

Visible Maths Pedagogy Project:

'Visible pedagogies and equitable outcomes in school mathematics'

Report from Year 1 (November 2017 to July 2018)

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1. About the project

1.1. Context:

The Visible Maths Pedagogy Project involved an academic researcher (AR) and two teacher researchers (TRs) working collaboratively to develop classroom practice. All three researchers share an interest in developing progressive teaching approaches and exploring issues of equity in mathematics classrooms. The project was based in Stoke Newington School, which is an Inner London school with 29% (above national average) of students designated as 'Pupil Premium' (a measure of socio-economic disadvantage) in 2016-17. The school had identified black Afro-Caribbean boys as an underachieving group.

The school's mathematics department had recently incorporated a series of problem-solving activities into its scheme of work and had taken a decision to move away from a rigid setting structure. The school had a focus on developing 'oracy', which in the mathematics department included encouraging students to express their reasoning through 'think, pair, share' strategies. The mathematics department introduced mixed attainment groupings in mathematics classes across Year 7 in 2017-18, with plans to extend this to Years 8 and 9 in subsequent years. The department were in the process of developing their scheme of work for Year 7 and met regularly to collaboratively plan lessons.

1.2. Aims:

The focus of the project was to develop strategies teachers can use to make their pedagogy more visible, i.e. to help all pupils recognise their intentions as teachers, when adopting progressive approaches to teaching secondary mathematics. In so doing, the project aimed to explore strategies that may be effective in reducing the large gaps in mathematics achievement existing between children from different socio-economic groups.

The initial research questions were:

- Which teaching strategies are successful in helping students develop their ability to decipher the recognition and realisation rules of the mathematics classroom?
- What impact do these strategies have on students' mathematical achievement and engagement, particularly for those from disadvantaged backgrounds?

This was a small-scale unfunded research project which served as a pilot study for a potentially larger-scale project. It provided an opportunity to try out and evaluate various research tools, with a view to incorporating these into the research design for a larger-scale future project. These included maintaining research journals, conducting peer observations, surveys and interviews of students and using video recordings of lessons to stimulate a critical reflection on practice (video-stimulated reflection).

1.3. Background:

Whilst other differences in achievement (e.g. boys and girls) have narrowed, socio-economic disadvantage remains the most decisive factor in determining success in school mathematics (Jorgensen 2016). This is despite recent government policies aimed at highlighting and reporting differences in attainment and, since the introduction of the 'pupil premium' in 2011, targeting resources at pupils eligible for Free School Meals. 'Progressive' teaching approaches, characterised by open-ended activities, collaboration between learners and an emphasis on developing problem-solving and reasoning skills, can lead to more equitable outcomes and greater levels of engagement amongst pupils (Boaler, 2008; Wright, 2016; 2017).

However, there is also a danger that their relatively unstructured nature can further disadvantage children from less wealthy backgrounds who are more likely to misinterpret the intentions of the teacher or to miss the point of the lesson. In a study in the United States, Lubienski (2004) describes how less-wealthy pupils

were further disadvantaged by progressive teaching approaches as the lack of structure meant they were less likely to recognise the intentions of the teacher, e.g. by failing to notice hints and clues and by missing the point of class discussions. Bernstein (2000) argues that children from disadvantaged backgrounds, because of their upbringing, are generally less able to decipher the 'rules of the game', i.e. the 'recognition rules' (identifying relevant meaning from tasks) and 'realisation rules' (formulating appropriate and legitimate responses).

1.4. Methodological approach:

We adopted a Participatory Action Research (PAR) methodology, which involves academics and teachers working collaboratively in order to reflect critically on existing practice and to explore the potential of alternative practices for transforming classroom practice. PAR aims to generate findings that are more relevant to practitioners, bring about positive social change and address issues of equity and social justice. Initial meetings involved critical reflection on existing practice in relation to the research literature with TRs reading and presenting selected papers for discussion. We subsequently discussed ways of making aspects of pedagogy more visible to students and devising strategies that enhance students' abilities to recognise the intentions of the teacher and which enable them to respond appropriately in order to realise success in mathematics. We made use of existing teaching materials that have been designed for use alongside progressive approaches to teaching mathematics. We chose to work with Year 7 students as the TRs taught parallel classes in Year 7.

We carried out a series of 'plan-teach-evaluate' action research cycles in order to try out strategies devised and agreed during meetings. We evaluated the impact of these strategies on the students through carrying out a short survey of students' responses and conducting short interviews with a small number of students (consent was gained from students and parents/carers for participation in these interviews). TRs were involved in designing these surveys and interviews and we discussed questions that should be asked and protocols that should be adopted. The focus of the surveys and interviews was on assessing students' success and engagement with mathematics, their dispositions towards learning, and their awareness of the pedagogical rationale employed by the teacher. PW visited the school in order to conduct meetings with the TRs.

There was an initial stage in which TRs engaged with relevant research literature selected from the field of critical mathematics education and used this to reflect critically on their own classroom practice. This was followed by two plan-teach-evaluate action research cycles in which we devised strategies, tried them out with students during a research lesson, evaluated their success, and further refined the strategies. The following research tools were tried out during the first year of the project:

- Research journals – used by TRs to record their reflections on discussions during meetings, to write a commentary on the development of their thinking and classroom practice over the year and to prompt discussion during research group meetings.
- Surveys – designed collaboratively and administered by TRs with all students in their Year 7 classes after the first research lesson.
- Interviews – semi-structured interviews conducted by TRs with three target students in each class (chosen from those identified as 'Pupil Premium').
- Peer observations – each research lesson was observed by one of the other two researchers who facilitated the video-recordings, drew up a time-line of key events and made brief notes.
- Video-stimulated reflection – each research lesson was video-recorded and extracts of this recording were used to stimulate discussion and critical reflection during subsequent research group meetings.

1.5. Research group meetings held during 2017-18

There were two initial meetings held on 1st November 2017 and 29th November 2017. The first of these was a planning meeting at which an overall structure for the project was considered and agreed. During the second of these meetings, the TRs presented two research papers (Boaler, 2018 and Lubienski, 2004 – previously identified by Pete) for discussion of how the findings were relevant to their classroom practice. A third paper (Wright, 2017) was presented by a student teacher who had been working within the school at the time.

The following research group meetings (each approximately one hour in duration) were held during the first year of the project:

1.5.1. Meeting 1 (16th January 2018)

The researchers discussed and finalised the plans for the research lessons to be taught by the TRs to their Year 7 classes on 31st January 2018 and observed by Pete. These were based on the Fraction Flags activity (already identified beforehand by the TRs). Two strategies were devised, i.e. ‘advocating’ and ‘separating’, to be used alongside the activity and a ‘think-pair-share’ teaching approach, and the purpose and rationale behind these strategies was discussed. We then considered how to make use of a survey to evaluate the success of the strategies. The final wording for the surveys and a protocol for its implementation were agreed.

1.5.2. Meeting 2 (7th February 2018)

Pete met with Alba to reflect on the first research lesson and evaluate the effectiveness of the first two strategies. We used the survey responses and watched video-clips of the research lesson (selected by Alba) to stimulate a critically reflective discussion.

1.5.3. Meeting 3 (7th February 2018)

Pete met with Tiago to reflect on the first research lesson and evaluate the effectiveness of the first two strategies. We used the survey responses and watched video-clips of the research lesson (selected by Tiago) to stimulate a critically reflective discussion.

1.5.4. Meeting 4 (20th March 2018)

Researchers met to reflect overall on the completion of cycle 1, in particular an evaluation of the two strategies and the extent to which the research tools (i.e. research journals, student surveys, video-stimulated reflection) helped in this evaluation. We also discussed the impact the project had up until that point on TRs’ classroom practice. Finally, we agreed a structure for cycle 2.

1.5.5. Meeting 5 (9th May 2018)

The researchers discussed and finalised the plans for the research lessons to be taught by the TRs to their Year 7 classes on 16th May 2018 and observed by each other. These were based on the Ratio Flags activity (already identified beforehand by the TRs). Three further strategies were devised, i.e. ‘scribing’, ‘annotating’ and ‘classifying’, to be used alongside the activity and various progressive pedagogies, and the purpose and rationale behind these strategies was discussed. We then considered how to make use of a series of student interviews (to be held in the days immediately following the research lesson) to evaluate the success of the strategies. The final wording for the interviews and protocols for their conduct were agreed.

1.5.6. Meeting 6 (13th June 2018)

All three researchers met to reflect on the second research lesson and evaluate the effectiveness of the three strategies tried out. We used the interview responses (transcribed by Pete) and watched video-clips of each research lesson in turn (selected by the TRs) to stimulate a critically reflective discussion.

1.5.7. Meeting 7 (2nd July 2018)

Researchers met to reflect overall on the completion of cycle 2, in particular an evaluation of the three strategies and the extent to which the research tools (i.e. research journals, interviews, video-stimulated reflection) helped in this evaluation. We also discussed the extent to which the outcomes of the project addressed the research questions and the impact the project had over the course of the first year on TRs' classroom practice. Finally, we considered how the project might develop into its second year.

1.6. The strategies

There were five strategies tried out during the first year of the project with the aim of making progressive pedagogies more visible to learners:

1.6.1. The 'advocating' strategy

During the use of 'think-pair-share', students were asked to present each other's ideas (rather than their own ideas). They were warned beforehand that they would need to listen in order to explain their partner's thinking. There was a follow-up activity in which TRs asked students why they thought this strategy was used. The rationale behind the strategy was to encourage students to engage with other people's ideas, communicate their own ideas clearly to others, to appreciate that there were multiple ways of solving a problem, and to prevent domination of the discussion by a limited number of students.

1.6.2. The 'separating' strategy

The teacher drew a line down the middle of the board, separating it into two columns (without revealing at this point the reason for doing so). Whilst students were presenting their solutions to a problem to the whole class, the teacher recorded (using notes to summarise if necessary) at least two 'reasoning' style responses on the left (without saying why) and at least two 'working-out' style responses on the right (again without saying why). This was followed by a discussion where the teacher asked students what distinguished the responses in the two columns, leading on to a discussion of the difference between 'reasoning' and 'working-out', and finally why students thought the strategy was used. The rationale behind the strategy was to help students appreciate the difference between reasoning and working-out and why it was important to be able to do so (e.g. to avoid providing working-out in response to a prompt to 'explain your reasoning').

1.6.3. The 'scribing' strategy

Whilst students were presenting their solutions to a problem to the whole class, the teacher wrote down on the interactive whiteboard (so it could be saved) exactly what individual students said regardless of whether they were correct or incorrect. This was followed up with a discussion in which the teacher highlighted any ambiguities or errors in what was written down, e.g. by deliberately misinterpreting it, and finally a discussion around why the strategy was used. The rationale behind the strategy was to challenge the assumption that everything the teacher writes down is correct, provide an opportunity for students to determine for themselves whether something is correct and recognise errors and misconceptions for themselves. The strategy was also designed to encourage students to develop their ability to verbalise accurately and to help the teacher to assess students' understanding.

1.6.4. The 'annotating' strategy

This was a follow-up to the 'scribing' strategy in which the teacher encouraged students to annotate a solution on the board (and then subsequently solutions in students' own books), using a different colour, to highlight how the solution might be improved. There was a particular focus on correcting errors, highlighting misconceptions and using more precise mathematical language where appropriate. This was followed by a discussion about why students thought the strategy was used. The rationale behind the strategy was to facilitate a discussion around common misconceptions and to help students appreciate the value of using more precise mathematical language alongside (more informal) everyday language in order to help them communicate their mathematical thinking accurately.

1.6.5. The 'classifying' strategy

Students were asked to solve a series of problems (in this case involving ratio) and then to sort them into three types of problems with similar characteristics. They were then asked to identify which methods they might use to solve other problems which were similar to the three types already encountered. Finally, students were asked why it is important to be able to identify/classify different types of problems before attempting to solve them. The rationale behind the strategy was to help students to recognise the type of (ratio) problem and hence decide which procedures need to be applied to solve unfamiliar problems in future, and to appreciate the importance/value of knowing which mathematical procedure to apply in solving unfamiliar problems (which is often the biggest challenge they face).

1.7. Survey and interview schedules

The following questions were agreed by the researchers for use in the surveys (administered at the end of the first research lesson during cycle 1) and in the interviews (conducted in the days immediately following the second research lesson during cycle 2).

1.7.1. Survey questions

Students were advised to read the introductory paragraphs of the survey before completing it. These outlined the aims of the research project, how the survey responses would be anonymised and used in the research, and that students could leave the survey blank if they did not wish to take part. Note the first question was purely for the purpose of assigning pseudonyms if this was felt necessary.

- *'Are you male or female (tick box)'*?
- *'How well do you think you've done in today's maths lesson?'* (students answer on scale of 1 to 5).
- *'How do you know?'* (provide space to explain).
- *'What do you think was the purpose of separating 'reasoning' and 'working-out' in the table?'*
- *'Why do you think the teacher asked you to explain your partner's thinking and not your own?'*

1.7.2. Interview questions

Before beginning the interviews, TRs first checked with students that they have had a chance to read the information leaflet. They were then given the opportunity to ask any questions they liked before starting. The following questions were designed as initial prompts to facilitate discussion between TRs and interviewees. In accordance with 'empathetic' interviewing, TRs were encouraged to ask follow-up questions to encourage students to expand on their ideas, e.g. 'That's interesting, tell me more about ...'

Initial questions asked (note that the third, fourth and fifth questions relate directly to the rationale for using the scribing, annotating and classifying strategies):

- *Did you enjoy today's (Wednesday's) lesson? Why?*

- *Did you notice anything different about today's (Wednesday's) lesson?*
- *Why do you think I wrote down exactly what students said even if it wasn't correct?*
- *How was it useful to you to annotate the answers?*
- *Why did I ask you to sort the problems into three types?*
- *How well did you do in today's lesson? How do you know?*

1.8. Data collection and analysis

The data collected included the written responses from the student surveys, audio-recordings of interviews conducted by TRs with students, and audio-recordings of the discussions between the three researchers during research group meetings. The audio-recordings were transcribed and fully anonymised, with pseudonyms being used for students and any third parties throughout the data analysis.

A thematic analysis of the data was carried out using a combination of deductive coding (derived from the theoretical background) and inductive coding (derived from a reading of the data). Different coding themes were developed and used for analysing the surveys, interviews and research group meetings. NVivo software was used to code the data and to facilitate the thematic analyses. A summary of findings, along with more detailed findings (and supporting evidence) from each thematic analysis, are included in subsequent sections of this report.

1.9. References to theory underlying research:

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique* (Revised ed.). Lanham, Maryland: Rowman & Littlefield Publishers.

Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal*, 34(2), 167-194.

Brydon-Miller, M., & Maguire, P. (2009). Participatory action research: contributions to the development of practitioner inquiry in education. *Educational Action Research*, 16(1), 79-93.

Jorgensen, R. (2016). The elephant in the room: Equity, social class, and mathematics. In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education* (pp.127-146). Charlotte, NC: Information Age

Lubienski, S. (2004). Decoding mathematics instruction: A critical examination of an invisible pedagogy. In J. Muller, B. Davies, & A. Morais, *Reading Bernstein, researching Bernstein* (p.91-122). London: Routledge.

Skovsmose, O., & Borba, M. (2004). *Research methodology and critical mathematics education*. In P. Valero, & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education* (pp. 207-226). Dordrecht: Kluwer Academic Publishers.

Wright, P. (2016). Social justice in the mathematics classroom. *London Review of Education*, 14(2), 104-118.

Wright, P. (2017). Critical relationships between teachers and learners of school mathematics. *Pedagogy, Culture and Society*, 25(4), 315-330

2. Summary of findings from thematic analyses

This section includes a summary of the themes emerging from each of the thematic analyses of:

- the student surveys administered at the end of the first research lesson (see Section 3);
- the interviews of target students conducted after the second research lesson (see Section 4);
- the seven research group meetings held during the first year of the project (see Section 5).

Note that more detailed findings (and supporting evidence) from each thematic analysis can be found in Sections 3, 4 and 5 of this report.

2.1. Thematic analysis of student surveys:

- Students generally felt positive about their level of success with most attributing this to completing lots of questions and getting them correct.
- Most students misinterpreted the teacher's intentions for using the 'separating' strategy giving reasons that did not reflect the primary reasons given by the TRs for using the strategy.
- Some students appeared to recognise the teacher's primary reasons for using the 'advocating' strategy, although the majority of students misinterpreted these as to enforce compliance, i.e. by ensuring students listened to others.

2.2. Thematic analysis of interviews:

- Students were generally beginning to develop a greater awareness of the teachers' intentions and were able to articulate some of their primary reasons for adopting the three strategies used in cycle 2.
- Students also commonly referred to other (potentially or partially valid) reasons not considered primary.
- 4 (out of 6) students articulated reasons closely reflecting the primary purpose of the 'scribing' strategy, i.e. to identify and address errors and misconceptions.
- 3 (out of 6) students articulated reasons closely reflecting the primary purpose of the 'annotating' strategy, i.e. to draw attention to errors and misconceptions in order to improve solutions.
- 3 (out of 6) students articulated reasons closely reflecting the primary purpose of the 'classifying' strategy, i.e. to recognise the correct procedures to apply in order to solve problems.
- A recurring theme across the interviews was a preoccupation amongst students with testing, with all 6 students at some point associating the strategies with revising for tests.
- All 6 students claimed to enjoy the research lessons, although they gave contrasting reasons for doing so – some attributed their enjoyment to familiarity with the work or finding it easy whilst others attributed this to progressive learning approaches such as problem-solving or working collaboratively.

2.3. Thematic analysis of research group meetings

The thematic analysis of the research group meetings was divided up according to five coding themes as follows.

2.3.1. Project research tools:

- Teacher researchers significantly developed their use of a range of research tools, e.g. surveys, interviews, video-stimulated reflection, and these were seen as complementary. Effective use of a research 'tool-kit', where the combination was seen as more powerful than merely a collection of research tools, to evaluate success of project strategies. Surveys and interviews, as well as videos, proved invaluable for stimulating critically reflective discussions.

- TRs took a lead in designing and administering surveys and interviews and this was seen as enhancing the trustworthiness of the findings, e.g. students more comfortable being interviewed by their own teachers, making effective use of 'empathetic' interviewing.
- Through explicit focus on designing and conducting surveys and interviews, and video-stimulated reflection, TRs began to incorporate the collection of evidence to evaluate teaching and learning into their routine practice in the classroom, drawing on research tools, albeit in less formal ways

2.3.2. Strategies tried out:

- The strategies focused on making clear the benefits of a range of progressive teaching and learning approaches including: working collaboratively, listening/explaining/justifying to other students, developing mathematical reasoning, identifying/learning from errors, identifying/addressing misconceptions, drawing out ambiguities, precise use of mathematical language, classifying/sorting problems. It is noticeable how this list reflects Swan's (2006) 'collaborative orientation' towards teaching and learning mathematics.
- Researchers were able to clearly articulate their rationale for using various progressive pedagogies and this was clearly reflected in the strategies.
- All strategies included (as a core element) a discussion with students of the teacher's pedagogical rationale, typically posing questions such as 'Why do you think that I ...?' in order to prompt discussion.

2.3.3. Critical research processes:

- The external stimulus provided by PW prompted critically reflective discussions of TRs' existing practice – this stimulus included posing probing questions and consistently relating ideas, suggestions and findings back to the theory and aims of the research project.
- The TR's detailed knowledge of their students and the classroom context meant that constraints on developing practice were identified and addressed and that a pragmatic approach was adopted to conducting the research.
- An alternative vision of practice was clearly articulated and was used to focus the researchers' planning and evaluation of strategies to try out.
- The researchers' critical reflection on practice was supported by using a range of triangulating evidence from videos, surveys, interviews and research journals.

2.3.4. Participatory Action Research characteristics:

- Collaborative relationships between researchers were apparent in the status deliberately afforded to TRs, e.g. in maintaining control over the videoing of lessons.
- The research project was participatory in that TRs played a leading role and took their own initiative in the design and implementation of the research tools and the evaluation of the strategies.
- Trialling and evaluating the strategies had a significant impact on TRs' classroom practice with many of the ideas taken up and used routinely with a range of classes.
- TRs developed a greater critical understanding of their own practice, becoming more aware of the rationale behind progressive pedagogies and beginning to question/critique previous assumptions, e.g. about what success in school mathematics entails.
- Through evaluating the effectiveness of the strategies, TRs developed a clear vision of how they might embed some of the ideas in their own practice (and that of others in the department) in order to make their pedagogy more visible to learners.

2.3.5. Impact on students:

- The following student outcomes of the project were seen as inter-connected and inseparable – each reinforced and was reinforced by the others: greater appreciation of the teacher's pedagogical rationale ⇔ greater appreciation of what it means to be successful ⇔ responding in more appropriate

ways to progressive pedagogies \Leftrightarrow greater levels of engagement of students with strategies \Leftrightarrow greater levels of engagement of students with progressive pedagogies.

- TRs and students initially had highly contrasting views of what success in school mathematics entails – both moved towards more of a shared understanding of what success in school mathematics looks like.

3. Thematic analysis of student surveys (cycle 1)

The surveys were given to students in both classes to complete at the end of the Fraction Flags lesson and 45 students completed them.

3.0. Coding scheme and categorised responses for student surveys

*Note the 'Number of responses' refers to the number to which each code was applied.

Code/category	Description	Number of responses*
Question 1: How well do you think you've done in today's maths lesson?		
1	Not well at all	1
2	Not well	5
3	OK	15
4	Quite well	18
5	Very well	6
Question 2: How do you know?		
The following categories are grouped as 'Students' dispositions towards learning':		
Good behaviour	Student attributes success to their own good behaviour.	2
High Effort	Student attributes success to their own high levels of effort.	2
Participation	Student attributes success to their own high levels of participation.	7
Working with others	Student attributes success to working well with other students.	5
Lack of participation	Student attributes lack of success to their own low levels of participation.	2
Poor behaviour	Student attributes lack of success to their own poor behaviour.	6
The following categories are grouped as 'Students' judgements about their work output':		
Answered questions	Student attributes success to answering teachers' questions correctly.	4
Correct answers	Student attributes success to getting correct answers to questions completed independently.	9
Work completed	Student attributes success to completing a large amount of work.	12
The following categories are grouped as 'Students' perceptions about how others saw them':		

Teacher approval	Student attributes success to the approval they receive from the teacher.	5
Lack of ability	Student attributes lack of success to their own lack of ability.	1
The following categories are grouped as 'Students' judgements about their level of understanding':		
Found work easy	Student attributes success to finding the work easy.	7
Found work difficult	Student attributes lack of success to finding the work difficult.	4
Needed help	Student attributes lack of success to needing to ask others for help.	1
Question 3: What do you think was the purpose of separating 'reasoning' and 'working-out' in the table?		
Articulates difference	Student refers to the difference between the two without referring to teachers' rationale for using the table.	6
Improves layout	Student refers to table helping students to improve the layout of their work.	7
Understand difference	Student refers to table helping students to see the difference between the two.	3
Understand method	Student refers to table helping students to understand the method.	9
Not understood	Student is not able to articulate a valid reason for separating the two or does not understand the question.	20
Question 4: Why do you think the teacher asked you to explain your partner's thinking and not your own?		
Understand other's ideas	Student refers to strategy helping them to understand each other's ideas and methods.	7
Check partner's understanding	Student refers to strategy as a means of checking their partner understood the work.	2
Engage with other's ideas	Student refers to strategy as encouraging them to engage with each other's ideas and methods.	1
Express your ideas	Student refers to strategy as encouraging them to express their ideas better.	2
Check listening to partner	Student refers to strategy as a means of checking they are listening to their partner.	24
Share ideas with others	Student refers to strategy as encouraging them to share their ideas and methods with each other.	6

Test your recollection	Student refers to strategy as a means of checking their recollection of the method.	1
Not understood	Student is not able to articulate a valid reason for explaining partner's thinking or does not understand the question.	7

3.1. Survey question 4 – ‘separating’ strategy

Students were asked to respond to the question “What do you think was the purpose of separating ‘reasoning’ and ‘working-out’ in the table?” (see strategy above)

Only 3 students (out of 45) gave a response that closely reflected the teacher’s rationale for separating these two in the table (despite this being the focus of a discussion during the lesson):

I think it was separated to show how to do it and why they did it.

So that we can understand how they’re different.

So that we understand the difference between the two.

Another 6 students, however, responded by describing the difference between the two, suggesting a partial awareness of the teachers’ rationale. These responses included:

Reasoning is to tell you what you need to know and working out is how you know.

The purpose of separating reasoning and working-out is that working out shows how to do it but reasoning explains how to do it.

Another 16 students gave responses that suggested a reason for separating them that did not reflect the rationale of the teacher, e.g. helping students to better understand the method or improve the layout of their work. There responses included:

So we can understand better how to use the methods.

Because the correct reasoning leads to the correct working-out and the correct working-out leads to the correct answer.

So the teacher can see your working out and how you got it.

The remaining 20 students either did not provide an answer or gave a response that suggests they did not understand the question, for example:

Because they are two different ways of working out.

Finding out different areas inside the shape.

I don’t know what this means.

3.2. Survey question 5 – ‘advocating’ strategy

Students were asked to respond to the question “Why do you think the teacher asked you to explain your partner’s thinking and not your own?” (referring to the Think-Pair-Share strategy employed). A similar question had already been posed and discussed during the lesson.

A significant number of responses (14 out of 45) suggested that the reasons given by students closely reflected the teachers' rationale for employing the strategy, i.e. to encourage students to share and engage with each other's ideas and methods. Examples include:

So that we understand other's point [of] view.

Because it helps you to understand different opinions on the maths problems and different paths to the answer.

Because you can get two different perspective[s] and it may help you finalise your idea.

However, approximately half of survey responses (21 out of 45) suggested that many students misinterpreted the teachers' intentions by viewing the strategy primarily as a means of enforcing listening. These include:

Because it shows you have good listening skills.

To see if you're listening to your friend.

Make sure you're listening and learning.

There were 3 (out of 45) responses that suggested some understanding of the teachers' rationale alongside enforcing listening (they were coded as both of the above):

So you can get more ideas and so you can improve your listening skill.

To show if we listen to our partner and understand them.

So we can listen better and know each other's ways of working stuff out.

The remaining 7 students either left the question blank or responded in such a way to suggest they hadn't understood the question.

3.3. Survey questions 2 and 3 (success in mathematics):

Students were asked to respond to the question: "How well do you think you've done in today's maths lesson? On a scale of 1 (Not well at all) to 5 (Very well indeed)." This was followed up by the question: "How do you know?"

More than half the students responded that they thought they did 'very well' (6) or 'quite well' (18). A further 15 said they thought they did 'OK'. Only 5 students said 'not well' and just 1 student said 'not well at all'. Students justified the level of their success with reference to various attributes (some referring to more than one in their response):

Most strikingly, 21 students (almost half) attributed the level of success to their output, e.g. the large amount of work completed (12), getting answers correct (9), answering the teacher's questions (4). Examples include:

Today I think I did well because I nearly finished the worksheet.

I completed most of the questions in a reasonable amount of time.

Got far on sheet, contributed in class.

Because all the questions I have done were correct and I can explain why in the questions this is right or wrong.

18 students referred to their dispositions towards learning, e.g. participation (9), behaviour (8), working well with others (5), effort (2). Examples include:

Because I didn't participate much.

Because my behaviour was appalling and I got sent out.

I know because I messed around then I changed and tried my hardest.

I know this because I was concentrating and I was working well with my partner.

12 students referred to their perceived level of understanding, e.g. they found the work easy (7), they found the work difficult (4), they needed help from others (1). Examples include:

Because I understood everything I learnt in today's lesson.

I know because of it was incredibly difficult and I got there in the end.

I had a lot of help from my partner and other people.

6 students referred to their self-image, e.g. affirmation from the teacher (5) perception of their own ability (1). Examples include:

Because my teacher said I was doing very well today in every task.

Because my teacher said well done.

Because I know I'm not that smart ...

4. Thematic analysis of student interviews (cycle 2)

4.0. Coding scheme for student interviews

*Note the 'Number of references' refers to the number of extracts of text to which each code was applied. The number in brackets refers to the number of interviewees for whom each code was applied to at least one response.

Code	Description	Number of references (sources)*
Teaching approaches (progressive pedagogy)		
Rich questioning	Student describes teachers' use of questioning to draw out understanding.	1 (1)
Problem-solving	Student describes the use of open-ended problems during the lesson.	12 (5)
Connecting topics	Student makes connections between different mathematical topics.	2 (2)
Focus on methods	Student refers to focusing on methods for solving problems rather than correct answers.	8 (4)
Explanation	Student refers to explaining and justifying solutions.	6 (3)
Collaborative work	Student refers to working collaboratively with other students.	11 (3)
Misconceptions	Student describes the use of misconceptions and errors to develop understanding.	9 (5)
Teaching strategies (visible pedagogy)		
Scribing	Student refers to features of the 'scribing' strategy (teacher writes down exactly what students say even if incorrect).	22 (6)
Annotating	Student refers to features of the 'annotating' strategy (teacher or student annotates a solution already displayed).	41 (6)
Classifying	Student refers to features of the 'classifying' strategy (students sort problems into different types).	31 (6)
Distinctions	Student refers to features of the 'distinctions' strategy (teacher draws out distinctions between reasoning and working-out by using a two-column table).	6 (2)
Students' experiences of learning		
Satisfaction	Student expresses satisfaction with their learning during the lesson.	10 (5)

Challenge	Student describes experiencing challenge during the lesson.	7 (3)
Lack of challenge	Student describes experiencing a lack of challenge during the lesson.	1 (1)
Frustration	Student expresses frustration with their learning during the lesson.	1 (1)
Fear/anxiety	Student describes experiencing fear or anxiety during the lesson.	2 (2)
Familiarity	Student refers to being familiar with the content of the lesson.	7 (3)
Confusion	Student describes experiencing confusion during the lesson.	4 (2)
Dispositions towards learning mathematics		
Enjoyment	Student articulates reasons for enjoying the lesson.	29 (6)
Lack of enjoyment	Student articulates reasons for not enjoying the lesson.	4 (1)
Perseverance	Students refers to their own perseverance during the lesson.	1 (1)
Motivation	Students refers to their own motivation during the lesson.	1 (1)
Empathy with others	Student exhibits empathy towards other learners.	11 (4)
Shared responsibility	Student describes sharing responsibility for others' learning.	8 (3)
Recognition rules		
Notices strategy	Student recalls the main strategy used by the teacher during the lesson.	9 (6)
Fails to notice strategy	Student does not recall the main strategy used by the teacher during the lesson.	7 (3)
Primary purpose	Student articulates a primary purpose (as identified by the teacher) for using the teaching approach in question.	21 (6)
Valid purpose	Student articulates a valid purpose (not identified as primary by the teacher) for using the teaching approach in question.	57 (6)
Invalid purpose	Student articulates an invalid purpose for using the teaching approach in question.	18 (4)
Realisation rules		
Successful	Student articulates reasons why they consider themselves successful during lesson.	23 (6)

Unsuccessful	Student articulates reasons why they consider themselves unsuccessful during lesson.	6 (2)
Appropriate behaviour	Student refers to behaviour which they consider appropriate during lesson.	5 (3)

4.1. General pedagogical awareness of students (matrix coding query)

Some differences between boys/girls, e.g. more instances of girls (38) describing valid purpose of strategies than boys (19), more instances of boys (13) describing invalid purpose of strategies than boys (5), and more instances of girls (16) recognising primary purpose of strategies than boys (5).

No significant differences between students with different heritage.

Some differences between Alba/Tiago's groups but may be explained by Alba having 2 girls/1 boy and Tiago 1 girl/2 boys.

However, be wary about counting instances – may simply reflect interviewers' willingness to persevere with helping students to articulate (more responses may indicate less able to articulate straight away). An example is Tiago's interaction with Marcus on the first strategy [need to check which one] (3 iterations in which he articulates a purpose succinctly) compared with Tiago's interaction with Mary (11 iterations where her responses tend to be less relevant and more repetitive). Number of instances might inform more detailed analysis but on its own would be reductive. Another example is Tiago's prolonged dialogue with Marcus on the second strategy when, despite being asked several times why annotating is useful, he is unable to provide a valid purpose.

However, the following data may be significant (as the above factors would balance out):

	Instances describing valid purpose of strategy	Instances describing invalid purpose of strategy
Boys	19	13
Girls	38	5

All students able to describe some valid purposes of strategies and one of the primary purposes of teaching strategies, although girls tended to do so in much more detail than boys, e.g. 5 instances of recognising a primary purpose for boys and 16 for girls.

Note that all six students referred to the lesson being videoed (5) or an extra person being in the room (4) when asked what they noticed that was different about the lesson – demonstrates that they do notice things.

4.2. Students' general dispositions towards learning (matrix coding query)

All 6 students articulated enjoyment of maths (boys = 14 instances, girls = 15 instances) and mostly considered themselves successful (boys = 12 instances, girls = 11 instances). There was only one girl of Turkish heritage (Mary) who expressed lack of enjoyment of maths or significant concerns over her own success.

4.3. Students' pedagogical awareness of different strategies (matrix coding query)

It would appear that students are more likely to articulate a valid purpose for those teaching strategies that are discussed after the event than for those discussed before the event (comparing the number of

instances where valid and invalid purposes are described) – note there were two teaching strategies that were discussed afterwards and only one beforehand (which is reflected in twice as many responses recognising the primary purpose).

However, this is contradicted by the number of students providing at least one response (numbers in brackets below) – there were more students who recognised at least primary purpose for the strategy discussed beforehand than for each strategy discussed afterwards. This is really dependent on the extent to which strategies are aggregated.

Strategy:	Describes VALID purpose	Describes INVALID purpose	Recognises a primary purpose
Discussion of strategies AFTERWARDS	41 (6)	10 (4)	12 (5)
Discussion of strategies BEFOREHAND	8(5)	6 (3)	7 (5)
Scribing students responses verbatim	17 (6)	3 (2)	6 (4)
Annotating solutions	24 (6)	7 (3)	6 (3)
Sorting and classifying problems	8 (5)	6 (3)	7 (5)

This section onwards rewritten to further draw out themes.

4.4. Students' awareness of the rationale behind specific teaching strategies

Alba and Tiago used three strategies for making their pedagogy more visible during the second action research cycle. All quotes below are in response to Alba/Tiago asking why students think they used the strategy. Note there are a number of occasions when students make reference to tests when explaining the rationale behind teaching strategies.

4.4.1. Scribing students' responses verbatim – 'scribing' (strategy 3)

The first of the three strategies in cycle 2 was to write down (on the interactive whiteboard so it can be saved) exactly what students said during whole class discussion (whether correct or incorrect). The aim was to draw out any ambiguity in what students say (e.g. deliberately misinterpret), highlight informal language and discuss mathematical alternatives. Alba/Tiago then planned to discuss with students afterwards why they wrote down exactly what students said and how this was aimed to help students to develop their ability to verbalise. They also planned to discuss why they recognised informal language but encouraged students to use precise mathematical language alongside.

The rationale/purpose for using the strategy (as discussed by Alba/Tiago/Pete in Meeting 5) was to:

- Challenge assumption that everything teacher writes is correct – students encouraged to consider for themselves if correct.
- Provides opportunity for students to recognise errors and misconceptions for themselves.
- Enables teacher to draw out ambiguity of language, highlight difference between informal/mathematical language, students appreciate value of using formal mathematical language alongside everyday language to verbalise their mathematical thinking.
- Enables teacher to assess students' understanding

Four of the six students appear to clearly recognise one of the primary purposes of the strategy, i.e. to identify and address errors and misconceptions, whilst the other **two** have only a partial recognition.

Marcus describes how the strategy can help bring errors/misconceptions out into the open so they can be addressed:

To maybe see like where we go ... because it's better if you say it out than keep it in because the teacher could help you and try and improve from wrong to right. (Marcus, #50)

Sophia describes how the strategy enables students to compare correct and incorrect answers and identify the differences between them (a primary purpose):

Cos then you can compare the correct and incorrect answers together and see, like, where you went wrong, and how you, you know, changed the answer to get the correct one. (Sophia, #34)

Sophia describes how this helped her to improve her own answers:

It did help me, um, a lot with the questions, and stuff, cos it showed me like what to do. And because when you were writing it down, it showed me what not to do, and then what to do, to get the right answer. (Sophia, #72)

Mary was the only student to refer to this strategy when asked what she noticed that was different about the lesson. She describes the strategy in detail:

And then sometimes, like yesterday, you did ... you just wrote down the questions, not the questions, but like the problems. ... You wrote down on the big board. And then you like asked us ... when like [another student] said something, you wrote down like everything they said and didn't change it. (Mary, #40)

Mary appeared to have a strong grasp of the rationale behind the approach, i.e. identifying mistakes and considering how to correct them, and contrasted this with normal teaching approaches:

... it was nice to like write down it, and then look at our mistakes. (Mary, #42) ... Because of, then we can, like, fix the mistakes. ... But usually people will like ... if we make a mistake, and then they just change it. They just tell you it's this, but they don't tell you why. And stuff. And then next time they do the same mistake. (Mary, #44)

Keira articulates how the strategy enables students to identify their own errors and share with others. She is the only one of the six students who articulates clearly the specific primary purpose of getting students to identify the errors themselves:

... it will be showing us that you think we've gone wrong a little bit. And by writing everything we've said, that will help, not just like the person, it will show everyone like where it went wrong. Instead of like you telling us, and that, we can learn from our mistakes. (Keira, #54)

Mary refers also to how this strategy could enable students to look back at their mistakes later on – not a primary purpose articulated by the teachers but a valid purpose nevertheless.

Mary refers to the fear/anxiety of being humiliated for making a mistake – perhaps this was why this strategy stuck in her mind as it was seen as a way of addressing misconceptions without having to admit to your own errors:

... and if other people didn't know they could learn as well. (Mary, #50) ... someone might be afraid to ask the question, they'd be like 'Is this a stupid question? I'm not going to ask the question'. Sometimes that does happen. Like if other people know it and they're just like putting their hand up, but you don't know it, then you're like afraid to ask the question, sometimes. And you think that something might happen, like someone might be like 'Oh, you don't know that!' And then, you know, in class, sometimes that happens. Like 'are you dumb?' Anyway sometimes they say that. (Mary, #52)

Ennis shows only a partial appreciation of the primary purpose of the strategy (to learn from errors):

So you help where they've gone wrong. And they can like learn from their mistakes and like write down the actual answer. (Ennis, #32)

Neal does not appear to have a strong grasp of this purpose (or any of the other primary purposes) of this strategy, although he articulates other valid purposes related to the primary purposes identified by Alba/Tiago. For example, Neal goes on to suggest the strategy is a way of focusing on the method rather than the answer and comparing methods to see if somebody else's method was easier (indirectly related to the primary purpose):

But then it doesn't really matter if it's correct or not. It's just their working out ... and maybe their working out might be correct, but it's just that they maybe done something wrong at the end. And then got maybe a different answer. (Neal, #38)

There were several other potentially valid purposes articulated by the students which were not identified as primary by Alba/Tiago (and therefore were not planned to be discussed during the lesson). By 'valid' I mean they might be used by other teachers or in other situations as a sensible justification for using the strategy.

Ennis describes a purpose of the strategy as providing an exemplar answer that students can use to assess and correct their own work:

... so other people can know how to do it. And so everyone could like ... everyone can understand from it, and see what they've done wrong, if they've got it wrong, and see what they've done correct, if they've got it correct. (Ennis, #28)

Similarly, Sophia sees the strategy as providing a reference point to help the teacher to explain errors more clearly to students:

... cos if she was incorrect then you'd be able to explain to her, like, more easily, to her, where she went wrong, if you wrote everything that she said down (Sophia, #32)

Neal describes the purpose as showing/explaining the correct method (without referring to addressing misconceptions):

... maybe they never got it right, but then someone worked out the same way but still got it right. And then you will show the person who did it how ... the way that they should do it correctly (Neal, #36)

Neal also sees it as a way of giving students some agency by acknowledging their approach to answering the question (without enabling them to spot their own mistakes, i.e. the aspect of agency that Alba/Tiago aimed to develop):

And like to just give people like a chance and to see what they could do. (Neal, #36) ... It's basically like for the students' perspective of like what they would do. How they would find the answer. (Neal, #38)

Marcus refers to the valid purpose of enabling students to look back at their work later in order to revise for a test (although he does refer to understanding as well as remembering as a way of preparing for tests):

If you write everything down then the students could take it in, and then maybe write down as well. (Marcus, #52) ... It helps them when they're in a test and maybe, before the test, they revise. And they see that working out, understand that questions in, may be in the test, and they can remember it without having to think or skip to the next question. (Marcus, #54)

Other primary purposes discussed by Alba/Tiago/Pete were not mentioned by the students, i.e. challenging the assumption that the teacher only ever writes down correct/perfect solutions, drawing out ambiguity of language, enabling students to verbalise their thinking by making appropriate use of informal and mathematical language.

4.4.2. Annotating solutions – ‘annotating’ (strategy 4)

The second strategy in cycle 2 (which followed on from the first) was for the teacher to annotate solutions on the board and then ask students to annotate their own solutions (to the same problem) in their books. Note that the primary purpose of the strategy was less clearly defined/articulated during the research group meetings and it was not specified clear beforehand what form these annotations should take (other than that they should be in a different colour and focus on improving the solution). Alba/Tiago planned to ask students afterwards why they thought it was useful to annotate solutions and to discuss the rationale for doing this.

The rationale/purpose for using the strategy (as discussed by Alba/Tiago/Pete in Meeting 5) was to:

- Facilitate a discussion focusing on errors, misconceptions, language and how to improve the solution (relates to the first strategy).
- Annotating with different colours enables students to draw a distinction between reasoning and their calculations/working-out (relates to the first strategy from cycle 1).

None of the six students referred to the purpose being to facilitate discussion, although Neal comes closest by highlighting how it can facilitate collaborative working leading to deepening understanding:

... because rather than if you just copied off your partner you could ... They said what to do and they got it correct. But they didn't really know how to do it properly. So then you would really, you would like show them how to do it, like, so they really understand. So they can do it another time in like the future, probably. (Neal, #46)

Three out of the six students do however appear to recognise how annotating can help to highlight errors/misconceptions and enable students to improve their solutions.

Neal describes how the strategy can help students to review their own methods/solutions by using annotations to show how to improve them:

Well, it just kind of helped me like to know what to do next time (Neal, #42) ... Because if the person had like a long way, you could annotate it and make it like a shorter way or easier way to do it, for the person. (Neal, #44) ... probably they have a method, but then that method might be a bit longer and it might not work for all questions. But in that method, how you explain it and like show the person how to do it, that would help them. Like that would show them a new method to do, and then make it easier for the person. (Neal, #50)

Sophia describes how annotating can help highlight where errors were made and hence identify ways of improving solutions to problems:

Well if you've improved the answer, then the annotations could tell you how you did that. (Sophia, #54) ... It would tell you like how you went wrong, and where you went wrong, and then it would tell you how to get to the right answer. (Sophia, #56)

The annotating strategy appears to have had a significant impact on Keira's thinking and she referred to the strategy in response to several different interview questions including 'Did you enjoy yesterday's lesson?', 'Did you notice anything different about yesterday's lesson?' and 'How well did you do in yesterday's lesson?'.

Keira describes how annotating enables errors to be highlighted as they arise, which helps avoid confusion later:

So when ... if we're answering a question, and we make like a mistake, we'll see like where we went wrong. Instead of like 'Ah, we'll have to go back' and think 'Ah, where did I go wrong?'

(Keira, #30) ... Cos like ... sometimes when you go wrong, you just mess up and then you forget everything. (Keira, #32)

Keira describes in detail how she applied the annotating strategy to solving a problem:

And so what I was annotating was like the numbers. So I wasn't just annotating any random words like 'make' or anything, I was like getting the key words and the key numbers and then annotating. And so like, I was annotating like 'How much blue paint does she use?', cos that's showing me 'what is the problem asking me?' (Keira, #60)

Whilst not identified as a primary purpose by Alba/Tiago/Pete, **both Keira and Ennis** describe how annotating can help pick out key aspects of the question.

Ennis describes annotation as being useful to pick out important information in the question:

Because you need to like pick out the pieces, like the important information. Because you have to read the question, cos it makes it like easier to understand. Ennis, #44) ... It would be useful because you could just easily like understand, and understand what each like part means. So it's easier for you to do the question and like write down like the important words like see what they mean. (Ennis, #46)

Ennis describes how annotation can make the meaning clearer and lead to a deeper understanding:

I think it was useful, because it like told the students like how it was done and like what it meant, what each like number and stuff meant, and what you had to do. (Ennis, #36) ... So you could do it slowly and understand the question more. (Ennis, #42)

Keira describes how annotating provides useful tips on key parts of the question:

So if we're doing it ... to like annotate the important, like crucial words, and stuff like that. (Keira, #10) ... so if I don't get the question, I just have to look at like the most important bits, not like ... I just look at the important bits and then I know like what I have to do from there. (Keira, #12)

Another potentially valid purpose of the strategy identified by **three of the six** students was to provide a reference point for future revision.

Keira describes how annotating could help remind students of the key information needed to solve similar problems at a later date:

So say I was working on ratio today [the day after the lesson], and like if I forgot a little bit, I can go and flick through my book and then look ... cos I've like underlined the key words, I'll look at that, I'll look at the main bit, but because I know which I've underlined, I know that 'Oh that's what I have to do', and carry on with my work. (Keira, 78)

Mary describes a valid purpose for annotating, i.e. to make sense of a solution when you read it back at a later date:

Because ... when you read back all your work, like that lesson you know what you're doing. But if you, let's say it's been like a month, and you don't remember what you were doing that lesson, you kind of need to know your workings out so you can understand what you did that lesson. Because you might forget. (Mary, #54)

Mary appears to relate this benefit to the need to prepare for tests:

... it's one day before a test, and you want to go over your notes but you don't know what that means ... you've got to go up to your teacher that day, if you like, if you've got maths first

period or second period. So you can, if you write in your book, then you can, when you go home, you can revise better. (Mary, #56)

Sophia describes annotating simply as note-taking that can help with revision in the context of preparing for tests (although she does not make clear the distinction between annotating and 'taking notes'):

It means where you take notes, like if you have a piece of text or a question, you can take notes on the most important bits. (Sophia, #42) ... say if you're going to do revision, then it gives you like ... annotating your work, it helps you in the future to understand the question more. (Sophia, #44) ... It was useful for me cos I know that when, like I'm going to do future tests, it will be really easy for me to understand the question and like remind me of the lesson that day. (Sophia, #48)

Marcus does not seem able to articulate a valid reason for annotating (despite being asked several times to do so) and appears to misinterpret annotation as part of the written solution rather than a commentary on it:

Was it about simplifying it? (Marcus, #56) ... I can't ... I don't really know. (Marcus, #58)

After being asked again, Marcus suggests that annotating is a way for the teacher to check the student has taken care to write the working out as well as the answer (misinterpreting annotation as working-out) and sees the purpose of the strategy as partly about compliance:

Maybe because annotating your answer could help you, like, in your book ... and you you've just wrote the answer, maybe you didn't do like the working out And the teacher asks you to work it out and you don't know how to work it out. And then the teacher would think that you didn't do it. (Marcus, #62) To know that you know what you're doing in the lesson, not just messing around. (Marcus, #64)

4.4.3. Sorting/classifying ratio problems – 'classifying' (strategy 5)

The third strategy in cycle 2, which formed the basis of the lesson's plenary, was to identify and classify three types of ratio problems that students had encountered during the lesson. This time Alba/Tiago planned to discuss with students the rationale for doing this, i.e. why it is important to be able to identify/classify different types of problems. Students were then asked to sort the problems into Type 1/Type 2/Type 3 and to discuss the differences with reference to examples of solutions displayed on the board.

The rationale/purpose for using the strategy (as discussed by Alba/Tiago/Pete in Meeting 5) was to:

- Enable students to recognise the type of (ratio) problem and hence decide which procedures need to be applied to solve unfamiliar problems.
- Enable students to appreciate the importance/value/challenge of knowing which mathematical procedure to apply in solving unfamiliar problems.

Three out of the six students appear to recognise one of the primary purposes of this strategy, i.e. to recognise which procedures to apply to solve problems.

Neal describes how recognising the type of problem can help students apply the correct method in solving problems in future:

So like we know which ... what method we're doing for it, it's either ... one of them were finding a ratio when the others are known. So if you do ... and then some of the questions in future come up like that, and then say that. And then you would really know how to do [them]. (Neal, #56)

When asked 'Why is it useful to you to know which type it is?', Neal claims that naming the type of problem can give you a clue how to solve it:

... 'find the ratio when the others are known'. So, in that question, if you didn't know before, if you never knew what that was, then now you know that all the ratios were known, but you need to find the one that wasn't. (Neal, #58)

Mary describes how classifying problems helps students identify the correct method to use straight away:

So we could know which ... which, like how to solve the questions. And like, instead of going through tons and tons of methods, it would just be like 'I know how to do this, we just do that method'. (Mary, #58)

Ennis describes how being able to recognise the type of problem enables you to solve the problem:

So we can know which one's which. So that like if ... for the first one, it's like, I think it's type 2, because it has numbers ... yeah numbers and words. And it tells you to find how much blue paint you use from the ratio. (Ennis, #48)

Ennis is able to classify other problems that Alba shows him into Type 1 and Type 2 and identifies key words in the question, e.g. 'simplify', that enable him to do so.

Ennis describes how this strategy is useful in a test situation (particularly for solving contextualised problems):

So you can like know which type is which, if you're on your test, and it says share the ratio into 1 to 3, you can understand that it's type 3, and you can know how to do the question. And if it's like a worded question you can know how to do that. (Ennis, #60)

Keira and Sophia both articulate a partially valid reason for the strategy, i.e. that it helps students appreciate that there are different types of problems, however they do not explain clearly how this is useful.

Keira suggests that classifying problems helps students gain an appreciation that there are different types of problems:

So we know what kind of like ratio questions they are, cos they're not just like one type of ratio question, they're different types. Because every question will be asking something different. (Keira, #84)

When asked how well she did in the lesson, Keira refers to successfully sorting the ratio problems, however she doesn't articulate clearly the purpose for doing so:

And because a couple of days ago we had that sheet, and it had like type 1, type 2, type 3, and that also helped me. Cos I was like looking what type 1 was, like simplifying. And I'd just look through, and I'd look through my answers and see that I've simplified, or I've found ... I've got one ratio and I have to find the other one. Yeah, I'd be like 'right type 2' or something like that. (Keira, #86)

Sophia appreciates that classifying helps appreciate that there is a difference between questions but she is unable to articulate clearly why this might be useful in future:

Cos they were like all different questions, it like shows you that we know the difference between each of the questions, and they're not all just like the same. (Sophia, #60) ... Um, well, because ... it's useful to understand cos ... er ... I don't know, it's just ... it will just help you in the future. (Sophia, #64)

Marcus articulates a less valid purpose of the strategy, i.e. to see whether students know how to sort problems in case such a task comes up in a test (which is unlikely):

I think it was to see if we knew what they were asking us and what it was about. And maybe ... it would maybe come up in a test and say 'Can you sort these word problems into different types?' (Marcus, #66)

Marcus describes how this strategy may help avoid making future errors (although he is not clear about how it can do so). There is an implication here that he is frustrated by making errors:

So, like when you're working it out you don't have to look back at it and say 'Ah, I've done the wrong thing'. But then if you know it already then you won't have to get frustrated about it. (Marcus, #72)

4.5 Enjoyment of learning maths

All six students describe how, in general, they enjoy the lesson. However, there are a variety of reasons given for why they enjoy learning maths.

Three of the six students attribute their enjoyment to finding the work easy or being familiar with the content.

Mary associates enjoyment of mathematics with solving easier problems and claims not to like the harder ones:

The simplifying ones I didn't like. But the other ones I did like, because when you're [inaudible] you think you understand it ... and it's easy. (Mary, #32)

Marcus associates his enjoyment of the lesson with familiarity with the content:

Yes, I enjoyed yesterday's lesson (Marcus, #16) ... because, even though we've done it before, it's still good to go over and check if you can still remember what you've already done. (Marcus, #16)

Keira says she enjoys learning particular topics in which she feels confident:

[I enjoyed the lesson] because, I like working like ... I really like ratio. (Keira, #10) ... Yeah. I like simplifying as well (Keira, #14) ... because I really like dividing and multiplication. Keira, #16)

Four of the six students give reasons for enjoying the lesson relating to their engagement with progressive pedagogies, particularly collaborative working and problem-solving. **Keira and Ennis** say they enjoyed learning new methods and techniques:

Keira also associates enjoyment of maths with learning new learning techniques

... and also like you gave us more like tips and techniques, like annotating, and that. (Keira, #10)

Ennis associates enjoyment of mathematics with learning new methods to solve a variety of problems:

[I enjoyed the lesson] because like, we have to like learn different methods of how to do it. Cos there were different types of questions, like unitary method, like sharing ratio, and like worded questions and stuff. (Ennis, #14) ... Because it's like a variety, you can like learn more ways of doing it. (Ennis, #18)

Neal associates enjoyment of the lesson with solving problems in a real-life context:

Well, I liked how you got to do with flags. And, well, your questions weren't really too tricky. Yeah. Well, yeah, just like how flags could help with maths as well. (Neal, #12) ... I just liked it because I had to work out, like ... imagine, well ... we knew one colour, and then we had to find out the other colour. (Neal, #16)

Neal seems to enjoy some of the more challenging problems:

I liked ... the questions were ... well, some of them were easy, but then some of them were tricky, I did like a lot of working out for it. (Neal, #64)

Sophia associates enjoyment of maths with working collaboratively with a partner:

Yeah, it was fun. Like I liked the partner work when you had to answer the starter. That was ... yeah, that was fun. (Sophie, #12)

Sophia associates enjoyment of maths with sharing ideas and providing mutual support:

... cos when I was talking to [another student], it helped me to explain my ideas, and then hear his ideas at the same time, and then we kind of just linked them together and then got the answer. (Sophia, #14) ... I just like explaining myself to someone, cos that's what I like to do instead of trying to figure it out by myself. (Sophia, #22)

Sophia also refers to the classifying strategy in explaining why she enjoyed the lesson (apparently because it involved group work):

I liked the ... the sharing, the group sharing, where you had to look at the questions, read them, and then put them into the groups. (Sophia, #16) ... it was just fun because we had to read it with the partners as well. And it was like just fun reading, re-reading over it again. (Sophia, #18)

4.6 Success in learning maths

All six students articulate how they experienced significant success during the lesson.

Four of the six students associate their success primarily with the number of questions completed correctly.

Sophia associates success with the number of questions completed and getting the answers correct:

I did pretty well, I got the starter correct. I got ... I didn't get to complete the ratio and flags one, but I got like to the second to last question. And then, with the sharing, I got them correct. (Sophia, #76) ... Cos, at the end of the classroom, you went through it, and you told us the answers ... and, yeah, I got them right. (Sophia, #82)

Similarly, Ennis associates success with completing questions and getting the answers correct. He appears to consider not understanding a question or being stuck as lack of success:

I think I did OK, but the last question was pretty hard and I really couldn't understand. (Ennis, #64) ... [I was successful] because I got most of the questions right, and I think it was pretty ... like I understood how to do it. And I wasn't really stuck, except for the last one ... I just understood how to do the questions. (Ennis, #78)

Marcus associates success by how far he got through the questions, getting correct answers and behaving appropriately during the lesson:

I think I did good. I got up to ... the Czech Republic flag [he later realised he meant Swedish flag]. (Marcus, #74) ... because I was concentrating. I got most of the ... I got all of the answers correct, from the ones that I did do. And I wasn't getting distracted, so yeah. (Marcus, #82)

Marcus describes not wanting to put his hand up out of fear of making a mistake highlighting how he attributes success to being correct:

I participated ... I didn't put my hand up a lot, which I usually do, so I was a bit scared to say something and think it was wrong. (Marcus, #46)

Mary associates being successful with understanding and completing questions, and appears to suggest that struggling with questions shows a lack of success:

The first half of the lesson was good. But then, because I didn't understand the, like ... I did understand it but like, I don't know, I didn't understand the flag questions, some of them. I had to spend a really long time on the questions, some of the questions. I did two questions, but those were the questions I understood. (Mary, #66)

When Mary is asked 'How did you know that you did well in the first part?', she replies:

Because I was happy about my answers (Mary, #70)

Mary also measures her success in relation to the achievement of other students:

It was a good lesson. The first half was good. The second half was not that good. And ... for some reason, I understand the things that [another student] doesn't understand, and he understands the things that I don't understand. (Mary, #74)

Neal and Keira appear to have a wider view of success in maths, associating their success more with engaging with effective learning strategies.

Although he mentions correct answers and using an appropriate methods, **Neal** appears to attribute his success more to working effectively with a partner:

Well, I did pretty good ... (Neal, #64) ... like because me and [another student] ... because like we did our own separate question. And then after we just worked it out together, after, to see if how we got the same answer, and then what method we did and see what's the easier method. (Neal, #66) ... so then you'd probably know that your question's correct but then you could ask other people as well, as in people next to you or behind you, to see if they done the same method as well, or a method easier than that one. (Neal, #68)

Keira associates her success with engaging with the teaching strategies and following the advice/guidance given by the teacher:

I think I did really good, because I was like annotating in my work. (Keira, #86) ... Well, because like when you was on ... when, like, I did, like ... when you were telling us to annotate, and like ... everything you did, like, I was doing as well. So like everything you were already doing, I knew like I was already doing that as well. (Keira, #96)

5. Thematic analysis of research group meetings (2017-18)

5.0. Coding theme used for thematic analysis of research group meetings

The following five coding themes were used for the thematic analysis of the research group meetings. Note 'RGM' is used throughout as an abbreviation for 'Research group meeting'.

5.0.1. Project research tools

- Research journals
- Student surveys
- Student interviews
- Peer observations
- Video-stimulated reflection

5.0.2. Strategies tried out

Cycle 1 strategies:

- Advocating
- Separating

Cycle 2 strategies:

- Scribing
- Annotating
- Classifying

5.0.3. Critical research processes

a) Pedagogical imagination

- Reflect on own practice
- Reflect on existing practice
- Relate theory to practice
- Consider alternative practice

b) Practical organisation

- Negotiate/design strategy to try out
- Consider constraints on practice
- Address constraints on practice

c) Explorative reasoning

- Plan how to use evidence
- Use evidence to evaluate strategies
- Relate evaluation to own practice
- Relate evaluation to existing practice
- Relate evaluation to alternative practice

5.0.4. PAR characteristics

a) Collaborative

- Consideration of relationships/roles

- Evaluation of relationships/roles

b) Participatory

- TRs involved in design of research
- TRs involved in evaluation of research
- TRs involved in evaluation of strategies
- TRs involved in analysis of data

c) Positive social change/Impact on TRs

- Impact on TRs' classroom practice
- Impact on TRs' thinking/beliefs
- Impact on TRs' agency/efficacy

5.0.5. Impact on students

- Appreciation of pedagogical rationale
- Engagement with project strategies
- Appreciation of how to achieve success
- Engagement with progressive pedagogies
- Appropriateness of responses to tasks
- Responses/engagement of all students
- Responses/engagement of target students

5.1. Project research tools

5.1.1. Research journals

RGM5 (#163-166) – Discussed using research journals to make notes during peer observations to inform reflection meetings.

RGM6 – Evaluated use of research journals. TC found them useful during peer observations to make descriptive notes on how three target students were responding (e.g. dialogue) and what the teacher was doing (#21-24). Colour-coded for language and pedagogies – AF found TC's notes useful when reviewing video of lesson (#25-30).

RGM7 – PW noticed AF and TC always had research journals at meetings – TC used mixture of research journal and electronic records for reflection (#57-60)

5.1.2. Student surveys

RGM1 – Collaborative design of surveys (used in cycle 1 but not in cycle 2), e.g. all suggesting wording for questions, TC proposing 5-point scale and AF raising questions about whether proposed questions are too leading (#94).

Negotiation over precise wording taking account of TC/AF's knowledge of students:

- *TC: Maybe, instead of saying 'successful' we should just say 'how well' or ... not 'how well' but ...*
- *AF: I just think that like the ... because their level, or their meaning of success varies within themselves, we won't necessarily know anyway, when we see their responses, whether that's them using their own degree of success or what we think that 'success' means, if that makes sense, so ...*
- *PW: OK. So, could we think ... could we phrase it differently? 'How well do you think you've done in this lesson?'*
- *AF: Yes, I think that's a bit more straightforward. That's got ... it's less heavy ... 'success' can mean a lot.*

(RGM1, #65-68)

RGM2/3/4/6/7 – Evaluation of surveys. Feeling that surveys gave overview of what majority of students were thinking (rather than a small vocal minority):

- *AF: I thought the surveys were a great tool for seeing an overview for the whole class.*

(RGM7, #63)

Surveys provided useful evidence for evaluating students' appreciation of teachers' pedagogical rationale. Structure, carefully thought-out questions and time to consider responses meant students' responses more likely to accurately reflect their feelings:

- *TC: I think the survey was very, surprisingly, good to have the results afterwards ... It, kind of, showed us that we can never assume or conclude anything because it might not be at all visible ... from their answers, we can actually see much better where they're coming from and what did they really get from the lesson. So that's a very good tool.*
- *PW: Is that because you're getting responses from everybody, and not just one or two who are most vocal? What is it that's powerful about the survey?*
- *TC: Because it's a more ... it's a structured element. They need to structure their reasoning, they need to elaborate more. And it gives us an idea of the entire group. And because the questions are ... were thought before the lesson, so they ... the answers we get are more meaningful.*

(RGM4, #59-61)

- *TC: I really like the idea of asking students about what they ... how they are feeling ...*
- *AF: ... in an impartial way.*

(RGM7, #66-67)

Surveys performed a different function to interviews and were not necessarily comparable. Consensus was that both should be used to provide stronger evidence to evaluate success of strategies:

- *AF: So, I think both were really interesting for different reasons, for different purposes, depending on what we're looking for. Doing both, at the same time, would be even better, because then we can get an idea of more.*
- *PW: Yeah. Well I guess, cos we did the survey at the start and the interview at the end ... If we'd done both at the start, and then both again at the end, we might really notice the differences between the two. Whereas, it's quite hard to compare what you got from the interviews at the end with what you got from the surveys at the beginning.*
- *AF: Yes. But good tools for different reasons, I think, really, really eye-opening ...*

(RGM7, #63-65)

Surveys prompted TRs to consider how to collect feedback from students on a more regular basis:

- *TC: I think it was really powerful and eye-opening, and it's something that I see myself doing more often. Just having a simple one question, or two, survey at the end of the lesson, for them to comment on what particular strategy I'm trying to introduce, to say what they think.*

(RGM7, #66)

5.1.3. Student interviews

RGM5 – Collaborative planning of interviews (used in cycle 2 but not in cycle 1) – PW sharing expertise on conducting interviews, e.g. adopting methods to put students at ease so they are willing to be more open in their answers:

- *PW: ... Some people refer to this as empathetic interviewing ... it's not like an interview where you have to ask the same questions ... like a job interview where everyone has to get the same questions, and you're not allowed to ask follow-up questions cos that's not fair. We want to just find out from them what they think. Therefore, as long as we ask them all the same initial questions, there's nothing wrong in, sort of, showing interest, and saying 'Oh, that's interesting, tell me more about that. What do you mean by that?' Yeah? I think you'll probably need to do that to try and encourage them to tell you more. So, don't be worried about follow-up questions.*
- *TC: But we're not allowed to be leading ... I mean, if they're not understanding, could you say: 'Remember the point in the lesson, when we talked about using "corners" instead of "vertices"?'*
- *PW: Yeah ... if you want to just draw their attention to something because you realise they've missed the point of the question ...*

(RGM5, #79-81)

- *PW: Just like in ... when we had the question in the survey about the table, you know, there's no problem in trying to make sure they understand what table you're talking about.*

(RGM5, #85)

- *PW: I think 'How well did you do in today's lesson?' should probably be the last one because I kind of think ... there's a potential for them to start getting defensive at that point.*
- *AF: Yeah.*
- *PW: So, you want them to be relaxed, and if they get defensive then, then they might not be open to the next question.*

(RGM5, #100-102)

Negotiation over precise wording with TRs taking a lead and PW facilitating by relating to aims of project – taking account of TC/AF's knowledge of students:

- *AF: I've just got here 'Why might it be important to listen to your partner's thinking?' Because that's in relation to the 'think-pair-share'.*
- *PW: So, we can tailor that to our new ones. So, we've got ...*
- *AF: 'Why might it be important ... why is it useful that I scribe? Why might that be useful for you, that I scribed your ideas?'*
- *TC: Or 'Why do you think I ...?' Should we start 'Why do you think I ...?'*
- *AF: ... 'Why do you think I scribed your ideas?'*
- *PW: I do like the 'Why do you think I ...?' type question. Cos this is all about, you know, teachers thinking that students understand when they don't really understand.*

(RGM5, #106-111)

RGM6/7 – Evaluation of interviews. TRs played leading role in evaluating interviews, e.g. AF concerns over being too leading:

- *AF: I had to be a bit leading, I felt. I don't know if my questioning ... some places I felt that it was. And I had to make that judgement between 'Do I make that question more leading to draw information out of them?' or 'Do I just accept that they don't know?'*

- *PW: I don't think that matters at all, cos I think the fact that the information only comes out when you have to really draw it out tells us something about it, yeah? How easy it is for them to articulate the information is important.*

(RGM6, #71-72)

The interviews were considered powerful in a different way to surveys, by allowing TRs to explore students' thinking in greater depth:

- *PW: Did you find the interviews powerful?*
- *AF: Really useful, really powerful ... because they zoned into each student a little bit more, and their thoughts.*

(RGM6, #166-167)

Interviews enabled TRs to evaluate the extent to which the strategies were successful, particularly for the target students:

- *PW: But this helped you to drill down a little bit?*
- *AF: Although some of the responses, some of their understanding about the strategies could definitely get better in future, they did generally get the gist. And it was very powerful, the effect that the tasks and the activities had on how they responded. And I thought that it was really ... it's surprising how much they took from it, from the tasks ... in a good way ... it makes me believe more in the power of, I suppose, of what we're doing.*

(RGM6, #170-171)

- *AF: I really noticed that, with the interviews of the three PPI students ... that was just as powerful. Their responses were really articulate. I mean this was after the discussion was explicit, but I still wasn't sure how much they would get from that. So, that was really interesting to see how much ... and it was a lot!*

(RGM7, #80)

TRs were generally successful at conducting interviews – by putting students more at ease they are likely to have got more meaningful responses than an external researcher would obtain:

- *PW: I mean I was really impressed with, actually, how you managed the discussions ... the way you've helped students to ... you know, you just: 'Well, tell me more ...' and I think, actually, it says something to me about ... why should researchers, who don't know the students, go in and ask them about the learning, when the teachers can have discussions with them? They're at ease cos they've already built up relationships with the teachers. Probably get much more out of that discussion than we would have got if I'd sat down with those three students and talked to them, I suspect.*

(RGM6, #182)

5.1.4. Peer observation

RGM1/5 -Planning how to carry out peer observations – PW carried out peer observations for cycle 1, TC/AF carried out peer observations of each other cycle 2. Discussed desirability of forewarning students that there would be a visitor in the class:

- *PW: Just to say, maybe the lesson before, somebody's coming in to just have a look at how well you're learning. It's a little research project, looking at how we can help people in general to learn better. So, there'll be a visitor in the class tomorrow taking some notes.*

(RGM1, #109)

Peer observer's role to assist with setting up, starting, stopping video and taking brief descriptive notes of key moments in the lesson that were relevant to project's aims, i.e. students' responses to strategies, and to produce a time-line to facilitate video-stimulated reflection. Care was taken to ensure peer observations were not about providing feedback or judging each other's teaching. Initially, peer observations were seen by PW as an alternative to videoing, but TF/AC decided to do both during cycle 2.

RGM2/3/6/7 – Evaluation of use of peer observations – useful for PW as external researcher to get a feel for the kind of students and contexts in the classes TC/AF were working with:

- *PW: I'm glad I came into the lessons ... because I could have done this without being here at all ... but kind of ... it's nice to have come in once to get a feel ...*
- *AF: ... for what the group's like.*
- *PW: ... for what the group's like, otherwise it's all very hypothetical and things ... and I think that's quite important.*

(RGM3, #11-13)

TC/AF comfortable being observed by peers (PW and each other), although there was some pressure felt in terms of addressing the aims of the project in the lesson due to other people's time invested:

- *TC: But, I think I'm used to being observed, so it doesn't make much difference. Obviously, I am aware that someone is there, some part of me is, but I'm still responding as I normally do.*

(RGM2, #82)

- *AF: For me ... because I know how challenging this group can be, behaviourally, I think my concern was that the behaviour would take too much attention away from the lesson. As it always is, not just with you being there, but because I knew that there was a focus on what we were trying to achieve. I really wanted those strategies to be played out in a fair way, so they would then take away something from that, right? So, I think that was my concern.*
- *PW: So, you couldn't sort of say: 'Oh well, it's not going to work this lesson, let's do it tomorrow instead'?*
- *AF: I couldn't ... exactly. Exactly. So, I knew that there was that ... no not pressure ... but I knew that there was that ... you know, I wanted them to take something away.*

(RGM3, #92)

TRs found conducting peer observations of jointly planned lessons beneficial in developing their practice:

- *TC: So, actually, seeing us teach the same lesson in a different way, I think that's also very powerful, for my own professional development.*

(RGM6, #172)

- *AF: I can honestly see what's going on and reflect on it. So, I think it would be just as powerful to do it on my own, especially if you're somebody who, you know, can be honest with yourself and evaluate critically. But ... there's an added bonus to having someone else in the room, who can give you a different perspective, perhaps.*

(RGM7, #88)

Peer observations, particularly notes and time-line produced, proved valuable for facilitating video-stimulated reflection:

- *PW: So, actually having the timeline and the surveys was useful to you when you were looking at the lesson and previewing it yourself?*
- *TC: More because I still had to observe the entire lesson, because I wanted to see some ... I could do it differently by just focusing on things that you've written down.*

(RGM2, #85-86)

- *TC: I think it's really beneficial ... to do it paired, so that you have another person observing. Because it's less time-consuming. Particularly, if we were to repeat it, we would be better probably on choosing the moments in the lesson. And we can have a discussion targeted so that we don't have to go over the video in its ...*
- *AF: ... entirety ...*
- *TC: ... entirety. So, which could have, like, the time-lines. As we could get better and better at doing it, we could just have those zoned-in discussions on the key moments that we focus on.*

(RGM7, #89-91)

- *AF: But it does make life easier if the person has got some key moments and you can go and view those key moments.*
- *PW: So that time-line thing was really useful?*
- *Yes. Yeah. It was really helpful when Tiago did that.*

(RGM7, #109-111)

5.1.5. Video-stimulated reflection

RGM1/5 – Planning how to use videos to stimulate critically reflective discussion (used in cycles 1 and 2). Less discussion of design in cycle 1 – more collaborative planning in cycle 2 when video-stimulated reflection meeting held with all 3 researchers.

Video-stimulated reflection (along with surveys/interviews) seen as alternative to traditional notions of research where research design and data collection determined exclusively by researchers – TRs retain ownership of video (which is not part of the data):

- *PW: So, we're kind of looking for alternatives to that, where the, sort of, control rests with you, if you like. And one way is we video, but you have control of the video, and you decide which bits to talk about in the reflection meeting.*

(RGM5, #68)

RGM5 – Discussion of the extent to which TC/AC should discuss the lessons (which they had peer observed) before the video-stimulated reflection meeting and focus of discussions – agreed not to have a 'full-blown discussion' (which might lead to valuable ideas not being recorded as part of the data):

- *AF: But it might be good to talk about it just a little bit, just whilst it's fresh. Yeah?*
- *PW: Yes, definitely ... And I guess the main focus ... I'll think up some questions, but the main focus will be: 'To what extent were the strategies successful, yeah?'*

(RGM5, #169)

RGM6 – Discussed and agreed format for video-stimulated reflection – TRs took it in turns to present a key moment from their video to prompt discussion:

- *PW: Yeah, so those are the two strategies, and that's what we'll focus on when we look at the videos. So, I don't know whether we want to look at one video first, both strategies. That would probably be easier, wouldn't it? Or we could look at one strategy, one video and then the other video.*
- *TC: Probably easier to ...*
- *AF: ... one video ...*
- *TC: ... one video at a time.*

(RGM6, #3-6)

RGM2/3 – video-stimulated reflection meetings – PW+TC/PW+AF respectively.

RGM6 – video-stimulated reflection meeting with PW+TC+AF

RGM4/7 – Overall evaluation of video-stimulated reflection.

Normalising the presence of the video by using it to video prior lessons (not part of the research project) meant students were already used to the camera and there was little ‘playing up’ for it.

- *AF: And ... yes, so I think having the camera the previous three lessons definitely helped ... keep things stable, if that makes sense, and normal ... normalise the situation rather than ... Because it is a bit different, you know, having a different adult in the room. But, you know, given the circumstances, I think they probably behaved and acted and performed the same way.*

(RGM3, #92)

Both TC and AF reported that the videos were a powerful tool, enabling them to recall key moments in the lesson, to notice things they hadn’t noticed during the lesson and to see things from a different perspective:

- *PW: I just wondered now, having thought back about longer term, what you think about that process of using the videos.*
- *TC: I think we both really thought it was useful ... as in a useful tool that we would like to use more. Not just for the project in particular, but for ... just grasping other aspects of our practices. I think it’s a very valuable tool.*
- *AF: Yeah. It’s really easy to forget we said when, you know, you’ve taught a few sequences of lessons, the specifics. And I think it just allowed us ... it made that so much easier, when we were reflecting, to look at specific points in the lesson, and how students received them, and how we responded to the students ... and just thinking of ways to do it the next time.*

(RGM4, #124-126)

- *TC: And then, actually, watching your video again, and again, gives me another opportunity to notice all of the details, because it’s the same lesson.*

(RGM6, #174)

- *PW: The literature calls it ‘video-stimulated recall’. I’d rather call it ‘video-stimulated reflection’. How did you find that, looking back on it now? At the time, you were very positive about it ... with more hindsight?*
- *AF: Well, for me, I really liked it the first time. It took me a while to watch the video back, the first time. But I think, once I did, I really bought in to it. ... this should be something that I can do, and should do, all the time, because I can see the benefits of it.*
- *PW: And you both did it the second time?*
- *AF: We did it the second time. Obviously, we didn’t have to at that point. But we did because we saw the benefits, the first time. It allowed me to look at everything from how I verbalized the questions ... to which students were paying attention in the class, so that I knew next time to really draw the attention of ... which students seemed to be responding more, so which students to then target the next time, who seem to be a bit quieter ... to how the discussion was going and who was engaged and how alert they seemed, as well, to being asked and to answering, potentially, a question. So, I think, yeah, it was great on so many levels. It gave me a lot of feedback that I used, I think. I’d like to use it more often.*

(RGM7, #82-85)

Previewing the video before the reflection meeting, and comparing to other evidence (time-line, surveys, interviews), enabled more critical reflection to take place:

- *TC: I think it’s, for me, what was really good was to be able to have the video before we are having this conversation. So, then I’m already thinking, and analysing, and making my own ...*

- *PW: So, you can present your ideas on your terms and not on somebody else's?*
- *TC: And not responding to any prompts before ... I'm still responding to your questions, but I was already thinking: 'How would I do it differently?' or 'What worked? And what does this answer in the survey mean?'*
- *PW: So, actually having the timeline and the surveys was useful to you when you were looking at the lesson and previewing it yourself?*
- *TC: More because I still had to observe the entire lesson, because I wanted to see some ... I could do it differently by just focusing on things that you've written down.*

(RGM2, #82-86)

TRs welcomed the opportunity to remain in control over the selection of the video clips from their own lesson to stimulate discussion, whilst taking account of key moments identified by the peer observer. They recognised the importance of considering these in the context of the whole lesson and viewing the complete video first before the reflective discussion:

- *TC: But I think, because of the way we've conducted these cycles, we had some discussions in mind and some strategies that we were focusing on. So, when we were ... when I was going over my video, I was looking for those moments in particular, because that's what we want to discuss later on, when the three of us met. ... But I think it's still valuable to have the video, and we're in control of the ... if we want to have, to watch it in full, we can do that.*
- *AF: I think you can use it both ways. It depends on the time available to you and the aim. It might be that you want to look at the whole lesson and see the impact that those moments, key moments, seem to be having on the atmosphere of the lesson as a whole. Because, actually, if we're talking about these strategies ... potentially engaging students more, we want to look at the big picture. We want to see whether that engagement is there for most of the lesson, you know. But, yeah. I think, if we want to look at key points, then it just makes that a lot easier.*
- *PW: So, it would be useful to watch and reflect, maybe, individually on the whole lesson first, before having that [discussion]?*
- *AF: I would like that, I think, also because it just refreshes my memory as well, I think, given ... especially if you're teaching the whole day, and you don't get the chance to reflect on it straight away, or talk about it straight away. But it does make life easier if the person has got some key moments and you can go and view those key moments.*

(RGM&, #106-109)

Although, there was a feeling that the process of videoing, whilst valuable, was time-consuming and should be used in a focused way:

- *TC: It's also very time-consuming, so it's something that cannot happen ...*
- *AF: too often ...*
- *TC: I mean, you made life easier for us, because you transcribed the parts. But if we were to do it individually, then it ...*
- *AF: ... it would take more time.*

(RGM4, #127-130)

5.2. Strategies tried out

5.2.1. Advocating (cycle 1)

During 'think-pair-share', the 'advocating' strategy involved asking students to present their partner's thinking to the whole group rather than their own and discussing with students the reason for doing so. Researchers articulated the rationale for the strategy as encouraging students to listen carefully to another person's answer, engage with their ideas, learn from other students, learn by working collaboratively:

- *TC: So ... that someone else is gaining from listening carefully and trying to say what someone else potentially ... has a better answer, or more clear, or more complete ...*
(RGM1, #22)
- *TC: But it's important that they create this routine of understanding ... of making this strong connection between 'it's important to have good listening skills because we want to hear what our partners are saying ... because we learn more from working together'.*
- *PW: So: 'We're more receptive to other ideas. We might actually develop our ideas by taking on board others and see how they sit with ours'?*
(RGM2, #36-37)
- *PW: We want them to see that, by listening, you know, you really have to engage with other people's ideas to be able to express them. So, it's that, sort of, being open to multiple ways of doing something and not just assuming that there's only one correct way, which ... you would have used yourself.*
(RGM2, #45)
- *AF: I used the 'sharing what your partner has heard' with my Year 9's, and my Year 8's, who are a Set 5, because I think it's really beneficial for them to really focus on what the other person's saying ...*
(RGM2, #66)

It was also justified as helping to prevent a small number of students from dominating discussions. Despite initial frustrations felt by some students, this aim appeared to have been realised over the course of the project:

- *PW: And they didn't hesitate, did they? They were ready to explain the other person's perspectives, which is quite impressive. Yes, so what you said to them, maybe because it's become routine, what you'd said to them before had prepared them to do this and they did it well. They must have been listening because ... they didn't volunteer to present their partner's ideas ... their partner wanted their own ideas to be presented, so they put their hand up, and they kind of dropped their partner in it because they had to be ready to explain. But they were ready to explain.*
- *TC: And some of them are more frustrated about that than others, for instance, some of them want to have more stage, like [Student A] or [Student B], so they would get a bit frustrated for not being able to say what they wanted to say ...*
- *PW: You mean because their partner isn't expressing exactly what they wanted to say as they would have wanted it?*
- *TC: Yes, or because they want to show it themselves. But then now I think the focus is already changing for them to ... they're really trying to explain to their partners because they know that's the rule of the game.*
(RGM2, #60-63)

Another reason articulated for the strategy was that it encouraged students to consider how to present their ideas more clearly to others:

- *PW: So, that highlights something else actually, coming back to the discussion we had earlier about the purpose of getting one person to explain the other person's reasoning ... as well as listening and taking on board other people's perspectives, there's something there about, if you know that somebody else has got to present your ideas for you, then you've really got to express your argument well and coherently. So, it's kind of ... it's encouraging students to think about how they express themselves and communicate their ideas clearly to their partner.*
(RGM2, #64)

5.2.2. Separating (cycle 1)

During a whole class discussion, whilst asking individual students to explain how they solved a rich mathematical problem, the 'separating' strategy involved recording contributions from students (in summarised note form where necessary) in one of two unlabelled columns on the board ('reasoning' and 'working out'). Students were then asked to discuss the distinction between the two columns and the rationale for making such a distinction. Researchers articulated the rationale for the strategy as enabling students to appreciate the difference between 'reasoning' and 'working out' and hence what the teacher (or examiner) is looking for when they ask them to explain their thinking or how they solved a problem:

- *AF: Well, it will help them to start thinking about ... the language, the reasoning, so our questioning, the way we question them. And prompt them to give actual reasoning rather than to describe the steps.*
(RGM1, #3)
- *PW: So, actually, what you're doing is you're writing the reasoning ones here, without telling them, and then another response you'll write it on this side, which is really working out. And then you, kind of, at the end, you say 'OK, why have I written some of them here ...?'*
- *AF: You get them to do the realising, basically ...*
- *TC: ... visually.*

(RGM1, #31-33)

Analysis of the students' responses during video-stimulated reflection encouraged researchers to reconsider the rationale for using the strategy, focusing more on the importance of prompting students to consider the reasoning involved in solving problems:

- *TC: ... Now, reflecting on the video, I would ask them: 'Why is it important for you to know this distinction?' ... with like a follow-up question ... 'So, why am I doing this?' ... 'What do I want you to see?' And then they're going to say 'The calculations and the reasoning, the ideas' ... So, they're separating the ideas from the working out. But then I could ask 'Why is it important to think about the ideas first ... and then go on to the calculations?' and 'What's the risk of just start[ing] working out without thinking about the ideas?' ... making them think about that.*

(RGM2, #11)

Evaluating the success of the strategies led researchers to discuss how they might improve the way in which they used the strategies to make them more effective:

- *AF: What I'm thinking from that is ... I could have written some of his reasoning down and some of his [working out] ... to make him aware that he's doing both, and the whole class, that he's doing both.*
- *PW: Yes, and ... there [you] might have drawn that distinction out even clearer because ... the same student is saying something, but some of the things he says are going over here and some of the things he says are going over here.*

(RGM3, #6-7)

5.2.3. Scribing (cycle 2)

During a whole class discussion, whilst asking individual students to present their solutions to a rich mathematical problem, the 'scribing' strategy involved the teacher writing down exactly what the student said, whether the answer was correct or incorrect, and discussing with students the reason for doing so. Researchers articulated the rationale for the strategy as challenging the notion that correct answers are necessarily more highly valued and that if the teacher writes something down it must be correct:

- *AF: Just write down what they're saying on the board and ...*
- *TC: ... in a very ... equal way ...*
- *AF: ... open ...*

- *TC: ... not giving any power to ...*
- *AF: ... any weighting, either, to any Yeah. And just sort of say: 'OK, why am I writing all these answers?' And hopefully they'll be thinking about: 'OK, they're all suggestions, and as important as each other'.*

(RGM4, #113-117)

- *TC: It will suggest ... a concern of them thinking that just because I'm writing it on the board ..., the teacher's writing, and that makes it correct. Which is not the case. I'm always thinking 'OK, I'm writing this, I hope they're not copying from what I'm doing on the board'.*

(RGM5, #11)

- *PW: But I think you were hinting at there might be a deliberate reason why 'I, as a teacher, have made the decision that I'm going to write the errors because I want to show the fact that sometimes I can be [wrong] ... just because I'm writing something doesn't mean it's correct.*

(RGM5, #18)

In writing down both incorrect and correct solutions, the teacher encourages students to judge for themselves whether a response is correct and reinforces the idea that incorrect answers should be welcomed as they provide opportunities for learning through identifying errors and improving solutions:

- *TC: Because there might be just a tiny problem in their reasoning, that gives a wrong answer. Or it could be that, later on ... I've used that correct, just the starting point is wrong. And, by asking them to volunteer more reasoning or working out, then we're just bringing everything together and we can have a discussion, generate a discussion about what's wrong, what's right even ...*
- *AF: I think it's really multi-faceted because ... it's like drawing their attention to the problem and expanding on the problem by writing it. It's drawing their attention to: 'Is this correct? Now that we're writing it down, does it look right? You judge whether this is correct or not.' And then, also, the idea of writing things down, and just making mistakes, and that's OK. And then you learn from that, and that's a journey. And I, as a teacher, am not going to tell you. You're going to decide for yourself through me writing it down.' So, there's so many things that I feel like they gain from that.*

(RGM5, #19-20)

The 'scribing' strategy also enables the teacher to draw students' attention to ambiguities in the use of language and common misconceptions:

- *PW: But one advantage in you doing it, as a teacher, is that you've got a chance to almost exaggerate the ambiguity. So, if they say something that could be taken the wrong way, then you make sure that you do take it the wrong way.*
- *TC: Yes, I do that ... There's one that I do very often, because it comes out very often ... if a student wants to do 200 divided by 4, they may say 4 divided by 200. So, I'll always write 4 divided by 200. And then some of them are starting to realise 'No, it's 200 divided by 4'. But I'm not making anything explicit, I'm just writing what they say on the board ... And then ... sometimes someone else is going to say: 'No, it's the other way around'. And that's a very, very common mistake that I find with my students.*

(RGM5, #26-27)

- *PW: Then there's a whole load of things ... drawing out ambiguity ... you're annotating what's already there, you're looking at how to improve what's written down ... you could draw attention to language, mathematical language and real-life language.*

(RGM5, #30)

- *PW: So, if they say: 'the corners of the square', you write down: 'corners of the square'. And then you have a discussion about: 'OK, you've used the word "corners", if you were to write in an exam ... or what*

would a mathematical way of writing this be? Rather than “corners”, what would you write? ... “Vertices”. So, why do we have these words “vertices” and “corners”, why don’t we just use “corners”?’ You know, having that discussion about ‘What is mathematical language for?’

- TC: ... And I think we also had this discussion ... ‘I want you to know these words. Why do I want you to know ... to use also these words, if yours are good enough? If I’m saying that yours are OK, why do I insist on also using this language? ... Because if you find this in a test, and you don’t make this association now, then you might not know what the question is asking you.’
- AF: ... It’s AFL as well, because it gives us a clear idea of whether their thinking is along the right track. If they’re able to verbalise what they’re thinking in the correct ... language, then we know they are thinking about the question correctly.

(RGM5, #32-34)

TRs believed that the scribing strategy had been successful in developing students’ appreciation of the value of engaging with incorrect responses:

- AF: I think some of the answers that that drew were to the point in the sense that straight away they were thinking: ‘OK, well other students can then judge for themselves whether that’s incorrect or correct before Miss tells us that it is’. So, it helps them start to consider, and think, for themselves independently. Second, they thought: ‘OK, well if it’s a mistake, then I need to spot that. And spotting it is a lot easier because Miss has written it on the board. It’s not just that I’m listening to someone saying it. It’s visible, it’s clear. And then I’m hopefully thinking about ways to improve and to fix that mistake, and to make it better’. So, I think they saw it from different angles, sort of the things we wanted them to get from it, the benefits. And hopefully if I do that in the future, which I have continued to do, that sense of frustration is removed from: ‘Why isn’t Miss saying anything about whether this is correct or not, yet? It’s because she wants us to think about this independently’.

(RGM6, #51)

- AF: And today I taught a lesson ... I taught them Period 1 ... and I reminded them, and I asked them: ‘Why is it that I’m not telling you?’ And they were able to respond and say: ‘This is why’.

(RGM6, #53)

The explicit discussions included in the ‘scribing’ strategy appear to have given students a greater appreciation of the pedagogical rationale behind it and hence enabled them to respond more appropriately:

- TC: All three [interviewees] get the idea of scribing, why the we do it on the board. And that’s something I’ve been using. It was not the first time that I used it with them. I ‘d never asked them ‘Why do I do it?’ so that’s the difference. ... It’s good to see that, when they’re responding now, they understand that.

(RGM6, #109)

- TC: From the first time that I started using them, they would interrupt themselves, like call out and try to correct it, and would say things like ‘Oh that’s wrong’, kind of defeating the purpose. And now they control themselves and they know that ... the fact that the teacher’s writing doesn’t make it right. Because it’s just scribing for someone else. ... I don’t know that for a fact, I’m interpreting what I see, that they don’t call out any more.

(RGM6, 111)

- AF: And, again, like you, I hadn’t made the scribing explicit. I obviously talk about the importance of writing the working-out, and your methodology, but I’ve never opened it up to the class as explicitly as I did at that time, in getting those answers from them.

(RGM6, #131)

- *AF: One of our strategies is to write down whatever they write, which is correct and incorrect. So, in that behaviour ... I'm not necessarily always doing the right thing, I'm doing the wrong thing for the right message, if that makes sense. So, we want them to be able to be more objective about things, and not just take everything that we do ... take it with a pinch of salt ... not take everything that we do literally.*
(RGM6, #138)

5.2.4. Annotating (cycle 2)

The 'annotating' strategy developed out of the 'scribing' strategy and focused on adding notes to solutions already displayed on the board (sometimes using a different colour) to highlight how they might be improved. Researchers articulated the rationale for the strategy as allowing for imprecise language, ambiguity and misconceptions to be addressed:

- *PW: you're annotating what's already there, you're looking at how to improve what's written down ... you could draw attention to language, mathematical language and real-life language.*
(RGM5, #30)
- *TC: The annotating strategy was the first time that I've used it explicitly, using a different colour, and the discussion from why it's useful.*
(RGM6, #113)

TRs used the strategy to model how students might annotate their own solutions to problems, which appeared to be an effective way of helping them to realise success:

- *AF: There's some of the strategies we used in the second cycle, like the annotating, cos ... making it visible was almost part of the modelling. So, I'm modelling it in a way that's really clear and explicit, and then they know the purpose of that is really significant. So, they buy into it more, I think, which is why that then seemed to pay off well in the class.*
- *TC: And what they have to gain from doing it ... their awareness of the game, for themselves, is clear. I think that's so important, to be able to be independent and to ... give more meaning to the work that they record in their books. I think that's essential for creating that link between ... the modelling, asking them to copy down from the board to their books, and annotate their answers with their own notes.*
(RGM7, #10-11)

5.2.5. Classifying (cycle 2)

The 'classifying' strategy involved asking students to identify the characteristics of a range of problems, which they have solved, identifying those with similar characteristics. They were then encouraged to sort other problems into similar types, before solving them, and discuss the rationale for doing so. Researchers articulated the rationale for the strategy as enabling students to recognise the key characteristics of problems that would enable them to apply the correct procedures for solving them:

- *PW: If it's about helping them to see the value of being able to identify types of problems, then you could, kind of ... they could sort them into three groups, they could have the problems, and they could work out which ones were a certain type of problem.*
(RGM5, #38)
- *PW: So, display the starter problems again, with the solutions still there, type 1, type 2, type 3. 'Can you sort the problems into type 1, type 2, type 3?'*
- *AF: That's what I was thinking, apart from ... do we tell them what the type of problems, or do we not, and leave it completely up to them to identify? ... Just 'type 1, type 2, type 3', where they don't get told: 'This is a "sharing", this is a "simplifying", this is a "find the missing part given another part"'. Because that might be quite nice, not telling them what the problem is, but just seeing the relationships between the problems, and linking them themselves.*

- *TC: In the starter, you mean?*
- *AF: Yes. So not telling them, but relating the subsequent problems to the starter problems, if that makes sense.*

(RGM5, #53-56)

5.3. Critical research processes

Note that the three critical research processes are based on those identified in Skovsmose and Borba's (2004) critical model of Participatory Action Research. These are pedagogical imagination, practical organisation and explorative reasoning.

a) Pedagogical Imagination

5.3.1. Reflect on own practice (Pedagogical Imagination)

Discussions of current practice focused on adopting progressive pedagogies and were prompted by PW's probing questions. TRs reflected on their current practice and the rationale behind it:

- *PW: What's the point of the think-pair-share? You know, I mean this is something that's been a focus for you.*
- *AF: Well, it will help them to start thinking about ... the language, the reasoning. So, our questioning, the way we question them, and prompt them to give actual reasoning rather than to describe the steps. Hopefully we'll get them to think about ... reasoning correctly and then they will do it themselves. So, I guess the think-pair-share acts as a modelling, but also as a prompt to think about it first by themselves ...*
- *PW: So, they think about it for themselves, they share their ideas with each other, then they kind of feedback to the rest of the class in some way, or to the teacher. So, what's the rationale behind that? Why is that better than just asking them directly?*
- *TC: I think that, by putting a problem on the board, which is just a fraction of the rest of the problems they need to solve, we're introducing the problem. ... And then we're showing, by the discussion, we're trying to elicit all of the information that is available in the problem. And we're modelling getting information from everyone in the group, and getting a possible answer, together. So, instead of the teacher saying ... (modelling on the board): 'This is how you solve this problem', we're trying to gather the ... how they would solve this problem, with what they already know. And then putting an answer together and using that as an example for what they would do next.*

(RGM1, #2-5)

Discussions included difficulties encountered in implementing progressive pedagogies and how TRs overcame these:

- *AF: I suppose the problem with think-pair-share is always one person, probably dominating more than another. So, I think the way that we try and tackle that, and we've sort of discussed this in department meetings, is we have them working with a pair next to them. But we also have them working with the person behind them. So, they're sort of getting exposed to different people, if say that pair isn't necessarily working with them equally, they have the opportunity to work with somebody else.*
- (RGM1, #18)
- *TC: I try to go around that by asking them to say ... they know that they're not going to say what they said in the conversation, they're going to say what the other person said.*
- (RGM1, #20)
- *TC: So ... that someone else is gaining from listening carefully and trying to say what someone else [said] ... that potentially has a better answer, or more clear, or more complete.*

5.3.2. Reflect on existing practice (Pedagogical Imagination)

In their discussion of existing practice, TRs demonstrated an appreciation that teachers' pedagogical decisions are often influenced by routines and assumptions, rather than a well thought-through rationale:

- *TC: I think that when we're planning, we're not thinking about why we're doing things in a certain way, but they make sense. And the pedagogy's there, we're just not making it visible to ourselves. It's, kind of, intuitively ...*
- *AF: ... it's assumed, yeah ...*
- *TC: ... we're following our own routines.*

(RGM4, #63-65)

TRs reflected on existing practice in relation to the aims of the project, i.e. how visible pedagogies are to students, and how this practice might account for some student disengagement:

- *AF: and we mentioned, as a third potential strategy ... this thing, that we always come back to sometimes quite naturally as teachers, as sometimes we do it, sometimes we don't ... 'Why am I not correcting you? Why am I not telling you the right answer?'*
- *PW: OK.*
- *AF: And I think sometimes we don't say that, sometimes we don't make that clear or visible to them. Sometimes we just, sort of, go 'let's elaborate' or maybe 'let's think about it in a different way'. Or we're moving on to a different student and saying: 'I'm going to come back to you', you know, 'let's see what [Name]'s has to say, I'm going to come back to you'. But we don't explain why we're doing that. I think maybe that could go a long way to not losing some of them? Because I think some of them may be switched off?*

(RGM4, #79-81)

- *AF: Or even, you know when sometimes the student is confident. And they'll say the answer, and they'll be like 'I know the answer, I'm going to switch off, I'm not going to be part of the discussion'.*

(RGM4, #90)

PW related TRs' reflections to his own experiences of teaching and observing other teachers' lessons:

- *PW: As someone who observes lots of lessons, you really notice this, that the way the teacher responds to students' answers ... you can see the students getting all sorts of hidden messages about mathematics from whether the teacher raises their eyebrows, or [says]: 'Mm, that's interesting, how about you?' ... They haven't said anything but ... the kids just take it in a particular way ... 'Oh, I've just got nothing valuable to contribute here'.*

(RGM4, #86)

5.3.3. Relate theory to practice (Pedagogical Imagination)

PW posed questions that reminded TRs of theories underpinning the project (in this case Bernstein) and encouraged them to take these on board in their discussions of how pedagogy can be problematic:

- *PW: what are the problems in terms of the recognition and ...? You remember the research talked about recognition and realisation rules, so they need to recognise what is expected of them and make an appropriate response. So, what will the problems be?*
- *TC: Laying out the work, I think that's a good one ... how they lay out the work. So how do they present a solution to the problem? That's a possible ... that's a problem.*

(RGM1, #9-10)

- AF: *So, using language clearly, is that something ... so how do we get them to do that?*
- TC: *... yes, language, reading language.*

(RGM1, #15-16)

PW also pointed out instances where the discussions of pedagogy related closely to the theories underpinning the project:

- PW: *So, we've got the scribing with four reasons for doing it. Now we might have the sorting. And I can see this as a separate strategy ... We often present these solutions already sorted for them, rather than getting them to identify, then sort them themselves. And then their discussion might be around: 'What might be the purpose of you sorting them?' You know: 'Why is that important?' Cos it is important. I mean, that's what Jo Boaler's research showed, wasn't it? The 'Open and closed mathematics' one ... that the main problem students have in the exam is not being able to identify that this is a question about simultaneous equations.*

(RGM5, #44)

5.3.4. Consider alternative practice (Pedagogical Imagination)

TRs articulated their vision of an alternative practice in which the strategies they were trying out became a routine part of practice and complemented their use of progressive pedagogies:

- AF: *I think it would be really nice, potentially, to have this, you know, embedded day-to-day, lesson-to-lesson, what we're doing, you know, this ... small but hopefully meaningful strategies. And then to have the learning journeys, sort of, keep tab of the big picture of the unit that they're learning as well. I don't know, maybe together they could be a strong tool.*

(RGM4, #67)

In evaluating the success of some of the strategies (in this case 'scribing'), TRs articulated an alternative vision of teaching and learning based on establishing a different environment, e.g. in which engaging in discussion around the different answers put forward was more important than getting answers correct:

- TC: *And some of the reasons why we might do that could be just saying: 'OK, I've already received the right answer, I want to hear some misconceptions. And I'm going to carry on asking until ...'*
- AF: *... Whereas when you don't give them that validation straight away, then they're more likely to sort of stay with you, maybe, and hear what someone else has to say? Or the way someone else has got to the answer? Maybe different to theirs? So, I don't know, there may be more invested in that way? Because they're not just like 'OK, well I've got it and I don't need to think about it in any other way'.*

(RGM4, #89-90)

- AF: *When we discussed it, we thought: 'exposing misconceptions ... owning the discovery'. We thought that was quite nice, you know, the journey of getting to the answer ...*
- PW: *So, do students see ... why that's more powerful in terms of retaining the understanding?*
- AF: *Yeah. We said 'moderating'. So, they moderate ... allowing them to call each other out on it ... gives them more of an opportunity to, kind of, take a step back and think about what the other person's saying. And maybe saying 'No, I disagree' or 'I agree', rather than the teacher just, sort of, going straight in and saying 'Yes' or 'No'.*

(RGM4, 92-94)

b) Practical Organisation

5.3.5. Negotiate/design strategy to try out (Practical Organisation)

Strategies to be tried out in the classroom were developed collaboratively through discussion and negotiation, sometimes with one researcher suggesting an idea, then others identifying potential issues to be addressed and building on the initial idea (in this case 'separating'):

- *PW: What strategy can we use to address this problem that ... they're not really aware of what we mean by reasoning, as opposed to working out?*
(RGM1, #25)
- *TC: I think for ... initially ...*
- *AF: ... the reasoning ...*
- *TC: ... you'd ask the question and then them to reflect on 'Why did I ask you this question and not necessarily ... give me just the answer?'*
- *PW: But what ...? This is just throwing ideas out here. Inevitably we're going to get some students giving working out and others giving reasoning. What about writing ... just thinking out loud here, you tell me if this is rubbish ... having a line down the board somewhere, or on the whiteboard, on one side writing responses some students give you, on the other side writing others. Not telling them which one is which.*
- *TC: And then asking them.*
- *PW: And then saying 'Why have I written some of those here? And why have I written some of them here? What's the difference between them?' ...*
- *TC: I think ... for a think-pair-share, in terms of reasoning, would there be enough things on ... each side? And ... would they be easily transcribed on the board?*
- *PW: Well, you might have to summarise it, yes?*
(RGM1, #27-37)
- *AF: I think you could have a couple of each, that would give you four points of reasoning ...*
- *PW: Because you don't have to write down everything. If you just made sure you had two of each ... and you might actually have taken down responses, but only written down two of each ...*
(RGM1, #44-45)

For some strategies (in this case 'scribing'), the idea appeared to develop almost spontaneously, in response to questioning:

- *PW: I just wonder if there are any other generic things we could do that were different, which would help students to see the pedagogy?*
- *AF: Just write down what they're saying on the board and ...*
- *TC: ... in a very ... equal way ...*
- *AF: ... open ...*
- *TC: ... not giving any power to ...*
- *AF: ... any weighting, either, to any Yeah. And just sort of say: 'OK, why am I writing all these answers?' And hopefully they'll be thinking about: 'OK, they're all suggestions, and as important as each other'.*
(RGM4, #112-117)

[note #113-117 above also used in 2.3]

Negotiations around some of the strategies focused on how to incorporate the idea within the constraints imposed by existing practice (in this case relating to 'classifying'):

- *PW: So, its great teaching them all these methods, but the thing you don't get if you teach in a very mechanistic way is you don't get that opportunity to say: 'Well, what sort of problem is this? How does*

it differ to this one? Which ones are the same?... A discussion around that would be quite interesting. The question is whether you've got time to do both.

(RGM5, #46)

- *AF: You could have a starter maybe, that has, like, six problems, six questions, on cards, maybe, cut out. And they can just, in pairs, decide what's what.*
- *TC: That's going to take a long time ...*
- *PW: They probably need to solve the problems first before they can sort them. We can probably sort them without solving them, but until they've solved them ...*
- *AF: What do you think if we make some very simple problems, OK just three questions, very simple, and they can just identify which one's which, three categories. And, also, that gives them a nice entry point into the main task ...*

(RGM5, #48-51)

[note this relates to 3.6 as well]

5.3.6. Consider/address constraints on practice (Practical Organisation)

Discussions between researchers included recognition that students' perceptions about what 'success' might look like would be influenced by the grading system used within the school:

- *TC: For instance, we have a grading system, and we ... sometimes I tell them: 'This is what secure means for this problem, this is advanced'.*

(RGM1, #63)

- *AF: I just think that ... because their level, or their meaning of success varies within themselves, we won't necessarily know anyway, when we see their responses, whether that's them using their own degree of success or what we think that 'success' means.*

(RGM1, #66)

TRs made use of their knowledge of students to propose more appropriate language for survey and interview questions:

- *TC: Maybe, instead of saying 'successful' we should just say 'how well' or ... not 'how well' but ...*

(RGM1, #65)

- *PW: OK. So ... could we phrase it differently? 'How well do you think you've done in this lesson?'*
- *AF: Yes, I think that's a bit more straightforward ... it's less heavy ... 'success' can mean a lot.*

(RGM1, #67-68)

Concerns over behaviour made it more difficult to focus on trying out the strategies, although TRs did not avoid trying these out with classes exhibiting more challenging behaviour:

- *AF: And doing this with a group that can be tough behaviourally is actually, hopefully, a big learning curve as well, for them and for me, because I can probably take away a lot from this, because it will be a lot easier for me to implement with all my other classes.*

(RGM3, #14)

- *AF: I think what happened is ... I thought about these questions, but I was thinking too much about the atmosphere in the classroom being calm enough to have some learning going on. So, I think that definitely, for me, that took away some of my attention from the problem at hand.*
- *PW: Yes, of course. It's so difficult to ... it's so hard to keep your line of thought when ... you know that you've got to keep an eye on what's going on over there and over there and over there.*
- *AF: Exactly.*

(RGM3, #49-51)

TRs recognised that their knowledge of classes, and close relationships built up with their students, would enable them to address any concerns over engagement of their students:

- *PW: But it's being pragmatic, isn't it? And it's making it work for the group you've got ... in the class in front of you. And it would be different for a different group ... but those are the kinds of decisions you have to make every day.*
- *AF: ... as a teacher, exactly. And that's fine. And that's where knowing your students comes in.*

(RGM3, 32-33)

There was some initial apprehension among TRs that students would be willing to engage with the strategies and recognise them as worthwhile activities:

- *PW: So, we were talking about ...?*
- *AF: ... about separating the table ... hesitation.*
- *PW: Why was there a ... hesitation to do that?*
- *AF: I think ... I probably just remember: 'Will I be able to get enough responses and keep their attention and engagement going to make this a meaningful discussion?' Given that, at that stage, you know, the few behaviour concerns in that class.*
- *PW: Because they wouldn't see the purpose of it?*
- *AF: And they wouldn't see the purpose ... and would the benefits outweigh the time spent on it?*

(RGM3, #45-50)

There was recognition amongst researchers that, as the strategies required students to adopt a significantly different approach to learning, they might not work effectively first time around:

- *PW: Well, I'm not suggesting anything, but I'm just thinking it's not surprising things don't work straight away but, having had the experience once, maybe now there's a chance to talk to students about the purpose of it, and then try it again, and then reflect on it with them, and talk about the purpose a bit more, try it again, and maybe ...*
- *PW: But really planning for it, I think, because it's hard to, I think, when you have a tough group, to ... until it becomes habit for both me and them, and then it's sort of a bit more smooth.*

(RGM3, #73)

(RGM3, #83)

There was also an appreciation that multiple strategies need not be tried out in a single lesson (as had been the case with research lessons):

- *TC: I would say that I would try to keep to one or two [strategies] in order not to lose the pace of the lesson, and for ... not to disengage because of having a discussion. So, I think there's a balance there, depending on the groups and depending on the lessons.*
- *PW: So, maybe in future if you're looking at strategies, you might do that over a two or three lesson series, so that you'd only, kind of, keep revisiting once in each lesson?*
- *AF: ... With other groups I need to be, sort of, more attentive of time and where it fits in. And a combination of: 'Let's have a discussion to draw your attention in', but: 'Let's now get on with exploring an activity or doing an activity, so that we don't lose each other's attention'.*

(RGM7, #49)

(RGM7, #53)

c) Explorative Reasoning

5.3.7. Plan how to use evidence (Explorative Reasoning)

Researchers discussed what the purpose was of collecting evidence, e.g. in relation to the survey and students' appreciation of what success in school mathematics looks like and how to achieve it:

- *TC: What is it that we want to capture? Is it their idea of ... the realisation of success in there?*
- *PW: Well, yes. Because they want ... we want them to see if they really know what success means in this lesson. So ... we've talked about what we think success means, so closely does that reflect what they think?*

(RGM1, #59-60)

There was discussion of what notes the observer should record in their research journal during the peer observations, i.e. what they might record in relation to the strategies:

- *PW: You could focus your notes on the strategies. You could talk about things that you would look for, yeah? So, you might decide ... well the language one is easy, isn't it? You might decide that the peer observer is going to make a note of all language that was used, whether it was mathematical or informal. So, if you're teaching, and Tiago hears some students at the back talking about "corners", he can make a note of that.*

(RGM5, #68)

Researchers discussed how the evidence (in this case the videos) would be used in the reflection meetings. There was some consideration of the extent to which TRs should discuss the evidence with each other in advance of the video-stimulated reflection meeting, and make notes in research journals, so that any insights generated from such discussions would be captured in the data:

- *PW: So, we'll do something quite similar to what we did individually ... We'll look at the videos in turn. And then we'll both be able to comment on it, because you'll both have been in the lesson. And we'll make use of the interviews as well.*
- *TC: Because we're not ... not only are we not going to reflect on the interviews, but we're not going to reflect on the ...*
- *AF: ... peer observations ...*
- *TC: ... on the peer observations, until that day?*
- *PW: Well, not in detail. I mean clearly you might want to say: 'How did it go ...?'*
- *AF: We can ...*
- *TC: We could even record it?*
- *PW: Well, if you do ... don't sit down and have long chats about it, is all I'm saying ...*
- *AF: But we can make notes?*
- *PW: You might want to make notes and then you can ... yes, you've still got your journals, hopefully, somewhere?*
- *AF: Yes ... yes.*
- *PW: If you have any discussions ... you can make notes in there, and then recount those at the reflection meeting. Just don't ...*
- *AF: ... We won't have a full-blown discussion ... no.*

(RGM5, #155-167)

5.3.8. Use evidence to evaluate strategies (Explorative Reasoning)

The videos provided strong evidence that enabled researchers to review the questions posed by the teacher and the responses of students during the whole class discussions, and to discuss the extent to

which these indicated whether the strategies were successful (in this case students had successfully generated their own terms in response to the 'separating' strategy):

- *PW: OK, pause it. So that was quite interesting because what was going on there is they were coming up with their own ideas for what they were going to call it ... They're all quite good ideas really: 'this one's got lots of numbers whereas this one's got lots of words' ... There's 'methods', there's 'instructions' ... I'm not quite sure about 'instructions' ... but they were kind of seeing differences themselves. You then ... well you picked up on 'working out' because that's, kind of, what we called it. So, it's good that that came up. And then you picked up on their idea of 'methods' and you gave it your own label 'reasoning', which is what we wanted to call it. So ... there's another sort of pedagogy going on there, isn't there, in that 'I'm recognising your language but, at the same time, I want to use particular terms alongside, that I want you to understand and know about'? Is that right?*
- *TC: Yes. It was trying to establish convention for one particular purpose, for the purpose of the survey, but also for later ... We can now use that term. And when they think about it, they're going to associate 'reasoning' to 'methods' to whatever they said, more like 'ideas', 'steps', and not with instructions, because they're going to separate it. And yes, I was kind of hoping that they would come up with the term. If they didn't, I would just ... as I did, I just wrote the method and then said: 'We can also call it reasoning', which is thinking 'OK, I really want you to name it the way you perceive it but also, in the future, we're going to have to use this word.'*

(RGM2, #15-16)

Researchers related the evidence from the videos to that from surveys and interviews (in these two cases relating to the 'advocating' and 'scribing' strategies), which acted as a way of evaluating the strategies from two different perspectives (triangulating data):

- *PW: OK, so up until that point ... you were asking them about the mechanics of the task, so it was like the routines of the task, yes? But your next question, I think, was more about the purpose of the task, yes?*
- *TC: Yes, so I asked him 'What are we trying to achieve there? Why do I ask you what your partner's thinking?'*
- *PW: And he replies: 'We have to have good listening skills' ... that's a reason for getting them to engage with each other's ideas and report each other's ideas.*
- *TC: Some of them already make that connection, looking at the survey responses ...*
- *PW: Any in particular?*

(RGM2, #33-39)

- *TC: This one ... 'To make sure you're listening'. But they're not saying why it is important. But someone said at some point ... 'So we can explain ...'*

(RGM2, #42)

- *PW: There's a few that say things like: 'We can share everyone's methods' ... number 21. Actually, number 2 is quite interesting, because they said: 'Test your listening and really express your ideas to help your partner answer questions'. So ... they haven't quite made that connection, have they? They're still maybe seeing it quite simplistically ... 'Oh, it's the teacher checking up on us that we are actually listening.'*

(RGM2, #45)

- *PW: And then ... how was that supported by the responses that students gave, do you think, in the interviews? ... So, the ones that took part in the discussion were ... were quite insightful and had noticed and worked out why you were doing it. In the interviews, how did that come across for the three that you were targeting?*
- *AF: I did ask a range of students, then. And I actually targeted some students who didn't have their hand up, just so I could see how engaged and insightful their answers were, as well, just to get a spread*

of responses. But yes, in terms of the students that we were targeting, with PPI students ... I thought that they'd actually took a lot in. So, what was it that I've got here? [Ennis] says in one of his responses, this is in 3.17 ...

(RGM6, #58-59)

- AF: ... Ennis said: 'So other people can know how to do it. So, they can see what they've done wrong. So, they can learn from their mistakes and write down an actual answer'. So, they seem to get the gist.

(RGM6, #61)

Evidence from the videos was used to formulate questions and to prompt discussions focused on evaluating the success of the strategies (in this case the 'separating' strategy):

- PW: So, you were using questioning a lot there ... and you sort of made it really clear why reasoning and working out are both important.
- AF: I am ... And the students that I asked ... so [Student A] is someone who is quite confident, and he's the one that gave me this earlier ... so I knew he'd be able to explain the starting point. But [Student B] is someone who is easily distracted, and if [Student B] understood the difference between the two, then most likely most other students had got some gist of the difference, as well ...
- PW: I just wonder ... how much of the questioning is about the strategy that you're using ... there wasn't much conversation about: 'Oh I can see why you've drawn a line down the page, and why you've used that to ... and why you've ...'. And maybe this would have been drawn out, like we said, if you had used the same student's response, but some of it went on one side and some of it went on the other.

(RGM3, #36-38)

5.3.9. Relate evaluation to own/existing practice (Explorative Reasoning)

TRs used the evaluation of the strategies to question their own practice, e.g. comparing the type of questions that they would normally have asked to those developed through the project:

- PW: Because the distinction here is you're asking: 'Why am I doing this?' rather than 'What's the purpose of ...?'
- TC: Yes, I sometimes do that as well. But, at this point, I wouldn't ... This is what I think I could, now reflecting on the video, I would ask them: 'Why is it important for you to know this distinction?'
- PW: So ... maybe having this focus means you've persevered with this type of questioning more than you would do normally?
- TC: ... Usually the kind of 'why' questions were more targeted at maths ... 'Why do you think this works in this way?' ... not necessarily 'Why does the teacher present the work to you in this way?'

(RGM2, #10-13)

- PW: So, it's allocating time specifically to talk about the pedagogy that you're using, which you wouldn't normally do. You might ask 'why' type questions, but you wouldn't spend ... how long did you spend having that discussion?
- AF: Probably about five minutes or so ... yeah.

(RGM4, #17)

- TC: And that's something I've been using [scribing]. It was not the first time that I used it with them. I'd never asked them: 'Why do I do it?' So, that's the difference ... I've never asked them 'Why?' It's good to see that, when they're responding now, they understand that.

(RGM6, #109)

- AF: And, again, like you, I hadn't made the scribing explicit. I obviously talk about the importance of writing the working-out, and your methodology, but I've never opened it up to the class as explicitly as I did at that time, in getting those answers from them.

(RGM6, #131)

- *PW: And I think as part of your normal teaching you might ask them: ‘Why am I saying “dimensions” and not “measurements”?’ Or: ‘... “dimensions” is a term you need to be familiar with because that’s what they might call them in the exams’.*

(RGM2, #23)

TRs articulated clearly the differences between the practices developed through the project and their previous practices and how these related to the evaluation of the strategies:

- *TC: That’s a major difference from last year ... allocating time, thinking times and discussion times in a lesson and embedding this routine ... [which] is, I think, clearly producing good results.*
- *AF: Yeah. I think having that intention planned into your planning, and then delivering that, and giving it the importance and buying into it as a teacher ... that will hopefully help that notion of transferring to the students that it is important. And: ‘Why is it important? And let’s have a think and discussion about that’.*
- *PW: So, this year you made more time for the think-pair-share, and things like that. But what you’re saying is ... as well as that ... making more time to then take it a step further and discuss why we’re doing think-pair-share?*
- *TC: Yeah. So that’s what we’re adding because of the project. That’s very clear to me that ... thinking for ourselves: ‘Why are we asking them to think about this task? What is the purpose?’ And that comes in when we’re planning the lessons. And then asking them, or thinking for ourselves: ... ‘What should be made visible to them in a certain activity?’ And that’s coming from what we’re doing, yeah ... thinking more. It’s not just asking the ‘why?’ or ‘how do you do this?’ questions, but it’s ‘Why am I doing this? Why are you doing this? Why am I asking you to do this?’*

(RGM4, #56-59)

- *PW: So, the annotating and the think-pair-share, those are things you might have done anyway, because that’s what you’re focusing on as a department. Is what you’re saying that the discussions that went with them, the strategies alongside them, through the project, have actually helped you to make those strategies more effective? Because the students ...?*
- *TC: Students are more engaged. Students are more up for doing them independently. They see the benefits. They see that it’s a tool that they can use independently, if they are struggling with a question, or if ...’ just in general, tackling a question, especially a problem-solving question. So, I think, making that part of our teaching does have a direct strong effect Having that discussion does have a direct strong effect, I’m seeing that, definitely.*

(RGM7, #54-55)

- *TC: And the question ‘Why does the teacher want to achieve that?’ is never present at first. They are more concerned with the outcome that they think the teacher is expecting, not so much with ‘What’s the pedagogy?’ So that’s why it’s kind of interesting when you ... when that’s made more clear to them.*

(RGM4, #12-13)

TRs also demonstrated an awareness of how their previous practice had influenced the ideas developed through the project (in this case how it might inform the collection of evidence through informal surveys and the value of doing this):

- *TC: I find myself using, more and more, scales of 1 to 5 with hands up. And that’s something I use whenever ... I want to know what they are thinking about this. And I ask them to be very honest. And it just gives me a good impression of how they’re feeling without necessarily using the whiteboards.*
- *PW: So, you’re doing that more because of this project?*
- *TC: No, not necessarily because of this project.*
- *PW: You’re saying that’s the sort of thing that you would do routinely?*

- *TC: Yes ... So that would be my 'go to' way of informing my immediate next steps ... That's how I've asked them to provide feedback. But I think that could be used also for the discussions, and that's something that I'm not using so far. So, if we have a discussion about making some pedagogy more visible, it might be a good idea, instead of doing a survey ... or if we're not planning to do any interviews ... we might want to ask them just to ... self-assess, how they feel about the discussion that we had ... Or ask them to write in their books something about ... if we didn't prepare for a survey, we could just write the question.*
- *PW: Just write a question on the board and give them all a piece of paper to write their response.*
- *TC: Yeah, that would be good.*
- *AF: I think it makes them feel appreciated, as well, and it makes them feel valued ...*
- *TC: ... their opinion matters.*

(RGM7, #71-79)

5.3.10. Relate evaluation to alternative practice (Explorative Reasoning)

Researchers used their evaluation of the strategies to consider how they might develop these further to move closer towards their vision of an alternative practice, e.g. how the 'scribing' strategy might be extended to enable students to appreciate the importance of valuing their own everyday/informal language alongside mathematically precise language:

- *PW: But from the perspective of what we're trying to do here, you might also say 'OK, so why am I not correcting you directly ... why am I saying 'Yes, that's OK and I'm going to call it this' ... what am I trying to get you to see?' ... It's a potential for something else we might do in the future in terms of focusing on a different hidden pedagogy. And what might be the danger of that staying hidden? ... What would be the value of drawing that out into the open and not leaving it just hidden and just implicit?*
- *TC: I think potentially ... they may not value their own contributions as much. So ... I'm correcting them in a very subtle way. They may not create the connection between the words ... "measurements" and "dimensions" are effectively related concepts. They may not appreciate the context where you would use one or the other.*
- *PW: So, they might dismiss their own ... or they might think you're dismissing their previous use of that word, and therefore they shouldn't use it any more. But that's not what you want them to do ... You want them to carry on using it but be aware that this is another term that they need to know about, which relates to it.*
- *TC: Because ... they trust me. I'm the teacher so they're probably going to do that more naturally than thinking 'OK, actually my name, the way I'm naming this, is also valid'.*

(RGM2, #23-26)

Researchers discussed how embedding strategies, such as those tried out during the project, into routine practice would give students a greater awareness of the teachers' pedagogical rationale and be able to articulate this more clearly:

- *AF: But obviously, in the ideal world, you'd do that so often that it just, sort of, becomes embedded and they know ... why we're doing what we're doing.*

(RGM3, #37)

- *AF: It's still early days, I think it's still a work-in-progress. I'm still probing a lot and, you know, eventually I'd like to see them, you know, become more fluent in their reasoning and explanations as time goes past.*

(RGM4, #27)

Researchers discussed how making pedagogy more visible, and hence raising students' awareness of the teachers' intentions, might enable stronger relationships and greater levels of trust to develop between teachers and students:

- *TC: And I think that ... having that discussion beforehand has probably seen a greater impact than having it afterwards ... they seem to be more aware ... they were more engaged and embracing more the idea of: 'OK, so that's why we're doing this'... I think that route translates to a more long-term effect of relationship building and knowing that the teachers do ask these questions because of something. So that's: 'Let's trust them now'.*

(RGM7, #7)

5.4. Participatory Action Research characteristics

Note the characteristics of PAR used in the analysis are derived from Brydon-Miller and Maguire (2009). They are collaborative, participatory and bringing about positive social change (in this case impact on TRs).

a) Collaborative

5.4.1. Consideration/evaluation of relationships/roles

The relationship between academic researcher and teacher researchers was an explicit focus for discussions during meetings with consideration given to issues of power and the aim of ensuring all partners had equal status in the research project (in contrast to more established approaches to research). An example was the decision that teacher researchers would retain control over the videos of lessons:

- *PW: When I initially thought about this, I thought: OK, what's the alternative to the traditional model of research, where either somebody comes in with an intervention and they say to you: 'Try this out in your classroom'? And then they video you doing it, and then they take the video off and they analyse it and they come back and tell you: 'This is what happened', yeah? So, we're kind of looking for alternatives to that, where the control rests with you, if you like. And one way is we video, but you have control of the video, and you decide which bits to talk about in the reflection meeting. And the other way of keeping ... the power with you, is where you ... rather than me come in, you take notes of each other.*

(RGM5, #68)

Care was taken to recognise and address power relationships that might affect the research, e.g. after the first cycle, it was not felt necessary for PW to attend the research lessons as this might influence what the TRs did in the classroom. There was some evidence that, since PW was only able to attend on pre-arranged days, this might put additional pressure on TRs to demonstrate positive outcomes from the research:

- *AF: I think my concern was that the behaviour would take too much attention away from the lesson. As it always is, not just with you being there, but because I knew that there was a focus on what we were trying to achieve. I really wanted those strategies to be played out in a fair way, so they would then take away something from that, right? So, I think that was my concern.*

(RGM3, #90)

[Note this quote also used in Section 1.4]

The relationships between TRs and students were also an explicit focus of discussion with a deliberate attempt to put students at ease by using TRs to administer surveys and interviews and adopting an 'empathetic interviewing' approach (see Sections 1.2 and 1.3).

- *PW: I mean I was really impressed with, actually, how you managed the discussions ... the way you've helped students to ... you know, you just: 'Well, tell me more ...' and I think, actually, it says something to me about ... why should researchers, who don't know the students, go in and ask them about the*

learning, when the teachers can have discussions with them? They're at ease cos they've already built up relationships with the teachers. Probably get much more out of that discussion than we would have got if I'd sat down with those three students and talked to them, I suspect.

(RGM6, #182)

[Note this quote also used in Section 1.3]

b) Participatory

5.4.2. Teacher Researchers involved in design of research

TRs played an active role in agreeing the research design, e.g. in negotiating and arriving at the final wording for surveys and interviews, drawing on their own detailed knowledge of the students (see extended dialogues between TRs and PW in Sections 1.2 and 1.3).

TRs posed questions that demonstrated an awareness of issues of the trustworthiness of the research findings, e.g. in relation to the possible leading nature of interview questions:

- *AF: Do we want to make it that explicit? ... Is that too leading as a question? I don't know. Is that giving it away too much in terms of 'well, because they want me to know the difference'? Because it's in the question, maybe?*

(RGM1, #94)

TRs also played an active role in deciding which video clips to review and agreeing the protocols for the video-stimulated reflections during meetings (see extended dialogue between TRs and PW in Sections 1.5).

TRs frequently took the initiative in trying out new ideas for research methods, e.g. in the way they took notes during peer observations to facilitate the video-stimulated reflections and begin to think about analysing the data:

- *AF: And at the same time, I made notes on pedagogies, and moments in the lesson where strategies were discussed. So, just for me, as reflection points, that we could then talk about.*

(RGM6, #15)

- *TC: I was taking notes ... focussing on the three students and noting what Alba was doing ... When I was doing it, I was just being very succinct about the notes ...*
- *PW: ... descriptive, what was happening?*
- *TC: Yes, this is what's happening, at that point. And then, whenever the strategies were being used ... I've kind of ... coded it somehow. And the dialogues around the teachers and students' responses ... I just took some brief notes to remind myself what was happening at the time. And then, when I did that one, I did it in more detail ... just as an exercise ... seeing what's going on at that time.*
- *PW: So, it's more than a time-line of key moments in the lesson ... you've actually started to do a little bit of analysis ... from the notes that you took, yeah?*
- *TC: Yeah, the only thing I did was just to ... to make the discussion today more focused ... just colour-coded.*

(RGM6, #22-26)

5.4.3. Teacher Researchers involved in evaluation of research

TRs played an active role in evaluating the use of research tools such as surveys and interviews (see extended dialogues between TRs and PW in Sections 1.2 and 1.3).

TRs demonstrated an awareness of how inferences might be drawn from the surveys and interviews:

- *AF: I think the questions are really good, as well, from the interview. I think they draw so many interesting things. You know, some of the things they draw is: 'OK, this is how much they've obviously taken from the task', and then other times it draws their insecurities, and other times it draws answers that they think we want to hear, rather than answers that they want to, actually, respond to. So, it's interesting. I think it's interesting in many ways.*

(RGM6, #181)

There was an acknowledgement among TRs that developing and trying out research tools involving gathering evidence helped them to see more clearly and critique what they were aiming to achieve during the lesson and the extent to which they were successful:

- *PW: A couple of things you're saying there. The first one is ... by doing the sort of things we're doing in the project, making the pedagogy more visible, you're actually almost having to think about why you're doing it yourself and becoming maybe a bit more critical of the way you're doing it ... And the second one about the survey is ... by focusing the questions specifically on what your objectives are in the lesson, it kind of gives much clearer focus to what those objectives are.*

(RGM4, #62)

- *AF: And I thought: 'How amazing that that ...' I mean, not to say that the tasks we did, they could always be refined more, but how amazing those simple tasks that we sat down to plan could lead to such a deeper understanding*

(RGM6, #171)

TRs also played a critical role in evaluating the video-stimulated reflection, seeing it as a powerful tool to become more critical of how students responded during lessons (see extended dialogue between TRs and PW in Sections 1.5).

TRs were able to identify how the various research tools complemented each other, e.g. having a peer observing the lesson enhanced the benefits of video-stimulated reflection:

- *PW: So ... what do you think would have been the difference between videoing yourself, watching it back, reflecting on your own, and the way we did it through a discussion with somebody else?*
- *AF: I think it's really useful to do it with somebody else. I think, you can offer your intention, you can offer your point of view, what you were thinking when you were delivering that task. And then you can get the other person's perspective of how they saw it from their side, as well. But I think ...*
- *PW: So, it's quite powerful that that person is there in the lesson?*
- *AF: Yes. And able to see the engagement of the students and their participation. But I think, I don't know, at this point in my teaching ... I can honestly see what's going on and reflect on it. So, I think it would be just as powerful to do it on my own, especially if you're somebody who, you know, can be honest with yourself and evaluate critically. But ... there's an added bonus to having someone else in the room, who can give you a different perspective, perhaps.*

(RGM7, #88)

[Note some of final paragraph also used in Section 1.4]

TRs were able to clearly articulate a justification for why it was beneficial to choose the clips of the videos to review (see extended dialogue between TRs and PW in Sections 1.5).

5.4.4. Teacher Researchers involved in evaluation of strategies

TRs identified for themselves ways in which they might have improved their implementation of the strategies (in this case the 'separating' strategy):

- *AF: What I'm thinking from that is ... I could have written some of his reasoning down and some of his [working out] ... to make him aware that he's doing both, and the whole class, that he's doing both.*
(RGM3, #6)

[Note this quote was also used in Section 2.2]

TRs took the initiative to share notes, from the first research lessons and the separate reflection meetings with PW, in order to reflect independently on how well the strategies had gone and possible reasons why:

- *AF: Well we just shared some notes on, I guess, how our lessons had gone and how our conversations regarding those lessons (with you) had gone (separately) as well. Then we kind of talked about which strategies worked really well for us ... what was it that was a challenge in achieving what we thought was the full potential of both strategies.*
(RGM4, #6)

TRs demonstrated an awareness of the complexity of the strategies and how they might go about developing their use over time to make them more effective:

- *AF: And I still feel from time to time that there are lessons where it takes a long time for them to grasp ideas. But I think that's because what we are trying to teach them is some rich stuff that really does need time, and it needs different approaches. And I think these strategies enable us to get there. And I think persistence and just, you know, moving, editing as you're going, and kind of fitting them around the right activities as well, but giving it the time that they need, I think does help.*
(RGM4, #54)

TRs showed an appreciation of how the approach to planning and evaluating the strategies had enabled them to more effectively evaluate the extent to which they were successfully;

- *AF: For me, what's been really useful is asking those explicit questions, and making those explicit points, both before a strategy is used, and after, and seeing the difference in the impact. So, when we've got together and collaboratively planned it, and reviewed the lesson, that's been really useful too, thinking about which way to pose the questions, or make the points, and the language we use. And then, asking students to then use the same language, or their own language, but to similar effect. So, I think, really, having that conversation of what I expect, and what each strategy means, and why we're doing it, and asking those 'why' questions has been really effective.*
(RGM7, #3)

5.4.5. Teacher Researchers involved in analysis of data

PW's analysis of data from the first year of the project was presented back to TRs for comment during the second year of the project (and hence will be reported on in the report from the second year).

c) Positive social change/impact on teacher researchers

5.4.6. Impact on Teacher Researchers' classroom practice

Trying out and seeing the benefits of the strategies for making progressive pedagogies more visible appears to have made TRs more predisposed towards doing this as part of their routine classroom practice:

- *TC: When I'm doing the future think-pair-share, I'm going to ask ... I'm going to see them in 10 minutes anyway, so the lesson's going to start with a think-pair-share ... because I can't resist asking them, 'Why do you think it's important to have good listening skills?'*
(RGM2, #69)

- *AF: And they did the table and I did the 'think-pair-share'. And I've done it like Tiago, I've used it consistently ever since then. And it felt really productive, getting their responses.*
(RGM4, #19)
- *AF: I think it's really interesting to see that, once that becomes more embedded, just how much further you can get into the depth of their ... of what they're doing, and deepening their understanding of the topic, because ... that's becoming more routine, I think.*
(RGM4, #21)
- *AF: I do from time to time bring up the whole 'making it explicit', just so it's not lost. Because I feel that it's still early stages. But I think, yes. And today I taught a lesson ... I taught them Period 1 ... and I reminded them, and I asked them: 'Why is it that I'm not telling you?' And they were able to respond and say: 'This is why'.*
(RGM6, #53)

TRs reported how the strategies used with their Year 7 classes (the classes they had chosen to focus on for the project) had also begun to influence the way they taught other classes:

- *AF: I used the 'sharing what your partner has heard' with my Year 9's, and my Year 8's, who are a Set 5, because I think it's really beneficial for them to really focus on what the other person's saying, because they do often miss it out because it's a Set 5. With my top set, obviously, I think they do that quite naturally, but it's really nice to sort of formalise it ... because they're not just getting really involved within their own heads, and they're opening up to someone else's ideas as well.*
(RGM3, #66)
- *PW: Do you think this has had an impact on your practice in general, with other classes as well?*
- *TC: Definitely, yeah. I think all of these strategies and all of the pedagogies that we've used ... we are much more aware that I need and should create discussion points to make them really explicit ... So, it's benefiting other classes and myself in my classroom.*
- *PW: Do you find you're doing that all the time, or just when you think: 'Oh, I should do some of that visible pedagogy stuff in my lesson today'?*
- *TC: ... So, I'm not planning, necessarily, for those moments, they're happening as I'm teaching and realising: 'OK, this is probably something that is not going to be very clear for them, unless I make it very clear right now.' And sometimes I'm not sure if it's going to be clear or not, it might be, but I ask them: 'Why might I ask you to do this? Why am I asking you to ...?' ... With my Year 10's ... I'm constantly asking: 'Why is it good to do this in this way? ... And I did that with the Year 9's as well as the Year 10's. So, that was very, very useful, as well. I think it has an impact in other classes, yeah.*
(RGM7, #43-46)
- *PW: And what about the impact on your practice? Have you noticed a similar thing to Tiago?*
- *AF: Yes, absolutely ... I was doing things like drawing students' attention ... to annotating, and how to answer an exam question. But I wasn't having that discussion about why we were doing what we were doing. So, I think, that's something that has become a lot more natural to my teaching ever since we've been doing the project ... And, like Tiago, I don't plan for them, I just have them instinctively in the lesson where I think they fit best, at that point. And, generally, it's worked really well.*
(RGM7, #52-53)

5.4.7. Impact on Teacher Researchers' thinking/beliefs

By thinking about how to make pedagogy more visible to students, TRs began to question their own rationale for using progressive pedagogies and the extent to which they had previously been aware of this:

- *PW: A couple of things you're saying there. The first one is ... by doing the sort of things we're doing in the project, making the pedagogy more visible, you're actually almost having to think about why you're doing it yourself and becoming maybe a bit more critical of the way you're doing it ... And the second one about the survey is ... by focusing the questions specifically on what your objectives are in the lesson, it kind of gives much clearer focus to what those objectives are.*

(RGM4, #62)

[Note: above quote also used in Section 4.3]

- *TC: Yes, I think definitely I'm thinking more critically about ... I think that when we're planning, we're not thinking about why we're doing things in a certain way, but they make sense. And the pedagogy's there, we're just not making it visible to ourselves. It's, kind of, intuitively ...*
- *AF: ... it's assumed, yeah ...*
- *TC: ... we're following our own routines. ... But then asking ourselves 'What do we want to make visible? What's important for them to realise?' And sometimes it's not planned at all, it happens in the middle of the lesson. So that's critical thinking about the pedagogy.*

(RGM4, #63-65)

[Note: Some of the above also used in Section 3.2]

TRs became more aware of the extent to which students often misinterpreted the teacher's intentions:

- *PW: So, you think that's because of the awareness it's raised for you, in just how little they actually understand generally about teachers' intentions? Perhaps an awareness that you didn't have before we focused on it as part of the project?*
- *TC: Definitely ... We need to ask them if they realise what we are doing, and then we can have a discussion about it. And that's one of the outcomes of the different things that we tried ... that sometimes we might try to guess and to read what they may be getting from the strategy, but until we ask them, we can't be certain.*

(RGM7, #47-48)

TRs began to reconsider their own notion of success in school mathematics:

- *AF: But it's interesting to consider, you know, is that success? Is that, you know, do we want them to kind of take that away as success?*

(RGM6, #138)

[Note: above quote also used in Section 5.3]

TRs became more aware of the impact of the project on building relationships of trust with students:

- *... I think that route translates to a more long-term effect of relationship building and knowing that the teachers do ask these questions because of something. So that's: 'Let's trust them now'.*

(RGM7, #7)

[Note: above quote also used in Section 3.10]

5.4.8. Impact on Teacher Researchers' agency/efficacy

There was some initial apprehension from TRs that the students might not respond positively to the strategies (see also Section 3.6):

- *AF: I think I was afraid when it happened the first time that it wouldn't have worked. And I think it actually worked because we gave them the time.*

(RGM4, #41)

However, through trying out and evaluating the strategies, TRs became much more confident that they would have a positive effect on students:

- *AF: And it was very powerful, the effect that the tasks and the activities had on how they responded ... It's surprising how much they took from it, from the tasks ... in a good way ... it makes me believe more in the power of what we're doing.*

(RGM6, #171)

TRs began to articulate a clear picture of how they might develop the project ideas in future and embed them into their classroom practice:

- *TC: So, already I can think of 1) how to be more clear with my questioning, 2) how can I improve it or add to it, so they can appreciate the implications of my teaching and their learning? So, why do I do things in a certain way? I'd like to imbed that into their thinking, I'd like them to think critically about 'Why does the teacher ask me to do this in this way?' Because that will enable them to read in other situations in the future.*

(RGM2, #69)

- *AF: I was thinking about this actually the other day ... it starts with us and our planning and the way we think about what we're presenting to the students first. And thinking critically, and 'What do I need? What am I doing that I need to then make visible to them in turn?' ... You can plan for it. I think we're in a really good position in the sense that we are planning a whole scheme of work for Year 7's at the moment and we are uploading lots of resources as we're going. And Tiago and I are both playing an active role with that in the department. So, we could use what we are doing here, when we are planning those activities, to add in those questions, make them visible on the board, so that other teachers ... We'd probably have to talk about it in department meetings to bring everybody else up to what we're doing and why we're doing that.*

(RGM4, #75)

5.5. Impact on students

5.5.1. Appreciation of pedagogical rationale

Initially there was some misunderstanding of the teachers' intentions with a common misperception that the pedagogies were primarily aimed at ensuring compliance with the teachers' wishes:

- *TC: And on the other strategy, of asking them to share what their partners had said, they kind of ... We were of the same conclusion that we did when we had our discussion ... that they understood [what] was meant to happen but the reasons why I wanted that to happen were not clear to them. And that was evident from the survey responses. So, they realised that they wanted more ... they thought I wanted that because ... just to make sure that they were listening, and that was not at all the case.*

(RGM4, #9)

- *AF: Because I think, similarly to Tiago's class, when I saw their survey responses, they'd said 'Well, I'm ... Miss is checking to see that I'm listening to my partner. That's why she's asking me, instead of my partner, for their own answer.'*

(RGM4, #21)

Explicit discussions of pedagogical rationale during the lessons enabled some students to develop an appreciation of the teachers' intentions:

- *TC: And then they came up with responses. It was not easy at first, but eventually they said it's good to listen to what your partner said. And I asked 'Why is it good to listen to them? What do you have to gain from them ... from listening to what your partner's saying?' And they realised that they might not ... might learn something that they don't know.*

(RGM4, #15)

- *TC: I think mine, only when I make it very explicit do they start thinking in terms of these questions. When I stop. When I ask them 'Why am I doing this right now? Why did I do this in this way?'*

(RGM4, #32)

- *PW: What's really interesting is they're noticing less that the teacher's doing something different. So that ... to me that kind of suggests how important it is to actually draw it out and make it explicit. Because otherwise they're not noticing.*

(RGM6, #76)

Over time, by making the teachers' intentions explicit through the strategies and classroom discussion, students were beginning to develop a greater appreciation of the teachers' pedagogical rationale:

- *PW: And they didn't hesitate, did they? They were ready to explain the other person's perspectives, which is quite impressive. Yes, so what you said to them, maybe because it's become routine, what you'd said to them before had prepared them to do this and they did it well.*

(RGM2, #60)

- *TC: But then now I think the focus is already changing for them to ... they're really trying to explain to their partners because they know that's the rule of the game.*

(RGM2, #63)

- *TC: I asked them the same question and they now realise that ... because it's to check if you're listening AND to ... because we can learn from our partners. And, so, they have ... they are much more aware of that. So that's being embedded now. So that's something positive that didn't happen from that lesson, but later on.*

(RGM4, #11)

- *AF: And I think, same to what you've seen with your class, they're now coming up with those reasons immediately without having to prompt or probe further, more. So, I think they're starting to understand the idea behind it.*

(RGM4, #21)

- *TR: It's not so much, kind of, like 'She's asking me this because she ... if I don't do this, I will be punished'. It's so much more that 'OK, she's asking me this because she actually learns something out of it that I might not know?'*

(RGM4, #34)

- *TC: From the first time that I started using them, they would interrupt themselves, like call out and try to correct it, and would say things like 'Oh that's wrong', kind of defeating the purpose. And now they control themselves and they know that ... the fact that the teacher's writing doesn't make it right. Because it's just scribing for someone else. ... I don't know that for a fact, I'm interpreting what I see, that they don't call out any more.*

(RGM6, #111)

- *AF: And you can ask them, you know: 'Why is it that I've asked you to do that?' And they will often respond. Sometimes they will still respond with just: 'Because you're checking that I'm listening', but more often than not they are expanding on that, saying: 'To check that I'm listening but also to check that I understand someone else's way of doing it'.*

(RGM7, #21)

- *TC: I think that the three ... the two boys definitely realised ... are realising more and more the purpose of the activities and the pedagogies involved. I would say the same is true for the rest of the students in the classroom. There are maybe one or two students that I don't think that it's producing the same kind of effect.*

(RGM7, #38)

Through focusing on the pedagogical rationale during meetings, which was necessary to evaluate whether or not students appreciated this rationale, TRs were becoming more aware of their own reasons for using particular pedagogies:

- *PW: Although, clearly, a spin-off is you're really thinking quite hard about what you're doing and drawing students in to, sort of ...*
- *AF: ... so thinking about ... sharing that.*
- *PW: ... thinking about what you're doing, yes.*

(RGM3, #44-46)

- *PW: We kind of, you know, engage with their idea but make sure we use the right word alongside it. Do they understand why we're doing that? Or do they just see that as a ...*
- *AF: ... correction ...*
- *PW: ... correction, just like their parents correct them when they say 'ain't'. They say 'No, say isn't'. You know, it's the same sort ... it's like nagging. Is it nagging or is it doing it for a pedagogical purpose?*

(RGM4, #100-102)

5.5.2. Engagement with project strategies

TRs reported how students were becoming more willing to engage with the project strategies, e.g. the advocating strategy:

- *TC: So, they would get a bit frustrated for not being able to say what they wanted to say ...*
- *PW: You mean because their partner isn't expressing exactly what they wanted to say as they would have wanted it?*
- *TC: Yes, or because they want to show it themselves. But then now I think the focus is already changing for them to ... they're really trying to explain to their partners because they know that's the rule of the game.*

(RGM2, #61-63)

- *PW: ... because they hadn't had that discussion before they did it, so maybe they weren't convinced of its ...?*
- *AF: I think they weren't invested in the idea enough.*

(RGM3, #59-60)

- *AF: I'm definitely doing a lot of the prompting. But I think there's higher levels of engagement with that. They see the point in having that discussion a bit more, I think.*

(RGM4, #33)

The target students (identified as disadvantaged), in particular, were willing to engage with the strategies and became more confident in doing so:

- *I think part of that is they're growing in maturity and I think the environment we've created where it's safe to have a discussion. ... I feel that, with those particular students, in general, when I was looking through their transcripts and listening to their interviews again, I got such a positive sense of how they came across and how well they'd engaged with the entire process.*

(RGM6, #79)

- *TC: Sophia had to, for the last task, the sorting activity, she actually completed all of it. So, she persevered through the lesson, she got it all right ...*

(RGM6, #80)

Increases in awareness of the pedagogical rationale appeared to be accompanied by increases in engagement with the strategies:

- *AF: I think it worked quite well with, like you said, there's some of the strategies we used in the second cycle, like the annotating ... making it visible was almost part of the modelling. So, I'm modelling it in a way that's really clear and explicit, and then they know the purpose of that is really significant. So, they buy into it more, I think, which is why that then seemed to pay off well in the class.*
- *TC: And what they have to gain from doing it. It's ... their awareness of the game, for themselves, is clear. I think that's so important, to be able to be independent and to have more ... give more meaning to the work that they record in their books ...*
- *AF: Yes, cos it's led to that conversation that we've had, that you know: 'This is ... what you're doing is an investment in understanding mathematics better, not just now, in the future of the time. And this is really valuable work, something to be proud of and something to invest in, by making annotations and improving it. And you can come back to it, and really value it, and I think take extra pride in your work', which hopefully gets them more excited ... about doing it well in the first place, as well.*

(RGM7, #10-12)

5.5.3. Appreciation of how to achieve success

Initially, students appeared to perceive success in terms of doing what they perceived the teacher expected them to do:

- *PW: 'Success is when we're kind of making the teacher happy and the teacher thinks we're behaving well', and things like that. That sort of came across in that. It sounds as though it's the same thing, doesn't it ... 'the teacher did that to make sure we're listening ... and if we're listening then we're being successful'.*
- *TC: ... They are more concerned with the outcome that they think the teacher is expecting, not so much with 'What's the pedagogy?'*

(RGM4, #12-13)

- *TC: Cos for ... at least, for Marcus, I'm still thinking that he's doing it to be compliant and not ... he doesn't see it as for his own benefit.*

(RGM6, #165)

This often manifested itself in students seeing success as completing lots of questions in a short space of time, getting answers correct, and finding the work easy, which contrasted with the researchers' perception of success:

- *PW: When you asked them the question: 'How do you know you've been successful?' I felt a lot of them were sort of saying 'I'm successful when I get the answers right ... I do lots of questions', rather than 'When I've really ... there's something I didn't know, and then I was challenged, and now I understand it better.'*

(RGM6, #114)

- *AF: And it's that ... when you're faced with a challenge, how you respond to it, more than 'Do you know the answer straight away?' or 'Do you know how to do the question straight away?'*
- *PW: So, you can't be successful unless you've experienced challenge ... ?*

- AF: Exactly.

(RGM6, #152-154)

Researchers discussed and developed a greater awareness of how students' perceptions of success differed to their own:

- PW: *They're still maybe seeing it quite simplistically ... 'Oh, it's the teacher checking up on us that we are actually listening. Because it's important to listen to other people. Because it's polite ... that's what well-behaved children do.' But, actually, we want them to see more than that, don't we? We want them to see that, by listening, you know, you really have to engage with other people's ideas to be able to express them.*

(RGM2, #45)

- AF: *And, also, during the interviews, when students said: 'I got the answers right', my immediate response to that was: 'OK, that's fine. Is there any other way, also?' Because it feels like that whole 'I've only done well, I've only been successful, I've only enjoyed this lesson because I've got the answers right. And it's like that's the 'end all' and 'be all', and it's so much more than that, obviously.*

(RGM6, #117)

- TC: *And, unless we tell them at the start of the lesson: 'This is something where you can measure your success'. ... we know what we would like them to be doing. When they start the lesson, they have their own idea of success and how they're going to measure it.*

(RGM6, #142)

- PW: *And yet they're seeing success as not having experienced challenge, because challenge means getting things wrong, and being stuck, not knowing what to do. And when they don't know what to do, they don't enjoy it, they don't see themselves as successful?*
- AF: *Yes. Fear of failure.*
- TC: *Failing a question is not being successful. Whereas learning ... 'OK, you've made a mistake, you're now making a note', so that's being successful. I think we need to make that explicit.*
- PW: *So, maybe ... I mean this might be something to focus on in future, you know, can we shift their idea of what being successful means, so that it is more in line with what we see as success? And, hopefully, what we see as success is more in their longer-term interest, what they need to do to be successful.*

(RGM6, #157-160)

There was some discussion as to whether it was desirable for students to see success as emulating what the teacher was doing during the lesson:

- AF: *I think it was Keira. She said: 'The other way that I knew I was right', she said, 'is because I was doing what you ... I had done what you'd done before'.*

(RGM6, #121)

- AF: *So, she's thought 'OK'. It's that reassurance, that: 'If Miss is doing it, and I've been doing it already, then it must be the right thing.'*

(RGM6, #125)

- PW: *Is it a good thing that they see success as emulating what the teacher's doing?*

(RGM6, #132)

- AF: *But it's interesting to consider, you know, is that success? Is that, you know, do we want them to kind of take that away as success? Because, obviously, one of our strategies is to write down whatever they write, which is correct and incorrect. So, in that behaviour, ... I'm not necessarily always doing the*

right thing, I'm doing the wrong thing for the right message, if that makes sense. So, we want them to be able to be more objective about things, and not just take everything that we do ... literally.

(RGM6, #138)

[last quote also used for 2.3]

5.5.4. Engagement with progressive pedagogies

TRs were seeing a general increase in students' engagement with progressive pedagogies, although it was not clear the extent that this could be attributed to the research project. There was a feeling that the more students saw themselves and others succeeding through progressive approaches to learning and discussing strategies, the more they engaged with and bought into these approaches:

- *AF: So, it's not so much that I'm asking them 'What's the correct answer?' and they're having to search for the correct answer. It's that 'This is a discussion, and we're sharing ideas, and you can tell me what you think because that's how we learn.' So, I think there they're buying more into the concept.*
(RGM4, #37)
- *AF: And I think the more we do it, I think the more they do buy into it, because there's even more engagement. And they know, when that discussion is happening, what it could lead to, because they've had that experience of listening, engaging with it, and that paying off in their work, I think now. So, I think it's that self-fulfilling prophecy of: 'I know if I fully engage with this, it can lead to this'.*
(RGM6, #88)
- *AF: I think it's several factors, yeah, it's them settling into school now that they've had time here. It's probably us as a department getting better at delivering mixed attainment lessons as well. But it's also, AF: I think, we can tell the difference, especially with our groups. The level of engagement is so much higher. And I think, because the quality of discussion that we're having with them, I think they want to know, they are invested. They know that this is having an effect. They can see, I think, the work that they're presenting, and their understanding, is getting better. So, I think it's that, sort of, momentum, positive momentum of: 'I know this is helping me, and I will pay attention and engage with it' ... on a subconscious level, probably.*
(RGM7, #26)
- *AF: Both of them [students who were not targeted] are more positive. Both of them are doing far more work and engage with work more than they were before. So, it's probably just a matter of time.*
(RGM7, #32)
- *AF: I think this is the other thing, they look at each other's work and I think they buy into it even more, as well. So, I think, when one learns ... when one of the pair learns to do it well, then they're seeing the other person ...*
- *PW: ... when they're seeing the other person being successful?*
- *AF: ... really successful, yeah. So, making notes, or responding, or asking questions. Yeah, I think that spurs them on even more, I think. They get to see what success looks like.*
(RGM7, #39-41)
- *PW: So, the annotating and the think-pair-share, those are things you might have done anyway, because that's what you're focusing on as a department. Is what you're saying that the discussions that went with them, the strategies alongside them, through the project, have actually helped you to make those strategies more effective? Because the students ...?*
- *TC: Students are more engaged. Students are more up for doing them independently. They see the benefits. They see that it's a tool that they can use independently, if they are struggling with a question, or if ...' just in general, tackling a question, especially a problem-solving question. So, I think, making*

that part of our teaching does have a direct strong effect Having that discussion does have a direct strong effect, I'm seeing that, definitely.

(RGM7, #54-55)

[Note last two points used also in 3.9]

There was recognition that the success of the project would not be seen immediately and that improvements in students' attitudes could only bring about long-term improvements:

- *PW: Let's suppose these strategies were really successful and it engaged the children and they started to realise what they need to do to move forward, that still doesn't mean there's going to be an immediate improvement in their mathematical understanding and attainment. It just means, hopefully, over the next few years, they're either not going to fall any further behind, or that they might even catch up a little bit. So, in a way, the only thing that we can notice straight away is the change in attitudes towards learning.*

(RGM7, #31)

5.5.5. Appropriateness of responses to tasks

Researchers discussed in detail the appropriateness of the language that students used in their responses, which appeared to be an important aspect of success:

- *PW: So, that was quite interesting because what was going on there is they were coming up with their own ideas for what they were going to call it ... they're all quite good ideas really, 'this one's got lots of numbers whereas this one's got lots of words' ... there's 'methods', there's 'instructions' ... they were kind of seeing differences themselves.*

(RGM2, #15)

- *PW: But you're also deliberately introducing new words to them that you want them to become familiar with, and you want that to become part of their vocabulary and something they use all the time in maths lessons to describe this particular thing.*
- *TC: That's funny that you mention it because earlier there was another episode where they were giving names for the dimensions and I really wanted them to use the term dimensions. And then they said 'measurements', which is good enough, but the word 'dimensions' is another clear example that they may be asked in a test ... 'What are the different dimensions?' So, again, yes, that's important ...*

(RGM2, #21-22)

Over time, TRs noticed that students were beginning to respond in more appropriate ways as they grew in confidence and developed a greater awareness of teachers' pedagogical rationale:

- *AF: They're gaining confidence, a few of the kids that maybe before would have been a little bit more disruptive as well so, as a result, are more focused. So that's really interesting to see, you know, that sometimes, as we kind of suspect maybe, behaviour can be because there's an underlying under-confidence there. So now that know what they're doing, they're sort of more of an incentive to feel, you know, 'I can give this a go' ... that sense of 'I can try'*

(RGM4, #23)

- *TC: But also ... because these were recurring strategies, they now seem to be aware of them. And they know: 'OK, so the teacher's going to ask me to ...'. Whenever we say: 'Be prepared to share ... your partner's reasoning', they are now more aware, definitely. There's a difference in the response from the beginning.*
- *PW: So, you're seeing a difference in how they respond to your teaching?*
- *TC: Completely. And the language that they use to present their partner's work ... Sometimes, without making that explicit now, which is kind of interesting. We no longer need to make it visible. They know*

that the conversation and collaboration is expected, and that they have to share, not just what they say, but what work that they've done to get there.

(RGM7, #18-20)

5.5.6. Responses/engagement of all students

There was some feeling that other students (rather than those targeted for interviews), including other disadvantaged students, might be the ones to benefit the most from the project:

- *AF: There are students in the class that I feel, that are not PPI, but are probably more in need of those tools as well, that are probably benefiting more. So, actually, the students I think, whilst they are a good reflection, there are other students as well that I feel that are benefiting.*
- *PW: What sort of students are they that could potentially benefit?*
- *AF: Lower attaining students. Students from, again, lower economic ... socio-economic backgrounds. Parents work long hours, from what I can tell. And less focused ... a little bit more distracted, little bit less mature as well, an understanding of what's needed for success. So, I think getting through to them is the hardest. But then also, when it does happen, the most rewarding for them. So, I think it's beneficial if you look at the bigger picture, for all of them really.*
- *TC: I agree with that. I think for all of the students who would benefit from ... or are benefiting from these strategies, not just PPI students. I think the kind of pedagogies that we are using are covering some holes or some unexplained parts of their learning, why they're doing things the way they are. And why are we asking them to do it in that way? So that's really important for all of them, I think.*

(RGM6, #84-87)

- *PW: So, when you talk about you're seeing an improvement in students' responses, is it all across the board, is it particularly true for disadvantaged students?*
- *TC: I would say it's across the board. It's not just for disadvantaged students ... But, in general, the class is responding well and better than in the beginning, they're more receptive.*

(RGM7, #24-25)

- *AF: But as a whole class, the discussions that we have are so lovely and so nice, and really engaged. I think that's probably the only way to be able to tell right now is how engaged they are.*

(RGM7, #28)

5.5.7. Responses/engagement of target students

There was some evidence that the target students were responding particularly well to the strategies:

- *But with [Sophia] and [Keira], especially with [Sophia], confidence is an issue. So, her being able to have a mature thoughtful discussion on some very subtle ideas was really positive. And I think it showed that she'd engaged, and she'd taken it on board, even if I didn't necessarily ask her.*

(RGM6, #79)

- *AF: They're both hard-working girls, both Sophia and Keira, and that resilience has really come through. And I think, because we're equipping them a little bit more with tools, and we're making those tools ever more explicit, it becomes a lot easier for them to pick up those tools and use them. So, yeah, that's the feeling that I get, that it is helping them.*

(RGM6, #82)

- *TC: I think Mary definitely understands all points of ... all of the pedagogies involved. So, she understands that taking notes is a good thing. She says, at some point, using her own words, that: 'It might be useful when I come to the day before the test. I can't find the teacher. I'll go through my notes and, if it's a long time after the actual lesson, I can then remember what I did wrong, and try to avoid to*

do it again'. She gets that very clearly. Also, she gets ... all three get the idea of scribing, why the we do it on the board.

(RGM6, #109)

- TC: But definitely the students that we interviewed, just in my classroom ... in my class, they all seemed to respond to that ... to the strategies. And they're very good at being ready to share and to verbalise other people's reasoning, which is good.

(RGM7, #25)

- TC: I think that the three ... the two boys definitely realised ... are realising more and more the purpose of the activities and the pedagogies involved ... And Mary, one of the focus students, I think she's been really on it. It's beautiful to watch. And she's constantly annotating and she's very good at presenting other.

(RGM7, #38)

Although there was still evidence to show that some target students did not fully understand the teacher's intentions all of the time:

- TC: It's interesting because Marcus says, at some point, that is because the teacher wants to see if I've done it. So, he does He takes notes because he thinks that I want to see it in his book. So, he doesn't appreciate the value of why annotating is important. But he does ... some of the pedagogy. So, he's one of the students that still is not fully understanding why I'm doing the things that I'm doing.

(RGM6, #113)