# A Novel Design of a Solid Oxide Fuel Cell-based Combined Cooling, Heat and Power Residential System in the UK

Xinjie Yuan, Yuanchang Liu, and Richard Bucknall

***Abstract*—This paper details the use of objective sizing techniques for a novel design of a residential solid oxide fuel cell (SOFC) combined cooling, heating and electrical power (CCHP) system for the UK market. The aim of the research is to determine the objective sizing of key parameters relating to the cooling, heating and power supply and demand, namely the number of cells in the SOFC, the effectiveness of the heat exchangers and the coefficient of performance (COP) of the absorption chillers. These parameters are determined taking into account the aspects of efficiency, economic and environmental impacts through use of the entropy-weighting approach and grey relationship analysis. The combination of these two approaches will help designers maximise efficiency of energy utilization and minimise emissions and costs of the system being examined. It is envisaged that electrical demand would be met by the fuel cell (FC) stacks while the most efficient use is made of heat that is generated by the FC through waste heat recovery to satisfy domestic hot water, freezers, space heating and space cooling. The demand of conventional electric freezers is innovatively designed to be fulfilled by heat exchangers and absorption chillers to further increase the efficiency of heat energy use. Due to the energy demand characteristics of the UK domestic sector, the proposed system structure, objective sizing values and operation control strategies - supported by MATLAB/Simulink software - are suited to the residential energy demands of a single household.**

***Index Terms*—SOFC; CCHP; heat pump; entropy weighting approach; grey relationship.**

## I. INTRODUCTION

NEW dwellings in the domestic sector in the UK require improved energy conservation measures to attain higher efficiencies when heated through improved heat retention [1]. Increasing thermal insulation in homes will dramatically reduce heating requirements [2], while at the same time utilising the main source of energy, natural gas, more efficiently. Another factor is that of rising ambient temperatures which could lead to increased energy demand for cooling. Adopting CCHP technology could be an effective approach to satisfying these demands. Domestic FCs are an attractive option for CCHP units due to their high efficiency, high power-to-heat ratio, low operation noise and technically simple maintenance requirements [3]. By using the same natural gas transmission infrastructure for most homes in the UK, the high temperature SOFC with high fuel flexibility would be appropriate for a domestic CCHP system. With regard to the government support, the UK provides fiscal and financial support mechanisms – tax support, feed-in tariff (FiT) and capital grants - for the promotion of the use of tri-generation systems [4]. Therefore, from the perspective of the government and users, SOFC-based CCHP systems in the UK could offer significant fuel cost and emission savings compared to conventional energy systems [5].

Studies pertaining to FC-based CCHP residential systems are still at the initial stage. At this stage, the objectives are mainly related to the performance of the system, namely efficiency, economic and environmental impacts. Single-objective analysis, parametric analysis [6], [7] & [8] and comparative analysis [9], [10], [11] & [12] are the most commonly used approaches. For multi-objective analysis, evolutionary algorithms are applied to solve problems of sizing [13], [14] & [15] and operation strategy [16], [17] & [18]. However, the subjective weight of different objectives applied in these works is largely influenced by the knowledge and experience of designers. Objective weighting is introduced in this paper to help designers maximize efficiency while minimizing emissions, fuel and system costs. There are several common methods: the variation coefficient [19], the principle component analysis [20], the vector similarity measures [21], the grey relationship analysis [22] and the entropy weighting approach [23]. The first three methods require large data sample sizes which are not usually available at the initial stage of system design [24]. In comparison, grey system theory enables the accuracy of the data analysis based on a smaller irregular data sample size with a certain grey level [25]. The innovative combination of the entropy-weighting approach will be to determine the objective weights for multiple objectives. It solves the inaccuracy of subjective weights caused by human judgement in the conventional grey relationship analysis. Therefore, the evaluation process is more objective and practically convenient.

In this paper, a novel single household SOFC-based CCHP system is considered [26], where the energy derived from the chemical reactions is directly converted into electrical energy and waste heat. By utilizing the heat exchangers, the absorption chillers and the heat pump, the waste heat is recovered for space cooling/heating and domestic hot water demands. It is proposed

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Manuscript received December 13, 2019.

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in this paper to increase the efficiency of fuel use further by replacing conventional electric freezers with heat exchangers and absorption chillers. This optimized following electric load (FEL) operation strategy ensures improved use of the waste heat from the proposed system structure. The selection of the number of cells is related to the system efficiency, fuel consumption, emissions, as well as SOFC capital investment, operation and maintenance costs. The effectiveness of heat exchangers and the COP of absorption chillers establish the relationship between the supply of and demand for cooling, heat and power. This paper considers the importance of each criterion objectively by innovative combination of the entropy-weighting approach and grey relationship analysis. Multi-criteria analysis is applied to the following: maximizing system efficiency while minimizing emissions, fuel and systems costs.

This paper is organised as follows: the proposed CCHP system is detailed in Section II. Section III presents the grey relationship analysis and the entropy weighting approach to rationalize the multi-objective problem. Section IV gives the data for the case study. The results and a comparative analysis are discussed in Section V with conclusions in Section VI.

## II. SYSTEM ILLUSTRATION

### A. *Proposed CCHP system*

The combined cooling, heating and power (CCHP) system is designed for a single non-pensioner household in the UK. The household electricity survey from Intertek [27] provides the data on annual domestic electricity usage. The time series starts from May 2010 and ends in July 2011. Since the sizing values of the system cannot be changed during operation, the design process is conducted according to the peak energy demands of the hottest and coldest days [27].

In Fig. 1, the system is comprised of an SOFC based CCHP with internal reforming units and heat recovery systems. All the electrical demand is satisfied by the SOFC power output. Heating and cooling demand is fulfilled by a heat pump, heat exchangers and absorption chillers using the waste heat energy of the SOFC exhaust.

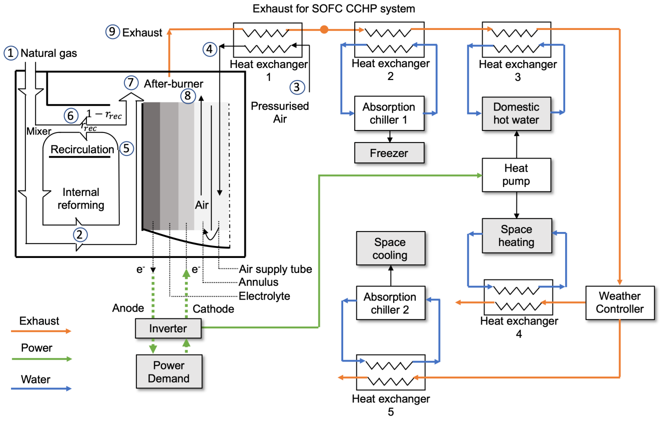

Fig. 1. The illustration of the proposed SOFC-CCHP system.

It is assumed that natural gas is the fuel. Part of the exhaust stream leaving the anode (node 5) is recirculated to the internal reforming unit. The remainder, combined with cathodic exhausts, passes through a catalytic after-burner (node 9). The temperature of the exhaust gas is then further increased to pre-heat the air inlet (node 3) through heat exchanger 1.

Heat exchanger 2 and absorption chiller 1 [28] are used together to satisfy freezer demand using the waste heat of the exhaust. Domestic hot water demand is delivered by heat exchanger 3. A weather sensor controller is set to adjust the usage of gases for space heating and cooling. Based on the characteristics of the relatively high heat to power demand ratio of the coldest day in the UK, waste heat is unable to satisfy all the heating demand when the SOFC is satisfying the electrical load. Therefore, a heat pump, powered by the FC, is applied when necessary.

### B. *Assumptions for modelling*

- SOFC modelling is 0-dimensional [29], where the current density in the axial direction of the tubular SOFC is an average value taken as being constant. The FC works in steady state and start-up is not considered.
- It is assumed that polarizations are mainly caused by electrochemical activation barriers, ohmic resistance and concentration polarizations [30].
- Fuel inlet is natural gas and all the methane is consumed in the water gas shift reaction. Fuel utilization factor is assumed to be 0.85 and the air utilization factor is 0.15 [31].
- The effectiveness of the counter-flow heat exchangers [32] and the coefficient of performance (COP) of the absorption chillers [33] are assumed to be between 0.4 and 0.8. The COP of the heat pump is taken as 3 [34].
- Based on the energy and mass balances, the SOFC heat loss is regarded as the difference between the power output, the energy of inputs (node 2 & 4) and outputs (node 7 & 8) in Fig. 1. Thermophysical properties of all fluids are assumed constant within the heat exchangers [35].

### C. *Optimized FEL operation strategy and constraints*

The optimised operation strategy follows two main principles of design as illustrated in Fig. 2: 1) energy balance - satisfy all the energy demands for different heating/cooling to power demand ratios and 2) objective functions - maximise system efficiency and minimise the costs of SOFC capital, operation and maintenance, fuel consumption, auxiliary components and emissions.

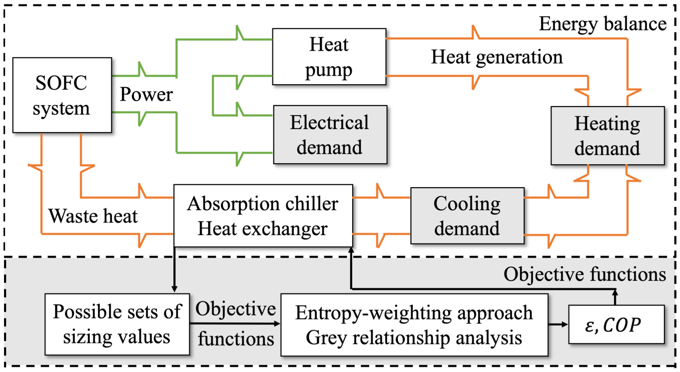

Fig. 2. Structure diagram of the evaluation of SOFC CCHP system.

In the UK domestic sector, the heating to power demand ratio in winter is much larger than that in summer [27]. Therefore, the design of sizing values is first based on the highest heat to power ratio in the 24-hour winter energy demand then the summer energy demand to obtain all the sizing values. As mentioned above, a heat pump powered by electricity from the SOFC is incorporated into this system. As implied in Fig. 2, the increase of SOFC power output leads to more waste heat, which decreases the power needed by the heat pump. The energy balance point should satisfy two constraints:

1) The waste heat from the SOFC exhaust should satisfy freezer and space cooling demand as stated in (1) and (2) as there is no electrically powered chiller in the system.

$$\varepsilon_{hx,2} \times \dot{Q}_{max,abc1} \times COP_{abc1} > Demand_{fre} \tag{1}$$

$$\varepsilon_{hx,5} \times \dot{Q}_{max,abc2} \times COP_{abc2} > Demand_{sc} \tag{2}$$

where $\varepsilon_{hx,2}$ and $\varepsilon_{hx,5}$ are the effectiveness of heat exchangers 2 and 5, $\dot{Q}_{max,abc1}$, $\dot{Q}_{max,abc2}$, $COP_{abc1}$ and $COP_{abc2}$ are the maximum heat transfer rate the coefficient of performance of absorption chillers 1 and 2, and $Demand_{fre}$ and $Demand_{sc}$ are the freezer and space cooling demands.

2) The heat energy recovered for the freezer, hot water and space heating demand and the heat energy provided by the heat pump should be equal to the total heating and freezer demand as stated in (3).

$$\dot{Q}_{fre} + \dot{Q}_{hw} + \dot{Q}_{sh} + COP_{hp} \times \dot{W}_{hp} = \sum Demand_{fre,\,hw,\,sh} \tag{3}$$

where $\dot{Q}_{fre}$, $\dot{Q}_{hw}$ and $\dot{Q}_{sh}$ are the actual heat transfer rates for freezer, hot water and space heating, $\dot{W}_{hp}$ is the power needed to drive the heat pump, $\sum Demand_{fre,\,hw,\,sh}$ is the total heating and cooling demand for these three energy requirements.

### D. Conventional separated production (SP) system

As a comparison with the proposed SOFC based CCHP system, the conventional separated production (SP) system without SOFC is illustrated in Fig. 3.

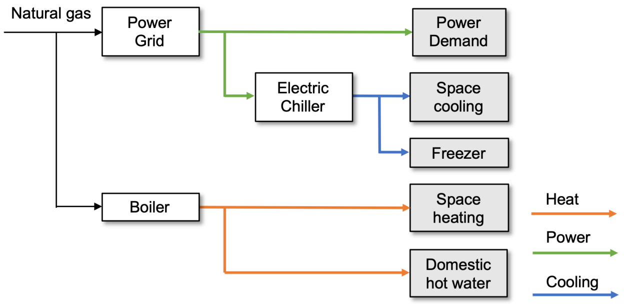

Fig. 3. The illustration of the conventional SP system.

In Fig. 3, it is assumed that the power grid and boilers are both supplied by burning natural gas for comparative analysis. Power demands are provided from the power grid through generation [36], transmission, distribution and transforming to the appropriate voltage [37]. Cooling demands are provided by electric chillers, including space cooling and freezers. The COP of the electric chiller is taken as 4 [38]. The boiler is used to provide heat energy demands including space heating and domestic hot water. The efficiency of the boiler is taken as 97% [39].

## III. METHODOLOGY

### A. Constraints and objective functions

The target sizing values in this project are obtained following two steps: 1) first determine the number of cells in the FC and 2) detail the parameters of the heat recovery system - the effectiveness of heat exchangers and COP of absorption chillers.

1) The number of cells cannot be changed during operation. Therefore, the appropriate number of cells is evaluated when all the electrical power, heating and cooling demands are provided by the SOFC electrical system power output. There are four main parameters during the evaluation: integrated efficiency ($f_{eff}$), SOFC cost rate ($f_{cost}$), fuel cost rate ($f_{fuel}$) and emission rate ($f_{emission}$). The integrated efficiency is as below:

$$f_{eff} = \frac{\sum_t \dot{W}_{stack}}{\sum_t (\dot{m}_{fuel} \times LHV)_{fuel,t}} \tag{4}$$

where $\dot{W}_{stack}$ is the power output of the SOFC stack, $\dot{m}_{fuel}$ is the mass flow rate of fuel inlet and $LHV$ is lower heating value of the fuel (the amount of heat energy released from combustion), and $t$ denotes the time slots of the chosen day.

The total cost rate ($f_{cost}$) of capital investment, operating and maintenance for the SOFC is evaluated in £/h by (5) below [40]:

$$f_{cost} = 0.77\frac{A_{cell} \times N_{cell} \times (2.96 \times T_{cell} - 1907) \times \frac{i_r \times (1+i_r)^n}{(1+i_r)^n - 1} \times \emptyset}{N} \tag{5}$$

where $A_{cell}$ and $N_{cell}$ are the area and number of cells. $T_{cell}$ is the operating temperature of FC, $i_r$ is the average level of the interest rate (assumed as 0.5% derived from the UK's past ten-year bank rates [41]), $\emptyset$ denotes the maintenance factor, assumed as 1.1, $N$ is the number of system operating hours (8760h/year) and $n$ is the system life (20 years) [40].

For the SOFC, the fuel cost rate ($f_{fuel}$) in £/s is another essential parameter that needs to be considered [42]:

$$f_{fuel} = c_{fuel} \times \dot{m}_{fuel} \tag{6}$$

where $c_{fuel}$ is the specific cost per kg of fuel.

The emission cost rate ($f_{emission}$) from the CO2 emissions in £/s is evaluated below [42]:

$$f_{emission} = c_{CO_2} \times \dot{m}_{fuel} \times r_{CO_2} \tag{7}$$

where $c_{CO_2}$ is the specific damage cost per kg of $CO_2$, $r_{CO_2}$ is the mass flow rate of $CO_2$ per kg of fuel input.

2) The significant parameters of the heat recovery system include the effectiveness of the heat exchangers and the coefficient of performance ($COP$) of the absorption chillers, the values of which have already been stated. As the cost of the SOFC is determined above, during the evaluation of the waste heat recovery system sizing values, the four criteria are integrated efficiency ( $f_{CCHP,eff}$ ), auxiliary device cost rate

($f_{aux}$), fuel cost rate ($f_{fuel}$) and emission cost rate ($f_{emission}$). The equations for fuel cost and emission rates remain the same as (6) and (7).

As the heating and cooling demands are satisfied by the proposed CCHP system, the integrated efficiency ($f_{CCHP,eff}$) is as follows:

$$f_{CCHP,eff} = \frac{\dot{W}_{ele}+\dot{Q}_{hw}+\dot{Q}_{fre}+\dot{Q}_{sh/sc}}{\dot{m}_{fuel}\times LHV} \tag{8}$$

where $\dot{W}_{ele}$, $\dot{Q}_{hw}$, $\dot{Q}_{fre}$ and $\dot{Q}_{sh/sc}$ are the electrical power, hot water, freezer and space heating demands in winter and summer, respectively.

The auxiliary device investment cost rate ($f_{aux}$) in (9) in £/h is the sum of the cost rate in £ of heat exchangers ($C_{hx}$) [43] in (10) and absorption chillers ($C_{abc}$) [44] in (11).

$$f_{aux} = \left(\sum_n C_{hx,n} + \sum_m C_{hx,m}\right) \times \frac{\frac{i_r\times(1+i_r)^n}{(1+i_r)^n-1}\times\emptyset}{N} \tag{9}$$

$$C_{hx} = 0.77(8500 + 409 \times A_{hx}^{0.85}) \tag{10}$$

$$C_{abc} = 0.77\left(540 \times \left(\frac{\dot{Q}_{abc}}{1000}\right)^{0.872}\right) \tag{11}$$

where $n$ and $m$ indicate the numbers of heat exchangers and absorption chillers respectively, $A_{hx}$ is the area of heat transfer and $\dot{Q}_{abc}$ is the total energy provided by the absorption chillers. $A_{hx}$ can be evaluated using either of two common approaches: logarithmic mean temperature difference (LMTD) and effectiveness-number of transfer units (ε-NTU) methods. LMTD is used when the inlet and outlet temperatures of both hot and cold fluids are known, which is not suitable for this project [45]. By comparison, in this case, ε-NTU [46] can help determine the actual heat transfer rate when only the inlet conditions of the hot and cold fluids are known. The heat transfer rates in the evaporator and generator can be determined from the application of the heat exchanger model combined with the COP value [47].

For the conventional SP system in Fig. 3, the investment cost functions in £ for the electric chiller $C_{ec}$ [44] in (12) and heat pump $C_{hp}$ [48] in (13) are given below. The unit cost of the boiler is set as £1000 [49].

$$C_{hp} = 0.77 \times 543.76\left(\frac{\dot{W}_{hp}}{1000}\right)^{0.8003} \tag{12}$$

$$C_{ec} = 0.77\left(482\left(\frac{\dot{W}_{ec}}{1000}\right)^{-0.07273} - 158.7\right)\frac{\dot{W}_{ec}}{1000} \tag{13}$$

where $\dot{W}_{hp}$ & $\dot{W}_{ec}$ are the power consumed by the heat pump and electric chiller, respectively.

### *B. Weights for multiple objective functions*

Grey relationship analysis is an important part of the grey system theory. It quantitatively describes the interaction between factors, of which those with the same trend of development have a closer interconnection [22]. The entropy-weighting approach is introduced to reduce the subjectivity during the evaluation and calculation processes [23]. The lower the information entropy of a factor, the greater the amount of information that this factor provides, and thus a higher effect on the whole system. The combination of these two methods can help designers objectively determine the importance and weights of various factors as shown in Fig. 4.

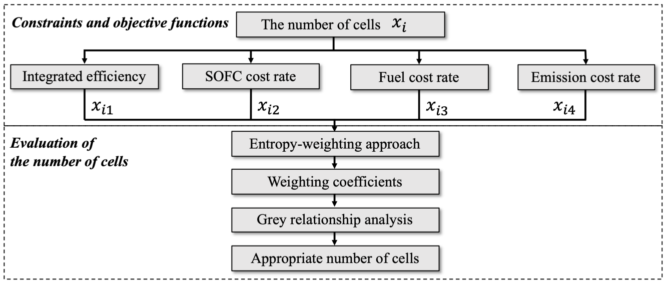

Fig. 4. The flow chart of evaluation of the appropriate number of cells.

From Fig. 4, the evaluation of the appropriate number of cells is done by maximising the integrated efficiency and minimising emissions, fuel and SOFC cost rates, using the entropy-weighting approach and grey relationship analysis. This process is influenced by the four key criteria discussed previously, denoted by $x_{i1}$ to $x_{i4}$ in Fig. 4. A matrix ($r \times 4$) can be obtained with 4 eigenvalues in (14), as below.

$$\mathrm{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{14} \\ x_{21} & x_{22} & \cdots & x_{24} \\ \vdots & \vdots & \vdots & \vdots \\ x_{r1} & x_{r2} & \cdots & x_{r4} \end{bmatrix} (i = 1,2,\cdots r,\ j = 1,2,\cdots 4) \tag{14}$$

where $x_{ij}$ in (14) denotes the value of the $i^{th}$ row representing the number of cells, determined by the maximum SOFC power output and the demand limit, $r$ being the number of cases, and the $j^{th}$ column representing the number of objectives, which is 4 in this case. The evaluation of the parameters of the heat recovery system follows a similar process as shown in Fig. 4, but with four new criteria as discussed previously.

Since different evaluation parameters have a variation of dimensions and units, the matrix should be standardised [22]. For integrated efficiency, the higher value is sought in (15) whilst the lower value is better for cost rates in (16). Therefore, for integrated efficiency, it should be standardised as follows:

$$Y_{ij} = \frac{x_{ij} - min_{i=1}^{r}(x_{ij})}{max_{i=1}^{r}(x_{ij}) - min_{i=1}^{r}(x_{ij})} \tag{15}$$

In comparison, the standardisation of the three cost rates is expressed as follows:

$$Y_{ij} = \frac{max_{i=1}^{r}(x_{ij}) - x_{ij}}{max_{i=1}^{r}(x_{ij}) - min_{i=1}^{r}(x_{ij})} \tag{16}$$

where $max_{i=1}^{r}(x_{ij})$ and $min_{i=1}^{r}(x_{ij})$ are the maximum and minimum values from all the rows of the $j^{th}$ column.

In this case, conditional entropy $H_j$ is the degree of dispersion of the $j^{th}$ factor conditioned on the discrete random number of cells [50].

$$H_j = -\frac{\sum_{i=1}^{r}\left[\frac{Y_{ij}}{\sum_{i=1}^{i} Y_{ij}}\times\log\left(\frac{Y_{ij}}{\sum_{i=1}^{i} Y_{ij}}\right)\right]}{\log(i)} \tag{17}$$

where $H_j$ is the conditional entropy of the $j^{th}$ column. However, in this case, it is found that (17) is not a valid expression when $Y_{ij} = 0$. Therefore, (18) is added as a constraint condition.

$$\lim_{\frac{Y_{ij}}{\sum_{i=1}^{r} Y_{ij}} \to 0} \frac{Y_{ij}}{\sum_{i=1}^{r} Y_{ij}} \times \ln\left(\frac{Y_{ij}}{\sum_{i=1}^{r} Y_{ij}}\right) = 0, \text{ when } \frac{Y_{ij}}{\sum_{i=1}^{r} Y_{ij}} = 0. \quad (18)$$

With the information entropy values of the four columns, the weights $w_j$ for the four objectives listed above can be calculated as follows [51]:

$$w_j = \frac{1-H_j}{c-\sum_{j=1}^{c} H_j} \quad (19)$$

The grey incidence coefficient $\gamma_{ij}$ is regarded as the proximity between the reference sequence $R_j$ and the standardised matrix $Y_{ij}$ [22].

$$\gamma_{ij} = \frac{min_{i=1}^{r} min_{j=1}^{c}(Y_{ij}-R_j)+\theta \times max_{i=1}^{r} max_{j=1}^{c}(Y_{ij}-R_j)}{(Y_{ij}-R_j)+\theta \times max_{i=1}^{r} max_{j=1}^{c}(Y_{ij}-R_j)} \quad (20)$$

where $\theta$ is the resolution coefficient to decrease the distortion caused by the large value of $min_{i=1}^{r} min_{j=1}^{c}(Y_{ij} - R_j)$. The degree of distortion is normally taken as 0.5 [22]. The reference sequence $R_j$ is the maximum of each column which can be defined as:

$$R_j = \max\,[Y_{1j,} Y_{2j,} \cdots, Y_{ij,} \cdots, Y_{cj}\,] \quad (21)$$

With the weighting coefficients and the grey incidence coefficient calculated above, the overall priority $p_i$ of the different number of cells can be expressed as follows [22].

$$p_i = \sum_{j=1}^{c} w_j \times \gamma_{ij} \quad (22)$$

The highest priority value $p_i$ indicates the best overall performance of the system considering all the factors.

## IV. CASE STUDY

In this section, the proposed SOFC based CCHP system and the objective sizing methods are evaluated based on the historic energy consumptions of a single non-pensioner household in the UK on the hottest and coldest day from May 2010 to July 2011 [27]. [36] [37] [52]z

The data input of both the proposed SOFC CCHP system and the conventional SP system lays the foundation of the comparative analysis. The general information on the power grid and corresponding prices are given in TABLE I.

TABLE I
GENERAL INFORMATION FOR THE POWER GRID AND PRICES

| Symbol | Quantity | Value | Ref. |
|---|---|---|---|
| Power grid Efficiency | Generation | 42.35% | [36] |
| | Transmission, distribution & transform | 85% to 96% | [37] |
| Price | Electricity | 14.78 pence/kWh | [52] |
| | Natural gas | 3.13 pence/kWh | [52] |

## V. RESULTS AND COMPARATIVE ANALYSIS

This section presents the results of the system design according to the theories discussed in Section II. Verification of the FC unit is conducted before sizing values are obtained. The sizing values include the number of cells and the parameters of the heat recovery system, including the number of cells in the SOFC, the effectiveness of heat exchangers and COP of absorption chillers.

### *A. Verification of FC unit*

In order to verify the FC modelling, the simulated results are compared with two sets of experimental data [53] & [54] under the same operating temperature of 1273K. The voltage of a single cell is determined mainly by the structure of the cell, material properties, molar fractions of fuel and water, current density and operating temperatures. Due to the limited amount of experimental data available, data points within the range from 150 $mA/cm^2$ to 500 $mA/cm^2$ [53] & [54] are compared in Fig.5.

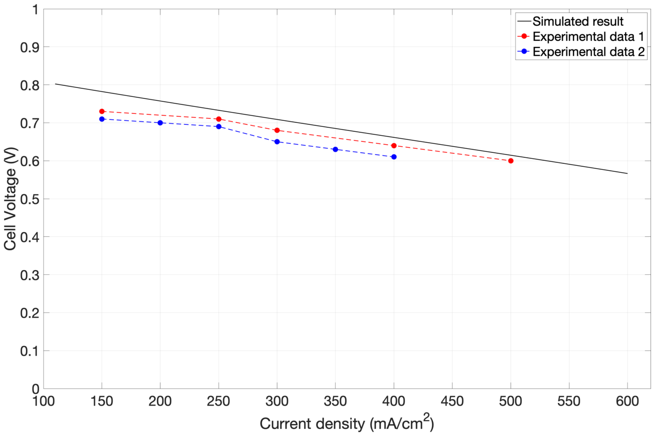

Fig. 5. Comparison of simulated results with experimental data 1 [53] & data 2 [54].

In Fig. 5, with 89% $H_2$ and 11% $H_2O$ (85% fuel utilization) as fuel and air as oxidant at 1273K [53] & [54], the relative error agreement is 6 to 8%. The choice of zero-dimension modelling is considered to be one of the reasons for this error. In practical experiments, the current density in the axial direction of the tubular SOFC varies, thereby influencing the over-voltage values. Another reason is that certain parameters, including the running times of the experiments, are not available. Overall, the model results shown in Fig. 5 correspond well with the experimental data.

In TABLE II, based on energy and mass balances, the molar compositions at nodes 1 to 9 in Fig. 1 can be obtained along with the corresponding temperatures. These are used for the evaluation of the sizing of the FC unit and the heat recovery system.

TABLE II
TEMPERATURE AND MOLAR COMPOSITION OF GASES AT NODE NO.1 TO 9 IN FIG. 1

| No. | $T_{cell}$ (K) | Molar composition (%) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $H_2$ | $CH_4$ | $CO$ | $CO_2$ | $H_2O$ | $N_2$ | $O_2$ |
| 1 | 298 | / | 98 | / | / | / | 2 | / |
| 2 | 1014 | 4.88 | 22.92 | 2.97 | 22.40 | 45.86 | 0.97 | / |
| 3 | 298 | / | / | / | / | / | 79 | 21 |
| 4 | 1066 | / | / | / | / | / | 79 | 21 |
| 5 | 1273 | 6.36 | / | 3.87 | 29.25 | 59.86 | 0.66 | / |
| 6 | 1273 | 6.36 | / | 3.87 | 29.25 | 59.86 | 0.66 | / |
| 7 | 1273 | 6.36 | / | 3.87 | 29.25 | 59.86 | 0.66 | / |
| 8 | 1273 | / | / | / | / | / | 81.57 | 18.43 |
| 9 | 1349 | / | / | / | 1.68 | 3.36 | 77.67 | 17.29 |

## B. *Sizing of FC unit*

The least number of cells is determined by the maximum power demand divided by the maximum power output of a single cell, at 36.25 $W$ at 747.8 $mA/cm^2$. Since the 24-hour power demand is a given value [27], more cells result in less power output from a single cell. Therefore, by decreasing the current density lower than 747.8 $mA/cm^2$ through having a larger number of cells means higher efficiency and lower fuel and emission cost rates, but higher SOFC cost rates. Objective weights for the four parameters – integrated efficiency, SOFC cost rate, fuel cost rate and emission cost rate obtained in (19) – are 0.1210, 0.6576, 0.1108 and 0.1108. The weight of the SOFC cost rate is high mainly because of its high dependence on the number of cells.

The priority value in Fig. 6 is an index concerning the performance of the whole system based on multi-objective functions. It first increases then decreases as the number of cells increases. When all the factors are considered, matching the required demand, the priority value is the highest. The integrated efficiency is 60.25% when the number of cells is 71. This evaluation lays the foundation for the CCHP system.

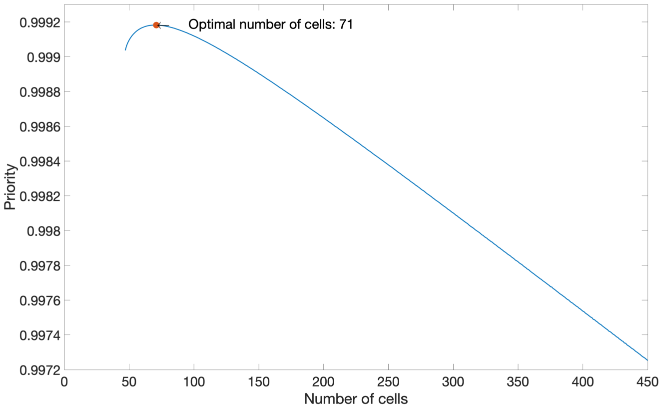

Fig. 6. The illustration of the evaluation of the number of cells based on grey relationship analysis and entropy weighting approach.

## C. *Sizing of heat exchangers and absorption chillers*

According to the demand on the coldest and hottest days in the UK [27], the effectiveness of the heat exchangers and the COP of absorption chillers are obtained based on the multi-objective functions.

From Fig. 7, different combinations of effectiveness of the heat exchangers and COP of the absorption chillers are evaluated by multi-objective functions. The priority values vary due to the changing effectiveness of the heat exchangers for the freezer, hot water, air pre-heating for the SOFC and the space heating.

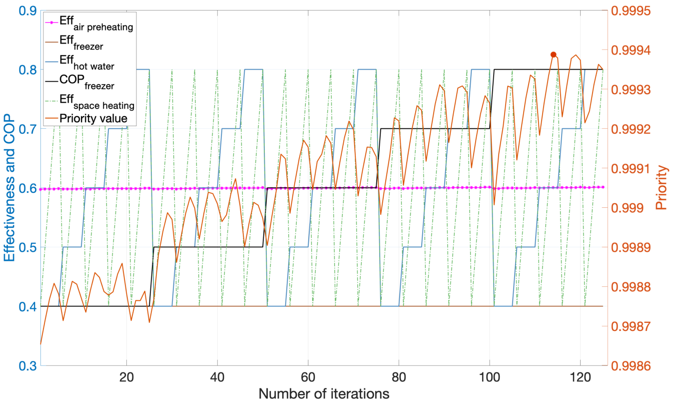

Fig. 7. The illustration of the evaluation of effectiveness of heat exchangers 1 to 4 and COP of absorption chiller 1 in winter based on grey relationship analysis and entropy weighting approach.

The weights for system efficiency, fuel and emission cost rates and auxiliary component cost rate obtained in (19) are 0.2497, 0.2503, 0.2503 and 0.2497, respectively. According to the demand on the coldest day, the effectiveness of heat exchangers 1 to 4 are 0.6004, 0.4000, 0.6000 and 0.7000, respectively. The COP of absorption chiller 1 is 0.8000. The weights of objective functions are similar, representing equal significance during the evaluation. The system efficiency reaches 90.96%.

Following a similar procedure as above, the effectiveness of heat exchanger 5 and the COP of absorption chiller 2 are 0.4000 and 0.6000 in summer as shown in Fig. 8.

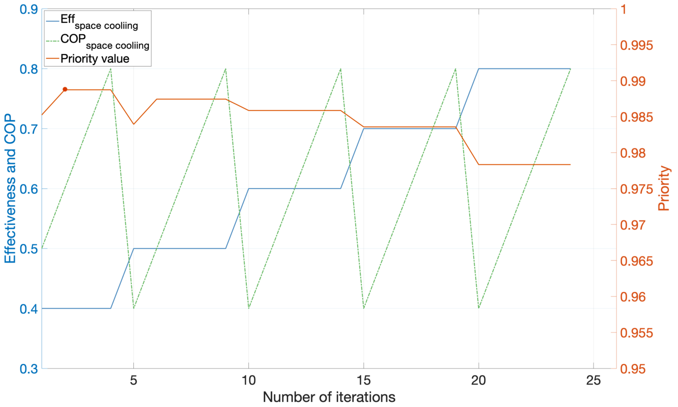

Fig. 8. The illustration of the evaluation of the effectiveness of heat exchanger 5 and COP of absorption chiller 2 in summer, based on grey relationship analysis and entropy weighting approach.

The system efficiency is 87.71%. The system efficiency of the SOFC CCHP system is higher than that of an SOFC electricity-only system. This is mainly through exploitation of waste heat. In the electricity-only situation, any heating and cooling requirements would need to be supplied from other sources. There will inevitably be losses associated with these sources. The maximum system efficiency of the SOFC CCHP system on the coldest day is slightly higher than the value attained on the hottest day. This is partly because of the characteristics of the temperate maritime climate in the UK. The high heating demand on the coldest day results in a higher heat to power demand ratio than on the hottest day. This leads to a higher effective use of waste heat and a further increase in overall system efficiency.

The optimised sizing values, including the number of cells, effectiveness of heat exchangers 1 to 5 and COP of absorption chillers 1 & 2, are obtained by the combination of grey relationship analysis and summarised in TABLE III.

TABLE III
SUMMARY OF OPTIMISED SIZING VALUES.

| | Function | Value | Effectiveness | COP |
|---|---|---|---|---|
| Number of cells | / | 71 | / | / |
| Heat exchanger 1 | Air preheating | / | 0.6004 | / |
| Heat exchanger 2 | Freezer demand | / | 0.4000 | / |
| Heat exchanger 3 | Hot water | / | 0.6000 | / |
| Heat exchanger 4 | Space heating | / | 0.7000 | / |
| Heat exchanger 5 | Space cooling | / | 0.4000 | |
| Absorption chiller 1 | Freezer demand | / | / | 0.8000 |
| Absorption chiller 2 | Space cooling | / | / | 0.6000 |

### D. *Comparative analysis*

Based on the optimised sizing values of the number of cells, effectiveness of heat exchangers 1 to 5 and absorption chillers 1 and 2, the SOFC CCHP system is analysed from multiple parameters. Compared with the conventional SP system, the overall efficiency, the investment cost, the energy cost including electricity and natural gas and the $CO_2$ emissions are compared in TABLE IV and TABLE V (below) for winter and summer. Variables termed variation and relative error agreement are applied in these two tables to identify the advantages and differences. It can be seen that the proposed SOFC CCHP system based on the two objective weighting approaches delivers some advantages but still has some disadvantages compared with the conventional SP system.

TABLE IV
COMPARATIVE ANALYSIS ON THE COLDEST DAY

| | SP system | SOFC CCHP system |
|---|---|---|
| Efficiency | 79% | 90.96% |
| Variation | / | +11.96% |
| Investment cost (£) | 1248 | 3060 |
| Relative error agreement | / | +145.19% |
| Energy cost (pence) | 292.46 | 73.98 |
| Relative error agreement | / | -74.70% |
| $CO_2$ Emission ($mg/s$) | 9.15 | 3.99 |
| Relative error agreement | / | -56.13% |

TABLE V
COMPARATIVE ANALYSIS ON THE HOTTEST DAY

| | SP system | SOFC CCHP system |
|---|---|---|
| Efficiency | 47% | 87.71% |
| Variation | / | +40.71% |
| Investment cost (£) | 1248 | 3060 |
| Relative error agreement | / | +145.19% |
| Energy cost (pence) | 101.23 | 28.97 |
| Relative error agreement | / | -71.38% |
| $CO_2$ Emission ($mg/s$) | 4.62 | 2.26 |
| Relative error agreement | / | -51.00% |

In TABLE IV, the efficiency of the SOFC CCHP system is 11.96% higher than that of the conventional system because of the use of waste heat recovery system. However, the difference is much smaller than the value in TABLE V, which is about 40.71% between the SP and CCHP system. This is due to the high efficiency of the boiler and high heat-to-power demand ratio in the UK in winter. Therefore, the efficiency of the SOFC CCHP system shows gains in both extreme weather conditions, particularly so in hot conditions.

The investment cost of the SP system is lower than that of the CCHP system due to the high costs of the SOFC system. However, with the improvement of the SOFC technology, the prices of SOFC systems are decreasing annually [55]. Another reason of the high investment costs is that this design is based on a single household. When it is promoted to multiple households, the per household investment of the system can be further reduced and the likely increased penetration rate will also bring cost reduction benefits. The SOFC system will be using the same natural gas transmission system as the SP system, so the SOFC's more fuel-efficient operation will make it economically competitive overall.

The energy costs in TABLE IV and TABLE V include the cost for electricity and natural gas. For the SP system, the electrical and cooling demands are provided by the power grid whilst the heat demand is fulfilled by natural gas using a boiler. For the SOFC CCHP system, the energy cost is the cost of natural gas due to the structure of the system. The energy cost of the CCHP system is 74.70% lower than that of the SP system on the coldest day and 71.38% on the hottest day. This variation is due to the high efficiency of heat use and the lower price of natural gas compared with the price of electricity in TABLE I.

Similarly, the $CO_2$ emissions of the CCHP system are 56.31% and 51.00% lower than the emissions emitted from the SP system. As natural gas has been selected in this paper as the fuel input, carbon dioxide is generated during the operation. In future work, bio natural gas [56] and hydrogen enriched natural gas will be used to further improve the sustainability and environmental friendliness of the proposed solution.

## VI. CONCLUSIONS

This paper investigates the design of a single household SOFC-based CCHP system using energy consumption data for the UK market, where the conventional electric freezer demand is replaced by heat exchangers and absorption chillers. The proposed system, using an optimised FEL operation strategy for the various components, makes optimum use of the waste heat of the SOFC. In order to avoid the subjectivity and inaccuracy of human judgement in a multi-objective analysis to maximise system efficiency and minimise emissions, fuel and system costs and key sizing values of the system are determined by a combination of objective weighting methods, namely the grey relationship analysis and the entropy weighting approach. The sizing values include the number of cells, effectiveness of heat exchangers and COP of absorption chillers. The number of cells is evaluated as 71. Based on the ratio of heating and cooling to power demand, the effectiveness of the five heat exchangers are 0.6004, 0.4000, 0.6000, 0.7000 and 0.4000. The COP of the absorption chillers 1 and 2 are 0.8000 and 0.6000. The maximum efficiency of the SOFC electrical-only system is

60.25%. The system efficiency attains 90.96% and 87.71% on the coldest and hottest days, respectively. Compared with the conventional SP system, the proposed SOFC based CCHP system has a higher efficiency (+11.96% & +40.71%), lower energy costs (-74.70% & -71.38%) and lower $CO_2$ emissions (-56.31% & -51.00%) on the coldest and hottest days. The investment cost can be decreased when this system is promoted to multiple households under both the grid-connected mode and the island mode. When an energy storage system is added, the power control for energy production and the power quality improvement should be further designed.

## ACKNOWLEDGMENT

The first author would like to thank China Scholarship Council (CSC) for supporting his studies at UCL. Thanks to Mr Konrad Yearwood for proof reading and valuable advice.

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