Consumer Privacy and Serial Monopoly*

V. Bhaskar
UT Austin

Nikita Roketskiy
UCL

January 7, 2021

Abstract

We examine the implications of consumer privacy when preferences today depend upon past consumption choices, and consumers shop from different sellers in each period. Although consumers are ex ante identical, their initial consumption choices cannot be deterministic. Thus, ex post heterogeneity in preferences arises endogenously. Consumer privacy improves social welfare, consumer surplus and the profits of the second-period seller, while reducing the profits of the first period seller, relative to the situation where consumption choices are observed by the later seller.

Keywords: consumer privacy, dynamic demand, endogenous screening, nonlinear pricing.

JEL Codes: D11, D43, L13

*Thanks to Konrad Mierendorff, Roland Strausz, two anonymous referees, the editor and seminar audiences at Bonn, ESSET Gerzensee, Humboldt University, EUI, EIEF, University of Bristol, Royal Holloway, QMUL and UCLA for very helpful comments. Bhaskar thanks the National Science Foundation, for its support via grant # SES 1629055. Roketskiy thanks the British Academy, the Leverhulme Trust and the Department for Business, Energy and Industrial Strategy for their support via grant # SRG19\190712.

1 Introduction

In many markets, consumers shop periodically from different suppliers, and their demand displays *inter-temporal substitutability.* If a consumer makes a large purchase at the supermarket, he is less likely to patronize his local butcher, baker or convenience store. Similarly, a consumer may shop one week at Walmart or Tesco, and another week at WholeFoods or Waitrose. Fox et al. (2004) and Thomassen et al. (2017) document evidence of multi-stop shopping for groceries in the US and in the UK respectively. Nor is this phenomenon restricted to grocery shopping. A car owner may use the official dealership for major repairs or annual service, but the local garage for more minor problems. If she opts for a full service at the dealership, she requires less from the local garage.

In other markets, multi-stop shopping is combined with *inter-temporal complementarity.* Consider the markets for computer hardware and software. Software often has minimum hardware requirements. A consumer who purchases a high-end computer is more likely to meet these requirements, and, as a result, would have a higher demand for software, as compared to a consumer who buys a low-end model.

A second set of examples arise when there is a single consumer who requires personalized services. A property owner who seeks to build a new house may need, in sequence, the services of an architect and an interior designer. In this case, the services of the two individuals are likely to be complementary, although there may be aspects where one service provider can substitute for the other.

This paper takes a first step towards the analysis of sequential multi-stop shopping. Our focus is on the implications of the privacy—as opposed to publicness—of consumption decisions, for allocative efficiency and the distribution of payoffs. To this end, we set out a model with two firms and a representative consumer, who interacts with the firms in sequence. For tractability, we make the simplifying assumption that the interaction only lasts two periods, with the firms interacting in a fixed order. This assumption is valid if the consumer's purchases at the upstream firm affect her demand from the downstream firm, but not vice-versa. Thus our model captures the essence of the interaction between the supermarket and the local shop, between the car dealership and the local garage, and between the hardware maker and the software supplier. It fits less the interaction between two supermarkets such as Walmart and WholeFoods, since demand linkages flow in both directions. This simplifying assumption allows us to present the analysis in terms of a two period model, since even interaction over an infinite horizon can be reduced to a sequence of two-period blocks.

Our analysis sheds light on two important policy questions. In recent years, there has been increased concern, especially in Europe, that supermarkets are putting local stores out of business, thereby decimating town centers and fragmenting local communities—see for example, the 2000 and 2008 reports of the Competition Commission in the UK. Indeed, the restrictions on supermarket opening hours on Sundays is motivated, in part, by such considerations. The second issue is consumer privacy, and its implications—our analysis presents a very different perspective from the existing literature.

Our analysis assumes that both upstream and downstream firm have monopoly power, that is tempered by the sequential nature of competition. This results in consumption distortions. When goods are substitutes, the leading seller has an incentive to oversell, thus reducing the demand for the rivals' products. Supermarkets and restaurants may encourage wasteful purchases and overconsumption—this way their patrons are less likely to visit the competitors in the future.

In the case of complementary goods, the distortion is in the opposite direction. The leading seller may undersell because he does not fully internalize the effect current sales have on the future demand. For instance, a hardware producer may not take into account the effect of its pricing decisions on the consumer's subsequent demand for software.

The extent of these distortions depends on what the sellers know about the consumer's past history. We contrast two cases. With public transactions, the downstream seller observes the consumer's purchases at the upstream firm. Under consumer privacy, the downstream seller has no information. Our main finding is that consumer privacy improves consumer welfare. It hurts the upstream firm, but improves the position of the downstream firm. The overall effect on utilitarian welfare is unambiguously positive. Privacy limits the distortions to consumption induced by the upstream firm, which more than offsets any distortions induced by the downstream firm.

We now proceed to a more detailed description of our model and results. We develop a two-period model in which a consumer's utility is quasi-linear, and depends on consumption in both periods, and is written as $u(x, y)$. When consumptions are substitutes, u is strictly submodular; when they are complements, u is strictly supermodular. To focus on the dynamic implications of endogenous choices, we assume that consumers are identical—differences in demand arise only due to differences in past consumption. Furthermore, we assume that a supplier at any date has monopoly power—e.g., because of search frictions—so that the market is characterized by serial monopoly.

We allow sellers to offer unrestricted non-linear prices and analyze the nature of inter-temporal competition. In the benchmark case, where consumption today is observed by tomorrow's seller, the first period seller induces over-consumption relative to the efficient allocation when goods are substitutes, and under-consumption when goods are complements. The intuition is straightforward: since the future seller will extract the buyer's surplus, the monopolist today seeks to induce the consumption level that maximizes the consumer's utility when she exercises her outside option tomorrow. This is larger than the efficient amount when goods are substitutes, and smaller when goods are complements.

Our main focus however, is on the more realistic case where neither the consumer's past consumption nor the past price offers are observed by the current seller. Our first result is that there cannot be a pure strategy equilibrium where consumption is deterministic, both for the case of complements and substitutes. The intuition for this is both simple and subtle: if first period consumption is deterministic and second period purchases are positive, then the consumer must be indifferent between the second-period seller's offer and her outside option, since the former fully extracts the consumer's surplus. However, in this case, seller 1 and the consumer can increase their joint surplus, either by increasing first period consumption or by reducing it. More generally, if the consumer is indifferent between her outside option and seller 2's offer in *any equilibrium*, pure or mixed, then seller 2 must exclude this consumer, i.e. seller 2's offer to this consumer must be zero. Consequently, in any equilibrium, first period consumption choices must be random, thereby generating private information and informational rents for consumers in the second period, as well as the required exclusion for the consumers with the lowest marginal willingness to pay. Even if consumers are ex ante identical, the first period seller offers a large set of quantities, giving rise to ex-post taste heterogeneity.

Our main finding is that equilibrium outcomes are essentially unique.[1] The first period monopolist offers a large menu, which ranges between the efficient quantity and that chosen in the observable consumption case. The consumer, who is indifferent between all bundles in the menu, chooses an item according to a continuous distribution with full support. This induces an endogenous screening problem in the second period, since the consumer has private information about her past consumption. We find that the consumer and the second-period seller benefit from unobservability, whereas the first-period seller loses, as compared to the observable consumption benchmark. Furthermore, consumer privacy unambiguously increases total welfare.

The remainder of this paper is organized as follows. Section 2 discusses the re-

[1] All equilibria have the same distribution over first and second period consumptions. They differ (possibly) only in terms of the distribution of payoffs between the first period seller and the consumer.

lated literature. Section 3 sets out the model, and examines two cases: where the second period seller observes past consumption, and where she only observes firm 1's offer. Section 4 analyzes private transactions and establishes existence and essential uniqueness of an equilibrium where consumer heterogeneity arises endogenously, and examines how privacy affects welfare and consumer surplus. The final section concludes. Proofs that are not presented in the body of the paper can be found in the appendix.

2 Related literature

Consumer privacy has become an important issue in the era of electronic records, big data and online shopping—see Acquisti et al. (2016) for a comprehensive survey of the topic. If past consumption decisions are observable, this may allow firms to identify the consumer's persistent type, thereby creating opportunities for price discrimination. When the consumer interacts repeatedly with the same firm, Taylor (2004) shows that a naive consumer may be exploited by the firm. However, if the consumer is sophisticated, then the firm may want to commit to not utilizing personal data, in order to avoid the ratchet effect, an argument that is also made by Villas-Boas (2004). Fudenberg and Tirole (2000) study behavior-based price discrimination, where the consumer's current choices reveal her relative preference for different brands.

These arguments also apply when upstream firms sell information on consumers to downstream firms, as noted by Calzolari and Pavan (2006). They show that the upstream seller offers the consumer full privacy if the preferences of the consumer and the downstream seller are additively separable in consumption over the two periods.

Tokis (2017) shows that when consumer valuations are imperfectly positively correlated or negatively correlated, the upstream firm may want to sell partial information on the consumer. Tokis allows the upstream firm to commit to a Bayesian experiment, and shows that an optimal experiment involves partial disclosure of the consumer's first period revealed valuation.

Ichihashi (2020) studies consumers' incentives to disclose private information when disclosure improves the product recommendations, at the cost of facilitating the firm's price discrimination. Dosis and Sand-Zantman (2019) consider a monopoly firm that sells a service. Data on the consumer's service utilization benefits a third party, but is costly for the consumer. Since the service provider cannot commit to a level of disclosure, when it has property rights over data, it will disclose all of it, without consideration of the costs to the consumer. When the consumer has property rights over data, she is unable to access the market for data, and hence no data is disclosed. Consequently, property rights should be assigned to the firm (resp. consumer) when the benefits of data usage are large (resp. small) relative to costs.[2]

Our model differs considerably from this literature, since we assume ex ante homogeneity of consumers. Heterogeneity therefore arises only because of differences in past consumption choices, and these are endogenous. In our setting, there is a conflict of interest between the upstream firm and the consumer, and also between the upstream firm and the downstream firm. The upstream firm will not voluntarily make a commitment to privacy, since making transactions public increases its monopoly power. Conversely, when transactions are private, this increases the consumer's bargaining power vis-a-vis both firms. Thus conceptually, our work is more closely related to models with hidden actions, e.g. work on static moral hazard with renegotiation (see Fudenberg and Tirole (1990) and Ma (1991)). Similarly, González (2004) and Gul (2001) analyze the hold-up problem with unobservable investment.

We now compare our model in more detail with Calzolari and Pavan (2006). When the goods offered by the two firms are either complements or substitutes, they provide *sufficient* conditions for the upstream firm to disclose a non-trivial amount of information to the downstream firm (as already noted, full privacy is optimal when utility is additively separable in the two goods). In contrast to their approach where the degree of consumer privacy is decided by the firms, we study what happens when consumer privacy is mandated by a third party (e.g., a regulator).

Our setup differs from Calzolari and Pavan (2006) in two important ways: we look at the environment with perfectly divisible goods and no ex ante private information, and we allow the consumer to purchase from the downstream firm even if she decided not to buy from the upstream firm (Calzolari and Pavan (2006) assume that the consumer cannot purchase from the downstream firm if she rejects the upstream firm's offer in favor of the outside option) .[3] First, under the latter assumption, the consumers may benefit from privacy even if they are homogeneous. Second, since goods are divisible, we are able to derive results on the direction of consumption distortion. For instance, when goods are imperfect substitutes, we show that the first seller induces overconsumption and the second seller induces underconsumption.

[2]Less directly related are the papers that consider externalities in selling data. Acemoglu et al. (2019) assume that consumer valuations are correlated. By buying information from consumer j, the firm can also learn about j's valuation. Consequently, data is sold too cheaply by consumers since they do not take into account the negative externality on other consumers. Jones and Tonetti (2020) examine the case of data that is broadly useful—individual medical records combined with genetic information are a case in point. Since data is a public good, it is under-provided.

[3]In our model, the sequential common agency is of "delegated" variety, whereas in Calzolari and Pavan (2006) it is closer to the "intrinsic" kind. Intrinsic and delegated common agency models usually have distinct predictions (Martimort and Stole, 2003, 2009).

Our model is related to models of common agency (see Bernheim and Whinston, 1986; Stole, 1991; Martimort, 1992; Martimort and Stole, 2002; and a survey by Martimort, 2006). The principals in these models correspond to our sellers, and the agent to the consumer. Bernheim and Whinston (1986) allow sellers offer extended menus. They show that there exists a pure strategy equilibrium where non-chosen items are priced "truthfully", that has good efficiency properties. In our context, pure strategy equilibria do not exist, even with unrestricted menus. The key difference is that the consumer's decisions are sequential in our model. When a consumer receives an offer from a seller today, she does not have the option of revising her purchases yesterday. We discuss this difference in more detail at the end of section 4.1. Furthermore, equilibrium outcomes are essentially unique in our setting, without recourse to refinements such as truthfulness.

The delegation principle for common agency models was introduced by Martimort and Stole (2002). Prat and Rustichini (1998) examine sequential common agency: the principals move sequentially, the offers they make to the agent are public, and the agent chooses after observing all offers. Thus the model they study is somewhat related to our model when transactions are public, although in our setting, the agent also chooses sequentially. Pavan and Calzolari (2009) extend the analysis of sequential common agency, and show that a version of the delegation principle applies. In our setup with quasi-linear utility and no ex ante private information, first period consumption is the only payoff-relevant variable that the consumer might report to the second period firm. This is indeed, what is allowed in our model. We discuss these connections further in the context of our results, see in particular Remark 12.

Finally, our work is also related to the literature on long-term bilateral contracts in a multilateral environment. Diamond and Maskin (1979) and Aghion and Bolton (1987) show that a buyer-seller pair today may induce inefficiency via long term contracts, in order to extract surplus from a future seller. Our contracts are static and the dynamics are induced by the agent's preferences. Furthermore, our focus on private transactions differs from this literature, which assumes that the future seller observes the past contract.

3 The model

Our model can be thought of as one with a single consumer, or as one with a continuum of ex ante identical consumers. For ease of presentation, we describe our model in terms of a single consumer—we will re-visit the alternative interpretation periodically. This consumer, who lives for two periods, visits seller 1 in the first period and seller 2 in the second period. Her utility is

$$u(x, y) - p - q,$$

where x and y are consumption in the first and second period respectively, and p and q are the payments made to sellers 1 and 2 respectively. The value of consumption in the second period depends on the level of consumption in the first period. We assume that u is strictly increasing, strictly concave and twice continuously differentiable. We assume throughout that either **A1** or **A2** holds:

A1: The two goods are complements, i.e. $u(x, y)$ is strictly supermodular.

A2: The two goods are imperfect substitutes, i.e. $u(x, y)$ is strictly submodular.

Notably, our key assumption is that preferences are *not* additively separable in the two goods, so that the cross-partial derivative of utility, $u_{12}(x, y)$, is never zero, and does not change sign. Given this assumption, we will dispense with the qualifiers "strictly" and "imperfect" in the remainder of this paper.[4]

We assume that each seller is a monopolist within the period, although he competes across periods with the other seller. Such monopoly power can arise even if there are many sellers in each period, if the consumer faces search costs, as in Diamond (1971). If we assume that a consumer cannot observe a firm's offer before visiting it, this gives rise to monopoly power for the seller who is visited.

Each seller has constant marginal cost k. Since a seller interacts with the consumer only for one period, he seeks to maximize his flow profit. We allow each seller to choose non-linear prices. Thus seller 1 chooses an arbitrary lower semi-continuous function $p : \mathbb{R}_+ \to \mathbb{R}_+$, where $p(x)$ is a price for a bundle of size x, and similarly, seller 2 chooses a lower semi-continuous function $q : \mathbb{R}_+ \to \mathbb{R}_+$.

The socially efficient level of consumption (x^*, y^*) is defined as follows. Define $y^*(x)$ as the value of y that solves $u_2(x, y) = k$, if this equation has a positive solution, and zero otherwise. Since $u_2(x, y)$ is strictly decreasing in y, there is a unique value $y^*(x)$ for every x. Consider values of x such that $y^*(x) > 0$; on this range, $y^*(.)$ is differentiable, strictly increasing when goods are complements and strictly decreasing when goods are substitutes. Let x^* be the value of x that solves $u_1(x, y^*(x)) = k$, and let $y^* := y^*(x^*)$. Since u is strictly concave, there is a unique solution, so that the socially efficient level of consumption (x^*, y^*) is unique.

We will consider two distinct information structures, which differ only in the information of firm 2.

[4] That is, the statement "goods are substitutes" should be read as "goods are imperfect substitutes", i.e. "u is strictly submodular". The statement "goods are complements" corresponds to the assumption "u is strictly supermodular".

- First period consumption is observed by firm 2.
- Transactions are private. Firm 2 does not observe either offers or consumer's choices from period 1.

Our equilibrium notion, across these models, is a version of Perfect Bayesian Equilibrium that satisfies the following conditions. First, the choices of the buyer and firm 2 are sequentially rational. Second, any deviation by seller 1 does not affect the buyer's beliefs about the pricing scheme that will be offered by seller 2. That is, beliefs satisfy the "no-signaling what you don't know" condition (Fudenberg and Tirole, 1991).

3.1 Observable consumption

Consider the situation where first-period consumption is perfectly observed by the second-period seller. First, observe that it does not matter whether the pricing scheme offered by seller 1 is observed by seller 2 or not. If it is observed, our equilibrium notion is equivalent to subgame perfect equilibrium. If the pricing scheme is unobserved by seller 2, this makes no essential difference. With quasi-linear utility, the price paid by the consumer in period one does not affect her incentives in period two. Thus the only period one variable that is payoff relevant for firm 2 is first period consumption.

Oberve that seller 1 acts as a Stackelberg leader: the quantity that seller 1 sells is chosen so as to maximize the joint payoff of seller 1 and the consumer, over the two periods, since seller 1 can extract all the surplus from the consumer. Since seller 2 is a monopolist in period 2, the consumer gets no surplus in the second period, and her payoff is equal to that from choosing the outside option, i.e., it equals $u(x, 0)$.[5] The consumer's value from consuming x equals

$$u(x, 0) - p.$$

If the consumer does not buy from the first period monopolist, her overall payoff equals $u(0, 0)$.[6] Since the first period seller optimally sets p so that

$$p(x) = u(x, 0) - u(0, 0),$$

the bundle that maximizes the first period profits when consumption is observable, x^o, satisfies

[5] Seller 2 chooses $y = y^*(x)$, and sets $q = u(x, y^*(x)) - u(x, 0)$. Thus the consumer's second period payoff $u(x, y^*(x)) - q$ equals $u(x, 0)$, since payments made to seller 1 are already sunk.

[6] Recall that her second period continuation payoff after any x always equals $u(x, 0)$.

$$u_1(x^o, 0) = k.$$

We shall assume throughout:

A3: $y^o := y^*(x^o) > 0$.

This assumption ensures that the first seller cannot serve the customer alone and the second seller plays a meaningful role in this market. An alternative formulation for this assumption is that $u_2(x^o, 0) > k$.

We define the *Stackelberg allocation* as the consumption pair (x^o, y^o).

Proposition 1. *Suppose that the first period consumption x is observed by the second period seller. In either case, the consumption profile corresponds to the Stackelberg allocation. If goods are substitutes, then $x^o > x^*$ so that the first period monopolist induces excessive consumption relative to the first best. If goods are complements, then $x^o < x^*$, so that the first period seller induces underconsumption relative to the first best. Second period consumption is always (conditionally) efficient.*

The proof of this proposition is intuitive. Assume that first period consumption x is observable. In the unique subgame perfect equilibrium, firm 2 chooses y to maximize $u(x, y) - u(x, 0) - ky$, which implies that the consumer's second period payoff is $u(x, 0)$. Thus firm 1 can charge a price of $u(x, 0) - u(0, 0)$, and will therefore maximize $u(x, 0) - kx - u(0, 0)$, proving the result. When goods are complements, the maximizer of $u(x, 0) - kx$ is strictly less than the maximizer of $u(x, y^*(x))$, since $y^*(x^o) > 0$. When goods are substitutes, the maximizer of $u(x, 0)$ is strictly more than x^*.

We note that both x^*, the efficient level, and x^o, the consumption level when it is observable, play an important role in the analysis when transactions are entirely private.

4 Private Transactions

We now analyze the main specification of our model, where seller 2 can observe neither the offer made by seller 1 nor the consumer's choice in the first period.

4.1 Non-existence of a pure strategy equilibrium

Our first result is that there does not exist a pure strategy equilibrium either when goods are substitutes or when they are complements. To gain some intuition for this result, consider the case where the goods are substitutes. Let us see why consuming x^o in the first period is not an equilibrium. Suppose firm 2 believes that the consumer has indeed consumed x^o. In this case, firm 2 would offer y^o, at a price $q = u(x^o, y^o) - u(x^o, 0)$. Given that this is firm 2's offer, the payoff of the coalition consisting of firm 1 and the consumer (Figure 1a, dashed line) from any $x < x^o$ is given by

$$u(x, y^*(x^o)) - q - kx.$$

Submodularity of preferences implies that the consumer strictly prefers $(y^*(x^o), q)$ to her outside option when $x < x^o$. The derivative of the above expression with respect to x is *negative* at $x = x^o$, since x^o maximizes $u(x, 0) - kx$ (Figure 1a, solid line). In other words, the Stackelberg outcome fails to be an equilibrium outcome when the first period action is not observable for a familiar reason—it is not a best response to the firm 2's action.

This raises the question: why is there not a pure strategy Nash equilibrium where the consumer chooses some $\tilde{x} < x^o$? In such a candidate equilibrium, firm 2 will offer $y^*(\tilde{x})$ at a price $q = u(\tilde{x}, y^*(\tilde{x})) - u(\tilde{x}, 0)$. Suppose now that firm 1 offers some $x \in (\tilde{x}, x^o]$. If the consumer accepts this offer, then it will be optimal for her to take the outside option in the second period. Consequently, the coalition consisting of firm 1 and the consumer will get a payoff of

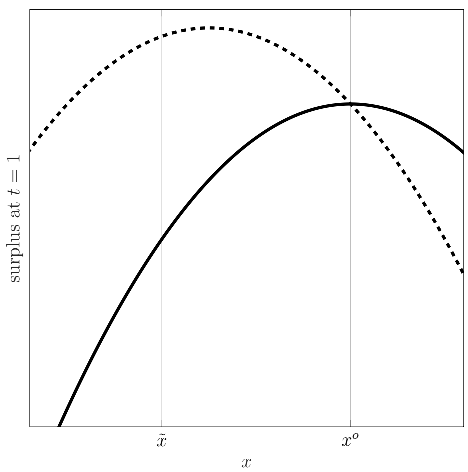
$$u(x, 0) - kx.$$

The derivative of this payoff with respect to x is strictly positive, since $x < x^o$ (Figure 1b, dashed line). In other words, there cannot be a pure strategy equilibrium with consumption below the Stackelberg level, since then there is an incentive to deviate upwards.

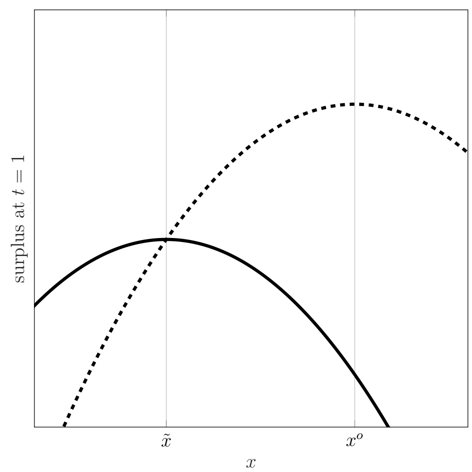
The fundamental problem is as follows. On the one hand, first period consumption x must maximize $u(x, y) - kx$, since the consumer's actual consumption in equilibrium is the pair (x, y). On the other hand, x must maximize $u(x, 0) - kx$, since the consumer's second period payoff in a pure strategy equilibrium must equal $u(x, 0)$, the payoff that she gets from the outside option. Since no single value of x can solve both these maximization problems when the cross-partial of utility is non-zero, there cannot be a pure strategy equilibrium.

Proposition 2. *If goods are imperfect substitutes or if they are complements, and assumption A3 is satisfied, there does not exist an equilibrium where the consumption in period 1 is deterministic.*

Proof. In our proof we allow sellers to choose menus, rather than just take it or leave it offers. In particular, allowing seller 2 to choose a menu does not solve the problem.



(a) Seller 1 sells x^o on path (solid line: value of buying x conditional on *not* buying in period 2, dashed line: value of buying x conditional on buying $y^*(x^o)$ in period 2);



(b) Seller 1 sells $\tilde{x} < x^o$ on path (solid line: value of buying x conditional on buying $y^*(\tilde{x})$ in period 2, dashed line: value of buying x conditional on *not* buying in period 2);

Figure 1: Consumer's value of purchasing first period consumption conditional on various choices in the second period.

Suppose, by contradiction, there exists an equilibrium in which first period consumption is deterministic. We denote this consumption by $\tilde{x}$. Let $q(y)$ be a menu offered by the second seller in this equilibrium. We allow q to be an arbitrary semi-continuous function.

Second period consumption is also deterministic and equals $y^*(\tilde{x})$, and the price paid equals $\tilde{q} = q(y^*(\tilde{x})) = u(\tilde{x}, y^*(\tilde{x})) - u(\tilde{x}, 0)$. This follows from the profit maximizing behavior of seller 2 given that he believes that first period consumption equals $\tilde{x}$. Let $\tilde{y} := y^*(\tilde{x})$.

Fixing the menu in the second period, let us define the sum of the payoffs of firm 1 and the consumer as a function of first period consumption

$$\Sigma(x) = \max_{y \geq 0}\{u(x, y) - q(y)\} - kx.$$

Note that value $\Sigma(x)$ is achieved when the consumer chooses optimally from the menu $q(y)$.

Since the consumer chooses $\tilde{x}$ and $\tilde{y}$ on equilibrium path, $\Sigma(x)$ must achieve a maximum at the same value of x as

$$u(x, \tilde{y}) - \tilde{q} - kx.$$

The latter function is strictly concave and differentiable with respect to x, therefore at the maximum the first order condition holds:

$$u_1(\tilde{x}, y^*(\tilde{x})) = k.$$

From this we conclude that $(\tilde{x}, \tilde{y}) = (x^*, y^*)$.

By sequential rationality of seller 2, $\tilde{q} = u(x^*, y^*) - u(x^*, 0)$ and, therefore, the sum of seller 1's and the consumer's equilibrium payoffs is

$$\Sigma(x^*) = u(x^*, 0) - kx^*.$$

We now argue that seller 1 has a profitable deviation, *no matter* what the menu $q(y)$ is as long as $q(0) = 0$. Since the option of not buying from seller 2 is always available to the consumer, seller 1 can offer the consumer the singleton menu of x^o rather than x^*. The joint payoff of seller 1 and the consumer from this deviation equals *at least* $u(x^o, 0) - kx^o$. Since x^o is the unique maximizer of $u(x, 0) - kx$, this deviation is profitable:

$$u(x^o, 0) - kx^o > u(x^*, 0) - kx^*.$$

This proves that the conjectured pure strategy equilibrium with deterministic consumption does not exist. □

This argument can be generalized beyond pure strategies: in equilibrium, the consumer cannot be indifferent between two different menu items offered by seller 2, as we show in Lemma 18 in Appendix A.

Remark 3. *With a continuum of anonymous consumers, the proposition implies that there cannot be an equilibrium where all the consumers consume the same amount in period one. In other words, even though the consumers are ex ante identical, they will be different when they confront firm 2, since there will be a non-degenerate distribution of first-period consumptions.*

The literature on static multilateral contracting has shown that for existence of pure strategy equilibria, it is essential that all sellers offer extended menus, with options that are not accepted by the buyer in equilibrium. These options must be appropriately priced, so that if seller 1 deviates, the buyer can exercise one of these options from seller 2. Martimort and Stole (2002) provide an example in a common agency context, while Rey and Whinston (2013) and Calzolari et al. (2017) provide another in the context of wholesaler retailer interaction with restrictions on the contracting space. Similarly, in classic common agency models (Bernheim and Whinston, 1986; Chiesa and Denicolò, 2009), it is essential that principals price non-chosen actions "truthfully"—otherwise, if they are restricted to take it or leave it offers, a pure strategy equilibrium may not exist.

In this context, it may be illuminating to see why the truthful equilibrium of the common agency model fails to be an equilibrium in our setting. In the static common agency version of our model, both sellers choose non-linear price schedules simultaneously; the consumer observes both schedules before choosing from each. In a truthful equilibrium, both sellers offer two-part tariffs with the per-unit price equal to marginal cost. Under these tariffs, the consumer chooses the efficient bundle (x^*, y^*). Note that this equilibrium is in pure strategies, and consumption is deterministic. Nonetheless, the equilibrium requires both firms to offer menus, rather than a single take it or leave it offer.[7] When the goods are imperfect substitutes,[8] the lump-sum fee that firm 2 charges satisfies:[9]

$$A_2 = u(x^*, y^*) - u(x^o, 0) - k[x^* - x^o] - ky^*.$$

[7] If firms were restricted to take it or leave it offers, no pure strategy equilibrium would exist. Indeed, even with other restrictions on the strategy sets of firms, such as take-it or leave-it offers, one may not have a pure strategy equilibrium or deterministic consumption when firms move simultaneously, as Inderst (2010) notes. In our setting, non-existence of a pure strategy equilibrium arises even though firms may choose arbitrary non-linear prices.

[8] The argument can be made similarly for the case when the goods are complements.

[9] Symmetrically, $A_1 = u(x^*, y^*) - u(0, y^\dagger) - k[y^* - y^\dagger] - kx^*$, where $y^\dagger$ is the maximizer of $u(0, y) - ky$.

Firm 2 cannot charge higher lump-sum fee since the consumer can always reject firm 2's offer and increase her consumption from firm 1 to $x^o > x^*$.

In our two-period dynamic model, the firms effectively choose actions simultaneously, since firm 2 does not observe firm 1's actions. However, the consumer must purchase from firm 1 before she sees the offer from firm 2. Consequently, in a candidate equilibrium where firm 1 chooses the tariff (A_1, k), and the consumer purchases x^* in period 1, firm 2 can increase its lump-sum fee to

$$A_2^{'} = u(x^*, y^*) - u(x^*, 0) - ky^*$$

and keep the per-unit price equal to the marginal cost. The new fee $A_2^{'}$ is chosen to make the consumer indifferent between buying y^* from firm 2 and not (under the premise that the amount purchased from firm 1 cannot be altered upon seeing the offer made by firm 2). The difference $A_2^{'} - A_2$ equals

$$[u(x^o, 0) - kx^o] - [u(x^*, 0) - kx^*] > 0$$

since x^o is the unique maximizer of $u(x, 0) - kx$. In other words, the common agency profile fails to be an equilibrium since the consumer cannot revise her choices at firm 1 after observing firm 2's offer. Nor is it an equilibrium for the firms to offer two-part tariffs, with marginal cost pricing and fixed fees A_1 and A_2' respectively; in this case, the consumer would do better by consuming x^o from firm 1 and nothing from firm 2.

4.2 Endogenous screening: An overview

Having established that the first period consumption must be random, we now construct an equilibrium with the following features. Seller 1 charges a two-part tariff (A, k), where A is the fixed fee, and the price per unit equals the marginal cost k. The consumer chooses period 1 consumption randomly, according to a continuous c.d.f. F that has support the interval $[\underline{x}, \overline{x}]$. Since first period consumption x affects the consumer's willingness to pay for the second period bundles, seller 2 offers an optimal screening menu. It is convenient to think of this menu in the form of a direct mechanism $(\hat{y}(x), \hat{q}(x))$, where the second period consumption $\hat{y}(x)$ is strictly monotone in x.[10] Under this menu, a consumer with first period consumption x gets

[10]We use direct mechanisms as an analytic tool and we *do not* claim that the set of direct mechanisms is sufficiently rich to span all equilibria in our model. For example, direct mechanisms do not account for items on the menu that are not chosen on equilibrium path; these items play an important role in our analysis, as shown in Section 4.5.

second period indirect utility $U(x)$,[11] where $U(x) - kx$ is constant on the interval $[\underline{x}, \overline{x}]$. Consequently, due to the two-part tariff in the first period, with marginal price k, the consumer is indifferent between all bundles in this interval, and it is optimal for her to randomize according to the distribution F. Finally, equilibrium is "essentially unique". In the case of complements, the equilibrium outcome is indeed unique. In the case of substitutes, if the second period seller caters to first period consumption levels smaller than $\underline{x}$, this may affect the division of payoffs between firm 1 and the consumer, by affecting the first-period outside option of the consumer.

Let us now examine how this randomization helps overcome the fundamental impossibility that arose with a pure strategy equilibrium. Consider, for illustrative purposes, the case of substitutes. Higher values of x correspond to "lower" types, and the worst type is the consumer who has consumed $\overline{x}$. Such a consumer is held to her outside option, and thus must be indifferent between accepting the offered second period bundle designed for her, $(\hat{y}(\overline{x}), \hat{q}(\overline{x}))$ and consuming $y = 0$. If $\hat{y}(\overline{x})$ was strictly positive, then the contradiction that arose in the pure strategy case would also arise here, since $\overline{x}$ cannot simultaneously maximize $u(x, \hat{y}(\overline{x})) - kx$ and $u(x, 0) - kx$. However, if $\hat{y}(\overline{x}) = 0$, then no contradiction arises, and indeed, that is the resolution to this problem. In other words, the induced distribution F ensures the exclusion of the consumer who has consumed $\overline{x}$.

More generally, given the allocation $\hat{y}(\tilde{x})$ for any type $\tilde{x}$, $\tilde{x}$ must maximize $u(x, \hat{y}(\tilde{x})) - kx$. That is, given the second period consumption, the first period bundle chosen by the consumer must maximize her payoff, net of the marginal cost. In other words, the second period consumption $\hat{y}(x)$ must satisfy:

$$u_1(x, \hat{y}(x)) = k, \forall x \in [\underline{x}, \overline{x}]. \tag{1}$$

Observe that this uniquely pins down the second period consumption $\hat{y}(x)$, and ensures that it is strictly decreasing when goods are substitutes, and strictly increasing when goods are complements.

To summarize, the critical features of any equilibrium are as follows:

1. $U(x) - kx$ is constant for every $x \in [\underline{x}, \overline{x}]$. This ensures that the consumer and seller 1 are indifferent as to which element of $[\underline{x}, \overline{x}]$ the consumer chooses.

2. The induced distribution of the first period consumption, F, is such that seller 2 finds it optimal to offer $U(x)$ for each $x \in [\underline{x}, \overline{x}]$.

[11] $U(x) := u(x, \hat{y}(x)) - \hat{q}(x)$, and the consumer's overall payoff equals $U(x) - kx - A$.

3. The second period consumption $\hat{y}(x)$ satisfies (1), and is therefore strictly decreasing in the case of substitutes, and strictly increasing in the case of complements.

4. Finally, the endpoints of the interval $[\underline{x}, \overline{x}]$ are pinned down by the characteristics of the solution to the monopoly screening problem. Since there is no distortion at the top, the second period consumption of the highest type—e.g., $\underline{x}$ in the case of substitutes—must be optimal given $\underline{x}$. Combined with point (1) above, this implies that $\underline{x} = x^*$, the first best level of consumption (when goods are complements, the highest type corresponds to $\overline{x}$ which must equal x^*). Since there is no informational rent at the bottom—e.g., for type $\overline{x}$ in the case of substitutes—her consumption level must maximize the joint payoff of the consumer and seller 1 given that she takes the outside option 0 in the second period. This implies $\overline{x} = x^o$. Thus, the first period consumptions span the range between first best and the equilibrium consumption in the case when the past history is observable, while second period consumptions lie between 0 and the first best consumption, y^*.

4.3 Equilibrium characterization

In this section we formalize the ideas presented in Section 4.2. We begin with the characterization of the continuation payoff in the second period and then we provide conditions that pin down the distribution of consumption in the first period.

Proposition 2 has established that in any equilibrium, the first period consumption must be random. Let X denote the support of the equilibrium distribution of the first period consumption F; X is a closed set, by definition. We shall also assume that every bundle in X is offered and chosen by the consumer.[12] Note that X cannot contain 0—in this case, seller 1's profits must equal zero, and this cannot be optimal for seller 1 and the consumer.

Denote a menu offered by seller 2 in equilibrium by $q(y)$ and the section of the menu that is chosen by the consumer on the equilibrium path by $(\hat{y}(x), \hat{q}(x))_{x \in X}$.[13] The second period indirect utility of the consumer after choosing bundle $x \in X$ in the first period is

$$U(x) := u(x, \hat{y}(x)) - \hat{q}(x).$$

[12] That is, we assume that the set of chosen bundles is closed, so that every $x \in X$ has an associated pair $(\hat{y}(x), \hat{q}(x))$ in the menu. This assumption is inessential, but simplifies the statement of some results.

[13] Note that $q(\hat{y}(x)) = \hat{q}(x)$ for all $x \in X$.

For any $x \notin X$, let

$$U(x) := \max_{y \geq 0}\{u(x,y) - q(y)\}.$$

Thus, U is specified by prescribing optimal choices for all non-chosen types.

The following lemma provides structure on the set X.

Lemma 4. *X is a compact interval.*

We now establish, using the arguments of Milgrom and Segal (2002), that U is absolutely continuous, and can therefore be written as the integral of its derivative. Lemma 4 above shows that the consumer's first period consumptions lie in a compact interval. The consumer's overall utility is quasi-linear and $u(x, y)$ is absolutely continuous in x, being concave. Furthermore, when goods are substitutes or they are complements, $u(.)$ satisfies the Spence-Mirrlees single crossing condition. Thus the conditions in Theorem 2, Corollary 1 and footnote 10 in Milgrom and Segal (2002) are satisfied, implying that U is absolutely continuous.

The sum of the payoffs for the consumer and seller 1 if the former chooses x is

$$\Sigma(x) := U(x) - kx.$$

When seller 1 sets the menu, he is able to extract the entire surplus net of the value of the first period outside option (that does not depend on the first period menu). Therefore, if x is purchased by the consumer in equilibrium, i.e., if $x \in X$, then $p(x) - kx = U(x) - kx - U(0) = \Sigma(x) - U(0)$. When prices are set according to the participation constraint, the seller's problem boils down to the question of which items to offer on the menu (or, equivalently, which items are prohibitively expensive). Seller's profit in equilibrium is

$$\int_X \Sigma(x)dF(x) - U(0).$$

By offering an alternative menu

$$\hat{p}(x) = \begin{cases} U(x) - U(0), \text{ if } x = x' \\ \infty, \text{ if } x \neq x' \end{cases}$$

the seller can obtain a profit equal to $\Sigma(x') - U(0)$. Therefore, the seller's optimality requires that for all $x \in X$ and for all $x' \geq 0$:

$$\Sigma(x) \geq \Sigma(x').^{14}$$

We now present a novel property of the consumer's indirect utility:

[14]Formally, for a given x', $\Sigma(x) \geq \Sigma(x')$ must hold almost always. However, since $\Sigma(\cdot)$ is absolutely continuous, this inequality must hold for all $x \in X$

Lemma 5. *Equilibrium second period indirect utility of the consumer $U(x)$ is differentiable at every $x \in X$.*

Remark 6. *The property that U is differentiable on X, the set of types that are chosen in equilibrium, follows from the endogeneity of types. Standard arguments in mechanism design, where types are exogenous, imply that U need not be everywhere differentiable, and there may be convex kinks. However, if U has a convex kink at x, then it would not be optimal for x to be chosen. Thus, differentiability follows from the endogeneity of types—this lemma applies more general than the specific context of our model.*

It is standard in theory of incentives that single-crossing and incentive compatibility implies weak monotonicity. However, lemma 5 allows a stronger result.

Lemma 7. *$\hat{y}(x)$ must satisfy*

$$u_1(x, \hat{y}(x)) = k.$$

Moreover, $\hat{y}(x)$ is strictly decreasing (resp. increasing) in x if goods are substitutes (resp. complements).

Proof. Since U is differentiable at $x \in X$, if x maximizes $\Sigma(.)$, it must satisfy

$$\Sigma'(x) = u_1(x, \hat{y}(x)) - k = 0.$$

Consider a case of substitutes—i.e., $u_{21} < 0$. If $x > \tilde{x}$ then $\hat{y}(x)$ must be strictly less than $\hat{y}(\tilde{x})$, or otherwise the expression for $\Sigma'(.)$ above will be strictly negative. Similarly, in the case of complements utility—i.e., if $u_{21} > 0$, $\hat{y}(x)$ must be strictly greater than $\hat{y}(\tilde{x})$. □

Let $\underline{x}$ denote the minimal element in X and $\overline{x}$ the maximal element. The following lemma shows that if individual rationality is satisfied for type $\overline{x}$ in the substitutes case, then it is satisfied for every other type—although familiar, the result is not immediate since the outside option $u(x, 0)$ is type dependent. A similar result is true for the case of complements.

Lemma 8. *If goods are substitutes, $U(x) - u(x, 0) \geq U(\overline{x}) - u(\overline{x}, 0)$ for all $x \in X, x \neq \overline{x}$. Moreover, under any profit maximizing second period contract, $U(\overline{x}) = u(\overline{x}, 0)$, and the individual rationality constraint binds for type $\overline{x}$. If goods are complements, the individual rationality constraint binds for type $\underline{x}$, and is slack for every other type in X.*

Proof. For $x < \overline{x}$, since type x can pretend to be $\overline{x}$, incentive compatibility implies that

$$U(x) \geq U(\overline{x}) + u(x, \hat{y}(\overline{x})) - u(\overline{x}, \hat{y}(\overline{x})).$$

Since $U(\overline{x}) \geq u(\overline{x}, 0)$,

$$U(x) - u(x,0) \geq [u(\overline{x},0) - u(x,0)] - [u(\overline{x}, \hat{y}(\overline{x})) - u(x, \hat{y}(\overline{x}))], \tag{2}$$

which is non-negative since goods are substitutes and $\hat{y}(\overline{x}) \geq 0$.

If $U(\overline{x}) > u(\overline{x}, 0)$, then a menu $(\hat{y}(x), \hat{q}(x))$ cannot be profit maximizing, since a uniform reduction in payoffs $U(x)$ by $U(\overline{x}) - u(\overline{x}, 0)$, achieved by raising $\hat{q}(x)$ by the same amount, preserves incentive compatibility and increases profits. To obtain the same result for the case of complements, replace $\overline{x}$ by $\underline{x}$ in the above argument. □

The following two lemmata identify $\underline{x}$ and $\overline{x}$. Recall that one of the bounds is identified using the fact that the highest type's consumption in the second period is efficient. In order to identify the other bound, we consider possible deviations by seller 1 and establish that the lowest type has to consume zero in the second period.

Lemma 9. *If goods are substitutes, $\overline{x}$ equals the value of x that maximizes $u(x, 0) - kx$—i.e., $\overline{x} = x^o$, and $\hat{y}(\overline{x}) = 0$. If goods are complements, $\underline{x}$ equals the value of x that maximizes $u(x, 0) - kx$—i.e., $\underline{x} = x^o$, and $\hat{y}(\underline{x}) = 0$.*

Proof. If goods are substitutes, since the second period participation constraint binds for the highest value of x that is offered by seller 1 and accepted by the consumer, the consumer is indifferent between $\hat{y}(\overline{x})$ and the consumption level 0 in the second period. By Lemma 18, there exists a profitable deviation for seller 1 unless $\hat{y}(\overline{x}) = 0$. Consequently, lemma 7 implies that $\overline{x}$ must equal the value of x that maximizes

$$u(x, 0) - kx,$$

so that $\overline{x} = x^o$. The proof for the case of complements is identical. □

Lemma 10. *If goods are substitutes, then $\underline{x} = x^*$; if goods are complements, then $\overline{x} = x^*$. In either case, $\hat{y}(x^*) = y^*$.*

Proof. Recall that $y^*(x)$ denotes the first best second period quantity conditional on any level of the first period consumption x. Suppose that goods are substitutes. On one hand, since there is no distortion at the top in the second period screening problem, seller 2 must offer $y^*(\underline{x})$ to the consumer who consumed $\underline{x}$ in the first period. On the other hand, Lemma 7 establishes that $\underline{x}$ must satisfy

$$u_1(\underline{x}, \hat{y}(\underline{x})) = k.$$

These two conditions imply
$$u_1(\underline{x}, y^*(\underline{x})) = k,$$
which means that $(\underline{x}, y^*(\underline{x}))$ satisfies the conditions for the first best allocation. The first best allocation is unique, therefore $\underline{x} = x^*$. When goods are complements, the "top" corresponds to $\overline{x}$, and the rest of the argument is the same. □

To summarize, the characterization in the above lemmata imply that $\overline{x} = x^o$ and $\underline{x} = x^*$ in the case of substitutes. When the goods are complements, $\overline{x} = x^*$ and $\underline{x} = x^o$.

To complete the description of the equilibrium, it remains to specify the distribution of first period consumption F that induces the second period consumption $\hat{y}(x)$ and consumer indirect utility $U(.)$ on the interval $[\underline{x}, \hat{x}]$, and this is set out in the following theorem:

Theorem 11. *There exists an equilibrium in which*

1. *Seller 1 offers a two-part tariff. The fixed fee equals seller 1's value added in the socially efficient consumption stream:*
$$u(x^*, y^*) - kx^* - u(0, y^*).$$
The per-unit price equals the marginal cost k.

2. *Seller 2 offers a menu that includes every bundle in $[0, y^*]$. The bundles in this menu are indexed by the first period consumption x. The price of a bundle $\hat{y}(x)$ is*
$$\hat{q}(x) = u(x, \hat{y}(x)) - kx - [u(\overline{x}, 0) - k\overline{x}].$$

3. *In the first period, the consumer randomly chooses the bundle according to a distribution F. In the second period, she chooses a consumption $\hat{y}(x)$ where x is her first period consumption.*

   If goods are substitutes, the support of the distribution F is $[x^*, x^o]$ and
$$F(x) = \exp\left[\int_x^{\overline{x}} \frac{u_{21}(z, \hat{y}(z))}{u_2(z, \hat{y}(z)) - k} dz\right].$$

   If goods are complements, the support of the distribution F is $[x^o, x^*]$ and
$$F(x) = 1 - \exp\left[\int_{\underline{x}}^{x} \frac{u_{21}(z, \hat{y}(z))}{k - u_2(z, \hat{y}(z))} dz\right].$$

Verification that the above distribution indeed accomplishes the task is set out in Appendix A.3, which completes the proof of the theorem.

Remark 12. *The theorem offers one implementation of the equilibrium outcome: firm 1 offers a menu of options to the consumer, who randomizes over consumption levels x according to F. An alternative implementation would have the firm randomizing over the singleton choices that it offers the consumer. That is, firm 1 makes a take it or leave it offer $(x, p(x))$ to the consumer, where x is random and has distribution F. The two implementations are outcome equivalent, yielding the same distribution over consumptions and realized profits of the two firms. This equivalence is an instance of the delegation principle stated in Martimort and Stole (2002). Indeed, there are infinitely many ways of generating the equilibrium outcome, since firm 1 may offer a restricted menu to the consumer, while randomizing between these restricted menus, leaving the consumer to randomize her choices within the restricted menu.*

Let us return to the interpretation of the model as one with a continuum of consumers, rather than just one. F now represents the distribution of first period consumptions—identical consumers end up consuming differently. This can be implemented by firm 1 offering a two-part tariff, and the consumers choosing different bundles, to give the distribution F. Alternatively, firm 1 could offer different consumers different deterministic bundles, thereby inducing the distribution F. The latter is probably more realistic, since it does not require the mass of consumers to coordinate their choices.

4.4 Uniqueness of equilibrium outcomes

All equilibria in this model have several common features. If the goods are complements, any equilibrium must have the same *outcome* as the equilibrium described in Theorem 11—that is the distribution of consumptions and the payoffs of the sellers and the consumer must be the same.[15] If goods are substitutes, there is a continuum of equilibrium outcomes; however these outcomes only differ because they distribute the surplus differently between seller 1 and the consumer—seller 2's payoff and the distribution of consumptions are invariant. Since the allocation is invariant across these equilibria, we say that the equilibrium outcome is *essentially unique*.

To formalize these ideas we characterize the objects that are invariant across all equilibria. We focus on the substitutes case, since equilibrium outcomes are unique when goods are complements.[16] Fix an equilibrium σ of the game. Let V_σ denote the

[15] As noted in remark 12, the equilibrium outcome can be generated in at least two different ways.
[16] Formally, the results below apply to both cases.

consumer's ex ante utility and let $\pi_{1,\sigma}$ denote the expected profit of seller 1 in this equilibrium. Let $\Sigma_\sigma := V_\sigma + \pi_{1,\sigma}$ denote the sum of payoffs of the consumer and seller 1. Also, let X_σ denote the set of the first-period consumptions chosen in equilibrium σ, and $U_\sigma(x)$ denote the information rent of the consumer after choosing x in the first period of equilibrium σ. Let $\tilde{\sigma}$ denote the specific equilibrium constructed in the previous section, the support of which is the largest possible set, $X_{\tilde{\sigma}} = [x^*, x^o]$. Note that by Lemmas 9 and 10, $X_\sigma \subset X_{\tilde{\sigma}}$.

Lemma 13. *For any equilibrium $\sigma : \Sigma_\sigma = \Sigma_{\tilde{\sigma}}$ and for any $x \in X_\sigma : U_\sigma(x) = U_{\tilde{\sigma}}(x)$.*

Proof. By Lemma 9 the maximal quantity offered and chosen in period one equals x^o in any equilibrium, and the informational rent that accrues to the consumer is zero in this case. Since x^o is in the support of every equilibrium, for any equilibrium σ

$$\Sigma_\sigma = u(x^o, 0) - kx^o = \Sigma_{\tilde{\sigma}}.$$

For all $x^* \in X_\sigma$ the following holds in equilibrium

$$\Sigma_{\tilde{\sigma}} = U_\sigma(x) - kx,$$

therefore $U_\sigma(x) = U_{\tilde{\sigma}}(x)$. □

In the light of this proposition, we write Σ and $U(x)$ for the payoffs that arise in *any* equilibrium. Let F denote the c.d.f. associated with $\tilde{\sigma}$, as defined in the previous section, and let f denote the associated density. The following theorem is our main result in this section—the reader should note that we will define robust equilibrium shortly.

Theorem 14. *1. If goods are complements, then the equilibrium outcome is unique.*

2. If goods are substitutes, the equilibrium outcome is essentially unique, and vary only in the division of payoffs between the consumer and seller 1. In every equilibrium:

(a) the distribution of the first-period consumption is F with the support on $[x^*, x^o]$;

(b) the items of the second-period menu that are chosen on equilibrium path are $\{(\hat{q}(x), \hat{y}(x))\}_{x \in [x^*, x^o]}$; and

(c) the sum of the equilibrium payoffs of seller 1 and the consumer, Σ, is invariant across equilibria.

The multiplicity of payoff division when goods are substitutes arises because seller 2 can add items that are larger than y^* to his menu. Since these items are never purchased in equilibrium, the prices can be chosen arbitrarily in a large set without affecting seller 2's profits. In particular, seller 2 can add $y^*(0)$ to his menu, at a price $q(0)$. As long as $q(0)$ is not too low, this does not affect the incentive constraints of any types that arise in equilibrium, i.e. in the set $[x^*, x^o]$. Furthermore, since the bundle $y^*(0)$ is never chosen on the equilibrium path, its price does not affect seller 2's profits. Indeed, the marginal price for purchasing above $y^*(x^*)$ can even be lower than marginal cost! That is, since the consumer always buys in period one, in equilibrium, and will never purchase more than $\hat{y}(x^*)$ in period 2, it does not cost firm 2 to price low for additional units.

However, the price $q(0)$ *does affect* the consumer's outside option in period 1—if he does not buy, then his payoff equals

$$\max\{u(0, \hat{y}(x^*)) - q(x^*), u(0, y^*(0)) - q(0)\}. \tag{3}$$

By making $q(0)$ small, firm 2 can raise the consumer's first period outside option, thereby reducing the fixed fee F that firm 1 can charge the consumer.

We now invoke the following perturbation of our model, that allows us to single out a unique $q(0)$, and thereby, unique equilibrium payoffs for all the agents. Suppose that there is a small probability $\epsilon > 0$ that the consumer is unable to shop in period one, and therefore consumes 0. Let us call this game $G(\epsilon)$. We define a *robust equilibrium* of our basic model, $G(0)$, as an equilibrium of $G(0)$ that is the limit of equilibria of a sequence of games, $G(\epsilon)$, as $\epsilon \to 0$. In this limit, even though $y^*(0)$ is not chosen on path, it is priced optimally.

Proposition 15. *Regardless of whether goods are substitutes or complements, there is a unique robust equilibrium outcome, so that the payoffs of all agents in the game are uniquely determined.*

Proposition 15, which shows uniqueness of robust equilibrium, contrasts with the results for static common agency. Under complete information, if goods are substitutes, there is a unique truthful equilibrium payoff allocation in static common agency. If goods are complements, multiplicity of the equilibrium payoffs arises (see Laussel and Le Breton, 2001).[17]

Finally, we may compare our results on consumer welfare with those obtained by Laussel and Le Breton (2001) for static common agency. The firms that sell

[17]Martimort (1992) finds similar results for a more complicated case of static common agency where the agent has private information. In the complements case, he finds that the allocation varies across equilibria, a possibility that is absent without private information.

substitutes are in competition with each other, and the consumer can exploit the competition to get a surplus in equilibrium. If goods are complements, the consumer does not get any surplus because the next best option to buying from both firms is not buying anything at all.

In our context, competition is sequential, and, hence, even when goods are substitutes, competition is muted—the buyer can no longer return to seller 1 when she finds the terms offered by seller 2 unattractive. Therefore, if transactions are observed, the consumer makes no surplus, regardless of whether goods are substitutes or complements.

If transactions are private, and goods are substitutes, the consumer does get a surplus, while if they are complements, she gets no surplus. Thus, the outcome is similar to that in static common agency, as in Laussel and Le Breton (2001), but the mechanism is more subtle. If the consumer rejects the offer of seller 1, her value from period 2 consumption increases. However, due to privacy, seller 2 does not observe this deviation, and, therefore, cannot exploit the consumer's hunger fully by adjusting the prices. This increases the value of the outside option when confronting seller 1, enabling the consumer to get a surplus. When goods are complements, this force does not operate—not buying from seller 1 *reduces* the consumer's value from the second period consumption, and consequently, she does not earn any surplus.

4.5 Privacy: Effects on welfare, consumer surplus and profits

Our analysis shows that consumers' privacy dramatically affects equilibrium outcomes. The equilibrium outcomes under privacy are very different from the equilibrium outcome in the benchmark, in which past consumption is observed by seller 2. This has distributional consequences, and also affects total (utilitarian) welfare. We find that under privacy:

(i) Social welfare, as measured by the sum of payoffs of all parties, is greater;

(ii) Consumer's utility is greater and seller 2's profit is larger;

(iii) Seller 1's profit is lower;

(iv) The sum of seller 1's profit and consumer utility is the same across the two regimes, privacy and no-privacy.

The most interesting of these results is that total welfare increases under consumer privacy. [18] To understand this, fix attention on the case where goods are substitutes. When transactions are public, there is excessive consumption in the first period, at x^o, in order to improve the second period outside option of the consumer, while second period consumption is conditionally efficient. Under private transactions, first period consumption becomes less distorted—indeed, it lies in the interval $[x^*, x^o]$. Although second period consumption is no longer constrained efficient, the improvement in first period efficiency more than offsets this.

Recall that both the consumer and the first period seller are indifferent between all on-path consumption sequences, including $(x^o, 0)$. This means that their joint payoff is the same across two regimes—it equals the sum of payoffs when the consumer chooses x^o in period one and her outside option in period two.[19] Any difference in welfare arises due to the difference in profit for the second period seller. Therefore, to develop intuition about the privacy-driven increase in welfare, we need to examine how consumer privacy affects the second seller.

For concreteness, suppose that the goods are substitutes. When the past consumption is private, the consumption in the first period is below x^o, and, therefore, the consumers exhibit higher demand for the second period good compared to the no-privacy scenario (more precisely, privacy boosts the distribution of willingness to pay for the second period good in the sense of first-order stochastic dominance). Thus, seller 2 unambiguously benefits from consumer privacy.

To look at this from a slightly different angle, consider an instance in which seller 1 sells $x < x^o$ in equilibrium. On the one hand, in order for seller 1 to be willing to sell less than x^o he must be compensated. The consumer with history $x < x^o$ will get information rent in the second period and seller 1 can include this rent into the price of x. On the other hand, despite the need to award information rent to the consumer with history x, seller 2 profits more from transacting with such a consumer compared to a consumer with *observed* history x^o (see Remark 17).

Finally, the consumer is better off because privacy increases her outside option in the first period. Since her choices are not observed by seller 2, the consumer can choose the outside option in period 1 without causing seller 2 to increase the prices in period 2. This increases the value of the first-period outside option as compared to the benchmark with observable consumption.

Our analysis highlights a novel source of conflict between the consumer and the

[18]Since preferences are quasi-linear, the sum of payoffs of the three parties is the appropriate notion of utilitarian welfare.

[19]When the past is observable, the consumer's overall payoff is the same whether she buys or does not buy in the second period.

upstream firm over privacy. Existing models of privacy are based on private information, and in such models, any conflict between the consumer and the upstream firm normally arises because the latter seeks to monetize the information it acquires from the consumer. Here, the upstream firm would like to make its transactions public for free. The consumer is worse off from the lack of privacy because it worsens her bargaining position vis-a-vis the upstream firm, rather than versus the downstream firm.

Let $\pi_i, i \in \{1, 2\}$ denote the profits of the two firms under privacy, and let V denote the consumer's payoff, let W denote social welfare. Let $\Sigma := \pi_1 + V$ denote the sum of payoffs of the consumer and firm 1. We append superscript o to each of these variables to denote their equilibrium values under the observable transactions benchmark—e.g. the consumer's payoff is V^o. The subscript σ associates a variable with equilibrium σ under privacy. We denote by $\tilde{\sigma}$ the specific equilibrium characterized in Theorem 11, where the second-period menu was minimal.

Theorem 16. *(i) The sum of payoffs of firm 1 and the consumer is invariant to the information structure, i.e.* $\forall\sigma : \Sigma_\sigma = \Sigma^o$*;*

(ii) Firm 1 has lower profits under privacy than under observable consumption, while the consumer has a greater payoff, i.e. $\forall\sigma : \pi_{1,\sigma} < \pi_1^o$ *and* $V_\sigma \geq V^o$*;*

(iii) Profits of firm 2 are greater under privacy, and its increased profits equal the increase in welfare, i.e. $\forall\sigma : W_\sigma - W^o = \pi_{2,\sigma} - \pi_2^o > 0$*; and*

(iv) When goods are complements, equilibrium payoffs are unique. When goods are substitutes, the sum of payoffs of the consumer and firm 1 (Σ_σ*), firm 2 profits (*$\pi_{2,\sigma}$*) and welfare (*W_σ*) are all invariant across equilibria, and only the division of* Σ_σ *between firm 1 and the consumer varies with* σ*.*

Proof. Note that the first-period surplus under privacy, Σ, can be evaluated at any point in the support of first period consumption. In particular, at x^o, the consumer takes her outside option in the second period, and so

$$\Sigma = u(x^o, 0) - kx^o.$$

This is *identical* with the total payoff of seller 1 and the consumer when consumption is observable—although the consumer purchases $y^o > 0$, seller 2 appropriates the difference $u(x^o, y^o) - u(x^o, 0)$, and hence the consumer's continuation payoff is $u(x^o, 0)$. We turn to the distribution of the total payoff between the two parties in the two cases. In the observable case, the consumer's payoff equals

$$V^o := u(0, 0).$$

In the unobservable case, the results now differ depending on whether goods are substitutes or complements. So we consider these in turn.

When goods are complements, the consumer who chooses the outside option in the first period, chooses the item $\hat{y}(x^o) = 0$ in the second period, and therefore gets a total payoff

$$V = u(0, 0).$$

This is exactly equal to V^o, and hence unobservability has no distributional effect on the first period payoffs when goods are complements.

When goods are substitutes, there is a continuum of equilibria that differ by the value of the outside option in the first period. Consider the equilibrium with the smallest such value—i.e., the equilibrium $\tilde{\sigma}$ characterized in Theorem 11. In this equilibrium, if the consumer chooses the outside option in the first period, she buys $\hat{y}(x^*) = y^*$ in the second period and, therefore, gets a total payoff

$$\underline{V} := V_{\tilde{\sigma}} = [u(0, y^*) - u(x^*, y^*)] + u(x^*, 0).$$

The difference in payoffs is

$$\underline{V} - V^o = [u(x^*, 0) - u(0, 0)] - [u(x^*, y^*) - u(0, y^*)] = \pi_1^o - \overline{\pi}_1 > 0,$$

where π_1^o denotes seller 1's profits in the observable case. The second equality in the above follows since the total payoff Σ is equal in the two cases. The strict inequality arises since u is strictly submodular.

Now consider the equilibrium with the largest value of the first-period outside option. Using the same argument we obtain that

$$\overline{V} := [u(0, y^*(0)) - u(x^*, y^*(0))] + u(x^*, 0) > \underline{V}.$$

and $\overline{\pi}_1 > \underline{\pi}_1$. For every $V \in [\underline{V}, \overline{V}]$, there exists an equilibrium in which the consumer's payoff is V and seller 1's profit is $\Sigma - V$.

We conclude that, in any equilibrium, the consumer is strictly better off when consumption is unobservable, and seller 1 is strictly worse off to exactly the same extent.

Since Σ is the same across all equilibria including the benchmark case of the observable past, the gain (loss) of seller 2 from unobservability of past consumption equals the increase (decrease) of the social welfare. Moreover, this gain (loss) is the same for all equilibria because the equilibrium distribution of consumption is the same.

To see that $\pi_2 > \pi_2^o$, note that if seller 2 offers a single item (q^o, y^o), it will be accepted with probability 1 because every consumer's type is better (in terms of marginal willingness to pay) than x^o. This offer is sub-optimal for firm 2—in fact, the firm finds it optimal to exclude the consumer who has chosen x^o. Hence the optimal menu must yield a strictly larger profit, i.e. $\pi_2 > \pi_2^o$. □

Our results imply that if the firm concerned (in our case, firm 1) has the choice regarding privacy policy, then it would choose no privacy, rather than privacy. This is detrimental to consumer interests, and also to the interests of downstream firms. Thus there is case for regulation—e.g., the government could legislate that the consumer has privacy unless she explicitly opts out.

The case for regulation is strengthened by the fact that the downstream firm benefits from consumer privacy and, yet, ex post, it would be willing to purchase the information from the upstream firm. Of course, this does not contradict our results because if the consumer expects a violation of his privacy, he would change his shopping behavior in period 1 and this would be detrimental to firm 2's profit.

The equilibrium consumption is distorted by the intertemporal competition between the sellers in an unusual way. If goods are substitutes, the consumer always over-consumes in the first period and under-consumes in the second. The realized social welfare is monotone in the first period consumption.

Remark 17. *Equilibrium social welfare conditional on first period consumption x is decreasing in x when goods are substitutes, and increasing when goods are complements.*

Proof. Equilibrium social welfare conditional on first period consumption x is

$$W(x) = u(x, \hat{y}(x)) - k(x + \hat{y}(x))$$

Taking a derivative, we obtain

$$\begin{aligned} W'(x) &= u_1(x, \hat{y}(x)) - k + \hat{y}'(x) u_2(x, \hat{y}(x)) - k\hat{y}'(x) \\ &= \hat{y}'(x) \left[u_2(x, \hat{y}(x)) - k \right], \end{aligned}$$

where the second line follows from the first period first order condition, equation (8). Since the second period consumption is always distorted, the term in square brackets is always positive, and hence W is increasing in x when $\hat{y}$ is increasing in x (i.e., when goods are complements), and decreasing in x when $\hat{y}$ is decreasing in x (i.e., when goods are substitutes). □

Table 1: Example

		x	y	W	π_1	π_2	V
	main model	Fig.2a	Fig.2c	6.99	[2.92, 3.08]	2.92	[1.00, 1.16]
$a = -1$	first best	1.00	1.00	7.00	-	-	-
	observable x	1.17	0.97	6.92	4.08	2.84	0.00
	main model	Fig.2b	Fig.2d	5.39	[1.36, 2.27]	1.31	[1.81, 2.72]
$a = -3$	first best	0.78	0.78	5.44	-	-	-
	observable x	1.17	0.58	5.10	4.08	1.02	0.00

Note: This example is computed using $u(x, y) = -3x^2 - 3y^2 + axy + 8x + 8y$ and $k = 1$.

4.6 An example

We now consider a numerical example, where u is given by:

$$u(x, y) = -3x^2 - 3y^2 + axy + 8x + 8y$$

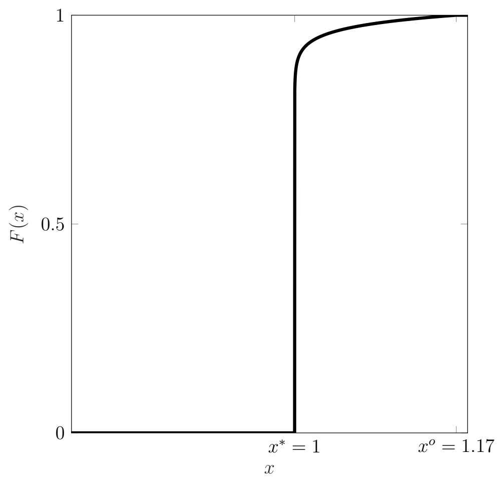
The parameter $a = u_{21}(x, y)$ is a measure of substitutability of the past and current consumption. We focus on the substitutes case, so that a is negative, and focus on two values, $a = -1$ and $a = -3$. The equilibrium values for the variables of interest are presented in Table 1. The equilibrium distributions of first period consumption for the two cases are given in Figure 2.

In these two cases, the equilibrium distribution is skewed to the left: most of the consumers consume an amount close to the socially efficient one. This is also reflected in the fact that efficiency loss in equilibrium, $W^* - W$, is small compared to the one in the benchmark model with observable consumption, $W^* - W^o$.

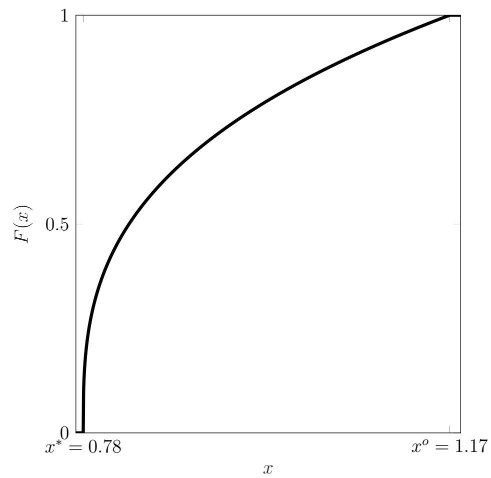
The distribution of the social welfare is also interesting: consumers and the second period seller obtain higher payoffs under privacy than when past consumption is observable. The profit comparison for the first period seller is the opposite of that: this seller's profit is reduced by privacy.

Note that the first seller's loss from privacy equals the consumer's gain and the increase in social welfare accrues fully to the second seller, as one would expect from the results in section 4.5. The example shows that the gain from privacy is much larger for the consumer than for seller 2.

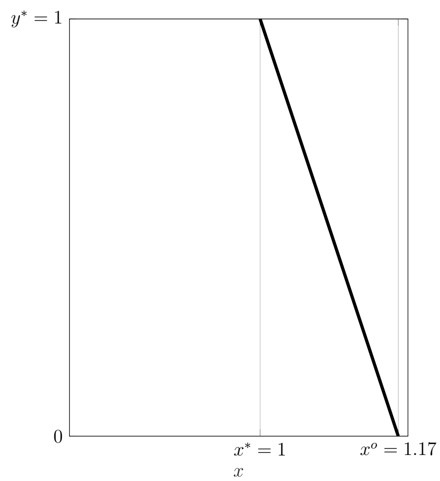
In the limit, when $a \to 0$, the goods become independent and the equilibrium distribution of consumption streams converges to a degenerate distribution (x^*, y^*). This result is not specific to the quadratic example and holds for a generic case of our model: when the cross derivative of $u(x, y)$ vanishes everywhere, $|x^o - x^*|$ converges



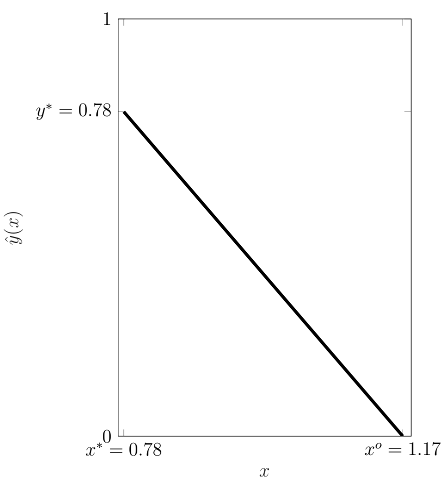
(a) $a = -1$;



(b) $a = -3$;



(c) $a = -1$;



(d) $a = -3$;

Figure 2: Distribution of first period consumption (top row) and second period consumption (bottom row). This example is computed using $u(x, y) = -3x^2 - 3y^2 + axy + 8x + 8y$ and $k = 1$.

to zero.

5 Concluding remarks

This paper has analyzed a model where the utility that the consumer derives from her purchases with her current supplier depends upon past consumption. Consider a consumer who shops at the supermarket on the weekend, and from local stores during the week. It is plausible that purchases from the supermarket affect her demand from the local store, but not the other way around. Consequently, the long term interaction between consumer and the two types of suppliers can be analyzed via a sequence of two-period models, as in this paper. This is also the case of auto-repair services, where the firms are the official dealer and the local garage, or the computer hardware-software example. In equilibrium, the supermarket faces consumers without private information and offers a two-part tariff to facilitate the creation of endogenous heterogeneity. Subsequently, the local store serves consumers with endogenous private information and offers a screening menu.

Our two-period model is less apt when the firms concerned are two supermarkets, both of which are frequented by the consumer. Purchases at either firm will affect demand in the other firm. One is led therefore to an infinite horizon model where flow utility in period t depends upon current consumption as well as consumption in the previous period. Such a model is significantly more complex than the present one, and its analysis is beyond the scope of this paper. Nonetheless, some of the conclusions of this paper carry over. When transactions are public, the consumption sequence is inefficient: when goods are substitutes (resp. complements) each firm induces overconsumption (resp. underconsumption). Furthermore, when transactions are private, consumption cannot be deterministic in any period, by the same argument as in our main model.

One question that has been raised by a referee is the following: what are the implications of allowing the consumer and firm 1 to re-contract after the consumer transacts with firm 2?[20] Let us examine the implications when purchases are observable. If we assume that the contracts between firm 1 and the consumer must be renegotiation proof, then for any consumption from seller 2, y, firm 1 must sell the conditionally efficient amount, which we can denote $x^*(y)$. Furthermore, since the consumer can always choose not to buy from firm 1, his payoff is no less than the payoff he gets when firm 2 is the Stackelberg leader and firm 1 is the follower.

[20] Of course, this only makes sense if the consumer has not already consumed the good purchased from seller 1 when she meets seller 2.

Consequently, firm 1 becomes a Stackelberg follower, which is worse than being the leader.

We have considered two extreme scenarios: full transparency and full privacy. There is a wide spectrum of possible privacy regimes in-between. It is plausible that the market outcome is likely to be affected by both how much and what kind of information is revealed to the downstream firm. For example, in some situations, the downstream firm may observe the prices charged by the upstream firm, but the quantities chosen by the consumer may be unobserved. This brings up interesting questions for future research, e.g., to what extent would the upstream firm limit the consumer's private information by limiting the set of options on its own menu?

We conclude with our policy finding: privacy improves welfare and consumer surplus, and mitigates the distortions due to seller monopoly power. However, in our model, upstream firms will not voluntarily protect consumer privacy, even when consumers are sophisticated, since making transactions public is a way for upstream firms to enhance their monopoly power. Indeed, if upstream firms could sell the information on the consumer's choices to the downstream firm, they would have every incentive to do so. Consequently, there is a role for regulation.

A Appendix

Lemma 18. *Suppose the consumer buys $\tilde{x}$ in the first period and chooses one of the two different available options—(q_1, y_1) and (q_2, y_2)—from the menu in the second period. If the consumer is indifferent between these two options—i.e., if*

$$u(\tilde{x}, y_1) - q_1 = u(\tilde{x}, y_2) - q_2,$$

then one of the sellers has a profitable deviation.

Proof. We prove this lemma for the case of submodular utility $u(x, y)$. The proof for the case of supermodular utility $u(x, y)$ is very similar and therefore omitted. Without loss of generality, let $y_1 < y_2$.

First, if $u_2(\tilde{x}, y_2) < k$, the seller in period 2 can modify his menu to weakly increase his profit. To see that, consider the set

$$X = \{x \mid u_2(x, y(x)) < k, \text{ and } (x, y(x)) \text{ is in the support of consumer's strategy}\}$$

If the seller replaces every item $y(x), x \in X$ with the item $y^*(x)$ (recall that $u_2(x, y^*(x)) = k$) and reduces the price of this item by $u(x, y(x)) - u(x, y^*(x))$, he will guarantee that a consumer with history $x \in X$ will purchase $y^*(x)$, and the profit from selling these items will be strictly higher since $u(x, y) - ky$ is maximized at $y^*(x)$. Therefore, we can restrict our attention to the case of $u_2(\tilde{x}, y_2) \geq k$.

The difference in profit between the two options that are offered in the second period are

$$q_2 - ky_2 - (q_1 - ky_1) = u(\tilde{x}, y_2) - u(\tilde{x}, y_1) - k(y_2 - y_1) > 0.$$

If the consumer chooses (q_1, y_1), seller 2 can reduce q_2 by arbitrarily small but positive amount and, thus, increase his profit.

If the consumer chooses (q_2, y_2), seller 1 has a profitable deviation. Indeed, if seller 1 induces $x > \tilde{x}$ instead of $\tilde{x}$, the consumer will choose either (q_1, y_1) or another, even smaller bundle. Similarly, if seller 1 induces $x < \tilde{x}$ instead of $\tilde{x}$, the consumer will choose either (q_2, y_2) or another, even larger bundle. Thus, the total payoff of the consumer and seller 1, $\Sigma(x)$, is bounded from below by

$$\Sigma(x) \geq \tilde{\Sigma}(x) := \begin{cases} u(x, y_1) - q_1 - kx, & \text{if } x \geq \tilde{x} \\ u(x, y_2) - q_2 - kx, & \text{if } x < \tilde{x} \end{cases}$$

and $\Sigma(\tilde{x}) = \tilde{\Sigma}(\tilde{x}) = u(\tilde{x}, y_1) - q_1 - k\tilde{x}$. The function $\tilde{\Sigma}(x)$ is continuous and goods are substitutes, hence

$$D_+\tilde{\Sigma}(\tilde{x}) - D_-\tilde{\Sigma}(\tilde{x}) = u_1(\tilde{x}, y_1) - u_1(\tilde{x}, y_2) > 0.$$

Therefore $\tilde{\Sigma}(x)$ cannot achieve the maximum at $\tilde{x}$, and so cannot $\Sigma(x)$. $\square$

A.1 Proof of Lemma 4.

First we show that consumptions lie in a compact interval. More specfically, when goods are substitutes, $X \subset [0, x^o]$; when goods are complements, $X \subset [0, x^*]$.

When goods are substitutes, the quantity of firm 1 that maximizes $\Sigma(x)$ is decreasing in the consumer's second period consumption. Since 0 is the lowest feasible second period consumption, it is never optimal for firm 1 to offer a quantity greater than x^o. When goods are complements, the quantity of firm 1 that maximizes $\Sigma(x)$, denoted $\tilde{x}(y)$, is increasing in the consumer's second period consumption, y, and solves $u_1(\tilde{x}(y), y) = k$. For any x, $\hat{y}(x) \leq y^*(x)$, since the allocation of any type can only be distorted downwards. Thus, the rationalizable choices for the firms are bounded above by x^* and y^* respectively.

Having show that X is bounded, and therefore, by definition closed, it remains to show that there cannot be any gaps. Suppose, to the contrary, that there exist a and $b > a$ in X such that $(a, b) \cap X = \emptyset$. If goods are substitutes, then the incentive constraint for type a, vis-a-vis type b, must bind under the profit maximizing menu offered by firm 2. If goods are complements, then the incentive constraint for type b vis-a-vis type a must bind. In either case, there exists a type who is indifferent between two available options offered by seller 2. Lemma 18 shows that this is inconsistent with an equilibrium, since at least one seller has a profitable deviation.

A.2 Proof of Lemma 5

First, we provide intuition for the proof. Standard arguments in mechanism design imply that at any point $x \in X$, the right-hand derivative exceeds the left-hand derivative. However, if this inequality is strict, and there was a convex kink at x, it would not be optimal for firm 1 and the consumer to choose x, i.e. x cannot belong to X. This intuition is made precise below.

Fix $x \in X$, and $(\hat{y}(x), \hat{q}(x))$. Note that we have extended U so that it is defined on an open interval $I \supseteq X$ rather than just the chosen points, X. For $z \in I - X$ let

$$U(x) := \max_{y \geq 0}\{u(x, y) - q(y)\}.$$

Thus, U is specified by prescribing optimal choices for all non-chosen types, and every point in X lies in the interior of I.

Consider the payoff of the consumer in the second period, $U(x + \delta)$—this is well defined for δ sufficiently small. Since the consumer with the first-period consumption $x + \delta$ can choose the contract chosen by the consumer with the first-period consumption x,

$$U(x + \delta) \geq u(x + \delta, \hat{y}(x)) - \hat{q}(x)$$

Thus, for $\delta > 0$

$$\frac{U(x + \delta) - U(x)}{\delta} \geq \frac{u(x + \delta, \hat{y}(x)) - u(x, \hat{y}(x))}{\delta}.$$

The above inequality implies

$$D_+U(x) := \liminf_{\delta \to 0+} \frac{U(x + \delta) - U(x)}{\delta} \geq u_1(x, \hat{y}(x)). \tag{4}$$

Since the inequality for $\delta < 0$ has a reversed sign, this yields

$$D^-U(x) := \limsup_{\delta \to 0-} \frac{U(x + \delta) - U(x)}{\delta} \leq u_1(x, \hat{y}(x)). \tag{5}$$

Now, the total payoff of seller 1 and consumer, $\Sigma(x)$, depends only upon $(\hat{y}(x), \hat{q}(x))$, and equals

$$\Sigma(x) = U(x) - kx.$$

Clearly, if $x \in X$, then, since x is chosen, it must maximize $\Sigma(x)$.

Define:

$$D^+U(x) := \lim \sup_{\delta \to 0+} \frac{U(x+\delta) - U(x)}{\delta},$$

$$D_-U(x) := \lim \inf_{\delta \to 0-} \frac{U(x+\delta) - U(x)}{\delta}.$$

The necessary conditions for x to maximize $\Sigma(x)$ are

$$\Sigma^+(x) = D^+U(x) - k \leq 0,$$

$$\Sigma^-(x) = D_-U(x) - k \geq 0.$$

These inequalities imply $D^+U(x) \leq D_-U(x)$. In conjunction with the inequalities (4) and (5), this implies that for any $x \in X$,

$$D^+U(x) = D_+U(x) = D^-U(x) = D_-U(x) = u_1(x, \hat{y}(x)).$$

Thus U is differentiable at x.

A.3 Proof of Theorem 11

We focus on the case where goods are substitutes. Since the argument is very similar when goods are complements, we omit it.

The proof consists of the following steps:

1. We derive necessary conditions for profit maximization in periods 1 and 2. We use these conditions to solve for the equilibrium menus and the distribution of the first period consumption F. When deriving the optimality conditions we conjecture and later verify that the distribution F is continuous.

2. We show that the solution we get for F is a cumulative distribution function—i.e., it is non-decreasing and right-continuous.

3. We show that the menus that we find induce the conjectured consumer choices (or using the language of the theory of incentives, the menus are incentive-compatible).

First, we verify that the proposed distribution F induces the second-period consumption $\hat{y}$ and indirect utility $U(x)$. By the envelope condition,

$$U(x) = U(\overline{x}) - \int_{x}^{\overline{x}} u_1(z, \hat{y}(z))dz.$$

The price charged by the seller 2 for the bundle $\hat{y}(x)$ is

$$\hat{q}(x) = u(x, \hat{y}(x)) - U(\overline{x}) + \int_{x}^{\overline{x}} u_1(z, \hat{y}(z))dz. \tag{6}$$

Hence the expected profit for this seller is

$$\int_{\underline{x}}^{\overline{x}} \left[u(x, \hat{y}(x))f(x) + u_1(x, \hat{y}(x))F(x) - k\hat{y}(x)f(x)\right] dx - U(\overline{x}).$$

Maximizing the above expression pointwise, we obtain that the distribution F must satisfy the first order condition

$$u_2(x, \hat{y}(x))f(x) + u_{21}(x, \hat{y}(x))F(x) - kf(x) = 0. \tag{7}$$

Note that it remains to check whether the solution to this first order condition satisfies consumer's participation constraints and incentive compatibility constraints. Lemma 20 accomplishes this task.

Seller 1 makes the consumer indifferent between the inside and the outside options, therefore, he charges a price for amount x that equals

$$p(x) = u(\underline{x}, \hat{y}(\underline{x})) - u(0, \hat{y}(\underline{x})) + \int_{\underline{x}}^{x} u_1(z, \hat{y}(z))dz.$$

In particular, the price for the bundle $\underline{x}$ equals

$$p(\underline{x}) = u(\underline{x}, \hat{y}(\underline{x})) - u(0, \hat{y}(\underline{x})).$$

Seller 1's profit from selling a bundle x has to be independent of x. Hence

$$u_1(x, \hat{y}(x)) - k = 0. \tag{8}$$

Equations (8) and (7) pin down unknown functions F and $\hat{y}$.

Lemma 19. *Suppose that F and $\hat{y}$ solve equations (8) and (7) with a boundary condition $F(\overline{x}) = 1$. Then F is a continuous c.d.f. and $\hat{y}$ is strictly decreasing (resp. strictly increasing) whenever goods are substitutes (resp. complements).*

Proof. By taking a derivative of equation (8) with respect to x we get

$$\hat{y}'(x) = -\frac{u_{11}(x, \hat{y}(x))}{u_{21}(x, \hat{y}(x))}$$

therefore $\text{sgn}(\hat{y}'(x)) = \text{sgn}(u_{21}(x, \hat{y}(x)))$.

The solution to equation (7) is

$$\ln F(x) = \int_x^{\overline{x}} \frac{u_{21}(z, \hat{y}(z))}{u_2(z, \hat{y}(z)) - k} dz \tag{9}$$

This solution is increasing in x if $u_2(x, \hat{y}(x)) \geq k$ for all $x \in [\underline{x}, \overline{x}]$. Lower bound $\underline{x}$ solves $u_2(\underline{x}, \hat{y}(\underline{x})) = k$ (see Hellwig, 2010). Moreover, since u is concave

$$\begin{aligned}
\frac{d}{dx} u_2(x, \hat{y}(x)) &= u_{21}(x, \hat{y}(x)) + u_{22}(x, \hat{y}(x))\hat{y}'(x) \\
&= u_{21}(x, \hat{y}(x)) - \frac{u_{11}(x, \hat{y}(x))}{u_{21}(x, \hat{y}(x))} u_{22}(x, \hat{y}(x)) \\
&= \frac{u_{22}(x, \hat{y}(x))u_{11}(x, \hat{y}(x)) - (u_{21}(x, \hat{y}(x)))^2}{-u_{21}(x, \hat{y}(x))} > 0.
\end{aligned}$$

Therefore, $u_2(x, \hat{y}(x))$ is (strictly) increasing in x and $u_2(x, \hat{y}(x)) \geq k$ for all $x \in [\underline{x}, \overline{x}]$. So far we established that $F(x)$ is a non-decreasing and everywhere except at $\underline{x}$.

To show that $F(x)$ is continuous at $\underline{x}$, we need to prove that

$$\lim_{x \to \underline{x}+0} \int_x^{\overline{x}} \frac{u_{21}(z, \hat{y}(z))}{u_2(z, \hat{y}(z)) - k} dz = -\infty. \tag{10}$$

Indeed, note that

$$\int_x^{\overline{x}} \frac{u_{21}(z, \hat{y}(z))}{u_2(z, \hat{y}(z)) - k} dz = \int_x^{\overline{x}} \frac{u_{12}(z, \hat{y}(z))}{u_2(z, \hat{y}(z)) - u_2(\underline{x}, \hat{y}(\underline{x}))} dz,$$

and let $\nu(x) = u_2(x, \hat{y}(x))$ and $x(\nu)$ be an inverse of it (recall that $\nu(x)$ is strictly monotone). Then, since u is strictly concave and strictly submodular and twice continuously differentiable, there exists $B > 0$ such that

$$\int_{x}^{\overline{x}} \frac{u_{12}(z, \hat{y}(z))}{u_2(z, \hat{y}(z)) - u_2(\underline{x}, \hat{y}(\underline{x}))} dz =$$

$$= \int_{\nu(x)}^{\nu(\overline{x})} \frac{1}{\nu - \nu(\underline{x})} \left[\frac{u_{12}^2(x(\nu), \hat{y}(x(\nu)))}{u_{12}^2(x(\nu), \hat{y}(x(\nu))) - u_{11}(x(\nu), \hat{y}(x(\nu))) u_{22}(x(\nu), \hat{y}(x(\nu)))} \right] d\nu <$$

$$< \int_{\nu(x)}^{\nu(\overline{x})} \frac{-B}{\nu - \nu(\underline{x})} d\nu.$$

Thus, $F(x)$ is continuous at $\underline{x}$.

To summarize, the solution $F(x)$ is a non-decreasing continuous function everywhere, $F(\underline{x}) = 0$ and $F(\overline{x}) = 1$, therefore $F(x)$ is a c.d.f. □

The next step of the proof is established by the following lemma, that shows that the consumer chooses option $\hat{y}(x)$ from the second period menu if she consumed x in the first period. Note that, among other things, this result ensures that the second period participation constraints hold: since $\hat{y}(x^o) = 0$, a consumer can always pretend that he consumed x^o in the first period and not buy anything in the second. Also, by Lemma 15 $\hat{y}$ is strictly monotone. This means that the participation constraint binds only for one type: $\overline{x}$.

Lemma 20. *If $\hat{y}(x)$ is decreasing and utility is submodular (or if $\hat{y}(x)$ is increasing and utility is supermodular), equation* (6) *implies*

$$u(x, \hat{y}(t)) - \hat{q}(t) \leq u(x, \hat{y}(x)) - \hat{q}(x)$$

for all $x, t \in [\underline{x}, \overline{x}]$.

Proof. Consider $x > t$. Since $\hat{y}(x)$ is decreasing and utility is submodular (or, alternatively, $\hat{y}(x)$ is increasing and utility is supermodular), we have

$$\hat{q}(t) - \hat{q}(x) = u(t, \hat{y}(t)) - u(x, \hat{y}(x)) + \int_t^x u_1(z, \hat{y}(z))dz \geq$$
$$u(t, \hat{y}(t)) - u(x, \hat{y}(x)) + \int_t^x u_1(z, \hat{y}(t))dz =$$
$$u(t, \hat{y}(t)) - u(x, \hat{y}(x)) + u(x, \hat{y}(t)) - u(t, \hat{y}(t)) =$$
$$u(x, \hat{y}(t)) - u(x, \hat{y}(x)).$$

The case with $x < t$ is identical. □

Finally, the value of the outside option for the consumer in the second period is $u(x, 0)$, therefore, if the consumer consumes $\overline{x}$ in the first period, her continuation utility in the second period is $U(\overline{x}) = u(\overline{x}, 0)$.

A.4 Proof of Theorem 14

Our proof hinges on two facts that have been established. First, for any equilibrium σ with support $X_\sigma : U_\sigma(x) = U(x)$, and second, $\hat{y}_\sigma(x)$, the bundle consumed in the second period in equilibrium σ by the consumer who consumed x in the past, is uniquely determined and coincides with that under $\tilde{\sigma} : \hat{y}(x)$. In other words, the payoff and the second-period consumption for any chosen first-period consumption is the same across all equilibria.

The argument is completed via the following lemma, that shows that any equilibrium distribution cannot have either gaps in the support or atoms (except for a possible atom at x^*, which is ruled out by Lemma 19).

Lemma 21. *Let σ be an equilibrium, and let G denote the c.d.f. of first period consumption corresponding to σ. The support of G equals $[x^*, x^o]$, and G cannot have atoms.*

Proof. Suppose, by contradiction, that G has a gap (x_1, x_2) in its support. Sequential rationality for the second-period seller implies that the consumer who consumed x_1 in the first period is indifferent between the item that she chooses in the second period and the item that is chosen by the consumer with a history x_2. This violates Lemma 18. Therefore, distribution G cannot have gaps in its support.

Suppose, by contradiction, that G has an atom at $\tilde{x} \neq x^*$. Hellwig (2010) establishes that in that case $\hat{y}(x)$—consumption in the second period as a function of the consumer's history—is discontinuous at $\tilde{x}$. Since G has no gaps in its support, by Lemma 7, $\hat{y}(x)$ must be continuous on $[x^*, x^o]$ hence the contradiction. □

A.5 Proof of Proposition 15

The proof of this proposition closely follows the proofs of Theorems 11 and 14, therefore, in the interest of brevity, we do not present the full argument.

Consider first the case where goods are substitutes. Suppose $\epsilon > 0$. The set of possible first period consumptions is $X(\epsilon) = \{0\} \cup [\underline{x}(\epsilon), x^o]$. The upper bound, x^o, is unaffected by the perturbation, but the lower bound is—namely $\underline{x}(\epsilon) > x^*$—since the second period consumption for this type is strictly below the conditional efficient one, given the mass point at 0. Let F_ϵ denote the distribution of first period consumptions, and for $x > 0$, let $f_\epsilon(x)$ denote the associated density function. For any $x \in X(\epsilon)$, the following equilibrium conditions must be satisfied:

$$u_1(x, \hat{y}_\epsilon(x)) = k \tag{11}$$

$$(u_2(x, \hat{y}_\epsilon(x)) - k) f_\epsilon(x) + u_{12}(x, \hat{y}_\epsilon(x)) F_\epsilon(x) = 0 \tag{12}$$

$$F_\epsilon(x^o) = 1 \tag{13}$$

$$F_\epsilon(\underline{x}(\epsilon)) = \epsilon \tag{14}$$

Since equations (11), (12) and (13) do not depend on ϵ, we conclude that $\hat{y}_\epsilon(x)$ and $F_\epsilon(x)$ coincide with the solutions identified in Theorem 11 on $[\underline{x}(\epsilon), x^o]$.

We use equation (14) to solve for $\underline{x}(\epsilon)$. Lemma 19 establishes that a continuous non-decreasing function $F(x)$ defined on $[x^*, x^o]$ solves (11), (12) and (13). Recall that $F(x^*) = 0$ and $F(x^o) = 1$. Therefore, by the intermediate value theorem, there exists $\underline{x}(\epsilon) \in [x^*, x^o]$ that solve equation (14) for any $\epsilon \in [0, 1]$. Moreover, $\underline{x}(\epsilon)$ converges to x^* as ϵ goes to zero.

In period 2, for any $\epsilon > 0$, the price $q(0)$ is no longer payoff irrelevant. Since type 0 is the "highest" type, firm 2 will offer this consumer the efficient allocation, $y^*(0)$. Furthermore, firm 2 will optimally set the price $q(0)$ at the maximum possible level, i.e. so that type 0 is indifferent between $y^*(0)$ at price $q(0)$ and $\hat{y}(\underline{x}(\epsilon))$ at price $q(\underline{x}(\epsilon))$. This implies that that the consumer's first period outside option indeed converges to $u(0, \hat{y}(x^*)) - q(x^*)$ when $\epsilon \to 0$—in the expression in (3), both elements in the set that the consumer maximizes over yield the same payoff.

When goods are complements, the argument is simpler.[21] First, when ϵ is small enough, $X(\epsilon) = \{0\} \cup [x^o, x^*]$. This follows from the fact that for small ϵ, it is optimal

[21] Since equilibrium is unique in $G(0)$ when goods are complements, robustness would follow from upper-hemicontinuity of the equilibrium correspondence, as a function of ϵ. Rather than take this route, we present a more direct argument.

for firm 2 to exclude type 0, who has failed to consume in period one. Consequently, the type with the lowest first period consumption, $\underline{x}$, is also allocated 0 in period 2, and must therefore be allocated x^o in period one. As in the substitutes case, the highest type in period one, $\bar{x}$, must be allocated $y^*(\bar{x})$, and so $\bar{x} = x^*$. Since the equilibrium satisfies the equations corresponding to 14, we may similarly conclude that $\hat{y}_\epsilon(x)$ and $F_\epsilon(x)$ coincide with the solutions identified in Theorem 11 on $[x^o, x^*]$.

References

Acemoglu, Daron, Ali Makhdoumi, Azarakhsh Malekian, and Asuman Ozdaglar, "Too much data: Prices and inefficiencies in data markets," Technical Report, National Bureau of Economic Research 2019.

Acquisti, Alessandro, Curtis Taylor, and Liad Wagman, "The economics of privacy," *Journal of Economic Literature*, 2016, *54* (2), 442–92.

Aghion, Philippe and Patrick Bolton, "Contracts as a Barrier to Entry," *The American Economic Review*, 1987, pp. 388–401.

Bernheim, B Douglas and Michael D Whinston, "Menu auctions, resource allocation, and economic influence," *The Quarterly Journal of Economics*, 1986, pp. 1–31.

Calzolari, Giacomo and Alessandro Pavan, "On the optimality of privacy in sequential contracting," *Journal of Economic Theory*, 2006, *130* (1), 168–204.

_ **, Vincenzo Denicolò, and Piercarlo Zanchettin**, "Exclusive dealing with distortionary pricing," *Mimeo*, 2017.

Chiesa, Gabriella and Vincenzo Denicolò, "Trading with a common agent under complete information: A characterization of Nash equilibria," *Journal of Economic Theory*, 2009, *144* (1), 296–311.

Diamond, Peter A, "A model of price adjustment," *Journal of economic theory*, 1971, *3* (2), 156–168.

_ **and Eric Maskin**, "An equilibrium analysis of search and breach of contract, I: Steady states," *The Bell Journal of Economics*, 1979, pp. 282–316.

Dosis, Anastasios and Wilfried Sand-Zantman, "The ownership of data," *Available at SSRN 3420680*, 2019.

Fox, Edward J, Alan L Montgomery, and Leonard M Lodish, "Consumer shopping and spending across retail formats," *The Journal of Business*, 2004, *77* (S2), S25–S60.

Fudenberg, Drew and Jean Tirole, "Moral hazard and renegotiation in agency contracts," *Econometrica*, 1990, *58* (6), 1279–1319.

_ **and** _ , "Perfect Bayesian Equilibrium and Sequential Equilibrium," *Journal of Economic Theory*, 1991, *53* (2), 236–260.

_ **and** _ , "Customer poaching and brand switching," *RAND Journal of Economics*, 2000, pp. 634–657.

González, Patrick, "Investment and screening under asymmetric endogenous information," *RAND Journal of Economics*, 2004, pp. 502–519.

Gul, Faruk, "Unobservable Investment and the Hold-Up Problem," *Econometrica*, 2001, *69* (2), 343–376.

Hellwig, Martin F, "Incentive problems with unidimensional hidden characteristics: A unified approach," *Econometrica*, 2010, *78* (4), 1201–1237.

Ichihashi, Shota, "Online privacy and information disclosure by consumers," *American Economic Review*, 2020, *110* (2), 569–95.

Inderst, Roman, "Models of vertical market relations," *International Journal of Industrial Organization*, 2010, *28* (4), 341–344.

Jones, Charles I and Christopher Tonetti, "Nonrivalry and the Economics of Data," *Americain Economic Review*, 2020.

Laussel, Didier and Michel Le Breton, "Conflict and cooperation: The structure of equilibrium payoffs in common agency," *Journal of Economic Theory*, 2001, *100* (1), 93–128.

Ma, Ching-to Albert, "Adverse selection in dynamic moral hazard," *The Quarterly Journal of Economics*, 1991, pp. 255–275.

Martimort, David, "Multi-principaux avec anti-selection," *Annales d'Economie et de Statistique*, 1992, pp. 1–37.

_ , "Multi-contracting mechanism design," *Econometric Society Monographs*, 2006, *41*, 57.

— **and Lars Stole**, "The revelation and delegation principles in common agency games," *Econometrica*, 2002, *70* (4), 1659–1673.

— **and —** , "Contractual externalities and common agency equilibria," *The BE Journal of Theoretical Economics*, 2003, *3* (1).

— **and —** , "Market participation in delegated and intrinsic common-agency games," *The RAND journal of economics*, 2009, *40* (1), 78–102.

Milgrom, Paul and Ilya Segal, "Envelope Theorems for Arbitrary Choice Sets," *Econometrica*, March 2002, *70* (2), 583–601.

Pavan, Alessandro and Giacomo Calzolari, "Sequential contracting with multiple principals," *Journal of Economic Theory*, 2009, *144* (2), 503–531.

Prat, Andrea and Aldo Rustichini, "Sequential common agency," Technical Report, Center for Economic Research, Tilburg University 1998.

Rey, Patrick and Michael D Whinston, "Does retailer power lead to exclusion?," *The RAND Journal of Economics*, 2013, *44* (1), 75–81.

Stole, Lars A, *Mechanism design under common agency*, Program in Law and Economics, Harvard Law School Cambridge, MA, 1991.

Taylor, Curtis R, "Consumer privacy and the market for customer information," *RAND Journal of Economics*, 2004, pp. 631–650.

Thomassen, Øyvind, Howard Smith, Stephan Seiler, and Pasquale Schiraldi, "Multi-category competition and market power: a model of supermarket pricing," *American Economic Review*, 2017.

Tokis, Konstantinos, "A mechanism design approach to the optimal disclosure of private client data," *Mimeo*, 2017.

Villas-Boas, J Miguel, "Price cycles in markets with customer recognition," *RAND Journal of Economics*, 2004, pp. 486–501.