Intergenerational Redistribution in the Great Recession*

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Abstract

The Great Recession saw sharp drops in labor earnings and in asset prices. How were the welfare losses from these declines distributed across different age groups? To answer this question we construct a stochastic overlapping-generations general equilibrium model in which households are subject to aggregate shocks that affect both earnings and asset valuations. A calibrated version of the model predicts that younger cohorts fare better than older cohorts when the equilibrium decline in the price of risky assets is large relative to the decline in wages. This finding emerges since the old partially rely on sales of risky assets to finance consumption, whereas the young accumulate wealth for life cycle reasons, and now purchase assets at depressed prices. In a calibrated version of our model, aggregate net worth declines by 20 percent relative to trend over a ten year interval, consistent with the experience of the U.S. economy. Average labor incomes decline 10 percent, but young households face even larger earnings declines. The model predicts that the Great Recession implied modest average welfare losses for households in the 20-29 age group, but very large welfare losses of around 10% of lifetime consumption for households aged 60 and older.

Keywords: Great Recession; Overlapping generations; Asset prices; Aggregate risk

JEL classification: E21, D31, D58, D91

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1 Introduction

The Great Recession was the largest contraction in the United States since the Great Depression. Aggregate output and household incomes fell ten percent below trend, while the prices of risky financial as well as real assets –especially stocks and real estate– declined twice as much as incomes. The goal of this paper is to explore the welfare consequences of a severe and long-lasting recession such as the Great Recession that features a sharp fall in labor earnings and a collapse in asset prices. Our main objective is to study how the welfare costs of such a recession vary across different age groups.

We argue that the welfare effects of large aggregate shocks are unevenly distributed across different generations. We document empirically that young households have little financial wealth, relative to their labor income, while older households are asset rich but have little human wealth, measured as the present discounted value of future labor income. In addition, young households who buy assets at depressed prices may gain from future asset price appreciation, while older households close to the end of the life cycle may die before prices can recover. A steep fall in asset prices therefore likely has more serious welfare implications for older households.

To empirically underpin our analysis, in the next section we use data from the Survey of Consumer Finances (SCF) to document how labor income and net worth vary over the life cycle. We confirm that older households indeed hold the vast majority of real and financial assets, whereas the young are financial wealth poor but human wealth rich. The same SCF data are then used to estimate the net worth losses associated with the decline in asset prices during the Great Recession, again focusing on how these losses vary with household age. To do so, we decompose net worth into different asset and liability classes, and impute losses by applying asset-class-specific price deflators to age-group-specific portfolios. Since we wish to emphasize that the asset price declines have been rather persistent, we focus on age-specific declines in overall net worth averaged over a ten year window from 2008 to 2017, rather than on the even sharper (but fairly transitory) decline in stock prices between mid 2007 and early 2009. We find that the average household experienced a decline in real net worth (relative to trend) of $112,300 during this period. These losses were heavily concentrated among older age groups: households aged 60-69 lost $183,600 on average.

These empirical observations suggest that the welfare losses from large economic downturns are unevenly distributed across different age groups in the population. However, a more complete welfare analysis requires forecasts for the future evolution of labor income and asset prices, and an understanding of how agents will optimally adjust savings and portfolio choice behavior in response to expected future wage and price changes. In the remainder of the paper, we therefore construct a stochastic general equilibrium model with overlapping generations and large aggregate shocks that affect both wages and endogenous asset prices. This model is designed with the explicit purpose of representing well the co-movement of incomes and asset prices in the Great Recession, and we use a version of the model calibrated to aggregate and micro data in 2007 to assess the distributional consequences of this specific severe macroeconomic downturn.[1] One question of particular interest that we can ask within the context of this model is whether young people conceivably benefit from becoming economically active in the midst of a large recession rather than in normal times.

The answers to these distributional questions crucially depend on the size of the decline in equilibrium risky asset prices, relative to the decline in income, in response to a negative aggregate shock. In the model, if middle-aged households have a strong incentive to sell their assets in the downturn (e.g., because they strongly value smooth consumption profiles), then equilibrium asset prices decline more strongly than income. This in turn benefits younger generations who buy these assets at low prices, potentially compensating them for the fall in earnings they experience. At the same time, we will present empirical evidence that younger households experience disproportionately large earnings losses in recessions, an observation that we will ensure holds true in our quantitative model as well.[2] Thus, the overall allocation of welfare losses from a recession depends crucially on the quantitative importance of asset price risk, the age differences in the exposure to this risk, and the age differences in the direct effect of recessions on labor income.

One challenge we face is to account simultaneously for the massive decline in the price of risky assets and a relatively constant real risk free interest rate during the Great Recession. We interpret this pattern as reflecting an increase in the equilibrium risk premium, and associate a model Great Recession with an increase in aggregate risk, in addition to a decline in labor incomes and dividends.[3] In particular, when the model economy enters the Great Recession-like state, a Great Depression-like event becomes a possibility. This time variation in aggregate risk activates a strong precautionary demand for risk-free bonds which supports bond prices at the same time that risky

[1] Note that the model is not designed to shed light on the underlying *causes* of the Great Recession. In addition, although we construct and calibrate the model with the Great Recession of the last decade in mind, its implied strong co-movement between incomes and risky asset prices is broadly consistent with the finding by Barro and Ursua (2008), Tables C1 and C2, that in *large* recessions, for the limited observations for the U.S. and for a larger sample of international large recessions, output and stock prices typically fall together. At the same time, as our calibration will make clear, the objective of the model is *not* to rationalize asset price dynamics of all classes of risky assets during typical moderate business cycles.

[2] The severe and persistent (over a decade) earnings losses of the young we model are consistent with the empirical literature that studies the labor market outcomes of young cohorts in deep recessions (see, e.g., Kahn 2010, Oreopoulos, von Wachter, and Heisz 2012, and Schwandt and von Wachter 2017).

[3] Bloom (2014) documents that macro uncertainty indeed rises strongly in recessions.

asset prices collapse. For plausible probabilities of a Great Depression the model delivers empirically realistic dynamics for the prices of both riskless and risky asset prices, making it a suitable laboratory for quantifying their welfare implications. At the same time it broadly rationalizes the age variation in portfolio composition observed in the SCF: older model households endogenously hold relatively safe portfolios coming into the Great Recession precisely because they are relatively asset-rich and human-wealth poor, and thus especially exposed to asset price fluctuations.

After having argued that our model economy paints an empirically plausible picture of the Great Recession along its key asset pricing and portfolio dimensions we turn to the normative evaluation of this event. In terms of welfare, a model Great Recession is associated with massive expected welfare losses for older households (on the order of 10% of remaining lifetime consumption), but much smaller losses (approximately 2% of lifetime consumption) for the young.

Our paper builds on two broad strands of the literature. First, methodologically, we use a stochastic overlapping generations (OLG) model with endogenous portfolio choice to study the implications of aggregate shocks for asset prices and intergenerational redistribution. The literature that analyzes asset prices and portfolio choice in stochastic life cycle economies includes Labadie (1986), Huffman (1987), Ríos-Rull (1994), Storesletten, Telmer, and Yaron (2004, 2007), Constantinides, Donaldson, and Mehra (2002), Kubler and Schmedders (2013), and, with a focus on housing, Piazzesi and Schneider (2012), Corbae and Quintin (2015), Favilukis, Ludvigson, and Nieuwerburgh (2017), as well as Kaplan, Mitman, and Violante (2017). Ríos-Rull (1996) investigates the properties of business cycles in this class of models, while Ball and Mankiw (2007), Smetters (2006), Krueger and Kubler (2004, 2006), Miyazaki, Saito, and Yamada (2009), Campbell and Nosbusch (2007), and Hasanhodzic and Kotlikoff (2013) analyze the allocation of aggregate consumption risk across different generations.

Second, in terms of economic substance, we study the distributional impact of a large shock to aggregate output and asset prices, focusing on the Great Recession as our application. Hur (2018), Peterman and Sommer (2019) and Menno and Oliviero (2016) also investigate the consequences of the Great Recession in life-cycle models, but focus, respectively, on the roles of borrowing constraints, social security, and house prices and mortgage debt for the distribution of welfare losses. Related, a number of papers study the distributional consequences across age cohorts of other types of large economy-wide shocks. Our analysis is similar in spirit to the study of Doepke and Schneider (2006a,b) who focus on the inflationary episode of the 1970s and, to a lesser extent, to the study of Meh, Ríos-Rull, and Terajima (2010). Other work employs OLG models to investigate the impact of large swings in the demographic structure of the population on factor and asset prices, as well as on the welfare of different age cohorts. Examples include Attanasio, Kitao, and Violante (2007), Krueger and Ludwig (2007), and Ríos-Rull (2001).

The remainder of this paper is organized as follows. In Section 2 we present the life-cycle facts on labor income, net worth, and portfolio allocations that motivate our quantitative analysis and that we use later to calibrate the model. In Section 3 we set up our model and define a recursive competitive equilibrium. Section 4 studies a simple three period version of our model in which the key asset price mechanism can be analyzed in the most transparent way. Section 5 is devoted to the calibration of the full quantitative model, and Section 6 reports the results from our thought experiment. Section 7 concludes. Details of the computational approach, proofs, additional theoretical results, and a discussion of the robustness of our findings to alternative parameterizations and modelling assumptions are relegated to the Appendix.

2 Data

In this section we document the life-cycle profiles for labor income, net worth, and portfolio composition that motivate our focus on heterogeneity along the age dimension and that will also serve as inputs for the calibration of the quantitative model. The need for detailed data on household portfolios leads us to use the Survey of Consumer Finances (SCF) as our primary data source. The SCF is the best source of micro data on the assets and debts of US households.[4] The survey is conducted every three years.

The 2007 survey captures the pre-recession peak in asset prices, and we use it to construct life-cycle profiles for labor income, total income, assets, debts, and net worth in the pre–Great-Recession state in Table 1. These profiles are constructed by averaging (using sample weights) across households partitioned into 10-year age groups. We divide total income into an asset income component and a residual non-asset-income component labeled labor income.[5] We measure net

[4]One advantage of the survey is that it over samples wealthy households, using a list based on IRS data. Because the SCF weighting scheme adjusts for higher non-response rates among wealthier households, it delivers higher estimates for average income and wealth, compared to other household surveys, such as the Current Population Survey (CPS) or the Panel Study of Income Dynamics (PSID).

[5]Asset income is defined as interest or dividend income (minus interest payment on debts), income from capital gains and asset sales, one-third of income from sole proprietorship other business or farm, and an imputation for rents from owner-occupied housing. We set these imputed rents equal to the value of primary residence times the rate of return on all other assets. This rate of return is computed as asset income (excluding imputed rents) divided by aggregate assets (excluding the value of primary residences and the value of vehicles).

Labor income is all other income: wage and salary income, two-thirds of income from sole proprietorship other business or farm, unemployment and workers' compensation, child support or alimony, income from social security other pensions or annuities, and income from other sources. Since labor income includes social security and defined worth as the value of all financial and non financial assets, less the value of all liabilities.[6]

From Table 1 we observe that in 2007 average nominal household income in the SCF was \$83,430, while average household net worth was \$555,660, for a net worth to income ratio of 6.66.[7] Average household assets amounted to \$659,000, with an average rate of return, measured as asset income divided by wealth, of 3.1%. Average household debts came to \$103,300, with an average interest rate on debts of 6.4%. The share of net asset income in total income was 16%. Young households had negative net asset income, despite having positive net worth, reflecting the higher average interest rate paid on debts relative to the rate earned on assets.

Table 1: Income and Wealth Over the Life Cycle (2007 SCF, \$1,000)

Age of Head	Total Income	Labor Income	Asset Income	Assets	Debts	Net Worth
All	83.43	70.07	13.36	659.00	103.34	555.66
20-29	38.83	39.68	-0.85	130.66	53.30	77.36
30-39	69.83	68.68	1.15	335.87	136.12	199.75
40-49	93.40	84.97	8.43	598.21	132.62	465.59
50-59	117.97	99.56	18.41	959.77	133.24	826.53
60-69	109.06	76.15	32.90	1,156.96	104.10	1,052.86
70+	57.56	34.46	23.11	756.76	28.48	728.28

Household income follows the familiar hump shape over the life cycle, while net worth peaks somewhat later. For 20- to 29-year-olds, average net worth is 1.9 times average labor income, while for households aged 70 and older, the corresponding ratio is 21.1. Thus, the old are much more exposed to fluctuations in asset prices than the young, and therefore endured much larger losses in net worth when asset prices collapsed in the Great Recession. We will ensure, through an appropriate calibration, that the life-cycle patterns of labor income and net worth in our structural OLG model are identical to the empirical profiles documented here.

While Table 1 suggests large losses for older households from a slump in asset prices, the risk

benefit pension income, even retired households will have non-trivial labor income, according to our definition.

[6]Our SCF-based measure of net worth excludes the present value of future pensions associated with defined benefit private pension plans and social security. Recall, however, that the pension income associated with these forms of wealth is part of our labor income measure. Thus, our calibration will capture how these programs impact discretionary savings.

[7]Since income questions refer to the previous calendar year, while questions about wealth are contemporaneous, we adjust income measures for CPI inflation between 2006 and 2007.

composition of net worth also varies substantially with age. To accurately estimate the losses in asset valuations by age group, we therefore further decompose portfolios by age group and examine the relative price changes across different asset classes. In Table 2 we decompose total net worth into risky net worth and safe net worth, where we define risky net worth as the value of stocks, residential real estate, non-corporate business, and non-residential property. We define safe net worth as the value of all other assets, less all debts.[8] In aggregate, risky net worth constitutes 93.9% of aggregate net worth. However, among 30- to 39-year-old, the corresponding ratio is 140.4%, while among those aged 70 or older, it is only 79.2%. These three ratios reflect three facts: (i) in the aggregate, net household holdings of safe assets are very small, (ii) younger households are short in safe assets, because they tend to have substantial mortgage debt (classified as a riskless liability) and only small holdings of riskless (and risky) financial assets, and (iii) older households have little debt and lots of assets, including a significant position in riskless financial assets.

Table 2: Portfolio Shares as a Percentage of Net Worth (2007 SCF)

Age of Head	(1) Stocks	(2) Res. real estate	(3) Noncorp. bus.	(4) Nonres. prop.	(5) Risky NW	(6) Bonds + CDs	(7) Cars	(8) Other assets	(9) Debts	(10) Safe NW
All	30.28	46.99	12.87	3.80	93.95	16.98	3.45	4.23	-18.60	6.05
20-29	13.20	77.67	43.31	1.28	135.46	13.66	15.26	4.51	-68.90	-35.46
30-39	26.27	96.47	12.73	4.97	140.44	13.80	9.73	4.19	-68.15	-40.44
40-49	30.41	57.62	12.55	3.81	104.38	15.17	4.44	4.49	-28.48	-4.38
50-59	32.70	42.40	13.53	3.72	92.35	17.02	2.79	3.96	-16.12	7.65
60-69	32.17	35.62	13.41	4.12	85.31	17.45	2.40	4.73	-9.89	14.69
70+	27.12	39.76	8.98	3.33	79.18	19.26	1.75	3.72	-3.91	20.82

Risky Net Worth (5) is equal to the sum of columns (1)+(2)+(3)+(4). Safe Net Worth (10) is the sum of columns (6)+(7)+(8)+(9). Total Net Worth is the sum of columns (5)+(10).

The distributions of net worth (Table 1) and its risky versus safe components (Table 2) jointly determine the direct allocation of capital losses across age groups when asset prices fell sharply during the Great Recession. To obtain a sense of the magnitude of age-specific capital losses during this time period we now estimate these losses using aggregate asset-class-specific price series to revalue age-group-specific portfolios.[9]

[8] Stocks include stocks held directly or indirectly through mutual funds and retirement accounts, and also include closely held equity. The category "Bonds + CDs" includes bonds (directly or indirectly held), cash, transaction accounts, CDs, and the cash value of life insurance.

[9] In Section 6.7 we compare the age distribution for net worth across the 2007, 2010, and 2013 waves of the SCF. However, changes in the cross-sectional age distributions over time reflect both the direct effect of asset price declines and also the endogenous effects of changes in saving rates in response to those capital losses. Our

To carry out this revaluation exercise we assume that 2007 SCF portfolios reflect the distribution of household net worth in the second quarter of 2007. We then revalue portfolios for each age group, and for each successive quarter, as follows. We price stock wealth using the Wilshire 5000 Index (the version that excludes reinvested dividends). We value residential real estate using the Case-Shiller National Home Price Index, which is a quarterly repeat-sales-based index. We price non-residential property using Greenstreet Advisors' Commercial Property Price Index, which is a monthly appraisal-based value-weighted index for the price of commercial real estate owned by real estate investment trusts. We price non-corporate business wealth using Flow of Funds data.[10]

Our goal is to measure the Great Recession declines in incomes and asset prices in a consistent fashion. We measure the Great Recession income decline as the deviation of real GDP per capita from trend, where trend GDP per capita grows at a constant 2 percent rate from 2007Q2 (see Section 5). We therefore also measure asset prices in real terms, and relative to the same 2 percent trend. We translate the nominal price series just described into real price series using the GDP deflator published by the BEA. We then compute deviations in these price series from a trend line benchmarked to 2007Q2 that grows at 2 percent per year. The logic for measuring price declines this way is that absent a recession one would expect asset prices to grow at a similar rate to GDP per capita.[11] We will focus on output declines and capital losses measured as averages over the 40 quarter period 2008Q1-2017Q4. We average over a ten year period for two reasons. First, ten years will be the period length in our calibrated model. Second, we want to emphasize the impact of relatively persistent asset price declines rather than the sharp but relatively transitory decline in stock prices observed in late 2008 and early 2009.

Table 3 applies real price changes relative to trend to the life-cycle profiles for aggregate net worth and its decomposition outlined in Tables 1 and 2. The first row of Table 3 reports average losses (in real terms, and relative to trend) on various components of net worth. The total loss reported is the sum of the losses on all risky assets.[12] The average household saw a

revaluation exercise here isolates the first effect.

[10] In particular, the Flow of Funds reports changes in market values for a variety of asset types by sector. We focus on the asset type "proprietors' investment in unincorporated business" for the household and non-profit sector.

[11] The real price of houses doubled between 1975 and 2006, consistent with a 2 percent trend annual growth rate (see Davis and Heathcote 2007, Figure 1). Real stock prices rose much faster over the same period, with the Wilshire 5000 Price Index averaging 5.5 percent annual growth.

[12] To quantify the decline in the total value of net worth (in contrast to the value of risky assets) requires making an additional assumption about the prices of the safe components of net worth (bonds, vehicles, other assets, and debts). If the aggregate value of safe assets (less debts) grows in real terms at the economy's trend growth rate, then the measured decline (relative to trend) in the real value of risky assets will equal the decline in the value of net worth. We will use this assumption later to justify targeting an aggregate net worth decline of 20.2% percent in our calibration.

price-change-induced decline in the value of their risky assets of $\$112,300$ between 2007Q2 and the 2008Q1-2017Q4 average, which amounted to 20.2 percent of 2007Q2 average net worth or 134.6 percent of 2007 average annual income. Asset prices reached their nadir in the first quarter of 2009, when losses averaged $\$169,700$ per household.

Table 3: CAPITAL LOSSES BY AGE GROUP

Real capital losses relative to trend from 2007Q2 to 2008Q1-2017Q4 ($1,000, 2007)								
Age of head	Stocks	Res. real estate	Noncorp. bus.	Nonres. prop.	Total	Total / net worth (%)	Total / income (%)	Total losses to 2009Q1
All	19.4	72.6	16.2	4.1	112.3	20.2	134.6	169.7
20-29	1.2	16.7	7.6	0.2	25.6	33.2	66.0	27.5
30-39	6.0	53.6	5.7	1.9	67.3	33.7	96.4	81.3
40-49	16.3	74.6	13.2	3.4	107.6	23.1	115.2	153.9
50-59	31.2	97.5	25.3	5.9	159.8	19.3	135.5	254.4
60-69	39.0	104.3	31.9	8.4	183.6	17.4	168.3	305.6
70+	22.8	80.5	14.8	4.7	122.7	16.9	213.2	190.7

Most of these capital losses are attributable to a decline in house prices. The trough for inflation-adjusted house prices was 2012Q1, by which date they had declined 37.3 percent relative to trend. Inflation-adjusted stock prices over the 2008:1-2017:4 period were 11.5% below trend on average, but this average masks dramatic variation within the period: stock prices were 49.4 percent below trend in 2009Q1, but 23.2 percent *above* trend by 2017Q4. Time series for real price changes relative to trend by asset type compared to 2007Q2 are reported in Table A-1 in Appendix A.

The remaining rows of Table 3 show that risky asset capital losses varied widely by age. Younger households lost much less, while those in the 60-69 age group lost the most: $\$183,600$ on average. At the same time, differences in portfolio composition were large enough to generate substantial age variation in returns. In particular, because younger households were more leveraged, they lost more as a percentage of their net worth: 20 and 30-year-olds lost around a third of net worth, while households older than 60 lost only 17 percent. In other words, absent age variation in portfolios, the losses experienced by younger households would have been smaller, and those experienced by older households would have been even larger.

Table 4 presents Great Recession losses on risky assets in nominal and real terms, in addition to our baseline relative to trend measure. As one would expect, asset price declines are smaller absent adjusting for trend growth, and smaller still when measured in nominal terms. Note, finally, that the decline in real net worth relative to trend was twice as large as the corresponding (and comparably measured) decline in GDP per capita.[13]

Table 4: Aggregate Capital Losses

Aggregate Wealth Losses from 2007Q2 to 2008Q1–2017Q4				
Risky asset losses ($1,000)			Net worth loss	GDP pc loss
Nominal	Real	Real rel. trend	rel. trend (%)	rel. trend (%)
17.4	61.6	112.3	20.2	10.0

3 The Model

The facts documented above guide our modelling choices. First, the substantial heterogeneity by household age in labor income and net worth requires an overlapping-generations life-cycle model. Our general equilibrium approach provides a theoretical link between the dynamics of income, consumption, and savings on the one hand, and asset prices on the other. Second, empirical portfolio allocations between risky and riskless assets display significant age heterogeneity, which translates into age variation in the sensitivity of net worth to aggregate shocks. This motivates us to consider models with both risky and safe assets. Third, the direct effect of recessions on labor income varies across age groups, largely reflecting the fact that younger workers are disproportionately likely to become unemployed.[14] This leads us to a model specification in which recessions change not only average earnings, but also the distribution of earnings by age.

3.1 Technology

A representative firm operates a Cobb-Douglas technology that takes as inputs a fixed factor K and labor L, and produces as output a non-storable consumption good Y. The firm's productivity varies with an aggregate shock z which drives aggregate fluctuations. Thus,

$$Y = z\, K^{\theta}\, L^{1-\theta},$$

where $\theta \in (0,1)$ is capital's share of output. Aggregate productivity z has finite support Z and evolves according to a Markov chain with transition matrix $\Gamma_{z,z'}$.

[13] At the end of 2017 asset prices were 7.3 percent below trend, and GDP per capita was 12.6 percent below trend.

[14] In Section B.4 we document the extent of age variation in labor income declines in the Great Recession.

We normalize $K = 1$. One interpretation of our assumption that capital K is in fixed supply is that K stands in for nonreproducible land or intangible capital. By making the stock of capital fixed, any changes in the demand for assets must translate into movements in asset prices rather than changes in the quantity of capital. This property is important given our focus on the welfare effects of large recessions that are accompanied by large asset price declines. In the standard frictionless business cycle model, by contrast, capital and consumption are the same good, and thus that model cannot generate any movements in the relative price of capital.

3.2 Endowments

Households live for I periods and then die with certainty.[15] Thus, the economy is populated by I distinct age cohorts at any point in time. Each age cohort is composed of identical households. In each period of their lives, households are endowed with one unit of time supplied to the market inelastically. Their age- and aggregate-shock-dependent labor productivity profile is given by $\{\varepsilon_i(z)\}_{i=1}^{I}$. Indexing the productivity profile to the aggregate shock allows us to capture heterogeneity across age groups in the impact of recessions on labor income. We normalize units so that $\sum_{i=1}^{I} \varepsilon_i(z) = 1$ for all $z \in Z$. Thus, aggregate labor supply is constant and equal to $L = 1$. This normalization also implies that aggregate output is given by $Y(z) = z$ for all $z \in Z$.

Labor markets are competitive, and therefore the economy-wide wage per labor efficiency unit supplied is equal to the marginal product of labor from the production technology: $w(z) = (1-\theta)z$. Note that because the aggregate supplies of capital and labor are exogenous, and the labor share of income is constant, fluctuations in z need not be interpreted simply as neutral shocks to multifactor productivity: they could equally well capture fluctuations in capital or labor productivity, or capital or labor utilization rates. Thus, our model is consistent with a range of alternative theories regarding the fundamental sources of business cycles.

3.3 Preferences

Households have standard time-separable preferences over stochastic consumption streams $\{c_i\}_{i=1}^{I}$ that can be represented by

$$E\left[\sum_{i=1}^{I}\prod_{j=1}^{i}\beta_j\, u(c_i)\right],$$

[15] We consider mortality risk as well as bequests in Subsection B.2 of the Appendix.

where β_i is the time discount factor between age $i-1$ and i (we normalize $\beta_1 = 1$). Age variation in the discount factor stands in for unmodeled changes in family size and composition, age-specific mortality risk, and any other factors that generate age variation in the marginal utility of consumption. We will calibrate the profile $\{\beta_i\}_{i=1}^{I}$ so that our economy replicates the life-cycle profile for net worth documented in SCF data in Section 2.

Expectations $E(.)$ are taken with respect to the underlying stochastic process governing aggregate risk. Finally, the period utility function is of the constant relative risk aversion form

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma},$$

where $1/\gamma$ is the intertemporal elasticity of substitution (IES).

3.4 Financial Markets

Agents trade financial assets to transfer resources over time. We consider two alternative market structures that differ in the set of assets that can be traded. In the benchmark market structure, households can trade both a risk-free bond and leveraged risky equity, and portfolio choice is *endogenous*. Denote by λ_i the share of savings allocated by a household of age i to stocks.[16] Our calibrated aggregate shock process will have the property that, conditional on the current state, only two values for next period productivity arise with positive probability; thus the economy with endogenous portfolio choice is equivalent to an economy in which a full set of Arrow securities is traded. We define an equilibrium for the Arrow securities economy in Appendix D, and in Appendix E we describe how to exploit this equivalence in the numerical methods we use to characterize equilibrium allocations. Note that while agents can structure portfolios to insure against aggregate risk in the endogenous portfolios economy, they cannot buy insurance ex ante against the date and state in which they become economically active. Thus, aggregate shocks will redistribute between existing and newly active households.

The second market structure also features two assets and an endogenous consumption-savings choice. However, the allocations $\{\lambda_i\}$ of household savings across the two assets are treated as

[16] In Appendix C we demonstrate that our model is isomorphic to a model in which households can also invest in residential real estate, as long as a) housing is not subject to trading frictions (such as adjustment costs or indivisibilities), b) there exists a competitive rental market, and c) households have Cobb-Douglas preferences over nondurable consumption and housing services. In that extended model, stock and house prices comove positively, as they did in the Great Recession. However, the extended model cannot speak to the experience of the 1970's when the prices of stocks and houses moved in opposite directions (see Piazzesi and Schneider, 2012).

exogenous parameters and calibrated to replicate the portfolio composition across risky and riskless assets by age observed in the SCF. Thus, households in this model effectively save in one mutual fund at each age, where the stock versus bond mix in the fund is age varying. Given total savings, the exogenous portfolio splits determine the relative dollar inflows into stock and bond markets, and the corresponding asset prices adjust to clear these markets. We will use this version of our model as a tool for calibrating all model parameters, and as a vehicle for assessing the quantitative importance of age variation in portfolios for our asset pricing and welfare results.

We will define competitive equilibrium recursively. The two alternative market structures differ only with respect to whether the division of household savings between stocks and bonds is specified exogenously or chosen optimally. We will therefore define a recursive competitive equilibrium only once, focusing on the economy with endogenous portfolios.

The aggregate state of the economy is described by the current aggregate shock z and the cross-sectional distribution $A = (A_1, \ldots, A_I)$ of shares of net worth plus asset income, which we label "gross wealth," where $\sum_{i=1}^{I} A_i = 1$. Newborn households enter the economy with zero initial wealth, so $A_1 = 0$. Individual state variables are a household's age i and its individual share of gross wealth, denoted by a.

The representative firm issues a constant quantity B of risk-free real bonds at a price $q(z, A)$ per unit. Each bond is a promise to pay one unit of the consumption good in the next period. We treat the supply of debt B as an exogenous time- and state-invariant *parameter* of the model. Dividends for the representative firm $d(z, A)$ are then given by aggregate capital income θz plus revenue from debt issuance $q(z, A)B$ less debt repayment B :

$$d(z, A) = \theta\, z - [1 - q(z, A)]\ B. \tag{1}$$

Note that returns to equity are risky, while the return to debt is safe and given by the reciprocal of the bond price. The supply of debt B determines the level of leverage in the economy: the higher is B, the more leveraged and risky are stocks. Let $p(z, A)$ denote the ex-dividend price of equity. Aggregate gross wealth is the value of aggregate payments to asset holders in the period, plus the ex-dividend value of equity:

$$W(z, A) = p(z, A) + d(z, A) + B = p(z, A) + \theta\ z + q(z, A)\ B,$$

where the second equality follows from the expression for dividends in equation (1).

3.5 Household Problem

Let $y_i(z, A, a)$ and $\lambda_i(z, A, a)$ denote the optimal household policy functions for total savings and for the fraction of savings invested in leveraged equity. Let $c_i(z, A, a)$ and $a'_i(z, A, a, z')$ denote the associated policy functions for consumption and for shares of next period gross wealth. The dynamic programming problem of the household reads as

$$v_i(z, A, a) = \max_{c,y,\lambda,a'} \left\{ u(c) + \beta_{i+1} \sum_{z' \in Z} \Gamma_{z,z'} \; v_{i+1}(z', A'(z'), a'(z')) \right\} \quad \text{s.t.} \tag{2}$$

$$c + y = \varepsilon_i(z)\, w(z) + W(z, A)\, a, \tag{3}$$

$$a' = \frac{\left(\lambda \frac{[p(z',A') + d(z',A')]}{p(z,A)} + (1-\lambda) \frac{1}{q(z,A)} \right) y}{W(z', A')}, \tag{4}$$

$$A' = G(z, A, z'). \tag{5}$$

The first constraint (3) is the household's budget constraint: consumption plus savings must equal labor earnings plus the household's share of gross wealth.[17] The second constraint (4) is the law of motion for the household's share of gross wealth. This constraint merits some additional explanation. Savings in equity are given by λy, and the gross return on these savings is given by $[p(z', A') + d(z', A')] / p(z, A)$. Savings in bonds are given by $(1 - \lambda)y$, and the gross return on these savings is $1/q(z, A)$. Thus, the numerator on the right-hand side of equation (4) is the gross value of the household portfolio at the beginning of next period. The household's *share* of next period gross wealth is this value divided by aggregate next period gross wealth, the denominator. The third constraint is the law of motion for the wealth distribution, which allows agents to forecast future prices, contingent on the sequence for future productivity. Let $G_i(z, A, z')$ denote the forecast for the share of next period gross wealth owned by age group i.

Definition 1. *A recursive competitive equilibrium is a value function and policy functions for each age, $v_i(z, A, a)$, $c_i(z, A, a)$, $y_i(z, A, a)$, $a'_i(z, A, a, z')$, $\lambda_i(z, A, a)$, pricing functions $w(z)$, $d(z, A)$, $p(z, A)$, $q(z, A)$, and an aggregate law of motion $G(z, A, z')$ such that:*

1. *Given the pricing functions and the aggregate law of motion, the value functions $\{v_i\}$ solve the recursive problem of the households, and $\{c_i, y_i, a'_i, \lambda_i\}$ are the associated policy functions.*

[17] In addition, consumption must be non-negative, and savings at age I must be non-negative.

2. *Wages and dividends satisfy*

$$w(z) = (1-\theta)\, z \qquad \text{and} \qquad d(z,A) = \theta\, z - [1 - q(z,A)]\, B.$$

3. *Markets clear*

$$\begin{aligned} \sum_{i=1}^{I} c_i(z,A,A_i) &= z \\ \sum_{i=1}^{I} \lambda_i(z,A,A_i)\, y_i(z,A,A_i) &= p(z,A) \\ \sum_{i=1}^{I} [1-\lambda_i(z,A,A_i)]\; y_i(z,A,A_i) &= B\, q(z,A). \end{aligned}$$

4. *The law of motion for the distribution of wealth is consistent with equilibrium decision rules*

$$G_1(z,A,z') = 0 \qquad \forall z' \tag{6}$$

$$G_{i+1}(z,A,z') = a_i'(z,A,A_i,z'), \qquad \forall z',\; i = 1, \ldots, I-1. \tag{7}$$

4 Developing Intuition: A Three Period Model

In order to develop intuition for the key mechanisms at work in our model, we now consider the special case of a three period economy ($I = 3$) designed to highlight (i) the key determinants of equilibrium asset price movements, relative to movements in output, and (ii) how asset price movements translate into differential welfare effects across generations.[18] We simplify the environment further by assuming that stocks are the only asset traded ($B = 0$ and $\lambda_i \equiv 0$), and that the aggregate shock is i.i.d and takes only two values: $Z = \{z_n, z_r\}$, where z_n denotes normal times, and z_r stands for a Great Recession-like state. Households do not value consumption when young and discount the future at a constant factor $\beta_2 = \beta_3 = \beta$ thereafter. In addition, households are productive only in the first period of their lives ($\varepsilon_1 = 1$ and $\varepsilon_2 = \varepsilon_3 = 0$). Given this set of assumptions, young households buy as many stocks as they can afford, and the old sell all stocks they own. Only the middle-aged make an non-trivial decision, namely how many shares to retain for old age. In a recession, lower stock prices have countervailing effects on the middle-aged's

[18] In Section B.1 we will use this model to compare our baseline level stationary process for aggregate productivity to an alternative specification featuring shocks to the growth rate. In Section B.2 we will use it to assess the qualitative importance of bequests for our argument.

choice. On the one hand, temporarily low current stock prices offer an incentive to reduce stock sales to exploit higher expected stock returns. On the other hand, consumption smoothing calls for selling a larger fraction of stocks, since asset sales are the only source of income for this group.

A 3 period OLG model is the simplest example in which the distribution of wealth A across different generations is a state variable.[19] Since young households start their lives with zero asset holdings and the total number of wealth shares sums to one, the wealth distribution is summarized by the wealth share of the old, for simplicity denoted by $A = A_3$. The wealth share share of middle-aged households is then given by $A_2 = 1 - A$. Their intertemporal Euler optimality condition for asset purchases reads as:

$$u'\left[c_2(z,A)\right] = \beta \sum_{z'} \Gamma_{z,z'} \frac{\left[p(z',A') + \theta z'\right]}{p(z,A)} u'\left[c_3(z,A;z',A')\right], \tag{8}$$

where consumption when middle-aged and old is given by

$$\begin{aligned} c_2(z,A) &= (1-A)\left[p(z,A) + \theta z\right] - G(z,A)p(z,A), \\ c_3(z,A;z',A') &= G(z,A)\left[p(z',A') + \theta z'\right]. \end{aligned}$$

Consumption of a middle-aged household is given by the wealth they enter the period with net of asset purchases $A' = G(z,A)$. In old age households simply consume the proceeds of their assets.

In equilibrium, the demand for shares by the young, $[1 - G(z,A)]$, must equal the number of shares that can be purchased with their total labor income $w(z)/p(z) = (1-\theta)z/p(z)$. Thus,

$$[1 - G(z,A)]\ p(z,A) = (1-\theta)z. \tag{9}$$

Equations (8)-(9) jointly determine the equilibrium pricing and policy functions $p(z,A)$, $G(z,A)$. Consumption and welfare at all ages is then determined by these functions (see Appendix F.1.)

In the remainder of the paper we measure the size of the decline in asset prices, relative to the

[19] In a two-period OLG model, all beginning-of-period wealth is held by the old, and thus this example cannot demonstrate the importance of the wealth distribution for asset prices. Furthermore, in a two-period model only the young make a consumption-savings decision and their Euler equation therefore prices assets. Thus asset prices can fall in a recession only if the young's consumption falls in a recession. Because recessionary stock prices declines reflect the fact that the young are taking a hit, the young cannot possibly gain from lower asset prices. In the three period model, in contrast, the middle-aged price the assets, and the welfare implications for the young can be positive, as demonstrated below.

decline in output z, by

$$\xi(A) = \frac{\log(p(z_r, A)/p(z_n, A))}{\log(z_r/z_n)},$$

where prices and thus the elasticity ξ, are functions of the wealth distribution A. An elasticity of $\xi = 2$, for example, indicates that the percentage decline in asset prices when the economy enters the recession is twice as large as the fall in output.

For *logarithmic utility* we can solve for the recursive competitive equilibrium in closed form, as Appendix G shows.[20] In equilibrium, asset prices are proportional to output z ($\xi(A) = 1$), the wealth distribution A does not respond to aggregate shocks, and consumption of all generations moves one for one with the shock. Since the aggregate shock is *iid*, the young, who do not value consumption in this version of the model, are exactly indifferent between being born in a recession versus being born in normal times.[21] The logic is that with log utility the young are compensated for their income losses by *precisely the same* decline in asset values, leaving their asset purchasing power unchanged.

In the data, however, asset prices fell much more than incomes, requiring an IES below one in the model. When the middle-aged are less willing to tolerate consumption changes over time, they are more tempted to *sell* assets in the recession state, and a larger equilibrium price decline is required to clear the market.[22] Furthermore, the wealthier are the asset-pricing middle-aged (i.e. the larger is $1 - A$), the larger is the downward pressure on asset prices in a recession and the larger is the equilibrium asset price decline. Thus, the size of asset price fluctuations is linked to the shape of the wealth distribution.

Figure 1 shows that the lower is the IES, $1/\gamma$ and the larger the wealth share of the middle-aged, the larger is the fall in asset prices, relative to output, in the recession.[23] For $\gamma = 2.75$, the calibrated value in the quantitative model, the elasticity of asset prices to output is $\xi(A) = 1.23$

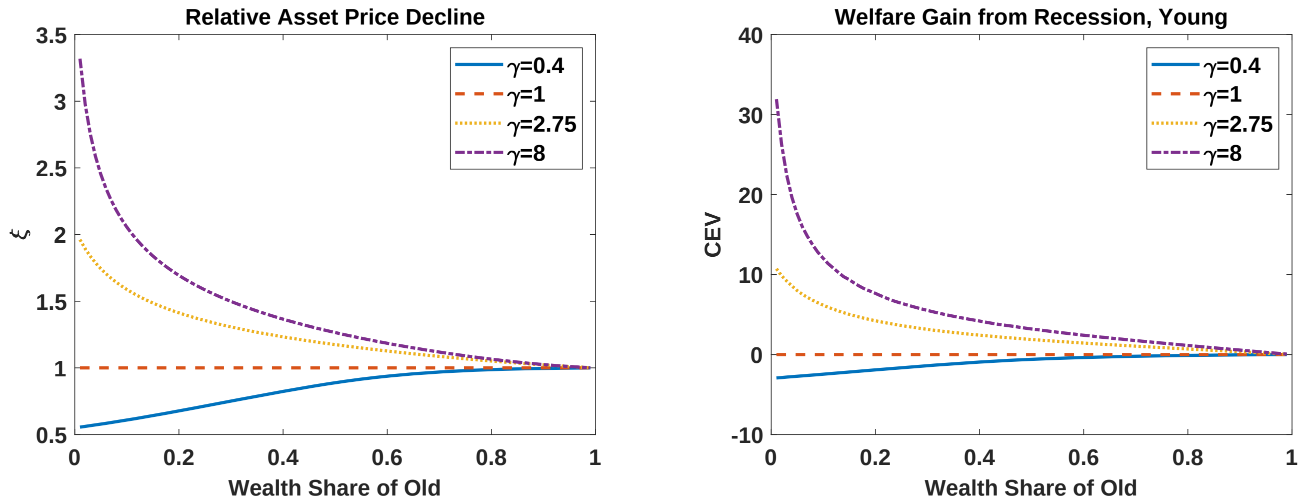
[20] This result is not specific to the three period example. Appendix G provides a full analytical solution of the log-utility case for arbitrarily many generations and for any Markov process for z,

[21] If output is positively serially correlated then the young suffer (mild) welfare losses because a recession today makes welfare losses as middle-aged more likely. The reverse logic applies if z is negatively correlated.

[22] In Appendix H we formally prove this intuition for the representative agent (RA) version of the model, by showing that given *iid* output shocks the asset price elasticity is exactly $\xi^{RA} = \gamma$.

[23] We choose parameter values for (β, θ) and the stochastic process for z consistent with the calibration of the full model in the next section. Specifically, $\theta = 0.3017$, and $\beta = \left[\Pi_{i=1}^{J}\beta_i\right]^{2/5}$ (where the β_i are the calibrated age-dependent ten-year time discount factors for the full model). The output loss in the recession z_r/z_n is calibrated to 10%. Since theoretical asset pricing results for the representative agent model and the two-period OLG model used above can only be established with *iid* shocks, we focus on this case and also assume that $z_n = z_r = 0.5$.

Figure 1: Asset Price Decline Relative to Output, Welfare



for a wealth distribution ($A = 0.41$) emerging after a long spell of high output. Thus, the price elasticity $\xi(A)$ in the three-generation OLG model is lower than in the RA model. Our quantitative model will generate an intermediate value of $\xi(A) = 2$.[24]

In this simple model the welfare consequences of a recession for the young directly depend on the magnitude of asset price movements, relative to labor income, as the right panel of Figure 1 shows.[25]. This asset price elasticity, determined by the IES and the wealth distribution, is therefore the crucial determinant for the distribution of welfare losses from the recession across generations. For example, when $\gamma = 2.75$ and $A = 0.41$, the young experience welfare gains from the recession worth 2.38% of lifetime consumption in this simple model.

This example demonstrates the crucial importance of the asset price channel for welfare, and the key role of the desire to smooth consumption through the recession (the IES) for its quantitative magnitude. The example also shows that these favorable asset price movements might induce welfare *gains* for the young from a recession. However, the simple model stacks the deck in favor of obtaining favorable welfare consequences for the young. First, the young do not consume today

[24] In Appendix I we show that as long as the IES $1/\gamma$ is smaller than one, in a two-period OLG model, $1 < \xi^{2OLG} < \xi^{RA} = \gamma$.

[25] Welfare consequences throughout this paper are measured as the percentage increase in consumption in all periods of a household's life that a household born in normal times would require to be indifferent to being born in a recession. Positive numbers therefore reflect welfare *gains* from a recession.

and thus are not directly affected by a decline in *current* aggregate consumption. Second, the labor income decline of the young is no larger than the decline in aggregate income. Third, the middle-aged have no labor income and hold no safe assets, forcing them to bear a disproportionate share of the burden of the recession. To relax these assumptions we now calibrate a quantitative version of the model to life-cycle profiles for labor income and wealth in the 2007 SCF, and insure that the model Great Recession features realistic age-dependent declines in labor income. All generations now value consumption, and make savings as well as portfolio decisions at all ages.

5 Calibration

We think of a model period as being 10 years. Agents are assumed to enter the economy as adults and live for $I = 6$ periods. The set of parameters characterizing households are risk aversion γ, the life-cycle profile for discount factors $\{\beta_i\}_{i=2}^{I}$, and the parameters governing labor endowments over the life-cycle profile $\{\varepsilon_i(z)\}_{i=1}^{I}$. The parameters defining capital's share of income and the partition of this income between bond and stockholders are θ and B. The technology parameters are the support Z and transition probability matrix Γ for the aggregate productivity shock z.

Our broad calibration strategy is to calibrate the aggregate endowment process (Z, Γ) directly from aggregate time series data and to jointly select (θ, B) such that the model reproduces the empirically observed average portfolio share of risky assets and the aggregate wealth to income ratio in the 2007 SCF. We choose the life cycle profiles $\{\beta_i, \varepsilon_i(z_n)\}$ so that the model-implied life cycle profiles for labor earnings and net worth align with their empirical 2007 counterparts. We set γ so that the decline in asset prices (relative to the fall in per capita income) in a model Great Recession matches that observed in the data. All parameter values are reported in Table 5 below. We now turn to the details of how these values are chosen.

5.1 Financial Market Parameters

We use a non stochastic version of the economy, in which the productivity shock is fixed at $z = 1$, to calibrate θ and B. Let $\bar{\lambda}$ denote the aggregate share of risky assets (stocks) in total household net worth. Let $\bar{W}$ denote the aggregate net worth to labor income ratio. Thus,

$$\bar{\lambda} = \frac{\bar{p}}{\bar{p} + \bar{q}B} \text{ and } \bar{W} = \frac{\bar{p} + \bar{q}B}{(1-\theta)} \tag{10}$$

where $\bar{p}$ and $\bar{q}$ are steady state stock and bond prices.

Table 5: PARAMETERS

Preferences and Technology				
$\theta = 0.3017$		$B = 0.0699$		$\gamma = 2.75$

Aggregate Risk

$$Z = \begin{pmatrix} 1.0152 \\ 0.9137 \\ 0.7217 \end{pmatrix} \qquad \Gamma_{z,z'} = \begin{pmatrix} 0.862 & 0.138 & 0.000 \\ 0.750 & 0.000 & 0.250 \\ 1.000 & 0.000 & 0.000 \end{pmatrix}$$

Age Varying Parameters

Age i	β_i	$\varepsilon_i(z_n)$ ($\times 100$)	λ_i (%)	$\varepsilon_i(z_r) - \varepsilon_i(z_n)$ (%)
20-29	1.00	74.6	136	-2.0
30-39	1.10	99.2	140	-4.3
40-49	1.05	122.7	104	0.6
50-59	1.10	143.8	92	-1.2
60-69	1.04	109.9	85	0.7
70+	0.93	49.8	79	12.1

Let $\bar{R} = 1/\bar{q}$ denote the steady state gross interest rate. In a non stochastic version of the model, the same interest rate $\bar{R}$ must apply to both stocks and bonds for agents to be indifferent about an interior portfolio split. Thus,

$$\bar{R} = \frac{1}{\bar{q}} = \frac{\bar{p} + \theta + \bar{q}B - B}{\bar{p}}. \tag{11}$$

Given three empirical measures for $\bar{\lambda}$ (where risky assets include real estate, as in the empirical section), $\bar{W}$, and $\bar{R}$, equations 10 and 11 can be used to solve for $\bar{p}$, B, and θ. From the 2007 SCF we measure $\bar{\lambda}$ and $\bar{W}$ by averaging across age groups.[26]

$$\begin{aligned} \bar{\lambda} &= 500.78/545.50 = 0.918 \\ \bar{W} &= 545.50/(10 \times 69.25) = 0.788 \end{aligned}$$

For the interest rate $\bar{R}$ we target an average weighted return across asset classes. Piazzesi, Schneider, and Tuzel (2007) report real returns on safe and risky assets of 0.75% and 4.75% per annum, where the latter is the return on an equally weighted portfolio of stocks returning 6.94% and

[26]In our model, each age group is assumed to be of equal size. Thus, given that we will later seek to replicate earnings and net worth for each age group, the appropriate aggregate targets are unweighted averages across age groups. Because these age groups are not of exactly identical size in the SCF, these aggregate targets do not correspond exactly to SCF population averages, but the differences are small.

housing returning 2.52%. Given that our period length is 10 years, this implies

$$\bar{R} = \bar{\lambda} \times 1.0475^{10} + (1 - \bar{\lambda}) \times 1.0075^{10} = 1.5485.$$

These values for $\bar{\lambda}$, $\bar{W}$, and $\bar{R}$ imply that $\theta = 0.3017$ and $B = 0.0699$. Model household net worth is 5.5 times annual model output, and thus net safe assets are $8.2 \times 5.5 = 45\%$ of annual output.

5.2 Intertemporal Elasticity of Substitution

The simpler version of the model explored in Section 4 indicated that the curvature parameter γ strongly influences the size of equilibrium asset price fluctuations. Given our isoelastic utility specification, γ plays two roles. First, $1/\gamma$ is the inter-temporal elasticity of substitution. In a recession, given positive expected income and consumption growth, the desire to smooth consumption intertemporally tends to drive asset prices down, and the price decline will be larger the larger is γ. Second, γ determines households' aversion to risk. When the economy enters the recession state, risk increases, and this will drive up the equilibrium equity premium. Thus, when the economy enters the recession, we expect the price of safe bonds to fall by less than the price of risky stocks.

We set the value for γ so that a model recession replicates the decline in aggregate household net worth observed during the Great Recession. Our interest is on the redistributional effects of large and persistent declines in income and asset prices. We therefore focus on replicating asset price declines measured as average deviations from trend over a 10 year post-recession period, which is also the length of a period in our calibrated model. The decline in mean real household net worth relative to 2007Q2, measured as the average deviation from trend over the 40 quarter period 2008Q1 to 2017Q4, was 20.2% (Table 4). The decline in real GDP per capita, measured over the same period and relative to the same 2% trend growth rate, was 10.0%.[27] Thus, the decline in asset prices associated with the recession was $20.2/10.0 = 2.0$ times as large as the decline in output. Given the process for the aggregate shock z described below, the model replicates this asset price decline exactly at $\gamma = 2.75$.[28]

[27] The average annual growth rate of US GDP per capita from 1969Q1 to 2007Q2 is 1.98 percent (NIPA, Table 7.1). We assume that both GDP per capita and asset prices are on trend in 2007Q2, which we take to be the date of the 2007 SCF survey. Recall that this survey is our baseline for measuring the Great Recession decline in household net worth.

[28] Estimates for the intertemporal elasticity of substitution $1/\gamma$ vary significantly, from values close to zero when estimated from macro data (see, e.g., Hall 1988) to values that can be above one when estimated from micro data on specific samples of households that are stockholders (see, e.g., Vissing-Jorgensen and Attanasio 2003). A value of $1/\gamma = 1/2.75$ is consistent with most estimates from macro data, and with estimates from micro data samples representative of the US population.

5.3 Aggregate Shocks

The aggregate shock z takes one of three values: $z \in Z = \{z_n, z_r, z_d\}$ The first state, z_n, corresponds to normal times. The second state, $z_r < 1$, is a Great-Recession-like state. The third state, $z_d < z_r$ is a Great-Depression-like disaster state. We set z_r/z_n so that transitioning from the normal to the recession state involves an output decline of 10.0%. We set z_d/z_n so that output in the disaster state is 28.9% below normal, the average deviation of GDP per capita from trend over the 5 year period 1932-1936.[29]

Since large recessions are rare events, there is limited empirical evidence to identify transition probabilities. To reduce the number of free parameters, we impose four restrictions on the transition matrix Γ. We assume that (i) the recession state can only be reached from the normal state ($\Gamma_{r,r} = \Gamma_{d,r} = 0$), and (ii) the disaster state can only be reached from the recession state ($\Gamma_{n,d} = \Gamma_{d,d} = 0$). There are two remaining transition parameters, the probability of entering a Great Recession, $\Gamma_{n,r}$, and the probability that a Great Recession turns into a Great Depression $\Gamma_{r,d}$.

We use these two parameters to target two moments. First, we want the US economy to spent a realistic fraction of time in Great-Recession or Great-Depression-like states. We define those states as times when GDP per capita is sufficiently below trend. Our times series for GDP per capita is from Maddison, and covers the 217 year period 1800 to 2016 (see Maddison Project 2013). Let Δ_t denote the deviation of GDP per capita from a Hodrick-Prescott trend in year t. We measure trend using a smoothing parameter $\omega = 6.25$ as advocated by Ravn and Uhlig (2002) for annual data We define the US economy as being in the recession or disaster state in year t if $\Delta_t < -2\%$. This threshold choice is guided by the goal of making the recession states relatively rare, so that the economy is mostly in normal times, while ensuring that the Great Recession counts as a recession.[30] Given the 2% recession threshold choice, the US economy has historically been in the recession or disaster state 14.75% of the time (32 years out of 217). This frequency serves as one target for the transition probabilities ($\Gamma_{n,r}, \Gamma_{r,d}$).

The second moment we target is a stable risk-free interest rate during a simulated model

[29] Here we set the constant trend growth rate equal to the average annual growth rate of US GDP per capita from 1929 to 1969, which is 2.50 percent. We assume the economy is on trend in 1929.

[30] The maximum deviation relative to trend during the Great Recession is -2.6% in 2009 and the maximum deviation during the Great Depression is -10.4% in 1933. These declines are much smaller than the declines relative to a constant growth trend reported above because the HP filter removes a lot of variance from the data. For this reason we do not use HP-filtered data to calibrate the size of aggregate shocks – doing so would greatly understate the welfare consequences of the Great Recession. At the same time, however, we have not used a linear filter over the entire 217 year Maddison sample to define the cyclical position of the economy because doing so would conflate low frequency changes in trend growth with cyclical fluctuations.

recession. Our motivation here is that the yield on 10-year Treasury inflation-indexed bonds – perhaps the closest empirical proxy to the bonds in our model – changed little between mid-2007 and mid-2009.[31] Given values for the levels of output in the recession and disaster states, and a value for risk aversion, the model-implied change in the risk free rate in a model recession depends on the perceived probability that a recession will turn into a disaster, $\Gamma_{r,d}$. The larger is this probability, the stronger is the precautionary motive to save and the lower the equilibrium interest rate. The model interest rate does not move on impact in our recession simulation given $\Gamma_{r,d} = 0.250$, implying that one in four large recessions turns in a disaster. This implies that the economy is in the disaster state 2.95% percent of the time.[32]

This output process has the property that model Great Recessions are large, rare events. Given a 10 year period length, the expected duration of a period of normal productivity is $10/0.138 = 72.5$ years. Risk is highest in the recession state, when the economy either recovers or deteriorates further. In particular, there are two types of model recessions. The first lasts for 10 years and does not involve a disaster. The second lasts for 20 years in total, with 10 years of recession followed by 10 years in the disaster state. Disasters are extremely rare, with one model disaster decade every $10/0.0295 = 339$ years.

Our stationary process for output necessarily implies negative serial autocorrelation for 10 year output growth rates. The implied serial correlation is -0.39, and the standard deviation of output growth (annualized and in percent units) is 0.87. The corresponding statistics from the Maddison data for the 1800 to 2016 period are -0.38 and 1.27.[33] Thus, the negative serial correlation in growth rates that is a feature of our output process is equally apparent in historical US data. The fact that our process generates slightly less growth rate volatility than observed empirically reflects the fact that the data feature frequent small fluctuations in addition to the large, rare recessions that are the only source of volatility in the model.

The stochastic output process we deduce from the data and which is the key driver of our asset pricing and welfare results shares important qualitative features with the processes estimated in the empirical and asset pricing literature on consumption disaster risk that started with Rietz (1988). Most closely related to ours is probably the paper by Nakamura, Steinsson, Barro, and

[31] Treasury Inflation-Protected Securities (TIPS) yields did fall more notably between mid-2009 and 2012, but this decline appears more secular than cyclical in nature.

[32] An alternative approach to calibrating the probability of transiting to the disaster state would be to define a disaster as corresponding to an output deviation from trend below a certain threshold. To deliver an unconditional disaster probability of 3% would require a disaster output deviation threshold of around -5%.

[33] This standard deviation is computed using the 207 over-lapping 10 year growth rates that can be computed using the Maddison data.

Ursá (2013). In their statistical model, a persistent Markov process determines whether or not the economy is in a disaster state. As long as the economy remains in the disaster state, consumption tends to decline, but when the disaster ends consumption gradually recovers towards a latent potential value. They estimate that potential consumption typically declines during a disaster, but does so more slowly than actual consumption. Thus, a significant portion of consumption losses during recessions is subsequently reversed. Even though our aggregate shock process is less rich than the one Nakamura et al. (2013) estimate, a transition to the recession state in our model is qualitatively similar to the onset of a disaster in their model. In particular, in both models this is a time when uncertainty about the aggregate state is high. In our model this is the case because there now is a chance that the economy transits to the disaster state (or of course fully recovers) in the next period. Another important similarity is that in both models, large negative shocks are not fully permanent, in contrast to models which specify a difference-stationary processes for output.[34] We discuss the implications of this modelling choice in Section B.1.

5.4 Life-Cycle Profiles

We set the life-cycle labor endowment profile $\{\varepsilon_i(z_n)\}_{i=1}^{I}$ to replicate the empirical 2007 SCF life-cycle profile for labor income, as described in Section 2. The life-cycle profile $\{\beta_i\}_{i=2}^{I}$ is then chosen so that the nonstochastic version of the model generates the 2007 SCF life-cycle profile for net worth, given the other determinants of life-cycle saving: the elasticity parameter γ, the profile for earnings, and the interest rate $\bar{R}$. In particular, note that household budget constraints in the deterministic version of the model can be written as

$$\begin{aligned} c_i &= (1-\theta)\varepsilon_i + \bar{R}y_{i-1} - y_i \text{ for } i = 1, \ldots, I-1 \\ c_I &= (1-\theta)\varepsilon_I + \bar{R}y_{I-1}, \end{aligned}$$

where y_i is total savings for age group i (net worth for age group $i+1$). The youngest age group corresponds to households aged 20-29, and the sixth and oldest age group corresponds to households aged 70 and above. We measure $\{(1-\theta)\varepsilon_i\}_{i=2}^{I}$ as 10 times average annual labor income of age group i, and $\{y_i\}_{i=1}^{I-1}$ as the average net worth of age group $i+1$. Because agents in our model enter the economy with zero initial wealth, we recategorize asset income for the youngest group in the SCF as labor income: thus, we set $\bar{R}y_0 = 0$ and set $(1-\theta)\varepsilon_1$ equal to 10 times average annual labor income for the youngest group plus $\bar{R}$ times the data value for y_0. We also

[34] However, a typical Nakamura et al. (2013) disaster does have a small permanent component, while recessions in our model are long lasting, but do not change trend output in the very long run.

set $y_I = 0$, since the oldest group does not save in our model. Given the sequences $\{(1-\theta)\varepsilon_i\}$, $\{y_i\}$, and $\bar{R}$, the budget constraints imply a life-cycle consumption profile, $\{c_i\}$. This profile can be used to back out the sequence of time discount factors that rationalizes the profile as reflecting optimal saving decisions.[35]

We summarize all life cycle parameters in Table 5, including the (annualized) time discount factors β_i (with β_1 normalized to 1), the life cycle profile of labor productivity in normal times, $\varepsilon_i(z_n)$, the portfolio shares for the exogenous portfolio economy (see Section 5.6), and the percentage change in ε by age when the model enters the recession state (see Section 5.5). Observe that β_i is generally larger than one. This reflects the fact that the data indicate strong income- and net worth growth over the life cycle between age 20-29 and age 50-59. However, β_i should not be interpreted solely as capturing pure time preference: it also incorporates the effects of age variation in family size on the marginal utility on consumption (Fernández-Villaverde and Krueger 2007).

5.5 Cyclical Declines in Earnings

We think of the age profile for labor income calibrated to 2007 SCF data (discussed above) as corresponding to the age profile for earnings in normal times $\{\varepsilon_i(z_n)\}_{i=1}^{I}$. We then measure the percentage decline in labor income by age group during the Great Recession in order to estimate how recession-induced earnings losses are distributed across the age distribution. In principle, we could measure these declines directly in the SCF, but the SCF sample size is relatively small, making it difficult to precisely estimate age-group-specific earnings declines. We therefore turn to the March CPS. We use the 2008 through 2017 survey years, containing data for income years 2007 through 2016. Our CPS measure of nonasset income is conceptually close to the SCF measure of labor income we used to calibrate the life-cycle profile for model earnings. This measure includes all CPS income components except for dividends, interest, rents, and one-third of self-employment income. The SCF and CPS life-cycle profiles for nonasset income align closely.

In the CPS, we compute each age-group's share of non-asset income in each year. We then measure $\varepsilon_i(z_r) - \varepsilon_i(z_n)$ for each age group as the difference between an average of age-group shares across income years 2008 through 2016 relative to age-group shares in income year 2007

[35] In the nonstochastic version of the model, the household's intertemporal first-order condition implies that $\bar{R}\beta_{i+1} = (c_{i+1}/c_i)^{\gamma}$. Note that the consumption profile is derived directly from household budget constraints and is pinned down by the data on labor income, net worth, and returns. Thus, the consumption profile is independent of preference parameters. However, supporting this consumption profile as an equilibrium outcome requires a specific discount factor profile $\{\beta_i\}$.

(the pre-recession year).[36] Thus, the way we measure age-group specific earnings declines is exactly analogous to how we measure the recession decline in aggregate income. Percentage changes in age-group-specific earnings are reported in Table 5. As expected, the age-earnings profile shifts in favor of older generations in a recession.[37] Households in their 20's and 30's see larger earnings losses than other groups. However, the group that really stands out are those aged 70 and older, who see a large increase in their share of aggregate earnings. In fact the increase in their relative share is so large that it dominates the decline in aggregate earnings, and thus the non-asset income of this group actually increases slightly when the economy enters a model recession: $\frac{\varepsilon_I(z_r)}{\varepsilon_I(z_n)} \times \frac{z_r}{z_n} = 1.121 \times 0.90 = 1.01$. The reason this group fares so well is that their non-asset income is dominated by social security and by other retirement income, and these sources of income are insulated from aggregate shocks.

Note that with a period length of 10 years, earnings losses are mechanically quite persistent in our model, consistent with the empirical literature that studies labor market outcomes of young cohorts in deep recessions (e.g. Kahn 2010, Oreopoulos et al. 2012, and Schwandt and von Wachter 2017). Kahn (2010) documents losses in earnings following the recession of the early 1980s lasting up to 15 years. Schwandt and von Wachter (2017) find effects that persist until 10 years into the labor market. For a moderate recession that raises unemployment 3 percentage points, they report a loss of cumulated earnings for labor market entrants of 60% of annual earnings. The loss we impose in our calibration (for the Great Recession in which the unemployment rate rose 6 percentage points) is 143% of annual earnings (see Table 5), a number that accords well with their estimates.

5.6 Calibration with Exogenous Portfolios

In our baseline model, agents choose both how much to save and how to divide savings between stocks and bonds. We also consider an alternative version of the model in which savings is a choice, but in which the portfolio split is exogenous. In this version, we introduce as new parameters age-varying portfolio splits $\{\lambda_i\}_{i=1}^{I}$, which we set equal to the age-group-specific shares of risky assets in net worth in the SCF (see Table 2, reproduced in Table 5). All other parameter values are identical to the baseline model.

[36] The March CPS was redesigned in 2014, and some questions related to income were changed. In that year a random subset of households was asked the old questions, while the rest were asked the new questions. We use the age-specific ratios of measured income across these two sub-groups to adjust measured income from 2014 onwards.

[37] The differentials across age groups are similar when considering a much narrower measure of labor income that only includes wage and salary income.

Note that the nonstochastic versions of the models with exogenous and endogenous portfolios are identical. In particular, absent aggregate shocks, model agents are indifferent about which portfolio mix to hold. Once aggregate shocks are introduced, the two economies are no longer identical. The advantage of the exogenous portfolios model, relative to the baseline, is that by forcing model agents to hold exactly the portfolios that actual households were holding in 2007, we generate a very realistic distribution of capital losses across age groups in a simulated model recession. The disadvantage of this approach is that it limits the ways in which agents can hedge aggregate risk ex ante, and the ways they can respond to shocks ex post.

6 Results for the Benchmark Economy

The calibration procedure just described delivers realistic life-cycle profiles for earnings and net worth. The calibration also ensures that when the shock resembling the Great Recession hits, it generates realistic declines in labor earnings and in aggregate asset values. These are necessary ingredients for our model to serve as a laboratory for studying the distributional effects of large recessions. Before exploring how the welfare costs of a recession vary by age, we first consider the model's implications for equilibrium asset price dynamics and for equilibrium portfolios.

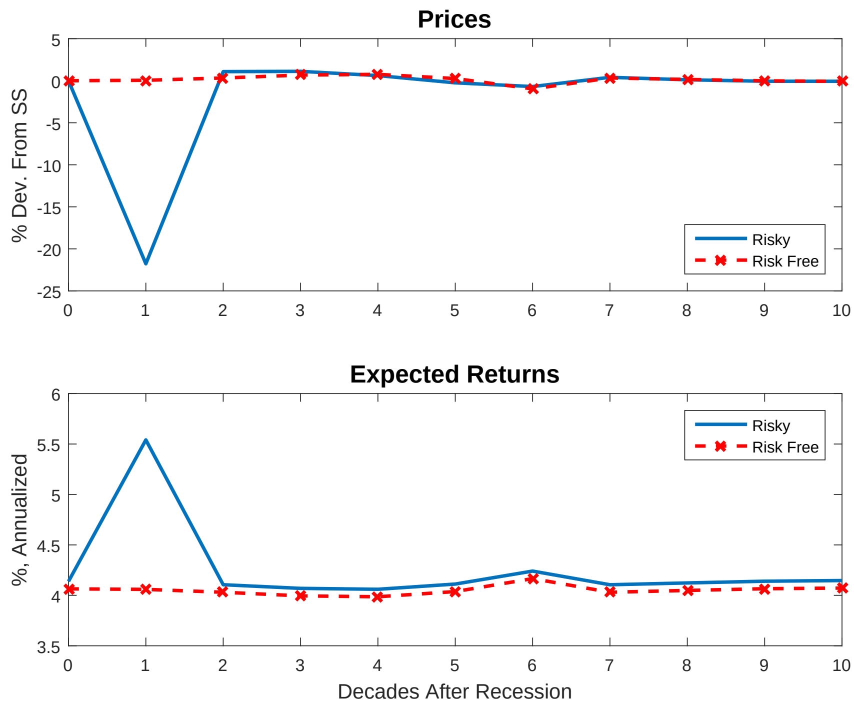
6.1 Asset Pricing Predictions for the Great Recession

Figure 2 plots asset prices along a model simulation in which the sequence for aggregate productivity features a long period of normal times, a Great-Recession-like event in period zero, and a return to normal times in subsequent periods. From the perspective of private household expectations, such a return to normal times was likely, but not certain. The top panel of the figure indicates that the price of stocks falls by 21.8 percent when the recession strikes, while the price of bonds barely moves.[38] Thus total wealth declines by exactly 20 percent in the recession, exactly as in the data.

The fact that equilibrium bond prices remain constant reflects two offsetting economic forces. On the one hand, expected consumption growth is positive in the recession state, since with a 75 percent probability, output will recover in the next period. Positive expected consumption growth constitutes an incentive for households to borrow, which pushes up the risk free rate. On the other hand, conditional on being in the recession state, there is a 25 percent chance that the economy will fall into a Great Depression. The fact that this disaster risk is so salient in the recession state

[38] Note again, as already discussed in the simple three-period model, that a low IES ($\gamma > 1$) is crucial for the finding that risky asset prices move more than output. With a unitary elasticity (log-utility) Proposition 2 in Appendix G shows that the elasticity of asset prices to output is exactly equal to one.

Figure 2: Equilibrium Asset Prices



means that the precautionary motive to accumulate safe assets is strong, putting upward pressure on the price of safe assets and thus downward pressure on the risk free rate.

The bottom panel of Figure 2 shows expected returns to stocks and bonds, on an annualized basis. The expected return to stocks always exceeds the expected return to bonds, indicating a positive equity premium.[39] The equity premium jumps notably when the recession hits, because in the recession state there is a large positive covariance between future stock returns and future consumption. In particular, households worry about very low stock returns in the positive probability event that the economy transits to the Great Depression state.[40] Thus they require a large risk premium to hold stocks in the Great Recession state.

To summarize, the decline in asset values in the model recession is driven by a sharp rise in the equilibrium risk premium and not to a rise in the equilibrium risk-free rate. This model-based narrative for asset price dynamics is consistent with the pattern observed empirically during the Great Recession. As documented in Section 2, the prices of risky assets – especially stocks and housing – fell sharply from mid-2007 to mid-2009 and remained low for a sustained period of time,

[39]We discuss standard asset pricing implications of the model in Section 6.6.

[40]When the recession simulated in Figure 2 is followed immediately by the disaster state, the stock price in the disaster state is 59.7 percent below its pre-recession value.

while the price of safe assets changed little.[41] In the recovery following the model recession, stock prices recover and overshoot their pre-recession level. This reflects the endogenous dynamics of the wealth distribution. When the shock hits, older households, especially those in the 60–69 age group, sell additional assets to fund consumption. Thus, after the recession, a larger share of aggregate wealth is held by younger cohorts, who are net buyers of assets, while less is held by older sellers. This translates into higher net asset demand after the recession and, consequently, overshooting stock prices. As younger cohorts age, the wealth distribution gradually shifts back toward older cohorts, who are net asset sellers. Thus, stock prices fall over time.

6.2 Portfolio Implications

In the benchmark economy, households choose their asset portfolios optimally to hedge against earnings and asset return risk. Since the underlying risk is aggregate in nature, somebody has to bear it, and endogenous portfolio choices can only achieve reallocation of this risk across different generations. Figure 3 plots the fraction of total savings in stocks by age in the model, both in normal times and in the recession period, as well as the fraction of total savings in risky assets by age in the 2007 SCF. Consistent with the data, younger households in the model find it optimal to hold riskier portfolios than older households, resulting in a broadly downward-sloping life-cycle profile for the risky portfolio share. For example, in normal times the youngest model age group has 161 percent of its savings in stocks, indicating a negative bond position equal to 61 percent of net worth, whereas the oldest model age group has a portfolio comprising 78 percent stocks and 22 percent bonds, almost identical to the data. The share of risky assets in total savings also declines with age in the SCF, indicating that the model offers a theoretical rationale for the observed tendency of households to shift toward safer portfolios as they age.

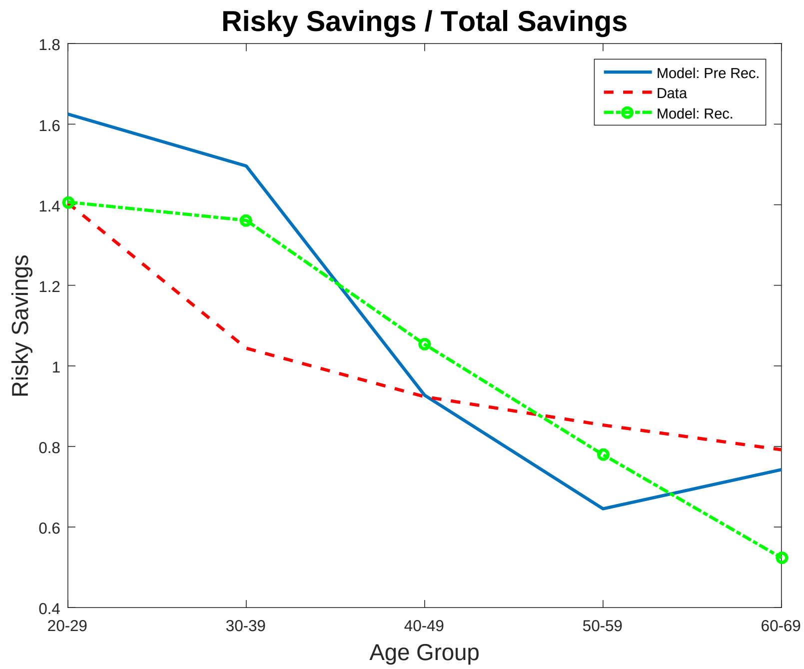
Why do model households gradually adopt safer portfolios as they age? First, note that markets are sequentially complete given that there are two freely traded assets and two possible next-period aggregate shock realizations z' conditional on each value for current output z.[42] Thus, all households *born prior to the recession* share risk perfectly in the recession period.[43] Recall that when the recession hits, stock prices fall by much more than output. Thus, younger households, who have little wealth relative to earnings, require a more leveraged portfolio to face the same

[41] In the context of our model, the possibility of a Great Depression is necessary to generate this result. Absent the Great Depression state (and thus absent time-varying aggregate risk), risk-free returns in the model would counterfactually shoot up in the Great Recession state.

[42] We exploit this fact in the computation of this version of the model; see Appendix E.

[43] This is shown in Figure!4, discussed below. Note that the OLG structure of the model prevents cohorts alive at a particular point in time from sharing aggregate risk with *past* or *future* generations.

Figure 3: Portfolio Shares in Risky Assets: Models and Data



consumption exposure to aggregate risk as older, wealthier households.[44]

Figure 3 indicates that, as the economy falls into a recession, the age-risk profile flattens, with the youngest households choosing to significantly reduce leverage. The logic is that in a model Great Recession, a Great Depression becomes possible, with an associated collapse in risky asset prices. This reduces the appetite of the young for risky assets.[45] Overall, the age variation in portfolios in the model is noticeably larger than the age variation in the SCF. The reason is that stock prices fall very sharply in the recession state – by more than two times as much as output. Older households, those in their 60's and 70's, who rely primarily on asset sales to finance consumption, would suffer disproportionate consumption losses if they did not hold a significant

[44] Note that with logarithmic preferences and symmetric earnings losses, prices fall exactly as much as output, human and financial wealth are therefore equally exposed to aggregate risk, and asset portfolios are age invariant (see Proposition 2 in Appendix G).

[45] Figure Figure 3 also shows that in normal times the 50-year-olds choose slightly *safer* portfolios than the 60-year-olds. The worst thing that can happen to this generation in normal times is to experience a recession in their 60s followed by a depression, whereas for the 60-year-olds the worst case scenario is a recession at the end of life. Conditional on already being in a recession, the worst case scenario for the 60 year-olds is a depression next period, whereas for the 50-year-olds it is a depression followed by normal times. Thus, in relative terms, life is riskier for the 50-year-olds in normal times than for their older brothers, whereas in a recession this ranking reverses. The portfolio positions of these two generations in normal times and in a recession reflect this age-dependent risk.

share of bonds in their portfolios. The flip side of the old holding a large position of bonds in equilibrium is that the young must take a significantly short bond position to clear the bond market.

It is possible that the discrepancy between the life cycle portfolio profile predicted by theory and the one observed in practice reflects economic forces absent in our model. Alternatively, it could be that leverage constraints prevent young households from debt-financing risky asset purchases on the scale that they would like to, as predicted by theory. The latter consideration is especially relevant for real estate, which is an important component of the stock of risky assets in the SCF data. In Section 6.4 we will therefore study a version of the model with exogenous portfolios in which we force households to hold precisely the portfolios observed in the 2007 SCF. This will allow us to explore the importance of portfolio allocations prior to the Great Recession for the distribution of welfare losses by age from this large macroeconomic shock. We now document the magnitude of these losses for the benchmark economy.

6.3 Welfare Losses from the Great Recession

Table 6 reports the welfare consequences of a model Great Recession event. We report two different measures of welfare, both assuming that the economy falls into the recession following a long period of normal times. The first column of the table measures the percentage change in consumption (in all future states and over all remaining periods of life) under a no-recession scenario needed to make households indifferent between the current aggregate state being z_n rather than z_r. The second column measures the percentage change in consumption in all remaining periods of life to make households indifferent between a life spent exclusively in normal times, and a life spent in the recession today followed by normal times for the rest of life. The key difference between these two welfare measures is that only the first includes the potential welfare losses households will experience if this recession is followed by a Great Depression.

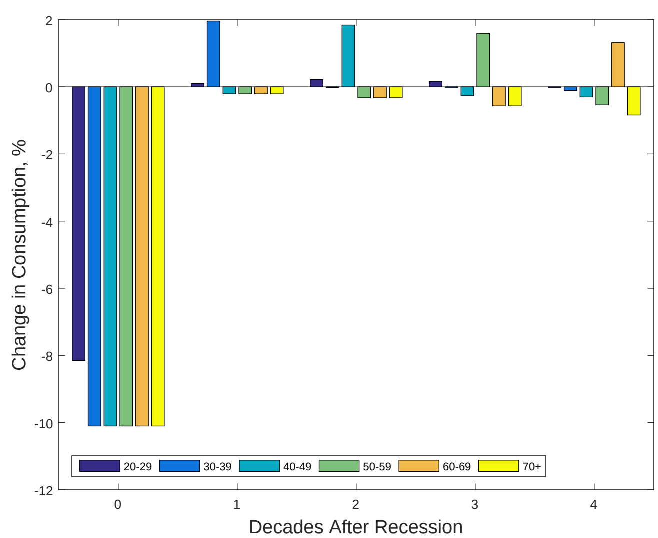
Table 6: Expected Vs. Realized Welfare Losses

Age Group	Expected	Realized
20-29	-1.97	-0.31
30-39	-4.60	-2.96
40-49	-5.60	-3.36
50-59	-7.49	-4.73
60-69	-9.68	-6.84
70+	-10.10	-10.10

Both welfare measures show that each group suffers welfare losses, but these losses are monotonically increasing in age, and differ dramatically in magnitude. Whereas the oldest cohort loses in excess of 10 percent of remaining lifetime consumption, the youngest cohort – the 20-year-olds who become economically active in the recession period – experience only moderate welfare losses. The welfare measure based on expected lifetime utility shows a loss of 2 percent of lifetime consumption, and the young roughly break even in terms of realized lifetime utility. Put differently, if it was not for the fact that a Great Recession makes a Great Depression possible, the young would be almost unaffected, in welfare terms.[46]

Figure 4 plots the consumption dynamics that underlie these welfare numbers. The figure plots changes in consumption, relative to the prerecession state, for each of the six model age groups, at each date along the simulation corresponding to Figure 2. The first set of bars shows the immediate age-specific consumption responses to the recession. In this period, consumption of all age groups except the youngest falls by slightly more than 10 percent, while the cohort that becomes active in the recession period sees a smaller consumption drop of only approximately 8 percent.

Figure 4: Consumption Dynamics by Age



For the oldest generation, the current recession period is the last period of life, and thus welfare losses (independent of the metric used) and current consumption losses coincide. The 10 percent loss for this group reflects the combined effects of a slight gain in labor earnings in the recession

[46]The realized welfare measure in general indicates smaller losses than the expected welfare metric, with the exception of the oldest age cohort, for whom the two measures coincide.

and the valuation loss they suffer on the stock component of their portfolio: stocks are more than 80 percent of this group's assets (as in the data), and stock prices fall by 21.8 percent. Households in the middle age groups see earnings losses, but are less reliant on immediate asset sales to finance consumption. Given the portfolios they have chosen, all cohorts 30 and older end up suffering the same immediate 10.0 percent consumption decline in the recession period, and these same cohorts experience equal (though smaller) consumption losses in each subsequent period. Recall again that markets are sequentially complete and thus all age groups that were economically active in the prerecession period share aggregate consumption risk equally.[47]

In contrast, the fact that the youngest group sees consumption fall by less than output indicates that this group absorbs less than an equal share of aggregate risk. In every subsequent period, the cohort that entered in the recession period enjoys consumption that is close to 2 percent higher than consumption of the same age group would have been in the absence of the Great Recession. The youngest age group fares so well, notwithstanding a very large hit to labor earnings when the recession hits, because they get to buy assets and thus shares of future dividends at temporarily and greatly depressed prices.[48] In Section 6.5 we will further investigate the importance of the asset price channel by considering a "partial equilibrium" version of our model in which recessions do not necessarily entail asset price declines.

6.4 Exogenous Portfolios

We now turn to the alternative model specification described in Section 5.6 in which we impose the risky versus safe asset portfolio shares observed in the 2007 SCF, rather than allowing households to choose their portfolios. The key advantage of this alternative model, relative to our baseline, is that it will more accurately replicate the empirical distribution of capital losses from price declines of risky assets when we simulate the Great Recession. The key disadvantage is that the path for

[47]That is, in Figure 4 the cohorts 40 and older experience the same consumption loss in period 1, whereas those 50 and older experience the same consumption loss in period 2 and so on.

[48]To what extent do the differential welfare losses documented so far reflect the fact that the current old will spend the remainder of their lives in the recession state, while younger households, in expectation, will spend the majority of their future lives in normal times? In order to address this issue, in Table A-11 in Appendix J, we report an alternative, wealth-based welfare loss measure. More precisely, we ask how much we must reduce *wealth* for each cohort in the no-recession state for households to be indifferent between life with or without a recession in the current period. We normalize these wealth changes by per capita consumption in normal times. Table A-11 indicates that a pattern of welfare losses emerges that is similar to our consumption-based measure: welfare losses are broadly monotonic in age in that older generations are made massively worse off by the recession, whereas the welfare losses for the youngest generations are relatively modest. Note that the wealth-based measure shows especially large losses for the 60-69 age cohort. It is at that age that life-cycle consumption peaks, and thus a given percentage of lifetime consumption loss translates into a large absolute wealth transfer required to compensate that group for a Great Recession-like event.

the equilibrium equity premium will now be a mechanical artefact of the exogenous portfolio splits that agents are forced to adopt and will not capture time-varying appetite for risk.

Table 7 compares asset price declines when a Great Recession hits in the baseline endogenous portfolio, and the alternative exogenous portfolio economy. The two models have two notable differences. First, the aggregate decline in asset prices is smaller in the exogenous portfolio model. Second, bond prices fall very sharply in that model – implying a counterfactual jump in the risk-free rate – while bond prices remain constant in the baseline model.

Table 7: Elasticity ξ of Asset Prices to Output

Asset	Baseline Model	Exogenous Portfolios
Wealth	2.00	1.88
Stock	2.18	1.91
Bond	0.00	1.56

To understand why asset prices are less sensitive to aggregate shocks with exogenous portfolios, recall that in this economy with fixed portfolios, the young are assigned safer portfolios and the old riskier portfolios with fewer safe bonds, relative to the baseline endogenous portfolios model (see Figure 3). This means that when the recession hits and risky asset prices fall, capital losses are smaller for younger cohorts relative to the baseline model, whereas capital losses for older cohorts are relatively larger. Thus, the wealth distribution in the recession period is tilted more strongly toward the young in the exogenous portfolio model, relative to the benchmark. This difference in the change of the wealth distribution translates into greater support for asset prices in the recession period in the exogenous portfolio model because younger cohorts are in the net saving part of their life cycle, whereas older cohorts are dissaving for life-cycle reasons.

Both stock and bond prices fall in the recession period in the exogenous portfolios economy because households are forced to buy stocks and bonds in fixed (but age-specific) proportions, so the extra risk they face in the recession state by construction cannot translate into an increase in the relative demand for safe assets as opposed to risky assets.[49]

Table 8 compares the age distribution of welfare losses in the exogenous portfolios economy to

[49]Why, then, do bond prices fall less than stock prices, even with exogenous portfolios? The reason is that the young hold riskier portfolios than the old, and thus a model recession does shift the wealth distribution toward the old (albeit less dramatically so than in the baseline model), and the old devote a larger share of their savings to bonds. Had we imposed counterfactually age-invariant portfolio shares for the risky asset, stock and bond prices would fall the same in percentage terms.

Table 8: Welfare Losses, Endogenous & Exogenous Portfolios (% lifet. cons.)

Age	Endogenous	Exogenous
20-29	-1.97	-2.78
30-39	-4.60	-4.34
40-49	-5.60	-3.49
50-59	-7.49	-7.36
60-69	-9.68	-11.61
70+	-10.10	-10.31

those in our baseline endogenous portfolio model. The broad pattern of welfare losses is similar, with the young still suffering less than the old. Two differences are noteworthy. First, the welfare losses for the cohort that enters the economy in the recession period are larger in the exogenous portfolios model, by 0.8 percentage points. Second, the 60-year-olds also lose more with exogenous portfolios, with losses now 11.6 percent of consumption, and the 40-year olds (who, as 30-year-olds, hold significantly safer portfolios than in the endogenous portfolio model) lose significantly less. In all three cases, the true welfare losses are likely intermediate between the two models. In particular, for the younger groups, the endogenous portfolio model potentially underestimates true welfare losses, since in this model the youngest group exploits temporarily depressed stock prices by choosing highly levered portfolios, where the degree of leverage exceeds that observed in the SCF prior to the Great Recession. On the other hand, the exogenous portfolio model likely exaggerates welfare losses for the youngest group, since in that model safe interest rates counterfactually rise in the recession state, and the young are forced to borrow. For the 60-year-olds, the endogenous portfolio model potentially underestimates welfare losses because this group's portfolio is too safe relative to the SCF (see Figure 3). At the same time, the exogenous portfolio model may exaggerate welfare losses because the 60-year-olds are not allowed to switch to safer portfolios when the recession hits and are therefore excessively exposed to the risk of stock price declines should a disaster strike.[50]

Finally, one interesting implication of our results is that although the endogenous portfolio model permits perfect sharing of aggregate risk between generations who enter the economy *prior* to the recession, it delivers worse risk sharing than the exogenous portfolios model between existing and *newborn* cohorts because equilibrium asset prices fall by more in the recession period.

[50] In the endogenous portfolio model, the 60-year-olds reallocate their portfolio toward safe assets in the recession state because this group will rely almost entirely on asset sales to finance consumption in the next period, when the disaster might strike, bringing a collapse in stock prices.

6.5 Quantifying the Asset Price Channel

How important are general equilibrium asset price movements in generating the age distribution for welfare losses from a model Great Recession reported in Table 8? We have highlighted the ability of the young to buy assets at depressed prices in the recession as the main reason for their rather small welfare losses, but is this intuition correct? To address these questions, we now revisit our cost-of-a-Great-Recession calculation under two alternative partial equilibrium scenarios in which we feed in exogenous paths for asset prices.[51] The two scenarios differ with respect to what happens to asset prices at date 0.

In our first "no price decline" scenario, there is no asset price recession: aggregate gross wealth W in the recession period is assumed equal to its value in the prerecession period, and the age distribution of this wealth A is also equal to the prerecession distribution. In our second "no price recovery" scenario, the Great Recession shock reduces aggregate gross wealth W by exactly the same amount as in the baseline general equilibrium model, and the recession also changes the age distribution of wealth shares A as in the baseline model. In both scenarios, from date 0 onward the gross return to saving that agents face is constant and equal to the return R to the aggregate market portfolio in the prerecession period. Note that this assumption implies no asset price recovery in the second scenario.

We assume that for all $t \geq 0$, agents can trade one-period Arrow securities that allow them to perfectly insure against aggregate risk. Given these assumptions, the household problem at age i and date 0 is to maximize expected remaining lifetime utility subject to a lifetime budget constraint. This budget constraint imposes that the discounted present value of date- and state-contingent consumption cannot exceed initial financial wealth $A^i W$ (which is scenario-specific) plus the discounted present value of labor income (which is common across both scenarios), where this present value depends discounts future earnings at rate R.

Under both scenarios, the optimal household allocation then amounts to setting consumption at each date and state to a fixed age-dependent fraction of date 0 lifetime income. Moreover, the welfare cost of a recession (defined exactly as in Section 6.3) is simply the age-specific percentage decline in the present value of expected lifetime resources (labor income plus initial wealth) that the recession entails (see Appendix K). Table 9 reports the welfare costs of a recession in the baseline general equilibrium model from Section 6.3 (first column of Table 6) and from the two

[51] As in Section 6.3, we imagine a scenario in which a long period of normal times is followed by a Great Recession-like shock at date 0. Also as in our previous experiment, this shock reduces current labor earnings and in addition shifts the probability distribution over future earnings.

partial equilibrium alternatives just described.

Table 9: Welfare Losses, Exogenous Asset Prices (% lifetime cons.)

Age	Benchmark	No Price Decline	No Price Recovery
20-29	-1.97	-4.86	-4.86
30-39	-4.60	-6.38	-9.61
40-49	-5.60	-5.94	-11.84
50-59	-7.49	-6.28	-12.16
60-69	-9.68	-4.79	-10.47
70+	-10.10	-1.98	-10.10

From Table 9 it is immediately obvious that the age distribution of welfare losses depends crucially on the asset pricing model. To understand these differences, it is useful to conceptualize the welfare consequences of the Great Recession for each age cohort as depending on (i) how the recession affects the discounted present value of their labor earnings, (ii) how it shapes the initial value of their financial portfolio, and (iii) how much they have to gain from a possible future asset price recovery. In the baseline model (column 1), all three forces are active, while the "no price recession" case isolates the first force, and the "no price recovery" experiment illustrates the combined impact of the first two effects.

The "No Price Decline" column of the table indicates that if we abstract from asset price movements, the age distribution of welfare costs would be to a significant degree reversed, relative to the baseline case in column 1. In particular, young and middle-age groups now experience large welfare losses, while the oldest two groups lose much less than in the benchmark case. Note that this result arises even though the younger age groups do not expect to live their whole lives in the recession state, while the 70+ group do, and the 60-year-olds have the not very pleasant prospect of a decade of recession followed by the risk of a disaster in their last decade of life. The reason for these changes in the distribution of the welfare losses when we shut down asset price movements is twofold. First, the young face somewhat larger percentage declines in labor earnings in the recession state. Second, the old rely much more on savings to finance consumption than labor earnings, and in the "No Price Decline" scenario, the recession does not dent the value of their savings. We conclude that abstracting entirely from asset price movements would give an extremely misleading picture of the age distribution of welfare losses from the Great Recession.

Moving to the "No Price Recovery" column now adds the welfare costs of capital losses from declining asset prices, assumed to be permanent in this scenario. This drastically increases the average welfare cost of the recession in the economy and also changes the age distribution of these costs. The cost for the youngest group is unaffected (relative to column 3) because this group starts its economic life with no assets, whereas the cost for the oldest group is as in the baseline general equilibrium model (column 2) because whether or not asset prices recover in the future is irrelevant for this group. The welfare costs are largest for 40- and 50-year-olds, who suffer both significant declines in labor earnings and large capital losses on their portfolios.

Comparing the "No Price Recovery" case to the baseline model shows that the expected asset price recovery that arises in general equilibrium cuts the welfare costs of the recession for younger age groups by more than half. In fact, because younger age groups are life-cycle savers, they actually suffer smaller welfare losses in the baseline model – in which prices fall and then recover – than in the "No Price Decline" model in which prices never move. This effect is most extreme for the newborn, who suffer no capital losses but benefit greatly from expected asset price appreciation.

Overall, the partial equilibrium thought experiments in this section make very clear that in order to assess the level and age distribution of welfare costs from the Great Recession, it is crucial to incorporate into the analysis the impact of asset price movements. Moreover, it is critical to make forecasts about future asset prices, in addition to specifying how asset prices move in the recession itself. The key advantage of our general equilibrium structural approach is that the theory itself generates joint predictions for the probability distribution over future labor income and future asset prices which imposes discipline over asset price dynamics, and their covariance with earnings and consumption.

6.6 Standard Asset Pricing Implications

Section 6.1 documented that the model captures well the two asset price facts that are central for quantifying the welfare consequences of the Great Recession by age: the large observed decline in risky asset prices and the relative constancy of the risk-free interest rate. We again want to stress that it is not our intent to build a model that offers a realistic account of movements in financial asset returns over typical business cycles. With that caveat in place, we now offer a broader discussion of our model's asset pricing properties.

Table 10 summarizes the mean and volatility of risky asset and risk-free bond returns, as well as their correlation. We also report their empirical counterparts, derived from the updated Shiller (1992) annual data on stock and bond returns. For both the model and the data, we calculate 10-year returns and report (annualized) means and standard deviations of log gross returns (not annualized).[52] In order to understand the importance of the possibility of Great Depression-like disasters for asset pricing moments, we also report model-based statistics that ignore these low-probability events. In addition to providing further insight into how our model works, one could also interpret these statistics as emerging from a version of our model in which the disaster state exists in the minds of model agents but never materializes in reality, or as offering a theoretical counterpart to empirical moments drawn from samples in which no disasters occur (such as the postwar United States).

Table 10: Statistics for 10-Year Returns

	Benchmark Model		
Asset	Average	Std. Dev.	Correlation
Stock	4.74%	20.4%	
Bond	4.47%	15.5%	0.89
	Model w/o Disasters		
Asset	Average	Std. Dev.	Correlation
Stock	4.51%	10.76%	
Bond	4.05%	0.46%	0.03
	Data		
Asset	Average	Std. Dev.	Correlation
Stock	6.62%	36.4%	
Bond	2.29%	30.4%	0.01

Comparing the top and bottom panels of Table 10, we observe that the model somewhat under-predicts the volatility of asset returns. It is consistent with the empirical observation that stock returns are more volatile than bond returns. Comparing the top and middle panels of the table indicates that the small possibility of slipping into a Great Depression, conditional on experiencing a Great Recession, is crucial for generating highly volatile asset returns. The table also suggests that stock and bond returns comove too strongly in the model relative to the data, but this comovement is driven exclusively by rare model disasters in which both stock and bond prices collapse and the real interest rate jumps.[53] In the one disaster in the data – the Great Depression – real interest rates were in fact high (see Table 2, column 11, in Hamilton (1987)).

[52]Because the model is solved at a 10-year frequency, return statistics are calculated directly based on a long simulation of model stock and bond returns. We simulate the model for a large number of periods because Great Depression-like disasters are rare but important for return volatility.

To calculate the empirical counterparts to model return moments, we use the series on Shiller's website, which span the period from 1871 to 2014. We derive 10-year gross returns as the product of 10-year contiguous annual gross returns based on the data from 1874 to 2014. Details of our calculations are contained in Appendix A.

[53]The empirical correlation of bond and stock is sensitive to the sample; see Appendix A.

Average returns to risky assets in the model exceed average returns to bonds, but if we interpret the risky asset in the model narrowly as corresponding to traded equity, then the model does not generate a large enough equity premium. The model premium is roughly 30 basis points per annum, compared to more than 400 basis points in the data.

Notwithstanding that our main objective is to study a specific historical episode, this relatively small equity premium seems to offer a case for adopting a more flexible preference specification, as in Epstein and Zin (1989) and Epstein and Zin (1991), allowing for a high risk aversion without imposing a low intertemporal elasticity of substitution. We now use a representative agent (RA) version of our model to explore this further.[54]

Table 11: Epstein-Zin Preferences in the Representative Agent Model

	Risk. Aver.	IES	Γ_{z_r,z_d}	$r(z_n, A_n)$	$r(z_r, A_r)$	Equity Prem.	Wealth Dec.
OLG (1)	2.75	1/2.75	0.250	4.06	4.06	0.27	-20.0
RA (2)	2.75	1/2.75	0.250	4.00	4.06	0.24	-21.2
RA (3)	50.00	1/2.75	0.250	2.41	-1.55	1.95	-17.9
RA (4)	50.00	1/2.13	0.002	3.19	3.19	1.33	-20.0

Row (1) is the baseline overlapping-generations model. Rows (2), (3) and (4) are representative agent economies with different values for risk aversion, for the IES, and for the probability of transiting from recession to disaster. All other parameters are identical to the baseline model, with $\beta = \frac{1}{1.5485}$. Risk free rates are $r(z, A) = 100 \times \left(q(z, A)^{-1} - 1\right)$. The equity premium is the unconditional average annualized excess return to stocks, in percent. Recession wealth declines are $100 \times \left(\frac{p(z_r,A_r)+q(z_r,A_r)B}{p(z_n,A_n)+q(z_n,A_n)B} - 1\right)$. In the representative agent model, the wealth distribution A is not a state variable. In the overlapping-generations economy, risk-free rates and wealth declines are computed assuming that A_n, the pre-recession wealth distribution, is the equilibrium distribution following a long period of normal times. The recession distribution, A_r, is given by the equilibrium law of motion $A_r = G(z_n, A_n, z_r)$.

The first two rows of the table indicate that under our baseline parameterization the RA and the OLG versions of the model have quite similar implications for average asset returns and for price declines in a model recession, indicating that we can safely use the representative agent version as a laboratory for exploring alternative specifications.

In the next row (RA 3) we retain all parameters at their baseline values except for the risk aversion coefficient, which we increase to 50. The model now delivers an equity premium that is an order of magnitude larger than under the baseline time-separable utility specification, though

[54]Solving the full overlapping-generations economy accurately for very high risk aversion and with large aggregate shocks is computationally challenging). Table 11 displays selected asset pricing statistics for this version of the model, for several different parameterizations.

it is still only half of the four percentage point premium in the data. However, a model Great Recession is now associated with a massively counterfactual fall in the risk-free rate. The intuition is that with high risk aversion, an increase in disaster risk translates into a very strong increase in the precautionary motive to accumulate safe assets. In our OLG model such an interest rate decline would allow the youngest generation to borrow at a negative risk-free rate in the recession state in order to finance risky asset purchases.

In the final row of the table (RA 4) we in addition permit a more flexible parameterization of the productivity process in which we set the probability that a recession turns into a disaster (Γ_{z_r,z_d}) and the intertemporal elasticity of substitution to jointly replicate (i) a flat risk free rate when the economy transits to the recession state, and (ii) a decline in wealth of 20 percent, as in the baseline OLG model. The model can successfully replicate these targets, but doing so requires dramatically reducing the disaster probability from 25 to 0.2 percent. Reducing the disaster risk in turn further reduces the model equity premium to only one third of its empirical counterpart.

We conclude that replicating the observed equity premium would require a fundamentally different model for aggregate risk. In the asset pricing literature, it is typically the growth rate for output, rather than its level, that is assumed to follow a stationary stochastic process. We explore such a process further in the Appendix.[55]

6.7 Movements in the Wealth Distribution in the Great Recession

The asset price movements and endogenous portfolio adjustments implied by the model induce shifts in the wealth distribution over the cycle, and thus in the model Great Recession (and subsequent recovery). In this section we discuss how the implications of the model along this dimension line up with data from the SCF. Using for 2007, 2010, and 2013 waves of the SCF, our preferred interpretation of the empirical time series is that 2007 captures the period before the Great Recession (with peak asset prices), that 2010 is a period in the midst of the recession (when asset prices have fallen but not recovered), and that 2013 reflects a time when the economy has at least partially recovered.

Table 12 reports the dynamics of net worth shares in both the model and in the SCF. For

[55]Such a process can potentially generate a large equity premium when (i) expected growth rates are highly volatile and positively serially correlated, and (ii) the IES and risk aversion are both larger than one (see e.g. Bansal and Yaron, 2004). Kaltenbrunner and Lochstoer (2010) show that in a production economy, iid shocks to the growth of output can translate into persistent changes in consumption growth. Note, however, that over the ten year time horizons we consider in this paper, growth rates in the United States are *negatively* serially correlated, contrary to what would be required for the long-run risk hypothesis to work.

both the baseline and exogenous portfolio model specifications, we report net worth shares in three consecutive model periods: (i) the prerecession state (where the economy has been in the normal state for many periods), (ii) the period in which the large recession strikes, and (iii) the post-recession period, assuming a recovery in output. Model gross wealth shares by age correspond exactly to the model state vector A. We measure net worth shares by age in the SCF by dividing each age group's average net worth by the unweighted sum of average net worth across groups.

Table 12: Dynamics of Net Worth Distribution: Models and Data

	Endogenous Portfolios			Exogenous Portfolios			Data: Net Worth SCF		
Age	Pre-Rec.	Rec.	Recov.	Pre-Rec.	Rec.	Recov.	2007	2010	2013
20-29	0.00	0.00	0.00	0.00	0.00	0.00	2.30	1.27	1.50
30-39	6.16	5.27	7.02	6.25	5.69	6.19	5.95	4.20	6.05
40-49	14.45	12.74	14.33	14.42	14.08	13.35	13.94	13.97	14.25
50-59	25.41	25.36	25.18	25.33	25.31	25.53	24.70	24.52	22.92
60-69	31.97	33.82	31.68	32.02	32.42	32.15	31.45	32.66	30.53
70+	22.00	22.81	21.80	21.97	22.50	22.78	21.67	23.38	24.74
total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

In the model, when the economy transits from normal times to the Great Recession state, the gross net worth shares of the 30- and 40-year-old age groups decline on impact, whereas those of the 60- and 70-year-olds age groups increase.[56] This shift in the wealth distribution toward older cohorts is driven purely by the fact that the young choose riskier portfolios in normal times (or are forced to hold riskier portfolios in the exogenous portfolios version of the model). When asset prices subsequently recover, the same life-cycle portfolio risk profile disproportionately benefits the young, and their wealth shares recover as the economy returns to the normal state.

In addition to these mechanical movements due to endogenous asset price fluctuations and age heterogeneity in portfolios, the baseline model with endogenous portfolios also predicts that the cohort that becomes economically active in a recession (who are in their 20's in the recession period and their 30's in the recovery period) benefits from being able to purchase assets at depressed prices and is therefore unusually asset-rich in the recovery period. The changes in net worth shares are larger in the endogenous portfolio specification, where the young choose more levered portfolios in the recession period and therefore gain more from the asset price recovery.

The net worth share dynamics observed in the data are similar to those predicted by the theory and generally lie in between the two alternative model specifications. The share of aggregate net

[56] Recall that the youngest age group is born with zero assets, and thus its wealth share is unaffected.

worth held by 30-year-olds declined from 5.95 percent in 2007 to 4.20 percent in 2010, whereas the share of the 70+ group increased from 21.67 percent to 23.38 percent. Subsequently, the net worth share of 30-year-olds rebounded to 6.05 percent in the 2013 SCF. This rebound is consistent with their having benefited from saving at depressed prices, though more time and data are required to assess whether the recent asset price recovery will permanently boost their wealth share.

7 Conclusion

These last findings bring us back to the main point of the paper: there is a silver lining for young households in the Great Recession stemming from asset prices dynamics. We have argued that this channel is quantitatively important and hence that it should be incorporated alongside labor market effects to assess the distribution of welfare losses from the Great Recession across age groups.

In Appendix B we demonstrate that our results are materially robust to modelling aggregate shocks as affecting the growth rate of productivity, rather than the level, to the inclusion of mortality risk and bequests, and to introducing simple forms of intra-cohort heterogeneity.

Our analysis could be further extended in several directions. First, we have modelled stocks and housing as a single risky asset, whereas those assets exhibited different price dynamics during the Great Recession, and stock ownership is much more concentrated than home ownership. A natural extension would be to model housing and mortgage debt explicitly. Second, while our focus has been on age differences in the welfare costs of large recessions, one could also explore other dimensions of heterogeneity, such as income and, especially, wealth in an even richer model of within-cohort heterogeneity than the one outlined in Section B.3. Such an extension would allow for an explicit modelling of credit constraints, differential access to credit (see e.g. Menno and Oliviero 2016 and Hur 2018), and potentially location-specific declines in housing values. Third, one could introduce idiosyncratic labor market risk, and explore how unemployment risk varies systematically by age, income, and wealth. Fourth, one could attempt to endogenize output by explicitly modelling capital and labor supply, which would in turn potentially allow for a deeper analysis of alternative drivers of the Great Recession. Finally, whereas our focus has been on the Great Recession, one could extend the analysis to study other episodes, such as the "dot com" bust in the early 2000s, or the asset price collapse in Japan in the early 1990s.

Knowledge about how the welfare costs of recessions are distributed across the age distribution can also help to inform the discussion of the appropriate policy response. Many of the policies that have been implemented in response to the Great Recession have redistributive consequences across generations. For example, financing an increasing share of the government budget through debt rather than taxation shifts the tax burden toward the young and future generations in life-cycle economies and benefits older age groups. Similarly, the Troubled Asset Relief Program (TARP) and large-scale asset purchases by the Federal Reserve were policies more or less explicitly designed to support asset prices. To the extent that these policies were successful, they disproportionately benefited wealthier households, who tend to be older.Given the age-asymmetric welfare losses we have estimated, a *distributional* argument can be made in favor of such policies.

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Appendix (for Review and Online Publication Only)

A Various Tables Referred to in the Text

Table A-1 summarizes the evolution of asset prices, over the last decade, for the broad asset classes used in the empirical analysis in Section 2.

Next we document in greater detail the empirical asset return statistics for stocks and bonds, over 10 year time intervals. As described in the main text, the Shiller data provide annual real returns on stocks and bonds from 1871 to 2014. From these annual gross real returns we can construct 10 year returns, for an arbitrary 10 year interval, by taking the product of 10 contiguous one year gross real returns. If we insist on non-overlapping 10 year time intervals, there are 5 different ways to construct our sample of 14 observations, starting in 1871 and ending in 2010, starting in 1872 and ending in 2011 and so forth. The following table reports the range of the mean and standard deviation of stock and bond returns, as well as their correlation, across these five different samples. We also calculate the same statistics based on a sample of yearly (and thus overlapping) 10 year returns, where the first observation uses annual return data from 1871 to 1880, the second observation data from 1872 to 1881 and so forth.

Finally, once we have constructed samples of 10 year gross returns R, we compute means and standard deviations based on the log of gross real returns $\ln(R)$, as is common in the literature (see e.g. Campbell,) where we note that

$$\ln(R_{t,t+10}) = \sum_{\tau=t}^{t+9} \ln(R_{\tau,\tau+1}) = \sum_{\tau=t}^{t+9} r_{\tau,\tau+1}$$

where $r_{\tau,\tau+1} = \ln(R_{\tau,\tau+1})$ is the annual net real return. In order to make the mean returns easier to interpret we annualize it through the transformation $\exp\left(E\left[\ln R\right]\right)^{\frac{1}{10}}$ and express it in percentage terms. In the main text we report statistics based on the 1974-2014 sample (the middle columns in table A12. As discussed in the main text, the range of possible empirical return statistics we could have reported is quite narrow for average returns and bond return volatility, slightly wider for stock return volatility and appreciably larger for the correlation between 10 year real stock and bond returns (which is why we do not emphasize that statistic in our discussion of the model-based results).

Table A-1: Real Price Declines Relative to 2007:2 by Risky Asset Class

	Stocks: Wilshire 5000	Res. Real Estate: Case Shiller	Noncorp Bus.: Flow of Funds	Nonres. Property: Greenstreet	Real Net Worth Rel. Trend	Real Net Worth
to 2007:3	-1.61	-1.77	-1.08	-0.59	-1.48	-0.99
to 2007:4	-2.04	-5.24	-3.12	-3.03	-3.60	-2.64
to 2008:1	-12.59	-9.43	-7.89	-7.62	-9.55	-8.19
to 2008:2	-11.56	-11.56	-12.27	-9.87	-10.89	-9.10
to 2008:3	-19.73	-14.16	-14.61	-15.07	-15.08	-12.95
to 2008:4	-42.82	-18.92	-19.24	-31.21	-25.52	-23.28
to 2009:1	-49.44	-23.69	-23.44	-37.44	-30.55	-28.10
to 2009:2	-43.88	-23.71	-28.01	-41.67	-29.62	-26.77
to 2009:3	-37.31	-23.04	-30.93	-41.30	-27.67	-24.38
to 2009:4	-31.96	-24.97	-31.64	-38.64	-26.95	-23.25
to 2010:1	-29.89	-27.54	-32.53	-36.32	-27.56	-23.50
to 2010:2	-29.43	-26.82	-30.91	-32.14	-26.72	-22.23
to 2010:3	-32.65	-27.77	-30.68	-30.06	-28.03	-23.24
to 2010:4	-26.21	-30.61	-30.51	-28.02	-27.31	-22.10
to 2011:1	-20.66	-33.03	-30.49	-26.34	-26.70	-21.05
to 2011:2	-20.57	-32.70	-30.92	-22.42	-26.43	-20.36
to 2011:3	-27.33	-33.03	-31.04	-22.00	-28.63	-22.36
to 2011:4	-28.16	-35.66	-30.58	-21.97	-30.05	-23.54
to 2012:1	-21.47	-37.27	-30.43	-22.32	-28.78	-21.76
to 2012:2	-22.46	-34.98	-29.72	-21.96	-27.90	-20.40
to 2012:3	-20.59	-34.06	-28.06	-21.49	-26.67	-18.64
to 2012:4	-20.24	-34.86	-27.35	-20.82	-26.82	-18.40
to 2013:1	-15.06	-34.57	-26.48	-20.28	-24.99	-15.94
to 2013:2	-10.53	-31.50	-24.78	-17.72	-21.86	-12.00
to 2013:3	-7.35	-29.82	-23.17	-18.02	-19.91	-9.36
to 2013:4	-3.02	-30.41	-22.32	-18.11	-18.77	-7.61
to 2014:1	-0.04	-30.70	-22.67	-17.91	-18.04	-6.32
to 2014:2	1.95	-29.50	-22.46	-16.71	-16.80	-4.43
to 2014:3	4.65	-29.05	-21.31	-15.32	-15.57	-2.53
to 2014:4	5.34	-29.75	-19.25	-13.29	-15.35	-1.79
to 2015:1	8.15	-29.94	-17.48	-11.63	-14.29	-0.08
to 2015:2	9.18	-28.68	-17.33	-10.15	-13.32	1.56
to 2015:3	4.24	-28.01	-16.78	-9.10	-14.39	0.81
to 2015:4	4.13	-28.38	-16.69	-7.68	-14.53	1.14
to 2016:1	-2.29	-28.56	-16.77	-7.14	-16.55	-0.76

The Flow of Funds also reports price changes for directly held corporate equities: this series aligns closely with the Wilshire 5000 index. It also reports a price series for residential real estate, based on the Loan Performance Index from First American Corelogic. This series closely tracks the Case-Shiller series. The house price series published by OFHEO (based on data from Fannie Mae and Freddie Mac) shows significantly smaller declines in house values.

Table A-2: Empirical Return Statistics

	Mean Return: $100\left[\exp\left(E\left[\ln R\right]\right)^{\frac{1}{10}}-1\right]$			Std.Dev.($\ln R$)		
	overlap $1871-2014$	non-overlap $1874-2014$	non-overlap [*min*, *max*]	overlap $1871-2014$	non-overlap $1874-2014$	non-overlap [*min*, *max*]
Equity	6.52	6.62	[6.43,6.62]	0.48	0.36	[0.35,0.55]
Bonds	2.36	2.29	[2.29,2.58]	0.31	0.30	[0.30,0.36]
Corr.	0.33	0.01	[0.01,0.52]			

B Robustness of Our Results

The purpose of this section is to investigate the robustness of our results to alternative modeling assumptions and parameterizations of the model. First, in Subsection B.1, we analyze the importance of the nature of the stochastic productivity process for the potency of the asset price channel, and in Subsection B.2 we discuss the importance of abstracting from bequests and inter-vivos transfers. In Subsection B.4, we assess the importance of the heterogeneous labor market experience across age cohorts in the Great Recession for the welfare results. Finally, Subsection B.3 evaluates the relative significance of the asset pricing and labor market effects when households differ in their wealth positions *within* age cohorts.

B.1 Level- or Growth-Rate Shocks?

In our model, aggregate output z is mean reverting, which implies that in a model recession, output and asset prices are expected to recover (although they might fall further if the economy enters the disaster state). Under what conditions are our results robust to introducing permanent shocks to z? To provide a transparent answer to this question, we turn back to the three-period version of our model from Section 4, but now assume that the *growth rate* of output $g' = z'/z$ follows a finite state Markov process with state space $\mathcal{G}$ and transition matrix $\Gamma_{g,g'}$. In Appendix F.2 we show that the optimality and market clearing conditions used to compute the equilibrium asset price function and law of motion are virtually identical to those in our benchmark model, but expressed in terms of the price-output ratio $\tilde{p}(g, A) = \frac{p(z,A)}{z}$ where the exogenous growth rate g between last period and today takes the place of the exogenous state variable z.

In this section we make three points. First, we show that the asset price decline relative to output in the model with stochastic growth rates is comparable to the decline in the model with trend-stationary output if and only if output growth over decades is negatively serially correlated, as it is in US data. Second, we show that the absolute welfare losses from a model recession are significantly larger in the stochastic growth economy for all but the oldest generation. Third, the *relative* welfare losses by age are quite similar across the growth-rate- and trend-stationary specifications. To do so we retain the same calibration as in Section 4, but now assume that output growth can take two equally likely values $g \in \{g_n, g_r\}$ that satisfy $g_n g_r = 1$ and $g_r/g_n = z_r/z_n$. Thus, the output decline when the economy falls into a Great Recession is as large as in the trend-stationary model and equal to $z_r/z_n - 1 = 10$ percent. The serial correlation of growth rates is determined by the persistence of both states, $\Gamma_{r,r} = \Gamma_{n,n}$. Real output per capita growth rates over longer time periods are negatively correlated in US data. For example, as discussed in Section 5.3, 10-year growth rates from the Maddison data have a serial correlation of -0.38, which the simple model replicates with $\Gamma_{r,r} = \Gamma_{n,n} = 0.31$.

In Figure A-2 in Appendix F.2 we display the relative asset price decline ξ as a function of the wealth distribution, both for the trend-stationary version of the model from Section 4 (with the same parameterization) and for the version with stochastic growth rates, for $\gamma \in \{1, 2.75, 8\}$. This is the analog of Figure 1 from Section 4. The main takeaway from the figure is that the elasticity of asset prices with respect to output movements is similar in the two specifications as long as the growth rate in the stochastic growth rate case is negatively correlated over time.[57] For example, with $\gamma = 2.75$ (as in the benchmark economy) and a wealth share of 40 percent for the old – the share after a long sequence of normal output / growth realizations – the price elasticity is 1.10 in the growth rate economy and 1.23 in the level economy.

Table A-3: Welfare Losses, Level vs. Growth Rate Risk (% lifetime cons.)

Age Group	Shocks to z	Shocks to z'/z
Young	2.4	-6.3
Middle	-4.5	-7.8
Old	-11.8	-10.8

As in the model with shocks to the level of productivity, larger asset price movements – induced either by a smaller IES ($1/\gamma$) or by a larger wealth share of the middle-aged – are associated with more benign welfare consequences for the young. Table A-3 summarizes the welfare losses from entering a recession, in both versions of the model, following a long sequence of good shocks. The table shows that if the output fall has a permanent component, then welfare losses for the young are larger than under the mean-reverting output scenario. In the stochastic growth version of the model, it is therefore hard to argue that the youngest generation might actually gain from a large recession. However, the fact that they suffer less than other generations *is* robust across both versions of the model, although the extent to which this is true is more pronounced in the benchmark-level-shock specification.

Finally, if 20-year output per capita growth rates were (counterfactually) *positively* correlated, then, at least for the time-separable Constant Relative Risk Aversion (CRRA) preferences with $\gamma > 1$ considered here, the model with growth rate shocks predicts asset price declines that are *smaller* than output declines ($\xi < 1$) and, consequently, welfare losses from a large recession that are *declining* with age. For example, if $\Gamma_{r,r} = \Gamma_{n,n} = 0.6$, implying a serial correlation of g equal to 0.2, we find that at a wealth share of 40 percent for the old, $\xi = 0.96$ and welfare losses for the young are 2.83 percent larger than losses for the old. Thus, this specification of the model reinforces the point that large asset price declines play a central role in mitigating young households' welfare losses from large recessions.

B.2 Introducing Bequests

Our baseline model abstracts from uncertain longevity and from bequests, which are sizeable in the United States.[58] We now explore the robustness of our findings to introducing these features,

[57] As before, the elasticity of prices to output is larger the greater is the wealth share of the price-elastic middle-aged. Also, for $\gamma = 1$, the percentage decline in asset prices is equal to that in output in both versions of the model, independent of the wealth distribution A. Thus, $\xi \equiv 1$ for both versions of the model when $\gamma = 1$. The same is true ($\xi \equiv 1$) in the stochastic growth rate model whenever the growth rates are i.i.d. See Appendix F.2.

[58] Barczyk, Fahle, and Kredler (2019) report statistics on bequests from Health and Retirement Survey (HRS) exit surveys from 2004 to 2012. The bequest distribution is highly skewed: the median estate is only \$20,000, while the mean is \$226,000, constituting 41% of mean net worth for 65-69 year old households in their sample.

again using the simple three-period model which offers the clearest intuition. We consider both accidental bequests associated with early death, and intentional bequests motivated by a "warm glow" motive for inter-generational transfers.[59] The basic structure of the model is the same as in Section 4: only the young earn labor income, and only the middle-aged and old consume. Now, however, middle-aged households survive to old age with exogenous probability $\psi \in (0, 1]$. The middle-aged can annuitize a fraction $\delta \in [0, 1]$ of their savings, so that a fraction δ of savings of the deceased passes proportionately to those who survive to old age, while a fraction $1 - \delta$ goes to the middle-aged as accidental bequests. The case $\delta = 1$ captures perfect annuity markets.

In addition to accidental bequests, surviving old households now decide how to split their resources at the start of the period between own consumption and intended bequests b_g. The utility function in old age over own consumption c_3 and intended bequests is

$$v_3(c_3, b_g) = \frac{c_3^{1-\gamma}}{1-\gamma} + \kappa \frac{b_g^{1-\gamma}}{1-\gamma} \tag{A-1}$$

where $\kappa \geq 0$ governs the strength of the warm-glow bequest motive. All bequests accrue to the middle-aged, who take expected inheritances as exogenously given. The discount factor that applies to old-age utility is now $\psi\beta$, reflecting both pure time preference and survival risk.

In Appendix F.3 we derive the Euler equation and asset market clearing conditions for this model, the analogues of equations (8) and (9) from Section 4. Let A denote the share of wealth of a representative surviving old household (prior to giving warm-glow bequests, but after payouts from annuities). Given the preferences described in $(A-1)$, old households optimally split resources $A\left[p(z, A) + \theta z\right]$ between consumption and bequests in fixed proportions. The Euler equation for asset purchases when middle-aged is now

$$c_2(z, A)^{-\gamma} \frac{\psi}{\psi + \delta(1-\psi)} = \beta\psi \sum_{z'} \Gamma_{z,z'} \left[\frac{p(z', G(z, A)) + \theta z'}{p(z, A)}\right] c_3(z', G(z, A))^{-\gamma} \tag{A-2}$$

The market clearing condition becomes

$$\left[1 - \frac{\psi G(z, A)}{\psi + \delta(1-\psi)}\right] p(z, A) = (1-\theta) z \tag{A-3}$$

From these equilibrium conditions it is clear that bequests have two basic effects in the model. First, they change the effective time discount factor and thus the incentive of middle-aged households to accumulate assets (eq. A-2). Second, bequests shift the distribution of resources toward the middle-aged which in turn affects equilibrium asset prices (eq. A-3). The extent to which these effects are present and quantitatively important depends on the the choices for the parameters β, κ, ψ, and δ.[60] However, the following proposition, proved in Appendix F.3, shows that alternative models deliver identical predictions for asset prices and welfare in a model recession as

[59] Introducing bequests due to perfect intergenerational altruism would connect past, present, and future cohorts in a single infinitely-lived dynasty, rendering the notion of redistribution across generations meaningless.

[60] Different choices for κ , ψ, and δ yield a range of alternative models of longevity risk and bequests, some of

(i) bequests leave the middle-aged with the same share of aggregate wealth in both models, and (ii) the middle-aged are equally keen to save.

Proposition 1. *Consider two versions of the three-period model that differ with respect to vector of parameters* $(\beta, \kappa, \psi, \delta)$*. These two models will have identical implications for asset price dynamics and for the welfare cost of recessions as long as they share common values for* $\tilde{\psi}$ *and* $\tilde{\beta}$*, where*

$$\begin{aligned}
\tilde{\psi} &= \frac{\psi + \delta(1-\psi)}{1+\kappa^{\frac{1}{\gamma}}} \\
\tilde{\beta} &= \beta\psi\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma}\left(\frac{\psi+\delta(1-\psi)}{\psi}\right)^{1-\gamma}
\end{aligned}$$

Here $\tilde{\psi}$ is inversely related to the strength of the bequest motive, which in turn determines the wealth of the middle-aged, while $\tilde{\beta}$ controls the strength of the middle-aged saving motive.[61] Examples of this proposition are given in the following corollaries:

Corollary 1. *Let Model A be the benchmark model (*$\psi_A = 1, \kappa_A = 0$*), and let Model B be an alternative with mortality and perfect annuity markets* $(\psi_B < 1, \delta_B = 1, \kappa_B = 0)$*. The two models have identical implications for asset prices and welfare in a recession if* $\beta_B = \beta_A \psi_B^{-\gamma}$.

Corollary 2. *Let Model A be a model with accidental bequests* $(\psi_A < 1, \delta_A = 0, \kappa_A = 0)$ *and let Model B be a model with no accidental bequests but with intended warm-glow bequests* $(\psi_B = \psi_A = \psi, \delta_B = 1, \kappa_B > 0)$*. The two models have identical implications for asset prices and welfare in a recession if* $\psi = \left[1+\kappa_B^{\frac{1}{\gamma}}\right]^{-1}$ *and* $\beta_B = \psi\beta_A$.

B.2.1 Back of the Envelope Calculation

The previous discussion has shown that mortality and bequests impact the incentive of middle-aged households to accumulate assets, and shift the distribution of resources toward the middle-aged. In Table A-4 we provide some illustrative, back of the envelope calculations for the potential effects of bequests on asset price dynamics and the welfare impact of a recession in our simple model.[62] We always recalibrate β such that $\tilde{\beta}$ is the same as in the benchmark economy.

which we explore in Table A-4. The benchmark model (labeled i) is an economy with no mortality risk and no bequests ($\psi = 1, \kappa = 0$). Introducing mortality risk, $\psi < 1$, a specification with $\delta = 1$ yields a model with perfect annuity markets (model ii), and one with accidental bequests if $\delta = 0$ (model iii). Model iv is a specification with no accidental bequests, but with a warm-glow bequest motive ($\kappa > 0$). Finally, in model v we allow for both accidental and intended bequests ($\psi < 1$, $\delta = 0$, $\kappa > 0$). The degrees of annuitization δ is irrelevant in models i and iv.

[61] In particular, $\tilde{\psi}$ is decreasing in perturbations to δ or ψ which increase the size of accidental bequests, and is decreasing in κ, which increases intentional bequests.

[62] We set $\psi = 0.71$, which is the conditional probability that a 65 year-old woman survives to age 80, as reported by The Hamilton Project (2015). We focus on the survival probability for women because for married households, the wife is typically the last spouse to die. Given this value for ψ, we set κ in model iv so that (intended) bequests in that model are equal to (accidental) bequests in model iii. In model v, $\psi = 0.71$ and $\delta = 0$ (as in model iii) while κ takes the same value as in model iv.

Table A-4: Asset Price Declines and Welfare: Models with Bequests

Model	Parameters						ξ (%)	Welfare (%)
	ψ	δ	κ	$\beta^{\frac{1}{20}}$	$\tilde{\beta}^{\frac{1}{20}}$	$\widetilde{\psi}$		
(i) Benchmark	1	–	0	1.042	1.042	1	1.23	2.38
(ii) Annuities	0.71	1	0	1.092	1.042	1	1.23	2.38
(iii) Accidental Beq.	0.71	0	0	1.059	1.042	0.71	1.28	1.85
(iv) Warm Glow	1	–	0.085	0.994	1.042	0.71	1.28	1.85
(v) Warm Glow + Acc.	0.71	0	0.085	1.011	1.042	0.50	1.33	1.44

First, as predicted by Corollary 1, the benchmark model (model i) and the model with annuities (model ii) have identical predictions for asset price volatility and the welfare impact of a model Great Recession, when the two models are calibrated to match the same wealth distribution and the same effective time discount factor. The same is true of the models with accidental bequests and warm-glow bequests (models iii and iv), numerically validating Corollary 2. Second, as the importance of bequests strengthens ($\widetilde{\psi}$ declines in Table A-4), wealth shifts to the asset-pricing middle-aged (the recipients of bequests). This translates into larger asset price declines in the recession, although quantitatively the effect is relatively small. Third, although the young benefit from being able to buy even cheaper assets in a recession when bequests are larger, they also expect to receive smaller bequests in the post-recession period, which reduces their expected welfare gains. In welfare terms this effect dominates and thus Table A-4 shows that as bequests become more important (lower $\widetilde{\psi}$), the welfare gains become smaller.[63] Quantitatively, the alternative models all deliver very similar welfare conclusions, ranging from 1.44% to 2.38%.[64] Thus, we conclude that the welfare effects of a recession for the young are likely to be similar across alternative bequest models, as long as those models are calibrated to deliver similar asset values and similar age distributions for wealth.[65]

B.3 Accounting for Intracohort Heterogeneity

Our baseline calibration replicates average income and wealth by age. However, it is well known that a large fraction of aggregate wealth is held by a relatively small fraction of households. Thus, one might wonder whether less wealthy households are likely to experience similar welfare losses across the life cycle. To address this question, we now consider an alternative version of the model

[63] Table A-10 in Appendix F.3 shows that increasing the effective discount factor, $\tilde{\beta}$, has only a minor impact on asset price volatility, but lowers the welfare gains from the recession for the young. This is because a higher discount factor shifts utility weights away from consumption when middle aged (when asset returns in the recovery from the recession are unusually high) and towards consumption when old.

[64] In an expanded version of Table A-4 in Appendix F.3 we also document results when β is held fixed. In this case the gains range from 1.24% to 2.76%; see Table A-10.

[65] De Nardi and Fella (2017) argue for a non-homothetic bequest function to rationalize the cross-sectional distribution of bequests. This feature, from which our discussion abstracts, might impact the intergenerational distribution of welfare, especially in extensions of our model that include intra-household heterogeneity.

in which there are two types of households: a wealthy type and a low-wealth type. We assume that the wealthy type accounts for a fixed fraction κ_y of aggregate labor earnings and passively holds a fixed fraction κ_a of aggregate debt and equity. It follows that the wealthy consume a fixed fraction $(1-\theta)\kappa_y + \kappa_a\theta$ of aggregate output at each date. Because the wealthy type is assumed to invest passively, assets are priced by the low-wealth type, and prices fluctuate such that this type always demands $(1-\kappa_a)$ shares and $\kappa_a B$ bonds.[66]

Since the wealthy type accounts for fixed fractions of earnings, asset holdings, and consumption, we can solve for equilibrium allocations by ignoring the wealthy group altogether and simply recalibrating the baseline model to target a lower wealth-to-labor-income ratio corresponding to $\bar{W}^P = \frac{(1-\kappa_a)}{(1-\kappa_y)}$. We identify the wealthy type with households in the top 10 percent of the net worth distribution in the 2007 SCF, which implies $\kappa_a = 0.678$, $\kappa_y = 0.334$, and $\bar{W}^P = 0.483\bar{W}$. Retaining our original targets for $\bar{R}$ and $\bar{\lambda}$ implies new values for the supply of bonds and for capital's share of $B^P = 0.0400$ and $\theta^P = 0.1729$.

Given the life-cycle profiles for labor income and net worth for the low-wealth group, we can follow the procedure outlined in Section 5 to reverse engineer the life-cycle profile $\left\{\beta_i^P\right\}$ that rationalizes the implied life-cycle profile for consumption for the low-wealth group. Because this group does less life-cycle saving, its consumption rises less rapidly over the life cycle, and the implied discount factors are therefore smaller. We will focus on the model specification in which agents choose portfolios endogenously, and report results for earnings losses that are asymmetric and symmetric across ages in the Great Recession. We retain our baseline value $\gamma = 2.75$.

Table A-5 compares the welfare losses associated with entering a recession in the low-wealth economy with those in the baseline model. The distribution of losses by age is qualitatively similar to that in the benchmark economy, but now the youngest generation suffers more significant welfare losses (4.3 percent rather than 2 percent). This reflects two factors. First, the young are now less patient, and thus care relatively less about the future, when asset prices recover. Second, and related, because they are less patient now the young hold less wealth that will benefit from price recovery in the first place. Thus, the young now evaluate the recession more similarly to hand-to-mouth consumers, for whom access to the asset market, and thus exposure to pricing channel is less important.

Comparing the welfare consequences across the different model specifications indicates that the welfare consequences of the Great Recession for the young are likely heterogeneous, depending on the relative importance of the labor market and asset market effects. Young households not especially severely hurt in the labor market (i.e., those who do not lose their job) and with significant (future) participation in the asset market might in fact have gained from the Great Recession (see e.g. last column of Table A-6). On the other hand, young households with more adverse labor market outcomes and with no strong attachment to financial markets faced welfare losses that were more similar to, but still smaller than older households, see the last two columns of Table A-5.

[66] An alternative would be a case in which the low-wealth type cannot hold stocks, as in Guvenen (2009).

Table A-5: Welfare Losses: Low-Wealth Calibration (% lifetime cons.)

Age Group	Baseline Wealth		Low Wealth	
	Asym. Δ Earnings	Sym. Δ Earnings	Asym. Δ Earnings	Sym. Δ Earnings
20-29	-1.97	-1.22	-4.33	-3.38
30-39	-4.60	-4.74	-6.41	-6.57
40-49	-5.60	-5.71	-6.99	-7.12
50-59	-7.49	-7.57	-8.13	-8.24
60-69	-9.68	-9.75	-9.70	-9.80
70+	-10.10	-10.15	-10.06	-10.13

B.4 The Importance of Asymmetric Earnings Losses

In Section 2 we documented that one salient feature of the Great Recession was that earnings losses were concentrated among young households, and argued that this empirical observation counteracts the potential benefits from the Great Recession through the asset price channel. In this section we assess the quantitative importance of the asymmetric age-earnings losses in the data. We now display results for an economy identical to the benchmark, but in which all age cohorts face counterfactual *symmetric* income losses of 10 percent, equal to the aggregate income loss (relative to trend) observed in the data.

From Table A-6 we see that abstracting from the especially bad labor market for young significantly lowers their welfare losses from the Great Recession, from close to 2 percent to 1.2. In fact, if one also abstracts from the fact that, in the model, a recession makes a Great Depression possible (by focusing on the realized welfare loss metric), then the youngest cohort actually gains (see the last column of Table A-6). Thus, as suggested by the simple three-period model discussed above, the large asset price decline in the recession would actually have left the young better off, had they not been simultaneously disproportionately affected by earnings losses. Even though this is a counterfactual scenario, it illustrates both the potency of the asset price movement channel as well as the importance of the severe downturn in labor income that the youngest cohorts experienced.

C The Economy with Housing

Residential real estate is the single most important component of household net worth. We now extend our baseline model to allow households to invest in three assets: stocks, bonds, and housing. The key finding will be that under a particular rescaling of parameter values, the welfare consequences of recessions in the economies with and without housing are identical. In what follows, the superscript H is used to differentiate parameter values from their counterparts in the original model without housing.

Table A-6: WELFARE LOSSES, ASYMMETRIC & SYMMETRIC EARNINGS Δ (% LIFET. CONS.)

Age Group	Expected Losses		Realized Losses	
	Asym. Δ Earnings	Sym. Δ Earnings	Asym. Δ Earnings	Sym. Δ Earnings
20-29	-1.97	-1.22	-0.31	0.44
30-39	-4.60	-4.74	-2.96	-3.11
40-49	-5.60	-5.71	-3.36	-3.48
50-59	-7.49	-7.57	-4.73	-4.80
60-69	-9.68	-9.75	-6.84	-6.90
70+	-10.10	-10.15	-10.10	-10.15

Preferences are given by

$$E\left[\sum_{i=1}^{I}\prod_{j=1}^{i}\beta_j^H\,\frac{x_i^{1-\gamma^H}-1}{1-\gamma^H}\right],$$

where x_i is a composite consumption bundle comprising nondurable consumption c and housing services s, with respective shares υ^H and $1-\upsilon^H$:

$$x_i = c_i^{\upsilon^H} s_i^{1-\upsilon^H}.$$

This Cobb-Douglas specification is consistent with extensive empirical evidence (see, for example, Davis and Ortalo-Magne 2011). We assume that the aggregate supply of housing services is state invariant and normalized to one. Housing is perfectly divisible, and there is a frictionless rental market. Thus, agents can separate the decisions of how much housing to consume versus how much housing to own for investment purposes. The technology for producing the nondurable good c is exactly the same as in the baseline model, and this technology produces z^H units of nondurable output. Now corporate debt is a promise to deliver one unit of the *composite* good x in the next period. The firm issues B^H units of this debt each period. We then have the following proposition.

Proposition 2. *If*

$$\begin{aligned}
(1-\theta) &= (1-\theta^H)\upsilon^H, \\
B &= \left(1+\frac{(1-\upsilon^H)}{\upsilon^H\theta^H}\right)B^H, \\
\{z\} &= \left\{\left(z^H\right)^{\upsilon^H}\right\}, \\
\{\beta_i\} &= \left\{\beta_i^H\right\}, \\
\{\varepsilon_i\} &= \left\{\varepsilon_i^H\right\}, \\
\gamma &= \gamma^H, \\
\Gamma &= \Gamma^H,
\end{aligned}$$

then in the economy without housing, the life-cycle consumption profiles and the law of motion for wealth are identical, state by state, to their counterparts in the economy with housing:

$$\begin{aligned} c_i(z, A, a) &= x_i(z^H, A, a) \\ G(z, A, z') &= G^H(z^H, A, (z^H)'). \end{aligned}$$

where G^H denotes the law of motion for wealth shares in the model with housing. It follows immediately that the welfare consequences of recessions in the two economies are identical for each age, in each aggregate state.

The key to this result is that, in this model, rents comove perfectly with output, and house prices comove perfectly with the value of nonhousing wealth. Let $p^h(z, A)$ denote the ex-rent price of housing, and let rents and the price of non-durable consumption be denoted by $r(z, A)$ and $p^c(z, A)$. Then in the model with housing,

$$\begin{aligned} p^h(z.A) &= \frac{(1-\upsilon^H)}{\upsilon^H \theta^H}\left[p(z, A) + q(z, A)B^H\right] \\ \frac{r(z, A)}{p^c(z, A)} &= \frac{(1-\upsilon^H)}{\upsilon^H} z^H. \end{aligned}$$

Thus, the housing asset offers the same returns as the market portfolio of corporate equity and debt. It follows that introducing housing does not affect households' ability to share risks across generations. At the same time, given Cobb-Douglas preferences and the implied constant expenditure shares for nondurable consumption and housing, introducing housing does not change the shapes of the life-cycle profiles for consumption or asset holdings.

Proof. In order to prove the previous result we begin by describing the decision problem in the model with housing. In a series of steps, we will then show that this problem is isomorphic to the decision problem in the model without housing.

Let $y_i(z, A, a)$ and $\lambda_i^e(z, A, a)$, $\lambda_i^h(z, A, a)$ denote the optimal household policy functions for total savings and for the fraction of savings invested in equity and housing. Let $c_i(z, A, a)$, $s_i(z, A, a)$, and $a_i'(z, A, a, z')$ denote the policy functions for nondurable consumption, housing consumption, and for shares of next period wealth. Let $p^c(z, A)$, $p^h(z, A)$, $p(z, A)$, $q(z, A)$, and $r(z, A)$ denote, respectively, the price of the nondurable consumption good, the price of housing, the price of stocks, the price of bonds, and the rental rate for housing, all relative to the composite good x. The dynamic programming problem of the household reads as

$$v_i(z, A, a) = \max_{c, y, \lambda^e, \lambda^h, a'} \left\{ u\left(c, s\right) + \beta_{i+1} \sum_{z' \in Z} \Gamma_{z, z'}\; v_{i+1}\left(z', A', a'\right) \right\} \qquad \text{s.t.} \tag{A-4}$$

$$
\begin{aligned}
p^c(z,A)c &+ y + r(z,A)s = \varepsilon_i(z)w(z) + W(z,A)a, && \text{(A-5)}\\
a' &= \frac{\left(\lambda^e \frac{[p(z',A')+d(z')]}{p(z,A)} + \lambda^h \frac{\left[p^h(z',A')+r(z',A')\right]}{p^h(z,A)} + (1-\lambda^e-\lambda^h)\frac{1}{q(z,A)}\right)\ y}{W(z',A')}, && \text{(A-6)}\\
A' &= G(z,A,z'). && \text{(A-7)}
\end{aligned}
$$

The aggregate value of start-of-period wealth in the model with housing is the value of aggregate payments to asset holders in the period $d(z) + r(z,A) + B$, plus the ex-dividend value of equity and housing $p(z,A) + p^h(z,A)$. Thus,

$$W(z',A') = p(z',A') + d(z') + p^h(z',A') + r(z',A') + B.$$

A recursive competitive equilibrium can be defined as in the baseline model.

1. Result on Rents: The agent's first-order condition with respect to the consumption of housing services implies

$$s_i(z,A,A_i) = \frac{(1-\upsilon)}{\upsilon r(z,A)} c_i(z,A,A_i)p^c(z,A). \tag{A-8}$$

Summing across age-groups,

$$\sum_{i=1}^{I} s_i(z,A,A_i) = \frac{(1-\upsilon)p^c(z,A)}{\upsilon r(z,A)} \sum_{i=1}^{I} c_i(z,A,A_i).$$

Imposing market clearing gives

$$r(z,A) = \frac{(1-\upsilon)}{\upsilon} p^c(z,A)z. \tag{A-9}$$

Let e denote total expenditure in units of the composite good:

$$e_i(z,A,A_i) = p_c(z,A)c_i(z,A,A_i) + r(z,A)s_i(z,A,A_i).$$

Substituting in A-8 gives

$$e_i(z,A,A_i) = \frac{1}{\upsilon} p_c(z,A)c_i(z,A,A_i). \tag{A-10}$$

Define aggregate consumption/output as

$$X(z,A) = \sum_i x_i(z,A,A_i).$$

In equilibrium

$$x_i(z, A, A_i) = c_i(z, A, A_i)^{\upsilon} \left(\frac{(1-\upsilon)}{\upsilon r(z, A)} c_i(z, A, A_i) p^c(z, A) \right)^{1-\upsilon}, \qquad \text{(A-11)}$$
$$= c_i(z, A, A_i) z^{\upsilon - 1}.$$

so aggregate composite consumption (aggregate output) is

$$\begin{aligned} X(z, A) &= \sum_i x_i(z, A, A_i), \\ &= z^{\upsilon-1} \sum_i c_i(z, A, A_i), \\ &= z^{\upsilon}. \end{aligned}$$

Solve for $p^c(z, A)$ by setting the value of aggregate composite consumption equal to aggregate expenditure (recall $p^x(z, A)$ is normalized to one):

$$\begin{aligned} z^{\upsilon} &= \sum_i e_i(z, A, A_i), \\ &= \frac{1}{\upsilon} p_c(z, A) z. \end{aligned}$$

Thus,

$$p^c(z, A) = \upsilon z^{\upsilon - 1}. \qquad \text{(A-12)}$$

It follows that

$$r(z, A) = \frac{(1-\upsilon)}{\upsilon} z p^c(z, A) = (1-\upsilon) z^{\upsilon}.$$

2. Result on House Prices: Recall that the numeraire here is the composite consumption good. Thus, dividends are given by

$$\begin{aligned} d(z, A) &= p^c(z, A)\theta z - B + q(z, A)B \\ &= \upsilon\theta z^{\upsilon} - B + q(z, A)B. \end{aligned}$$

where the second line follows from equation (A-12). Consider the following two assets: a claim to aggregate capital income (unlevered equity) and housing. The respective returns to the two assets are

$$\begin{aligned} \frac{p(z', A') + d(z', A') + B}{p^u(z, A)} &= \frac{p^u(z', A') + \upsilon\theta\,(z')^{\upsilon}}{p^u(z, A)} \\ \frac{p^h(z', A') + r(z', A')}{p^h(z.A)} &= \frac{p^h(z', A') + (1-\upsilon)\,(z')^{\upsilon}}{p^h(z.A)}. \end{aligned}$$

Note that the income streams associated with these two assets are in fixed proportions. It follows immediately that

$$p^h(z.A) = \frac{(1-\upsilon)}{\upsilon\theta} p^u(z, A).$$

3. Result on Portfolio Choice: Given that the return to housing is equal to the return to unlevered equity, we can write the law of motion for individual wealth as

$$
\begin{aligned}
a' W(z', A') &= \left(\lambda^e \frac{[p(z', A') + d(z')]}{p(z, A)} + \lambda^h \frac{[p(z', A') + d(z') + B]}{p(z, A) + q(z, A)B} + (1 - \lambda^e - \lambda^h) \frac{1}{q(z, A)} \right) y \\
&= \left[\left(\lambda^e + \frac{\lambda^h p(z, A)}{p(z, A) + q(z, A)B} \right) \frac{[p(z', A') + d(z')]}{p(z, A)} + \right. \\
&\qquad \left. \left((1 - \lambda^e - \lambda^h) + \frac{\lambda^h q(z, A)B}{p(z, A) + q(z, A)B} \right) \frac{1}{q(z, A)} \right] y \\
&= \left[\widetilde{\lambda} \frac{[p(z', A') + d(z')]}{p(z, A)} + \left(1 - \widetilde{\lambda} \right) \frac{1}{q(z, A)} \right] y,
\end{aligned}
$$

where

$$
\widetilde{\lambda} = \lambda^e + \frac{\lambda^h p(z, A)}{p(z, A) + q(z, A)B}.
$$

Note that (i) there is no reference to house prices or rents in this law of motion, and (ii) there is only one meaningful portfolio choice for agents, given that the return to housing is a linear combination of the returns to equity debt.

4. Expression for Aggregate Wealth: Aggregate wealth can be written as

$$
\begin{aligned}
W(z, A) &= p(z, A) + d(z) + p^h(z, A) + r(z, A) + B \\
&= p(z, A) + q(z, A)B + p^h(z, A) + \upsilon\theta z^\upsilon + (1 - \upsilon) z^\upsilon \\
&= p(z, A) + q(z, A)B + \frac{(1 - \upsilon)}{\upsilon\theta} \left(p(z, A) + q(z, A)B \right) + \left(\upsilon\theta + 1 - \upsilon \right) z^\upsilon.
\end{aligned}
$$

Define

$$
\begin{aligned}
\tilde{p}(z, A) &= \left(1 + \frac{(1 - \upsilon)}{\upsilon\theta} \right) p(z, A) \\
\tilde{B} &= \left(1 + \frac{(1 - \upsilon)}{\upsilon\theta} \right) B \\
\tilde{d}(z, A) &= \left(1 + \frac{(1 - \upsilon)}{\upsilon\theta} \right) \upsilon\theta z^\upsilon - \tilde{B} + q(z, A)\tilde{B}.
\end{aligned}
$$

In terms of this notation, aggregate wealth is given by

$$
W(z, A) = \tilde{p}(z, A) + q(z, A)\tilde{B} + \left(1 + \frac{(1 - \upsilon)}{\upsilon\theta} \right) \upsilon\theta z^\upsilon. \tag{A-13}
$$

5. Final Household Problem: We can now write the agent's problem without any reference to housing as

$$
v_i(z, A, a) = \max_{c, y, \widetilde{\lambda}, a'} \left\{ \frac{x^{1-\gamma}}{1 - \gamma} + \beta_{i+1} \sum_{z' \in Z} \Gamma_{z,z'} \, v_{i+1} \left(z', A', a' \right) \right\}
$$

subject to

$$x + y = \varepsilon_i(z)(1-\theta)\upsilon z^{\upsilon} + W(z, A)a \tag{A-14}$$

$$a' = \frac{\left(\widetilde{\lambda}\frac{\left[\tilde{p}(z',A')+\left(1+\frac{(1-\upsilon)}{\upsilon\theta}\right)\upsilon\theta(z')^{\upsilon}-\tilde{B}+q(z',A')\tilde{B}\right]}{\tilde{p}(z,A)} + \left(1-\widetilde{\lambda}\right)\frac{1}{q(z,A)}\right)\ y}{W(z', A')}, \tag{A-15}$$

taking as given laws of motion for z and A, and where $W(z', A')$ is given by A-13. It is clear that this model is identical to the model without housing defined in the text, as long as parameter values in the model without housing are the following functions of parameters in the model with housing, where the latter are now denoted with superscript H :

$$\begin{aligned} 1-\theta &= (1-\theta^H)\upsilon^H \\ z &= \left(z^H\right)^{\upsilon} \\ B &= \left(1 + \frac{(1-\upsilon^H)}{\upsilon^H\theta^H}\right) B^H. \end{aligned}$$

∎

D Exploiting Sequentially Complete Markets in the Economy with Endogenous Portfolios

Since in our applications the number of values the aggregate state z^{prime} can take tomorrow is two for every state z today, markets are sequentially complete when households can freely trade a bond and a stock, even though z can take three values. We exploit this for the purposes of characterizing equilibrium prices numerically. In particular, after having solved for equilibrium allocations (as described in the next section), we can easily construct prices of state-contingent claims (Arrow securities). We then reconstruct the equilibrium prices of conventional stocks and bonds as additional (effectively redundant) assets. For completeness, we here supply the equilibrium definition for the economy with a full set of Arrow securities.

Let $a_i(z')$ be shares of stock purchased by a household of age i. These shares represent a claim to fraction $a_i(z')$ of the capital stock if and only if aggregate state z' is realized in the next period. The state of the economy is the distribution of shares of stock A, given the current period shock z. We denote the state-contingent stock prices $P(z, A, z')$.

With this asset market structure, the maximization problem of the households now reads as

$$v_i(z, A, a) \quad = \quad \max_{c\geq 0, a'(z')} \left\{ u(c) + \beta_{i+1} \sum_{z'\in Z} \Gamma_{z,z'}\ v_{i+1}\left(z', A'(z'), a'(z')\right)\right\} \tag{A-16}$$

$$\text{s.t.} \qquad c + \sum_{z'} \left[a'(z') - a\right]\ P(z, A, z') = \varepsilon_i(z)w(z) + d(z)a \tag{A-17}$$

$$A'(z') = G(z, A, z') \tag{A-18}$$

with solution $c_i(z, A, a)$, $a_i'(z, A, a, z')$.

Definition 1. *A recursive competitive equilibrium with complete markets are value and policy functions* $\{v_i, c_i, a_i'\}$*, pricing functions* w, d, P*, and an aggregate law of motion* G *such that:*

1. *Given the pricing functions and the aggregate law of motion, the value functions* $\{v_i\}$ *solve the recursive problem of the households and* $\{c_i, a_i'\}$ *are the associated policy functions.*

2. *Wages and dividends satisfy*

$$w(z) = (1-\theta)z \qquad \textit{and} \qquad d(z) = \theta z. \tag{A-19}$$

3. *Markets clear*

$$\sum_{i=1}^{I} c_i(z, A, A_i) \;=\; z \tag{A-20}$$

$$\sum_{i=1}^{I} a_i'(z, A, A_i, z') \;=\; 1 \qquad \forall z' \in Z. \tag{A-21}$$

4. *The aggregate law of motion is consistent with individual optimization*

$$\begin{aligned} G_1(z, A, z') &= 0 \\ G_{i+1}(z, A, z') &= a_i'(z, A, A_i, z') \qquad \forall z',\ i = 1, ..., I-1. \end{aligned} \tag{A-22}$$

We now describe how we reconstruct returns and prices for conventional stocks and bonds, given the prices of state-contingent shares, exploiting the equivalence between the two market structures when the aggregate shock takes only two values. Let $W(z, A)$ denote the value of the (unlevered) firm after it has paid out dividends. This is equal to the price of all state-contingent shares:

$$W(z, A) = \sum_{z' \in Z} P(z, A, z'). \tag{A-23}$$

In the presence of state-contingent shares, risk-free bonds and levered stocks are redundant assets, but they can still be priced. We now compute these prices $q(z, A)$ and $p(z, A)$ as functions of the state-contingent prices $P(z, A, z')$ and $W(z, A)$. There are two ways of securing one unit of the good unconditionally in the next period. One could either buy one unit of the risk-free bond at price $q(z, A)$ or instead buy a bundle of state-contingent shares for each possible z', setting the state-specific quantity to $1/\left[W(z', G(z, A, z')) + \theta z'\right]$ so as to ensure a gross payout of one in each state. A no-arbitrage argument implies that the cost of the two alternative portfolios must be identical:

$$q(z, A) = \sum_{z'} \frac{P(z, A, z')}{W(z', G(z, A, z')) + \theta z'}. \tag{A-24}$$

With the bond price in hand, the stock price can immediately be recovered from the condition that the value of the unlevered firm (in the economy with state-contingent shares) must equal the value of levered stocks and risk-free bonds:

$$p(z, A) = W(z, A) - q(z, A)B. \tag{A-25}$$

E Computational Appendix

Even for a moderate number of generations, the state space is large: $I-2$ continuous state variables (plus z). Since we want to deal with large shocks, local methods should be used with caution. We therefore use global approximation on sparse grids, thereby respecting the size of our aggregate shock while avoiding the curse of dimensionality. The baseline model takes advantage of the fact that our two assets span the space of possible shocks, therefore allowing us to solve the model via a planning problem using the Negishi algorithm. The fixed-portfolio economies do not solve such a planning problem, so we must directly solve for the competitive equilibrium.

E.1 Endogenous Portfolio Economy

In our economy with endogenous trade in stocks and bonds, there are two assets and two values for the aggregate shock. Thus, the set of agents who are active at dates $t-1$ and t select portfolios that pool date t risk perfectly and share the same growth rate for the marginal utility of consumption. But it is impossible for agents active at date $t-1$ to share risk with agents who enter the economy at date t, and thus shocks at t reallocate resources between the newborn and existing cohorts.

The computational challenge for characterizing equilibrium allocations is to characterize this reallocation. As Brumm and Kubler (2013) show, the key condition that pins down the share of the newborns is their lifetime budget constraint that must be satisfied. Thus, computing a competitive equilibrium amounts to solving for a law of motion for the consumption share of newborn agents, with the property that the present values of lifetime income and consumption are equal with zero initial wealth. We now formally describe this way of characterizing competitive equilibrium.

Let the aggregate state be z and the vector $\boldsymbol{\lambda} = \boldsymbol{\lambda}_1, ..., \boldsymbol{\lambda}_I$ where $\lambda_i \in [0, 1]$ for all i. Let $\boldsymbol{\lambda}$ define age group i's share of aggregate output as follows for $i = 1, ..., I$:

$$c_i(z, \boldsymbol{\lambda}) = \boldsymbol{\lambda}_i z \tag{A-26}$$

Let $\boldsymbol{\lambda}' = G(z', \boldsymbol{\lambda})$ define a law of motion for $\boldsymbol{\lambda}$ and thus a resource-feasible allocation. We will use the notation $\boldsymbol{\lambda}'_i = G_i(z', \boldsymbol{\lambda})$.

Allocations in the competitive equilibrium with endogenous portfolios are defined by one particular specification for $G(z', \boldsymbol{\lambda})$.

The numerical challenge is to characterize $G_1(z', \boldsymbol{\lambda})$ which defines $\boldsymbol{\lambda}'_1$. Given $G_1(z', \boldsymbol{\lambda})$, we will see that the remaining $G_i(z', \boldsymbol{\lambda})$, for $i = 2, ..., I$, defining consumption shares for agents of ages $i = 2, ..., I$ in the next period are given by

$$\boldsymbol{\lambda}'_i = G_i(z', \boldsymbol{\lambda}) = \beta_i^{\frac{1}{\gamma}} \frac{(1 - G_1(z', \boldsymbol{\lambda}))}{\sum_{i=1}^{I-1} \beta_{i+1}^{\frac{1}{\gamma}} \boldsymbol{\lambda}_i} \times \boldsymbol{\lambda}_{i-1} \qquad i = 2, ..., I. \tag{A-27}$$

The logic for this specification, as we will see shortly, is that it guarantees that all agents share the same state-contingent inter-temporal marginal rate of substitution. Note that, by construction

$$\sum_{i=1}^{I} \boldsymbol{\lambda}_i' = 1$$

We now describe how we solve for the function $G_1(z', \boldsymbol{\lambda})$ corresponding to the competitive equilibrium.

Define, for $i = 1, ..., I - 1$

$$\begin{aligned} p_i(z, z', \boldsymbol{\lambda}) &= \pi(z'|z) \frac{\Pi_{l=1}^{i+1} \beta_l c_{i+1}(z', \boldsymbol{\lambda}')^{-\gamma}}{\Pi_{l=1}^{i} \beta_l c_i(z, \boldsymbol{\lambda})^{-\gamma}} \\ &= \pi(z'|z) \frac{\Pi_{l=1}^{i+1} \beta_l \left(\boldsymbol{\lambda}_{i+1}' z'\right)^{-\gamma}}{\Pi_{l=1}^{i} \beta_l \left(\boldsymbol{\lambda}_i z\right)^{-\gamma}} \\ &= \pi(z'|z) \frac{(1 - G_1(z', \boldsymbol{\lambda}))}{\sum_{i=1}^{I-1} \beta_{i+1}^{\frac{1}{\gamma}} \boldsymbol{\lambda}_i} \left(\frac{z'}{z}\right)^{-\gamma}. \end{aligned} \tag{A-28}$$

where the second line substitutes in the consumption sharing rule (A-26), and the third line uses the law of motion (A-27). Note from the third line that $p_i(z, z', \boldsymbol{\lambda})$ is independent of i.

Next define functions $B_i(z, \boldsymbol{\lambda})$ as follows, starting from $i = I$, and moving sequentially down to $i = 1$:

$$\begin{aligned} B_I(z, \boldsymbol{\lambda}) &= c_I(z, \boldsymbol{\lambda}) - w_I(z) \\ B_{i-1}(z, \boldsymbol{\lambda}) &= c_{i-1}(z, \boldsymbol{\lambda}) - w_{i-1}(z) + \sum_{z' \in \mathcal{Z}} p(z, z', \boldsymbol{\lambda}) B_i(z', \boldsymbol{\lambda}') \\ B_1(z, \boldsymbol{\lambda}) &= c_1(z, \boldsymbol{\lambda}) - w_1(z) + \sum_{z' \in \mathcal{Z}} p(z, z', \boldsymbol{\lambda}) B_2(z', \boldsymbol{\lambda}'), \end{aligned} \tag{A-29}$$

where in each case $\boldsymbol{\lambda}' = G(z', \boldsymbol{\lambda})$.

Claim Given the sharing rule (A-26) and the law of motion $G_i(z', \boldsymbol{\lambda})$ for $i \geq 2$ (A-27), the allocation defined by a function $G_1(z', \boldsymbol{\lambda})$ is a competitive equilibrium in the stock economy if and only if the implied $B_1(z, \boldsymbol{\lambda}) = 0$ for all $z, \boldsymbol{\lambda}$.

Proof First, note that allocations in the stock economy are identical to those in an economy in which agents trade two Arrow securities, each of which pays out if and only if one particular value for the aggregate shock z is realized. In an economy with trade in Arrow securities, the conditions defining a competitive equilibrium are: (i) security prices reflect state-contingent marginal rates of substitution, (ii) the agent's budget constraints are satisfied at each age, where financial wealth at age $i = 1$ is zero, and (iii) the aggregate resource constraint is satisfied. These conditions are all satisfied by the allocation described above: condition (i) is equation (A-28), condition (ii) is equations (A-29), and condition (iii) is satisfied by virtue of equation (A-26).

Note finally that for computational purposes it is not necessary to carry around the entire vector $\boldsymbol{\lambda}$ since $\sum_{i=1}^{I} \boldsymbol{\lambda}_i = 1$. Thus, a sufficient state vector is $\left(z, \{\boldsymbol{\lambda}_i\}_{i=1}^{I-1}\right)$. The law of motion A-27 is still sufficient to define consumption for all age groups in the next period.

We now move to solving for the unknown function G_1 described above. In order to solve the model we implement the following algorithm:

1. Initiate a grid of $\mathcal{L} = \{\boldsymbol{\lambda}^j\}_{j=1}^{J}$, where each $\boldsymbol{\lambda}^j$ is an $I-1$ dimensional vector. These will be the collocation points and are in practice sets on a Smolyak grid.

2. For each $z \in \mathcal{Z}$, and each $j = 1, ..., J$, guess a value $G_1(z, \boldsymbol{\lambda}_j)$. Use these guesses to construct an interpolating function $\hat{G}_1(z, \boldsymbol{\lambda})$ for any vector $\boldsymbol{\lambda}$. In practice, we use Chebyshev polynomials in this step.

3. For the $(\#\mathcal{Z})^{I-1}$ possible histories through which a newborn agent could live, and for each $j = 1, ..., J$, use $\hat{G}_1$ to construct consumption allocations, Arrow securities prices, and budget errors as described above. The interpolation is necessary because the vector of weights will typically not lie on the grid after one period passes.

4. Steps [2] and [3] create $\#\mathcal{Z} \times J$ equations (the budget errors for each shock value and each collocation vector) in the same number of unknowns (the values of G_1 for each shock and collocation vector). We use a nonlinear root finder to solve this system of equations.

E.2 Fixed Portfolio Economy

Relative to the methods described in Krueger and Kubler (2004, 2006), there are two additional complications in the present model. The first is that, while the sparse grids used there are subsets of $(I-1)-$dimensional cubes, wealth shares used in this paper are defined on the $(I-2)$ dimensional *simplex*. We deal with this issue by defining the state space in levels of wealth rather than in shares, and then we map a generation's level of wealth into a share when evaluating the Euler equations. The second complication is that the prices of the assets cannot be read off the first-order conditions in this model[67] but must instead adjust so that the excess demand for stocks and bonds is zero in both cases. We now describe our algorithm for solving the model.

1 Solve for the steady state prices and wealth *levels*: $\overline{p}, \overline{q}, \overline{W} = (\overline{W}_2, \overline{W}_3, ..., \overline{W}_{I-1}, \overline{W}_I)$. As described above, we work with an endogenous state space of dimension $I-1$ rather than $I-2$ and then map wealth levels into wealth shares.

2 Create a sparse grid around the steady state wealth distribution. Call this grid $\mathbb{W}$. We verify ex-post that the wealth distribution stays within this hyper-cube along the simulation path.

[67] One can do this in production economies where factor prices equal marginal productivities

3 We start with an outer loop over prices (this loop was unnecessary in Krueger and Kubler (2004)). At an outer loop iteration n we have guesses from the previous iteration for Chebyshev coefficients $(\alpha_z^{p,n}, \alpha_z^{q,n})$ for the prices that are used to compute the values of prices (p, q) for each realization of z and each point $W \in \mathbb{W}$ in the endogenous state space. We denote the vector of price values by $(\psi_{z,W}^{p,n}, \psi_{z,W}^{q,n})_{W\in\mathbb{W}}$. The Chebyshev coefficients $(\alpha_z^{p,n}, \alpha_z^{q,n})$ also determine the pricing functions on the entire state space, denoted by $(\widehat{\psi}_z^{p,n}, \widehat{\psi}_z^{q,n})$, somewhat abusing notation.[68]

4 Given approximate pricing functions in the inner loop, we iterate over household policies. In this loop we generate both the savings policy function and the law of motion for the wealth distribution consistent with approximate price functions $(\widehat{\psi}_z^{p,n}, \widehat{\psi}_z^{q,n})$. The savings policy is indexed by generation and *current* state z, and so the current guess of the savings policy function at policy iteration m when the price iteration is n is determined by Chebyshev coefficients of the form $(\alpha_{z,i}^{y,n,m})$. These can be used to compute the optimal savings level at each grid point W and is denoted by $(\psi_{z,i,W}^{y,n,m})$. As in the previous step, the Chebyshev coefficients also determine the entire approximating savings functions $(\widehat{\psi}_{z,i}^{y,n,m})$. The law of motion for wealth is a function of savings, current prices, and future prices; it must therefore be indexed by current state z, generation i, and future state z'. Similarly, the Chebyshev coefficients $(\alpha_{z,i,z'}^{G,n,m})$ are used to compute the law of motion $(\psi_{z,i,z',W}^{G,n,m})$ for all points $W \in \mathbb{W}$ and to generate the approximating functions $\widehat{\psi}_{z,i,z'}^{G,n,m}$.

5 At this point we loop over each value of z and each point in $W \in \mathbb{W}$ and solve the $I-1$ Euler equations for the $I-1$ optimal savings levels, $y_{i,z,W}$. The Euler equations that we solve to generate the updated savings levels are:

$$u'(c_i(y_{i,z,W}; W, z)) = \beta_i \mathbb{E}_z \widehat{R}_i^{n,m}(z') u'(\widehat{c}_{i+1}(W_+(z'), z'^{n,m}),$$

where

$$\widehat{R}_i^{n,m}(z') = \left(\lambda_i \frac{\widehat{\psi}_{z'}^{p,n}(\psi_{z,z',W}^{G,n,m}) + \theta z' + \overline{B}\widehat{\psi}_{z'}^{q,n}(\psi_{z,z',W}^{G,n,m}) - \overline{B}}{\psi_{z,W}^{p,n}} + \frac{(1-\lambda_i)}{\psi_{z,W}^{q,n}}\right)$$

$$W_+(z') = \begin{bmatrix} W_{+,2}(z') \\ \dots \\ W_{+,I} \end{bmatrix} = \begin{bmatrix} \left(\lambda_1 \frac{\widehat{\psi}_{z'}^{p,n}(\psi_{z,z',W}^{G,n,m}) + \theta z' + \overline{B}\widehat{\psi}_{z'}^{q,n}(\psi_{z,z',W}^{G,n,m}) - \overline{B}}{\psi_{z,W}^{p,n}} + \frac{(1-\lambda_1)}{\psi_{z,W}^{q,n}}\right) y_{1,z,W} \\ \dots \\ \left(\lambda_{I-1} \frac{\widehat{\psi}_{z'}^{p,n}(\psi_{z,z',W}^{G,n,m}) + \theta z' + \overline{B}\widehat{\psi}_{z'}^{q,n}(\psi_{z,z',W}^{G,n,m}) - \overline{B}}{\psi_{z,W}^{p,n}} + \frac{(1-\lambda_{I-1})}{\psi_{z,W}^{q,n}}\right) y_{I-1,z,W} \end{bmatrix}$$

[68] Note that this notation implies $\widehat{\psi}_z^{p,n}(W) = \psi_{z,W}^{p,n}$.

$$
\begin{aligned}
c_1(y_{1,z};W,z) &= (1-\theta)z\epsilon_1(z) - y_{1,z,W} \\
&\quad \text{for } i = 1,...,I-1: \\
c_i(y_{i,z};W,z) &= (1-\theta)z\epsilon_i(z) + \left(\widehat{\psi}^{p,n}_{z,W} + \theta z + \overline{B}\,\widehat{\psi}^{q,n}_{z,W}\right)\frac{W_i}{\sum_{l=2}^{I} W_l} - y_{i,z,W} \\
&\quad \text{for } i = 1,...,I-2: \\
\widehat{c}_{i+1}(W_+(z'),z'^{n,m}) &= (1-\theta)z\epsilon_{i+1}(z') + W_{+,i+1}(z') - \widehat{\psi}^{y,n,m}_{z',i+1}(W_+(z')) \\
\widehat{c}_I(W_+(z'),z'^{n,m}) &= (1-\theta)z\epsilon_I(z') + W_{+,I}(z').
\end{aligned}
$$

Note that in the calculation of the c_i's, we switch from using wealth levels to using wealth shares to satisfy the requirement that only the latter are truly minimal state variables. This is another difference relative to the previous use of Smolyak polynomials in Krueger and Kubler (2004).

With the new savings in hand, we update the savings policies as $\psi^{y,n,m+1}_{z,i,W} = y_{i,z,W}$. The law of motion for wealth levels is updated via

$$
\psi^{G,n,m+1}_{z,i,z',W} = \left(\lambda_i \frac{\widehat{\psi}^{p,n}_{z'}(\psi^{G,n,m}_{z,z',W}) + \theta z' + \overline{B}\widehat{\psi}^{q,n}_{z'}(\psi^{G,n,m}_{z,z',W}) - \overline{B}}{\psi^{p,n}_{z,W}} + \frac{(1-\lambda_i)}{\psi^{q,n}_{z,W}}\right)\psi^{y,n,m+1}_{i,z,W}.
$$

6 If $\max_{W\in\mathbb{W}}\max_z\max_{z'}\max_i|\psi^{y,n,m+1}_{z,i,W} - \psi^{y,n,m}_{z,i,W}|$ is below an acceptable tolerance level, then we proceed to step [7]. Otherwise we return to [4] with the updated savings functions and aggregate law of motion for wealth, but now indexed by step $m+1$. We now generate new Chebyshev coefficients $\alpha^{y,n,m+1}_{z,i}$ by solving the system $\widehat{\psi}^{y,n,m+1}_{z,i}(W) = \psi^{y,n,m+1}_{z,i,W}$ for each $W \in \mathbb{W}$.

7 For each point in the grid $\mathbb{W}$ and each value z, we check the market clearing conditions. If:

$$
\max_{W\in\mathbb{W}}\max_z \left|\sum_{i=1}^{I-1}\frac{\psi^{y,n,m+1}_{z,i,W}\lambda_i}{\psi^{p,n}_{z,W}} - 1\right| + \left|\sum_{i=1}^{I-1}\frac{\psi^{y,n,m+1}_{z,i,W}(1-\lambda_i)}{\psi^{q,n}_{z,W}} - \overline{B}\right|
$$

is below an acceptable tolerance level we stop. Otherwise, we update our guess of prices $\psi^{p,n+1}_{z,W} = \sum_{i=1}^{I-1}\lambda_i\psi^{y,n,m+1}_{z,i,W}$ and $\psi^{q,n+1}_{z,W} = \sum_{i=1}^{I-1}(1-\lambda_i)\psi^{y,n,m+1}_{z,i,W}/\overline{B}$ and return to step [3]. We now generate new Chebyshev coefficients $(\alpha^{p,n+1}_{z,i}, \alpha^{q,n+1}_{z,i}$ by solving $\widehat{\psi}^{p,n+1}_{z}(W) = \psi^{p,n+1}_{z,W}$ and $\widehat{\psi}^{q,n+1}_{z}(W) = \psi^{q,n+1}_{z,W}$ for each value of z and each $W \in \mathbb{W}$.

E.3 Numerical Accuracy

The analytical results available for the endogenous portfolio economy, for the case when $\gamma = 1$ and the age profile of earnings does not vary with z, provide us with a useful test case to assess the numerical accuracy of our computational results. We now compare our numerical results with the theoretical prediction from Proposition 2 in the main text, item by item. We make the following observations from our simulations:

1 The distribution of wealth shares is constant along the simulation. This is shown for the computed model in Table A-7.

Table A-7: WEALTH SHARES WITH $\gamma = 1$

	30-39	40-49	50-59	60-69	70-79
Expansion	6.1%	14.2%	25.3%	32.2%	22.3%
Recession	6.1%	14.2%	25.3%	32.2%	22.3%

2 Aggregate wealth is proportional to the aggregate shock. Specifically:

$$p(z, \bar{A}) + q(z, \bar{A})\bar{B} = 0.5501z.$$

3 The theoretical expressions for stock and bond prices hold with a maximal error of 0.002% in our simulation.

4 According to the proposition, with $\gamma = 1$ portfolio shares λ_i are age invariant and proportional to $\frac{p(z,\bar{A})}{z}$. The shares for each generation in the boom and recession states are shown to be age invariant in Table (A-8). The maximal deviation from the theoretical value is 0.001%.

Table A-8: PORTFOLIO SHARES WITH $\gamma = 1$

	20-29	30-39	40-49	50-59	60-69
Boom	0.9178	0.9178	0.9178	0.9178	0.9178
Recession	0.9178	0.9178	0.9178	0.9178	0.9178

5 Consumption profiles normalized by total output z (that is, consumption shares) are displayed in the first two columns of Table A-9. Note that they are independent of z, as the proposition indicates. They are also equal to the theoretical consumption shares characterized by Proposition 2 and displayed in the last column of Table A-9.

Table A-9: CONSUMPTION AS FRACTION OF OUTPUT

Age Group	Expansion	Recession	Theory
20-29	5.33%	5.33%	5.33%
30-39	8.92%	8.92%	8.92%
40-49	12.51%	12.51%	12.51%
50-59	20.55%	20.55%	20.55%
60-69	27.96%	27.96%	27.96%
70+	24.74%	24.74%	24.74%

6 The equity premium in theory is 0.6642% and the simulated equity premium is 0.6634%.

F 3-Period Economy: Details and Additional Results

In this appendix we provide details of the analysis of the three period model, first for the case where aggregate productivity z follows as stationary Markov process in levels (studied in section 4 of the paper), and then for the case of stochastic growth rates of z (as investigated in section B.1).

F.1 Details of the 3-Period Economy with Stationary Productivity

As discussed in the main text, the two functional equations fully characterizing the recursive equilibrium in the 3-period OLG economy are given by the intertemporal Euler equation

$$u'\left[(1-A)\left(p(z,A)+\theta z\right)-G(z,A)p(z,A)\right]= \beta\sum_{z'}\Gamma_{z,z'}\frac{[p(z',A')+\theta z']}{p(z,A)}u'\left[G(z,A)\left(p(z',A')+\theta z'\right)\right], \quad \text{(A-30)}$$

and the (rewritten) asset market clearing condition that the labor income of the young equals that cohort's purchases of shares in the risky asset:

$$[1-G(z,A)]\; p(z,A)=(1-\theta)z. \quad \text{(A-31)}$$

These two equations determine the equilibrium price $p(z,A)$ of the risky asset and the law of motion of the wealth distribution $A'=G(z,A)$. The young do not consume by assumption, and consumption of the middle-aged and old are given by

$$c_3(z,A) = A\left[p(z,A)+\theta z\right], \quad \text{(A-32)}$$
$$c_2(z,A) = (1-A)\left[p(z,S)+\theta z\right]-G(z,A)p(z,A), \quad \text{(A-33)}$$

and expected lifetime utility is therefore determined as

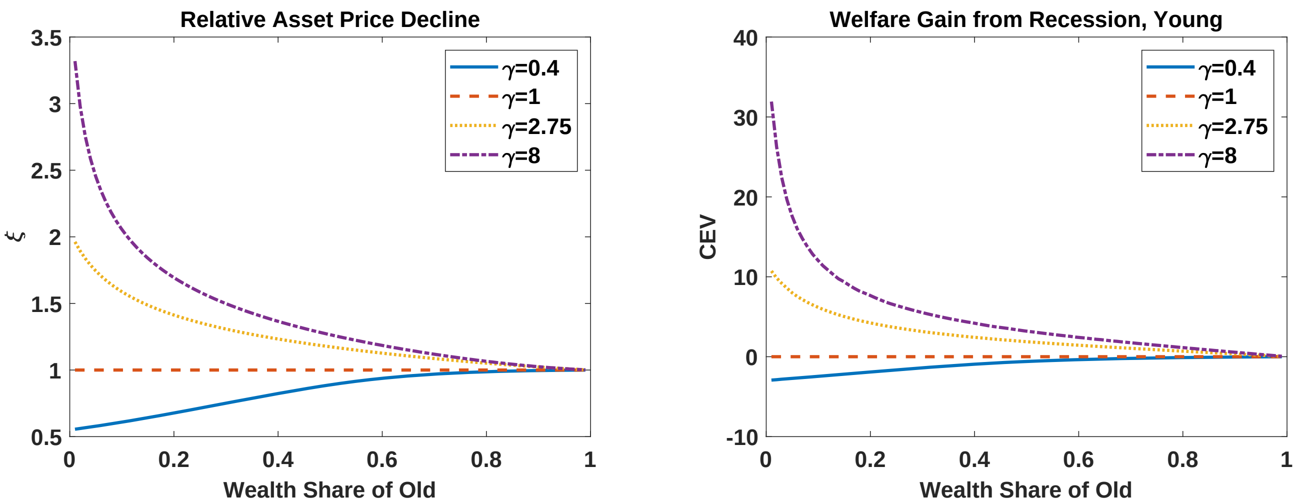
$$v_3(z,A) = u\left[c_3(z,A)\right], \quad \text{(A-34)}$$
$$v_2(z,A) = u\left[c_2(z,A)\right]+\beta\sum_{z'}\;\Gamma_{z,z'}\;u[c_3(z',G(z,A))], \quad \text{(A-35)}$$
$$v_1(z,A) = \beta\sum_{z'}\;\Gamma_{z,z'}\;v_2\left[z',G(z,A)\right]. \quad \text{(A-36)}$$

The welfare losses or gains from a great recession for the young are therefore derived from comparing $v_1(z_n,A)$ and $v_1(z_r,A)$, and the magnitude of the relative asset price decline is measured as the percentage decline in the price of the risky asset, relative to that of output:

$$\xi(A)=\frac{\log(p(z_r,A)/p(z_n,A))}{\log(z_r/z_n)}$$

In Figure A-1 (which is also Figure 1 in the main text) we plot, in the left panel, the elasticity of asset prices to income, $\xi(A)$, against the share of wealth held by old households at the beginning of the period. The right panel display the welfare consequences of young households from the recession, as defined in the main text. The different lines correspond to different values of the IES, and positive numbers represent welfare gains from the recession, negative values welfare losses.

Figure A-1: Asset Price Decline Relative to Output, Welfare



F.2 Details of the 3-Period Economy with Stochastic Growth Rates

In the model with stochastic growth rate, assume that the growth rate (between today and next period) of aggregate productivity

$$g' = \frac{z'}{z}$$

follows a finite state Markov chain with state space $\mathcal{G}$ and transition matrix $\Gamma_{g,g'}$. Now the state space of the economy consists of the current growth rate g and again the wealth distribution, represented by the share of wealth (risky assets) of the old A coming into the period. To compute the model it is easier to work with variables that are deflated by current productivity z. Therefore define

$$\begin{aligned}
\tilde{p}(g, A) &= \frac{p(z, A)}{z} \\
\tilde{c}_m(g, A) &= \frac{c_m(z, A)}{z} \\
\tilde{c}_o(g, A) &= \frac{c_o(z, A)}{z}
\end{aligned}$$

As before, the Euler equation can now be written as

$$[(1 - A)\left(p(z, A) + \theta z\right) - G(z, A)p(z, A)]^{-\gamma} = \beta \sum_{z'} \Gamma_{z,z'} \frac{[p(z', A') + \theta z']}{p(z, A)} \left[G(z, A)\left(p(z', A') + \theta z'\right)\right]^{-\gamma}$$

and dividing by $z^{-\gamma}$ yields

$$\begin{aligned}
&\left[(1-A)\left(\frac{p(z,A)}{z}+\theta\right)-G(z,A)\frac{p(z,A)}{z}\right]^{-\gamma} \\
= \quad &\beta\sum_{z'}\Gamma_{z,z'}\left(\frac{z'}{z}\right)^{1-\gamma}\frac{[p(z',A')/z'+\theta]}{p(z,A)/z}\left[G(z,A)\left(\frac{p(z',A')}{z'}+\theta\right)\right]^{-\gamma}.
\end{aligned}$$

In terms of the deflated price, and now making the switch in the state variable from z to g, this yields the Euler equation

$$\begin{aligned}
&[(1-A)(\tilde{p}(g,A)+\theta)-G(g,A)\tilde{p}(g,A)]^{-\gamma}= \\
&\qquad\qquad \beta\sum_{g'}\Gamma_{g,g'}(g')^{1-\gamma}\frac{[\tilde{p}(g',A')+\theta]}{\tilde{p}(g,A)}[G(g,A)(\tilde{p}(g',A')+\theta)]^{-\gamma},
\end{aligned}$$

or

$$1=\beta\sum_{g'}\Gamma_{g,g'}(g')^{1-\gamma}\frac{[\tilde{p}(g',A')+\theta]}{\tilde{p}(g,A)}\left[\frac{G(g,A)(\tilde{p}(g',A')+\theta)}{(1-A)(\tilde{p}(g,A)+\theta)-G(g,A)\tilde{p}(g,A)}\right]^{-\gamma}.$$

The previous asset market clearing condition $(A-31)$ now reads as

$$[1-G(g,A)]\tilde{p}(g,A)=(1-\theta),$$

and thus again we have two functional equations which determine the equilibrium price function $\tilde{p}(g,A)$ and law of motion for wealth $G(g,A)$.

In figure A-2 we again plot the elasticity of asset prices to income, $\xi(A)$, against the share of wealth held by old households at the beginning of the period, but now for both the economy with stochastic productivity levels and stochastic productivity growth rates. We observe that, for fixed risk aversion the elasticity of asset prices with respect to output movements is qualitatively similar in the two specifications (again, as long as growth rates in the stochastic growth rate case are negatively correlated over time), and that the elasticity of prices to output movements increases with the wealth share of middle-aged households.

F.3 Details of the 3-Period Economy with Stochastic Death and Bequests

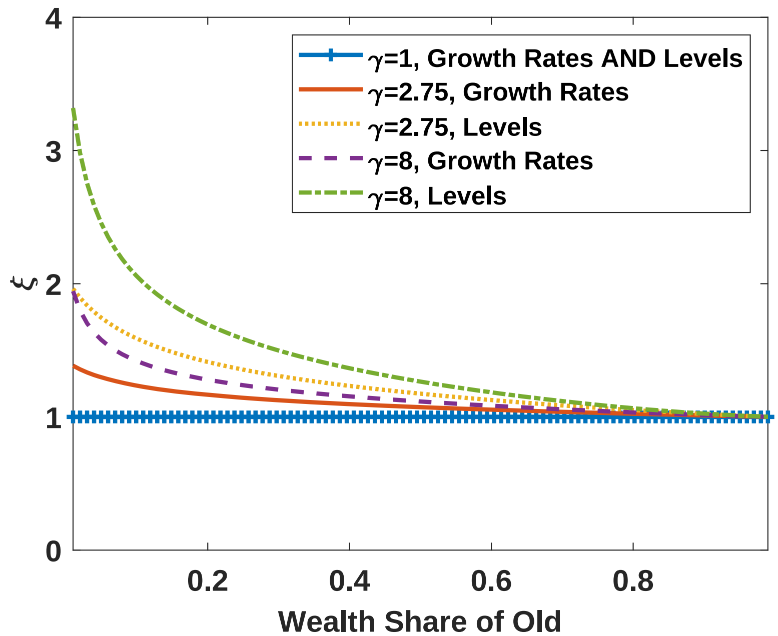
In this appendix we derive the Euler equation and the market clearing condition for the model with stochastic lifetime and bequests in Section B.2. Let A_3 denote the share of aggregate stock, per capita, in the hands of the surviving old at the start of age 3, before any warm glow bequests. There is one unit of stocks, so the market clearing condition for stocks is

$$A_2+\psi A_3=1.$$

Let the state variable capturing the wealth distribution be summarized by $A=A_3$. The household problem at age 3 for a surviving household is

$$\max_{c,b_g}\left\{\frac{c_3(z,A)^{1-\gamma}}{1-\gamma}+\kappa\frac{b_g(z,A)^{1-\gamma}}{1-\gamma}\right\},$$

Figure A-2: Relative Asset Price Decline: Level vs. Growth Rate Shocks



subject to

$$c_3(z,A) + b_g(z,A) = A\left[p(z,A) + \theta z\right].$$

The first order condition deliver optimal consumption and bequests as:

$$\begin{aligned} c_3(z,A) &= \frac{1}{1+\kappa^{\frac{1}{\gamma}}} A\left[p(z,A)+\theta z\right], \\ b_g(z,A) &= \frac{\kappa^{\frac{1}{\gamma}}}{1+\kappa^{\frac{1}{\gamma}}} A\left[p(z,A)+\theta z\right], \end{aligned}$$

and associated utility in the last period of life given by:

$$\begin{aligned} \frac{c_3(z,A)^{1-\gamma}}{1-\gamma} + \kappa \frac{b_g(z,A)^{1-\gamma}}{1-\gamma} &= \frac{1}{1-\gamma}\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma} \left(A\left[p(z,A)+\theta z\right]\right)^{1-\gamma} \\ &= \frac{\left[1+\kappa^{\frac{1}{\gamma}}\right]}{1-\gamma} c_3(z,A)^{1-\gamma}. \end{aligned}$$

Lifetime utility as of age 1 is

$$E_{(z,A)}\left[\beta\left(\frac{c_2(z',A')^{1-\gamma}}{1-\gamma}\right) + \beta\psi\frac{c_3(z'',A'')^{1-\gamma}}{1-\gamma} + \beta\psi\kappa\frac{b_g(z'',A'')^{1-\gamma}}{1-\gamma}\right].$$

Accidental bequests (per recipient household of age 2) correspond to the value of stocks plus dividends of those agents who do did not survive to age 3:

$$b_a(z, A) = (1-\psi)(1-\delta)A\left[p(z, A) + \theta z\right].$$

Total bequests received by each age 2 household are the sum of warm glow bequests plus accidental bequests:

$$\begin{aligned} b(z, A) &= b_a(z, A) + \psi b_g(z, A) \\ &= \left((1-\psi)(1-\delta) + \psi\frac{\kappa^{\frac{1}{\gamma}}}{1+\kappa^{\frac{1}{\gamma}}}\right)A\left[p(z, A) + \theta z\right]. \end{aligned}$$

After making bequests, stocks per capita for the old is

$$\left(1 - \frac{\kappa^{\frac{1}{\gamma}}}{1+\kappa^{\frac{1}{\gamma}}}\right)A = \frac{1}{1+\kappa^{\frac{1}{\gamma}}}A.$$

Stocks (per capita) for the age 2 group after receiving warm-glow bequests is

$$1 - \psi A + \psi\frac{\kappa^{\frac{1}{\gamma}}}{1+\kappa^{\frac{1}{\gamma}}}A = 1 - \frac{\psi}{1+\kappa^{\frac{1}{\gamma}}}A.$$

The budget constraint for the age 2 group is therefore

$$c_2(z, A) = \left[1 - \frac{\psi}{1+\kappa^{\frac{1}{\gamma}}}A\right]\left[p(z, S) + \theta z\right] - sp(z, A),$$

where

$$a' = \left(\frac{\psi + \delta(1-\psi)}{\psi}\right)s.$$

Note that with $\delta = 1$, $a' = \frac{1}{\psi}s$, while with $\delta = 0$, $a' = s$. So we can write the budget constraint in terms of a' as

$$c_2(z, A) = \left[1 - \frac{\psi}{1+\kappa^{\frac{1}{\gamma}}}A\right]\left[p(z, A) + \theta z\right] - \frac{\psi a'}{\psi + \delta(1-\psi)}p(z, A).$$

The equilibrium consistency condition is

$$a' = A' = G(z, A).$$

The Lagrangian is

$$\mathcal{L}(z, A) = \max_{a'}\left\{\begin{array}{c} u\left(\left[1 - \frac{\psi}{1+\kappa^{\frac{1}{\gamma}}}A\right]\left[p(z, A) + \theta z\right] - \frac{\psi a'}{\psi+\delta(1-\psi)}p(z, A)\right) \\ +\beta\psi\sum_{z'}\Gamma_{z,z'}\frac{1}{1-\gamma}\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma}\left(a'\left[p(z', G(z, A)) + \theta z'\right]\right)^{1-\gamma} \end{array}\right\},$$

with optimality condition

$$\left(\left[1-\frac{\psi}{1+\kappa^{\frac{1}{\gamma}}}A\right][p(z,A)+\theta z]-\frac{\psi a'}{\psi+\delta(1-\psi)}p(z,A)\right)^{-\gamma}p(z,A)\frac{\psi}{\psi+\delta(1-\psi)}$$
$$=\beta\psi\sum_{z'}\Gamma_{z,z'}\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma}[p(z',G(z,A))+\theta z']\left(a'\left[p(z',G(z,A))+\theta z'\right]\right)^{-\gamma},$$

or in consumption terms

$$c_2(z,A)^{-\gamma}\frac{\psi}{\psi+\delta(1-\psi)}=\beta\psi\sum_{z'}\Gamma_{z,z'}\left[\frac{p(z',G(z,A))+\theta z'}{p(z,A)}\right]c_3(z',A'\cdot)^{-\gamma}$$

The market clearing condition is that wealth per capita for age 2 before they receive warm glow bequests is the wealth they saved plus the accidental bequests received

$$\begin{aligned}1-\psi G(z,A) &= \frac{(1-\theta)z}{p(z,A)}+(1-\psi)(1-\delta)s(z,A)\\ &= \frac{(1-\theta)z}{p(z,A)}+(1-\psi)(1-\delta)\frac{\psi}{\psi+\delta(1-\psi)}G(z,A),\end{aligned}$$

which implies

$$p(z,A)=\frac{(1-\theta)z}{1-\frac{\psi}{\psi+\delta(1-\psi)}G(z,A)}.$$

These are the Euler equation and the market clearing conditions stated in the main text.

F.3.1 Proof of Proposition 1 and Corollaries 1 and 2

The equilibrium conditions for the general version of the 3-period economy are

$$\begin{aligned}&\left(\left[1-\frac{\psi}{1+\kappa^{\frac{1}{\gamma}}}A\right][p(z,A)+\theta z]-\frac{\psi G(z,A)}{\psi+\delta(1-\psi)}p(z,A)\right)^{-\gamma}p(z,A)\frac{\psi}{\psi+\delta(1-\psi)}\\ =\;&\beta\psi\sum_{z'}\Gamma_{z,z'}\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma}[p(z',G(z,A))+\theta z']\left(G(z,A)\left[p(z',G(z,A))+\theta z'\right]\right)^{-\gamma}\end{aligned}\tag{A-37}$$

$$p(z,A)=\frac{(1-\theta)z}{1-\frac{\psi}{\psi+\delta(1-\psi)}G(z,A)}\tag{A-38}$$

Define a new state variable

$$\tilde{A}=\frac{\psi A}{\psi+\delta(1-\psi)}$$

and let $H(z,\tilde{A})$ define the law of motion for $\tilde{A}$:

$$\tilde{A}' = H(z,\tilde{A}) = \frac{\psi}{\psi+\delta(1-\psi)} G\left(z, \tilde{A}\frac{\psi+\delta(1-\psi)}{\psi}\right)$$

Now rewrite the asset-marking clearing condition eq. (A-38) in terms of the new state variable $\tilde{A}$ and let $q(z,\tilde{A})$ denote the new market-clearing price function:

$$\begin{aligned} p(z,A) &= \frac{(1-\theta)z}{1-\frac{\psi}{\psi+\delta(1-\psi)}G\left(z,\tilde{A}\frac{\psi+\delta(1-\psi)}{\psi}\right)} \\ &= \frac{(1-\theta)z}{1-H(z,\tilde{A})} = q(z,\tilde{A}) \end{aligned} \tag{A-39}$$

We can now rewrite the household first-order condition eq. (A-37) as

$$\begin{aligned} &\left(\left[1-\frac{\psi+\delta(1-\psi)}{1+\kappa^{\frac{1}{\gamma}}}\tilde{A}\right]\left[q(z,\tilde{A})+\theta z\right]-H(z,\tilde{A})q(z,\tilde{A})\right)^{-\gamma} q(z,\tilde{A})\frac{\psi}{\psi+\delta(1-\psi)} \\ = \;& \beta\psi\sum_{z'}\Gamma_{z,z'}\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma}\left[q(z',H(z,\tilde{A}))+\theta z'\right]\left(H(z,\tilde{A})\frac{\psi+\delta(1-\psi)}{\psi}\left[q(z',H(z,\tilde{A}))+\theta z'\right]\right)^{-\gamma} \end{aligned}$$

which simplifies to

$$\begin{aligned} &\left(\left[1-\frac{\psi+\delta(1-\psi)}{1+\kappa^{\frac{1}{\gamma}}}\tilde{A}\right]\left[q(z,\tilde{A})+\theta z\right]-H(z,\tilde{A})q(z,\tilde{A})\right)^{-\gamma} q(z,\tilde{A}) \\ = \;& \beta\psi\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma}\left(\frac{\psi+\delta(1-\psi)}{\psi}\right)^{1-\gamma}\sum_{z'}\Gamma_{z,z'}\left[q(z',\tilde{A}')+\theta z'\right]\left(H(z,\tilde{A})\left[q(z',\tilde{A}')+\theta z'\right]\right)^{-\gamma} \end{aligned} \tag{A-40}$$

An equilibrium of this economy is a pair of functions $q(z,\tilde{A})$ and $H(z,\tilde{A})$ that satisfy eqs. (A-39) and (A-40). Let

$$\begin{aligned} \tilde{\psi} &= \frac{\psi+\delta(1-\psi)}{1+\kappa^{\frac{1}{\gamma}}} \\ \tilde{\beta} &= \beta\psi\left[1+\kappa^{\frac{1}{\gamma}}\right]^{\gamma}\left(\frac{\psi+\delta(1-\psi)}{\psi}\right)^{1-\gamma} \end{aligned}$$

The two equilibrium conditions can now be written as

$$\left(\left[1-\tilde{\psi}\tilde{A}\right]\left[q(z,\tilde{A})+\theta z\right]-H(z,\tilde{A})q(z,\tilde{A})\right)^{-\gamma} q(z,\tilde{A}) \tag{A-41}$$

$$= \tilde{\beta}\sum_{z'}\Gamma_{z,z'}\left[q(z',\tilde{A}')+\theta z'\right]\left(H(z,\tilde{A})\left[q(z',\tilde{A}')+\theta z'\right]\right)^{-\gamma} \tag{A-42}$$

$$q(z,\tilde{A}) = \frac{(1-\theta)z}{1-H(z,\tilde{A})} \tag{A-43}$$

The parameters $(\beta, \kappa, \psi, \delta)$ now appear in the equilibrium conditions only through the quasi-parameters $\tilde{\psi}$ and $\tilde{\beta}$. It is immediate that any two economies with potentially different values for the vector of parameters $(\beta, \kappa, \psi, \delta)$ will exhibit identical dynamics for the asset price $q(z, \tilde{A})$ and identical dynamics for the variable $\tilde{A} = \frac{\psi A}{\psi+\delta(1-\psi)}$ if and only if the two economies share the same values for $\tilde{\psi}$ and $\tilde{\beta}$ (in addition to common values for the capital share parameter θ, for risk aversion γ, and for the process for aggregate risk, $\Gamma_{z,z'}$).

It remains to verify that the welfare cost of a recession can also be expressed as a function only of the quasi-parameters $\tilde{\psi}$ and $\tilde{\beta}$. Expected lifetime utility for an age 1 individual given state (z, A) is

$$\begin{aligned} U(z,A) &= \frac{\beta}{1-\gamma} E_{(z,A)} \left[\left(\left[1 - \frac{\psi}{1+\kappa^{\frac{1}{\gamma}}} A' \right] [p(z',A') + \theta z'] - \frac{\psi}{\psi+\delta(1-\psi)} A'' p(z',A') \right)^{1-\gamma} \right] \\ &\quad + \frac{\beta^2 \psi}{1-\gamma} E_{(z,A)} \left[\left[1 + \kappa^{\frac{1}{\gamma}} \right] \left(\frac{1}{\left[1+\kappa^{\frac{1}{\gamma}} \right]} A'' \left[p(z'',A'') + \theta z'' \right] \right)^{1-\gamma} \right]. \end{aligned}$$

where (z', A') denotes the state in the next period (i.e., at age 2) and (z'', A'') denotes the state in the period after that (at age 3).

In terms of the transformed state variable $\tilde{A}$, expected utility can be expressed as

$$\begin{aligned} U(z,\tilde{A}) &= \frac{\beta}{1-\gamma} E_{(z,\tilde{A})} \left[\left(\left[1 - \frac{\psi+\delta(1-\psi)}{1+\kappa^{\frac{1}{\gamma}}} \tilde{A}' \right] \left[q(z',\tilde{A}') + \theta z' \right] - \tilde{A}'' q(z',A') \right)^{1-\gamma} \right] \\ &\quad + \frac{\beta^2 \psi}{1-\gamma} E_{(z,\tilde{A})} \left[\left[1 + \kappa^{\frac{1}{\gamma}} \right] \left(\frac{1}{\left[1+\kappa^{\frac{1}{\gamma}} \right]} \frac{\psi+\delta(1-\psi)}{\psi} \tilde{A}'' \left[q(z'',\tilde{A}'') + \theta z'' \right] \right)^{1-\gamma} \right] \\ \frac{U(z,\tilde{A})}{\beta} &= \frac{1}{1-\gamma} E_{(z,\tilde{A})} \left[\left(\left[1 - \tilde{\psi} \tilde{A}' \right] \left[q(z',\tilde{A}') + \theta z' \right] - \tilde{A}'' q(z',A') \right)^{1-\gamma} \right] \\ &\quad + \frac{1}{1-\gamma} \tilde{\beta} E_{(z,\tilde{A})} \left[\left(\tilde{A}'' \left[q(z'',\tilde{A}'') + \theta z'' \right] \right)^{1-\gamma} \right]. \end{aligned}$$

It follows that $\frac{U(z,\tilde{A})}{\beta}$ depends on the vector of parameters $(\beta, \kappa, \psi, \delta)$ only through the quasi-parameters $\tilde{\psi}$ and $\tilde{\beta}$. Thus, the welfare cost of entering the economy in a recession, which is defined as the solution ω to the equation

$$U(z_n(1-\omega), \tilde{A}) = U(z_r, \tilde{A}),$$

also depends on the parameters $(\beta, \kappa, \psi, \delta)$ only via $\tilde{\psi}$ and $\tilde{\beta}$.

Corollary 1. Let Model A be our baseline model ($\psi_A = 1$) and let Model B be an alternative with mortality risk ($\psi_B = 1$) but perfect annuity markets ($\delta_B = 1$). Note that $\tilde{\psi}$ is immediately identical across models (given $\kappa_A = \kappa_B = 0$). It follows that the two models have identical implications for asset price dynamics and welfare in a model recession as long as

$$\tilde{\beta} = \beta_A = \beta_B (\psi_B)^\gamma .$$

Corollary 2. Let Model A be a model with accidental bequests but no warm-glow motive ($\delta_A = 0, \kappa_A = 0$) and let Model B be an alternative with no accidental bequests but an active warm-glow motive ($\delta_B = 1, \kappa_B > 0$). Given $\psi_A = \psi_B = \psi < 1$, the two models have identical implications for asset price dynamics and welfare as long as

$$\begin{aligned} \tilde{\psi} &= \psi = \frac{1}{1 + (\kappa_B)^{\frac{1}{\gamma}}}, \\ \tilde{\beta} &= \beta_A \psi = \beta_B \psi \left[1 + (\kappa_B)^{\frac{1}{\gamma}}\right]^\gamma \psi^{\gamma - 1} = \beta_B \psi^\gamma \psi^{-\gamma} = \beta_B. \end{aligned}$$

F.3.2 Additional Results for the Bequest Economy

Table A-10: Extended Table: Asset Price Declines and Welfare: Models with Bequests

Model	Parameters						ξ (%)	Welfare (%)
	ψ	δ	κ	$\beta^{\frac{1}{20}}$	$\tilde{\beta}^{\frac{1}{20}}$	$\widetilde{\psi}$		
(i) Benchmark	1	–	0	1.042	1.042	1	1.23	2.38
(ii)' Annuities	0.71	1	0	1.042	0.994	1	1.23	2.76
(ii) Annuities (recal.)	0.71	1	0	1.092	1.042	1	1.23	2.38
(iii)' Accidental Beq.	0.71	0	0	1.042	1.024	0.71	1.28	1.97
(iii) Accidental Beq. (recal.)	0.71	0	0	1.059	1.042	0.71	1.28	1.85
(iv)' Warm Glow	1	–	0.085	1.042	1.092	0.71	1.26	1.38
(iv) Warm Glow (recal.)	1	–	0.085	0.994	1.042	0.71	1.28	1.85
(v)' Warm Gl. & Acc Beq	0.71	0	0.085	1.042	1.073	0.5	1.33	1.24
(v) Warm Glow + Acc. (recal.)	0.71	0	0.085	1.011	1.042	0.50	1.33	1.44

G Economy with Logarithmic Utility

If the economy is populated with households with logarithmic utility that have life cycle endowment profiles that do not depend on the aggregate shock, then we can solve for a recursive competitive equilibrium in closed form.

Proposition 2. *Assume (i) the period utility function is logarithmic ($\gamma = 1$), and (ii) relative earnings across age groups are independent of the aggregate state, $\varepsilon_i(z) = \varepsilon_i \ \forall z$. Then there exists a recursive competitive equilibrium in the economy with endogenous portfolio choice with the following properties:*

1. *The distribution of wealth A is constant over time. Denote this distribution $\overline{A} = (\overline{A}_1, \dots, \overline{A}_I)$. Then*
$$G_{i+1}(z, \overline{A}, z') = a_i'(z, \overline{A}, \overline{A}_i, z') = \overline{A}_{i+1} \qquad \forall z, z', \ \forall i = 1, \dots, I-1.$$
2. *Aggregate wealth is proportional to the aggregate shock:*
$$p(z, \overline{A}) + q(z, \overline{A})B = \Psi z \qquad \forall z,$$
where Ψ is a constant that does not depend on the value for B.
3. *Stock and bond prices are given by*
$$\begin{aligned} p(z, \overline{A}) &= p(z) = \Psi z - B \frac{z}{R} \sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \\ q(z, \overline{A}) &= q(z) = \frac{z}{R} \sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \qquad \forall z, \end{aligned}$$
where $R = (\Psi + \theta)/\Psi$ is the nonstochastic steady state gross interest rate.
4. *Asset portfolios are identical across age groups:*
$$\lambda_i(z, \overline{A}, \overline{A}_i) = \lambda(z) = \frac{p(z)}{\Psi z} \qquad \forall z, \ \forall i = 1, \dots, I-1.$$
5. *Consumption and savings at each age are proportional to the aggregate shock:*
$$\begin{aligned} c_i(z, \overline{A}, \overline{A}_i) &= \left[(1-\theta)\varepsilon_i + \theta \overline{A}_i + \left(\overline{A}_i - \overline{A}_{i+1}\right) \Psi\right] z, \\ y_i(z, \overline{A}, \overline{A}_i) &= \overline{A}_{i+1} \Psi z \qquad \forall z, \ \forall i = 1, \dots, I-1. \end{aligned}$$
6. *The equity premium is given by*
$$\sum_z \Pi_z \left\{ \sum_{z'} \Gamma_{z,z'} \frac{[p(z') + d(z')]}{p(z)} - \frac{1}{q(z)} \right\} = R \sum_z \frac{\Pi_z}{z} \left\{ \frac{\sum_{z' \in Z} \Gamma_{z,z'} z' - \left(\sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \right)^{-1}}{1 - \frac{B}{R\Psi} \sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'}} \right\}$$
where Π_z denotes the unconditional probability distribution over z.

Corollary 1. *If z is iid over time, then stock and bond prices are proportional to the aggregate shock and the average equity premium is given by* $R\left(\sum_z \frac{\Pi_z}{z}\sum_z \Pi_z z - 1\right) / \left(1 - \frac{B}{R\Psi}\sum_z \frac{\Pi_z}{z}\right)$.

Corollary 2. *In the limit as $\Gamma_{z,z} \to 1\ \forall z$ (perfectly persistent shocks), $q(z) \to R^{-1}$ and $p(z) \to \Psi z - BR^{-1}$.*

We will verify that the conjectured expressions for prices and allocations satisfy households' budget constraints, households' intertemporal first-order conditions, and all the market clearing conditions.

1. Market Clearing Recall that $\sum_{i=1}^{I} \varepsilon_i = 1$, $\overline{A}_1 = 0$ and $\sum_{i=1}^{I} \overline{A}_i = 1$. It is then straightforward to verify that the expressions in Proposition 2 for $\lambda_i(z)$, $c_i(z, \overline{A}, \overline{A}_i)$, and $y_i(z, \overline{A}, \overline{A}_i)$ satisfy the market clearing conditions for goods, stocks, and bonds.

2. Budget Constraints Given identical portfolios across age groups, all households earn the return to saving. Substituting in the candidate expressions for prices (Property 3) and portfolio shares (Property 4), the gross return to saving conditional on productivity being z_{-1} in the previous period and z in the current period is

$$\begin{aligned}
\frac{\lambda(z_{-1})\left[p(z) + d(z)\right]}{p(z_{-1})} + \frac{1 - \lambda(z_{-1})}{q(z_{-1})} &= \frac{p(z) + d(z)}{z_{-1}\Psi} + \frac{z_{-1}\Psi - p(z_{-1})}{z_{-1}\Psi q(z_{-1})} \\
&= \frac{p(z) + d(z)}{z_{-1}\Psi} + \frac{B}{z_{-1}\Psi} \\
&= \frac{z\left(\Psi + \theta\right)}{z_{-1}\Psi}.
\end{aligned}$$

Given this expression for returns, consumption for a household of age i is

$$c_i(z, \overline{A}, \overline{A}_i) = (1 - \theta)\varepsilon_i z + y_{i-1}(z_{-1}, \overline{A}, \overline{A}_i)\frac{z\left(\Psi + \theta\right)}{z_{-1}\Psi} - y_i(z, \overline{A}, \overline{A}_i).$$

Substituting in the candidate expression for $y_i(z, \overline{A}, \overline{A}_i)$ gives

$$c_i(z, \overline{A}, \overline{A}_i) = z\left[(1 - \theta)\varepsilon_i + \theta\overline{A}_i + \left(\overline{A}_i - \overline{A}_{i+1}\right)\Psi\right],$$

which is the conjectured expression for equilibrium consumption (Property 5). Thus, the conjectured allocations satisfy households' budget constraints.

3. Optimal Savings and Portfolio Choices It remains to verify that agents' intertemporal first-order conditions with respect to stocks and bonds are satisfied. For bonds we have

$$\frac{q(z)}{c_i(z, \overline{A}, \overline{A}_i)} = \beta_{i+1}\sum_{z' \in Z}\Gamma_{z,z'}\frac{1}{c_{i+1}(z', \overline{A}, \overline{A}_{i+1})} \qquad \forall i = 1, ..., I - 1.$$

Substituting in the expression for consumption,

$$q(z) = \kappa_{i+1}(\overline{A}) \sum_{z' \in Z} \Gamma_{z,z'} \frac{z}{z'} \qquad \forall i = 1, ..., I-1, \tag{A-44}$$

where

$$\kappa_{i+1}(\overline{A}) = \beta_{i+1} \frac{(1-\theta)\varepsilon_i + \theta \overline{A}_i + \left(\overline{A}_i - \overline{A}_{i+1}\right) \Psi}{(1-\theta)\varepsilon_{i+1} + \theta \overline{A}_{i+1} + \left(\overline{A}_{i+1} - \overline{A}_{i+2}\right) \Psi}.$$

For stocks, we have

$$\begin{aligned} p(z) &= \kappa_{i+1}(\overline{A}) \sum_{z' \in Z} \Gamma_{z,z'} \frac{z}{z'} \left(p(z') + \theta z' + q(z')B - B\right) \\ &= \kappa_{i+1}(\overline{A}) \sum_{z' \in Z} \Gamma_{z,z'} z \left(\Psi + \theta - \frac{B}{z'}\right) \qquad \forall i = 1, ..., I-1. \end{aligned} \tag{A-45}$$

Adding the two first-order conditions for stocks and bonds gives

$$\begin{aligned} p(z) + q(z)B &= z\Psi = \kappa_{i+1}(\overline{A}) \sum_{z' \in Z} \Gamma_{z,z'} \left[z \left(\Psi + \theta - \frac{B}{z'}\right) + \frac{z}{z'} B \right] \\ &= \kappa_{i+1}(\overline{A}) z \left(\Psi + \theta\right) \qquad \forall i = 1, ..., I-1. \end{aligned} \tag{A-46}$$

This equation is satisfied as long as

$$\kappa_{i+1}(\overline{A}) = \frac{\Psi}{\Psi + \theta} = \frac{1}{R} \qquad \forall i = 1, ..., I-1$$

Given this expression for $\kappa_{i+1}(\overline{A})$, it is immediate that the expressions for asset prices (Property 3) satisfy the households' first-order conditions for stocks and bonds (equations A-44 and A-45):

$$\begin{aligned} p(z, \overline{A}) &= p(z) = z\Psi - B \frac{z}{R} \sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'}, \\ q(z, \overline{A}) &= q(z) = \frac{z}{R} \sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \qquad \forall z. \end{aligned}$$

4. Equity Premium   We can derive a near-closed-form expression for the equity premium (up to the endogenous value for wealth Ψ). Let Π_z denote the unconditional probability of

aggregate productivity being z. The average equity premium is defined as

$$
\begin{aligned}
& \sum_z \Pi_z \left\{ \sum_{z'} \Gamma_{z,z'} \left[\frac{p(z') + d(z')}{p(z)} \right] - \frac{1}{q(z)} \right\} \\
= \; & \sum_z \Pi_z \left\{ \sum_{z'} \Gamma_{z,z'} \left[\frac{p(z') + \theta z' + q(z')B - B}{p(z)} \right] - \frac{1}{q(z)} \right\} \\
= \; & \sum_z \frac{\Pi_z}{p(z)} \left\{ \sum_{z'} \Gamma_{z,z'} \left[z' \left(\Psi + \theta \right) \right] - B - \frac{\sum_{z' \in Z} \Gamma_{z,z'} z \left(\Psi + \theta - \frac{B}{z'} \right)}{\sum_{z' \in Z} \Gamma_{z,z'} \frac{z}{z'}} \right\} \\
= \; & \left(\Psi + \theta \right) \sum_z \frac{\Pi_z}{p(z)} \left\{ \sum_{z'} \Gamma_{z,z'} z' - \frac{1}{\sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'}} \right\} \\
= \; & R \sum_z \frac{\Pi_z}{z} \left\{ \frac{\left(\sum\limits_{z' \in Z} \Gamma_{z,z'} z' - \left(\sum\limits_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \right)^{-1} \right)}{\left(1 - \frac{B}{R\Psi} \sum\limits_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \right)} \right\}
\end{aligned}
$$

, where $R = \frac{(\Psi+\theta)}{\Psi}$. If z is *iid* so that $\Gamma_{z,z'}$ is equal to the unconditional probability $\Pi_{z'}$, this simplifies to

$$
\sum_z \Pi_z \left\{ \sum_{z'} \Pi_{z'} \left[\frac{p(z') + d(z')}{p(z)} \right] - \frac{1}{q(z)} \right\} = R \frac{\left(\sum\limits_z \frac{\Pi_z}{z} \sum\limits_z \Pi_z z - 1 \right)}{\left(1 - \frac{B}{R\Psi} \sum\limits_z \frac{\Pi_z}{z} \right)}.
$$

5. Solving for Ψ and $\overline{A}$ Equations (A-46) are the first-order conditions for pricing claims to capital for a nonstochastic life-cycle economy, in which the constant asset price is Ψ and the constant asset income is θ. The $I-1$ equations (A-46) combined with $\overline{A}_1 = 0$ and the market clearing condition $\sum\limits_{i=1}^{I} \overline{A}_i = 1$ can be used to solve numerically for $\left\{ \overline{A}_i \right\}_{i=1}^{I}$ and Ψ. This system of equations is the one used to calibrate the nonstochastic version of our model economy. There we set θ to replicate a target interest rate $R = \left(\Psi + \theta \right) / \Psi$, and we set the life-cycle profile $\{\beta_i\}_{i=2}^{I}$ to replicate the empirical distribution for wealth by age, which determines both the aggregate start-of-period wealth $(\Psi + \theta)$ and its age distribution $\left\{ \overline{A}_i \right\}_{i=1}^{I}$.

H Asset Prices in the Representative Agent Economy

Suppose the representative agent invests an exogenous fraction λ of savings in stocks and fraction $1-\lambda$ in bonds. Let $c(z,a)$ and $y(z,a)$ denote optimal consumption and savings as functions of the aggregate shock z and individual start-of-period wealth a, and let $p(z)$ and $q(z)$ be the equilibrium prices for stocks and bonds. Note that there is no need to keep track of aggregate wealth as a state: by assumption, the supply of capital is constant and equal to one. Thus, prices can only depend on z.

The dynamic programming problem for a household is

$$v(z,a) = \max_{c\geq 0, y} \left\{ u(c) + \beta \sum_{z' \in Z} \Gamma_{z,z'} \; v\left(z', a'\right)\right\}$$

subject to

$$c + y = (1-\theta)z + \left(p(z) + d(z) + B\right) a$$

and the law of motion

$$a' \left[p(z') + d(z') + B\right] = \left(\frac{\lambda \; [p(z') + d(z')]}{p(z)} + \frac{(1-\lambda)}{q(z)}\right) \; y.$$

The solution to this problem yields decision rules $c(z,a)$, $y(z,a)$ and $a'(z', y(z,a))$ is the associated value for next period wealth.

Given the preferences and technology described above, the market clearing conditions are simply

$$\begin{aligned} \lambda y(z,1) &= p(z), \\ (1-\lambda) y(z,1) &= q(z) B, \\ c(z,1) &= z. \end{aligned}$$

The individual and aggregate consistency condition is

$$a'(z', y(z,1)) = 1.$$

Now suppose the process for z is a two-state Markov chain. There are just two equity prices to solve for: $p(z) \in \{p(z_l), p(z_h)\}$. The two market clearing conditions for stocks and bonds imply a parametric relationship between $q(z)$ and $p(z)$:

$$q(z) = \frac{p(z)(1-\lambda)}{\lambda B}.$$

Thus, stock and bond prices must be equally sensitive to aggregate shocks. The realized gross real return to saving is given by

$$\lambda \frac{[p(z') + d(z')]}{p(z)} + (1-\lambda)\frac{1}{q(z)} = \frac{p(z') + \lambda\theta z'}{p(z)},$$

where the second equality follows from substituting in $d(z') = \theta z' + q(z')B - B$ and the expression for $q(z)$, as a function of $p(z)$. Thus, the equilibrium equity prices are defined by the solutions to the two intertemporal first-order conditions:

$$
\begin{aligned}
p(z_h)u'(c(z_h,1)) &= \beta \sum_{z' \in Z} \Gamma_{z_h,z'} \left[u'(c(z',1)) \left[\lambda \theta z' + p(z') \right] \right], \\
p(z_l)u'(c(z_l,1)) &= \beta \sum_{z' \in Z} \Gamma_{z_l,z'} \left[u(c(z',1)) \left[\lambda \theta z' + p(z') \right] \right],
\end{aligned}
$$

which, using the market clearing condition for consumption and the CRRA preference specification, can be written as

$$
\begin{aligned}
p(z_h) z_h^{-\gamma} &= \beta \Gamma_h \; z_h^{-\gamma} \left[\lambda \theta z_h + p(z_h) \right] + \beta (1 - \Gamma_h) z_l^{-\gamma} \left[\lambda \theta z_l + p(z_l) \right], \\
p(z_l) z_l^{-\gamma} &= \beta \Gamma_l \; z_l^{-\gamma} \left[\lambda \theta z_l + p(z_l) \right] + \beta (1 - \Gamma_l) z_h^{-\gamma} \left[\lambda \theta z_h + p(z_h) \right],
\end{aligned}
$$

where $\Gamma_h = \Gamma_{z_h,z_h}$ and $\Gamma_l = \Gamma_{z_l,z_l}$. From the second pricing equation,

$$
p(z_h) = \frac{\beta \Gamma_h \lambda \theta z_h + \beta (1 - \Gamma_h) \frac{z_l^{-\gamma}}{z_h^{-\gamma}} \left(\lambda \theta z_l + p(z_l) \right)}{(1 - \beta \Gamma_h)}.
$$

Substituting this into the first pricing equation,

$$
p(z_l) = \frac{\beta \Gamma_l \; z_l^{-\gamma} \lambda \theta z_l + \beta (1 - \Gamma_l) z_h^{-\gamma} \left(\lambda \theta z_h + \frac{\beta \Gamma_h \lambda \theta z_h + \beta (1 - \Gamma_h) \frac{z_l^{-\gamma}}{z_h^{-\gamma}} \lambda \theta z_l}{(1 - \beta \Gamma_h)} \right)}{z_l^{-\gamma} \left(\frac{(1 - \beta)(1 + \beta(1 - \Gamma_h - \Gamma_l))}{(1 - \beta \Gamma_h)} \right)}.
$$

Since the expression for $p(z_h)$ is symmetric, we can take the ratio to express the ratio of prices across states as a function of fundamentals:

$$
\frac{p(z_l)}{p(z_h)} = \frac{z_l}{z_h} \left(\frac{(1 - \Gamma_l) \, z_h^{1-\gamma} z_l^{\gamma - 1} + (\beta + \Gamma_l - \beta \Gamma_h - \beta \Gamma_l)}{(1 - \Gamma_h) \, z_l^{1-\gamma} z_h^{\gamma - 1} + (\beta + \Gamma_h - \beta \Gamma_h - \beta \Gamma_l)} \right).
$$

Note that λ and θ have dropped out here: the *ratio* of stock prices across states does not depend on either λ or θ, though the *levels* of prices do. If aggregate shocks are *iid*, then $1 - \Gamma_l = \Gamma_h$ and the expression above simplifies to

$$
\frac{p(z_l)}{p(z_h)} = \left(\frac{z_l}{z_h} \right)^{\gamma}.
$$

It is straightforward to verify that the same result is obtained even without the *iid* assumption in two special cases: $\gamma = 1$ or $\beta = 1$.

I Asset Prices in the Two-Period Overlapping-Generations Economy

In this appendix we study the simplest OLG framework in which households live for only two periods: $I = 2$. We use this example to discuss how the curvature parameter γ affects the elasticity ξ of price changes to output changes in OLG economies. To make that discussion most transparent, we focus on an economy with only the risky stock ($B = 0$ and $\lambda_i \equiv 0$) and assume that households only earn labor income in the first period of life: $\varepsilon_1 = 1$ and $\varepsilon_2 = 0$. Since young households start with zero assets, all wealth is therefore held by old agents. As a consequence, the wealth distribution is degenerate (and time invariant) in this economy. As in the representative agent model, the only state variable is the exogenous shock $z \in \{z_l, z_h\}$.

Consumption of young and old households is given by

$$\begin{aligned} c_1(z) &= (1-\theta)z - p(z) \\ c_2(z) &= \theta z + p(z), \end{aligned}$$

and the stock market price is determined by the intertemporal Euler equation

$$p(z)\ [(1-\theta)z - p(z)]^{-\gamma} = \beta \sum_{z' \in \{z_L, z_h\}} \Gamma_{z,z'}\ [\theta z' + p(z')]^{-\gamma}\ [\theta z' + p(z')]\,. \tag{A-47}$$

No closed-form solution is available for the functional equation $p(z)$ that solves equation $(A-47)$ outside of the special cases $\gamma = 0$ and $\gamma = 1$. However, taking a first order approximation of the Euler equation around the point $z_l/z_h = 1$ we can show

Proposition 3. *To a first order approximation*

$$\begin{aligned} \xi^{2p} &\approx \frac{\gamma(1-\theta)}{1-\theta\frac{(R-\gamma)}{(R-1)}} \\ &= \xi^{RA} \times \frac{1-\theta}{1-\theta\frac{(R-\gamma)}{(R-1)}}, \end{aligned}$$

where $R = \frac{\theta+p}{p} > 1$ is the steady state gross return on the stock.[69]

We prove this proposition in section below. Note first that for $\gamma = 1$, this formula is exact (as shown in the previous section) and delivers $\xi^{2p} = \xi^{RA} = 1$: prices fall by exactly as much as output in a downturn. Second, ξ^{2p} is increasing in γ, and thus for $\gamma > 1$ we have $\xi^{2p} > 1$. Third, $\xi^{2p} < \xi^{RA}$. Thus, as long as the intertemporal elasticity of substitution $1/\gamma$ is smaller than one, asset prices fall by more than output in a recession, but by less than in the corresponding representative agent economy with infinitely lived households.

The reason is as follows. Consumption of the current old generation must decline in the recession since the price of the asset, the only source for old-age consumption, is lower in the bad

[69] For θ such that $R = \beta^{-1}$, the expression simplifies to $\xi^{2p} \approx \frac{\gamma(\beta+1)}{\gamma\beta+1}$.

than in the good aggregate state of the world. Moreover, for $\gamma > 1$, consumption of the old is more sensitive to aggregate shocks than consumption of the young:

$$\frac{c_1(z_h)}{c_1(z_l)} < \frac{z_h}{z_l} < \frac{c_2(z_h)}{c_2(z_l)}.$$

The second inequality reflects the fact that $c_2(z_h)/c_2(z_l) = p_h/p_l > z_h/z_l$ (since $\xi^{2p} > 1$), while the first inequality follows from market clearing: $(c_1(z_h) + c_2(z_h)) / (c_1(z_l) + c_2(z_l)) = z_h/z_l$. The fact that aggregate risk is disproportionately borne by the old explains why stock prices are less volatile in this economy than in the analogous representative agent economy. Recall that stocks are effectively priced by younger agents, because the supply of stocks by the old is inelastic at any positive price. Because the old bear a disproportionate share of aggregate risk, the young's consumption fluctuates less than output. Thus, smaller price changes (relative to the representative agent economy) are required to induce them to purchase the aggregate supply of equity at each date.

One might wonder whether it is possible that $c_1(z_h) < c_1(z_l)$, so that newborn households would potentially prefer to enter the economy during a recession rather than during a boom. The answer turns out to be no: while stock prices fall by more than output in the event of a recession, they never fall by enough to compensate the young for their decline in labor earnings. The logic for this result is straightforward. In a two-period OLG economy, stock prices are defined by the inter-temporal first-order condition for young households (equation A-47). With *iid* shocks, the right-hand side of this condition is independent of the current value for z. Taking the ratio of the two pricing equations across states, the ratio of stock prices across states is given by

$$\frac{p_h}{p_l} = \left(\frac{c_1(z_h)}{c_1(z_l)}\right)^{\gamma}.$$

The advantage to the young from entering the economy during a recession is that they buy stocks cheaply, $p_h/p_l > 1$. But the optimality restriction above then implies that $c_1(z_h)/c_1(z_l) > 1$, so the young must suffer low consumption if they enter during a recession. Intuitively, low prices are needed to induce the young to buy stocks when the marginal utility of current consumption is high. But a high marginal utility of consumption requires low consumption for these households.

This example reveals that for the young to potentially gain from a recession, we need people to live for at least three periods, while the previous example with logarithmic preferences indicates that we also require $\gamma > 1$. That is why, in the main text, we focus on the 3 period version of the model in which both desired results can emerge simultaneously.

I.1 Proof of Proposition 3

Let $\widetilde{p} = \frac{p(z_h)}{p(z_l)}$, $\widetilde{z} = \frac{z_h}{z_l}$, and $\widetilde{x} = \frac{z_l}{p_l}$. In terms of these variables, the intertemporal first-order conditions, conditional on the current state being z_l and z_h are, respectively,

$$\begin{aligned} ((1-\theta)\widetilde{x} - 1)^{-\gamma} &= \beta\Gamma_{z_l,z_l}\ (\theta\widetilde{x} + 1)^{1-\gamma} + \beta\Gamma_{z_l,z_h}\ (\theta\widetilde{z}\widetilde{x} + \widetilde{p})^{1-\gamma} \\ \widetilde{p}\left((1-\theta)\widetilde{z}\widetilde{x} - \widetilde{p}\right)^{-\gamma} &= \beta\Gamma_{z_h,z_l}\ (\theta\widetilde{x} + 1)^{1-\gamma} + \beta\Gamma_{z_h,z_h}\ (\theta\widetilde{z}\widetilde{x} + \widetilde{p})^{1-\gamma}. \end{aligned}$$

Our goal is to solve for $\widetilde{p}$ as a function of $\widetilde{z}$. However, except for the special case $\gamma = 1$, this system of equations cannot be solved in closed form. So instead we will linearize these equations and look for an approximate solution for relative prices as a linear function of relative productivity. We proceed as follows:

1. Take first-order Taylor-series approximations to these two first-order conditions around the nonstochastic steady state values for $\widetilde{p}$, $\widetilde{z}$, and $\widetilde{x}$, which we denote P, Z, and X (where $Z = P = 1$). This gives a system of two equations in three first-order terms $(\widetilde{x} - X)$, $(\widetilde{z} - Z)$, and $(\widetilde{p} - P)$:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{pmatrix} \begin{pmatrix} (\widetilde{x} - X) \\ (\widetilde{z} - Z) \\ (\widetilde{p} - P) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{A-48}$$

where

$$\begin{aligned} A_{11} &= -\gamma \left((1-\theta)X - 1\right)^{-\gamma-1}(1-\theta) - \\ &\qquad \left((1-\gamma)\beta\Gamma_{z_l,z_l}(\theta X + 1)^{-\gamma}\theta + (1-\gamma)\beta\Gamma_{z_l,z_h}(\theta X + 1)^{-\gamma}\theta\right) \\ A_{12} &= -(1-\gamma)\beta\Gamma_{z_l,z_h}(\theta X + 1)^{-\gamma}\theta X \\ A_{13} &= -(1-\gamma)\beta\Gamma_{z_l,z_h}(\theta X + 1)^{-\gamma} \\ A_{21} &= -\gamma \left((1-\theta)X - 1\right)^{-\gamma-1}(1-\theta) - \\ &\qquad \left((1-\gamma)\beta\Gamma_{z_h,z_l}(\theta X + 1)^{-\gamma}\theta + (1-\gamma)\beta\Gamma_{z_h,z_h}(\theta X + 1)^{-\gamma}\theta\right) \\ A_{22} &= -\gamma \left((1-\theta)X - 1\right)^{-\gamma-1}(1-\theta)X - (1-\gamma)\beta\Gamma_{z_h,z_h}(\theta X + 1)^{-\gamma}\theta X \\ A_{23} &= \gamma \left((1-\theta)X - 1\right)^{-\gamma-1} + \left((1-\theta)X - P\right)^{-\gamma} - (1-\gamma)\beta\Gamma_{z_H,z_H}(\theta X + 1)^{-\gamma}. \end{aligned}$$

2. Use the first equation in A-48 to solve for $(\widetilde{x} - X)$ as a linear function of $(\widetilde{z} - Z)$ and $(\widetilde{p} - P)$:

$$(\widetilde{x} - X) = -\frac{A_{12}}{A_{11}}(\widetilde{z} - Z) - \frac{A_{13}}{A_{11}}(\widetilde{p} - P).$$

Then substitute this solution into the second equation in (A-48), and solve for $(\widetilde{p} - P)$ as a function of $(\widetilde{z} - Z)$:

$$\begin{aligned} (\widetilde{p} - P) &= -\frac{A_{21}}{A_{23}}(\widetilde{x} - X) - \frac{A_{22}}{A_{23}}(\widetilde{z} - Z) \\ &= -\frac{A_{21}}{A_{23}}\left(-\frac{A_{12}}{A_{11}}(\widetilde{z} - Z) - \frac{A_{13}}{A_{11}}(\widetilde{p} - P)\right) - \frac{A_{22}}{A_{23}}(\widetilde{z} - Z). \end{aligned}$$

Thus,

$$\xi^{2p} \approx \frac{\widetilde{p} - P}{\widetilde{z} - Z} = \frac{A_{21}A_{12} - A_{22}A_{11}}{A_{23}A_{11} - A_{21}A_{13}}.$$

3. Now assume productivity shocks are *iid*, so that $\Gamma_{z_l,z_h} = \Gamma_{z_h,z_h} = \Gamma_{z_h}$ and $\Gamma_{z_l,z_l} = \Gamma_{z_h,z_l} = 1 - \Gamma_{z_h}$. Under this *iid* assumption, $A_{11} = A_{21}$ and thus

$$\xi^{2p} \approx \frac{A_{21}A_{12} - A_{22}A_{21}}{A_{23}A_{11} - A_{21}A_{13}} = \frac{A_{12} - A_{22}}{A_{23} - A_{13}} = X\frac{\gamma(1-\theta)}{((X - X\theta - 1) + \gamma)}$$

Recall that X is the inverse of the steady state stock price, so we can equivalently write this elasticity in terms of the steady state gross interest rate R, where $R = \theta X + 1$:

$$\xi^{2p} \approx \frac{\gamma(1-\theta)}{1 - \theta\frac{(R-\gamma)}{(R-1)}}. \tag{A-49}$$

This is the expression given in the text. Note that for $\gamma = 1$, $\xi^{2p} = 1$.

4. We want to show that $1 < \xi^{2p} < \xi^{RA}$ for $\gamma > 1$. First, note that in any equilibrium, a positive stock price implies $R > 1$. Then

$$\frac{1}{\xi^{2p}} = \frac{1 - \theta\frac{(R-\gamma)}{(R-1)}}{\gamma(1-\theta)} = \frac{1}{\gamma}\left(1 + \frac{(\gamma-1)\frac{\theta}{R-1}}{(1-\theta)}\right) > \frac{1}{\gamma} = \frac{1}{\xi^{RA}}.$$

Thus, $\xi^{2p} < \xi^{RA}$.

Given that $\xi^{2p} = 1$ when $\gamma = 1$, showing that ξ^{2p} is strictly increasing in γ is sufficient to prove that $\xi^{2p} > 1$:

$$\frac{\partial}{\partial\gamma}\left(\frac{\gamma(1-\theta)}{1 - \theta\frac{(R-\gamma)}{(R-1)}}\right) = (\theta - 1)(R-1)\frac{R\theta - R + 1}{(R - R\theta + \theta\gamma - 1)^2}$$

It follows that ξ^{2p} is strictly increasing in γ if and only if $R > \frac{1}{1-\theta}$. But in any equilibrium, positive consumption for the young requires exactly this condition:

$$(1-\theta) - \frac{\theta}{R-1} > 0 \Leftrightarrow R > \frac{1}{1-\theta}.$$

We conclude that $\xi^{2p} > 1$.

5. In the special case in which θ is such that $R = \frac{1}{\beta}$, the expression for ξ^{2p} simplifies further. The steady state value for R is an endogenous variable and has to satisfy the steady state version of the intertemporal first-order condition, where the steady state stock price is $\theta/(R-1)$:

$$\frac{\theta}{R-1}\left((1-\theta) - \frac{\theta}{R-1}\right)^{-\gamma} = \beta\left(\theta + \frac{\theta}{R-1}\right)^{-\gamma}\left(\theta + \frac{\theta}{R-1}\right). \tag{A-50}$$

When $\beta = \frac{1}{R}$, equation (A-50) can be solved in closed form to give $R = \frac{1}{1-2\theta}$. Thus, we have $R = \frac{1}{\beta} = \frac{1}{1-2\theta}$, which implies $\theta = \frac{1}{2}(1-\beta)$. Substituting $R = \frac{1}{\beta}$ and $\theta = \frac{1}{2}(1-\beta)$ into equation (A-49) gives

$$\xi^{2p} \approx \frac{\gamma(1-\theta)}{1 - \theta\frac{(R-\gamma)}{(R-1)}} = \frac{\gamma(\beta+1)}{\gamma\beta + 1},$$

J Wealth-Based Welfare Measures

Table A-11: Wealth-Based Welfare Losses (%)

	Portfolio & Earnings Model			
	Endogenous Port.		Exogenous Port.	
Age Group	Asym.	Sym.	Asym.	Sym.
20-29	-3.55	-2.21	-5.00	-4.30
30-39	-11.99	-12.43	-11.31	-8.78
40-49	-19.35	-19.77	-12.06	-13.33
50-59	-32.25	-32.65	-31.72	-30.08
60-69	-41.90	-42.21	-50.24	-50.98
70+	-26.62	-26.72	-27.17	-31.50

K Welfare with Fixed Prices

In this appendix we provide the details of our calculations for the partial equilibrium thought experiments in section 6.5 of the main text.

K.1 No Asset Price Recession

In the first scenario, there is a recession at 0 but nothing happens to asset prices. So in the recession period, the wealth distribution is again A_{-1} and aggregate start of period wealth is

$$W_{-1} = p(z_H, A_{-1}) + \theta z_H + q(z_H, A_{-1})B$$

Thus, the age distribution of start of period wealth in the recession period is exactly what it would have been given no recession in the general equilibrium model.

Now, starting with their age-specific wealth, $A^i_{-1}W_{-1}$, agents at each age maximize expected lifetime utility looking forward, taking as given the stochastic process for z with the corresponding implications for their earnings. They believe that at each date and in each state, one period ahead gross returns will be R, where R is defined below.

We compare welfare conditional on a recession at date 0 to welfare without a recession at date 0. When there is no recession at date, households also have start of period wealth $A^i_{-1}W_{-1}$, but they have higher labor earnings (and a different probability distribution over future earnings). They take as given the same gross returns R looking forward.

The date 0 constraints for an agent of age i with (without) a recession are

$$\begin{aligned} c_i + y_i &= \varepsilon_i(z_R)(1-\theta)z_R + A^i_{-1}W_{-1} \\ c_i + y_i &= \varepsilon_i(z_R)(1-\theta)z_H + A^i_{-1}W_{-1} \end{aligned}$$

where y_i is savings. We will later assume savings are divided between $z'-$contingent Arrow securities, and that the cost of buying one of each of these securities (thereby delivering one unit of consumption tomorrow) is $1/R$.

K.2 Permanent Asset Price Recession

In the second scenario, the distribution of start of period wealth in the recession moves just as it does in the baseline general equilibrium model. Call this recession distribution, A_R, where A_R is the distribution of start of period wealth conditional on a long period of normal times prior to date 0, followed by a recession at date zero. Aggregate start of period wealth is

$$W_R = p(z_R, A_R) + \theta z_R + q(z_R, A_R)B$$

Thus, the age distribution of start of period wealth in the recession period is exactly what it would have been given a recession in the general equilibrium model.

Now, starting with their age-specific wealth, $A_R^i W_R$, agents at each age maximize expected lifetime utility looking forward, taking as given the stochastic process for z with the corresponding implications for their earnings. They believe that at each date and in each state, one period ahead returns will be R (as in the other scenario).

The date 0 constraints for an agent of age i with (without) a recession are

$$\begin{aligned} c_i + y_i &= \varepsilon_i(z_R)(1-\theta)z_R + A_R^i W_R \\ c_i + y_i &= \varepsilon_i(z_H)(1-\theta)z_H + A_{-1}^i W_{-1} \end{aligned}$$

K.3 Returns and Risk Sharing

One issue is that if there is no uncertainty about stock or bond returns, and equities offer a return premium, everyone will want to short bonds, if they are allowed to do so. Thus we need to either impose exogenous portfolios or assume that both assets must pay the same constant return from the recession period onward. In either case it seems reasonable to set the return to saving equal to the aggregate return to assets in the repeated normal state, i.e.,

$$R = \frac{p(z_H, A_{-1}) + d(z_H, A_{-1}) + B}{p(z_H, A_{-1}) + q(z_H, A_{-1})B}.$$

A second issue is that with constant returns, there is no way to pool aggregate risk. This introduces an asymmetry relative to the baseline model, which we probably don't want. Suppose we introduce Arrow securities, which pay one unit of consumption if a particular aggregate state is realized, and which are priced at actuarially fair rates, so the price of consumption in z' given z

today is

$$q(z,z') = \frac{pr(z'|z)}{R}$$

Thus the price of one unit of consumption for sure is simply

$$\sum_{z'} q(z,z') = \frac{1}{R}$$

Given these assumptions, the household problem at age i and date 0 given initial productivity z_0 and initial start of period wealth distribution $A^i W$ is

$$\max_{\left\{c_j(z^{0+j-i})\right\}} \left\{ u(c_i(z_0)) + \sum_{j=i+1}^{I} \left(\beta_i \times ... \times \beta_{j-1}\right) \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) u(c_j(z^{0+j-i})) \right\}$$

subject to

$$c_i(z_0) + \sum_{j=i+1}^{I} \frac{1}{R^{j-i}} \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) c_j(z^{0+j-i}) \leq LTI^i_{z_0}$$

$$LTI^i_{z_0} \equiv A^i W + \varepsilon_i(z_0)(1-\theta)z_0 + \sum_{j=i+1}^{I} \frac{1}{R^{j-i}} \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0)\varepsilon_i(z\left(z^{0+j-i}\right))(1-\theta)z\left(z^{0+j-i}\right)$$

where z^{0+j-i} is a possible history from date zero to date $0+j-i$, and $\pi_0(z^{0+j-i}|z_0)$ is the corresponding probability conditional on z_0 (which matters because shocks are not *iid*) and where $z\left(z^{0+j-i}\right)$ is just the last element of the sequence. Note that I have written the consumption prices straight into the budget constraint.

The first order conditions are

$$\begin{aligned} u'(c_i(z_0)) &= \lambda \\ \left(\beta_i \times ... \times \beta_{j-1}\right) \pi_0(z^{0+j-i}|z_0) u'(c_j(z^{0+j-i})) &= \lambda \frac{1}{R^{j-i}} \pi_0(z^{0+j-i}|z_0) \end{aligned}$$

which imply

$$c_j(z^{0+j-i}) = \left[R^{j-i}\left(\beta_i \times ... \times \beta_{j-1}\right)\right]^{\frac{1}{\gamma}} c_i(z_0)$$

We also have

$$\begin{aligned} c_i(z_0) + \sum_{j=i+1}^{I} \frac{1}{R^{j-i}} \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) c_j(z^{0+j-i}) &= LTI^i_{z_0} \\ c_i(z_0) + \sum_{j=i+1}^{I} \frac{1}{R^{j-i}} \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) \left[R^{j-i}\left(\beta_i \times ... \times \beta_{j-1}\right)\right]^{\frac{1}{\gamma}} c_i(z_0) &= LTI^i_{z_0} \\ c_i(z_0)\left(1 + \sum_{j=i+1}^{I} \frac{1}{R^{j-i}} \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) \left[R^{j-i}\left(\beta_i \times ... \times \beta_{j-1}\right)\right]^{\frac{1}{\gamma}}\right) &= LTI^i_{z_0} \end{aligned}$$

or

$$c_i(z_0) = \chi^i_{z_0} LTI^i_{z_0}$$

where

$$\chi^i_{z_0} = \frac{1}{\left(1 + \sum_{j=i+1}^{I} \frac{1}{R^{j-i}} \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) \left[R^{j-i}\left(\beta_i \times ... \times \beta_{j-1}\right)\right]^{\frac{1}{\gamma}}\right)}$$

But note here that in fact nothing in $\chi^i_{z_0}$ depends on the history, and the probabilities add to one, so we can write

$$\chi^i_{z_0} = \chi^i = \frac{1}{1 + \sum_{j=i+1}^{I} \left(R^{j-i}\right)^{\frac{1}{\gamma}-1} \left(\beta_i \times ... \times \beta_{j-1}\right)^{\frac{1}{\gamma}}}$$

K.4 Lifetime Utility and Welfare Calculations

Lifetime utility is

$$\begin{aligned}
& u(c_i(z_0)) + \sum_{j=i+1}^{I} \left(\beta_i \times ... \times \beta_{j-1}\right) \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) u(c_j(z^{0+j-i})) \\
= \; & u(c_i(z_0)) + \sum_{j=i+1}^{I} \left(\beta_i \times ... \times \beta_{j-1}\right) \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) u\left(\left[R^{j-i}\left(\beta_i \times ... \times \beta_{j-1}\right)\right]^{\frac{1}{\gamma}} c_i(z_0)\right) \\
= \; & \left(1 + \sum_{j=i+1}^{I} \left(\beta_i \times ... \times \beta_{j-1}\right) \sum_{z^{0+j-i}} \pi_0(z^{0+j-i}|z_0) \left(\left[R^{j-i}\left(\beta_i \times ... \times \beta_{j-1}\right)\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}\right) \frac{\left(\chi^i LTI^i_{z_0}\right)^{1-\gamma}}{1-\gamma} \\
= \; & \left(1 + \sum_{j=i+1}^{I} \left(\beta_i \times ... \times \beta_{j-1}\right) \left[\left(R^{j-i}\right)^{\frac{1-\gamma}{\gamma}} \left(\beta_i \times ... \times \beta_{j-1}\right)^{\frac{1-\gamma}{\gamma}}\right]\right) \frac{\left(\chi^i LTI^i_{z_0}\right)^{1-\gamma}}{1-\gamma}
\end{aligned}$$

Note that lifetime utility depends on the initial state z_0 and the initial start of period wealth distribution $A^i W$ only through the term $LTI^i_{z_0}$.

What is the welfare cost of entering in a recession? Define it as the solution ω to

$$\begin{aligned}
& \left(1 + \sum_{j=i+1}^{I} \left(\beta_i \times ... \times \beta_{j-1}\right) \left[\left(R^{j-i}\right)^{\frac{1-\gamma}{\gamma}} \left(\beta_i \times ... \times \beta_{j-1}\right)^{\frac{1-\gamma}{\gamma}}\right]\right) \frac{\left(\chi^i LTI^i_{z_H}\right)^{1-\gamma}}{1-\gamma} (1+\omega)^{1-\gamma} \\
= \; & \left(1 + \sum_{j=i+1}^{I} \left(\beta_i \times ... \times \beta_{j-1}\right) \left[\left(R^{j-i}\right)^{\frac{1-\gamma}{\gamma}} \left(\beta_i \times ... \times \beta_{j-1}\right)^{\frac{1-\gamma}{\gamma}}\right]\right) \frac{\left(\chi^i LTI^i_{z_L}\right)^{1-\gamma}}{1-\gamma}
\end{aligned}$$

or

$$\begin{aligned}
\left(LTI^i_{z_H}\right)^{1-\gamma} (1+\omega)^{1-\gamma} &= \left(LTI^i_{z_L}\right)^{1-\gamma} \\
\log\left(LTI^i_{z_H}\right) + \log(1+\omega) &= \log\left(LTI^i_{z_L}\right) \\
\omega &\approx \log\left(\frac{LTI^i_{z_L}}{LTI^i_{z_H}}\right)
\end{aligned}$$

This entity is straightforward to calculate. In particular, for scenario 1 (no asset price recession) the calculation is

$$\omega_1 \approx \log\left(\frac{A^i_{-1}W_{-1}+\varepsilon_i(z_L)(1-\theta)z_L+\sum_{j=i+1}^{I}\frac{1}{R^{j-i}}\sum_{z^{0+j-i}}\pi_0(z^{0+j-i}|z_L)\varepsilon_i(z\left(z^{0+j-i}\right))(1-\theta)z\left(z^{0+j-i}\right)}{A^i_{-1}W_{-1}+\varepsilon_i(z_H)(1-\theta)z_H+\sum_{j=i+1}^{I}\frac{1}{R^{j-i}}\sum_{z^{0+j-i}}\pi_0(z^{0+j-i}|z_H)\varepsilon_i(z\left(z^{0+j-i}\right))(1-\theta)z\left(z^{0+j-i}\right)}\right)$$

For scenario 2 (asset price recession) the calculation is

$$\omega_2 \approx \log\left(\frac{A^i_{R}W_{R}+\varepsilon_i(z_L)(1-\theta)z_L+\sum_{j=i+1}^{I}\frac{1}{R^{j-i}}\sum_{z^{0+j-i}}\pi_0(z^{0+j-i}|z_L)\varepsilon_i(z\left(z^{0+j-i}\right))(1-\theta)z\left(z^{0+j-i}\right)}{A^i_{-1}W_{-1}+\varepsilon_i(z_H)(1-\theta)z_H+\sum_{j=i+1}^{I}\frac{1}{R^{j-i}}\sum_{z^{0+j-i}}\pi_0(z^{0+j-i}|z_H)\varepsilon_i(z\left(z^{0+j-i}\right))(1-\theta)z\left(z^{0+j-i}\right)}\right)$$

We can also translate the answers into dollar numbers, by computing $\omega LTI^i_{z_H}$.

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