Simulation of statistically accurate time-integrated dynamic speckle patterns in biomedical optics: supplemental document

1. EIGENVALUES AS A FUNCTION OF INTEGRATION TIME

Figure S1 shows the first five eigenvalues of $g_1(\tau)$ for $p = 1$ and $\alpha = 1$. At very short integration times the measurement is dominated by one mode. As the integration time increases this dominance decreases and the modes have an increasingly equal contribution to the time-integrated measurement, which results in blurring.

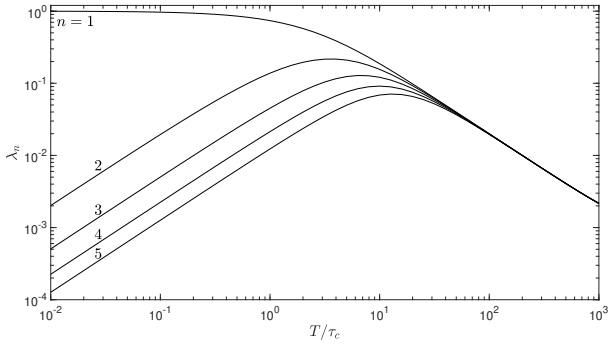

Fig. S1. The first five eigenvalues of $g_1(\tau)$ for $p = 1$, $\alpha = 1$, $\tau_c = 0.37$ ms and variable ratios of T/τ_c.

2. COMPUTATIONAL PERFORMANCE AND FURTHER ALGORITHMIC DETAIL

A major consideration when using this simulation technique is that larger ratios of T/τ_c require a larger N value in order to sample $g_1(\tau)$ with sufficient accuracy. This accuracy can be ensured by running the presented validations for a particular set of simulation parameters. Additionally, a larger N value will increase the computational demand of the simulation; the choice of N (and also the number of eigenvalues to retain) therefore reflects a compromise between accuracy and performance.

We note that for each heterogeneous field simulation, λ_n is a 2D matrix of size N_{label} x N which need only be computed once. This λ_n 2D matrix is then distributed across N fully developed speckle patterns according to a N_{label} x 600 x 600 logical 3D matrix (each layer of this 3D matrix acts as a binary mask for each tissue label and for each fully developed speckle pattern). The edges that exist in the simulated 2D-TIDSP between two different tissue labels are then smoothed by the CTF. The accuracy and spatial resolution of the simulation could be increased by increasing the value of N_{label} that is used, which would allow for gradual spatial transitions to be modelled more explicitly. However, this would increase the computational expense of the simulation and also rely on a sufficiently accurate partitioning of the initial image.

Using MATLAB 2020a on a PC with 32 GB RAM and a 2.6 GHz processor with $N = 1000$ and $N_{\text{label}} = 5$, 0.3 s is required to compute the λ_n matrix used in the presented *in vivo* simulation, and a further 86 s is required to simulate each of the 30 2D-TIDSPs. If sufficient memory were available, it would be preferable to use pre-computed libraries of fully-developed speckle patterns in the above calculation. Additionally, GPU acceleration and lower level programming languages could also be employed.

3. PARAMETERISATION OF THE COHERENCE FACTOR

A limitation of this simulation framework is that it does not allow for the parameterisation of β. Although it is trivial to simulate the effect of two orthogonal polarisation states by taking the sum of two 2D-TIDSPs, simulating the effect of other factors that have an effect on β (such as coherence length and stability of the laser light source, stray light, detector stability and sample coupling) is more complex and would be an interesting further study. We have successfully modelled the CTF of a coherent imaging system, but variations in speckle to pixel size ratio could be further modelled by upsampling or downsampling of the simulated 2D-TIDSPs [1].

4. NOISE AND EXPERIMENTAL VALIDATION

The addition of specific models of measurement noise (such as that due to shot noise, read noise, dark noise and hot pixels [2]) to simulated 2D-TIDSPs, together with experimental validation of this combined technique, is outside the scope of this work but would be a useful and interesting further study. However, we have robustly validated our technique against previously published solutions for the exact probability density function of time-integrated intensity for coherent light [3]. Furthermore, we have validated our technique (both in terms of intra-image and inter-image statistics) against recently published solutions for speckle contrast for different forms of homogeneous field [4].

5. *IN VIVO* SIMULATION

Figure S2 shows a magnified view of the 200 x 200 pixel ROI delineated by the white dashed square in Fig. 4(b). This figure depicts an edge between a large vessel and surrounding parenchyma.

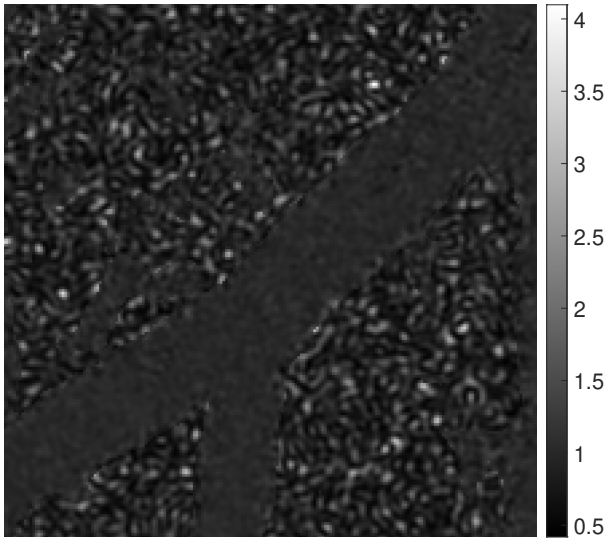

Fig. S2. Magnified view of the 200 x 200 pixel ROI delineated by the white dashed square in Fig. 4(b).

REFERENCES

1. S. J. Kirkpatrick, D. D. Duncan, and E. M. Wells-Gray, "Detrimental effects of speckle-pixel size matching in laser speckle contrast imaging," Opt. Lett. **33**, 2886–2888 (2008).
2. H. M. Varma, C. P. Valdes, A. K. Kristoffersen, J. P. Culver, and T. Durduran, "Speckle contrast optical tomography: A new method for deep tissue three-dimensional tomography of blood flow," Biomed. Opt. Express **5**, 1275–1289 (2014).
3. J. Goodman, *Statistical Optics* (Wiley, Hoboken, New Jersey, 2015), 2nd ed.
4. C. Liu, K. Kiliç, S. E. Erdener, D. A. Boas, and D. D. Postnov, "Choosing a model for laser speckle contrast imaging," Biomed. Opt. Express **12**, 3571–3583 (2021).