Submitted to the BiNs Workshop to be held on 22-23 JULY 2004 IN SEOUL, KOREA

EQUILIBRIUM ANALYSIS OF TRIP CHAINS IN CONGESTED NETWORKS

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ABSTRACT

In this paper, we develop a model of travel in a chain of trips joining several locations through a congested network. We develop a microscopic analysis of individual benefits obtained by spending time at each of the locations and costs incurred through travel between them. This is combined with a macroscopic equilibrium model of travel during congested peak periods to show how individuals' travel choices are influenced by the congestion that result from corresponding choices made by others. We show how different travellers can achieve identical net utilities by making different combinations of choices within the equilibrium. The resulting model can be used to investigate the effect on travel behaviour and individual utility of various transport interventions, and we illustrate this by considering the effect of a peak-period charge that eliminates congestion.

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1. Introduction

The decision entailed in undertaking a trip from one location to another is influenced by the benefits that would be gained by remaining at the origin, the conditions that are encountered in the making the trip, and the benefits gained through arriving at the destination. A traveller who undertakes a trip will generally do so because the benefits of reaching the destination outweigh the losses associated with leaving the origin and the costs of making the trip. In making a trip, the traveller will use a public access transport system of a kind that can become congested at peak times. Thus the collective behaviour of travellers influences the conditions experienced by each of them in travelling.

From the point of view of the individual travellers, the requirement to travel arises from the range of locations at which different activities can be undertaken. This view of travel leads to a microscopic analysis of trip-making behaviour by individuals. On the other hand, the collective effect of travellers can be to cause congestion that will impact on the travellers themselves through increased journey times and decreased convenience of travel. This view leads to a macroscopic analysis of travel in congested networks.

In the present paper, we consider how these two distinct approaches to the analysis of travel behaviour can be combined into a self-consistent model. This brings a utility-based analysis of trip chains that describes the requirement to travel together with an equilibrium-based analysis of the choices that are available to travellers, and in particular those of departure-time and route.

According to this model, benefits are obtained through time spent at the different locations, whilst costs are incurred through travel between them. This represents the role of travel as a means to the end of gaining access to facilities at a range of locations and hence emphasises its nature as a derived demand. In this model, we represent the benefit of attendance in the form of utility that depends on timing and duration of attendance. The resulting trip chaining analysis is based upon consideration of the individuals' requirements for travel, and is framed in terms of the benefits that they gain through having travelled. We represent the cost of travel between locations, including congestion delays that are incurred as a consequence of travel during peak periods, in the form of travel time. The present approach combines these elements in a single framework, for which we present analytical results. We apply this analysis to a simple example and show how in equilibrium, different travellers can achieve identical net utility through different combinations of utility and travel cost by scheduling their travel at different times.

2. A model of trip making

2.1 Introduction

We consider a population of travellers that is homogeneous in respect of their travel needs and their trip making decisions. Suppose that each of these travellers undertakes a chain of trips that starts and ends at the same location (home) and visits a series of locations that we take in the first instance to be predetermined. The timing of these trips depends on the timeliness of attendance at each of the origin and the destination locations in respect of the benefits that accrue to the individuals. The duration of each trip depends on the traffic conditions that are encountered, and can be estimated by use of a traffic model. We suppose that travel between locations is undertaken when the benefit of attendance at the destination surpasses that of remaining at the origin when allowance is made for the time and cost of travel. The timing of the trip is then a resolution of the tension between the benefits for the individual of being at the origin and at the destination. This is balanced against the cost of travel through the network, which varies according to the congestion caused collectively by travellers. Hence the departure rates and consequent levels of congestion in the network are endogenous to the present analysis.

2.2 Analysis of trip-making

Consider a single trip j $(1 \le j \le J)$ made by a traveller that forms part of the day-long chain of J trips. Suppose that the traveller departs from location $j - 1$ on trip j at time s_j and consequently arrives at location j at time $\tau_j(s_j)$ so that the duration of this trip is $\tau_j(s_j) - s_j$. The arrival time for the trip is determined from the departure time by use of a traffic model according to the conditions that are encountered. The total travel time during a trip chain of this kind is then $\Sigma_j [\tau_j(s_j) - s_j]$.

We suppose that the time from t to s spent at location j confers a benefit to an individual, which we represent as $f_j(t, s)$. For convenience, we express this in terms equivalent to savings in travel time. This benefit can depend separately and jointly on each of the start time t, the duration of attendance $s - t$, and the end time s. In a trip chain, the start time t at location j is given by the arrival time $\tau_j(s_j)$ of journey j, and the end time s is given by the departure time s_{j+1} of journey $j+1$. Thus the benefit derived from attendance at location j is $f_j[\tau_j(s_j), s_{j+1}]$ and the benefit accumulated during a trip chain of this kind is $\Sigma_j f_j[\tau_j(s_j), s_{j+1}]$.

The net benefit to an individual of undertaking a trip chain of this kind with departure times $\mathbf{s}$ is then

$$V(\mathbf{s}) = f_0[0, s_1] + \sum_{j=1}^{J} f_j[\tau_j(s_j), s_{j+1}] - [\tau_j(s_j) - s_j] \tag{1}$$

where by convention we set $s_{J+1} = 24$ h so that the final part of the day is spent at location J.

Variations in departure time s_j will affect the benefit that is obtained at the origin location $j - 1$, the duration of the journey, and the arrival time at the destination location and hence the benefit obtained there.

2.3 Analysis of network equilibrium

Suppose that the departure rate of individuals on trip j $(1 \le j \le J)$ at time s is $e_j(s)$. In equilibrium, the value $V(\mathbf{s})$ achieved by each individual is identical – otherwise, some would have an incentive to change their departure times. Following Heydecker and Addison (2004), we note that while the departure rate is non-zero, the value of net utility V is invariant with respect to time so that

$$e_j\left(s_j\right) > 0 \quad \Rightarrow \quad \frac{\partial V}{\partial s_j} = 0 \quad \left(1 \le j \le J\right) \tag{2}$$

Let the partial derivatives of the functions $\mathbf{f}$ be $\partial f_j(t, s)/\partial t = f_j^1$ and $\partial f_j(t, s)/\partial s = f_j^2$. We note that whenever marginal increases in attendance at location j confer benefits, $f_j^1 \le 0$ (for greater benefit with earlier arrival) and $f_j^2 \ge 0$ (for greater benefit with later departure). Then we can express the equilibrium condition as

$$e_j\left(s_j\right) > 0 \quad \Rightarrow \quad \left(f_{j-1}^2\left(s_j\right) + f_j^1\left[\tau_j\left(s_j\right)\right]\dot{\tau}_j\left(s_j\right)\right) - \dot{\tau}_j\left(s_j\right) + 1 = 0 \quad \left(1 \le j \le J\right). \tag{3}$$

Rearranging this gives

$$e_j\left(s_j\right) > 0 \quad \Rightarrow \quad \dot{\tau}_j\left(s_j\right) = \frac{1 + f_{j-1}^2\left(s_j\right)}{1 - f_j^1\left[\tau_j\left(s_j\right)\right]} \quad \left(1 \le j \le J\right). \tag{4}$$

Now flow propagation (see, for example, Heydecker and Addison, 1996) on trip j ($1 \le j \le J$) means that the arrival rate $g_j(t)$ at location j satisfies

$$e_j\left(s_j\right) = g_j\left[\tau_j\left(s_j\right)\right]\dot{\tau}_j\left(s_j\right) . \tag{5}$$

Thus the equilibrium departure profile $e_j(s)$ from location j–1 ($1 \le j \le J$) is generated by the arrival profile $g_j(t)$ at location j using the flow propagation relationship (5) together with the invariance relationship (4) as:

$$e_j\left(s_j\right) = \left(\frac{1 + f_{j-1}^2\left(s_j\right)}{1 - f_j^1\left[\tau_j\left(s_j\right)\right]}\right) g_j\left[\tau_j\left(s_j\right)\right] \quad \left(1 \le j \le J\right) . \tag{6}$$

The arrival rate profile $g_j(t)$ at location j can be found from a suitable traffic model together with knowledge of departure profiles $e_j(s)$ from location j - 1 at times for which $\tau(s) \le t$: Mun (2001) has investigated the suitability of models for this purpose.

In order for travel on journey j to be confined to a bounded interval of time, either the marginal value of attendance at the origin should decrease over time, or the marginal value of attendance at the destination should increase over time, or both of these. Equilibrium is achieved through variations in travel time between origin and destination that are complementary to these variations in utility.

2.4 Volume of travel

The volume of travel that takes place on each trip j is determined as the time integral of the departure rate. Travel starts when the net utility V of a trip made in uncongested conditions rises to the equilibrium value. Travel then continues until the net utility of further trips falls below the equilibrium value, even when the network is uncongested. Thus the volume E_j of travel of journey j is given by $E_j = \int_s e_j(s)\, ds \ (1 \le j \le J)$, where the integrand has non-zero value only within a certain departure time interval for which travel is possible at the equilibrium cost. Thus the start time of the interval during which travel takes place determines the end time, and together with the inflow profile they determine the volume. For conservation of flow along the trip chain, we require that the volume E_j be equal for all trips $j \ (1 \le j \le J)$.

The present model thus provides a relationship between the volume of trips made and the net utility V achieved through making them, as given by (1). We expect that as the volume of travel increases, so congestion will increase travel costs and also reduce the time available to gain benefit from attendance at the locations, so that utility decreases on each of these grounds. This relationship can be used in conjunction with a demand function $D(V)$ to establish a demand-performance equilibrium in which

$$E_j = D(V) \ (1 \le j \le J) \tag{7}$$

where V is given by (1).

3. Example calculations

By way of example, we consider a two-tip tour from home to work and back. We suppose that the marginal value of time at home is high early on during the day, and then falls to a constant value near the start of the morning peak period, representing a reluctance to depart too early: this is illustrated in Figure 1. Similarly, we suppose that there is a premium on attendance at work during core hours, and that some reward is given for flexible working beyond those: a marginal value of time at work that achieves this is illustrated in Figure 2.

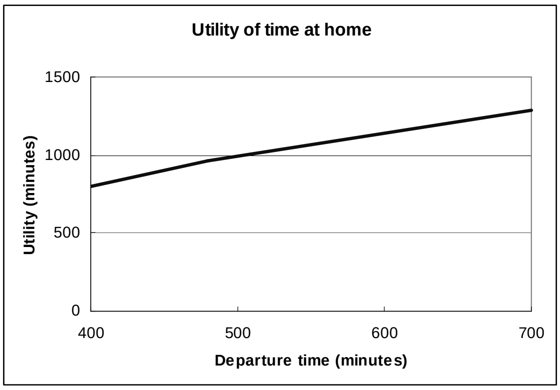


Figure 1: Utility of time at home

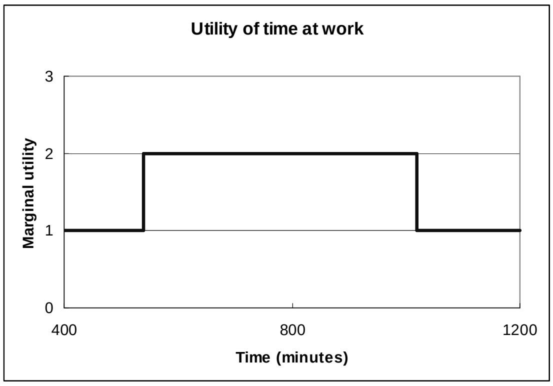


Figure 4: Marginal wage rate at work

Let the marginal value of time at home, expressed in units of travel time saved, be 2.0 before 08:00 and 1.5 after that. Thus the utility $f_0(s)$ of remaining home until time s is $f_0(s) = 1.5\, s + 0.5\, \mathrm{Min}(s,\ s_0)$, where $s_0 = 8$ h . Suppose that the value of time at work is determined by three elements: the time of arrival, the duration of stay, and the time of departure. Let the wage rate, expressed in units of equivalent travel time saved, be 1.0 outside the core hours of 09:00 to 17:00, and 2.0 within these core hours. Thus the benefit $f_1(t, s)$ of working from time t to time s is given by $f_1(t, s) = s - t + \mathrm{Min}(s, s_1) - \mathrm{Max}(t, t_1)$, where $s_1 = 9$ h and $t_1 = 17$ h . Because time at home is valued at 1.5 throughout the day after 08:00, the return journey from work to home is influenced by that.

According to this specification, we have the following relationships for the marginal utilities that affect the two journeys:

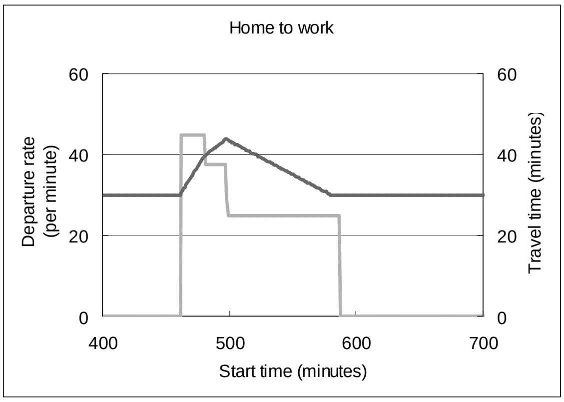
$$f_0^2(s) = \begin{cases} 2.0 & s < s_0 \\ 1.5 & s > s_0 \end{cases},$$

$$f_1^1(t) = \begin{cases} -1.0 & t < t_1 \\ -2.0 & t > t_1 \end{cases} \text{ and } f_1^2(s) = \begin{cases} 2.0 & s < s_1 \\ 1.0 & s > s_1 \end{cases}, \text{ and}$$

$$f_2^1(t) = -1.5\,. \tag{8}$$

We suppose that the free-flow travel time for each journey is 30 minutes, and that the capacity of the network in each direction is 1800 vehicles/h .

When the start time of departures from home to work is 07:40 (460 minutes), the end time of departures is 09:40 (580 minutes) and the total volume of travel is 3,600 trips. In order to achieve the same volume for the return journey, departures from work to home start at time 16:00 (960 minutes), and end at time 18:00 (1,080 minutes). The departure profiles and consequent travel times that achieve equilibrium are shown in Figures 3 and 4: the initial peak in the profile of departures from home is a consequence of the higher marginal value of time at home before 08:00 (480 minutes).



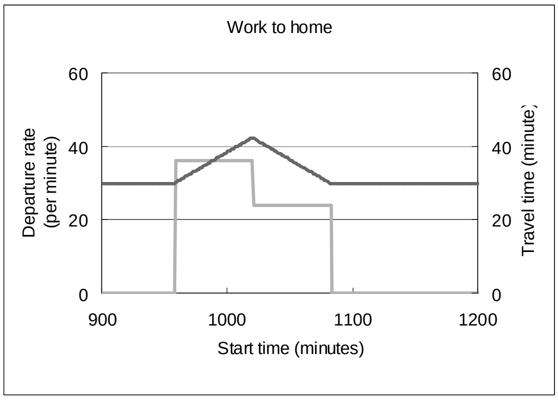


Figure 3: Journey from home to work

Figure 4: Journey from work to home

The travel time for departure from home at 07:40 is 30 minutes, and the utility of having remained home until that time is equivalent to having saved 920 minutes of travel time. An individual who departs at this time will then arrive at work at 08:10: if they remain only until the time of the first departure from work to home, at 16:00, they will gain utility equivalent to a saving of 890 minutes of travel time. Departing for home at that time gives a travel time of 30 minutes, resulting in arrival at home at time 16:30 where they will gain a further 675 minutes of utility during the remainder of the day. The net utility for this individual is then the sum of the two home utilities plus the work utility minus the two travel costs, giving 2,425 minutes, which will be identical for all travellers in this homogeneous group. These results are summarised in Table 1.

Table 1: Utility obtained by an individual departing at the start of the morning peak

	Home		Travel	Work			Travel	Home	
	Duration (minutes)	Depart	Duration (minutes)	Arrive	Duration (minutes)	Depart	Duration (minutes)	Arrive	Duration (minutes)
Time	460	07:40	30	08:10	470	16:00	30	16:30	450
Utility	920		-30		890		-30		675

Consider now another individual who departs from home at the most congested time of 08:16 (496 minutes) and will arrive at work at 09.00 after a travel time of 44 minutes. Because in this example each of the marginal utilities (8) of attendance does not depend on other start and end times, the journey from home to work can be equilibrated separately from that from work to home. Compared with the earliest departing individual, this one will gain an equivalent of 64 minutes of additional utility at home, but will lose 14 minutes through increased travel time and a further 50 equivalent minutes of wages through later arrival. The results of this variation in departure time from home are shown in Table 2. This illustrates that in equilibrium, different individuals achieve identical net utilities through different interrelated combinations of costs and benefits by timing their journeys and activities differently.

Table 2: Utility obtained by an individual departing at the height of the morning peak

	Home		Travel	Work			Travel	Home	
	Duration (minutes)	Depart	Duration (minutes)	Arrive	Duration (minutes)	Depart	Duration (minutes)	Arrive	Duration (minutes)
Time	496	08:16	44	09:00	420	16:00	30	16:30	450
Utility	984		-44		840		-30		675

Finally, suppose that a time-varying charge is levied for travel during the morning peak, and that this is calculated to achieve the same arrival profile but without any congestion. In order to do this, the travel time should be identical for all travellers and the total utility maintained at the same constant value through variations in the charge. For a traveller who departs home at 08:16, the travel time of 30 minutes will lead to arrival at 08:46. Compared with the case shown in Table 2, this traveller will then save 14 minutes travel cost and consequently gain 14 minutes utility as payment for employment: the charge required to render this in equilibrium is then equivalent to 28 minutes of travel time rather than the 14 minutes that are saved. The results of these calculations are shown in Table 3. In this case, the individual has a shorter journey and also earns more at work, but passes both of these benefits on through the congestion charge.

Table 3: Utility obtained by an individual under congestion-eliminating charging

	Home		Travel	Work			Travel	Home	
	Duration (minutes)	Depart	Duration (minutes)	Arrive	Duration (minutes)	Depart	Duration (minutes)	Arrive	Duration (minutes)
Time	496	08:16	30	08:46	434	16:00	30	16:30	450
Utility	984		-30		854		-30		675

4. Summary

The present model provides a representation of the way in which travellers choose their departure times in trip chains according to the benefits that they gain through attendance at different locations and the travel conditions that they encounter between them. Choices made by individuals can, in equilibrium, include a range of possible balanced combinations of costs of travel and benefits obtained in attending the locations. We have shown how this can be used to investigate the effects of changes that might be made to travel provision (for example, to the free-flow travel time, capacity or monetary charges made for use of the network) and to the marginal utilities of attendance at the locations. The influence of changes of these kinds on trip making behaviour can then be explored by using the present model in conjunction with a demand relationship as part of a demand-performance equilibrium model.

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