Essays on Incomplete Market and Aggregate Fluctuation

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I, Seungcheol Lee, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work. Chapter 1 was undertaken as joint work with Morten Ravn and Ralph Luetticke. Chapter 3 was undertaken as joint work with Edward Crawley.

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Abstract

This thesis consists of three chapters on incomplete markets and aggregate fluctuations. Chapter 1 examines the impact of frictional financial intermediation in a heterogeneous agents new Keynesian (HANK) model. An incentive problem restricts banking sector leverage and gives rise to an equilibrium spread between the returns on savings and debt. The interest rate spread that impacts on the wealth distribution and movements in it subjects borrowers and savers to different intertemporal prices. The model generates a financial accelerator that is larger than in a representative agent setting, derives mainly from consumption rather than investment, and works through a countercyclical interest rate spread. Credit policy can mute this mechanism while stricter regulation of banking sector leverage inhibits households' ability to smooth consumption in response to idiosyncratic risk. Thus, although leverage restrictions stabilize at the aggregate level, we find substantial welfare costs.

In Chapter 2, we show that it is optimal to pay more attention to employment stabilization when both a heterogeneity of households and the matching friction exist even though the price adjustment cost is substantial. This implies that the optimal policy needs to strike a balance between price adjustment and employment stability rather than to pursue complete price stabilization in order to make the poor better off. In addition, we study the effect of a time-varying transfer rule on the volatility of inflation and employment with respect to a volatile job separation shock. We find that the Ramsey planner pays less attention to employment stabilization when a countercyclical transfer rule is in operation.

Chapter 3 analyzes the transmission mechanism of monetary policy to consumption in New Keynesian models with heterogeneous households. We show that

in these models the countercyclical nature of profits, empirically false, plays a large role in amplifying the intertemporal substitution channel. On the other hand, the interest rate exposure channel, empirically large, plays a small role. We suggest expanding the role of the interest rate exposure channel, while dampening the amplification effect of countercyclical profits, is of primary quantitative importance in future work.

Impact Statement

This thesis presents results that are of academic interests and have policy implications. Chapter one, Financial Friction: Macro vs Micro volatility, shows how wealth inequality matters for the impact of frictions in financial intermediation. When households differs in wealth, the financial accelerator works mainly through consumption rather than through investment. This new finding on the financial accelerator highlights the effectiveness of a credit policy. In addition, the heterogeneity in households wealth leads a macropduential regulation to involve a trade-off between macroeconomic stabilization and microeconomic volatlity. The micro consumption instability implies additional welfare loss of regulation policy on banks' leverage, which should be carefully considered by regulation authorities.

Chapter two, Optimal Monetary Policy with TANK and SAM, is motivated by a possible trade-off between the two objectives of the Federal Reserve: price stability and maximum sustainable employment. Specifically, this chapter deals with optimal path of inflation and employment when we consider both a heterogeneity in households and the friction in labor market. My model offers a theoretical ground for policy makers to pay more attention to employment stabilization when they consider both the poor and the friction in labor market.

The third chapter, Decomposition of Monetary Policy Transmission with Heterogeneous households, addresses the issue of monetary policy transmission via the lens of Auclert (2019) in the theoretical HANK models. This chapter clarifies that the HANK models with predetermined variables are also able to decompose a transitory monetary policy shock into the redistribution channels with small errors. In addition, it is shown that the share of heterogeneous income channel is large while

that of interest exposure channel is small in the standard new Keynesian models, which is empirically false. This finding calls for further researches on theoretical HANK models in order to understand the monetary policy transmission better in terms of the redistribution channels that have close links to micro data.

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Introduction

This thesis is composed of three chapters, connected by a common theme: incomplete markets and aggregate fluctuation. I use macroeconomic models to bring new insights to monetary, fiscal, credit, and macroprudential policy in incomplete markets setting.

In Chapter 1, we consider aggregate fluctuations in setting with idiosyncratic risk, nominal rigidity and frictional financial intermediation. We introduce financial intermediation into a heterogeneous agents new Keynesian (HANK) model in order to research the impact of the financial friction on consumption. A central aspect of our analysis is the importance of movement in the spread between the savings and lending rates facing households. The countercyclical movement in the spread induce a key role for consumption in the transmission mechanism. An increase in the interest rate on debt relative to the return on savings will hold back indebted households' consumption relative to wealthier households. The increase in the spread also implies that more households remain at the kink in their budget constraints where their marginal propensities to consume are very high. Another key insight of our analysis is the impact of macroprudential regulation that limits bank leverage. A stricter banks net worth regulation results in the higher spread and a larger share of households at the kink in their budget constraint, thus, inducing micro volatlity. Hence, we find that such policies are associated with a trade-off between micro and macro volatility. We find that the average welfare loss from a 25% reduction in banking sector leverage is 1.4% of life-time consumption.

In Chapter 2, we analyzes the optimal choices of a social planner with respect to aggregate shocks in the environment of heterogeneous households, sticky prices

and matching friction. We show that it is optimal to pay more attention to employment stabilization than to price adjustment cost when both the rule-of-thumb households and the matching friction exist even though the price adjustment cost is substantial. This implies that the optimal policy needs to strike a balance between price adjustment and employment stability rather than to pursue complete price stabilization in order to make rule-of-thumb consumers better off. We implement two comparisons so as to investigate whether only the combination of TANK and SAM results in the deviation from the complete price stabilization. Lastly, we study the effect of a time-varying transfer rule on the volatility of inflation and employment with respect to a more volatile job separation shock. We find that the Ramsey planner pays less attention to employment stabilization when a countercyclical transfer rule is in operation and more attention if a procyclical transfer rule is in operation.

In Chapter 3, we analyze the transmission mechanism of monetary policy to consumption in New Keynesian models with heterogeneous households. This chapter uses the lens of the monetary policy decomposition in Auclert (2019). The method assumes that a transitory monetary policy shock has no persistent effects, which do not hold up in many models. However, we show that the decomposition works pretty well even in some models with predetermined variables such as capital or the entire distribution of assets. We begin our analysis with a standard TANK model in which a proportion of households live hand-to-mouth. We find that the decomposition accounts for 95 percent of the change in consumption even in the TANK with capital. In addition, the decomposition also works well in a HANK model with an entire wealth distribution under standard calibrations. Specifically, we show that in these models the countercyclical nature of profits, empirically false, plays a large role in amplifying the intertemporal substitution channel. On the other hand the interest rate exposure channel, empirically large, plays a small role. We suggest expanding the role of the interest rate exposure channel, while dampening the amplification effect of countercyclical profits, is of primary quantitative importance in future work.

Chapter 1

Financial Frictions: Macro vs Micro Volatility

1.1   Introduction

Since the Global Financial Crisis (GFC) much research effort has gone into examining the consequences of imperfections in financial markets for the functioning of the economy. This paper contributes to this literature by showing how wealth inequality deriving from market incompleteness and idiosyncratic risk matter for the impact of frictions in financial intermediation. When households differ in wealth, the financial accelerator mainly works through consumption, and macroprudential regulation involves a trade-off between macroeconomic stabilization and microeconomic volatility. These properties are fundamentally different from those in representative agent economies in which the financial accelerator impacts mainly on investment and macroprudential regulation involves a trade-off between the level of income and macroeconomic volatility. Our results highlight the importance of the spread between the interest rates on household savings and debt for consumption dynamics across the wealth distribution.

Frictionless financial markets allow resources to flow to their most productive uses and provide the economy with immunity to the propagation of shocks deriving from cross-agent differences in their evaluation of intertemporal trade-offs. This cornerstone of economic theory serves as a useful benchmark but a number of its key implications stand in stark contrast with empirical evidence. The frictionless

model, for example, implies that households are perfectly insured against idiosyncratic income risk. An extensive empirical literature has challenged this implication and documented that household consumption is sensitive to household-specific income shocks.[1] The frictionless model also implies that central bank purchases of assets should be neutral, an implication that seems strongly challenged by the evidence of the impact of unconventional policies in the aftermath of the GFC.[2]

Such findings have motivated extensive research examining the impact of financial frictions. One line of work has considered aggregate fluctions in settings with idiosyncratic risk, incomplete markets, and frictional goods and/or labor market. In this line of work, frequently referred to as HANK, lack of insurance markets and borrowing constraints inhibit agents' ability to smooth out adverse income shocks, which makes the distribution of marginal propensities to consume a key statistic. Another line of work has instead investigated frictional financial intermediation. This literature typically retains the representative agent assumption, focusing on how agency problems in the financial sector impact on macroeconomics outcomes. A key result is that financial intermediaries matter for macroeconomic (in)stability. In particular, financial frictions may amplify the impact of shocks on the economy due to a financial accelerator that leads to exaggerated investment responses. Moreover, financial intermediaries may be a source of instability due to shocks to their balance sheets.

In this paper, we shift the attention of the financial frictions literature to its impact on consumption when households differ in wealth. For this purpose, we introduce financial intermediation into a heterogeneous agents new Keynesian (HANK) setting. The economy is composed of a financial sector, a corporate sector, a household sector and a government. There are nominal rigidities on the supply side, while households are subject to uninsurable idiosyncratic income risk. Banks intermediate between savers (households) and borrowers which are either firms or

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[1]See, for example, Cochrane (1991), Mace (1991), Blundell et al. (2008), or recently Fagereng et al. (2016)

[2]See, for example, Gagnon et al. (2011), Krishnamurthy and Vissing-Jørgensen (2011), Chen et al. (2012) or Gambacorta et al. (2014).
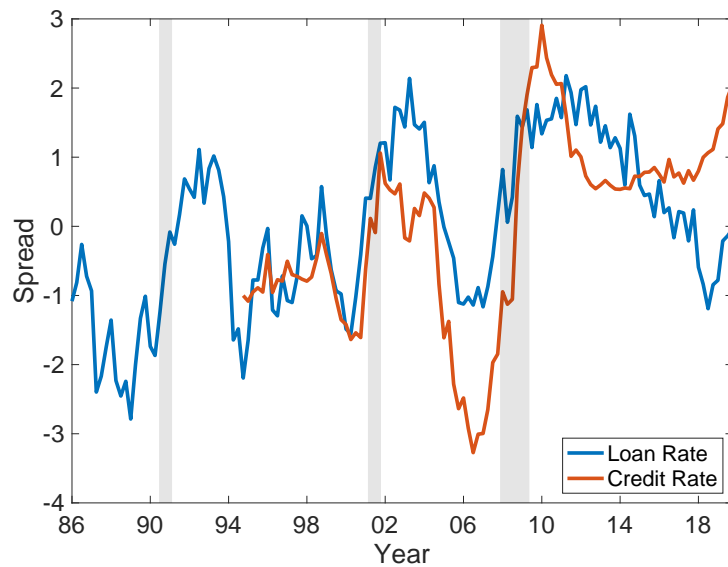
Notes: Demeaned spread in percentage points. Loan Rate: Rate on personal loans at commercial banks (2 year loan) minus 2-year Treasury rate. Credit Rate: Commercial bank rate on credit card plans minus 3-month T-Bill rate. Retrieved from FRED, Federal Reserve Bank of St. Louis. Shaded areas correspond to recessions according to the NBER.

Figure 1.1: Borrowing Penalty

households. This intermediation is hampered by an agency problem which limits banks' investment in assets.

A central aspect of our analysis is the importance of movements in the spread between the savings and lending rates facing households. In our analysis, this spread derives from banks' incentive problem that limits their investment in assets to a certain fraction of their net worth as well as from a resource cost of issuing unsecured consumer debt. Figure 1.1 shows two (demeaned) measures of this spread, the difference between the interest rate on personal loans and the two-year Treasury rate and the difference between the interest rate on credit card debt and the three-month T-bill rate. NBER recessions are indicated by the shaded grey areas. Both measures of the spread increase abruptly and significantly in recessions and tend to decline during expansions. Such countercyclical movements in the spread are consistent with the predictions of our model to the extent that banking sector net worth is procyclical, a property that we show holds in response to recessionary technology shocks, monetary policy shocks, and "capital quality" shocks.

Such movements in the spread induce a key role for consumption in the transmission mechanism. In fact, we show that the model generates a financial accelerator that mainly works through consumption rather than investment. To see why, note that movements in the spread between savings and lending rates imply a differential impact of shocks on households depending on their net asset positions. First, due to potentially binding borrowing constraints or kinks in the budget constraints, marginal propensities to consume differ across the wealth distribution. Secondly, an increase in the interest rate on debt relative to the return on savings will hold back indebted households' consumption relative to wealthier households. For indebted households, recessions will therefore tend to induce strong consumption reductions which we show dominate in the aggregate because wealthy households are able to smooth out income shocks.

Consider the response of the economy to declining net worth of banks. Lower net worth means that banks have less capacity to invest in the corporate sector inducing an increase in the spread of the return on bank assets over the deposit rate. The deflationary pressures lead the central bank to cut deposit rates which gives households with positive net asset positions less incentive to save. Poorer households, however, face increasing interest rates on consumption loans due to the higher spread forcing these households to reduce their consumption. Similarly, adverse productivity shocks or contractionary monetary policy shocks reduce banking sector net worth that through the interest-rate-spread channel differentially impacts on households according to their net assets. In each of these cases, the increase in the spread implies that more households remain at the kink in their budget constraints where their marginal propensities to consume are very high. Because of the large consumption responses, we show that the output responses to shocks are amplified relative to representative agent economies. Hence, while the model has a financial accelerator, it derives from consumption mainly.

Another key insight of our analysis is the impact of macro prudential regulation that limits bank leverage. The literature usually argues that such regulation trades off increased stability of the economy with lower average activity. We show that the

trade off is different in the heterogeneous agent economy. Because of the impact on leverage, stricter macro prudential regulation increases the spread between consumer debt and the return on deposits. A higher spread means that a larger share of households find themselves at the kink in the budget constraint at zero wealth, that debt is costlier for households with negative net asset positions, and that it is more attractive for households with positive net asset positions to avoid becoming indebted. Through each of these channels, household consumption becomes more sensitive to idiosyncratic risk thus inducing micro volatility. Hence, we find that such policies are associated with a trade-off between micro and macro volatility, a trade-off that is felt throughout the wealth distribution including by less wealthy households who are proportionally more harmed by higher cost of borrowing. We find that the average welfare loss from a 25% reduction in banking sector leverage is 1.4% of life-time consumption.

Our analysis adds to the rapidly expanding HANK literature.[3] This literature has so far concentrated upon examining how frictions in goods and labor markets, such as nominal rigidities or matching frictions, combine with incomplete markets to produce new insights about macroeconomic fluctuations and economic policy. Parts of this literature has included further frictions such as asset illiquidity, differences in the returns on debt and savings (see, for example, Bayer et al. (2019); Kaplan et al. (2018)), or constraints on monetary policy (see, for instance, Farhi and Werning (2016); Korinek and Simsek (2016)) but this literature has not considered the impact of financial intermediation. Our model with frictional banking makes the cost of borrowing endogenous and this has fundamental consequences for the transmission mechanism. Such endogenous changes in the spread between interest rates changes the fraction of households with high marginal propensities to consume. This is a key sufficient statistic for many shocks and policies, see for example Auclert (2019) or Auclert et al. (2018). In addition to this, movements in the spread imply that households are differently exposed to shocks depending on their

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[3]See, for example, Auclert (2019); Bayer et al. (2019); Broer et al. (2019); Gornemann et al. (2012); Guerrieri and Lorenzoni (2017); McKay et al. (2016); Kaplan et al. (2018); Ravn and Sterk (2017).

net asset position. Our analysis also adds to the the literature on financial frictions. The latter has highlighted the importance of the financial accelerator for business cycles.[4] We show that the financial accelerator becomes more powerful in a model with consumer credit by directly affecting consumption. The key role of banks' net worth for the propagation of shocks has led to the literature on macro prudential regulation. We add to this literature by showing that tighter regulation of banks negatively affects household insurance, which has first order effects on welfare.

Complementary to our work, Fernández-Villaverde et al. (2020) combine a financial sector à la Brunnermeier and Sannikov (2014) with heterogeneous households. They show that the interaction between the demand of bonds by the financial sector and the precautionary supply of bonds by households produces significant endogenous aggregate risk when solved globally. Our focus is very different and shows instead how financial frictions interact with the wealth distribution through the interest rate spread.

The remainder of the paper is structured as follows. We present the model in the next Section. Thereafter, we discuss the calibration and some implications for the links between the financial sector and the wealth distribution in Section 2. Section 3 investigates the transmission mechanism of the model. Section 4 looks into the impact of macroprudential regulation. Finally, we conclude in Section 5.

1.2 Model

The economy is composed of a financial sector, a corporate sector, a household sector and a government sector. The model combines nominal rigidities on the supply side, incomplete markets and idiosyncratic risk amongst the households, and financial frictions in the financial sector. We will show that this model has important implications for the transmission mechanism and for the impact of macro prudential regulation.

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[4]See Bernanke et al. (1999) for a survey and more recently, for example, Gertler and Kiyotaki (2010); Gertler and Karadi (2011); Brunnermeier and Sannikov (2014).

1.2.1 Households

There is a continuum of measure one of ex-ante identical households indexed by *i*. Households are infinitely lived, have time-separable preferences and derive utility from consumption c_{it} and disutility from working l_{it}. Households switch randomly between being workers or rentiers. Workers supply labor competitively and are subject to idiosyncratic earnings risk. Rentiers receive a share of the profits made by the corporate and the financial sectors but do not participate in the labor market. The rentiers delegate all intertemporal firm decisions to risk neutral managers. We assume that the claims to the pure rents cannot be traded as an asset.

Preferences are time separable and given as:

$$\mathbf{U}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{it}^{1-\mu}}{1-\mu} - \chi \frac{l_{it}^{1+1/\gamma}}{1+1/\gamma} \right] \tag{1.1}$$

where $\mathbb{E}_s x_{it}$ denotes the expectation of x_{it} conditional on all information available at date $s \leq t$. β is the subjective discount factor, $\mu \geq 0$ is the inverse of the intertemporal elasticity of substitution, $\chi > 0$ is a constant, and $\gamma \geq 0$ is the Frisch labor supply elasticity.

Households maximize subject to sequences of budget constraints and borrowing constraints:

$$c_{it} + b_{it+1} \leq \mathbf{R}\left(b_{it}, R_{S,t}, R_{L,t}\right) b_{it} + (1-\tau)\left(w_t h_{it} l_{it} + \mathbb{I}_{h_{it}=0}\mathscr{F}_t\right), \tag{1.2}$$

$$b_{it+1} \geq -\underline{\mathbf{b}} \tag{1.3}$$

b_{it+1} denotes financial net assets chosen in period *t*. Households can save by either purchasing risk-free government bonds, $b_{G,it+1}$, or by making bank deposits, $b_{D,it+1}$. If the household wishes to borrow, it can take out a bank loan, $b_{L,it+1}$, but only up to the borrowing limit, $-\underline{\mathbf{b}} \leq 0$. The interest rate schedule is given as:

$$\mathbf{R}\left(b_{it}, R_{S,t}, R_{L,t}\right) = \begin{cases} R_{S,t} & \text{if } b_{it} = b_{D,it} + b_{G,it} \geq 0 \\ R_{L,t} & \text{if } b_{it} = b_{L,it} < 0 \end{cases} \tag{1.4}$$

where $R_{S,t} = R_{S,t}^N/\pi_t$ is the gross saving rate, $R_{S,t}^N$ is the gross nominal interest rate and $\pi_t = P_t/P_{t-1}$ is the gross inflation rate (P_t is the price of the consumption good).

The gross real interest rate on outstanding debt is given by $R_{L,t} \geq R_{S,t}$. Note that since $R_{L,t} \geq R_{S,t}$, a household will never want to hold assets and have debt simultaneously.

All households pay the same constant proportional tax rate τ on their income. Rentiers' income is given by their share of the profits from firms and banks, $\mathscr{F}_t$. Working households' labor income is given by $w_t h_{it} l_{it}$ where w_t is the real wage per efficiency unit of labor and $h_{it} l_{it}$ is effective labor supply. h_{it} denotes idiosyncratic labor productivity which evolves according to a log-AR(1) process (conditional upon the worker having had the same labor force status last period):

$$
h_{it} = \begin{cases} \exp\left(\rho_h \log h_{it-1} + \varepsilon_{h,it}\right) & \text{with probability } 1 - \zeta \quad \text{if} \quad h_{it-1} \neq 0, \\ 1 & \text{with probability } \iota \quad\quad \text{if} \quad h_{it-1} = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1.5}
$$

where $\rho_h \in (-1, 1)$. $\varepsilon_{h,it}$ is assumed to be iid normally distributed with variance σ_h^2. Here $\zeta \in (0, 1)$ denotes the probability that a worker becomes a rentier while $\iota \in (0, 1)$ is the probability rentiers become workers.[5] A rentier that reverts to becoming a worker starts with median productivity, $h_{it} = 1$.

Suppose that $\zeta \simeq 0$. In this case, workers choose assets and labor supply according to the first-order necessary conditions:

$$
l_{it}^{1/\gamma} = \frac{1}{\chi} c_{it}^{-\mu} (1 - \tau) w_t h_{it} \tag{1.6}
$$

$$
c_{it}^{-\mu} = \begin{cases} \beta \mathbb{E}_t c_{it+1}^{-\mu} R_{S,t+1} & \text{if} \quad b_{i,t+1} > 0, \\ \beta \mathbb{E}_t c_{it+1}^{-\mu} R_{L,t+1} & \text{if} \quad b_{i,t+1} \in (-\underline{\mathbf{b}}, 0). \end{cases} \tag{1.7}
$$

Constrained households, $b_{it+1} = -\underline{\mathbf{b}}$, or at the kink, $b_{it+1} = 0$, instead choose consumption as

$$
c_{it} = \mathbf{R}(b_{it}, R_{S,t}, R_{L,t}) b_{it} + (1 - \tau)\left(w_t h_{it} l_{it} + \mathbb{I}_{h_{it}=0} \mathscr{F}_t\right) \tag{1.8}
$$

Hence, consumption choices differ across households depending on their net assets. Those constrained by the borrowing limit or with zero assets will have unit

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[5]Hence the share of rentiers amongst households is given as $\zeta / (\zeta + \iota)$. We will assume that this is very small.

marginal propensities to consume. Borrowers and savers will have different intertemporal marginal rates of substitution due to the wedge between interest rates on savings and on debt. Indebted households will for that reason choose higher consumption growth than savers (i.e. lower current consumption relative to resources). These differences in consumption spill over to labor supply with poorer more indebted households supplying more labor (relative to current income) than their richer cousins.

1.2.2 Firms

There are three types of firms in the economy: (a) intermediate goods producers who hire labor services and rent capital to produce goods, (b) final goods producers who differentiate intermediate goods and sell them to goods bundlers, and (c) capital goods producers, who turn bundled final goods into capital goods.

When profit maximization decisions in the firm sector require intertemporal decisions (i.e. in price setting and in producing capital goods), we assume for tractability that the rentiers delegate the decision power to a mass-zero group of risk neural managers who are compensated by a share in profits.[6] They do not participate in any asset market and have the same discount factor as all other households. Since managers are a mass-zero group in the economy, their consumption does not show up in any resource constraint and all profits go to the rentiers (whose $h = 0$).

1.2.2.1 Final Goods Producers and Goods Bundlers

Households, capital producers and the fiscal authority purchase bundled goods from competitive firms. These firms and assemble the good using inputs of final goods. Their technology is:

$$Y_t = \left(\int_j y_{jt}^{1-1/\eta} dj \right)^{1/(1-1/\eta)} \tag{1.9}$$

---

[6]Since we solve the model by a first-order perturbation in aggregate shocks, the assumption of risk-neutrality only serves as a simplification in terms of writing down the model. With a first-order perturbation we have certainty equivalence and fluctuations in stochastic discount factors become irrelevant for price setting.

where y_{jt} denotes the input of final good of variety j which is purchased at price p_{jt}. $\eta > 1$ is the elasticity of substitution between the final goods. The demand for final goods variety j is therefore given as:

$$y_{jt} = \left(p_{jt}/P_t\right)^{-\eta} Y_t \qquad (1.10)$$

where $P_t = \left(\int_j p_{jt}^{1-\eta} dj\right)^{1-\eta}$ is the price index. The resource constraint is then:

$$Y_t = C_t + I_t + G_t + Y_t^{ad} \qquad (1.11)$$

where $C_t = \int_i c_{it} di$ is aggregate consumption, I_t denotes investment, and G_t is government consumption. Y_t^{ad} denotes some further resource costs specified below.

Final goods are produced by a continuum of monopolistically competitive firms. Producer j buys the intermediate good at the nominal price MC_t. We assume price adjustment frictions à la Rotemberg (1981). Under this assumption, the firms' managers maximize the expected present value of real profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Y_t \left\{ \left(\frac{p_{jt}}{P_t} - mc_t\right) \left(\frac{p_{jt}}{P_t}\right)^{-\eta} - \frac{\eta}{2\kappa_Y} (\log \frac{p_{jt}}{p_{jt-1}})^2 \right\}, \qquad (1.12)$$

Here $mc_t = MC_t/P_t$ are real marginal costs, $\kappa_Y > 0$ captures price adjustment costs with $\kappa_Y \to \infty$ denoting flexible prices. We focus on symmetric equilbria in which all firms set the same prices. Imposing symmetry, the first-order necessary condition for optimal prices is given as:

$$\log\left(\pi_t\right) = \beta \mathbb{E}_t \log\left(\pi_{t+1}\right) \frac{Y_{t+1}}{Y_t} + \kappa_Y \left(mc_t - \frac{\eta-1}{\eta}\right), \qquad (1.13)$$

where π_t is the gross inflation rate of final goods and $\frac{\eta}{\eta-1}$ is the target markup.

1.2.2.2 Intermediate Goods Producers

Intermediate goods producers are competitive and operate constant returns technologies given as:

$$M_t = \mathscr{Z}_t H_t^{\alpha} \left(\xi_t K_t\right)^{1-\alpha}, \qquad (1.14)$$

where $\mathscr{Z}_t$ is total factor productivity which follows an autoregressive process in logs. ξ_t denotes the quality of capital so that $\xi_t K_t$ is the effective quantity of capital

at time t. ξ_t also follows an autoregressive process. $\alpha \in (0, 1]$ is the labor share of income and H_t is the effective labor input:

$$H_t = \left(\int_i l_{it} h_{it} di \right) \tag{1.15}$$

Let mc_t be the relative price at which the intermediate good is sold to final goods producers. Labor is rented period-by-period on a competitive spot market. Labor demand satisfies the first-order condition:

$$w_t = \alpha mc_t \mathscr{Z}_t \left(\frac{\xi_t K_t}{H_t} \right)^{1-\alpha} \tag{1.16}$$

After production, the firms have $(1 - \delta) K_t$ units of capital left which sell at the normalized (relative) price of 1 per unit, where $\delta \in (0, 1)$ is the depreciation rate. All profits and the remaining capital stock are then paid to the firms' owners. Then the firm acquires new units capital, K_{t+1}, at the price Q_t per unit which are used for production the next period. Capital purchases are financed through issuing $b_{F,t}$ units of equity at the price of Q_t each, i.e.:

$$Q_t K_{t+1} = Q_t b_{F,t} \tag{1.17}$$

After production, the firms pay out all remaining value in the firm to their equity owners. The return offered to the current equity holders is given as:

$$R_{K,t} = \frac{(r_{K,t} + Q_t - \delta) \xi_t}{Q_{t-1}} \tag{1.18}$$

where $r_{K,t}$ is the marginal product of capital:

$$r_{K,t} = \alpha mc_t \mathscr{Z}_t \left(\frac{H_t}{\xi_t K_t} \right)^{\alpha} \tag{1.19}$$

1.2.2.3   Capital Goods Producers

New capital goods are produced by competitive firms. They purchase I_t of bundled goods and transform these into ΔK_{t+1} units of new capital goods according to:

$$I_t = \frac{\psi_k}{2} (\Delta K_{t+1}/K_t)^2 K_t + \Delta K_{t+1}. \tag{1.20}$$

where $\psi_k \geq 0$ captures adjustment costs.

The first-order necessary condition is:

$$\frac{\Delta K_{t+1}}{K_t} = \frac{Q_t - 1}{\psi_k},$$

(1.21)

so that the capital stock is rising (falling) whenever $Q_t > 1$ ($Q_t < 1$).[7]

1.2.3 Banks

Our modeling of the banking sector extends Gertler and Karadi (2011) to include unsecured consumer lending but otherwise follows their setup. A continuum of banks of measure Z, indexed by $z \in (0, Z)$ provides financial intermediation services. Banks are owned by the rentiers but they delegate management to risk neutral bankers who discount future utility at the rate of β. Bankers start life with a start-up fund and build up net worth during their banking careers. Every period a fixed fraction $\theta \in (0, 1)$ of the managers die and replaced by new ones. As in Gertler and Kiyotaki (2010), an agency problem constrains bankers ability to leverage net worth and induces interest rate wedges.

Banks intermediate between households and the corporate sector and between different types of households. The activities of the banks can be summarized in two stages. In the first stage, banks raise deposits ($b_{D,t+1}^z$) from savers. In the second stage, banks use the deposits and their net worth (n_t^z) to invest in equity ($b_{F,t}^z$), bought at price Q_t per unit, and make loans to households ($b_{L,t+1}^z$). The bank's balance sheet follows as:

$$Q_t b_{F,t}^z + b_{L,t+1}^z = n_t^z + b_{D,t+1}^z$$

(1.22)

The gross interest rate on deposits, $R_{D,t+1}$, has to equal the return on government bonds, $R_{S,t+1}$. The return on equity purchases is $R_{K,t+1}$. Bankers can freely choose whether to invest in consumption loans or in corporate sector equity and there is no default risk associated with either asset. The return to banks from consumer loans therefore needs to be $R_{K,t+1}$. Hence, the law of motion of net worth is given as:

$$n_{t+1}^z = \left(R_{K,t+1} - R_{S,t+1}\right)\left(Q_t b_{F,t}^z + b_{L,t+1}^z\right) + R_{S,t+1} n_t^z$$

(1.23)

---

[7]We assume that capital goods producers are each small and thus ignore their externality on the future cost of capital goods production.

We assume that banks face additional costs of supplying loans to households. In particular, making loans to households induces an additional cost that we assume is proportional to the number of units of loans issued. One can think of these as costs of checking whether the size of the loan requested by a household is compatible with the borrowing limit. These costs are passed on to borrowers, i.e.:

$$R_{L,t} = \varphi R_{K,t} \tag{1.24}$$

where $\varphi \geq 1$.

As in Gertler and Kiyotaki (2010), the banker can divert a fraction $\lambda \in (0,1)$ of its assets. Should this happen, depositors declare bankruptcy, the bank closes, and the depositors recover the remaining fraction of $1 - \lambda$ of assets. Thus, bankers will refrain from diversion only if the following constraint is satisfied:

$$\mathbf{V}_t^z \geq \lambda \left(Q_t b_{F,t}^z + b_{L,t+1}^z \right) \tag{1.25}$$

where $\mathbf{V}_t^z$ denotes the value of the bank given as:

$$\mathbf{V}_t^z = \max \mathbb{E}_t \sum_{i=0}^{\infty} (1-\theta)\, \theta^i \beta^{i+1} n_{t+1}^z \tag{1.26}$$

$\mathbf{V}_t^z$ can be expressed as:

$$\mathbf{V}_t^z = \upsilon_{b,t} \left(Q_t b_{F,t}^z + b_{L,t+1}^z \right) + \upsilon_{n,t} n_t^z \tag{1.27}$$

where

$$
\begin{aligned}
\upsilon_{b,t} &= \mathbb{E}_t \left[(1-\theta)\beta \left(R_{K,t+1} - R_{S,t+1} \right) + \beta \theta x_{t,t+1} \upsilon_{b,t+1} \right] \tag{1.28}\\
\upsilon_{n,t} &= \beta \mathbb{E}_t \left[(1-\theta) R_{S,t+1} + \theta g_{t,t+1} \upsilon_{n,t+1} \right]
\end{aligned}
$$

$\upsilon_{b,t}$ is the value of a marginal extra unit of bank assets, and $\upsilon_{n,t}$ is the value of a marginal unit of net worth. $x_{t,t+1} = \left(Q_t b_{F,t}^z + b_{L,t+1}^z \right) / \left(Q_{t-1} b_{F,t-1}^z + b_{L,t}^z \right)$ is the growth rate of bank assets and $g_{t,t+1} = n_{t+1}^z / n_t^z$ is the growth rate of net worth. Both $x_{t,t+1}$ and $g_{t,t+1}$ are identical across banks (see below) and therefore not indexed by z. Thus, $\upsilon_{b,t}$ and $\upsilon_{n,t}$ are also equalized across banks.

We will assume that the incentive constraint is binding so that banks will be unable to invest sufficiently to close the gap between the return on assets and the

interest they offer on savings, ie. $R_{K,t} \geq R_{S,t}$. Imposing that the constraint binds, implies that:

$$Q_t b^z_{F,t} + b^z_{L,t+1} = \phi_t n^z_t \tag{1.29}$$

where ϕ_t is given by

$$\phi_t = \frac{\upsilon_{n,t}}{\lambda - \upsilon_{b,t}} \tag{1.30}$$

It then follows that:

$$n^z_{t+1} = \left(\left(R_{K,t+1} - R_{S,t+1} \right) \phi_t + R_{S,t+1} \right) n^z_t \tag{1.31}$$

and therefore $x_{t,t+1} = (\phi_t/\phi_{t-1}) g_{t,t+1}$ and $g_{t,t+1} = \left(\left(R_{K,t+1} - R_{S,t+1} \right) \phi_t + R_{S,t+1} \right)$ which both are the same across banks as conjectured.

Let $\omega/(1-\theta)$ be the fraction of banking sector value that is injected to new bankers. Aggregating across banks, banking net worth, $N_t = \int_z n^z_t dz$, then obeys the law of motion:

$$N_t = \theta \left[\left(R_{K,t} - R_{S,t} \right) \phi_{t-1} + R_{S,t} \right] N_{t-1} + \omega \left(Q_t b_{F,t-1} + b_{L,t} \right) \tag{1.32}$$

1.2.4 Government

1.2.4.1 Monetary Policy

We assume that monetary policy is conducted by setting the nominal interest rate according to a Taylor-type rule:

$$R^N_{S,t+1} = \overline{R}^N_S \left(\frac{\pi_t}{\overline{\pi}} \right)^{\kappa_\pi} \exp \left(\varepsilon^m_t \right) \tag{1.33}$$

where $\overline{R}^N_S$ is the long-run level of the short term nominal interest rate, $\overline{\pi}$ is an inflation target, and $\kappa_\pi > 0$ is the interest rate response to deviations of inflation from its target. ε^m_t is a monetary policy shock. It follows an AR(1) process with persistence $\rho_m \in (0,1)$ and iid innovations that are normally distributed with mean 0 and variance σ^2_m.

1.2.4.2 Fiscal Policy

The fiscal authority manages government debt, purchases of final goods and is in charge of tax collection. The government budget constraint is given as:

$$B_{G,t+1} = R_{S,t} B_{G,t} + G_t - T_t + DC_t$$

where $B_{G,t+1}$ is the amount of debt issued in period t and T_t are tax revenues:

$$T_t = \tau \left(w_t H_t + \mathscr{F}_t \right)$$

DC_t denotes the net costs of carrying out credit policy which are specified below.

In order to anchor government debt and impose government solvency, we assume that government purchases of goods are governed by the feedback rule:

$$\frac{G_t}{\overline{G}} = \left(\frac{G_{t-1}}{\overline{G}} \right)^{\rho_G} \left(\frac{B_{G,t}}{\overline{B}_G} \right)^{-\gamma_G}$$

where $\overline{G} > 0$ is a constant denoting the long-run level of government spending, and $\rho_G \in (-1, 1)$ allows for partial adjustment of government spending. The last term captures how deviations of government debt from its target, $\overline{B}_G$, triggers spending adjustments. We assume that $\gamma_G > 0$ so that the government cuts spending when debt is rising in order to improve the primary budget balance with the aim of stabilizing debt dynamics.

1.2.4.3 Credit Policy

The central bank may also facilitate lending, which we call credit policy. Let S_t^g be the value of assets intermediated via government assistance and let S_t be the total value of intermediated assets: i.e.,

$$S_t = S_t^p + S_t^g \tag{1.34}$$

where $S_t^p = Q_t b_{F,t} + b_{L,t+1}$ is the total value of privately intermediated assets. To conduct credit policy, the central bank issues government debt to households that pays the deposit interest rate $R_{S,t+1}$ and then lends the funds to non-financial firms and households at the market lending rates $R_{K,t+1}$ and $R_{L,t+1}$, respectively. Importantly, the government always honors its debt but it involves an efficiency costs.

In particular, the central bank credit involves an efficiency cost of τ_I per unit supplied. Hence, the government does not have an incentive to completely replace banks.

Suppose that central bank funds the fraction of ψ_t of intermediated assets: i.e.,

$$S_t^g = \psi_t S_t \tag{1.35}$$

The cost of this policy is $\tau_I \psi_t S_t$. Its net earning from intermediation in any period t equals to $(R_{K,t} - R_{S,t})S^g_{t-1}$.[8] Considering this government activity with banks intermediation, we can rewrite equation (1.34) to obtain

$$S_t = \phi_t N_t + \psi_t S_t = \phi_{c,t} N_t \tag{1.36}$$

where ϕ_t is the leverage ratio for privately intermediated funds and $\phi_{c,t}$ is the leverage ratio for total intermediated funds.

$$\phi_{c,t} = \frac{1}{1 - \psi_t} \phi_t \tag{1.37}$$

The central bank injects credit in response to movements in credit spreads as the following feedback rule:

$$\frac{\psi_t}{\bar{\psi}} = \left[\frac{E_t(R_{K,t+1} - R_{S,t+1})}{\bar{R}_K - \bar{R}_S} \right]^v . \tag{1.38}$$

According to this rule, the central bank expands credit as the spread increases relative to its steady state value.

1.2.5 Market clearing

Let $\Theta_t(b,h)$ denote the joint distribution of assets and productivity across households. The market clearing condition for the savings market reads:

$$\int_{b^* > 0} b^*(b,h) \Theta_t(b,h) \, \mathrm{d}b\mathrm{d}h = B_t = B_{D,t+1} + B_{G,t+1} \tag{1.39}$$

where $b^*(b,h)$ is the policy function that solves the households' savings problem, $B_{D,t+1}$ and $B_{G,t+1}$ denote aggregate supply of bank deposits and government bonds, respectively.

The credit market clearing condition is:

$$N_t + B_{D,t+1} = Q_t K_{t+1} + \int_{b^* < 0} b^*(b,h) \Theta_t(b,h) \, \mathrm{d}b\mathrm{d}h$$

which states that credit supply from banks and saving households are equal to the credit demand from firms and borrowing households. The market for capital goods has to clear:

$$\frac{\Delta K_{t+1}}{K_t} = \frac{Q_t - 1}{\psi_k}$$

Clearing of goods market implies that:

$$\left(1 - \frac{\varepsilon}{2\kappa_Y} (log(\pi_t))^2 \right) Y_t = C_t + I_t + G_t + \tau_I \psi_t S_t + B_{L,t} A_t$$

---

[8]The surcharge on consumer loans is wasted.

where τ_I is the cost parameter from the government intermediation and $A_t = (\varphi - 1)R_{K,t}$ is the wasted intermediation cost. The government budget constraint, taking into account credit policy,

$$G_t + R_{S,t}B_{G,t} + \tau_I \psi_t S_t = T_t + B_{G,t+1} + (R_{K,t} - R_{S,t})\psi_{t-1}S_{t-1}$$

is then satisfied by the Walras' law whenever the credit, deposit, goods, labor, capital and capital service markets clear.

1.3   Calibration

We solve the model by first-order perturbation, using the method of Bayer and Luetticke (2018). We calibrate the model so that one period corresponds to a quarter. Table 1.1 contains the parameter values of the calibration.

   We assume that the intertemporal elasticity of substitution is equal to 1, a value in the range of empirical estimates from studies of household consumption such as Attanasio and Weber (1993) or studies of aggregate data such as Eichenbaum et al. (1988). We set the Frisch elasticity equal to one, a standard value in macro literature even if slightly above the consensus view from the labor literature. We calibrate χ, the weight on the disutility of labor to target a value of labor supply equal to one third. The intertemporal discount factor is calibrated by targeting an annual capital-output ratio of 2.5. Together with other parameters, this implies $\beta = 0.986$ indicating that households engage in quite substantial amounts of precautionary savings.

   We assume that the output elasticity to labor, α, is equal to 67 percent. The depreciation rate is assumed to be two percent per quarter while capital adjustment costs, ψ, are calibrated to target a volatility of investment to output of 3 in response to TFP shocks. The parameter η, the elasticity of substitution between goods varieties, is calibrated to induce a long-run mark-up of five percent. The price stickiness parameter κ_Y is calibrated by exploiting that the slope of the Phillips curve in the Rotemberg model can be related to the average price contract length implied by this slope in a Calvo model. Using this, we calibrate κ_Y so that it is consistent with an average contract length of four quarters.

Calibrated Parameters	Description	Value
Households		
β	Discount factor	0.986
χ	Disutility weight of labor	7
μ	Relative risk aversion	1
γ	Frisch elasticity of labor supply	1
ρ_h	Persistence of income shocks	0.98
σ_h	Variances of income shocks	0.06
Production		
α	Labor share	0.67
Capital		
ψ_k	Capital adjustment cost	6.0
δ	Capital depreciation	0.02
Final Goods		
κ_Y	Price stickiness	0.09
μ^Y	Markup	0.05
Intermediation		
λ	Divertible fraction of capital	0.381
θ	Bank survival ratio	0.972
ω	Transfer to the entering bankers	0.002
τ_I	Government intermediation cost	0.001
φ	Consumer loan cost	1.015
Monetary and Fiscal Rules		
κ_π	Reaction to inflation	1.5
γ_G	Reaction to debt	0.1
ρ_G	Persistence in fiscal rule	0.9
Aggregate Shocks		
ρ_{CQ}, σ_{CQ}	Persistence, standard deviation	0.66, 0.01
ρ_{TFP}, σ_{TFP}	Persistence, standard deviation	0.90, 0.01
ρ_{MP}, σ_{MP}	Persistence, standard deviation	0.50, 0.001

Table 1.1: Model Parameterization

		Baseline
Households (%)		
(At kink)	(%)	8.9
(Borrowers)	(%)	24.3
Return on capital	(R_K,% quarterly)	1.14
Saving interest rate	(R_S,% quarterly)	0.75
Lending interest rate	(R_L,% quarterly)	2.69
Spread	(bp, quarterly)	194

Table 1.2: Steady state statistics
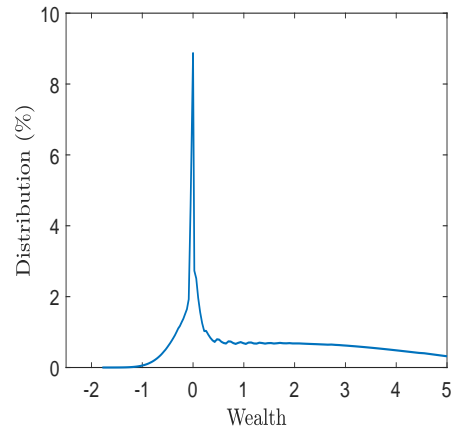
Figure 1.2: Distribution of wealth (wealth: b / Y)

We follow the calibration of Gertler and Karadi (2011), assuming that bankers can divert around 38 percent of the bank's assets and that the survival rate is 97.2 percent per quarter (so that their planning horizon is approximately 10 years). This implies a leverage ratio of 3.5. Even though some banks in the data have higher leverage ratios, which often comes from housing finance, the leverage ratio for corporate and non-corporate business sectors is closer to two in the data. We also follow Gertler and Karadi (2011) in assuming that the transfer to new banks correspond to 0.2 percent of the banking assets.

The inflation coefficient in the Talyor rule is 1.5, a standard value in the literature. We assume that the central bank pursues price stability and set $\overline{\pi} = 1$. To ensure government solvency, government spending reacts to debt, $\gamma_G = 0.1$, and also features inertia, $\rho_G = 0.9$. We set the level of long-run government debt, $\overline{B}_g$, to target a ratio of bank deposits to total gross savings in bonds and deposits to 0.85. This value is consistent with the equivalent share in the Survey of Consumer Finances.

We then target moments of the wealth distribution when calibrating the borrowing limit **b** and the spread on unsecured consumer loans determined by φ. In the Survey of Consumer Finances, roughly 20 percent of households are borrowers and 10 percent of households have close to zero wealth (putting them at the kink in

their budget constraint). To match these we set **b** equal to 2 times average income and φ so that the spread of the interest rate on consumer loans over the deposit rate is 1.94 percent point per quarter while the spread over the return on equity is 1.55 percent point per quarter. At the annual rate, the calibrated spread over the savings rate is 8.1 percent point which is marginally lower than e.g. the calibration of Kaplan et al. (2018) who assume a spread of 10 percent. The spread implies issuing a consumption loan induces a resource cost of 1.5 percent of the loan amount which is passed on to the households.

For the idiosyncratic income risk, we assume that $\rho_h = 0.98$ and $\sigma_h^2 = 0.06^2$. These values correspond to estimates for net household (after tax and transfers) income from the Survey of Income and Program Participants (for the 1984-2013 sample), see Bayer et al. (2019).[9]

These parameter values imply the following wealth distribution and interest rate schedule, see Table 1.2 and Figure 1.2. It is noticeable that there is a mass point in the wealth distribution at zero wealth that derives from the kink in the budget constraint of households induced by the spread of the lending rate over the savings rate. It is also clear that there is a considerable mass of households with close to zero wealth. This is induced by the relatively high variance of idiosyncratic income shocks which induce movements to/from the zero wealth state. The left tail of the wealth distribution is very thin due to the utility cost suffered by households who are prevented from taking on additional debt. Indeed, there are almost no households at the borrowing limit in the stationary distribution. Thus, the high MPC households all derive from the interest rate spread rather than mechanically from the borrowing constraint.

The economy is subject to 3 aggregate shocks, which are TFP, monetary policy, and capital quality shocks. TFP shocks are persistent with an autocorrelation of 0.9 and have a standard deviation of 1%. Monetary shocks are less persistent with an autocorrelation of 0.5 and have a standard deviation of 10 basis points. Capital quality shocks are somewhat persistent with an autocorrelation of 0.66 and have a

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[9]These estimates control for purely transitory income shock and for a deterministic component.

standard deviation of 1%.

1.4 The Transmission Mechanism

We first investigate whether the set-up has new implications for the impact of shocks on the economy. We look at three shocks: Two shocks that has attracted much attention in the business cycle literature, technology shocks and monetary policy shocks, and a capital quality shock that the financial frictions literature has focused attention on. We also examine the stabilizing role of central bank provided credit supply.

We show that the model introduces a new mechanism which has been overlooked in the literature: The impact of endogenous movements in the spread of the interest rate on consumer credit relative to the savings rate. This spread moves countercyclically in response to each of these shocks because of the impact on banking sector net worth. Such countercyclical movements in the spread imply that households face different trade-offs depending on their net wealth position giving rise to divergent consumption responses to shocks along the wealth distribution. Moreover, a higher spread increases the consumption response to shocks deriving from households with zero (or close to zero) net wealth. Through these channels, consumption account for a larger fraction of aggregate adjustments to shocks.

Hence, we move the attention from the spread between the return on corporate debt and savings rates, much studied in the financial frictions literature, to the spread between the interest on consumer debt and deposit rates. This spread matters when agents differ in wealth and face uninsurable idiosyncratic risk.[10]

Capital quality shocks:

We first look at the capital quality shock that Gertler and Karadi (2011) argued an important factor in the Great Financial Crisis. Figure 1.3 illustrates the impact of a one percent shock to ξ_t which simultaneously lowers productivity and increases the depreciation rate. In order to understand the importance of household heterogeneity, we show both the impact of the shock in the baseline model (in blue) and

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[10]We provide robustness checks to other formulations of the lending rate in Appendix B.
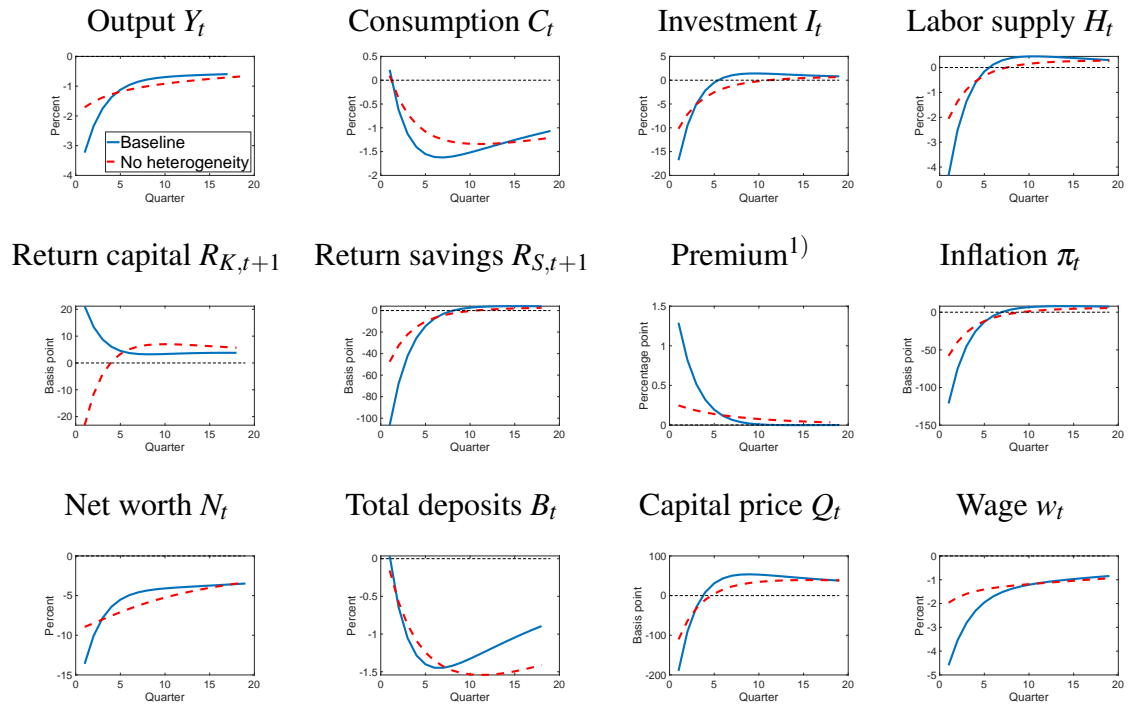
1) $E_t(R_{L,t+1} - R_{S,t+1})$ in Baseline and $E_t(R_{K,t+1} - R_{S,t+1})$ with No heterogeneity.

Figure 1.3: Aggregate effects of a capital quality shock

in a representative agent economy (RANK) where there is no idiosyncratic earnings risk, the wealth distribution is degenerate, and all households are savers (red dotted line).

The decline in capital quality sets off fire-sales of capital and produces a sudden steep decline in the price of new capital, Q_t. The drop in the price of capital worsens banks' balance sheets, and because of their leveraged positions, forces them to cut back on investment in equity and in consumer loans. This sets in motion a process through which reductions in banks' investments lowers the capital price which lower banking sector net worth inducing a further fall in investment etc. In equilibrium, the price of capital falls by approximately two percent and net worth by 14 percent on impact. Both of these responses are substantially larger than in the model without heterogeneity in which the capital price declines by around one percent and net worth by close to 10 percent.

The capital quality shock has a large and persistent impact on the economy. In the initial period, output declines 3.2 percent and 6 quarters later, aggregate output
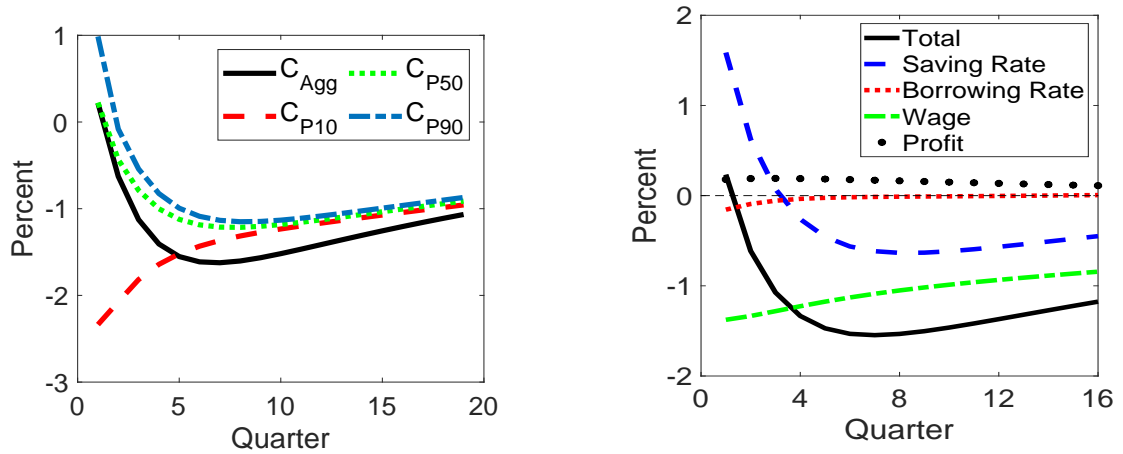
A) Consumption responses by wealth   B) Decomposition of aggregate consumption

Notes: Panel A plots the consumption response for households at 10th, 50th, 90th percentile of the wealth distribution.

Panel B plots the decomposition of aggregate consumption into the effect of each price sequence by using household policy functions.

Figure 1.4: Transmission to consumption: Capital quality shock

is still one percent below its steady-state value. The decline of output in the first period is amplified by a factor of two in the HANK economy relative to the RANK economy. It is also noticeable that while aggregate investment accounts for why output falls in the very first period, thereafter consumption accounts for much more of the fall in aggregate spending.

Recall that declining net worth in the banking sector implies that the return on its assets (equity and consumption loans) must rise relative to the price of funding (the return on savings). The savings rate is dictated by the Taylor rule and the deflationary impact of the capital quality shock leads the central bank to cut nominal and real returns on savings. In a representative agent economy, the increase in the spread is accomplished by the interest on corporate loans falling less than the interest rate on consumer deposits. Allowing for household heterogeneity changes this. In particular, the interest rate on consumption loans has to rise because, otherwise, the bank would stimulate the demand for credit forcing it to cut further back on corporate loans. Thus, not only does the spread between the return on bank assets and the price of funds increase, but the interest rates on savings and on loans move

in opposite directions.

These movements in interest rates impact differentially on households across the wealth distribution. Figure 1.4, Panel A, illustrates the consumption paths for households in the 10th percentile (who are indebted), 50th percentile, and 90th percentile of the wealth distribution, together with aggregate per capita consumption. The impact on the median and wealthy households' consumption is very mild. These households have savings allowing them to smooth consumption in response to the decline in real wages. Moreover, the drop in the return on savings motivates these households to substitute towards current consumption. In combination, this implies that richer households' consumption moves little. Indebted households instead get hit not only by lower real wages but also have to pay higher interest rates on their debt inducing a strong decline in consumption. Moreover, the negative wealth effect spurs an increase in labor supply which reinforces the drop in real wages that derive directly from the shock. Lower real wages hit poorer households hard also because many of these households have low productivity. For the 10th percentile, consumption drops by almost two-and-a-half percent.

In Figure 1.4, Panel B, we decompose the aggregate response of consumption into the impact of the savings rate, the lending rate, the wage rate, and profits. While the saving rate contributes positively to consumption, the lending rate contributes negatively to consumption. The wage rate is the main channel that makes consumption fall accounting for a large fraction of the decline in consumption at all horizons.

Added to this, the increase in the spread induces a rise in the share of zero wealth households who have high marginal propensities so consume. Figure 1.5 shows that the average MPC of the economy increases by 13% in response to the capital quality shock. This together with the direct impact on borrowers implies that aggregate consumption falls significantly more in the heterogenous agent model than in the representative agent economy.

An additional way to quantify the importance of borrowing households for aggregates is to look at a counterfactual economy with zero borrowing limit. In this economy all households supply deposits and are barred from taking out consumer loans. Figure 1.6 compares our baseline economy to such an economy without
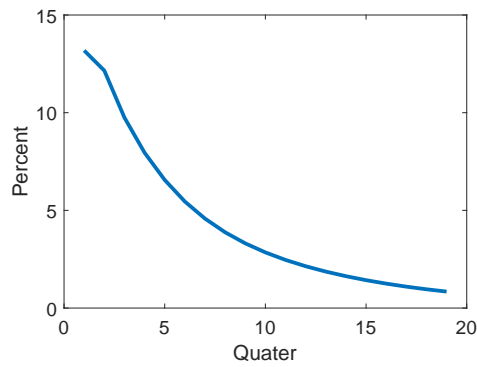
Figure 1.5: Impulse response of the average MPC to capital quality shock

borrowing. The counterfactual economy behaves similar to the RANK model and induces a much smaller output response to the shocks and a much milder reduction in consumption. Thus, our results derive from allowing households to demand loanable funds from banks.
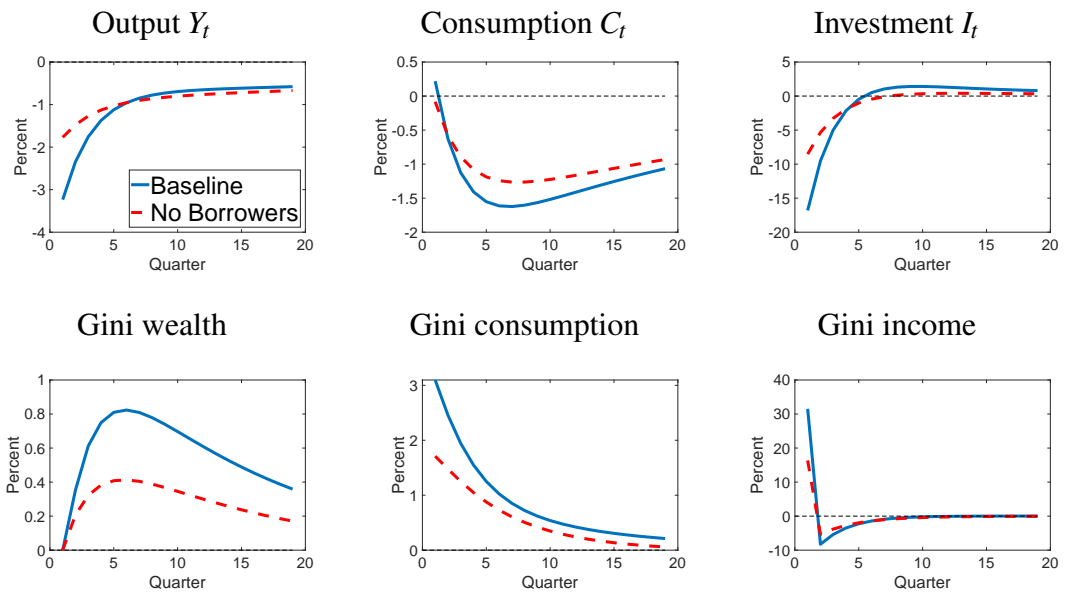
Figure 1.6: Aggregate and distributional effects of a capital quality shock w/o borrowers

Finally, it is worth noticing that the countercyclical lending rate also matters for inequality, see Figure 1.6. With borrowers, the Gini coefficient of consumption increases by three percent and stays elevated for three years. Without borrowers,

in contrast, the Gini coefficient of consumption only increases by less than two percent.

Technology shocks:
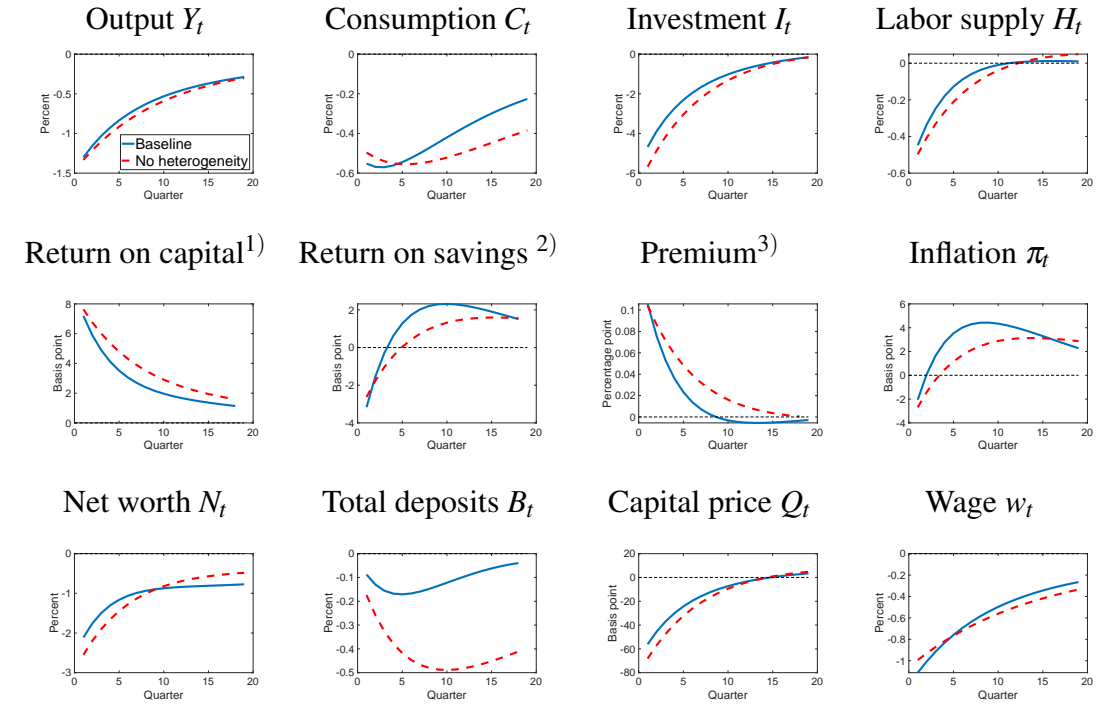
1) $R_{K,t+1}$, 2) $R_{S,t+1}$, 3) $E_t(R_{L,t+1} - R_{S,t+1})$ in Baseline and $E_t(R_{K,t+1} - R_{S,t+1})$ with No heterogeneity.

Figure 1.7: Aggregate effects of a TFP shock

The key insights from above carry over to the impact of technology shocks. Figure 1.7 shows the adjustment of the economy to a one percent decline in TFP. This shock is recessionary, lowers the productivity of labor and of capital, and produces a fall in the price of new capital. Aggregate output falls and along with it the economy sees declining investment and consumption. As in the standard financial frictions model, the impact of technology shocks are mildly amplified due to the rise in the interest rate premium that follows from the declining banking sector net worth.

At the level of the aggregate output, the introduction of incomplete markets has little impact on the financial accelerator. However, introducing household heterogeneity again introduces a more important role for consumption in the macroe-
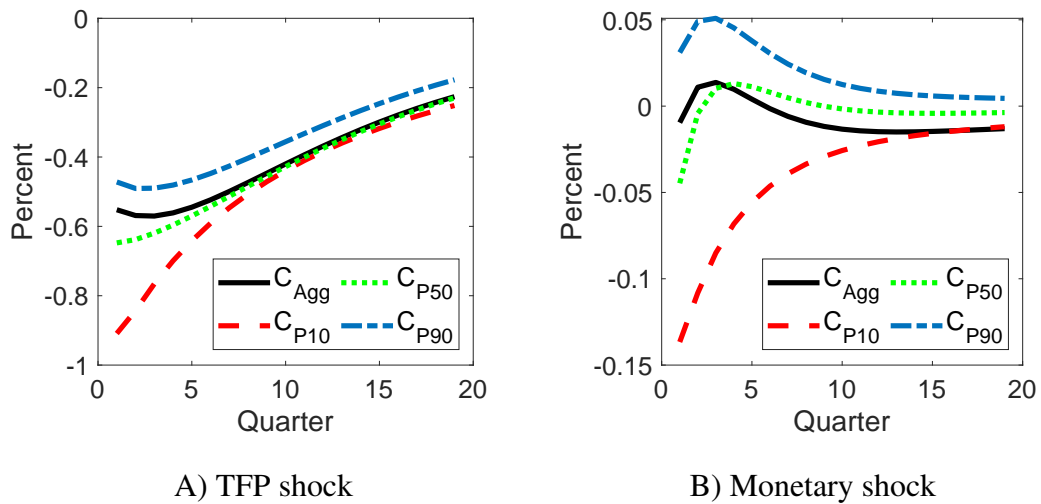
Figure 1.8: Consumption impulse responses by wealth percentiles

conomic adjustment process, and especially so in the shorter run. The roots of this derive again from the countercyclical spread between savings and lending rates, and the impact thereof on consumption adjustments along the wealth distribution. Declining net worth in the banking sector induces lower interest rates on deposits but higher interest rates on loans. The results of this, see Figure 1.8 Panel A, is a sharp drop in consumption for indebted households that is almost twice the size of the aggregate decline. This result derives from poorer households suffering from both poorer credit conditions and from lower income forcing them to reduce consumption strongly while richer households smooth out the income shock and are induced to substitute towards current consumption due to the drop in the real return on savings.

Monetary policy shocks: Concerns about differential impact of shocks along the wealth distribution that we have pointed to above are common as far as popular discussions of monetary policy are concerned. Yet arguments are often centered around how common changes in interest rates impact differentially on households according to their portfolio composition. We add to this that the spread in interest rates faced by borrowers and savers also increases when the central bank raises short term nominal interest rates.

Output Y_t Consumption C_t Investment I_t Labor supply H_t

Return on capital[1)]   Return on savings [2)]   Premium[3)]   Inflation π_t

Net worth N_t Total deposits B_t Capital price Q_t Wage w_t
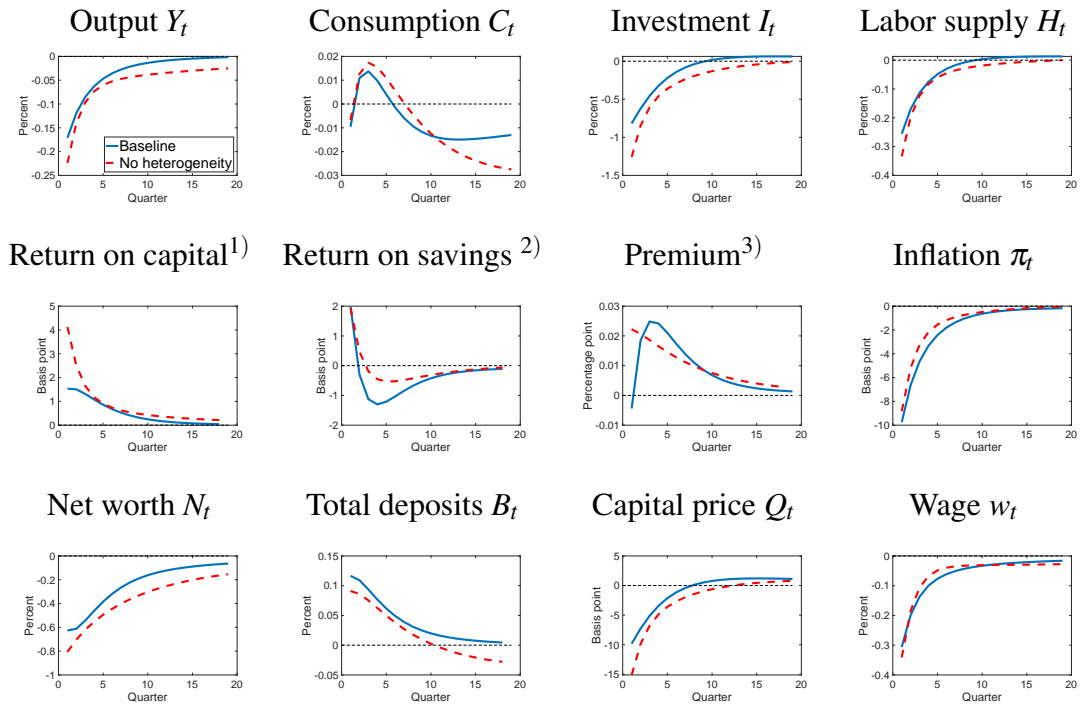
1) $R_{K,t+1}$, 2) $R_{S,t+1}$, 3) $E_t(R_{L,t+1} - R_{S,t+1})$ in Baseline and $E_t(R_{K,t+1} - R_{S,t+1})$ with No heterogeneity.

Figure 1.9: Aggregate effects of a monetary policy shock

Figure 1.9 illustrates the response of the economy to a contractionary monetary policy shock assuming that the shock follows an autoregressive process with persistence 0.5. The impact of monetary policy shocks on aggregate output in the heterogeneous agents economy are very close to those that arise in the representative agent model but we confirm again that much of the adjustment mechanism is accounted for aggregate consumption rather than investment.[11] The contractionary monetary policy shock increases the cost of funds for banks by forcing up the short term real interest rate on deposits. The price of capital falls, banks see their net worth decline and this forces the spread to increase.

Thus, relative to standard intuition, the model adds the insight that borrowers are harder hit by the increase in interest rates than in models without financial frictions. The product of this is that households are affected differently by the monetary

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[11]In a different setup with household portfolio choice, Luetticke (2018) finds consumption to be more responsive to monetary shocks as well because of the positive (negative) covariance between the distributional consequences and marginal propensities to invest (consume).

policy shock. Richer households earn higher real returns on their savings. The consumption of the median household is approximately unaffected by the contraction in the economy (see Figure 1.8, Panel B) and at the 90th percentile consumption actually rises. In contrast, indebted households face higher cost of credit leading to a strong reduction in their consumption. On average, the impact on richer households dominate the aggregate response yet consumption accounts for more of the macroeconomic adjustment than in the representative agent economy.

Credit policy: Given the results above, it is interesting to ask whether credit policy can help alleviate the amplification of shocks that occur through the countercyclical movements in the spread between savings and lending rates. Gertler and Karadi (2011) show that such a policy can mute the financial accelerator by stabilizing the impact of shocks on the spread which in their setting amplifies shocks through the investment response. As we have argued above, in the incomplete markets setting, much of the adjustment process occurs through consumption. Hence credit policy, while still stabilizing, may have different effects.
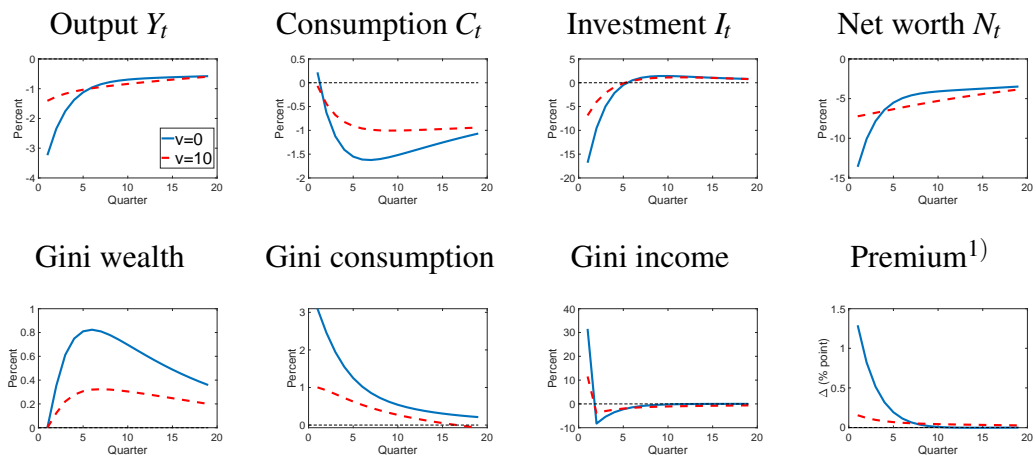
Notes: $v = 0$ corresponds to baseline and $v = 10$ corresponds to active credit policy with a semi-elasticity of credit supply to the spread set equal to 10.

1) $E_t(R_{L,t+1} - R_{S,t+1})$.

Figure 1.10: Aggregate and distributional effects of credit policy

Here we focus on the impact of credit policy after a capital quality shock. As above, we feed in a one percent decline in ξ_t. Our specification of credit policy

implies that the central bank provides more liquidity when the corporate lending rate increases relative to the savings rate. Hence, as the shock hits the economy and spreads rise due to declining banking sector net worth, the central back steps in with credit supply. We show the impact of this in Figure 1.10 in which we illustrate the impulse responses for both the baseline economy (without credit policy) and for a specification where the semi-elasticity of credit supply to the spread, v, is set equal to 10.

We find that credit policy has a large stabilizing role in this economy. The impact effect on aggregate output is more than halved when the central bank supplies credit in response to the widening interest rate spread. The spread itself declines very significantly as does the fire sales of bank assets (and therefore banking sector net worth). The policy stabilizes both the decline in investment produced by the capital quality shock and, importantly, removes the large amplification that derives from aggregate consumption.

The credit policy stabilizes the interest rate spread. Therefore, it restores lending to households after an adverse capital quality shock which enables indebted households to avoid having to cut their consumption dramatically. Moreover, this policy also mutes the increase in the share of hand-to-mouth households that arise in the absence of this policy. When there is insurance against idiosyncratic risk, as in the representative agent economy, credit policy only stabilizes due to investment being less adversely affected by the shock to banking sector net worth. We find that the stabilization is much larger in the HANK economy, which shows the importance of the spread between the interest rates on savings and borrowing facing households.[12]

Finally, it is noticeable that the smaller recession with credit policy translates into a smaller response of inequality. The Gini coefficient of consumption increases by 70% less. The effects on wealth inequality are similar.

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[12]See Appendix Figure A.2 for the impact of this policy in the counterfactual representative agent economy.

1.5 Macroprudential Regulation

The credit supply policy discussed above is an effective means of dampening the amplification of shocks through financial accelerator-type mechanisms. Such policies, however, imply resource costs on the part of the central bank and may also induce further incentive problems if banks ex-ante take into account how excessive risk taking on their part may be mitigated by central bank actions.

An alternative option is to regulate the banking sector in such a way that the amplification mechanism is neutralized. Here we will consider on such macroprudential regulation implemented through limiting banks' ability to leverage their net worth. By restricting leverage, shocks to the economy have less impact on banking sector net worth that stabilizes the impact of shocks.

The standard trade-off from introducing such regulation is that stabilization of the financial accelerator comes at the cost of lower steady-state output (since banks become more restricted in their investment activities). Here we will show that the trade-off is very different in the incomplete markets set-up. It involves a different trade-off between "micro volatility" and "macro stability" while long run steady-state output costs may be close to zero.

Long-run effects:

We first consider the long-run impact of regulating the banking sector. To be specific, we suppose that the regulator restricts ϕ, banking sector leverage, by 25 percent relative to its baseline value (3.47). This policy corresponds to what the market would impose on financial intermediaries that can divert 55.8 percent of the banks' capital (as compared to 38.1 percent in the baseline). Thus, the regulator imposes much stricter standards than the market forces.

Table 1.3 reports the long-run impact of this regulation on both aggregate variables as well as on distributional indicators. For a point of comparison, we also report the impact of the regulation in a counterfactual representative agent economy. The most direct effect of the macroprudential regulation is to increase the spread between the return on bank investments and the deposit rate because of constraints imposed on the intermediary in its attempts to profit from high returns on

	Heterogeneity		No Heterogeneity	
	Baseline	Low Leverage	Baseline	Low Leverage
Leverage	3.47	2.60	3.47	2.60
	Interest rates			
Return on capital (R_K, %)	1.14	1.17	1.60	1.74
Return on savings (R_S, %)	0.75	0.47	1.40	1.40
Lending interest rate (R_L, %)	2.69	2.73	-	-
	Aggregates			
Output	1.120	1.125	0.941	0.923
Capital	11.20	11.11	8.19	7.73
Labor	0.360	0.364	0.324	0.324
Consumption	0.738	0.733	0.652	0.640
	Household distribution			
At kink (%)	8.9	16.9	-	-
Borrowers (%)	24.3	30.9	-	-
Gini Wealth	0.921	0.961	-	-
Gini Consumption	0.259	0.273	-	-
Gini Income	0.320	0.325	-	-

Notes: We compare the baseline steady state to one with low leverage (a high divertibility parameter $\lambda = 0.558$). The last two columns do so for the model with a representative household.

Table 1.3: Steady state: Baseline and low leverage

investment opportunities. In equilibrium, the spread between the interest rate on loans and the deposit rate increases by 128 basis points (annualized) for consumer loans and 124 basis points for corporate loans.

The increase in the spread has distributional consequences because it exaggerates the kink in the budget constraint for zero wealth households. Figure 1.11 shows

A) Distribution of wealth (b / Y)
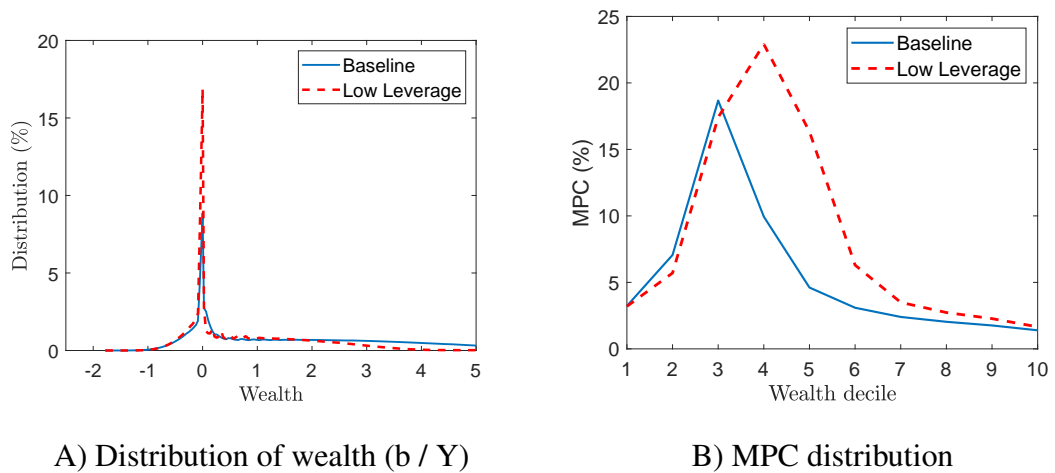
B) MPC distribution

Figure 1.11: Distributions: Baseline and low leverage

the long-run wealth distributions in the baseline economy and in the low leverage economy. It is noticeable that the spike at zero wealth is much higher when leverage is restricted. Indeed, we find that the share of households with zero wealth increases from just below 9 percent in the baseline to almost 17 percent in the low leverage economy. The share of borrowing households also increases (from 24 percent to 31 percent) but this is mainly due to transitions from zero wealth to marginally negative wealth produced by the mass point of households with zero assets.

Interestingly, the regulation of banks' leverage has no output costs. On the contrary, we find a small increase (0.4 percent) in aggregate output that derives from a combination of an increase in labor supply of 1.1 percent and a minor fall in the aggregate capital stock of 0.8 percent. This contrasts with the representative agent model in which there is a significant drop in output (of 2 percent) produced by a drop in the capital stock of almost 6 percent (induced by a decline in banks' financing of investment projects). In this economy, the return on savings in the steady-state is determined by the rate of time preference, $\beta^{-1} - 1$. Thus, a higher spread is reflected in the return on capital only and for that reason the macro prudential regulation induces a lower capital stock.

These effects are very different under incomplete markets. Here, the higher risk of being stuck at zero wealth gives households with positive wealth a precautionary

1) $R_{K,t+1}$, 2) $E_t(R_{L,t+1} - R_{S,t+1})$.
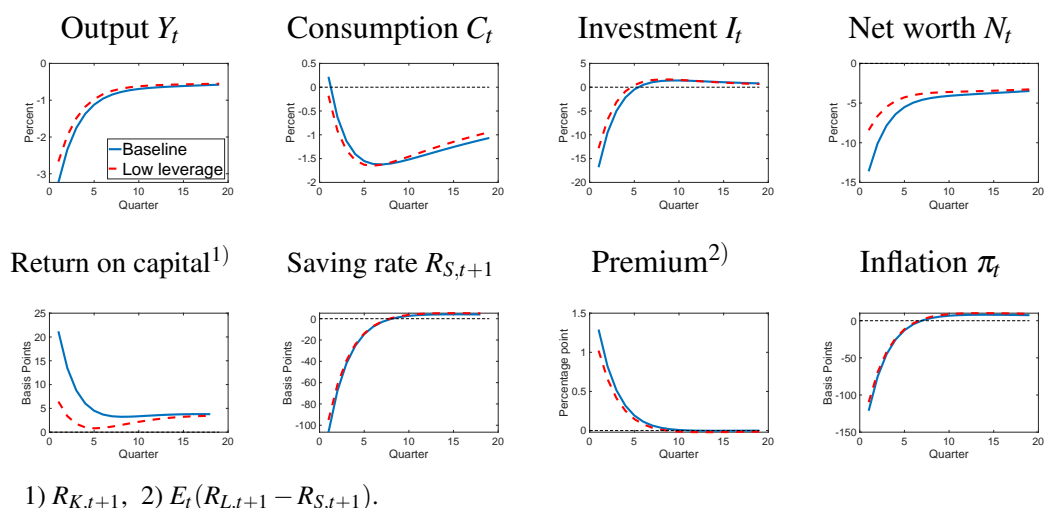
Figure 1.12: Impulse responses to capital quality shock with low leverage

savings motive, which puts downward pressure on the return on savings. Moreover, lower wealth households increase their labor supply, which increases the return on capital. In equilibrium these forces imply that although the spread increases, the impact on the aggregate capital is marginal and aggregate output rises. Given the savings desire of wealthier households, the increase in the spread is, in contrast to the representative agent economy, accomplished by the combination of a strong decline in the savings rate and marginally higher returns on equity investment and on consumer loans.

Thus, the common wisdom about the long-run output costs of macroprudential regulation is challenged in this model because of labor supply responses amongst poorer households and savings choices made by wealthier households.

Volatility: The aim of the macroprudential regulation is to lower the sensitivity of the economy to shocks. In Figure 1.12 we illustrate the impulse response functions of the economy to a one percent capital quality shock comparing the regulated economy with the baseline calibration. Restricting banks' leverage stabilizes aggregate output especially in the short run because net worth falls much less in response to the shock. This also implies a much smaller impact of the capital quality shock on aggregate investment. Yet, consumption falls more in the first 6 quarters.

Table 1.4 quantifies these effects by reporting selected 2nd moments of the

economy computed from simulations of the model in response to all three aggregate shocks (and idiosyncratic shocks). The regulatory intervention lowers aggregate output volatility as measured by the standard deviation by almost 10 percent and the relative volatility of investment by 14 percent. The relative volatility of consumption, by contrast, increases by 7 percent. These numbers are similar to those that arise in the representative agent economy thus indicating that macroprudential regulation appears to be as effective at stabilizing the economy against the financial accelerator as in the earlier literature.

However, this macro stabilization comes at a large cost in terms of micro volatility. The increase in the spread induced by the more restrictive regulatory framework induces a large increase in the sensitivity of household consumption to income shocks. Recall that households at the borrowing limit and at zero wealth have large marginal propensities to consume. Because of income shocks and wealth mobility, households will move in and out of these high MPC states. Figure 1.11 Panel B shows the average MPC for each wealth decile in the baseline economy and in the economy with lower leverage.[13] The macroprudential regulation induces a large increase in the MPCs for a significant fraction of households. For the median wealth households, the MPC rises from approximately 5 percent to close to 20 percent.

It follows from Figure 1.11 Panel B that a by-product of macroprudential regulation is to increase volatility at the micro level. Figure 1.13 shows the volatility of household consumption computed as the standard deviation of consumption over a 5 years horizon.[14] Panel A reports this measure in the absence of aggregate shocks while Panel B allows for aggregate shocks as well. Regardless of whether one allows for aggregate shocks or not, household consumption volatility increases sharply across the wealth distribution when banking sector leverage is lower. The ir-

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[13]The MPCs are computed by examining the response of consumption to a one percent (of income) transfer across the wealth distribution.

[14]The figure shows the average standard deviation of quarterly growth rates of household consumption for a simulation of length five years computed over 100.000 individuals and then averaged over wealth deciles.

	Baseline (A)	Low Leverage (B)	(B/A - 1)
Heterogeneity			
STD(Y) (%)	4.00	3.62	-9.5%
STD(C)/STD(Y)	0.98	1.05	7.1%
STD(I)/STD(Y)	3.18	2.74	-13.9%
No Heterogeneity			
STD(Y) (%)	3.79	3.57	-5.8%
STD(C)/STD(Y)	1.08	1.11	2.8%
STD(I)/STD(Y)	3.08	2.83	-8.1%

1) We report standard deviations of aggregate variables as $100 * \log(X/X^{SS})$ in response to TFP, monetary, and capital quality shocks.

2) We target an output volatility of 4%, which corresponds to the volatility of US real output per capita (1954-2015) (after taking logs and controlling for a linear trend). We adjust the standard deviation of the CQ shock to 0.5% to hit this target.

Table 1.4: Volatility of aggregate variables

relevance of aggregate shocks for this picture derives from the much higher variance of idiosyncratic income shocks than aggregate shocks. Quantitatively, the increase in consumption volatility is very large with the mean household experiencing a 10 percent increase in household consumption volatility, with even larger increases for wealthier households.

Welfare: Given these results, we then ask whether macroprudential regulation is beneficial for welfare or not. We compare the welfare across quintiles of the wealth distribution for the baseline calibration and for the economy in which banking sector leverage is lowered by 25 percent. To capture the effects of aggregate volatility on welfare we solve the model by second order perturbation. We do not take into account transitional costs but it so turns out that this is not so relevant for our analysis because of the moderate impact on the aggregate capital stock that we discussed above.
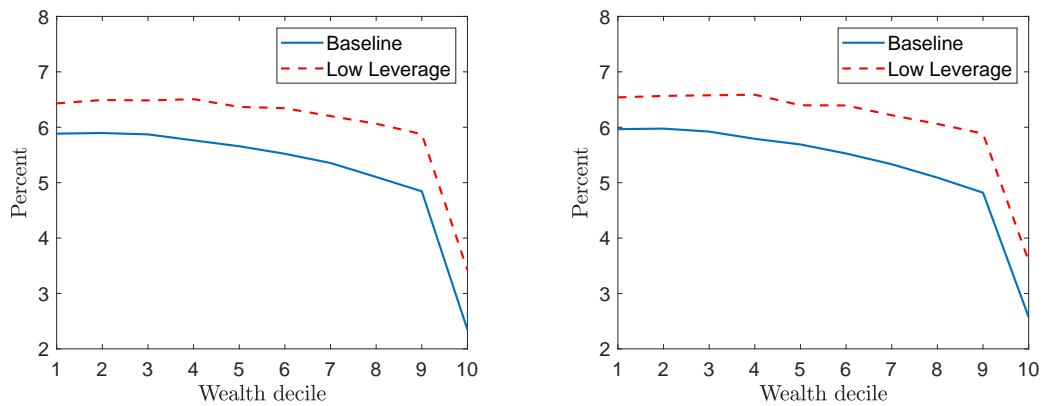
A) Only idiosyncratic shocks   B) Aggregate and idiosyncratic shocks

Notes: Volatility refers to the average standard deviation of quarterly growth rates of household consumption for a simulation of length five years computed over 100.000 individuals.

Figure 1.13: Micro consumption volatility by wealth deciles

We report the results in Table 1.5 that shows the welfare gains/losses for households across the wealth distribution from moving to a world with less leverage. We report the results both with and without aggregate shocks. In the absence of aggregate shocks, the average welfare loss is 1.1% of life-time consumption. All households prefer the steady state with higher leverage because it implies a lower lending rate and a higher saving rate. The welfare losses are largest for households in the top 20% of the wealth distribution because of the lower return on their savings. With aggregate shocks, the average welfare loss is 1.4% of life-time consumption. The difference between the regime with low and high leverage hence becomes even larger in the presence of aggregate shocks. While low leverage reduces the volatility of aggregate output, the relative volatility of aggregate consumption increases and the absolute volatility of consumption for some households increases as well.

For poor households, macroprudential regulation requires a trade-off: Higher costs of borrowing vs. less aggregate volatility. It turns out that in our calibration, poor households prefer less regulation. The benefit of lower output volatility does not translate into lower consumption volatility, because the fraction of households with high marginal propensities to consume increases markedly, see Figure 1.11 Panel B. Hence, consumption responds more to aggregate and idiosyncratic shocks

	Consumption Equivalent Variation	
	only idiosyncratic shocks	aggregate and idiosyncratic shocks
1. Wealth quintile	0.53%	0.82%
2. Wealth quintile	0.56%	0.71%
3. Wealth quintile	0.62%	0.74%
4. Wealth quintile	0.78%	0.96%
5. Wealth quintile	3.15%	3.88%
Aggregate	1.10%	1.39%

We report the fraction of life-time consumption that households are willing to give up to stay in the baseline economy relative to a counterfactual economy with 25% less leverage.

Table 1.5: Welfare costs of macruprudential regulation

for the majority of households.

The reason why we find welfare costs of macroprudential regulation is that it increases the interest rate spread which exaggerates the kink in the budget constraint and hinders households' ability to smooth out adverse shocks. Thus, the policy involves a trade-off between stabilizing the aggregate economy but destablizing at the household level. Credit policy based stabilization does not involve such a trade-off and therefore appears more palatable.

1.6 Conclusion

In this paper we have considered the impact of frictional financial intermediation in a HANK setting. Heterogeneity between households implies that banks intermediate not only between the household sector and the corporate sector, as in most analyses of financial intermediation, but also between different types of households some of whom are savers others borrowers. We adopt commonly used arguments for incentive problems in the banking sector that induce an inverse relationship be-

tween banking sector net worth and the spread between the interest earned on the banks' assets (corporate investments and household loans) and liabilities (household deposits).

The spread between the return on household savings and the interest on household debt affect the long-run wealth distribution. This happens primarily through the spread generating a mass point in the wealth distribution at zero wealth. Thus, financial sector efficiency has long-run implications beyond those emphasized in the representative agent literature, which mostly relate to aggregate investment.

We have derived three major results. First, with household heterogeneity, the financial accelerator works through consumption and tends to be larger than in representative agent settings. This result derives from countercyclical movements in the savings-lending interest rate spread that induces differential consumption responses of households with positive and negative net assets. Contractionary shocks harm indebted households not only because of lower income but also because the interest rate on their debt increases. Furthermore, higher spreads exaggerate the mass point in the wealth distribution of households that have large marginal propensities to consume. In combination, these forces introduce a key role for consumption in the adjustment of the economy to shocks.

Secondly, credit policy – central bank purchases of assets when the interest rate spread rises – is shown to be a very effective tool for stabilizing the financial accelerator. Such a policy stabilizes the countercyclical movements in the spread and removes the amplification of shocks that derive from the consumption adjustments. Indeed, we find that such a policy is much more effective in the heterogeneous agents economy than in the representative agent economies usually focused upon in the financial intermediation literature.

Third, we show that macroprudential regulation has very different effects than usually emphasized in the literature. We consider regulation of banking sector leverage with the aim of muting the financial accelerator. The standard trade-off considered from such regulation is that it comes at the cost of lower average activity. This does not necessarily happen in the incomplete markets setting because savings re-

spond to the policy. Indeed, we find little impact on aggregate output. Instead, the cost of this regulation is that it hampers households' ability to smooth out idiosyncratic risk (because of the rising interest rate spread). We find that this induces significant welfare costs.

Our work suggests several promising avenues for future research. First, we introduce a wedge between the return on household debt and corporate investments by assuming a simple resource cost of issuing household loans. It would be interesting to consider the implications of household default risk as a source of this spread. It would also be interesting to examine long-term debt such as mortgage contracts. The short term pass-through to mortgage rates from policy rates may be smaller especially because mortgages often are issued with fixed rates. On the other hand, due to household leverage, the mechanisms that we have described may be even stronger in such a setting.

Chapter 2

Optimal monetary policy with TANK and SAM

2.1 Introduction

In the aftermath of the Great Recession, the interest in the effect of households' inequality or heterogeneity on the implementation of monetary policy has burgeoned among researchers and policymakers. For example, Kaplan et al. (2018) and Auclert (2019) study the role of households heterogeneity in the transmission mechanism of monetary policy to consumption. Nuño and Thomas (2019) and Bhandari et al. (2020) investigate the optimal monetary or fiscal policy with heterogeneous assets and consumption. However, very little normative research has been done on the relation between mandates of monetary policy and heterogeneity in households.

As is well known, the two objectives as mandated by the Congress in the Federal Reserve Act are promoting (1) maximum employment, which means all Americans that want to work are gainfully employed and (2) stable prices for the goods and services we all purchase. According to Reis (2013), the current state of knowledge leans towards there being a Phillips curve representing a short run trade-off relation between price stability and employment stability such that giving up some price stability can increase the real stability of an economy.[1] In this context, I ex-

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[1] On the other hand, Blanchard and Galí (2007) argue that stabilizing inflation is equivalent to stabilizing output gap and they call this property the *divine coincidence*.

amine the optimal dynamics of inflation and employment when there is a simple form of households inequality.

Specifically, this paper analyzes the optimal choices of a social planner with respect to aggregate shocks in the environment of heterogeneous households, sticky prices and matching friction. I allow for a heterogeneity in households following Campbell and Mankiw (1989) and Galí et al. (2007). There are two types of households, optimizers and rule-of-thumb consumers[2]. I assume that the rule-of-thumb consumers do not save or borrow and they consume their income in a hand-to-mouth way. On the other hand, optimizers have an access to the government bond market and a profit income from the ownership of firms. The model is also characterized by price adjustment cost of Rotemberg (1982a) and matching frictions in the labor market, as described by Mortensen and Pissarides (1999). Due to the characteristics of nominal rigidity with heterogeneous households and the friction in labor market, I call the baseline model in this paper as TANK (Two Agent New Keynesian) and SAM (Searching and Matching friction) following the terminology of Ravn and Sterk (2020).

In this paper, the design of optimal monetary policy follows the Ramsey approach in which the optimal path of all variables is obtained by maximizing agents' welfare subject to the relations describing the competitive economy. I enlarge the planner's state space with additional co-state variables in order to rewrite the planner's problem in a recursive stationary form following Marcet and Marimon (2019). Then, I assume that, at t=0, the benevolent planner has been operating for an infinite number of periods. In choosing optimal policy, the planner is assumed to honor commitments made in the past. Woodford (2003) refers to this form of policy commitment as 'optimal from the timeless perspective'. The steady states and the dynamics implied by the Ramsey equilibrium are solved by the approach of Schmitt-Grohé and Uribé (2012).

The main contribution of this paper is to show that it is optimal to pay more

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[2]Hand-to-mouth consumers. I use rule-of-thumb consumers (optimizers) and spenders (savers) interchangeably.

attention to employment stabilization than to price adjustment cost when both the rule-of-thumb households and the matching friction exist even though the price adjustment cost is substantial. For instance, after a negative TFP shock, it is optimal to decrease employment and output *less* at the higher costs of price adjustment and job posting when there are *more* rule-of-thumb consumers. This implies that the optimal policy needs to strike a balance between price adjustment and employment stability rather than to pursue complete price stabilization in order to make rule-of-thumb consumers better off. We find similar results with respect to a government spending or a job separation shock. This result runs in parallel with the conclusion of Debortoli et al. (2019), which shows that a welfare loss functions should sometimes be given a greater weight on measures of economic activity than that on inflation.

We implement two comparisons so as to investigate whether only the combination of TANK and SAM results in the deviation from the complete price stabilization. To start with, we contrast the baseline model with a TANK accompanied by hours worked rather than the matching friction.[3] We find that rule-of-thumb consumers in TANK with hours worked do not lead to deviations from price stabilization with respect to a TFP or a government spending shock.[4] Since rule-of-thumb consumers can better smooth their consumption through the adjustment of hours worked, the social planner has less of an incentive to increase their consumption on impact through the stabilization of labor supply. As a result, the social planner chooses to avoid price adjustment cost with respect to the aggregate shocks. Next, the baseline model is compared with a TANK accompanied by a constant transfer.[5] We, again, find an almost complete price stabilization after incorporating the transfer policy. As the heterogeneity in the steady state consumption level is removed with the transfer, the planner has almost no incentive to deviate from price stabilization since the consideration for rule-of-thumb consumers diminishes.

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[3]SAM versus Hours Worked (extensive margin versus intensive margin).

[4]We cannot consider a job separation shock for this comparison since there is no extensive margin of labor supply in TANK with hours worked.

[5]Without versus with a transfer policy (with versus without heterogeneous consumption level).

Lastly, we study the effect of a time-varying transfer rule on the volatility of inflation and employment with respect to a more volatile job separation shock. A transfer rule from optimizers to rule-of-thumb consumers is considered with respect to an AR(2) job separation process. We find that the Ramsey planner pays less attention to employment stabilization when a countercyclical transfer rule is in operation and more attention if a procyclical transfer rule is in operation. This result may imply that an appropriate fiscal rule can replace the role of monetary policy in stabilizing employment. In addition, we identify that more volatile job separation shock calls for more active monetary policy regardless of the type of a transfer rule.

The remainder of the paper is structured as follows. After reviewing related literature, I start in Section 2.2 by describing a model of TANK and SAM. In Section 2.3, I present calibration and the related data. In addition, I describe how to compute steady state and present the corresponding values. Section 2.4 turns to the dynamics of the Ramsey planner. Firstly, I show the second moment of choice variables with respect to series of productivity, government spending and job separation shocks. Next, I identify the optimal monetary policy that maximizes social welfare after the aggregate shocks when rule-of-thumb consumers are incorporated into the model. I show that optimal monetary policy is characterized with the deviation from complete inflation stabilization when rule-of-thumb consumers exist regardless of the substantial price stickiness.

2.1.1 Literature review

Siu (2004) and Schmitt-Grohé and Uribé (2004) show that the optimal volatility of inflation is near zero even for a small amount of price stickiness. In other words, monetary policy should not be used to stabilize debt. Faia (2009) analyzes the design of optimal monetary policy for a framework with sticky prices and matching frictions in the labor market. The paper shows that monetary authority has to strike a balance between stabilizing inflation and reducing unemployment rate because search externalities generate a trade-off between unemployment and inflation when the Hosios condition is not satisfied. Thomas (2008) shows that optimal monetary policy implies a case against price stability when nominal wage bargaining is

staggered.

In contrast to the representative households model, Mankiw (2000) argues the necessity of rule-of-thumb consumers in building a model of analyzing fiscal policy. Galí et al. (2007) incorporate this heterogeneity of households in their model so as to show that consumption rises in response to an increase in government spending. They assume one labor union in the labor market. Furthermore, they assume that the steady state consumption of two households are same in order to draw their conclusion. Bosca et al. (2011) analyzes the effects of introducing rule-of-thumb consumers and consumption habits into the labor market search model. They also use one labor union assumption and argue that the wage can be determined by multi-person Nash bargaining. Bilbiie (2008) shows, using the two-equation general equilibrium model with rule-of-thumb consumers, that the share of spenders has a nonlinear effect on the monetary policy effectiveness.

Menna (2016) studies optimal monetary and fiscal policy with rule-of-thumb consumers and intensive margin of labor supply when many fiscal instruments are available. He argues that monetary policy stabilizes inflation while fiscal policy play a role in attenuating the effect of productivity shocks on income distribution. Bhandari et al. (2020) research fluctuations in macroeconomic aggregates and cross-sectional income and wealth distribution in a heterogeneous agent model with incomplete market and sticky nominal prices. They show that the Ramsey planner uses inflation to offset inequality-increasing shocks to the cross-sectional distribution of labor earnings. Nuño and Thomas (2019) study optimal monetary policy in an incomplete-markets model with non-contingent nominal assets and costly inflation. They show that the optimal policy under commitment features a positive initial inflation, followed by a gradual decline towards zero inflation. Gerke et al. (2020) quantify the effect of forward guidance with a transfer rule and rule-of-thumb households. They argue that when the households exhibit a sufficient countercyclical redistribution, this results in a dampening of the power of forward guidance.

With regard to shocks in the job market, Ravn and Sterk (2017) show that

higher risk of job loss and worsening job finding prospects during unemployment depress goods demand because of a precautionary saving motive. Larkin (2019) investigates the interaction between household asset and labor market choices in the face of a lower job separation rate. He argues that the Great Recession following a tranquil labor market environment amplified the negative response of consumption. Zhang (2017) studies the effect of shocks to unemployment benefits and matching efficiency in order to explain the high and persistent unemployment rate in the United States during and after the Great Recession.

There are several studies on the estimation of the share of the rule-of-thumb consumer. Jappelli (1990) identifies the credit-constrained households as those that have had their request for credit rejected by financial institutions or those who may not apply for loans because they perceive that they will be refused. He finds the the share of credit-constrained households was 19% by using 1983 SCF data. For 1989-1998 survey data, Lyon (2003) finds that the share of credit-constrained households is around 20%. On the other hand, Grant (2007) finds that around 30% of households are constrained based on Consumer Expenditure Survey (CEX) data.

2.2   Model

I construct a model which combines the price adjustment cost of Rotemberg (1982a), matching frictions in the labor market following Mortensen and Pissarides (1999) and household heterogeneity as in Mankiw (2000). The economy is made up of households that consume and work, firms that produce output, and a monetary authority and a government in charge of the nominal interest rate and tax, respectively.

2.2.1   Households

Households are of two types: There is a continuum of mass $1 - \xi (\in [0,1])$ of "optimizers", who save and own firms. The remaining fraction ξ of "rule-of-thumb" consumers who do not own any assets nor have any liabilities. All households are infinitely-lived, discount the future at the same factor $\beta \in [0,1]$. A household

$i(\in [o,r]^6)$ maximizes intertemporal preferences and derives utility from consumption of private goods, $c_{i,s}$.

A preference for the household is given as:

$$U_{it} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_{i,s}^{1-\sigma} - 1}{1-\sigma} \right] \qquad (2.1)$$

where $c_{i,s}$ denotes aggregate consumption of type i households in final goods, σ is the inverse of the intertemporal elasticity of substitution. The value function for savers and spenders are denoted as $W_{o,s}$ and $W_{r,s}$, respectively. The consumption level of an individual household is a constant-elasticity-of-substitution aggregator of a basket of consumption goods, $c_{i,s}^j$:

$$c_{i,s} = \left(\int_j (c_{i,s}^j)^{(1-1/\varepsilon)} dj \right)^{1/(1-1/\varepsilon)} \qquad (2.2)$$

where $\varepsilon > 1$ is the elasticity of substitution between goods varieties. Workers who are not employed get unemployment benefit $\mu < w_s$.

2.2.1.1 Optimizers

Optimizers earn and share all the income of its members maximizing the sum of their expected utilities. They have to meet the following flow-budget constraint:

$$c_{o,s} + \frac{B_{o,s}}{p_s} = w_{o,s} n_{o,s} + \mu(1-n_{o,s}) + \Pi_{o,s} - \frac{T_{o,s}}{p_s} + R_s \frac{B_{o,s-1}}{p_s} \qquad (2.3)$$

$w_{o,s}$ is real labor income. Members of unemployed households, denoted by, $1 - n_{o,s}$, receive an unemployment benefit, μ. Wage is specified by the contract signed between the worker and the firm. In addition, it is the result of a Nash bargaining process. Savers also invest in non-state contingent nominal bonds B_s, which pay a gross nominal interest rate R_s next period. They receive profit $\Pi_{o,s}$ from the firm sector which they own and pay lump sum taxes $T_{o,s}$. The price level is denoted by p_s. Savers choose the set of processes $\{c_{o,s}, b_{o,s}\}_{s=0}^{\infty}$ by taking as given the set of processes $\{p_s, w_{o,s}, R_s\}$ and the initial wealth $b_{o,-1}$ in order to maximize their lifetime utility subject to their budget constraint. Let $\lambda_{o,s}$ be the Lagrangian multiplier on the savers' budget constraint, then, the first order conditions can be written as:

$$1 = E_s \left[\Lambda_{s,s+1} \frac{R_s}{\pi_{s+1}} \right] \qquad (2.4)$$

---

[6]o: optimizers or savers, r: rule-of-thumb consumers or spenders.

where $\Lambda_{s,s+t} = \beta^t \frac{\lambda_{o,s+t}}{\lambda_{o,s}}$, and

$$\lambda_{o,s} = c_{o,s}^{-\sigma_c} \tag{2.5}$$

A No-Ponzi condition on wealth is also required.

2.2.1.2 Rule-of-thumb consumers

The remaining measure of $(1 - \xi)$ households, rule-of-thumb consumers does not save and consumes all their disposable income. Hence, they face the following budget constraint:

$$c_{r,s} = w_{r,s}n_{r,s} + \mu(1 - n_{r,s}) - t_{r,s} \tag{2.6}$$

where μ is real unemployment benefits received by unemployed spenders and $t_{r,s} = \frac{T_{r,s}}{P_s}$. A fraction $n_{r,s}$ of the spenders is employed and a fraction $u_{r,s} = 1 - n_{r,s}$ is unemployed, thus, the number of employed spenders is ξn_s. I assume that spenders do not receive profits of firms.

2.2.2 Firms

Firms produce output in a monopolistic competitive market. They have to pay a fixed cost (κ) to open up a vacancy. Firms choose the number of employees (n_s) and vacancies (v_s) as well as prices, p_s^j, to maximize the discounted sum of future profits by taking the wage as given as seen in Ravn and Sterk (2020).

$$Max \, \Pi_{j,t} = E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ \frac{p_s^j}{p_s} y_s^j - w_s n_s - \kappa v_s - \frac{\psi}{2} \left(\frac{p_s^j}{p_{s-1}^j} - 1 \right)^2 y_s \right\} \tag{2.7}$$

subject to

$$y_s^j = \left(\frac{p_s^j}{p_s} \right)^{-\varepsilon} y_s = a_s n_s \tag{2.8}$$

$$n_s = (1 - \rho)n_{s-1} + q_s v_s \tag{2.9}$$

where w_s is real wage, $\frac{\psi}{2} \left(\frac{p_s^j}{p_{s-1}^j} - 1 \right)^2 y_s$ denotes the cost of adjusting prices, a_s is a stochastic term representing random technological progress ($lna_s = \rho_a lna_{s-1} + \varepsilon_{a,t}$). The variable q_s represents the vacancy filling probability. The value of firms is denoted as V_s.

The marginal cost of firms is mc_s, which is the Lagrange multiplier associated with the constraint (2.8). We derive first order conditions with respect to p_s^j, n_s and v_s. We focus on symmetric equilibria in which all firms set the same price. Then, we get the following firms' optimality conditions:

$$\frac{\kappa}{q_s} = mc_s a_s - w_s + \beta E_s \left\{ \frac{\lambda_{o,s+1}}{\lambda_{o,s}}(1-\rho)\frac{\kappa}{q_{s+1}} \right\} \qquad (2.10)$$

$$[(1-\varepsilon) + mc_s \varepsilon] = \psi(\pi_s - 1)\pi_s - \beta E_s \left\{ \frac{\lambda_{o,s+1}}{\lambda_{o,s}}\psi(\pi_{s+1} - 1)\pi_{s+1}\frac{y_{s+1}}{y_s} \right\} \qquad (2.11)$$

2.2.3 Labor market

The labor market structure follows the standard search and matching framework. Matching firms and workers is a time-consuming process and is, thus, costly. In addition, firms need to find exactly one worker to produce goods.

2.2.3.1 Timing

The measure of unemployed workers after matching at the previous period is denoted by u_{s-1}. A share of the employed workers, ρn_{s-1}, is separated exogenously. Thus, the job searcher, e_s, can be denoted as

$$e_s = u_{s-1} + \rho n_{s-1}$$

Firms post vacancies v_s to match with the job searcher e_s by a matching function $m(e_s, v_s)$, which shows constant return to scale. The market tightness is represented by $\theta_s = v_s/e_s$. Each worker and firm take the labor market tightness as given. The number of employed workers in the current period is given by

$$n_s = (1-\rho)n_{s-1} + m(e_s, v_s)$$

where $m(e_s, v_s) = m e_s^{\varphi} v_s^{1-\varphi}$. The newly employed workers are assumed to start to work immediately. Thus, employment results from firms' and workers' search behavior. The employed workers enter into production: $y_s = a_s n_s$.

2.2.3.2 Bargaining and wage schedule

I assume that the worker delegates a labor union to negotiate a contract in wages once a vacancy-offering firm and a job-seeking worker match, following Galí et al. (2007) and Bosca et al. (2011).[7] This union merges the surplus from employment of both households and uses this merged surplus in the bargaining of wages.[8] The implication of this one labor union assumption is that all workers experience the same employment rates ($n_{o,s} = n_{r,s} = n_s$) and receive the same wages ($w_{o,s} = w_{r,s} = w_s$). Thus, the Nash bargaining process maximizes the weighted product of the parties' gains from employment

$$\max_{w_s}(\lambda_s^h)^\zeta (F_s^j)^{1-\zeta} = \max_{w_s}\left[(1-\xi)\frac{\lambda_{o,s}^h}{\lambda_{o,s}} + \xi\frac{\lambda_{r,s}^h}{\lambda_{r,s}}\right]^\zeta (F_s^j)^{1-\zeta} \qquad (2.12)$$

where $\zeta \in [0,1]$ is workers' bargaining power. $\lambda_{o,s}^h \ (= \frac{\partial W_{o,s}}{\partial n_{o,s}})$, $\lambda_{r,s}^h$ [9] $(= \frac{\partial W_{r,s}}{\partial n_{r,s}})$ and F_s^j $(= \frac{\partial V_s}{\partial n_s})$ are the marginal value of employment for savers, spenders and firms respectively. The terms in the square bracket represent the surplus of workers, while the latter is the surplus of firm. Particularly, $\frac{\lambda_{o,s}^h}{\lambda_{o,s}}$ and $\frac{\lambda_{r,s}^h}{\lambda_{r,s}}$ represent the earning premium of employment over unemployment for savers and spenders, respectively.

The optimal real wage under lump-sum taxation is derived by the solution[10] of the Nash bargaining problem.

$$w_s = \zeta[mc_s a_s + \beta(1-\rho)E_s\left(\frac{\lambda_{o,s+1}}{\lambda_{o,s}}\kappa\theta_{s+1}\right)]$$

$$+ (1-\zeta)\mu + (1-\zeta)[(1-\rho)\xi\beta E_s(1-\eta_{s+1})\frac{\lambda_{r,s+1}^h}{\lambda_{r,s+1}}\left(\frac{\lambda_{o,s+1}}{\lambda_{o,s}} - \frac{\lambda_{r,s+1}}{\lambda_{r,s}}\right)]$$

$$(2.13)$$

where η_s is the job finding rate. The wage is interpreted as a weighted average of the feasible wage and the reservation wage. The third term of the right-hand-side in (2.13) is a part of the reservation wage and can be interpreted as an inequality in

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[7]Even though savers and spenders have different outside options, the steady state wage in TANK is same with that in the representative households (RANK) model.

[8]I implement a robustness test using an ad hoc wage rule that is similar to the rule in Den Haan et al. (2018). Refer to Appendix B.4 for details.

[9]$\lambda_{r,s}^h = C_{r,s}^{-\sigma}(w_s - \mu) + \beta(1-\rho)E_s[(1 - m\theta_{s+1}^{1-\varphi})\lambda_{r,s+1}^h]$

[10]Refer to the Appendix B.2 for the derivation.

utility. Note that this term vanishes in the steady state; thus, both steady state wages in TANK and in RANK are same. Spenders can not smooth consumption over time, but they can make use of the fact that a matching today is, to some extent, expected to go on in the future. This yields an expected labor income that can be consumed tomorrow. Therefore, they use the margin that wage bargaining provides them with to enhance their lifetime utility by narrowing the difference in utility over savers.

2.2.4 The optimal policy plan

This section is devoted to specify a set-up for the optimal policy plan.

Definition 1. For given stochastic process $\{a_s, g_s\}_{s=0}^{\infty}$ and for given p_{-1}, b_{-1}, plans for the control variables $\{\Xi_s\}_{s=o}^{\infty} = \{c_{o,s}, c_{r,s}, n_s, v_s, \pi_s, b_s, mc_s, R_s, \tau_s\}_{s=0}^{\infty}$ and for the Lagrangian multipliers $\{\Phi_s\}_{s=0}^{\infty} = \{\phi_{1,s}, \phi_{2,s}, \phi_{3,s}, \phi_{4,s}, \phi_{5,s}, \phi_{6,s}, \phi_{7,s}\}$ describe a first best constrained allocation if they solve the following optimization problem:

$$
\begin{aligned}
Min_{\{\Phi_s\}_{s=0}^{\infty}} Max_{\{\Xi_s\}_{s=0}^{\infty}} \ E_0 \sum_{s=0}^{\infty} \beta^s E_s & \left\{ (1-\xi)\frac{c_{o,s}^{1-\sigma}-1}{1-\sigma} + \xi\frac{c_{r,s}^{1-\sigma}-1}{1-\sigma} \right. \\
& + \phi_{1,s}[w_s n_s + \mu(1-n_s) + \frac{\Pi_s}{1-\xi} + R_{s-1}\frac{b_{s-1}}{\pi_s(1-\xi)} - \tau_s - c_{o,s} - \frac{b_s}{1-\xi}] \\
& + \phi_{2,s}[-c_{o,s}^{-\sigma} + \beta E_s c_{o,s+1}^{-\sigma}\frac{R_s}{\pi_{s+1}}] \\
& + \phi_{3,s}[w_s n_s + \mu(1-n_s) - \tau_s - c_{r,s}] \\
& + \phi_{4,s}[-\frac{\kappa}{m}\theta_s^{\varphi} + mc_s a_s - w_s + (1-\rho)\beta E_s\{\frac{\lambda_{o,s+1}}{\lambda_{o,s}}\frac{\kappa}{m}\theta_{s+1}^{\varphi}\}\} \\
& + \phi_{5,s}[-\{(1-\varepsilon) + mc_s\varepsilon\} + \psi(\pi_s-1)\pi_s - \beta E_s\{\frac{\lambda_{o,s+1}}{\lambda_{o,s}}\psi(\pi_{s+1}-1)\pi_{s+1}\frac{y_{s+1}}{y_s}\}] \\
& + \phi_{6,s}[\tau_s + b_s - g_s - \mu(1-n_s) - R_{s-1}b_{s-1}/\pi_s] \\
& \left. + \phi_{7,s}[-n_s + (1-\rho)n_{s-1} + me_s^{\varphi}v_s^{1-\varphi}] \right\}
\end{aligned}
$$

(2.14)

where $y_s = a_s n_s$, $u_s = 1 - n_s$, $e_s = u_{s-1} + \rho n_{s-1}$, $\theta_s = v_s/e_s$ and w_s is given by (2.13).

I assume the social planner has the utilitarian utility function. Thus, he maximizes the weighted average of households' utility. With regard to the constraints, the first two are the budget constraint and the Euler equation of savers, respectively. The next is the spenders' budget constraint. Firms' two optimality conditions with

respect to employment and price are followed. The last two constratints are the budget constraint of government and the transition equation for employment, respectively.

This problem as specified in (2.14) is non-reculsive because of future expectations of control variables. A way to transform the same problem in a recursive stationary form is to extend the planner's state space with the Largrangian multipliers associated with the forward looking control variables, as developed by Marcet and Marimon (2019). Such Lagrangian multipliers have an important meaning in that the planner commits to the pre-announced policy plan if he obeys the values.

The technical characteristics of the equilibrium concept with a timeless perspective adopted here over the standard Ramsey problem is that the optimality conditions with regard to the equilibrium here are time-invariant. On the contrary, the equilibrium conditions in the first period are different from those applying to later periods under the usual Ramsey equilibrium definition.

2.3 Steady state

In this section, I calibrate parameter values from data and literature. Next, I describe how to compute the steady state following Schmitt-Grohé and Uribé (2012). Finally, I compare steady states in representative agents (RANK) models with those in the baseline model (TANK).

2.3.1 Calibration

The fraction of rule-of-thumb consumer is set to 0.2, which is a close to the average share of credit-constrained households in the Survey of Consumer Finance[11] (SCF) data. This calibration value is much smaller than 0.5 which can be found in Campbell and Mankiw (1989) and Galí et al. (2007). Figure 2.1 displays the share[12] of credit-constrained households in the SCF data, which can be a proxy for the share of rule-of-thumb consumers. I follow the definition of credit-constrained households as in Jappelli (1990). The credit-constrained consumers are defined as

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[11]1989-2016 survey years

[12]Weights of households are considered.

those who have had their request for credit rejected by financial institutions (turned down) or those who do not apply because they perceive that they will be refused loans (discouraged). The number of credit-constrained households increases after the Great Recession and drop significantly in the 2016 survey year. The average share of the constrained households is 0.20 during 1989-2016 survey years, which is quite considerable.
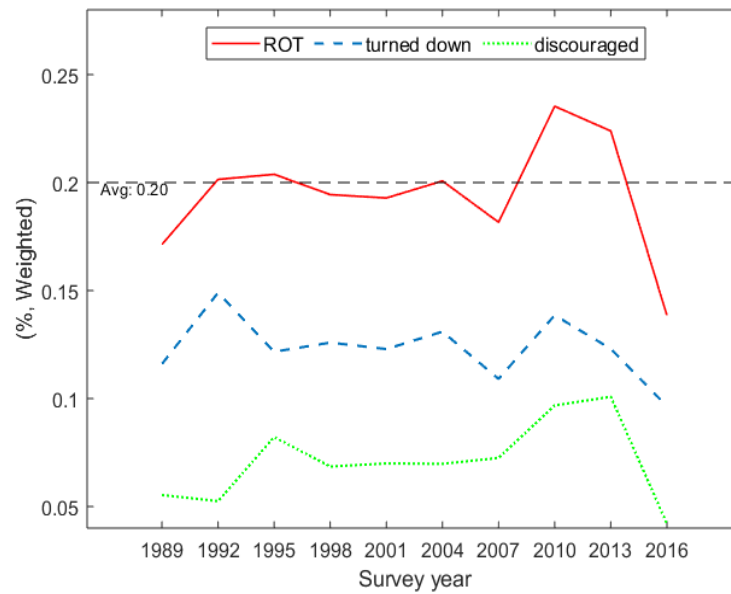
Figure 2.1: Share of ROT consumers (SCF, weighted))

I follow Faia (2009) for most other calibrations. First of all, we assume zero government supply in order to focus on the optimal monetary policy. The discount factor (β) is 0.99 for both households. The parameter on consumption in the utility function, σ, is set to equal to 2. The value for the price elasticity of demand, ε, is 6. The cost of adjusting prices, ψ, is 20 that corresponds to around 0.5 of inflation sensitivity to margianl cost following Lubik and Schorfheide (2004).

The parameter of the homogeneous matching function is set to 0.4. The bargaining power of workers are also assumed to be 0.4[13]. The steady state job filling rate is set to 0.7. The exogenous separation rate, ρ, is set to 0.1. I target the steady state government spending to output ratio = 22% according to Cui (2016). Gerke

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[13]Thus, I assume that the Hosios condition is satisfied.

Parameters	Description	Value
ξ	Share of rule-of-thumb consumers	0.2
Preferences		
β	Discount factor	0.9902
σ	Relative risk aversion	2
Production		
ε	Elasticity of demand	6
ψ	Price adjustment cost	20
labor market frictions		
φ, ζ	Matching technology parameter, workers' bargaining power	0.4
$q(\theta)$	Steady state firm matching rate	0.7
ρ	Exogenous separation rate	0.1
μ	Unemployment benefit	0
Government		
g^{ss}/y^{ss}	Steady state ratio of government spending to output	0.22
τ	Countercyclical (procyclical) transfer policy parameter	-0.10 (0.10)
Shocks		
$\daleth_a, \sigma_a$	Persistence, standard deviation (TFP)	0.95, 0.01
$\daleth_g, \sigma_g$	Persistence, standard deviation (gov. spending)	0.90, 0.01
$\daleth_x, \sigma_x$	Persistence, standard deviation (job separation)	0.91, 0.01
γ_1, γ_2	AR(2) parameter for a job separation process	1.70, -0.75

Table 2.1: Calibrated parameters

et al. (2020) identify the value of a transfer policy parameter as 0.15 (0.50) for an estimated (calibrated) model. I use an ad hoc value for the parameter between the two values.

The aggregate productivity shock process, a_s, follows an AR(1) and is calibrated so that it persists at 0.95, which is a standard value. The logs of government consumption (g) and job separation rate (ρ) obey the following exogenous process, $ln(\iota_s/\iota) = \daleth_\iota ln(\iota_{s-1}/\iota) + \varepsilon_s^\iota$. $\daleth_\iota$ and σ_ι where $\iota = \{g, \rho\}$. $\daleth_g$ and $\daleth_x$ are set to 0.90 and 0.91 following Galí et al. (2007) and Ravn and Sterk (2017), respectively. Standard deviations of all shocks are calibrated to 0.01 for the purpose of comparison. I also consider an AR(2) process for a job separation shock in order to consider a

more volatile shock in a job market. The parameters for the process are calibrated to lead the job separation rate to peaking during the fifth period after a shock.

2.3.2 Computing steady state

2.3.2.1 Computing method

This sections describes the steps for solving for the steady state implied by the model as in Schmitt-Grohé and Uribé (2012). Given a policy rule, the process $pol_s = \{i_s, \tau_s\}$, the equilibrium conditions of the model can be denoted as

$$E_s \mathbf{C}(x_s, y_s, pol_s, s_s, x_{s+1}, y_{s+1}, pol_{s+1}, z_{s+1}) = 0$$

$$z_{s+1} - z_s = \daleth(z_s - z) + \eta \sigma \varepsilon_{s+1}$$

where x_s ($n_x \times 1$) is a a vector of predetermined endogenous variables, y_s ($n_y \times 1$) is a vector of nonpredetermined endogenous variables, pol_s is a policy instruments vector of the government, z_s ($n_z \times 1$) is a vector of predetermined exogenous variables and ε_s is an $n_\varepsilon \times 1$ vector of exogenous i.i.d innovations with mean zero. The matrix of autoregressive parameters is denoted by $\daleth$ and σ is a scalar which represents the amount of uncertainty.

To highlight the part of the planner's optimization problem that is related to computing the optimal policy from the timeless perspective, the Lagrangian is rewritten as

$$\mathscr{L} = \cdots + U_s + \beta E_s U_{s+1} + \beta^{-1} \Lambda'_{s-1} \mathbf{C}_{s-1} + \Lambda'_s E_s \mathbf{C}_s + \beta E_s \Lambda'_{s+1} \mathbf{C}_{s+1} + \cdots$$

where $U_s = U(c_{o,s}, c_{r,s})$, $\mathbf{C}_s = \mathbf{C}(x_s, y_s, pol_s, s_s, x_{s+1}, y_{s+1}, pol_{s+1}, z_{s+1})$ and Λ_s is the vector of Lagrangian multipliers.

Let w_s represent the vector of variables that the social planner chooses in period s. The vector Ω_s is given by

$$\Omega'_s = [x'_{s+1}, y'_s, pol'_s]$$

The first order condition of the social planner with respect to Ω_s is given by $\frac{\partial \mathscr{L}}{\partial \Omega_s}$.

In a deterministic steady state, the equilibrium conditions of the optimal policy are written as

$$P(x, y, pol; z) + Q(x, y, pol; z)\Lambda = 0$$

and

$$\mathbf{C}(x, y, pol; s) = 0$$

where $P(x, y, pol; z)'$ is the steady state of $\frac{\partial U_s}{\partial \Omega_s} + \beta \frac{\partial U_{s+1}}{\partial \Omega_s}$, $Q(x, y, pol; z)'$ is the steady state of $\beta^{-1} \frac{\partial \mathbf{C}_{s-1}}{\partial \Omega_s} + \frac{\partial \mathbf{C}_s}{\partial \Omega_s} + \beta \frac{\partial \mathbf{C}_{s+1}}{\partial \Omega_s}$ and $C(x, y, pol; z)'$ is the steady state of $\mathbf{C}_s$.

The aim is to get the optimal steady state values of x_s, y_s and pol_s given that the steady state value of z_s is known. Let $\delta(pol)$ be the function that gauges the distance between the steady state value of $\frac{\partial \mathscr{L}}{\partial \Omega_s}$ and zero, given that *pol* solves the equilibrium conditions of the model. Then, the steady state of the optimal policy is a vector pol^* such that $\delta(pol^*) = 0$. In addition, we get the value of Lagrangian multipliers at the steady state as $\Lambda^* = -Q(x^*, y^*, pol^*, 0)^{-1} P(x^*, y^*, pol^*, 0)$

2.3.2.2 Steady states in RANK and in TANK

The table 2.2 displays the steady state values in both TANK and RANK. The level of optimizers' consumption in TANK is similar to that of representative households in RANK. The consumption level of rule-of-thumb consumers is about 60% of optimizers'. Other steady state values are almost the same in both models. It is assumed that the supply of government bonds is zero for convenience.

Variables	TANK	RANK
Consumption	0.36	0.36
(Optimizer)	0.40	-
(Rule of thumb)	0.23	-
output (employment)	0.72	0.72
wage	0.53	0.53
inflation	1.00	1.00
nominal interest rate (%, Quarterly)	0.99	0.99

Table 2.2: Steady state comparison

2.4 Dynamics

In this section, we find the optimal dynamics of the economy. The focus is on impulse responses with respect to productivity, government spending and job separa-

tion shocks. The first two shocks are the main drivers of business cycle fluctuations in industrialized economies. A job separation shock is also considered in order to analyze dynamics during periods of job market turmoils such as sharp increases of unemployment due to the Great Recession or the recent COVID-19 emergency. First I compute optimal second moments with respect to the aggregate shocks, then I discuss the dynamics through corresponding impulse responses.

2.4.1 Moments

I adopt the procedure described in Schmitt-Grohé and Uribé (2004). I compute J simulations of length T periods and take the arithmetic average of the moments. Schmitt-Grohé and Uribé (2004) show that a first order approximation is good as long as the simulation period is not very long and set J=500 and T=100. I adopt the same values. All shocks are drawn from the standard normal distribution.

Shocks	TFP		Gov spending		Job separation		All[1]	
Variables	T	R	T	R	T	R	T	R
Inflation(π)	0.06	0.00	0.03	0.00	0.03	0.01	0.08	0.01
Employment(n)	0.92	0.96	0.11	0.12	0.73	0.75	1.22	1.26
$std(n)/std(\pi)$	**14.8**	423.0	**3.6**	238.3	**21.7**	95.2	**15.5**	152.4
T/R[2]	3.5	-	1.5	-	**22.8**	-	10.2	-
Consumption	4.59	4.65	0.82	0.81	1.51	1.54	5.03	5.07
(OPT)	4.24		0.67	-	1.39	-	4.63	-
(ROT)	7.07		1.87	-	2.40	-	7.88	-
Output	3.23	3.28	0.11	0.12	0.73	0.75	3.40	3.43
Wage	3.31	3.39	0.19	0.28	0.68	0.70	3.47	3.54
Interest rate	0.26	0.30	0.10	0.10	0.23	0.28	0.37	0.43

1) 3 shocks are considered all together

2) $\left(\frac{std(n)}{std(\pi)}\right)^T / \left(\frac{std(n)}{std(\pi)}\right)^R$ (%)

Table 2.3: Second moments comparison (%)

In TANK, optimal inflation volatility significantly rises against that in RANK

with respect to all shocks. This result is different from the standard[14]. In addition, relative volatility of employment ($std(n)/std(\pi)$) drops substantially. This implies the stronger preference of the planner in favor of employment stabilization over price stabilization when rule-of-thumb consumers are considered. Since the rule-of-thumb consumers have neither the ownership of firms nor the access to government bonds, they stabilize their consumption only by employment. Thus, the planner is likely to allow higher price adjustment cost in return for relatively more stabilized employment. With respect to job separation shocks, we find a deviation from price stabilization even in RANK. More importantly, the relative volatility of employment over inflation in TANK against in RANK ($(\left(\frac{std(n)}{std(\pi)}\right)^T / \left(\frac{std(n)}{std(\pi)}\right)^R)$) is the highest with regard to the shocks. This implies that the role of monetary policy on stabilization of employment can be limited when it comes to a shock in the job market.
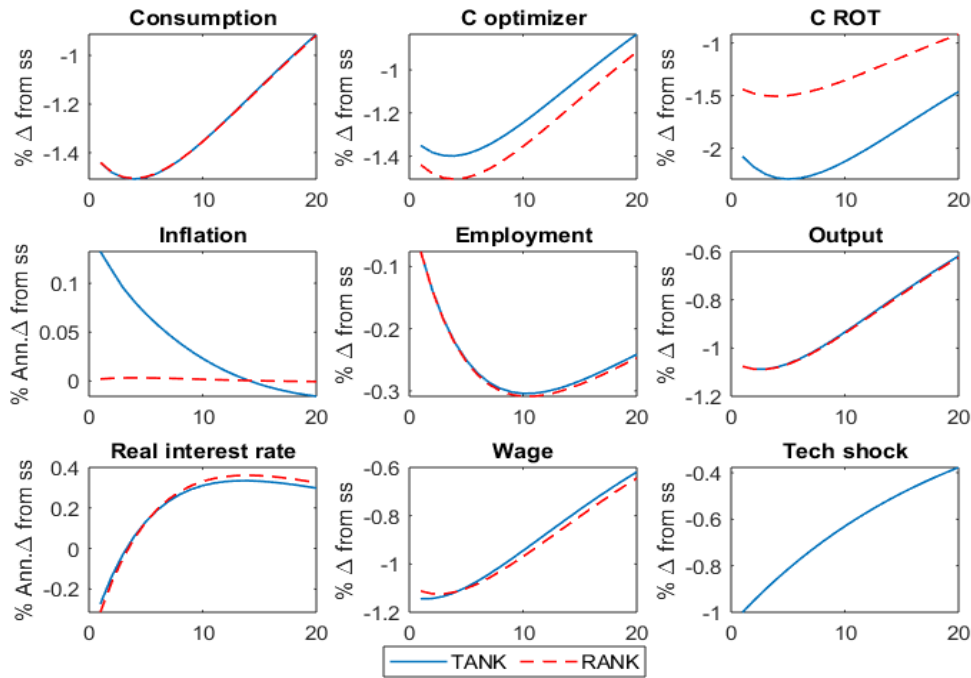
2.4.2   TANK vs RANK with lump-sum tax

In order to consider the role of rule-of-thumb households in the dynamics, I compare the dynamics in the baseline model (TANK) with those in the representative households model (RANK) with regard to a TFP, a government spending and a job separation shock.

Figure 2.2 shows the impulse responses after a negative TFP shock depending on the heterogeneity of households. The blue line shows the impulse responses to a technology shock when households are heterogeneous; the dotted red line shows the impulse responses when they are homogeneous. Note that both consumptions of optimizer and rule-of-thumb consumers in RANK represent the same aggregate consumption.

The one of main differences is the response of inflation. It is optimal to deviate from complete price stabilization at the higher costs of price adjustment and job posting when there are *more* rule-of-thumb consumers. The reason for this is that, for a negative TFP shock, it is optimal to decrease employment and output *less* when there are *more* rule-of-thumb consumers even though the costs of price adjustment

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[14]We can see the standard results in Section 2.4.3.1 with hours worked instead of search and matching friction.

Note) Inflation and interest rate: annualized percentage change from the steady state.
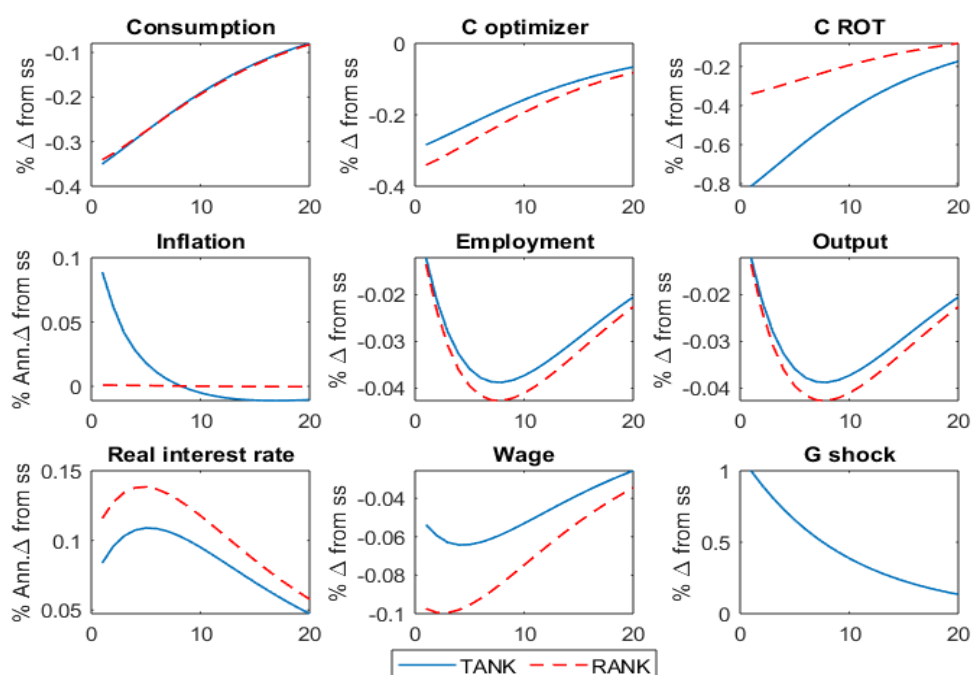
Figure 2.2: TANK vs RANK (TFP shock and lump-sum tax)

and job posting become higher. This implies that the optimal policy needs to strike a balance between price and employment stability if there exists heterogeneity in households even though the Hosios condition is satisfied. Faia (2009) shows that the Ramsey optimal response of inflation with respect to a TFP shock is almost complete price stabilization in her representative households model if the Hosios condition is satisfied.[15] However, Figure 2.2 shows that a fraction of rule-of-thumb consumers provides grounds for the deviation from complete price stabilization in return for less decrease of employment.

We identify the impulse responses of the same variables with respect to a positive government shock in Figure 2.3. With heterogeneous households, the shock leads the social planner to allow less of a decrease in wages on impact so as to consider the rule-of-thumb consumers, which is related to a positive inflation and a smaller decrease in employment. On the contrary, the response of inflation in RANK shows nearly zero inflation when the social planner commits to the optimal

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[15]RANK in this paper shows the similar result.

policy, which is similar to Siu (2004) and Schmitt-Grohé and Uribé (2004).[16]
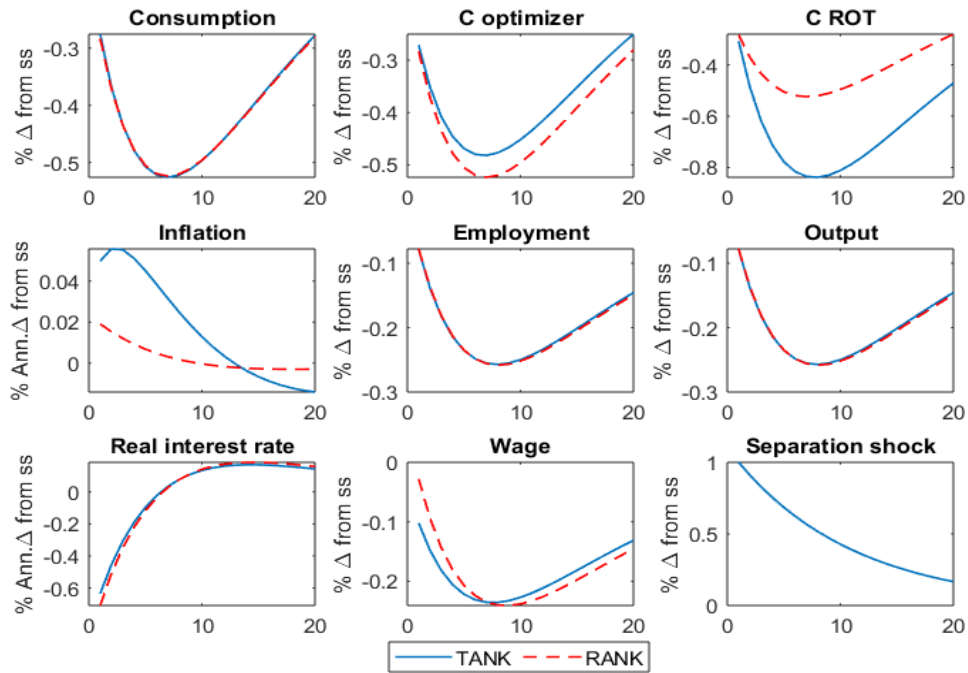

Note) Inflation and interest rate: annualized percentage change from the steady state.

Figure 2.3: TANK vs RANK (Gov. spending shock and lump-sum tax)

Figure 2.4 displays the impulse responses to a positive job separation shock.[17] First of all, the increase of inflation in TANK is smaller both absolutely and relatively than that with respect to a TFP or a government spending shock. This represents the fact that a relatively smaller price adjustment cost is allowed for the stabilization of employment in TANK with regard to a job separation shock than to the other shocks. In addition, the response of wage on impact in TANK drops more in the first two years, which is related to the smaller increase in inflation.

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[16]In Faia (2009), a deviation from a complete price stabilization occurs even in a representative household model under the setting of zero bond supply and positive unemployment benefit when the Hosios condition is not satisfied.

[17]Refer to the Appendix B.6 for impulse responses to a negative shock on matching efficiency (m).

Note) Inflation and interest rate: annualized percentage change from the steady state.

Figure 2.4: TANK vs RANK (Job separation shock and lump-sum tax)

2.4.3 Comparison between TANKs

I compare the second moment and impulse responses from different models in order to verify whether the only combination of rule-of-thumb consumers and search and match friction (TANK-SAM) leads to deviation from price stabilization. Firstly, I contrast the effect of search and matching friction with that of hours worked. Next, I analyze the effect of a transfer to rule-of-thumb consumers so as to demonstrate the role of heterogeneity in consumption level.

2.4.3.1 Comparison 1: SAM vs HW

To begin with, we compare two TANK models in order to analyze the effect of search and matching friction on impulse responses. Specifically, we build a standard TANK model with hours worked[18] (TANK-HW) instead of the search and matching friction.

The standard TANK model is adopted from Galí et al. (2007).[19] The house-

---

[18]Intensive margin of labor supply.

[19]Refer to Appendix B.5 for details.

holds have a common separable preference such as

$$U(c_{is}, n_{is}) = \frac{c_{is}^{1-\sigma}}{1-\sigma} - \psi \frac{n_{is}^{1+\gamma}}{1+\gamma} \tag{2.15}$$

where both ψ and γ are calibrated to 1. With the separable preference and the assumption of labor unions, the hours worked for each households is a function of wage and marginal utility of each households.

$$n_s = \Big[\frac{\varepsilon_w - 1}{\varepsilon_w}\Big(\frac{\xi}{c_{r,s}^{\sigma}} + \frac{1-\xi}{c_{o,s}^{\sigma}}\Big)w_s\Big]^{1/\gamma} \tag{2.16}$$

Table 2.4 compares the second moments in TANK-SAM with those in TANK-HW with regard to government spending shocks. Remarkably, the relative volatility of employment over inflation $(std(n)/std(\pi))$ drops much less in TANK-HW than in TANK-SAM against that in each corresponding RANK model. This represents the fact that the social planner still pays much attention to the stabilization of inflation than to that of employment in TANK-HW even though the same fraction of rule-of-thumb consumers is incorporated in the model.

Variables	SAM		Hours Worked (HW)	
	TANK	RANK	TANK	RANK
Inflation(π)	0.03	0.00	0.00	0.00
Employment(n)	0.11	0.12	0.32	0.31
$std(n)/std(\pi)$	**3.58**	238.32	**67.48**	201.51
Consumption	0.82	0.81	0.14	0.00
(Optimizer)	0.67	-	0.12	-
(Rule of thumb)	1.87	-	0.24	-
Output	0.11	0.12	0.32	0.31
Wage	0.19	0.28	0.01	0.01
Nominal interest rate	0.10	0.10	0.03	0.03

Table 2.4: Second moments comparison (%, government spending shocks)

Figure 2.5 and 2.6 display a comparison between two TANKs in regard to a TFP shock and a government spending shock, respectively. The blue solid line represents the impulse responses in TANK-SAM, while the red dotted line represents

the same in TANK-HW. We find that rule-of-thumb consumers in TANK-HW do
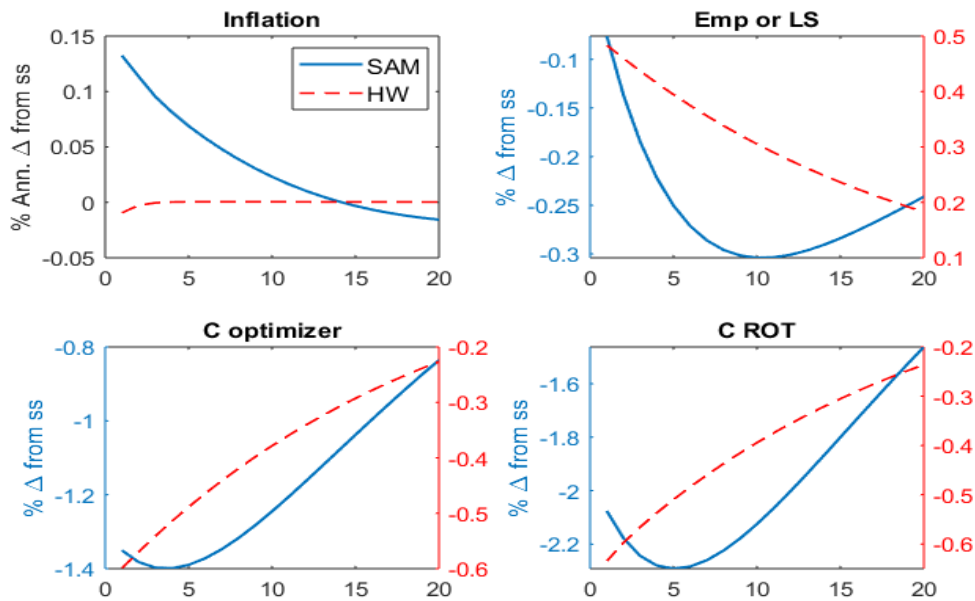
Note) Inflation: annualized percentage change from the steady state.
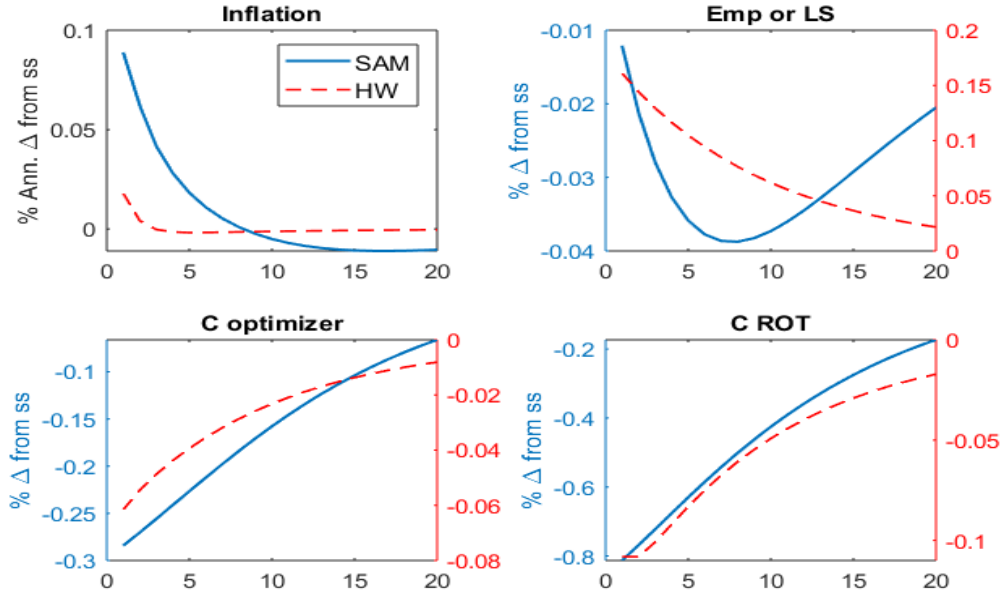
Figure 2.5: 3 TANKs comparison (SAM vs HW, TFP shock (-1%))

not lead to deviation from price stabilization with respect to both shocks. Since rule-of-thumb consumers can better smooth their consumption through the adjustment of hours worked, the social planner has less of an incentive to increase the their consumption on impact through the adjustment of wages. As a result, the social planner is more willing to avoid price adjustment costs with respect to the TFP shock.[20] We also discover the smaller response of inflation on impact of a government spending shock in TANK-HW.

2.4.3.2 Comparison 2: Baseline vs a model with transfers

In this section, we discover the effect of heterogeneity in consumption level on impulse responses by considering a transfer policy. Specifically, we consider a constant transfer to rule-of-thumb consumers in order to remove the difference in the steady state level of each household's consumption. Thus, the budget constraints

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[20]The result of almost complete price stabilization can also be found with a KPR preference.
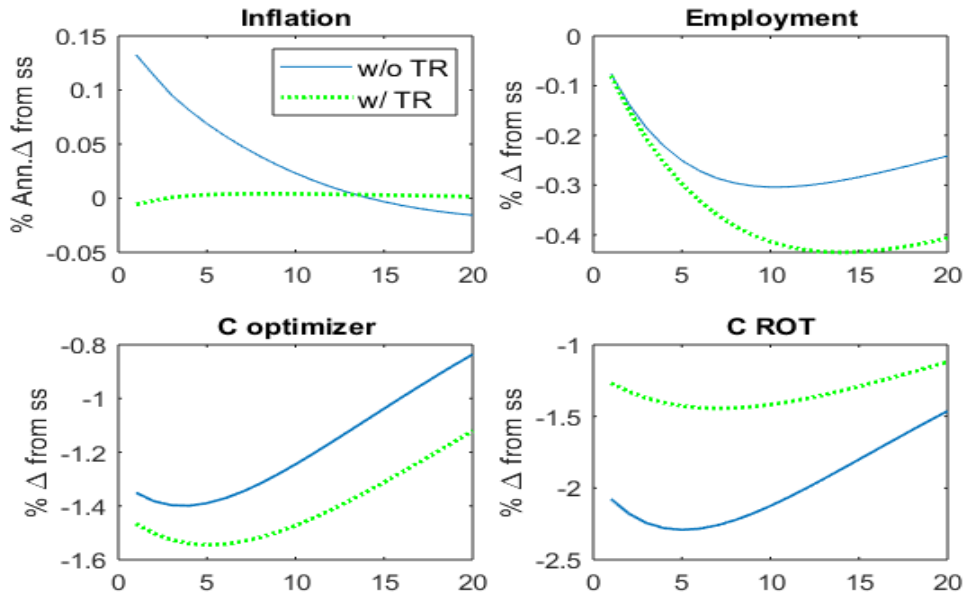
Note) Inflation: annualized percentage change from the steady state.

Figure 2.6: TANKs comparison (SAM vs HW, Gov. spending shock (1%))

of the two types of households are given by

$$
\begin{aligned}
c_{os} + \frac{B_s}{1-\xi} &= w_s n_s + \mu(1-n_s) + \frac{\Pi_s}{1-\xi} + \frac{R_{s-1} B_{s-1}}{\pi_s(1-\xi)} - T_s \\
c_{rs} &= w_s n_s + \mu(1-n_s) - T_s + \frac{TR}{\xi}
\end{aligned}
\tag{2.17}
$$

where the transfer is calibrated to make the two types of consumption equal in the steady state. Figure 2.7 and 2.8 display impulse responses of TANKs depending on the transfer policy. All the other things are the same in both TANKs. We again find almost complete price stabilization after introducing the transfer policy. As the heterogeneity of consumption level in the steady state is removed by the transfer, the planner has almost no incentive to deviate from price stabilization and reduce the employment volatility since the necessity of consideration for rule-of-thumb consumers diminishes. This exercise shows that the heterogeneous level of consumption is imperative for the result of deviation from the complete price stabilization with respect to aggregate shocks.

Inflation

Employment

C optimizer

C ROT

Note) Inflation: annualized percentage change from the steady state.

Figure 2.7: TANKs comparison (without vs with transfer, TFP (-1%))

Inflation

Employment

C optimizer

C ROT
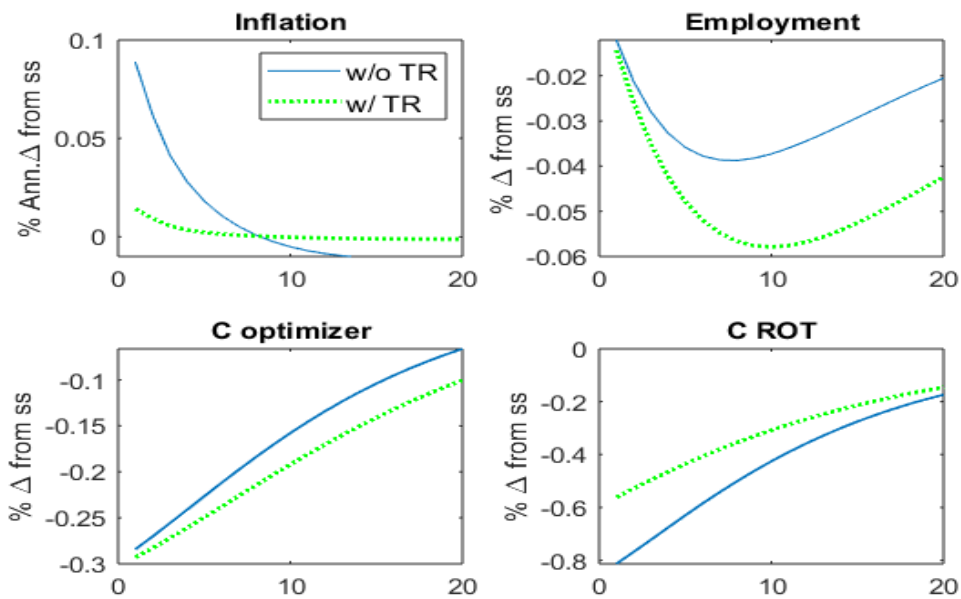
Note) Inflation: annualized percentage change from the steady state.

Figure 2.8: TANKs comparison (without vs with transfer, Gov. spending (1%))

2.5 Transfer policy and job separation shocks

We study the effect of time-varying transfer rules on the dynamics with regard to job separation shocks since we find that the stabilizing effect of monetary policy

can be limited when it comes to a job separation shock in section 2.4. We consider a countercyclical (procyclical) transfer[21] to rule-of-thumb consumers and identify the characteristics of dynamics after job separation shocks. In order to reflect reality more accurately, we adopt a distortionary income tax rate, τ_{is}, and a positive level of government debt. In this context, we have the following budget constraints of each household and the government, and the transfer rule.

$$
\begin{aligned}
c_{os} + \frac{B_s}{1-\xi} &= (1-\tau_{is})(w_s n_s + \mu(1-n_s) + \frac{\Pi_s}{1-\xi}) + \frac{R_{s-1}B_{s-1}}{\pi_s(1-\xi)} \\
c_{rs} &= (1-\tau_{is})(w_s n_s + \mu(1-n_s)) + \frac{TR_s}{\xi} \\
T_s + B_s &= G_s + \frac{R_{s-1}B_{s-1}}{\pi_s} + TR_s(1+c_{TR}) \\
TR_s &= \tau(y_s - y^{ss}) + TR^{ss}
\end{aligned}
\tag{2.18}
$$

where $T_s = \tau_{is}(w_s n_s + \mu(1-n_s) + \Pi_s)$, c_{TR}[22] is the cost of transfer and the degree of countercyclical (procyclical) transfers is governed by $\tau \leq (\geq)0$, which rebates income from optimizers, whenever aggregate output is different from steady state. The steady state level of debt is 74%[23] of the annual output. In addition, the transfer in the steady state (TR^{ss}) is set to zero.

We examine the effect of a job separation rate that follows an AR(2) process in order to reflect the fact that the job separation peaks a few periods later after a recession starts.[24] Specifically, we consider the following process of job separation,

$$
\rho_s = \gamma_1 \rho_{s-1} + \gamma_2 \rho_{s-2} + \varepsilon_{\rho s}
\tag{2.19}
$$

where $\gamma_1 > 1$ and $\gamma_2 < 0$.[25] Note that the job separation with the AR(2) process results in a stronger and more prolonged job separation than a AR(1) process.

Table 2.5 shows volatilities with multiple series of job separations shocks in TANK depending on the type of the transfer rule and the shock process. First of

---

[21]Gerke et al. (2020) study a similar transfer rule.

[22]This is set to zero unless stated.

[23]The average government debt to GDP ratio in the US during the last 30 years.

[24]I reflect that the rate of layoffs and discharges (non-farms) peaks in the fifth quarter after the Great recession begins in the US.

[25]$\gamma_1 + \gamma_1 < 1$, $\gamma_2 - \gamma_1 < 1$ and $|\gamma_2| < 1$ are also required for a non-explosive process.

all, the countercyclical transfer rule results in the highest relative volatility of employment over inflation $(std(n)/std(\pi))$ for both processes of job separation shocks. The transfer rule acts favorably toward the welfare of rule-of-thumb consumers and the planner does not need to bear higher price adjustment cost in return for more stabilized employment. This may imply that an appropriate transfer rule can replace the insurance role of monetary policy. On the other hand, the procyclical transfer rule calls for the relatively larger consideration of employment stabilization since the rule goes against the welfare of spenders.

Transfer rule	Countercyclical		Time-invariant		Procyclical	
Job separation	AR(2)	AR(1)	AR(2)	AR(1)	AR(2)	AR(1)
Inflation(π)	0.03	0.01	0.06	0.03	0.11	0.05
Employment(n)	3.42	1.27	3.42	1.28	3.43	1.24
$std(n)/std(\pi)$	**112.84**	**150.55**	52.71	50.35	**31.16**	**27.00**
Consumption	4.71	1.88	4.75	1.91	4.78	1.85
(Optimizer)	5.10	2.05	4.76	1.91	4.41	1.71
(Rule of thumb)	2.23	0.86	4.69	1.88	7.24	2.84
Output	3.42	1.27	3.42	1.28	3.43	1.24
Wage	0.69	0.22	0.55	0.20	0.56	0.22
Nominal Interest rate	1.11	0.33	1.03	0.31	0.95	0.29

Table 2.5: Second moments (%) with transfers and job separation processes

Figure 2.9 and 2.10 represent impulse responses depending on transfer rules and job separation processes after a job separation shock. The countercyclical transfer rule leads to the most stable inflation responses, due to more stabilized consumption of the rule-of-thumb consumers in regard to the job separation shock. Likewise, the procyclical transfer rule results in the most volatile inflation responses. Inflation drops on impact of the shock since it is helpful for increasing the profit income when a countercyclical transfer rule is in operation, which is favorable to savers. Correspondingly, when a procyclical transfer rule is in operation, inflation rises.

The AR(2) job separation process results in more volatile and prolonged tur-
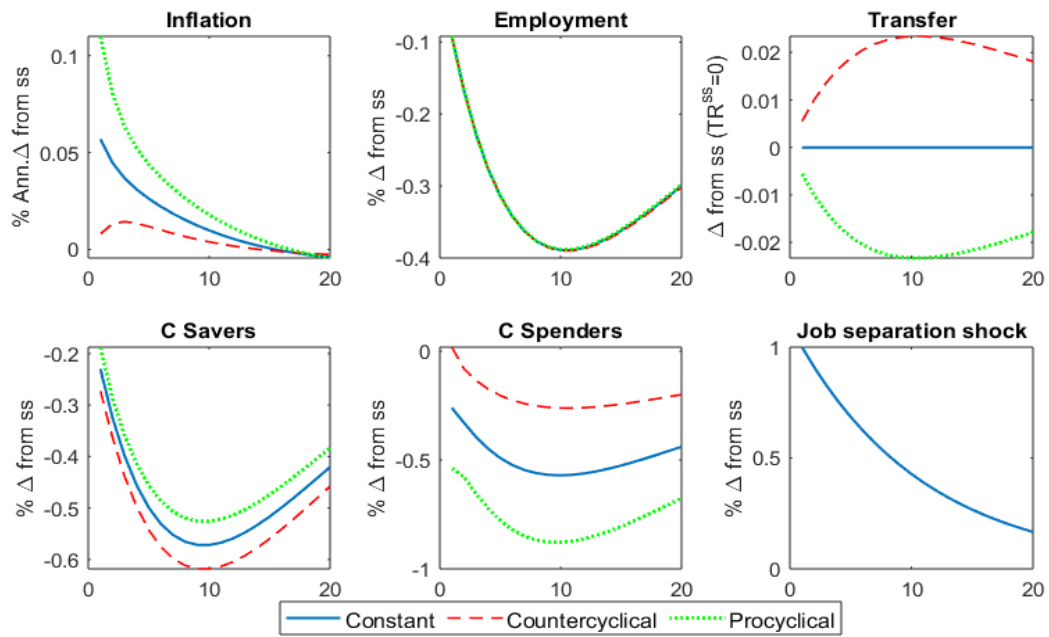
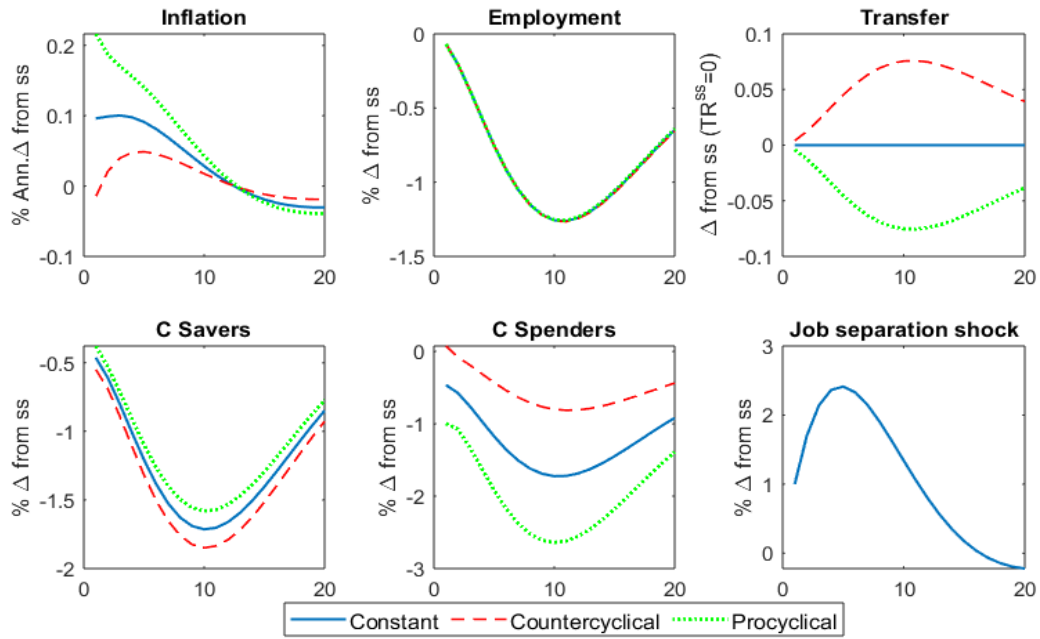Note) Inflation: annualized percentage change from the steady state.

Figure 2.9: Transfer rules and Job separation (AR(1))

moils in job market. As a result, the economy experiences a much deeper and longer recession, which is less favorable to the consumption of both households. The importance of an appropriate transfer rule becomes larger, especially for credit-constrained households since the rule leads to more stabilized consumption of spenders with much smaller costs of price adjustment.

Lastly, we consider an effect of the redistribution cost by setting a positive value of c_{TR}. Figure 2.11 represents impulse responses with respect to an AR(1) job separation shock when a countercyclical transfer rule with a redistribution cost in operation. The redistribution cost requires higher volatilities of inflation and both types of consumption for the same size of transfer. This exercise shows that the efficiency of transfer policy is important for monetary policy to focus on price stability.

2.6 Conclusion

We find that it is optimal to pay more attention to employment stabilization than to price adjustment when both the rule-of-thumb households and the matching friction

Note) Inflation: annualized percentage change from the steady state.

Figure 2.10: Transfer rules and Job separation (AR(2))
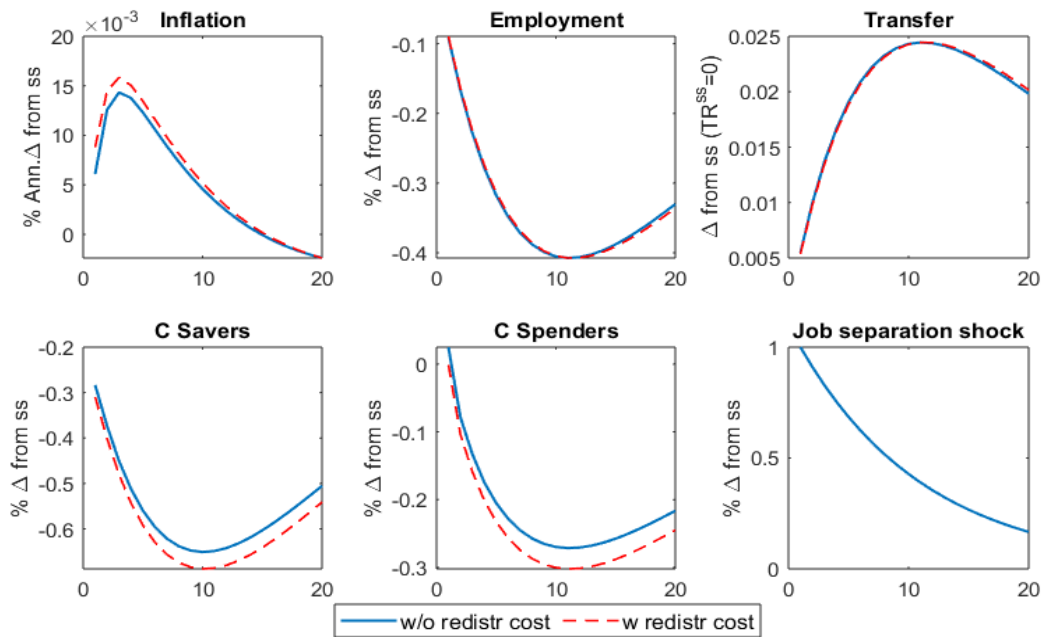
Note) Inflation: annualized percentage change from the steady state.

Figure 2.11: Countercyclical transfer rule with a redistribution cost, $c_{TR} = 0.5$

exist although the price adjustment cost is substantial. The reason for this result is that rule-of-thumb consumers can smooth their consumption only by employment

since they do not have access to government bond market or profits income. This implies that the optimal policy needs to strike a balance between price adjustment and employment stability rather than to pursue complete price stabilization in order to make rule-of-thumb consumers better off.

The size of relative employment stabilization in return for price adjustment in TANK depends on the type of the aggregate shock. The relative standard deviation of employment over inflation in TANK is the smallest with respect to government spending shocks. On the other hand, the relative volatility of employment is a little bit larger with respect to a TFP shock or a job separation. In addition, the relative volatility of employment over inflation in TANK against in RANK is the highest with regard to the job separation shocks. This implies that the role of monetary policy on the stabilization of employment can be limited when it comes to a shock in the job market.

We also identify that the search and matching friction accompanied by heterogeneous consumption is essential for deriving the results through two comparisons. To start with, we compare the baseline model (TANK-SAM) with a model of rule-of-thumb consumers accompanied by hours worked (TANK-HW). We find that spenders in a model with hours worked do not lead to deviation from price stabilization with regard to a TFP or a government spending shock. Since rule-of-thumb consumers can better smooth their consumption through the adjustment of hours worked, the social planner has less of an incentive to increase the their consumption upon the impact of a shock through the adjustment of wage. As a result, the social planner is more willing to avoid price adjustment cost with respect to the aggregate shocks. Next, the baseline model is compared with a model of a constant transfer. We again find almost no inflation response to aggregate shocks after incorporating the transfer policy. As the heterogeneity in consumption in the steady state falls due to the transfer, the planner has less of an incentive to deviate from price stabilization since the consideration for rule-of-thumb consumers diminishes.

Lastly, we study the role of a time-varying transfer policy with respect to a more volatile and prolonged job separation shock. A countercyclical transfer may

replace the role of monetary policy in terms of stabilizing employment. In addition, a more volatile job separation shock calls for more stabilized employment at the higher cost of price adjustment regardless of the type of a transfer rule.

An interesting extension would be to consider the implications for optimal policy when there are shocks on labor supply schedule, for instance, in times of the Great Recession or the recent COVID-19 emergency. Werning et al. (2020) study labor supply shocks that trigger changes in aggregate demand larger than the shocks themselves with a model of two sectors and incomplete markets. I expect that the multi-sector and incomplete markets would call for more active optimal monetary policy with respect to, for example, a job separation shock.

Chapter 3

Decomposition of Monetary Policy Transmission with Heterogeneous Households

3.1 Introduction

What is the mechanism via which a monetary policy shock affects consumption? In standard representative agent New Keynesian (RANK) models this is through the intertemporal substitution channel: when real interest rates decline, the price of consumption today drops relative to the price in the future, so households choose to consume more today. However, recent evidence from microdata has brought this mechanism into question.[1] A number of new models claim to match both the micro and macro data better.

This paper uses the lens of the monetary policy decomposition presented in Auclert (2019) to analyze some standard modeling approaches. The advantage of Auclert's decomposition is that it can be very closely tied to the micro data. However, strictly it requires assumptions that do not hold up in many models. Our paper also analyzes how useful the decomposition is in these cases. In particular the method assumes that a transitory monetary policy shock has no persistent effects.

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[1] In particular household marginal propensity to consume appear to be much higher than RANK models would suggest (e.g. Parker et al. (2013) among many others) and the elasticity of intertermporal substitution is likely small (e.g. Best et al. (2018))

This is clearly the case in any model with no predetermined variables: transitory shocks cannot be propagated into the future because there are no state variables that can carry information with them. Such models include the standard RANK model without capital as well as the beseline two agent New Keynesian (TANK) model we consider in this paper. We break this assumption in two ways. First, we add capital, a predetermined variable, to our TANK model which results in persistence of a monetary policy shock due to the slow movement of the capital stock. Second, in our heterogeneous agent New Keynesian (HANK) model, the entire distribution of assets is a predetermined state variable with potentially important dynamics.

3.1.1   Findings

We begin our analysis with a standard TANK model in which a proportion of households live hand-to-mouth, have no debt, and earn only labor income.[2] As there is no debt neither the interest rate exposure nor the Fisher channel play a role. We find instead a large role being played by the earnings heterogeneity channel: firm profits are countercyclical in the New Keynesian model, so the poor households see significantly more income variation over the business cycle than the wealthy. This is an important finding and draws into question some of the key results in the HANK literature so far. That profits are countercyclical is not empirically true, so this transmission mechanism does not fit the evidence.[3]

Without a large earnings heterogeneity channel, the standard TANK model would continue to lean very heavily on the intertemporal substitution channel. Our next iteration of the model shows a potential for a very different transmission mechanism, one that fits the microdata but that current models do not quantitatively capture. We allow the hand-to-mouth households in our TANK model to maintain a debt up to some fraction of their steady state income. When interest rates are low they will be able borrow a little more, and when they are high a little less, thus providing an interest rate exposure channel through which monetary policy acts.

We find the income channels (both aggregate and heterogeneous) act as a mul-

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[2]We closely follow Debortoli and Gali (2018) here.

[3]Broer et al. (2019) come to a similar conclusion.

tiplier for both the intertemporal substitution and the interest rate exposure channels. As we decrease the elasticity of intertemporal substitution, the intertemporal substitution channels diminishes along with the income multipliers of it. The interest rate exposure channel, on the other hand, remains the same size. This suggests it may be possible to create a monetary policy model in which intertemporal substitution plays no role at all, but which nonetheless fits the macrodata. This is something we plan to tackle in future work.

The rest of the paper investigates how useful Auclert's decomposition is in models with predetermined variables. Our first such model extends the TANK model with capital. We find the decomposition fails when we have no convex capital adjustment costs, but that with standard parameterizations of these costs the decomposition accounts for over 95 percent of the change in consumption.

Finally we consider a one-asset HANK model.[4] This breaks with the assumption required for Auclert's decomposition in two ways. First, as with other HANK models the entire distribution of assets is a predetermined state variable, allowing for potential persistence following a monetary policy shock. We show that although the distribution of assets is predetermined, a monetary policy shock has little persistence and the shock is decomposed reasonably well by the Auclert's sufficient statistics.

Overall we believe significant progress has been made in understanding the transmission mechanism of monetary policy recently, both empirically and theoretically. However, we find there is still a great deal of divergence between the two and believe going forward models should target the interest rate exposure and aggregate income channels, and give the intertemporal substitution and earnings heterogeneity channel a smaller role.

3.2 Transmission Channels

We will make heavy use of the monetary policy partial equilibrium decomposition described in Auclert (2019). He makes the assumption that for an individual a one

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[4]Our model closely relates to the two asset model presented in Bayer et al. (2019)

time shock to nominal interest rates looks like i) a transitory change in income, ii) a one off change in the price level, iii) a change in the real interest rate. Here we provide a brief description of each of the five channels he identifies and then give some indication as to the conditions under which they sum to the aggregate change in consumption. For more detail please refer to Auclert's paper.

3.2.1 Aggregate Income Channel

The aggregate income channel measures how much consumption changes due to the change in aggregate income, under the assumption that all incomes move proportionally. The size of this channel is given by:

$$AggInc = \mathbb{E}_i\left(\text{MPC}_i Y_i\right) \frac{dY}{Y} \tag{3.1}$$

where MPC_i is the marginal propensity to consume of household i and the expectation is taken over all households.[5] That is the aggregate income channel is the income weighted marginal propensity to consume multiplied by the change in aggregate income.

3.2.2 Earnings Heterogeneity Channel

A monetary policy shock may not change the income of every household proportionally. If households with high MPCs see relatively larger income changes than households with low MPCs, then overall the channel through which monetary policy affects consumption through income will be larger than measured by the aggregate income channel. The total income channel is simply the expectation of each household's MPC multiplied by their own change in income, $\mathbb{E}_i\left(\text{MPC}_i dY_i\right)$. The earnings heterogeneity channel is measured as the residual of the total income channel after taking away the aggregate income channel:

$$EarnHet = \mathbb{E}_i\left(\text{MPC}_i dY_i\right) - \mathbb{E}_i\left(\text{MPC}_i Y_i\right) \frac{dY}{Y} \tag{3.2}$$

---

[5]Strictly this is the marginal propensity to consume out of income *after* accounting for labor response. In our models hours are either rationed or do not depend on wealth, so this definition of MPC coincides with the more standard definition.

we can also understand this channel as the covariance between the individual MPC and the difference of individual income change from average income change, $Cov_i(\text{MPC}_i, \ dY_i - Y_i\frac{dY}{Y})$.

3.2.3 Fisher Channel

Inflation has the effect of changing the real value of nominal assets and debts. The Fisher channel measures how this affects aggregate consumption, making the assumption that households individual MPCs apply to this change in wealth. The key household level measure here is the net nominal position (NNP), that is the sum of all nominal assets net of nominal debts for each household. The size of the channel is then:

$$Fisher = -Cov_i(\text{MPC}_i, \text{NNP}_i)\frac{dP}{P} \tag{3.3}$$

where P is the price level.

3.2.4 Interest Rate Exposure Channel

The interest rate exposure channel measures how much households change their consumption due to unhedged interest rate exposure (URE). Unhedged interest rate exposure is defined as the difference between all maturing assets (including income) and maturing liabilities (including planned consumption), and is therefore the quantity of saving that is planned to be invested at this periods interest rate. When this period's real interest rate goes up, this effectively increases the budget constraint of those households who have positive unhedged interest rate exposure. Under certain conditions these households will increase their consumption by their MPC multiplied by the change in their budget constraint. That is:

$$IRE = Cov_i(\text{MPC}_i, \text{URE}_i)\frac{dR}{R} \tag{3.4}$$

where R is the real interest rate.

3.2.5 Intertemporal Substitution Channel

Finally the intertemporal substitution channel measures how much households will shift their consumption between time periods due to the change in the real interest

rate.

$$IntSubs = -\mathbb{E}_i\left(\frac{1}{\sigma_i}(1-\text{MPC}_i)C_i\right)\frac{dR}{R} \tag{3.5}$$

where $\frac{1}{\sigma_i}$ is the household's elasticity of intertemporal substitution and C_i is their consumption this period.

3.2.6 Aggregation

Auclert (2019) shows that these five channels sum exactly to the aggregate change in consumption following a monetary policy shock under some conditions.

$$dC = \overbrace{\mathbb{E}_i[MPC_iY_i]\frac{dY}{Y}}^{AggInc} + \overbrace{Cov_i(MPC_i, dY_i - Y_i\frac{dY}{Y})}^{EarnHet} \overbrace{-Cov_i(MPC_i, NNP_i)\frac{dP}{P}}^{Fisher}$$
$$\underbrace{+Cov_i(MPC_i, URE_i)\frac{dR}{R}}_{IRE} \underbrace{-\frac{1}{\sigma}\mathbb{E}_i[(1-MPC_i)c_i]\frac{dR}{R}}_{IntSubs}$$

The conditions are as following. First, preferences must be separable. Second, a monetary policy shock has a purely transitory effect, changing income and the real interest rate for one period only, while effecting a one time change in the price level. For New Keynesian models with no predetermined variables, such as the standard consumption New Keynesian model or the baseline two agent New Keynesian model presented below, this is the case. Models with capital, or where the distribution of wealth persists into the next period such as the HANK model presented below, do not fit this decomposition. We will measure the error as the difference between the sum of the five channels and the actual change in consumption.

3.3 A TANK Model in which the Decomposition Works Exactly

3.3.1 Model Overview

We begin our analysis with a baseline two agent New Keynesian (TANK) model. Our baseline TANK model is composed of two types of agents, Ricardian and non-Ricardian, along with a continuum of intermediate goods firms, a perfectly competitive final goods firm, and a monetary policy authority. The model is closely

related to the standard New Keynesian model with Calvo pricing frictions, the main difference being the addition of the non-Ricardian households. A key addition in our model, compared with other TANK models, is to allow for the non-Ricardian households to hold a non-zero quantity of short term nominal debt (owed to the Ricardian households) so that we have non-trivial levels for households' unhedged interest rate exposure (URE) and net nominal positions (NNP).

The advantage of starting our analysis with this model is that it contains no predetermined variables, and therefore the conditions for our partial equilibrium decomposition hold exactly. As well as being a useful starting point to build upon, it also highlights how the transmission mechanism works in TANK models (and HANK models more generally), showing just how important the earnings heterogeneity channel is in these models.

3.3.2 Households

A proportion λ of households, which we shall call non-Ricardian, live hand-to-mouth, consuming all their income in each period. The remaining $(1-\lambda)$, which we shall call Ricardian, are unconstrained optimizing agents. Following Debortoli and Gali (2018), and in order to keep the supply side as simple as possible, we assume the markup on wages (see below) is high enough that households supply as much labor as demanded by the firms.

3.3.2.1 Ricardian Households

Each period Ricardian households choose how much to consume, C_t^R, in order to maximize their life time (separable) utility:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{\left(C_t^R\right)^{1-\sigma}}{1-\sigma}-\frac{\left(N_t^R\right)^{1+\psi}}{1+\psi}\right)$$

where N_t^R is their hours worked. They are subject to the budget constraint:

$$P_tC_t^R + (R_t^n)^{-1}B_{t+1} = N_t^RW_t + P_tD_t + B_t$$

where P_t is the price level in period t, R_t^n is the gross nominal interest rate between t and $t+1$, B_t is the quantity of bonds bought at time $t-1$ paying one unit of nominal

currency in period t, W_t is the nominal wage per unit of labor in period t and D_t is the real dividend payed by firms in period t. All firm profit goes to the Ricardian households and this is shared equally between them.

The Euler equation for these Ricardian households IS:

$$\left(C_t^R\right)^{-\sigma} = \beta \mathbb{E}\left(R_t^n \frac{P_t}{P_{t+1}} \left(C_{t+1}^R\right)^{-\sigma}\right) \tag{3.6}$$

3.3.2.2 Non-Ricardian households

Non-Ricardian households are more impatient than the Ricardian households and as a result are up against their borrowing limit. They can borrow nominal bonds up to the point where their expected *real* payment in the next period is equal to a fixed fraction Ω of their steady state income. Each period they optimize their period utility:

$$\frac{\left(C_t^{NR}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^{NR}\right)^{1+\psi}}{1+\psi}$$

subject to their budget constraint:

$$C_t^{NR} \leq N_t^{NR} \frac{W_t}{P_t} + \left((R_t^n)^{-1} \frac{\mathbb{E}_t P_{t+1}}{P_t} - \frac{\mathbb{E}_{t-1} P_t}{P_t}\right) \Omega \bar{N}_{NR} \overline{W/P} \tag{3.7}$$

where $\overline{W/P}$ and $\bar{N}_{NR}$ are the steady state real wage and hours worked by non-Ricardian households.

3.3.2.3 Household Aggregation and Wage Schedule

With the non-Ricardian proportion of households equal to λ, total consumption is:

$$C_t = \lambda C_t^{NR} + (1-\lambda)C_t^R \tag{3.8}$$

Hours are equally rationed between both types of household such that:

$$N_t = N_t^{NR} = N_t^R \tag{3.9}$$

The real wage is set according to the demand schedule:

$$\frac{W_t}{P_t} = \mathscr{M}^\omega (C_t)^\sigma (N_t)^\psi \tag{3.10}$$

where $\mathcal{M}^{\omega} > 1$ can be interpreted as the gross average markup of wages. We assume $\frac{W_t}{P_t} \geq \left(C_t^R\right)^{\sigma} \left(N_t^R\right)^{\psi} \geq \left(C_t^{NR}\right)^{\sigma} \left(N_t^{NR}\right)^{\psi}$ so that households always provide the hours demanded by the firms.[6]

3.3.3 Firms

The production side of the economy follows the standard New Keynesian model with Calvo price adjustment. The firm side of the economy is identical to that presented in Gali (2008) except for the fact that firms choose both labor and capital (and thus their production function has constant returns to scale) each period. This simplifies the analysis a little, as all firms share the same marginal cost. In our base model we hold the aggregate quantity of capital constant, but including it here allows for easy extension to the model with investment.

3.3.3.1 Final Goods Firm

The final goods firm produces a final consumption good, Y_t, from intermediated inputs, $X_t(j)$ for $j \in [0, 1]$ using the technology:

$$Y_t = \left(\int_0^1 X_t(j)^{1-\frac{1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Profit maximization yields the demand schedule $X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$ where P_t is the price of the final good. Competition also imposes a zero profit condition that yields $P_t = \left(\int_0^1 P_t^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$.

3.3.3.2 Intermediate Goods Firm

There is a continuum of intermediate goods firms, indexed by $j \in [0, 1]$ each of which uses both labor and capital each period according to the production function:

$$X_t(j) = AK(j)^{\alpha} N_t(j)^{1-\alpha}$$

As our focus is on monetary policy shocks, we assume the levels of technology (A) and capital (K) to be constant. Constant returns to scale results in the marginal cost

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[6]This demand schedule follows Debortoli and Gali (2018) and is close to the wages a union representing both types of household would set. We also tried allowing wages to be set by the market. This results in counter-factual results such as non-Ricardian and Ricardian households moving their hours worked in opposite directions during a recession.

being equal for all firms.

The probability that a firm is able to adjust its price in any period is equal to $1 - \theta$. A firm that is able to adjust its price in period t will choose a price P^* to maximize the current market value of profits it will make while the price remains effective. That is firm j solves the problem:

$$\max_{P^*} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \{ \Lambda_{t,t+k} X_{t+k}(j) (P_t^* - MC_{t+k} P_{t+k}) \} \tag{3.11}$$

subject to the demand constraints:

$$X_{t+k}(j) = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \tag{3.12}$$

where $\Lambda_{t,t+k} = \beta^k \left(\frac{c_{t+k}^R}{c_t^R} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$ is the stochastic discount factor for nominal payoffs, for the Ricardian households who own the profits from the firms.

The first order condition arising from (3.11) and (3.12) is:

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} X_{t+k}(j) \left(P_t^* - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k} P_{t+k} \right) \right\} = 0 \tag{3.13}$$

Finally, with only a fraction $1 - \theta$ of firms changing their prices in any given period, the aggregate price level moves according to:

$$P_t = \left(\theta P_{t-1}^{1-\varepsilon} + (1 - \theta)(P_t^*)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

3.3.4 Monetary Policy

We assume the central bank follows a simple log-linear Taylor rule with weight on inflation only:

$$\frac{R_t^n}{\bar{R}^n} = \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} exp(v_t) \tag{3.14}$$

where $\bar{R}^n$ and $\bar{\Pi}$ are the nominal interest rate and inflation in the steady-state, respectively. In line with the transitory nature of the experiment we are running, we assume no persistence in v_t.

3.3.5 Equilibrium

As our baseline model has no investment, the goods market clearing condition is:

$$Y_t = C_t \tag{3.15}$$

and the total capital and labor used must equal that available, $\int_0^1 K_t(j)dj = \bar{K}$ and $\int_0^1 N_t(j)dj = N_t$.

3.3.6 Steady State

We will study small fluctuations around the zero inflation steady-state. As hours are allocated evenly between the two types of households we have that the share of hours worked by non-Ricardians is $\bar{n}_{NR} = \lambda$, and that by Ricardians is $\bar{n}_R = 1 - \lambda$. The steady state consumption shares ($\bar{c}_{NR} = \lambda \overline{C^{NR}}/\overline{Y}$ and $\bar{c}_R = (1 - \lambda)\overline{C^R}/\overline{Y}$) are less simple, both because Ricardians earn all the income from the firms and they get paid interest from the non-Ricardian households' debt. In steady-state the markup over marginal cost is equal to $\frac{\varepsilon}{\varepsilon-1}$, and the real wage is equal to the marginal productivity of labor adjusted down by this markup, $(1-\alpha)\frac{\varepsilon-1}{\varepsilon}\frac{\overline{Y}}{\overline{N}}$.

Using this steady-state wage, along with the non-Ricardian budget constraint (3.7) we can identify the steady-state proportion of non-Ricardian consumption:

$$\bar{c}_{NR} = \lambda \left(1 - \Omega(1-\beta)\right)\frac{\varepsilon - 1}{\varepsilon}(1 - \alpha) \tag{3.16}$$

3.3.7 Log-linearized Model

We use small letters to indicate percentage changes from steady-state values and then linearize around the steady-state. We begin with the basic building blocks of the New Keynesian model. First the Euler equation for Ricardian households, linearized from equation (3.6), becomes:

$$c_t^R = \mathbb{E}_t c_{t+1}^R - \frac{1}{\sigma}(r_t^n - \mathbb{E}_t \pi_{t+1}) \tag{3.17}$$

The New Keynesian Phillips curve, derived from the pricing equation (3.13), is:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)\tilde{y}_t \tag{3.18}$$

where the output gap, $\tilde{y}_t$, in this case with fixed technology and capital is just the percentage deviation of output from steady-state output. In addition, the linearized monetary policy rule is as the following.

$$r_t^n = \phi_\pi \pi_t + v_t$$

Unlike the standard New Keynesian model, these three are not enough to pin down the model as the Euler equation (3.17) is determined by Ricardian households, while total consumption and production involves the non-Ricardians too. We have the aggregation conditions from equations (3.8), (3.9) and (3.15):

$$c_t = \bar{c}_{NR} c_t^{NR} + \bar{c}_R c_t^R \qquad (3.19)$$

$$n_t = \bar{n}_{NR} n_t^{NR} + \bar{n}_R n_t^R \qquad (3.20)$$

$$\tilde{y}_t = c_t \qquad (3.21)$$

and the non-Ricardian budget condition from equation (3.7):

$$(1 - \Omega(1 - \beta)) c_t^{NR} = w_t + n_t^{NR} + \Omega \left(\pi_t - \mathbb{E}_{t-1} \pi_t \right) - \beta \Omega (i_t - \mathbb{E}_t \pi_{t+1}) \qquad (3.22)$$

where w_t is the real wage in period t. Note $\pi_t - \mathbb{E}_{t-1} \pi_t$ represents unexpected inflation between $t - 1$ and t and relates to the net nominal position of the non-Ricardian households. The expected return on nominal bonds, $r_t = r_t^n - \mathbb{E}_t \pi_{t+1}$, would be the real interest between t and $t + 1$ if such a market existed and relates to the unhedged interest rate exposure of the non-Ricardian households. In the case where there is no debt ($\Omega = 0$), both these components of the budget constraint disappear. Further note that in this model $\mathbb{E}_{t-1} \pi_t$ will always be equal to zero, so the model has no predetermined variables.

The first order condition for hours worked, equation (3.10), along with the equal allocation of hours, give:

$$w_t = \sigma c_t + \psi n_t \qquad (3.23)$$

$$n_t^R = n_t^{NR} \qquad (3.24)$$

Finally the connection between hours worked and the output gap is given by:

$$\tilde{y}_t = (1 - \alpha) n_t \qquad (3.25)$$

σ	1.0	Inverse EIS
ψ	1.0	Inverse Frisch Elasticity
ϕ_π	1.5	Taylor Rule Coefficient
θ	0.667	Calvo stickiness parameter
β	0.97	Discount Factor
α	0.33	Capital Share
ε	6.0	Elasticity of sub. between goods
λ	0.2	Share of Keynesian Households
Ω	0.0	Keynesian Debt as Share of Income
δ	0.1	Depreciation (capital model only)

Table 3.1: Baseline Calibration

Note capital does not appear in the linearized production function because of the fixed capital assumption.

The final baseline model consists of the Taylor rule, equation (3.14), along with the equations (3.17) through (3.25) and the identity $r_t = r_t^n - \mathbb{E}_t \pi_{t+1}$.

3.3.8   Calibration

We calibrate to standard parameters based on annual periods. Baseline parameters are shown in table 3.1. We will vary some of these to see how the size of the different transmission mechanisms changes with them.

3.4   Results from the Baseline TANK Model

As there are no predetermined variables in our baseline TANK model the decomposition of transmission mechanisms described in section 3.2 works exactly. Here we look at how monetary policy divides into the five different channels according to the proportion of non-Ricardian households, as well as the extent to which they are able to take on debt.[7]

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[7]Refer to Appendix C.1 for details.

3.4.1 Model with No Debt

To start we consider the model in which the non-Ricardian households cannot hold any debt, as is standard in TANK models.[8] The transitory nature of the shock means that expected inflation next period is zero, and hence a one percent decrease in the nominal rate translates exactly into a one percent decrease in the real rate (if it were to trade).

Figure 3.1 shows the size of each transmission channel following a one percentage point decrease in the nominal interest rate, where the proportion of non-Ricardian households is on the x-axis. Note that both the interest rate exposure channel and the Fisher channel are absent in this model as there is no debt between the two types of agent. The left side of the graph shows the transmission channels when there are very few non-Ricardian households. As has been well documented[9] the intertemporal substitution model dominates in this case, seen here in the division of transmission channels along the y-axis corresponding to the RANK model. We have set the elasticity of intertemporal substitution equal to one, so a one percentage point decrease in the real interest rate increases consumption of the Ricardian households by exactly one percent. This is seen as the intercept with the y-axis, divided into a large intertemporal substitution channel of size β, and a small aggregate income channel of size $1 - \beta$.

Moving along the x-axis increases the proportion of non-Ricardian households. The size of the intertemporal substitution channel decreases in line with the consumption share of Ricardian households. As Ricardian households own all the capital as well as the profits from the firms, their consumption share falls more slowly than their share of households. As we introduce non-Ricardian households the size of the aggregate income channel increases, as the average MPC across households grows. While this aggregate income channel is substantial, it ends up being dominated by the earnings heterogeneity channel. This channel is both less intuitive and economically more questionable. It arises because during a boom, the extra

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[8]For examples see Debortoli and Gali (2018), Galí et al. (2007) and Broer et al. (2019)
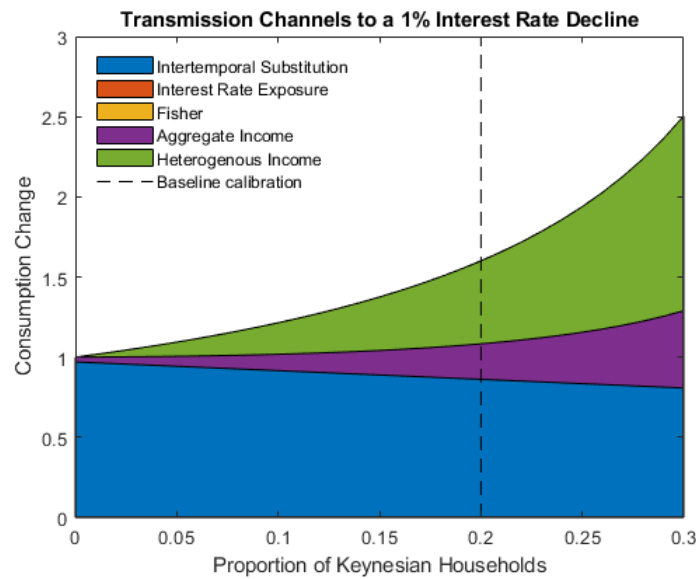
[9]e.g. Kaplan et al. (2018)

Figure 3.1: Changing the Proportion of non-Ricardian households, $\sigma = 1$

income is not distributed equally between the non-Ricardian and Ricardian house-holds, but instead goes predominantly to the non-Ricardian households. This is due to the fact that when the output gap is positive, markups above marginal cost are small, so workers get paid closer to their marginal product while the profits of the firms are reduced. When the proportion of non-Ricardian households reaches 0.3 this earnings heterogeneity channel actually dominates both of the other channels.

This feature of the standard New Keynesian model, that markups are low dur-ing a boom and high during a recession, is not backed by empirical evidence and has led some away from price frictions and toward nominal wage frictions.[10]. While we are sympathetic to this approach, for this paper we maintain the sticky price assump-tion to stay close to the existing HANK literature. One way to remove this earnings heterogeneity channel completely would be to divide the income from capital and profits proportionally between the Ricardian and the non-Ricardian households. In that model, the total consumption change would remain unchanged as the number of non-Ricardian households increased, with the intertemporal substitution channel decreases proportional to the share of Ricardian's in the economy and the aggregate

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[10]This point is emphasized in Broer et al. (2019) and motivates modeling choices in Auclert and Rognlie (2018)

income channel making up the remainder. While the channels would be different, in this model the aggregate dynamics would be *identical* to the RANK model.

3.4.2 Introducing Debt

In this section we analyze what happens when the non-Ricardian households are allowed to take on debt equal to some fraction of their steady state income. For the remainder of this section we will fix the proportion of non-Ricardian households at 0.2, giving an economy-wide MPC of just over 20 percent. This number is chosen both because it is close to a number of the current theoretical HANK models, and a larger number causes indeterminacy problems for some parameterizations.[11] However, we accept 0.2 is on the low end of empirical estimates.[12] In figure 3.1 from the previous section, there is a dotted line drawn where the proportion of non-Ricardian households equals 0.2. This shows the size of the transmission channels for this section when there is no debt.

Figure 3.2 shows how the size of the transmission channels change with the level of debt held by the non-Ricardian households, with the three panels showing this for decreasing elasticity of substitution.[13] Starting with the top panel, we consider how the model behaves with an elasticity of substitution equal to one. The intercepts with the y-axis exactly correspond with the intercepts with the dotted line from figure 3.1. This is the size of the transmission channels when a proportion 0.2 of households are non-Ricardian and these households have no debt. As in the previous section, the intertemporal substitution channel is slightly below one, while the income channels also play a significant role due to presence of non-Ricardian households. However, with no debt at the intersection with the y-axis both the interest rate exposure and Fisher channels are zero.

As the quantity of debt that the non-Ricardian households can take on increases, both the interest rate exposure and Fisher channel start to become quan-

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[11]See Gali et al. (2004) for a detailed discussion on determinacy of TANK models.

[12]A large literature aims to estimate MPC. See Johnson et al. (2006), Parker et al. (2013), Fagereng et al. (2016) and Crawley and Kuchler (2019) for a small selection of examples.

[13]The elasticity of substitution is equal to $1/\sigma$, so the three panels in figure 3.2 represent an elasticity of substitution of 1, 0.5 and 0.33 respectively.
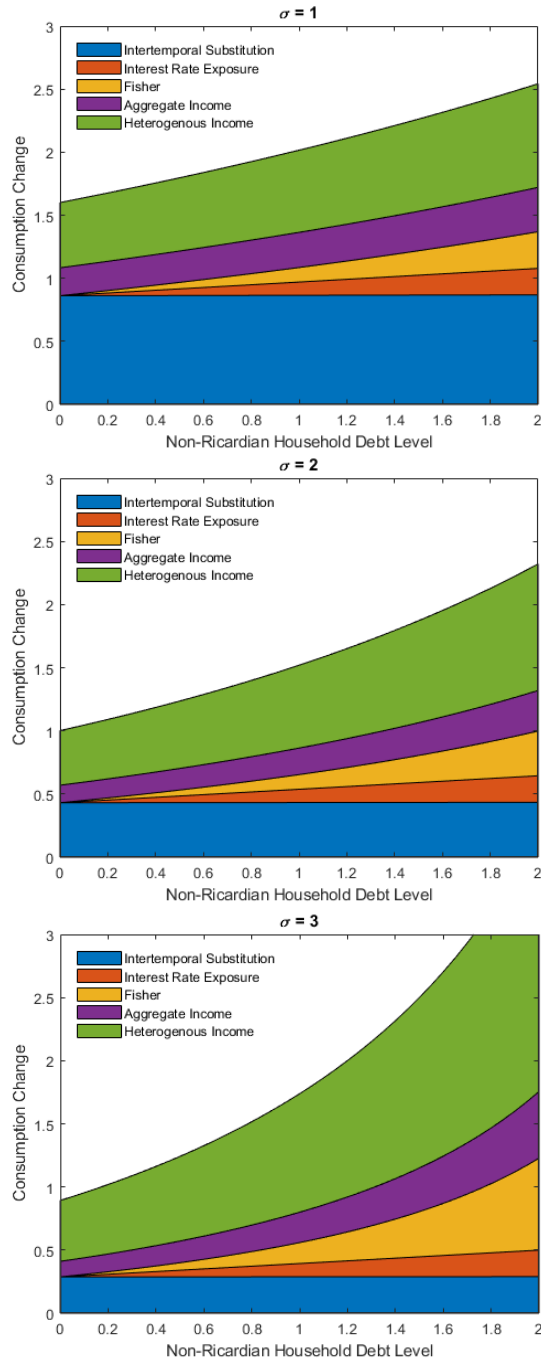
Figure 3.2: Changing the Debt of non-Ricardian households

titatively important. Still looking at the top panel of figure 3.2, we see both of these channels growing, but they are still dominated by the intertemporal substitution channel. The two income channels grow in exact proportion to the other three channels, acting as a constant multiplier of the other three channels, no matter the quantity of debt. It may be useful to think of the transmission of monetary policy acting in stages. First aggregate demand is directly affected by the intertemporal substitution and interest rate exposure channels. The size of these channels depends only on the change in the interest rate, and is not changed as output and inflation change. The size of the Fisher channel is proportional to the amount of nominal debt, multiplied by the size of the overall change in income.[14] Finally the income channels are each a constant proportion of the total income change. We can think of intertemporal substitution and interest rate exposure as providing the initial 'kick', which is then augmented by the Fisher and income channels.

The center and bottom panels of figure 3.2 show the same channels, but when the elasticity of substitution is 0.5 and 0.33 respectively. The size of the intertemporal substitution channel is reduced in the same proportion, by 0.5 and 0.33 as the Ricardian households are now less happy to shift consumption between periods. However, the interest rate exposure channel remains exactly the same size as before. It is determined by the change in the borrowing cost along with the size of the debt, both of which are unchanged. The aggregate income channel is also exactly the same multiple of the other channels in all three panels, as is the Fisher channel.[15] The earnings heterogeneity multiplier grows significantly with σ. This is because the markup, and hence firm profits, become *more* countercyclical with higher σ. Again, this feature of the standard New Keynesian model is undesirable and leads us here to be unable analyze the model under low elasticities of substitution that we believe to be more empirically reasonable.[16]

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[14]This is because inflation is proportional to the output gap in this model.

[15]The aggregate income multiplier is constant across both debt levels and intertemporal elasticity. The Fisher multiplier varies by debt level, but for any particular debt level it does not vary with intertemporal elasticity

[16]See Havranek (2015) for a meta-study for EIS estimates.

This brings us to a broader point: the calibration of the elasticity of intertemporal substitution (EIS) in the standard New Keynesian model has been chosen to match aggregate data, despite the little micro evidence we have suggesting a much lower level. Figure 3.2 shows why, in the absence of debt, a large EIS is required: with no debt the intertemporal substitution channel is the only 'kick' to aggregate demand, so if this is small we need very large multipliers to get a sizable consumption response to monetary policy. If we make the EIS small, we need something else to take its place. Interest rate exposure is another 'kick', that empirical evidence has shown could be large,[17]. By introducing interest rate exposure, we allow our models to use more micro-founded estimates of the EIS while still generating the kinds of aggregate responses estimated in the macro data.

3.5 Relaxing the Fixed Capital Assumption

We now relax the assumption of fixed capital and allow for investment. If there are no costs to investment, then households will invest until the new capital stock gives rise to the changed interest rate, which will result in a very persistent change in the interest rate. We will need convex investment adjustment costs to avoid this persistence, and hope to show that reasonable calibrations result in little change in the capital stock and hence low interest rate persistence.

3.5.1 The Model

The model is identical to the baseline model, except for the fact that the Ricardian households are now able to invest in capital as well as nominal bonds. Aggregate investment at time t, INV_t, along with the level of capital at time t, K_t, together determine the capital level at time $t+1$:

$$\text{INV}_t = \Phi\left(\frac{K_{t+1}}{K_t}\right) K_t \qquad (3.26)$$

where $\Phi(1) = \delta$ is the per period depreciation, $\Phi'(1) = 1$ and $\Phi''(1) = \psi_K > 0$ represents convex capital adjustment costs. It is the fact that capital in period $t+1$ is *predetermined* in period t that differentiates this model from the baseline model

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[17]See Auclert (2019) and Crawley and Kuchler (2019).

in terms of breaking the assumptions required for Auclert's decomposition to hold. In steady state the investment share of income is $\overline{inv} = \frac{\varepsilon-1}{\varepsilon}\frac{\delta\alpha}{1/\beta-(1-\delta)}$.[18]

3.5.2 Changes Relative to the Linear Baseline Model

Given nominal interest rate and inflation expectations, the individual optimization problems for both the Ricardian and non-Ricardian households, as well as firms, remains identical to the baseline model. That results in equations (3.17), (3.22) and (3.23) remaining unchanged. Differences occur in aggregation.

As the natural level of output (output that would occur with flexible prices) is no longer constant, the output gap, $\tilde{y}$, is no longer equal to output. The model needs equations to define the natural level output and the output gap, along with an adjusted New Keynesian Phillips curve:[19]

$$y^n = \frac{\alpha(1+\psi)}{\frac{\sigma(1-\alpha)}{\overline{c}_{NR}+\overline{c}_R}+\psi+\alpha}k_t + \frac{(1-\alpha)\sigma\frac{\overline{inv}}{\overline{c}_{NR}+\overline{c}_R}}{\frac{\sigma(1-\alpha)}{\overline{c}_{NR}+\overline{c}_R}+\psi+\alpha}inv_t \tag{3.27}$$

$$\tilde{y}_t = y_t - y^n \tag{3.28}$$

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\frac{\sigma}{\overline{c}_{NR}+\overline{c}_R}+\frac{\psi+\alpha}{1-\alpha}\right)\tilde{y}_t \tag{3.29}$$

Furthermore, the aggregate production function, equation (3.25), now includes capital:

$$y_t = \alpha k_t + (1-\alpha)n_t \tag{3.30}$$

Aggregation of output now includes the investment share, so equation (3.19) is replaced by:

$$y_t = \overline{c}_{NR}c_t^{NR} + \overline{c}_R c_t^R + \overline{inv}\,inv_t \tag{3.31}$$

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[18]This comes from equating the steady-state return from investment with $1/\beta$, the steady-state real interest rate, and using the fact that in equilibrium the total income allocated to capital is equal to $\frac{\alpha}{1-\alpha}$ times the total income allocated to labor. For other steady-state ratios, equation (3.16) remains the same, but now $\overline{c}_R = 1 - \overline{inv} - \overline{c}_K$, taking account of the fact that investment now takes a chunk out of output which is no longer equal to aggregate consumption.

[19]Natural output is derived from the fact that under flexible prices, the markup over marginal cost will be constant ($\frac{\varepsilon}{\varepsilon-1}$). Investment is taken as given.

The law of motion for capital is introduced to the model:

$$\delta \ inv_t = k_{t+1} - (1-\delta)k_t \tag{3.32}$$

As is the equation for the shadow price of capital, q_t, determined by the convexity of adjustment costs:

$$q_t = \psi_C(k_{t+1} - k_t) \tag{3.33}$$

Finally we require an equation to equate the expected return on investment with the expected real return on nominal bonds:

$$r_t + q_t = \beta(1-\delta)\mathbb{E}_t q_{t+1} + (1 - \beta(1-\delta))(\mathbb{E}_t(w_{t+1} + n_{t+1}) - k_{t+1}) \tag{3.34}$$

3.5.3   Results from the Model with Investment

For our partial equilibrium decomposition to approximate the aggregate consumption change, the shock to income, interest rates and inflation must be close to transitory. This poses a serious challenge for a model with capital, which is a slow moving variable. Figure 3.3 shows the problem. The figure displays the path of capital following a one percentage point negative shock to the nominal interest rate for different levels of capital adjustment convexity. Immediately we can see capital is a very persistent variable, with more than half of the increase in capital still present after six years. When there is no convexity in the capital adjustment costs ($\psi_c = 0$), the one percentage point decrease in the nominal rate results in a *large* positive increase in the quantity of capital. For typical values of ψ_c, often between one and three, the change is an order of magnitude smaller, while unsurprisingly capital remains unchanged in the case of infinite capital adjustment costs. This suggests the case of infinite adjustment costs will be similar to the fixed capital model. As we will see later this is mostly true, but the presence of positive investment makes the measurement of URE more subtle.

As figure 3.3 makes clear, capital is highly persistent. However, we can see in figure 3.4 that in the cases where $\psi_c \geq 1$ the nominal interest rate path along with the consumption path for non-Ricardian and Ricardian households is close to
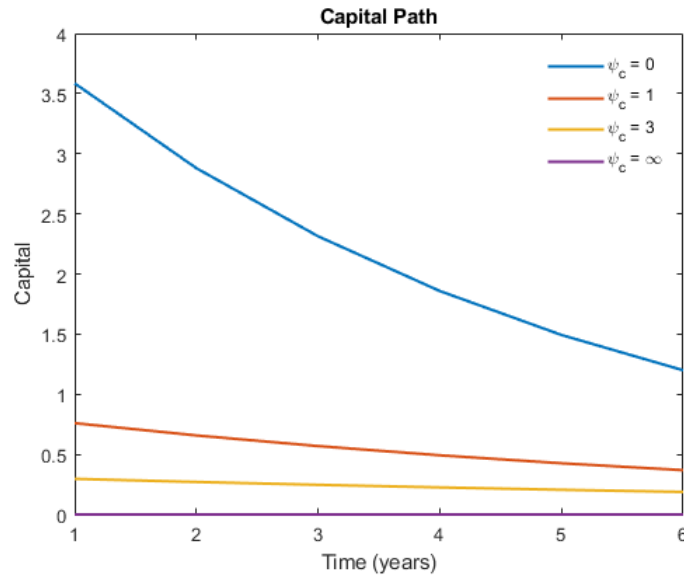
Figure 3.3: Path of Capital following a 1% nominal interest rate shock

transitory. First consider the paths from figure 3.4 in which there are no convex adjustment costs to capital. In this case a one percent decrease in the nominal rate is highly persistent, as the level of capital fully adjusts such that the marginal product of capital is lower by the change in the interest rate and this change persists. All this extra investment in the first period dramatically (and unrealistically) increases wages, which the non-Ricardian households consume that period. Ricardian households invest most of their extra income, which allows them to maintain a higher path of consumption going forward.

When convex adjustment costs are introduced things look very different. Ricardian housesholds can no longer increase capital to maintain consumption going forward, because the adjustment costs kick in. As a result the change in nominal interest rate is almost entirely transitory (in the case where $\psi_c = \infty$ this is exactly true), as is the change in consumption behavior of both types of household. Again, due to the countercyclical behavior of profits in the standard New Keynesian model, non-Ricardians react much more to the change than Ricardians.

In order to quantify how large of a deviation the model with capital is from the assumptions needed for our decomposition to work exactly, table 3.2 shows the percent difference between the true consumption change and that estimated using

the partial equilibrium decomposition. The table shows the error for total consumption, as well as the error individually calculated for both the Ricardian and non-Ricardian households. First note that the method correctly estimates the consumption changes for the non-Ricardian households. This is because their behavior only depends on current income and the persistence can only affect consumption through the intertemporal substitution channel and wealth effects, which in their case are always zero. The error for Ricardian households (and overall consumption change) is unsurprisingly large when there are no convex adjustment costs, but this quickly comes down for standard calibrations if ψ_c between 1 and 3. Furthermore, as the value of σ rises, and the intertemporal substitution channel gets relatively smaller, this error diminishes.

The existence of capital raises the question of whether we should be including a wealth effect as a separate channel through which monetary policy operates. With convex capital adjustment costs, a decrease in the interest rate will increase the value of existing capital and hence the wealth of capital holders. Indeed this is also the case in the baseline fixed capital model. In that model the stream of income to the Ricardians from the capital is offset by the stream of consumption generated by it. While the Ricardians increase their wealth when the price of capital increases, this is exactly offset by the increase in the value of their planned consumption. That is the increase in wealth does not allow them in increase their consumption in every period, it is instead an artifact of the fact that with a lower interest rate consumption today is relatively cheaper, that is the wealth effect is entirely subsumed in the intertemporal substitution effect.

The model with capital does not allow such an easy interpretation of the change in wealth, even in the case with infinite capital adjustment costs. This is because the Ricardians are consistently investing to offset depreciation. In our decomposition this saving counts as unhedged interest rate exposure because their return on investment will be equal to the real interest rate. However, investments that were already planned will not be subject to this higher price - it is the marginal investments that suffer from the convex adjustment costs. If we change our definition of unhedged
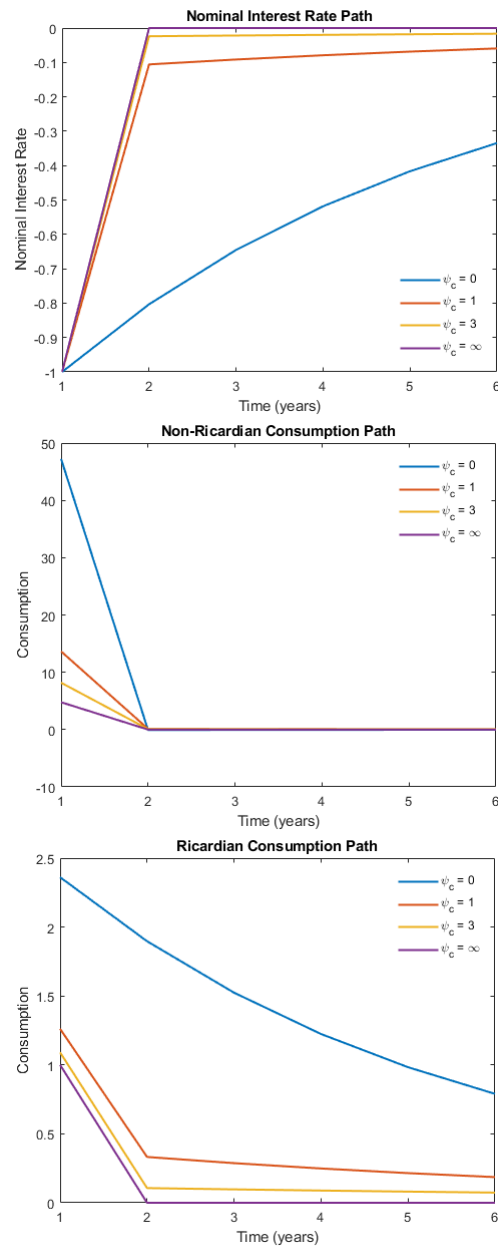
Figure 3.4: Paths following a 1% nominal interest rate decrease

ψ_c	Total Consumption	Ricardian Consumption	Keynesian Consumption
0	-18.3 %	-57.3 %	0.0 %
1	-8.8 %	-18.5 %	0.0 %
3	-4.0 %	-7.0 %	0.0 %
∞	-0.6 %	-0.8 %	0.0 %

Table 3.2: Percentage Error of Decomposition

interest rate exposure to exclude planned investment, then partial equilibrium de-composition gives no error for the model with infinite adjustment costs.

The change in the value of existing assets is shown in figure 3.5. With no ad-justment costs the value of assets remains constant over time as the consumption asset is freely exchangeable with capital next period. Similarly, with infinite ad-justment costs the price of a unit of capital next period moves one for one with the interest rate. For values in between the price of assets jumps up in period one fol-lowed by a persistent period in which capital adjusts back down to the steady state, and hence assets prices are low due to convex adjustment costs.

Overall the partial equilibrium decomposition works reasonably well with the addition of capital in the TANK model. However, in our model firms are risk neutral and it is clear that models in which firms are also able to have unhedged exposures to inflation and interest rates could complicate the transmission mechanism in quan-titatively important ways. This may be especially true with the introduction of the banking sector, which has been shown emprirically to hold large unhedged interest rate exposures.[20]

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[20]See Landier et al. (2013). While empirical evidence suggests households are negatively exposed to interest rate hikes, the financial sector seems to be positively exposed. This suggests the transmis-sion of monetary policy may be very different in times when the banking sector is working well to those when the banking sector is in crisis (when interest rate declines may not be as effective).
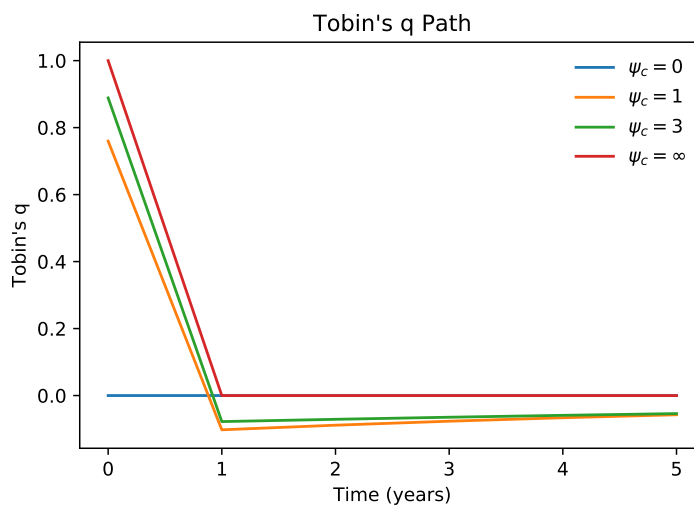
Figure 3.5: Path of Tobin's q following a 1% nominal interest rate shock

3.6 A Simple HANK Model

3.6.1 The Model

The model we study is a one asset version of the HANK model presented in Bayer et al. (2019). We also follow the solution method presented in that paper.

3.6.1.1 Households

In a given period household i has labor productivity h_{it}, chooses their consumption c_{it} and hours worked n_{it}. Households have a standard separable preferences and act in order to maximize their expected utility:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta_i^t (u(c_{it}) - \chi \nu(n_{it}))$$

where $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ and $\nu(n) = \frac{n^{1+\psi}}{1+\psi}$. Workers are assumed to be more impatient than entrepreneurs ($\beta_w < \beta_e$). The household's budget constraint is as in the following

$$c_{it} + b_{it+1} \leq \frac{R_{t-1}^n}{\Pi_t} b_{it} + (1 - \tau_t)[s_{it} \frac{W_t}{P_t} h_{it} n_{it} + (1 - s_{it}) \mathscr{F}_t], \qquad (3.35)$$

where $s_{it} = 1$ if household i is a worker and $s_{it} = 0$ otherwise. $\mathscr{F}_t$ represents profits from firms. Households' net nominal positions (NNP) and unhedged interest rate exposure (URE) correspond to b_i and $(1 - \tau_t)[s_i \frac{W_t}{P_t} h_i n_i + (1 - s_i) \mathscr{F}_t] + b_i - c_i$, respectively, in the steady state.

Households consume a consumption bundle formed according to a Dixit-Stiglitz aggregator:

$$c_{it} = \left(\int_{j=0}^{1} c_{ijt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The price of each good is p_{jt} resulting in the aggregate price level $P_t = \left(\int_{j=0}^{1} p_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ with demand for each good:

$$c_{ijt} = \left(\frac{p_{jt}}{P_t} \right)^{-\varepsilon} c_{it}$$

Household labor productivity evolves according to a $\log-AR(1)$ process, with a fixed probability that the household becomes an entrepreneur, receives no labor income, but instead collects a share of the firm profits:

$$h_{it} = \begin{cases} \exp(\rho_h h_{it-1} + \varepsilon_{it}^h) & \text{with prob } 1-\zeta \text{ if } h_{it-1} \neq 0 \\ 0 & \text{with prob } \iota \text{ if } h_{it-1} = 0 \\ 1 & \text{otherwise} \end{cases}$$

That is a non-entrepreneur switches to an entrepreneur state with probability ζ, while an entrepreneur switches to a non-entrepreneur with unit labor productivity with probability ι. In the entrepreneur state the household receives a fixed share of the economic profits of the firms, Π_t, and these rents are not tradeable. Households must pay a same fraction of their respective income as a tax τ_t in return for interest rate income from government bonds.

3.6.1.2 Price Setting

Prices are set by risk-neutral managers who form a group of measure zero.[21] We assume Rotemberg (1982b) pricing frictions, leading to a New Keynesian Phillips curve:

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \beta_e \mathbb{E}_t \left(\log\left(\frac{P_{t+1}}{P_t}\right) \frac{Y_{t+1}}{Y_t} \right) + \kappa \left(MC_t - \frac{\varepsilon-1}{\varepsilon} \right)$$

---

[21] Assuming the price setters are risk neutral makes the optimal price setting problem tractable without taking away from the important economics of the model.

where Y_t is total output in period t, MC_t is the real marginal cost and κ measures the size of the Rotemberg price frictions. In equilibrium all goods will have the same price.

3.6.1.3 Fiscal Policy

Our model assumes the government imposes an income tax on labor income as well as profits. Moreover, government implements the following government spending and tax rules.

$$\frac{G_t}{\bar{G}} = \left(\frac{G_{t-1}}{\bar{G}}\right)^{\rho_g} \left(\frac{B_t}{\bar{B}}\right)^{-\gamma_G}$$
$$\frac{\tau_t}{\tau} = \left(\frac{B_t}{\bar{B}}\right)^{\gamma_T}$$

where $\bar{G}$, $\bar{B}$ and $\bar{\tau}$ are government spending, debt and tax rate in the steady state, respectively. The government budget constraint reads

$$B_{t+1} = \frac{R_{t-1}}{\Pi_t} B_t + G_t - T_t \tag{3.36}$$

where $T_t = \tau_t(w_t N_t + \mathscr{F}_t)$.

3.6.1.4 Monetary Policy

As in the TANK model, we assume the central bank follows the Taylor rule given in equation (3.14).

3.6.1.5 Equilibrium

The labor market clears at the competitive prices. The government bond market clearing condition is as following.

$$\int b^d(b,h;\pi_t,R_t)\Theta_t(b,h)dbdh = B_{t+1} \tag{3.37}$$

where $\Theta_t(b,h)$ is the joint distribution of savings and the productivity of individual household. The aggregate resource constraint represents the clearing of goods market.

$$Y_t = C_t + G_t$$

The government budget constraint is then satisfied whenever the goods, labor and government bonds markets clear by the Walras' law.

3.6.2 Calibration

I set discount factors for entrepreneurs and workers as 0.99 and 0.975, respectively in order to target the average MPC of 0.224, which is equivalent to that in TANK. This value of the average MPC is also close to that in Auclert (2019). The large fraction of borrowing constrained households and their high MPC leads to this relatively high average MPC. The borrowing constraint in HANK is set to 0, thus, households buy government bonds or they are at the borrowing constraint without savings.[22] I set the weight on disutility of labor supply to 6 so as to target the aggregate labor supply of 40% out of the time endowment.

3.6.3 Results from the HANK model

3.6.3.1 Is a Monetary Policy Shock Transitory in the HANK Model?

While the aggregate level of government debt at the start of each period will only change in the initial period, the distribution of wealth may propagate through time leading to errors in our decomposition. Figure 3.6 shows the impulse response of a one percentage point decline in the nominal interest rate. It is clear the responses of aggregate consumption to the monetary policy shock, along with the output response, are transitory in nature. However, the inflation impulse response functions show some persistence, possibly due to the fact that the monetary policy decline acts as a wealth transfer from the wealthy to the poor, so higher interest rates are required to dampen demand back down to the steady state level. The one percentage point decline in nominal interest rates leads to about 2 percentage point increase in consumption.

3.6.3.2 Which Transmission Channels Are Important?

We rewrite the decomposition in section 3.2 in terms of the percentage change of aggregate consumption and the corresponding sufficient statistics for each channel. The heterogeneous income channel is written under the assumption that the elasticity of agent i's relative income to aggregate income is constant over households.

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[22]The reason for no borrowers is that the model cannot accommodate the change of the borrowing constraint depending on the change of interest rate in transition.

Parameters	Description	Value	targets
Households			
β_e	Discount factor of entrepreneurs	0.990	
β_w	Discount factor of workers	0.975	MPC: 0.224
σ	Elasticity of intertemporal substitution	1	
ψ	Inverse of Frisch elasticity	1	
χ	Weight on disutility of labor supply	6	L^{agg} : 0.40
$\underline{b}$	Borrowing limit	0	
Income process			
ρ_h	Persistence of productivity shock	0.979	
σ_h	Variance of productivity shock	0.059	
$P(E\ or\ B)$	Prob. to become entrepreneurs or bankers	0.0005	
$P(W)$	Probability to become workers again	0.0625	
Final Goods			
κ	Price stickiness	0.0883	Once a year
μ	Markup	0.05	5%
Government			
$\frac{B}{4Y}$	Government debt/annual output	50%	
τ	income tax rate	0.2	
ρ_g	persistence of spending	0.95	
γ_G	reaction to debt	0.02	
γ_T	reaction to debt	0.02	
Monetary policy			
ϕ_π^M, ρ_R	reaction to inflation, persistence	1.5, 0.0	

Table 3.3: Calibrated Parameters
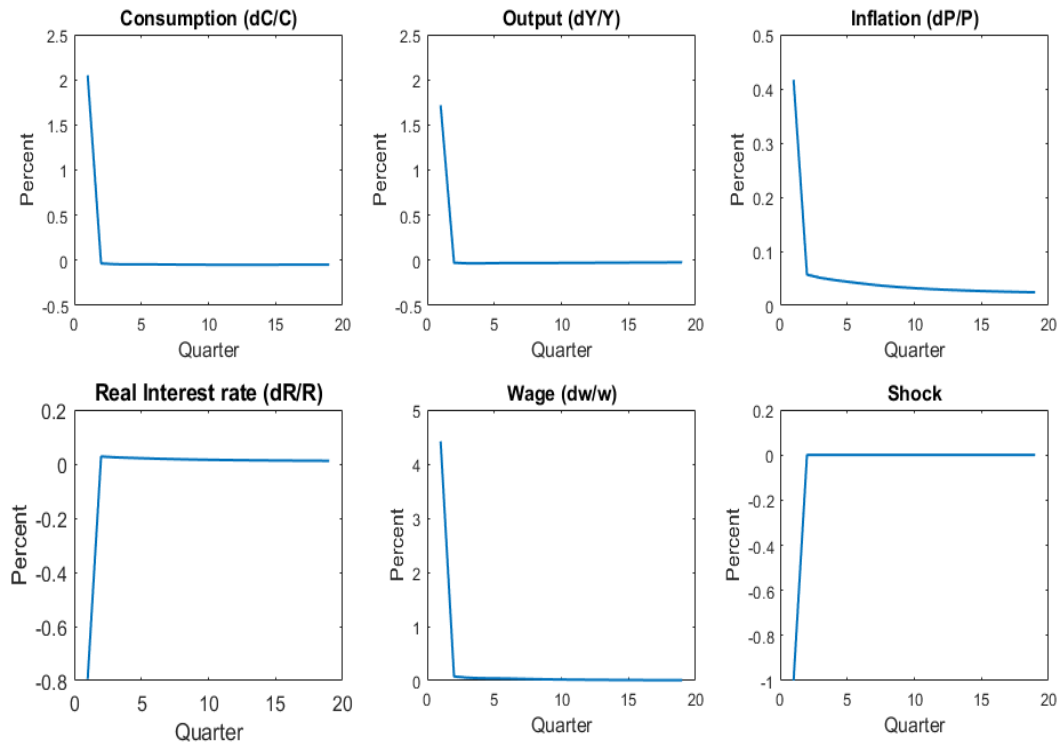
Figure 3.6: IRFs following a 1% Nominal Interest Rate Decline for the HANK Model

$$\frac{dC}{C} = \overbrace{\mathbb{E}_i[MPC_i \frac{Y_i}{C}] \frac{dY}{Y}}^{AggInc:\ \mathcal{M}} + \overbrace{\gamma Cov_i(MPC_i, \frac{Y_i}{C}) \frac{dY}{Y}}^{EarnHet:\ \gamma \mathcal{E}_Y} \overbrace{- Cov_i(MPC_i, \frac{NNP_i}{C}) \frac{dP}{P}}^{Fisher:\ -\mathcal{E}_{\mathcal{P}}}$$

$$\underbrace{+ Cov_i(MPC_i, \frac{URE_i}{C}) \frac{dR}{R}}_{IRE:\ \mathcal{E}_R} \underbrace{- \frac{1}{\sigma} \mathbb{E}_i[(1-MPC_i) \frac{c_i}{C}] \frac{dR}{R}}_{IntSubs:\ -\frac{\mathcal{L}}{\sigma}}$$

Table 3.4 displays sufficient statistics for the consumption response on impact of an expansionary monetary policy shock from the HANK model. We set the signs of the statistics considering the fact that output and price increase while real interest rate decreases for the monetary shock. First of all, the model succeeds in generating sufficient statistics with same signs as in the data, while the respective extent is somewhat different.

The aggregate income channel is summarized by $\mathcal{M}$ that we calculate to be 0.21. This means that if income for all households in the economy increased by 1%, aggregate consumption growth would increase by 21 basis points. $\gamma \mathcal{E}_Y$, which is calculated to be 0.29, implies that the consumption goes up by 29 basis point for

	$\mathcal{M}$	$\gamma\mathscr{E}_Y$	$-\mathscr{E}_p$	$\mathscr{E}_R$	$-\mathscr{S}$
Model	0.21	0.29	0.47	-0.47	-0.83
Data					
Danish registry data[1]	0.52	0.10[3]	0.75	-0.26	-0.49
SHIW[2]	0.57	0.17[3]	0.07	-0.11	-0.55
PSID[2]	0.08	0.14[3]	0.02	-0.05	-0.97
CE[2]	0.14	0.17[3]	0.11	-0.09	-0.90

1) From Crawley and Kuchler (2019). Refer to Appendix C.4 for details.

2) From Auclert (2019). Refer to Appendix C.5 for details.

3) $\gamma = -3.4$ (from Patterson (2019)) is applied.

Table 3.4: Sufficient statistics in the Model and Literature

the 1% increase of the aggregate income through the earning heterogeneity channel. The size of $\gamma\mathscr{E}_Y$ in the model is larger than those in data, which hints that the countercyclical profits in the New Keynesian model with the sticky price may produce a relatively large earning heterogeneity channel.

The $-\mathscr{E}_p$ is estimated to be 0.47 suggesting that a one-time increase in the price level of 1% increases aggregate consumption growth by 47 basis points due to the consumption rise of the poor. For example, the households at the borrowing constraint increase their consumption with respect to the price change since they face the lower tax rate due to the tax rule, while they do not lose their wealth from inflation. We estimate $\mathscr{E}_R$ and $-\mathscr{S}$ to be negative 0.47 and negative 0.83, respectively. This suggests that a 1% *decrease* in the interest rate raises aggregate household expenditure by 47 and 83 basis points through the interest rate exposure[23] and intertemporal substitution channel, respectively.

Table 3.5 shows how the period 1 consumption response divides into the five channels we have identified, as well as an error term that subsumes the persistent behavior. We see that the intertemporal substitution and heterogeneous income chan-

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[23]As with the Fisher channel, the poor increase their consumption in response to the drop of interest rate since they face the lower tax rate due to the decrease of the gross government debt.

nels are both relatively large. The aggregate income channel, along with the interest rate exposure and Fisher channels play a small role in the monetary policy transmission mechanism for this model.
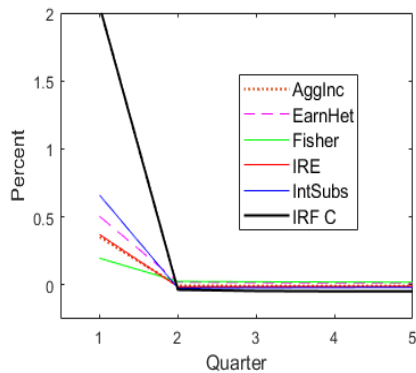

Aggregate Income	17.2%
Earnings Heterogeneity	24.6%
Fisher	9.6%
Interest Rate Exposure	18.1%
Intertemporal Substitution	32.2%
Error	0.5%

Figure 3.7   Table 3.5

Figure 3.8 exhibits the decomposition of the monetary policy transmission depending on the size of the inverse elasticity of intertemporal substitution, σ.[24] First of all, we find that the total size of impulse response of consumption tends to go up proportional to the size of σ when it is larger than 1. By looking at each channel, the intertemporal substitution and interest rate exposure channels are replaced by the other channel as σ rises. Lastly, all channels are ended up being dominated by the earnings heterogeneity channel when the size of CRRA is large enough, which is similar to the result in TANK.

3.7 Conclusion

Our paper shows that the transmission mechanism of monetary policy can look very different in a model with heterogeneity in household behavior. Our view of the empirical evidence is that the interest rate exposure channel is likely to be of primary quantitative importance. However, we have shown in this paper that such a mechanism is of limited importance in the standard TANK and HANK models in use today.

We believe much progress has been made recently in understanding the role

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[24]Refer to C.6 for different calibrations of β and $\frac{B}{Y}$
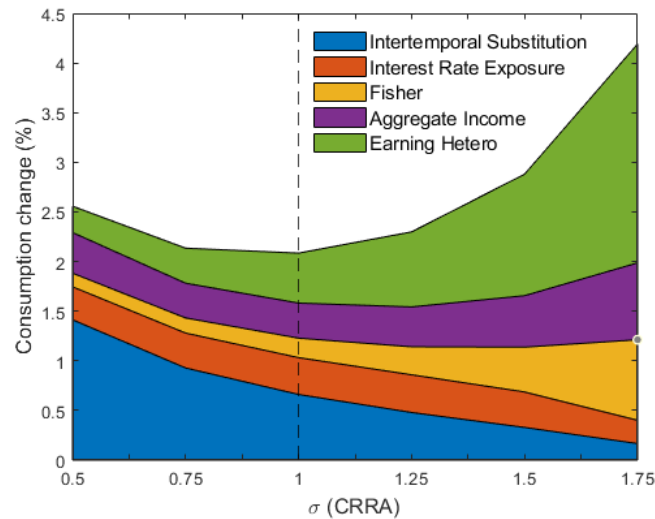
1) IRFs of consumption on impact of 1% monetary policy shock.

2) Black dotted line represents the baseline calibration.

Figure 3.8: Decomposition of consumption responses depending on σ

of consumption behavior in macroeconomic models. While there are clear gaps in our understanding, a path forward bridging both empirical results and theory is within sight. At present the dynamics of inflation, in our paper taken from the New Keynesian Phillips curve, remains a separate area of research to which we have little to add.

We believe future research should focus on reducing the countercyclical profits of New Keynesian models which leads to a large earnings heterogeneity channel in our models. Furthermore, finding models with small intertemporal substitution channels, while still maintaining determinacy, is of primary importance.

Appendix A

Appendix to Chapter 1

A.1 Further Impulse Responses

Figure A.1 reports the labor supply response of households at the 10th, 50th, and 90th percentile of the wealth distribution.

(a) Capital quality shock   (b) TFP shock   (c) Monetary shock
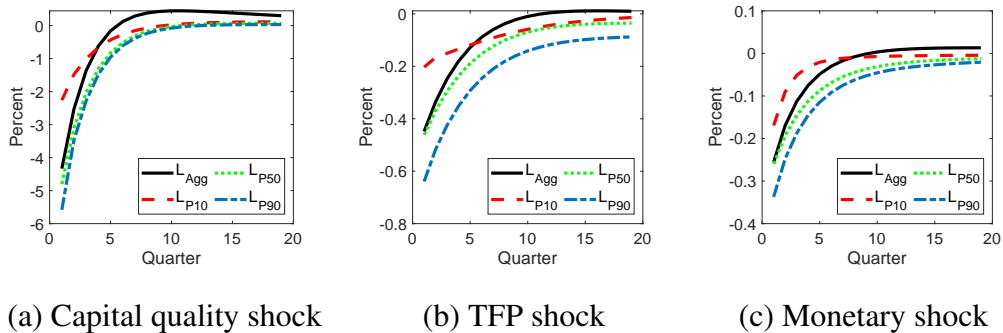
Figure A.1: Labor supply responses by percentiles

Figure A.2 reports the aggregate effects of credit policy in response to a capital quality shock for the economy without household heterogeneity.

A.2 Alternative Models of the Lending Rate

We consider two alternative formulations of the borrowing penalty that applies to household borrowing. We consider 1) an additive penalty that does not depend on the banking premium, and 2) the case of a penalty that is proportional to the banking premium. Our results are qualitatively robust and amplified in the latter case.
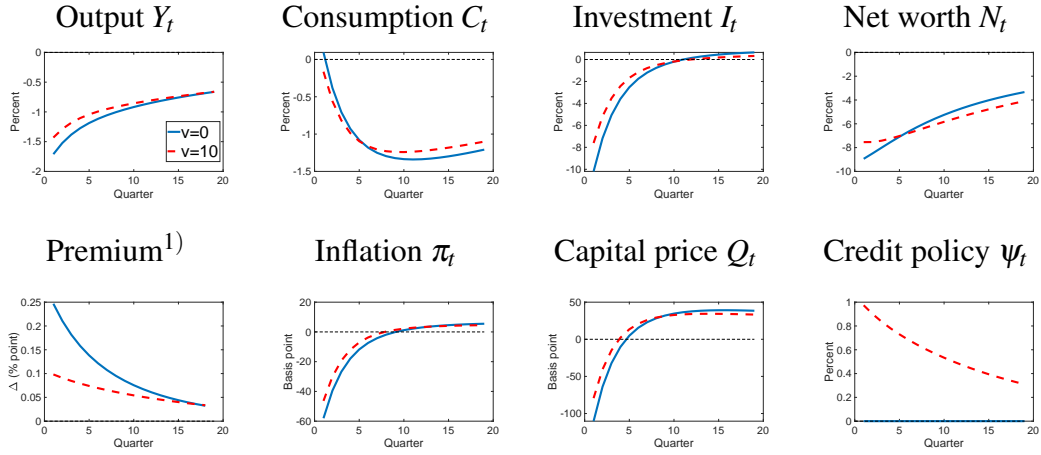
1) $E_t(R_{K,t+1} - R_{S,t+1})$.

Figure A.2: Aggregate effects of credit policy without household heterogeneity

A.2.1 Additive Borrowing Penalty

In this section, we assume that borrowing penalty is constant and added return on capital.[1] This means, we specify:

$$
\mathbf{R}\left(b_{it}, R_{S,t}, R_{K,t}\right) = \begin{cases} R_{S,t} & \text{if} \quad b_{it} = b_{D,it} + b_{G,it} \geq 0 \\ R_{K,t} + A & \text{if} \quad b_{it} = b_{L,it} < 0 \end{cases} \tag{A.1}
$$

Table A.1 presents the steady state distributions and interest rates for the baseline calibration and the low leverage calibration. We choose A such that we have the same lending rate as in the baseline model and hence the baseline steady state is unchanged. The low leverage steady state and its impact on consumption volatility are almost identical to the main text, see Figure A.3 and A.4.

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[1]We assume that this cost is wasted, thus, banks obtain the same lending interest rate $R_{K,t}$ from both firms and borrowing households.

	Baseline	**Low Leverage**	
		Additive penalty	Proportional penalty
Leverage	3.47	2.60	2.60
		Interest rates	
Return on capital (R_K, %)	1.14	1.17	1.17
Return on savings (R_S, %)	0.75	0.47	0.47
Lending interest rate (R_L, %)	2.69	2.73	3.97
		Aggregates	
Output	1.120	1.125	1.120
Capital	11.20	11.11	11.07
Labor	0.360	0.364	0.362
Consumption	0.738	0.733	0.734
		Household distribution	
At kink (%)	8.9	16.9	43.2
Borrowers (%)	24.3	30.9	0.8
Gini Wealth	0.921	0.961	0.940
Gini Consumption	0.259	0.273	0.269
Gini Income	0.320	0.325	0.321

Notes: We compare the baseline steady state to one with low leverage (a high divertibility parameter $\lambda = 0.558$).

Table A.1: Steady state: Baseline and low leverage with additive penalty

A.2.2 Borrowing Penalty Proportional to Spread

In this section, we assume that the borrowing penalty is proportional to the premium charged by banks, which is the difference between the interest rates on deposits and
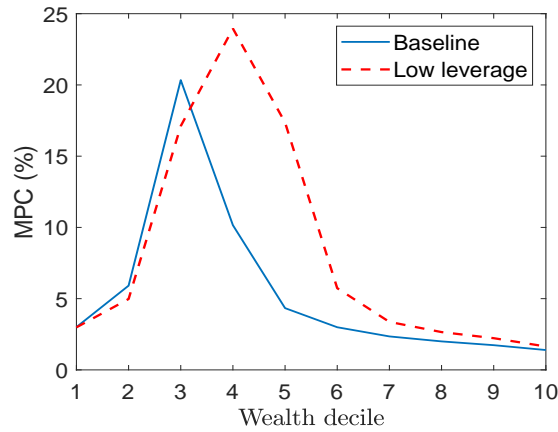
Figure A.3: MPC by wealth decile and leverage
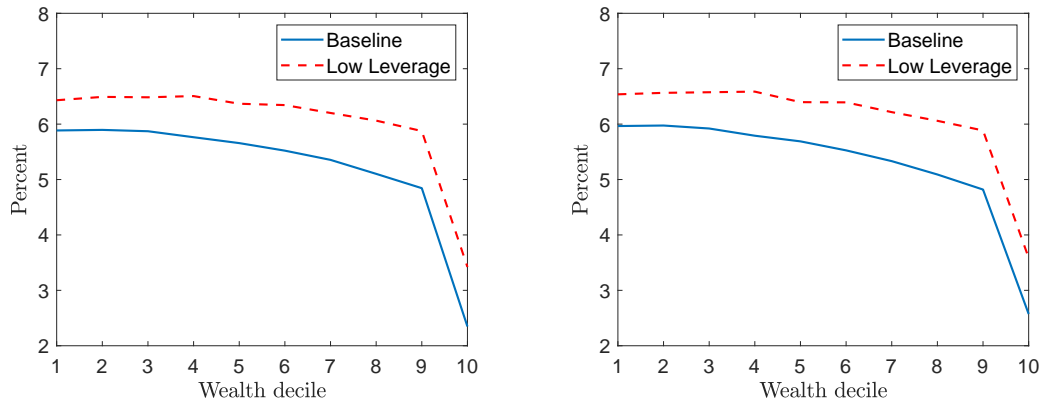
Only idiosyncratic shocks   Aggregate and idiosyncratic shocks

Notes: Volatility refers to the average standard deviation of quarterly growth rates of household consumption for a simulation of length five years computed over 100.000 individuals.

Figure A.4: Micro consumption volatility by wealth deciles

capital.[2] This means, we specify:

$$\mathbf{R}\left(b_{it}, R_{S,t}, R_{K,t}\right) = \begin{cases} R_{S,t} & \text{if} \quad b_{it} = b_{D,it} + b_{G,it} \geq 0 \\ A(R_{K,t} - R_{S,t}) & \text{if} \quad b_{it} = b_{L,it} < 0 \end{cases} \tag{A.2}$$

This formulation captures the idea that the marginal cost of issuing consumer loans might increase with lower leverage. Now, the lending rate responds substantially more to aggregate shocks. As a result, the output response to a capital quality

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[2]We assume that this cost is wasted, thus, banks obtain the same lending interest rate R_t^k from both firms and borrowing households.

shock almost doubles, see Figure A.5. This is driven by a stronger fall in consumption. Aggregate consumption falls by 1.5 percent, and consumption of borrowers falls by 15 percent, see Figure A.6 panel A. Looking at the decomposition of aggregate consumption in Figure A.6 panel B, the lending rate now explains a sizable fraction of the fall of aggregate consumption. Hence, our results on the transmission mechanism are even stronger.

When it comes to macroprudential regulation, a reduction of 25 percent in leverage increases the spread between the interest rates on savings and debt by 616 basis points (annualized), see Table A.1. This leads to a sizable increase in marginal propensities to consume, see Figure A.7. The volatility of consumption, therefore, increases substantially more across steady states, see Figure A.8. However, for poor households, the volatility of consumption with aggregate shocks is lower with low leverage. Aggregate stabilization is more important here because of the strong incidence of aggregate shocks on borrowers.
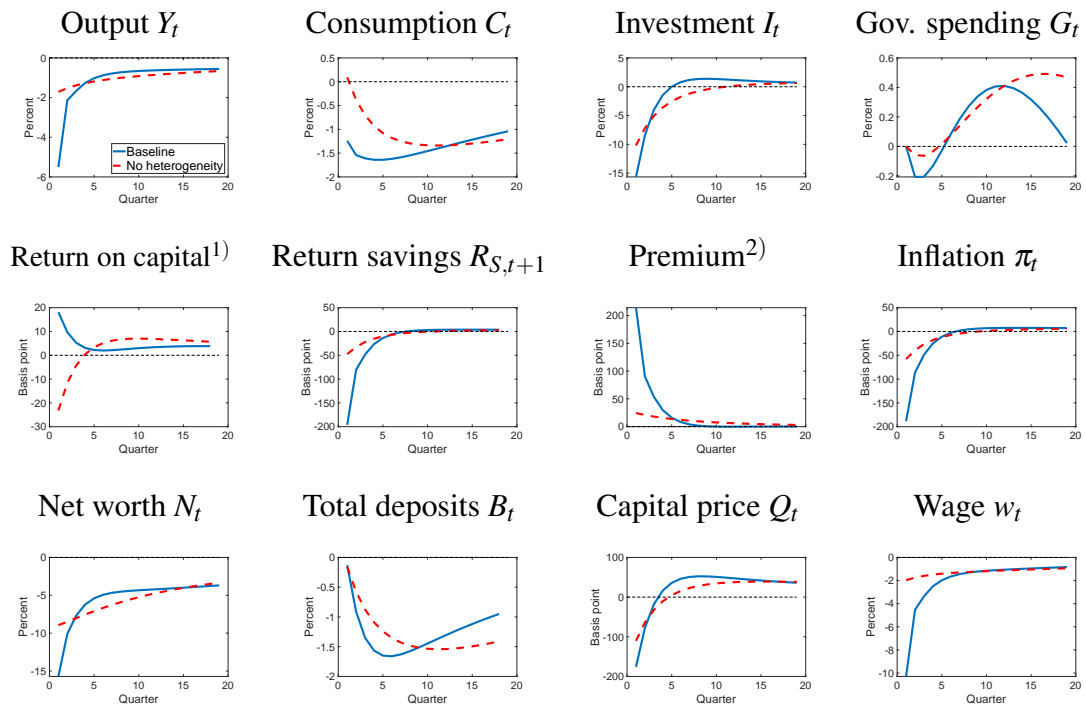
1) $R_{K,t+1}$

2) $E_t(R_{L,t+1} - R_{S,t+1})$ in Baseline and $E_t(R_{K,t+1} - R_{S,t+1})$ with No heterogeneity.

Figure A.5: Aggregate effects of a capital quality shock
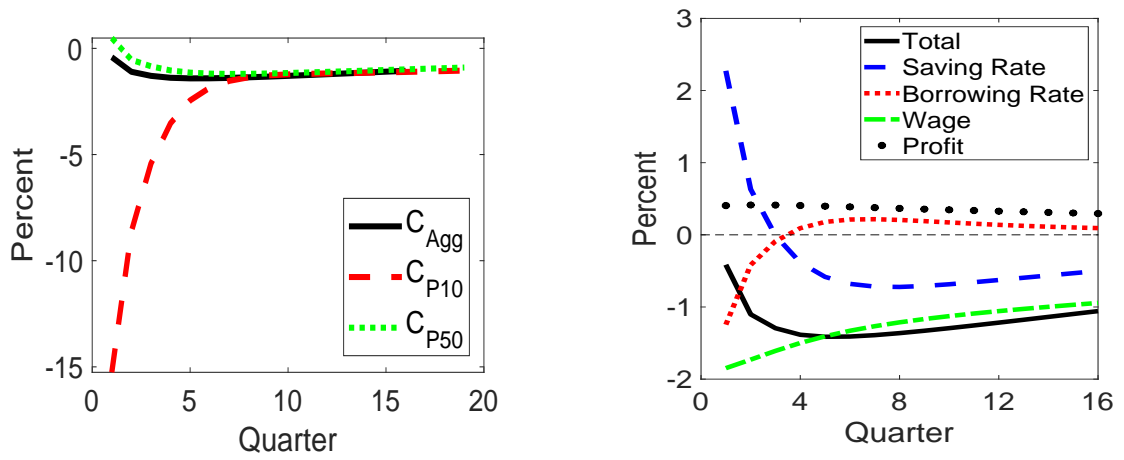
A) Consumption responses by wealth   B) Decomposition of aggregate consumption

Figure A.6: Transmission to consumption: Capital quality shock
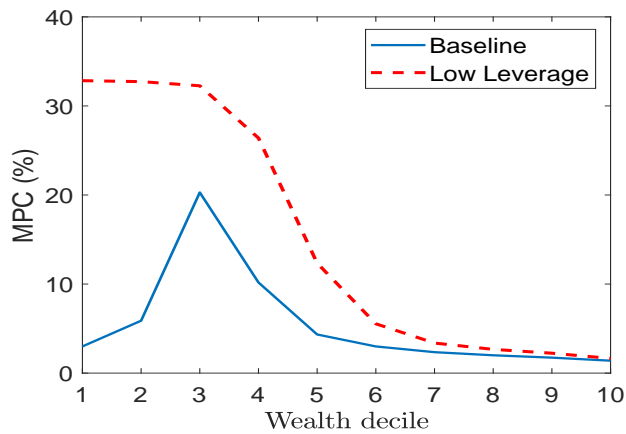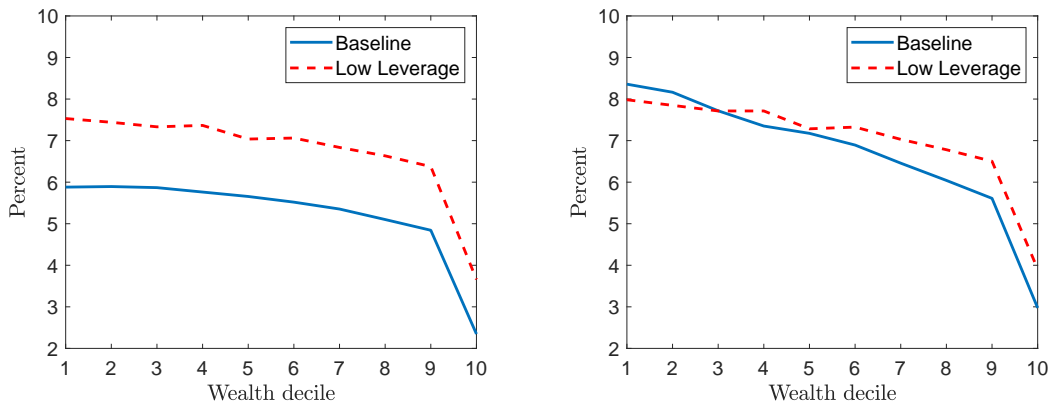
Figure A.7: MPC by wealth decile and leverage

<div align="center">

Only idiosyncratic shocks   Aggregate and idiosyncratic shocks

</div>

Notes: Volatility refers to the average standard deviation of quarterly growth rates of household consumption for a simulation of length five years computed over 100.000 individuals.

<div align="center">

Figure A.8: Micro consumption volatility by wealth deciles

</div>

Appendix B

Appendix to Chapter 2

B.1 Equilibrium conditions

B.1.1 Households

$$c_{o,s} + \frac{B_{o,s}}{p_s} = w_{o,s}n_{o,s} + \mu(1 - n_{o,s}) + \frac{\Phi_{o,s}}{p_s} - \frac{T_{o,s}}{p_s} + R_s\frac{B_{o,s-1}}{p_s}$$

$$c_{o,s}^{-\sigma} = \beta E_s[c_{o,s+1}^{-\sigma}\frac{R_s}{\pi_{s+1}}]$$

$$c_{r,s} = w_{r,s}n_{r,s} + \mu(1 - n_{r,s}) - \frac{T_{r,s}}{p_s}$$

B.1.2 Firms

$$\frac{\kappa}{q_s} = mc_sz_s - w_s + \beta E_s\{\frac{\lambda_{o,s+1}}{\lambda_{o,s}}(1 - \rho)\frac{\kappa}{q_{s+1}}\}$$

$$[(1 - \varepsilon) + mc_s\varepsilon] = \psi(\pi_s - 1)\pi_s - \beta E_s\{\frac{\lambda_{o,s+1}}{\lambda_{o,s}}\psi(\pi_{s+1} - 1)\pi_{s+1}\frac{y_{s+1}}{y_s}\}, \; y_s = z_sn_s$$

B.1.3 Labor market

Under lump-sum tax regime,

$$w_s = \zeta[mc_sa_s + \beta(1 - \rho)E_s\eta_{s+1}(\frac{\lambda_{o,s+1}}{\lambda_{o,s}}\frac{\kappa}{q_{s+1}}))]$$

$$+ (1 - \zeta)[\mu + (1 - \rho)\xi\beta E_s(1 - \eta_{s+1})\frac{\lambda_{r,s+1}^h}{\lambda_{r,s+1}}(\frac{\lambda_{o,s+1}}{\lambda_{o,s}} - \frac{\lambda_{r,s+1}}{\lambda_{r,s}})]$$

Under income tax regime,

$$w_s = \zeta [mc_s a_s + \beta(1-\rho)E_s\Big(\frac{\lambda_{o,s+1}}{\lambda_{o,s}}\frac{\kappa}{q_{s+1}}(1-(1-\eta_{s+1})\frac{(1-\tau^i_{s+1})}{(1-\tau^i_s)}))\Big]$$

$$+ (1-\zeta)\Big[\mu + (1-\tau^i_s)^{-1}(1-\rho)\xi\beta E_s(1-\eta_{s+1})\frac{\lambda^h_{r,s+1}}{\lambda_{r,s+1}}\Big(\frac{\lambda_{o,s+1}}{\lambda_{o,s}} - \frac{\lambda_{r,s+1}}{\lambda_{r,s}}\Big)\Big]$$

$$n_s = (1-\rho)n_{s-1} + me_s^\varphi v_s^{1-\varphi}, \quad e_s = u_{s-1} + \rho n_{s-1}, \quad u_s = 1-n_{s-1}$$

B.1.4 Government

$$\tau_s + b_s = g_s + \mu(1-n_s) + R_{s-1}b_{s-1}/\pi_s$$

B.2   Real Wage Schedule

B.2.1   Lump-sum tax

The Nash bargaining process maximizes the weighted product of the parties' surpluses from employment

$$\max_{w_s}(\lambda_s^h)^{\zeta}(F_s^j)^{1-\zeta} = \max_{w_s}\left[(1-\xi)\frac{\lambda_{o,s}^h}{\lambda_{o,s}} + \xi\frac{\lambda_{r,s}^h}{\lambda_{r,s}}\right]^{\zeta}(F_s^j)^{1-\zeta} \tag{B.1}$$

The optimality condition with respect to wage gives the following

$$\lambda_s^h = \frac{\zeta}{1-\zeta}F_s^j \tag{B.2}$$

Since the Bellman equations for savers (i=o) and spenders (i=r) are

$$W_{i,s} = \max_{c_{i,s}}\left\{\frac{c_{i,s}^{1-\sigma_c}}{1-\sigma_c} + \beta E_s W_{i,s+1}\right\} \tag{B.3}$$

subject to their budget constraints ((2.3) and (2.6)), respectively, the value of additional employment for savers and spenders ($\frac{\partial W_{i,s}}{\partial n_s}(=\lambda_{i,s}^h)$) can be written as

$$\lambda_{i,s}^h = \lambda_{i,s}(w_s - \mu) + \beta E_s\{(1-\rho)(1-\eta_{s+1})\lambda_{i,s+1}^h\} \tag{B.4}$$

We know that F_s^j is equal to $\frac{\kappa}{q_s}$, thus, we can rewrite the optimality condition (B.2) as the following:

$$\frac{\zeta}{1-\zeta}\frac{\kappa}{q_s} = (w_s - \mu) + \beta E_s\{\frac{\lambda_{o,s+1}}{\lambda_{o,s}}(1-\rho)(1-\eta_{s+1})\frac{\lambda_{o,s+1}^h}{\lambda_{o,s+1}}\}$$
$$+ \beta E_s\{\frac{\lambda_{r,s+1}}{\lambda_{r,s}}(1-\rho)(1-\eta_{s+1})\frac{\lambda_{r,s+1}^h}{\lambda_{r,s+1}}\}$$
$$= (w_s - \mu) + (1-\rho)\frac{\beta}{\lambda_{o,s}}E_s(1-\eta_{s+1})\left[(1-\xi)\lambda_{o,s+1}^h + \xi\frac{\lambda_{o,s}}{\lambda_{r,s}}\lambda_{r,s+1}^h\right] \tag{B.5}$$

The terms in square bracket in the above equation are equal to

$$(1-\xi)\lambda_{o,s+1}^h + \xi\frac{\lambda_{o,s+1}}{\lambda_{r,s+1}}\lambda_{r,s+1}^h + \xi\frac{\lambda_{o,s}}{\lambda_{r,s}}\lambda_{r,s+1}^h - \xi\frac{\lambda_{o,s+1}}{\lambda_{r,s+1}}\lambda_{r,s+1}^h$$
$$= \lambda_{o,s+1}\lambda_{s+1}^h + \xi\lambda_{r,s+1}^h\left(\frac{\lambda_{o,s}}{\lambda_{r,s}} - \frac{\lambda_{o,s+1}}{\lambda_{r,s+1}}\right)$$
$$= \frac{\zeta}{1-\zeta}\lambda_{o,s+1}\frac{\kappa}{q_{s+1}} + \xi\lambda_{r,s+1}^h\left(\frac{\lambda_{o,s}}{\lambda_{r,s}} - \frac{\lambda_{o,s+1}}{\lambda_{r,s+1}}\right)$$

Therefore, the right hand side of the equation (B.5) is equal to

$$(w_s - \mu) + (1-\rho)\beta E_s (1 - \eta_{s+1}) \frac{\lambda_{o,s+1}}{\lambda_{o,s}} \frac{\zeta}{1-\zeta} \frac{\kappa}{q_{s+1}}$$

$$+ (1-\rho)\beta E_s (1 - \eta_{s+1}) \xi \left[\frac{\lambda_{r,s+1}^h}{\lambda_{r,s+1}} \left(\frac{\lambda_{r,s+1}}{\lambda_{r,s}} - \frac{\lambda_{o,s+1}}{\lambda_{o,s}} \right) \right]$$

After rearranging terms in (B.5), we can finally get the expression for real wage as:

$$w_s = \zeta \left[mc_s a_s + \beta(1-\rho) E_s \eta_{s+1} \left(\frac{\lambda_{o,s+1}}{\lambda_{o,s}} \frac{\kappa}{q_{s+1}} \right) \right)]$$

$$+ (1-\zeta)\left[\mu + (1-\rho)\xi\beta E_s (1 - \eta_{s+1}) \frac{\lambda_{r,s+1}^h}{\lambda_{r,s+1}} \left(\frac{\lambda_{o,s+1}}{\lambda_{o,s}} - \frac{\lambda_{r,s+1}}{\lambda_{r,s}} \right) \right]$$

(B.6)

where ξ is the fraction of spenders.

B.2.2 Income tax

Under the income tax regime,

$$\lambda_s^h = \frac{\zeta}{1-\zeta}(1 - \tau_s^i)F_s^j$$

Therefore, the wage under the income tax regime can be written as following:

$$w_s = \zeta \left[mc_s a_s + \beta(1-\rho) E_s \left(\frac{\lambda_{o,s+1}}{\lambda_{o,s}} \frac{\kappa}{q_{s+1}} (1 - (1 - \eta_{s+1}) \frac{(1-\tau_{s+1}^i)}{(1-\tau_s^i)}) \right) \right]$$

$$+ (1-\zeta)\left[\mu + (1-\tau_s^i)^{-1}(1-\rho)\xi\beta E_s (1 - \eta_{s+1}) \frac{\lambda_{r,s+1}^h}{\lambda_{r,s+1}} \left(\frac{\lambda_{o,s+1}}{\lambda_{o,s}} - \frac{\lambda_{r,s+1}}{\lambda_{r,s}} \right) \right]$$

(B.7)

B.3 The effect of fiscal policy

In this section, we consider the role of fiscal policy with a positive debt level and a tax policy. we compare the results in the baseline model with those in the model with a fiscal policy. Figure B.1 and B.2 show impulse responses to a TFP- and a government spending-shock in TANKs depending on the level of debt. Blue (red dashed) line displays impulse responses in the base line (a model with positive debts).
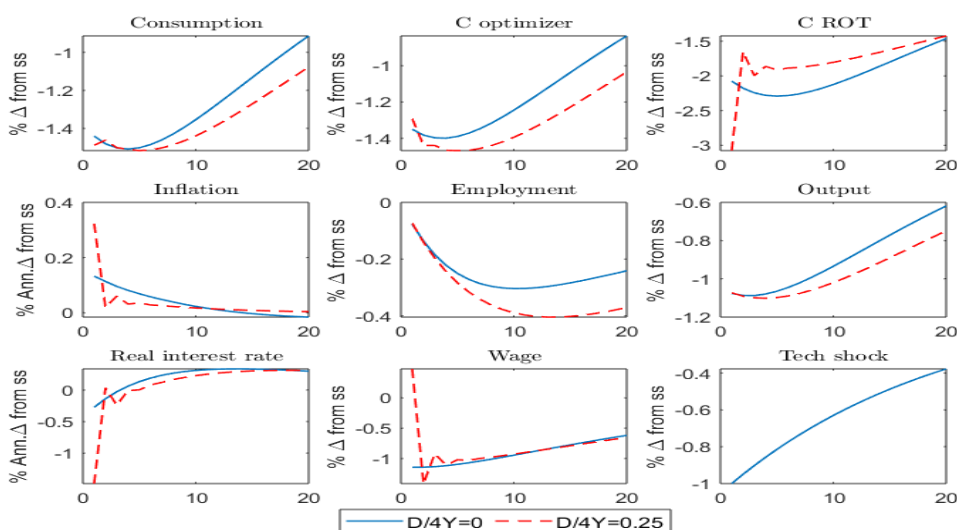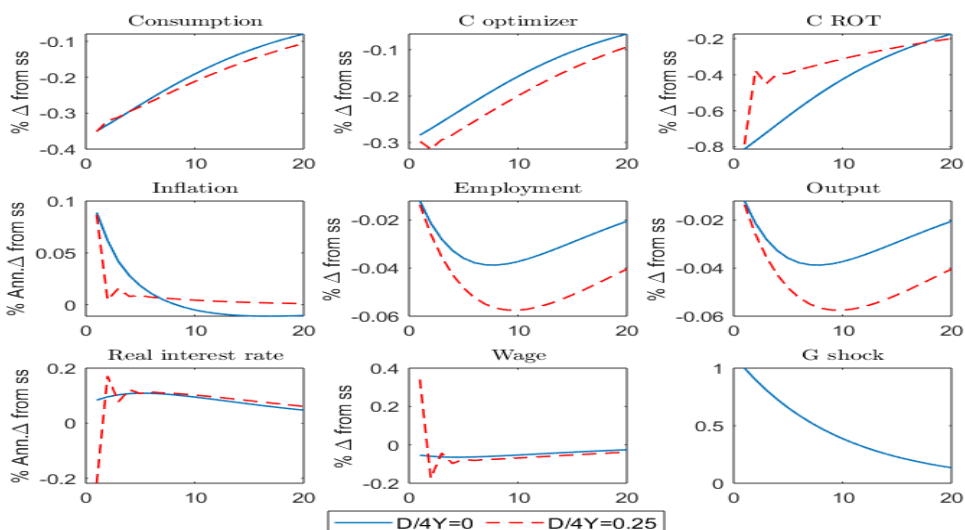

Figure B.1: TANK vs TANK (without vs with fiscal policy, TFP)

Figure B.2: TANK vs TANK (without vs with fiscal policy, Gov. spending)

B.4 Ad hoc wage rule instead of Nash bargaining

Individually different level of asset holdings would affect the worker's bargaining position and thus wage. In order to check the problem of multi-worker Nash bargaining, I consider the following ad hoc wage rule, which is similar to that in Den Haan et al. (2018).

$$w_s = w^{ss} \left(\frac{a_s}{a^{ss}}\right)^{\omega_a} \left(\frac{mc_s}{mc^{ss}}\right)^{\omega_{mc}} \tag{B.8}$$

Figure B.3 and B.4 show impulse responses to a TFP shock or a government spending shock, respectively. I calibrate both ω_a and ω_{mc} to 1 in order to prevent sticky wage. We find that both ways of wage determination lead to the same result in regard to the optimal response of inflation.
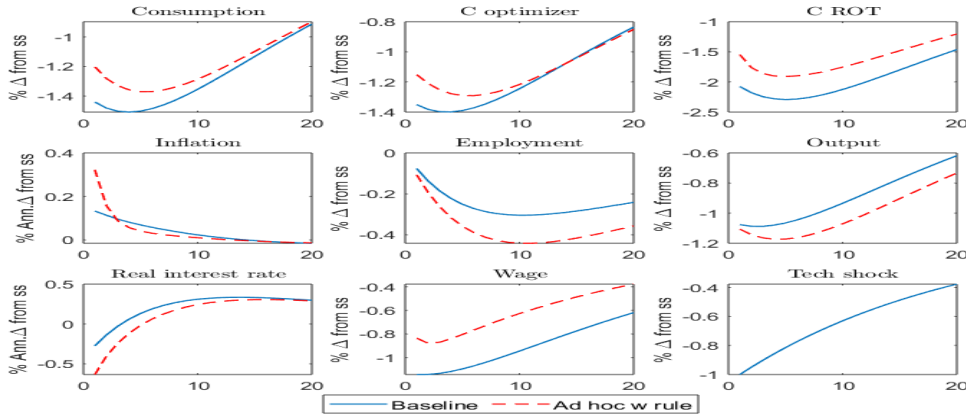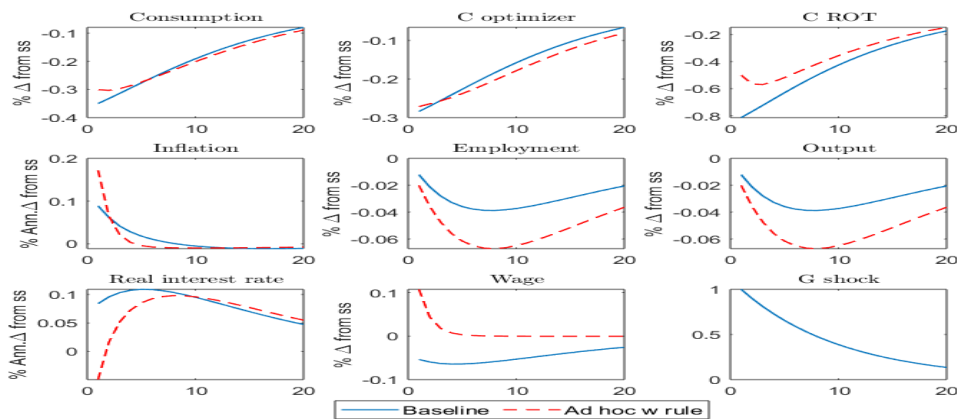

Figure B.3: Nash bargaining vs Wage rule (TFP shock)

Figure B.4: Nash bargaining vs Wage rule (Gov. spending shock)

B.5   A standard TANK model with hours worked

This appendix describes a standard TANK model as in Galí et al. (2007). The period utility function, which is common to households, is a separable preference such as

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi\frac{N_t^{1+\gamma}}{1+\gamma} \tag{B.9}$$

A continuum of unions exists and each of which represents workers of a certain type. Effective labor input hired by firm j is a CES function of the quantities of the different labor types employed,

$$N_t(j) = \left(\int_o^1 N_t(j,i)^{\frac{\varepsilon_w-1}{\varepsilon_w}} di \right)^{\frac{\varepsilon_w}{\varepsilon_w-1}} \tag{B.10}$$

where ε_w is the elasticity of substitution across different types of households. The fraction of rule-of-thumb and optimizing consumers is uniformly distributed across worker types. Each period, a typical union sets the wage for its workers in order to maximize the objective function

$$\xi\left[\frac{w_t(z)N_t(z)}{C_{r,t}^\sigma} - \frac{N_t(z)^{1+\gamma}}{1+\gamma}\right] + (1-\xi)\left[\frac{w_t(z)N_t(z)}{C_{o,t}^\sigma} - \frac{N_t(z)^{1+\gamma}}{1+\gamma}\right]$$

subject to a labor demand schedule

$$N_t(z) = \left(\frac{w_t(z)}{w_t}\right)^{-\varepsilon_w} N_t$$

The first order condition of this problem can be written as follows after invoking symmetry.

$$\left(\frac{\xi}{C_{r,t}^\sigma N_t^\gamma} + \frac{1-\xi}{C_{o,t}^\sigma N_t^\gamma}\right)w_t = \frac{\varepsilon_w}{\varepsilon_w - 1} \tag{B.11}$$

I assume that the wage markup $\mu_w (= \frac{\varepsilon_w}{\varepsilon_w-1})$ is sufficiently large so that the condition $w_t \geq C_{j,t}^\sigma N_t^\gamma$ for j=r,o are satisfied for all t. Both conditions guarantee that both type of households will be willing to meet firms' labor demand at the prevailing wage. The problem of intermediate goods firm is as following.

$$\max_{\{p_s^j\}} E_s \sum_{s=0}^\infty \beta^s \frac{c_{o,t+s}^{-\sigma}}{c_{o,t}^{-\sigma}} \left\{ \left(\frac{p_s^j}{p_s} - mc_s\right)y_s^j - \frac{\phi}{2}\left(\frac{p_s^j}{p_{s-1}^j} - 1\right)^2 y_s \right\}$$

where $y_s^j = \left(\frac{p_s^j}{p_s}\right)^{-\varepsilon} y_s$ and $mc_s = \frac{w_s}{a_s}$ is the real marginal cost.

Figure B.5 and B.6 compare impulse responses in TANK of the search and matching friction with those in TANK of hours worked. We find that the inflation response in TANK with hours worked more closes to price stabilization since rule-of-thumb consumer can better smooth their consumption through adjustment of hours worked.
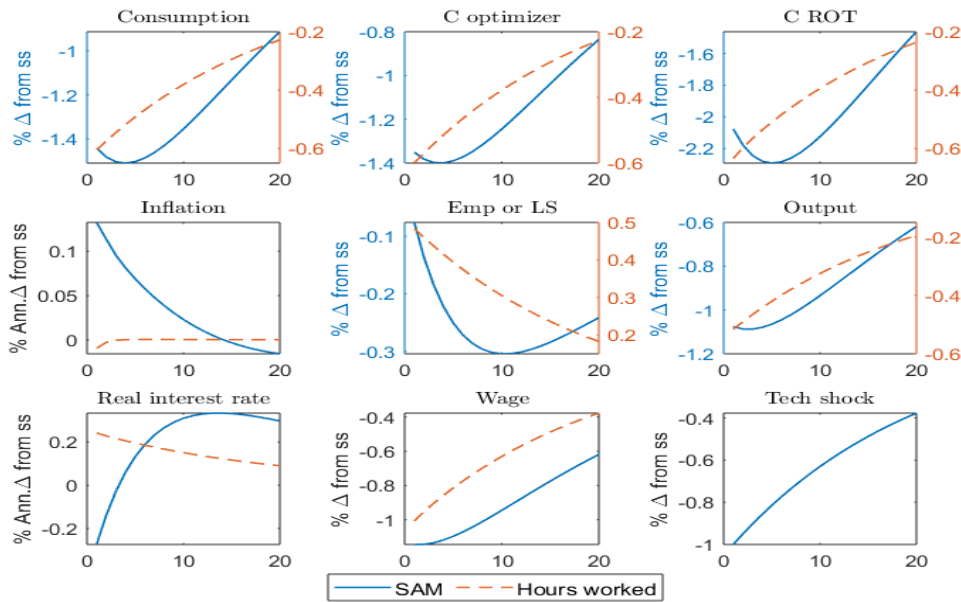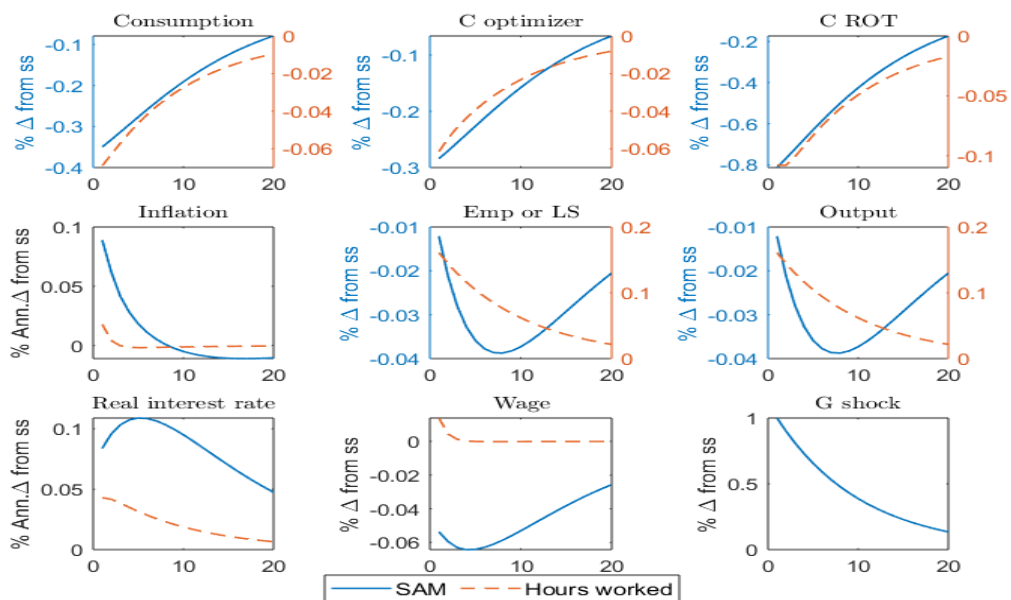

Figure B.5: TANK vs TANK (TFP shock and lump-sum tax)

Figure B.6: TANK vs TANK (G shock and lump-sum tax)

B.6   Impulse responses to a shock on matching efficiency

Figure B.7 shows impulse responses with respect to a negative shock on the matching efficiency (m) in the matching function. The persistence is 0.91 that is same value in the job separation shock. The negative shock on matching efficiency leads to larger responses of inflation than those with respect to a job separation shock in both TANK and RANK.
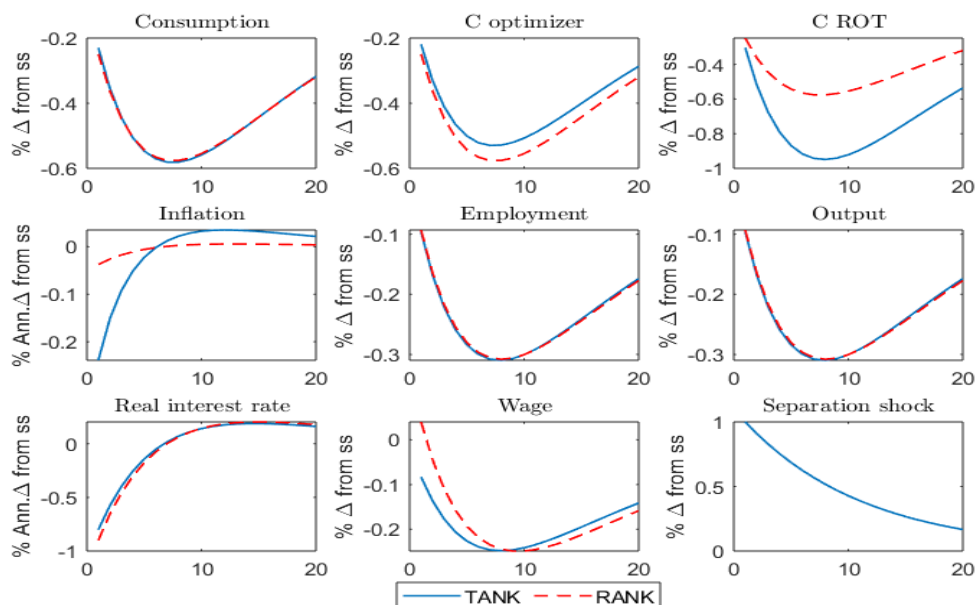

Figure B.7: TANK vs RANK (matching efficiency shock)

Appendix C

Appendix to Chapter 3

C.1 Decomposition in TANK

Auclert (2019) shows that the aggregate consumption response to a monetary policy shock can be decomposed as the sum of five channels: Aggregate income, earning heterogeneity, interest rate exposure, Fisher and intertemporal substitution channels. We calculate each channel in TANK as the following.

1. $AggInc = E_i(MPC_iY_i)\dfrac{dY}{Y} = (MPC_{NR}\bar{y}_{NR} + MPC_R\bar{y}_R)\dfrac{dY}{Y}$

where $MPC_{NR} = 1$, $MPC_R = 1 - \beta$, $\bar{y}_{NR} = \dfrac{\lambda \bar{Y}_{NR}}{\bar{Y}}$ and $\bar{y}_R = \dfrac{(1-\lambda)\bar{Y}_R}{\bar{Y}}$.

2. $EarnHet = E_i(MPC_i dYi) - E_i(MPC_iY_i)\dfrac{dY}{Y}$

$$= MPC_{NR}dY_{NR}\dfrac{\lambda}{\bar{Y}} + MPC_R dY_R\dfrac{1-\lambda}{\bar{Y}} - AggInc$$

3. $IRE = E_i(MPC_i URE_i)\dfrac{dR}{R}$

$$= (MPC_{NR}URE_{NR} + MPC_R URE_R)\dfrac{dR}{R}$$

where $URE_{NR} = -\beta\Omega\bar{y}_{NR}$ and $URE_R = -URE_{NR}$.

4. $Fisher = -E_i(MPC_i NNP_i)\dfrac{dP}{P}$

$$= (MPC_{NR} NNP_{NR} + MPC_R NNP_R)\dfrac{dP}{P}$$

where $NNP_{NR} = -\Omega \bar{y}_{NR}$ and $NNP_R = -NNP_{NR}$.

5. $IntSub = E_i(\dfrac{1}{\sigma_i}(1 - MPC_i)C_i)\dfrac{dR}{R}$

$$= \dfrac{1}{\sigma}[(1 - MPC_{NR})\bar{c}_{NR} + (1 - MPC_R)\bar{c}_R]\dfrac{dR}{R}$$

where $\bar{c}_{NR} = \dfrac{\lambda \bar{C}_{NR}}{\bar{C}}$ and $\bar{c}_R = \dfrac{(1-\lambda)\bar{C}_R}{\bar{C}}$.

C.2 Equilibrium conditions

(Consumption Euler equation)

$$(C_t^R)^{-\sigma} = \beta E_t[(C_{t+1}^R)^{-\sigma}\dfrac{R_t^n}{\Pi_{t+1}}]$$

(Production)

$$Y_t = A_t \bar{K}^\alpha N_t^{1-\alpha} \text{ or } Y_t = A_t K_t^\alpha N_t^{1-\alpha}$$

(Labor supply)

$$w_t = \mathscr{M}^\omega (C_t)^\sigma (N_t)^\psi$$

(Non-Ricaridan's budget constraint)

$$C_t^{NR} = N_t^{NR} w_t + \left(I_t^{-1}\dfrac{\mathbb{E}_t P_{t+1}}{P_t} - \dfrac{\mathbb{E}_{t-1}P_t}{P_t} \right) \Omega \bar{N}_{NR}\overline{W/P}$$

(Final goods market clearing)

$$Y_t = C_t \text{ or } Y_t = C_t + INV_t$$

(Labor market clearing)

$$N_t = N_t^{NR} = N_t^R$$

(No arbitrage condition)

$$E_t \left[R_t - \frac{R_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t} \right] = 0 \text{ where } R_t = \frac{R_t^n}{\pi_{t+1}} \text{ and } R_t^k = w_t \frac{\alpha}{1-\alpha} \frac{L_t}{K_t}$$

(Capital goods market clearing)

$$Q_t = 1 + \psi_C \left(\frac{\Delta K_{t+1}}{K_t} \right)$$

(Taylor rule)

$$\frac{R_t^n}{\bar{R}^n} = \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} exp(v_t)$$

(New Keynesian Phillips Curve, log-linearized)

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(y_t - y_t^n) \text{ where } y_t^n = 0 \text{ or } \frac{\alpha(1+\psi)}{\sigma(1+\alpha)\frac{1}{\bar{c}} + \alpha + \psi} k_t + \frac{\sigma(1-\alpha)\frac{i\bar{n}v}{\bar{c}}}{\sigma(1-\alpha)\frac{1}{\bar{c}} + \alpha + \psi}$$

C.3 Log-linearization

We show the log-linearization for some non-standard equations.

C.3.1 Non-Ricardian budget constraint (eq.(3.7))

$$c_t^{NR} = \frac{\bar{n}_{NR}\bar{w}}{\bar{c}^{NR}}(n_t^{NR} + w_t) + (\pi_t - E_{t-1}\pi_t - \frac{E_t\pi_{t+1} - r_t^n}{\bar{R}^n})\Omega\bar{n}_{NR}\bar{w}$$

$$= \frac{1}{(1 - \Omega(1-\beta))}(w_t + n_t^{NR} + \Omega((\pi_t - E_{t-1}\pi_t) - \beta(r_t^n - E_t\pi_{t+1})))$$

C.3.2 Natural level of output (eq.(3.29))

From the definition of the price markup,

$$\mathcal{M}_t^p = \frac{(1-\alpha)K_t^\alpha N_t^{-\alpha}}{\mathcal{M}^\omega C_t^\sigma N_t^\psi}$$

We find the log-linearized natural level of output when the log-linearized markup is equal to zero.

$$m_t^p = \alpha k_t - (\alpha + \psi)n_t - \sigma c_t$$

$$= \alpha k_t - \frac{\alpha + \psi}{1-\alpha}(y_t - \alpha k_t) - \sigma(\frac{1}{\bar{c}}y_t - \frac{i\bar{n}v}{\bar{c}}inv_t)$$

$$y_t^n = \frac{\alpha(1+\psi)}{\sigma(1+\alpha)\frac{1}{\bar{c}} + \alpha + \psi}k_t + \frac{\sigma(1-\alpha)\frac{i\bar{n}v}{\bar{c}}}{\sigma(1-\alpha)\frac{1}{\bar{c}} + \alpha + \psi}$$

C.3.3 No arbitrage condition (eq.(3.34))

By equating the expected return on investment with the expected real return on nominal bonds,

$$E_t\left[R_t - \frac{R_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t}\right] = 0$$

Since $\bar{r}^k = \frac{\alpha}{1-\alpha}\frac{\bar{w}\bar{N}}{\bar{K}} = \bar{r} - (1-\delta) = \frac{1}{\beta} - (1-\delta)$,

$$E_t(q_t + r_t) = E_t(\frac{\frac{\alpha}{1-\alpha}\frac{\bar{w}\bar{N}}{\bar{K}}[w_t + n_t - k_t] + (1-\delta)q_{t+1}}{\bar{r}^b})$$

$$= \beta(1-\delta)E_t q_{t+1} + (1 - \beta(1-\delta))E_t(w_{t+1} + n_{t+1} - k_{t+1})$$

C.4 Danish registry data

Crawley and Kuchler (2019) use income and expenditure data comes from Danish administrative panel data during 2003-2015, of which sample contains millions of households, in order to estimate the sufficient statistics. They use after tax and transfer income and imputed expenditure. To be specific, the expenditure is imputed from income minus contribution to pension schemes and the changes in (non-pension, non-housing) net worth.

In addition, the marginal propensity to consume (MPC), the net nominal position (NNP) and unhedged interest rate exposure (URE) are required for estimating the sufficient statistics. Firstly, they calculate a regression coefficient of transitory consumption ($\tilde{c}$) with respect to transitory income ($\tilde{y}$) over a year, $\psi = \frac{Cov(\tilde{c}, \tilde{y})}{Var(\tilde{y})}$, marginal propensity to expenditure (MPX) out of transitory income. The MPX (ψ) can be interpreted as "if income is higher by one unit this year due to transitory factors, then consumption this year will be expected to be higher by ψ unit", which is an equivalent to the standard interpretation of MPC. Next, NNP and URE for the various sectors in the Danish economy is calculated from the registry data as well as the financial accounts from the national accounts statistics.[1] NNP for households is calculated as financial assets minus liabilities. As financial assets, they include bank deposits as well as the market value of securities (excluding shares). Liabilities include all debt to financial institutions as well as publicly administered student debt, tax debt and other debt to government bodies. URE is calculated as annual savings (i.e. after-tax income minus expenditure) plus maturing assets minus maturing liabilities. As maturing assets, they include all bank deposits thereby assuming that they are floating rate. They assume a maturity of 5 years for securities held by households, and therefore include 20% of the value of securities. Regarding liabilities, they assume that all bank debt is floating rate.[2] For mortgage debt, they calculate the stock of debt which is due to have interest rate reset over the com-

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[1]NNP and URE can only be calculated in the period 2009-2015 due to mortgage information being sufficiently detailed in the previous years.

[2]On average 95% of bank debt from households is floating rate according to the interest rate statistics collected by Danmarks Nationalbank since 2013.

ing 12 month. Their measure of maturing liabilities only includes the loans which contractually are due to have their interest rate reset over the coming 12 months.

C.5 Data from three surveys in Aulcert (2019)

In order to calculate sufficient statistics, Auclert (2019) uses three survey data, the Italian Survey of Households Income and Wealth (SHIW), the U.S. Panel Study of Income Dynamics (PSID) and the U.S. Consumer Expenditure Survey (CE). All samples in the survey data contain thousands of households.

C.5.1 Income and Consumption

Pre-tax income is used in the PSID and the CE, while post-tax income is used in the SHIW. For the measure of consumption, SHIW covers both nondurable and durables, while PSID and CE includes only nondurables and food, respectively. Ideally, the income should includes income from all sources: labor, dividend and interest rate. The consumption also should includes all expenditure on durable goods, rents, and interest payment if data are available.

C.5.2 MPC

The individual MPC information is available in the SHIW data. For PSID and CE data, he estimates the MPC by stratifying the population in terciles.

C.5.3 NNP and URE

NNP is identified as the difference between directly held nominal assets (mainly deposits and bonds) and directly held nominal liabilities (mainly mortgages and consumer credit). URE conceptually measures the total resource flow that a household to invest over the first period of his consumption plan. Thus, URE is the sum of after tax income and NNP minus consumption. With respect to maturity of assets and liabilities, he assumes that a) time and saving deposits have a duration of two quarters, b) adjustable-rate mortgages have a duration of three quarters, and c) debt outstanding on credit cards has duration of two quarters since detailed maturity information is typically absent.

C.5.4 Measurement error

Each survey has it own strengths and weakness. The CE has excellent information on consumption and liabilities, but very poor information on assets. Both the PSID and the SHIW appear to considerably undermeasure consumption.

C.6 Decomposition with other calibrations
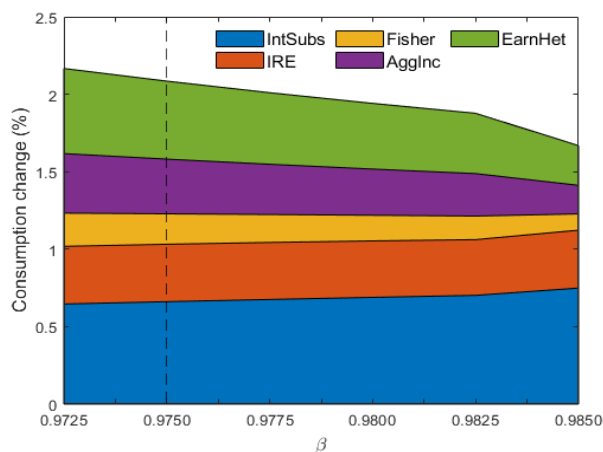
1) IRFs of consumption on impact of 1% monetary policy shock

2) Black dotted line represents the baseline calibration.

Figure C.1: Depending on the size of β_w
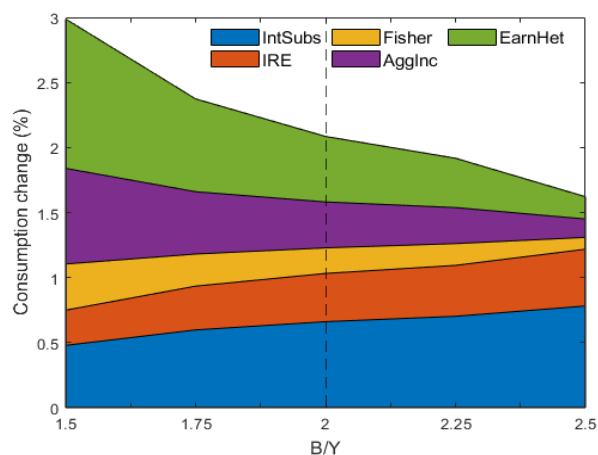
1) IRFs of consumption on impact of 1% monetary policy shock

2) Black dotted line represents the baseline calibration.

Figure C.2: Depending on the size of $\frac{B}{Y}$

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