Stress–strain–temperature relationship for concrete

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Highlights:

- Discuss the limitations of the stress–strain curves of concrete in Eurocode 2
- Take into account the coupled effects between stress and expansion
- Propose a stress–strain–temperature relationship of concrete using equations
- Highlight the capability of the newly-developed stress–strain–temperature relationship

Abstract:

When concrete structures are subjected to load and temperature simultaneously, it is essential to take into account the coupled effects between stress and expansion. However, due to incomplete understanding, such coupled effects have only been incorporated into current Eurocode 2 (EC2) stress–strain curves by means of empirical correlations. These empirical correlations at different target temperatures are presented in tables that do not allow to clearly identify the correlation chosen to obtain the specific values. A further limitation of these tables is that the relationships cannot be used to evaluate the performance of concrete structures during the cooling phase. In this paper, a physically-based model of the coupled effects between stress and expansion is used to define the strain corresponding to the compressive strength, and thus to develop a simple formulation for stress–strain–temperature relationship of concrete. The results are then compared with the EC2 stress–strain–temperature table. The expression of stress–strain–temperature relationship developed in this paper successfully agrees with the stress–strain curves of concrete in EC2 used for the heating phase. More importantly, the proposed stress–strain–temperature relationship can also be applicable for design purposes of concrete structures during the cooling phase.

Keywords: stress–strain–temperature relationship; structural response; total strain model; load-induced thermal strain; performance-based design.

1. Introduction

When explicitly analysing the performance of concrete structures during the heating and cooling phases of a fire, it is necessary that the stress–strain–temperature of the concrete material be well-understood and well-defined [1, 2]. Since the 1970s, numerous studies have been conducted to understand the behaviour of concrete at elevated temperatures. In light of recent failures of concrete structures in fires [2], it is clearly essential to attain more in-depth knowledge of stress–strain–temperature relationships for concrete that are applicable to both heating and cooling phases.

In a fire, structural elements are subject to load and heat simultaneously; therefore, the stress–strain–temperature relationship of concrete must take into account the coupled effects between stress and expansion while the temperature increases [3, 4]. Unfortunately, while the properties of some materials such as concrete and steel seem to have been relatively well defined, the coupled effects between stress and expansion of concrete have not been adequately accounted for in the total strain model used for design purposes in EC2 [5]. Instead, the current total strain has been defined as a combination of four main strain components: (i) free thermal strain; (ii) stress-related strain; (iii) creep strain; and (iv) transient thermal strain or load-induced thermal strain (LITS) - in which the LITS has been principally developed by best-fittings to the experimental data [6-12], thus limiting the applicability of the associated total strain models [3]. Additionally, the LITS has also typically been implicitly incorporated into the design stress–strain curves through the mechanical strain and tabled in standard by suggesting the strain values corresponding to the compressive stresses at target temperatures [5, 13].

As a result, the applicability of stress–strain curves in EC2 is limited to the test data collected, and types of concrete studied. More importantly, the current stress–strain curves in EC2 are only valid for the heating phase [13]. Consequently, even the LITS is implicitly incorporated into the design stress–strain curves, the accuracy of mechanical behaviour such as stress and deformation is still questionable, primarily when the stress–strain curves introduced in EC2 are used [14] or when temperatures are greater than 500°C [9]. Therefore, it is necessary to develop a stress–strain–temperature relationship for concrete which has explicit links to the physical bases of the coupled effects between stress and expansion of concrete.

This paper proposes a new stress–strain–temperature relationship of concrete based on the generic stress–strain behaviour and the physically-based strain model that incorporates the coupled effects between stress and thermal expansion. The results of the model are compared to the stress–strain curves in EC2 at different target temperatures. Also, the applicability of the proposed stress–strain–temperature relationship for both the heating and cooling phases is demonstrated.

2. Total strain model and the stress–strain–temperature relationship in EC2

To examine the performance of structures at high temperature, the relationship of stress, strain and temperature of materials must be clearly defined [15]. In concrete structures, the stress–strain–temperature relationship for concrete is represented mathematically by a best-fit to stress and strain data collected from unstressed, thermal steady-state tests. The tests are usually conducted as follows:

- The samples are heated to target temperature unrestrained and unloaded;
- The target temperature is maintained for 2-3 hours to ensure uniformity of temperature within the sample;
- The samples are then loaded until failure;
- The stress and strain data at the target temperature is then recorded throughout loading;

The stress–strain relationships developed from the unstressed steady-state tests do not incorporate any of the coupled effects of load and heat. When concrete structural elements subjected to high temperature, the thermal loads can induce thermal stresses if the elements are not free to expand. In turn, the resulted stresses will affect the thermal deformation of the structural elements [3, 4, 16]. To incorporate these coupled effects, a total strain model is then introduced, and the coupled effects between stress–expansion behaviour are taken into account through the introduction of the load-induced thermal strain (LITS) [5], as follows:

$$\varepsilon_{tot} = \varepsilon_\sigma + \varepsilon_{th} + \varepsilon_{LITS} + \varepsilon_{cr} \tag{1}$$

where, ε_{tot} is the total strain of element, ε_σ is the stress-dependent strain, ε_{th} is the free thermal strain, and ε_{LITS} is the load-induce thermal strain. The term ε_{cr} represents the creep strain which is another phenomenon that needs to be accounted for. When temperature increase is rapid, like in the case of a fire, the creep strain ε_{cr} is much smaller relative to the other strains and thus assumed zero in the total strain model [9]. Equation (1) becomes:

$$\varepsilon_{tot} = \varepsilon_M + \varepsilon_{th} = \varepsilon_\sigma + \varepsilon_{LITS} + \varepsilon_{th} \tag{2}$$

$$\varepsilon_M = \varepsilon_\sigma + \varepsilon_{LITS} \tag{3}$$

where ε_M is the mechanical strain.

The stress–strain–temperature relation of concrete is then represented as σ–ε_M–T and can be used for the design purpose [17]. It can be clearly seen from Equations (2) and (3), that the adequacy of σ–ε_M–T relationship depends on an appropriate description of ε_{LITS} as a function of temperature and stress. However, current LITS correlations cannot fully explain the physical meanings of LITS [3, 8, 13]. Consequently, the accuracy and applicability of the developed mechanical strain (ε_M) and the current σ–ε_M–T in EC2 need to be further analysed.

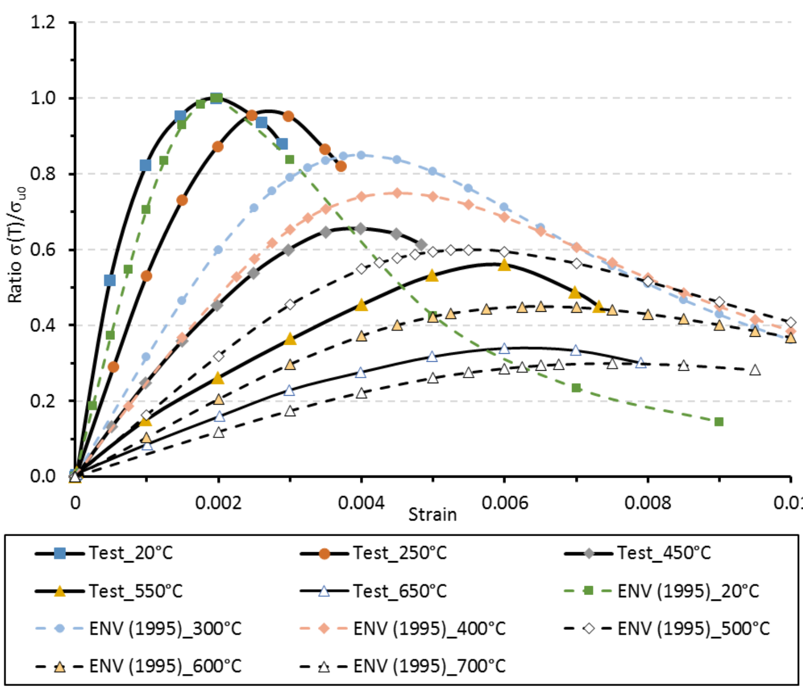

Figure 1. Comparison of the actual stress–strain curves of concrete at different target temperatures conducted by Schneider [18] and European Standard [19].

Table 1. Strain and stress values of the stress–strain relationships of normal weight concrete at elevated temperatures used in ENV [19] and EC2 [5].

T [°C]	**The corresponding strain of compressive strength ($\varepsilon_{c1,T}$) ($\times 10^{-3}$)**			**Maximum strain ($\varepsilon_{cu1,T}$) ($\times 10^{-3}$)**		**σ_{uT}/σ_{u0}**	
	ENV		**EC2**	**ENV**	**EC2**	**ENV and EC2**	
	Range	**Rcm.**	**Rcm.**	**Rcm.**	**Rcm.**	**Sil.**	**Cal.**
20	2.5	2.5	2.5	20.0	20.0	1	1
100	2.5 – 4.0	3.5	4.0	22.5	22.5	1	1
200	3.0 – 5.5	4.5	5.5	25.0	25.0	0.95	0.97
300	4.0 – 7.0	6.0	7.0	27.5	27.5	0.85	0.91
400	4.5 – 10.0	7.5	10.0	30.0	30.0	0.75	0.85
500	5.5 – 15.0	9.5	15.0	32.5	32.5	0.60	0.74
600	6.5 – 25.0	12.5	25.0	35.0	35.0	0.45	0.60
700	7.5 – 25.0	14.0	25.0	37.5	37.5	0.30	0.43
800	8.5 – 25.0	14.5	25.0	40.0	40.0	0.15	0.27
900	10.0 – 25.0	15.0	25.0	42.5	42.5	0.08	0.15
1000	10.0 – 25.0	15.0	25.0	45.0	45.0	0.04	0.06
1100	10.0 – 25.0	15.0	25.0	47.5	47.5	0.01	0.02
1200	—	—	—	—	—	—	—

*Rcm. = Recommended; Sil. = Siliceous aggregates; Cal. = Calcareous aggregates.

Table 1 shows the recommended strain values corresponding to the compressive strength for the generic behaviour of concrete, as shown in Equation (4). In addition, it also shows the maximum strain ($\varepsilon_{cu1,T}$). Figure 1 shows the stress–strain curve for samples that are (i) first heated to a target temperature unrestrained and unloaded and then (ii) loaded to failure to obtain each of the curves shown. In ENV [19], the recommended strain values corresponding to the compressive strength are extracted from these plots and, therefore, do not account for LITS. In EC2 [5], the recommended strain values corresponding to compressive strength are much higher than those recommended in ENV [19]. The additional strain is introduced to account for LITS, but no underlying representation of how LITS has been accounted for is detailed [13]. Consequently, the extracted Young's modulus values from stress–strain curves in EC2 are much smaller than those that are determined using ENV [5, 19].

In both cases, the recommended compressive strength and corresponding strain are used as inputs to the function proposed by Schneider [17] (Equation (4)) to obtain the stress–strain relation.

$$\sigma(T) = \frac{\varepsilon(T)}{\varepsilon_{c1,T}} \times \frac{3 \times \sigma_{uT}}{2 + \left(\frac{\varepsilon(T)}{\varepsilon_{c1,T}}\right)^3} \tag{4}$$

where σ_{uT} is the compressive strength and $\varepsilon_{c1,T}$ is the corresponding strain at the compressive strength. In addition, the value for a maximum strain (at crushing) is defined as $\varepsilon_{cu1,T}$. It is important to note that following the point corresponding to the compressive strength, the test data (Figure 1) are available only up to post-peak stresses of approximately $0.8\sigma_{uT}$; therefore there seems little experimental basis to extrapolate $\varepsilon_{cu1,T}$.

Test data [13, 20] have shown that this formulation seems to provide a satisfactory means for the safe design of concrete structures in fire. Nevertheless, it can be argued that further improvement is required. The current approach is capable of establishing a conservative approach to failure, but the stress distribution and deformation of structures cannot be determined correctly if the EC2 curves are used [14]. It is thus necessary to develop the stress–strain–temperature of concrete in which the actual performance of concrete materials and the coupled effects between stress and expansion can be adequately and explicitly taken into account.

3. Developing the σ–ε_M–T relationship for concrete

3.1 Mechanical strain (ε_M) of concrete at elevated temperatures

The first step of developing the mechanical strain using for the σ–ε_M–T of concrete is understanding the coupled effect between stress and expansion. By using fundamental thermodynamics and continuum mechanics laws, Le *et al.* [3] developed an expression that defines the effects of stress on thermal expansion coefficient of solid material when it is subject to simultaneous heat and load conditions. When the concrete is subjected to load and temperature change simultaneously, the energy balance can be summarised as Equation (5) [4, 21]:

$$\rho_0 c_e \frac{\partial T}{\partial t} = T.\frac{\partial \sigma}{\partial T}.\frac{\partial \varepsilon}{\partial t} \tag{5}$$

where, ρ_0 is the density of concrete and c_e is the specific heat. By using Hooke's law to determine the change of stress respect to temperature ($\partial\sigma/\partial T$), Equation (5) can be revised to:

$$\rho_0 c_e \frac{\partial T}{\partial t} = -ET\left(-\frac{\partial \varepsilon}{\partial T} - \frac{\sigma}{E^2}.\frac{\partial E}{\partial T}\right).\frac{\partial \varepsilon}{\partial t} = -\alpha ET\frac{\partial \varepsilon}{\partial t} \tag{6}$$

The thermal expansion coefficient of concrete subjected to simultaneous heat and load conditions is, therefore, a summation of its free linear expansion coefficient and stress-dependent thermal expansion coefficient, as shown in Equation (7):

$$\alpha = \alpha_0 - \frac{\sigma}{E^2}\frac{\partial E}{\partial T} \tag{7}$$

By substituting the newly-defined thermal expansion coefficient into the total strain model, the coupled behaviour of stress and expansion can then be revised and incorporated into a total strain model. A revised total strain model of concrete under simultaneous heat and load conditions was then proposed to take into account the coupled effects, as follows:

$$\varepsilon_{tot} = \varepsilon_\sigma + \alpha.\Delta T = \varepsilon_\sigma + \left(\alpha_0 - \frac{\sigma(T)}{E(T)^2}\frac{\partial E}{\partial T}\right)\Delta T \tag{8}$$

The physically-based model of the LITS is defined by substituting the Equation (8) into Equation (2), thus:

$$\varepsilon_{LITS} = -\frac{\sigma(T)}{E(T)^2}\frac{\partial E}{\partial T}\Delta T \tag{9}$$

where, $E(T)$ is the elastic Young's modulus of concrete at the target temperature, $\partial E/\partial T$ is the rate of change of elastic Young's modulus respect to temperature, and ΔT equals (T - T_{amb}).

Substituting the LITS expression of Equation (9) into Equation (3), we have:

$$\varepsilon_M(T) = \varepsilon_\sigma(T) + \varepsilon_{LITS}(T) = \varepsilon_\sigma(T) - \frac{\sigma(T)}{E(T)^2}\frac{\partial E}{\partial T}\Delta T \tag{10}$$

It should be noted that the expression for the stress-dependent strain ($\varepsilon_\sigma(T)$) does not distinguish between elastic and plastic strain, and can be determined from the unstressed steady-state test as described in Section 2.

3.2 The σ–ε_M–T relation of concrete

The generic behaviour used in this paper has long been used in EC2, making it the most widely used generic model for concrete stress–strain relationship. This generic behaviour was initially proposed by Schneider [17] and has been approved by a draft committee of RILEM experts. This generic behaviour of concrete material agrees well with the actual stress–strain curves of concrete at elevated temperatures [17, 18, 22-25]. Also, it has been demonstrated to capture quite satisfactorily experimental results during the heating phase and thus has been generally accepted by authorities and regulators [13, 26]. The mechanical strain used in the generic behaviour of EC2 has implicitly included the coupled effects between the stress and expansion through the LITS correlation [8]. In this paper, the mechanical strain at a target temperature (ε_M), as in Equation (10), can be explicitly calculated because $\varepsilon_\sigma(T)$, $E(T)$ and $\partial E/\partial T$ are already determined from the unstressed steady-state test.

While the relationship between stress and strain is well established in the elastic region, an empirical formulation to cover both elastic and plastic regions is still necessary. Most empirical formulations will converge to the simple linear relationship in the elastic region and provide a fit to the evolution of the stress–strain curve past this region. As can be seen in Figure 1, the generic function with the suggested strain values in ENV [19] has a good agreement with the actual empirical stress–strain curves for concrete [18]. The generic stress–strain relationship, as shown in Equation (4) for normal strength concrete (NSC) could be revised by replacing the value of strain $\varepsilon_M(T)$ and $\varepsilon_{M1,T}$ as follows:

$$\sigma(T) = \frac{\varepsilon_M(T)}{\varepsilon_{M1,T}} \times \frac{3 \times \sigma_{uT}}{2 + \left(\frac{\varepsilon_M(T)}{\varepsilon_{M1,T}}\right)^3} \tag{11}$$

where, $\sigma(T)$ is stress, $\varepsilon_M(T)$ is mechanical strain, σ_{uT} is the compressive strength, $\varepsilon_{M1,T}$ is strain corresponding to the compressive strength σ_{uT}. The proposed strain value corresponding to the compressive strength at a target temperature ($\varepsilon_{M1,T}$) can then be calculated as follows:

$$\varepsilon_{M1,T} = \varepsilon_{c1,T} - \frac{\sigma_{uT}}{E(T)^2}\frac{\partial E}{\partial T}.\Delta T \tag{12}$$

The stress–strain–temperature of concrete during the heating phase can be then re-written as Equation (13):

$$\sigma(T) = \frac{\varepsilon_M(T)}{\varepsilon_{c1,T} - \frac{\sigma_{uT}}{E(T)^2}\frac{\partial E}{\partial T}.\Delta T} \times \frac{3 \times \sigma_{uT}}{2 + \left(\frac{\varepsilon_M(T)}{\varepsilon_{c1,T} - \frac{\sigma_{uT}}{E(T)^2}\frac{\partial E}{\partial T}.\Delta T}\right)^3} \tag{13}$$

During the cooling phase, $\partial E/\partial T$ can be considered as zero because the elastic Young's modulus is approximately constant when the temperature decreases [27]. Therefore, the coupled effects between stress and expansion disappear in the stress–strain–temperature relationship of concrete during the cooling phase. The stress–strain–temperature relationship obtained from the unstressed steady-state test could be used to calculate the stress–strain evolution. It should be noted that the value of strain corresponding to the compressive strength used for cooling phase should be collected directly from the unstressed steady-state test or the recommended value of strain from ENV [19], as shown in Table 1. An example of a stress–strain relationship at 400°C is summarized in Table 2.

Figure 2 provides a plot of Equation (13), where the coupled effects of stress and temperature are taken into account. Given the explicit dependency of the stress–strain curve with temperature, it is essential to represent this relationship as a function of temperature. Thus the conventional two-dimensional plot needs a third dimension. This approach has been followed in the past by others to represent multiple dependencies of the stress–strain curve [15, 28].

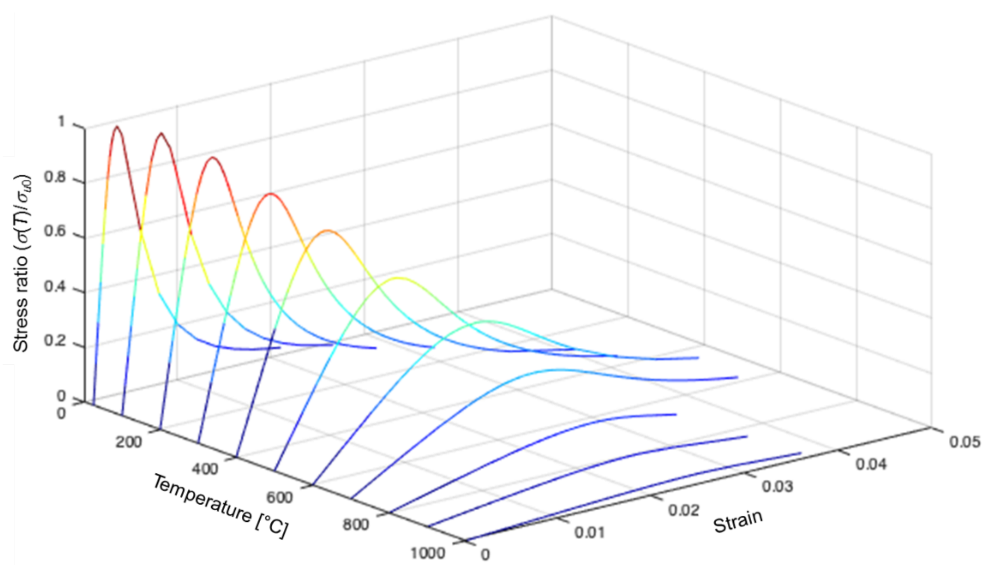

Figure 2. Proposed stress–strain–temperature relation of concrete taken into account coupled effects between stress and expansion.

Table 2. An example of the stress–strain relationship of concrete at 400°C using the proposed model.

Ratio σ(400°C)/σ_{u0}	ε_{LITS}	$\varepsilon_{c1,T}$	$\varepsilon_{M1,T}$
0	0	0	0
0.436	0.0011	0.0030	0.0041
0.558	0.0015	0.0040	0.0055
0.609	0.0016	0.0045	0.0061
0.689	0.0018	0.0055	0.0073
0.717	0.0019	0.0060	0.0079
0.736	0.0019	0.0065	0.0084
0.747	0.0020	0.0070	0.0090
0.750	0.0020	0.0075	0.0095
0.747	0.0020	0.0080	0.0100
0.738	0.0019	0.0085	0.0104
0.724	0.0019	0.0090	0.0109
0.707	0.0018	0.0095	0.0113
0.640	0.0017	0.0110	0.0127
0.616	0.0016	0.0115	0.0131
0.591	0.0015	0.0120	0.0135
0.541	0.0014	0.0130	0.0144
0.494	0.0013	0.0140	0.0153
0.450	0.0012	0.0150	0.0162

4. Discussion

As the proposed stress–strain–temperature relationship was developed from the generic behaviour used in EC2 with the recommended values of strain from ENV, this proposed model is directly compared to the stress–strain–temperature relationship reported in EC2. As can be seen from Table 2 and Figure 3, the strain corresponding to the compressive strength calculated by Equation (13) and the recommended values by EC2 is similar for the entire range of temperature from ambient to 1000°C. This good agreement strongly indicates that the proposed stress–strain–temperature relationship can be calculated, instead of using the fixed values in EC2 for different types of concrete.

Table 3. Comparison of the strain values corresponding to compressive strength.

Strain corresponding to compressive strength	**Temperature [°C]**										
	25	**100**	**200**	**300**	**400**	**500**	**600**	**700**	**800**	**900**	**1000**
EC2	0.0025	0.0040	0.0055	0.0070	0.0100	0.0150	0.0250	0.0250	0.0250	0.0250	0.0250
Proposed model	0.0025	0.0042	0.0055	0.0076	0.0095	0.0127	0.0173	0.0196	0.0220	0.0222	0.0250

The good agreement between the proposed stress–strain–temperature relationship and the stress–strain curves provided in EC2 for concrete shows that the proposed stress–strain–temperature relationship could be used as an alternative method to develop mechanical properties of concrete at elevated temperatures during the heating phase. Also, such predictive capability of the proposed σ–ε_M–T relationship of concrete in this paper offers a significant advantage over the table of stress–strain relation at high temperature in EC2 [5]. The main advantage is that this σ–ε_M–T relationship allows to extend the application of Equation (13) to different types concrete if the generic behaviour and the unstressed test data of the concrete are known. As long as the generic function of a stress–strain relationship is collected from the unstressed steady-state test, the corresponding mechanical strain (ε_M) used for a designed stress–strain–temperature relationship could be practically determined using Equation (10) for any given applied stress (σ) at the target temperature. In addition, the coupled effects between stress and expansion of concrete subjected to load and temperature could be explicitly captured by a physically-based model rather than by other best-fittings to the limited experimental data. The LITS model developed for this proposed stress–strain–temperature relationship could be used to explicitly consider the coupled effects between stress and expansion in modelling the behaviour of concrete structures during the heating phase. It should be noted that much better agreements between the finite element model and experimental data are usually obtained when LITS is explicitly considered in the model [29]. By developing the stress–strain–temperature as discussed in this paper, the strain corresponding to the compressive strength can be explicitly calculated with a clear justification of how the LITS is being incorporated. The predictive capacity of the LITS has also been demonstrated by total strain predictions (Equation (8)) that agree well with available test data for concrete [3, 9, 11].

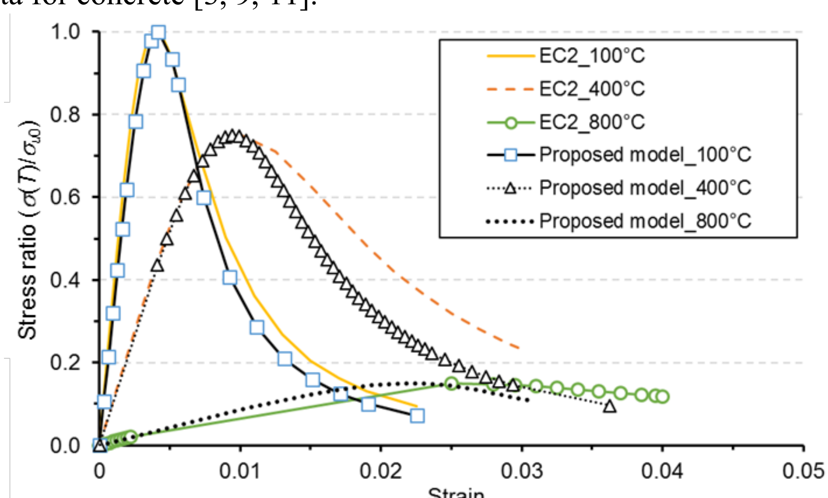

Figure 3. Comparison of the design stress–strain curves between the proposed model and EC2 [5].

An added strength of the proposed stress–strain–temperature relationship of concrete is that it can be used to evaluate the performance of concrete structures during the cooling phase. This simulation purpose cannot be achieved by using the EC2 stress–strain curves because they have implicitly included the LITS, which is irrecoverable during the cooling phase [30, 31]. As demonstrated in Equation (9), the newly-developed LITS model is a function of temperature (ΔT), stress ($\sigma(T)$), Young's modulus of concrete at target temperature ($E(T)$), the rate of change of Young's modulus respected temperature ($\partial E/\partial T$). The disappearance of the LITS during cooling phase could be explained by the fact that Young's modulus of concrete is only slightly reduced since the start of the cooling phase [27]. As a result, the $\partial E/\partial T$ is approximately zero, and thus the contribution of the LITS is essentially negligible during the cooling phase, as mentioned in Section 3.2. The stress–strain–temperature relationship used for concrete structures during the cooling phase could be achieved from the unstressed test, where the LITS does not present.

However, the use of the current physically-based model of the coupled effect between stress and expansion to develop the LITS has the drawback that, beyond the elastic region, it still depends on an empirical formulation. Thus, the strain could be different when using these equations for different concretes in the region where the concrete no longer behaves as an elastic material. This is an only slight disadvantage since at least the total strain model using the physically-based model of the LITS can capture the strain of concrete subjected to load and temperature for the entire range of temperature [3].

5. Summary

In this paper, the current total strain model and the table of stress–strain curves in EC2 are discussed. The stress–strain curves of concrete collected from the unstressed tests cannot be used directly for design purposes of concrete structures during the heating phase because the coupled effects between stress and expansion do not appear during this testing procedure. Currently, the total strain model used for design purpose in EC2 relies purely on a regression analysis. The strain values corresponding to compressive strength at target temperatures used in EC2 are presented without a transparent justification of how the coupled effects between stress and expansion are being incorporated. Thus, the stress–strain curves of concrete in EC2 can only be used for the heating phase; they are not applicable for the cooling phase. It is, therefore, necessary to incorporate into the Eurocode 2 formulations a physically-based approach considering the mentioned coupled effects. The stress–strain–temperature relationship of concrete could be then used for the design of concrete structures during both the heating and cooling phases.

By using a physically-based model for the coupled effects between stress and expansion in concrete subjected to simultaneous heat and load, an equation is proposed to explicitly calculate the strain value corresponding to compressive strength. The expression is shown to agree well with the stress–strain curves of concrete in EC2 used for the heating phase. In addition, this stress–strain–temperature relationship can also be used for design purposes during the cooling phase. Thus, this approach can now be extended to different types of concrete (including the modern high performance concretes) as long as the generic stress–strain behaviour of concrete, Young's modulus E, and $\partial E/\partial T$ during the unstressed steady-state test are known for such concrete.

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