Thermal States as Universal Resources for Quantum Computation with Always-On Interactions

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Measurement-based quantum computation utilizes an initial entangled resource state and proceeds with subsequent single-qubit measurements. It is implicitly assumed that the interactions between qubits can be switched off so that the dynamics of the measured qubits do not affect the computation. By proposing a model spin Hamiltonian, we demonstrate that measurement-based quantum computation can be achieved on a thermal state with always-on interactions. Moreover, computational errors induced by thermal fluctuations can be corrected and thus the computation can be executed fault tolerantly if the temperature is below a threshold value.

Introduction.—A quantum computer can solve certain problems considered hard for a classical computer with an exponential speedup [1]. Standard quantum computing uses unitary evolution as a basic mechanism for information processing. Another paradigm is measurement-based quantum computing (MBQC), in which one processes quantum information by single-particle operations and measurements only, on a nontrivial entangled state [2]. Such entangled states serve as universal resources of MBQC [3]. The first identified universal resource was the cluster state. It can be obtained as the unique ground state of a Hamiltonian with five-body interactions [4], but can never occur as the unique ground state of any two-body Hamiltonian [5]. Fortunately, there exist universal resources that are the unique ground states of two-body Hamiltonians, albeit with particles of local Hilbert space larger than that of a qubit. These two-body Hamiltonians include the tricluster model [6], an Affleck-Kennedy-Lieb-Tasaki (AKLT)-like model [7], the two-dimensional AKLT model [8,9] and a quadratic Hamiltonian of continuous variables [10]. However, in order to use the ground state of a system as a universal resource, one usually needs to switch off interactions of the system sequentially [6,7,9,11]. Otherwise, the desirable quantum correlations could be destroyed due to the time evolution of the state via interactions. Therefore, in previous proposals, MBQC based on ground states requires not only single-particle operations and measurements but also a good control of interactions. In this Letter, we find that it is possible to remove this extra requirement; i.e., MBQC can be performed with always-on interactions.

To this end, we propose a two-dimensional (2D) system and a three-dimensional (3D) system, whose ground states are universal resources for MBQC. We show that 2D and 3D systems can be generalized to a family of similar models. These spin models may be realized in physical

systems such as cold atoms [12], polar molecules [13], trapped ions [14] and Josephson junction array [15]. We construct a ground state as a universal resource for MBQC by showing that the ground state can be converted into a cluster state by single-particle operations and measurements [9,16]. In practice, one obtains a thermal state instead of the ground state as a universal resource for MBQC. Thus an energy gap is needed to protect the state from thermal fluctuations, which is indeed the case in our model. However, it is not clear how high a temperature could be tolerated before the state would no longer be a universal resource of MBQC. Therefore, we investigate their thermal states and find that computational errors in MBQC induced by thermal fluctuations can be corrected as long as the temperature is below a certain value.

2D system.—The 2D system is shown in Fig. 1, which is a hexagonal lattice with one more particle on each edge. The system is composed of spin-3/2 particles, in which particles on edges are called bond particles, while others are called center particles. Particles are combined by two types of interactions

$$V_{\text{line}} = \Delta(S_{\mathbf{c}}^x A_{\mathbf{b}}^x + S_{\mathbf{c}}^y A_{\mathbf{b}}^y + S_{\mathbf{c}}^z A_{\mathbf{b}}^z), \tag{1}$$

$$V_{\text{dash}} = \Delta(S_{\mathbf{c}}^x B_{\mathbf{b}}^x + S_{\mathbf{c}}^y B_{\mathbf{b}}^y + S_{\mathbf{c}}^z B_{\mathbf{b}}^z), \tag{2}$$

where $S_{\mathbf{c}}^\alpha$ is the spin operator of the corresponding center particle, and $A_{\mathbf{b}}^\alpha$, $B_{\mathbf{b}}^\alpha$ are operators of the corresponding bond particle [17]. Operators of bond particles satisfy commutation relations $[I_{\mathbf{b}}^\alpha, J_{\mathbf{b}}^\beta] = i\delta_{IJ}\epsilon_{\alpha\beta\gamma}I_{\mathbf{b}}^\gamma$ and $\vec{I}_{\mathbf{b}}^2 = 3/4$, where I, $J = A$, B, and α, β, $\gamma = x$, y, z. Therefore, $A_{\mathbf{b}}^\alpha$ and $B_{\mathbf{b}}^\alpha$ are two sets of independent spin-1/2 operators.

The Hamiltonian of the system is $H = \sum_{\mathbf{r}} h_{\mathbf{r}}$, where $h_{\mathbf{r}} = \Delta\vec{S}_{\mathbf{r}} \cdot \vec{I}_{\mathbf{r}}$, $\vec{I}_{\mathbf{r}} = \vec{I}_{\mathbf{r+1}} + \vec{I}_{\mathbf{r+2}} + \vec{I}_{\mathbf{r+3}}$, and **r** denotes
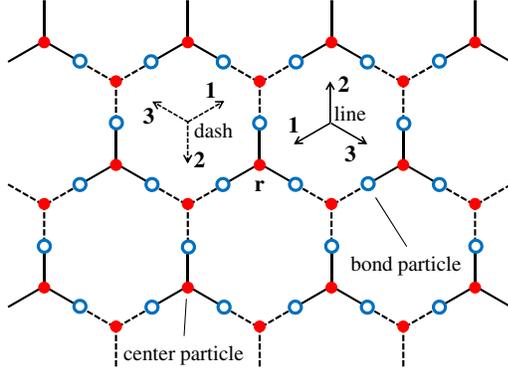
FIG. 1 (color online).   The two-dimensional system composed of spin-3/2 particles. The system is a hexagonal lattice, where center particles (red round) are located on vertices, while bond particles (blue ring) are located on edges. There are two kinds of interactions between center particles and bond particles, V_{line} (line) and V_{dash} (dash line). $\mathbf{r}$ denotes the position of a center particle, and the vectors between the center particle and its three interacting bond particles are $\mathbf{1}$, $\mathbf{2}$, $\mathbf{3}$ respectively.

the position of a center particle, $\mathbf{r} + \mathbf{a}$ denotes the position of one bond particle interacting with the center particle $\mathbf{r}$. Here, $\{\mathbf{a}\}$ depends on $I \in \{A, B\}$ as shown in Fig. 1.

Ground state and energy gap.—We can rewrite $h_{\mathbf{r}}$ as $h_{\mathbf{r}} = \Delta(\vec{T}_{\mathbf{r}}^2 - \vec{S}_{\mathbf{r}}^2 - \vec{I}_{\mathbf{r}}^2)/2$, where $\vec{T}_{\mathbf{r}} = \vec{S}_{\mathbf{r}} + \vec{I}_{\mathbf{r}}$. Here, $\vec{S}_{\mathbf{r}}$, $\vec{I}_{\mathbf{r}}$ and $\vec{T}_{\mathbf{r}}$ all satisfy commutation relations of spin operators. Therefore,

$$h_{\mathbf{r}} = \frac{\Delta}{2}[T_{\mathbf{r}}(T_{\mathbf{r}} + 1) - S_{\mathbf{r}}(S_{\mathbf{r}} + 1) - I_{\mathbf{r}}(I_{\mathbf{r}} + 1)], \quad (3)$$

where $S_{\mathbf{r}} = 3/2$ and $I_{\mathbf{r}} = 1/2$ or $3/2$. When $I_{\mathbf{r}} = 1/2$, $T_{\mathbf{r}} = 1, 2$. When $I_{\mathbf{r}} = 3/2$, $T_{\mathbf{r}} = 0, 1, 2, 3$. One can get the minimum energy by taking $I_{\mathbf{r}} = 3/2$ and $T_{\mathbf{r}} = 0$, which means the ground state, $|g\rangle_{\mathbf{r}}$, of $h_{\mathbf{r}}$ has a total spin 0. The energy difference between the ground state and the first excited state is Δ. Because these $h_{\mathbf{r}}$ are independent with each other, the ground state of the whole system is $|G\rangle = \bigotimes_{\mathbf{r}} |g\rangle_{\mathbf{r}}$ and protected by an energy gap Δ. The energy gap only depends on the interaction constant, and does not vanish in the thermodynamic limit.

POVM and GHZ state.—As the first step of MBQC on the ground state, the POVM $\mathbf{I} = \sum_{\alpha=x,y,z} F^{\alpha\dagger} F^{\alpha}$ is performed on center particles. Here, $F^{\alpha} = (S_{\mathbf{r}}^{\alpha 2} - 1/4)/\sqrt{6}$, which projects the center spin into the subspace spanned by two states with maximum spin component in the α direction. Because the ground state $|g\rangle_{\mathbf{r}}$ has a total spin 0, all three spin-$I_{\mathbf{r}+\mathbf{a}}$ are antiparallel with the center spin $S_{\mathbf{r}}$. Therefore, the POVM projects the state $|g\rangle_{\mathbf{r}}$ into a GHZ state, e.g., for the outcome z, the output state is $|\text{GHZ}\rangle_{\mathbf{r}} = (|\tilde{0}000\rangle + |\tilde{1}111\rangle)/\sqrt{2}$, where $|\tilde{0}\rangle = -|S_{\mathbf{r}}^z = 3/2\rangle$, $|\tilde{1}\rangle = |S_{\mathbf{r}}^z = -3/2\rangle$ are the state of the center spin, and $|0\rangle$ ($|1\rangle$) is the eigenstate of $I_{\mathbf{r}+\mathbf{a}}^z$ with eigenvalue $-1/2$ ($1/2$). The state $|g\rangle_{\mathbf{r}}$ is an isotropic state. Therefore, all outcomes are equivalent to the outcome z up to a set of single-particle

operations $U(\hat{\alpha}) = \exp[i\vec{T} \cdot \vec{n}(\hat{\alpha})]$, where α is the outcome and $n(\hat{\alpha}) = \hat{\alpha} \times \hat{z} \arcsin(|\hat{\alpha} \times \hat{z}|)/|\hat{\alpha} \times \hat{z}|$. Then, the state of the whole system after POVMs and single-particle operations is $|\{\text{GHZ}\}\rangle = \bigotimes_{\mathbf{r}} |\text{GHZ}\rangle_{\mathbf{r}}$, which can also be described by a set of stabilizers, $W_{\mathbf{r}} = X_{\mathbf{r}} \prod_{\mathbf{a}=1,2,3} 2I_{\mathbf{r}+\mathbf{a}}^x$ and $W_{\mathbf{r},\mathbf{r}+\mathbf{a}} = 2Z_{\mathbf{r}} I_{\mathbf{r}+\mathbf{a}}^z$ for all $\mathbf{r}$ and $\mathbf{a}$. $|\{\text{GHZ}\}\rangle$ is the eigenstate with eigenvalue 1 of all of these stabilizers. Here, X, Y and Z are Pauli operators of the qubit $\{|\tilde{0}\rangle, |\tilde{1}\rangle\}$.

Cluster state and universality of the ground state.—By measuring physical quantities $A_{\mathbf{b}}^x B_{\mathbf{b}}^z$ and $A_{\mathbf{b}}^z B_{\mathbf{b}}^x$ on bond particles, the state $|\{\text{GHZ}\}\rangle$ can be projected [18,19] (or "fused") into a hexagonal cluster state, which has the same lattice with center particles. Eigenstates of $A_{\mathbf{b}}^x B_{\mathbf{b}}^z$ and $A_{\mathbf{b}}^z B_{\mathbf{b}}^x$, which are measurement basis, can be found in Ref. [17]. Considering a product of stabilizers

$$W_{\mathbf{r}} \prod_{\mathbf{a}=1,2,3} W_{\mathbf{r}+2\mathbf{a},\mathbf{r}+\mathbf{a}} = X_{\mathbf{r}} \prod_{\mathbf{a}=1,2,3} Z_{\mathbf{r}+2\mathbf{a}} (4A_{\mathbf{r}+\mathbf{a}}^x B_{\mathbf{r}+\mathbf{a}}^z), \quad (4)$$

one can get a new stabilizer by replacing $A_{\mathbf{b}}^x B_{\mathbf{b}}^z$ with outcomes. In Eq. (4), we have taken the case $I_{\mathbf{r}+\mathbf{a}}^x = A_{\mathbf{r}+\mathbf{a}}^x$ as an example, and the result is the same for $I_{\mathbf{r}+\mathbf{a}}^x = B_{\mathbf{r}+\mathbf{a}}^x$. The new stabilizers define a hexagonal cluster state on center particles up to a Pauli frame, which can be corrected by single-particle operations [2]. The hexagonal cluster state is a universal resource for MBQC [3]. Then, universal MBQC can be performed on center particles.

3D system and topology protected cluster state.—Following the idea of 2D system, we propose a 3D system, whose ground state is also a universal resource for MBQC. The system is shown in Fig. 2(a), which is composed by spin-2 particles and spin-3/2 particles, where center particles are spin-2 particles and bond particles are spin-3/2 particles. The interactions between particles are the same as Eq. (1) and (2). Therefore, the Hamiltonian of the 3D system has the same form as 2D system, $H = \sum_{\mathbf{r}} h_{\mathbf{r}}$, $h_{\mathbf{r}} = \Delta \vec{S}_{\mathbf{r}} \cdot \vec{I}_{\mathbf{r}}$, where $\vec{I}_{\mathbf{r}} = \vec{I}_{\mathbf{r}+1} + \vec{I}_{\mathbf{r}+2} + \vec{I}_{\mathbf{r}+3} + \vec{I}_{\mathbf{r}+4}$. Here, $\{\mathbf{r} + \mathbf{a}\}$ denote four bond particles interacting with the center particle $\mathbf{r}$.

In the 3D system, one can get the minimum energy of $h_{\mathbf{r}}$ by taking $I_{\mathbf{r}} = 2$ and $T_{\mathbf{r}} = 0$ in Eq. (3). Therefore, in the 3D system, the ground state of each $h_{\mathbf{r}}$ is an isotropic state with a total spin 0. The energy difference between the ground state and the first excited state is Δ, which means the 3D system is also gapped.

The ground state of the 3D system can be reduced to a 3D cluster state, as shown in Fig. 2(b). Firstly, center particles are measured as $\mathbf{I} = \sum_{k=1}^{7} F^{\dagger}(\hat{\alpha}_k) F(\hat{\alpha}_k)$, which is a POVM with seven outcomes. Here, $F(\hat{\alpha}_k) = \sqrt{N_k} P(\hat{\alpha}_k)$, and

$$P(\hat{\alpha}) = |\hat{\alpha}; 2\rangle\langle\hat{\alpha}; 2| + |\hat{\alpha}; -2\rangle\langle\hat{\alpha}; -2| \quad (5)$$

projects the center spin into the subspace spanned by two states with maximum spin component in the $\hat{\alpha}$ direction. $|\hat{\alpha}; m\rangle$ is the eigenstate of $\hat{\alpha} \cdot \vec{S}_{\mathbf{c}}$ with eigenvalue m.

$N_k = 1/3$ for $k \leq 3$ and $N_k = 3/8$ for $k \geq 4$. The seven directions are shown in Fig. 2(c) and 2(d). Because four spins $\{I_{\mathbf{r}+\mathbf{a}}\}$ are all antiparallel with the center spin-$S_{\mathbf{r}}$, the output states of the POVM are GHZ states. These GHZ states are equivalent to the GHZ state of outcome z, up to single-particle operations $U(\hat{\alpha})$. Therefore, POVMs on center particles, with $U(\hat{\alpha})$ together, can transform the ground state to a state stabilized by $W_{\mathbf{r}} = X_{\mathbf{r}}\prod_{\mathbf{a}}2I_{\mathbf{r}+\mathbf{a}}^x$ and $W_{\mathbf{r},\mathbf{r}+\mathbf{a}} = 2Z_{\mathbf{r}}I_{\mathbf{r}+\mathbf{a}}^z$, where $\mathbf{a} = 1, 2, 3, 4$.

Measuring physical quantities $A_{\mathbf{b}}^x B_{\mathbf{b}}^z$ and $A_{\mathbf{b}}^z B_{\mathbf{b}}^x$, one can generate a new set of stabilizers $X_{\mathbf{r}}\prod_{\mathbf{a}}Z_{\mathbf{r}+2\mathbf{a}}$, which defines a 3D cluster state on center particles, as shown in Fig. 2(b). On the 3D cluster state, quantum correlations are protected topologically and fault tolerant quantum computing can be simulated using topological error correction [20].

Thermal computational errors and error correction.— We have proved the ground states of 2D and 3D systems are universal resources for MBQC. However, in practice, a system cannot reach the exact ground state, but rather a thermal state at finite temperature. Thermal fluctuations can reduce the quantum correlations on ground states and induce computational errors on the cluster state, which will be used for MBQC. The thermal state is the Gibbs state $\rho = Z^{-1}e^{-\beta H}$, where $Z = \mathrm{tr}e^{-\beta H}$, $\beta = 1/T$ is the temperature, and ρ can be rewritten as $\rho = \prod_{\mathbf{r}}\rho_{\mathbf{r}}$. Here $\rho_{\mathbf{r}} = Z_{\mathbf{r}}^{-1}e^{-\beta h_{\mathbf{r}}}$ is the Gibbs state of $h_{\mathbf{r}}$. After the POVM and $U(\hat{\alpha})$, the state $\rho_{\mathbf{r}}$ is transformed into
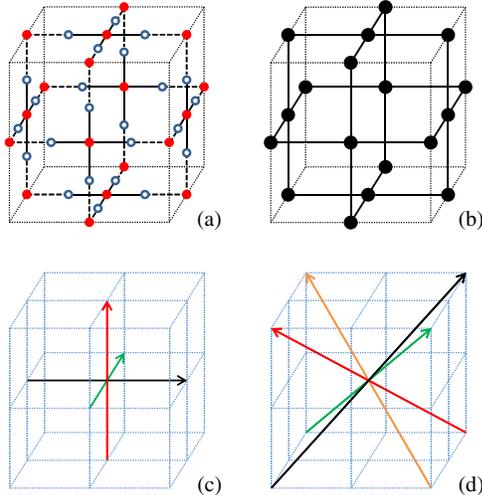

FIG. 2 (color online). The three-dimensional system. (a) The elementary cell of the system. The system is composed of spin-2 particles and spin-3/2 particles. Spin-2 particles are center particles (red round), and spin-3/2 particles are bond particles (blue ring). (b) The elementary cell of the three-dimensional topology protected cluster state, which can be prepared by single-particle operations and measurements on the ground state of the 3D system. (c) Three directions for $k \leq 3$ of the POVM on spin-2 particles, which are orthogonal with each other and passing face centers of a cube. (d) Four directions for $k \geq 4$ of the POVM on spin-2 particles, which are along body diagonals of the same cube.

$\sigma_{\mathbf{r}} = F\rho_{\mathbf{r}}F^{\dagger}/\mathrm{tr}(F\rho_{\mathbf{r}}F^{\dagger})$, where $F = F^z$ for the 2D system and $F = F(\hat{z})$ for the 3D system. At an absolute zero temperature, $\sigma_{\mathbf{r}} = |\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|_{\mathbf{r}}$ is the desired GHZ state. Here, $|\mathrm{GHZ}\rangle_{\mathbf{r}}$ is a GHZ state of four qubits for the 2D system and five qubits for the 3D system.

The post-POVM state $\sigma_{\mathbf{r}}$ at finite T is only approximately a GHZ state, i.e., is equivalent to a perfect GHZ state affected by errors. The probability of an error occurring on the post-POVM state is $\epsilon = 1 - \mathcal{F}$, where $\mathcal{F} = \mathrm{tr}(\sigma_{\mathbf{r}}|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|_{\mathbf{r}})$ is the fidelity of the GHZ state, as shown in Fig. 3. Those errors are propagated under the measurements of the bond particles and subsequent correction operations [21]. The resulting error superoperators act on the yet unmeasured center particles, and have the following properties: (i) there is one independent error superoperator $\mathcal{E}_{\mathbf{r}}$ for every $\mathbf{r}$, (ii) $\mathcal{E}_{\mathbf{r}}$ acts at the locations $\mathbf{r}$ and $\{\mathbf{r} + 2\mathbf{a}, \forall\mathbf{a}\}$. Where the center particles are measured in the X-basis for the purpose of topological error correction on the 3D cluster state (in most of the cluster), there arise two further simplifications: (iii) All errors are equivalent to Z errors or the identity, and (iv) Correlations between errors on neighboring center particles can be discarded. The latter arises because errors at $\mathbf{r}$ and at $\mathbf{r} + 2\mathbf{a}$ are corrected by different error-correction procedures running independently of another [20].

On the 3D cluster state, for each $\mathbf{r}$, the resulting error is $\mathcal{E}_{\mathbf{r}} = E_1 \circ E_2$, with $E_1 = (1 - p_1) + p_1[Z_{\mathbf{r}}]$ and $E_2 = (1 - p_2 - p_3) + p_2/4\sum_{\mathbf{a}}[Z_{\mathbf{r}+2\mathbf{a}}] + p_3/6\sum_{\mathbf{a},\mathbf{a}'}[Z_{\mathbf{r}+2\mathbf{a}}Z_{\mathbf{r}+2\mathbf{a}'}]$. Therein, the error probabilities p_1, p_2 and p_3 depend on the temperature T. If $p_3 \ll p_1$, $p_2 \ll 1$, then the local errors are almost independent and the error level is described by an effective local error probability $p \simeq p_1 + p_2 + 2p_3$. Error-correction is possible if
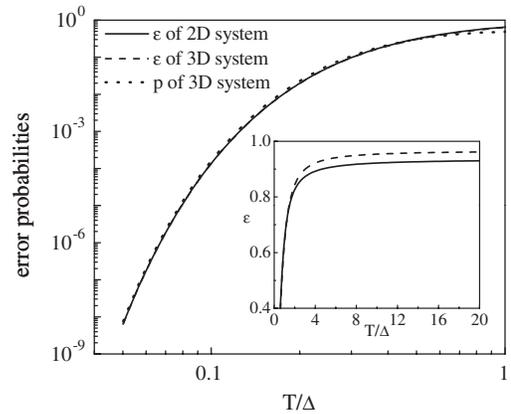

FIG. 3. Error probabilities ϵ on a GHZ state and p on the cluster state as functions of the temperature. For 2D system the GHZ state is a four-qubit state, and for 3D system the GHZ state is a five-qubit state. ϵ are almost the same for 2D and 3D systems when the temperature $T < \Delta$. When $T/\Delta = 0.2$, $p = 3\%$, which means errors induced by a lower temperature are tolerable by using the topological error-correction algorithm on the 3D system.

$p < 3\%$ [22], which translates into a threshold temperature $T_t = 0.2\Delta$; See Fig. 3. At that point, p_1, $p_2 \sim 10^{-2}$ and $p_3 = 10^{-6}$, justifying the assumption of uncorrelated local errors.

MBQC with always-on interactions.—In practical application, one can convert the initial state, usually a thermal state, to a cluster state one qubit at a time. Once we need the qubit **r**, we can apply POVMs on the center particle **r** and its neighboring center particles $\{\mathbf{r} + 2\mathbf{a}\}$. Based on outcomes of POVMs, single-particle operations $U(\hat{a})$ are chosen. Then, bond particles $\{\mathbf{r} + \mathbf{a}\}$ are measured, and outcomes are used to correct the Pauli frame of qubit **r**. No further operation is needed on any other particle in order to convert the center particle **r** to a qubit on the cluster state.

With always-on interactions, we need to consider the time evolution driven by the time-independent Hamiltonian. Since the initial state is not converted to the cluster state simultaneously, there are some untouched particles. They remain in the initial thermal state, which is close to the ground state due to the existence of the energy gap. Other particles evolve with the Hamiltonian, and their quantum correlations will be changed under time evolution. Fortunately, the time evolution is periodic with a period $4\pi/\Delta$ for the 2D system and $2\pi/\Delta$ for the 3D system. Then, one can perform operations on these particles at the revival time of quantum correlations, $t = 4n\pi/\Delta$ and $t = 2n\pi/\Delta$ for 2D and 3D systems, respectively, where $t = 0$ is the time of the first operation on the particle and its interaction particles, and $n = 0, 1, 2, \ldots$. If we assume only one operation can be performed on each particle at one revival time, particles can be measured out before $n = 6$. Therefore, the MBQC can be performed on our proposed systems with always-on interactions. Here, operations are required to occur precisely. We remark that errors due to timing imprecision can also be analyzed similarly to thermal errors.

Discussion.—In summary, we proposed a 2D and a 3D gapped system, whose ground state is entangled based on a factorized Hamiltonian. With a factorized Hamiltonian, quantum computing can be performed without the need to switch off interactions. The ground state can be reduced to a deterministic cluster state, in contrast to AKLT like systems where cluster states are obtained with stochastic structures. Errors induced by thermal fluctuations can be corrected as long as the temperature is below a critical threshold. There are other choices of $A_\mathbf{b}^\alpha$ and $B_\mathbf{b}^\alpha$ that satisfy conditions of spin-1/2 operators. By replacing the center particle with different spin systems, i.e., spin-$m/2$ particles, one can get different spatial connectivities that each qubit is connected to m other qubits in the cluster state. Thereby, it can be generalized to 3D and more complicated configurations.

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[17] $A_\mathbf{b}^x = (S_\mathbf{b}^{-2} + S_\mathbf{b}^{+2})/4\sqrt{3}$, $A_\mathbf{b}^y = i(S_\mathbf{b}^{-2} - S_\mathbf{b}^{+2})/4\sqrt{3}$, $A_\mathbf{b}^z = S_\mathbf{b}^z(13 - 4S_\mathbf{b}^{z2})/12$ and $B_\mathbf{b}^x = (S_\mathbf{b}^x + 4S_\mathbf{b}^z S_\mathbf{b}^x S_\mathbf{b}^z)/4\sqrt{3}$, $B_\mathbf{b}^y = (S_\mathbf{b}^y + 4S_\mathbf{b}^z S_\mathbf{b}^y S_\mathbf{b}^z)/4\sqrt{3}$, $B_\mathbf{b}^z = S_\mathbf{b}^z(4S_\mathbf{b}^{z2} - 7)/6$. The eigenstate with $A_\mathbf{b}^x B_\mathbf{b}^z = (1 - 2\mu)/4$ and $A_\mathbf{b}^z B_\mathbf{b}^x = (1 - 2\nu)/4$ is $|\mu, \nu\rangle = (|3/2\rangle + i^{2\nu}|1/2\rangle + i^{2\mu}| - 1/2\rangle - i^{2\mu}i^{2\nu}| - 3/2\rangle)/2$, where μ, $\nu = 0$, 1 and $S_\mathbf{b}^z|m\rangle = m|m\rangle$.

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