Nonperturbative Entangling Gates between Distant Qubits Using Uniform Cold Atom Chains

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We propose a new fast scalable method for achieving a two-qubit entangling gate between arbitrary distant qubits in a network by exploiting dispersionless propagation in uniform chains. This is achieved dynamically by switching on a strong interaction between the qubits and a bus formed by a nonengineered chain of interacting qubits. The quality of the gate scales very efficiently with qubit separations. Surprisingly, a sudden switching of the couplings is not necessary. Moreover, our gate mechanism works for multiple gate operations without resetting the bus. We propose a possible experimental realization in cold atoms trapped in optical lattices and near field Fresnel trapping potentials.

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Introduction.—Universal quantum computation can be achieved by arbitrary local operations on single qubit and one two-qubit entangling gate [1]. In an array of spins an entangling gate between neighboring qubits can be accomplished by letting them interact. However, for nonneighboring qubits, a direct interaction is normally not possible unless there is a separate common bus mode [2] or flying qubits. In realizations without an additional bus mode, such as with cold atoms in optical lattices, gates usually occur in parallel between all neighboring pairs [3]. Thus, designing bus modes for logic gates between arbitrary and distant pairs of qubits is of utmost importance and various unconventional examples of buses are continuously being proposed [4,5]. One possible realization is to have both the qubits and the bus composed of the same physical objects, generally called spin chains. The quality of an unmodulated spin chain, even as a data bus, is affected by dispersion [6]. Thus, in order to have a quantum gate between two qubits through such buses [5,7–9], delocalized encodings over several spins [10], delicately engineered couplings [11] or very weak couplings between qubits and the bus [5] is thought to be necessary. Recently, a new scheme based on tuning the couplings between qubits and the bus has been proposed [12] for fast and high-quality state transmission, which we here exploit for achieving an entangling gate between arbitrarily distant qubits.

Cold atoms in optical lattices are now an established field for testing many-body physics [13–17]. In particular, chains of atoms in Mott insulator regime (one atom per site) are being built experimentally [16,17], paving the way for realizing spin Hamiltonians [18]. With recent cooling methods, the required temperatures for observing magnetic quantum phases has become reachable [19]. In this framework, a series of multiple two-qubit gates, acting globally and simultaneously, has been proposed [20] and realized [15]. Could the same framework solve the problem of realizing quantum gates between any two selected neutral atom qubits? This is still an outstanding problem, unless one uses the physical movements of neutral atoms to each other's proximity [21] which has its own complexity.

Recently, single site addressing in an optical lattice setup has been experimentally achieved [17]. Furthermore, local traps have been proposed for individual atoms using near field Fresnel diffraction (NFFD) light [22]. A new approach for scalable quantum computation has been suggested [23] through a combination of local NFFD traps, for qubits, and an empty optical lattice, for mediating interaction between them. Since the interaction is achieved through controlled collisions between delocalized atoms it may suffer a high decoherence when qubits, on which the gate is applied, are far apart [15].

In this Letter we put forward a scalable dynamical scheme for achieving high-quality entangling gates between two distant qubits, suitable for subsequent uses without resetting. We do not demand encoding, engineering, or weak couplings: we only need switchable couplings between qubits and the bus. We also propose an application, based on a combination of NFFD traps and optical lattices.

Introducing the model.—Let us describe our bus as a chain of spin $1/2$ particles interacting through

$$H_M = J\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \lambda\sigma_n^z\sigma_{n+1}^z), \quad (1)$$

where σ_n^α ($\alpha = x, y, z$) are Pauli operators acting on site n, J is the exchange energy, and λ is the anisotropy. The qubits A and B, on which the gate acts, sit at the opposite sides of the bus, labeled by sites 0 and $N+1$, respectively. The interaction between the bus and the qubits is $H_I = J_0\sum_{n=0,N}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \lambda\sigma_n^z\sigma_{n+1}^z)$, where

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the coupling J_0 can be switched on or off. For the moment the anisotropy λ is set to zero. Initially the qubits are prepared in the states $|\psi_A\rangle$ and $|\psi_B\rangle$ and decoupled from the bus which is in the state $|\psi_M\rangle$, an eigenstate of H_M, for instance the ground state. Since H_M commutes with the parity operator $\prod_{n=1}^{N}(-\sigma_n^z)$ and with the mirror-inversion operator, the state $|\psi_M\rangle$ has a definite parity $(-1)^p$, for some integer p, and is mirror symmetric. At time $t = 0$ the coupling J_0 is switched on and the whole system evolves under the effect of the total Hamiltonian $H = H_M + H_I$, i.e., $|\Psi(t)\rangle = e^{-iHt}|\psi_A\rangle|\psi_M\rangle|\psi_B\rangle$. In Ref. [12] it was shown that by tuning J_0 to an optimal value $J_0^{\text{opt}} \simeq 1.05JN^{-1/6}$ the mirror-inversion condition [7] is nearly satisfied resulting in a fast high-quality transmission. In fact, when $|\psi_B\rangle$ is initialized in either $|0\rangle \equiv |\uparrow\rangle$ or $|1\rangle \equiv |\downarrow\rangle$ an arbitrary state of A is transmitted almost perfectly to B after time $t^* \simeq (0.25N + 0.52N^{1/3})/J$.

The Hamiltonian H is mapped to a free-fermionic model by Jordan-Wigner transformation $c_n = \Pi_{k=0}^{n-1}(-\sigma_k^z)\sigma_n^-$ [where $\sigma_n^{\pm} = (\sigma_n^x \pm \sigma_n^y)/2$] followed by a unitary transformation $d_k = \sum_n g_{kn}c_n$. The total Hamiltonian finally reads $H = \sum_k \omega_k d_k^\dagger d_k$ where the explicit form of g_{kn} and ω_k are given in [24,25]. The dynamics in the Heisenberg picture is given by $c_n(t) = \sum_m U_{nm}(t)c_m$ where $U_{nm}(t) = \sum_k g_{kn}g_{km}e^{-i\omega_k t}$. When the perfect transmission condition, i.e., $J_0 = J_0^{\text{opt}}$, is satisfied we have $|U_{0,N+1}(t^*)|^2 \simeq 1$, and thus we set $U_{0,N+1}(t^*) = e^{i\alpha_N}$. The phase α_N is irrelevant to the quality of transmission, but is the heart of our proposal for obtaining an entangling gate between A and B. We define $|\Psi_{ab}\rangle = |\Psi(0)\rangle$ with $|\psi_A\rangle = |a\rangle$ and $|\psi_B\rangle = |b\rangle$ where $a, b = 0, 1$. When J_0 is switched on the whole system evolves and at $t = t^*$ the states of A and B are swapped, while the bus takes its initial state $|\psi_M\rangle$, as a result of the mirror-inverting dynamics. Therefore, an almost perfect transmission is achieved with an overall phase ϕ_{ab}, namely $e^{-iHt^*}|\Psi_{ab}\rangle \approx e^{i\phi_{ab}}|\Psi_{ba}\rangle$. The explicit form of ϕ_{ab} follows from the dynamics depicted above with the freedom of setting $\phi_{00} = 0$. For instance,

$$e^{-iHt^*}|\Psi_{10}\rangle = e^{-iHt^*}c_0|\Psi_{00}\rangle \simeq U_{0,N+1}(-t^*)c_{N+1}|\Psi_{00}\rangle = (-1)^{p+1}e^{-i\alpha_N}|\Psi_{01}\rangle \equiv e^{i\phi_{10}}|\Psi_{01}\rangle \quad (2)$$

defines $\phi_{10} = (p + 1)\pi - \alpha_N$ while $\phi_{01} = \phi_{10}$ due to the symmetry of the system. With similar argument we get $\phi_{11} = \pi - 2\alpha_N$. Therefore, the ideal mirror-inverting dynamics defines a quantum gate G between A and B, which reads $G|ab\rangle = e^{i\phi_{ab}}|ba\rangle$ in the computational basis. Independent of the value of α_N when the pair A, B is initially in the state of $|++\rangle$, where $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, the application of the gate G results in a maximally entangled state between A and B. Furthermore, the phase α_N is found to be equal to $\frac{\pi}{2}(N + 1)$.

Since the dynamics is not perfectly dispersionless, $|U_{0,N+1}(t^*)|$ is not exactly 1, gate G is not perfectly unitary. In fact, the dynamics of the qubits is described by a completely positive map, $\rho_{0,N+1}(t) = \mathcal{E}_t[\rho_{0,N+1}(0)]$, which can be written as $\langle i|\rho_{0,N+1}(t)|j\rangle = \sum_{k,l}\mathcal{E}_{ij,kl}(t) \times \langle k|\rho_{0,N+1}(0)|l\rangle$. To quantify the quality of the gate we calculate average gate fidelity $\mathcal{F}_G(t) = \int d\psi\langle\psi|G^\dagger\mathcal{E}_t[|\psi\rangle\langle\psi|]G|\psi\rangle$ where the integration is over all two-qubit pure states. Using the results of [26,27] we get

$$\mathcal{F}_G(t) = \frac{1}{5} + \frac{1}{20}\sum_{i,j,k,l} G_{ik}^*\mathcal{E}_{ij,kl}(t)G_{jl}, \quad (3)$$

where $\mathcal{E}_{ij,kl}(t) = \langle i|\mathcal{E}_t[|k\rangle\langle l|]|j\rangle$ are numerically evaluated.

In Fig. 1(a) we plot the time evolution of the average gate fidelity for a bus of length $N = 100$ initially in its ground state: $\mathcal{F}_G(t)$ displays a marked peak at $t = t^*$. The scaling of the gate fidelity with N is shown in Fig. 1(b) where we remarkably see that $\mathcal{F}_G(t^*)$ exceeds 0.9 even for chains up to $N = 100$ and decays slowly. The inset of Fig. 1(b) shows that our dynamics is fast.

Our dynamical gate works properly for arbitrary initial states of the bus with fixed parity. Ideally after each gate application the parity of the bus remains unchanged making it perfect for reusing. Let us initially set the bus in its ground state and define $F_M(t)$ as the fidelity between the ground state of H_M and the density matrix of the bus at time t. To see how the quality of the gate operation is affected by k subsequent uses of the bus, we compute $\mathcal{F}_G(kt^*)$ and $F_M(kt^*)$ which are shown in Table I for $k = 1, \ldots, 8$ subsequent uses.

Application.—We now propose an application of the above gate mechanism for a scalable neutral atom quantum computer. We consider a network of qubits each encoded in two degenerate hyperfine levels of a neutral atom, cooled and localized in a separate NFFD trap [22]. In Fig. 2(a) we show a single atom confined in a NFFD trap. The position of the minimum of the trapping potential is controlled by varying the aperture radius [22] through microelectromechanical system technology, as proposed in [23]. Local unitary operation on each qubit may be applied through an extra fiber, along with the NFFD trapping fiber [23], as shown in Fig. 2(a). The qubits in the network are connected

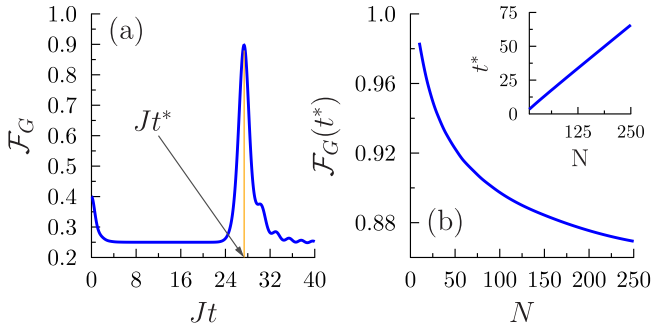

FIG. 1 (color online). (a) Evolution of the average gate fidelity for a chain of $N = 100$ and $J_0 = 0.5J$. (b) $\mathcal{F}_G(t^*)$ as a function of N. The inset shows the optimal time versus N.

TABLE I. $\mathcal{F}_G(kt^*)$ and $F_M(kt^*)$ for k subsequent uses of the bus without resetting ($N = 8$).

k	1	2	3	4	5	6	7	8
$\mathcal{F}_G(kt^*)$	0.984	0.961	0.939	0.918	0.898	0.879	0.861	0.844
$F_M(kt^*)$	0.966	0.926	0.884	0.840	0.795	0.748	0.701	0.654

by a bus realized by cold atoms in an optical lattice, prepared in the Mott insulator regime [16,17], interacting through an effective spin Hamiltonian equation (1) [18]. For the moment we assume that the two qubits interact with the atoms in the ending sites of the lattice, as shown in Fig. 2(b). To switch on the interaction H_I between the qubits and the bus we have to move the minimum of NFFD trapping potential slightly higher such that the qubits move upwards and sit at a certain distance from the ends of the lattice. By controlling such a distance one can tune the interaction coupling to be J_0^{opt}.

Consider now the optical lattice size to be larger than the distance between the qubits A and B [see Fig. 2(c)]. If we simply switch on the interaction between qubits and two intermediate sites (L, R) of the optical lattice, the two external parts of the lattice deteriorate the quality of the gate. To preserve the gate quality we need to separate the bus, extended from L to R, from the rest of the optical lattice. This can be done as shown in Fig. 2(c) by adiabatically shining a localized classical laser beam (with strength Ω and detuning $\Delta \gg \Omega$) on the atoms sitting on sites $L - 1$ and $R + 1$, to drive them off resonance. This effectively generates a Stark shift $\delta E = \Omega^2/\Delta$ between the two degenerate ground states, which can be treated as a local magnetic field in the z direction on sites $L - 1$ and $R + 1$. Keeping Ω/Δ small one can control the strength Ω and detuning Δ such that δE becomes larger than J. When $\delta E \gg J$ the bus is separated from the external parts of the optical lattice. Moreover, since Hamiltonian (1) has a finite gap $\propto J/N$, as δE adiabatically increases the bus moves into its ground state, meanwhile splitting up from the rest. In Fig. 3(a) we plot $F_M(t)$ over the course of adiabatic cutting when the whole lattice is initially in its ground state. At the end of the process the bus results in its ground state and the gate operation can be accomplished as discussed above. After the operation of the gate one may want to glue the previously split optical lattice and bring it back into its ground state. This can be done easily by adiabatically switching off δE as shown in Fig. 3(a).

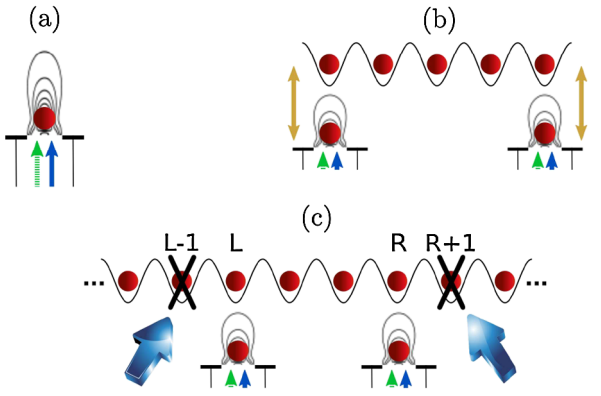

FIG. 2 (color online). (a) Local NFFD trap with two optical fibers, one for trapping (solid blue) and one for unitary single qubit operations (dashed green). (b) Schematic interaction between qubits (local traps) and the ending sites of the bus (optical lattice). (c) Adiabatic cutting of the bus into three parts.

Time scale.—We now give an estimation of t^* in the worst case scenario where A and B sit on the boundary of the lattice, which typically consists of $N \simeq 100$. The typical values for J in optical lattices are few hundred Hertz (e.g., $J = 360h$ Hz in [28]). From the inset of Fig. 1(b) we get $Jt^* \simeq 30$ for $N = 100$ and thus $t^* \simeq 13$ ms which is well below the typical decoherence time of the hyperfine levels ($\simeq 10$ min [29]). Though there are some recent realizations of entangling gate faster than ours [30], they are less versatile as they design a specific, isolated gate which is not part of an extended system. Considering this

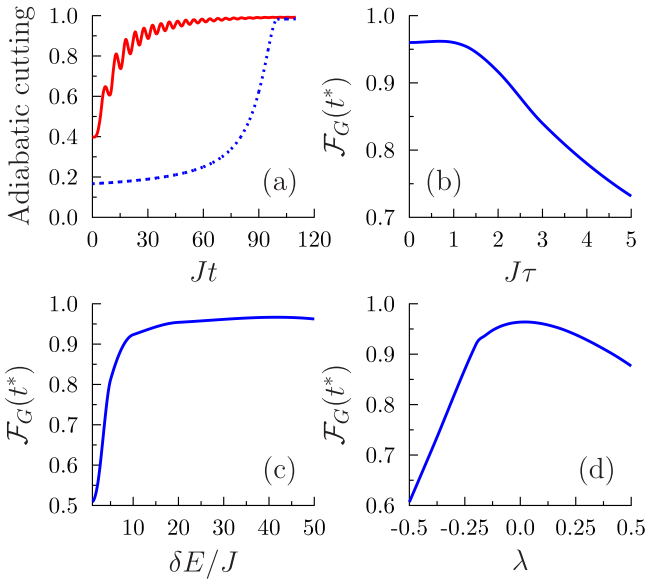

FIG. 3 (color online). (a) Evolution of $F_M(t)$ under the adiabatic cutting (solid red). Evolution of the fidelity between the ground state of the whole optical lattice with the state of the split one under the adiabatic gluing (dashed blue). $\delta E/J$ is linearly varied between 0 and 30 over the time $100/J$. (b) $\mathcal{F}_G(t^*)$ vs switching time τ over which J_0 is linearly switched on from 0 to J_0^{opt}. (c) $\mathcal{F}_G(t^*)$ vs $\delta E/J$ after adiabatic cutting. (d) $\mathcal{F}_G(t^*)$ as a function of anisotropy λ. The length is set to $N = 16$.

latter kind of architecture, our mechanism is much faster than the perturbative methods [5,24], and operates at the time scale of $\mathcal{O}(N/J)$.

Imperfections.—Cold atom systems are usually clean and almost decoherence free; however, in the above proposed setup there might be some sources of destructive effects which may deteriorate the quality of our scheme. In particular, we consider (i) gradual switching of J_0, (ii) imperfect cutting of the chain when δE is not large enough, and (iii) the existence of interaction terms in the Hamiltonian which alter its noninteracting free-fermionic nature. In Fig. 3(b) we show $\mathcal{F}_G(t^*)$ when J_0 is gradually switched on from 0 to J_0^{opt} according to $J_0(t) = J_0^{\mathrm{opt}} t/\tau$, as a function of switching time τ. It is indeed of general relevance that a plateau over which $\mathcal{F}_G(t^*)$ remains constant is observed, even for τ as long as $1/J_0$. In Fig. 3(c) we plot $\mathcal{F}_G(t^*)$ as a function of the energy splitting δE on which the cutting process is based. As it is clear from Fig. 3(c), when $\delta E > 10J$ the bus is well isolated from the external parts. We have also studied the effect of the anisotropy λ, possibly entering H_M and H_I, due to imperfect tuning of laser parameters [18]. In Fig. 3(d) we plot $\mathcal{F}_G(t^*)$ as a function of λ and observe weak deterioration of the gate quality as far as $|\lambda| < 0.2$.

Conclusions.—In this Letter, we have proposed a scalable scheme for realizing a two-qubit entangling gate between arbitrary distant qubits. In our proposal, qubits are made of localized objects which makes single qubit gates affordable. The qubits interact dynamically via an extended unmodulated bus which does not need to be specifically engineered and actively serves to operate as a nonperturbative fast entangling gate. Provided the coupling between the qubits and the bus is properly tuned, the dynamical evolution of the whole system is essentially dispersionless, thus allowing several subsequent uses of the bus without resetting. Surprisingly, a sudden switching of the coupling is not necessary and our fast dynamical gate mechanism is not altered by a possibly gradual switching. Our proposal is general and can be implemented in various physical realizations. Specifically, we have proposed an application based on neutral atom qubits in an array of separated NFFD traps connected by an optical lattice spin chain data bus, which both are accessible to the current technology.

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[1] M. J. Bremner *et al.*, Phys. Rev. Lett. **89**, 247902 (2002).
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[3] D. Jaksch *et al.*, Phys. Rev. Lett. **82**, 1975 (1999).
[4] K. Stannigel *et al.*, Phys. Rev. Lett. **105**, 220501 (2010); G. Burkard and A. Imamoglu, Phys. Rev. B **74**, 041307 (2006).
[5] N. Y. Yao *et al.*, Phys. Rev. Lett. **106**, 040505 (2011); N. Y. Yao *et al.*, arXiv:1012.2864.
[6] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003); A. Bayat and S. Bose, Phys. Rev. A **81**, 012304 (2010).
[7] M. H. Yung and S. Bose, Phys. Rev. A **71**, 032310 (2005); M. H. Yung, S. C. Benjamin, and S. Bose, Phys. Rev. Lett. **96**, 220501 (2006).
[8] S. R. Clark, C. Moura Alves, and D. Jaksch, New J. Phys. **7**, 124 (2005).
[9] A. V. Gorshkov *et al.*, Phys. Rev. Lett. **105**, 060502 (2010).
[10] T. J. Osborne and N. Linden, Phys. Rev. A **69**, 052315 (2004).
[11] C. Albanese *et al.*, Phys. Rev. Lett. **93**, 230502 (2004).
[12] L. Banchi *et al.*, Phys. Rev. A **82**, 052321 (2010).
[13] W. S. Bakr *et al.*, Nature (London) **462**, 74 (2009).
[14] G. K. Brennen *et al.*, Phys. Rev. Lett. **82**, 1060 (1999).
[15] O. Mandel *et al.*, Nature (London) **425**, 937 (2003).
[16] M. Greiner *et al.*, Nature (London) **415**, 39 (2002).
[17] J. F. Sherson *et al.*, Nature (London) **467**, 68 (2010); W. S. Bakr *et al.*, Science **329**, 547 (2010); M. Karski *et al.*, New J. Phys. **12**, 065027 (2010).
[18] L. M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).
[19] P. Medley *et al.*, arXiv:1006.4674.
[20] H. Briegel *et al.*, J. Mod. Opt. **47**, 415 (2000).
[21] J. Cirac and P. Zoller, Nature (London) **404**, 579 (2000).
[22] T. N. Bandi, V. G. Minogin, and S. N. Chormaic, Phys. Rev. A **78**, 013410 (2008).
[23] M. Nakahara, T. Ohmi, and Y. Kondo, arXiv:1009.4426.
[24] A. Wójcik *et al.*, Phys. Rev. A **72**, 034303 (2005).
[25] L. Campos Venuti *et al.*, Phys. Rev. A **76**, 052328 (2007).
[26] P. Zanardi and D. A. Lidar, Phys. Rev. A **70**, 012315 (2004).
[27] M. A. Nielsen, Phys. Lett. A **303**, 249 (2002).
[28] S. Trotzky *et al.*, Phys. Rev. Lett. **105**, 265303 (2010).
[29] J. Bollinger *et al.*, IEEE Trans. Instrum. Meas. **40**, 126 (1991).
[30] T. Wilk *et al.*, Phys. Rev. Lett. **104**, 010502 (2010); L. Isenhower *et al.*, Phys. Rev. Lett. **104**, 010503 (2010).