

Pricing CDSs' capital relief

Positions in credit default swaps (CDSs) are eligible instruments to reduce some Basel III capital requirements. The value of this benefit should be reflected in the price. *Chris Kenyon and Andrew Green* incorporate this into a pricing model for CDSs, and show it may account for more than half the spread

The Basel Committee on Banking Supervision capital requirements known as Basel 2.5 and III – and their legal implementations such as the European Capital Requirements Directive (CRD IV) and more recently by the US Federal Reserve Board – set out specific capital charges for counterparty default risk and credit valuation adjustment (CVA) variation. The rules laid out by the committee allow for banks to reduce these charges by taking positions in appropriate credit default swaps (CDSs) and dealers are looking to mitigate the charge (see Thompson & Dahinden, 2013).

This use of CDSs, above their use for default protection, should have an effect on their prices. Some believe this contributes to a kind of feedback loop – a ‘doom loop’ in the more colourful vernacular – whereby uncollateralised exposures prompt banks to buy protection, and hence spreads to widen, which in turn cause the exposures to grow, and so on. But few attempts have been made at quantifying this phenomenon (see Cameron, 2012, and Carver, 2011). Avoiding this loop is one reason proposed for local Basel III exemptions in CRD IV.

This article aims to address this, by incorporating capital mitigation into CDS prices. This is done by directly considering a third leg in the instrument, aside from the premium and default protection legs: the capital relief leg. The fair spread is affected, seemingly substantially, by the consideration of CDSs' role in capital mitigation. Plausible model parameters applied to vanilla interest rate swap trades suggest it may account for as much as half the spread once capital relief is included.

Capital charges are implemented by local regulators using a two-tier model, according to whether a given bank has permission to use an internal model method (IMM) or not. The effect on the charge – and hence CDS spreads – of the two methods is partly through a scaling factor dependent on asset correlations – internal parameters for IMM banks, regulator proscribed otherwise. In high-correlation – stressed – scenarios, IMM banks may in fact be losing out to their less sophisticated competitors when it comes to cost of capital.

In theory, making strong assumptions such as complete markets and zero CDS-bond basis, CDSs can be replicated by shorting the underlying name's bonds and using a risk-free cash account (Carr, 2005). If capital relief really were priced in by a protection seller, there would be no buyers since the product could be replicated.

In practice, these assumptions do not hold. Basel III itself recognises the basis, for instance, and requires it to be modelled, and so capital relief can be priced in. When they do, there is no longer a unique clearing price, as each bank's capital calculation and consumption and relief will be different. This is a first attempt at what we expect to be a broader theme in pricing theory – including the effects of the capital costs, and relief, of buyers and sellers. Capital consumption is not just an internal question, and we can expect it to cause differential pricing in future. It also challenges the use in Basel III of CDS spreads to derive market-implied

default probabilities, without considering the effect that using these very CDSs to hedge it will have.

CDSs with capital relief

We consider a bank buying a CDS on some reference entity to provide capital relief for the Basel 2.5 default risk and Basel III CVA charge. To handle its own counterparty risk (see Brigo & Capponi, 2010) the trade is perfectly collateralised, and the counterparty and reference entity are assumed to have zero default correlation. Together these mean that the capital charge for the CDS trade itself can be considered negligible.

The fair CDS spread sets the premiums equal to default protection plus capital relief:

$$PremLeg = ProtLeg + ReliefLeg(Reg(bank), entity, AC) \quad (1)$$

The premium and protection legs depend on the recovery, *rec*, while the capital relief leg depends on both the default reference entity and the protection-buying bank regulatory status *Reg()*. This is either on an IMM basis, if the bank's regulator has approved, or according to a standardised formula, possibly depending on asset class (AC). For example, under the current exposure methodology (CEM) set percentages are specified for the exposure at default (EAD) for interest rate, foreign exchange, equities, precious metals and other commodity trades (see table A). There is no explicit dependence on the buyer's deals with the reference entity because that is implicitly included in the choice of the CDS notional, tenor, etc. But portfolio effects in Basel III CVA value-at-risk capital calculation are included.

Equation (1) uses survival probabilities on all three legs because capital relief is only valuable while the reference entity has not defaulted. For an IMM bank, the CDS rate is used in both the premium leg and the relief leg because CVA VAR uses observed CDS spreads (not capital-adjusted CDS spreads). So IMM banks are, for this item, at a relative disadvantage to banks on standardised approaches whose CVA VAR formula does not use observed CDS spreads.

We now expand each leg in equation (1):

$$\begin{aligned} PremLeg_{a,b}(c) &= c \mathbb{E} \left[D(0, \xi) \left(\xi - T_{\beta(\xi)-1} \right) I_{\{T_a < \xi < T_b\}} \right] \\ &+ \sum_{i=a+1}^b c \mathbb{E} \left[D(0, T_i) \tau_i I_{\{\xi \geq T_i\}} \right] \\ &= c \int_{T_a}^{T_b} P(0, t) \left(t - T_{\beta(t)-1} \right) \mathbb{Q}(\xi \in [t, t + dt]) \\ &+ c \sum_{i=a+1}^b P(0, T_i) \tau_i \mathbb{Q}(\xi \geq T_i) \end{aligned} \quad (2)$$

$$\begin{aligned} ProtLeg_{a,b}(LGD) &= \mathbb{E} \left[I_{\{T_a < \xi \leq T_b\}} D(0, \xi) LGD \right] \\ &= LGD \int_{T_a}^{T_b} P(0, t) \mathbb{Q}(\xi \in [t, t + dt]) \end{aligned} \quad (3)$$

$$\begin{aligned}
& \text{ReliefLeg}_{a,b}(K(\cdot)) \\
&= \int_{T_a}^{T_b} \mathbb{E} \left[D_{cap}(0,t) H \left(K_{relief}(t, cI_{\{IMM\}}), t \right) I_{\{\xi \geq t\}} \right] dt \quad (4) \\
&= \int_{T_a}^{T_b} P_{cap}(0,t) H \left(K_{relief}(t, cI_{\{IMM\}}), t \right) \mathbb{Q}(\xi \in [t, t+dt])
\end{aligned}$$

where a, b are protection limit times for the CDS; ξ is the default time; $\beta(\xi)$ is the number of next coupon payment after time ξ ; c is the CDS spread; $D(0, t)$ is the stochastic risk-free discount factor from zero to t ; $D_{cap}(0, t)$ is the stochastic capital discount factor from zero to t ; $P(0, t)$ is the risk-free zero-coupon bond with maturity t ; $P_{cap}(0, t)$ is the capital zero-coupon bond with maturity t ; $K_{relief}(t, cI_{\{IMM\}})$ is the capital relief from unit notional of CDS protection at time t ; $H(\cdot, t)$ is the instantaneous cost of capital at t ; τ_i is the year fraction for i th premium payment; and $\mathbb{Q}(\cdot)$ are the survival probabilities at time zero.

Equations (2) and (3) are standard (Brigo & Mercurio, 2006) under the assumptions given above, while equation (4) is new to capture the capital relief obtained from CDS contracts. Depending on the circumstances, it is possible that not all the capital relief is priced in. We assume zero transaction costs, for example, for changing levels of capital.

The fair CDS spread from equations (1), (2), (3) and (4) is:

$$c = \frac{LGD \int_{T_a}^{T_b} P(0,t) \mathbb{Q}(\xi \in [t, t+dt]) + \int_{T_a}^{T_b} P_{cap}(0,t) H \left(K_{relief}(t, cI_{\{IMM\}}), t \right) \mathbb{Q}(\xi \geq t)}{\int_{T_a}^{T_b} P(0,t) (t - T_{\beta(t-1)}) \mathbb{Q}(\xi \in [t, t+dt]) + \sum_{i=a+1}^b P(0, T_i) \tau_i \mathbb{Q}(\xi \geq T_i)} \quad (5)$$

where $LGD = 1 - rec$ is the loss given default. For CDS buyers with IMM approval, equation (5) has the fair CDS spread appearing on both sides of the equation as it is used in the CVA capital charge. Equation (5) then requires a non-linear numerical solution.

Capital pricing

For simplicity, we start from the point of view of a non-IMM bank and consider only credit risk capital. This leads to a default capital cost (DCC) and a CVA capital cost (CVC). We do not include market risk or operation risk, etc. Where there are ambiguities in the Basel documents, we use UK regulations for details. We go into depth on the derivation of the regulatory equations for the CVA capital charge to understand the portfolio effects on a non-IMM bank, and how an IMM bank's portfolio characteristics can result in different capital charges.

Basel III specifies the capital required at any given date. However, the cost of capital for a trade is the lifetime capital cost, not the cost of the trade-date capital requirement. We consider all capital costs in terms of lifetime cost. Of course, this lifetime depends on the lifetimes of the counterparties. We take the point of view that the bank (or trader) considers costs as a going concern and so uses counterparty default time as the end of the trade if this occurs prior to maturity. It would be possible to include own-default time, funding, etc, in future work.

■ **CVA capital charge.** We start from the standardised CVA risk capital charge in paragraph 104 of Basel Committee (2011), noting that this is not a risk-weighted asset but capital directly:

$$\begin{aligned}
& K_{CVC} \\
&= 2.33\sqrt{h} \left\{ \left(\sum_i 0.5w_i (M_i EAD_i - M_i^{hedge} B_i) - \sum_{ind} w_{ind} M_{ind} B_{ind} \right)^2 \right. \\
&\quad \left. + \sum_i 0.75w_i^2 (M_i EAD_i^{total} - M_i^{hedge} B_i)^2 \right\}^{1/2} \quad (6)
\end{aligned}$$

A. CEM potential future exposure notional add-on dependency on maturity and asset class under paragraph 92 of Basel Committee (2006)

Maturity	IR	FX/gold	Equities	PM	OC
Less than one year	0.0%	1.0%	6%	7%	10%
One to five years	0.5%	5.0%	8%	7%	12%
Greater than five years	1.5%	7.5%	10%	8%	15%

Note: PM = precious metals other than gold; OC = other commodities, for example, West Texas Intermediate oil

where h is the one-year risk horizon in units of years, that is, $h = 1$; w_i is the risk weight of the i th counterparty based on external rating (or equivalent); EAD_i is the EAD of counterparty i , discounted at a 5% rate including effective maturity; B_i is the notional of purchased single-name CDS hedges, discounted as above; B_{ind} is the notional of purchased index CDS hedges, discounted as above; w_{ind} is the risk weight of index hedge using one of seven weights using the average index spread, and w_i that of the single-name hedge; M_i is the effective maturity of transactions with counterparty i (for non-IMM this is notional weighted average, and is not capped at five years); M_i^{hedge} is the maturity of hedge instrument with notional B_i ; and M_{ind} is the maturity of index hedge ind (see Pykhtin, 2012).

Equation (6) can be reproduced from two sources: first, a 99% one-sided standard normal distribution with mean zero gives the 2.33 factor; second, there is an assumption that all counterparties have a correlation of 25%. Taking equation (6) with no hedging, we have:

$$\begin{aligned}
K^2 &\propto \left(\sum_i 0.5w_i M_i EAD_i \right)^2 + \sum_i 0.75w_i^2 M_i^2 EAD_i^2 \\
&= \left(\frac{1}{2} \sum_i \sigma_i \right)^2 + \frac{3}{4} \sum_i \sigma_i^2 \\
&= \frac{1}{4} n^2 \sigma^2 + \frac{3}{4} n \sigma_i^2 \\
&= \sigma^2 \left(\frac{1}{4} n^2 + \frac{3}{4} n \right) \quad (7)
\end{aligned}$$

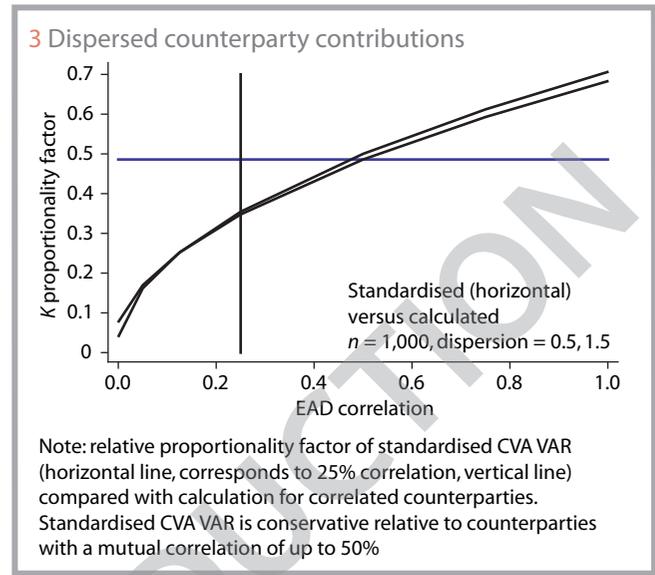
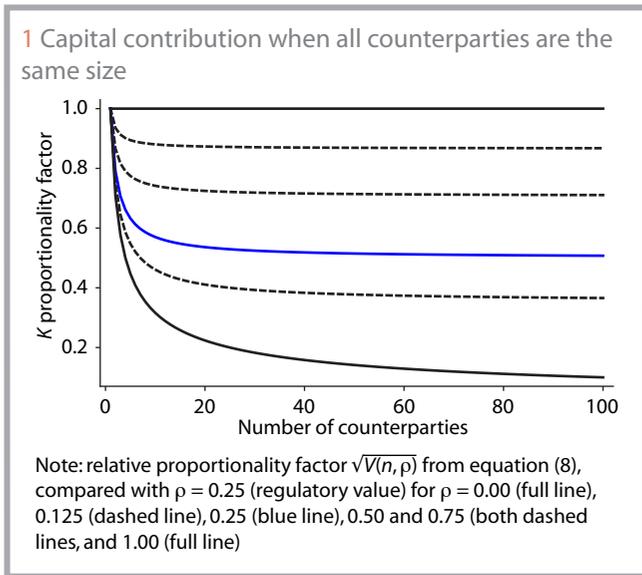
where we have written $\sigma_i = w_i M_i EAD_i$ and assumed all the σ_i are equal. Now consider the variance $V(n, \rho)$ of n random variables with mutual correlation ρ :

$$\begin{aligned}
V(n, \rho) &= \sum_{i=1}^n \sum_{j=1}^n \text{cov}(i, j) \\
&= \sum_{i=1}^n \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=i+1}^n \rho_{i,j} \sigma_i \sigma_j \\
&= n\sigma^2 + n(n-1)\rho\sigma^2 \\
&= \sigma^2 (\rho n^2 + n(1-\rho)) \quad (8)
\end{aligned}$$

Hence equation (6) assumes $\rho = 1/4$, after making similar assumptions about σ_i for the n random variables. As n increases, the proportionality factor for K quickly converges to $1/2 = \sqrt{1/4}$ as n^2 soon dominates n .

We can now ask how CVA VAR capital depends on the portfolio distribution.

■ **Portfolio effects.** If a bank is under the standardised CVA risk capital charge, equation (6) holds. This makes strong assumptions about portfolio correlation, which can be seen by comparing equations (7) and (8). We can look at the capital effect of portfo-



lio size, homogeneity and correlation using equation (8).

Figure 1 shows how the proportionality factor for K in equation (6), $\sqrt{V(n, \rho)}$ from equation (8), depends on n and ρ . Portfolios with higher ρ will be charged lower capital with the standardised formula than if their actual CVA correlation was used. Figure 1 shows a homogeneous portfolio, that is, when all counterparty exposures are identically distributed normal random variables. The regulatory value $\rho = 0.25$ of the mutual correlation of these normal distributions means that each counterparty's capital effect is close to half its stand-alone effect.

To investigate the effect on the proportionality factor K of the distribution of counterparty exposure sizes, we assume that this distribution is lognormal and alter its parameters. We keep the average size, as measured by the average σ , or $\sum w_i M_i EAD_i$, constant and alter the dispersion parameter σ_D of the portfolio distribution D of exposures:

$$D \sim e^{\mu_D - \sigma_D^2/2 + \sigma_D N}$$

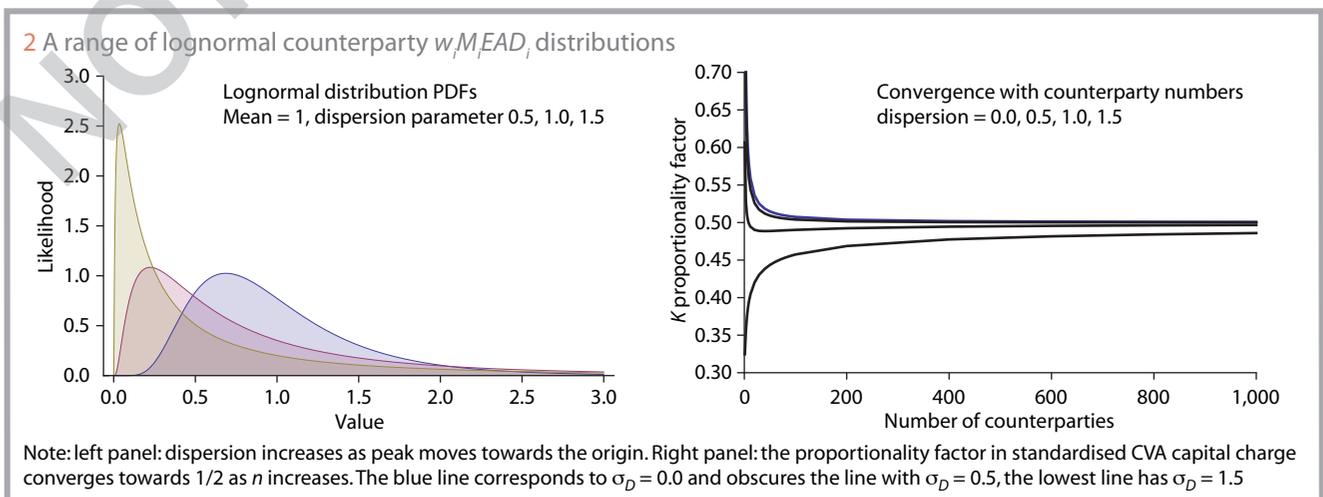
where N is a standard normal distribution, and set $\mu_D = 0$ arbitrarily. Individual counterparty sizes are taken as quantiles of the distribution D . Since the counterparties are now not of equal size, the exact equivalence with $\rho = 0.25$ no longer holds.

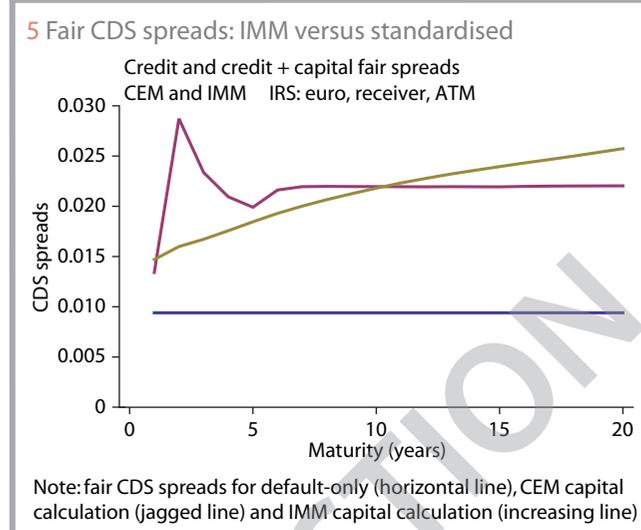
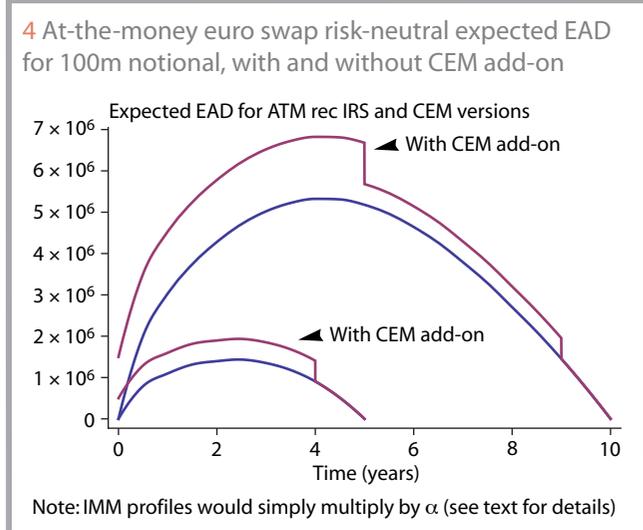
Figure 2 shows how the proportionality factor in the standardised CVA VAR capital charge converges as the number of counterparties n increases. With a range of dispersions of the counterparty sizes, $\sigma_D = 0.5, 1.0, 1.5$, we see that the proportionality factor converges to around 1/2 for reasonable numbers of counterparties, that is, around 1,000. The dispersion parameter of $\sigma_D = 1.5$ gives a long tail of counterparty sizes, and the other cases model more concentrated portfolios of counterparty sizes.

Figure 3 compares the standardised CVA VAR proportionality factor for $n = 1,000$ and sets of counterparties with differing correlations from the uniform case. The standardised calculation is conservative for correlations up to about 50% and not thereafter. This range of K between benign and high-correlation crisis scenarios is captured by the fact that IMM banks must use the sum of stressed and non-stressed capital charges. So non-IMM banks may actually have the advantage here. We assume for simplicity that the sum of the stressed and non-stressed parameters' effects is the same as the non-IMM K factor.

■ **Default capital charge.** The Basel III default risk charge is mostly unchanged from Basel 2.5 (Basel Committee, 2006).

The effective maturity, M , is capped at five years. An IMM bank multiplies the EAD of a netting set by a constant factor α ,





while a non-IMM bank using the CEM uses add-ons based on notionals. The one-year default probability, PD , is floored at 3 basis points and is calculated on a historical basis, according to default experience, mapping to external data, and statistical default models taking account of at least five years.

CDS examples

We now consider examples to gauge the proportion of the CDS spread that is paying for capital relief and not default protection using equation (5). We use the relation that the CVA on a trade is equal to the cost of a corresponding contingent CDS contract. In general, CVA on swaps can be recursive depending on close-out conventions (Burgard & Kjaer, 2011). We consider the non-recursive version for simplicity.

For our IMM examples, to get the dependence on market observed CDS spread into the calculations that are based on the standardised CVA capital charge formula, we scale the weight w by the ratio of the observed-implied to calculated-implied default probabilities at M .

■ **Interest rate swap.** As a basic financial instrument, we pick an at-the-money vanilla euro interest rate swap (IRS). The key thing we need to calculate is the EAD as this feeds into all the capital calculations, as well as the usual CVA.

Assuming for simplicity that default is independent of interest rates, the expected exposure at any future time S discounted to the present is given by the corresponding swaption price. Practically we use prices of collateralised swaptions, which have the same effect of creating prices independent of counterparty risk. We use the inverse risk-free discount factor to S to get the forward premium, which is the expected future exposure. Practically we could obtain forward premiums directly from the market, but for examples we use a swaption implied volatility surface with data from Bloomberg.

Note that risk factors dynamics behind EAD profiles for DCC must pass historical backtesting. The underlying risk factors are explicitly permitted to be calibrated to either market-implied or historical data. For a detailed discussion, see chapter 11 of Kenyon & Stamm (2012).

Figure 4 shows the expected EAD profiles for a receiver IRS with and without CEM add-ons that are linked to notionals and to remaining maturities. With current low interest rates, the add-ons are significant fractions of the profiles.

Figure 5 shows the fair CDS spreads for default only, CEM

B. Parameters for IRS examples

Parameter	Value	Source/motivation
Alpha α	1.3	Middle of range
Hazard rate	0.0156	Five-year observed CEM CDS is 0.02
Recovery rate	0.40	Typical
Historical default probability	0.0024	Global BBB from S&P
Cost of capital	0.10	Choice
Minimum capital	0.10	Typical
Discounting	Cost of capital	Choice

Note: the hazard rate makes the five-year observed CDS rate 2% assuming capital is priced in and this is calculated by CEM. The 2% is chosen to roughly line up with Markit BBB five-year generic CDS spreads (see Vazza et al, 2011)

capital calculation and IMM capital calculation. The jaggedness of the CEM calculation derives directly from the changes in add-on with increasing swap maturity – one year and below there is no add-on. It is also a function of the current low interest rate regime, so the add-on appears large. Table B provides the parameters for the example, based on typical bank minimum capital requirements, and discounting at its cost of capital, as is natural from a corporate finance perspective.

■ **Comparisons.** Table C shows the division of observed CDS spreads, on a stand-alone basis, into default protection, Basel 2.5 default capital and Basel III CVA capital, for five-year IRSs. Results for both non-IMM and IMM banks are shown. The part of the CDS spread attributable to default protection is less than half, and this holds across a range of ratings, or equivalently observed CDS spreads. This proportion is lower than in earlier examples because there are higher weightings and the S&P long-term default probability increases quickly as rating decreases.

Table C is constructed by solving for a hazard rate such that the CDS price including Basel 2.5 and Basel III capital relief is at par. For the non-IMM case, this can be simply done using equation (5). For CDS buyers with IMM approval, equation (5) has the fair CDS spread appearing on both sides of the equation because the CDS spread is also used in the CVA capital charge. We used a non-linear solver to get the appropriate spreads in that case.

Table D includes the effect of the scaling factor K , which depends on the protection buyer's counterparties. Figure 2 shows that this quickly converges to 0.5 so we use that value. We see

C. Breakdown of observed CDS spreads for five-year IRSs into default protection and capital relief (DCC and CVC)

	Parameters				CEM			IMM		
	CDS bp	Rec %	S&P bp	w_i %	Default %	DCC %	CVC %	Default %	DCC %	CVC %
A	90	38	8	0.8	27	42	31	38	36	26
BBB	130	38	24	1	18	55	27	29	48	23
BB	290	37	90	2	29	47	25	38	42	20
B	510	36	448	3	34	45	21	41	41	18
CCC	1,170	33	2,600	10	33	36	32	37	35	28

Note: calculation is on a stand-alone basis, that is, not including the scaling K for the number of counterparties of the protection buyer and their distribution. Observed CDS spreads are generic from Markit. Five-year chosen as liquid CDS point. OC and rates capital relief are very close at this maturity (only), so only rates results are shown

that there is a significant reduction in the part due to Basel III capital relief as expected and now the default protection part of the CDS spread is closer to half.

Conclusion

Under Basel III, and previously under Basel 2.5, CDSs provide capital relief. If capital relief is priced into CDS prices, then a new model is required to price CDSs and derive market-implied default probabilities. We have presented a CDS model that addresses these requirements, now with three legs: premium, protection and capital relief. We do not know how much capital relief is actually priced in. This will be determined by market expectations of when regulations will come into force, which exceptions will be present, market incompleteness (replication costs) and competition between CDS sellers. Since the market is incomplete (for example, CDS bond basis, difficulties of bond shorting, etc), pricing in capital relief is possible.

All banks measure capital use, and collateralised CDSs consume minimal capital. Nonetheless, the capital relief they afford to protection buyers can markedly affect CDS pricing. It is not enough to simply measure the capital consumption of a trade to understand the effect of capital on the trade price.

We have shown that capital relief pricing has a potentially significant effect on CDS spreads, easily reaching 50% of the observed CDS spread. Both the IMM status of the CDS buyer and the asset class that the CDS buyer is obtaining capital relief on have major effects, especially for shorter maturities. Portfolio effects are relatively easy to include for non-IMM banks because the proportion-

D. Non-IMM (CEM) versus IMM (approximate) including the scaling factor $K = 0.5$ (see figure 2) for the portfolio effect that the protection buyer sees because of their counterparties

Rating	CEM			IMM		
	Default %	DCC %	CVC %	Default %	DCC %	CVC %
A	32	50	18	44	41	15
BBB	21	64	16	33	54	13
BB	33	53	14	42	47	11
B	38	50	12	45	45	10
CCC	39	42	19	43	41	16

Note: parameters as in table C

ality factor K quickly asymptotes to 0.5 and this is robust against different counterparty size concentrations. Institutions on the systemically important banks list (Financial Stability Board, 2012) may see different prices because they have higher minimum capital requirements and this is known to their counterparties.

Unlike our non-IMM bank calculations, our IMM-approved bank calculations are approximate in many ways and should be taken cautiously. Including the observed CDS spread in the standardised CVA calculation via default probability ratio is an approximation. Detailed IMM analysis is an area for future investigation.

For simplicity, we assumed that counterparty default is independent of interest rates, that is, neglecting wrong-way (and right-way) risk. Moving to a model including a dependence mechanism is straightforward using simulation.

Our model including capital relief can be used to obtain bounds on market-implied hazard rates and adjust observed CDS spreads for capital relief. Given the potential ambiguity of CDS interpretation between default and capital, their direct regulatory use in Basel III for CVA VAR capital may need reassessment. We see this article as the first in a new wave of pricing where capital effects are included directly, as opposed to simply calculating capital consumption. ■

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References

Basel Committee on Banking Supervision, 2006

International convergence of capital measurement and capital standards
June

Basel Committee on Banking Supervision, 2011

Basel III: a global regulatory framework for more resilient banks and banking systems
Available at www.bis.org/publ/bcbs189.pdf

Brigo D and A Capponi, 2010

Bilateral counterparty risk with application to CDSs
Risk March, pages 85–90, available at www.risk.net/1594872

Brigo D and F Mercurio, 2006

Interest rate models: theory and practice
Springer, second edition

Burgard C and M Kjaer, 2011

Partial differential equation representations of derivatives with bilateral counterparty risk and funding costs
Journal of Credit Risk 7, pages 75–93

Cameron M, 2012

New CRD IV draft exempts sovereign trades from CVA capital charge
Available at www.risk.net/2157060

Carr P, 2005

Replicating a defaultable bond
Bloomberg/NYU Courant Institute

Carver L, 2011

A recipe for disaster?
Risk November, pages 16–20, available at www.risk.net/2120808

Financial Conduct Authority, 2013

Prudential sourcebook for banks, building societies and investment firms
August

Financial Stability Board, 2012

Update of group of global systemically important banks
Available at www.financialstabilityboard.org/publications/r_121031ac.pdf

Kenyon C and R Stamm, 2012

Discounting, Libor, CVA and funding: interest rate and credit pricing
Palgrave Macmillan, 2012

Pykhtin M, 2012

Model foundations of the Basel III standardised CVA charge
Risk July, pages 60–66, available at www.risk.net/2189065

Thompson T and V Dahinden, 2013

Counterparty risk and CVA survey. Current market practice around counterparty risk regulation, CVA management and funding
Deloitte and Solum Financial Partners

Vazza D, N Kraemer, E Gunter,

N Richhariya and A Sakhare, 2011
Annual US corporate default study and rating transitions
Available at www.standardandpoors.com/ratings/articles/en/us/?articleType=HTML&assetID=1245331026864