Hole-Hole Interaction Effect in the Conductance of the Two-Dimensional Hole Gas in the Ballistic Regime

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On a high-mobility two-dimensional hole gas (2DHG) in a GaAs/GaAlAs heterostructure we study the interaction correction to the Drude conductivity in the ballistic regime, $k_B T\tau/\hbar > 1$. It is shown that the "metallic" behavior of the resistivity ($d\rho/dT > 0$) of the low-density 2DHG is caused by the hole-hole interaction effect in this regime. We find that the temperature dependence of the conductivity and the parallel-field magnetoresistance are in agreement with this description, and determine the Fermi-liquid interaction constant F_0^σ which controls the sign of $d\rho/dT$.

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It is well known that electron-electron interaction gives rise to a quantum correction to the classical (Drude) conductivity [1,2]. Its manifestation can be quite different in the two regimes which relate the quasiparticle interaction time, $\hbar/k_B T$, to momentum relaxation time, τ: diffusive ($k_B T\tau/\hbar < 1$) and ballistic ($k_B T\tau/\hbar > 1$). So far the interaction correction to the conductivity of two-dimensional (2D) systems has been studied only in the diffusive regime, which is applicable to low-mobility (small τ) systems.

The role of interactions has now been intensely discussed, after the observation of the metallic behavior in some low-density, high-mobility 2D systems [3], where the diffusion approximation becomes invalid even at low T. Recently, a theory of the interaction correction in the ballistic and intermediate regimes has been developed [4]. Stimulated by this theory, we examine the role of the hole-hole interaction effects in a 2D hole gas (2DHG) which shows a metallic $\rho(T)$. We analyze the temperature dependence of the conductivity and positive magnetoresistance in parallel field, and show that these two main features of the metallic state can be explained by the interaction effect. It has the same origin as the logarithmic correction studied earlier [1], but now manifests itself in the ballistic regime.

The interaction theory [4] considers elastic (coherent) electron scattering from the modulated density of other electrons (Friedel oscillation) caused by an impurity with a short-range potential. The phase of the Friedel oscillation $\Delta p \propto \exp(i2k_F r)$ is such that the wave scattered from the impurity interferes constructively with the wave scattered from the oscillation (Fig. 1(a)), leading to the quantum correction to the Drude conductivity σ_0. The model [4] gives several predictions to be tested experimentally. First, the logarithmic correction in the diffusive regime of multiple-impurity scattering becomes a linear temperature dependence for the case of single-impurity scattering at $k_B T\tau/\hbar > 1$:

$$\delta\sigma(T) = \sigma_0\left(1 + \frac{3F_0^\sigma}{1 + F_0^\sigma}\right)\frac{k_B T}{E_F}, \qquad (1)$$

where F_0^σ is the Fermi-liquid interaction parameter in the triplet channel. The coefficient in the temperature dependence originates from two contributions: one due to exchange processes (Fock) and another due to direct interaction (Hartree). Second, for a wide range of parameter F_0^σ the model allows the change of the sign of $d\rho/dT$ with parallel magnetic field—the effect seen in recent experiments. The magnetic field suppresses the correction in the triplet channel in Eq. (1), resulting in a universal, positive correction to the Drude conductivity in magnetic field, σ_0^B, and hence the "insulating" behavior of $\rho(T)$:

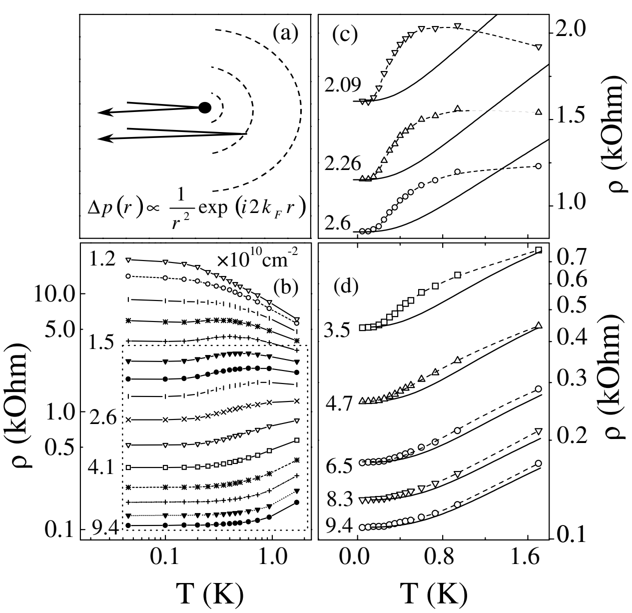

FIG. 1. (a) Diagram of electron scattering by an impurity and the Friedel oscillation it produced. (b) Temperature dependence of the resistivity at different hole densities near the crossover in the sign of $d\rho/dT$. (c),(d) Resistivity on the metallic side of the crossover (symbols), and contribution to $\rho(T)$ due to phonon scattering (solid lines).

$$\delta\sigma = \sigma_0^B \frac{T}{T_F} \quad \text{at } B \geq B_S. \tag{2}$$

Here B_S is the field corresponding to the full spin polarization of the 2D system, $B_S = 2E_F/g^*\mu_B$, where g^* is the Landé g factor, μ_B is the Bohr magneton, and T_F is the Fermi temperature. Note that the same functional dependence as in Eq. (1) was derived in [5], where the authors consider the temperature effect on the screening of the impurity potential. The model [5] has been applied to the analysis of the linear $\rho(T)$ of a 2D hole gas in GaAs [6]. It was shown there that in accordance with Eq. (1) $\delta\sigma(T)/\sigma_0$ scales as T/T_F, although the change of the slope with carrier density disagrees with that expected from [5]. It is important to mention that model [5] considers only the Hartree potential of interacting electrons and ignores the Fock contribution. As a result, it predicts only the positive sign of $d\rho/dT$ and does not allow the change in the sign of $d\rho/dT$ with magnetic field.

The experiments have been performed on a (311)A GaAs/AlGaAs heterostructure with a peak mobility of 6.5×10^5 cm^2 V^{-1} s^{-1}, which shows the crossover from "metal" to "insulator" at $p \sim 1.5 \times 10^{10}$ cm^{-2} [Fig. 1(b)]. A standard four-terminal lock-in technique has been used for resistivity measurements at temperatures down to 50 mK, with currents of 1–10 nA to avoid electron heating. The hole density p in the metallic region is varied by the gate voltage in the range (1.4–9.4) $\times 10^{10}$ cm^{-2}, corresponding to the interaction parameter $r_s = 10$–17. The effective mass has been determined from the analysis of the Shubnikov–de Haas oscillations, and found to be density independent within the experimental accuracy: $m^* = (0.38 \pm 0.02)m_e$. This value coincides with the previously reported value $m^* = (0.37 - 0.38)m_e$ [7]. In [8,9] the metallic character of a higher density 2DHG in GaAs was explained in terms of inelastic scattering between two hole subbands, split due to spin-orbit interaction. In our low-density structures the effect of the band splitting is negligible.

Figure 1(b) represents the temperature dependence of the resistivity, with the metallic region under study marked by a box. The increase of the resistivity with T can be simply due to phonon scattering, which cannot be ignored in GaAs structures with piezoelectric coupling even at temperatures below 1 K. In Figs. 1(c) and 1(d), curves $\rho(T)$ for different densities are plotted together with the theoretical dependence presented as $\rho(T) = \rho_0 + \rho_{ph}$, where $\rho_0 = \sigma_0^{-1} = \rho(T = 0)$ is the residual resistivity due to impurity scattering, obtained by extrapolation to $T = 0$, and ρ_{ph} is the result of the calculations for the phonon scattering in GaAs heterostructures [10]. The latter is represented as $\rho_{ph}(T) = \frac{a(T/T_0)^3}{1+c(T/T_0)^2}$, where parameters a and c depend on the carrier density, effective mass, and crystal properties, and $T_0 = k_B^{-1}\sqrt{2m^* S_t^2 E_F}$, where S_t is the transverse sound velocity. This relation corresponds to the intermediate temperature range between the Bloch-Gruneisen, $T < T_0$, and the linear, $T > T_0$, regimes, for the case of nonscreened phonon scattering. (The criterion $T < T_0/\pi$ [11] for the screened phonon scattering is not satisfied for the majority of our data.) We calculate a, c, T_0, and thus ρ_{ph} at each hole density according to Ref. [10] using $m^* = 0.38m_e$.

One can see that at the highest p, phonon scattering can fully explain the experimental dependence $\rho(T)$. However, with decreasing density another contribution develops, which dominates at low T and low densities. Figure 2(a) shows this contribution obtained by subtracting that of phonon scattering. The peaklike shape of $\rho(T)$, with the maximum at $T_{\max} \approx 0.3T_F$, is in qualitative agreement with numerical calculations [12] and experiment [13].

In order to compare the results in the low-temperature range of $\rho(T)$ with Eq. (1), we replot in Fig. 2(b) the data in the conductivity form: $\Delta\sigma(T) = [\rho(T) - \rho_{ph}(T)]^{-1} - \rho_0^{-1}$. The condition for the ballistic regime $k_B T\tau/\hbar \geq 1$ is satisfied in our structure at $T > 50$–100 mK, and a linear fit of $\Delta\sigma(T)$ gives the value of parameter F_0^σ (Fig. 2(c)). The following comments can be made on this result. First, the interaction constant is negative, and this provides the metallic slope in $\rho(T)$. Second, its absolute value decreases with increasing density, which is in agreement with the expectation that the ratio of the exchange to kinetic energy of quasiparticles decreases to zero at large densities. Third, one can see that the measured value does not exceed 0.42 and, when extrapolated to the density of the crossover from metal to insulator ($p \sim 1.5 \times 10$ cm^{-2}), is much

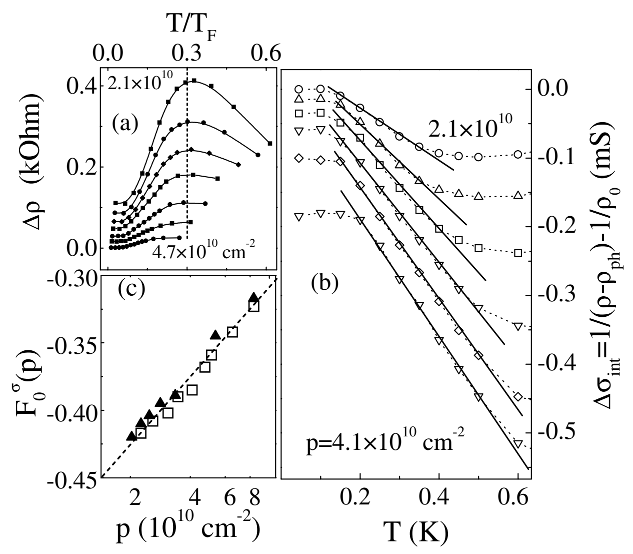

FIG. 2. (a) Impurity scattering contribution $\Delta\rho$ against dimensionless temperature at different p. [For clarity, curves in (a) and (b) are offset vertically from zero value at $T = 0$.] (b) The same data as in (a) but in conductivity form, with linear fitting. (c) Fermi liquid parameter versus hole density. Open symbols show the result obtained from the analysis of $\rho(T)$ at zero magnetic field; closed symbols show the result from the analysis of the parallel-field magnetoresistance, shown in Fig. 4(b).

smaller than the value of $|F_o^\sigma| = 1$ expected for the Stoner instability [4]. This implies that our description of the metallic system as a weakly interacting Fermi liquid is self-consistent.

Let us now turn to the increase of resistance with parallel field shown in Fig. 3(a), which is similar to that observed earlier on the 2DHG [15]. It was recently shown that the hump in $\rho(B_{\|})$ corresponds to the magnetic field B_S of full spin polarization of the 2DHG [16]. For an overall view of the magnetoresistance, we will first analyze it as is done for 2D electrons in a Si metal-oxide-semiconductor field-effect transistor [17] and GaAs heterostructure [18]. This analysis is based on the model [14] of positive magnetoresistance at $T = 0$, which considers the effect of parallel field on impurity scattering. Figure 3(b) shows $\rho(B_{\|})/\rho(B_{\|} = 0)$ as a function of dimensionless magnetic field B/B_S, with B_S found as a fitting parameter. Its value is shown by the dashed line in Fig. 3(a) and corresponds to the position of the hump. In accordance with [18], all the data collapse on one curve, which is close to the theoretical dependence apart from the region near B_S where one can expect a contribution from another mechanism [19]. Using the value of B_S and the fact that m^* is density independent, one can obtain the effective g factor, $g^* = 2E_F/\mu_B B_S$, whose dependence on the density is shown in the inset of Fig. 3(b). The g factor decreases with decreasing density—similar behavior was recently observed for 2D electrons in GaAs [20].

Figure 4 (inset) shows the temperature dependence of the magnetoresistance, where one can see that $B_{\|}$ drives the ‘‘metallic’’ state into insulator. To compare this result with the prediction given by Eq. (2) we analyze the temperature dependence of the resistivity at field B_S. (In this analysis we neglected altogether the phonon contribution,

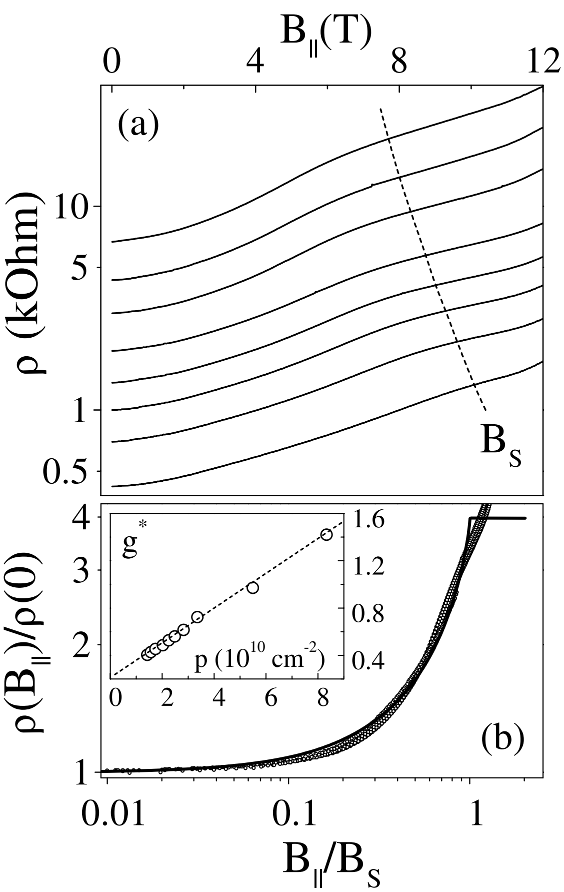

FIG. 3. (a) Dependence of the resistivity on parallel magnetic field at $T = 50$ mK and $p = (1.43, 1.57, 1.75, 2.03, 2.26, 2.49, 2.83, 3.36) \times 10^{10}$ cm^{-2}, from top to bottom. (b) Scaled data, with an added curve $\rho(B_{\|})$ for $p = 8.34 \times 10^{10}$ cm^{-2}; solid line is the result of the model [14]. Inset: dependence of the effective g factor on the hole density, obtained from the value of B_S.

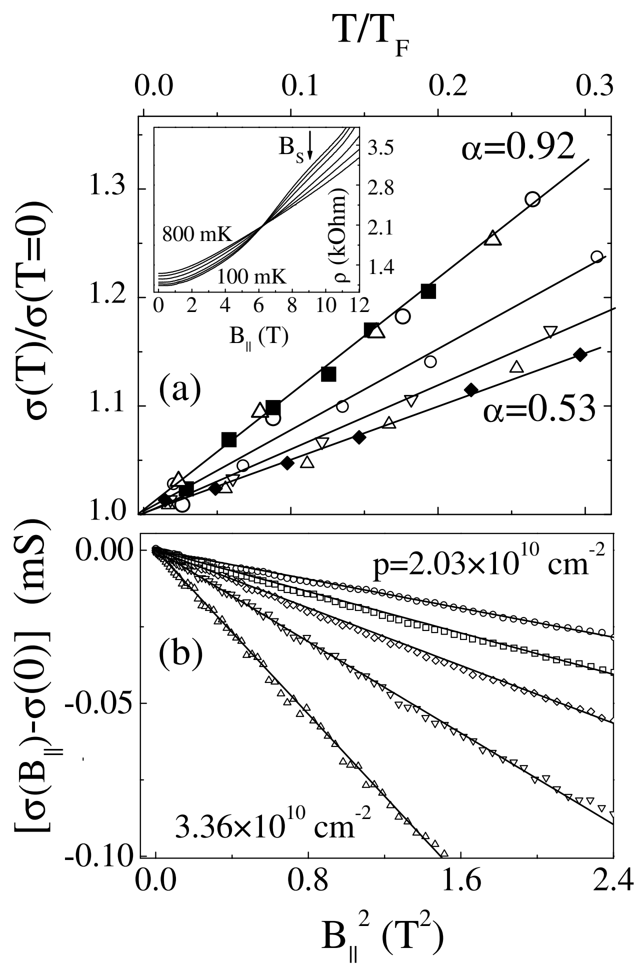

FIG. 4. (a) Temperature dependence of the conductivity at $B_{\|} = B_S$ for different hole densities. Coefficient $\alpha = 0.53$ is obtained for $p = (2.49, 2.83) \times 10^{10}$ cm^{-2}; $\alpha = 0.62$ for $p = 2.26 \times 10^{10}$ cm^{-2}; $\alpha = 0.74$ for $p = 2.03 \times 10^{10}$ cm^{-2}; and $\alpha = 0.92$ for $p = (1.43, 1.57, 1.75) \times 10^{10}$ cm^{-2}. Inset: $\rho(B_{\|})$ for $p = 2.26 \times 10^{10}$ cm^{-2}, at different temperatures: $T = 0.1$, 0.2, 0.3, 0.45, 0.6, 0.8 K. (b) Magnetoconductivity against $B_{\|}^2$, at $T = 0.6$ K for densities $p = (2.03, 2.26, 2.49, 2.83, 3.36) \times 10^{10}$ cm^{-2}.

as at $T < 0.5$ K it becomes less than 5%, due to the fourfold resistance increase at $B_{||} = B_S$.) The resulting dependences, shown in Fig. 4(a), are indeed linear. By extrapolation to $T = 0$, we find the value of the Drude conductivity σ_0^B and determine the slope α of the straight lines. Its value is close to the expected one, $\alpha = 1$, although we find that agreement is better for smaller p, where α increases to 0.92. This can be attributed to the fact that in the real system the scatterers are not pointlike, as assumed in the theory. However, with decreasing density and increasing Fermi wavelength, $\lambda_F \propto p^{-1/2}$, the approximation of short-range scatterers becomes more applicable. The fact that impurity scattering in our structure with a spacer of 500 Å is dominated by a short-range potential agrees with earlier findings that in high-mobility structures with large spacers it is the background impurities which determine the mobility [21].

A detailed quantitative analysis of the magnetoresistance at finite temperature can be done at small fields where the discussed model gives a simple prediction for the magnetoconductivity $\Delta\sigma = \sigma(B_{||}, T) - \sigma(0, T)$ in the ballistic regime. The analytical expression for weak fields, such that $x = \frac{Ez}{2k_BT} \ll 1 + F_0^\sigma$, is given in [22]

$$\Delta\sigma(B_{||}) = \frac{2F_0^\sigma}{1 + F_0^\sigma}\sigma_0 \frac{T}{T_F} K_b\left(\frac{Ez}{2T}, F_0^\sigma\right), \tag{3}$$

where $Ez = g_o\mu_B B_{||}$, g_o is the bare g factor, and $K_b(x, F_0^\sigma) \approx x^2 f(F_0^\sigma)/3$, $f(z) = 1 - \frac{z}{1+z}[\frac{1}{2} + \frac{1}{1+2z} - \frac{2}{(1+2z)^2} + \frac{2\ln(2(1+z))}{(1+2z)^3}]$. In fact, a direct comparison of the exact expression for $K_b(x, F_0^\sigma)$ and its low-field asymptote shows that they coincide in a wider range of magnetic field. We use this asymptote at $x \leq 1 + F_0^\sigma$, where it gives an error of less than 2%, provided $0.25 \leq F_0^\sigma \leq 0.45$.

In Fig. 4(b) we plot the magnetoconductivity at $T = 0.6$ K as a function of $B_{||}^2$ for fields satisfying the above condition. We use σ_0 obtained in the above analysis at $B_{||} = 0$. As the bare g factor is not known, instead of g_o we use the value of g^* determined from the analysis of $\rho(B_{||})$, at the lowest T. After that, the only unknown parameter in the slope of $\Delta\sigma(B_{||}^2)$ is F_0^σ. We extract its value and in Fig. 2(c) compare it with that determined earlier from $\rho(T)$ at zero field, where good agreement between the two approaches is seen.

It is interesting to note that in the previous study at lower hole densities [23], with $p = (1.2\text{–}1.7) \times 10^{10}$ cm^{-2}, the interaction correction appeared to be suppressed, both in the Hall coefficient and the phase breaking rate, when compared with estimations from the diffusion theory [1]. The origin of this becomes clear if one takes into account that in the previous work $k_BT\tau/\hbar = 0.1\text{–}1$, and the system was, in fact, in the intermediate regime where the temperature dependence of the Hall coefficient is expected to be small [24]. In that regime the interaction correction was negligible compared with the correction of weak localization. In the ballistic regime studied now, with $k_BT\tau/\hbar = 1\text{–}10$, the interaction correction develops and dominates weak localization.

In conclusion, we have studied the temperature dependence of the conductivity and the magnetoresistance in parallel field of a low-density 2D hole gas in the metallic phase, near the crossover in the sign of $(d\rho/dT)$. We have demonstrated that the metallic character of $\rho(T)$ and the positive magnetoresistance are caused by the hole-hole interaction in the ballistic limit $k_BT\tau/\hbar > 1$. We have found the Fermi-liquid constant F_0^σ, which determines the sign of $\rho(T)$. Its value near the crossover appears to be significantly smaller than expected for the ferromagnetic instability.

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