CHAPTER 8
EVERY STUDENT COUNTS: LEARNING MATHEMATICS ACROSS THE CURRICULUM

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INTRODUCTION

In the twenty-first century, mathematical confidence and functionality are considered to be of crucial importance to individuals and to national economies, which is reflected in national policies, as well as in the status of international performance comparisons such as PISA and TIMSS[1]. However, as a subject specialist in another subject, you might well think you simply can't afford the teaching time to deliberately embed mathematics in your lesson. In this chapter we try to show how, by being aware of how your subject harnesses mathematical thinking, even if only in low-key ways, you can support students in making confident and informed use of that – but we also encourage you to keep talking to the teachers of mathematics in your school, so they become more aware of what your students are meeting and when. That way, students can begin to make meaningful connections across the curriculum, and enhance their grasp of, and interest in, your own subject.

The National Curriculum for England (2014) describes the centrality of mathematics in the curriculum as follows:

Mathematics is a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject (DfE, 2014, p3).

Numeracy, including statistical literacy, is a part of effective mathematical functioning and can be defined as the "ability to use mathematics in everyday life" (National Numeracy, 2017). The digital world for which we are now preparing learners requires high levels of engagement with many and varied tools and resources unknown to earlier generations and so we have a responsibility to support learners to engage with those new demands and

[1] The Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) compare educational systems and outcomes for participating countries.

opportunities, including for working in mathematical ways wherever there is a need to do so.

Because mathematics is so pervasive across the curriculum and beyond, all teachers need a fundamental knowledge of both how to teach mathematical ideas within their subject area, and how learners learn them. Teaching mathematics is, like all teaching, a highly skilled undertaking, but specialist and non-specialist teachers alike can take up a wide range of support and advice available to them to support this endeavour: see the Further Resources at the end of the chapter.

OBJECTIVES

By the end of this chapter, you should have:

- An understanding of how to help learners grasp core mathematical concepts
- Analysed the opportunities for developing learners' mathematics skills, knowledge and understanding in your subject
- Recognised the value and roles of mathematics and mathematical thinking in multiple contexts, subjects and everyday life
- Understood some ways of developing positive attitudes to mathematics in learners.

Check how the information in this chapter enables you to meet the requirements for your first year of teaching.

THE NATURE OF MATHEMATICS

Learners have a variety of views about what mathematics is, and of their relationship with it. We take the view that learning and teaching mathematics is fundamentally about recognising pattern and structure (sometimes in abstract forms) and working with this recognition in reasoned ways that both stem from, and are applied to, the world around us: that includes other curriculum areas!

Mathematics helps us to make sense of that world and provides ways of thinking about and solving both practical and abstract problems. It is not 'magic' that appears out of nowhere nor is it disconnected from anything else. So a priority for young people engaging with mathematical ideas is that mathematics should be approached in ways which 'make sense'. Users of mathematics in other subjects might not be confident about the underlying reasons for something, for example, the form that 'standard deviation' of a set of data takes – but it is important to communicate to learners that there *is* good reason - perhaps that they could ask a mathematics teacher about – and that, for applications, what is most important is understanding and interpreting the information that measure gives.

Not all teachers are mathematically fluent or confident – and, unfortunately, some have had negative experiences of learning mathematics themselves. It would be very sad if those negative experiences were also projected onto future generations of learners, so as a teacher, it is important to try to model positive attitudes to mathematics and to demonstrate how to approach situations where you are mathematically challenged: Has anyone met this idea before? What does it mean? If you *are* faced with questions about the mathematics that you cannot answer, tell learners you will talk with colleagues and then come back to them.

Ideally, a school will have policies that make it clear in your schemes of work where learners are meeting mathematical ideas for the first time in your subject, and where they have already met them in their mathematics curriculum. It is helpful for learners, if they have met ideas elsewhere, if you know *how* those have been approached, and find this out from learners; if this is the first time they have met a mathematical idea, that's fine. In either case, talk with a mathematics teacher if it is not clear to you how best to approach the idea, ideally in advance of working with a mathematical concept in class.

For example, in Geography, students might well be meeting the mathematical representations such as stacked bar charts, or chloropleths that show area as proportional to number, for the first time. Support learners to make sense of them. In other cases, learners will have met underlying ideas but maybe in a different guise. For instance, by KS4, graphs arising from experimental data are treated differently in each of Biology, Chemistry, and Physics, because of the nature of the data represented. However, in mathematics lessons learners are more usually dealing with continuous functions that require representation with a smooth curve. In another example, 'lines of best fit' in Mathematics GCSE only relate to directly proportional relationships, but in Business or Science or History the 'best fit' from a limited number of data points might well be a curve. Such issues of interpolation and extrapolation occur across the curriculum and learners can benefit from talking about their experiences in other parts of the curriculum. Further, there is symbiosis here: practical work in any subject that requires measuring or recording or representation of data.

One area that can often challenge learners is the use of formulae. Try to help them make sense of the underlying *meaning* of the formula, ask them to estimate values for the quantities involved, use learners to explain to the class the approach they prefer to use, listening for the vocabulary and sense-making they are drawing on, so that you build support rather than cause further confusion.

And always, if in doubt, talk with mathematics colleagues, and don't feel guilty about doing so: your conversation also helps them to appreciate learners' needs beyond the mathematics classroom, which they in turn can draw upon in their own teaching. That way, everyone benefits: you will be more confident about how you are working mathematically with learners; your colleagues teaching mathematics will also develop their repertoire of where else in the curriculum learners are engaging with mathematical ideas. Your colleagues can also enrich learners' experiences and support them to link their learning across the curriculum, which is key to working confidently with mathematics.

THE CURRICULUM IN MATHEMATICS

It is helpful for teachers who use mathematics in their subjects to have some familiarity with learners' prior mathematical experiences. The mathematics National Curriculum (DfE 2014, p3) aims to ensure that all learners:

- *become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that learners develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;*
- *reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;*
- *can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.*

Note this goes well beyond 'doing the mathematics'. When learners are *fluent* in mathematics they are able to recognise when to use their mathematical knowledge, can use it flexibly and efficiently, and apply it to a range of situations and problems. Allied with this, of course, is a familiarity with an increasing range of mathematical facts, procedures and concepts. By the time they arrive at secondary school, most learners will be reasonably confident with the four operations (+, -, x, ÷) for whole numbers of any size, for many decimals, and for some fractions. They will have met a range of measures and statistical representations, and worked with 2-D and 3-D shapes. But they won't necessarily be confident in working with all those in new situations

In mathematics, learners need both 'procedural fluency' and 'conceptual understanding'. This is because it is not sufficient only to be able to recall the procedure for carrying out a calculation: often when solving a problem or applying that knowledge the learner needs a deeper understanding in order to recognise and apply mathematics appropriately. Cross-curricular teaching such that learners can really understand the mathematics they are using will help learners to develop robust procedures and build confidence.

Importantly for teachers of other subjects, the second and third aims of the mathematics National Curriculum encapsulate the kinds of mathematics-related skills in young people need for the development of statistical literacy. For example, learners can engage in analyses of fair trade issues, of global neonatal mortality rates, of athletics records…. In your subject areas, learners might be engaged in problems related to how something changes, or has changed, with time, what quantities are interdependent and in what way, whether we can formulate relationships in such as a way as to be able to predict…. In order to support

learners in coming to function mathematically within your subject, your need to have some understanding of where it fits within their mathematics curriculum trajectory.
Now undertake Task 8.1.

<start task>

Task 8.1 Identifying the mathematics required in your subject

Look at the curriculum for one of your teaching groups and identify the parts where your learners might need to use mathematical skills and knowledge. Use the mathematics National Curriculum (https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study) to help you to identify when they meet those in their mathematics lessons: have they met them at primary school? Or in KS3?

Now meet with someone who teaches your, or similar, learners mathematics to discuss how the identified skills and knowledge are addressed. Curriculum mismatches, whether in timing or approach, need not matter if they are understood. If your learners' first encounter with a mathematical concept, tool or skill is in your lessons, both you and their mathematics teacher need to be aware of that. Similarly, if there are reasons for a variety of approaches, it is helpful to learners if those are understood by all the teachers who are working with that material. For example, is the plotting of a set of experimental points on a graph to be followed by joining them with straight line segments, or a curve, or a line of best fit, or…? And why? Make a note of any outstanding tensions.

Discuss this information with your mentor and store the information in your professional development portfolio (PDP) to refer as you plan your lessons.

<end task>

As with any teaching, your knowledge does need to be secure with respect to your own subject knowledge, including its mathematical aspects. However, even this is not sufficient; you also need 'Pedagogical Content Knowledge' (PCK)[2]. This is the knowledge that makes the discipline accessible to learners. It is one thing, for example, to know how to calculate, for example, a percentage decrease population during the Black Death (which is content knowledge); it is quite another to be able to explain this to learners in a way that makes sense to them, using appropriate examples and diagrams and in a way that draws on appropriate prior learning.
Pedagogical content knowledge for teaching mathematical aspects of the curriculum in school includes an understanding of an appropriate subset of:

- Conceptions and common misconceptions in mathematics;

[2] Shulman, L. S. 1987. Knowledge and teaching: foundations of the new reform. *Harvard Educational Review* no. 57 (1):1-22.

- Progression in learning mathematics, including some familiarity with the mathematics curriculum that enables appropriate approaches in other areas;
- Use of multiple appropriate representations, common models, concrete-pictorial-abstract approaches to developing concepts;
- Appropriate and meaningful use of mathematical digital technologies, and a variety of other tools, for teaching and learning using mathematics;
- Importance of meaningful, progressive and rigorous language in learning mathematics: the language, including symbolism, of mathematics is a very compact and precise one, and that precision needs to be learnt over time in order to communicate mathematics effectively. Much of that can be done via teacher modelling, and the building up of learners' experiences accompanied by appropriate teacher expectations. For example, 'sums' are a particular sort of calculation that uses addition: if another sort of process is indicated, or it is not yet decided, it is helpful to refer to that as purely a 'calculation'.
- Knowledge of how young people interact with mathematics, their typical progression, ways of thinking, acting and being with mathematical aspects of your subject, and their affective needs as they do so.

We now discuss these different types of pedagogical knowledge briefly, using examples that should be familiar and applicable to those who main role is teaching other subjects in the curriculum.

CONCEPTIONS AND MISCONCEPTIONS IN MATHEMATICS

Estimating, and considering whether one's answer makes sense, are important skills for calculating. Some learners think that 'multiplying makes things bigger', or 'to multiply by 10 all I need to do is add a zero on the end of a number'. This is not surprising given that their first experiences are with numbers greater than one, where this is true. This is an example of a *partial conception*, where the learner is seeking to generalise from their experience but does not yet have sufficient experience to understand that they are incorrect in their assumption. Other misconceptions may emerge because the learner misunderstands the mathematics. For example learners might interpret a distance-time graph comprising straight-line segments as a journey that goes up and down hills. Alternatively, misconceptions might arise where learners have experienced poor, or limited, explanation on the part of the teacher.

Far from setting out to be troublesome, young people will try to make sense of mathematical ideas. However, given their much more limited life experience, it is not surprising that the sense they make is often initially different from that made by adults, and particularly mathematically-confident adults. Processes of 'cognitive conflict' whereby learners come to see their present understanding is not fit for purpose in wider context, can often underpin changes in conception – important knowledge for the wider adult (mathematical) community as it is not always clear why learners misunderstand some ideas.

Finding this out can often be achieved by providing opportunities to work in situations where the learner has already developed sense-making – perhaps here, with money – and provoking that cognitive conflict. In the case of multiplying by ten, for example, it is clear that the price of ten £4.99 DVDs is not £4.990!

PROGRESSION IN LEARNING MATHEMATICS

Topics in mathematics are deeply connected, and new learning almost always relies on previous learning within the same and other topics. For example, drawing a graph from data in science requires an understanding of the number line. It draws on: skills learnt plotting points using ordered pairs; calculations using a variety of small, large, integer and decimal numbers; multiplication facts; a need to recognise pattern; and an understanding that a graph is a representation of an infinite number of two-dimensional points whose coordinates have a particular relationship. Producing a histogram in Business Studies draws on some of these skills, but also further requires an understanding of the difference between discrete and continuous data and a recognition of the differences and similarities between graphical representations and the ways graphs are used in both algebra and statistics. Work across the school curriculum draws on different aspects of learners' graphing skills and the need to recognise and apply these skills appropriately.

REPRESENTING MATHEMATICS

One of the most powerful tools in mathematics is the ability to represent a mathematical situation in different forms in order either to solve it or to explore and explain its different features. Using the previous example of a scientific relationship, the specific case of the mathematical function $2x + 3$ can be expressed algebraically as $y = 2x + 3$ and learners at key stage 3 might be expected to have worked with such functions in mathematics lessons. We can also say that 'the y value is double the value of x and 3 more', to make sense of it. We can form a table of some of the pairs of points satisfying this relationship and we can plot them on a graph shown in Figure 8.1 using, for example, a free graph-plotting tool such as Desmos (www.desmos.com). But in physics, a very similar relationship exists between a temperature F measured in degrees Fahrenheit, and a temperatures C measured in degrees Celsius: $F = 1.6C + 32$. In both examples, these letters represent algebraic variables, an important mathematical idea that all learners are required to understand by the end of Key Stage 4. Learners need support to build their confidence that they can deal with different letters but the same underlying ideas. Ideally that support will be provided in both mathematics and physics lessons.

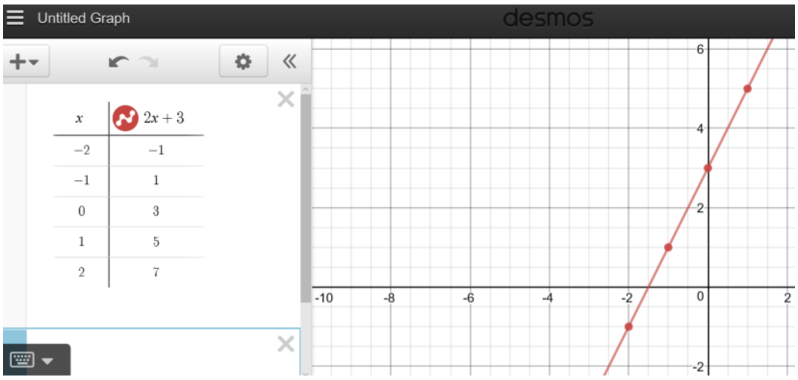

Figure 8.1: Graph and table for *y = 2x + 3* (*Image produced with permission using Desmos https://www.desmos.com/)*

Using such a tool produces a quick and accurate graph – but also the opportunity to gain a deeper understanding of the relationship, and importantly, the links between the geometric features of the graph and its algebraic or tabular forms.

Another powerful visual representation in mathematics is the use of a bar model, which requires proportional reasoning. The example in Figure 8.2 demonstrates how to find 60% of £50 using bar modelling. If you ask learners how they would approach such a calculation, they might well draw on such a representation.

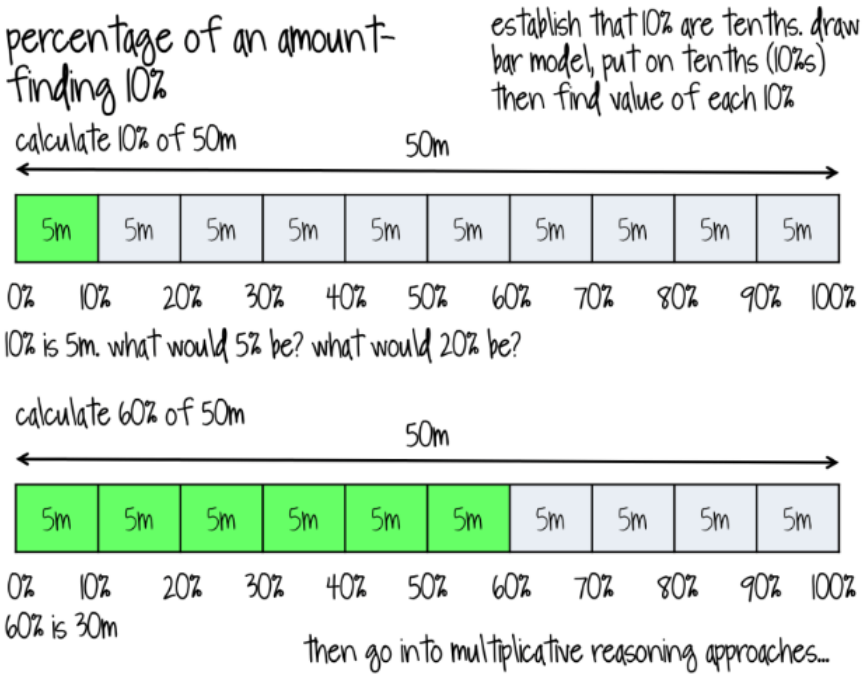

Figure 8.2: Using bar modelling to find 60% of £50 (screen shot from www.greatmathematicsteachingideas.com)

CONCRETE – VISUAL – ABSTRACT PEDAGOGY

Primary schools and, increasingly, secondary schools, commonly use a *concrete – visual – abstract*[3] approach to mathematics. In this approach learners spend time using physical objects to explore the structure of mathematics problems before moving on to use images as a 'bridge' to learning how to manipulate situations symbolically. For example, learners might use Dienes blocks (place value apparatus), counters, straws, rods or cards before moving on to using a diagrammatic or digital resource, and finally to a confident grasp of the underpinning abstract concepts. That process cannot be rushed if young people are to develop robust conceptual understanding – and you will often see advanced level mathematics learners using diagrams to give them access to the mathematics underpinning a situation in a way that the written or physical situation does not afford: the diagram absorbs some of the mental attention needed to grasp the whole, and so allows freer and more productive use of remaining brainpower. It is therefore important that learners do not think they should have 'grown out of' using diagrams or other representations, but instead be encouraged to recognise them, when appropriate, as valuable tools for thinking.

Learners can also draw their own informal representations, for example when the situation calls for sharing a quantity in a given ratio, or dealing with the mathematics of a mechanical or technical situation. The evidence is that all learners, not just lower attainers, need time with both concrete materials and visual representations to explore and model problems before working on the mathematics more abstractly. Some other subject areas use quite demanding ideas of, for example, ratio and proportion: it is helpful to learners to allow them to work with these in their preferred way. So if learners appear to have difficulty grasping mathematical ideas you're using, it might be that using a more visual approach will make links with what they *can* grasp. What you would like to achieve is meaning-making in your subject area!

APPROPRIATE AND MEANINGFUL USE OF DIGITAL TECHNOLOGIES

There is an overwhelming choice of digital resources available for teachers (and learners) to use in and with mathematics and it is sometimes challenging to determine if and when they are worthwhile. We have pointed above to Desmos, if you need to produce graphs, and both Excel or Geogebra (www.geogebra.com) can be harnessed to represent and analyse data. Learners require a scientific calculator for GCSE Mathematics, and it is reasonable to expect them to have that available for use in other subject areas as well. If you are dealing with comparatively complex formulae, e.g. calculating the radius needed for a chloropleth, learners might need support in using their calculator appropriately, but again, try to help them make sense of what they are doing and then they will retain that.

LANGUAGE FOR LEARNING MATHEMATICS

Learning and teaching mathematics uses language in a highly specialised manner. When considering the vocabulary of mathematics there are technical words that are used only in mathematics, and there are words that have different meanings in mathematics than they

[3] This is based on Jerome Bruner's conception of the *enactive, iconic and symbolic* (Bruner 1966).

do in everyday life. For example, when two shapes are ‘similar’ in mathematics we mean that they have a specific defined relationship, with the same shape but possibly different size, not that they are ‘a bit the same’. Some aspects of grammar, such as ‘and’, ‘or’ also operate differently and usually more rigorously in mathematics, and that rigour has to be developed and maintained if the concision of mathematical language is to retain its usefulness, but don’t be afraid to use mathematically related vocabulary particular to your subject as well: for example, learners in their mathematics lessons will have had limited experience of using experimental data, but in context, the notions of ‘dependent’ and ‘independent’ variable are powerful and precise. Explaining mathematics-related tools therefore requires careful use of precise language by both by teachers and learners.

Now undertake Task 8.2.

<start task>

Task 8.2 Analysing the mathematics in learners’ subject work

Pick a topic from the scheme of work for your subject where learners will need to use some mathematics, no matter how basic. After you have taught the topic, choose the written work of some learners to analyse in more detail:

- What mathematics did learners use in the topic?
- What evidence do you have that they understood it?
- What difficulties did they have? Did they reveal any misconceptions about the mathematics?
- Use some of the ideas in the previous parts of this chapter to help you think about how you could improve the teaching of this topic in the future.

You might like to refer to the work you did in Task 8.1 to help you.

Discuss this information with your mentor and store the information in your PDP to refer as you plan your lessons.

<end task>

In terms of the mathematical skills and processes learners are likely to use across the curriculum, being efficient in carrying out aspects of mathematical calculations is only part of the picture. As already mentioned, it is insufficient for learners to develop knowledge of facts if they don’t also have a conceptual understanding of what they are doing. Within calculations, the mathematics curriculum sets out to ensure that for each of the four arithmetic processes (+, -, x, ÷) learners have a standard method they can apply efficiently and reliably with a variety of numbers, including fractions. However, the curriculum also encourages estimation and mental methods as a first resort, and teachers of other parts of the curriculum have a key role to play here, in modelling their own adult ways of dealing with numbers, and the discipline-specific ways they have of checking that answers to calculations make sense. In so doing, they are supporting learners in developing powerful ways of harnessing mathematical thinking in a variety of situations – in becoming genuine *users* of mathematics.

Mathematical reasoning underpins mathematical problem solving – including the application of learners' mathematical knowledge in other areas. Their reasoning can be built up from asking for straightforward explanations towards the eventual achievement of sustained chains of fairly formal deductive reasoning, but underpinning all of those are consistent questioning of the mathematical thinking learners are engaging in: 'Tell us how you sorted that out'; 'Can you explain why you did that?' 'How do you know?' 'What information did you use to come to that answer?' And importantly, engaging in such questioning whether or not the solution is a valid one, so that over time, learners themselves come to question their chain of thinking at every stage.

Once learners are confident to work in such ways informally and orally, they are then in a position to begin to communicate that thinking in writing. And here we come to another central aim of the overall curriculum - that learners should come to be able to *communicate*, in this case mathematically, in a variety of ways: orally, diagrammatically, graphically, algebraically, and via formal written deductive reasoning, among others. Mathematical communication in its most efficient form is very concise – but in order to maintain rigour, that concision needs precise use, and careful development of learners' mathematical communication, often employing standard English terminology in parallel with technical words ('the gradient, or amount of slope of the graph, tells us...'; 'How would you explain that in English?') so that learners begin to make the cognitive links between technical descriptions and meaning.

It is fine for learners to experience *challenge* within the range of their mathematical experiences, and learn how more experienced others, notably their teachers, cope with that, by sharing and critiquing their ideas so far, by revisiting possible related facts and processes, by going away and 'thinking about it', and by harnessing the support of others with perhaps different viewpoints or knowledge. *Use errors positively* – as an opportunity to learn – and if one learner has a misconception, you can be fairly sure they are not the only one: be sure to value that opportunity overtly, so that learners come to see how it can be harnessed for positive ends. Sometimes learners think best when they are quiet and on their own, but often a combination of individual thought and cooperative or *collaborative* approaches is the most fruitful, and the range of teachers working with learners in such ways can all contribute to a growing appreciation of that.

SUMMARY AND KEY POINTS

- For non-specialists in particular, this can all seem rather daunting, but positive approaches that emphasise conceptual understanding and on working together to make sense of the mathematics, making links within and beyond mathematics, embracing the opportunities offered by misconceptions and by getting stuck, can build up both teacher and learner confidence over time. That way, your consciousness of the mathematical needs of your subject area enhance your teaching, and your learners' confidence and interest in it.

- For those who use mathematics in the curriculum, there are high quality, evidence-based websites that support such approaches; the mathematics subject professional associations also offer a range of subject-specific support and resources (see below).
- Finally, learners benefit from having teachers who ask more knowledgeable others if they are stuck, reflect with colleagues on their teaching of mathematics-linked parts of their curriculum, and know the subject, their learners and the resources available, harnessing them in positive ways to enjoy working mathematically themselves. It is amazing how those beginner teachers, who come to use mathematics in the classroom with some trepidation, can build up their own enjoyment and confidence by working in these ways, and everyone benefits.

Record in your professional development portfolio (PDP), how the information in this chapter enables you to meet the requirements for your first year of teaching.

FURTHER RESOURCES

Available support

- The Association of Teachers of Mathematics (www.atm.org.uk) and The Mathematical Association (www.m-a.rg.uk) jointly publish *Primary Mathematics*, as well as a range of other classroom-focused periodicals and other resources.
- The National Centre for Excellence in the Teaching of Mathematics (www.ncetm.org.uk) is government-funded to support evidence-based good practice in schools, and especially to coordinate high quality teacher development for that.
- NRICH www.nrich.mathematics.org provides challenging and engaging activities at all levels 5-18 to develop mathematical thinking and problem-solving skills that show rich mathematics in meaningful contexts.

Appendix 2 lists subject associations and teaching councils.

Books in the Learning to Teach series which you may find helpful are as follows:

Capel, S., Leask, M. and Younie, S. (eds.) (2019) *Learning to Teach in the Secondary School: A Companion to School Experience,* 8th edn, Abingdon: Routledge.

This book is designed as a core textbook to support student teachers through their initial teacher education programme.

Capel, S., Leask, M. and Turner, T. (eds.) (2010) *Readings for Learning to Teach in the Secondary School: A Companion to M Level Study*, Abingdon: Routledge.

This book brings together essential readings to support you in your critical engagement with key issues raised in this textbook.