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Competition among Sellers in Securities Auctions[†]

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We study simultaneous security-bid second-price auctions with competition among sellers for potential bidders. The sellers compete by designing ordered sets of securities that the bidders can offer as payment for the assets. Upon observing auction designs, potential bidders decide which auctions to enter. We characterize all symmetric equilibria and show that there always exist equilibria in which auctions are in standard securities or their combinations. In large markets the unique equilibrium is auctions in pure cash. We extend the model for competition in reserve prices and show that binding reserve prices never constitute equilibrium as long as equilibrium security designs are not call options. (*JEL* D44, D82, G10)

In economic and financial markets, rights to control and exploit assets with the potential to generate cash flows are often sold through auctions. While in some auctions the winner's future profits from the asset cannot be used as a basis for payment, so only cash bids are possible, in many cases it is possible to observe and verify the ex post cash flows. This means that bidders can submit their bids in the form of securities, the values of which are contingent on future cash flows from the asset. Securities auctions are common in the real world: examples include government sales of oil leases, lead-plaintiff, wireless spectrum, and highway building auctions. Many other transactions, though not formally auctions, are similar to securities auctions in nature: examples include corporate takeovers, intercorporate asset sales, book publishing, advertising, and venture capital markets.[1]

In this paper we analyze equilibrium auction mechanisms when sellers compete for potential bidders by altering security designs of their auctions, rather than reserve prices, as in the prior literature. Specifically, we consider an environment in which

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[1] Payments are combinations of an upfront cash payment and a royalty payment that is contingent on future cash flows in oil lease auctions (Kenneth Hendricks and Robert H. Porter 1988; Kenneth Hendricks, Joris Pinkse, and Robert H. Porter 2003) and the book publishing industry (Richard E. Caves 2003). Contingency-fee auctions to choose the lead plaintiff in class action suits are discussed by Jill E. Fisch (2001). In wireless spectrum auctions bids are commitments, which sometimes lead to defaults (Charles Z. Zheng 2001; Paul Milgrom 2004). Peter M. DeMarzo, Ilan Kremer, and Andrzej Skrzypacz (2005) list these and other examples of securities auctions.

there are multiple sellers and potential buyers. Initially, each buyer has no information about his valuations of the assets except for his prior beliefs, which are identical for all assets. The sellers propose designs of their auctions to potential buyers, and each buyer decides which auction to enter. For ease of exposition, we start with the assumption that security design of the auction is the only choice for the seller, and then extend the choice set to include a reserve price or auction format. The problem we study is relevant in contexts in which both competition among auctioneers for a limited number of potential bidders and bids in the form of securities are important. In order to aid in the intuition of the model, consider the following two real-world examples of competition among sellers in securities auctions:

Example 1 (**Oil Lease Auctions**)**:** Rights to develop oil fields are sold in formal auctions by the US federal government as well as governments of multiple states such as Alaska, Wyoming, and Colorado. A system of selling oil lease rights through auctions is also in place in other countries such as Canada and Brazil. Importantly, payments are combinations of an upfront cash amount, called a bonus, and a royalty payment based on the percentage of the cash value of oil produced. The bonus is determined in a competitive auction, and the royalty payment is set by the state (if the lease is sold by the state government) or federal (if the lease is sold by the federal government) law. There is a limited set of potential bidders (most importantly, major oil-producing firms), and due diligence is costly.[2] As a result, for each seller it is important to provide sufficient surplus so that a major oil-producing firm undertakes costly due diligence and participates in the auction, and the size of this surplus depends on the auction format used by alternative sellers. If the government of a region introduces an abnormally high royalty rate relative to the other regions, the auctions of this government will attract fewer bidders.[3]

Example 2 (**Auctions of Companies**)**:** Private companies and divisions of public companies are often sold through an auction process, where the advisor of the selling company acts as de facto auctioneer.[4] For each target there is a limited set of potential buyers, and due diligence is costly in terms of both time and money. As a result, each target competes with ex ante similar firms that might be acquired in the future for the resources of potential bidders. Payments are often made in the form of buyer's equity and debt. Hence, to the extent that the target's advisor can commit to accept bids only of a certain form (e.g., all cash or all equity), this setting is also an example of competition among sellers in securities auctions.

Even though cash and securities auctions might seem similar, they provide different incentives for the bidders. While in cash auctions the value of the bid is

[2] For example, Hendricks, Pinkse, and Porter (2003) state that hiring a geophysical company to "shoot" a seismic survey costs approximately $12 million, while getting more detailed information on a smaller area costs an additional amount between $500,000 and $1 million (all costs are in 1982 dollars). Even though costs are often shared between several firms, they still represent significant amounts.

[3] Competition effects are known to the sellers. For example, the US Government Accountability Office states: "All else equal, more investment dollars will flow to regions in which the government take is relatively low, where there are large oil and gas deposits that can be developed at relatively low cost, and where the fiscal system and government are deemed to be relatively more stable." (US Government Accountability Office (GAO), "Oil and gas royalties: A comparison of the share of revenue received from oil and gas production by the federal government and other resource owners," GAO-07-676R, May 1, 2007.)

[4] See Robert G. Hansen (2001) for the description and model of the typical auction process.

independent of the bidder's identity, in securities auctions it is not the case. If two bidders offer the same securities, the security submitted by the higher type[5] is worth more. As a result, a higher type must pay more than in a cash auction to separate himself from lower types. This intuition underlies the result first obtained by Hansen (1985) that auctions in equity yield higher expected revenues to the seller than cash auctions. This result is generalized by Matthew Rhodes-Kropf and S. Viswanathan (2000), who show that securities auctions yield higher expected revenues than cash auctions, and DeMarzo, Kremer, and Skrzypacz (2005), who study the general class of securities auctions and determine that an auction in steeper securities yields higher revenues. Because potential bidders make endogenous participation decisions, the role of the security design of the auction in our framework is twofold. On the one hand, the seller wants to design the set of securities in a way that maximizes her revenues for a given number of bidders. This effect is achieved by making the security design of the auction steeper. On the other hand, she wants to leave the bidders a sufficiently large surplus to deter them from taking the outside option, whose value is determined endogenously by the security designs adopted by other sellers. This is achieved by making the security design flatter.

We characterize all symmetric Bayes-Nash equilibria of the game, determined by both the equilibrium security designs adopted by the sellers and the bidders' selection and bidding strategies. If the desire to attract additional bidders is sufficiently valuable relative to the desire to extract the surplus from a given set of bidders, auctions in pure cash constitute an equilibrium design. In this case, bidders compete by offering cash bids, and the one who submits the largest amount wins the auction. On the contrary, if the desire to extract the surplus from a given set of bidders is sufficiently valuable relative to the desire to attract additional bidders, auctions in call options constitute an equilibrium security design. Bidders compete by offering call options on future cash flows from the asset, and the one who bids a security with the lowest strike price wins the auction. When the trade-off is nontrivial, there exist multiple equilibrium security designs.

Having characterized equilibrium security designs, we study which of them are the most intuitive. We prove that for any set of security designs that can be ordered in steepness from sufficiently flat to sufficiently steep, there exists an equilibrium security design that belongs to this set. This implies that there always exist equilibrium security designs in mixes of standard securities. For example, if equity is too steep to be an equilibrium security design, then there exists an equilibrium in which the auctions are in combinations of cash and equity. Payments of this form are observed in oil lease auctions, book publishing, and corporate acquisitions.

Our paper not only rationalizes auctions in combinations of simple securities but also attempts to shed light on how parameters of the market structure determine the steepness of the payment mechanisms using numerical examples. First, a larger number of potential bidders or a smaller number of sellers typically makes the equilibrium security designs steeper, as competition concerns become less important. This suggests that relatively low royalty rates[6] in oil lease auctions in

[5] Here and hereafter a higher type means a bidder with a higher valuation of the asset being sold.

[6] US GAO, "Oil and gas royalties: A comparison of the share of revenue received from oil and gas production by the federal government and other resource owners", GAO-07-676R, May 1, 2007.

the United States and Canada can be partially explained by competition among state and federal governments for limited resources of oil firms. Second, an important determinant of the equilibrium security designs is the size of the market. Specifically, as the number of sellers and potential bidders goes up in a way that their ratio remains constant, the equilibrium security designs appear to become flatter. Intuitively, larger size of the market implies that a seller's deviation to a flatter security design becomes more profitable, simply because more bidders are likely to switch to a more bidder-friendly auction when there are more alternative auctions. We prove that in the limit, as the size of the market grows to infinity, the unique equilibrium security design is pure cash. Thus, the model implies that auctions in large markets will always be conducted in cash even if bids can be contingent on future cash flows from the asset. Third, we show in numerical examples that an increase in mean bidder valuation appears to lead to flatter equilibrium security designs. Because higher mean valuation typically leads to greater surpluses of both the seller and the winning bidder, the model suggests that flatter securities are, on average, associated with greater synergies from the auction as a whole as well as greater surpluses of both the seller and the winning bidder, which is broadly consistent with empirical evidence on corporate takeovers.

Because a large body of the existing literature focuses on competition among auctioneers in reserve prices, we extend our model by allowing each seller to choose a reserve price in addition to the security design of her auction. Even though commitment to a reserve price can improve the revenues of the seller in the monopolistic framework, binding reserve prices never constitute an equilibrium outcome as long as the equilibrium security design is not call options. The intuition behind this result comes from the difference between the effect of security designs and reserve prices on efficiency of the auction. While a higher reserve price has a detrimental effect on efficiency, a steeper security design does not. This is because in any securities auction the asset is always sold to the bidder with the highest valuation, while in an auction with a binding reserve price there is a positive probability that the asset is not transferred even though the transaction is efficient. As a result, an increase in the steepness of securities is always preferred to an increase in the reserve price, so in equilibrium reserve prices can be binding only if the auction is in call options, the steepest possible securities. This finding is consistent with surprisingly low reserve prices in US oil lease auctions (R. Preston McAfee and Daniel Vincent 1992).

Our paper is related to two strands of literature. First, it is a part of the literature that studies competition among sellers in auction procedures. Michael Peters and Sergei Severinov (1997); Roberto Burguet and József Sákovics (1999); and Angel Hernando-Veciana (2005) focus on competition among auctioneers in reserve prices. Specifically, Peters and Severinov (1997) study the market with infinitely many sellers and potential buyers, Hernando-Veciana (2005) considers large but finite markets, and Burguet and Sakovics (1999) study the case of two sellers. Among these papers, our model is most closely related to the first model in Peters and Severinov (1997), as we focus on the case when buyers learn their valuations after making entry decisions. Unlike ours, the above papers focus on auctions in cash only. McAfee (1993) and Peters (1997, 2001) study competition in mechanisms without restricting their set to auctions. However, mechanisms in these papers cannot be contingent on future cash flows, so transactions in securities are not allowed.

Second, our paper is related to the literature on securities auctions, originated with Hansen (1985) and John G. Riley (1988). Zheng (2001), Rhodes-Kropf and Viswanathan (2005), and Simon Board (2007) study auctions in which budget-constrained bidders use external resources to finance their bids. Even though bids are expressed in the form of cash, they are effectively securities, because the winner has an option to default. Rhodes-Kropf and Viswanathan (2000) generalize the result of cash inferiority to the set of standard securities but show that the existence of pooling equilibria may destroy this result. DeMarzo, Kremer, and Skrzypacz (2005) extend early results to a general setting which allows for virtually all possible security designs and also consider auctions without commitment to a particular security design. Shimon Kogan and John Morgan (2010) study auctions in pure debt and pure equity under moral hazard. However, in all these papers the number of bidders is exogenous. In our model competition among sellers leads to endogenous participation decisions of the bidders, which has important consequences for the equilibrium security designs.

The rest of the paper is organized as follows. We start with Section I that describes the setup of the model and solves the benchmark case of a monopolistic seller. In Section II, we characterize all symmetric equilibria of the model and study their properties. In Section III, we discuss model implications for the relation between steepness of securities, market structure, and synergies. In particular, we prove that in large markets the unique equilibrium is auctions in pure cash. Section IV extends the model by allowing the sellers to compete both in security designs and reserve prices. In Section V, we discuss several other extensions of the model. Finally, Section VI concludes. All proofs are given in the Appendix.

I. The Model

A. *Setup*

The setup of the model is an extension of DeMarzo, Kremer, and Skrzypacz (2005) for multiple sellers and endogenous entry.[7] There are n_b ex ante identical risk-neutral potential bidders who are interested in acquiring an asset. The asset can be rights to manage a particular project, such as a firm, an innovation, or developing an oil field. There are n_s risk-neutral sellers of ex ante identical indivisible assets, which have zero intrinsic value for the sellers.[8] Each seller sells her asset in an auction. Each bidder is interested in acquiring only a single asset (either due to limited capability of managing simultaneous projects or because two similar projects in the bidder's portfolio would make him overexposed to a particular project-type risk). In addition, each bidder cannot enter more than one auction. This assumption can be justified by high costs (in terms of both time and money) of undertaking due diligence about each asset being sold. The winner of each auction is required to make an investment $x > 0$, which is common knowledge and equal for all bidders

[7] The particular choice to model endogenous entry via competition among sellers for bidders is without loss of generality: similar trade-offs between benefits and costs of entering an auction determine equilibrium security designs in any other endogenous entry framework. A practical advantage of the model with seller competition, however, is that its predictions are related to (and can be checked against) simple and observable characteristics of the market structure, such as the number of sellers, bidders, etc.

[8] The model can be easily generalized to the case of positive intrinsic values.

and assets.[9] Conditional on investment x made by bidder i, asset j yields a stochastic future payoff Z_{ij}. For simplicity, in what follows we omit index j unless ambiguity arises and think of project payoffs within the bounds of a single auction. The interest rate is normalized to zero.

Bidders do not have any information about the project payoff before entering an auction, except for its ex ante distribution, which is the same across bidders.[10] After deciding which auction to enter, each bidder i privately learns his valuation V_i of the asset in the auction he entered. We do not directly model costs of due diligence analysis that bidders have to go through in order to obtain their valuations. However, because this process is long in practice,[11] it is natural to assume that after V_i has been obtained, the bidder cannot switch to another auction. Following DeMarzo, Kremer, and Skrzypacz (2005), we make the following assumptions regarding valuations and payoffs:

ASSUMPTION A: *For any auction with k bidders, $k = 1, \ldots, n_b$, the private valuations $V = (V_1, \ldots, V_k)$ and payoffs $Z = (Z_1, \ldots, Z_k)$ satisfy the following properties:*

(*i*) *V_i are i.i.d. with density $f(v) > 0$ on the support $[v_L, v_H]$;*

(*ii*) *Conditional on V_i, Z_i is distributed on $[0, \infty)$ with density $h(z|v)$ which is twice differentiable in z and v, and functions $zh(z|v)$, $|zh_v(z|v)|$, and $|zh_{vv}(z|v)|$ are integrable;*

(*iii*) *(Z_i, V_i) satisfy the Strict Monotone Likelihood Ratio Property (SMLRP): $h(z|v)/h(z|v')$ is increasing in z if $v > v'$.*

Part (i) of Assumption A implies that the focus is on the auctions with independent private valuations. This assumption underlies most models of auction theory, so it is a natural assumption in modeling auctions with securities as well. While common value aspects are likely to be important in many real-world examples of securities auctions, in many settings the common component of valuations is common knowledge among bidders. For example, in a typical process of an auction for a company (see Hansen 2001 for a description), all bidders obtain access to the same information about the company being sold, which implies that the common component of valuations is likely to be learned by all bidders. In this context, one can think about a bidder's valuation V_i as a sum of the common component, which is common knowledge, and the private component, which is the bidder's private information. In Section V we discuss a possible extension of the model for the case of common values that are not common knowledge. Part (ii) of Assumption A assumes integrability conditions that allow the interchange of the expectation and derivative operators. Finally, part (iii) means that Z_i and V_i are strictly affiliated.[12] Intuitively, V_i is always a good signal about Z_i.

[9] Section V discusses the endogenous choice of x by the bidder.
[10] Section V discusses how the results are likely to be affected if this assumption is relaxed.
[11] For example, in case of corporate acquisitions due diligence typically takes several months (Peter Howson 2003).
[12] See Milgrom and Robert J. Weber (1982) for a discussion of strict affiliation.

Denote the cumulative distribution functions corresponding to $f(v)$ and $h(z\,|\,v)$ by $F(v)$ and $H(z\,|\,v)$, respectively. Without loss of generality, the private valuations can be normalized so that

$$\mathbb{E}[Z_i\,|\,V_i] \;-\; x \;=\; V_i. \tag{1}$$

This allows us to interpret valuation V_i as the NPV of the project, which is assumed to be nonnegative.

This paper focuses on auctions in which bids are securities. In other words, participants of an auction compete for the object by offering contingent claims on the future asset payoff. These contingent claims can be expressed as functions $S(z)$, where z is the asset payoff. Following DeMarzo, Kremer, and Skrzypacz (2005), we formalize the notion of feasible security bids:

DEFINITION A: *A feasible security bid is described by a function* $S(z) \geq 0$, *such that* $S(z)$ *and* $z - S(z)$ *are weakly increasing.*

Definition A puts two restrictions on the set of feasible securities that are standard in security design literature.[13] First, condition $S(z) \geq 0$ can be interpreted as a liquidity or limited liability constraint for the seller. In particular, it implies that the seller cannot commit to invest her own resources into the asset, which rules out the solution of Jacques Crémer (1987) in which the seller simply buys out the best bidder. Second, a feasible security bid requires both the seller's and bidder's payments to be weakly increasing in the realized payoff z. This assumption ensures that each auction is efficient. It can be justified using the "sabotage" argument: if monotonicity is not satisfied, then one of the parties could be better off destroying the output.[14] Unlike DeMarzo, Kremer, and Skrzypacz (2005) we do not make an assumption that $S(z) \leq z$. Hence, our definition allows for a slightly wider set of possible security bids. Specifically, in addition to all widely used "pure" securities like equity, debt, convertible debt, and call options, it allows for pure cash bids, for which $S(z)$is independent of z, as well as partial cash bids.

For any security S, denote its expected value conditional on the bidder's valuation as

$$ES(v) \;\equiv\; \mathbb{E}_z[S(Z_i)\,|\,V_i \;=\; v]. \tag{2}$$

Thus, if bidder with valuation v offers security S to the seller, their expected payoffs are $v - ES(v)$ and $ES(v)$, respectively. It can be shown that $ES(v)$ is twice differentiable, and $0 < ES'(v) < 1$ for any security except $S(z) = c$ and $S(z) = c + z$

[13] See, e.g., David C. Nachman and Thomas H. Noe (1994); DeMarzo and Darrell Duffie (1999); DeMarzo, Kremer, and Skrzypacz (2005).

[14] Specifically, suppose that each party can destroy any share of the project return before it gets verifiable. Suppose that there exists $z' > z$ such that $z' - S(z') < z - S(z)$. Then, if z' is realized, the winner will find it optimal to destroy $\Delta z = z' - z$ of the project return. As a result, ex ante it will be beneficial for the seller to modify the security so that $z' - S(z') = z - S(z)$. Then, the winning bidder gets the same payment, so his bidding and participation incentives are unaffected. However, the seller is strictly better off. Similarly, the ability of the seller to destroy the output justifies monotonicity of $S(z)$. See Oliver Hart and John Moore (1995) for additional details of this argument.

for $c \geq 0$.[15] This result illustrates the main difference of a securities auction from the cash auction: the value of the bidder's payment depends on his identity.

For the auction mechanism to be well defined, the seller needs to formulate the allocation procedure. In other words, the seller has to come up with a set of securities that can be used by the bidders and specify the "best" security for any subset of this set. This is formalized using the definition of an ordered set of securities:

DEFINITION B (*DeMarzo, Kremer, and Skrzypacz* 2005): *An ordered set of securities is defined by the function* $S(s,z)$ *for* $s \in [s_L, s_H]$, *such that:*

(*i*) *for every* s, *set element* $S(s,\cdot)$ *is a feasible security;*

(*ii*) $ES_s(s,v) > 0$ *for all* v, *where* $ES(s,v) \equiv \mathbb{E}_z[S(s,z) \mid V_i = v]$;

(*iii*) $ES(s_L, v_L) \leq v_L$ *and* $ES(s_H, v_H) \geq v_H$.

For some set to be an ordered set of securities it must satisfy three properties. According to property (i), each element of the set has to be a feasible security. Property (ii) requires that the value of security be increasing in s for any valuation v. That is, for security S_1 to be ranked higher than security S_2, it must be valued higher for any possible NPV of the project. Finally, property (iii) guarantees that an ordered set of securities covers a sufficient range of bids.

The assumption that the selling mechanism specifies the ordered set of securities is important due to two reasons. First, the assumption allows us to retain the auction-like characteristic of the selling mechanism. If the set of securities is not ordered, comparison of bids is difficult because their ranking depends on the beliefs of the seller about the bidders' types. In contrast, when the securities are ordered, one security is always ranked higher than the other, so the auction procedure is well defined. Second, as DeMarzo, Kremer, and Skrzypacz (2005) show, even if the set of securities is not ordered and off-equilibrium beliefs of the seller are required to satisfy the D1 refinement, the equilibrium selling mechanism still retains features of an auction with an ordered set of securities. Specifically, they consider the following "informal" auction: bidders submit bids from a nonordered set of securities, the seller chooses the most attractive bid ex post, and the winner gets the asset and makes the payment. They show that the unique symmetric equilibrium satisfying the D1 refinement is equivalent to a first-price auction in which bidders bid with the flattest securities possible.

The paper focuses on second-price sealed-bid auctions. Each bidder submits a security bid $S(s,z)$ from the ordered set of securities prespecified by the seller. Then, the bidder who submitted the highest-ranked security wins the asset and pays according to the second highest security. Second-price auctions are strategically equivalent to English auctions in our setting.[16]

[15] This result can be shown using the proof of DeMarzo, Kremer, and Skrzypacz (2005) (see their Lemma 1) with a slight difference that in our setting $S(z)$ is dominated not by z, but by $S(0) + z$. This difference is because our model allows for a wider set of feasible security bids.

[16] We have also solved the model with first-price sealed-bid auctions. All results still hold albeit under additional technical assumptions on the joint distribution of valuations and project payoffs.

Consider an auction with k bidders and an ordered set of securities $S(s,z)$. The following lemma shows that bidding behavior in a second-price security-bid auction is similar to that in a cash auction: it is optimal to bid the security that corresponds to the bidder's true value.

LEMMA 1: *Consider an auction with $k \geq 2$ bidders and an ordered set of securities $S(s,z)$. The unique equilibrium in weakly undominated strategies is to bid according to $s(v)$, such that*

$$ES(s(v),v) = v. \tag{3}$$

The equilibrium strategy $s(v)$ is increasing in v.

As DeMarzo, Kremer, and Skrzypacz (2005) show, the seller's expected revenues from the auction are related to the steepness of securities used in bidding. What does steepness of a security mean? Comparison of the slopes is inappropriate since slopes of two securities can be ranked differently in different intervals of project payoffs. For example, the slope of debt is higher than the slope of equity if cash flows are low but lower if cash flows are high. What is important for steepness are the slopes of the two securities at the point where they cross each other. Intuitively, security S_1 is steeper than security S_2 if it is more sensitive to deviations at the crossing point. Formally:

DEFINITION C (*DeMarzo, Kremer, and Skrzypacz* 2005): *Security S_1 strictly crosses security S_2 from below if $ES_1(v^*) = ES_2(v^*)$ implies $ES_1'(v^*) > ES_2'(v^*)$. An ordered set of securities $\mathbf{S}_1$ is steeper than an ordered set $\mathbf{S}_2$ if for all $S_1 \in \mathbf{S}_1$ and $S_2 \in \mathbf{S}_2$, S_1 strictly crosses S_2 from below.*

Finally, we specify the timing of the model. At Stage 1, each of n_s sellers decides on the security design of her auction. In other words, each seller j specifies an ordered set of securities $S_j(s,z)$ that can be used by the bidders. At Stage 2, upon observing the security designs offered by the sellers, potential bidders make their entry decisions.[17] At Stage 3, after committing to auctions, bidders in each auction j learn their valuations. At Stage 4 they submit their bids, choosing securities from the prespecified set $S_j(s,z)$. The bidder who submitted the highest-ranked security wins the auction, obtains the asset and makes the investment. Finally, the project returns are realized and the winner of each auction pays according to the security submitted by the second-ranked bidder.[18] The timeline of the model is presented in Figure 1.

[17] In many real-world settings competition for potential bidders is dynamic rather than static. For example, in the market for corporate takeovers a potential acquirer may choose not to bid for a target expecting that a more valuable auction will occur in the future. For tractability we assume that all auctions take place simultaneously. However, the intuition of our model is also applicable to models with dynamic competition.

[18] Note that if there is only one bidder, the selling format is not defined since there is no second highest bidder. For completeness, we assume that in this case the bidder pays according to a security submitted by the lowest possible bidder, v_L.

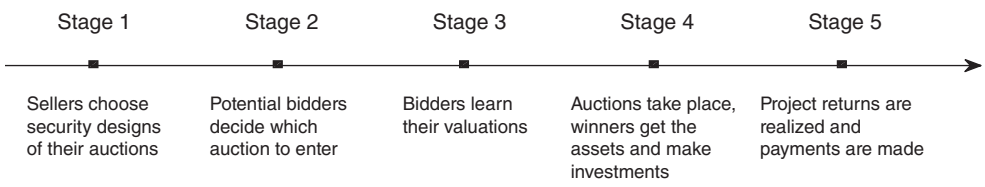

FIGURE 1. TIMING OF THE MODEL

The paper focuses on symmetric Bayes-Nash equilibria. Specifically, we make three assumptions corresponding to each stage of the game.[19] First, as discussed above, at Stage 4 we focus on symmetric equilibria in each auction. Although second-price auctions can have other equilibria, clearly, the symmetric equilibrium in weakly dominant strategies is the most natural one. Second, we assume that in equilibrium the sellers offer the same security designs. Again, as the sellers are ex ante identical, this is a natural assumption. Finally, we focus on equilibria in which potential bidders use the same selection rule to choose among auctions. Given that the sellers offer the same security designs, this implies that in equilibrium each potential bidder randomizes over all auctions with equal probabilities. Note that equilibria in which the bidders use the same selection rule are not the only possible ones. For example, when n_b is a multiple of n_s, equilibria in which the bidders choose among auctions in pure strategies such that each auction gets exactly n_b/n_s bidders are also natural. Because in real life most securities auctions are anonymous with respect to bidder identities, so that potential bidders are likely to make participation decisions noncooperatively, we focus on equilibria in which the bidders use the same selection rule.

B. *Benchmark Case: Monopolistic Seller*

As a benchmark, consider the case in which there is no competition among sellers. Then, all potential bidders enter the unique auction. Suppose that the seller's choice of the ordered set of securities is S. If the bidder with valuation v wins the auction, he pays according to the security submitted by the bidder with the second-ranked valuation, denoted by y. Hence, the expected value of his payment is $ES(s(y,S),v)$. The seller's ex ante expected revenues are

$$U^s(S) = n_b \int_{v_L}^{v_H} F(v)^{n_b-1} \mathbb{E}_y\left[ES(s(y,S),v) \mid v\right] dF(v). \tag{4}$$

where $\mathbb{E}_y\left[ES(s(y,S),v) \mid v\right] \equiv \mathbb{E}_y\left[ES(s(y,S),V^{(1)}) \mid V^{(1)} = v\right]$ is the interim (after learning his valuation but before bidding) expected payment of a bidder with valuation v, conditional on his winning the auction, and $V^{(1)}$ is the first-order statistic of n_b bidder valuations. Similarly, let $U^b(v,S)$ and $U^b(S)$ denote the bidder's interim and ex ante (before learning his valuation) expected surpluses from the auction.

[19] See Peters and Severinov (1997) for similar assumptions in the context of competition among sellers in reserve prices.

In the monopolistic seller framework, as DeMarzo, Kremer, and Skrzypacz (2005) establish, the seller's revenues are higher when the auction is in steeper securities. Intuitively, this result follows from an application of the linkage principle (Milgrom and Weber 1982). By definition, steeper securities are more sensitive to changes in the valuation at the crossing point. Hence, in an auction with a steeper set of securities it is more costly for a bidder to separate himself from a marginally lower type: even if they bid the same security, a marginally higher type pays more. This intensifies competition among bidders and thereby increases the seller's revenues. To show this, rewrite the value of the winner's payment given valuation of the second highest bidder as

$$ES(s(y,S),v) = y + ES(s(y,S),v) - ES(s(y,S),y) \tag{5}$$
$$= y + \int_y^v ES_v(s(y,S),\omega)\, d\omega.$$

Consider two ordered sets of securities, S and $\tilde{S}$, such that S is steeper than $\tilde{S}$. By definition of steepness, $ES_v(s,v) > E\tilde{S}_v(s,v)$ whenever $ES(s,v) = E\tilde{S}(s,v)$. Therefore, $ES(s(y,S),v)$ is greater than $E\tilde{S}(s(y,\tilde{S}),v)$ everywhere except at $v = y$. As a result, the seller always prefers steeper security designs. As call options cross any non-call option security from below, auctions in call options, where higher bids correspond to lower strike prices, yield the highest revenues:[20]

PROPOSITION 1: *If the market consists of only one seller, the seller strictly prefers auctions in steeper securities. Therefore, the equilibrium security design of the auction is call options, where higher bids correspond to lower strike prices.*

If there is competition among sellers, however, this intuition is no longer valid. Suppose, for example, that the first of the two sellers conducts the auction in cash, while the second one's security design choice is call options. Clearly, for *any given* number of bidders the seller who offers a securities auction is going to get higher expected revenues. Thus, bidders who participate in the securities auction get lower expected surplus compared to participants in the cash auction with the same number of bidders. A bidder who can choose which auction to enter will clearly prefer to compete in the cash auction. Therefore, the number of bidders in the securities auction will, on average, be lower. The trade-off between the ability to extract more profits for any fixed number of bidders and the loss of profits due to a lower expected number of bidders is what determines the equilibrium security design in a competitive seller setting.

II. Solution of the Model

This section provides the solution of the model for the general case of $n_s > 1$ sellers when bidders decide endogenously which auction to enter. The model is solved

[20] There is a technical point that sets of call options can be incomparable in steepness with other sets that include call options. However, it can still be shown that auctions in call options yield the highest possible revenues. See the proof of Corollary to Proposition 1 in DeMarzo, Kremer, and Skrzypacz (2005) for details.

by backward induction. First, we solve for the optimal entry strategies given that all sellers but one choose the same security design. Then, we consider the seller's security design decisions and characterize all symmetric equilibria.

A. *Bidding Strategies*

Consider an auction with k bidders, who can submit bids from an ordered set of securities $S(s,z)$. By Lemma 1, each bidder i's equilibrium bidding strategy $s(v_i,S)$ is given by (3). Since the within-auction symmetric equilibrium is efficient, a bidder with valuation v_i wins with probability $F(v_i)^{k-1}$. In this case, he pays according to the security submitted by the second highest bidder, whose expected value is $\mathbb{E}_y[ES(s(y,S),v_i)\,|\,v_i]$. The corresponding interim expected bidder's surplus is equal to

$$U^b(v_i,k,S) \;=\; F(v_i)^{k-1}\big(v_i \;-\; \mathbb{E}_y[ES(s(y,S),v_i)\,|\,v_i]\big). \tag{6}$$

Because distribution of the second-order statistic of k random variables conditional on the first-order statistic being v_i is the same as distribution of the first-order statistic of $k-1$ random variables whose distribution is truncated at v_i, we can write (6) as

$$U^b(v_i,k,S) \;=\; \int_{v_L}^{v_i} \big(v_i \;-\; ES(s(y,S),v_i)\big)\, d\big(F(y)^{k-1}\big).$$

To obtain the bidder's ex ante expected surplus from an auction with k bidders, we integrate $U^b(v_i,k,S)$ over the distribution of v_i:

$$U^b(k,S) \;=\; \int_{v_L}^{v_H}\int_{v_L}^{v_i} \big(v_i \;-\; ES(s(y,S),v_i)\big)\, d\big(F(y)^{k-1}\big)\, dF(v_i). \tag{7}$$

In addition, let $V(k)$ denote the interim expected total surplus:

$$V(k) \;\equiv\; \begin{cases} k\displaystyle\int_{v_L}^{v_H} F(v_i)^{k-1} v_i\, dF(v_i), & \text{if } k \;\geq\; 1 \\ 0, & \text{if } k \;=\; 0. \end{cases} \tag{8}$$

Because the security design does not affect efficiency of the auction, $V(k)$ is independent of S.

Before considering the bidders' participation decisions, we prove the following lemma:

LEMMA 2: *The interim expected total surplus $V(k)$ is an increasing and concave function of $k = 0,1,2,\ldots$. The bidder's interim expected surplus $U^b(v,k,S)$ is a decreasing and convex function of $k = 1,2,\ldots$ for any valuation $v \in [v_L,v_H]$ and any ordered set of securities S.*

The results established in Lemma 2 are intuitive. First, since the auction is efficient, a higher total number of bidders leads to a higher valuation of the winner.

Hence, as k increases, the expected total surplus $V(k)$ goes up. Concavity of $V(k)$ means that each additional bidder increases the total surplus by a smaller amount than the previous additional bidder. This is intuitive, because the valuation of the highest bidder is affected less by addition of each subsequent bidder. Second, an increase in k intensifies competition among bidders. This is the intuition behind the result that $U^b(v,k,S)$ is decreasing in k. Convexity of $U^b(v,k,S)$ with respect to k means that each additional bidder intensifies competition by a smaller amount than the previous additional bidder.

B. *Entry Decisions*

Now consider the potential bidders' participation decisions. We focus on symmetric equilibria in which all sellers choose the same security design and all bidders use the same selection rule to choose among auctions. Given this, it is sufficient to consider deviations from a symmetric equilibrium by a single seller. Specifically, suppose that the ordered set of securities in all but one auctions is $\tilde{S}$, while the ordered set of securities in the remaining auction is S. Let $q(S,\tilde{S})$ denote the probability with which a bidder selects this auction. Because we are looking for symmetric equilibria, the probability with which each bidder picks any auction with security design $\tilde{S}$ must be the same. As there are $n_s - 1$ such auctions, this probability is equal to $\left(1 - q(S,\tilde{S})\right)/(n_s - 1)$.

Consider a potential bidder, who believes that all other bidders make entry decisions using the rule described above. If he enters the auction with security design S, his expected utility is

$$\sum_{k=1}^{n_b} \binom{n_b - 1}{k - 1} q(S,\tilde{S})^{k-1} (1 - q(S,\tilde{S}))^{n_b - k} U^b(k,S). \tag{9}$$

The intuition behind (9) is the following. The bidder faces $n_b - 1$ potential competitors, each of whom enters the auction with probability $q(S,\tilde{S})$. Hence, the bidder faces exactly $k-1$ competitors with probability $\binom{n_b - 1}{k-1} q(S,\tilde{S})^{k-1} (1 - q(S,\tilde{S}))^{n_b - k}$, in which case his ex ante expected surplus from such auction is $U^b(k,S)$. Summing up over all possible realizations of k yields (9).

Similarly to (9), if the bidder enters an auction with security design $\tilde{S}$, his expected utility is

$$\sum_{k=1}^{n_b} \binom{n_b - 1}{k - 1} \left(\frac{1 - q(S,\tilde{S})}{n_s - 1}\right)^{k-1} \left(1 - \frac{1 - q(S,\tilde{S})}{n_s - 1}\right)^{n_b - k} U^b(k,\tilde{S}). \tag{10}$$

To select among auctions using mixed strategies, the bidder's payoff from choosing any auction must be the same. Hence, $q(S,\tilde{S})$ must satisfy the following equation:

$$\begin{aligned} &\sum_{k=1}^{n_b} \binom{n_b - 1}{k - 1} q^{k-1} (1 - q)^{n_b - k} U^b(k,S) \\ &= \sum_{k=1}^{n_b} \binom{n_b - 1}{k - 1} \left(\frac{1 - q}{n_s - 1}\right)^{k-1} \left(1 - \frac{1 - q}{n_s - 1}\right)^{n_b - k} U^b(k,\tilde{S}), \end{aligned} \tag{11}$$

where for simplicity we omit the arguments of $q(S,\tilde{S})$. The following lemma establishes several intuitive properties of $q(S,\tilde{S})$:

LEMMA 3: *Suppose that among n_s auctions, $\tilde{S}$ is the ordered set of securities in $n_s - 1$ auctions, and S is the ordered set of securities in the remaining auction. Then, the probability with which each potential bidder enters the remaining auction, $q(S,\tilde{S})$, satisfies the following properties:*

(*a*) *$q(S,\tilde{S})$ is uniquely defined by* (11)*;*

(*b*) *$q(S_1,\tilde{S}) < q(S_2,\tilde{S})$ if S_1 is steeper than S_2;*

(*c*) *$q(S,\tilde{S}_1) > q(S,\tilde{S}_2)$ if $\tilde{S}_1$ is steeper than $\tilde{S}_2$;*

(*d*) *$q(S,S) = 1/n_s$.*

C. *Equilibrium Security Designs*

Finally, we consider the choice of security designs by the sellers. Consider a seller who chooses the ordered set of securities S when all the other sellers choose the ordered set of securities $\tilde{S}$. Let $U^s(S,\tilde{S})$ denote the ex ante expected revenue of this seller

$$U^s(S,\tilde{S}) = \sum_{k=0}^{n_s} \binom{n_b}{k} q^k (1-q)^{n_b-k} U^s(k,S), \tag{12}$$

where $U^s(k,S)$ is the seller's expected revenue from an auction with k bidders. Using (7),

$$U^S(k,S) = \begin{cases} k\int_{v_L}^{v_H}\int_{v_L}^{v} ES\big(s(y,S),v\big)\, d\big(F(y)^{k-1}\big)\, dF(v), & \text{if } k \geq 1 \\ 0, & \text{if } k = 0. \end{cases} \tag{13}$$

The characterization of symmetric equilibria requires finding a solution to the fixed point problem in the function space. Fortunately, the problem can be substantially simplified to a problem in which the seller chooses only a single parameter, the probability with which a bidder enters her auction. To see this, notice that the total surplus of each auction is split between k bidders and the seller and does not depend on the security design of the auction

$$U^s(k,S) + kU^b(k,S) = V(k),$$

for all k and S. Using this and $\binom{n_b}{k} = (n_b/k)\binom{n_b-1}{k-1}$, we can write (12) as:

$$\begin{aligned} U^s(S,\tilde{S}) &= \sum_{k=0}^{n_b} \binom{n_b}{k} q^k (1-q)^{n_b-k} V(k) \\ &\quad - n_b q \sum_{k=1}^{n_b} \binom{n_b-1}{k-1} q^{k-1} (1-q)^{n_b-k} U^b(k,S). \end{aligned} \tag{14}$$

The structure of (14) is intuitive. The first term represents the expected total surplus from the auction, which depends on the security design only through entry probabilities. The second term represents the bidders' part of the expected total surplus. Here, $n_b q$ is the expected number of the bidders in the auction, and the sum is the expected utility of a bidder from participating in the auction with a random number k of competitors. By the market equilibrium condition (11), the expected surplus of a bidder from participating in the auction must be equal to his outside option of entering an auction with security design $\tilde{S}$. Therefore, (14) is equal to

$$(15)\qquad \sum_{k=0}^{n_b}\binom{n_b}{k}q^k(1-q)^{n_b-k}V(k) - n_b q\sum_{k=1}^{n_b}\binom{n_b-1}{k-1} \times \left(\frac{1-q}{n_s-1}\right)^{k-1}\left(1-\frac{1-q}{n_s-1}\right)^{n_b-k}U^b(k,\tilde{S}).$$

From (15) we can see that the seller's expected surplus depends on the security design of her auction only via the participation probability $q(S,\tilde{S})$. Thus, we can alternatively write $U^s(S,\tilde{S})$ as $U^s(q,\tilde{S})$, so the seller's optimal security design problem can be reformulated in terms of choosing the probability with which each bidder decides to participate in the auction. Mathematically, the seller's optimization problem is to maximize $U^s(q,\tilde{S})$ over $q \in [q_L(\tilde{S}), q_H(\tilde{S})]$, where $q_L(\tilde{S})$ and $q_H(\tilde{S})$ are the lowest and highest participation probabilities the seller can achieve by altering security design of her auction. Since the bidder's expected surplus is the highest when the auction is in cash and lowest when the auction is in call options, $q_L(\tilde{S}) = q(S_{call},\tilde{S})$ and $q_H(\tilde{S}) = q(S_{cash},\tilde{S})$, where S_{call} and S_{cash} are the ordered sets of cash amounts and call options, respectively.

Because $q(S^*,S^*) = 1/n_s$ by Lemma 3, for S^* to be the equilibrium security design it is necessary and sufficient that

$$\frac{1}{n_s} = \arg\max_{q\in[q(S_{call},S^*),\,q(S_{cash},S^*)]} U^s(q,S^*).$$

The following proposition establishes existence of a symmetric equilibrium and characterizes all equilibrium security designs:

PROPOSITION 2: *There always exists an equilibrium in which all sellers choose the same security design and all potential bidders choose among auctions using the same selection rule. Let $\phi(S)$ denote the following function of the ordered set of securities S:*

$$(16)\quad \phi(S) = \sum_{k=0}^{n_b}\binom{n_b}{k}\frac{(n_s-1)^{n_b-k-1}(kn_s-n_b)}{n_s^{n_b-1}}V(k) - n_b\sum_{k=1}^{n_b}\binom{n_b-1}{k-1}\frac{(n_s-1)^{n_b-k}}{n_s^{n_b-1}}U^b(k,S) + n_b\sum_{k=1}^{n_b}\binom{n_b-1}{k-1}\frac{(n_s-1)^{n_b-k-2}((k-1)n_s-n_b+1)}{n_s^{n_b-1}}U^b(k,S).$$

(*i*) *If* $\phi(S_{cash}) \geq 0$, *then auctions in cash is an equilibrium security design. Higher bids correspond to higher cash amounts.*

(*ii*) *If* $\phi(S_{call}) \leq 0$, *then auctions in call options is an equilibrium security design. Each seller gets a call option on the asset. Higher bids correspond to lower strike prices.*

(*iii*) *If* $S \neq S_{cash}$ *and* $S \neq S_{call}$, *then security design* S *is an equilibrium security design if and only if* $\phi(S) = 0$.

The equilibrium security designs are determined by the seller's trade-off between making the auction more attractive to bidders in order to increase participation and extracting higher surplus from a given set of bidders. In equilibrium both incentives are captured by function $\phi(S) = U_q^s(1/n_s, S)$, which is the marginal value from inviting additional bidders by making the auction marginally more attractive to them when the security design of all auctions is S. It consists of three terms. The first term corresponds to an increase in the total expected surplus from the auction due to marginally higher participation. The two other terms represent the change in the bidders' expected surplus from the auction. The second term is the change in the bidders' expected surplus due to higher participation conditional on the same outside option. The third term is the change in the bidders' expected surplus due to the change in the outside option conditional on the same participation in the auction. If $\phi(S) > 0$, then each seller has incentives to deviate from security design S by decreasing the steepness of the ordered set of securities. Even though decreasing the steepness reduces the surplus that the seller extracts from a given set of bidders, expected revenues of the seller will be higher because of higher participation. The opposite occurs if $\phi(S) < 0$.

Proposition 2 establishes that the equilibrium security designs can be either interior or boundary. The first two cases represent boundary equilibria. If $\phi(S_{cash}) \geq 0$, participation is so important that a seller wants to increase participation even if all auctions are in cash. As a result, auctions in cash is an equilibrium security design. If $\phi(S_{call}) \leq 0$, then gains from higher participation are relatively unimportant: A seller wants to extract more surplus at the cost of lower participation even if all auctions are in call options. As a result, auctions in call options is an equilibrium security design. In particular, this is the unique equilibrium in the benchmark case studied in Section IB. The last case of Proposition 2 corresponds to interior equilibria. The equilibrium sets of securities are such that the seller extracts zero benefits from altering the security design in a way that marginally affects participation.

Proposition 2 characterizes equilibria in which all sellers use the same security designs. While equilibria in which security designs across auctions are different may exist, symmetric equilibria are the most intuitive in the market consisting of ex ante identical sellers. More importantly, equilibria in which the security designs of all auctions are the same always weakly dominate other equilibria in terms of the total market welfare.[21] As such, the existence of symmetric equilibria implies the market's ability to operate efficiently in the sense of maximizing total market welfare.

[21] To see this, let $\{q_j^*\}_{j=1}^{n_s}$ be the set of equal probabilities of entry when all auctions have the same security designs: $q_j^* = 1/n_s$, $j = 1, \ldots, n_s$, and let $\{q_j\}_{j=1}^{n_s}$ be the probabilities of entry consistent with an outcome in which

D. *Equilibria with Standard Securities*

Even though Proposition 2 characterizes all symmetric equilibria in the model, it does not always specify what securities they involve. Specifically, it does not say anything about how equilibrium security designs look when $\phi(S_{cash}) < 0 < \phi(S_{call})$. This section demonstrates that the set of all symmetric equilibria always contains equilibria in which auctions are in mixes of standard securities, such as equity, cash, and call options.

In particular, Proposition 3 establishes a more general result: for any set $\mathbb{S}$ of security designs that can be ordered in steepness from sufficiently flat to sufficiently steep (for example, $\mathbb{S}$ can include mixes of cash and equity with various proportions of equity in the mix), there always exists an equilibrium security design that belongs to set $\mathbb{S}$.

PROPOSITION 3: *Consider any set of security designs* $\mathbb{S} = \{S(\lambda, s, z),\ \lambda \in [\lambda_L, \lambda_H]\}$ *such that* $S(\lambda, \cdot, \cdot)$ *is steeper than* $S(\lambda', \cdot, \cdot)$ *if and only if* $\lambda > \lambda'$, *and* $S(\lambda, s, z)$ *is continuous in* λ. *Let* $\phi(\lambda) \equiv \phi(S(\lambda, \cdot, \cdot))$. *If* $\phi(\lambda_L) \leq 0$ *and* $\phi(\lambda_H) \geq 0$, *there exists an equilibrium security design that belongs to* $\mathbb{S}$.

To shed light on the intuition behind Proposition 3, consider the set of security designs that can be ordered in steepness by parameter $\lambda \in [\lambda_L, \lambda_H]$. Then, $S(\lambda_L, s, z)$ and $S(\lambda_H, s, z)$ are the flattest and steepest security designs in the set, respectively. If set $S(\lambda_L, s, z)$ is sufficiently flat ($\phi(\lambda_L) < 0$) and set $S(\lambda_H, s, z)$ is sufficiently steep ($\phi(\lambda_H) > 0$), then there must be a point between λ_L and λ_H at which a seller has no incentives to alter the steepness parameter λ of her auction. By Proposition 2, the corresponding $S(\lambda, s, z)$ is an equilibrium security design.

Because cash and call options are the flattest and steepest securities, respectively, Proposition 3 implies that there always exist equilibria in which auctions are conducted in combinations of standard securities, such as cash, equity, and call options. For example, if equity is a sufficiently steep security design, there exists a symmetric equilibrium in which the seller fixes the proportion of equity in the bid, and bidders compete by offering more cash and correspondingly more equity. Similarly, if equity is sufficiently steep and debt is sufficiently flat, then there exists a symmetric equilibrium in which the auction is in combinations of debt and equity where, for example, each seller fixes the ratio of expected value of debt to expected value of equity in bids. We discuss these two types of equilibria in more detail in the next section.

III. Model Implications

In this section we analyze several implications of the model. We study equilibria in which auctions are in combinations of standard securities. First, we consider equilibria in which bids are combinations of cash and equity. In this case, a seller

security designs of some of the auctions are different. Applying Jensen's inequality to $V(k)$, we get:

$$\frac{1}{n_s}\sum_{i=1}^{n_s} V(q_j^*) \;=\; V\Big(\frac{1}{n_s}\sum_{j=1}^{n_s} q_j^*\Big) \;=\; V\Big(\frac{1}{n_s}\sum_{j=1}^{n_s} q_j\Big) \;\geq\; \frac{1}{n_s}\sum_{i=1}^{n_s} V(q_j).$$

fixes the proportion of cash in the bid, and bidders compete for the asset by offering more cash and correspondingly more equity. Second, we consider equilibria in which bids are combinations of debt and equity. In this case, a seller fixes the proportion of expected revenue from each security in the bid, and bidders compete for the asset by offering more debt and more equity while keeping the ratio constant. An implication of the model is that transactions in standard securities and their combinations can be rationalized as equilibrium outcomes in the model when the set of securities is unrestricted. Sections IIIA and IIIB examine how different parameters of the market structure affect the equilibrium security designs in these two examples. In particular, Section IIIA suggests that equilibrium security designs are steeper if there are more potential bidders and fewer competing sellers in the market. Section IIIB suggests that equilibrium security designs become flatter when the size of the market goes up for any fixed ratio of potential buyers to sellers. We prove that in the limit, as the size of the market increases to infinity, the unique equilibrium auction formats are simple cash auctions. Finally, Section IIIC suggests that implications of our model are consistent with empirical evidence on the relation between means of payment and buyers' and sellers' announcement returns in mergers and acquisitions and intercorporate asset sales.

Consider the following two ordered sets of security designs:

Set of Security Designs $\mathbb{S}1$.—Bids are combinations of cash and equity. The seller fixes the proportion of cash in the bid:

$$\lambda_L = 0;\ \lambda_H = 1; \tag{17}$$

$$S(\lambda, s, z) = \lambda s z + (1 - \lambda)s.$$

Set of Security Designs $\mathbb{S}2$.—Bids are combinations of debt and equity. The seller fixes the ratio of expected revenue from debt to that of equity in a way that this ratio does not depend on the bid amount s:

$$\lambda_L = 0; \quad \lambda_H = 1; \tag{18}$$

$$S(\lambda, s, z) = \min\{z, s\} + \sigma(\lambda, s)z,$$

$$\text{where } \sigma(\lambda, s) \text{ is such that } \frac{\mathbb{E}[\min(z, s) \,|\, v : s(v; \sigma(\cdot)) = s]}{\mathbb{E}[\sigma(\lambda, s)z \,|\, v : s(v; \sigma(\cdot)) = s]} = \frac{1 - \lambda}{\lambda}.$$

Set $\mathbb{S}1$ is bounded by the ordered bids in pure cash from below and by the ordered bids in pure equity from above. Bids in combinations of cash and equity are observed in many markets, such as oil lease auctions, book publishing, intercorporate asset sales, and corporate acquisitions. If $\lambda \in (0, 1)$, intermediate security designs are parameterized by λ in such a way that the expected proportion of equity to cash from bidder with valuation v is $\lambda(x + v)/(1 - \lambda)$.

In set $\mathbb{S}2$, each bid is restricted to have a fixed ratio of debt to equity expected value.[22] Bids in debt, equity or their combinations are observed in markets for venture capital, where an entrepreneur solicits venture capital financing, or in wireless spectrum auctions.[23] If $\lambda = 0$, the set of possible bids is debt securities, which are ordered by the face value of debt that the bidder promises to repay after project returns are realized. If $\lambda = 1$, the set of possible bids consists of pure equity securities. Intermediate values of λ specify ordered sets of securities that consist of various mixes of debt and equity, such that the ratio of debt to equity expected value is fixed. Note that this parametrization is nonlinear, as it indirectly defines the function $\sigma(\lambda, s)$ of debt face value through bid s (i.e., the share of equity that the bidder offers to the seller).

We choose the following benchmark values (to be changed one at a time in comparative statics): the project investment is $x = 100$; the valuations v_i are distributed uniformly over $[v_L, v_H] = [45, 85]$; and the project's cash flows are $z_i = \theta(x + v_i)$, where θ is distributed lognormally with mean 1 and volatility of 50 percent. The benchmark number of sellers and buyers is 2 and 4, respectively. In all our examples, each of sets $\mathbb{S}1$ and $\mathbb{S}2$ contains at most one equilibrium security design of the model.

A. *Security Bids and Number of Potential Bidders per Seller*

One of the important observable characteristics of the market structure is the number of potential bidders per seller. An increase in the number of potential bidders keeping the number of sellers constant decreases competition among sellers for potential bidders. As a result, equilibrium security designs should intuitively become steeper as the sellers prefer to shift the focus to extracting surplus from a given set of bidders rather than inviting additional bidders into the auction. Similarly, steeper equilibrium security designs result from a decrease in the number of sellers keeping the number of potential bidders constant.

Figure 2 (panels A and B) shows comparative statics of the equilibrium with respect to changes in the total number of bidders n_b keeping the number of sellers fixed at 2. Panel A of the figure plots the equilibrium expected proportion of equity in the cash-equity and debt-equity auctions. If there are few potential bidders ($n_b \leq 4$), the unique equilibrium is auctions in cash. Intuitively, when the number of potential bidders is low, the seller's marginal value of each additional bidder is high, so inviting additional bidders into the auction is the most important concern for the sellers. As a result, the sellers offer participants higher surplus by conducting auctions in cash. Auctions in pure debt securities will be the equilibrium outcome only if the sellers are not allowed to conduct auctions in flatter securities. As the number of potential bidders goes up, the trade-off between participation and surplus extraction becomes less trivial. As a result, interior equilibria arise. Panel A of Figure 2

[22] Note that this example violates some of our technical assumptions: first, security bids are not feasible for low realizations of project returns; second, strictly speaking, set $S(\lambda_2, s, z)$ may not be steeper than set $S(\lambda_1, s, z)$, $\lambda_1 < \lambda_2$, because a security from the first set may cross a security from the second set twice. This simplified example, however, provides a tractable numerical solution and general intuition in "debt-equity" security mixes. An alternative that corrects for these violations, but is more difficult to solve, is to substitute unlevered equity for levered: $\min(z, s + \sigma(\lambda, s)z)$.

[23] In venture capital deals both debt and equity securities are often used (Steven N. Kaplan and Per Stromberg 2003). In wireless spectrum auctions bids are payment obligations, and the winners sometimes default (Zheng 2001; Milgrom 2004).

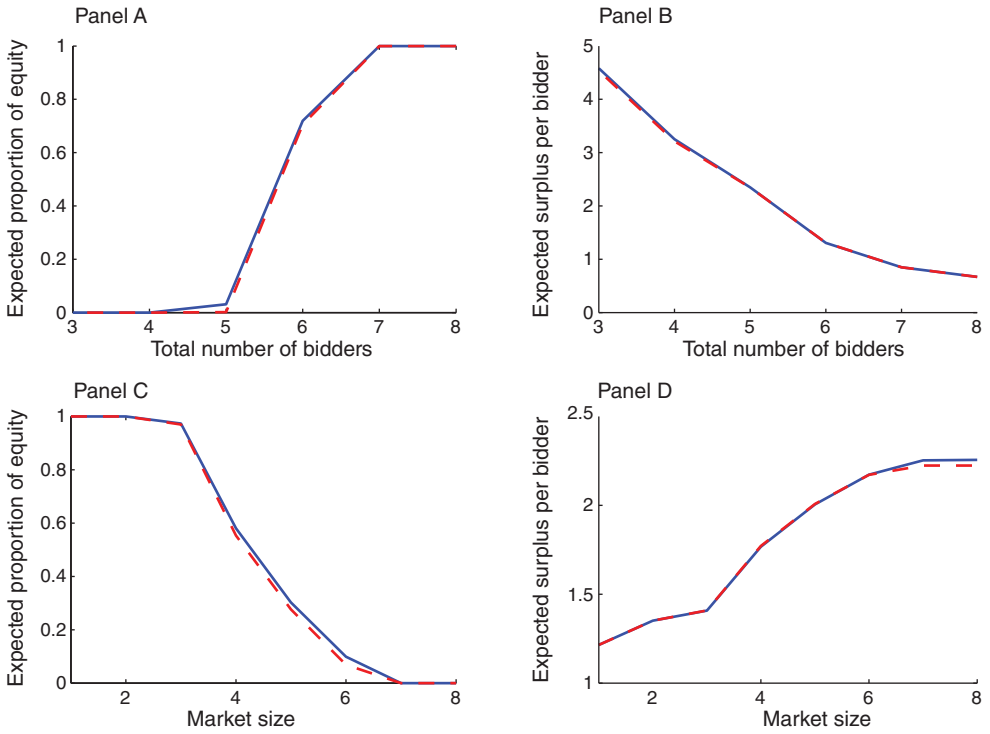

FIGURE 2. COMPARATIVE STATICS OF THE OPTIMAL SECURITY DESIGN WITH RESPECT TO MARKET CHARACTERISTICS

Notes: The figure shows the equilibrium proportion of ex ante expected revenues paid in the form of equity in the total ex ante expected seller's revenues and the expected surplus per bidder for benchmark parameters of Section III as a function of total number of potential bidders keeping number of sellers fixed at 2 (panels A and B) and as a function of market size keeping $n_b/n_s = 4$ (panels C and D). Solid line corresponds to cash-equity equilibria ($\mathbb{S}1$), and dashed line corresponds to debt-equity equilibria ($\mathbb{S}2$). Points at which the equilibrium proportion of debt or equity are equal to 1 are not equilibria of the model but would be equilibria if the set of security designs admissible to the sellers were restricted to either $\mathbb{S}1$ or $\mathbb{S}2$.

shows two of these equilibria: the equilibrium in which auctions are in the mixes of cash and equity (which exists when $n_b \geq 5$) and the equilibrium in which auctions are in the mixes of debt and equity (which exists when $n_b \geq 6$). Finally, as the number of potential bidders goes up even more, neither $\mathbb{S}1$ nor $\mathbb{S}2$ contains equilibria of the model, because each seller prefers to deviate from auctions in pure equity by switching to a steeper security design. However, as a direct corollary to Proposition 3, pure equity would be the equilibrium security design if the set of security designs admissible to the sellers were restricted to either $\mathbb{S}1$ or $\mathbb{S}2$.

Panel B of Figure 2 plots the ex ante expected surplus of a bidder. As the total number of bidders goes up, the expected surplus of each bidder goes down. Intuitively, the larger is the total number of bidders, the lower is the probability that a particular bidder wins. Hence, the expected surplus of a bidder is lower. Not shown on the graph, the total expected surplus of all bidders also goes down.

The result that greater competition among sellers leads to flatter equilibrium security designs is consistent with empirical evidence on the cross section of royalty rates in oil lease sales. Many studies show that US and Canadian federal and state governments receive one of the lowest government takes in the world, where government take is defined as the percentage of the cash value from produced oil that

goes to the government.[24] While there can be multiple factors that account for these differences, our model points to one of them—competition among US and Canadian state and federal governments for a limited amount of resources of oil-producing companies. Specifically, if government of a state increases the required government take substantially, a potential bidder will have fewer incentives to undertake costly due diligence of oil fields sold by this state, as it has an outside option of putting more effort in other oil lease auctions. This competition effect is likely to drive the equilibrium government take below what is observed in many other countries where all oil-producing rights are sold monopolistically by federal governments.[25]

B. *Security Bids and Size of the Market*

Another important characteristic of the market structure is the size of the market. Figure 2 (panels C and D) shows comparative statics of the equilibrium with respect to a change in the total number of bidders n_b and sellers n_s keeping their ratio n_b/n_s fixed at 4. Panel C of the figure plots the equilibrium proportion of equity in the cash-equity ($\mathbb{S}1$) and debt-equity ($\mathbb{S}2$) equilibria. As the size of the market goes up, the equilibrium proportion of equity in cash-equity and debt-equity auctions declines. Intuitively, the greater is the size of the market, the easier it is for a seller to invite an additional bidder to her auction by deviating to a flatter design. Hence, larger markets are associated with flatter security designs. As the size of the market becomes sufficiently large, equilibrium security designs become pure cash. Panel D of Figure 2 plots the ex ante expected surplus of a bidder as a function of the size of the market. As the size of the market increases, the expected surplus of a bidder goes up. This happens for two reasons. First, the equilibrium security design becomes more bidder friendly. Second, an increase in the size of the market leads to greater variation in the number of bidders participating in each auction. Because by Lemma 2 the bidder's surplus is convex in the number of bidders in the auction, larger variance and fixed at n_b/n_s average level of competition means that the ex ante expected surplus of each bidder increases even if the security design of the auction is the same.

Proposition 4 shows that the convergence of the equilibrium security designs to pure cash in large markets is a general result that holds in an unrestricted space of security designs:

PROPOSITION 4: *Suppose that the ratio of the number of potential bidders and sellers is* $n_b/n_s = n$, *and the market is large* $(n_s \to \infty)$. *Then, the unique symmetric equilibrium is auctions in cash.*

The result obtained in Proposition 4 and supported by Figure 2 suggests that transactions in securities are more likely to be observed in small markets, while transactions in large markets are always undertaken in cash. These implications are

[24] This percentage depends on both the royalty rate of the oil lease auction and the fiscal regime and corresponds to the slope of the buyer's payment as a function of the value of the asset. For a review of cross-country studies of government takes see US GAO, "Oil and gas royalties: A comparison of the share of revenue received from oil and gas production by the federal government and other resource owners," GAO-07-676R, May 1, 2007.

[25] For simplicity, in this argument we ignore competition among federal governments of different countries. However, the argument is valid as long as assets in one country are more similar than assets in different countries.

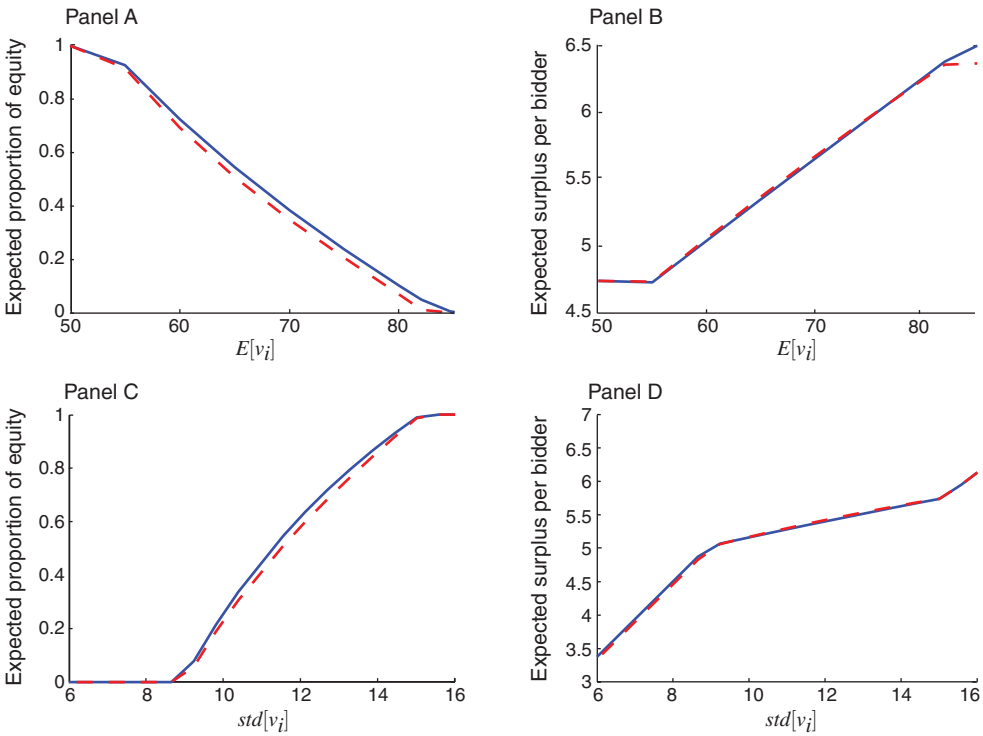

FIGURE 3. COMPARATIVE STATICS OF THE OPTIMAL SECURITY DESIGN WITH RESPECT TO PARAMETERS OF DISTRIBUTION OF BIDDER VALUATIONS

Notes: The figure shows the equilibrium proportion of ex ante expected revenues paid in the form of equity in the total ex ante expected seller's revenues and the expected surplus per bidder for benchmark parameters of Section III as a function of the mean (panels A and B) and variance (panels C and D) of distribution of bidder valuations. Solid line corresponds to cash-equity equilibria ($\mathbb{S}1$), and dashed line corresponds to debt-equity equilibria ($\mathbb{S}2$). Points at which the equilibrium proportion of debt or equity are equal to 1 are not equilibria of the model but would be equilibria if the set of security designs admissible to the sellers were restricted to either $\mathbb{S}1$ or $\mathbb{S}2$.

consistent with empirical observations. Transactions in securities are observed in "unique" environments, in which there are few sellers of similar goods, while transactions in large markets are typically in cash. For example, in the motion pictures industry profit-sharing compensation schemes are more common for more experienced actors (Darlene C. Chisholm 1997), conditional on the same revenues from the movie and a number of other characteristics of the movie and the actor. As the market for more experienced actors is arguably smaller both on the demand and on the supply side, this observation is consistent with our results.

C. *Security Bids and Distribution of Valuations*

In this section we look at how the parameters of the distribution of valuations affect the equilibrium security designs and expected surpluses of the parties in our examples. Figure 3 shows comparative statics of the equilibrium in our numerical example with respect to changes in the mean and the standard deviation of the valuation, $\mathbb{E}[v]$ and $std[v]$. Panel A of the figure plots the equilibrium proportion of equity in cash-equity and debt-equity auctions for different values of $\mathbb{E}[v]$. As

$\mathbb{E}[v]$ increases, the expected proportion of equity declines, so equilibrium security designs become flatter. An increase in mean increases the marginal value from inviting an additional bidder to the auction. While it also leads to greater incentives to extract surplus from a given set of bidders, the former effect dominates the latter, so the expected proportion of equity in the payment declines. Panel B of Figure 3 plots the ex ante expected surplus of a bidder as a function of mean $\mathbb{E}[v]$. While it is monotonically increasing, the magnitude is different depending on whether the equilibrium security design is boundary or interior. Specifically, an increase in $\mathbb{E}[v]$ has a larger effect on the bidder's expected surplus if the equilibrium is interior. This is because in this range greater mean not only leads to greater total surplus from the auction but also makes equilibrium security designs more bidder friendly. A prediction of Figure 3 is that the use of flatter securities should, on average, be associated with higher synergies from the auction as well as higher surpluses of both the seller and the winning bidder. This prediction is consistent with broad evidence that in corporate takeovers deals in cash are associated with significantly higher acquirer's announcement-induced abnormal returns and takeover premiums (which correspond to the winning bidder's and seller's surpluses, respectively) than deals in stock.[26] This evidence is usually explained by asymmetric information about the value of the target[27] or tax concerns (e.g., Brown and Ryngaert 1991). Here, the negative relation between steepness of securities used in transaction and synergies arises because of differences in the sellers' and buyers' endogenous bargaining powers. If expected synergies are high ($\mathbb{E}[v]$ is high), marginal value of inviting an additional bidder to the auction process is high. As a result, sellers compete among each other for potential bidders by offering auctions in flatter securities, so in equilibrium high expected synergies are associated with adoption of flat securities, such as cash.

Panels C and D of Figure 3 plot the equilibrium proportion of equity and the corresponding surplus of a bidder as a function of the standard deviation of v. We keep the uniform distribution and alter the range of values that valuations can take.[28] As the distribution of valuations becomes more dispersed, the equilibrium proportion of equity increases. This is because the deviation by the seller from cash or debt to equity intensifies competition among bidders much less if the variance of valuations and the resulting distance between first- and second-highest valuations is large. As a result, for high variance of v, bidders' participation decisions respond

[26] Evidence that targets receive higher offer premiums in cash offers than in stock offers is provided by, among others, Yen-Sheng Huang and Ralph A. Walkling (1987); Carla Hayn (1989); and Henri Servaes (1991) on deals in the United States, Julian R. Franks, Robert S. Harris, and Colin Mayer (1988) on deals in the United Kingdom, and B. Espen Eckbo and Herwig Langohr (1989) on deals in France. Evidence that acquirer's announcement-induced abnormal returns are lower for stock offers than for cash offers is provided by, among others, Nickolaos G. Travlos (1987); James W. Wansley, William R. Lane, and Ho C. Yang (1987); David T. Brown and Michael D. Ryngaert (1991); and Servaes (1991).

[27] The standard explanation is that of Stewart C. Myers and Nicholas S. Majluf (1984). Other explanations based on asymmetric information include Hansen (1987); Michael J. Fishman (1989); Elazar Berkovitch and M. P. Narayanan (1990); and B. Espen Eckbo, Ronald M. Giammarino, and Robert L. Heinkel (1990). See Sandra Betton, Eckbo, and Karin S. Thorburn (2008) for a review.

[28] Figure 3 presents the results for the case when the mean valuation does not change with the change in variance. An alternative is to also change the mean valuation in a way that keeps the expected total surplus from an auction constant. However, the results are not significantly affected, so we do not report them here.

less to a deviation by the seller, so sellers have more incentives to increase steepness of the security design in the auction.

IV. Incorporating Reserve Prices

Most of the literature that studies competition among auctioneers for potential bidders focuses on competition in reserve prices, which is either assumed exogenously[29] or derived endogenously from competition in general mechanisms.[30] However, our model did not allow for the seller to choose a reserve price, i.e., to commit not to sell the asset if the security submitted by the winning bidder is not high enough. Indeed, we assumed that the lowest security was such that bidders of all types earned nonnegative profits: $ES(s_0, v_L) \leq v_L$. In this section we extend our model by allowing the sellers to choose reserve prices in addition to security designs. We show that the equilibrium is robust to the addition of reserve prices. In fact, as long as call options is not an equilibrium security design, all equilibrium auction designs in the extended model are characterized by (i) equilibrium security designs of the main model and (ii) nonbinding reserve prices.

In the context of securities auctions, a reserve price is defined by a reserve security, that is, the minimum security that the bidders are allowed to bid:

DEFINITION D: *Consider an auction in which bidders make bids from an ordered set of securities* $S(s,z)$, $s \in [s_L, s_H]$. *A reserve price is a security* $r \in [s_L, s_H]$ *such that the bidders are allowed to bid only securities above the reserve price:* $S(s, \cdot)$, $s \geq r$.

The definition of the reserve price in the context of securities auctions is intuitive. For example, in an auction in pure equity, a reserve price is a minimum fraction of the asset that the bidders are allowed to bid. Similarly, in an auction in pure debt, a reserve price is a minimum face value of the debt security that the bidders are allowed to bid. Notice that the definition of the reserve price in the context of securities auctions is consistent with the definition of the reserve price in cash auctions. Indeed, if an ordered set of securities is simple cash bids, then a reserve price is a minimum amount of cash that bidders are allowed to bid.

Consider a seller who sets $r \in [s_L, s_H]$ as a reserve price. If no bidder makes a bid above r, the seller does not sell the asset. If there is at least one bidder who makes a bid above r, the asset is sold and the winning bidder pays according to the security which is the highest between r and the bid of the second highest bidder. For any r, no bidder with valuation below v_r can make a positive profit, where v_r is such that:

$$ES(r, v_r) = v_r.$$[31]

[29] Peters and Severinov (1997); Burguet and Sákovics (1999); Hernando-Veciana (2005).

[30] For example, McAfee (1993).

[31] Note that the cutoff type v_r and, equivalently, the payment of the cutoff type $ES(r, v_r) = v_r$ strictly increases with r. Hence, similar to cash auctions, finding the optimal reserve price in a security auction is equivalent to choosing the cutoff type or the value of the lowest acceptable bid. For any such v_r, the seller computes the corresponding reserve price r by inverting $ES(r, v_r) = v_r$.

Because the asset is transferred only if the valuation of the highest bidder is above v_r, the interim expected total surplus from the auction is equal to

$$V(k, v_r) = \begin{cases} k\int_{v_r}^{v_H} F(v_i)^{k-1} v_i \, dF(v_i), & \text{if } k \geq 1 \\ 0, & \text{if } k = 0. \end{cases}$$

By analogy with (7), the interim expected bidder's surplus from the auction is equal to

$$U^b(k, S, r) = \int_{v_r}^{v_H} \int_{v_L}^{v_i} \left(v_i - ES\big(\max(r, s(y, S)), v_i\big)\right) d\big(F(y)^{k-1}\big)\, dF(v_i).$$

As in the main model, consider a seller who chooses ordered set of securities S and reserve price r, when all other sellers choose ordered set of securities $\tilde{S}$ and reserve price $\tilde{r}$. For a potential bidder to select among auctions using mixed strategies, his payoff from choosing all options must be the same. As a result, the probability of choosing the deviator $q(S, r, \tilde{S}, \tilde{r})$ satisfies

$$\sum_{k=1}^{n_b} \binom{n_b - 1}{k - 1} q^{k-1} (1 - q)^{n_b - k} U^b(k, S, r) = \sum_{k=1}^{n_b} \binom{n_b - 1}{k - 1} \left(\frac{1 - q}{n_s - 1}\right)^{k-1} \left(1 - \frac{1 - q}{n_s - 1}\right)^{n_b - k} U^b(k, \tilde{S}, \tilde{r}). \tag{19}$$

Hence, the deviating seller's ex ante expected revenues are equal to

$$U^s(S, r, \tilde{S}, \tilde{r}) = \sum_{k=0}^{n_b} \binom{n_b}{k} q^k (1 - q)^{n_b - k} V(k, r) - n_b q \sum_{k=1}^{n_b} \binom{n_b - 1}{k - 1} \times \left(\frac{1 - q}{n_s - 1}\right)^{k-1} \left(1 - \frac{1 - q}{n_s - 1}\right)^{n_b - k} U^b(k, \tilde{S}, \tilde{r}). \tag{20}$$

For (S^*, r^*) to be the equilibrium auction design in this extended version of the model it is necessary and sufficient that

$$(S^*, r^*) \in \underset{S, r \in [s_L, s_H]}{\arg\max}\, U^s(S, r, S^*, r^*).$$

The following proposition shows that the equilibrium security designs of the main model are robust to the addition of reserve prices:

PROPOSITION 5: *Suppose that auctions in call options is not an equilibrium security design. Then, the set of equilibrium auction formats is* $\{(S, r) : S \in \mathbb{S}, r : v_r \leq v_L\}$, *where* $\mathbb{S}$ *is the set of equilibrium security designs in the main model, given in Proposition* 2.

According to Proposition 5, if call options is not an equilibrium security design, all equilibrium auction designs in the extended model are characterized by equilibrium security designs of the main model and nonbinding reserve prices. The key force that drives this result is the difference between how security designs and reserve prices affect the auction. A binding reserve price has a detrimental effect on efficiency of the auction, because there is a positive probability that the asset will not be transferred to the bidder even though the transfer is efficient. Unlike reserve prices, auctions in securities allow the seller to increase her expected revenue without affecting efficiency of the auction. This is because in the absence of a reserve price, in any securities auction the asset is always sold to the bidder with the highest valuation. As a result, each seller prefers to extract the surplus from the auction by conducting her auction in securities rather than committing to a binding reserve price. This result is consistent with the evidence of significant royalty rates and surprisingly low reserve prices in oil lease auctions in the United States (McAfee and Vincent 1992).

Proposition 5 implies that reserve prices will not be implemented as long as an auctioneer can extract additional surplus by increasing steepness of the security design. There are three cases when reserve prices will be implemented. First, binding reserve prices can arise if the auctions are conducted in call options. Because call options are the steepest possible securities, a seller cannot extract higher surplus by further altering security design of the auction and thus may choose a binding reserve price. Second, binding reserve prices can arise if a seller's outside option is above the lowest possible valuation v_L. In this case, selling the asset is not efficient when the valuation of the highest bidder is below the seller's outside option. As a result, in equilibrium the sellers will commit to binding reserve prices that prohibit the sale whenever it is not efficient. Finally, binding reserve prices can arise whenever future cash flows from the asset are not contractible, and hence only cash bids are feasible. In this case, if a seller sets the reserve price above her valuation, she increases her surplus while affecting the efficiency of the auction. Internet auctions are the case in point. While most of the goods sold in these auctions have large market size and thus deals are done in cash, sellers of some of the more unusual items could have benefited if they could add security elements to the deal, such as percentage of the next sale for goods whose price has a high probability of growing in the future. However, the inability to verify and contract on the future cash flows of a remote bidder prohibits security design choice in this setting.

V. Extensions

In this section we discuss five potential extensions of the model.

Choice of Auction Format.—The main setting of the model assumes that the format of the auction as the second-price auction is fixed and the sellers can choose only the security designs of their auctions. Clearly, this is a simplification, as usually the sellers also can choose between various formats of the auction. In this section we show that the assumption that we make is not restrictive.

Specifically, suppose that each seller s can choose the auction procedure $M_s \in \{FP, SP\}$ in addition to the security design of her auction. If the seller chooses $M_s = SP$, then the format of the auction is second price, as in the main setting of the

model. In contrast, if the seller chooses $M_s = FP$, then the format of the auction is first price. In this case, the winning bidder pays according to the security submitted by him rather than by the bidder with the second-highest valuation. The following proposition shows that as long as the equilibrium security design in the main model is not call options, all equilibria of the main model will also be equilibria in the setting in which the sellers can also choose between first- and second-price auctions:

PROPOSITION 6: *If $S^* \neq S_{call}$ is an equilibrium security design in the main model, then a pair (S^*, SP) is also an equilibrium in the extended model, in which the sellers can choose the auction format in addition to the security design.*

Proposition 6 implies that equilibria found in the main model are robust to the assumption that the auction format is fixed. The force that drives this result is that conditional on the mechanisms used by all competing sellers, each seller's mechanism affects her expected surplus only through the probability with which each bidder enters her auction. Allowing a seller to choose the auction format in addition to its security design expands the interval of entry probabilities from which a seller can choose by changing her selling procedure. As a result, if the equilibrium in the main model is interior, then it is preserved in the extended version of the model. Interestingly, even if the equilibrium in the main model is boundary at cash auctions, then it is still preserved in the extended version of the model because of the revenue equivalence of cash auctions. However, if the equilibrium security design is call options, it may not be preserved in the extended model. As DeMarzo, Kremer, and Skrzypacz (2005) show, the first-price auction in call options generates higher revenues for the seller than the second-price auction in call options. Hence, if the equilibrium in the main setting is boundary at call options, each seller may find it optimal to deviate to the first-price auction in order to extract more surplus from bidders.

Relaxing the Liquidity Constraint.—In our main setting we assume that $S(z) \geq 0$, meaning that the sellers cannot reimburse the winners. This condition guarantees that the call option is the steepest possible security. Indeed, if the liquidity constraint is relaxed, then securities that allow for reimbursement (i.e., they have $S(0) < 0$) can be steeper than call options and thus can extract higher surplus from the bidders. Relaxing the liquidity constraint does not destroy equilibria as long as the equilibrium security design in the main setting is not call options. The intuition is similar to that of Proposition 6. Relaxing the liquidity constraint expands the lower boundary of the interval of entry probabilities from which a seller can choose by altering her security design. Therefore, as long as the equilibrium of the model is interior or boundary at cash, it is preserved in the extended version of the model. Thus, competition among sellers provides one reason why securities with reimbursement are almost never observed in practice. Another likely reason is non-contractibility of investment. As DeMarzo, Kremer, and Skrzypacz (2005) argue in their Proposition 6, reimbursement can be ruled out if investment X is not contractible.

Partial Valuation Uncertainty.—The situation in which the bidders are ex ante identical might not be a good description of many securities auctions. Indeed, in some merger markets, as well as markets in entertainment and the sports industry,

bidders have partially revealed information about the value of the asset at the time they decide to enter an auction. When the bidders have different beliefs about valuations of assets sold by different sellers, this creates an ex ante *clientele* for each seller by isolating groups of buyers who continue to pursue their preferred assets. As a result, competition among sellers is softened, which is likely to induce sellers to set steeper security designs in equilibrium.

Common Values.—While the assumption of independent private values is standard in many models of auctions, it may not be appropriate for some environments in which securities auctions are used. For example, it is likely that bidders' valuations in oil lease auctions have a significant common component: if one bidder has information that concentration of oil in a given region is high, it increases not only his valuation, but also, if made known to other bidders, valuations of everyone.

The presence of the common value component does not affect the main trade-off in the model. An interesting question is whether the common value component makes equilibrium security designs more or less steep compared to the private value model. In the *pure* common value setting, regardless of the number of bidders k participating in an auction, the interim expected total surplus is constant.[32] As a result, informally, compared to the main model with the same distribution of valuations, each additional bidder brings less revenue to the seller when participation in the auction increases: increased competition means that the seller is able to capture a larger share of the total surplus, but the expected size of the surplus does not grow with the number of bidders. As a result, for the same security design as in the private value framework, the incentive to attract more bidders is now dominated by the incentive to increase steepness of the security design. Therefore, in equilibrium in the pure common value framework the sellers are likely to use steeper security designs.

Moral Hazard.—Separation of ownership and control, which occurs when securities are used as bids in the auctions and the buyer is responsible for the exploitation of the asset, raises an additional problem of value destruction through moral hazard. If the winning bid is independent of the realization of future cash flows, then the buyer's and seller's incentives are aligned. However, if the auction is in securities, this is not the case. For example, in an oil lease auction a change in the royalty rate is likely to affect not only the number of bidders who choose to participate in the auction but also the investment of personal resources by the winner. If the royalty rate is high, the winner of the auction has little incentive to invest, as most of the upside is captured by the government. Kogan and Morgan (2010) study auctions in pure debt and pure equity under moral hazard, and show that the seller has to trade off the security design effect against the moral hazard effect.

While many possible specifications of moral hazard could be offered, the message is the same in every case. As long as contingent contracts between the winning bidder

[32] $V(k) = \int_{v_1,\dots,v_k} (1/k)\sum_{i=1}^{k} v_i\, dF(v_1)\dots dF(v_k) = \int_{v^{(1)},\dots,v^{(k)}} (1/k)\sum_{i=1}^{k} v^{(i)}\, dF\left(v^{(1)},\dots,v^{(k)}\right)$, where $v^{(i)}$ is ith order statistic of k random variables. Switching summation and integration operators back and forth, $V(k) = (1/k)\sum_{i=1}^{k}\int_{v_L}^{v_H} v f_{(i)}(v)\, dv = (1/k)\int_{v_L}^{v_H} v\sum_{i=1}^{k} f_{(i)}(v)\, dv = \int_{v_L}^{v_H} v f(v)\, dv = \bar{v}$ for every k, as the density of ith order statistic is $f_{(i)}(v) = kf(v)\binom{k-1}{i-1}F(v)^{(k-1)-(i-1)}\left(1-F(v)\right)^{i-1}$, so that $\sum_{i=1}^{k} f_{(i)}(v) = kf(v)$ and equal to $V_{PV}(1)$.

and the co-owning seller destroy incentives to invest in the project, the main trade-off of the benchmark model is augmented with the seller's need to provide these incentives. As a result, auctions in cash and securities that do not distort incentives much are more likely to be conducted. Unlike the model without moral hazard, symmetric equilibria in the model with moral hazard will, in general, be inefficient. Because auctions in cash never distort incentives of the buyer, they can be considered as reasonable policy choices to maximize the total welfare of market participants.

VI. Concluding Remarks

In this paper we study the competitive design of auction formats when payments can potentially depend on realizations of future cash flows. This question has not been studied in the literature before: research on competitive mechanism design has focused on the case of cash payments, and prior research on securities auctions has considered only the case of exogenous number of bidders in the auction. Our contribution is threefold. First, we characterize all symmetric equilibria that arise in the model. If bidders' participation decisions are endogenous, then the result of the prior literature that a seller prefers to run an auction in the steepest possible securities can be violated. When there is competition among sellers, equilibrium security designs reflect the seller's trade-off between extracting surplus from a given number of bidders and attracting additional bidders by making the auction more appealing than her rivals' auctions. If one of the forces dominates, the equilibrium security design is either the flattest possible (pure cash) or the steepest possible (call options). If neither of the forces dominates, the equilibrium security designs include combinations of standard securities empirically observed in many auctions.

Second, we study how the parameters of market structure affect the steepness of equilibrium security designs. There are two important determinants of the equilibrium security designs: (i) the ratio of potential bidders to sellers; and (ii) the size of the market. We show in a numerical example that the relative abundance of potential bidders appears to increase the steepness of equilibrium security designs, while larger size of the market appears to decrease their steepness. We prove that in the limit, as the size of the market grows to infinity, in the unique equilibrium auctions are conducted in pure cash. In addition, the equilibrium security designs appear to depend on expected synergies from the auction in a way consistent with empirical evidence.

Finally, we compare competition in security designs with competition in reserve prices, which was the focus of prior research. To do this we extend the model by allowing each seller to choose a reserve price in addition to security design. We show that as long as call options is not an equilibrium security design in the main model, the equilibrium auction formats are characterized by equilibrium security designs of the main model and nonbinding reserve prices.

Our analysis makes an assumption that the selling mechanism specifies an ordered set of securities that can be submitted by the bidders. This assumption is important because it retains the auction-like feature of the selling mechanism. One may ask what happens if the sellers can use general mechanisms that can be contingent on the realization of the project payoff. More precisely, suppose that each seller can commit to any selling mechanism that specifies allocation of the asset and payments as functions of the bidders' messages and the realization of the project payoff.

Clearly, auctions with securities is one class of mechanisms of this kind, but not the only one. While this game is considerably more general than the one we study in this paper, the main trade-off remains the same: choosing a more bidder-friendly selling mechanism reduces the surplus that the seller obtains from a given number of bidders but increases the participation by attracting the bidders from alternative sellers. Analysis of this general framework can shed light on a number of important questions. For example, under what conditions is the equilibrium in which all sellers use securities auctions preserved as an equilibrium in a model where each seller is allowed to deviate to any alternative selling mechanism? Are auctions in pure cash preserved as an equilibrium selling mechanism in large markets in this generalized framework? We leave these interesting questions for future research.

Appendix A. Proofs

The proofs of all lemmas and Propositions 5 and 6 are available from the *AER* online Appendix. Here, we provide the proofs of Propositions 2–4.

PROOF OF PROPOSITION 2:

First, we prove that for any security design $\tilde{S}$ of other auctions, there exists a unique best response $q^{BR}(\tilde{S})$ that maximizes $U^s(q,\tilde{S})$ on $[q(S_{call},\tilde{S}),q(S_{cash},\tilde{S})]$. Let $P(q,n,k)$ denote $\binom{n}{k}q^k(1-q)^{n-k}$. Then, we can write the seller's ex ante expected surplus as:

$$\text{(A1)}\qquad U^s(q,\tilde{S}) = \sum_{k=0}^{n_b} P(q,n_b,k)V(k) - n_b q \sum_{k=1}^{n_b} P\left(\frac{1-q}{n_s-1},n_b-1,k-1\right)U^b(k,\tilde{S}).$$

Then, the first derivative of $U^s(q,\tilde{S})$ with respect to q is equal to

$$\text{(A2)}\qquad U^s_q(q,\tilde{S}) = \sum_{k=0}^{n_b} P_q(q,n_b,k)V(k) - n_b \sum_{k=1}^{n_b} P\left(\frac{1-q}{n_s-1},n_b-1,k-1\right)U^b(k,\tilde{S}) + \frac{n_b q}{n_s-1}\sum_{k=1}^{n_b} P_q\left(\frac{1-q}{n_s-1},n_b-1,k-1\right)U^b(k,\tilde{S}).$$

The first-order conditions imply that at the interior solution $U^s_q(q^{BR}(\tilde{S}),\tilde{S})=0$. The second derivative of $U^s(q,\tilde{S})$ with respect to q is equal to

$$\text{(A3)}\qquad U^s_{qq}(q,\tilde{S}) = \sum_{k=0}^{n_b} P_{qq}(q,n_b,k)V(k) + \frac{2n_b}{n_s-1}\sum_{k=1}^{n_b} P_q\left(\frac{1-q}{n_s-1},n_b-1,k-1\right)U^b(k,\tilde{S}) - \frac{n_b q}{(n_s-1)^2}\sum_{k=1}^{n_b} P_{qq}\left(\frac{1-q}{n_s-1},n_b-1,k-1\right)U^b(k,\tilde{S}).$$

Consider the second term on the right-hand side of (A3). Since probabilities $P(q,n-1,k-1)$ sum up to one over k, for any q and n, $\sum_{k=1}^{n} P_q(q,n-1,k-1)=0$. Moreover, $P_q(q,n-1,k-1)<0$ for $k<q(n-1)+1$ and $P_q(q,n-1,k-1)>0$, otherwise. Consider:

$$\begin{aligned}(\text{A4}) \qquad & \sum_{k=1}^{n} P_q(q,n-1,k-1)U^b(k,\tilde{S}) \\ & = \sum_{k=1}^{[q(n-1)]+1} P_q(q,n-1,k-1)\big(U^b(k,\tilde{S})-U^b\big([q(n-1)]+1,\tilde{S}\big)\big) \\ & \quad + \sum_{k=[q(n-1)]+2}^{n} P_q(q,n-1,k-1)\big(U^b(k,\tilde{S})-U^b\big([q(n-1)]+1,\tilde{S}\big)\big),\end{aligned}$$

where $[\cdot]$ denotes the integer part of the argument. Because $U^b(k,\tilde{S})$ is decreasing in k, $U^b(k,\tilde{S})-U^b\big([q(n-1)]+1,\tilde{S}\big)$ is positive for $k<q(n-1)+1$ and negative otherwise. The right-hand side of (A4) consists of two terms. The first term sums up multiples of nonpositive and nonnegative numbers $\big(P_q(q,n-1,k-1)$ and $U^b(k,\tilde{S})-U^b\big([q(n-1)]+1,\tilde{S}\big)\big)$. The second term sums up multiples of positive and negative numbers $\big(P_q(q,n-1,k-1)$ and $U^b(k,\tilde{S})-U^b\big([q(n-1)]+1,\tilde{S}\big)$, respectively$\big)$. Hence,

$$\sum_{k=1}^{n} P_q(q,n-1,k-1)U^b(k,\tilde{S})<0.$$

Therefore, the second term on the right-hand side of (A3) is negative.

Consider a series $\{P_{qq}(q,n,k),k=0,\ldots,n\}$. Since $\sum_{k=0}^{n} P(q,n,k)=1$, $\sum_{k=0}^{n} P_{qq}(q,n,k)=0$. Moreover,

$$P_{qq}(q,n,k)=\binom{n}{k}q^k(1-q)^{n-k}\left(\left(\frac{k}{q}-\frac{n-k}{1-q}\right)^2-\frac{k}{q^2}-\frac{n-k}{(1-q)^2}\right),$$

which implies that there are numbers k_1 and $k_2>k_1$ such that $P_{qq}(q,n,k)$ is positive for all $k<k_1$ and $k>k_2$, and negative for all $k\in(k_1,k_2)$. It follows that

$$\sum_{k<k_1,k>k_2} P_{qq}(q,n,k)=-\sum_{k_1<k<k_2} P_{qq}(q,n,k).$$

Define probability distributions $G_1(k)$ and $G_2(k)$ by:

$$\mathbb{P}_{G_1}(K=k)=\begin{cases}\dfrac{P_{qq}(q,n,k)}{\sum_{k<k_1,k>k_2} P_{qq}(q,n,k)}, & \text{if } k<k_1 \text{ and } k>k_2 \\ 0, & \text{otherwise;}\end{cases}$$

$$\mathbb{P}_{G_2}(K = k) = \begin{cases} \dfrac{-P_{qq}(q,n,k)}{\sum_{k<k_1,k>k_2} P_{qq}(q,n,k)}, & \text{if } k \in (k_1,k_2) \\ 0, & \text{otherwise.} \end{cases}$$

Notice that by construction $G_2(k)$ crosses $G_1(k)$ from below. Observe that

$$\sum_{k=0}^{n} P(q,n,k)k = qn \quad \Rightarrow \quad \sum_{k=0}^{n} P_{qq}(q,n,k)k = 0$$

$$\Rightarrow \sum_{k<k_1,k>k_2} P_{qq}(q,n,k)k = -\sum_{k_1<k<k_2} P_{qq}(q,n,k)k,$$

which implies that $G_1(k)$ and $G_2(k)$ have the same mean. In addition, because $G_2(k)$ crosses $G_1(k)$ from below, $\int_0^x G_1(t)\,dt \geq \int_0^x G_2(t)\,dt$ for all x. Thus, $G_2(k)$ second-order stochastically dominates $G_1(k)$ (see, e.g., Proposition 6.D.2 in Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Green 1995). Therefore, for any concave function $\psi(K)$,

$$\sum_{k_1<k<k_2} \frac{-P_{qq}(q,n,k)}{\sum_{k<k_1,k>k_2} P_{qq}(q,n,k)} \psi(k) \geq \sum_{k>k_1,k<k_2} \frac{P_{qq}(q,n,k)}{\sum_{k<k_1,k>k_2} P_{qq}(q,n,k)} \psi(k)$$

$$\Rightarrow \sum_{k=0}^{n} P_{qq}(q,n,k)\psi(k) \leq 0.$$

By Lemma 2, both $V(k)$ and $-U^b(k,S)$ are concave in k. Thus, the first and the last terms of the right-hand side of (A3) are negative. Hence, $U^s(q,\tilde{S})$ is strictly concave in q for any $\tilde{S}$. Therefore, the maximizer $q^{BR}(\tilde{S})$ on $[q(S_{call},\tilde{S}), q(S_{cash},\tilde{S})]$ is unique.

Consider the case $\phi(S_{cash}) \geq 0$. Because $U^s(q,S_{cash})$ is concave in q,

$$\frac{1}{n_s} = q(S_{cash},S_{cash}) \in \underset{q\in[q(S_{call},S_{cash}),\,q(S_{cash},S_{cash})]}{\arg\max} U^s(q,S_{cash}).$$

Hence, cash auctions is an equilibrium outcome. Consider the case $\phi(S_{call}) \leq 0$. Again, because $U^s(q,S_{call})$ is concave in q, auctions in call options is an equilibrium outcome. Finally, consider the last case. If an ordered set of securities S satisfies $\phi(S) = 0$, then $U^b(q,S)$ reaches its maximum at $q = 1/n_s$. Hence, any ordered set of securities S that satisfies $\phi(S)$ is an equilibrium security design. If $\phi(S_{cash}) < 0$ and $\phi(S_{call}) > 0$, then there exists S such that $\phi(S) = 0$ as constructed in the proof of Proposition 3.

PROOF OF PROPOSITION 3:

Continuity of $S(\lambda,s,z)$ ensures that function $\phi(\lambda)$ is a real-valued continuous function on the interval $[\lambda_L,\lambda_H]$. In addition, $\phi(\lambda_H) \geq 0$ and $\phi(\lambda_L) \leq 0$. Therefore,

by the intermediate value theorem, on the interval $[\lambda_L, \lambda_H]$ there exists a point $\tilde{\lambda}$ at which $\phi(\tilde{\lambda}) = 0$. Hence, there exists an equilibrium security design that belongs to set $\mathbb{S}$.

PROOF OF PROPOSITION 4:

Consider $\phi(S)/n_s$, where $\phi(S)$ is given by (16). As $n_s \to \infty$, the first term of $\phi(S)/n_s$ converges to

$$\text{(A5)}\quad \lim_{n_s\to\infty} \frac{1}{n_s}\sum_{k=0}^{n_s n}\binom{n_s n}{k}\frac{(n_s-1)^{n_s n-k-1}(kn_s-n_s n)}{n_s^{n_s n-1}}V(k) \;=\; \sum_{k=0}^{\infty}\frac{n^k e^{-n}}{k!}\,(k-n)V(k),$$

by applying the formula for the density function of the Poisson distribution as the limit of the number of successful Bernoulli events with the probability of success $1/n_s$. Plugging in the expression for $V(k)$, we get:

$$\sum_{k=0}^{\infty}\frac{n^k e^{-n}}{k!}(k\;-\;n)\int_{v_L}^{v_H} kvF(v)^{k-1}f(v)\,dv$$

$$=\;\int_{v_L}^{v_H} e^{-n}v\frac{f(v)}{F(v)}\left(\sum_{k=0}^{\infty}\frac{\big(nF(v)\big)^k}{k!}\,k(k\;-\;n)\right)dv.$$

The term in the brackets is equal to

$$e^{nF(v)}\sum_{k=0}^{\infty}\frac{\big(nF(v)\big)^k e^{-nF(v)}}{k!}\,k^2\;-\;ne^{nF(v)}\sum_{k=0}^{\infty}\frac{\big(nF(v)\big)^k e^{-nF(v)}}{k!}\,k$$

$$=\;e^{nF(v)}\big(nF(v)\;+\;n^2F(v)^2\;-\;n^2F(v)\big),$$

as the linear combination of the first and the second moment of the Poisson distribution with parameter $nF(v)$. Hence, the first term converges to

$$\text{(A6)}\qquad n\int_{v_L}^{v_H} ve^{-n\left(1-F(v)\right)}\big(1\;+\;nF(v)\;-\;n\big)f(v)\,dv.$$

The second and the third term of $\phi(S)/n_s$ converge to, correspondingly:

$$\text{(A7)}\quad \lim_{n_s\to\infty} n\sum_{k=1}^{n_s n}\left(\begin{array}{c} n_s n\;-\;1 \\ k\;-\;1\end{array}\right)\frac{(n_s\;-\;1)^{n_s n-k}}{n_s^{n_s n-1}}U^b(k,S) \;=\; \sum_{k=1}^{\infty}\frac{n^k e^{-n}}{(k\;-\;1)!}\;U^b(k,S);$$

$$\lim_{n_s\to\infty} n\sum_{k=1}^{n_s n}\left(\begin{array}{c} n_s n\;-\;1 \\ k\;-\;1\end{array}\right)\frac{(n_s\;-\;1)^{n_s n-k-2}\big((k\;-\;1)n_s\;-\;n_s n\;+\;1\big)}{n_s^{n_s n-1}}\,U^b(k,S)$$

$$=\;\sum_{k=1}^{\infty}\frac{n^k e^{-n}}{(k\;-\;1)!}\;\lim_{n_s\to\infty}\frac{(k\;-\;1\;-\;n)n_s\;+\;1}{(n_s\;-\;1)^2}\;=\;0.$$

Consider auctions in cash. A well-known result from the theory of cash auctions is

$$U^b(k, S_{cash}) = \int_{v_L}^{v_H} (1 - F(v))F(v)^{k-1}\, dv.$$

Then, the limit of the second term of $\phi(S_{cash})/n_s$ is equal to

$$\text{(A8)} \quad \sum_{k=1}^{\infty} \frac{n^k e^{-n}}{(k-1)!} \int_{v_L}^{v_H} (1 - F(v))F(v)^{k-1}\, dv$$

$$= \int_{v_L}^{v_H} e^{-n} \frac{(1 - F(v))}{F(v)} \left(\sum_{k=0}^{\infty} \frac{(nF(v))^k}{k!} k \right) dv = n \int_{v_L}^{v_H} e^{-n(1-F(v))}(1 - F(v))\, dv.$$

Combining (A6) with (A7) and (A8), we conclude that $\phi(S_{cash})/n_s$ converges to

$$n \int_{v_L}^{v_H} e^{-n(1-F(v))}\big(v(1 + nF(v) - n)f(v) - 1 + F(v)\big)\, dv$$

$$= n \int_{v_L}^{v_H} d\big(-e^{-n(1-F(v))}(1 - F(v))v\big) = ne^{-n}v_L.$$

Therefore, in large markets $\phi(S_{cash}) > 0$, so auctions in cash is an equilibrium in large markets. Because for any S, $U^b(k, S)$ is larger than or equal to $U^b(k, S_{cash})$, in large markets, $\phi(S) > 0$ for any S. Therefore, there is no symmetric equilibrium in which auctions are in noncash securities.

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