

## High precision beam momentum determination in a synchrotron using a spin resonance method

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**Abstract.** In order to measure the mass of the  $\eta$  meson with high accuracy using the  $dp \rightarrow {}^3\text{He}\eta$  reaction, the momentum of the circulating deuteron beam in the Cooler Synchrotron COSY of the Forschungszentrum Jülich has to be determined with unprecedented precision. This has been achieved by studying the spin dynamics of the polarized deuteron beam. By depolarizing the beam through the use of an artificially induced spin resonance, it was possible to evaluate its momentum  $p$  with a precision of  $\Delta p/p < 10^{-4}$  for a momentum of roughly 3 GeV/c.

### 1 Introduction

For numerous experiments, knowing the beam momentum in an accelerator with highest accuracy is essential. Obvious examples are investigations of production reactions very close to the thresholds as well as particle mass determinations on the basis of reaction kinematics. We present a technique that allows one to determine the momentum of a deuteron beam that is suitable for its use in a precise measurement of the  $\eta$  meson mass.

Measurements of the mass of the  $\eta$  meson performed at different experimental facilities over the last decade have resulted in very precise results which differ by up to 0.5 MeV/c<sup>2</sup>, i.e., by more than eight standard deviations. The experiments that are no longer considered in the PDG tables [1] generally involve the identification of the  $\eta$  as a missing-mass peak produced in a hadronic reaction. In order to see whether this is an intrinsic problem, and to clarify the situation more generally, a refined measurement of the  $dp \rightarrow {}^3\text{He}\eta$  reaction was proposed [2] at the Cooler Synchrotron COSY of the Forschungszentrum Jülich [3].

After producing the  $\eta$  mesons through the  $dp \rightarrow {}^3\text{He}\eta$  reaction using a hydrogen cluster-jet target [4], the  ${}^3\text{He}$  were detected with the ANKE magnetic spectrometer [5] that is located at an internal target position of the storage ring. The  $\eta$  mass can be extracted from pure kinematics through the determination of the production threshold. This requires one both to identify the threshold and to measure accurately the associated beam momentum.

For the new  $\eta$  mass proposal [2], the decision was taken to measure at thirteen fixed energies. To determine the mass using this kinematic method with a precision that is competitive with other recent measurements, i.e.,  $\Delta m_\eta < 50$  keV/c<sup>2</sup> [1], the associated beam momenta have to be fixed

with an accuracy of  $\Delta p/p < 10^{-4}$ . This requires the thirteen beam momenta in the range of 3100 – 3200 MeV/c to be measured to better than 300 keV/c.

Generally at synchrotron facilities like COSY, the velocity of the beam particles, and hence the beam momentum, is determined from the revolution frequency combined with the absolute orbit length. The precision of this method is limited to  $\Delta p/p \approx 10^{-3}$  and therefore the beam momentum must be determined in some other way.

A method proposed for electron colliders more than thirty years ago to overcome this problem [6] has been very successfully applied at the VEPP accelerator of the BINP at Novosibirsk to measure the masses of a wide variety of mesons from the  $\phi$  to the  $\Upsilon$  [7]. The precise beam momentum determination was achieved by using an artificial spin resonance, induced by a horizontal *rf* magnetic field from a solenoid to depolarize a vector polarized accelerator beam. The beam depolarizes when the frequency of the externally applied field coincides with that of the spin precession in the ring. The position of the depolarizing resonance depends purely upon the revolution frequency and the kinematical factor  $\gamma$  of the beam particles. The measurement of the revolution and depolarizing frequencies together allows the evaluation of  $\gamma$  and hence the beam momentum  $p$ .

There is no in-principle reason why the induced depolarization approach should not be equally applicable to other beam particles with an intrinsic spin, such as protons or deuterons. In fact, the effects have recently been confirmed at COSY in studies of the spin manipulation of both polarized proton [8] and deuteron beams [9]. This is the methodology that we are pursuing at COSY for the measurement of the  $\eta$  mass. For the first time in 2007 it was possible in a test measurement to reach an accuracy in the beam momentum calibration of  $\Delta p/p < 10^{-4}$  using the technique with a coasting beam but no internal target [10].

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Here we describe how the method can be used in a standard beam time under normal experimental conditions in the presence of a thick internal target.

## 2 Spin in synchrotron and artificially induced depolarizing resonance

In contrast to the case of a spin-half fermion such as an electron or proton, the deuteron is a spin-one boson that can be placed in three magnetic sub-states  $m = -1, 0, +1$ , and the resulting polarization phenomenology is more complex. Eight independent parameters are necessary to characterize a spin-one beam, three for the vector polarization and five for the tensor. However, only the vector polarization is used in the present experiment for the spin resonance method since it can be measured with higher precision than the tensor with the beam polarimeter. The motion of the spin vector  $\mathbf{S}$ , defined in the rest frame of the particle, in a circular accelerator, synchrotron or storage ring, is given by the Thomas-BMT equation [11]. In a synchrotron without horizontal magnetic fields and where the electric field is always parallel to the particle motion, the spin motion is only a function of the transverse magnetic fields  $\mathbf{B}_\perp$  of the accelerator. The deuteron spin precesses around the stable spin direction, which is given by the vertical fields of the guiding dipole magnets of the synchrotron. The number of spin precessions during a single circuit of the machine, the spin tune  $\nu_s$ , is proportional to the particle energy. In the coordinate basis of the moving particle, the spin tune is given by

$$\nu_s = G \gamma, \quad (1)$$

whereas, taking into account the extra spin precession during a single circuit of the machine, this becomes  $\nu_s = 1 + G \gamma$  in the laboratory frame. Here  $G = (g - 2)/2$  is the gyromagnetic anomaly of the particle, where  $g$  is the gyromagnetic factor. For deuterons the gyromagnetic anomaly is  $G_d = -0.1429872725 \pm 0.0000000073$ .

The beam polarization can be perturbed by a horizontal magnetic field in the synchrotron and, if the frequency of the perturbation coincides with the spin precession frequency, the beam depolarizes. A horizontal  $rf$  field from a solenoid or even a dipole can lead to  $rf$ -induced depolarizing resonances. The spin resonance frequency is given by [6]

$$f_r = (k + \gamma G) f_0, \quad (2)$$

where  $f_0$  is the revolution frequency of the beam,  $\gamma G$  is the spin tune, and  $k$  is an integer. If the  $rf$  frequency of the perturbation is close to  $f_r$  then the polarization of the beam is maximally influenced. In our experiment we only considered the first spin resonance, i.e.,  $k = 1$ , due to the frequency range of the  $rf$  solenoid. The kinematic  $\gamma$ -factor and thus the beam momentum can be determined on the basis of Eq. 2 purely by measuring both the revolution and spin-resonance frequencies.

## 3 Revolution and spin-resonance frequency

### 3.1 Experimental conditions

For a high precision experiment it is crucial that the beam momentum remains stable throughout the whole of the accelerator cycle. In a typical cycle of an ordinary scattering experiment at ANKE, the beam is first injected into COSY and accelerated to the nominal momentum. The  $rf$  cavity is then switched off to provide a coasting beam that fills the ring uniformly. This then gives constant count rates which reduces the dead time of the data acquisition system (DAQ). But, because of the energy losses of the charged beam particles due to electromagnetic processes as the beam passes repeatedly through the target, the momentum changes and this leads to a shift in the revolution frequency [12]. For a deuteron beam and a hydrogen cluster-jet target with a density of  $\rho = 1 \times 10^{15} \text{ cm}^{-2}$ , the revolution frequency would change by up to 103 Hz over a 180 s long cycle, corresponding to a shift in beam momentum of 2.2 MeV/c.

To compensate for this effect and to guarantee a constant beam momentum over the whole data-taking cycle, a second cavity, the barrier bucket ( $bb$ ) cavity, was switched on after the  $rf$  cavity was switched off. Using the  $bb$  cavity a beam with a constant momentum over the whole cycle was produced that filled roughly 80 – 90% of the ring homogeneously and thus achieved the necessary reduction in the dead time of the DAQ.

The thirteen closely spaced energies studied near the  $\eta$  threshold were divided into two so-called supercycles, each consisting of eight different beam energy settings. Each supercycle was used for five days of continuous Schottky data taking to study the long term stability of COSY and to take data in parallel for the  $\eta$  meson mass determination. The reason for choosing supercycles instead of independent measurements at fixed energy was to guarantee the same experimental conditions for each of the beam energies. In this way the systematic uncertainties could be investigated in more detail, as will be discussed in Sec. 3.2 and 3.3.

Before starting each of the five day blocks, the individual beam energies were measured using 36 s accelerator cycle lengths. After the injection of the beam into COSY, the stored deuterons were accelerated to the first nominal beam energy of the supercycle using the regular COSY  $rf$  cavity. At  $t = 3.7$  s this cavity was switched off and at  $t = 4$  s the  $bb$  cavity was put into operation to compensate for the beam energy losses. At  $t = 20$  s the depolarizing  $rf$  solenoid was switched on for five seconds and this was followed by a beam polarization measurement for five seconds using the EDDA detector. At  $t = 36$  s the cycle was ended. This procedure was repeated at the same beam energy but with different  $rf$  solenoid frequencies in order to obtain the spin-resonance spectrum. After completion of this first sub-measurement, the next beam energy of the supercycle was chosen and the corresponding spin resonance spectrum measured until complete data had been taken at all the energies of the supercycle.

After measuring the spin resonance spectrum, the supercycle was switched on for five days of continuous data taking to investigate the long term stability of the COSY accelerator. For this study the polarization measurements were omitted and total cycle lengths of 206 s were used. After injection, acceleration with normal cavity  $h = 1$  and starting the  $bb$  cavity, Schottky measurements were performed over the time interval of  $t = 14 - 196$  s. The eight beam energies in one supercycle resulted in a total time of 1648 s, after which the supercycle was repeated. After the five days of data taking, the individual beam energies were remeasured again in order to control systematic effects.

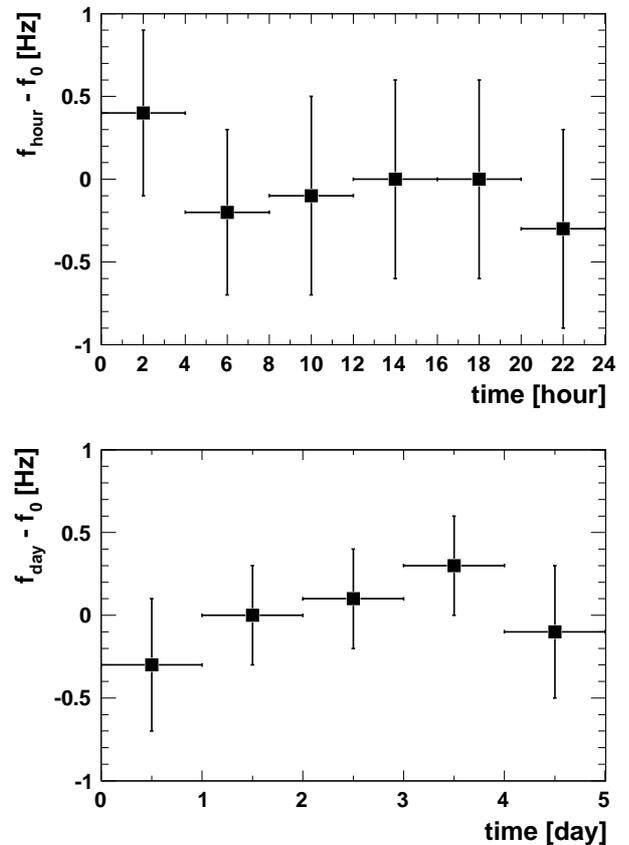
Both in the beam energy determination, as well as later in the Schottky data-taking time, one had to be assured that the measurements within the cycles were started sufficiently long after the ramping of the COSY dipole magnets for the acceleration of the beam. Otherwise, the not-yet-stable magnetic fields would lead to deviations in the values determined for the beam momentum. Detailed measurements of the beam energy by the spin-resonance method as a function of the time in the cycle showed that the experimental situation is already stable ten seconds after the start of the cycle [13]. Therefore, the  $rf$  solenoid field and the Schottky data-taking were started 20 and 14 s after injection, respectively. As a further check, measurements showed that the same beam energy was observed close to the end of the cycle as at the beginning [13] (see Fig 2). Thus it is valid to investigate the beam energy at one fixed time during the cycle and to take the resulting value as representative for the whole cycle.

### 3.2 Determination of the revolution frequency $f_0$ via the Schottky noise measurements

The revolution frequency  $f_0$  was measured by using the Schottky noise of the deuteron beam. For this measurement of the Schottky noise, the beam pick up and the spectrum analyzer of the stochastic cooling system of COSY were used. During the whole beam time the Schottky spectra were recorded every 30 s so that altogether nearly 15000 distributions were collected and sorted by energy, i.e., by flat top. The position of the Schottky distribution of the circulation frequency is stable for the whole cycle time and the full width at half maximum is in the region of 40 – 50 Hz for all energies. From all the Schottky spectra at one fixed energy of the five days of data taking that were measured under the same conditions an average revolution frequency was estimated. The statistical uncertainty is below 0.2 Hz for all energies and depends on both the number of measured Schottky spectra and on the distribution variations.

The  $bb$  cavity compensates beam target energy loss effects and should ensure that the revolution frequency remains constant. The large number of Schottky measurements allowed us to study the long term stability and to identify the magnitude of the variations of the revolution frequency at COSY. Therefore all the Schottky spectra at one energy from one day were analyzed and the mean revolution frequency of that day was calculated. In addition

the revolution frequencies for these data were calculated for every four hours to study the daily variation of the circulation frequency. The differences between the revolution frequencies of every four hours and the mean frequency of the day are presented in the upper part of Fig. 1. To study the variation of the revolution frequency over the five days of data taking, the same procedure was carried out for the Schottky data measured over this period. The differences between the mean revolution frequencies of every day and the mean frequency of the whole five days of data taking are presented in the bottom part of Fig. 1. The horizontal bars represent the time intervals for which the revolution frequency was evaluated.



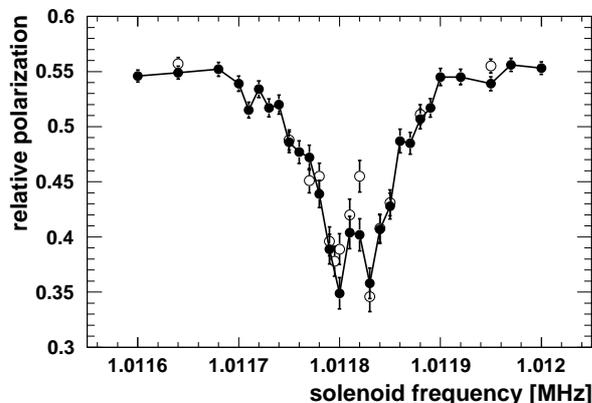
**Fig. 1.** Stability of the revolution frequency  $f_0$ . In panel (a) the differences between the revolution frequencies for every four hours and the mean revolution frequency of the day are shown. In panel (b) the differences between the revolution frequencies for each day and the mean revolution frequency of the five days of Schottky data taking are shown. From these figures it is clear that the revolution frequency at COSY is very stable, with variations below 1 Hz at a circulation frequency of  $f_0 \approx 1.4$  MHz.

The analysis shows that the revolution frequency at COSY over one day and also over five days is very stable. The variations of the revolution frequency are very small, being on the order of 1 Hz at a circulation frequency of  $f_0 \approx 1.4$  MHz. In sum, it was possible to determine the revolution frequencies for all energies with a statistical uncertainty below 1 Hz. Nevertheless a much larger system-

atic uncertainty of  $\Delta f_0 = 6$  Hz dominated the precision, and this arose from the preparation of the Schottky spectrum analyzer used.

### 3.3 Determination of the spin resonance frequency $f_r$ via an induced spin resonance

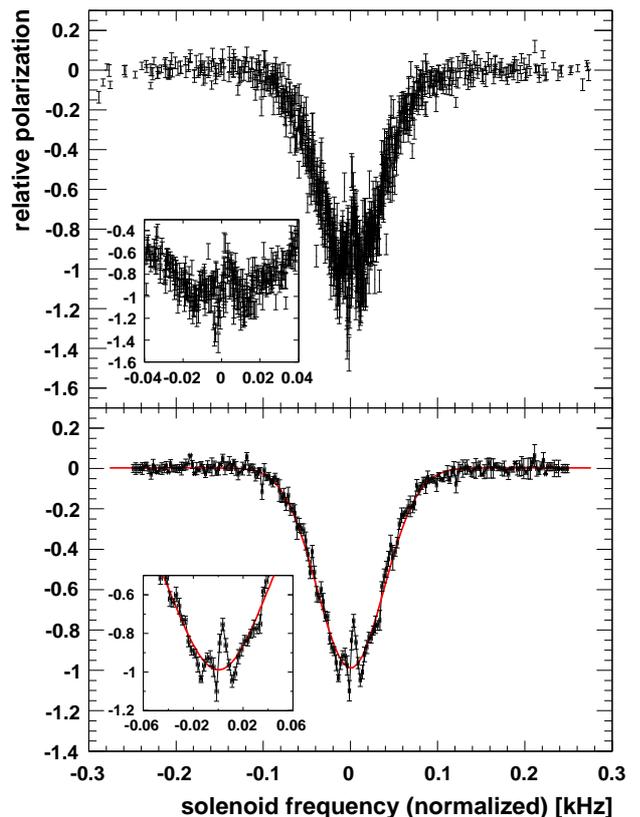
For all thirteen energies the spin resonance spectrum was measured twice, once before and once after the five days of Schottky data taking, as described in Sec.3.1. The relative polarization of the beam was measured with the EDDA detector [14]. For our purposes absolute calibrations of this device at the different energies was not required; a quantity proportional to the polarization is sufficient.



**Fig. 2.** Spin resonance measurements at one energy (closed circles). The open symbols represent results obtained for an extended cycle time, where the perturbing solenoid was switched on after 178 s.

An example of a spin resonance spectrum at one energy is shown in Fig. 2. Far away from the spin resonance, at 1.0116 MHz and 1.0120 MHz, a high beam polarization was measured. In contrast, if the frequency of the solenoid coincided with the spin precession, the beam was maximally depolarized. The full width at half maximum was in the region of 80-100 Hz for all energies. Unlike the earlier spin resonance test measurement with a coasting beam, i.e., no cavities and no internal target [10], the spin resonance spectra are not smooth. The structures, especially the double peak in the center, are caused by the interaction of the deuteron beam with the *bb* cavity.

However, to study in more detail the shape of the spin resonance spectra, all 26 spectra were fitted with gaussians and then shifted along the abscissa so that the mean value was zero. They were then scaled to produce a uniform height. The resulting normalized spin resonance spectrum is shown in the upper part of Fig 3. This is symmetric around zero and smooth, except for the structure at the center. This region is shown in greater detail in the lower part of Fig. 3. A structure with a symmetric double peak and an oscillation was observed in the center of the spin resonance. However, it is important to note that the gaussian

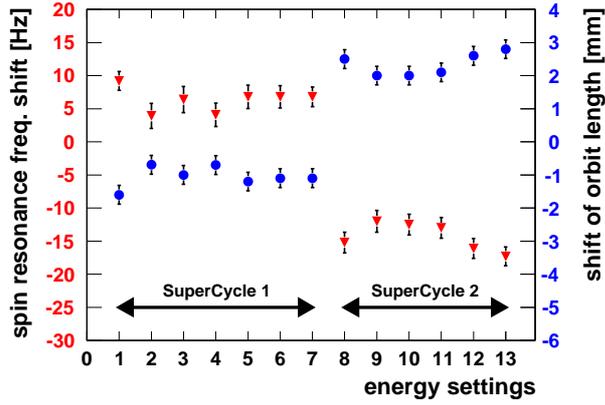


**Fig. 3.** Panel (a): The spin resonance spectra normalized by a gaussian. Panel (b): The same but with a binned abscissa in addition. The spin resonance shape is symmetric about zero and smooth except in the center, where a double peak structure is seen. This arises from the synchrotron oscillations of the beam particles caused by the interaction of the deuteron beam with the *bb* cavity. The inserts show the resonance valley in greater detail.

mean value, i.e., the spin resonance frequency, is not influenced by this structure. This was checked by making a fit where the data points at the center were neglected. The spin resonance frequencies  $f_r$  for all energies were extracted from the spin resonance spectra by a gaussian fit which gave  $\chi^2/\text{ndf}$  in the region of 2–3. The statistical uncertainties of the spin resonance frequencies are on the order of 1–2 Hz at  $f_r \approx 1.01$  MHz.

It is important for the interpretation of the spin resonance measurements to know to what extent the positions of the observed spin resonance frequencies are stable over the finite accelerator cycle in the presence of a thick internal target. Therefore, in a special measurement the switch-on of the *rf* solenoid was delayed from 20 s to 178 s in order to investigate the position of the spin resonance frequency close to the end of a long cycle. The observed data (open symbols of Fig. 2) showed a resonance position which agreed with the data taken at the beginning of the cycle to within 2 Hz.

In Fig. 4 the shifts between the first and second spin resonance measurements are shown as red triangles for all thirteen energies. The frequencies in the first supercycle decrease by between 4 and 10 Hz for all energies, whereas



**Fig. 4.** (Color online) The spin resonance frequencies were measured twice, once before and once after the five days of data taking. The red triangles present the shift of the spin resonance frequency  $f_r$  from the first to the second measurement. These shifts correspond to changes in the orbit length, which are shown as blue circles. For the first supercycle, the spin resonance frequencies decrease between the two measurements by 4 – 10 Hz, which corresponds to a increase in the orbit length in the range of 0.7 – 1.6 mm. For the second supercycle an increase of the spin resonance in the range of 12 – 17 Hz was observed, i.e., a decrease in the orbit length in the range of 2.0 – 2.8 mm.

for the second supercycle they increase in the range of 12 – 17 Hz. These systematic shifts of the frequencies in the same direction indicate slight changes in the COSY settings. Because the revolution frequency is stable, as described in Sec. 3.2, the change is attributed to a shift in the orbit length  $s$ .

The velocity  $v$  of the particle is the product of the revolution frequency and the orbit length  $v = s f_0$ . Using Eq. 2, the orbit length can be calculated from the revolution and the spin resonance frequencies:

$$s = c \left[ \frac{1}{f_0^2} - \left( \frac{G_d}{f_r - f_0} \right)^2 \right]^{\frac{1}{2}}, \quad (3)$$

which allows the orbit lengths to be extracted with a precision better than 0.3 mm for every flat top. Since the nominal COSY circumference is 183.4 m, this gives a relative accuracy of  $\Delta s/s \leq 2 \times 10^{-6}$ . The precision is dominated by the uncertainty of the spin resonance frequency. The shift in the spin resonance frequency corresponds to a change in the orbit length of up to 3 mm, which is presented for all energies in Fig. 4 as blue circles. The shifts of the spin resonance frequencies of the first supercycle suggest an increase in the orbit length in the range of 0.7 – 1.6 mm and to a decrease in the range of 2.0 – 2.8 mm for the second supercycle.

To determine the precise beam momenta, the mean value of the two spin resonance measurements for every energy was calculated. These mean values differ by up to 10 Hz from the single spin resonance measurements. Nevertheless, in view of the observed shift of the spin resonance frequency, a very conservative systematic uncertainty of  $\Delta f_r = 15$  Hz was assumed.

## 4 Results

The deuteron kinematic  $\gamma$ -factor and the beam momenta were calculated according to Eqs. 4

$$\gamma = \frac{1}{G_d} \left( \frac{f_r}{f_0} - 1 \right)$$

$$p = m_d \beta \gamma = m_d \sqrt{\gamma^2 - 1} \quad (4)$$

from the knowledge of the revolution and the spin resonance frequencies. The accuracies to which both of the frequencies are determined are dominated by systematic effects. The revolution frequency measured by the Schottky spectrum analyzer has an uncertainty of  $\Delta f_0 = 6$  Hz, corresponding to one in the beam momentum of 50 keV/c. The error in the determination of the spin resonance arises from the small variations of the orbit length and  $\Delta f_r = 15$  Hz corresponds to an uncertainty in the beam momentum of 164 keV/c. Because these systematic uncertainties are independent, they are added quadratically to give a total uncertainty  $\Delta p/p \leq 6 \times 10^{-5}$ , i.e., a precision of 170 keV/c for beam momenta in the range of 3100 – 3200 MeV/c. This is over an order of magnitude better than ever reached before for a standard experiment in the COSY ring. The measurement of the beam momentum differed by up to 5 MeV/c from the nominal requested momentum.

Two further quantities, the beam momentum smearing  $\delta p/p$  and the smearing of the orbit length  $\delta s/s$ , can be extracted from the spin resonance spectra. The width of the spin resonance spectra depends only on the strength of the resonance and the momentum smearing. In our case a resonance strength of  $\epsilon = 3.2 \times 10^6$  leads to a spin resonance with a FWHM width of 9.1 Hz. This is much smaller than the observed width of 80-100 Hz, which is therefore dominated by the momentum spread of the beam. Assuming a gaussian distribution in the revolution frequency with a FWHM = 40 – 50 Hz, and neglecting all other contributions, the width of the spin resonance distribution requires a momentum spread of  $(\delta p/p)_{rms} \approx 2 \times 10^{-4}$ . This upper limit on the beam momentum width corresponds to a smearing of the orbit length of  $(\delta s/s)_{rms} \approx 4 \times 10^{-5}$ .

The momentum spread could be checked from the frequency slip factor  $\eta$ , which was measured at each energy. Using  $\delta p/p = 1/\eta \times (\delta f_0/f_0)$  this leads for example at  $p^{nominal} = 3.1625$  GeV/c to  $(\delta p/p)_{rms} = 1.4 \times 10^{-4}$ , which is consistent with the value obtained from the resonance distribution.

## 5 Conclusions and outlook

We have shown how to determine the momentum of a deuteron beam in a circular accelerator with high precision using the spin resonance technique developed at the VEPP accelerator for electron beams. We have studied the depolarization of a polarized deuteron beam at COSY through an induced spin resonance for thirteen different beam energies. This was done under standard experimental conditions, i.e., with cavities, in particular the *bb* cavity, and

a thick internal cluster-jet target. The momenta and other beam properties were found by measuring the position of the spin resonance and revolution frequencies.

It was possible to determine the beam momenta with an accuracy of  $\Delta p/p \leq 6 \times 10^{-5}$ , i.e., the thirteen momenta in the range 3100 – 3200 MeV/c were measured with precisions of  $\approx 170$  keV/c, a feat never before achieved at COSY. The actual precision was limited by the systematic variations of the orbit length and the characteristics of the Schottky spectrum analyzer.

The orbit length could be extracted from the revolution and spin resonance frequencies with an accuracy of  $\Delta s/s \leq 2 \times 10^{-6}$ . Thus for COSY, with a circumference of 183.4 m, the orbit length could be measured with a precision below 0.3 mm. This may allow one to obtain a better knowledge of the orbit behavior in COSY. In addition, the distributions in the beam momentum and orbit length could be extracted from the data. The results are also sensitive to the synchrotron oscillations which lead to synchrotron side-band resonances and so these can also be studied with the spin resonance method. These results were achieved using a deuteron beam, but there are no in-principle reasons why the depolarization technique should not be applicable to proton beams at COSY with same success.

In summary, the spin resonance method is a powerful beam diagnostic tool for circular accelerators, synchrotrons or storage rings to investigate and determine beam properties. In our particular case it should eventually allow the mass of the  $\eta$  meson to be measured with a precision of  $\Delta m_\eta \leq 50$  keV/c<sup>2</sup>.

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