What *is* the dynamical hypothesis?

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Abstract: Van Gelder's specification of the dynamical hypothesis does not improve on previous notions. All three key attributes of dynamical systems apply to Turing machines and are hence too general. However, when a more restricted definition of a dynamical system is adopted, it becomes clear that the dynamical hypothesis is too underspecified to constitute an interesting cognitive claim.

Van Gelder claims that the dynamical hypothesis entails three key properties, but all three properties apply to Turing machines, the paradigmatic nondynamical system.

1. Quantitative in state. "A system is quantitative in state when there is a metric over the state set such that behavior is systematically related to distances as measured by that metric" (sect. 3.3, para. 3).

This is true of a Turing machine. Define the following metric: the distance between two states is the minimal number of steps between them. The behavior of the Turing machine systematically relates to this metric (at each step, the machine will step to a neighboring state in this metric). This does not, of course, imply that all neighboring states are equally *accessible,* but this holds true for dynamical systems as well, where one cannot, for instance, simply reverse the direction of time.

2. Quantitative state/time interdependence. "A system is quantitative in *time* when time is a quantity; that is, there is a metric over the time set such that system behavior is systemically related to distances as measured by that metric . . . *amounts* of change in state are systematically related to *amounts* of elapsed time" (sect. 3.3, para. 5).

This is also true of a Turing machine. The standard metric over discrete times (such that the distance between $t = m$ and $t = n$ is $|n - m|$). Plus the distance metric over space just mentioned will suffice. System behavior is again systematically related to time in this sense. Also, this metric is neither trivial, nor only occasionally or accidentally related to system behavior. Contrary to van Gelder's claims, the notion of computation embodied by Turing machines has central interest in the time course of computation: computational complexity theory (Garey & Johnson 1979) is a fundamental topic in computer science. Algorithms are evaluated not only in terms of *effectiveness,* but also in terms of *efficiency;* that is, questions are standardly evaluated not only in terms of computability but also in terms of tractability. This concern naturally carries through to computational accounts of cognition (e.g., Falkenhainer & Forbus 1989). Furthermore, within the framework of the computational hypothesis, there are models that have sought specifically to capture the time course of human behavior. Recent examples of this are Anderson and Matessa's (1997) production-rule system of serial memory, which seeks to model latencies or the careful evaluations of competing models of analogy with respect to response time predictions by Keane et al. (1994).

3. Rate dependence. "Rates of change depend on current rates of change" (sect. 3.3, para. 6). As stated, this is a tautology, because it is not clear what separates "rates of change" from "current rates of change."

Van Gelder elaborates: "In these systems, variables include both basic variables and the rates of change of those variables" (sect. 3.3, para. 6). This seems completely mysterious, because we are given no analysis of what it is for a system to *include* a variable.

Van Gelder does note that "a *variable* is simply some entity that can change. . . . The *state* of the system is simply the state or value of all its variables at a time" (sect. 3.1, para. 1). From this it seems that state is just defined extensionally in terms of an arbitrary set of variables. If so, given any concrete object, we can define a system by a set of variables associated with that object and then define a new system including these variables and their rates of change. The latter system will be dynamical, according to the criterion of rate dependence. For any concrete object whatever (including the brain), at any level of analysis whatever, it seems that we can trivially satisfy the third criterion just by adding additional variables by fiat. So we seem to be no further forward.

What alternative analysis might be more appropriate? Van Gelder's Table 1 gives seven previous definitions of dynamical systems. Of these, 1 and 2 are tied directly to their physical realization, and hence not relevant in this more general context, whereas 5, 6, and 7 are trivially satisfied by Turing machines (essentially because Turing machines evolve deterministically over time).

However, consider definition 3 that a dynamical system is "a smooth manifold together with a vector field" (Casti 1993). Because this definition requires that the state space be smooth, the Turing machine *is* ruled out, because it has a discrete state space.

In brief, definition 4 states that dynamical systems are *continuous* deterministic systems, but once we realize that this is the fundamental claim, then it is clear that the dynamical hypothesis is simply too underspecified to be of any interest.

The computational hypothesis does not *merely* say that the mind is discrete at a high level of analysis. Instead, it applies a theory of symbolic computation of enormous theoretical richness and practical power. However, the dynamical hypothesis *does* merely state that the system is continuous – it says nothing about how it works, aside from the trivial truth that it should be studied using the diverse tools of dynamical systems theory. In short, the dynamical hypothesis has the same status that a putative "discrete hypothesis" concerning the mind would have had before Turing, von Neumann, and development of digital, symbolic computation: that is, it would be almost completely devoid of substance.