Influence of Initial Void Ratio on Critical State Behaviour of Poorly Graded Fine Sands

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Abstract

The application of critical state framework depends on the presence of either a unique critical state line (CSL). However, many authors have observed that a transitional behaviour occurs in certain granular intermediate soils in which the fines content as well as the initial void ratio have a significant effect on the location of this line. This work investigates these effects in poorly graded granular soils as previous studies reported that this type of soil does not exhibit transitional behaviour. Results from this study reveal that poorly graded fine-grained sand exhibits transitional behaviour similar to that of intermediate soils and subsequently, it can be grouped into ranges of initial densities in which each group approaches a unique CSL in v:lnp' space. A unique state boundary surface is identified by determining the critical state parameters from each group and using these parameters to normalize the stress paths; this shows that the critical state framework can be successfully applied to this type of soil.

Keywords

Fabric/structure of soil, Laboratory test, Sand.

Introduction

Characterizing the behaviour of soil at high strains, or what is known as the critical state, is very important. This concept was first introduced by Casagrande [1], who discovered that during shearing, sand in a loose state decreases in volume, whereas sand in a dense state increases in volume until a critical void ratio is achieved. In their study on the yielding of soils, Roscoe et al. [2] concluded that each soil specimen has a particular critical pressure at which it will shear at constant volume when subjected to shear distortion. Furthermore, according to the critical state theory [3], when continuously distorted until they flow as frictional flow, soils or other granular materials will approach a well-defined critical state. Although the critical state was initially associated with clay, many authors have attempted to apply this concept to sandy soils [4,5]. However, the application of the critical state to sands is challenging owing to the difficulty of defining a virgin consolidation line for such soils [6] as well as the fact that compression in sand is influenced by stresses developed at the contact between the particles [7]. Nevertheless, some attempts to apply this concept to sands have been successful [8, 9].

In order to understand the micro-scale behaviour of sandy soils, Thornton [27] have suggested a correlation between the macro-scale angle of shearing resistance and inter-particle friction than the experimental work of Skinner [28]. In the small strain region, Clayton & Heymann [33] shown that particle shape has a significant impact on stiffness. In a more recent study Cavarretta et. al. [29] developed new technologies to quantify particle shape and surface roughness accurately and to measure particle contact stiffness and inter-particle friction. Authors studied micro-mechanical measurements with conventional soil mechanics tests (oedometer and triaxial) to provide experimental data that relate the nature of the constituent particles with the macro-scale mechanical response of the soil. Authors reported a clear relationship between the inter-particle friction and the particle surface roughness. However, the macro-scale experiments indicated that_while the material response may be slightly dependent on the surface roughness and friction, the influence of particle shape is more significant. Senetakis et al [30] in a similar study, reported that, repeating interparticle shearing tests showed a small reduction in the friction angle, which might be attributed to some damage at the asperities during the first shearing.

Furthermore, Zhao et. al. [34] investigated the particle breakage under one-dimensional compression and concluded that a dense specimen has lower failure probability and less extensive failure modes than a loose specimen around the yield stress. Additionally, authors reported that the influence of initial density on particle survival probability diminishes after substantial particle breakage. However, a particle failure event leads to higher compression in a loose specimen and generates more fines through further comminution of existing fragments.

In their study, on the micro-mechanics of crushable aggregates, McDowell, G. R. and Bolton, M. D. [31] have shown that shearing is far more effective at breaking particles than isotropic or k_0 loading. Similarly, Coop et. al. [32] performed a series of ring shear tests and concluded that particle breakage continues to very large strains, far beyond the reach of triaxial equipment. These authors reported that particle breakage is accompanied by volumetric compression, and occurs even for tests at modest confining stresses. If an apparent critical or constant-volume state is seen in a triaxial test, as Chandler [25] assumed, it can only be as a result of counteracting dilative strains from particle rearrangement and compressive strains from particle breakage. Luzzani & Coop [26] found that even quartz sands at low-stress levels were subjected to small amounts of particle breakage. In a recent study by Rezaeian et al [35], large samples of a dynamically compacted well-graded granular material for base and sub-basehave were tested in a large triaxial equipment and the results show that the soil suffers from breakage at all confining pressures and that increasing confininsg stresses increase breakage.

Recent studies [10, 8, 11–13] on intermediate soils reported that soils with a mixture of particles ranging between sand and clay that were remoulded in laboratory conditions did not approach a unique critical state line (CSL). Shipton and Coop [10] showed that each type of soil produced a wide range of CSLs, and the authors proposed that these lines are influenced by the presence of fine particles within the soil mass. Kwa and Airey [12] experimentally investigated different mixes of sand and fines, and found that in all cases, two unique lines were approached in which the denser samples tended to approach a parallel line located below that of the samples with higher void ratios. The relation between the initial void ratio and the CSL was investigated by Ferreira and Bica [8]. The authors reported that each initial void ratio produces different parallel CSLs in v:lnp' space.

However, the above studies focused on intermediate soils and no study has investigated poorly graded granular soils. Pestana and Whittle [14] as well as Lade and Yamamuro [15] reported that the compression behaviour of cohesionless poorly graded soils at high strains is not affected by the initial void ratio.

Nevertheless, Ventouras [16] conducted a series of tests varying the fines content up to 15%, on a small range of high void ratios (loose samples) and found parallel compression lines. The author suggested that, by analogy, the CSL would show similar behaviour for this poorly graded quarzitic sand. Similar results were obtained by Shipton [17] in clays. This is an indication that these materials have a transitional behaviour, however no conclusions were made by the researchers [16, 17] as a more thorough campaign of tests is needed to verify the non-uniqueness of the CSL.

To address this lack of knowledge, in this study, a series of triaxial tests were conducted on poorly graded and sub-angular fine sand. A large range of void ratios were used, covering the spectrum between the maximum and minimum void ratios. All tests were sheared to large strains in order to try to achieve a null volumetric strain. Furthermore, a normalisation regime, similar to the approach proposed by Ferreira and Bica [8] was adopted to distinguish the influence of the initial void ratio on critical state behaviour of poorly graded fine sands.

Materials and Procedures

The sand used in this study, which was named "Akdeniz" by the Geological Department, Cyprus, is fine-grained aeolianite obtained from a sandpit (4 km inland) located at the western part of the Cyprus Island. Fig. 1 shows the particle size distribution of this sand. The Akdeniz sand can be defined as a poorly graded sand as 90% of its particles have sizes in the range of 0.12–0.3 mm. Table 1 presents some physical properties of this sand, as well as properties of other types of sand mentioned in this study. An optical microscope with a data link to a desktop computer was used to create high-contrast, digital images, each showing around 50 grains. It can be observed from Fig. 2 that grains are mostly angular with rough surfaces.

Initially, the soil sample was air dried to determine the dry mass. Samples with different initial void ratios measuring 100 mm in height x 50 mm in diameter were prepared using a split mould at the top of a triaxial pedestal. A negative pressure of 20 kPa was applied to remove the mould and complete the triaxial set up. Different methods were employed to obtain wider ranges of densities between the minimum and maximum void ratios (0.505 – 0.895) of the sand used in this study as the preparation method does not affect the location of the CSL [18, 19]. The less dense specimens (group N1) were prepared by pouring sand and water with no tamping to ensure minimum density. The denser samples (group N2) were prepared using the same amount of water used for group N1; however, they were prepared in three layers and each layer was tamped 25 times using a modified compaction rod. To achieve even higher densities, the dry compaction method was used to prepare a third group of samples (group N3) by pouring sand in five layers with no addition of water and tamping each layer 25 times. As can be seen in Table 2 such adoption of different preparation method enabled to achieve relative densities ranging from 13.57% to 97.69%. Next, a small reduction in air pressure was carried out, allowing water from the backpressure system to saturate the sample. Then, the initial dimensions of the specimen were determined, and a small confining stress was applied in the triaxial chamber to circulate water in the specimen to improve saturation. The initial dimensions were measured directly after removing the mould, and the initial void ratio, e_0 was calculated using four different approaches proposed by Shipton and Coop [10]. The mean of e_0 was determined from the values obtained from the four methods and the accuracy was estimated using the maximum deviation of any value of e_0 from the mean, discarding any clearly anomalous value. A constant effective pressure of 30 kPa was applied to the specimens. All specimens, except group N3, were saturated under back pressure of 300 kPa and the saturation was completed when a B value of 95% or higher was reached. The specimens were then consolidated isotropically up to the desired confining pressure and then sheared under drained conditions up to 20% axial strain. In total, 17 drained triaxial tests were conducted on the poorly graded Akdeniz sand samples with a wide range of initial void ratios. The tests conducted on the specimens and the properties of each test are listed in Table 2.

Experimental results and discussion

Fig. 3 shows stress-strain and volumetric change curves of each group of specimens N1 (Fig. 3a), N2 (Fig. 3b) and N3 (Fig. 3c) . It can be seen that, for each group, an increase in confining pressures lead to an increase in shear strength. Looking at the volumetric behaviour of each group it is easy to realise that the reduction in the initial void ratio results in less compressibility of specimens. Furthermore, it can be seen that most of the low confinement specimens on all groups suffer from strain localisation, having a well-defined failure plane, represented by the abrupt change from a large dilation to a fairly constant volume at large strains. In order to obtain a better estimate of the critical state of those samples, the hyperbolic model was used in the volumetric strain curves, by using the following equation:

$$\varepsilon_v = \frac{\varepsilon_a}{a` + b` \varepsilon_a} \tag{1}$$

In this equation a` and b` are constants determined using the original data. Dashed lines on the fig.3 show the extrapolations up to 22% strain that was determined by using the hyperbolic fit.

The ends of the test points in v:lnp' space are plotted as shown in Fig. 4 and were used to determine the CSLs reached by the triaxial samples. The standard engineering pressures used in this study do not permit the determination of the normal compression line (NCL) of quartz sand, which is traditionally used to normalise shearing data, therefore the equivalent pressure on the CSL (Eq. 2) was used to normalise the shearing data. The NCL and CSL determined by Dos Santos et al. [20] and Ferreira & Bica [8] are also plotted in Fig. 4, which shows a slope very similar to the slope of the CSLs obtained from the groups of N1, N2 and N3

$$p'_{cs} = exp^{\frac{(\Gamma - \nu)}{\lambda}}. \tag{2}$$

It is possible to group the tests and use several ranges between the minimum and maximum void ratios tested. For the sand investigated in this study, three CSL lines were used to represent the tested ranges; one for each type of the initial preparation group. These lines are defined by the general equation given in Eq. (3) and the ranges of the initial void ratios corresponding to each group are listed in Table 2, whereas the CSL parameters for each group are listed in Table 3.

$$\nu = \Gamma - \lambda \ln p'. \tag{3}$$

Although the different groups approached different CSLs in v:lnp’ space, they have essentially the same CSL in q:p’ space. Fig. 5 shows the typical stress ratios (q/p’) plotted against the axial strain for all three groups. Although the samples were prepared using different methods, all samples exhibited similar behaviour. A typical increment of up to the peak stress is followed by a plateau indicating a constant load rate. As noted by many authors, the critical state is assumed to be reached when no further change occurs in the volumetric strain or stress. It can be observed from Fig. 5 that all tests reached the plateau after approximately 15% strain. In each case, all samples reached a constant stress ratio around the selected critical state gradient (M) value of 1.4, which indicates that the grading of the CSL in q:p’ space is not affected by the initial void ratio. Thus, the concept of a unique CSL is still valid for granular soils; this line can be clearly observed in the critical state on q : p` diagrams of responses of all specimens in Fig. 6.

Fig. 7 shows typical stress–dilatancy data based on the total strains of the three groups of tests, where q/p’ is plotted against dƐv/dƐs. It can be observed that all samples exhibited fairly similar behaviour and dilation occurred at a lower stress, whereas as the strain increases, the behaviour changes back to the zero dilation axis, indicating that the sample has reached the critical state. In contrast, the samples showed less tendency to dilate at higher pressures. In addition, test results indicate that the value of q/p’ at the critical state is 1.4 for all groups; according to the stress–dilation analysis, all the tests terminated fairly close to the critical state, and despite the different initial void ratios and different preparation methods, the M value is the same for all samples.

The resulting normalisation shown in Fig. 8 defines a clear state boundary surface on the dry side of the critical state (Hvorslev surface), as extremely high-pressure tests are required to define the state boundary surface on the wet side. This unique state boundary surface achieved by normalising shear data confirms the hypothesis that there are many parallel CSLs in v:lnp’ space even for poorly graded sand; however, the behaviour of samples from any initial void ratio can be predicted if it is associated with the appropriate CSL. It has been previously suggested that the transitional behaviour is due to different reasons such as fines content [19] or fabric structure [12]. However, the findings of this study show that although these factors affect the location of the CSL, the initial void ratio strongly influences transitional behaviour.

The non-uniqueness of the CSL must be as a result of change in the fabric due to variations in the initial void ratio, as the tested sand is poorly graded. Moreover, no change in the specific volume such as the inter-granular void ratio [24] can result in a unique line in v:lnp' space. Some authors have associated this transitional behaviour to the fines content; however, this behaviour is also associated with poorly graded sand with no fines content, as shown in the results.

Fig. 1 shows the particle size distribution of the soils mentioned above; sand exhibiting transitional behaviour is identified as poorly graded (intermediate) and having fines. Furthermore, the initial void ratio seems to influence the location of the CSL. In the study by Altuhafi and Coop [21] on well-graded sand, which has a transitional behaviour, the researchers explained that this behaviour is directly related to particle breakage under compression and emphasized that the lower the initial void ratio, the smaller the particle breakage. They concluded that this is the reason poorly-graded granular materials (such as the sand investigated in this study) tend to follow a unique CSL, which contradicts the results obtained in this study. In contrast to the above findings, it can be seen on Fig. 1 that the grain size distribution curves after testing of a specimen even at 500 kPa lead to particle breakage. Even though the applied confining stresses are low it was reported by the McDowell, G. R. & Bolton, M. D. [31] that shearing is far more effective at breaking particles than isotropic compression. Furthermore such breakage of grains at low confinement have been confirmed in there studies of Luzzani & Coop [26] and Coop et. al. [32]. Moreover, Rezaeian et. al. [35], samples were tested up to 400 kPa confinement and, in this study, the samples were subjected to 1000 kPa.

Although the initial void ratio can affect the degree of particle breakage, it does not necessarily account for the unique CSL obtained in this study. In constrast, many studies have shown that particle breakage may lead to a shift in the CSL location [7,8,9]. This high degree of particle breakage in poorly-graded sand may account for such transitional behaviour considering that the sample will have higher amount of fines after yielding such that it can be categorised as well-graded soil. Furthermore, the shape of the particles is another factor to be considered. Cho et al. [22] concluded that the void ratio range tends to increase when the roundness of the particle decreases, which will have an impact on packing of particles and hence affects breakage of the sand. It can be argued that breakage is likely to be more pronounced in these samples as they are poorly graded. Therefore, similar to what was reported by Senetakis et al [30], repeating interparticle shearing tests showed a small reduction in the friction angle, which might be attributed to some damage at the asperities during the first shearing. it is likely that the surface conditions of grains in the shearing zone will also change during the shearing of specimens in this study. A sieve analysis of the samples before and after shearing can be seen on Fig. 1, confirming that particles have been broken during shearing. The occurrence of breakage bring the grain size distribution state closer to the state of intermediate soils, having smaller fines content, similar to Ventouras [16] and Shipton [17].

The resulting transitional behaviour from poorly-graded sand emphasizes the incompleteness of the traditional critical state theory and explains the role that fabric plays in reaching the critical state, which was suggested by Li and Dafalias [23] who proposed the anisotropic critical state theory (ACST). This theory includes another condition, the fabric anisotropy variable, to the critical state. However, further research is required in this area.

Conclusion

The behaviour pf poorly graded fine grained Akdeniz sand conforms well to the critical state soil mechanics framework. The findings of this study indicate that transitional behaviour in sand is not only exhibited by intermediate soils but also by poorly graded soils. It has been widely suggested by many authors that such transitional behaviour is due to variations in the grain sizes within the soil strata. Nevertheless, results from this study show that transitional behaviour can occur even in poorly graded sand. Furthermore, the initial void ratio, rather than the grain size distribution, strongly influences transitional behaviour. This result also agrees with results reported by Dos Santos et. al. [20], Ventouras [16] and Shipton [17] in which the authors concluded that the observed transitional behaviour is due to the initial void ratio. Furthermore, the numerous parallel CSLs observed were successfully grouped using the approach proposed by Ferreira and Bica [8], and a single λ, M was obtained for the formation of the state boundary surface. The Roscoe surfaces of the specimens could not be determined due to low stress levels used in the tests. Therefore, specimens prepared in the same way should be tested at higher confining pressures to determine the complete state boundary surfaces.

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Figures captions

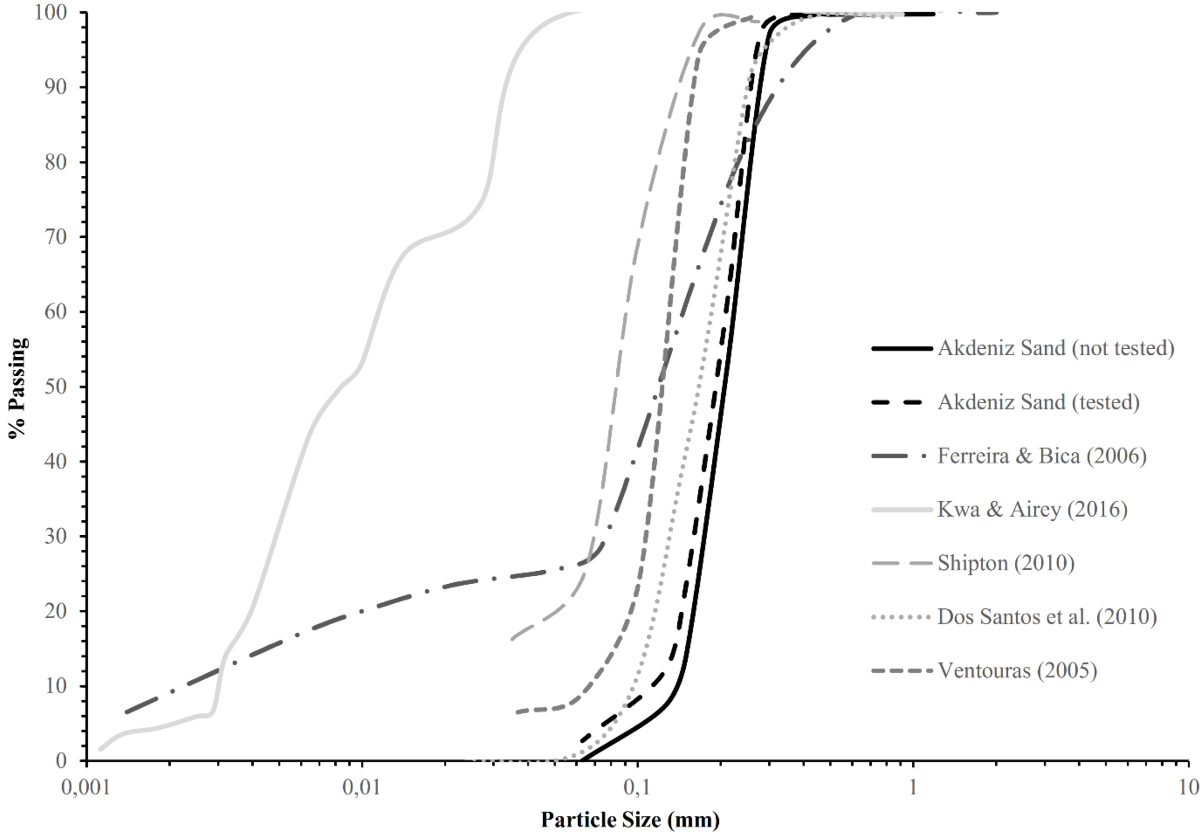

Fig. 1 Grain size distribution of the sands reported in the literature

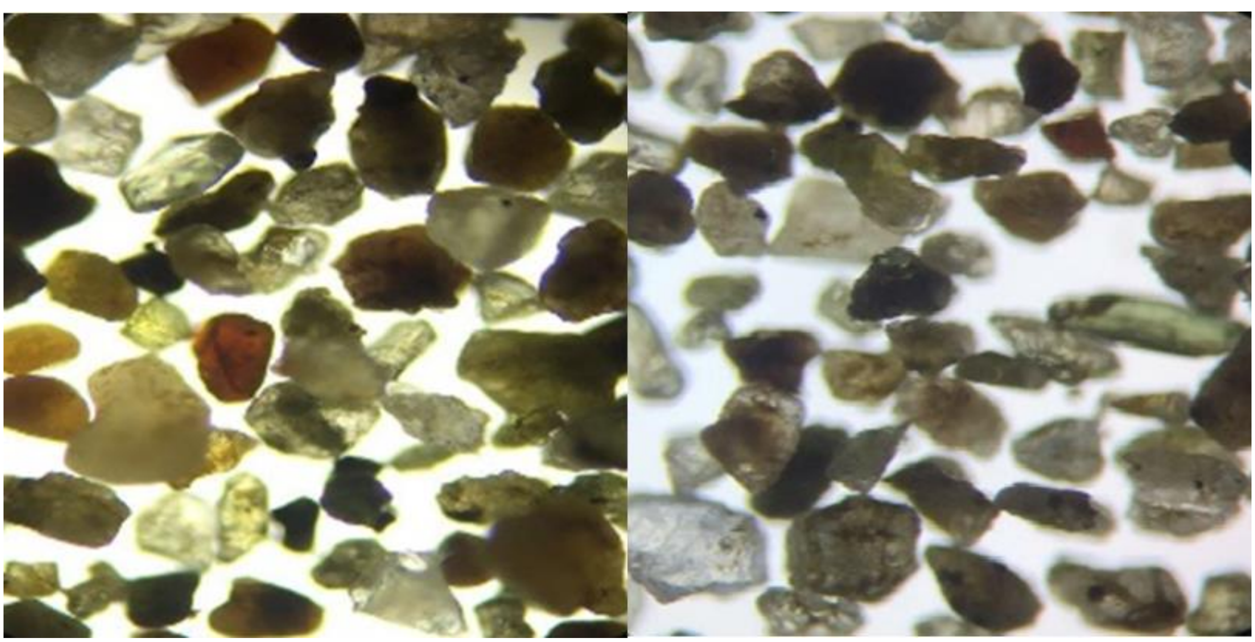

Fig. 2 Optical microscopy images of poorly graded Akedeniz sand.

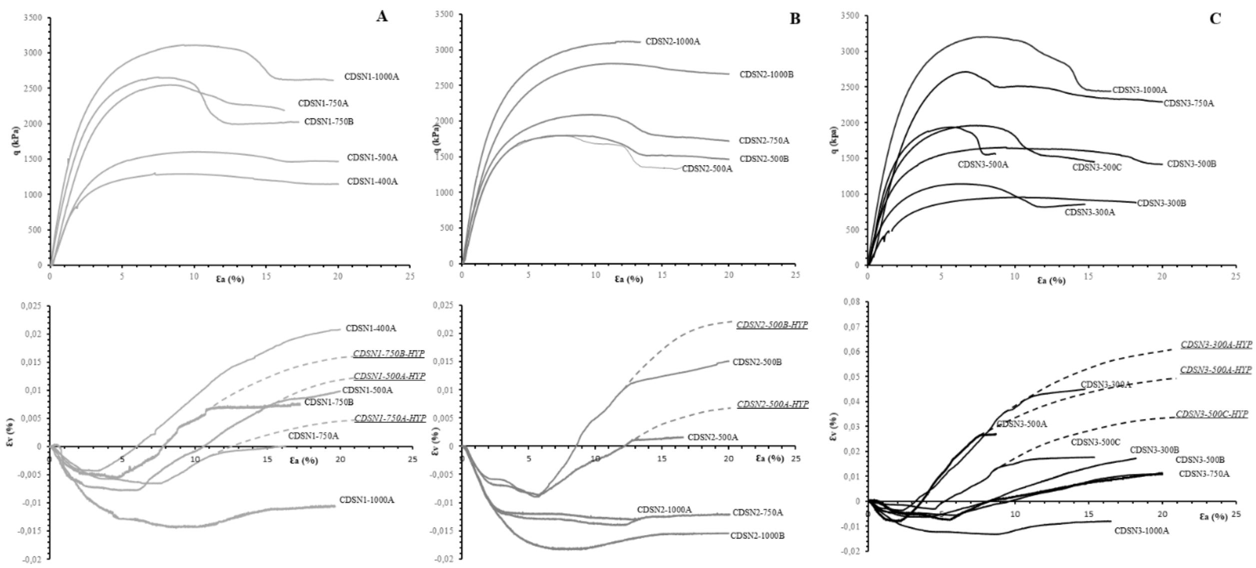

Fig. 3 Triaxial tests on remoulded Akdeniz sand (a) group N1, (b) group N2 and (c) group N3

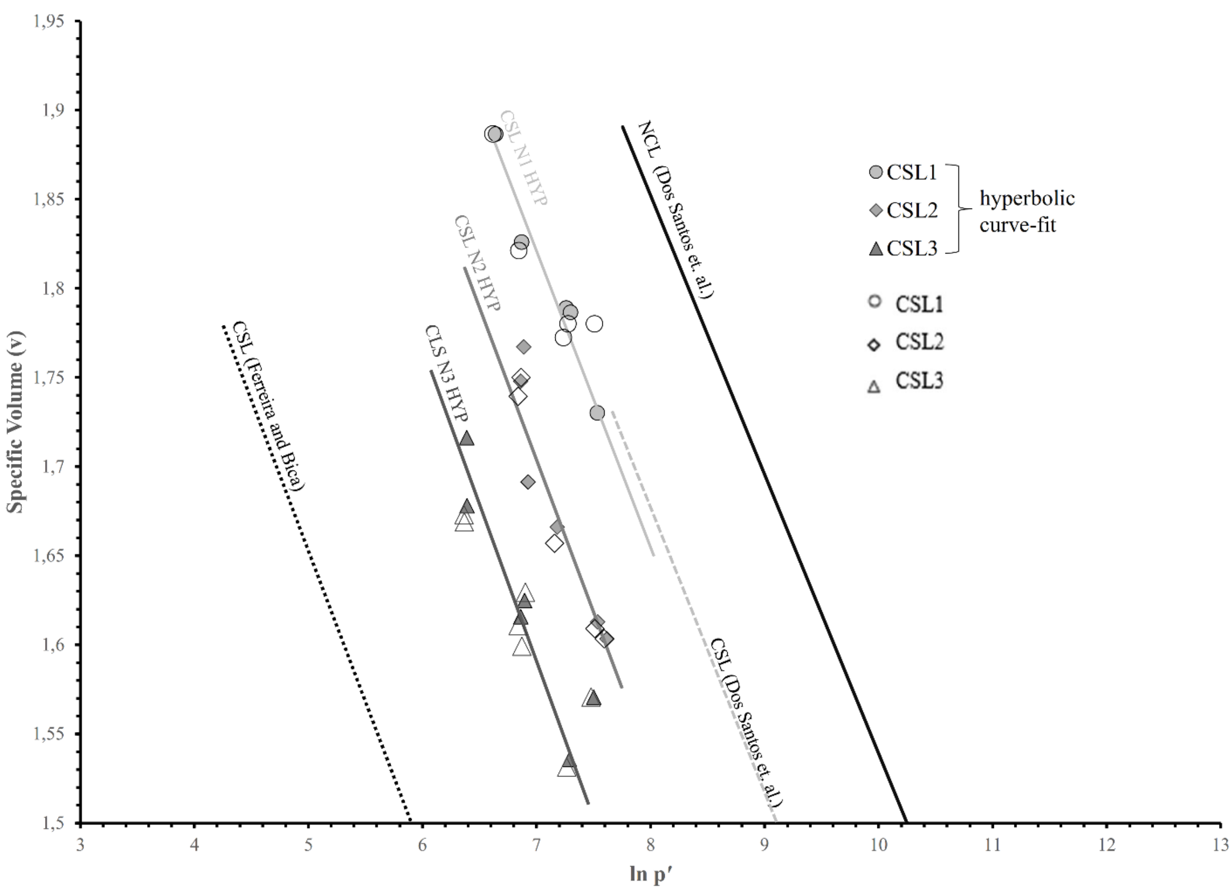

Fig. 4 Critical states in the volumetric plane for groups CSL of N1, N2, N3, Dos Santos et. al. and Ferreira and Bica

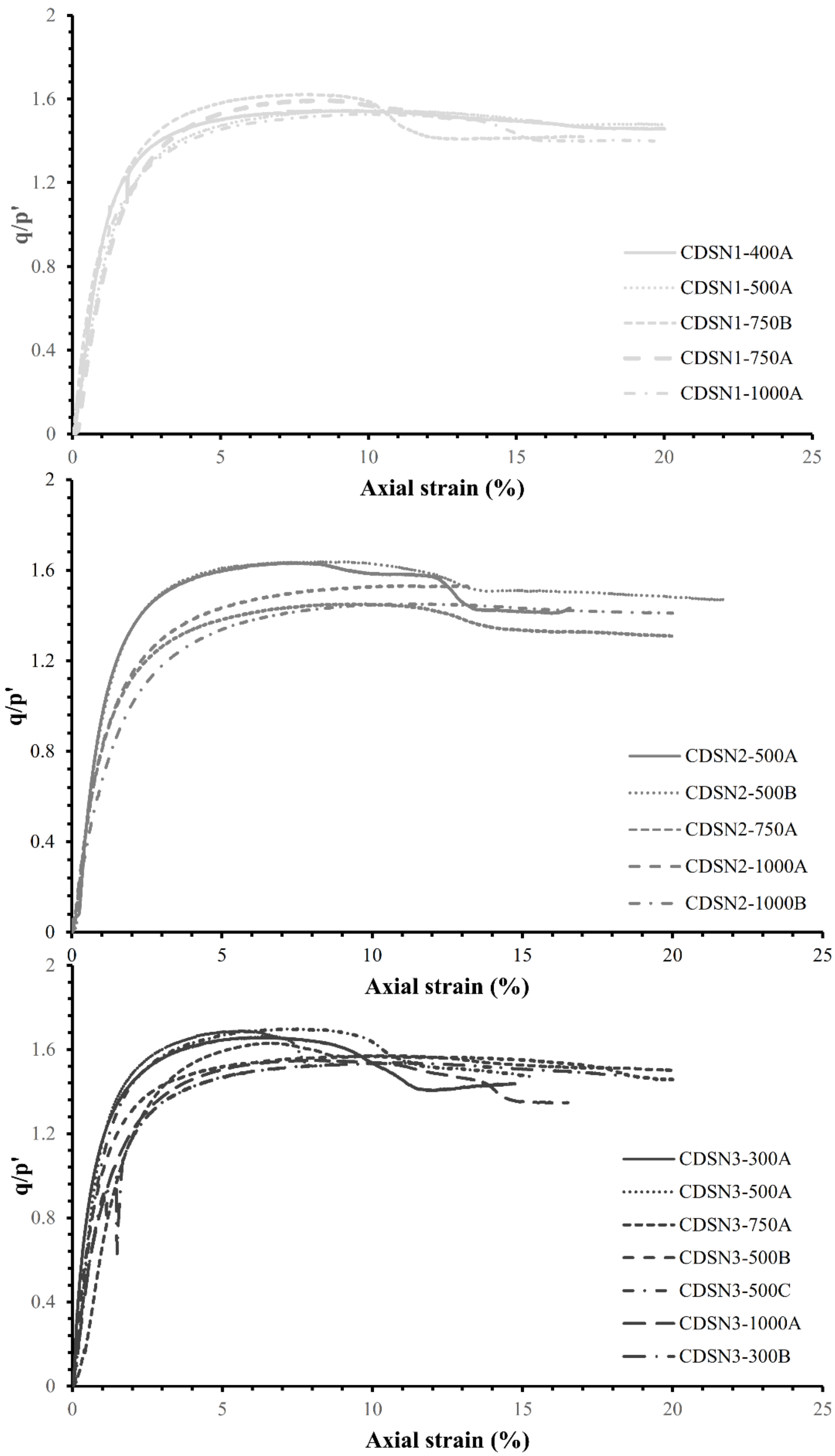

Fig. 5 Stress–strain shearing data for: (a) group N1, (b) group N2 and (c) group N3

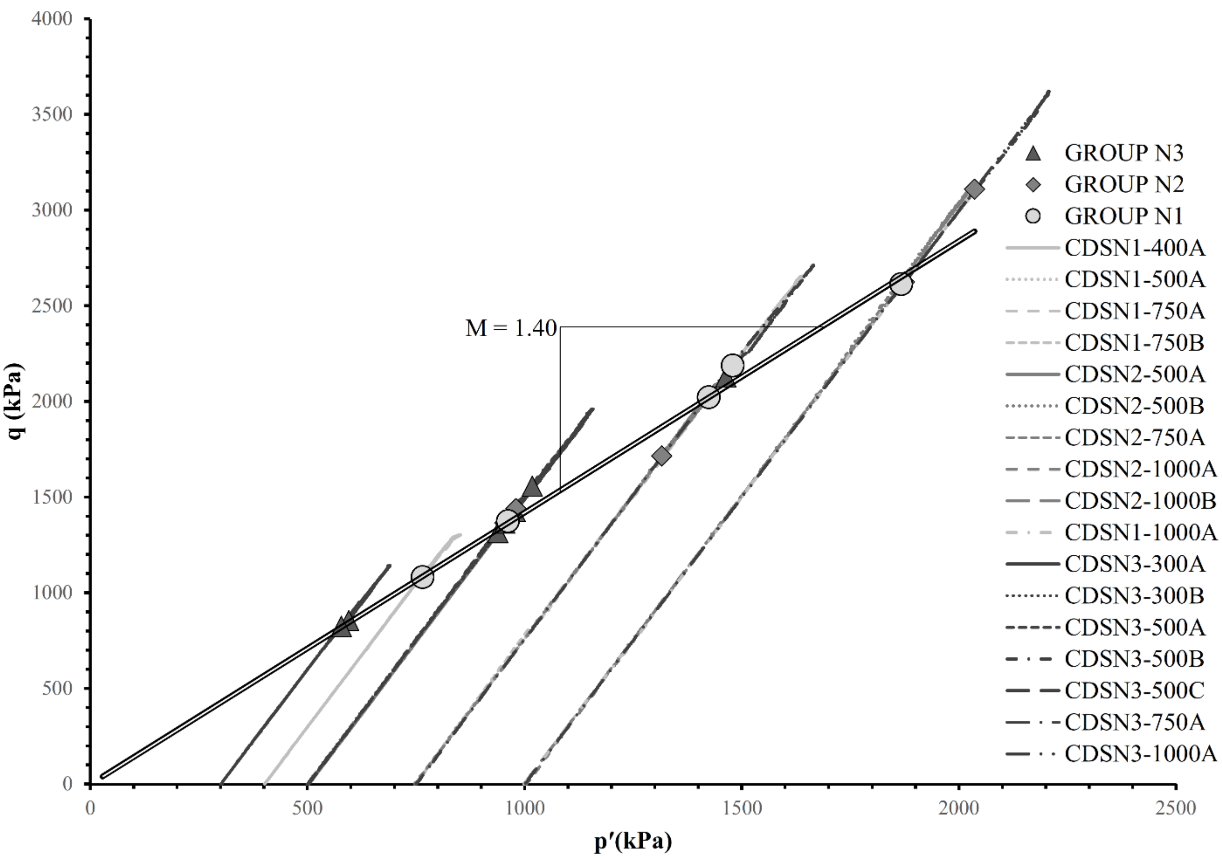

Fig. 6 Critical state on q : p` diagrams of groups N1, N2, N3

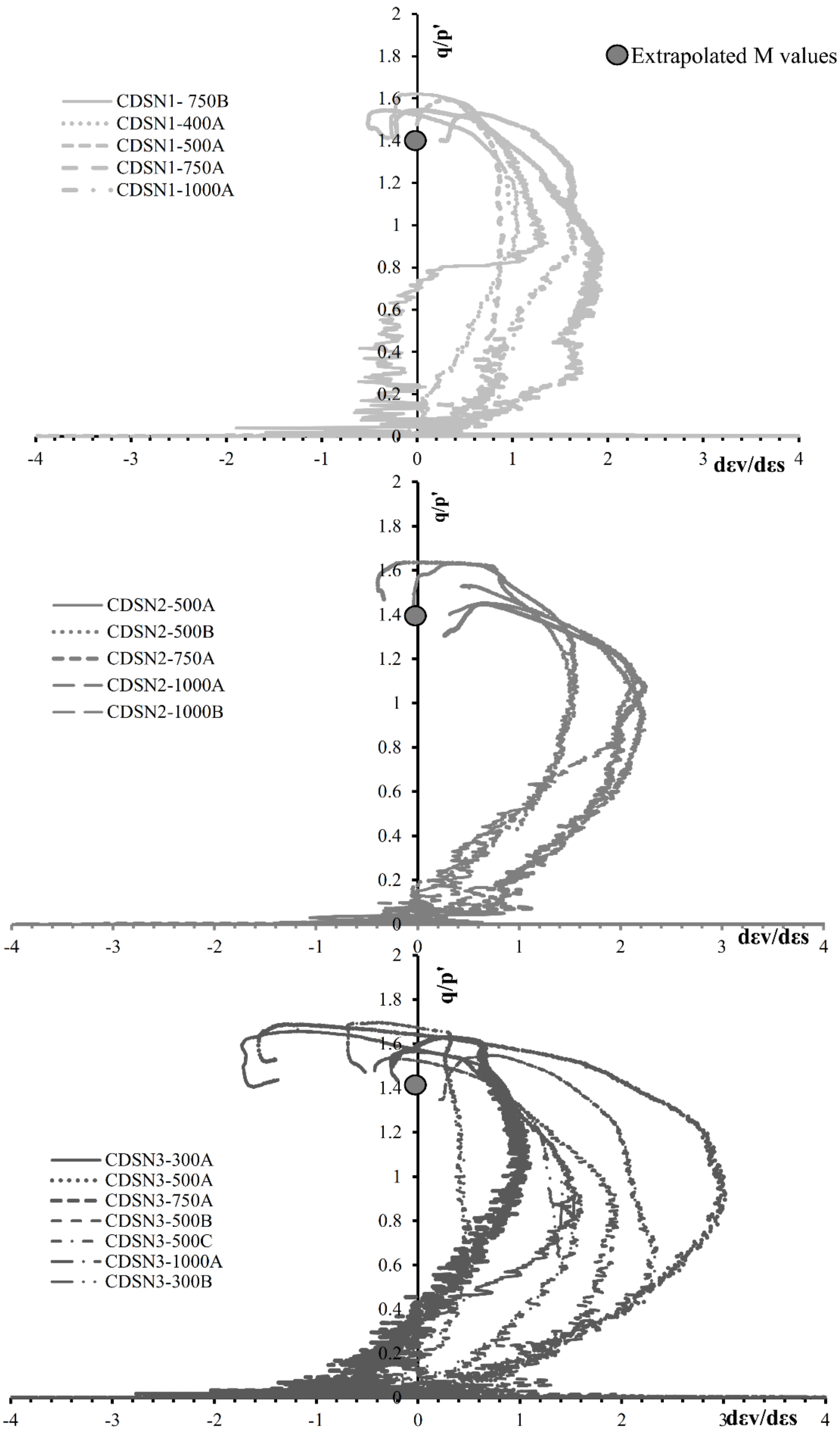

Fig. 7 Stress–dilatancy data for triaxial tests: (a) group N1, (b) group N2 and (c) group N3

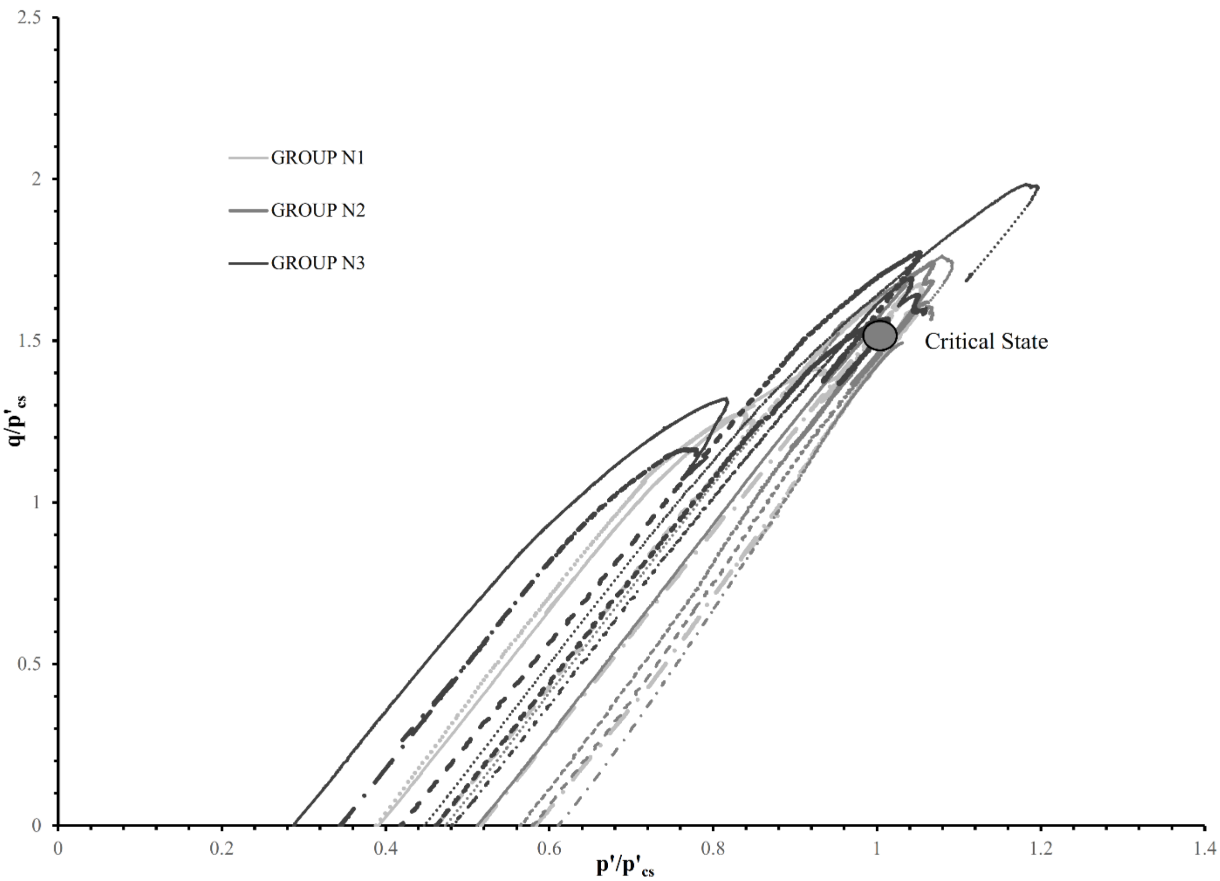

Fig. 8 State paths for all three groups

Table captions

Table 1 Physical properties of tested sand and other sands reported in the literature

	Akdeniz Sand	Dos Santos et al. (2010)	Ferreira and Bica (2006)	Ventouras (2010)	Shipton (2010)	KWA and Airey (2016)
D10	0.12	0.10	0.03	0.06	0.04	0.00
D30	0.16	0.12	0.08	0.11	0.07	0.01
D60	0.23	0.18	0.16	0.12	0.09	0.01
Coefficient of Uniformity	1.92	2.10	5.33	2.00	2.57	4.00
Coefficient of Curvature	0.93	1.00	1.33	1.68	1.56	0.69
Specific Gravity	2.67	2.62	2.65	2.65	2.71	2.65

Table 2 List of triaxial tests conducted and their properties

No.	***Short Name***	***Group***	**Consolidation Pressure (kPa)**	***V after Shear***	***V after Sat.***	***V after Con.***	***Relative density %***
1	CDSN1-400A		400	1.886	1.867	1.842	13.59
2	CDSN1-500A		500	1.821	1.832	1.8	24.36
3	CDSN1-750A		750	1.78	1.809	1.779	29.74
4	CDSN1-750B		750	1.772	1.784	1.759	34.87
5	CDSN1-1000A	N1	1000	1.73	1.773	1.749	37.44
6	CDSN2-500A		500	1.739	1.756	1.736	40.77
7	CDSN2-500B		500	1.75	1.733	1.722	44.36
8	CDSN2-750A		750	1.657	1.713	1.677	55.90
9	CDSN2-1000A		1000	1.603	1.671	1.624	64.49
10	CDSN2-1000B	N2	1000	1.609	1.695	1.681	54.87
11	CDSN3-300A		300	1.673	1.609	1.601	75.38
12	CDSN2-300A		300	1.668	1.662	1.64	65.38
13	CDSN3-500A		500	1.629	1.616	1.586	79.23
14	CDSN3-500B		500	1.61	1.622	1.593	77.44
15	CDSN3-500C		500	1.599	1.611	1.571	83.08
16	CDSN3-750A		750	1.531	1.586	1.514	97.69
17	CDSN3-1000A	N3	1000	1.57	1.625	1.583	80.00

Table 3 Critical state parameters

Group	Γ	λ	***M***
N1	2.95	0.169	1.4
N2	2.94	0.163	1.4
N3	2.93	0.170	1.4
Botucatu Sand *	2.49	0.177	1.4
Osorio sand ‡	2.90	0.156	1.22

*Ferreira and Bica (2006); ‡ Dos Santos et. al. (2010)