KVA: capital valuation adjustment by replication

Credit (CVA), debit (DVA) and funding (FVA) valuation adjustments are now familiar concepts, but banks also pay for capital. Here, Andrew Green, Chris Kenyon and Chris Dennis introduce a capital valuation adjustment (KVA) to pricing by extending the Burgard-Kjaer semi-replication method, considering that capital may reduce funding needs and hedging transactions themselves generate capital requirements

Capital is a legal requirement for financial institutions holding derivatives, and the requirements have increased over the past few years (Basel Committee on Banking Supervision 2011; Dodd and Frank 2010), so it is surprising few papers include it in derivatives pricing (Hull and White 2014; Kenyon and Green 2013, 2014a,b). Here, we extend the hedging framework of Burgard and Kjaer (2011b, 2013a) and Kenyon and Kenyon (2013) to price the capital requirements of derivatives trades by replicating their cost, together with the costs of credit and funding. Thus, we present a capital valuation adjustment (KVA) alongside the existing adjustments for credit and funding.

Capital pricing appears challenging because there are hundreds of pages of regulations, lifetime costs are required (not just spot) and calculations at different levels of granularity must be combined. For example, for counterparty credit risk and credit valuation adjustment (CVA), capital netting sets are important, while the whole portfolio is needed to determinate the stressed period for market risk for SVAR calculation. Table A gives a brief list of capital regulations in Basel III and the calculations required. However, this complexity does not introduce anything fundamentally new.

The two truly new elements in KVA, as compared with CVA or FVA, are that hedging trades themselves generate capital requirements and that capital may be used for funding. We include the capital requirements of hedges simply by always calculating the requirement of the entire portfolio. Furthermore, we introduce a parameter, ϕ, to represent the fraction of capital, K, used for funding. Capital used for funding represents the use of funds from issued equity capital. Clearly $\phi \in [0, 1]$.

Funding with capital reduces funding requirements. However, Basel III appears to prohibit explicitly linking capital issuance and its inverse to trading strategies. Thus, while a derivative can be funded by explicitly issuing and buying back bonds, specific derivatives or strategies cannot be funded by explicitly issuing and buying back capital. Of course, allocating varying amounts of capital to derivatives over their lifetime is required as their capital requirements change. In the numerical examples we consider two cases: the base case ($\phi = 0$) and full use of capital for funding ($\phi = 1$). We do not discuss the practicalities of different choices, but leave this for further research. Systematic and theoretical consequences are dealt with elsewhere (Kenyon and Green 2014a,b).

The main contribution of this paper is to extend the pricing picture by including the cost of capital, the KVA, in derivatives pricing by replication. Given the increased regulatory focus on capital post-crisis, continuing regulatory developments and their cost, this is long overdue.

A. Typical categories of capital regulations, alternatives and type of calculation required

Classification	Alternatives	Calculation type
Counterparty credit risk	*EAD calculation*	
	CEM	Function of netting set value
	Standardised	Function of netting set value
	Internal model method	Exposure profile
	Weight calculation	
	Standardised	External ratings
	FIRB	Internal & external ratings
	AIRB	Internal & external ratings, internal LGDs
CVA capital	Standardised	Function of EAD
	Advanced	VAR/SVAR on regulatory CVA or CS01
Market Risk	Standardised	Deterministic formulas
	Internal model method	VAR+SVAR

Extending semi-replication to capital

To include the cost of (regulatory) capital in pricing alongside credit and funding valuation adjustments, we extend the semi-replication argument of Burgard and Kjaer (2013a). This paper uses the notation of Burgard and Kjaer, and additions (table B). The sign convention is the value of a cash amount is positive if received by the issuer. As with Burgard and Kjaer, we seek to find the economic or shareholder value of the derivative portfolio, $\hat{V}$. Note also that here, as with Burgard and Kjaer (2013a), we neglect balance sheet feedback effects as they are hard to realise, particularly when existing debt is issued at fixed coupons or spreads. Burgard and Kjaer studied balance sheet feedback in Burgard and Kjaer (2011a).

The dynamics of the underlying assets are:

$$dS = \mu_s S\,dt + \sigma_s S\,dW \tag{1}$$

$$dP_C = r_C P_C\,dt - P_C\,dJ_C \tag{2}$$

$$dP_i = r_i P_i\,dt - (1 - R_i) P_i\,dJ_B \quad \text{for } i \in \{1, 2\} \tag{3}$$

On default of the issuer, B, and the counterparty, C, the value of the derivative takes the following values:

$$\hat{V}(t, S, 1, 0) = g_B(M_B, X) \tag{4}$$

$$\hat{V}(t, S, 0, 1) = g_C(M_C, X) \tag{5}$$

The two g functions allow a degree of flexibility to be included in the model around the value of the derivative after default. The usual

B. A summary of the notation, which is also common with Burgard and Kjaer (2013)	
Parameter	Description
$\hat{V}(t,S)$	The economic value of the derivative or derivative portfolio
V	The risk-free value of the derivative or derivative portfolio
U	The valuation adjustment
X	Collateral
K	Capital requirement
Π	Replicating portfolio
S	Underlying stock
μ_S	Stock drift
σ_S	Stock volatility
P_C	Counterparty bond (zero recovery)
P_1; P_2	Issuer bond with recovery R_1; recovery R_2 (note $R_1 \neq R_2$)
$\mathrm{d}\bar{\beta}_S$; $\mathrm{d}\bar{\beta}_C$; $\mathrm{d}\bar{\beta}_X$; $\mathrm{d}\bar{\beta}_K$	Growth in the cash account associated with stock; counterparty bond; collateral; capital (all prior to rebalancing)
r; r_C; r_i; r_X; r_F	Risk-free rate; yield on counterparty bond; issuer bond; collateral; issuer bond (one-bond case)
M_B; M_C	Close-out value on issuer default; counterparty default
α_C; α_i	Holding of counterparty bonds; issuer bond
δ	The stock position
γ_S	Stock dividend yield
q_S; q_C	Stock repo rate; counterparty bond repo rate
J_C; J_B	Default indicator for counterparty; issuer
g_B; g_C	Value of the derivative portfolio after issuer default; counterparty default
R_i; R_C	Recovery on issuer bond i; counterparty derivative portfolio
λ_C; λ_B	Effective financing rate of counterparty bond $\lambda_C = r_C - r$; spread of a zero-recovery zero-coupon issuer bond (for bonds with recovery the following relation holds: $(1-R_i)\lambda_B = r_i - r$ for $i \in \{1,2\}$)
s_F; s_X	Funding spread in one bond case $s_F = r_F - r$; spread on collateral
$\gamma_K(t)$	The cost of capital (the assets comprising the capital may themselves have a dividend yield and this can be incorporated into $\gamma_K(t)$)
$\Delta\hat{V}_B$; $\Delta\hat{V}_C$	Change in value of derivative on issuer default; on counterparty default
ϵ_h	Hedging error on default of issuer (sometimes split into terms independent of and dependent on capital: $\epsilon_h = \epsilon_{h_0} + \epsilon_{h_K}$)
P	$P = \alpha_1 P_1 + \alpha_2 P_2$ is the value of the own bond portfolio prior to default
P_D	$P_D = \alpha_1 R_1 P_1 + \alpha_2 R_2 P_2$ is the value of the own bond portfolio after default
ϕ	Fraction of capital available for derivative funding

assumption is that:

$$\begin{aligned} g_B &= (V-X)^+ + R_B(V-X)^- + X \\ g_C &= R_C(V-X)^+ + (V-X)^- + X \end{aligned} \tag{6}$$

where:

$$x^+ = \max\{x, 0\} \tag{7}$$

$$x^- = \min\{x, 0\} \tag{8}$$

We assume the funding condition:

$$\hat{V} - X + \alpha_1 P_1 + \alpha_2 P_2 - \phi K = 0 \tag{9}$$

where the addition of ϕK represents the potential use of capital to offset funding requirements. The growth in the cash account positions (prior to rebalancing) are:

$$\mathrm{d}\bar{\beta}_S = \delta(\gamma_S - q_S) S\,\mathrm{d}t \tag{10}$$

$$\mathrm{d}\bar{\beta}_C = -\alpha_C q_C P_C\,\mathrm{d}t \tag{11}$$

$$\mathrm{d}\bar{X} = -r_X X\,\mathrm{d}t. \tag{12}$$

In the portfolio, Π, we account for two different sources of regulatory capital requirements: the derivative and the replicating portfolio. Positions in the stock and counterparty bond will themselves attract a capital requirement, hence:

$$K \equiv K(t, V, \text{‘market risk’}, X, C, \delta, \alpha_C) \tag{13}$$

that is, regulatory capital associated with the derivative is a function of the derivative portfolio value, its sensitivities through market risk capital, the collateral account value and the rating of the counterparty. The capital associated with the hedge portfolio is a function of the position in stock and bond. This reflects that regulatory capital applies to the whole derivative portfolio and not individual trades or counterparties. Some elements of the regulatory capital framework need to be attributed to portfolios from an overall net position. For example, market risk capital is calculated on the net position of all derivatives, while CVA capital under the standardised approach is calculated across all counterparties.

The change in the cash account associated with the capital position is:

$$\mathrm{d}\bar{\beta}_K = -\gamma_K(t) K\,\mathrm{d}t \tag{14}$$

This treats capital as borrowed from shareholders to support derivative trading activities. The cost of capital is thus the cost of the return expected by shareholders for putting their capital at risk. In essence, the derivatives business borrows the capital and pays cash profits to the shareholders at a given rate. We do not include a $\mathrm{d}J_B$ term for any impact of issuer default. This reflects that capital available to compensate creditors on issuer default is already part of the recovery rate

R_B.[1] We have implicitly assumed the counterparty rating is constant; the model could be extended for rating transitions.

Using Ito's lemma, the change in the value of the derivative portfolio is:

$$\begin{aligned} \mathrm{d}\hat{V} = \frac{\partial \hat{V}}{\partial t}\,\mathrm{d}t + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 \hat{V}}{\partial S^2}\,\mathrm{d}t + \frac{\partial \hat{V}}{\partial S}\,\mathrm{d}S \\ + \Delta\hat{V}_B\,\mathrm{d}J_B + \Delta\hat{V}_C\,\mathrm{d}J_C \end{aligned} \tag{15}$$

Assuming the portfolio, Π, is self-financing, its change in value is:

$$\begin{aligned} \mathrm{d}\Pi = \delta\,\mathrm{d}S + \delta(\gamma_S - q_S)S\,\mathrm{d}t + \alpha_1\,\mathrm{d}P_1 + \alpha_2\,\mathrm{d}P_2 \\ + \alpha_C\,\mathrm{d}P_C - \alpha_C q_C P_C\,\mathrm{d}t - r_X X\,\mathrm{d}t - \gamma_K K\,\mathrm{d}t \end{aligned} \tag{16}$$

Adding the derivative and replicating portfolio together, we obtain:

$$\begin{aligned} \mathrm{d}\hat{V} + \mathrm{d}\Pi = \bigg[&\frac{\partial \hat{V}}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 \hat{V}}{\partial S^2} + \delta(\gamma_S - q_S)S \\ &+ \alpha_1 r_1 P_1 + \alpha_2 r_2 P_2 + \alpha_C r_C P_C \\ &- \alpha_C q_C P_C - r_X X - \gamma_K K\bigg]\,\mathrm{d}t \\ &+ \epsilon_h\,\mathrm{d}J_B + \bigg[\delta + \frac{\partial \hat{V}}{\partial S}\bigg]\,\mathrm{d}S \\ &+ [g_C - \hat{V} - \alpha_C P_C]\,\mathrm{d}J_C \end{aligned} \tag{17}$$

where:

$$\begin{aligned} \epsilon_h &= [\Delta\hat{V}_B - (P - P_D)] \\ &= g_B - X + P_D - \phi K \\ &= \epsilon_{h_0} + \epsilon_{h_K} \end{aligned} \tag{18}$$

is the hedging error on issuer default. In the final line the hedging error has been split into a term that does not depend on capital, ϵ_{h_0}, and a term that does depend on capital, ϵ_{h_K}. Note that $\epsilon_{h_K} \neq -\phi K$ as the bond position is itself dependent on capital through the funding requirements, as defined by equation (9). If $\phi = 0$ then the hedge error is as in Burgard and Kjaer.

Assuming replication of the derivative by the hedging portfolio, except on issuer default, gives:

$$\mathrm{d}\hat{V} + \mathrm{d}\Pi = 0 \tag{19}$$

We make the usual assumptions to eliminate the remaining sources of risk:

$$\delta = -\frac{\partial \hat{V}}{\partial S} \tag{20}$$

$$\alpha_C P_C = g_C - \hat{V} \tag{21}$$

and this leads to the PDE:

$$\begin{aligned} 0 = \frac{\partial \hat{V}}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 \hat{V}}{\partial S^2} - (\gamma_S - q_S)S\frac{\partial \hat{V}}{\partial S} \\ - (r + \lambda_B + \lambda_C)\hat{V} + g_C\lambda_C + g_B\lambda_B \\ - \epsilon_h\lambda_B - s_X X - \gamma_K K + r\phi K \\ \hat{V}(T, S) = H(S) \end{aligned} \tag{22}$$

where the bond-funding equation (9) has been used, along with the yield of the issued bond, $r_i = r + (1 - R_i)\lambda_B$, and the definition of ϵ_h in equation (18) to derive the result:

$$\begin{aligned} &\alpha_1 r_1 P_1 + \alpha_2 r_2 P_2 \\ &\quad = rX - (r + \lambda_B)\hat{V} - \lambda_B(\epsilon_h - g_B) + r\phi K \end{aligned} \tag{23}$$

Note this paper assumes zero bond-CDS basis throughout.

Writing the derivative portfolio value, $\hat{V}$, as the sum of the risk-free derivative value, V, and a valuation adjustment, U, and recognising that V satisfies the Black-Scholes PDE:

$$\begin{aligned} \frac{\partial V}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - (\gamma_S - q_S)S\frac{\partial V}{\partial S} - rV = 0 \\ V(T, S) = 0 \end{aligned} \tag{24}$$

gives a PDE for the valuation adjustment:

$$\begin{aligned} \frac{\partial U}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} - (\gamma_S - q_S)S\frac{\partial U}{\partial S} - (r + \lambda_B + \lambda_C)U \\ = V\lambda_C - g_C\lambda_C + V\lambda_B - g_B\lambda_B \\ + \epsilon_h\lambda_B + s_X X + \gamma_K K - r\phi K \\ U(T, S) = 0 \end{aligned} \tag{25}$$

Formally applying the Feynman-Kac theorem gives:

$$U = \mathrm{CVA} + \mathrm{DVA} + \mathrm{FCA} + \mathrm{COLVA} + \mathrm{KVA} \tag{26}$$

where:

$$\begin{aligned} \mathrm{CVA} = -\int_t^T \lambda_C(u)\mathrm{e}^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))\,\mathrm{d}s} \\ \times \mathbb{E}_t[V(u) - g_C(V(u), X(u))]\,\mathrm{d}u \end{aligned} \tag{27}$$

$$\begin{aligned} \mathrm{DVA} = -\int_t^T \lambda_B(u)\mathrm{e}^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))\,\mathrm{d}s} \\ \times \mathbb{E}_t[V(u) - g_B(V(u), X(u))]\,\mathrm{d}u \end{aligned} \tag{28}$$

$$\begin{aligned} \mathrm{FCA} = -\int_t^T \lambda_B(u)\mathrm{e}^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))\,\mathrm{d}s} \\ \times \mathbb{E}_t[\epsilon_{h_0}(u)]\,\mathrm{d}u \end{aligned} \tag{29}$$

$$\begin{aligned} \mathrm{COLVA} = -\int_t^T s_X(u)\mathrm{e}^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))\,\mathrm{d}s} \\ \times \mathbb{E}_t[X(u)]\,\mathrm{d}u \end{aligned} \tag{30}$$

[1] We also considered using capital to offset losses on counterparty default, which would lead to a term in $\mathrm{d}J_C$ in equation (14). Symmetry would suggest that if capital can be used for funding then it could also offset losses on default. If we consider capital to be an exogenous resource and ignore the impact on the balance sheet, as is done in this paper, this is appealing. However, we rejected this as unrealistic. Losses do directly affect the balance sheet and hence capital. To fully understand the interrelationship between counterparty default and capital requires a full balance sheet model.

$$\text{KVA} = -\int_t^T e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))\,ds} \times \mathbb{E}_t[(\gamma_K(u) - r(u)\phi)K(u) + \lambda_B \epsilon_{h_K}(u)]\,du \tag{31}$$

FCA above contains only the classical non-capital dependent hedging error, while the capital-dependent terms have been grouped in KVA. Alternatively, we could have grouped the additional term in the KVA integral in the FCA integral to reflect the offset with funding:

$$\text{FCA}' = -\int_t^T (\lambda_B(u)\mathbb{E}_t[\epsilon_h(u)] - r(u)\phi\mathbb{E}_t[K(u)]) \times e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))\,ds}\,du \tag{32}$$

$$\text{KVA}' = -\int_t^T \gamma_K(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))\,ds} \times \mathbb{E}_t[K(u)]\,du \tag{33}$$

Whatever arrangement is selected, the capital elements resolve to calculate integrals over the capital profile $\mathbb{E}_t[K(u)]$, which is strictly positive. We consider capital profile generation next.

Regulatory capital requirements and costs

Regulatory capital is a portfolio-level requirement. Consider an interest rate swap that is traded, unsecured, with a corporate client. This trade has market risk, counterparty credit risk and CVA capital requirements associated with it. To hedge the market risk the trading desk enters a collateralised offsetting swap with the street. This hedge trade generates a little counterparty credit risk and CVA capital, but drastically reduces the market risk capital.[2] When pricing derivatives it is no longer sufficient to look at the impact of only the new trade. Instead, pricing increments of hedge packages is more appropriate. This will be illustrated in our numerical examples.

KVA itself, like CVA and FVA, has market risk sensitivities. The counterparty credit risk (CCR) term is driven by the exposure at default (EAD), and hence by the exposure to the counterparty. Capital requirements increase as exposure rises, irrespective of any impact on credit quality. KVA hedging involves using trades to generate retained profits to offset additional capital requirements arising from market moves. However, KVA hedges will again generate capital requirements, but because capital is generally a small fraction $O(1/10)$, requirements converge quickly.

Table A gives an overview of Basel III (Basel Committee on Banking Supervision 2011) capital requirements calculation and we refer readers there for full details. Here, we will only consider the three main capital requirements to which most derivative trades are subject. That is, market risk capital, counterparty credit risk capital and CVA capital:

$$K = K_{\text{MR}}\left(u, \frac{\partial V}{\partial S}\right) + K_{\text{CCR}}(u, V, C, X) + K_{\text{CVA}}(u, V, C, X) \tag{34}$$

[2] Note a similar situation has been avoided in the context of CVA capital for CDS spread hedges. Qualifying CDS positions designated as CVA hedges are exempt from further capital requirements.

Market risk capital is a capital requirement to offset losses due to market movements for traded products (Basel Committee on Banking Supervision 2006), so we write it as a function of the sensitivity of the unadjusted value, V. CCR is capital covering non-payment by counterparties and is calculated for over-the-counter derivatives as:

$$\text{RWA} = w \times 12.5 \times \text{EAD} \tag{35}$$

where w is the weight and EAD is the (regulatory) exposure at default of the counterparty. CVA capital was introduced by Basel III. Its European Implementation (CRD-IV) exempts EU-domiciled corporate counterparties, but is otherwise required.

■ **Cost of capital** The cost of capital represents the percentage return on regulatory capital that must be paid to shareholders. Thus the authors consider it an internal parameter set by the Bank's board in consultation with shareholders. Similar to funding, the cost of capital is idiosyncratic. A proxy is the return-on-equity (ROE) target in banks' annual reports. Reboul *et al* (2014) suggested a typical group ROE target might be 10%.

Numerical examples

Here, we provide example results to allow the impact of KVA to be assessed and compared with existing valuation adjustments, which were calculated using numeric integration of equations (27) through (31). We chose to calculate the case of semi-replication with no shortfall at own default, equivalent to 'strategy 1' in Burgard and Kjaer (2013a). We also chose the first issuer bond to have zero recovery, and use it to invest or fund the difference between $\hat{V}$ and V. The P_2 bond position has recovery $R_2 = R_B$ and is given by the funding constraint in (9). Hence:

$$\alpha_1 P_1 = -U \tag{36}$$

$$\alpha_2 P_2 = -(V - \phi K) \tag{37}$$

Using these hedge ratios gives the value of the issuer bond portfolio in default as:

$$P_D = -R_B(V - \phi K) \tag{38}$$

and hence $\epsilon_h = \epsilon_{h_0} + \epsilon_{h_K}$ is then given by:

$$\epsilon_h = (1 - R_B)[V^+ - \phi K] \tag{39}$$

where here:

$$\epsilon_{h_0} = (1 - R_B)V^+ \quad \text{and} \quad \epsilon_{h_K} = -(1 - R_B)\phi K$$

This choice gives the following formulas for CVA, DVA, FCA and KVA for regular bilateral closeouts:

$$\text{CVA} = -(1 - R_C)\int_t^T \lambda_C(u) e^{-\int_t^u (\lambda_B(s)+\lambda_C(s))\,ds} \times \mathbb{E}_t[e^{-\int_t^u r(s)\,ds}(V(u))^+]\,du \tag{40}$$

$$\text{DVA} = -(1 - R_B)\int_t^T \lambda_B(u) e^{-\int_t^u (\lambda_B(s)+\lambda_C(s))\,ds} \times \mathbb{E}_t[e^{-\int_t^u r(s)} ds(V(u))^-]\,du \tag{41}$$

C. Counterparty data for the examples

Counterparty rating	bp	Standardised risk weight (%)	CVA risk weight w_i (%)
AAA	30	20	0.7
A	75	50	0.8
BB	250	100	2
CCC	750	150	10

D. XVA values for a 10-year, GBP, payer interest rate swap

						KVA				
ϕ	Swap	Rating	CVA	DVA	FCA	MR	CCR	CVA	Total	IR01
0	Pay	AAA	−4	39	−14	−262	−3	−9	−253	9.5
0	Pay	A	−10	38	−14	−256	−8	−10	−259	9.6
0	Pay	BB	−31	33	−12	−234	−14	−22	−279	10.0
0	Pay	CCC	−68	24	−9	−185	−16	−87	−342	11.3
1	Pay	AAA	−4	39	−14	−184	−2	−6	−170	9.5
1	Pay	A	−10	38	−14	−180	−4	−7	−176	9.6
1	Pay	BB	−31	33	−12	−166	−7	−16	−198	9.9
1	Pay	CCC	−68	24	−9	−134	−8	−63	−260	11.0

Results are quoted in basis points of the trade notional. The first column, ϕ, specifies the use of capital for funding

$$\mathrm{FCA} = -(1-R_B)\int_t^T \lambda_B(u)\mathrm{e}^{-\int_t^u(\lambda_B(s)+\lambda_C(s))\,\mathrm{d}s} \times \mathbb{E}_t[\mathrm{e}^{-\int_t^u r(s)\,\mathrm{d}s}(V(u))^+]\,\mathrm{d}u \tag{42}$$

$$\mathrm{KVA} = -\int_t^T \mathrm{e}^{-\int_t^u(\lambda_B(s)+\lambda_C(s))\,\mathrm{d}s} \times \mathbb{E}_t[\mathrm{e}^{-\int_t^u r(s)\,\mathrm{d}s}K(u)(\gamma_K(u)-r_B(u)\phi)]\,\mathrm{d}u \tag{43}$$

where we have used the fact that $r(u)+(1-R_B)\lambda_B(u) = r_B(u)$. This shows that in the event ϕ is non-zero then the capital cost is reduced by the bank funding rate.

Since we consider an interest rate swap, interest rates are now assumed to be stochastic and so appear inside expectations. The derivation follows the same steps as for derivatives based on stocks (omitted here for space; see SSRN version).

Capital requirements: market risk uses the standardised approach; CCR uses the current exposure method for EAD, and the standardised approach with external ratings for weights; CVA uses the standardised approach with the approximation for large numbers of counterparties:

$$K^i_{\mathrm{CVA}} \approx \tfrac{2.33}{2}\sqrt{h}\omega_i M_i \mathrm{EAD}_i^{\mathrm{total}}$$

We assume the issuer holds a minimum capital ratio requirement of 8% and the issuer cost of capital, γ_K, is 10%.

The examples are calculated using a single 10-year GBP interest rate swap with semi-annual payment schedules. The fixed rate on the swap is 2.7%, ensuring the unadjusted value is zero at trade inception. The issuer credit spread is a flat 100 basis points across all maturities and the issuer recovery rate is assumed to be 40%.

We calculate all valuation adjustments for four different counterparty ratings and spread combinations: AAA, A, BB and CCC. The spreads are given in table C alongside the risk weight. The counterparty recovery rate is assumed to be 40%.

Table D presents the results for a 10-year, GBP, payer interest rate swap. Setting aside the market risk component of the capital, we see the KVA from CCR and CVA terms gives similar adjustments to existing CVA, DVA and FCA terms, demonstrating KVA is a significant contributor to the price of the derivative.

The market risk is assumed to be unhedged, so this KVA component is relatively large, because under the standardised approach, the capital requirement on a 10-year transaction of this type is scaled to a 60bp move in rates. Practical application would calculate the market risk capital requirement over all trades in a portfolio, including hedges, and attribute them to trade level. The cases where capital is used for funding, $\phi = 1$, show a reduction in capital costs.

To assess the impact of hedging, the second example consists of two back-to-back interest rate swaps (see table E). The hedge trade is the exact mirror of the primary trade and perfectly collateralised, so it has no CVA, DVA, FCA or collateral valuation adjustment (ColVA). The market risk capital will be zero as the swaps match exactly and hence can be removed from market risk capital under Basel. However, the other KVA terms are not zero. In addition, although the portfolio has no market risk capital requirement, it has an open market risk position from the valuation adjustment terms and the portfolio IR01 is given in the final column.

If, instead of using a back-to-back hedge, the net portfolio market risk is eliminated at trade inception with a static hedge, the portfolio will have an IR01 of zero at the start. However, it still attracts market risk capital as the trade and hedge do not match exactly. This is illustrated in table F. Again, for the hedge trade, we assume perfect collateralisation.

E. XVA values for a 10-year, GBP, payer interest rate swap with an identical, but perfectly collateralised back-to-back hedge

						KVA				
ϕ	Swap	Rating	CVA	DVA	FCA	MR	CCR	CVA	Total	IR01
0	Pay	AAA	−4	39	−14	0	−3	−9	9	0.6
0	Pay	A	−10	38	−14	0	−8	−10	−3	0.7
0	Pay	BB	−31	33	−12	0	−14	−22	−45	1.1
0	Pay	CCC	−68	24	−9	0	−16	−87	−156	2.4
1	Pay	AAA	−4	39	−14	0	−2	−6	13	0.6
1	Pay	A	−10	38	−14	0	−4	−7	3	0.7
1	Pay	BB	−31	33	−12	0	−7	−16	−32	1.0
1	Pay	CCC	−68	24	−9	0	−8	−63	−125	2.1

Results are quoted in basis points of the trade notional. The market risk capital is now zero because the trade and hedge perfectly offset

F. XVA values for a 10-year, GBP, payer interest rate swap with perfectly collateralised hedge, adjusted to offset the portfolio IR01

						KVA					Hedge change
ϕ	Swap	Rating	CVA	DVA	FCA	MR	CCR	CVA	Total	IR01	(%)
0	Pay	AAA	−4	39	−14	−17	−4	−12	−13	0	7
0	Pay	A	−10	38	−14	−20	−11	−13	−30	0	8
0	Pay	BB	−31	33	−12	−28	−20	−31	−88	0	12
0	Pay	CCC	−68	24	−9	−45	−22	−127	−249	0	24
1	Pay	AAA	−4	39	−14	−12	−3	−8	−1	0	6
1	Pay	A	−10	38	−14	−13	−5	−9	−14	0	7
1	Pay	BB	−31	33	−12	−18	−9	−23	−59	0	11
1	Pay	CCC	−68	24	−9	−29	−12	−92	−187	0	21

Results are quoted in basis points of the trade notional. The residual IR01 is now zero, but market risk capital is non-zero as the trade and hedge do not perfectly offset each other from a capital perspective. The last column gives the adjustment to the hedge trade notional required to obtain the IR01 of zero.

The impact of allowing capital to be used as funding, $\phi = 1$, is significant. KVA from CCR is reduced by approximately half, while that from CVA is reduced by around one-third. Where the market risk capital is non-zero, the KVA associated with it is also reduced by approximately one-third.

Conclusions

We have presented a unified model for valuation adjustments, which includes the impact of capital, and introduced a new 'XVA' term called KVA. Practical examples of KVA on an interest rate swap have demonstrated how significant capital costs are, and that KVA is broadly similar in size to the other components of XVA. The use of capital to reduce funding requirements ($\phi = 1$) results in reductions in KVA of around one-third to a half (we grouped all capital-related effects into KVA in the examples). However, it is unclear in practice whether this option will be available to a derivatives trading desk. In as much as this reflects a divergence of practice from actual effects, some reassessment may be required.

The implication of the introduction of KVA is that just like CVA and FVA, it should be managed and hedged. KVA can be aligned with the counterparty, and clearly has contingency on the survival of the counterparty and issuer. The most appropriate approach would be to manage KVA alongside CVA and FVA at portfolio level in a central resource-management desk. **R**

Andrew Green is the head of the CVA/FVA quantitative research team at Lloyds Banking Group in London. Chris Kenyon and Chris Dennis are directors in the same team. **The authors would like to thank Lincoln Hannah, Glen Rayner and the two anonymous referees for their contributions. The views expressed are those of the authors only and no other representation should be attributed.**
**Email: andrew.green2@lloydsbanking.com,
chris.kenyon@lloydsbanking.com,
chris.dennis2@lloydsbanking.com**

REFERENCES

Basel Committee for Banking Supervision, 2006
International convergence of capital measurement and capital standards
June

Basel Committee for Banking Supervision, 2011
Basel III: a global regulatory framework for more resilient banks and banking systems

Burgard C and M Kjaer, 2011a
In the balance
Risk November, pages 72–75

Burgard C and M Kjaer, 2011b
Partial differential equation representations of derivatives with bilateral counterparty risk and funding costs
Journal of Credit Risk 7, pages 75–93

Burgard C and M Kjaer, 2013
Funding strategies, funding costs
Risk December, pages 82–87

Dodd C and B Frank, 2010
Dodd-Frank Wall Street Reform and Consumer Protection Act HR 4173, www.sec.gov/about/laws/wallstreetreform-cpa.pdf

Hull J and A White, 2014
Risk neutrality stays
Risk October, page 48

Kenyon C and A Green, 2013
Pricing CDSs' capital relief
Risk October, pages 62–66

Kenyon C and A Green, 2014
Regulatory costs break risk neutrality
Risk September, pages 76–80

Kenyon C and A Green, 2014
Regulatory effects remain
Risk October, page 49

Kenyon C and R Kenyon, 2013
DVA for assets
Risk February, pages 72–75

Reboul P, B Perrin, G Morel, and J Peace, 2014
Corporate and investment banking outlook
Technical report, Roland Berger and Nomura