Performance Analysis of Rateless-Coded Non-Orthogonal Multiple Access over Nakagami-*m* Fading Channels with Delay Constrains

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***Abstract*—To achieve efficient and reliable data transmission in a communication system in time varying conditions, a downlink non-orthogonal multiple access system based on rateless codes (NOMA-RC) is proposed in this paper. The NOMA-RC system can continuously send superimposed signals to the users according to the decoding results within a limited time. After a user decodes the signals successfully, it will send an acknowledgement to the transmitter. Then the system may adjust the message to be transmitted to improve the decoding probability for the remaining users. The performance of the NOMA-RC system with delay constrains is analyzed over Nakagami-*m* fading channels. The simulation results show that the NOMA-RC system is capable of reducing the transmission time and improving system efficiency compared to orthogonal multiple access system based on rateless codes.**

I. Introduction

With the rapid growth requirements of the fifth generation (5G) mobile communications in terms of the spectrum efficiency and the number of user connections, it is becoming more difficult for an orthogonal multiple access (OMA) system to cope with this trend [1]. Alternatively, non-orthogonal multiple access (NOMA) technology is an effective way to improve the capacity and the spectrum efficiency of a system, which is considered as a promising multiple access scheme in the next generation mobile communication networks [2]. The NOMA system helps to achieve large-scale heterogeneous data traffic in further mobile networks.

There has been a significant research on the NOMA technology in the recent years. NOMA has been combined with some key technologies such as multiple-input multiple-output (MIMO), Polar codes etc. to improve the system performance [3]–[5]. However, fixed rate codes are commonly used for the physical layer's channel coding in the existing literature. NOMA systems under time varying channel conditions are also not well studied yet. For a communication system with fixed rate codes, it needs to estimate the channel states frequently, then to select an appropriate coding rate and modulation mode to transmit the message [6]. Unfortunately, it is difficult for such systems to obtain high performance over time varying channels.

Rateless codes automatically adjusts the rate in adverse conditions without knowing the channel state information (CSI) in advance [7]–[10]. The transmitter can continuously generate many inter-dependent encoded symbols. If a receiver fails to decode the message, the sender will add some new symbols to jointly decode it. When the total mutual information accumulated by the receiver exceeds the information entropy, it will decode the message with a high probability [10]. Finally, an acknowledgement (ACK) message from the receiver will be returned through the feedback links. Since rateless codes are proposed, they have been widely used in relay communication systems and orthogonal frequency division multiplexing systems [11]–[13]. Most of the rateless codes are developed in Rayleigh fading channels, while the existing contributions lack the use of rateless codes in Nakagami-*m* fading channels. More specifically, the Nakagami-*m* fading channels can be reduced into different channels according to the size of *m*. For example, when $m = 1$, it is a Rayleigh fading channel. It also can be converted to a Rice fading channel with a fading parameter *k* when $m = (k+1)^2 \big/ (2k+1)$ is used.

A NOMA system based on rateless codes (NOMA-RC) is proposed with the limitation of delay, and its performance is analyzed and discussed over Nakagami-*m* fading channels. The transmitter first superimposes the message from the users to generate some composite signals, and then sends them to each user in a broadcast manner. Next, a receiver exploits successive interference cancellation (SIC) algorithm to eliminate interference from the other users and then decodes its information. When there is a successful user, the system will re-allocate the power and supplement some symbols to the remaining users according to the ACK from the user. This helps to increase the successful decoding probability for the users. The NOMA-RC system has the ability to achieve good performance by adjusting either the transmission power or the maximum delay for different application scenarios. For example, under conditions of limited delay, the system can improve its performance by increasing the transmission power. For low transmitting power devices such as small satellites and unmanned aerial vehicles etc., we also choose to increase the transmission time to ensure the system reliability. For Nakagami-*m* fading channels, the transmission time for two users is analyzed in the NOMA-RC system with delay constrains in detail. The simulation results show that when compared with the conventional OMA system, the proposed NOMA-RC system can reduce the transmission time and improve the transmission efficiency of the system.

II. System Model

There are commonly multiple users around a base station (BS) which generates plenty of different symbols using rateless encoders, and all the users can continuously receive observations from the channels to jointly decode these symbols. When the accumulated mutual information of a user is greater than or equal to the entropy of the source H_0, the user can decode the information [10]. If a user decodes successfully, its ACK will be transmitted to the BS through a feedback channel. Then the BS may adjust the power factors and superimposed signals according to the decoding results of the users. The channel gain between a user U_i and the BS is denoted as $|h_i|^2, \forall 1 \le i \le m_0$, where m_0 is the total number of users. We assume that the channels between the users and the BS are independent and follow Nakagami-*m* distribution. Thus, the probability density function (PDF) and cumulative distribution function (CDF) of $|h_i|^2$ are expressed as [14]

$$f(x) = \frac{m^m x^{m-1}}{\Omega_i^m \Gamma(m)} \exp\left(-\frac{mx}{\Omega_i}\right), x \ge 0, \quad (1)$$

$$F(x) = 1 - \exp\left(-\frac{mx}{\Omega_i}\right) \sum_{k=0}^{m-1} \frac{1}{k!} \left(\frac{mx}{\Omega_i}\right)^k, x \ge 0, \quad (2)$$

where m determines the kind of fading channel. The parameter Ω_i is the average received power of U_i which satisfies [15]

$$\Omega_i\,(\mathrm{dBm}) = P_t\,(\mathrm{dBm}) + K\,(\mathrm{dB}) - 10\tau \lg\left(\frac{d}{d_0}\right), \quad (3)$$

where K is the constant coefficient related to each antenna element and the average channel loss, P_t is denoted as the transmission power of the BS, τ is the path loss exponent and d is the distance from a user to the BS. The message from the users is encoded by rateless encoders to generate some symbols which are then multiplied with corresponding power factors and form a composite signal to be transmitted. The signal received by U_i is given as

$$y_i = h_i \left(\sum_{i=1}^{m_0} \sqrt{a_i P_t} x_i\right) + n_i, \quad (4)$$

where n_i denotes the additive Gaussian white noise and a_i is the allocated power factor. It satisfies [16]

$$a_i = \left(2^{R_i} - 1\right) \left(\rho_i + \sum_{j=i+1}^{m_0} \left(2^{R_i} - 1\right) \rho_j \prod_{k=i+1}^{j-1} 2^{R_k}\right) \Big/ P_t, \quad (5)$$

where $\rho_i = -\frac{P_{\sigma^2}}{\Omega_i \log(1-\varepsilon_i)}$, ε_i is the outage probability and P_{σ^2} is the average power of the noise. Besides, we have $a_1 \ge a_2 \ge \cdots \ge a_i \ge \cdots \ge a_{m_0}, \forall 0 \le a_i \le 1$.

III. Communication Process

We take two users as an example to describe the communication process in quasi-static fading channels. The whole communication process is divided into two stages. When the BS simultaneously transmits superimposed signals to two users, the first stage will be terminated when there is a successful decoding user. In the second stage, the BS may adjust the transmission power of the remaining user according to the feedback of the successful one.

Since the number of participating users in each phase is different, the states of the channels will also change. Referring to the channel model in [10], we assume that the channel is a quasi-static fading channel which means that the CSI remains unchanged in the same phase, but the states between different phases are independent of each other. The required time for the near user U_n to decode the far user U_f is

$$T_{n(f)} = \frac{H_0}{\log_2\left(1 + r_{n(f)}\right)}, \quad (6)$$

where $r_{n(f)}$ is the signal to interference noise ratio (SINR) for U_n to decode U_f, which is expressed as $r_{n(f)} = \frac{P_t a_f |h_n|^2}{P_{\sigma^2} + P_t a_n |h_n|^2}$. The parameters a_n and a_f are the power factors of U_n and U_f, respectively. The user U_n can use the SIC algorithm to eliminate the signal from U_f and then decode its own information. The required time for U_n to decode its own message is

$$T_{n(n)} = \frac{H_0}{\log_2\left(1 + r_{n(n)}\right)}, \quad (7)$$

where $r_{n(n)} = \frac{P_t a_n |h_n|^2}{P_{\sigma^2}}$. We set the scenario of U_n to successfully decode U_i as $D_{n(i)}$. For U_n, only when $D_{n(f)}$ and $D_{n(n)}$ occur simultaneously, the ACK of the user can be sent to the BS. So the time for U_n to successfully decode is $T_{n,1} = \max\left(T_{n(f)}, T_{n(n)}\right)$. For U_f, it treats the information of U_n as interference signals. So it is $r_{f(f)} = \frac{P_t a_f |h_f|^2}{P_{\sigma^2} + P_t a_n |h_f|^2}$. The time for $D_{f(f)}$ is

$$T_{f(f)} = \frac{H_0}{\log_2\left(1 + r_{f(f)}\right)}. \quad (8)$$

The decoding time in the first stage is $T_1 = \min\left(T_{n,1}, T_{f(f)}\right)$. Then, the BS continues to transmit the related information to the remaining user until it achieves success. So the total required time for the two users is

$$T_2 = \frac{H_0 - T_1 \log_2(1 + r_{i,1})}{\log_2(1 + r_{i,2})} + T_1, \quad (9)$$

where $r_{i,1}$ is the SINR of the failed user $U_i\,(i = n, f)$ in the first stage and $r_{i,2}$ is the SINR of U_i in the second stage.

IV. Decoding Time for Different Stages

This section introduces two transmission schemes. They are an ideal transmission scheme and a delay constrain transmission scheme, respectively. Besides, the transmission time is analyzed in the paper.

A. Ideal transmission scheme

The BS generates an infinite number of different symbols using rateless encoders. A user has the ability to complete the decoding with a probability of getting close to 1 as long as it accumulates enough symbols. Thus, both users decode successfully when the NOMA-RC system does not have the transmission time constrains. There are following two cases to be discussed.

1) U_n first, U_f second: User U_n needs to decode U_f first, and then decodes its own message in the first stage. For U_n, it needs to satisfy the following expression:

$$T_1 \log_2 (1 + r_{n,1}) \ge H_0, \tag{10}$$

where $r_{n,1} = \min \left(r_{n(f)}, r_{n(n)} \right)$.

Theorem 1. *The closed-form expression for the PDF of $r_{n,1}$ can be given by*

$$f_{r_{n,1}}(x) = \begin{cases} \exp\left(-\frac{mcx}{\Omega_n(a-bx)} - \frac{mcx}{\Omega_n b}\right) \frac{cm^m}{\Omega_n^m \Gamma(m)} \\ \left(\begin{array}{c} \frac{a(cx/(a-bx))^{m-1}}{(a-bx)^2} \sum\limits_{k=0}^{m-1} \frac{1}{k!} \left(\frac{mcx}{\Omega_n b}\right)^k \\ + \frac{(cx/b)^{m-1}}{b} \sum\limits_{k=0}^{m-1} \frac{1}{k!} \left(\frac{mcx}{\Omega_n(a-bx)}\right)^k \end{array} \right), 0 \le x < \frac{a}{b}. \\ 0, x > \frac{a}{b}. \end{cases}, \tag{11}$$

where $a = P_t a_f,\ b = P_t a_n,\ c = P_{\sigma^2}$, and Ω_n is the average received power of U_n.

Proof: According to (1), (2) and $r_{n(f)} = \frac{a|h_n|^2}{c + b|h_n|^2}$, the PDF and CDF of $r_{n(f)}$ can be given by

$$f_{r_{n(f)}}(x) = \begin{cases} \left(\begin{array}{c} \frac{ac}{(a-bx)^2} \frac{m^m (cx/(a-bx))^{m-1}}{\Omega_n^m \Gamma(m)} \\ \times \exp\left(-\frac{mcx}{\Omega_n(a-bx)}\right) \end{array} \right), 0 \le x < \frac{a}{b}. \\ 0, x > \frac{a}{b}. \end{cases}, \tag{12}$$

and

$$F_{r_{n(f)}}(x) = \begin{cases} 1 - \left(\begin{array}{c} \sum\limits_{k=0}^{m-1} \frac{1}{k!} \left(\frac{mcx}{\Omega_n(a-bx)}\right)^k \\ \times \exp\left(-\frac{mcx}{\Omega_n(a-bx)}\right) \end{array} \right), 0 \le x < \frac{a}{b}. \\ 1, x > \frac{a}{b}. \end{cases} \tag{13}$$

respectively.

We assume that $r_{n(f)} = \frac{a|h_n|^2}{b|h_n|^2 + c} = x_1$, $r_{n(n)} = \frac{b|h_n|^2}{c} = y_1$ and $r_{n,1} = \min \left(r_{n(f)}, r_{n(n)} \right) = z$. For x_1 and y_1 are two related variables, we can obtain

$$\begin{aligned} \mathrm{P}(Z \le z) &= \mathrm{P}(\min(x_1, y_1) \le z) \\ &= 1 - \mathrm{P}\left(x_1 > z, x_1 > \frac{az}{b(1+z)} \right). \end{aligned} \tag{14}$$

We can discuss it in the following two situations.
(1) If $z > \frac{az}{b(1+z)}$, it is

$$F_{r_{n,1}}(z) = 1 - \mathrm{P}\left(x_1 > z, x_1 > \frac{az}{b(1+z)} \right) = F_{r_{n(f)}}(z). \tag{15}$$

(2) If $z \le \frac{az}{b(1+z)}$, it is

$$\begin{aligned} F_z(z) &= 1 - \mathrm{P}\left(x_1 > z, x_1 > \frac{az}{b(1+z)} \right) \\ &= \mathrm{P}\left(x_1 \le \frac{az}{b(1+z)} \right) = F_{r_{n(f)}}\left(\frac{az}{b(1+z)} \right). \end{aligned} \tag{16}$$

Thus, we can obtain the PDF and CDF of $r_{n,1}$, respectively, are,

$$f_{r_{n,1}}(z) = \begin{cases} \left(\begin{array}{c} \frac{ac}{(a-bz)^2} \frac{m^m (cz/(a-bz))^{m-1}}{\Omega_n^m \Gamma(m)} \\ \times \exp\left(-\frac{mcz}{\Omega_n(a-bz)}\right) \end{array} \right), \frac{a}{b} > z > \frac{a-b}{b}. \\ \left(\begin{array}{c} \frac{c(1+z)^2}{a} \frac{m^m (cz/b)^{m-1}}{\Omega_n^m \Gamma(m)} \times \\ \exp\left(-\frac{mcz}{b\Omega_n}\right) \frac{a}{b(1+z)^2} \end{array} \right), z \le \frac{a}{b} - 1. \end{cases} \tag{17}$$

$$F_{r_{n,1}}(z) = \begin{cases} 1 - \left(\begin{array}{c} \sum\limits_{k=0}^{m-1} \frac{1}{k!} \left(\frac{mcz}{\Omega_n(a-bz)}\right)^k \\ \times \exp\left(-\frac{mcz}{\Omega_n(a-bz)}\right) \end{array} \right), \frac{a}{b} > z > \frac{a-b}{b}. \\ 1 - \left(\begin{array}{c} \sum\limits_{k=0}^{m-1} \frac{1}{k!} \left(\frac{mcz}{b\Omega_n}\right)^k \\ \times \exp\left(-\frac{mcz}{b\Omega_n}\right) \end{array} \right), z \le \frac{a}{b} - 1. \end{cases} \tag{18}$$

Proposition 1. The decoding time in the first phase is determined by U_n. The required time for U_n to successfully decode is $t_1 = \frac{H_0}{\log_2(1 + r_{n,1})}$. The PDF of t_1 is

$$g_{r_{n,1}}(t_1) = -\frac{2^{H_0/t_1} \ln 2 H_0}{t_1^2} f_{r_{n,1}}\left(2^{H_0/t_1} - 1 \right). \tag{19}$$

Theorem 2. *In the first stage, the information accumulation for U_f per unit time is $y = \log_2(1 + r_{f,1})$. The PDF of y is given as*

$$f_y(y) = 2^y \ln(2) \left(\begin{array}{c} \frac{acm^m (c(2^y-1)/(a-b(2^y-1)))^{m-1}}{\Omega_f^m \Gamma(m)(a-b(2^y-1))^2} \\ \times \exp\left(-\frac{mc(2^y-1)}{\Omega_f(a-b(2^y-1))}\right) \end{array} \right), y \ge 0, \tag{20}$$

where Ω_f is the average received power of U_f.

Proof: The SINR of U_f in the first phase is $r_{f,1} = \frac{P_t a_f |h_f|^2}{P_t a_n |h_f|^2 + P_{\sigma^2}}$, the PDF of $r_{f,1}$ is

$$f_{r_{f,1}}(x) = \begin{cases} \left(\begin{array}{c} \frac{acm^m (cx/(a-bx))^{m-1}}{\Omega_f^m \Gamma(m)(a-bx)^2} \\ \times \exp\left(-\frac{mcx}{\Omega_f(a-bx)}\right) \end{array} \right), 0 \le x < \frac{a}{b}. \\ 0, x > \frac{a}{b}. \end{cases} \tag{21}$$

Using $y = \log_2(1 + r_{f,1})$, the PDF of y is given as

$$f_y(y) = 2^y \ln 2 f_{r_{f,1}}(2^y - 1), y \ge 0. \tag{22}$$

Substituting (21) into (22), (20) can be achieved.

Proposition 2. The remaining information is $w = H_0 - t_1 y$. Using (19) and (20), the PDF of w is

$$f_w(w) = \int_{A_1}^{T_s} \frac{1}{t_1} g_{r_{n,1}}(t_1) f_y\left(\frac{H_0 - w}{t_1} \right) dt_1, H_0 \ge w \ge 0, \tag{23}$$

where $A_1 = \frac{c\left(2^{H_0/T_s} - 1\right)}{a - \left(2^{H_0/T_s} - 1\right) b}$.

Theorem 3. *In the second phase, the information accumulated by U_f per unit time is $u = \log_2\left(1 + r_{f,2}\right)$, The closed-form expression for the PDF of u is given by*

$$f_u\left(u\right) = 2^u \ln 2 \left(\begin{array}{c} \frac{cm^m\left(c\left(2^u-1\right)/P_t\right)^{m-1}}{P_t \Omega_f^m \Gamma(m)} \\ \times \exp\left(-\frac{cm\left(2^u-1\right)}{P_t \Omega_f}\right) \end{array} \right), u > 0. \quad (24)$$

Proof: When U_n completes the decoding in the first stage, the BS only needs to send the messages to U_f in the second stage. Then, the BS will re-adjust the transmission power factor for U_f with $a_f = 1$ and $a_n = 0$. The SINR of U_f in the second stage is $r_{f,2} = \frac{P_t |h_f|^2}{P_{\sigma^2}}$, thus we can achieve the PDF of $r_{f,2}$ as

$$f_{r_{f,2}}\left(x\right) = \left(\frac{cm^m\left(cx/P_t\right)^{m-1}}{P_t \Omega_f^m \Gamma\left(m\right)} \exp\left(-\frac{cmx}{P_t \Omega_f}\right) \right), x \geq 0. \quad (25)$$

Similar to Theorem 2, (24) can be achieved according to $u = \log_2\left(1 + r_{f,2}\right)$.

Proposition 3. The extra time for the remaining information to U_f is $s = \frac{w}{u}$. Using (23) and (24), the PDF of s is

$$g_s\left(s\right) = \int_0^\infty u f_u\left(u\right) f_w\left(us\right) du, s \geq 0. \quad (26)$$

The time for two users to successfully decode is $t_2 = t_1 + s$. Using (19) and (26), the PDF of t_2 is

$$g_{r_{f,2}}\left(t_2, \infty\right) = \int_0^\infty g_{r_{n,1}}\left(t_1\right) g_s\left(t_2 - t_1\right) dt_1, t_2 > 0. \quad (27)$$

2) U_f first, U_n second: In the first stage, the information accumulation for U_n per unit time is $y = \log_2\left(1 + r_{n,1}\right)$. Using (17), the PDF of y is given as

$$f_y\left(y\right) = 2^y \ln 2 f_{r_{n,1}}\left(2^y - 1\right), y \geq 0. \quad (28)$$

Theorem 4. *When U_f is the first one to achieve success, the time in the first stage is determined by U_f which is $t_1 = \frac{H_0}{\log_2(1 + r_{f,1})}$. The closed-form expression for the PDF of t_1 is given by*

$$g_{r_{f,1}}\left(t_1\right) = -\frac{2^{H_0/t_1} \ln 2 H_0}{t_1^2} \left(\begin{array}{c} \frac{acm^m\left(c\left(2^{H_0/t_1}-1\right)/\left(a-bx\right)\right)^{m-1}}{\Omega_f^m \Gamma(m)\left(a-b\left(2^{H_0/t_1}-1\right)\right)^2} \\ \times \exp\left(-\frac{mc\left(2^{H_0/t_1}-1\right)}{\Omega_f\left(a-b\left(2^{H_0/t_1}-1\right)\right)}\right) \end{array} \right), \quad (29)$$

where $t_1 \geq \frac{H_0}{\log_2\left(1 + \frac{a}{b}\right)}$.

Proof: According to Proposition 1 and (21), (29) can be obtained.

Combining (28) and (29), the PDF of $w = N_0 - t_1 y$ is expressed as follows:

$$f_w\left(w\right) = \int_{A_1}^{T_s} \left(\begin{array}{c} \frac{2^{((H_0 - w)/t_1)} \ln 2}{t_1} g_{r_{f,1}}\left(t_1\right) \\ \times f_{r_{n,1}}\left(2^{((H_0-w)/t_1)} - 1\right) \end{array} \right) dt_1, H_0 \geq w \geq 0. \quad (30)$$

The SINR of U_n in the second stage is $r_{n,2} = \frac{P_t |h_n|^2}{P_{\sigma^2}}$, we can achieve the PDF of $r_{n,2}$ as

$$f_{r_{n,2}}\left(x\right) = \left(\frac{cm^m\left(cx/P_t\right)^{m-1}}{P_t \Omega_n^m \Gamma\left(m\right)} \exp\left(-\frac{cmx}{P_t \Omega_n}\right) \right), x \geq 0. \quad (31)$$

The extra time for the remaining information to U_n is $s = \frac{w}{u}$, where $u = \log_2\left(1 + r_{n,2}\right)$. According to (30) and (31), the PDF for s is given by

$$g_s\left(s\right) = \int_0^\infty u 2^u \ln 2 f_{r_{n,1}}\left(2^u - 1\right) f_w\left(us\right) du, s \geq 0. \quad (32)$$

Using (29) and (32), the PDF of $t_2 = t_1 + s$ is

$$g_{r_{n,2}}\left(t_2, \infty\right) = \int_0^\infty g_{r_{f,1}}\left(t_1\right) g_s\left(t_2 - t_1\right) dt_1, t_2 \geq 0. \quad (33)$$

B. Transmission Scheme with Delay Constrains

There is always the longest tolerance time T_s in an actual system. It means that the BS will not transmit symbols for the failed frame message all the time. If the message cannot be decoded within T_s, the transmitter will discard it or clear the previous symbols and add some new ones again. Thus, there are four cases for the two users in a limited transmission time which are discussed below.

1) Case 1 : U_n succeeded but U_f failed

The probability of U_n decoding successfully but U_f failing in t_1 is

$$P_{n\overline{f}}\left(t_1\right) = \int_0^{t_1} g_{r_{n,1}}\left(t\right) dt \left(1 - \int_0^{t_1} g_{r_{f,1}}\left(t\right) dt\right). \quad (34)$$

Combining (19), (27) and (29), the probability that U_f can decode in t_2 and the mean time are given by

$$P_{f|n\overline{f}}\left(t_2\right) = \int_0^{T_s} \left(\begin{array}{c} g_{r_{n,1}}\left(t_1\right)\left(1 - \int_0^{t_1} g_{r_{f,1}}\left(t\right) dt\right) \\ \times \int_{t_1}^{t_2} g_{r_{f,2}}\left(t, T_s\right) dt \end{array} \right) dt_1, \quad (35)$$

and

$$E_{n\overline{f}}\left(T_s\right) = \int_0^{T_s} t_2 P_{f|n\overline{f}}\left(t_2\right) dt_2. \quad (36)$$

respectively.

2) Case 2: U_f succeeded but U_n failed

The probability of U_f decoding successfully but U_n failing in t_1 is

$$P_{f\overline{n}}\left(t_1\right) = \int_0^{t_1} g_{r_{f,1}}\left(t\right) dt \left(1 - \int_0^{t_1} g_{r_{n,1}}\left(t\right) dt\right). \quad (37)$$

Combining (19), (29) and (31), the probability that U_n can decode in t_2 is

$$P_{n|f\overline{n}}\left(t_2\right) = \int_0^{T_s} \left(\begin{array}{c} g_{r_{f,1}}\left(t_1\right)\left(1 - \int_0^{t_1} g_{r_{n,1}}\left(t\right) dt\right) \\ \times \int_{t_1}^{t_2} g_{r_{n,2}}\left(t, T_s\right) dt \end{array} \right) dt_1, \quad (38)$$

The mean time is

$$E_{f\overline{n}}\left(T_s\right) = \int_0^{T_s} t_2 \times P_{n|f\overline{n}}\left(t_2\right) dt_2. \quad (39)$$

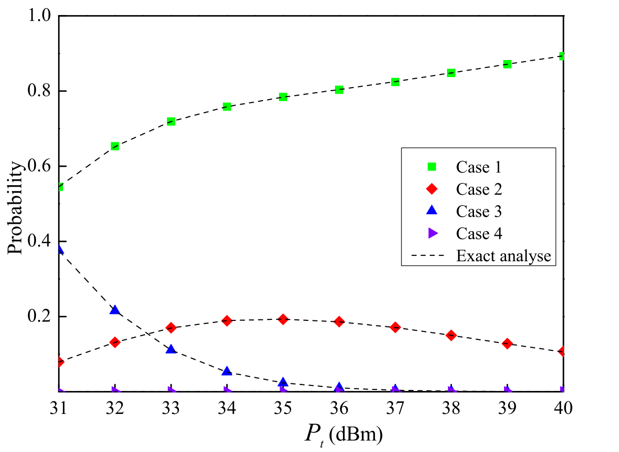

Fig. 1. Decoding probability versus the transmission power in the first stage with $m = 1, T_s = 5, H_0 = 1$.

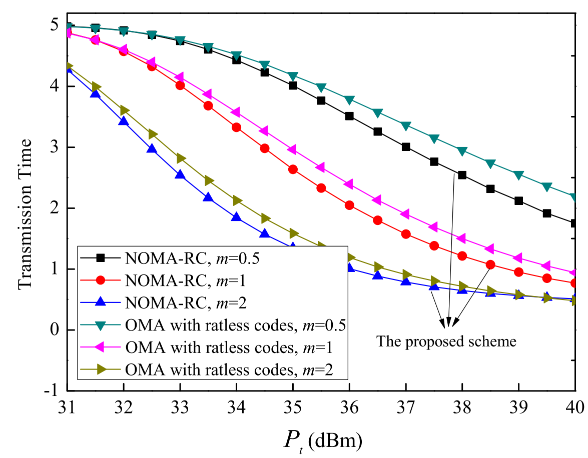

Fig. 2. Comparison of the average total transmission time between the proposed NOMA-RC and conventional OMA systems over Nakagami-m channels for $T_s = 5, H_0 = 1$.

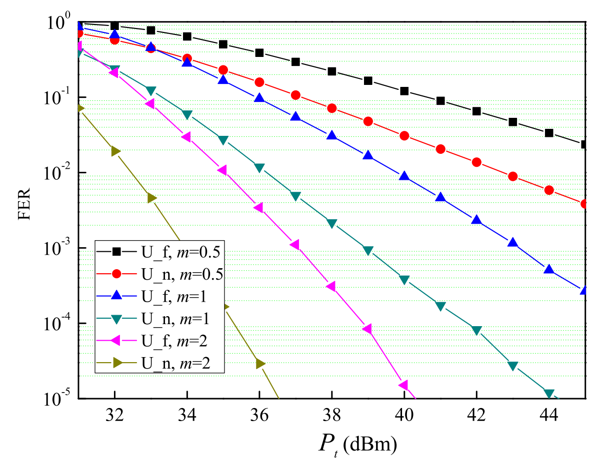

Fig. 3. FER comparison of the two users for different m with for $T_s = 5, H_0 = 1$.

3) Case 3: Both U_n and U_f succeeded simultaneously
Both the probability of Case 3 and the mean time are 0.

Proof: Please see Appendix A.

4) Case 4: Both U_n and U_f failed
The probability of Case 4 and the mean time for the two users can be obtained as

$$P_{\overline{n}\overline{f}}(T_s) = \left(1 - \int_0^{T_s} g_{r_{n,1}}(t)dt\right)\left(1 - \int_0^{T_s} g_{r_{f,1}}(t)dt\right), \tag{40}$$

and

$$E_{\overline{n}\overline{f}}(T_s) = T_s P_{\overline{n}\overline{f}}. \tag{41}$$

respectively.

In T_s, the probability of at least one failed user is

$$P_{\overline{n}|\overline{f}}(T_s) = 1 - P_{n|f\overline{n}}(T_s) - P_{f|n\overline{f}}(T_s) - P_{nf}(T_s). \tag{42}$$

The mean time is provided as

$$E_{\overline{n}|\overline{f}}(T_s) = T_s P_{\overline{n}|\overline{f}}(T_s). \tag{43}$$

Combining (36), (39) and (43), the average time for the system with delay constrains to deal with the two users is

$$\overline{T_2} = E_{n\overline{f}}(T_s) + E_{f\overline{n}}(T_s) + E_{nf}(T_s) + E_{\overline{n}|\overline{f}}(T_s). \tag{44}$$

V. Simulation Results

The performance of the NOMA-RC system is presented by the simulation experiments over Nakagami-m channels. This section analyzes and discusses the average transmission time and frame error rate (FER) performance of the system. Assuming that there are two users U_n and U_f, and their distance from the BS are set as $d_1 = 1000\,\mathrm{m}$ and $d_2 = 500\,\mathrm{m}$, respectively. Other simulation parameters are as follows: $K{=}-38.757$, $\tau = 3.71$, $d_0 = 1$, $P_{\sigma^2}{=}-108\,\mathrm{dBm}$, $a_f = 0.2$, $a_n = 0.8$.

It can be observed from Fig. 1 that Case 1 increases as P_t increases, while Case 4 continues decreasing. User U_n can achieve success quickly in the first stage, while the successful probability of U_f is relatively lower. If the two users decode simultaneously, they need to accumulate the same information entropy at the same time. From Appendix A, we can obtain that the probability of this event is 0. Furthermore, the simulation results are consistent with the theoretical results derived from (34), (37), (40) and (47).

Fig. 2 shows the transmission time required for the NOMA-RC and OMA with rateless codes systems to deal with the two users over Nakagami-m channels. When P_t is small, the decoding failure probability of each user is relatively higher. The system is forced to terminate transmitting the message due to maximum delay constrains. Thus, the average transmission time is longer. As P_t increases, the more information will be accumulated by the users per unit time and less time is required to complete the decoding. Furthermore, the curves will drop faster as m increases. Compared with the OMA system, the NOMA-RC system can simultaneously transmit the data of multiple users in time varying channels, so less transmission time is required for the same m.

A user can achieve success by accumulating mutual information continuously. In other words, it can complete the decoding by extending the transmission time even at low

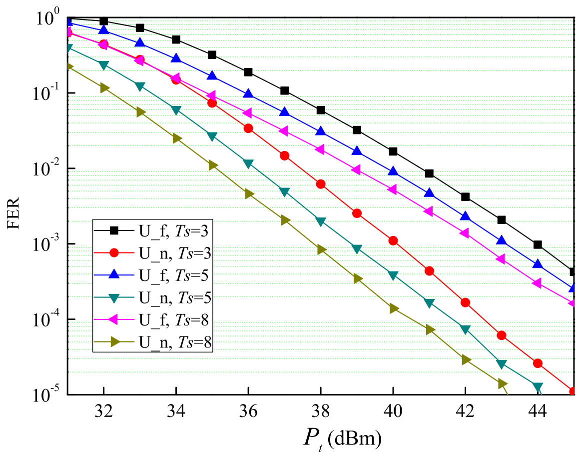

Fig. 4. FER comparison of the two user for different T_s with $H_0 = 1$, $m = 1$.

SNR. The NOMA-RC system has to make a response within the time T_s due to maximum transmission time limitation. Fig. 3 and Fig. 4 show the FER performance of the users over Nakagami-*m* channels and with different T_s, respectively. As T_s increases, the more mutual information the users accumulate, the higher successful probability they will achieve. For example, when the transmission power is 38 dBm, the FER of the system with T_s=3 and T_s=8 are 8.39×10^{-4} and 6.2×10^{-3}, respectively. Thus, improving channel conditions and extending transmission time are the effective ways to improve the FER performance. Especially for devices with limited storage energy, we can extend the time to ensure the reliability of communication. For some applications with low latency requirements, the system can reduce the time by increasing the transmission power.

VI. Conclusion

This paper proposes a NOMA transmission scheme based on rateless codes over Nakagami-*m* fading channels. The symbols of each user generated by a rateless encoder are multiplied by a power factor in the BS. They are superimposed to form several composite signals which are simultaneously transmitted to each user. The NOMA-RC system is capable of utilizing the rateless encoders to generate many different symbols for each user and enhance the anti-noise ability. Besides, each user can jointly decode by accumulating previously received symbols to improve the spectrum efficiency. Experiments show that the NOMA-RC system can reduce the transmission time and help to improve the FER performance. In addition, it can also adjust the maximum transmission time or transmission power to meet different requirements according to different application scenarios.

Appendix A

If U_n and U_f achieve success simultaneously at $t_1 \in (0, T_s]$, we can obtain

$$\begin{aligned} Q_{nf}(t_1) &= P\left(\min\left(r_{n(f)}, r_{n(n)}\right) = r_{f,1}, t_1\right) \\ &= P\left(\gamma_{n(f)} = \gamma_{f,1}, t_1\right) P\left(\gamma_{n(f)} < \gamma_{n(n)}\right) + \\ &\quad P\left(\gamma_{n(n)} = \gamma_{f,1}, t_1\right) P\left(\gamma_{n(f)} > \gamma_{n(n)}\right). \end{aligned} \tag{45}$$

As $|h_n|^2$ and $|h_f|^2$ are two independent and continuous random variables, we can consider the two parameters $r_{n(f)}$ and $r_{f,1}$ as independent and continuous random variables with $r_{n(f)}=\frac{a|h_n|^2}{b|h_n|^2+c}$ and $r_{f,1}=\frac{a|h_f|^2}{b|h_f|^2+c}$.

$$\begin{aligned} &P\left(\gamma_{n(f)} = \gamma_{f,1}, t_1\right) \\ &= P\left(\gamma_{n(f)} = \gamma_{f,1}\right) P(t_1) \\ &= \int_{\gamma_{n(f)}}^{\gamma_{n(f)}} \int_0^{\infty} f\left(\gamma_{n(f)}, \gamma_{f,1}\right) d\gamma_{n(f)} d\gamma_{f,1} P(t_1) \\ &= 0. \end{aligned} \tag{46}$$

where $f\left(\gamma_{n(f)}, \gamma_{f,1}\right)$ is the joint density function of $\gamma_{n(f)}$ and $\gamma_{f,1}$. Similarly, as $r_{n(n)}=\frac{b|h_n|^2}{c}$ and $r_{f,1}$ are also independent and continuous random variables, we can deduce $P_{nf}\left(\gamma_{n(n)} = \gamma_{f,1}, t_1\right)$=0.

Finally, we can express that the probability of the two users achieving success simultaneously in T_s as

$$P_{nf}(T_s) = \int_0^{T_s} Q_{nf}(t_1)\, dt_1 = 0. \tag{47}$$

And the mean time is $E_{nf}(T_s) = 0$.

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