Submitted to *Decision Analysis*
manuscript DA-11-18-ART-0107.R2

The Valuator's Curse: Decision Analysis of Overvaluation and Disappointment in Acquisition

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Initial valuations of entrepreneurial ventures offering uncertain payoffs can often be over-valued by investors; namely, the expected payoff post acquisition is smaller than the expected payoff prior to acquisition when the investor harbors uncertainties about various components of the business. Common explanations involve irrationality such as psychological preference for potential over realized payoffs (Tormala et al. 2012). We provide a different, rational explanation which we term the valuator's curse. It is similar in nature to the winner's curse in auctions (Thaler 1988) and the optimizer's curse in decision analysis (Smith and Winkler 2006), but the source of the curse is neither from the competitive effects of an auction-type mechanism nor from the optimization effects in a choice among alternatives. Rather the effect is generated from the nonlinear evaluation of the payoffs, even though the inputs to the evaluation are unbiased. We formalize the valuator's curse and discuss its implications to entrepreneur's learning. The valuator's curse proves a boon to the entrepreneur as it leads to larger capitalizations.

Key words: Over-valuation, disappointment, valuator's curse

1. Introduction

Recent years have observed many privately held startups offering their shares to the world through initial public offerings (IPOs) (Economist 2019a). Although many are loss making and their promise of future profitability is highly uncertain, investors nevertheless place remarkably high valuations to acquire the company. For instance, Snap Inc, the parent company of Snapchat was valued $24bn when it went public in spring 2017; the ride-hailing giant Uber completed its IPO valued at $82bn in May 2019; while the office subleasing firm WeWork was once valued at $45bn pre-IPO. The pre-acquisition optimism was not sustained: Snap Inc and Uber experienced post-acquisition disappointment as their valuations tumbled shortly afterwards; while in the case of WeWork, the IPO was unsuccessful altogether and postponed indefinitely (Moore and Bradshaw 2019, Economist 2019b).

Similar to 'buyer's remorse' of home buyers, the incidences of pre purchase overvaluation and post acquisition disappointment is considered to be a consistent phenomenon. Money managers report that new stock offerings in IPO markets tend to fall below opening price in the first year (Fisher 2014). While evidence is not conclusive that IPOs are over-valued on average, anecdotal evidence of over-valuation was significant enough to warrant formal academic studies (e.g., see Gompers and Lerner (2003) and the references therein).

Theoretical explanations regarding overvaluation exist in the setting of auctions, i.e., the winner's curse (Thaler 1988), and project selection optimization, i.e., the optimizer's curse (Smith and Winkler 2006). However, the overvaluation we highlight in the investment of risky ventures derives neither from the competitive effects of an auction, as in the winner's curse, nor from the optimization effects in a choice among risky alternatives, as in the optimizer's curse. Much less is known about the possible causes of over-valuation in entrepreneurial ventures.

The most common explanation of overvaluation point to irrationality of decision makers. For example, Tormala et al. (2012) demonstrate that people often exhibit a preference for *potential* over *realized* achievements, suggesting that investors prefer promise of profits rather than realized ones. In one of their eight studies, for instance, participants showed a propensity to offer a higher salary to a potential NBA player than a player with five years experience when their performance statistics were identical (for the potential player, the statistics were reported as expected values; for the player with five years experience, they were reported as observed values). Tong et al. (2018) show that when faced with a choice under uncertainty, decision makers will overestimate the value of their preferred alternatives and deviate from rational choice. Another common psychological explanation involves debate around hubris, or overconfidence in one's ability. Hubris leads to founding of new ventures against a high base rate of failures (Hayward et al. 2006), more investments in innovation projects (Galasso and Simcoe 2011), while Hogarth and Karelaia (2012) argues that excess entry results from fallible judgment not overconfidence. However, few papers in the literature provide explanations for overvaluation based on rational decisions.

In this paper, we bridge that gap by presenting a rational explanation employing an analytical model. In particular, we take a decision theoretic approach and focus on the role of uncertainty, which we term the valuator's curse. From the eyes of an outside investor, an enduring difference between valuing a venture pre- and post-acquisition is the *level of uncertainty*. For example, when gauging a company's chance of becoming profitable or the demand for its innovative new product, investors would have greater uncertainty about their estimates prior to acquisition than post acquisition. Yet it is unclear as to why an investor would value a risky venture more highly when uncertainty is high (pre-acquisition) as opposed to when it is low (post-acquisition). While it may be plausible if the investor is risk-seeking, or if downside of investment is limited as in conventional situations of real options (Dixit 1992), neither is the case here – investors of risky ventures have a comparable level of risk aversion as the others (Gladwell 2010, Holm et al. 2013), and there is no safety net to guard against the downside when investing in risky start-up ventures.

We characterize two examples that are often associated with a risky investment to an investor: (a) venture with unknown chance of reaching profit, and (b) venture with an innovative product for unknown consumer preference. We examine a stylized model for each and analyze its expected value and its dependence on level of uncertainty about the unknown parameters. We find using Bayesian conjugate pair analysis that the expected value for both models *increases* with uncertainty – that is, the more you know (post-acquisition) the less valuable the venture appears. This result is driven by the fact that the payoff functions for risky investments are nonlinear in the unknown variables when evaluating uncertain ventures. This decision theoretic explanation is also robust to moderate levels of risk-aversion of investors.

From an entrepreneur's perspective, the valuator's curse appears to be at odds with the traditionally held belief that learning to reduce uncertainty is valuable. The value of information is positive, however, only in decision-making contexts because information helps improve decision quality and improves the profitability to the eye of the entrepreneur. Absent any decision, conveying information to a potential investor to reduce their uncertainty hurts the company's value to the eye of the investor. Therefore, we find that if the entrepreneur's objective is to grow the company long term, the entrepreneur should learn to benefit from improved decisions; if the entrepreneur's objective is to maximize the value to the eye of an outside valuator (and assuming investors will know what the entrepreneur knows through due diligence process), the entrepreneur should exploit the valuator's curse by deciding to not learn. In other words, the valuator's curse proves a boon to the entrepreneur, as it leads to larger capitalizations.

The paper is organized as follows. In the following section, we formalize the valuator's curse by presenting the two stylized models (§§2.1-2) and discussing its robustness to investor's risk aversion (§2.3). In §3, we revisit the value of learning in a decision making setting (§3.1) and when decision is not present (§3.2) and discuss implications to an entrepreneur. We conclude in §4.

2. Formalization of Valuator's Curse

This section examines, from the eye of an outside investor, the expected value of an innovative venture with uncertain payoffs. Because these firms build new product or enter a new market, it is uncertain whether they will reach profitability within a time/budget, or what the demand for their new product would be.

We present two stylized models for valuing a venture with (a) unknown success probability and (b) unknown consumer preferences for its new product. Employing Bayesian conjugate pairs, we examine the impact of uncertainty on the valuation and its cause.

To simplify illustration, we consider a monopoly firm, and myopic consumers who are not strategic and do not learn. While we do not explicitly model network externality, its effect can be captured by appropriate transformation of the demand functions. Finally, we will initially assume that the valuator is risk-neutral in §§2.1-2, but discuss the effect of risk-aversion in §2.3.

2.1. A venture with unknown probability of success

We begin by introducing a stylized example of an entrepreneurial venture that has profit potential but unknown chance of success. To simplify the setting further, an entrepreneur is given a coin to toss, with unknown probability of heads q^*. If he tosses heads (success), the venture is successful and results in profit $\pi > 0$; the venture earns 0 reward if he tosses tails (failure).[1] The investor has a belief q about the true success probability q^*,

$$q = \begin{cases} q^* + \epsilon & \text{w/prob. } 0.5, \\ q^* - \epsilon & \text{w/prob. } 0.5, \end{cases}$$

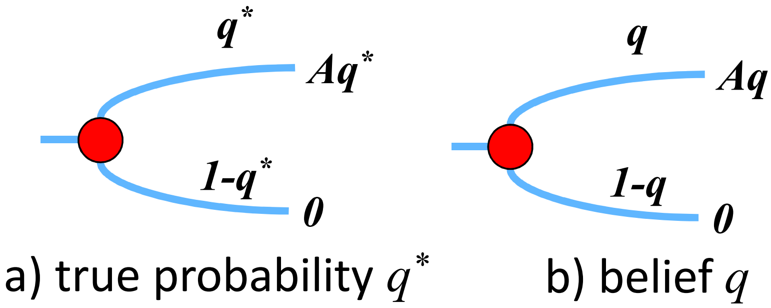
where ϵ denotes the level of inaccuracy in his beliefs. The belief is *unbiased* in the sense that the valuator does not overstate or understate his success probability on average. If $\epsilon = 0$,

[1] The value of 0 can be generalized; here we assume 0 for illustrative simplicity.

the valuator knows the true probability. Thus, ϵ is the valuator's measure of uncertainty about q^*. The valuator does not get to choose ϵ (or q^* for that matter).

The profit $\pi(q^*)$ depends on the true success probability, q^*. If the chance of success is higher, it may imply a higher revenue derived from positive network effects or may imply ease of running the business resulting in lower additional cost (or further investment). As such, the profitability from the venture is higher if successful. On the other hand, if the chance of success is lower, it indicates less revenue derived from potential network effects or greater operating difficulty and higher added cost (or further investment), cutting into the profitability after success and concomitantly lowering the reward from the venture. For simplicity, we assume the reward for heads is $\pi(q^*) = A \cdot q^*$ for some positive constant A. This canonical example is displayed in the left panel of Figure 1. Since the entrepreneur does not know q^*, his valuation of the lottery is based on his belief about q^*, shown on the right panel of Figure 1.

Figure 1 **The reward depends on the success probability q^*, and the expected reward depends on the belief q.**

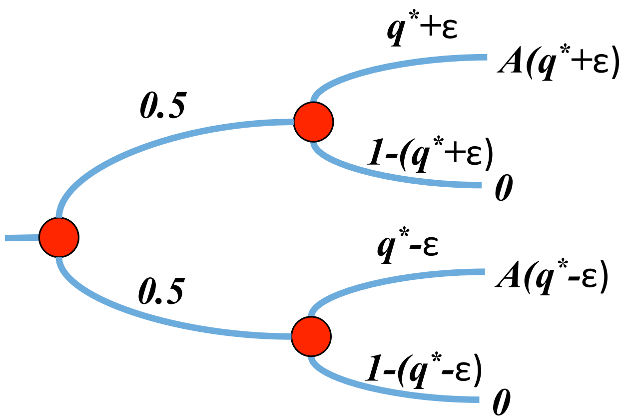


We next compute the expected profit seen from the eye of the valuator, which reflects the valuation of the business venture. The expected profit (valuation) based on the unbiased belief, $\mathbb{E}[\pi] = \mathbb{E}\Big[\mathbb{E}[\pi|q]\Big]$, is depicted graphically in Figure 2: with probability 0.5, $q = q^* + \epsilon$, and with probability 0.5, $q = q^* - \epsilon$. We have,

$$\mathbb{E}[\pi|q] = q \cdot Aq + (1-q) \cdot 0 = Aq^2,$$

$$\begin{aligned}
\mathbb{E}[\pi] &= \mathbb{E}\Big[\mathbb{E}[\pi|q]\Big] = 0.5[A(q^*+\epsilon)^2] + 0.5[A(q^*-\epsilon)^2] \\
&= 0.5A[2(q^*)^2 + 2\epsilon^2] = A(q^*)^2 + A\epsilon^2 \\
&= \mathbb{E}[\pi|q^*] + A\epsilon^2.
\end{aligned}$$

Figure 2 **The expected reward depends on the belief.**



Examining the role of uncertainty ϵ, we find that having a greater level of belief uncertainty, $\epsilon > 0$, leads the investor to expect a higher profit than when he holds the belief with certainty, $q = q^*$. Equivalently, the expected profit decreases as the uncertainty level ϵ decreases (e.g., after acquiring the firm). We will term this decrease in uncertainty leading to greater estimation of value, the *valuator's curse.*

Valuations increasing with uncertainty is also observed when the decision maker is risk-seeking, or in the setting of valuing real options (Dixit 1992). Neither is the case in this context. Rather, the valuator's curse is driven by the fact that the payoff function has a *nonlinear* dependence on the unknown parameter q. Specifically, in this example the payoff function $\pi(q)$ is convex in q, and the valuator's curse is consistent with Jensen's inequality.

This canonical example can be generalized via employing Bayesian conjugate pair analysis. The coin toss is based on a Bernoulli random variable, X, where the success probability is q^*. The valuator has a prior belief on q^* represented by a random variable q with a Beta distribution with parameters α and β. As before, we assume an unbiased prior, i.e., $\mathbb{E}(q) = \alpha/(\alpha+\beta) = q^*$. Rather than the uncertainty being captured by ϵ, it is measured by the variance $\mathbb{V}(q) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$. The expected profit is given in the following result. (All proofs appear in Appendix B.)

Proposition 1 **(Expected Profit based on Belief)**.

$$\mathbb{E}_q[\pi] = A\Big((q^*)^2 + \mathbb{V}(q)\Big).$$

The following corollary illustrates the valuator's curse.

Corollary 1 **(Impact of Uncertainty in Belief on Expected Profit)**. *For any given unbiased belief, i.e.,* $q^* = \frac{\alpha}{\alpha+\beta}$*, having greater uncertainty about the belief (greater variance* $\mathbb{V}(q)$*) increases valuation.*

In the valuator's curse, although the valuator has an unbiased belief about the probabilities of the unknown, it results in biased beliefs about the payoffs, where positive outcomes appear more beneficial than negative outcomes appear costly. We next examine a separate setting common in venture valuation to examine whether the valuator's curse is present.

2.2. A venture with unknown consumer preferences for its new product

Consider an entrepreneurial venture bringing an innovative new product to the market. With no existing previous sales record, it does not have perfect knowledge of the consumer valuations. We assume that the entrepreneur knows that consumer willingness-to-pay v are distributed as $f(v|\phi)$ (e.g., through experience or data on related products), where ϕ denotes an unknown parameter (e.g., mean) of the distribution. The investor has an unbiased prior belief about ϕ, which is distributed according to $g(\phi)$.

Price of the product p as well as its marginal cost c is set by the firm and hence exogenous to an outside valuator. Given a selling price p, consumers will purchase the product if their valuation $v > p$, and not otherwise. Let us represent the demand, $D(p|\phi) = \int_p^\infty f(v|\phi)dv$. Thus, the expected profit derived from the new product is:

$$\begin{aligned} E_\phi\{(p-c)D(p|\phi)\} &= E_\phi\left\{(p-c)\int_p^{+\infty} f(v|\phi)dv\right\} \\ &= (p-c)\int_p^{+\infty}\left[\int_{-\infty}^{+\infty} f(v|\phi)g(\phi)d\phi\right]dv. \end{aligned} \tag{1}$$

We next evaluate this expected profit employing two different consumer valuation distribution $f(\cdot)$. For each, we employ Bayesian conjugate pairs to examine the impact of uncertainty in beliefs on the expected profit.

2.2.1. Normally distributed $f(\cdot)$. Assume that consumer valuations v are normally distributed with an unknown mean ϕ but with a known precision r (or variance $1/r$ – we represent variation in terms of precision rather than variance because the formulas for updating are easier). This distribution may be appropriate for utility products that most people find some value in having, but differ to some (known) degree. That is, $v \sim \mathcal{N}(\phi, 1/r)$, where ϕ is the unknown parameter.

The valuator holds a prior belief about ϕ which is represented by a normal distribution with mean μ and precision τ, i.e., $\phi \sim \mathcal{N}(\mu, 1/\tau)$. So, τ is the measure of uncertainty. The next proposition states the expression for the expected profit.

PROPOSITION 2 **(Expected Profit based on Belief)**. *If $v \sim \mathcal{N}(\phi, 1/r)$ and belief $\phi \sim \mathcal{N}(\mu, 1/\tau)$ then*

$$E_\phi\{(p-c)D(p|\phi)\} = (p-c)(1-H(p)), \tag{2}$$

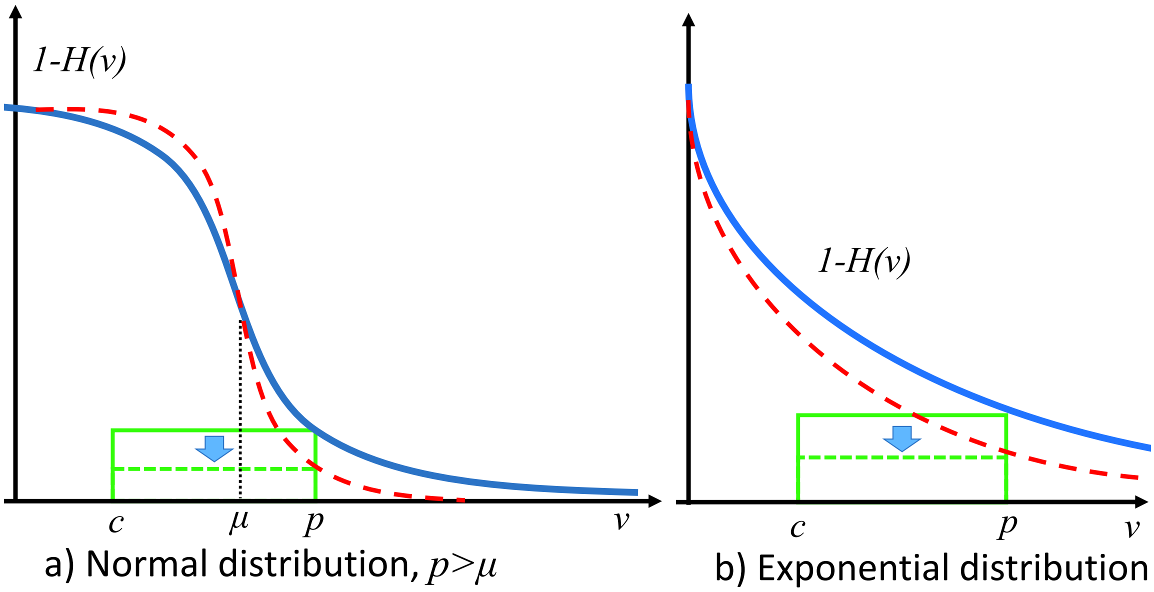
where H is the cumulative distribution function of a normally distributed random variable with mean μ and variance $\frac{1}{\tau}+\frac{1}{r}$, i.e., $\mathcal{N}\left(\mu, \frac{1}{\tau}+\frac{1}{r}\right)$.

The following corollary establishes the valuator's curse.

COROLLARY 2 **(Impact of Uncertainty in Belief on Expected Profit)**. *If the price p is greater than (equal to, less than) the prior mean μ, the expected profit decreases (remains the same, increases) in the precision of belief τ.*

The results above establish the valuator's curse for $p > \mu$: the expected profit decreases as uncertainty about the mean consumer valuation shrinks (after acquisition). This is displayed for the normal distribution in the left panel of Figure 3. The valuator's curse occurs because the profit function is nonlinear. The conditional nature of Corollary 2, depending as it does on the relation between μ and p, shows that the valuator's curse is not ubiquitous but common.

Figure 3 Expected demand curves and profits.



Note. The solid (dashed) curves represent "expected demand" curves with greater (less) variance. The solid (dashed) areas inside the rectangle represents the expected profits with greater (less) variance.

2.2.2. Exponentially distributed $f(\cdot)$. Assume instead that the valuations v are exponentially distributed in the consumer population, but the valuator does not know the mean

λ^{-1} of this exponential distribution. This distribution may be appropriate for auxiliary products that most people would find no value in, but a few find high degree of value. The valuator's belief is characterized by a Gamma distribution with parameters (α,β), where mean is α/β and variance is $\frac{\alpha}{\beta^2}$. The valuator relies on his belief about λ to make profit projections.

Given price p, and marginal cost c, the expected profit (valuation) derived from the new product based on those beliefs is given by the following proposition.

Proposition 3 **(Expected Profit based on Belief)**. *If $v \sim Expon(\lambda)$ and belief $\lambda \sim Gamma(\alpha,\beta)$ then*

$$E_\lambda\{(p-c)D(p|\lambda)\} = (p-c)\left(\frac{\beta}{p+\beta}\right)^{\alpha}. \tag{3}$$

The following corollary establishes the valuator's curse.

Corollary 3 **(Impact of Uncertainty in Belief on Expected Profit)**. *The expected profit decreases in variance, ceteris paribus.*

In this situation, regardless of price p, the expected profit decreases with less uncertainty. The intuition is graphically illustrated in the right panel of Figure 3. Again, the valuator's curse is driven by the profit function's nonlinear dependence on the unknown λ, and in particular the non-linearity is often such that positive outcomes are more beneficial than negative outcomes are costly.

Observing both panels of Figure 3, we can see that the valuator's curse is driven by the fact that the right tail of the expected demand curve increases in the uncertainty. We acknowledge its special significance when valuing a new product launch given the important role that the high value consumers play in the entrepreneurial setting.

2.3. Robustness to Investor's Risk Attitude

So far, we have discussed the valuator's curse with the assumption that the investor is risk neutral. In this section, we examine the impact of risk attitude. To avoid redundancy, we will employ the canonical model of §2.1.

Let I represent the investment opportunity when q^* is known, and let II represent the investment opportunity when there is uncertainty about q^*. If there is no uncertainty about the value of q^*, the expected value of the investment opportunity for a risk neutral investor is, as shown in §3, just $E(I)=(q^*)^2A$. If, on the other hand, the investor does not know the exact value of q^*, but has a belief q such that $q=q^*+\epsilon$ with probability 0.5, and $q=q^*-\epsilon$ with probability 0.5, then the expected value of the investment opportunity for a risk neutral investor is $E(II)=[(q^*)^2+\epsilon^2]A>(q^*)^2A$. As noted, a risk neutral investor will have a preference for greater uncertainty: the bigger ϵ, the better.

We are interested in the effects of risk aversion, whether risk aversion mitigates the preference for uncertainty, and if so, exactly how much risk aversion is needed to negate the preference for uncertainty. Suppose instead of being risk neutral, the investor has exponential utility with risk tolerance γ:

$$u(x)=1-e^{-x/\gamma}.$$

If the investor knows the value of q^*, then the expected utility of the coin toss is

$$E(u(I))=q^*(1-e^{-q^*A/\gamma})=q^*-q^*e^{-q^*A/\gamma}.$$

If the investor does not know q^*, but has a belief q such that $q=q^*-\epsilon$ with probability 0.5, and $q=q^*+\epsilon$ with probability 0.5, then the expected utility of the coin toss is

$$\begin{aligned}E(u(II)) &= 0.5(q^*+\epsilon)(1-e^{-(q^*+\epsilon)A/\gamma})+0.5(q^*-\epsilon)(1-e^{-(q^*-\epsilon)A/\gamma})\\ &= q^*-\left[0.5(q^*+\epsilon)e^{-(q^*+\epsilon)A/\gamma}+0.5(q^*-\epsilon)e^{-(q^*-\epsilon)A/\gamma}\right].\end{aligned}$$

The next result demonstrates the impact of risk tolerance γ.

PROPOSITION 4 **(Impact of Risk Attitude)**. *For any q^* and ϵ, there exists γ^* such that if $\gamma < \gamma^*$, $E(u(I)) > E(u(II))$.*

In other words, when the valuator is sufficiently risk-averse, the valuator's curse disappears and he expects a greater reward with more accuracy. Proposition 4 also shows that valuator's curse is robust as long as the investor is not too risk-averse, which is the case when the investor has a diverse-enough portfolio.

3. Implication for Learning for Entrepreneurs

Traditionally held wisdom dictates that gathering information to reduce uncertainty or learn is valuable in the eye of the entrepreneur. The *valuator's curse* identified in the previous section appears to be at odds with this notion as it shows that from the eye of the outside investor, the value of the venture decreases when uncertainty is reduced. In this section, we disentangle this apparent contradiction. To avoid redundancy, we will employ the new product launch setting with normally distributed customer valuations.

Suppose that the entrepreneur can learn to reduce doubt about ϕ via sampling, i.e., selling to customers in a test market or working with consumers in a focus group, representing the sampling process. To avoid the complications of dealing with censored data, we assume that during the sampling phase, the entrepreneur learns the valuations v_i, not just the outcome of the purchase decision. Each sample item v_i is an independent draw. Through sampling, the entrepreneur observes consumer valuations $v_1, v_2, \ldots, v_n$, and then updates his beliefs to $g(\phi|v_1, v_2, \ldots, v_n)$. If n is the sample size, let $S_n = v_1 + v_2 + \cdots + v_n$ is a sufficient statistic for $v_1, v_2, \ldots, v_n$. So, (1) becomes:

$$\begin{aligned} E_{\phi|S_n}\{(p-c)D(p|\phi, S_n)\} &= E_{\phi|S_n}\left\{(p-c)\int_p^{+\infty} f(v|\phi)dv\right\} \\ &= (p-c)\int_p^{+\infty}\left[\int_{-\infty}^{+\infty} f(v|\phi)g(\phi|S_n)d\phi\right]dv. \qquad (4) \end{aligned}$$

After observing the sample, the posterior distribution of beliefs is normal with updated mean $\hat{\mu}_n = \frac{\tau}{\tau+nr}\mu + \frac{r}{\tau+nr}S_n$, or a weighted average of the prior mean and the sample mean. The precision of the posterior distribution is $\tau + nr$. As $n \to \infty$, the posterior mean $\hat{\mu}_n$ approaches $\frac{S_n}{n}$, the sample mean, and the variance, $\frac{1}{\tau+nr}$, converges to zero. We have:

$$f(v|\phi) = \sqrt{\frac{r}{2\pi}}e^{-\frac{r(v-\phi)^2}{2}}, \;\; g(\phi|S_n) = \sqrt{\frac{\tau+nr}{2\pi}}e^{-\frac{\tau+nr}{2}\left(\phi - \frac{\mu\tau+rS_n}{\tau+nr}\right)^2}.$$

After sampling n and observing S_n, we have

$$E_{\phi|S_n}\{(p-c)D(p|\phi,S_n)\} = (p-c)(1-H_n(p|S_n)), \tag{5}$$

where $H_n \sim \mathcal{N}\left(\frac{\mu\tau+rS_n}{\tau+nr}, \frac{1}{\tau+nr}+\frac{1}{r}\right)$. It is clear that as n increases, the variance of H_n will decrease. However, the mean changes based on S_n, a random variable. For example, if the observed value of S_n is large, then the expected profit would increase after sampling; whereas if the observed value of S_n is small, then the expected profit would decrease after sampling. We next discuss the distribution for S_n.

Two accepted views on sampling v_i

There are two accepted methods[2] of sampling each v_i (and therefore S_n) when simulating learning (Lai 1987). Similar to the distinction between risk and uncertainty (Knight 1921), one resembles learning to reduce risk by confirming a belief, whereas the other resembles learning to reduce uncertainty in an exploratory setting. In the first approach, sometimes referred to as the *classical method*, it is assumed that there is a true underlying value ϕ and that each sample v_i is selected from $v_i \sim N(\phi, 1/r)$. That is, the belief of the entrepreneur does not influence the true distribution and the sample will be selected so that $v_i = \phi + \epsilon_i$ where $\epsilon_i \sim N(0, 1/r)$ (see e.g., Bertsimas and Mersereau 2007). Thus, $S_n = v_1 + \cdots + v_n =$

[2] Other related simulation methods that builds on these are discussed in Appendix A.

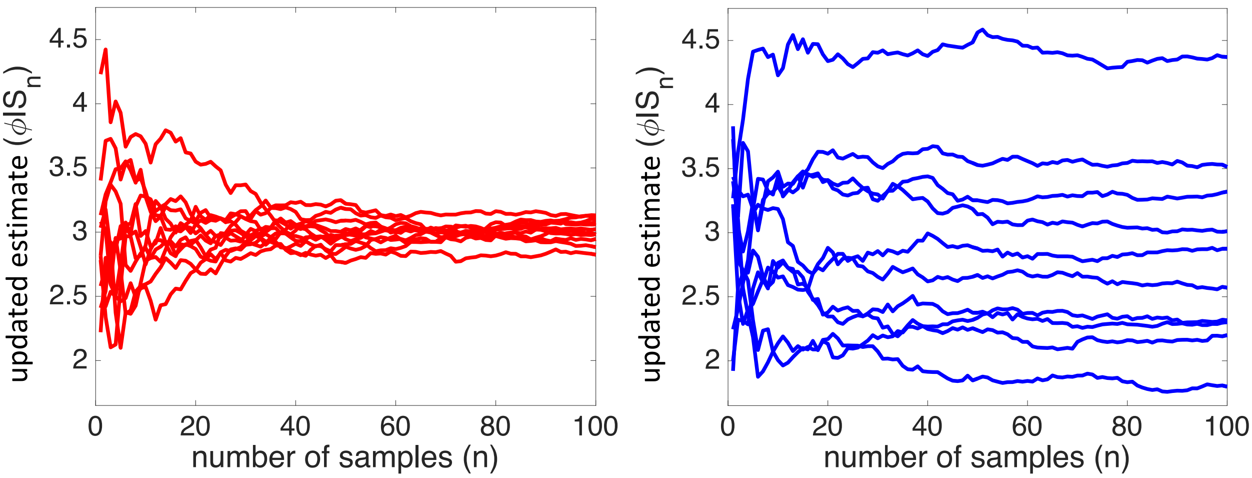
$n\phi + (\epsilon_1 + \cdots + \epsilon_n)$. If the entrepreneur has an unbiased belief so that $\phi = \mu$, then $S_n \sim N(n\mu, n/r)$. This approach is useful for examining how an objective (e.g., expected profit) may be affected by the quality of the information (e.g., accuracy and bias) initially available about ϕ.

In the second approach, sometimes referred to as the *Bayesian method*, a $\hat{\phi}$ is randomly drawn from the initial belief distribution $N(\mu, 1/\tau)$ (e.g., Caro and Gallien 2007) and each sample v_i is then drawn from $v_i \sim N(\hat{\phi}, 1/r)$. This is similar to the assumption underlying expected value of information calculations (Clemen and Reilly 2001). Here, the random realization of $\hat{\phi}$ is assumed to be the true underlying value. The entrepreneur's belief influences the simulation process as it relies on the assumption that the predictive Bayesian distribution (in our case, the $\phi \sim N(\mu, 1/\tau)$) is correct. Thus, $S_n = v_1 + \cdots + v_n = n\hat{\phi} + (\epsilon_1 + \cdots + \epsilon_n)$, where $\epsilon_i \sim N(0, 1/r)$. Since $\hat{\phi} \sim N(\mu, 1/\tau)$, S_n is distributed normally with $E(S_n) = n\mu$ and $Var(S_n) = n^2/\tau + n/r$.

In the first approach, the entrepreneur's updated belief $\phi | v_1, \ldots, v_n$ approaches ϕ as the sample size increases. This is consistent with the behavior of the sample average $\frac{S_n}{n} \sim N(\phi, \frac{1}{nr})$, which converges to $\frac{S_n}{n} \to \phi$ as $n \to \infty$. In the second approach, as the sample size increases, the updated belief approaches the value $\hat{\phi}$ that was initially selected from $N(\mu, 1/r)$. This is consistent the sample average $\frac{S_n}{n} \sim N\left(\mu, \frac{1}{\tau} + \frac{1}{nr}\right)$, which converges to $\frac{S_n}{n} \to \phi \sim N(\mu, 1/\tau)$ as $n \to \infty$, i.e., it approaches a distribution. Figure 4 illustrates 10 sample paths using the two different approaches. The first approach (left panel) exhibits *convergence* and *consistency*, while the second approach (right panel) exhibits *convergence*, but *not consistency*.

In the following, we will use both sampling methods to examine the impact of learning in a decision making setting (§3.1) and a passive setting (§3.2).

Figure 4 Sample paths of classical approach (left) and the Bayesian approach (right) to sampling.



Note. Parameters: $\mu = \tau = r = 3$.

3.1. Decision Making Setting

The notion that information is valuable is valid in a *decision-making* setting where the decision maker can use information to improve decisions (e.g., to improve future profitability). The inherent assumption in this setting is that the decision-maker is the long-term stake-holder of the firm.

Let us consider the pricing decision. We next define the expected value of sample information (EVSI). Without information ($n = 0$), the firm would set price that would maximize the expected profit in (2), or in other words,

$$\begin{aligned} p^* &= \arg\max_p \ \mathbb{E}_\phi \left[(p-c) \int_p^\infty h(v|\phi) dv \right] \\ &= \arg\max_p \ (p-c)(1-H(p)), \end{aligned}$$

yielding a value of $V(p^*)$. Observe here that as long as the marginal cost c is not too low, the optimal price $p^* > \mu$. This indicates that valuator's curse would continue to exist in a decision-making setting (Corollary 2). In other words decision making may influence the magnitude of the valuator's curse, but it does not determine whether or not it would exist.

If the firm were to make the decision after acquiring sample information $S_n = v_1 + \cdots + v_n$, the expected value V_n relies on maximizing the expression (5), or

$$\begin{aligned} V_n &= \mathbb{E}_{S_n}\left\{\max_p \ \mathbb{E}_{\phi|S_n}\left[(p-c)\int_p^\infty h_n(v|\phi,S_n)dv\right]\right\} \\ &= \mathbb{E}_{S_n}\left\{\max_p \ (p-c)(1-H_n(p|S_n))\right\} \end{aligned}$$

Definition 1 (Expected Value of Sample Information). The expected value of sample information (EVSI) is defined by $V_n - V(p^*)$.

Proposition 5 **(Positive EVSI)**. *Regardless of whether* $S_n \sim N(n\mu, n/r)$ *or* $S_n \sim N(n\mu, n^2/\tau + n/r)$*, EVSI is positive for* $n > 0$.

In this price-setting context, learning has value because more information allows the firm to optimize their pricing. The next result shows the properties of EVSI.

Corollary 4 **(Property of EVSI)**. *If* $S_n \sim N(n\mu, n/r)$*, then EVSI is (i) concave increasing in* n $\forall\tau$*; (ii) decreasing in* τ $\forall n$*; and (iii) submodular in* (n,τ).

While it is difficult to analytically characterize the properties when $S_n \sim N(n\mu, n^2/\tau + n/r)$, we numerically observe the same properties for EVSI. Namely, EVSI is increasing in n but with diminishing marginal returns (part (i)); decreasing with the precision of the prior belief τ because the more informed your belief (larger τ implies greater precision), the less valuable is an additional sample (part (ii)); and submodular in (n,τ) – that is, the value of additional sample diminishes when you have greater precision in your beliefs; similarly the value of increasing precision in beliefs diminishes when you have a larger sample size.

3.2. Passive Setting

We now examine a passive valuation setting where the valuator does not make a pricing decision. In this setting, the price p is considered exogenous either because the price has already been set, or is not a decision for the valuator. Suppose that in the investor's due diligence process, the entrepreneur is legally obligated to disclose all his knowledge truthfully to the questions of the investors. In other words, the value seen from the eyes of the investor corresponds to the entrepreneur's payoff.

To find the value of learning n, we need to find the expression for $E_\phi\{(p-c)D(p|\phi)\}$ by integrating out S_n. The following shows the impact of n.

PROPOSITION 6 **(Expected Profit after Sampling n)**.

(i) If $S_n \sim N(n\mu, n/r)$,

$$\mathbb{E}_{S_n}\mathbb{E}_{\phi|S_n}\{(p-c)D(p|\phi,S_n)\} = E_\phi\{(p-c)D(p|\phi)\} = (p-c)(1-G_n(p)),$$

where $G_n \sim \mathcal{N}\left(\mu,\ \frac{nr}{(\tau+nr)^2} + \frac{1}{\tau+nr} + \frac{1}{r}\right)$.

(ii) If $S_n \sim N\left(n\mu, \frac{n^2}{\tau} + \frac{n}{r}\right)$,

$$\mathbb{E}_{S_n}\mathbb{E}_{\phi|S_n}\{(p-c)D(p|\phi,S_n)\} = \mathbb{E}_\phi\{(p-c)D(p|\phi)\} = (p-c)(1-H(p)),$$

where $H \sim N\left(\mu, \frac{1}{\tau} + \frac{1}{r}\right)$.

DEFINITION 2 (EXPECTED VALUE OF SAMPLE INFORMATION). The expected value of sample information (EVSI) absent decision making is the difference between $\mathbb{E}_{S_n}\mathbb{E}_{\phi|S_n}\{(p-c)D(p|\phi,S_n)\}$ and the expected profit (2).

PROPOSITION 7 **(Nonpositive EVSI)**. *if* $S_n \sim N(n\mu, n/r)$*, EVSI is negative and decreasing in n; if* $S_n \sim N\left(n\mu, \frac{n^2}{\tau} + \frac{n}{r}\right)$*, EVSI is* 0.

We find that when decision is not involved, learning is never beneficial and can only hurt the entrepreneur.

Discussion

The main insight regarding entrepreneur's learning is the following. If the entrepreneur's objective is to grow the company long term, it would face decisions under uncertainty. In this setting, learning benefits the entrepreneurs because it improves their decision making. Consequently, it would be optimal for the entrepreneur to determine how much to learn based on the cost of learning.

On the other hand, suppose the entrepreneur's objective is to sell the firm in the short term to an outside investor. An interested outside investor would conduct due diligence to learn everything about what the entrepreneur knows. In such situation, it would be optimal for the entrepreneur to exploit the valuator's curse and maximize the firm's value to the eye of the buyer by choosing not to learn. In other words, the valuator's curse proves a boon to the entrepreneur as it leads to larger valuations.

4. Conclusion

In this paper, we have presented the notion of valuator's curse, which provides a rational decision-theoretic explanation to the empirically observed phenomenon of over-valuations of new ventures and post-IPO disappointment, buyer's remorse of home buyers, and a preference for uncertainty as in the studies by Tormala et al. (2012). We have illustrated the valuator's curse in two steps. First, we showed via two disparate examples that the payoff functions associated with risky ventures are nonlinear in the unknown parameters; second, we showed that this nonlinearity can cause the expected payoff from the risky ventures to increase in the level of uncertainty.

The valuator's curse is different from the winner's curse in auctions in that it is not generated by sets of beliefs; and it is different from the optimizer's curse in decision making in that it is not generated from the expectation of extreme order statistics. The valuator's curse is driven by the general nonlinearity in the payoffs associated with valuing risky ventures.

From the entrepreneur's perspective, we found that if the entrepreneur's objective is long term ownership, there is benefit in acquiring information as it improves better operational decision making. In a shorter term sales situations, however, we find that *not* learning is beneficial as it takes advantage of the valuator's curse. That is, the valuator's curse proves a boon to the entrepreneur as it leads to larger valuations.

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Appendix A: Related Simulation Method

A related simulation method is when each sample v_i is drawn from an iteratively updated $\hat{\phi}_i$ for $i = 1, \dots, n$, i.e.,

$$\begin{aligned}
v_1 &\sim N\left(\phi_1, \frac{1}{r}\right), \text{ where } \phi_1 \sim N\left(\mu_1, \frac{1}{\tau_1}\right), \\
v_2 &\sim N\left(\phi_2, \frac{1}{r}\right), \text{ where } \phi_2 \sim N\left(\frac{\tau_1\mu_1 + rv_1}{\tau_1 + r}, \frac{1}{\tau_1 + r}\right) \equiv N\left(\mu_2, \frac{1}{\tau_2}\right), \\
&\vdots \\
v_n &\sim N\left(\phi_n, \frac{1}{r}\right), \text{ where } \phi_n \sim N\left(\frac{\tau_{n-1}\mu_{n-1} + rv_{n-1}}{\tau_{n-1} + r}, \frac{1}{\tau_{n-1} + r}\right) \equiv N\left(\mu_n, \frac{1}{\tau_n}\right).
\end{aligned}$$

LEMMA A.1 **(Distribution of S_n)**. *The ex-ante sum of sample values, S_n is normally distributed with mean $n\mu$ and variance $\frac{n^2}{\tau} + \frac{n}{r}$, i.e., $S_n \sim N\left(n\mu, \frac{n^2}{\tau} + \frac{n}{r}\right)$.*

There are also hybrid forms of simulations, which combine the classical and Bayesian methods. For example, in the hierarchical Bayesian simulation approach, $\hat{\phi_1}$ would be chosen from $N(\mu, 1/\tau)$ and then m samples of v_{1i} are drawn from $N(\hat{\phi_1}, 1/r)$; then another $\hat{\phi_2}$ would be chosen from $N(\mu, 1/\tau)$ and another m samples of v_{2i} would be drawn from $N(\hat{\phi_2}, 1/r)$. In such case, one can show that $S_n \sim N\left(n\mu, \frac{n}{\tau} + \frac{n}{r}\right)$, and the value is strictly decreasing in n.

Appendix B: Proofs

Proof of Proposition 1.

$$\begin{aligned}
\mathbb{E}_q[\pi] &= \mathbb{E}_q\{q \cdot Aq\} = A \cdot \mathbb{E}_q[q^2] \\
&= A \cdot \Big([\mathbb{E}(q)]^2 + \mathbb{V}(q)\Big) = A \cdot \Big((q^*)^2 + \mathbb{V}(q)\Big) \\
&= \left(\frac{\alpha}{\alpha+\beta}\right)^2 + \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \quad \square
\end{aligned}$$

Proof of Corollary 1. Clearly follows from Proposition 1. $\square$

Proof of Proposition 2. We will proceed as follows. We will first solve the integral in the square brackets in (4), which is a generalization of that in (1), to integrate out the ϕ, then integrate with respect to v to arrive at our expected profit expression. (i) We have

$$\left[\int_{-\infty}^{+\infty} f(v|\phi)g(\phi|S_n)d\phi\right] = \int_{-\infty}^{+\infty} \sqrt{\frac{r}{2\pi}} e^{-\frac{r(v-\phi)^2}{2}} \sqrt{\frac{\tau+nr}{2\pi}} e^{-\frac{(\tau+nr)(\phi-\frac{\mu\tau+rS_n}{\tau+nr})^2}{2}} d\phi$$

$$= \int_{-\infty}^{+\infty} \sqrt{\frac{r}{2\pi}} \sqrt{\frac{\tau+nr}{2\pi}} e^{-\frac{1}{2}\left[r(v-\phi)^2 + (\tau+nr)(\phi-\frac{\mu\tau+rS_n}{\tau+nr})^2\right]} d\phi$$

$$= \int_{-\infty}^{+\infty} \sqrt{\frac{r}{2\pi}} \sqrt{\frac{\tau+nr}{2\pi}} e^{-\frac{1}{2}\left[(r+(\tau+nr))\phi^2 - 2((\tau+nr)\frac{\mu\tau+rS_n}{\tau+nr}+rv)\phi + (rv^2+\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2(\tau+nr))\right]} d\phi$$

$$= \int_{-\infty}^{+\infty} \sqrt{\frac{r}{2\pi}} \sqrt{\frac{\tau+nr}{2\pi}} e^{-\frac{r+(\tau+nr)}{2}\left[\phi^2 - 2\left(\frac{(\tau+nr)\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)+rv}{r+(\tau+nr)}\right)\phi + \left(\frac{rv^2+\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2(\tau+nr)}{r+(\tau+nr)}\right)\right]} d\phi$$

$$= \int_{-\infty}^{+\infty} \sqrt{\frac{r}{2\pi}} \sqrt{\frac{\tau+nr}{2\pi}} e^{-\frac{r+(\tau+nr)}{2}\left[\left(\phi - \frac{rv+(\tau+nr)\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)}{r+(\tau+nr)}\right)^2 + \left(\frac{rv^2+\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2(\tau+nr)}{r+(\tau+nr)}\right) - \left(\frac{rv+(\tau+nr)\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)}{r+(\tau+nr)}\right)^2\right]} d\phi$$

$$\begin{aligned}
&= e^{-\frac{r+(\tau+nr)}{2}\left[\left(\frac{rv^2+\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2(\tau+nr)}{r+(\tau+nr)}\right) - \left(\frac{rv+(\tau+nr)\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)}{r+(\tau+nr)}\right)^2\right]} \\
&\quad \times \sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)} \left\{\int_{-\infty}^{+\infty} \sqrt{\frac{r+(\tau+nr)}{2\pi}} e^{-\frac{r+(\tau+nr)}{2}\left(\phi - \frac{rv+(\tau+nr)\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)}{r+(\tau+nr)}\right)^2} d\phi\right\} \qquad \text{(A-1)}
\end{aligned}$$

$$= \sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)} e^{-\frac{1}{2}\left[rv^2 + \left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2(\tau+nr) - \frac{(rv+(\tau+nr)\left(\frac{\mu\tau+rS_n}{\tau+nr}\right))^2}{r+(\tau+nr)}\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)} e^{-\frac{1}{2}\left[\left(r - \frac{r^2}{r+(\tau+nr)}\right)v^2 - \left(\frac{2r(\tau+nr)\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)}{r+(\tau+nr)}\right)v + \left(\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2(\tau+nr) - \frac{(\tau+nr)^2\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2}{r+(\tau+nr)}\right)\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)} e^{-\frac{1}{2}\left[\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)v^2 - 2\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)v + \left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2\left(\frac{(\tau+nr)r}{r+(\tau+nr)}\right)\right]}$$

$$= \sqrt{\frac{1}{2\pi}\frac{r(\tau+nr)}{r+(\tau+nr)}} e^{-\frac{1}{2}\frac{r(\tau+nr)}{r+(\tau+nr)}\left[v^2 - 2\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)v + \left(\frac{\mu\tau+rS_n}{\tau+nr}\right)^2\right]} = \sqrt{\frac{1}{2\pi}\frac{r(\tau+nr)}{r+(\tau+nr)}} e^{-\frac{1}{2}\frac{r(\tau+nr)}{r+(\tau+nr)}\left(v - \left(\frac{\mu\tau+rS_n}{\tau+nr}\right)\right)^2},$$

where the expression inside the brackets in (A-1) amounts to 1 as it integrates a normal density function.

(ii) Substituting this expression back into the original expression for expected profit (1), we have

$$(p-c)\int_p^{+\infty}\left[\int_{-\infty}^{+\infty} f(v;\phi)g(\phi|S_n)d\phi\right]dv = (p-c)\int_p^{+\infty}\sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)} e^{-\frac{1}{2}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)\left(v-\left(\frac{\mu\tau+rS_n}{\tau+nr}\right)\right)^2} dv$$
$$= (p-c)(1-H(p)),$$

where $H \sim \mathcal{N}\left(\frac{\mu\tau+rS_n}{\tau+nr}, \frac{1}{\tau+nr}+\frac{1}{r}\right)$. Thus, we have our expression for (5), and by setting $n=0$, we have expression (2). □

Proof of Corollary 2. The result follows directly from Proposition 2. □

Proof of Proposition 3. We have,

$$\begin{aligned} E_\lambda\{(p-c)D(p,\lambda)\} &= E_\lambda\left\{(p-c)\int_p^\infty \lambda e^{-\lambda v}dv\right\} = E_\lambda\{(p-c)e^{-\lambda p}\} = \int_0^\infty (p-c)e^{-\lambda p}\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}d\lambda \\ &= (p-c)\frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty \lambda^{\alpha-1}e^{-(p+\beta)\lambda}d\lambda = (p-c)\frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty\left(\frac{u}{p+\beta}\right)^{\alpha-1}e^{-u}\frac{d\lambda}{(p+\beta)} \\ &= (p-c)\left(\frac{\beta}{p+\beta}\right)^\alpha\frac{\int_0^\infty u^{\alpha-1}e^{-u}du}{\Gamma(\alpha)} = (p-c)\left(\frac{\beta}{p+\beta}\right)^\alpha. \end{aligned}$$

After observing valuations $v_1, \ldots, v_n$ drawn independently from exponential distribution with mean λ^{-1}, the prior belief about λ updates to a Gamma distribution with parameters $(\alpha+n, \beta+\sum_{i=1}^n v_i)$, and we have,

$$E_{\lambda|(v_1,\ldots,v_n)}\{(p-c)D(p,\lambda)\} = (p-c)\left(\frac{\beta+\sum_{i=1}^n v_i}{p+\beta+\sum_{i=1}^n v_i}\right)^{\alpha+n} = (p-c)\left(\frac{\beta+S_n}{p+\beta+S_n}\right)^{\alpha+n}. \quad \square$$

The expression (3) follows from $n=0$.

Proof of Corollary 3. Observe that keeping the mean $\alpha/\beta = \gamma$ constant, variance is $\frac{\alpha}{\beta^2} = \frac{\gamma}{\beta}$. Expression (3) becomes, for any $p>0$, $p\left(\frac{\beta}{p+\beta}\right)^{\gamma\beta}$, which is decreasing in β. □

Proof of Proposition 4. If A/γ is large (or γ is small), indicating high risk aversion, we have

$$\begin{aligned} E(u(I)) > E(u(II)) &\Leftrightarrow 0.5(q^*+\epsilon)e^{-(q^*+\epsilon)A/\gamma} + 0.5(q^*-\epsilon)e^{-(q^*-\epsilon)A/\gamma} > q^*e^{-q^*A/\gamma} \\ &\Leftrightarrow 0.5(e^{-\epsilon A/\gamma}+e^{\epsilon A/\gamma})q^*e^{-q^*A/\gamma} + 0.5\epsilon(e^{-\epsilon A/\gamma}-e^{\epsilon A/\gamma})e^{-q^*A/\gamma} \\ &\Leftrightarrow (e^{-\epsilon A/\gamma}+e^{\epsilon A/\gamma}-2) > \epsilon(e^{\epsilon A/\gamma}-e^{-\epsilon A/\gamma}) \\ &\Leftrightarrow e^{-\epsilon A/\gamma}(q^*+\epsilon)+e^{\epsilon A/\gamma}(q^*-\epsilon) > 2q^*. \qquad \square \end{aligned}$$

Proof of Proposition 5. Clear from the definition of EVSI. □

Proof of Corollary 4. We first show that (a) $EVSI(n,\tau)$ is decreasing in τ (part (ii)), then (b) $EVSI(n,\tau)$ is supermodular (part (iii)), and finally (c) $EVSI(n,\tau)$ is concave increasing in n (part (i)).

(a) (part (ii)). First, observe that as $n\to\infty$, $H_n \sim N\left(\frac{\mu\tau+rS_n}{\tau+nr}, \frac{1}{\tau+nr}+\frac{1}{r}\right) \to H \sim N\left(\mu, \frac{1}{r}\right)$. Thus, $p^n(S_n,\tau)\to p^* \equiv \arg\max_p \Big[(p-c)(1-H(p))\Big]$, which depends neither on τ nor S_n. It suffices to show that $EVSI(n,\tau)$ is decreasing in τ for large n. We have

$$\begin{aligned} EVSI(\infty,\tau) &\triangleq (p^*-c)(1-H(p^*)) - (p^0(\tau)-c)\Big(1-H(p^0(\tau))\Big) \\ &= (p^*-c)\bar{\Phi}(\sqrt{r}(p^*-\mu)) - (p^0(\tau)-c)\bar{\Phi}\Big(\sqrt{r}(p^0(\tau)-\mu)\Big), \end{aligned}$$

where $\bar{\Phi}(\cdot)$ denotes the complementary *cdf* of the standard normal distribution. Observe that the first term does not depend on τ, and that the second term depend on τ only through its dependence on $p^0(\tau)$. We will next show that $p^0(\tau)$ decreases in τ.

Recall that $p^0(\tau) \equiv \arg\max_p \Big[(p-c)(1-H_0(p))\Big]$, where $H_0 \sim N\left(\mu, \frac{1}{\tau}+\frac{1}{r}\right)$. Equivalently, we have $\arg\max_p \pi(p,\xi) \triangleq (p-c)(1-\Phi(\sqrt{\xi}(p-\mu)))$, $\xi \equiv \frac{r\tau}{r+\tau}$ is an increasing function of τ. Let $p(\xi)$ denote the value of p at which the maximum occurs; that is,

$$\max_p \pi(p,\xi) = \pi(p(\xi),\xi).$$

A condition that guarantees $p(\xi)$ is decreasing in ξ is that $\pi(p,\xi)$ is *submodular*, or equivalently, $\frac{\partial^2\pi(p,\xi)}{\partial p\partial\xi}<0$ (Ross 1983, p.6). Taking the partial derivatives, as long as $p>\mu$

$$\begin{aligned} \frac{\partial\pi(p,\xi)}{\partial p} &= (p-c)[-\phi(\sqrt{\xi}(p-\mu))\sqrt{\xi}] + (1-\Phi(\sqrt{\xi}(p-\mu))), \\ \frac{\partial^2\pi(p,\xi)}{\partial p\partial\xi} &= \frac{\partial}{\partial\xi}\Big[1-\Phi(\sqrt{\xi}(p-\mu)) - (p-c)\phi(\sqrt{\xi}(p-\mu))\sqrt{\xi}\Big] \\ &= -\phi(\sqrt{\xi}(p-\mu))(p-\mu)\cdot\frac{1}{2\sqrt{\xi}} - (p-c)\left[\sqrt{\xi}\phi'(\sqrt{\xi}(p-\mu))\cdot\frac{p-\mu}{2\sqrt{\xi}} + \frac{1}{2\sqrt{\xi}}\phi(\sqrt{\xi}(p-\mu))\right] \\ &= -\frac{\phi(\sqrt{\xi}(p-\mu))(p-\mu)}{2\sqrt{\xi}} - \frac{(p-c)\phi(\sqrt{\xi}(p-\mu))}{2\sqrt{\xi}} - \frac{\phi'(\sqrt{\xi}(p-\mu))(p-\mu)(p-c)}{2} \\ &= -\frac{\phi(\sqrt{\xi}(p-\mu))}{2\sqrt{\xi}}((p-\mu)+(p-c)) - \frac{\phi'(\sqrt{\xi}(p-\mu))(p-\mu)(p-c)}{2} < 0. \end{aligned}$$

Thus, $p^0(\tau)$ is decreasing in τ, which implies that $EVSI(\infty,\tau)$ (and $EVSI(n,\tau)$ $\forall n$) is decreasing in τ.

(b) (part(iii)) We will show by induction that EVSI$(n+1,\tau)$ − EVSI(n,τ) is decreasing in τ.

(base case) Because $EVSI(n,\tau)$ is decreasing in τ for any n, we have that EVSI$(1,\tau)$ is decreasing in τ. This is equivalent to EVSI$(1,\tau)$ − EVSI$(0,\tau)$ decreasing in τ because EVSI$(0,\tau)=0$.

(induction step) Recall that EVSI(n,τ) is defined based on $H_n \sim N\left(\frac{\tau\mu+rS_n}{\tau+nr}, \frac{1}{\tau+nr}+\frac{1}{r}\right)$. Let $\tau' \equiv \tau+nr$ and $\mu' \equiv \frac{\tau\mu+rS_n}{\tau+nr}$. Then we have $H_n \sim N(\mu', 1/\tau'+1/r)$, and $H_{n+1} \sim N\left(\frac{\tau\mu'+rv}{\tau+r}, \frac{1}{\tau'+r}+\frac{1}{r}\right)$, which is equivalent to the expression EVSI$(1,\tau')$ − EVSI$(0,\tau')$. Thus, EVSI$(n+1,\tau)$ − EVSI(n,τ) is decreasing in τ. $\forall \tau_2 > \tau_1$,

$$\begin{aligned}
&EVSI(n+1,\tau_2) - EVSI(n,\tau_2) < EVSI(n+1,\tau_1) - EVSI(n,\tau_1)\\
\Leftrightarrow\ &EVSI(n+1,\tau_2) - EVSI(n+1,\tau_1) < EVSI(n,\tau_2) - EVSI(n,\tau_1)\\
\Leftrightarrow\ &EVSI(n+1,\tau_2) + EVSI(n,\tau_1) < EVSI(n+1,\tau_1) + EVSI(n,\tau_2),
\end{aligned}$$

where the final inequality is the definition of submodularity (**?**, p. 6).

(c) (part (i)) We will first prove that EVSI(n,τ) is increasing in n, then show that it is concave in n. First, EVSI$(1,\tau)>0$ because information has value. This is equivalent to EVSI$(1,\tau)$− EVSI$(0,\tau)>0$, since EVSI$(0,\tau)=0$. If this is true, then EVSI$(n+1,\tau)$− EVSI$(n,\tau)>0$ $\forall n$, by transforming variables $\tau' \equiv \tau+nr$ and $\mu' \equiv \frac{\tau\mu+rS_n}{\tau+nr}$. Thus, EVSI$(n,\tau)$ is increasing in n.

Next, we prove concavity by showing that submodularity of EVSI(n,τ) implies concavity of EVSI(n,τ). We have that for any $\tau'>\tau$,

$$EVSI(n+1,\tau') - EVSI(n,\tau') < EVSI(n+1,\tau) - EVSI(n,\tau). \tag{A-2}$$

EVSI$(n+1,\tau')$ and EVSI(n,τ') employ distributions $H_{n+1} \sim N\left(\frac{\tau'\mu+rS'_{n+1}}{\tau'+(n+1)r}, \frac{1}{\tau'+(n+1)r}+\frac{1}{r}\right)$ and $H_n \sim N\left(\frac{\tau'\mu+rS'_n}{\tau'+nr}, \frac{1}{\tau'+nr}+\frac{1}{r}\right)$ respectively. EVSI$(n+1,\tau)$ and EVSI(n,τ) employ distributions $H_{n+1} \sim N\left(\frac{\tau\mu+rS_{n+1}}{\tau+(n+1)r}, \frac{1}{\tau+(n+1)r}+\frac{1}{r}\right)$ and $H_n \sim N\left(\frac{\tau\mu+rS_n}{\tau+nr}, \frac{1}{\tau+nr}+\frac{1}{r}\right)$ respectively.

Since the inequality (A-2) holds for any $\tau'>\tau$, it must also holds for $\tau'=\tau+r$ and $S'_n = S_n + v - \mu$. Changing the variables accordingly, the distributions of EVSI$(n+1,\tau')$ and EVSI(n,τ') can be rewritten as:

$$\begin{aligned}
H_{n+1} &\sim N\left(\frac{(\tau+r)\mu+r(S_{n+1}+v-\mu)}{\tau+r+(n+1)r}, \frac{1}{\tau+r+(n+1)r}+\frac{1}{r}\right) = N\left(\frac{\tau\mu+r(S_{n+1}+v)}{\tau+(n+2)r}, \frac{1}{\tau+(n+2)r}+\frac{1}{r}\right),\\
H_n &\sim N\left(\frac{(\tau+r)\mu+r(S_n+v-\mu)}{\tau+r+nr}, \frac{1}{\tau+r+nr}+\frac{1}{r}\right) = N\left(\frac{\tau\mu+r(S_n+v)}{\tau+(n+1)r}, \frac{1}{\tau+(n+1)r}+\frac{1}{r}\right),
\end{aligned}$$

which correspond to distributions for EVSI$(n+2,\tau)$ and EVSI$(n+1,\tau)$ respectively. Concavity follows since,

$$\begin{aligned}
&EVSI(n+1,\tau') - EVSI(n,\tau') < EVSI(n+1,\tau) - EVSI(n,\tau)\\
\Rightarrow\ &EVSI(n+2,\tau) - EVSI(n+1,\tau) < EVSI(n+1,\tau) - EVSI(n,\tau). \qquad \square
\end{aligned}$$

Proof of Proposition 6. (i) Since $S_n \sim \mathcal{N}(n\mu, n/r)$, we have

$$
\begin{aligned}
&E_\phi\{(p-c)D(p)\} = E_{S_n}\left[E_{\phi|S_n}\{(p-c)D(p)\}\right] \\
&= E_{S_n}\left[(p-c)\int_p^{+\infty}\sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)}e^{-\frac{1}{2}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)(v-\left(\frac{\mu\tau+rS_n}{\tau+nr}\right))^2}dv\right] \\
&= (p-c)\int_{-\infty}^{+\infty}\left[\int_p^{+\infty}\sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)}e^{-\frac{1}{2}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)(v-\left(\frac{\mu\tau+rS_n}{\tau+nr}\right))^2}dv\right]\sqrt{\frac{1}{2\pi}\cdot\frac{r}{n}}e^{-\frac{1}{2}\frac{r}{n}(S_n-n\mu)^2}dS_n \\
&= (p-c)\int_p^{+\infty}\sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)}\sqrt{\frac{1}{2\pi}\frac{r}{n}}\left[\int_{-\infty}^{+\infty}e^{-\frac{1}{2}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)(v-\left(\frac{\mu\tau+rS_n}{\tau+nr}\right))^2}e^{-\frac{1}{2}\frac{r}{n}(S_n-n\mu)^2}dS_n\right]dv. \quad \text{(A-3)}
\end{aligned}
$$

We will now solve for the expression inside the square bracket of (A-3), which equals

$$
\begin{aligned}
&\int_{-\infty}^{+\infty}e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left((v-\frac{\mu\tau}{\tau+nr})-\frac{r}{\tau+nr}S_n\right)^2+\frac{r}{n}(S_n-n\mu)^2\right]}dS_n \\
&=\int_{-\infty}^{+\infty}e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left[(v-\frac{\mu\tau}{\tau+nr})^2-2(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}S_n+\left(\frac{r}{\tau+nr}S_n\right)^2\right]+\frac{r}{n}(S_n^2-2n\mu S_n+(n\mu)^2)\right]}dS_n \\
&=\int_{-\infty}^{+\infty}e^{-\frac{1}{2}\left[\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]S_n^2-2\left[\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu\right]S_n+\left[\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2\right]\right]}dS_n \\
&=\int_{-\infty}^{+\infty}e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left\{S_n^2-2\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}S_n+\frac{\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right\}}dS_n \\
&=\int_{-\infty}^{+\infty}e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left(S_n-\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right)^2}\times \\
&\qquad e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left\{\frac{\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]^2\right\}}dS_n \\
&=e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left\{\frac{\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]^2\right\}}\frac{1}{\sqrt{\frac{1}{2\pi}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]}} \\
&\times\sqrt{\frac{1}{2\pi}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]}\int_{-\infty}^{+\infty}e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left(S_n-\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right)^2}dS_n \\
&=\frac{1}{\sqrt{\frac{1}{2\pi}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]}}e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left\{\frac{\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]^2\right\}},
\end{aligned}
$$

where again the integral is equal to 1 because it is integrating out a normal probability density function.

Combining with the expression $\sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)}\sqrt{\frac{1}{2\pi}\cdot\frac{r}{n}}$ inside the integral in (A-3),

$$
\frac{\sqrt{\frac{1}{2\pi}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)}\sqrt{\frac{1}{2\pi}\cdot\frac{r}{n}}}{\sqrt{\frac{1}{2\pi}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]}}e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left\{\frac{\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]^2\right\}}
$$

$$= \sqrt{\frac{1}{2\pi}\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}\frac{r}{n}}{\frac{r(\tau+nr)}{r+\tau+nr}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]}\, e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left\{\frac{\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]^2\right\}}.$$

Evaluating the expression in the exponent,

$$e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}\right]\left\{\frac{\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]^2\right\}}$$

$$= e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2-\frac{\left(\frac{r(\tau+nr)}{r+(\tau+nr)}(v-\frac{\mu\tau}{\tau+nr})\frac{r}{\tau+nr}+\frac{r}{n}n\mu\right)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]}$$

$$= e^{-\frac{1}{2}\left[\frac{r(\tau+nr)}{r+\tau+nr}(v-\frac{\mu\tau}{\tau+nr})^2+\frac{r}{n}(n\mu)^2-\frac{\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)^2\left(v-\frac{\mu\tau}{\tau+nr}\right)^2\left(\frac{r}{\tau+nr}\right)^2+2r\mu\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)\left(v-\frac{\mu\tau}{\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)+(r\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]}$$

$$= e^{-\frac{1}{2}\left[(v-\frac{\mu\tau}{\tau+nr})^2\left[\frac{r(\tau+nr)}{r+\tau+nr}-\frac{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)^2\left(\frac{r}{\tau+nr}\right)^2}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]+\frac{r}{n}(n\mu)^2-\frac{2r\mu\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)\left(v-\frac{\mu\tau}{\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\frac{(r\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]}$$

$$= e^{-\frac{1}{2}\left[(v-\frac{\mu\tau}{\tau+nr})^2\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]+\frac{r}{n}(n\mu)^2-\frac{2r\mu\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)\left(v-\frac{\mu\tau}{\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\frac{(r\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]}$$

$$= e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{(v-\frac{\mu\tau}{\tau+nr})^2+\left[\frac{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}\right]\left(\frac{r}{n}(n\mu)^2-\frac{2r\mu\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)\left(v-\frac{\mu\tau}{\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}-\frac{(r\mu)^2}{\frac{r(\tau+nr)}{r+(\tau+nr)}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right)\right\}}$$

$$= e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{(v-\frac{\mu\tau}{\tau+nr})^2+\left(\frac{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}\right)\frac{r}{n}(n\mu)^2-\frac{2r\mu\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)\left(v-\frac{\mu\tau}{\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)}{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+(\tau+nr)}\right)}-\frac{(r\mu)^2}{\frac{r}{n}\frac{r(\tau+nr)}{r+(\tau+nr)}}\right\}}$$

$$= e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{(v-\frac{\mu\tau}{\tau+nr})^2+\left(\frac{r}{\tau+nr}\right)^2(n\mu)^2+\frac{(\frac{r}{n})^2(n\mu)^2}{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}-\frac{(r\mu)^2}{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}-\frac{n}{r}2r\mu\frac{r}{\tau+nr}\left(v-\frac{\mu\tau}{\tau+nr}\right)\right\}}$$

$$= e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{(v-\frac{\mu\tau}{\tau+nr})^2+\left(\frac{rn\mu}{\tau+nr}\right)^2-\frac{2\mu nr}{\tau+nr}\left(v-\frac{\mu\tau}{\tau+nr}\right)\right\}}$$

$$= e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{v^2-\frac{2\mu\tau}{\tau+nr}v+\left(\frac{\mu\tau}{\tau+nr}\right)^2+\left(\frac{rn\mu}{\tau+nr}\right)^2-\frac{2\mu nr}{\tau+nr}v+\frac{2\mu nr(\mu\tau)}{(\tau+nr)^2}\right\}}$$

$$= e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{v^2-\left[\frac{2\mu\tau}{\tau+nr}+\frac{2\mu nr}{\tau+nr}\right]v+\frac{(\mu\tau)^2+(rn\mu)^2+2\mu^2nr\tau}{(\tau+nr)^2}\right\}} = e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{v^2-2\mu v+\frac{(\mu\tau+rn\mu)^2}{(\tau+nr)^2}\right\}}$$

$$= e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{v^2-2\mu v+\frac{\mu^2(\tau+nr)^2}{(\tau+nr)^2}\right\}} = e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{(v-\mu)^2\right\}}.$$

Putting it altogether, we have

$$\begin{aligned} E_\phi\{(p-c)D(p)\} &= (p-c)\int_p^{+\infty}\sqrt{\frac{1}{2\pi}\left[\frac{\frac{r(\tau+nr)}{r+(\tau+nr)}\frac{r}{n}}{\frac{r(\tau+nr)}{r+\tau+nr}\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]}\, e^{-\frac{1}{2}\left[\frac{\frac{r}{n}\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)}{\left(\frac{r(\tau+nr)}{r+\tau+nr}\right)\left(\frac{r}{\tau+nr}\right)^2+\frac{r}{n}}\right]\left\{(v-\mu)^2\right\}} dv \\ &= (p-c)(1-H(p)), \end{aligned}$$

where $H\sim\mathcal{N}\left(\mu, \frac{nr}{(\tau+nr)^2}+\frac{1}{\tau+nr}+\frac{1}{r}\right)$.

(ii) We have $S_n \sim\mathcal{N}(n\mu, \frac{n^2}{\tau}+\frac{n}{r})$. In the derivation of the above expression, replace the variance term $\frac{n}{r}$ with $\frac{n^2}{\tau}+\frac{n}{r}$. This leads to the expression for $H_n\sim N\left(\mu, \frac{rn}{\tau(\tau+nr)}+\frac{1}{\tau+nr}+\frac{1}{r}\right)$. One can see that

$$\frac{rn}{\tau(\tau+nr)}+\frac{1}{\tau+nr}+\frac{1}{r} = \frac{1}{\tau+nr}\left(1+\frac{nr}{\tau}\right)+\frac{1}{r} = \frac{1}{\tau+nr}\left(\frac{\tau+nr}{\tau}\right)+\frac{1}{r} = \frac{1}{\tau}+\frac{1}{r}. \qquad \square$$

Proof of Proposition 7. (i) Clear from the illustration of Figure 3 that EVSI is decreasing in n. (ii) The EVSI expression does not depend on n. $\square$

Proof of Lemma A.1. We show by induction. It is easy to show that $E(S_n) = n\mu$ by applying law of iterative expectation. We will focus on showing that $Var(S_n) = n^2/\tau + n/r$. The expressions is useful for expressing μ_n and τ_n in terms of μ_1 and τ_1.

$$\mu_n = \frac{\tau_1\mu_1}{\tau_1 + (n-1)r} + \frac{r}{\tau_1 + (n-1)r}(v_1 + \cdots + v_{n-1}), \quad \tau_n = \tau_1 + (n-1)r. \tag{A-4}$$

(Base Case) $n = 2$. We have $Var(v_1 + v_2) = Var(v_1) + Var(v_2) + 2Cov(v_1, v_2)$. Using the law of iterated variance, we have

$$\begin{aligned}
Var(v_1) &= Var(E(v_1|\phi_1)) + E(Var(v_1|\phi_1)) = Var(\phi_1) + \frac{1}{r} = \frac{1}{\tau_1} + \frac{1}{r}, \\
Var(v_2) &= Var(E(v_2|\phi_2)) + E(Var(v_2|\phi_2)) = Var(\phi_2) + \frac{1}{r}.
\end{aligned}$$

The expression for $Var(\phi_2)$ requires employing law of iterated variance again,

$$\begin{aligned}
Var(\phi_2) &= Var(E(\phi_2|v_1)) + E(Var(\phi_2|v_1)) = Var\left(\frac{\tau_1\mu_1 + rv_1}{\tau_1 + r}\right) + \frac{1}{\tau_1 + r} = \frac{r^2}{(\tau_1 + r)^2}Var(v_1) + \frac{1}{\tau_1 + r} \\
&= \frac{r^2}{(\tau_1 + r)^2}\left(\frac{1}{\tau_1} + \frac{1}{r}\right) + \frac{1}{\tau_1 + r} = \frac{r}{(\tau_1 + r)^2} + \frac{r^2}{\tau_1(\tau_1 + r)^2} + \frac{1}{\tau_1 + r} = \frac{\tau_1 r + r^2 + \tau_1(\tau_1 + r)}{\tau_1(\tau_1 + r)^2} \\
&= \frac{r(\tau_1 + r) + \tau_1(\tau_1 + r)}{\tau_1(\tau_1 + r)^2} = \frac{(\tau_1 + r)(\tau_1 + r)}{\tau_1(\tau_1 + r)^2} = \frac{1}{\tau_1}.
\end{aligned}$$

Thus, $Var(v_2) = \frac{1}{\tau_1} + \frac{1}{r} = Var(v_1)$.

We next find the expression for $Cov(v_1, v_2)$. Observe that v_2 depends on v_1 as follows,

$$v_2 = \phi_2 + \xi = \left[\frac{\tau_1\mu_1}{\tau_1 + r} + \frac{r}{\tau_1 + r}v_1 + \epsilon_2\right] + \xi,$$

where $\xi \sim N\left(0, \frac{1}{r}\right)$, and $\epsilon_2 \sim N\left(0\frac{1}{\tau_1 + r}\right)$. Since $Var(v_1) = Var(v_2) = \frac{1}{\tau_1} + \frac{1}{r}$, $Corr(v_1, v_2) = \frac{r}{\tau_1 + r}$. Thus,

$$\begin{aligned}
Var(v_1 + v_2) &= Var(v_1) + Var(v_2) + 2Cov(v_1, v_2) = Var(v_1) + Var(v_2) + 2SD(v_1) \cdot SD(v_2) \cdot Corr(v_1, v_2) \\
&= \frac{1}{\tau_1} + \frac{1}{r} + \frac{1}{\tau_1} + \frac{1}{r} + 2\left(\frac{1}{\tau_1} + \frac{1}{r}\right)\frac{r}{\tau_1 + r} = \frac{4}{\tau_1} + \frac{2}{r}.
\end{aligned}$$

(Induction step). Now suppose that $Var(v_1 + \cdots + v_{n-1}) = \frac{(n-1)^2}{\tau_1} + \frac{n-1}{r}$. We have

$$Var(v_1 + \cdots + v_n) = Var(v_1 + \cdots + v_{n-1}) + Var(v_n) + 2Cov([v_1 + \cdots + v_{n-1}], v_n).$$

We next find the expressions for $Var(v_n)$ and $Cov([v_1 + \cdots + v_{n-1}], v_n)$. Using the law of iterated variance,

$$Var(v_n) = Var(E(v_n|\phi_n)) + E(Var(v_n|\phi_n)) = Var(\phi_n) + \frac{1}{r} = \frac{1}{\tau_1} + \frac{1}{r}.$$

where expression for $Var(\phi_n)$ is found by using iterated variance rule again,

$$\begin{aligned} Var(\phi_n) &= Var(E(\phi_n|\mu_n)) + E(Var(\phi_n|\mu_n)) = Var\left(\frac{\tau_{n-1}\mu_{n-1} + rv_{n-1}}{\tau_{n-1}+r}\right) + \frac{1}{\tau_{n-1}+r} \\ &= Var\left(\frac{\tau_1\mu_1}{\tau_1+(n-1)r} + \frac{r(v_1+\cdots+v_{n-1})}{\tau_1+(n-1)r}\right) + \frac{1}{\tau_1+(n-1)r} \\ &= \frac{r^2}{(\tau_1+(n-1)r)^2} Var(v_1+\cdots+v_{n-1}) + \frac{1}{\tau_1+(n-1)r} \\ &= \frac{r^2}{(\tau_1+(n-1)r)^2}\left(\frac{(n-1)^2}{\tau_1} + \frac{n-1}{r}\right) + \frac{1}{\tau_1+(n-1)r} \\ &= \frac{\tau_1 r(n-1) + r^2(n-1)^2 + \tau_1(\tau_1+(n-1)r)}{\tau_1(\tau_1+(n-1)r)^2} = \frac{(\tau_1+(n-1)r)(\tau_1+(n-1)r)}{\tau_1(\tau_1+(n-1)r)^2} = \frac{1}{\tau_1}, \end{aligned}$$

where the third equality is due to (A-4), and fifth equality is because of the induction assumption.

We next find the expression for $Cov([v_1+\cdots+v_{n-1}], v_n)$. We examine the correlation. v_n depends on all previous $n-1$ samples drawn as follows,

$$v_n = \phi_n + \xi = [\mu_n + \epsilon_n] + \xi = \left[\frac{\tau_1\mu_1}{\tau_1+(n-1)r} + \frac{r}{\tau_1+(n-1)r}(v_1+\cdots+v_{n-1}) + \epsilon_n\right] + \xi,$$

where $\xi \sim N\left(0, \frac{1}{r}\right)$ and $\epsilon_n \sim N\left(0, \frac{1}{\tau_n}\right)$. Since $Corr(v_1+\cdots+v_{n-1}, v_n) = \frac{r}{\tau_1+(n-1)r}\cdot\frac{SD(v_1+\cdots+v_{n-1})}{SD(v_n)}$, we have

$$\begin{aligned} Cov(v_1+\cdots+v_{n-1}, v_n) &= Corr(v_1+\cdots+v_{n-1}, v_n)\cdot SD(v_1+\cdots+v_{n-1})\cdot SD(v_n) \\ &= \left[\frac{r}{\tau_1+(n-1)r}\cdot\frac{SD(v_1+\cdots+v_{n-1})}{SD(v_n)}\right] SD(v_1+\cdots+v_{n-1})\cdot SD(v_n) \\ &= \frac{r}{\tau_1+(n-1)r}\cdot Var(v_1+\cdots+v_{n-1}) = \frac{r}{\tau_1+(n-1)r}\left(\frac{(n-1)^2}{\tau_1} + \frac{n-1}{r}\right) \\ &= \frac{r}{\tau_1+(n-1)r}\,\frac{r(n-1)^2+\tau_1(n-1)}{\tau_1 r} = \frac{n-1}{\tau_1}. \end{aligned}$$

where the third to last equality is due to the induction assumption. Thus,

$$\begin{aligned} Var(S_n) &= Var([v_1+\cdots+v_{n-1}]+v_n) = Var(v_1+\cdots+v_{n-1}) + Var(v_n) + 2Cov([v_1+\cdots+v_{n-1}], v_n) \\ &= \frac{(n-1)^2}{\tau_1} + \frac{n-1}{r} + \frac{1}{\tau_1} + \frac{1}{r} + \frac{2(n-1)}{\tau_1} = \frac{(n-1)^2+2(n-1)+1}{\tau_1} + \frac{n}{r} = \frac{n^2}{\tau_1} + \frac{n}{r}. \qquad \square \end{aligned}$$