\phi = \{\alpha, \mu, \lambda, \sigma, \gamma, \eta_\omega, \psi_{0,u}\}$ to be estimated i.e seven parameters: given the are targeting three moments for each year of their 30 year dataset, the model is over-identified.

Finally, the authors construct an instrument for hours worked, $\hat{h}_{u,t}, \hat{h}_{s,t}$ to allow for potential endogeneity between relative hours worked and relative wages. While such endogoneity would be irrelevant if the sole goal was to match the model to the data, I presume the authors employ this strategy so that they can more credibly give the parameters economic interpretations, i.e. as elasticities of substitution, and hence use the model for counter-factual analysis. The exogenous factor inputs used in model estimation are therefore $\hat{X}_t = (K_{st,t}, K_{eq,t}, \hat{h}_{u,t}, \hat{h}_{s,t})$. Estimation then proceeds in three steps:

[13]In fact, as a robustness check, Krusell et al. (2000) do allow for a non-zero covariance between the two efficiency shocks and differing variances, but the estimated covariance is very small and there is little difference between the estimated variances so they opt for a benchmark estimation with zero covariance and a common variance.

[14]The authors set $\kappa_{eq} = 0.125$ and $\kappa_{st} = 0.05$ following Greenwood et al. (1997).

[15]The authors estimate η_ε via an ARMA regression of q_t.

1. Draw S values of the vector of shocks to labour efficiency, ω_t^j, and of the forecast error in expected price gains of capital equipment, ϵ_t^j,(where j indexes the realization of the shock) to get S realizations of $f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi)$ from the model for each time period t,

2. Use these S realizations to obtain the following moments:

$$m_s(\hat{X}_t, \phi) = \frac{1}{S}\sum_{i=1}^{S} f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi)$$
$$V_s(\hat{X}_t, \phi) = \frac{1}{S-1}\sum_{i=1}^{S}(f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi) - m_s(\hat{X}_t, \phi))(f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi) - m_s(\hat{X}_t, \phi))'$$

3. Minimise the following objective function:

$$\begin{aligned} l_s(\hat{X}_t, \phi) = &\frac{1}{2T}\sum_{t=1}^{T}\Big\{(Z_t - m_s(\hat{X}_t, \phi))'V_s(\hat{X}_t, \phi) \\ &\times (Z_t - m_s(\hat{X}_t, \phi)) + ln(det(V_s(\hat{X}_t, \phi)))\Big\} \end{aligned} \tag{3.28}$$

In a companion paper to Krusell et al. (2000), Ohanian et al. (1997) look at how successfully the estimation approach above identifies the true parameters of the model in Monte Carlo simulations, and find very small median and mean biases in estimators even when using relatively few simulations in estimation i.e. for $S = 10$. They find that for $S = 50$ the mean bias is “essentially zero”.

3.4.2 Incorporating Frictions into KORV Estimation

I proceed in two steps to incorporate the sequential auction model of Cahuc et al. (2006) into the KORV production set-up. First I separately estimate the parameters of the sequential auction model, which include job contact rates for employed workers of each skill type $\lambda_{1,i,t}$ and the parameters of their match distribution. Appendix E examines identification of these parameters in greater detail, proving exact identification of the job contact rates using the empirical strategy outlined here and showing evidence from Monte Carlo simulations that my strategy for estimating the parameters of the match quality distribution also successfully identifies the true parameters of the model.

In the second part of my estimation approach, I estimate the parameters of the KORV production function in a way the incorporates the changes to labour market frictions implied by my estimation of the sequential auction model of the intermediate goods sectors. This step is, in econometric terms, a minor modification of the original approach of Krusell et al. (2000), as presented above, that uses two key outputs from the sequential auction model: the average match quality and wage of each skill type which are both identified up to a common scaling factor, which is the price of the intermediate good produced in a given skill sector. The rest of this section describes each of these steps in greater detail.

Sequential Auction Estimation: Job Contact Rates

The monthly job contact rate for employees, $\lambda_{1,i,t}$, is chosen so that the model matches the empirical proportion of individuals continuously employed in a year who have more than one employer (denoted $\tau_{i,t}$).This moment is given in the model by equation (3.29)

$$\tau_{i,t} = 1 - \int_{\nu_{inf_{i,t}}}^{\nu_{max}} (1 - \lambda_{1,i,t}\bar{F}_{i,t}(\nu))^{12} dL_{i,t}(\nu) \tag{3.29}$$

In Appendix E, I show that this expression is independent of the match quality distribution - this can be seen by change of variable in the integration - meaning I can estimate job contact rates separately of distributional parameters. The expression is also an increasing monotonic function of $\lambda_{1.i}$ which implies this parameter is indeed identified when I estimate it by simulated method of moments, as set out in equation (3.30) (where $\hat{x}$ denotes the empirical counterpart of model moment x).

$$\lambda^*_{1,i,t} = \underset{\lambda_{1,i,t}}{\operatorname{argmin}} \big(\tau_t(\lambda_{1,i,t}) - \hat{\tau}_t\big)^2 \tag{3.30}$$

Sequential Auction Estimation: Distribution of Match Heterogeneity

I assume that sampling distribution of match heterogeneity can be characterised by a lower truncated log normal distribution, and therefore can be fully described by three parameters: the mean and variance parameters, $\zeta^{\nu}_{i,t}$, $\eta^{\nu}_{i,t}$, and lower truncation point, $\nu_{inf_{i,t}}$. Note that by estimating the lower bounds directly I bypass the need to estimate job contact rates for the unemployed or replacement rates. This follows because my principal interest interest is to estimate the distribution of wages and match quality for workers in the intermediate goods market; unemployment conditions influence these variables through the lower bound of the match quality distribution only.

Given I only have data on employees, and not employers, a natural option to estimate $\zeta^{\nu}_{i,t}$ and $\eta^{\nu}_{i,t}$ is to use moments of the wage distribution for workers of each skill type $i \in u, s$. Note, however, that all wages of a given skill type are scaled by the price of the intermediate good, p_i (see equation (3.17)), which depends on the parameters of the KORV production function that I have yet to estimate. I therefore require the moment of the wage distribution that I will target to be scale invariant, and so choose the variance of log wages.

As both $\zeta^{\nu}_{i,t}$ and $\eta^{\nu}_{i,t}$ have a positive monotonic impact on match quality dispersion in the model, they will not be separately identified using the variance of log wages. I therefore set the value of $\zeta^{\nu}_{i,t}$ to target the mean of the sampling distribution, $\mathbb{E}^{F_{i,t}}(\nu)$, to an arbitrary fixed value ($= 1$). Note that this also avoids introducing a "black-box" source of skills biased technological change via an increase in the relative means of the sampling distribution of match quality $\mathbb{E}^{F_{s,t}}(\nu)/\mathbb{E}^{F_{u,t}}(\nu)$ (Krusell et al. (2000) impose that the relative labour efficiency of skilled to unskilled workers is constant for the same reason) but doesn't rule out an endogenous increase in the mean of the cross section distribution of match quality $\mathbb{E}^{L_{i,t}}(\nu)$. The variance parameter of the sampling distribution of match quality, $\eta^{\nu}_{i,t}$, is therefore left free to match the dispersion of log wages within a skill type i in the model to it's empirical counterpart.

Finally I must estimate the lower bound of the distribution of match quality,

$\nu_{inf_{i,t}}$. Provided the bargaining parameter is sufficiently high, a worker at a match of quality $\nu = \nu_{inf_{i,t}}$ will earn the lowest wage in the model's wage distribution, denoted $\underline{w}_{i,t}$, where $\underline{w}_{i,t} = \nu_{inf_{i,t}} \times p_{i,t}$.[16] Since all wages are scaled by the price of the intermediate good, $p_{i,t}$, which will not be estimated at this stage, rather than target the lower bound of the wage distribution I target the ratio of the lower bound to the median wage: $\underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}})$. When it comes to the empirical counterpart of this moment, I choose to use the average wages of workers in the bottom two percentiles of the wage distribution (again relative to the median) rather than the lower bound of the empirical wage distribution as this is likely to subject to significant measurement error.

In summary, I estimate the parameters of the sampling distribution, $\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$, by solving the minimisation problem shown in equation (3.31), where $\hat{x}$ denotes the empirical counterpart of model moment x, and W is the weighting matrix.[17]

$$
\begin{aligned}
&(\ \zeta^{\nu *}_{i,t}, \eta^{\nu *}_{i,t}, \nu^{*}_{inf_{i,t}}) = \underset{\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}}{\operatorname{argmin}} \ (m_t - \hat{m}_t)^T W (m_t - \hat{m}_t) \qquad (3.31) \\
&m_t \equiv \big(var_{\log(w_{i,t})}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \\
&\mathbb{E}^{F_{i,t}}(\nu)(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}) \big)
\end{aligned}
$$

I calculate the moments of the wage distribution in the model from a given guess for parameters by generating a sample of workers using the cross section distributions in equations (3.19) and (3.20) to give the match quality of the workers' employers and outside options, and then using equation (3.17) to then

[16] In the model, the minimum wage in the population of workers will be paid to workers with match quality equal to the lower bound of the match distribution in the model when $\beta > \frac{\lambda_1}{\rho + \delta + two\lambda_1}$. This condition is derived from observing first that the wage expression in equation (3.17) is always decreasing in ν^-, so the lowest wage observable wage will certainly belong to those who have come from unemployment i.e. who have $\nu^- = \nu_{inf}$. Such workers will have a wage precisely equal to ν_{inf} when they are matched with the lowest match quality firms i.e. $\nu^+ = \nu^- = \nu_{inf}$. Finally a sufficient condition for this to be the lowest wage in the population provided the derivative of the wage expression is positive for this worker and the second derivative is always positive. The latter condition always holds, and former holds when β is greater than the threshold shown above.

[17] The weighting matrix W, is chosen so I effectively minimise the percentage deviation of model moments from their empirical moments, which avoids the scale of absolute moment deviations biasing estimates i.e. $W = I.\frac{1}{\hat{m}}$.

generate the wages of these workers.

Sequential Auction Estimation: Other Parameters to be Calibrated

In the absence of matched employee and employer data, I set the bargaining parameter to $\beta = 0.95$. I find lower levels of the bargaining parameters mean the model struggles to hit the level of the labour share and rise in the graduate wage premium seen in the data. This occurs because the sequential auction part of the model sets an upper bound on the labour share in the overall model, since incorporating a final goods sector with the KORV production function will always decrease the labour share, relative to its level in the intermediate goods sector, due to the presence of capital inputs. The upper bound on the labour share implied by the sequential auction results may be close to or even below the empirical labour share that I am targeting if the bargaining parameter is set too low. This issue is explored quantitatively in Appendix D. Although this calibrated bargaining parameter value appears high compared to some results in the micro literature, for example Cahuc et al. (2006), many of these estimates come from structural models that do not feature capital and so are not directly comparable to ours. Finally, I arbitrarily set the monthly discount rate to 0.004.

Adding Sequential Auction Results to KORV Estimation

I adopt essentially the same empirical approach as Krusell et al. (2000) i.e matching the models predictions for the evolution of the graduate wage premium and labour share to their empirical counterparts and imposing a zero rate-of-return (RoR) difference between capital structure and capital equipment in the model. However, I make two key modifications to incorporate results from the sequential auction estimation.

The first modification comes about because the unskilled and skilled labour inputs are not simply hours worked by the two types but rather the amount of intermediate goods from each skill type sector. The inputs U_t and S_t therefore become as defined in equation (3.10) where I multiply the labour inputs that KORV use (total hours in efficiency units) by the average match quality in each skill sector, which are derived from estimation of the sequential auction part of my model. I denote estimated average match equality as $\mathbb{E}^{\hat{L}_{i,t}}(\nu) \equiv \int_{\nu_{\hat{inf}_{i,t}}}^{\nu_{max}} \nu \hat{\ell_{i,t}}(\nu)$ (where hats denote estimated variables/parameters).

Second average wages for a given skill type i are no longer simply the marginal product of that skill type in production of the final good, but determined as specified in equation (3.23). I decompose this expression into two parts, as shown below.

$$\begin{aligned}
\mathbb{E}^{L_{i,t}}(w_{i,t}) \quad &= p_{i,t} \times \mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1) \\
\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1) \equiv &\int_{\underline{\nu}}^{\nu_{max}} \Bigg[\nu - [1 + \kappa_{1,i}\bar{F}(\nu)]^2 \times \\
&\int_{\nu_{inf}}^{\nu} \frac{(1-\beta)[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\bar{F}(x)]}{[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\beta\bar{F}(x)][1 + \kappa_{1,i}\bar{F}(x)]^2} dx \Bigg] \ell_i(\nu) d\nu
\end{aligned} \tag{3.32}$$

Thus average wages are calculated by multiplying the price of the intermediate good $p_{i,t}$ (which equals its marginal product in the production of the final good) by the average wages in the intermediate good sector when the price of the intermediate good is normalised to one $E(w_{i,t}, p_{i,t} = 1)$. I estimate $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$ and $\mathbb{E}^{L_{i,t}}(\nu)$ by using estimation results from the sequential auction part of my model. Specifically, I simulate a sample of workers, using equations (3.19) and (3.20) to generate the match quality of these workers' employers and outside options and equation (3.17) to generate their wages, and then taking the average wage and match quality of the simulated sample of workers.

A short hand way of stating the points made above is that, to adapt KORV's original methodology to including an intermediate goods sector with a sequential auction labour market, I scale the labour input of skill type i used in KORV by a 'productivity scale' which is my estimate of $\mathbb{E}^{L_{i,t}}(\nu)$ and I calculate average wages by multiplying the marginal product of a given intermediate good in the KORV production function by a 'wage scale' that is my estimate of $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$. Otherwise, estimation of the parameters of the KORV

Table 3.1: Parameter Estimates: KORV vs Replication

Parameter	KORV findings	Replication Estimates
α	0.117	0.121
γ	-0.495	-0.459
σ	0.401	0.39

production function proceeds exactly as described in section 3.4.1.

3.5 Results

This section starts by verifying that I can replicate the results provided in Krusell et al. (2000) when I use their estimation strategy and data. I then show results from estimation of the sequential auction model of the intermediate goods sector, and finally I show the impact of incorporating search frictions into the KORV production process. When considering this impact my focus will be on how, if at all, estimates of capital skill complementarity change and how that changes explanations for the rises in the graduate wage premium.

3.5.1 Replication of KORV methodology

I am able to replicate results from KORV both in terms of fit to the author provided data - see Figure 3.7 for my fit to the data and Figure 3.8 for the equivalent Figure in Krusell et al. (2000) - and parameter estimates - see Table 3.1.[18] When I include more recent data in my replication of the KORV methodology, rather than using only their original sample period of 1963-1992, I find the model again fits the data well - see Figure 3.9.[19] Table 3.2 shows inclusion of more recent data decreases estimates of capital skill complementarity (as captured by the difference in the elasticity of substitution between unskilled labour and capital equipment, denoted $\varepsilon_{U,K_{eq}}$, and the elasticity of substitution between skilled labour and capital equipment, denoted $\varepsilon_{S,K_{eq}}$).[20] An explanation for the reduction in estimated capital skill complementarity

[18]Note that KORV provide estimates of α, σ, γ only so I focus on these parameters in Table 3.1.

[19]I have to switch from author provided data to publicly available data to extend the time period, which is why the parameter estimates for the original sample period shown in Table 3.2 differ from those in Table 3.1.

[20]$\varepsilon_{U,K_{eq}} \equiv \frac{1}{1-\sigma}$, $\varepsilon_{S,K_{eq}} \equiv \frac{1}{1-\gamma}$.

Figure 3.7: Replication of KORV

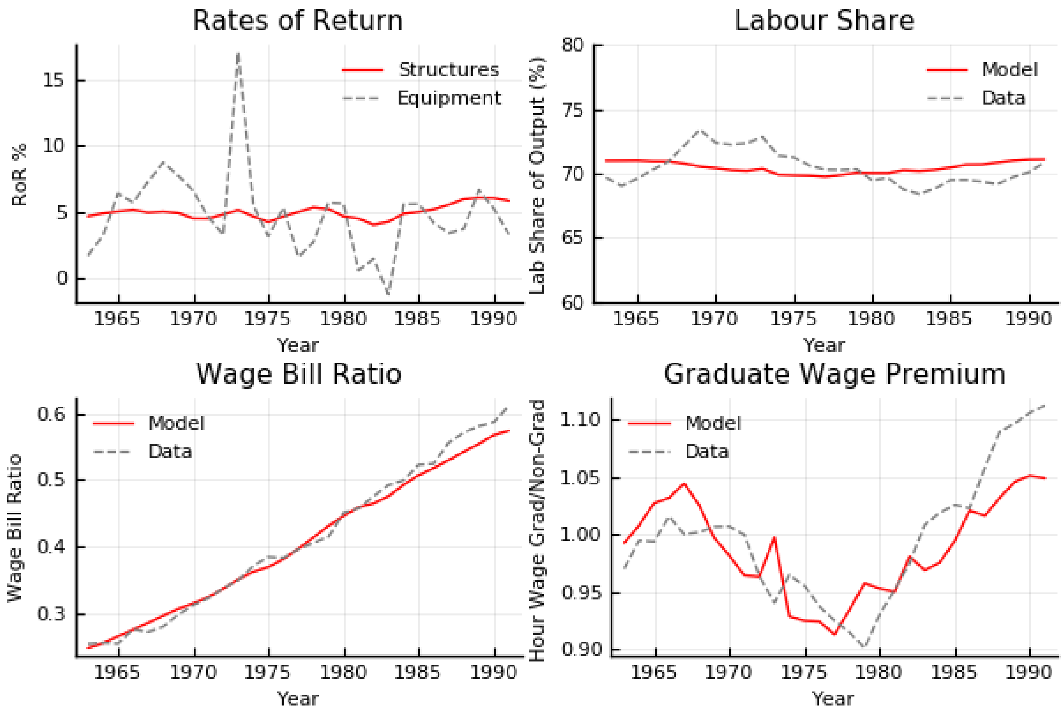


Table 3.2: Parameter Values with Extended Sample Period

Parameter	Original Sample Period	Extended Sample Period
λ	0.925	0.92
μ	0.767	0.866
α	0.099	0.094
γ	-0.477	-0.313
σ	0.454	0.439
$\varepsilon_{S,K_{eq}}$	0.677	0.762
$\varepsilon_{U,K_{eq}}$	1.833	1.781
CSC Strength: $\boldsymbol{\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}}$	**1.156**	**1.02**

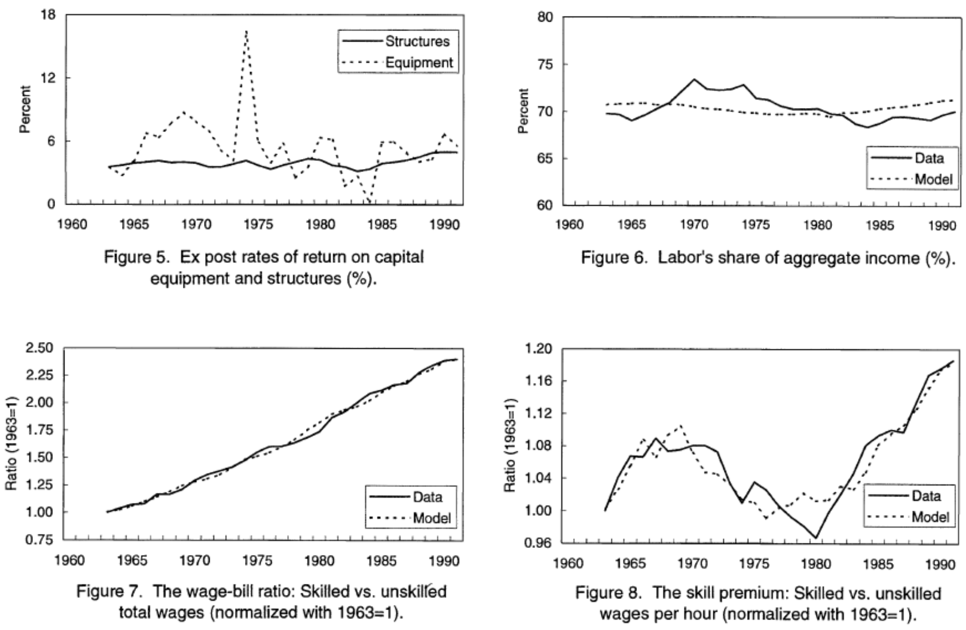
could be that the growth in the graduate wage premium remains steady after 1992, despite an sharp acceleration in capital equipment growth (see Figure 3.1); to reconcile these two patterns the model requires a lower estimate of capital skill complementarity than in the original sample period.

3.5.2 Sequential Auction Results

Sequential Auction Results: Job Contact Rates

The first row of Figure 3.10 shows my estimates of job contact rates for unskilled and skilled workers, in absolute and relative terms (I plot a six year rolling average of estimated relative contact rates to emphasise the trend). The

Figure 3.8: KORV's original fit



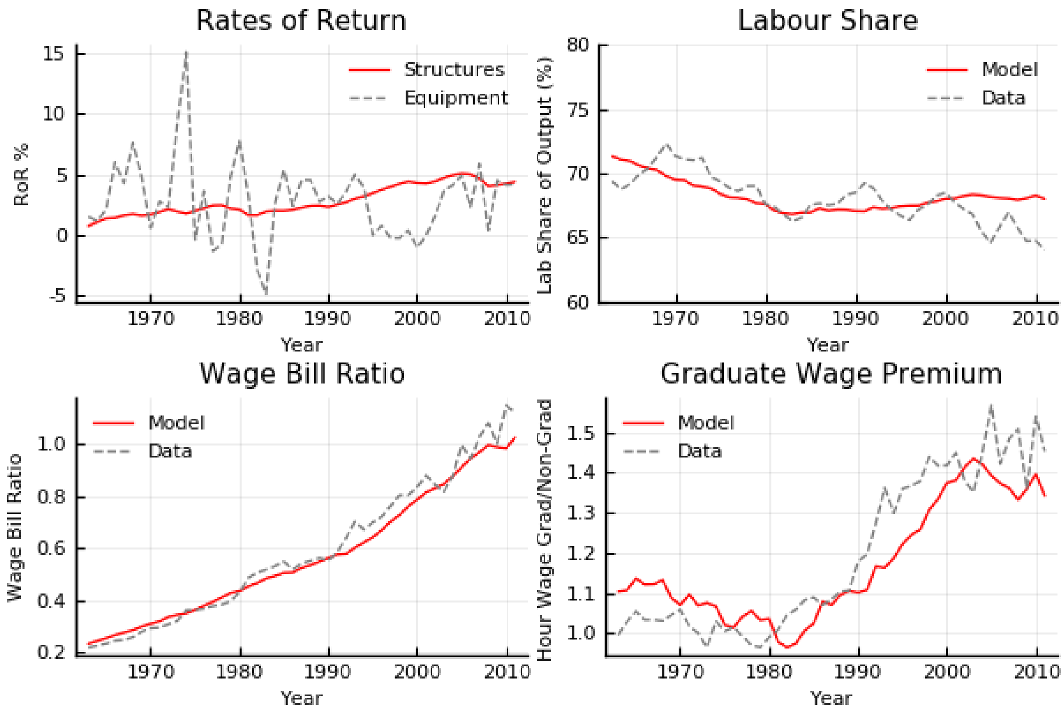
Notes: I am unable to directly provide model predictions using KORV's parameter estimates, as the author's only provide a subset of the relevant estimates, so I directly reproduce the figure from Krusell et al. (2000) showing the fit of their model to the data.

second row of the same figure shows the empirical targets these estimates are based on - the proportion of individuals with more than one (non concurrent) employer in a year ("multiple employer rate") - and the corresponding model moments.

I am able to exactly match the model moments to their empirical counterparts. Estimated job contact rates do not exactly track the data on multiple employer rates because job contact rates are not the sole determinant of the multiple employer rate: job destruction also plays a role, as shown in equation (3.29). The intuition here is that workers who exit and enter the labour market more frequently will spend more time at the bottom of the job ladder and hence move employers more often.

However, overall the trend is for estimated job contact rates for skilled workers to decrease over time relative to those of unskilled workers, which mirrors the trend in the multiple employer rate.

Figure 3.9: Replication of KORV: Extended Sample Period

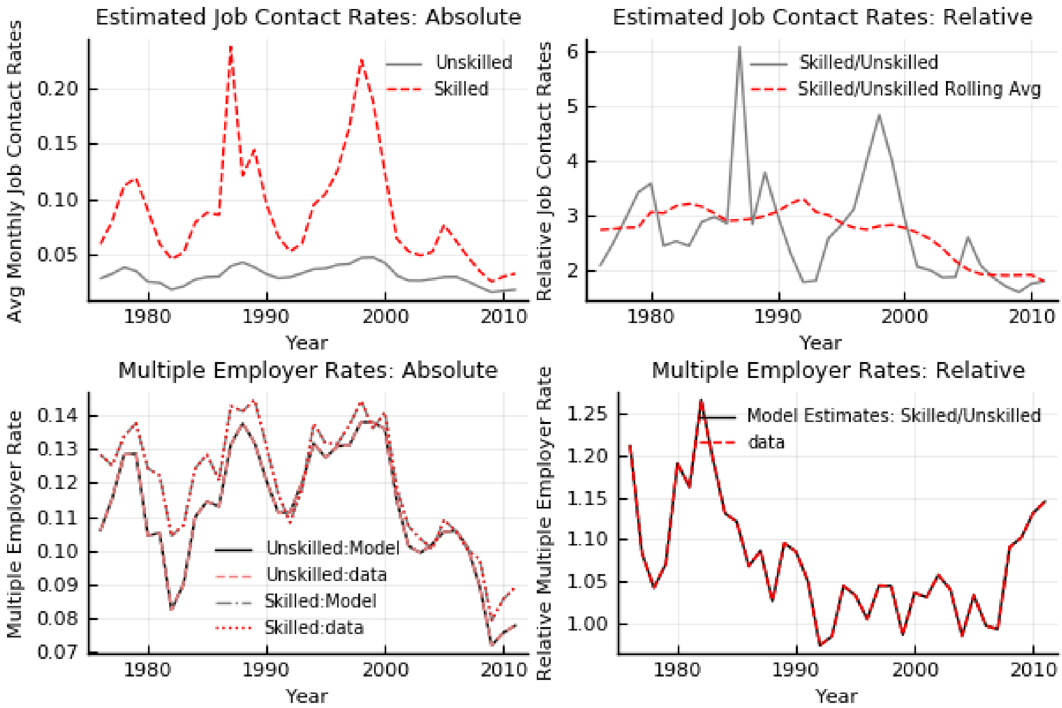


Sequential Auction Results: Distribution of Match Quality

For each of my two skill types, I estimate three parameters of the match quality distribution, which is assumed to take a truncated log normal form: the mean, variance and lower bound parameters, $\zeta^{\nu}_{i,t}$, $\eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ respectively. I am able to match the model to the targeted empirical moments precisely in the case of both $\eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ where the relevant targets are log wage variance and the ratio of average wages of workers in the bottom two percentiles of the wage distribution to median wages respectively - the top row of figures 3.11 and 3.12 show parameter estimates for $\eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ respectively, and the bottom rows show the close fit of model moments to the data. Estimates of $\zeta^{\nu}_{i,t}$ are in a sense less relevant since they are simply set at the level necessary to keep the mean of the sampling distribution of match quality constant at an arbitrary target ($\mathbb{E}^{F_{i,t}}(\nu) = 1$).

The estimated variance parameter of the sampling distribution of match quality, $\hat{\eta}^{\nu}_{i,t}$, for skilled workers increases over time relative to the equivalent parameter for unskilled workers, mirroring changes in the empirical target

Figure 3.10: Job Mobility



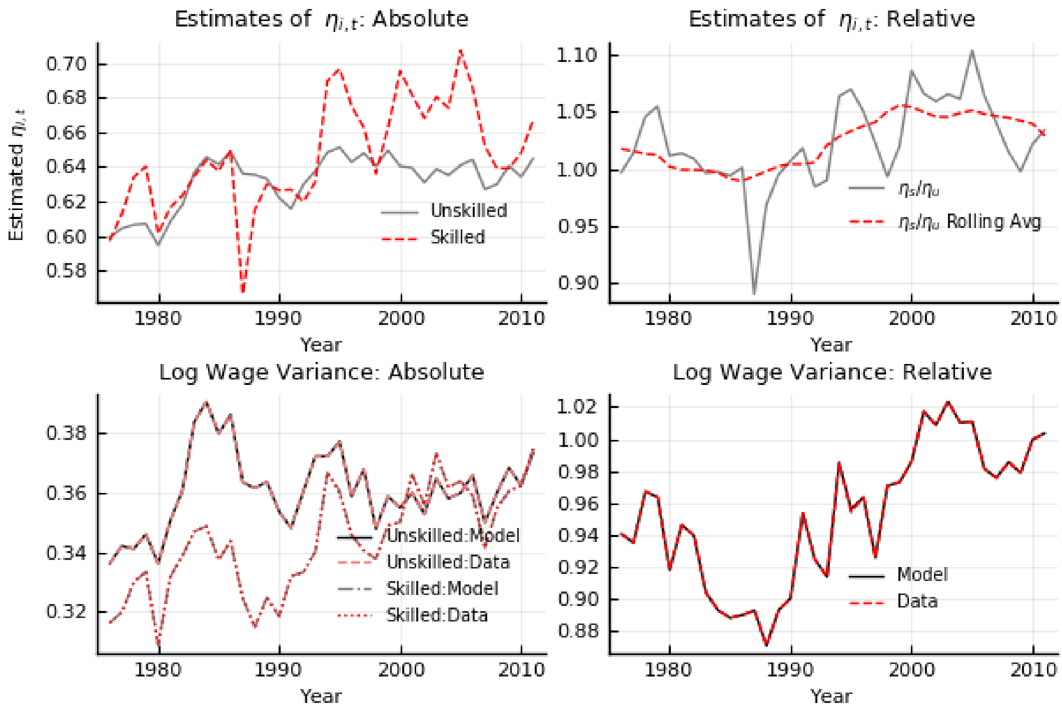
(residual log wage variance). Estimates of the lower bound of the match quality distribution, $\hat{\nu}_{inf_{i,t}}$ decrease in relative terms for skilled workers, again mirroring the trend in the empirical target.

3.5.3 Impact of Sequential Auction Estimates on KORV results

As argued in section 3.4.2, the results of my estimation of the sequential auction structure of the intermeidate goods market can be fully characterised by two series for the purposes of estimating the parameters of the KORV production function. The first series is the 'productivity scale', which is my estimate of the average match quality by skill $\mathbb{E}^{L_{i,t}}(\nu)$ that I use to scale labour inputs. The second series is the'wage scale', $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$, which relates to average wages of skill type i via the identity $\mathbb{E}^{L_{i,t}}(w_{i,t}) = p_{i,t}\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$. These series are plotted in absolute and relative terms in Figure 3.13, with a rolling 6 year average of the relative series added to emphasise the relevant trends.

Reflecting trends in job contact rates, the relative productivity and wage scaling factors of the high skill workers are increasing, albeit very mildly, until

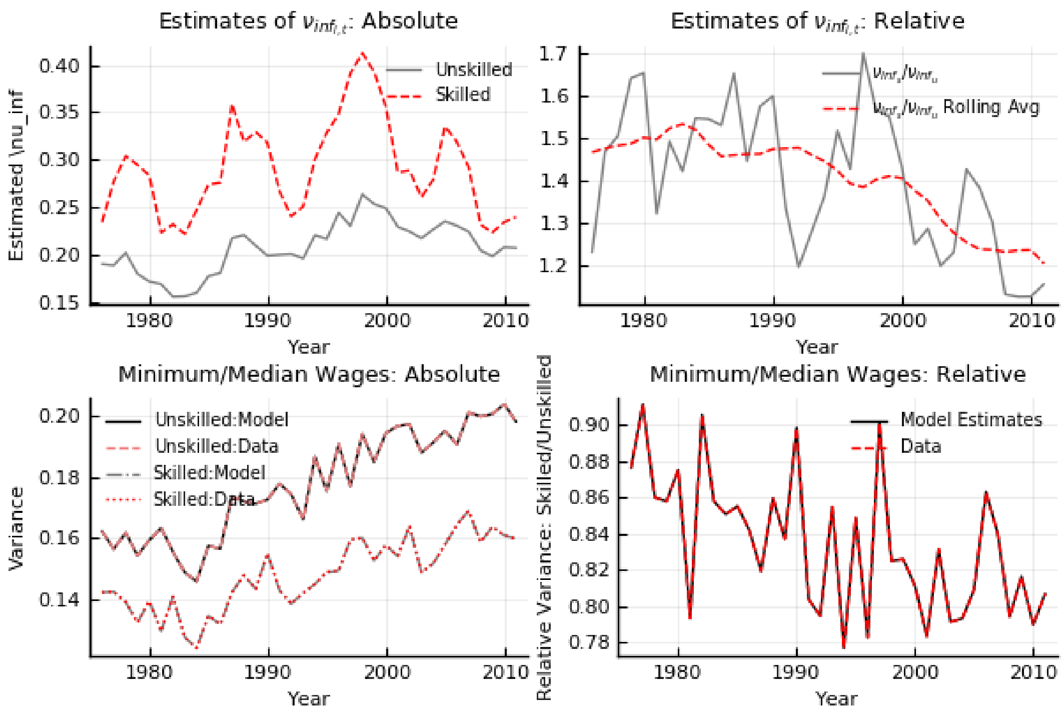
Figure 3.11: Match Quality Dispersion



around the early 1990s, but then decrease after this. However, this trend is not strong enough to significantly change estimates of the parameters in the KORV production function as shown in Table 3.3. In particular, the estimate of capital skill complementarity (the difference between the elasticity of substitution between unskilled labour and capital equipment and skilled labour and capital equipment, $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$ is very similar. The model with frictions seems to fit the data slightly less accurately than the original KORV formulation as shown in Figure 3.14.

The model with frictions is still entirely reliant on the capital skill complementarity channel to generate an increase in the graduate wage premium, as can be seen by examining model predictions when I shut down this channel by imposing $\sigma = \gamma$: Figure 3.15 shows that both the model with frictions and without in fact predict large falls in the graduate wage premium, due to the increase in the relative supply of graduates, when there is no capital skill complementarity.

Figure 3.12: Match Quality Lower Bound



3.6 Conclusion

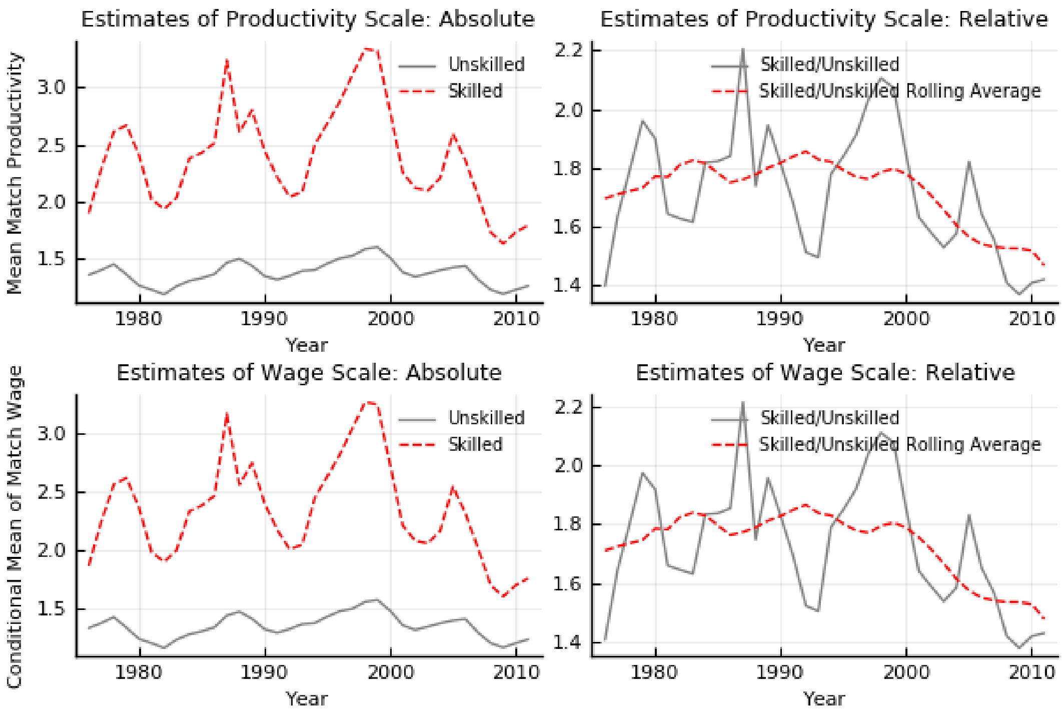
I developed an empirically testable model that combines the production framework specified in Krusell et al. (2000) with the sequential auction model of Cahuc et al. (2006). This model has the potential to identify the contribution of institutions, frictions and technology to growing wage inequality, though I focus on the latter two dimensions in this paper.

The empirical contribution of this paper is to examine whether adding a

Table 3.3: KORV parameter values: the importance of frictions

Parameter	With Frictions	Without Frictions
λ	0.507	0.568
μ	0.644	0.806
α	0.083	0.091
γ	-0.188	-0.209
σ	0.33	0.352
$\varepsilon_{S,K_{eq}}$	0.841	0.827
$\varepsilon_{U,K_{eq}}$	1.493	1.544
CSC Strength: $\boldsymbol{\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}}$	**0.651**	**0.716**

Figure 3.13: Scale Factors

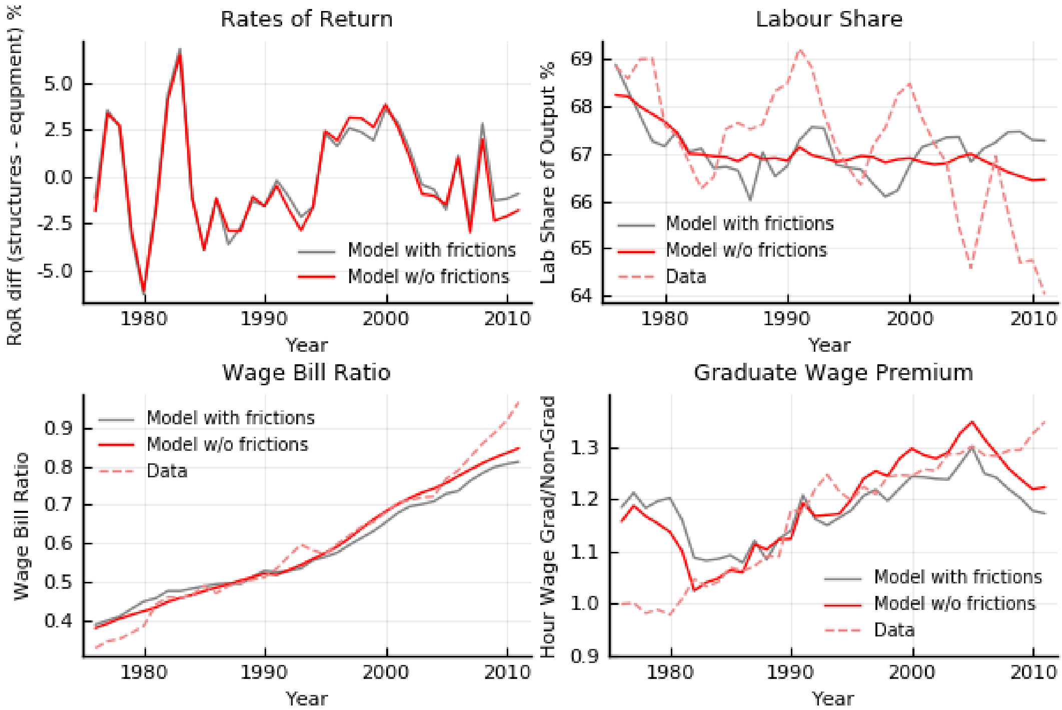


sequential auction labour market structure to Krusell et al. (2000) materially changes estimates of capital-skill complementarity. If I maintain consistency with Krusell et al. (2000) by using CPS labour market data only i.e. where no employer side information is used, I find that estimates of capital-skill complementarity are not significantly changed by allowing for labour market frictions.

This reflects the fact that my empirical measures of job market frictions, taken as a whole, do not move decisively in favour of either skilled or unskilled workers. Contrary to my expectations, both measures of job mobility rates and job destruction rates move in favour of unskilled workers relative to skilled workers, however, this is partly offset by an increase in estimated match quality dispersion for skilled workers which means climbing the job ladder brings greater rewards and boosts their average pay.

To maintain consistency and comparability with Krusell et al. (2000) I do not use any direct data on firm heterogeneity, which limits my ability to identify changes to the distribution of match heterogeneity and also constrains identifi-

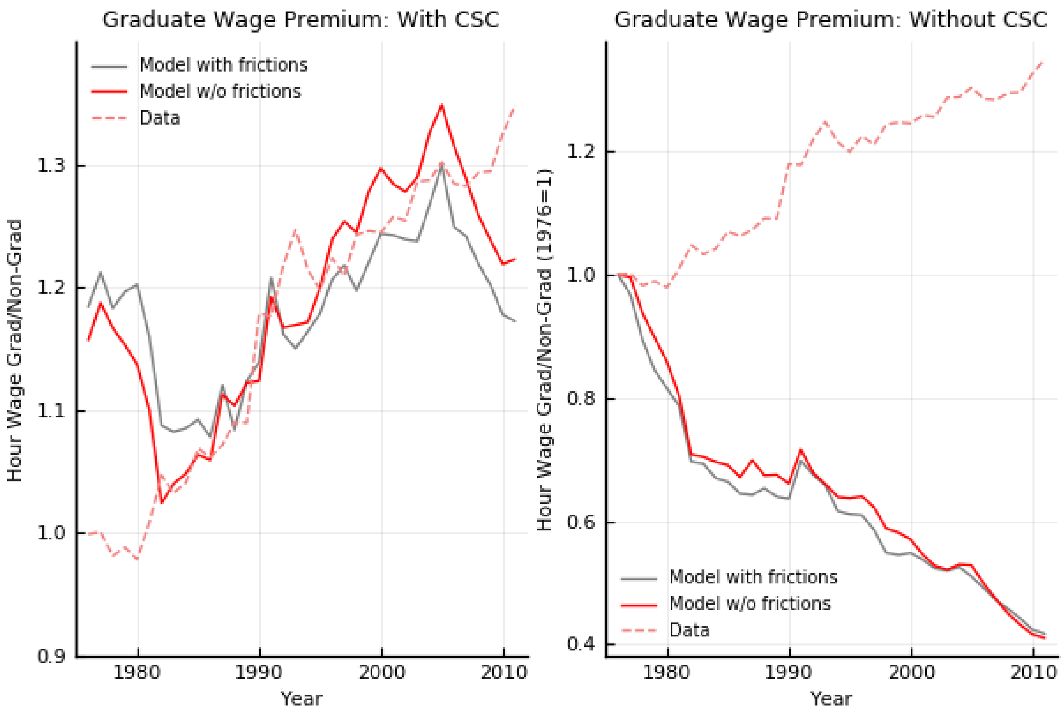
Figure 3.14: Model Fit: With and Without Frictions



cation of bargaining parameters, which could be one way to capture changes to the institutional environment. Adapting this framework to matched employer employee data is therefore a promising line of future research.

I also make the simplifying assumption that job contact rates are exogenous in my model, in keeping with Cahuc et al. (2006). However, this means changes in the use of capital, and hence the relative demand for skilled labour, have no impact on the job market frictions these workers face. Adding endogenous vacancy creation to the model, while representing a significant theoretical and empirical challenge, would therefore shed light on the links and feedback mechanisms between wage inequality, technology and labour market frictions.

Figure 3.15: Model Fit: No Capital Skill Complementarity



Chapter 4

Minimum Wages and Asset Accumulation

4.1 Introduction

Minimum wages are often motivated by concerns over inequality and poverty, however their impact on consumption inequality, a key outcome for assessing welfare impacts, has not received significant attention in either the structural or empirical literature. One reason for this is that the structural literature on minimum wages draws extensively on models with search frictions, as in van den Berg and Ridder (1998), Flinn (2006) and Engbom and Moser (2017), which typically assume risk neutral agents. The assumption of risk neutrality hinders analysis of the impact of the minimum wage on consumption inequality because, in risk neutral models, workers are indifferent to (mean-preserving) variation in consumption over time and across different employment states. Models with risk neutral workers are therefore unable to offer well defined predictions regarding consumption, and typically assume workers consume all income so consumption inequality is directly equated with income inequality.

In this chapter, I propose an on-the-job search model with capital skill complementarity, as per Chapter 2, but now with risk averse workers who can self-insure via asset accumulation. Adding these features allows me to examine the impact of minimum wages on consumption inequality.

While it is not the goal of this paper, including asset accumulation could also provide useful insights into the distribution of gains and losses from the minimum wage, since ownership of firms' equity can be endogonized.

I find that workers increase their savings to self-insure themselves against increased unemployment risk as the minimum wage increases. Their ability to self-insure means decreases in consumption inequality from the minimum wage continue to occur at relatively high minimum wage levels i.e. even when unemployment is rising. In a model where workers have no access to savings increasing the minimum wage to such levels would increase consumption inequality because increased unemployment risk has a more significant pass-through to consumption inequality.

I build on the same structural literature as in Chapter 2 so I do not include a fulll literature review here. I am aware of aware of only one other study, Aaronson et al. (2012), to look at the impact of the minimum wage on the consumption and savings/debt decisions of workers. Aaronson et al. (2012) provide difference-in-difference estimates of the short term spending response of households affected by a minimum wage hike. They find a \$1 hourly minimum wage hike increases quarterly household income by \$250 and quarterly household spending by \$700 in the short term. The authors attempt to reconcile those findings with a life cycle model where they model the minimum wage hike as a temporary deterministic increase to an exogenous income process. This is very different from the approach of this chapter, which is to consider the steady state consumption impacts of a permanent change in the minimum wage, allowing for endogenous changes in wages, unemployment and job mobility rates.

This approach builds on a broader literature that combines search frictions with asset accumulation, e.g. Andolfatto (1996), Krusell et al. (2010) and Lise (2011). However, this literature has not explicitly considered the role of the minimum wage in this setting.

The rest of this chapter is organised as follows. Section 4.2 will present my model, and Section 4.3 sets out my calibration strategy. Section 4.4 presents results from simulating the steady state impact of minimum wages on asset accumulation and consumption inequality, and Section 4.5 concludes.

4.2 The Model

4.2.1 Model Environment

Model Environment: Workers

There are two skill types of workers, unskilled and skilled, with skill indexed by $j \in u, s$. The fraction of the worker population of skill type j is denoted ℓ_j, and I normalise the total population to one. Unlike in Chapter 2, there will no heterogeneity in worker ability within the two skill groups. All workers and firm owners have a common discount factor, $\beta \in (0, 1)$. Workers can insure through risk free assets, a, but cannot borrow, and have constant relative risk aversion (CRRA) preferences over consumption, c:

$$u(c) = \frac{c^{1-\iota}}{1-\iota}, \ \iota > 0 \tag{4.1}$$

The budget constraint facing a worker takes the general form: $c + \frac{a'}{1+r} = y + a$, where a' represents the next period asset holdings of the worker, and y and r are the current period income and the risk-free rate of return respectively.

Model Environment: Production Structure

As in Chapter 2, I have two stages of production. First there is an intermediate goods sector with search frictions, where I maintain the typical assumptions of the search literature (no capital and constant returns to scale production in labour inputs). Second, I include a final good sector with a production function that combines intermediate goods with capital, and features imperfect substitutability of all factors and capital skill complementarity as per Krusell et al. (2000) (henceforth referred to as the “KORV” production function).

There will be a segmented intermediate goods sector for each worker skill type ($j \in u, s$). Firms in these intermediate sectors can be thought of as hiring agencies for the final goods firm, that face search frictions and wage bargaining.

Model Environment: Final Good Firms

Final goods are produced using capital structures, K_{st}, capital equipment, K_{eq}, and the intermediate goods produced by unskilled and by skilled workers, denoted by U and S respectively:

$$\begin{aligned} Y &= AG(K_{st}, K_{eq}, U, S) \\ &= AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \end{aligned} \tag{4.2}$$

with $\sigma, \rho < 1$ and $\alpha, \lambda, \mu \in (0, 1)$. The elasticity of substitution between the intermediate good produced by unskilled workers and capital equipment, denoted by $\varepsilon_{u,k_{eq}}$, equals $1/(1-\sigma)$. The elasticity of substitution between the intermediate goods produced by unskilled and skilled workers, denoted $\varepsilon_{u,s}$, is also given by $1/(1-\sigma)$. The elasticity of substitution between the skilled intermediate input and capital equipment, denoted by $\varepsilon_{s,k_{eq}}$, is given by $1/(1-\rho)$. The parameter, α, together with λ, determine the capital share of output, and μ determines the output share of unskilled intermediate good sectors.

The production function will exhibit capital skill complementarity, meaning capital equipment will be more substitutable with the intermediate good produced by unskilled workers than with the intermediate good produced by skilled workers (i.e. $\varepsilon_{u,k_{eq}} > \varepsilon_{s,k_{eq}}$), whenever $\sigma > \rho$. This is exactly what Krusell et al. (2000) find to be the case and I will use their parameter estimates (I discuss my calibration approach further in section 4.3).

Model Environment: Intermediate Goods Sectors

There is a separate intermediate goods sector for each worker type $j \in \{u, s\}$, and one intermediate firm for each worker in the economy. This implies the fraction of intermediate goods firms in sector j equals the fraction of type j workers in the total worker population, ℓ_j. I assume all intermediate firms sell competitively to the final good firm.

I assume constant returns to scale in intermediate good sectors, with the output of a given intermediate sector j equal to the employment rate of type j workers multiplied by their population density ℓ_j and hours worked $\bar{h}$. This implies $U = \ell_u(1 - e_u^{ue})\bar{h}$ and $S = \ell_s(1 - e_s^{ue})\bar{h}$, where e_j^{ue} is the unemployment rate of a type j worker. I include hours worked as the KORV production function was originally specified with labour input measured in terms of total hours, however, I assume both worker types are full-time, i.e. work a fixed 40 hour week, and do not model the intensive margin of labour supply. Intermediate goods sectors are completely segmented in the sense that a type j firm can only ever employ a type j worker and vice versa.

Model Environment: Search Frictions and Wage Bargaining in the Intermediate Goods Sectors

I assume that both unemployed and employed workers randomly search for jobs. The homogeneity of intermediate goods firms means workers exist in one of three states: unemployed; employed but not yet poached by another employer ('not-poached'); or employed and poached ('poached'). The employment state for a worker of skill type j is denoted as $\Upsilon_j \in \{ue, np, p\}$, where the indices $\{ue, np, p\}$ represent the unemployed, not-poached and poached employment states respectively.

The number of newly formed job matches is given by matching function $M(S_j, V_j)$, where S_j is the effective number of type j job searchers (unemployed and not-poached workers) and V_j is the number of type j vacancies. I assume that unemployed workers search more intensely than non-poached workers so that $S_j = N_j^{ue} + \chi_j N_j^{np}$, where N_j^{ue} is the number of unemployed type j workers, N_j^{np} is the number of not-poached workers, and χ_j is the search intensity rate for employees relative to the unemployed ($\chi > 0$). Once a worker is poached they stop searching as all firms are the same.

Defining $\theta_j \equiv V_j/S_j$ as labour market tightness, the contact rate is $q(\theta_j) \equiv M(S_j, V_j)/V_j$ for type j firms, and $(\theta_j q(\theta_j),\ \chi_j \theta_j q(\theta_j))$ for type j unemployed and not-poached workers respectively. The fraction of type j workers who are poached is denoted by e_j^p and the fraction who are not-poached by e_j^{np} (with the residual fraction unemployed denoted by e_j^{ue}). The share of effective job searching workers that are not-poached is denoted as $s_j^{np} \equiv \frac{\chi_j e_j^{np}}{\chi_j e_j^{np} + e_j^{ue}}$, and the share that are unemployed as $s_j^{ue} \equiv 1 - s_j^{np}$. Finally matches are destroyed with exogenous probability, δ_j.

I follow the approach of Cahuc et al. (2006) where all firms and workers engage in Nash bargaining. For unemployed workers matched with a firm, who then become 'not-poached' workers in my terminology, standard Nash bargaining takes place. This bargaining is subject to the constraint that the bargained wage must be at least as large as the legally binding minimum wage, m_w. Note that the bargained wage will depend on the asset holdings, a, of the worker since these determine the value of remaining in unemployment and of entering employment.

When a not-poached worker makes contact with another employer, becoming a poached worker, they also engage in Nash bargaining but this time the bargain is between the incumbent and poaching employer and the worker, as in Cahuc et al. (2006). The rival employers bid-up the wage until the value of employing a poached worker to the firm equals the value of carrying a vacancy. Free entry will drive the latter to zero, due to the existence of a fixed vacancy cost κ_j. As type j firms are a priori identical, the poaching firm will offer the same wage as the incumbent (which we will see is the price of the intermediate good) leaving the worker indifferent between the two rival firms. As in Chapter 2, I arbitrarily assume the worker moves with probability one to a poaching firm conditional on making contact with them. This assumption means job contact rates, which are unobservable in the data, are equal to job mobility rates, which are observable.

4.2.2 Behaviour in the Model Economy

Behaviour: workers

A worker of a given type j exists in one of three employment states: unemployed and receiving flow income b, not-poached and receiving the higher of the Nash bargained wage w_j^b and the minimum wage m_w, or poached and receiving wage w_j^p. The expected lifetime utility of being in each of these employment states with asset holidings, a, is denoted by $V_j^{ue}(a)$, $V_j^{np}(a)$, and $V_j^p(a)$ respectively.

Workers face a trivial labour market participation decision, as per Chapter 2, but now also must choose how much assets to carry forward to the next period, a', given their current asset level, a, and employment state. The Bellman equations for a unemployed, not-poached and poached worker are therefore:

$$V_j^{ue}(a) = \max_{a'} \left\{ u(b + a - \frac{a'}{1+r}) + \beta[\theta_j q(\theta_j) V_j^{np}(a') + (1 - \theta_j q(\theta_j)) V_j^{ue}(a')] \right\} \tag{4.3}$$

$$V_j^{np}(a) = \max_{a'} \left\{ u(\max(w_j^b(a), m_w) + a - \frac{a'}{1+r}) + \beta\big[\delta_j V_j^{ue}(a') + \right. \tag{4.4}$$

$$\left. (1 - \delta_j)[\chi \theta_j q(\theta_j) V_j^p(a') + (1 - \chi \theta_j q(\theta_j)) V_j^{np}(a')]\big] \right\}$$

$$V_j^p(a) = \max_{a'} \left\{ u(w_j^p + a - \frac{a'}{1+r}) + \beta[\delta_j V_j^{ue}(a') + (1 - \delta_j) V_j^p(a')] \right\} \tag{4.5}$$

Equation (4.3) tells us that an unemployed worker of skill level j receives benefits, b, in the current period and in the next period either gets a job offer with probability $\theta_j q(\theta_j)$, which they will always accept and so become a not-poached worker, or remains unemployed with probability $1 - \theta_j q(\theta_j)$. Equation (4.4) tells us that a not-poached worker gets the higher of the Nash bargained wage or the minimum wage in the current period and in the following period loses their job with probability δ_j , gets poached with probability $(1 - \delta_j)\chi\theta_j q(\theta_j)$ or remains not-poached with probability $(1 - \delta_j)(1 - \chi\theta_j q(\theta_j))$. Finally equation (4.5) tell us that a poached worker gets a wage w_j^p in the current period and the next period either loses their job with probability δ_j or remains employed as a poached worker (since they have already reached the top of the job ladder) with probability $1 - \delta_j$.[1]

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[1] I show later that poached workers are paid a wage equal to the price of the intermediate good they produce, which is independent of the worker's asset holdings. The price of the intermediate good is equal to the marginal product of the intermediate good, which will

The optimal savings policy functions derived from these Bellman equations are denoted $\{\psi_j^{ue}(a),\ \psi_j^{np}(a),\ \psi_j^{p}(a)\}$. These, combined with transition rates between employment states, also imply the steady state distribution of assets by employment state: $\{f_j^{ue}(a), f_j^{np}(a), f_j^{p}(a)\}$, where $f(a)$ denotes the pdf of the asset distribution.

Behaviour: Final Good Producers

The final good producer's profit maximisation problem is as follows, where we normalise the price of the final good to one:

$$\max_{K_{st},K_{eq},U,S} \Pi = AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} - p_u U - p_s S - r_{st}K_{st} - r_{eq}K_{eq} \tag{4.6}$$

As in Krusell et al. (2000), I impose a no arbitrage condition between capital equipment and capital structures. This implies that the net of depreciation rental rates for capital equipment and structures must be equal to some common interest rate, r, which implies their gross rental rates, r_{eq} and r_{st}, are related as follows: $r_{eq} - \delta_{eq} = r_{st} - \delta_{st} = r$, where δ_{eq} and δ_{st} are the depreciation rates for capital equipment and structures respectively.[2] I assume the final goods sector is competitive so factors of production are paid their marginal products, as shown in equations (4.7) through to (4.10).

$$p_u = A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}}\mu U^{\sigma-1} \tag{4.7}$$

$$p_s = A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} \times(1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma-\rho}{\rho}}(1-\lambda)S^{\rho-1} \tag{4.8}$$

$$r_{eq} = A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} \times(1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma-\rho}{\rho}}K_{eq}^{\rho-1} \tag{4.9}$$

---

always exceed the minimum wage in equilibrium. If this were not the case intermediate firms would be loss making and leave the market, until the price of the intermediate good is bid up by the final good producer to the level of the minimum wage (Inada conditions guarantee this point will be reached)

[2]When it comes to calibrating the model I will assume that both net of depreciation rates equal the natural rate of interest $r = \frac{1}{\beta} - 1$.

$$r_{st} = \alpha A K_{st}^{\alpha-1}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \tag{4.10}$$

Behaviour: Intermediate Goods Producers

Intermediate firms are either inactive, generating zero expected liftetime utility for their owners (we refer to the expected lifetime utility of firm ownership as the firm's value), or exist in one of three active states: (i) carrying a vacancy, with a firm value denoted by J_j^v (ii) employing a not-poached worker who has assets a (recall assets determine bargained wages), with a firm value denoted by $J_j^{np}(a)$, and (iii) employing a poached worker at a wage w_j^p, with a firm value denoted by J_j^p. The corresponding bellman equations are:

$$J_j^v = -\kappa_j + \beta[q(\theta_j)\{s_j^{ue}\int J_j^{np}(a)f_j^{ue}(a) + (1-s_j^{ue})J_j^p\} + (1-q(\theta_j))J_j^v] \tag{4.11}$$

$$J_j^{np}(a) = p_j - \max(w_j^b(a), m_w) + \tag{4.12}$$
$$\beta\Big[(1-\delta_j)\big\{\chi\theta_j q(\theta_j)J_j^p + (1-\chi\theta_j q(\theta_j))J_j^{np}(\psi_j^{np}(a))\big\} + \delta_j J_j^v\Big]$$

$$J_j^p = p_j - w_j^p + \beta[(1-\delta_j)J_j^p + \delta_j J_j^v] \tag{4.13}$$

Equation (4.11) tells us that a firm in intermediate good sector j carrying a vacancy pays a vacancy cost, κ_j, in the current period and in the next period makes contact with an unemployed worker with asset holdings a with probability $q(\theta_j)s_j^{ue}f_j^{ue}(a)$, makes contact with an employed worker with probability $q(\theta_j)(1-s^{ue})$, or remains carrying a vacancy with probability $1-q(\theta_j)$. Equation (4.12) tells us that a firm employing a not-poached worker with assets a gets profits $p_j - \max(w_j^b(a), m_w)$ in the current period and in the next period remains employing that worker (whose asset level evolves according to their optimal savings choice $\psi_j^{np}(a)$) with the probability $(1-\delta_j)(1-\chi\theta_j q(\theta_j))$, loses the worker to a rival firm with probability $(1-\delta_j)\chi\theta_j q(\theta_j)$, or the job is destroyed with probability δ_j. Finally equation (4.13) tells us a firm employing a poached worker gets profit $p_j - w_j^p$ in the current period and in the next period the job is either destroyed with probability δ_j or they remain employing the poached worker with probability $1-\delta_j$.

Free entry into markets by inactive firms will drive the value of holding a vacant job, J_j^v, to zero, and competition between employers drives the value of employing a poached worker to the value of holding vacancy e.g. $J_j^p = 0$ too. The free entry condition ($J_j^v = 0$) and poaching condition ($J_j^p = 0$) imply the poached wage, w_j^p equals the price of the intermediate good p_j.

Using these conditions, and substituting 4.12 into 4.11, I get the following no entry condition:

$$\begin{aligned}
\kappa_j =& \beta q(\theta_j) s_j^{ue} \int J_j^{np}(a) f_j^{ue}(a) \\
\Rightarrow \frac{\kappa_j}{\beta q(\theta_j) s_j^{ue}} =& p_j - \int \max(w_j^b(a), m_w) f_j^{ue}(a) da \\
&+ \int \left[\beta(1-\delta_j)(1-\chi\theta_j q(\theta_j)) J_j^{np}(\psi_j^{np}(a))\right] f_j^{ue}(a) da
\end{aligned} \tag{4.14}$$

Inactive firms will enter the market, by posting a new vacancy, until the discounted expected profits from hiring a not-poached worker (RHS of equation (4.14)) equal the discounted expected vacancy cost (LHS of the equation). The discounting of expected profits reflects both the discount factor and the risk that the worker will be exogenously separated from the firm (with probability δ_j) or be poached by another firm (with probability $\chi\theta_j q(\theta_j)$).

The Nash bargained wage is determined in the standard maximisation problem, shown in equation (4.15).

$$w_j^b(a) = \underset{w_j^b(a)}{\operatorname{argmax}} (V_j^{np}(a) - V_j^u(a))^{\phi_j} (J_j^{np}(a))^{1-\phi_j} \tag{4.15}$$

The asymmetry between the risk neutrality of the managers of intermediate firms and risk aversion of workers means the first order condition of the Nash bargaining problem yields a polynomial in $w_j^b(a)$, after substitution of the relevant value functions (equations (4.4) and (4.12)) into equation (4.15). The order of this polynomial is determined by the degree of relative risk aversion ι in the utility function given in equation (4.1).

4.2.3 Equilibrium

One condition for a steady state equilibrium in the model, which I will formally define later, is that the labour market is in steady state. This requires the following equations to hold:

$$\delta_j(1 - e_j^{ue}) = \theta_j q(\theta_j) e_j^{ue} \tag{4.16}$$

$$\theta_j q(\theta_j) e_j^{ue} = (\delta_j + (1 - \delta_j)\chi_j \theta_j q(\theta_j)) e_j^{np} \tag{4.17}$$

Equation (4.16) equates inflows into unemployment (LHS of the equation) to outflows (RHS), where the inflow consists of employees losing their jobs, with probability δ_j, and the outflow is unemployed workers gaining jobs, with probability $\theta_j q(\theta_j)$. Similarly equation (4.17) equates the inflow in of workers into the not-poached state (LHS) with the outflow (RHS), where the inflow consists of unemployed workers gaining employment with probability $\theta_j q(\theta_j)$, and the outflow is not-poached workers either losing their job, with probability δ_j, or becoming poached, with probability $(1 - \delta_j)\chi_j \theta_j q(\theta_j)$.

I denote the labour market tightness and unemployment level satisfying these conditions as θ_j^{ss} and $e_j^{ue^{ss}}$ respectively. I derive a supply function for intermediate goods, shown in equation (4.18), from these steady state conditions and the no entry condition in the intermediate good sector. The corresponding demand equation comes from the first order conditions of the final good producer's profit maximisation problem, and is shown in equation (4.19).

$$\begin{aligned} p_j^s = {} & \frac{\kappa_j}{\beta q(\theta_j^{ss}) s_j^{ue}} \\ & + \int [\max(w_j^b(a), m_w) - \beta(1 - \delta_j)(1 - \chi \theta_j q(\theta_j)) J_j^{np}(\psi_j^{np}(a))] f_j^{np}(a) \end{aligned} \tag{4.18}$$

$$p_j^d = \frac{\partial Y}{\partial (1 - e_j^{ue^{ss}})} \tag{4.19}$$

The intersection of this system of equations determines equilibrium in the intermediate goods market for a given interest rate.

4.2.4 Equilibrium Definition

Note that in my baseline calibration and for simulated results I assume a small open economy, and hence solve the model for a constant interest rate, r. I therefore do not impose an asset clearing condition as part of the equilibrium definition.

Definition 2. *The recursive stationary equilibrium consists of:*

(i) *a set of worker value functions* $\{V_j^{ue}(a),\ V_j^{np}(a),\ V_j^{p}(a)\}$ *and the individual decision rules for asset holdings* $\{\psi_j^{ue}(a),\ \psi_j^{np}(a),\ \psi_j^{p}(a)\}$ *for all workers;*

(ii) *the distribution of asset holdings for each worker and for each employment state:* $f_j^{ue}(a)$, $f_j^{np}(a)$ *and* $f_j^{p}(a)$*) and a set of employment states* $\{e_j^{ue}, e_j^{np}, e_j^{p}\}$.

(iii) *a set of firm value functions* $\{J_j^{v}, J_j^{np}, J_j^{p}(a)\}$*,and vacancies,* v_j*, for all intermediate goods firms;*

(iv) *a choice of capital equipment, capital structures,unskilled and skilled intermediate goods* (K_{eq}, K_{st}, U, S) *by the final good producer*

(v) *prices* $\{p_j,\ w_j^{b}(a),\ w_j^{p}\}$ *; which satisfy:*

1. *Consumer Optimisation:*
   Given the job-finding probabilities and prices, the individual decision rules $\{\psi_j^{ue}(a),\ \psi_j^{np}(a),\ \psi_j^{p}(a)\}$ *satisfy conditions 4.3, 4.4 and 4.5.*

2. *Final Good Producer Optimisation:*
   Given prices and job contact rates, the final good producer demands capital equipment and structures, K_{eq} *and* K_{st} *and intermediate goods* U *and* S *to satisfy the FOCs 4.7 through to 4.10 .*

3. *Steady State in the Intermediate Good Sector:*
   The no-entry condition, 4.14, and steady state conditions 4.16 and 4.17 are met.

4. *Intermediate Goods Market Clearing:*

   Demand and supply for each intermediate good must be equal, implying conditions 4.18 and 4.19 hold for all intermediate good sectors $j \in u, s$.

5. *Wage Determination:*

   not-poached workers are paid the higher of the Nash bargained wage wage $w_j^b(a)$ *and the minimum wage,* m_w*, and poached workers are paid the competitive wage,* $w_j^p = p_j$

6. *Consistency:*

   Given employment and vacancy rates, the job contact rates determined by the matching function are consistent with those used in the worker and firm optimisation problems.

4.2.5 Solution Algorithm

For a fixed world interest rate, r, we:

1. Guess unemployment rate e_{j0}^{ue} for each skill type $j = u, s$. Use this guess to calculate the implied amount of intermediate goods produced by unskilled and skilled workers (U and S).

2. Solve the final good firms FOCs to get the final good firms' use of capital equipment and structures K_{eq} and K_{st} and the price of intermediate goods p_u and p_s that are consistent with the implied levels of U and S calculated above.

3. Use the conditions 4.16 and 4.17 to derive vacancy levels necessary for the unemployment guess $e_{j,0}^{ue}$ to be consistent with steady state in the labour market. This then implies employment transition probabilities for the unemployed and employed via the matching function: $\theta_j q(\theta_j)$ and $\chi_j \theta_j q(\theta_j)$ respectively.

4. Use the price of intermediate goods and employment transition probabilities calculated above to solve workers' value functions (computational

details are specified below) and Nash bargained wage, $w_j^b(a)$. Wage of not-poached worker is whatever is highest of this bargained wage and minimum wage

   (a) A guess and verify process is necessary within this step i.e. I first guess the bargained wage at each asset level, use this to solve for workers' and intermediate firms' value functions, and then update the guess of the bargained wage using equation (4.15).

5. Use the asset policy rules $\{\psi_j^{ue}(a), \psi_j^{np}(a), \psi_j^{p}(a)\}$ derived in above step and employment transition probabilities $\theta_j q(\theta_j)$ and $\chi_j \theta_j q(\theta_j$ to construct transition matrix P, and solve for the invariant asset distributions $f_j^{ue}(a)$, $f_j^{np}(a)$ and $f_j^{p}(a)$.

6. Use the bargained wage function $w_j^b(a)$, invariant asset distribution $f_j^{ue}(a)$ and price of intermediate goods p_j to compute an updated unemployment guess, e_{j1}^{ue} for $j \in \{u, s\}$, by solving the free entry condition 4.14.

7. Update and repeat iteration until convergence of unemployment guess.

I implement this solution algorithm using the following computational specifications. First, I solve workers value functions using value function iteration (VFI), over an asset grid with 250 points. I then solve for the invariant asset distribution using by interpolating the policy rules obtained in the VFI step over a finer asset-grid with 5000 points. The time period is monthly (though I present some wage results in hourly format for comparison with the minimum wage).

4.3 Calibration

4.3.1 Calibration Strategy

I will take all but one of the parameters of the final good production function from Krusell et al. (2000). This means applying parameters estimated under the assumption of competitive labour markets to my model that assumes labour market frictions. However, results from Chapter 3 of this thesis suggest the parameter estimates obtained by Krusell et al. (2000) are robust to allowing for labour market frictions. This provides some reassurance that applying their parameter estimates to a model with search frictions is not unreasonable. There is a separate issue that the estimates that Krusell et al. (2000) provide are based on calibration to the US economy, and I will be calibrating my model to the UK. However, given similarities in labour market trends in the US and UK and, relatively open capital markets between the two countries, this again does not seem unreasonable as a calibration approach.

I use the matching function specification, and parameter, from Hagedorn and Manovskii (2008b) - $M(u, v) = uv/(u^\gamma + v^\gamma)^{1/\gamma}$, which ensures job contact rates are bounded between zero and one. I focus on estimating: (i) TFP, (ii) the share parameter, μ, in the KORV production function,and (iii) recruitment costs κ_u, κ_s. I denote the parameters to be estimated as $\Phi = (A, \mu, \kappa_u, \kappa_s)$. The remaining parameters are taken from the literature and are denoted by Ω.

I estimate the parameters in Φ by simulated method of moments (SMM), targeting median wages and unemployment rates for non-graduates and graduates. The absolute magnitudes of median wages help to discipline the TFP parameter, A, and their relative magnitudes will discipline the output share parameter, μ. Finally, unemployment rates are an obvious, and widely used, way to pin down the costs of vacancy creation in the model (κ_u, κ_s).

The SMM approach I use is summarised in equation (4.20), where $\hat{M}$ denotes a vector of the empirical moments given above, and $M(\Phi, \Omega)$ denotes the model predictions of these moments for given choice of estimated and calibrated parameters.[3] All of the empirical moments are taken from Labour

[3]The weighting matrix W, is chosen so I effectively minimise the percentage deviation of model moments from their empirical moments, which avoids the scale of absolute moment deviations biasing estimates i.e. $W = I.\frac{1}{\hat{M}}$.

Force Survey data for 2013-14.

$$\Phi^* = \underset{\Phi}{\operatorname{argmin}}(M(\Phi,\Omega) - \hat{M})'W(M(\Phi,\Omega) - \hat{M}) \tag{4.20}$$

4.3.2 Estimation Results

Table 4.1 summarises the ability of my model to match its empirical targets. Given the model is just identified (I have four parameters to estimate and target four moments), it is not surprising that I hit the empirical targets more or less exactly. Table 4.2 shows the parameters I estimate using SMM. The share parameter μ is most relevant for hitting relative wages of unskilled and skilled workers in my model and as expected, given a positive skill premium in the data, its estimated value allocates more output share to skilled workers. It is perhaps counter-intuitive that the estimated recruitment costs are higher for unskilled workers than skilled; however this is compensating for the fact that job separation rates are higher for unskilled workers in the data and the minimum wage is more significant for these workers relative to their median wage. Therefore without the difference in recruitment costs, the unemployment gap between unskilled and skilled workers would be counter-factually large.

The parameters that I take from the literature, directly from the data, set at their statutory levels or set by assumption are shown in Table 4.3. I calibrate the model to data from 2013-14, as this precedes the significant increases in the minimum wage that started in 2014-15 and are planned to end when the minimum wage reaches 60% of the median wage in 2020-21. I assume unemployment income is paid at a fixed rate that is common for all workers. [4]

[4] Unlike in many other jurisdictions, the main form of unemployment benefits in the UK is paid at a flat rate, as under my baseline calibration, rather than as a fixed percentage of previous earnings. Of course, workers may have access to other forms of insurance: Chapter 3 of this thesis considers minimum wage impacts when workers can self-insure themselves through asset accumulation.

Table 4.1: Estimation Results

Moment	Model Moment	Empirical Moment	% Deviation (Model - Data)
Median Hourly Wage: Unskilled	9.53	9.5	0.27
Median Hourly Wage: Skilled	15.82	15.71	0.74
Unemployment: Unskilled	0.07	0.07	0.29
Unemployment: Skilled	0.03	0.03	-0.01

Table 4.2: Estimated Parameters

Parameter	Description	Value
μ	Share parameter determining skill premium in KORV production function	0.389
A	Total Factor Productivity	9.475
κ_u	Hiring cost: unskilled workers	1393.96
κ_s	Hiring cost: skilled workers	1038.18

Table 4.3: Calibrated Parameters

Parameter	Description	Source	Value
δ_u	Job destruction rate: unskilled	LFS 2013q4-2014q3	0.011
δ_s	Job destruction rate: skilled	LFS 2013q4-2014q3	0.007
χ_u	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.112
χ_s	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.075
b	Monthly Unemployment benefits (job seekers allowance)	Legislative level 2013-14	313.492
m_w	Hourly minimum wage	Legislative level 2013-14	6.31
σ	Elasticity of substitution between unskilled and skilled workers	Krusell et al. (2000)	0.401
ρ	Elasticity of substitution between skilled workers and capital equipment	Krusell et al. (2000)	-0.495
α	Capital Structures Parameter	Krusell et al. (2000)	0.117
λ	Input share parameter for capital equipment and skilled labour	Krusell et al. (2000)	0.3
γ	Matching Parameter	Hagedorn and Manovskii (2008a)	0.407
β	Monthly discount factor for workers and firms	By assumption	0.996
ϕ_u	Nash Bargaining Parameter for unskilled workers	By assumption	0.5
ϕ_s	Nash Bargaining Parameter for skilled workers	By assumption	0.5

4.3.3 Non-targeted Empirical Moments

Table 4.4 compares the model's predictions to a range of empirical moments we have not explicitly targeted. The model predicts smaller mark-ups and a higher labour share of income than the model I developed in Chapter 2. One possible explanation for this is that the ability to self-insure improves workers outside options (the expected lifetime utility of being in unemployment) and hence leaves them in a stronger bargaining position with firms.

I also examine the model's predictions for asset-accumulation both by skill level (rows 5 and 6 of Table 4.4) and for wealth inequality (rows 6 and 7). The model gets the right sign of the correlation between education and wealth but, significantly underestimates its magnitude. The model also under-predicts the degree of right tail inequality in the wealth distribution, as measured by the share of total wealth held by the top 1% of the wealth distribution. However, the model only has two sources of risk, wage and unemployment, and is not designed to capture many of the savings motives usually emphasised in the literature, i.e. bequests, pension savings, and ill-health, so these results are not entirely surprising.

Table 4.4: Non-targeted Macro Moments

Moment	Model Moment	Empirical Moment
Labour Share of GVA[1]	0.82	0.76
Mark-Up Ratio[2]	1.01	1.5
Net Capital Stock/GVA[3]	1.78	2.6
Median Wealth Unskilled[4]	£66,896	£84,644
Median Wealth Skilled[4]	£69,803	£211,200
Top 10% Wealth Share[5]	0.35	0.52
Top 1% Wealth Share[5]	0.13	0.2

[1] Bank of England, includes self-employed labour income (imputing it as compensation per employee multiplied by number of self-employed). GVA=Gross Value Added

[2] Empirical moment taken from De Loecker and Eeckhout (2018), model moment is calculated analagously (as described in text).

[3] UK National accounts, ONS.

[4] Data from Wealth and Asset Survey (WAS), ONS. WAS defines total net wealth as the sum of four components and is net of all liabilities: net property wealth, net financial wealth, private pension wealth.

[5] UK Data from World Inequality Database. Based on net personal wealth is the total value of non-financial and financial assets (housing, land, deposits, bonds, equities, etc.) held by persons aged over 20, minus their debts.

4.4 Results

I first present results from the model without a minimum wage in order to build intuition in the underlying model mechanisms. I then present results on the comparative static impacts of increasing the minimum wage. All simulated impacts of the minimum wage described in this section are equilibrium outcomes conforming to the equilibrium definition provided in section 4.2.4. These results therefore reflect steady state impacts only and do not include any transition dynamics.

4.4.1 Results: No Minimum Wage

I focus here on savings decisions by workers since this is the key contribution of this chapter, relative to Chapter 2. These savings decisions are driven by the earnings risk workers face; Figure 4.1 shows how earnings vary by the employment state (unemployed, not-poached and poached), skill and asset holdings of the worker. The model predicts a positive relationship both between a not-poached worker's wage (determined by standard Nash bargaining) and their asset holdings, and between workers' wages and their skill type. Both results are driven by my choice of bargaining parameter (recall I set $\Phi = 0.5$ for both skill types). However, the positive relationship between a not-poached worker's wage and their asset holdings is only significant at low levels of assets; at higher levels the relationship is largely flat, which is consistent with results in the literature e.g. Andolfatto (1996).[5]

Figure 4.2 plots the savings policy functions of workers by employment state and skill. First, for all skill types, unemployed workers have the lowest propensity to save and poached workers the highest. This is in keeping with results from Lise (2011) in that those at the top of the job ladder have the most to lose and so have a greater precautionary savings motive. The dispersion in savings policies across employment states is greatest for skilled workers; an intuitive result given that they face the greatest income risk.

[5] see Appendix G for discussion of the relationship between the not-poached worker's wage and their asset holdings and skill type, and how the bargaining parameter influences this relationship.

Figure 4.1: Wages in the Model

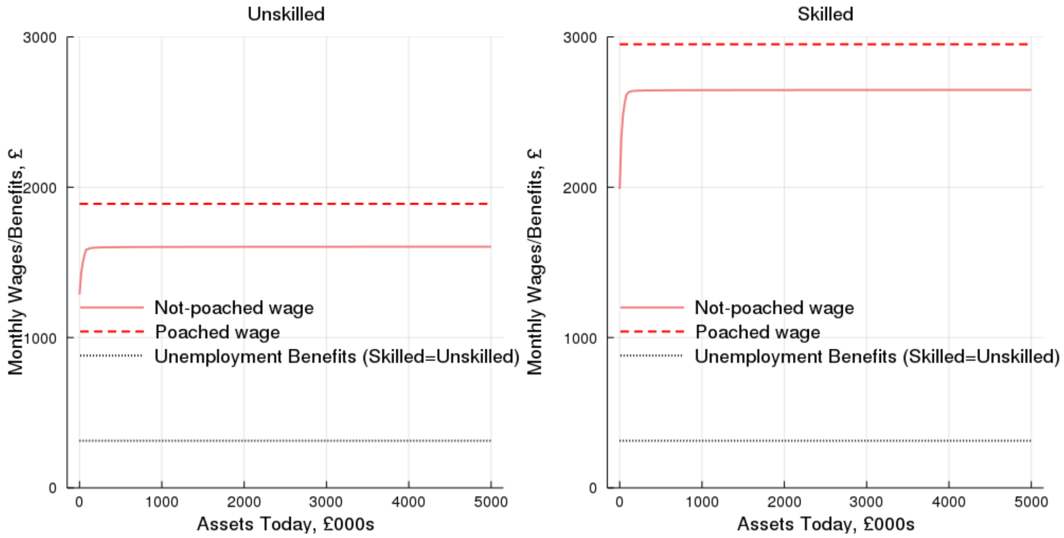
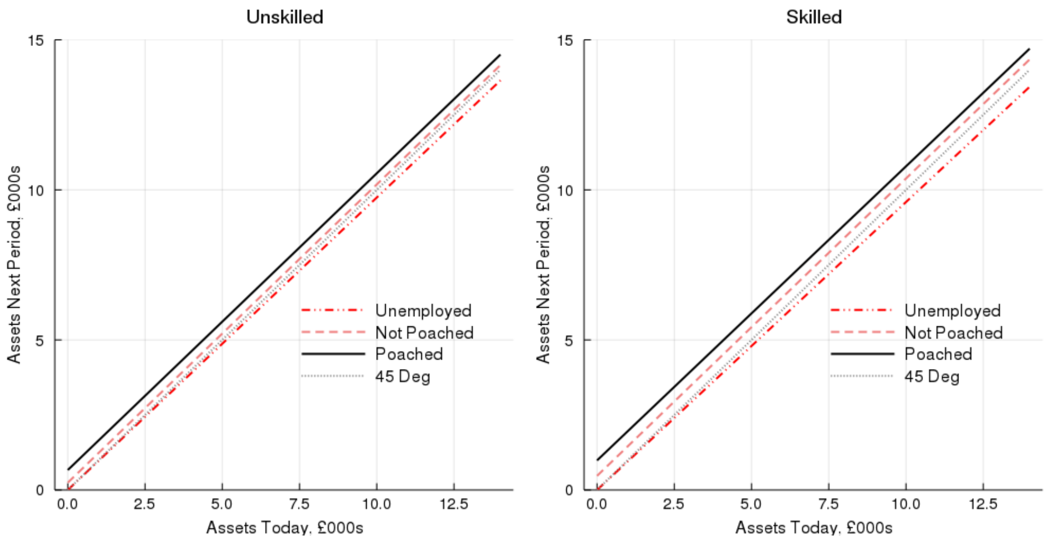


Figure 4.2: Savings Policy Functions



Notes: The asset grid in this figure is truncated so that differences in policy functions are visible. This has the side-effect of giving the false appearance that policy functions do not converge.

4.4.2 Results: Minimum Wage Impacts

I again start by considering earnings risk, and how this varies with the minimum wage, before presenting the key results of this paper; the impact of the minimum wage on savings and hence on consumption inequality.

Minimum Wage Impacts: Unemployment, Wage and Earnings Risk

The largest earnings risk in the model comes from the threat of unemployment, as suggested in Figure 4.1. The impact of the minimum wage on equilibrium unemployment rates in the model is shown in Figure 4.3. As way of comparison with the results of Chapter 2, Figure 4.4 compares the unemployment response in the baseline model developed here ("Series 1" in the Figure) to the unemployment response in the model in Chapter 2 (i.e. with no savings but ability heterogeneity: "Series 2"). Figure 4.4 also includes the unemployment response of a model with no savings and no ability heterogeneity ("Series 3") so that we can distinguish the impact of including savings and removing heterogeneity in ability. The results show that the difference between the unemployment response in this chapter and in Chapter 2 is entirely driven by the lack of ability heterogeneity; including savings in the model does not change the response of unemployment to the minimum wage.

I now consider how the minimum wage affects the cross sectional variance of wages and earnings faced by workers, conditional on their skill type (earnings is defined as unemployment benefits for unemployed workers and wages for employed workers).[6] The variance in wages for a given skill type of worker is in principle driven by two sources of wage dispersion. First, wages vary across the different employment states of workers (not-poached or poached). Second, wages of not-poached workers vary with their asset holdings, which

[6] The cross sectional variance in wages across workers of a given skill type j is shown in equation (4.21) (where Υ represents the employment state of a worker and w denotes their wage) and the cross sectional variance of earnings is given in equation (4.22) (where ω denotes earnings).

$$Var(w_j) = \int^{\Upsilon\in\{np,p\}} \int^{a} w_j^{\Upsilon}(a)^2 f_j^{\Upsilon}(a)da\, d\Upsilon - \left(\int^{\Upsilon\in\{np,p\}} \int^{a} w_j^{\Upsilon}(a) f_j^{\Upsilon}(a)da\, d\Upsilon\right)^2 \tag{4.21}$$

$$Var(\omega_j) = \int^{\Upsilon\in\{ue,np,p\}} \int^{a} \omega_j^{\Upsilon}(a)^2 f_j^{\Upsilon}(a)da\, d\Upsilon - \left(\int^{\Upsilon\in\{ue,np,p\}} \int^{a} \omega_j^{\Upsilon}(a) f_j^{\Upsilon}(a)da\, d\Upsilon\right)^2 \tag{4.22}$$

Figure 4.3: Unemployment Response

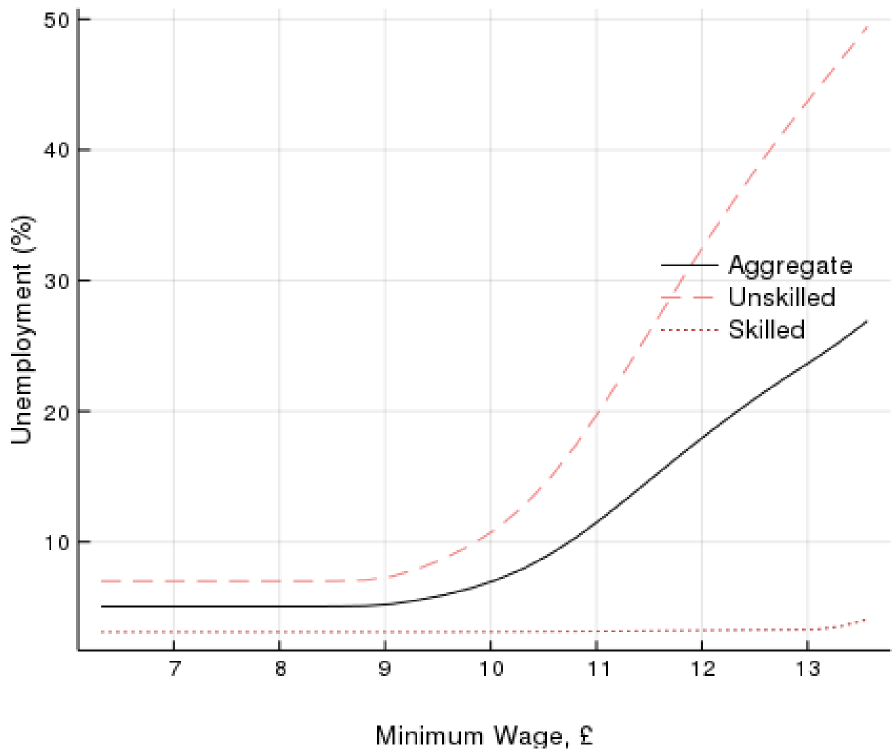
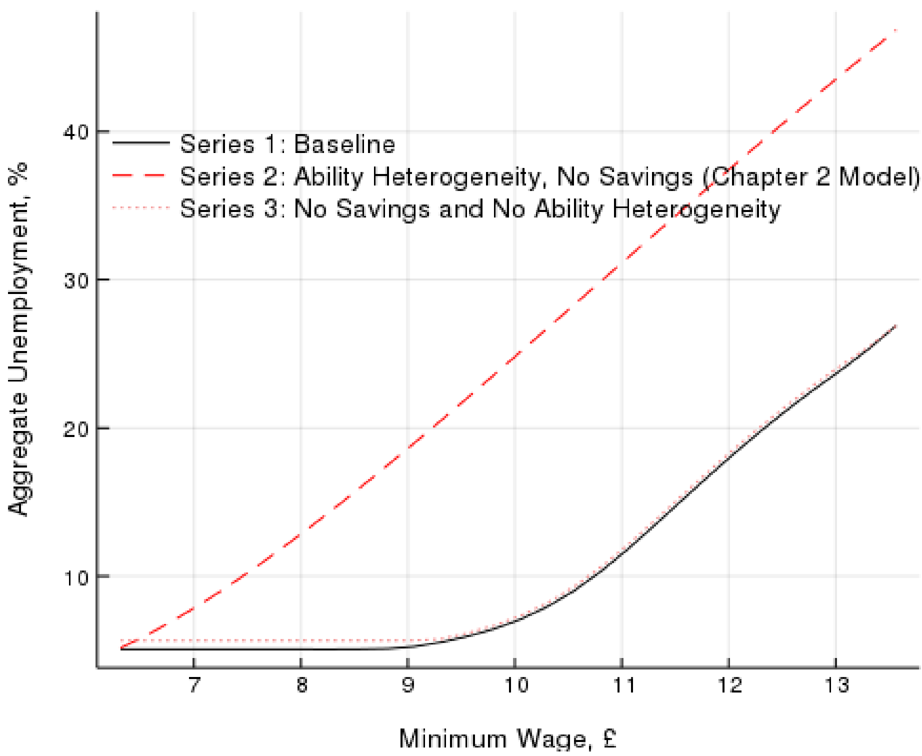


Figure 4.4: Comparison of Unemployment Responses



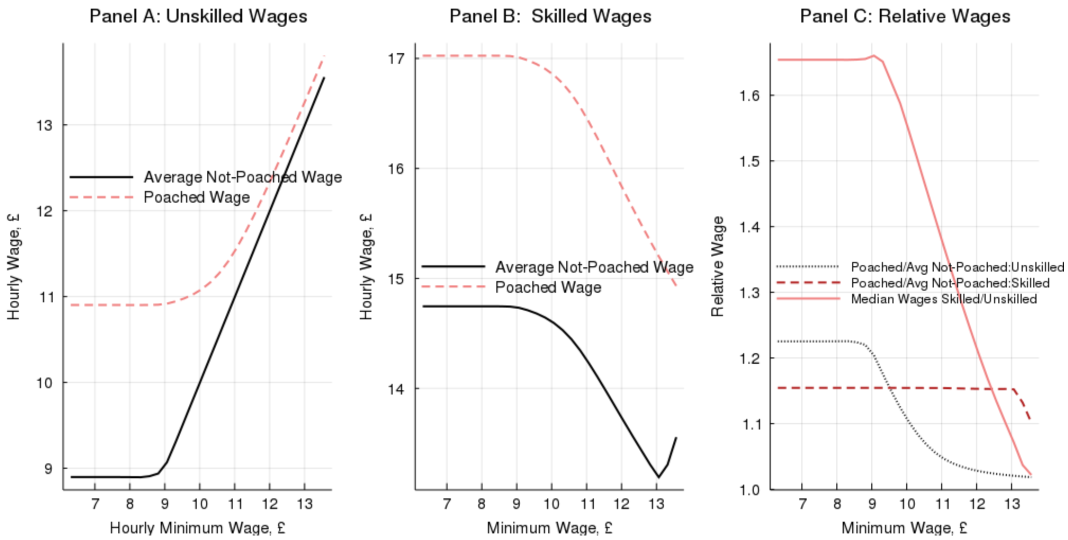
will be distributed according to the non-degenerate invariant distribution of asset holdings. However, we have seen above, i.e. in Figure 4.1, that wages of not-poached workers do not significantly vary with asset holdings, except at low levels, so the variation in wages across employment states will be the principal source of wage/earnings dispersion for a given skill type of worker.

Figure 4.5 shows the impact of the minimum wage on the level of the poached wage and the average not-poached wage received by unskilled and skilled workers.[7] We see that, at low levels, the minimum wage binds only on not-poached unskilled workers. The minimum wage generates a positive spillover for poached unskilled workers because it increases the unemployment rate of unskilled workers, and therefore raises the marginal product and price of the intermediate good produced by unskilled workers. However, the increased unemployment of unskilled workers generates a negative spillover on the wages of not-poached and poached skilled workers, as shown in Panel B of Figure 4.5. This reflects the levels of elasticity of substitution between factor inputs in the KORV production function as determined by the parameter values used in my calibration. The combined effect of these minimum wage impacts is a relatively sharp decline in the skill premium, as shown in Panel C of Figure 4.5.

Figure 4.6 shows the impact of the minimum wage on the cross sectional variance of earnings and wages faced by unskilled and skilled workers. The minimum wage uniformly decreases the variance of wages for unskilled workers. However, it also increases the wage levels for not-poached and poached unskilled workers relative to unemployment benefits, which, combined with the increase in unemployment, causes a uniform increase in the variance of earnings for unskilled workers. We have seen that the increased unemployment of unskilled workers reduces the wage received by not-poached and poached skilled workers. This means skilled workers initially see their earnings risk fall in response to small increases in the minimum wage, due the decreasing gap

[7]The average wage of a not-poached worker of skill type j, denoted $\bar{w}_j^{np}$, is defined as $\bar{w}_j^{np} \equiv \int^a w_j^{np}(a) f_j^{np}(a) da$.

Figure 4.5: Wage Response By Skill



between their unemployment benefits and wages. However, the earnings risk faced by skilled workers increases significantly once the minimum wage is high enough to directly bind their wages, which is driven by the increase in their unemployment rate and increase in their wage levels. In contrast the variance of their wages decreases uniformly.

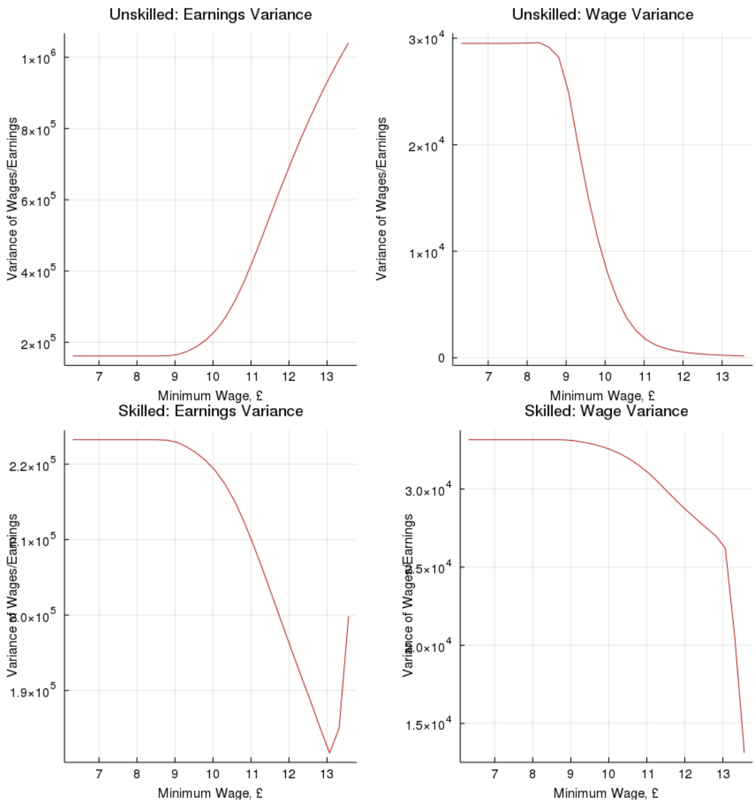
To summarise, the minimum wage sharply decreases the variance in wages faced by unskilled workers but, because of its positive impact on unemployment and average wages, eventually causes earnings risk for unskilled workers to rise. The unemployment response of unskilled workers has spillover impacts on the earnings and variance of wages faced by skilled workers, causing their earnings risk to intially fall before rising steeply when minimum wages are high enough to directly bind their wages.

Minimum Wage Impacts: Savings

Figure 4.7 shows how the average steady asset holdings of unskilled and skilled workers varies with the minimum wage, where the average is taken across the invariant distribution of asset holdings and employment states.[8] We see that, unsurprisingly, the asset holdings of unskilled workers are signif-

[8] Specifically, Figure 4.7 plots $\bar{a}_j(m_w) \equiv \int^{\Upsilon \in \{ue,np,p\}} \int^a a f_j^{\Upsilon}(a) da \, d\Upsilon$, and $\bar{a}(m_w) = \sum^{j \in \{u,s\}} \bar{a}_j(m_w) \ell_j$.

Figure 4.6: Variance of Wages and Earnings, by Skill

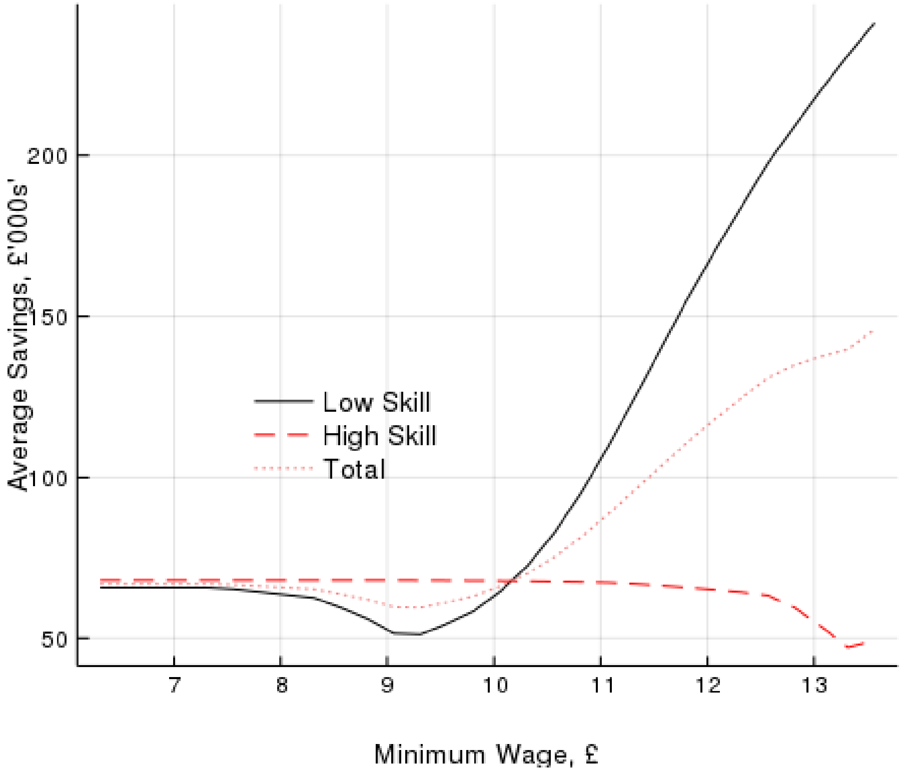


icantly more responsive to minimum wages than skilled workers. Two forces shape the savings response of unskilled workers to higher minimum wage levels: the mechanical decrease in the variance of their wages, and the increase in the variance of their earnings which is caused both by a higher unemployment rate and by an increasing gap between unemployment benefits and wage levels. Initially the decrease in the variance of unskilled workers' wages means they reduce their precautionary savings. However, when the minimum wage is increased to higher levels unskilled workers increase their savings due to the increase in the variance of their earnings.

Skilled workers also decrease their savings initially due to the gradual decrease in the variance of their earnings shown in Figure 4.6. At much higher minimum wage levels the increase in skilled workers' unemployment rates induces them to increase their savings too.

Figure 4.8 provides more detail on the savings response of workers to changes in the minimum wage by showing how the policy function response of workers

Figure 4.7: Savings Response By Skill

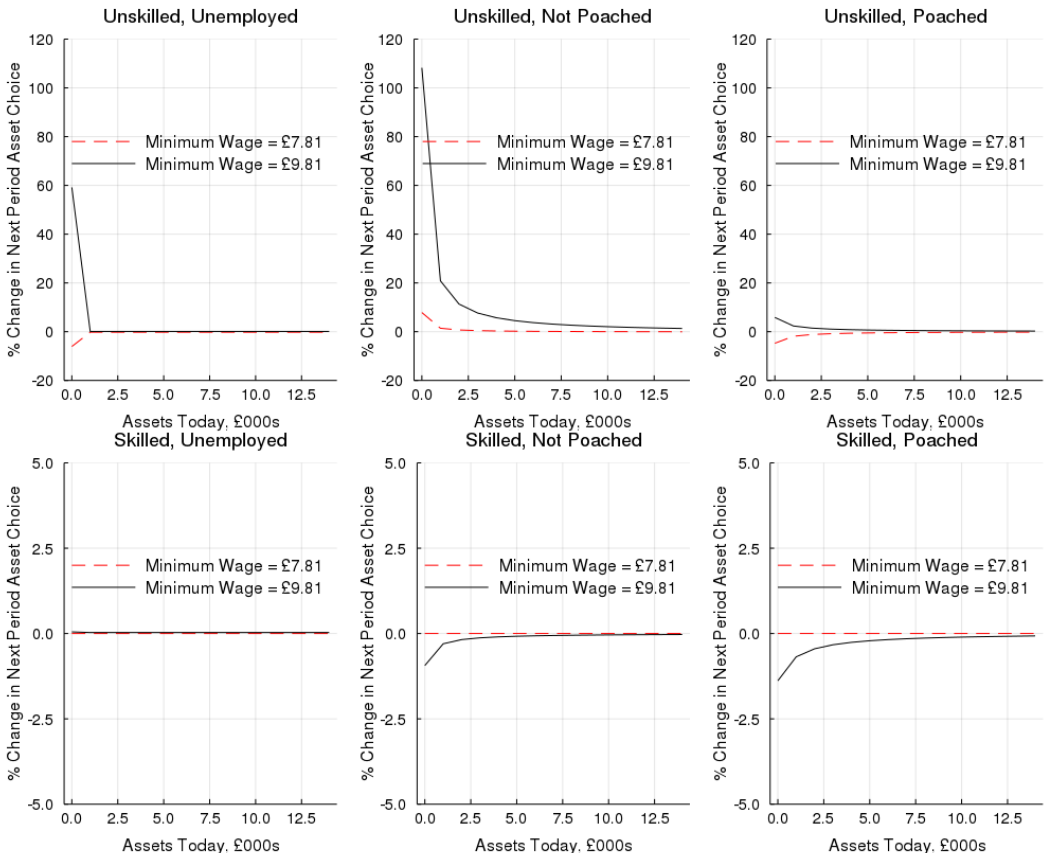


varies with their skill level and employment state. Each subplot shows the percentage change in the workers' choice of next period assets a' (as a function of assets held today, a) relative to their asset choice when the minimum wage is set to its 2013 value (£6.31).[9]

Three findings stand out. First, not-poached unskilled workers are the most responsive to minimum wage changes, which is not surprising given that the minimum wage directly binds their wages but only has an indirect impact on unskilled poached workers and on all skilled workers (except at very high minimum wage values where it is binding for both skill types of workers). Second, moderate increases in the minimum wage induce both the unemployed and poached unskilled workers to save less, due to the decrease in the variance of wages, but more significant increases induce them to save more due to increases in the variance of earnings. In contrast, not-poached unskilled workers save

[9]Specifically, Figure 4.8 plots $\Delta\psi_j^{\Upsilon}(a|m_w) \equiv \frac{\psi_j^{\Upsilon}(a|m_w)-\psi_j^{\Upsilon}(a|m_w^{2013})}{\psi_j^{\Upsilon}(a|m_w^{2013})}$ for each value of the minimum wage m_w, and for all employment states, $\Upsilon \in \{ue, np, p\}$ and skill types of workers.

Figure 4.8: Changes to Savings Policy Functions

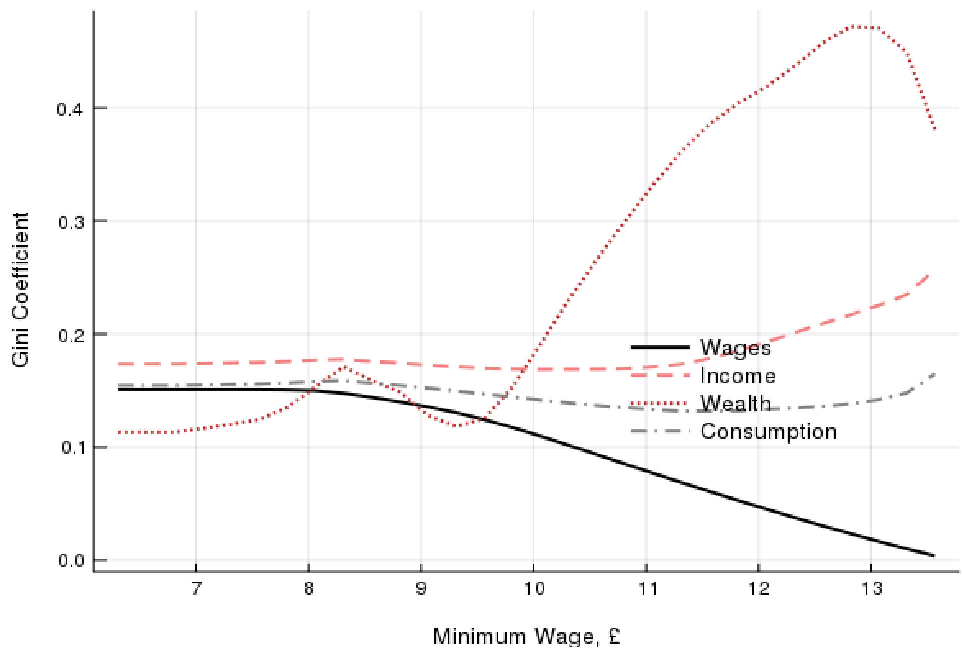


Notes: Each subplot shows the percentage change in the workers choice of next period assets relative to their asset choice when the minimum wage is set to its 2013 value (£6.31).

more in response to both moderate and higher minimum wage increases, suggesting that the increase in the variance of earnings is more relevant to them than the reduction in the variance of wages. Finally, the savings decisions of skilled workers respond only to the higher of the minimum wage values I consider. Both not-poached and poached skilled workers decrease their savings at these minimum wage values because of the decrease in the variance of their earnings caused by the negative spillover impact of higher unskilled unemployment on their wage levels.

To summarise, moderate minimum wage increases causes unskilled workers to decrease their levels of precautionary savings. However, at higher minimum wage levels unskilled workers increase their savings in response to increases in their earnings risk. This pattern is mirrored at higher minimum wage levels

Figure 4.9: Inequality Response to Minimum Wage



for skilled workers, though they decrease their savings at lower minimum wage levels because of spillover impacts from the increased unemployment of unskilled workers. These savings responses are important to understanding the aggregate inequality responses, which are discussed below.

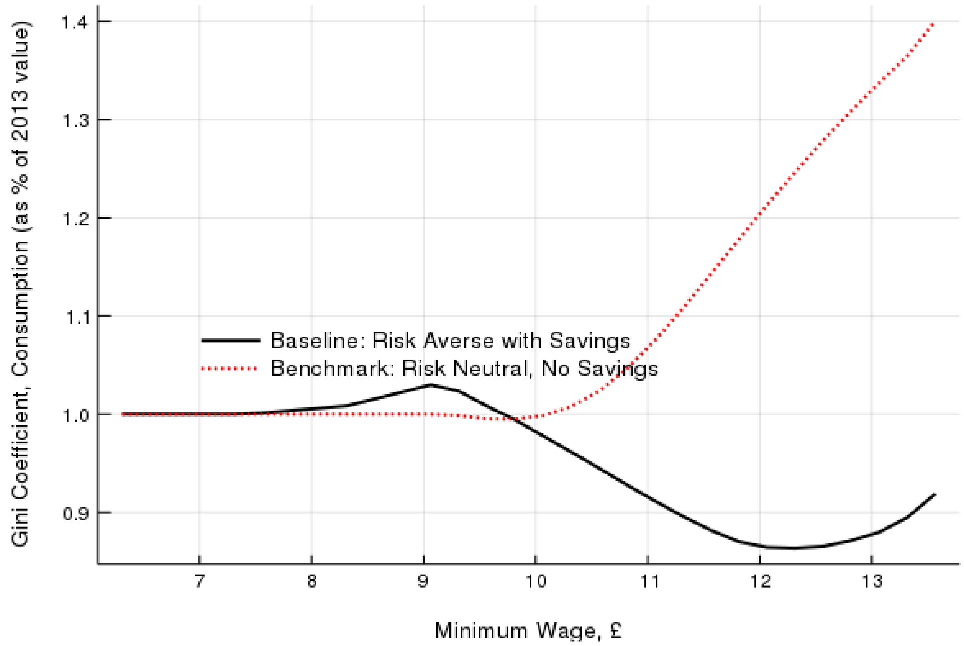
Minimum Wage Impacts: Inequality

Figure 4.9 shows how the gini coefficients for wages, income, wealth and consumption vary with the level of the minimum wage in my model. These measures of inequality are calculated across all workers in the economy i.e. they do not condition on skill type. Wage inequality uniformly decreases with the minimum wage, which reflects a fall in wage dispersion within worker skill types (see Figure 4.6) and a fall in the wage-skill premium induced by the minimum wage (see panel C of Figure 4.5). Income inequality initially falls because of this decrease in wage inequality but then rises as the unemployment rate of unskilled workers increases. Initially wealth inequality rises because unskilled workers decrease their savings from an average level that was already below that of skilled workers. As the unemployment impact of the minimum wage increases unskilled workers increase their savings causing wealth inequality to fall as the average savings level of unskilled workers catches up with the average savings level of skilled workers. As the minimum wage is increased further, wealth inequality increases as the savings of unskilled workers surpass those of skilled workers and continue to rise.

Finally, consumption inequality and income inequality both have a "U" shaped relationship with the minimum wage, which is the net impact of the fall in wage inequality and increases in unemployment rates. However, the turning point of this relationship occurs at a significantly lower minimum wage value for income inequality than for consumption inequality. This reflects the ability of workers to self-insure themselves against increased unemployment risk using asset accumulation. This is a key result of my model since in models without asset accumulation, consumption inequality would increase much sooner.

Figure 4.10 shows the response of consumption inequality in the baseline model developed here compared to a benchmark model with the same production and labour market structure but risk neutral workers with no access to savings (this is the model of Chapter 2, without ability heterogeneity within skill types). In this benchmark model, the response of income inequality and consumption inequality are the same. For moderate levels of the minimum wage, the decrease in wage inequality is almost exactly offset by an increase in unemployment risk to leave consumption inequality in the model without savings broadly flat. At higher minimum wage values, the increase in unemployment risk dominates causing consumption inequality to rise significantly. In contrast, consumption inequality in the model developed here is heavily shaped by the savings responses discussed above. There is a small initial rise in consumption inequality, which mirrors the initial increase in wealth inequality and is driven by the fall in unskilled workers' savings. However, as the minimum wage increases, consumption inequality falls significantly and doesn't start rising until the minimum wage is increased to relatively high values i.e. above £12. The minimum wage therefore appears to be more effective at reducing consumption inequality when one allows for workers to self-insure

Figure 4.10: Inequality Response to Minimum Wage, under different models



with asset accumulation than in models where this is ruled out. [10]

4.5 Conclusion

The introduction of minimum wages, and increases to their value, are often motivated by concern over inequality. A crucial dimension of inequality, at least as it pertains to welfare, is consumption inequality. However existing structural models of the minimum wage tend to assume risk neutral agents who can't save, and have no desire to do so. This limits the scope for analysis of the impact of minimum wages on consumption inequality, since in such models consumption inequality is synonymous with income inequality.

This chapter has developed a model of the minimum wage that features on-the-job search and asset accumulation by workers, alongside a production function with several margins of substitution between factor inputs. I analysed the labour market effects of a minimum wage in a similar model (though without asset accumulation) in Chapter 2 of this thesis. This chapter has built on this analysis by showing that allowing for asset accumulation implies

[10]This conclusion also holds when considering consumption inequality conditional on skill type, rather than inequality for the entire population of workers - see Appendix H.

the minimum wage is more effective at reducing consumption inequality than equivalent models with risk neutral workers would suggest. This is because savings allow workers to self-insure themselves against increases in unemployment and earnings risk generated by the minimum wage, limiting the pass through of these risks to consumption.

However, this conclusion comes with two important caveats. First, my analysis is based on the steady state impact of minimum wages and so does not include the impact of any transition dynamics. This could be significant if an increase in the minimum wage significantly increases consumption inequality along the transition path as workers adjust their savings. However, both unemployment and savings would adjust gradually along the transition path to equilibrium so it is certainly not a given that consumption inequality would increase.

The second caveat is that I have considered the minimum wage in isolation of other policy instruments like taxes and transfers. Considering the efficacy of the mininum wage as a redistributive instrument compared to other policies represents a potentially useful extension to the analysis presented in this chapter.

Chapter 5

General Conclusions

This thesis contributes to the literature on job market frictions, wage inequality and minimum wages. The main methodological contribution of Chapters 2 and 3 is to develop a model that combines search frictions with a richer production framework than typically found in the literature. Specifically I develop a model that features search frictions and on-the-job search and a production technology with several margins of substitution between factor inputs. In my final chapter, I extend this framework further by allowing for workers that are risk averse and can self-insure themselves against wage and unemployment risk using asset accumulation. These contributions have allowed me to explore new aspects of policy debates - e.g. nonlinearities in the unemployment response to the minimum wage, and the role of asset accumulation in determining the impact of the minimum wage on consumption inequality - and examine new explanations for rising wage inequality.

Chapter 2 considered whether there are likely to be significant nonlinearities in the relationship between the minimum wage and unemployment. In the model I develop, nonlinearities are driven by: (i) endogenous nonlinearities in labour demand that arise both from using a multi-input production function and from endogenous vacancy creation; and (ii) exogenous nonlinearities in the distribution of ability across workers. When calibrated to match the UK economy, the model suggests a nonlinear unemployment reaction that bites well within the range of minimum wage levels planned in the UK over the next two years. Of the endogenous mechanisms driving this nonlinearity, I find the quantitative impact of imperfect substitution between workers of differing abilities - a factor not featured in most search models of the minimum wage - is significantly larger than the impact of endogenous vacancy creation.

The research agenda developed in this chapter could be extended in both theoretical and empirical dimensions. The model could be usefully extended by including firm heterogneity, which would allow us to asses the “cleansing” impact of the minimum wage i.e. whether the minimum wage is likely to clear the market of lower productivitiy firms and allow higher productivity firms to expand (Mayneris et al. (2014)). The quantitative implications of the model could be tested empirically to a greater degree by examining whether recent significant increases in the minimum wage in the UK show evidence of a nonlinear impact i.e. with higher employment impacts in regions where the bite of the minimum wage is higher.

Chapter 3 uses a similar combination of frictional labour markets and a production function with capital skill complementarity to propose a model that can help to quantify the relative importance of institutions, labour market frictions and technology in explaining inequality trends. It makes a contribution to the empirical literature on wage inequality, which has tended to focus either on technological explanations for wage inequality, as in Krusell et al. (2000) and Katz and Murphy (1992), or institutional factors as in Card and DiNardo (2002), but hasn’t assessed both factors jointly. Equally relatively little attention has been paid to the impacts of changes in labour market frictions on wage inequality. I developed an empirically testable model that can jointly asses the importance of changes to labour market frictions, institutions and technology. This model combines the production technology in Krusell et al. (2000) with labour market frictions in the form of a sequential auction model of on-the-job search as described in Cahuc et al. (2006).

I took this model to the data to test whether estimates of capital skill complementarity in Krusell et al. (2000) are robust to the inclusion of labour market frictions and find that they are: both models (with and without frictions) produce similar estimates of the strength of capital skill complementarity and are reliant on this channel to match the observed increase in the graduate wage premium.

A useful empirical extension to this line of research would be to estimate the model with matched employee and employer (MEE) data, which would allow identification of changes to bargaining parameters by skill and over time, allowing a better assesment of the impact of changes to institutional settings. Using MEE data would also likely yield more robust estimates of the impact of changes to firm heterogeneity on wage inequality.

Chapter 4 returned to the minimum wage model of Chapter 2 and considered the impact of allowing for asset accumulation by risk averse workers. I found that the workers' ability to self-insure via asset accumulation plays an important role in determining the response of consumption inequality to minimum wage increases.The model predicts that the minimum wage achieves reductions in consumption inequality even when set at relatively high levels that cause unemployment to significantly increase. In a model without savings, increasing the minimum wage level to such levels would increase consumption inequality because increased unemployment risk has a more significant pass-through to consumption inequality.

Introducing asset accumulation into a structural model of the minimum wage also has potential to shed light on other issues such as who pays for the minimum wage, since firm ownership is endogenously determined when asset accumulation is explicitly modeled. While I focused on a baseline of a small open economy, where the interest rate is fixed at a given level, it would be useful to consider the general equilibrium impacts of minimum wage increases under asset market clearing. In this setting, for example, one could examine whether changes to workers' savings in response to minimum wage increases affects the extent to which firms substitute capital for labour.

Appendix A

Parameter Impacts on Model Moments

While the lack of closed form solutions in the model prevents proof of identification, it is nevertheless instructive to explore how varying the magnitude of the parameters I estimate affects the simulated model moments. I do this in Figure A.1, which looks at the impact of varying each of the estimated parameters by plus and minus 25% from its value in my baseline estimation.

First, and somewhat reassuringly, none of the parameters *individually* appear to have observationally equivalent impacts on the model moments i.e. each produce a distinct range of impacts. Of course this does not imply identification, where the more relevant question is whether jointly varying a combination of parameters has observationally equivalent impacts on the model moments as varying any single parameter. Nonetheless, it is instructive to consider how my parameters individually effect the various model moments.

η_u and η_s are the parameters that determine the dispersion of the log normal ability distribution of unskilled and skilled workers respectively. As expected, they have positive monotone impacts on the model moments related to wage dispersion: the standard deviation of log wages, and the p90/50 and p50/10 ratios of the wage distributions. The dispersion parameters also affect median wages, albeit only weakly.

It is notable that increasing dispersion parameter for skilled workers increases the proportion of employees covered by the minimum wage , but not for unskilled workers. A priori this relationship is ambiguous: increase dispersion shifts mass from the centre of the wage distribution to the left and right tails, which means some workers who were paid the minimum wage go into unemployment, lowering the coverage rate, but also shifts mass from slightly higher up the wage distribution closer to the minimum wage, raising the coverage rate. For unskilled workers, the unemployment effect is relatively strong and offsets the inflow of somewhat higher paid workers into the minimum wage. For skilled worker, the latter effect is dominant so minimum wage coverage increases with the dispersion of ability.

The elasticity of substitution between heterogeneous workers of differing abilities also monotonically increases the unemployment rate for a given skill group, but monotonically decreases measures of wage dispersion. The unemployment impact is as expected: as workers become more substitutable, the presence of a fixed minimum wage (it is set at it's 2013-14 level in my baseline calibration) causes greater unemployment of low skilled workers. The intuition behind the decrease in wage dispersion is that as it becomes easier to substitute workers in production there is less of a premium for scarcity, which decreases wage dispersion.[1] The ease of substitution between workers within a given skill type also has strong positive impact on the proportion of these workers being paid the minimum wage for both skill groups.

The bargaining parameter is the most directly relevant on the model predictions for minimum wage coverage, in the sense that, as expected, it has a relatively strong monotone negative effect on minimum wage coverage, but only a relatively weak impact on other moments.

Perhaps surprisingly, the cost of vacancy posting has a strong positive effect on minimum wage coverage and a relatively weaker impact on unemployment. This reflects the presence of on-the-job search in the model. Increasing the

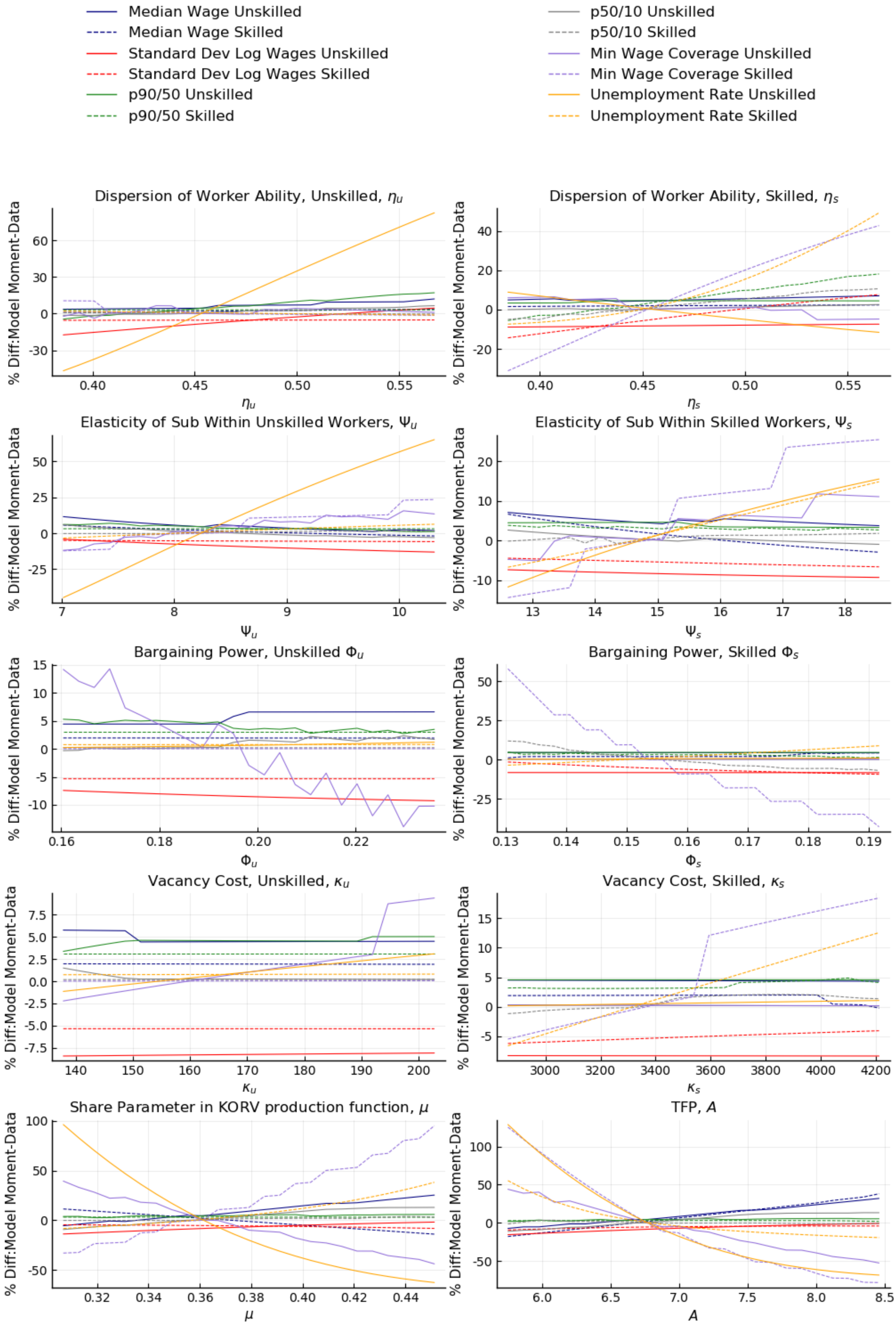
[1]This relationship is also ambiguous a priori: increasing the elasticity of substitution decreases the scarcity premium both for scarce high ability workers, and scarce low ability workers. The latter impact could in theory be dominant and raise wage dispersion but does not in my calibration.

vacancy cost means lower job contact rates for unemployed and employed workers (job contact rates for employees are directly proportional to contact rates for unemployed workers by assumption). Since job contacts raise employees' wages, a reduction in the contact rate shifts the wage distribution to the left and so increases minimum wage coverage. The relatively weak impact of vacancy costs on unemployment reflects the fact that a substantial part of unemployment is caused by the impact of the minimum wage on demand for intermediate goods by final good producers and that the cost of vacancy creation does not have a significant impact on this relationship.

Finally the TFP and the share parameter, A and μ in the KORV production function have the expected impacts: TFP increases wages and employment for both unskilled and skilled workers, whereas μ, which determines the output share of the unskilled intermediate sectors, improves wage and employment outcomes for the unskilled at the expense of the skilled.

The assumption that ability is log normally distributed plays an important role in allowing me to use the chosen empirical moments to discipline my parameter estimates, especially with regard to using the minimum wage coverage rate to discipline the bargaining parameter, as discussed in Flinn and Heckman (1982). The model would likely be severely under-identified if we allowed the ability distribution to take a more flexible non-parametric form. However, the model's ability to match the wage distribution very closely - see Figure 2.3 - suggets the assumption of a log normal distribution is a reasonable one.

Figure A.1: Parameter Impacts on Model Moments



Appendix B

Minimum Wage impacts: Wage Spillovers

I define minimum wage spillovers to mean any change in the shape of the wage distribution above the minimum wage, other than a purely mechanical truncation effect. [1]

As discussed above, the fact that the minimum wage acts as a side constraint on the Nash bargained wage rules out wage spillovers due to pure bargaining impacts. However, changes to labour demand in the model generate both *within* and *between* group spillovers, where group is defined both by the skill level of the worker and their heterogeneous ability type. I again index these groups by j.

The imposition of a minimum wage, m_w, generates within group spillovers for type j workers whenever the minimum wage is in the range $w_j^b < m_w < p_j$. In this scenario, not poached type j workers receive the minimum wage rather than their Nash bargained wage i.e. there is no spillover within a given employment state. The resulting wage increase means intermediate firms reduce their vacancy creation relative to a counterfactual scenario with no minimum wage, which increases the unemployment rate until the equilibrium condition

[1] see e.g. Flinn (2002) - a mechanical truncation impact of the minimum wage occurs whenever the minimum wage decreases aggregate unemployment, and therefore increases the wage density for all remaining employed workers even if their employment levels are unchanged.

shown in equation **??** again holds. This generates a wage spillover for poached workers who see their wage increase, since it equals the marginal product of the intermediate good they produce which rises as employment falls. Thus despite their wage initially exceeding the minimum wage, poached workers will still see their wage increase due to the imposition of the minimum wage i.e. there is a positive spillover. However, reduced vacancy creation decreases job-to-job mobility rates, so although the poached workers see their wages increase, the density of such workers decrease. The net impact on the within group wage distribution depends on the relative magnitude of the positive spillover from the increase in the poached wage and the negative spillover from reduced job mobility rates.

Between group spillovers are generated because raising the price of one type of worker via the minimum wage always alters demand for all other types. The direction of spillovers between workers of different skill and ability types will depend on the degree of complementarity in a given calibration of the production function.

Appendix C

Sources of Nonlinearities

I isolate the sources of the nonlinear unemployment response in my model by simulating results from three alternative models. Each of these alternative models has a different feature removed from the baseline model. The three features that cause the nonlinear unemployment response in my baseline model are imperfect substitution between factor inputs, endogenous vacancy creation and the non-uniform distribution of workers' abilities. Accordingly, the first of the three alternative models I discuss here has perfect substitution between factor inputs, where factor inputs are high and low skill labour of varying ability types. This alteration removes imperfect substitution of factor inputs as a driver of the nonlinear unemployment response (but otherwise the model is as per my baseline model). The second model I discuss removes endogenous vacancy creation from the baseline model but keeps all other factors the same. Finally, I discuss a version of my model with a uniform distribution of worker abilities, again keeping all other features as per my baseline model. In each case, I will describe exactly how the change is implemented, how it affects the solution algorithm I use to solve for equilibrium in the model, and any necessary changes to calibrated parameters.

C.0.1 Sources of Nonlinearities: Imperfect Substitution

I consider the impact of allowing imperfect substitution by constructing an alternative model with perfect substitution between factor inputs. In this alternative model, the final good production function becomes $Y =$

$\sum_{i=1}^{M} x_{u,i}y_{u,i}+\sum_{i=1}^{M} x_{s,i}y_{s,i}$, where as before $y_j = (1-e_j^{ue})\ell_j$. The only price consistent with non-zero equilibrium employment rate in the intermediate goods sector is $p_j = x_j$. If the price is above this point, final good producers will not demand any y_j. If $p_j < x_j$ final good producers demand an infinite amount of y_j, which is not consistent with equilibrium as this implies zero unemployment at which point recruitment costs for intermediate firms are infinite. In this environment, which is the standard production function assumed in most search models, minimum wages have a cliff-edge impact: when m_w exceeds x_j, employment of that type falls to zero.

The solution algorithm I use to solve this alternative model is as follows (italicised text emphasises differences from our baseline algorithm):

1. Guess the unemployment rate e_{j0}^{ue}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$.

2. Set $p_j = max(m_w, x_j)$. *Unlike in baseline model, p_j is now independent of the unemployment guess e_{j0}^{ue}*

3. Use the conditions 4.16 and 4.17 to derive vacancy levels necessary for the unemployment guess $e_{j,0}^{ue}$ to be consistent with steady state in the labour market. This then implies employment transition probabilities for the unemployed and employed via the matching function: $\theta_j q(\theta_j)$ and $\chi_j\theta_j q(\theta_j)$ respectively.

4. Use employment transition probabilities from above and condition that poached worker is paid $w_j^p = max(p_j, m_w)$ to solve worker value functions and Nash bargained wage using equations 4.3 to 4.5 and 4.15 respectively. Wage of not-poached worker is whatever is highest of this bargained wage and minimum wage.

5. Update employment guess:

   (a) *If $m_w > x_j$, then set $e_{j1}^{ue} = 1$. Note in baseline model, $e_j^{ue} = 1$ is not consistent with equilibrium as the intermediate good has infinite marginal product at zero.*

(b) If $m_w <= x_j$, use wage levels from above steps to give an updated unemployment guess, e^{ue}_{j1}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$ that simultaneously solves free entry condition 4.14 for the intermediate firm and the final good firm's FOC i.e. equations 2.18 and $p_j = x_j$.

6. Repeat iteration until convergence of unemployment guess.

In the simulations presented in Figures 2.5 and 2.6 for this alternative model, I use exactly the same parameters as under my baseline calibration with one exception. I impose that the ability levels x_j in this alternative model are exogenously set at the price of intermediate goods in the baseline model when there is no minimum wage. This ensures that in the absence of the minimum wage, this alternative model predicts the same wage and unemployment levels as in the baseline model.

C.0.2 Sources of Nonlinearities: Endogenous Vacancy Creation

In my baseline model, intermediate firms respond to a binding increase in the minimum wage by reducing vacancy creation. Remaining vacancies are filled at a higher rate, which reduces recruitment costs (and increases the price of the intermediate good) until the point where the expected profits from issuing a vacancy are again zero. In the alternative model considered here, I effectively assume the supply of vacancies is completely inelastic so that, in the absence of a minimum wage, contact rates for unemployed and not-poached workers are fixed at a level $\lambda^*_{0,j}$ and $\lambda^*_{1,j}(=\chi_j\lambda^*_{0,j})$ respectively which imply an unemployment rate e^{ue*}_j (I will describe how I calibrate $\lambda^*_{0,j}$ shortly). When the minimum wage is imposed then, if $p_j(=\partial Y/\partial y_j(e^{ue*}_j)) \geqslant m_w$ then the unemployment rate and contact rates are unchanged. If $p_j(=\partial Y/\partial y_j(e^{ue*}_j)) < m_w$ then the unemployment rate and contact rates exogenously adjust until the marginal product of the intermediate good is raised to the level of the minimum wage. This model therefore does not include the fall in recruitment costs from endogenous vacancy creation as a force restoring the intermediate goods market to equilibrium following a minimum wage change.

The solution algorithm I use to solve this alternative model is as follows (italicised text emphasises differences from our baseline algorithm).

1. Guess the unemployment rate $e^{ue}_{j0} = e^{ue*}_{j}, \forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$.

2. Use this guess to construct the aggregate output of intermediate goods produced in the unskilled and skilled intermediate sectors (these aggregate outputs, U and S, are defined in equation **??**).

3. Solve the final good firm's FOCs (equations 4.9 and 4.10) to get their optimal choice of capital equipment and structures, K_{eq} and K_{st} that is consistent with the implied levels of U and S from above and firm optimisation. Then derive the price of each intermediate good p_j that is consistent with firm optimisation at the unemployment guess $e^{ue}_{j,0}$ using the FOCs in equations 4.7 and 4.8.

4. Use the steady state condition that $\delta_j(1 - e^{ue}_j) = \lambda_{0,j} e^{ue}_j$, to derive job contact rates for the unemployed and not-poached workers as a function of the unemployment guess e^{ue}_{j0}.

5. Use job contact rates from above and condition that poached worker is paid $w^p_j = max(p_j, m_w)$ to solve worker value functions and Nash bargained wage using equations 4.3 to 4.5 and 4.15 respectively. Wage of not-poached worker is whatever is highest of this bargained wage and minimum wage.

6. Update employment guess:

    (a) *If $m_w > p_j$, then set e^{ue}_{j1} at the level that equates $p_j(= \partial Y/\partial y_j(e^{ue}_j))$ with m_w.*

    (b) *If $m_w <= p_j$, then $e^{ue}_{j1} = e^{ue}_{j0}$.*

7. Repeat iteration until convergence of unemployment guess.

In the simulations presented in Figures 2.5 and 2.6 for this alternative model, I use exactly the same parameters as under my baseline calibration with one exception. I impose that the contact rates for the unemployed and employed, $\lambda^*_{0,j}$ and $\lambda^*_{1,j}$, equal the endogenously determined contact rates in the baseline model when there is no minimum wage. This ensures that in the absence of the minimum wage, this alternative model predicts the same wage and unemployment levels as in the baseline model.

C.0.3 Sources of Nonlinearities: Non-uniform distribution of ability types

In this alternative model, I do not alter any of the fundamental mechanisms of the baseline model but simply impose that $x \sim U(x_{min}, x_{max})$ where the boundaries of this interval are the same as under my baseline calibration, which are the same for unskilled and skilled workers. The equilibrium definition and solution algorithm remain as in the main body of this chapter.

Appendix D

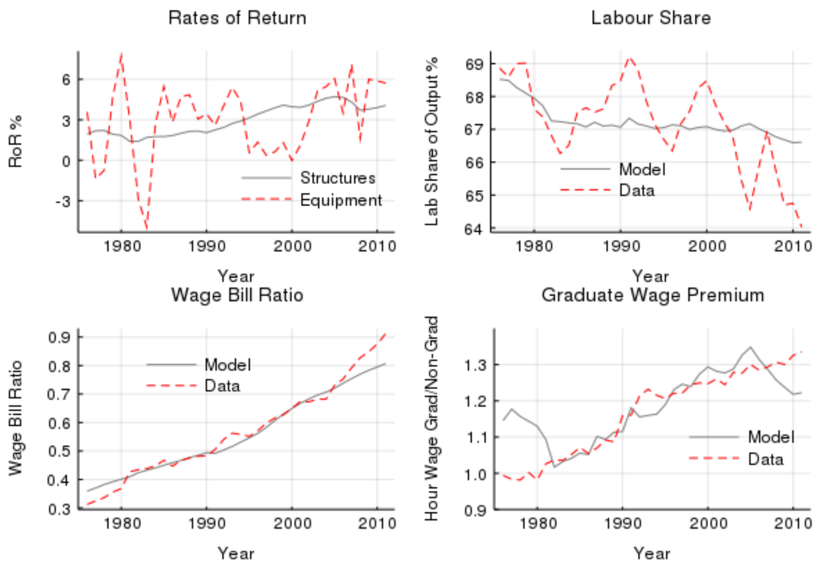
Model Results and Bargaining Power

This appendix discusses the sensitivity of my results to the choice of the bargaining parameter. In my baseline estimation I impose a high level of bargaining power for both worker types ($\beta_u = \beta_s = 0.95$). I find that when I set the bargaining parameter at significantly lower levels, i.e. 0.75 or 0.5, and estimate the parameters of the KORV production function there is an acute tension between the model's ability to match both the rise in the graduate wage premium and the level of the labour share of output. The rest of this appendix explains this tension and its quantitative impact. Overall I find that only when I assume a relatively high bargaining parameter are I able to satisfactorily match the relevant trends in the data.

I first consider the intuition for why there might be a tension between matching the rise in the graduate wage premium and level of the labour share at lower levels of the bargaining parameter. First recall that the original, competitive, version of the KORV model is relatively successful at matching both the rise in the graduate wage premium and the labour share: see Figure D.1. When I introduce the sequential auction model into this set-up, average wages will now be lower than the marginal product of labour if the bargaining parameter is significantly less than unity and for realistic job contact rates. In other words, the labour share will be lower in the model with frictions than in the original

KORV environment for a given set of production function parameters. When I estimate the KORV parameters in my frictional labour market model, and have a low level of bargaining power, the estimation approach compensates for the downwards pressure this puts on the labour share by making labour more important (and capital less important) in the production of output. However, this jeapordises the ability of the model to match the graduate wage premium since the increased use of capital equipment is the main channel that pushes the wage premium up.

Figure D.1: KORV with perfect competition



To illustrate this quantitative impact of this tension, let us consider estimates of the KORV production function parameters when I set the bargaining parameter to 0.5 for both unskilled and skilled workers - see Table D.1 and Figure D.2. The estimate of α, the exponent of capital structures (K_{st}), hits the zero lower bound, and it also delivers lower levels of λ, the coefficient of capital equipment since this too increases the labour share. However a lower level of λ limits the channel of capital skill complementarity - see equation 3.9 - and means that though the model can fit the labour share to a reasonable approximation, it completely misses the rise in GWP: see Figure D.2. Indeed the fit is much worse than that of the purely competitive set-up in KORV: see Figure D.1.

Table D.1: KORV parameter values: bargaining parameter impact

Parameter	No Frictions (KORV)	Baseline ($\beta = 0.95$)	$\beta = 0.5$
λ	0.568	0.507	0.17
μ	0.806	0.644	0.401
α	0.091	0.083	0.0
γ	-0.209	-0.188	0.043
σ	0.352	0.33	-0.166

Increasing the bargaining parameter from 0.5 (Figure D.2 is based on this) to 0.95 improves the results significantly - see Figure D.3. While much of the micro evidence points to much lower levels of the bargaining parameter, generally such estimates are highly model dependent.

Figure D.2: KORV with frictions: $\beta = 0.5$

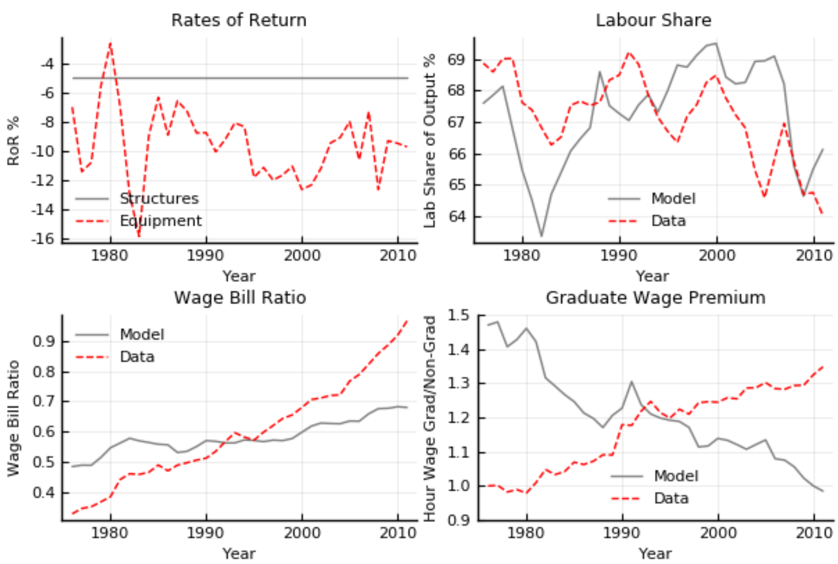
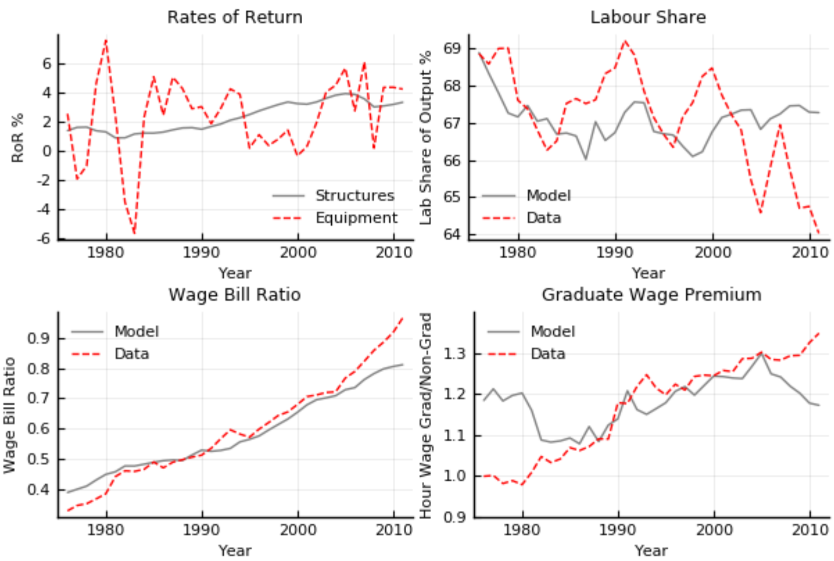


Figure D.3: KORV with frictions - baseline bargaining power: $\beta = 0.95$



Appendix E

Identification

There are two sets of parameters to identify in my model: the parameters of the KORV production function, and those in the sequential auction model of the labour market. While Krusell et al. (2000) do not explicitly discuss identification in their paper, they do refer to the results of a companion empirical paper Ohanian et al. (1997) which shows their estimation strategy is succesful at identifying the true parameters in Monte Carlo simulations. As my estimation of the parameters of the KORV production function very closely follows their method, and is done separately and subsequently to estimation of the sequential auction parameters, I do not repeat that exercise here and instead rely on their identification results.

The sequential auction structure of the labour market in my model is no different from Cahuc et al. (2006), however I use employee reported data (from the CPS) to estimate the relevant parameters, whereas Cahuc et al. (2006) used matched-employee-employer (MEE) data. I chose to use CPS data because a key motivation for this paper is to test the robustness of findings in Krusell et al. (2000) to incorporating frictions; I therefore sought to maintain as much consistency as possible to their estimation approach which used CPS data for wages and labor input. However, the MEE data that Cahuc et al. (2006) use plays a key role in their identification strategy so it is worth considering whether the parameters I wish to identify in the sequential auction model are identified when using employee data only.

First bargaining parameters by worker skill level are much more difficult, if not impossible, to identify without some form of employer information. In the absence of such data, match output or suplus becomes much more difficult to estimate and hence reliable estimates of bargaining parameters are not readily available. This is why I choose to set bargaining parameters by assumption.[1] The remaining objects of interest in the sequential auction model are job contact rates, (note job destruction rates come straight from the data) and the distribution of match quality, where I will consider the possibility of both non-parametric and parameteric identification.

E.0.1 Job Contact Rates

There are two job contact rates in the sequential auction model for each skill type of worker: those for the unemployed and employed: $\lambda_{0,i}$ and $\lambda_{1,i}$ respectively. $\lambda_{0,i}$ determines the unemployment rate and, because it influences the outside option of workers, the minimum match quality of firm that a worker will accept an offer at. However, the unemployment rate does not play a role in the estimation of the KORV parameters (since labour input is total hours worked by workers and is taken straight from the data) or in the estimation of any other parameters in the sequential auction model, and I will estimate the lower bound of acceptable match quality directy, as described in the next section. I therefore have no need to estimate $\lambda_{0,i}$.

I instead focus on estimation of $\lambda_{1,i}$, which is key for determining both average match quality, and average wages of worker of a given skill type. Both variables play a role in estimating the parameters of the KORV production function, as described in section 3.5.3.

I estimate $\lambda_{1,i}$ using SMM and targeting the proportion of continuous employed workers in a given year who have moved employers at least once. I denote this proportion τ_i. In the model, the expression for this moment is given in equation E.1, which is obtained by substituting the expression for the

[1] The analysis of Appendix D suggests the labour share in the economy is informative about the average bargaining power of all workers, though would not help to estimate bargaining parameters by skill level.

cross section distribution of match quality in equation 3.21 into equation 3.29.

$$\tau_i = 1 - \int_{\nu_{inf_i}}^{\nu_{max}} (1 - \lambda_{1,i,}\bar{F}_i(\nu))^{1two} \frac{1 + \kappa_{1,i}}{[1 + \kappa_{1,i}\bar{F}_i(\nu)]^2} f_i(\nu) d\nu \tag{E.1}$$

As I are estimating $\lambda_{1,i}$ separately, and prior to, the estimation of the match quality distribution F, I require that equation E.1 is independent of F. This can be proven by integrating by change of variable i.e. if I let $r = \bar{F}_i(\nu)$ so that $\frac{dr}{d\nu} = -f(\nu)$ the expression for τ becomes as shown in equation E.2, which is independent of F.

$$\tau_i = 1 - \int_0^1 (1 - \lambda_{1,i,t} r)^{12} \frac{1 + \kappa_{1,i}}{[1 + \kappa_{1,i} r]^2} dr \tag{E.2}$$

It is then a simple matter to show that this expression is monotonically increasing in $\lambda_{1,i}$ (recalling that $\kappa_{1,i} \equiv \lambda_{1,i}/\delta_i$), which given the quadratic objective function in SMM proves identification of $\lambda_{1,i}$.

E.0.2 Distribution of Match Quality

There are two considerations when discussing identification the distribution of match quality. First, I need to consider whether the distibution can be non-parametrically identified or not. I will argue that it can be, but only by relying heavily on the structure of the model. Therefore when it comes to estimation I prefer to assume a log normal distribution of match quality. I will estimate the parameters of the match quality distribution by targeting moments of the empirical wage distribution. However, higher order moments of the wage distribution in the model are not tractable, hindering an analytical proof of identification, so I instead present evidence from Monte Carlo simulations that my estimation strategy can identify the “true” parameters of the match quality distribution.

I start by showing that, in theory, the match distribution could be identified non-parametrically. Consider the expression for the wage earned by worker of skill type i, whose current employer has match quality ν^+ and whose outside option match quality (the second highest quality match they've had contact with) is ν^- as shown in equation E.3. It is immediately clear that if I were to use the empirical wage distribution to try and identify ν^+ I encounter the problem that wages depend not only on ν^+ but ν^- so I can't simply invert equation E.3 to back out the quality of the current match, ν^+. Note I assume that I do know the other parameter values in the equation due to the identification arguments presented above for job contact rates, and because other parameters either come straight from the data, like job destruction rates, or are set by assumption, like bargaining parameters.

$$\phi(p_i, \nu^-, \nu^+) = p_i \left(\nu^+ - (1-\beta) \int_{\nu^-}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx \right) \tag{E.3}$$

However, I do know that all employees who were unemployed in the previous period and then get a job (I refer to these workers as entrants) have a common level of ν^-, which equals ν_{inf_i}, the lower bound of the match quality distribution. Entrants will therefore be paid the wage shown in equation E.4.

$$\phi(p_i, \nu_{inf_i}, \nu^+) = p_i \left(\nu^+ - (1-\beta) \int_{\nu_{inf_i}}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx \right) \tag{E.4}$$

I argued previously that if the bargaining parameter is high enough to guarantee that wages are an increasing function of the employer's match quality (which is the case in my baseline), then ν_{inf} is identified as the lower bound of wages in the empirical wage distribution. Therefore, in principle, I could identify the distribution of ν by inverting equation 3.13 for each wage in the empirical distribution of entrants' wages. This inversion can be done as follows: I start by letting $w = \phi(p_i, \nu_{inf_i}, \nu^+)$ and differentiating w with respect to ν^+ to get:

$$\begin{aligned}
\frac{dw}{d\nu+} &= p_i\left[1-(1-\beta)\frac{\rho+\delta_i+\lambda_{1,i}\bar{F}_i(\nu^+)}{\rho+\delta_i+\lambda_{1,i}\beta\bar{F}_i(\nu^+)}\right] \\
&= p_i\left[\frac{\beta(\rho+\delta_i)+(two\beta-1)\lambda_{1,i}\bar{F}_i(\nu^+)}{\rho+\delta_i+\lambda_{1,i}\beta\bar{F}_i(\nu^+)}\right] \\
\implies \frac{d\nu+}{dw} &= \frac{1}{p_i}\frac{\rho+\delta_i+\lambda_{1,i}\beta\bar{F}_i(\nu^+)}{\beta(\rho+\delta_i)+(two\beta-1)\lambda_{1,i}\bar{F}_i(\nu^+)} \qquad \text{(E.5)}
\end{aligned}$$

Further note that under the assumption I have made about the bargaining parameter, a worker's wage is an increasing function of the match quality of their employer (ν_+), which implies that $\bar{F}_i(\nu^+) = \bar{F}_i^w(w(\nu^+))$. This is helpful since, while $\bar{F}_i(\nu^+)$ is not observable in the data, $\bar{F}_i^w(w(\nu^+))$ is. Substituting $\bar{F}_i(\nu^+) = \bar{F}_i^w(w(\nu^+))$ into equation E.5 I can then derive an expression for $\nu+$ in terms of w by solving this differential equation.

However, this relies heavily on the structure of the model and, moreover, on part of the structure - the entrant wage distribution - that was not a particular focus of Cahuc et al. (2006). I therefore choose to make a parametric assumption for the distribution of match quality, and assume it is log normal.

I must now show that I can identify the parameters of this log normal distribution i.e. the mean parameter, ζ_i, the variance parameter, η_i and the lower bound ν_{inf_i}. Recall that my estimation of these parameters is based on a SMM approach as summarised in equation E.6, where $\underline{w_i}$ is the lowest wage in the wage distribution, $Q^{50}_{w_i}$ is the median wage and $\mathbb{E}^{F_{i,t}}(\nu)$ is the mean of the match quality sampling distribution, which will be targeted at a fixed value (I impose $\mathbb{E}^{F_{i,t}}(\nu) = 1$).

$$\begin{aligned}
(\,\zeta_{i,t}^{\nu*}, \eta_{i,t}^{\nu*}, \nu^*_{inf_{i,t}}) &= \underset{\zeta_{i,t}^{\nu},\eta_{i,t}^{\nu},\nu_{inf_{i,t}}}{\operatorname{argmin}} \; (m_t-\hat{m}_t)^T W (m_t-\hat{m}_t) \qquad \text{(E.6)}\\
m_t \equiv \big(&var_{\log(w_{i,t})}(\zeta_{i,t}^{\nu}, \eta_{i,t}^{\nu}, \nu_{inf_{i,t}}), \underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta_{i,t}^{\nu}, \eta_{i,t}^{\nu}, \nu_{inf_{i,t}}),\\
&\mathbb{E}^{F_{i,t}}(\nu)(\zeta_{i,t}^{\nu}, \eta_{i,t}^{\nu}, \nu_{inf_{i,t}})\big)
\end{aligned}$$

Proof of identification is hindered by the lack of tractability of the higher order moments of the wage distribution i.e. consider the expression for the second moment of the wage distribution $\mathbb{E}(w_i^2)$, which is given in equation E.7 and already incorporates two simplifying assumptions: $\beta = 0$ and $\rho = 0$.

$$\mathbb{E}(w_i^2) = p_i^2 \int_{\underline{\nu}}^{\nu_{max}} \Bigg[\nu^2 - 2[1+\kappa_{1,i}\bar{F}_i(\nu)]^2 \times \int_{\nu_{inf}}^{\nu} \{q - \int_q^{\nu} (\kappa_{1,i}\bar{F}_i(x)dx)\} \frac{dq}{1+\kappa_{1,i}\bar{F}_i(q)}\Bigg] \frac{1+\kappa_{1,i}}{[1+\kappa_{1,i}\bar{F}_i(\nu)]^2} f_i(\nu)d\nu \tag{E.7}$$

Given the intractable nature of this expression, I test whether my estimation procedure correctly identifies the true parameters of the model by Monte Carlo methods. That is I simulate a cross-section sample of wages for 50,000 workers (i.e. slightly less than the 60,000 that feature in the CPS) from the model with an arbitrary choice of parameters (henceforth the "true" parameters). I then estimate the model using this simulated data to see if I recover the true parameters.

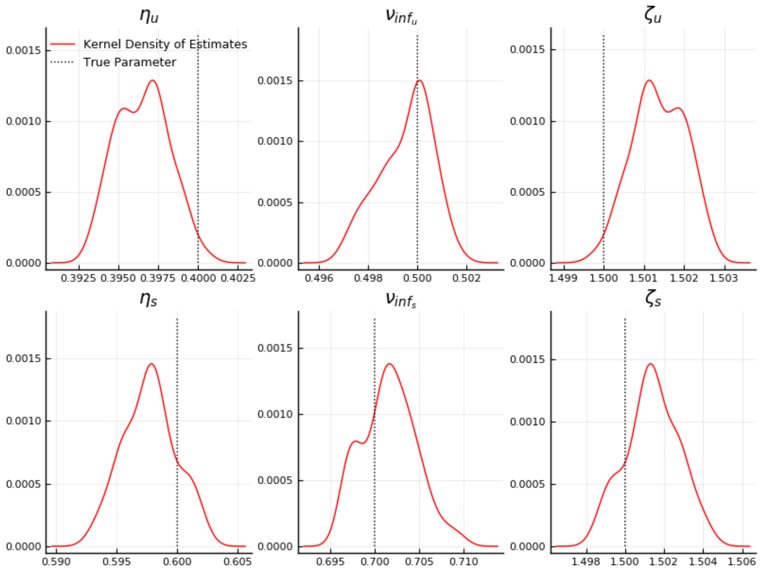
Before considering results, recall that I estimate the lower bound of the match quality distribution by targeting the ratio of the lower bound of wages in my sample relative to the median. As argued above this gives exact identification of the $\nu_{inf,i}$. I therefore feed the true parameter for the lower bound of the match quality into my estimation procedure directly, since it is exactly identified, rather than the mininum of simulated wages i.e. I set $\underline{\hat{w}_{i,t}}/Q^{50}_{\hat{w}_{i,t}}$ - the empirical moment I are targeting - to $\nu_{inf_{i,t}}/Q^{50}_{\hat{w^{sim}}_{i,t}}$, where the superscript sim denotes simulated wage data.[2]

The results of my Monte Carlo simulation exercise - where I estimate 50 sets of parameters (corresponding to 50 simulations of data from the true model) - are shown in Figure E.1. I see that my estimation strategy is reasonably

[2]As in my actual empirical estimation of the sequential auction parameters, I normalise the price of the intermediate good, p_i to one when performing the Monte Carlo test of identification.

successful in recovering the true parameters, though not perfect: while there are some biases in the estimates, in each case they are very small in size.

Figure E.1: Monte Carlo Analysis of Identification



Appendix F

Robustness

This section tests the robustness of parameter estimates of the KORV production function in my model to changes to my empirical strategy for estimating the parameters of the sequential auction model of the intermediate goods markets. In particular, I consider the impact of: (i) estimating the lower bound of the match quality distribution by targeting the average wage of workers in the first percentile of the wage distribution (rather than average wage of the bottom two percentiles) - see column 4 of Table F.1, (ii) estimating the lower bound of the match quality distribution by targeting the average wage of workers in bottom 5 percentiles - see column 5, (iii) estimating the variance parameter of the (log normal) sampling distribution of match quality by targeting residual wage variance, where I will now control for age as well as race, sex and years of education in calculating this residual variance - see column 6, (iv) estimating the variance parameter of the sampling distribution by targeting the interquartile range of residual log wages, rather than the variance - see column 7

Table F.1: KORV parameter values with frictions: robustness

(1) Parameter	(2) Without Frictions	(3) With Frictions: Baseline	(4)	(5)	(6)	(7)
λ	0.568	0.507	0.504	0.506	0.507	0.505
μ	0.806	0.644	0.457	0.405	0.277	0.488
α	0.091	0.083	0.083	0.083	0.083	0.083
γ	-0.209	-0.188	-0.186	-0.187	-0.188	-0.184
σ	0.352	0.33	0.329	0.331	0.33	0.322
$\varepsilon_{S,K_{eq}}$	0.827	0.841	0.843	0.842	0.841	0.844
$\varepsilon_{U,K_{eq}}$	1.544	1.493	1.49	1.495	1.493	1.474
CSC Strength: $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$	**0.716**	**0.651**	**0.646**	**0.653**	**0.651**	**0.63**

Appendix G

Bargained wages, Wealth and Skill

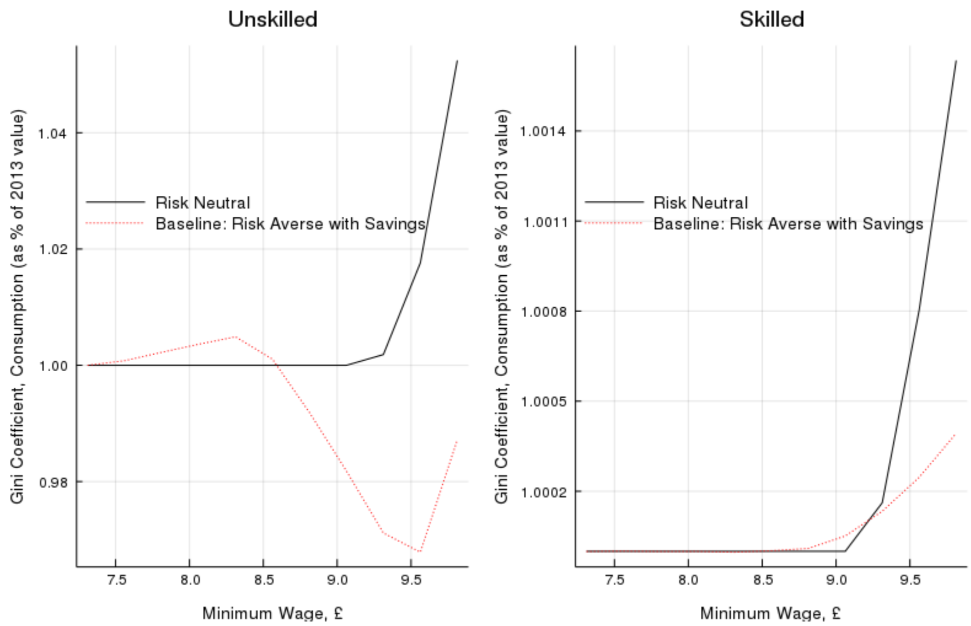
If I had opted for pure monoposny model, i.e with $\beta = 0$, then not-poached wages (effectively reservation wages) would be less than unemployment benefits for both types of workers as both worker types would be willing to pay a price to enter the labour market so that they can eventually earn the poached wage. skilled workers would be willing to pay a higher price, as they have a higher poached wage, and hence would have lower reservation wages then low skill workers.

Further, the fact that workers would receive less in their not-poached state than in unemployment would mean the not-poached wage decreases with wealth for both worker skill types, under pure monopsony. This is because increasing wealth has two opposing effects on the not-poached wage level: on the one hand it increases unemployed workers expected lifetime utility, which means they require a higher wage to enter employment. On the other hand, it also increases their lifetime utility from being employed at a given wage which puts downward pressure on the reservation wage. If not-poached wages are always paid less than the unemployment benefit - as is the case under pure monopsony - decreasing marginal utility means the gain in lifetime utility from being unemployed with a higher asset level is less than the gain when workers are not-poached, so the not-poached wage decreases with wealth.

Appendix H

Consumption Inequality Conditional on Skill Type

Figure H.1: Inequality Response to Minimum Wage



None of these changes to my empirical strategy make a significant difference to my results.

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