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Natural Heterogeneity Prevents Synchronization of Fridges With Deterministic Frequency Control

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ABSTRACT Appliances that cycle on and off throughout the day, such as fridges, freezers, and air-conditioners can collectively provide second-by-second electricity supply-demand balancing known as frequency response. Previous studies have shown that deterministic temperature set-point control of a homogeneous population of such appliances can cause herding behavior with detrimental effects on the system. Here, we use computational modeling to establish the minimum population heterogeneity required to prevent herding problems without requiring centralized or stochastic control. We discover a linear relationship between the benefits that fridges can provide and their number. The impact on system benefits and on fridge temperatures of varying fridge frequency sensitivity is also explored, and a viable range for sensitivity (the control parameter) is proposed. Our approach involves simulating a large heterogeneous population of frequency-sensitive fridges using 12 months' GB system data from National Grid. We compare the historic frequency response from other response providers with their response in our fridge simulations to determine the benefits of the fridge population response. We find that a fridge population can offer a valuable demand-side response service to the electricity system operator, requiring neither the expensive infrastructure of centralized control nor the regular intervention of stochastic control for temperature cycle desynchronization.

INDEX TERMS Demand-side management, frequency control, frequency response, modeling, power system modeling, power system stability, refrigerators, simulation, thermostatically-controlled loads.

I. INTRODUCTION

As our electricity supply becomes more dependent on volatile resources such as the wind and sun, flexibility becomes increasingly important for maintaining system security at palatable costs [1]–[3]. Power system frequency is the frequency of the alternating current produced when rotating turbines drive synchronous generators that produce electrical energy. The System Operator (SO) is required to maintain the frequency very close to the nominal value at all times. The SO does this by employing flexible generation, demand and storage to balance supply and demand by monitoring the system frequency and adjusting their input/output accordingly. One type of this service is known as frequency response[1] or frequency regulation. When the frequency falls below the nominal value (50Hz in GB) frequency response providers increase generation (or decrease demand), and *vice versa* when the frequency rises above nominal.

Thermostatically-controlled loads (TCLs), such as fridges, freezers, air-conditioners and hot-water tanks can be flexible demand resources because users only value the average temperature they deliver, rather than the exact times they switch on or off. We will show that using the power system frequency (which TCLs can sense using basic equipment) to determine the operating set points of each TCL, a population could collectively provide frequency response to the system with negligible impact on their individual operation.

The technology to control an appliance using the system frequency was patented in 1979 by Schweppe [7], and research into the potential role of TCLs for system balancing began in the 1980s [8]–[11]. Recent years have seen a renewed interest in the possible use of TCLs for frequency response, with prominent papers such as [12]–[21]. The challenge is to control potentially millions of domestic appliances cheaply, efficiently and reliably for the benefit of the

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[1]See, for example, [4], [5] for a description of the different types of frequency response and balancing services more widely. During 2017-18 (and similarly in previous years) the SO in Great Britain (GB) paid approximately £140M for response [6]

power system without impacting user experience. Various control schemes have been proposed, generally classed as either (predominantly) *centralized* or *decentralized*. Their relative advantages and disadvantages are discussed in [22]. Our choice of decentralized control is largely due to the financial burden and security risks associated with the very large secure communications infrastructure required for centralized control.

For decentralized control each TCL has its own built-in control mechanism that (in our case) changes the operating temperature set points based on the grid frequency. The challenge of decentralized control is to prevent the synchronization of TCL temperature cycling following frequency disturbances, since all TCLs are responding to the same signal with closed-loop control. Synchronization can cause power system frequency fluctuations and instabilities if many TCLs are switching on/off at almost the same time, causing sudden large imbalances in the supply-demand balance[2]. Indeed, populations of TCLs have exhibited synchronization problems in simulations in previous work [20]–[26].

A popular approach to preventing synchronization problems is to introduce stochasticity to the control scheme, such as in [15], [21], [27]–[31]. For example, switching might be governed by probability rates, or time spent off following a frequency event may be determined probabilistically. Two potential disadvantages are envisaged with (at least highly frequent) stochastic switching. Firstly, naive randomized switching could lead to multiple switch events in short succession, which can cause wear on the appliance (although this can be avoided depending on the control). Secondly, regular stochastic switching could garner negative public opinion if there are fears about appliances acting randomly.

Our work proposes that stochastic control is not necessary to prevent herding behavior, so long as the population of TCLs is sufficiently heterogeneous. There are examples of simulations of heterogeneous TCLs with deterministic control that do not exhibit herding behavior, such as in [13], [22]. Until now, no one has asked how much heterogeneity is required to prevent synchronization problems. In this paper we answer this question for a population of fridges with deterministic control, and discuss the plausibility of the existence of this minimum level in the real world. In particular, we consider heterogeneous temperature set points, asymptotic fridge heating and cooling temperatures, heating/cooling coefficients, and (resulting from the above) different fridge on and off durations and duty cycles.

The main contributions of this paper are as follows. Firstly, to determine the minimum level of parameter heterogeneity required to prevent synchronization problems without resorting to stochastic or centralized control. We show that the required level is very small, and well within what could naturally be expected to occur. Secondly, we find the relationship between the number of frequency-sensitive fridges on the system and the amount of frequency response from other providers they displace. Thirdly, we investigate the impact of varying the sensitivity of the fridge temperature set points to the frequency (our control parameter) on the level of displaced response and also on fridge operating temperatures. The remainder of this paper is structured as follows. In Section II we describe our assumptions, model, and choice of parameters. In Section III we present the results of our analysis, and in Section IV we offer a discussion and conclusions.

II. METHODS

Rather than simulate every fridge in our simulated frequency-sensitive population individually, we reduce computation time and resource by simulating 10,000 groups of fridges. Within each group all fridges start (and remain) at the same initial conditions and have the same parameter values. Different groups have different initial conditions and parameter values (see Section II-D for full details). We model 1M fridges (i.e. 100 per group unless stated otherwise[3]). There are an estimated 10.1M households in GB with a fridge, and 18.6M with a fridge-freezer [5]. Each combination of parameter values is explored with 36 simulations spanning 12 months during 2015-16, and each simulation runs for 10 days with 1s resolution. We use various historic data from the GB SO National Grid and model what would have happened had a fridge population been frequency responsive. We are able to model the amount of response provided by others when the fridges are contributing compared to how much was provided historically, incorporating natural variations in demand and stored kinetic energy. This is a large improvement on modeling TCLs in isolation responding to a one-off frequency event found in previous papers, such as, for example [13], [23], [32].

A. ASSUMPTIONS

As in [22] we make the following assumptions:

(i) All parameters remain constant over time;
(ii) Fridges respond to frequency deviations with a 1s detection delay and with negligible measurement error;
(iii) There is no influence from the fridge door being opened or by the addition/removal of food[4];
(iv) Temperature is sensed and set points defined with machine precision;
(v) Fridges consume constant power when on and zero power when off;
(vi) Power system frequency is the same everywhere on the network and there are no inter-area oscillations [33].

The implications of these assumptions are discussed in Section IV.

[2]Normally the total load from a large number of such appliances is approximately constant.

[3]We can vary the number of fridges in the population (between 0.2M - 10M) by changing the power consumption of each group.

[4]This is likely to be a conservative assumption, since such events are likely to increase the heterogeneity of fridge temperatures.

TABLE 1. Notation.

Symbol	Description	Units
Dem	total measured demand	MW
$\text{Dem}_{\omega 0}$	demand at nominal frequency	MW
Dem_{resp}	original demand response	MW
Dem^*_{resp}	simulated demand response	MW
Gen_{resp}	original generator response	MW
Gen^*_{resp}	simulated generator response	MW
E_k	total stored kinetic energy	MVAs
f	historic system frequency	Hz
f^*	simulated system frequency	Hz
Imb^*_{tot}	simulated total imbalance	MW
Imb_{under}	original underlying imbalance	MW
N	number of fridges	-
p	power consumption per (switched on) fridge	W
sdf	standard deviation factor	-
$\text{SD}^i_{\max}$	maximum standard deviation for parameter i	-
t	time	s
$T(t)$	temperature	°C
T^0_-	nominal lower temperature set point	°C
T^0_+	nominal upper temperature set point	°C
T_-	lower temperature set point	°C
T_+	upper temperature set point	°C
TCL_{resp}	TCL (fridge) response	MW
T_{OFF}	asymptotic heating temperature (room temperature)	°C
T_{ON}	asymptotic cooling temperature	°C
α	heating/cooling coefficient	s^{-1}
β	temperature set point sensitivity to frequency	$°\text{C Hz}^{-1}$
Δt	time step size	s
σ_i	standard deviation of parameter i	-
τ_{ON}	on cycle duration	s
τ_{OFF}	off cycle duration	s

B. SIMULATION

We use the model and simulation method described fully in [22], which we outline here. Notation are summarized in Table 1.

1) INPUTS

There are four types of input data spanning our 12-month simulation period during 2015-16, provided by National Grid[5].

[5]Unless stated otherwise, data were provided confidentially. However, for box and whisker plots of all data used see Appendix C in [5].

Kinetic energy data are an estimate for the total stored kinetic energy in MVAs (megavolt-ampere seconds) [34], related to total system inertia. (Half-hourly) values are calculated by summing the kinetic energy of all running synchronized generators with an estimate of kinetic energy from demand. Typical kinetic energy values are within 2×10^4 - 4×10^4 MVAs. When the stored kinetic energy of the system is low (such as during times of low demand and high renewable output) the system is more sensitive to supply-demand imbalances and fast-acting response provision is more valuable.

Demand data in MW are a sum of the power leaving the electricity transmission system, including any power exports through the interconnectors to other countries. Half-hourly energy demand data are publicly available [35].

Historic system frequency data in hertz at a 1s resolution had previously undergone a cleaning process that took advantage of readings from multiple locations and are publicly available [35]. During the 360 days' data used in the simulations, multiple large frequency events occurred and are captured in this dataset. Figure C3 in Appendix C3 of [5] indicates the amount of frequency data outside of the range 49.8Hz - 50.2Hz (present in most of the 10-day simulation periods).

Response holdings are the amount of frequency response delivery in MW that National Grid expects each second as a function of grid frequency and available providers. There are three types of response holdings: primary and secondary response (1s and 11s delay, respectively, when frequency is below 49.985Hz), and high response (1s delay when frequency is above 50.015Hz). The data allow us to simulate the total frequency response from other providers historically, and crucially, how the response would have been different had a population of TCLs been providing response to the system.

2) ALGORITHM

Step 1 is performed once for all time steps $t \in \{1, 2, \ldots 864000\}$ of duration $\Delta t = 1\text{s}$ for each 10-day simulation period. Steps 2 - 7 form an iterative loop. To initialize the simulation we set the simulated frequency $f^*(1) = f(1)$, where the * notation is used to distinguish simulated variables from their historic values.

STEP 1: CALCULATE THE UNDERLYING SUPPLY-DEMAND IMBALANCE

To simulate the impact of a fridge population on the power system frequency we need to understand the underlying behavior that gave rise to the historic system frequency. This is done for all time steps before the simulation starts. The underlying supply-demand imbalance, $\text{Imb}_{under}(t)$ (in MW), is estimated accounting for frequency-sensitive demand and the actions of historic frequency response provision. By 'frequency-sensitive demand' we are referring to the increase (or decrease) in synchronous demand when frequency rises (or falls) due to the speeding up (or slowing down) of synchronous machines [36]. National Grid infers an effect of 2.5% change in demand as frequency deviates from

50Hz [36];

$$\mathrm{Dem}\big(f(t)\big) = \mathrm{Dem}_{\omega 0}(t)\big(1 + 0.025(f(t) - 50)\big). \qquad (1)$$

where $\mathrm{Dem}\big(f(t)\big)$ is historic measured demand and $\mathrm{Dem}_{\omega 0}(t)$ is the estimate for the historic demand *had the frequency been 50Hz at all times* (found by rearranging (1)). The historic response from demand is given by

$$\mathrm{Dem}_{resp}(f(t), t) = \mathrm{Dem}\big(f(t)\big) - \mathrm{Dem}_{\omega 0}(t). \qquad (2)$$

The historic response from generation, $\mathrm{Gen}_{resp}(f(t), t)$, is estimated using the response holdings data described above, and running the simulation without a frequency-sensitive fridge population. $\mathrm{Dem}_{resp}(f(t), t)$ and $\mathrm{Gen}_{resp}(f(t), t)$ are then used to calculate the historic underlying imbalance $\mathrm{Imb}_{under}(t)$. For brevity we refer the reader to [22] for details.

STEP 2: CALCULATE THE NEW RESPONSE FROM THE HISTORIC PROVIDERS

For each time step we start by using the response holdings data to simulate the actions of the historic frequency response providers based on the updated frequency from the previous time step. We denote this by $\mathrm{Gen}^*_{resp}\big(f^*(t - \Delta t), t\big)$.

STEP 3: UPDATE FRIDGE TEMPERATURE SET POINTS

A fridge switches on/off when its temperature hits its upper/lower temperature set points T_+ and T_-, respectively. Following [13] we establish a linear relationship between the temperature set points and the power system frequency:

$$T_\pm\big(t\big) = T^0_\pm - \beta\big(f^*(t - \Delta t) - 50\big) \qquad (3)$$

where control parameter $\beta > 0$ determines the sensitivity of the temperature set points to frequency deviations and $T^0_\pm$ are the ‘normal’ temperature set points (we use 2°C and 7°C as in [27]). Fig. 1 illustrates this concept.

STEP 4: CALCULATE THE NEW FRIDGE TEMPERATURES AND ON/OFF STATES

Denote by $T(t)$ the temperature of a fridge at time t, the cooling/heating coefficient by α, and the asymptotic temperatures that the fridge would reach if left on or off indefinitely by T_{on} or T_{off}, respectively. Then as in [21],

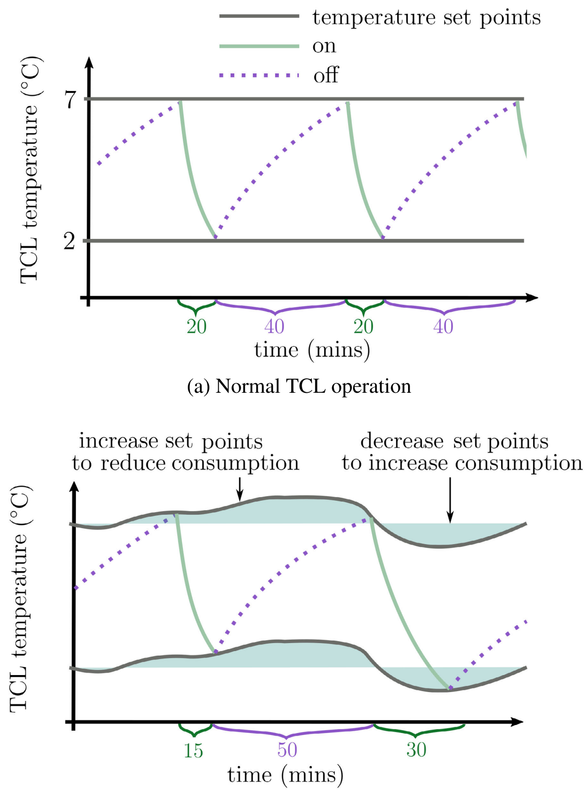
$$\dot{T}(t) = \begin{cases} \alpha\big(T_{\mathrm{on}} - T(t)\big) & \text{when a fridge is on} \\ \alpha\big(T_{\mathrm{off}} - T(t)\big) & \text{when a fridge is off.} \end{cases} \qquad (4)$$

Note that this is equivalent to the model used in papers such as [25], where in their notation, $\alpha = \frac{1}{CR}$, $T_{\mathrm{off}} = \theta_{amb}$ and $T_{\mathrm{on}} = \theta_{amb} - PR$, where C is the thermal capacitance, R is the thermal[6] resistance, θ_{amb} is the ambient temperature and P is the cooling power of the TCL when switched on.

The temperature T of a fridge is given by solving (4);

$$T(t) = (T(t - \Delta t) - T_{\mathrm{on}})e^{-\alpha\Delta t} + T_{\mathrm{on}} \;\; \text{when on} \qquad (5a)$$

[6]Fridge operation and set point control are also illustrated dynamically in the accompanying video available at http://ieeexplore.ieee.org.



(a) Normal TCL operation

(b) Frequency-sensitive TCL operation

FIGURE 1. Illustrative example of how TCLs such as fridges can support system frequency through temperature set point control.

$$T(t) = (T(t - \Delta t) - T_{\mathrm{off}})e^{-\alpha\Delta t} + T_{\mathrm{off}} \;\; \text{when off} \qquad (5b)$$

(assuming no switching). If $T(t)$ lies outside the interval (T_-, T_+) then a switch on or off should have occurred during $[t - \Delta t, t]$. The exact time of the switch on/off is estimated and the temperature is recalculated from the switch time to the end of the time step using linear interpolation. For the precise details see Appendix B in [5].

The initial temperatures and on/off states of the 10,000 groups of fridges are set by evenly distributing the groups around the on/off temperature cycle.[7]

STEP 5: CALCULATE RESPONSE FROM THE FREQUENCY-SENSITIVE FRIDGE POPULATION

Response from the TCLs (in our simulation fridges), $\mathrm{TCL}_{resp}(f^*(t - \Delta t), t)$, is given by the number of switched on fridges multiplied by their individual power consumption p, subtracted from the expected population demand if it were not frequency-sensitive.[8]

[7]The groups of fridges are evenly distributed over time in the cycle rather than temperature due to the nonlinear heating and cooling rates. For a longer explanation see [5], [22].

[8]Expected population demand is the average fridge duty cycle (proportion of cycle switched on) multiplied by the power consumption of one fridge p and by the number of fridges in the population.

STEP 6: CALCULATE TOTAL IMBALANCE

At each time step we calculate the total underlying supply-demand imbalance $\mathrm{Imb}^*_{tot}(t)$;

$$\begin{aligned}\mathrm{Imb}^*_{tot}(t) = \mathrm{Imb}_{under}(t) &+ \mathrm{Gen}^*_{resp}\big(f^*(t-\Delta t), t\big) \\ &- \mathrm{TCL}_{resp}(f^*(t-\Delta t), t) \\ &- \mathrm{Dem}^*_{resp}(f^*(t), t). \end{aligned} \tag{6}$$

STEP 7: UPDATE SYSTEM FREQUENCY

We use the total system imbalance from step 6 to update the system frequency, based on the linear approximation for $\dot{f}^*$ from [36];

$$f^*(t) = f^*(t-\Delta t) + \frac{50\mathrm{Imb}^*_{tot}(t)}{2E_k(t)}\Delta t \tag{7}$$

where E_k is total stored kinetic energy in MVAs. The algorithm now updates to the next time step and iterates through steps (2) - (7).

C. ANALYSIS

The most valuable way to measure the impact of the frequency-sensitive fridge population is to ask how the provision of frequency response from other providers is affected. If it is lower then the fridges have a positive effect on the system. If, however, other actors have to provide more response, then the fridges destabilize the system and do more harm than good. We measure this impact with our variable 'cumulative response savings' (*respSave* in MWh), which we define as the sum of the difference between historic frequency response provision and response provided by the historic providers in the simulation with the fridges present:

$$\begin{aligned} respSave(t) = \sum_{\hat{t}=1}^{t} \Big\{ &|\mathrm{Gen}_{resp}\big(f(\hat{t}-\Delta t), \hat{t}\big)| \\ &- |\mathrm{Gen}^*_{resp}\big(f^*(\hat{t}-\Delta t), \hat{t}\big)| \Big\}. \end{aligned} \tag{8}$$

We use the absolute value of response because it can be positive or negative, depending on whether the frequency is above or below 50Hz. If the fridges are acting beneficially to the system then the cumulative response savings will grow over time, and if they cause more harm than good then the cumulative savings will become negative.

D. PARAMETER CHOICES

The mean of each heterogeneous parameter is shown in Table 2. We take the mean of T_{off}, T^0_- and T^0_+ from [27], the duty cycle from [37], and the asymptotic cooling temperature T_{on} results from these choices. Heating/cooling rate α is chosen for a total cycle of 45 minutes, similar to [37]. The on and off durations τ_{on} and τ_{off}, respectively, are derived from the other parameters. The parameters without heterogeneity are the number of fridges in the population, $N = 1$M; the power consumption of each fridge when switched on, $p = 70$W [20], [27]; and the control parameter $\beta = 2.4°\mathrm{C.Hz}^{-1}$ (see equation (3)) ensures that food will not freeze within the statutory operating range of the system frequency. We explore the impact of varying N and β in Section III.

Table 4.3 in [5] contains a survey of ten approaches to parameter heterogeneity in the literature, and this was used to inform our choices for introducing heterogeneity. In the absence of data on realistic distributions for the parameters, we use normal distributions. For each heterogeneous parameter, we increase the heterogeneity by choosing a 'maximum value' for the standard deviation, $\mathrm{SD}_{\max}$, and simultaneously multiply each parameter's $\mathrm{SD}_{\max}$ by a *standard deviation factor*[9], sdf $\in [0, 1]$. That is to say, the standard deviation of parameter i is given by

$$\sigma_i = \mathrm{sdf} \times \mathrm{SD}^i_{\max}. \tag{9}$$

For example, room temperature T_{off} in GB is unlikely to vary across the country by more than a few degrees Celsius, and a range of $\pm 4°$C would be a reasonable, if conservative maximum range[10]. We expect 99.7% of the population to lie within this range if $3\sigma = 4$, and so to achieve this for our simulations with the greatest heterogeneity (sdf $= 1$), we set $\mathrm{SD}_{\max} = 1.33$ for T_{off}. For the asymptotic cooling temperature T_{on} we let $3\sigma = 6$ similarly to [13], [37], [38]. Rather than treat the temperature set points T^0_- and T^0_+ independently, which could result in fridges with impossibly short or overly long cycle times, we instead treat their difference. For each fridge, T^0_- and $(T^0_+ - T^0_-)$ are selected independently from normal distributions and T^0_+ is calculated from their sum. Fridge cooling rates vary depending on the age and model of the appliance and the amount of food inside. We allow the cooling/heating rate α to vary such that $3\sigma = \frac{\bar{\alpha}}{2}$ in the largest diversity case. In the absence of data, we consider a maximum range of $\bar{\alpha} \pm 50\%$ to be a highly conservative estimate.

III. RESULTS

Our first aim is to determine the minimum amount of heterogeneity (the minimum sdf) required to prevent the fridges from destabilizing the system frequency. Homogeneous populations of TCLs have shown to synchronize when responding deterministically to a frequency signal [20]–[26]. [22] shows that a homogeneous population will often cause negative cumulative response savings (meaning that the other response providers have to provide more response when the fridges are attempting to 'help'), but that some heterogeneity can alleviate these problems.

Fig. 2 shows the cumulative response savings at the end of each 10-day simulation for different values of sdf (note the logarithmic x-axis due to the very small level of sdf required) for a population of 1M fridges. Below sdf $\approx 10^{-4}$ the results are similar to the homogeneous population (sdf $= 0$). Increasing the sdf to approximately $10^{-3.5}$ results in

[9]Note that the sdf is called the 'diversity factor' δ in [5], [22].

[10]Note that this is the mean $\pm 20\%$, as used by [13], [37], [38]. We mention percentages for comparison with other references, but of course in the context of temperature in °C, percentage differences have no sensible meaning. The actual size of the maximum range will be shown to have little bearing on our results.

TABLE 2. Mean and SD_{max} for each parameter (lower four parameters result from choices for the upper set).

Parameter	Mean	SD_{max}	Units
T_{ON}	-26	2.00	°C
T_{OFF}	20	1.33	°C
T^0_-	2	0.50	°C
$T^0_+ - T^0_-$	5	0.33	°C
$\alpha \times 10^5$	18.08	3.01	s^{-1}
p	70	0	W
β	2.4	0	°C.Hz^{-1}
T^0_+	7.00	0.60	°C
τ_{ON}	15.15	3.27	mins
τ_{OFF}	30.00	6.91	mins
duty cycle	33.55	2.69	%

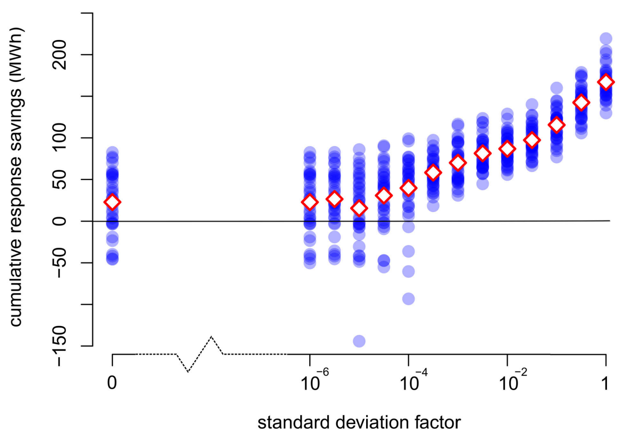


FIGURE 2. Final cumulative response savings after each 10-day simulation (blue circles) and means (red diamonds) for different sdf. The heterogeneity eradicates negative outcomes for sdf > 10^{-4}.

TABLE 3. Implications of sdf = 10^{-2} for each parameter.

Parameter	Standard Deviation	99.7% Interval			Units
T_{ON}	0.020	−26	±	0.060	°C
T_{OFF}	0.133	20	±	0.040	°C
T^0_-	0.005	2	±	0.015	°C
$T^0_+ - T^0_-$	0.003	5	±	0.010	°C
$\alpha \times 10^5$	0.030	18.08	±	0.090	s^{-1}
T^0_+	0.006	7	±	0.018	°C
τ_{ON}	0.033	15:09	±	5:89	min:s
τ_{OFF}	0.069	30:00	±	12:44	min:s
duty cycle	0.027	33.55	±	0.081	%

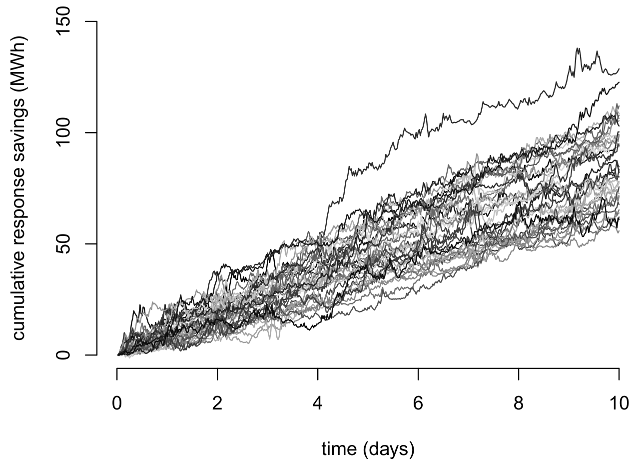


FIGURE 3. Cumulative response savings over time for each of the 36 10-day simulations for 1M fridges with sdf = 10^{-2}.

positive savings in all simulations, which continue to increase in value as the sdf increases. We find that when the number of fridges in the frequency-sensitive population is increased to 10M the negative cumulative savings are also eliminated by increasing the sdf from 10^{-4} to $10^{-3.5}$. Therefore we consider $10^{-3.5}$ to be the minimum heterogeneity requirements for preventing the detrimental behavior exhibited by less-heterogeneous populations. For this level of parameter heterogeneity, the simulations show that the probability of synchronization is less than 1s per year since in each of the 36 10-day periods studied none of the populations were closer to synchronization than at the start.

To understand the significance of the minimum sdf required to prevent negative cumulative response savings, Table 3 shows the standard deviation and 99.7% confidence interval for each heterogeneous parameter for (a much more conservative) sdf = 10^{-2}. The intervals are all very small, well within what might reasonably be expected to occur in the real world. For example, a wide spread of mean internal dwelling temperature in English households were observed in [39], with a standard deviation of 2.47°C; several orders of magnitudes greater than our minimum requirement of 0.133°C. Hence, even in the absence of stochastic or centralized control, the natural diversity that exists within a population of fridges, room temperatures and temperature set point preferences can be expected to prevent the synchronization problems exhibited in simulations of homogeneous populations. Fig. 3 shows the accumulation of cumulative response savings over time for sdf = 10^{-2}. We see a roughly linear increase over time, with a large variation between simulations. This is largely due to the variations in the underlying supply-demand imbalance and stored kinetic energy of the system over a year, highlighting the need to simulate electricity grid operation over multiple time periods to consider a range of real system conditions. This figure can be contrasted with Fig. 13a in [22] (where sdf = 0) to see how the heterogeneity has eliminated negative response savings.

An important question to ask is how changing the control parameter β (introduced in equation (3)) would affect the amount of cumulative response savings. Fig. 4 shows the final cumulative response savings for sdf = 10^{-2} and 1M

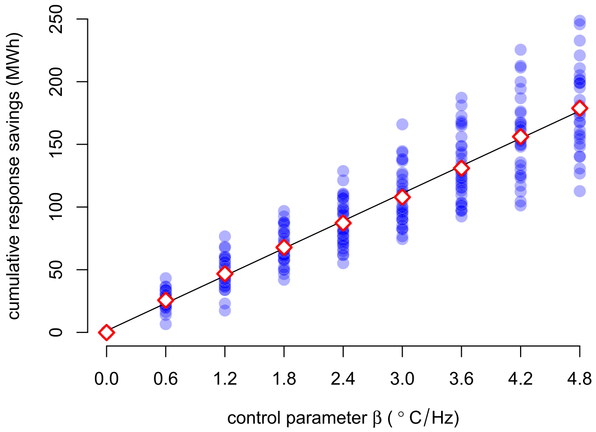


FIGURE 4. Final cumulative response savings after each 10-day simulation (blue circles) for different β. Means shown with red diamonds. Linear trend line estimated from the means.

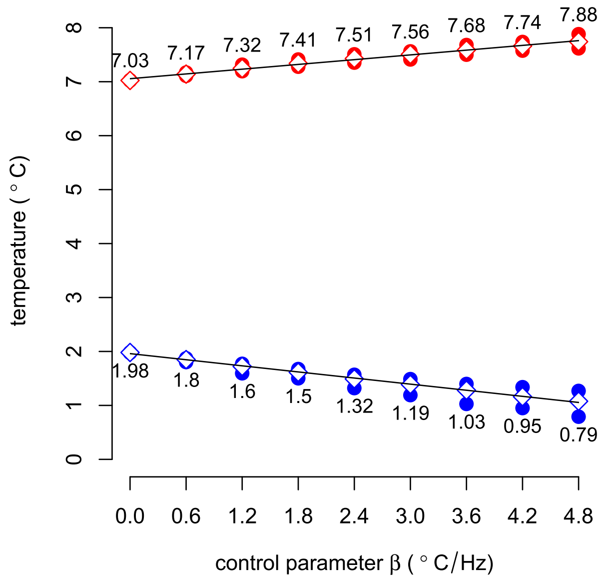


FIGURE 5. Minimum, maximum (blue circles) and mean (blue diamonds) of the lowest temperature reached by any fridge over all simulations and likewise for the highest temperature ever reached (red). The 'worst' values (for the fridges) are labeled.

fridges for different β. A trend line has been drawn from the means for each value of β, and indicates that for every 1°C.Hz^{-1} increase in β the savings increase by approximately 36.5MWh. Although the savings increase linearly with β, our choice of β is limited by its impact on fridge temperatures. The higher β, the further away fridges operate from their 'normal' temperature set points $T^0_\pm$. Fig. 5 shows the minimum, maximum and mean of the lowest temperature reached by any fridge over all simulations (blue) and the same for the highest temperature ever reached (red). The 'worst' values in each case are labeled. These values are approximately linearly dependent on β. If a fridge normally operates between 2-7°C, a user may not be happy with potential operation between, say, 0.79-7.88°C, and therefore $\beta = 4.8$ would be too high. We propose that $\beta \in [2, 4]$ would be a suitable choice that achieves reasonable response savings ($\approx$ 75-150MWh for 1M fridges) without compromising user experience (no more than $\approx$ 1°C outside of the normal operating range).

We would like to ensure that our results are robust to changing the number of fridges in the frequency-sensitive population and to understand the benefits of increasing the population. Previously we have been simulating 10,000 groups of 100 (identical) fridges (1M in total). Fig. 6 shows the final cumulative response savings when we vary the number of fridges between 20 and 1000 per group, i.e. 0.2M-10M in total[11]. We see that the relationship is linear: increasing the number of fridges by 1M increases the cumulative response savings by approximately 80MWh. This relationship is possibly surprising, as one might have expected a decreasing rate of return for using more fridges, or an initially increasing rate of return at the low end of the scale. For context, the original total response provided by other response providers over the 10-day periods ranged between

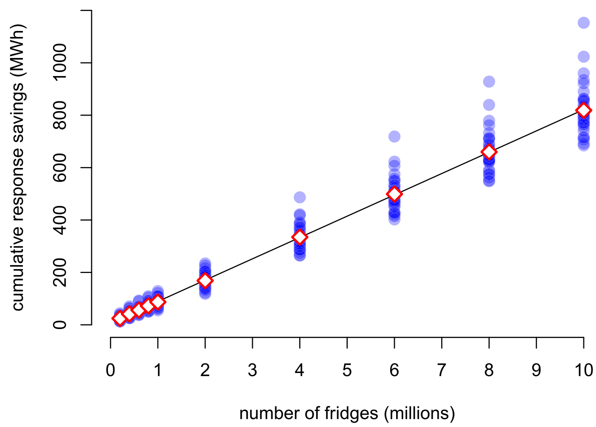


FIGURE 6. Final cumulative response savings for different population sizes when sdf = 10^{-2}, $\beta = 2.4$.

8,004MWh and 12,172MWh with a mean of 9,357MWh (approximately).

IV. DISCUSSION AND CONCLUSIONS

We agree with [40] that "while some [demand response] schemes are already in place, it can be expected that TCLs will play a much more important role in providing a fast and accurate source of flexibility in the future electricity grid". Control schemes for TCLs to provide frequency response have become a popular research topic in recent years. Our simulations are unique in modeling not just a frequency-sensitive TCL population, but also existing frequency response providers whose response is affected by the behavior of the TCLs. Previous work has considered the impact of TCLs responding to an initial large frequency

[11] Recall that there are approximately 10.1M households in GB with a fridge, which is why we explore this range.

deviation on the grid frequency over the subsequent few hours [13], [17], [21], or the response over several hours of a TCL population (in isolation) to a varying input signal [14], [16]. Capturing how other response providers react to changes in system frequency under realistic system conditions allows us to accurately determine the amount of response that could be displaced by a population of fridges. Our simulations run continuously for 10 days, and cover 1 year to account for the variations in system conditions typically found over a year.

A homogeneous population of frequency-sensitive TCLs has been shown to exhibit temperature cycle synchronization that can destabilize the power system frequency [13], [20]–[26]. In this paper we established the minimum level of parameter heterogeneity required to prevent harmful herding behavior in a population of fridges, and found it to be far less than we would expect to occur naturally. We elicited the linear relationship between the parameter controlling the sensitivity of the fridges to frequency deviations (β) and the benefits the fridges could provide to the system. We also studied the impact on fridge temperatures of varying β. This allowed us to propose a range for a suitable choice of β. We also determined the linear relationship between the number of frequency-sensitive fridges and reduction in frequency response required from other providers (MWh). By considering the needs of the system as a whole we have been able to capture what is most important for grid stability - providing a service that can contribute to overall provision, and displacing, for example, the most expensive or most polluting existing providers of frequency response.

Although many real-world attributes are captured in our simulations, such as the response of other providers, national demand and total stored kinetic energy, it is important to consider the potential implications of our simplifying assumptions (Section II-A). We have assumed that system frequency is identical across the electricity grid, when in reality fluctuations have an origin and spread across the network. TCLs respond to the frequency at their location, and thus could potentially prevent local fluctuations from becoming grid-scale. Imbalances at the distribution network level are becoming increasingly important to manage [41], and so modeling fridges on a network could show greater advantages for the system than presented here. We have also assumed that all parameters remain constant over time. This ignores any fridge door opening and food addition or removal, as well as changes in room temperature or long-term changes such as appliance efficiency reduction. Opening the door and adding/removing fridge contents are random events, with some correlation around meal times, and will typically diversify the temperatures and on/off states. The minimum heterogeneity requirements for synchronization prevention indicate that even if many people were to use their fridge at a similar time, the differences in room temperatures, food temperatures and durations that doors remain open would be highly unlikely to counteract the natural population diversity and cause synchronization. Nevertheless, the clustering of these activities around meal times should be explored before implementing a large-scale rollout. Other assumptions which require further investigation are the absence of measurement error, which could be tested with a small number of appliances and sensors, and the energy consumption of each fridge, which in reality would exhibit a small spike as they switch on.

Certain areas remain open questions for further examination. Data covering 12 months during 2015-16 was used, but it is possible for the system to experience greater frequency fluctuations than occurred during this period, and simulation of power outages could be beneficial. As explained above, we estimate that the probability of synchronization with the minimum heterogeneity requirements is less than 1s/year. Further work is recommended to improve this bound and to determine analytically how it decreases with sdf. One could also investigate the impact of modeling fridges on a network, with all of the spatial effects involved. Is a similar minimal amount of heterogeneity required to prevent synchronization for other types of TCL, such as air-conditioners, hot-water tanks and heat pumps? Our results are likely to be most applicable to those with similar on/off durations. A small-scale trial could test the parameter assumptions made here, and determine the most effective and affordable control equipment to fit/retro-fit into TCLs. From an economic standpoint, the response capabilities and costs of TCLs should be compared with those of other response providers (and potential providers), incorporating how the value of system flexibility will increase in future [1]–[3]. Finally, greater policy research into achieving a large-scale roll-out of frequency-sensitive TCLs is needed, including, for example, participation incentives (either market or government-led) or mandates to motivate the development of this service.[12]

In conclusion, this analysis shows that a population of fridges has the potential to provide a valuable frequency response service to the GB System Operator without affecting the needs of the appliance users, and without requiring centralized or stochastic control, because the existing heterogeneity in the fridge population is much larger than the minimum required for stability established in this paper. Modeling the impact on the actions of other response providers offers valuable insights into the benefits of such a service, contextualizing the scale of the potential response service in relation to the current needs of the GB power system.

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[12] The UK Department for Business, Energy & Industrial Strategy (BEIS) recently ran a consultation on proposals for setting standards for smart appliances. See https://www.gov.uk/government/consultations/proposals-regarding-setting-standards-for-smart-appliances for details.

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