288

MEASURING VISIBILITY TO URBAN FUNCTIONS WITH SOCIAL MEDIA DATA

YAO SHEN1,2, STEPHEN LAW3,4, KAYVAN KARIMI4

1 Department of Urban Planning, Tongji University, 1239 Siping Road, China

2 The Bartlett Centre for Advanced Spatial Analysis, University College London, 90 Tottenham Court Rd, London, W1T 4TJ, UK

3 Alan Turing Institute, 96 Euston Road, London, NW1 2DB, UK

4 Space Syntax Laboratory, Bartlett School of Architecture, University College London, Gower Street, WC1E 6BT, UK

ABSTRACT

Urban public space is not only characterised by the connection of visible spaces, but also featured by the visible functions, thereby forming the different spatial atmosphere that humans can interpret. In this article, public spaces are conceptualised as a set of viewsheds/isovists that are further used to generate visibility graph (VG) of mutual visibility between spatial locations and functional places. By adding land-use locations into the visibility graph, the current Visibility Graph Analysis (VGA) measures in the space syntax model can be extended to form a novel approach to investigate the fine-scaled spatio-functional interactions though public space in the dense built environment. In so doing, a framework called 'Function Visibility Graph Analysis (FVGA) is introduced, in which a series of measures are introduced to reflect various respects of viable landscape. The public spaces are further grouped to function spaces, for describing the functional identity for urban spaces with reference to the compositions of the visibility for various types of functions. All these measures of the graph are accomplished of comparing location to location within a system, comparing systems with different spatial layouts or land-use patterns. It is shown that FVGA can be effective to assess the feasibility of detail urban design and land-use allocations.

KEYWORDS

Visibility graph, land-use system, spatial configuration, isovist analysis, urban design

1. INTRODUCTION

In this study, the concept of 'isovist' of space – the convexly visible space from a location is extended to a fruitful concept of 'functional isovist' – the geometrically inter-visible urban activities from a location, so that the visual experience of people through open spaces in the city can be quantified and mapped more comprehensively. This extension, not only produces valuable insights to traditional visual graph analysis, but minifies its potential boundary effects and enhances its predictability on human flows and description of the transformation of the visible urban structures featured by urban form and function simultaneously.

One essential challenge in modern urban design is discovering how the change of people's traveling behaviours and the cognitive experience in public space are impacted by detailed designs of the built environment. Intervisibility is an important concept in accessing the existing/planned fine-scale architectural landscapes forming the distributed potentials of face-to-face encounters, as well as linking the built morphologies with the aggregate travel patterns and other socioeconomic outcomes across spaces. The development of this idea can be traced back to the period when the concepts of 'isovist'

(Tandy 1967) or 'viewshed' (Amidon and Elsner 1968; Lynch 1960) were born. Due to its close relation to the concept of ecological visual perception (Gibson 1979), isovist/viewshed captures the 'visible volume' from places and by which it builds the linkage between a place and people's behaviour in it, including how people potentially perceive and use it (Benedikt 1979). The interplay between these visible volume across places provides a scope to unfold the relationship between architectural forms and the visual structures of spatial narratives (Benedikt 1979; Batty 2001), and aggregated behaviours (Turner et al. 2001; Turner 2003).

Following the tradition that was begun by Benedikt (1979), the analyses of urban visibility patterns conventionally relied on local morphological properties of the isovist for every location, including, size, perimeter, volume compactness, etc., or its network properties such as clustering coefficient or degree, to represent the visually perceivable narratives from place to place (e.g., Benedikt and Burnham 1985; Fisher 1995; Lake et al. 1998; Batty 2001). By contrast, visibility graph analysis, derived from the isovist research, emphasised more on the visual interrelationship between places, providing insights on how isovists socially or aesthetically function by uncovering the hidden visibility structures behind spatial layouts (Turner et al. 2001; O'Sullivan and Turner 2001). Despite the novelty of existing visibility analyses, they have still limited to addressing the complexity of spatial configuration by neglecting the essential roles of other urban attractors embedded in spatial fabrics. This leads to the difficulty of interpreting the visually perceived deviations triggered by urban regeneration without a direct change of the built form, constraining the relevant applications in urban design practice.

The purpose of this article is to develop a framework of methods addressing the interrelated visual characteristics between one location and another (re)rendered by the urban form and the dynamically shifting functions, simultaneously. Based on the functional visibility graph, a rebuilt visibility graph aided with the volunteered location information of urban functions, numerous measures of functional visibility across scales are proposed to capture aspects of the perception of the built environment. By looking at the transformation of the functional visibility structures of a central area in Tianjin, China, this study shed lights on the effects of spatio-functional interaction on visual experience and spatial performance at the architectural level associated with the urban traveling behaviours. The delivered approach in this study complements the existing configurational analysis of the visualscape and opens the discussion on the future visibility analysis in the modern digital society.

2. CONCEPTUALISATION OF FUNCTIONAL VISIBILITY GRAPH

There are serval ways of computing the isovist areas via constructing a visibility graph in architectural environment. Raster-based method was delivered to define isovist and its fields (Ratti and Richens 2001). In this method, image processing technologies enable users to taking into account the vertical change of urban form. The other method is gird-based in which built form is abstracted as a regular tessellation for subsequent analyses (Turner et al. 2001; Batty 2001). The grid method is ideal for constructing the visibility graph with vertices and edges which representing the vantage points in space and the visual connections between any visible link between any pair of vantage points. The advantages of this method also include that it can capture the network properties of space more comprehensively with various cost metrics, which has been adopted in the geographical and configurational studies, such as Euclidean distance, topological steps, angular change, etc. Meanwhile, the visual problem is converted to a connectivity problem in graph analysis. An example has been shown in Fig. 1(a), in which a visual network is built by removing the invalid edges from the fully interconnected visibility graph and the isovist of one location can be defined as the area covered by the nodes linking directly to it with visibility edges. It is easy to add a new layer of network to make a visibility graph with form and function information at the same time. Within this layer, an urban function or activity location is treated as a node, and the inter-visual relationships between form and function nodes are defined as edges. This modified visibility graph is called as functional visibility graph in this study, where there are two type of nodes, form nodes and function nodes representing the space and activity locations, respectively, and one type edges showing the inter-visibility between them. From a formal perspective, a visibility graph can be defined as a graph, $G^{s}(V^{s}, E^{s})$, containing the space vertices V^{s} and edges, E^{s} , connecting any pairs of mutually visible nodes; while a functional visibility graph, $G^{f}(V^{s/f}, E^{s/f})$ where the vertices, $V^{s/f}$, including the space and function nodes, $V^{s/f} = \{v^s, v^f\}$, and the edges, $E^{s/f}$, interlinking two nodes with direct visibilities, which can be space nodes or function nodes, $E^{s/f} = \{e^{ss}, e^{ff}, e^{sf}\}$.

As shown in Fig. 1, in the functional visibility graph, we can compute an isovist for a function location (Fig. 1(b)). By overlapping the isovist areas for two functions, we can obtain a new area from which both two function activities can be simultaneously seen. In other words, this area is the place having better visibility to functions than other places in this experimental example (Fig. 2(c)). The visibility graph can be classified as two groups: the first-order graph and the second-order graph. The former one is for generating a collection of isovist, and the latter one is for producing an interconnected graph of isovists, which enables the calculation of 'steps' – the number of isovists – required along a path from one place to another (Turner et al. 2001). In this sense, visibility is conceptualised as cost for space traverses through spaces. By mapping the average visibility cost from every place to other destinations, a global view on the visibility performance of the space in question can be obtained. In a basic T-shape model, the distribution of mean visibility steps to spaces highlights the corners that are visually shallower than other places, particularly those located close to the boundary of the T-shape obstacle in the middle (Fig. 2(a)). In the models with one and two functional locations, the mean visibility steps to functions via the rebuilt functional visibility graphs pick the right corners as the visibility cores to urban activities through the spatial configuration. By rebuilding the visibility graph and changing the focused destinations, we can observe how urban functional landscape can stretch the visualscape by comparing the patterns of mean visibility step depth in various scenarios.



Fig. 1 The pattern of connections for a simple spatial configuration in a first-order visibility graph(a); the pattern of connections for a simple function location in a first-order visibility graph (b); and the pattern of co-connections for two simple function locations in a first-order visibility graph (c)

Through analyses of functional visibility graph, we can have a sense of functional-visual shallowness/depth of the spatial configuration, which is essential for evaluating the land-use plans in the spatial context where they will be implemented in. Unlike the visibility graph built with an assumption that all the spaces are featureless, functional visibility graph implies more flexibility of addressing various features of function nodes, for instance, the popularity, floor size, property rights, demographic characteristics, etc. Through questioning the 'functional accessibility' patterns hidden behind functional visibility graph, people can expect a series of more contemporary and direct answers regarding the interaction between urban form and function visually in a fine scale manner than what produced by former visibility analysis.



Fig. 2. The pattern of angular mean depth for a simple configuration(a); the pattern of angular mean depth from a function location(b); the pattern of angular mean depth from two function locations (c)

3. DATASETS AND METHODS

03.01 Towards a point-path-plane understanding of functional visibility landscape

The working flow from data collection to result interpretation is shown in Fig. 3. Four main steps are included. In the first step, we collect the spatial and functional information required and processing them as the ideal formats for the subsequent steps. For instance, the social media check-ins records should be aggregated to the points-of-interest they are tagged; then, they will be spatially assigned to their nearest boundaries of public spaces. In the second step, space nodes and functional nodes that are mutually visible are interlinked by visibility edges to form a functional visibility graph as described in the previous section. Two parameters are required at this stage to identify the resolution and scale focused for the subsequent analyses, including cell size and radius which can be defined based on different cognitive cost, such as the metric distance or topo-geometric depth which has been widely used in space syntax analysis. Due to the fact that the shape of public spaces might vary significantly from place to place, the space nodes generated by a regular cell grid might cause discontinuity or loss of the functional visibility graph. To avoid this awkward, another strategy is employed to allocate randomised vantage points with a mean distance equal to the fixed cell size so that the space coverage and its configurational change can be fully captured. After the functional visibility graph is formulated, the metric/topo-geometric connection properties will be assigned at every space node so that scalerelated visibility analysis. Here, we use angular change as the cognitive cost along the shortest visible path followed the work conducted by Turner (2003) who extended the angular analysis of axial maps to visibility graph analysis and verified its significance on improving the people movement forecasting. Based on the settings of cell size and radius, visibility graph is analysed with a series of basic measures, including the visible function size, the function evenness and mean shortest path depth. The former two reflects two essential aspects of visible function popularity: its size and complementarity effects. Their formal definitions will be introduced in the following section. These three basic concepts are further packaged as several integrated indices capturing the narrative of visibility along the traveling paths, point-based visible function centrality patterns, and the visible function regions of clustered places with similar compositions of visible function centrality in the step of result interpretation. These three principle visibility measures, reflecting interplay between different basic measures, highlight a pathpoint-planar knowledge on functional visibility landscape by using different data mining and analysis approaches.



Fig. 3. The research design for function visibility graph analysis (FVGA)

03.02 Analysing functional visibility graph

In this study, we measure graph structural properties of the built functional visibility graph. Apart from the centrality measures widely adopted in graph theory, we focus more on the functional characteristics from the point of view of each space vertex in functional visibility graph across scales, which includes the visible function size, visible function evenness, and mean shortest path depth. These three aspects are all scale-related, and the interplays between these core elements can be applied to visibility analysis of visualscape in the future work with specific focuses. By plotting the immediate functional visibility measures along the route(s) in question, people can have a timely continuous sense of visibility experience for different modes of traveling spaces. If urban spaces are segmented according to the composition of functional visibility to various complementary urban activities, a sense of visible function region across spaces can be shown. And if all these dimensions are integrated, the landscape of functional visibility can be formed to reveal the variation of overall visible potential to functions.

03.02.01 Three basic measures

Visible function size

Visible function size of a space vertex i is the amount of the function nodes j in the set of vertices, $V(d_{ij})$, visually connected though the visibility edges with a cumulative edge length less than d_{ij} . If we continuously consider the typology (k = {1,2,3,...,K}) and weighting element (W_j) of urban activities happening on the function vertex j, the form of visible function size (S_i) can be expressed mathematically as the equation below. This index represents the magnitude of functional information through the visibility graph at a radius d_{ij} .

$$S_i = \sum_k S_i^k = \sum_k \sum_{j \in V(d_{ij})} W_j^k \tag{1}$$

Visible function evenness

Visible function evenness of a space vertex i equals to the entropy implied by the compositions of visible function size to different urban activities in type k with a cumulative edge length less than d_{ij} . Entropy gives a sense of richness of functional information, which is related to the co-presence between different activities visible, the visible diversity. Visible functional evenness complements the visible function size as it represents the order of the visibility that is less size-related. The formal way to measure visibility evenness (E_i) in this study can be expressed as:

$$E_i = \frac{-\sum_k P_i^k \log P_i^k}{\log K} \tag{2}$$

$$P_i^k = \frac{\bar{s}_i^k}{\sum_k \bar{s}_i^k} = \frac{\frac{\bar{s}_i^k}{\sum_i \bar{s}_i^k}}{\sum_k \frac{\bar{s}_i^k}{\sum_i \bar{s}_i^k}}$$
(3)

where, P_i^k denotes the probability of urban activities in type k visible from space vertex i relative to other types of activities, and \bar{S}_i^k refers to the normalised visible function size of a specific type of activity k. Normalisation is a pre-condition for making the presence of different activities comparable and giving the right value to entropy index.

Mean shortest angular path depth

Mean shortest path depth, L_i , measures the weighted mean angular step depth required from the original space node to the reachable function verities though the angularly shortest paths connecting them. It can be a direct sum of the mean shortest path depth to every type of function nodes, l_{ij}^k . This measure reflects the visibility cost required from space to functions.

$$L_{i} = \sum_{k} L_{i}^{k} = \sum_{k} \sum_{j} l_{ij}^{k} = \sum_{k} \sum_{j} \frac{\sum_{j \in V(d_{ij})} l_{ij}^{k}}{\sum_{j \in V(d_{ij})} w_{e}^{k}}$$

$$\tag{4}$$

Here, l_{ij}^k denotes to the cumulative angular change along the shortest angular path connecting the original space node i to the function node j in type k, and w_e^k refers to the angle of each individual connection e along that path. In this equation, the angular distance is normalised as a ratio of two angular measures, which returns a value of step depth, reflecting the relative efficiency of the geometric grids and enabling the comparison between different systems, even though the resultant outputs have no units and any absolute meaning in reality.

0.3.02.02 Interplay between measures

Viso-functional connectivity

Viso-functional connectivity measures the immediate visible function size from a space vertex i. It is equivalent to the weighted function size reachable in the isovist area from the place i. From a mathematical view, this can be represented as the visual function size at 1 angular step depth:

$$C_{i} = S_{i}^{1} = \sum_{k} \sum_{j \in V(L_{ij})} W_{j}^{k} , \{L_{ij} = \frac{l_{ij}}{w_{e}} = 1\}$$
(5)

This index could be very useful to quantify the perceivable spatial narratives feed by urban from and functions simultaneously, particularly when a route, a set of subsequent view points, is selected. In so doing, planners and designers can access the dynamic meanings of the built environment for individuals when they pass the important place in design.

Functional visibility

Functional visibility indexes the extent to which urban functions are visually reachable from the place i, which illustrates the closeness between a set of function and a place in question. It incorporates all the three basic measures as one metric:

$$V_i = \frac{S_i^{E_i}}{L_i^{\sigma}} \tag{6}$$

where V_i denotes to the functional visibility index, which is hereby computed as the interaction between the benefit and cost. The benefit is related to the combined effects of the visually reachable function size and its diversity. The cost, on the other hand, is modelled as the visual distance. By putting the benefit and cost in a gravity-like form, we measure the resultant score of functional visibility as the ratio of the functional benefit to the visual cost for accessing it. Here, we adopt an inverse distance function with an exponential parameter, σ , to reflect the distance decay effects. It is fixed as 1 in this study due to the absence of proper data to calibrate it.

This metric, as suggested by its form, reflects the visible spatial potentials projected in the detail urban form and function on the architectural scale. If a geographical point of view is applied, this index could be understood as a specific type of place centrality, capturing the spatial interaction between urban functions through the visual interlinks. Through mapping this, designers can obtain a structural sense of the visible function centrality then know where is more visually and functionally proximity to other places than others.

Visible function closeness

This index is a disaggregated version of the functional visibility by the types of city functions, in which the factor of the visual function evenness is neglected. It can be defined as below. It enable a more comprehensive scrutiny on every aspect of the functional visibility whereby reserve the room for the subsequent analysis.

$$V_i^k = \frac{S_i^k}{L_i^{k^{\sigma}}}$$
(7)

Visible function regions

Based on the introduced visible function closeness to different functions in question, we can partition the architectural space to different functional regions maintaining more common characteristic functions inside the boundaries than that outside them. The method we use here is network-based consensus clustering method (Lancichinetti and Fortunato 2012), a widely used method in biometrics research. As a young member in the family of community detection featured by its advantage on handling multi-dimensional dataset and automatically selecting the optimised number of clusters by maximising the modularity of the consensus network in each interaction process.

4. A PRELIMINARY APPLICATION IN TIANJIN

A central area sized with a 1km*1km geographical window is selected for case study in this research. It is located in the city centre where urban functions are agglomerated and projected along the spatial grids. The selected area covers the place where the well-known pedestrian shopping avenue, the Binjiang Road, is located, and the preserved historic districts with sophisticated system of urban public space, the colonial areas in modern urbanisation process are also covered.

The data source adopted in this work including the point-based social media data, the detail shape file representing the architectural environment and the gate count survey result. The first two datasets are used for computing the FVGA measures. The formats of these datasets are shown in Fig. 4. Here, we use the amount of check-ins for each point-of-interest (POI) as its importance. All the POIs are classified

in 11 main non-residential types according to the check-in habits reflected by the statistical analysis, including retail, catering, hotel, office, culture, transport, recreation, education, public service, hospital, and park (Shen and Karimi 2016).



Fig. 4. (a) A 1km by 1km area within central Tianjin. (b) The check-ined POIs dataset within the proposed study area. (c) Map of gate counts.

FVGA indices at R1

We first explore the FVGA metrics at 1 angular step, as they are the immediate visibility properties that people perceived. The results are documented in Fig. 5. The map of visible function size at R1 successfully captures the well-known high street, Binjiang street, where many shops are clustering (red colour in Fig.5 (a)). This street, however, is not the highland supporting the high degree of the mixture of urban activities. The centre for activity mixture shirt towards the other two main roads located on the east. It is also noted that, the circular square, is a place can easily observer various other urban activities rather than only shopping. The result of the function visibility at R1 combines the effects of the visual function size and evenness, illustrating the street intersections where people can see dense and diverse activities at their first glance. If an infinite radius is set, the map of function visibility highlights two areas at the southeast and northwest of the square are the global centres with lowest visually-cognitive cost to different active functions.



Fig. 5. (a) Visible function size at R1 (b) Visible function evenness at R1 (c) Function visibility at R1; (d) Function visibility at Rn

FVGA indices at the radius based on metric distance metric

It is noted that there might be boundary effect for the VGA when the study area is selected in an arbitrary manner. As we can see in Fig. 5 (d), the detected centre is very likely to be located in the geometrical centre, though it shows spatial heterogeneity clearly. Therefore, we calibrate the radius setting based on the metric distance metric, and record the visualisation results in Fig. 6, in which 300m is picked up as the radius.

The results are very distinguished from that shown in Fig. 5. The map of visible function size shows an area for the agglomeration of activities. The mixture of these activities based on 300 meters, demonstrates a large area in the east of main high street – Binjiang Street, interconnecting different corners. Comparing this with the map shown in Fig. 5(b), it suggests that the mixture of functions is a two-folded phenomenon: it could be perceived by people angularly as well as by them metrically when they travel around a place. The map of mean depth showing the intersections between main roads and the areas with less functions. The function visibility at 300m finally shows the areas scored higher than other places from all aspects discussed. This distribution captures the areas from the main shopping streets to their neighbouring active streets whereby forming a walkable active community. It is far different from what we got in Fig 5(c) and (d).



Fig. 6. Functional visibility graph analysis at 300 metres. (a) Visible function size; (b) Visible function evenness; (c) Mean angular step; (d) Function visibility.

In order to validate the effectiveness of different indices at different forms of radii quantitatively, we test the correlative relationships between proposed indices and the gate count data. The results are recorded in Tab. 1 where the metrics from the VGA model, the FVGA model at R1, and FVGA at R300 are compared according to their predictability on the aggregated human flows. It is noted that FVGA indices based on metric distance (300m) are all statistically significant for estimating the variation of gate count data. For VGA model, visual integration at Rn is a significant variable, and the visible function size at R1 and function visibility at Rn are significant in the FVGA model based on angular distance. But the performance of these factors indicated by the adjust R2 scores is not as good as those factors in the FVGA model based on a local metric distance (300m). All these findings showcase that the FVGA model based on a local metric distance metric might be more appropriate that that based on other settings. It is also demonstrated that the delivered FGVA model can improve the performance of the traditional FVGA model.

	Correlation to observed gate counts		
VGA model	Adjusted R2	t-ratio	p-value
Isovist	0.054	2.03	0.0453*
Clustering coefficient	0.004	0.54	0.5877
Visual integration	0.166	4.14	<0.0001*

Tab. 1. Results by regressing observe human's movement against predicted outcomes

FVGA model (based on angular distance)

Visible function size at R1	0.125	3.35	0.0012*
Visible function evenness at R1	0.021	1.47	0.1446
Function visibility at Rn	0.151	3.91	0.0002*

FVGA model (based on metric distance)

Visible function size at 300 m	0.614	13.79	<0.0001*
Visible function evenness at 300 m	0.474	8.91	< 0.0001*
Mean angular step at 300 m	0.142	-3.63	<0.0001*
Function visibility at 300 m	0.759	16.59	<0.0001*

Visible function regions

The results of the visible function regions (at 300m) are shown in Fig. 7. There are 7 clusters are detected by maximising the modularity of the consensus network created based on 11 networks with the visible function closeness metrics computed following the way recorded in equation 7. Cluster 1 and 7 are both the business areas dominated by retail, recreation, while they can be easily distinguished by the clustering of hotels and catering activities. Cluster 3 is also an active areas characterised by the transport nodes, parks, cultural sites and hospitals that are more visually perceivable than other places. Cluster 4 is less active than cluster 3, though it can be featured by the clustering of similar activities as cluster 3. Cluster 5 and 6 are both highlighted by the higher visibility scores to education than that to others, but the former one can be also captured as its visual exposure to transport-related services, and the latter one can be further featured by its visual connectivity to hospitals and public service providers. The structure of these clusters could be changed as the shift of layouts or the evolution of micro-economy processes.



Fig. 7. Result of the groups in terms of the visual function closeness for different active functions in 2014

5. CONCLUSIONS

This paper introduced a new framework to analyse the functional visibility graph, the visibility graph to urban functions based on the spatial and functional configuration. A series of indices are delivered to capture different aspects of the functional visibility. Some of them are centrality-like, representing the structural properties that sensitive to the change of spatial grids and land-use allocation. Some of them are membership-based, indicates the 'communities', disconnected areas indicated by the visibilities to various urban functions, showing the process how sense of place could be reshaped by the space and its content – urban functions.

The results in this work validate the effectiveness of the proposed metrics preliminarily. The good predictability of these factors on the actual urban movement indicates the importance of urban functions in shaping our visual landscape which in turn reshape our traveling patterns. The framework proposed in this work, due to its sensitivity to the change of spatial layout and function locations, is of great value for the planning and design for the architectural built environment.

REFERENCES

Amidon, E. L., & Elsner, G. H. (1968). Delineating landscape view areas... a computer approach. Res. Note PSW-RN-180. Berkeley, CA: US Department of Agriculture, Forest Service, Pacific Southwest Forest and Range Experiment Station. 5 p, 180.

Batty, M. (2001). Exploring isovist fields: space and shape in architectural and urban morphology. Environment and planning B: Planning and Design, 28(1), 123-150.

Benedikt, M. L. (1979). To take hold of space: isovists and isovist fields. Environment and Planning B: Planning and design, 6(1), 47-65.

Benedikt, M. L., & Burnham, C. A. (1985). "Perceiving architectural space: from optic rays to isovists", Persistence and Change Eds WH Warren, RE Shaw.

Fisher, P. F. (1995). An exploration of probable viewsheds in landscape planning. Environment and Planning B: Planning and design, 22(5), 527-546.

Gibson, J. J. (1966). The senses considered as perceptual systems.

Lake, M. W., Woodman, P. E., & Mithen, S. J. (1998). Tailoring GIS software for archaeological applications: an example concerning viewshed analysis. Journal of Archaeological Science, 25(1), 27-38.

Lynch, K. (1960). The image of the city (Vol. 11). MIT press.

O'Sullivan, D., & Turner, A. (2001). Visibility graphs and landscape visibility analysis. International Journal of Geographical Information Science, 15(3), 221-237.

Ratti, C., & Richens, P. (1999). Urban texture analysis with image processing techniques. In Computers in Building (pp. 49-64). Springer, Boston, MA.

Shen, Y., & Karimi, K. (2016). Urban function connectivity: characterisation of functional urban streets with social media check-in data. Cities, 55, 9-21.

Tandy, C. R. V. (1967). The isovist method of landscape survey. Methods of landscape analysis, 9-10.

Turner, A. (2003). Analysing the visual dynamics of spatial morphology. Environment and Planning B: Planning and Design, 30(5), 657-676.

Turner, A., Doxa, M., O'sullivan, D., & Penn, A. (2001). From isovists to visibility graphs: a methodology for the analysis of architectural space. Environment and Planning B: Planning and design, 28(1), 103-121.