A practical system for improved efficiency in frequency division multiplexed wireless networks

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Abstract

Spectral efficiency is a key design issue for all wireless communication systems. Orthogonal frequency division multiplexing (OFDM) is a very well-known technique for efficient data transmission over many carriers overlapped in frequency. Recently, several papers have appeared which describe spectrally efficient variations of multi-carrier systems where the condition of orthogonality is dropped. Proposed techniques suffer from two weaknesses: Firstly, the complexity of generating the signal is increased. Secondly, the signal detection is computationally demanding. Known methods suffer either unusably high complexity or high error rates because of the inter-carrier interference. This work addresses both problems by proposing new transmitter and receiver architectures whose design is based on using the simplification that a rational Spectrally Efficient Frequency Division Multiplexing (SEFDM) system can be treated as a set of overlapped and interleaving OFDM systems.

The efficacy of the proposed designs is shown through detailed simulation of systems with different signal types and carrier dimensions. The decoder is heuristic but in practice produces very good results which are close to the theoretical best performance in a variety of settings. The system is able to produce efficiency gains of up to 20% with negligible impact on the required signal to noise ratio.

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a well-known technique for efficient data transmission. OFDM is at the core of communications technologies such as Digital Audio Broadcasting (DAB) and Digital Video Broadcast (DVB), wireless broadband networks such as Worldwide Interoperability for Microwave Access (WiMAX) and long term evolution (LTE) systems. In OFDM, data is transmitted using a number of orthogonal carrier frequencies. Recently many authors have proposed non-orthogonal systems or Spectrally Efficient Frequency Division Multiplexing (SEFDM) systems. OFDM symbols are sent on carrier frequencies separated by F and the symbols remain constant for time T (the symbol period) with $TF = 1$. This ensures no sub-channel interference. For SEFDM, $TF = \alpha < 1$ and, while there will necessarily be sub-channel interference, the key advantage is that the available spectrum can be used more efficiently.

This paper suggests design for a simple to implement transmitter and receiver/decoder for SEFDM systems. The transmitter design and the decoder design are interlinked. The key insight is to see SEFDM as a small number of interleaved OFDM systems. The design can increase spectral efficiency by 20% using similar techniques to traditional OFDM and with little compromise to the required signal to noise ratio for the system. The designs require only slightly more complexity in the receiver and transmitter. The decoder design is not optimal but is, instead, designed to be fixed (and low) complexity heuristic with "good enough" performance. It is shown in simulation that gains significantly above 20% are unlikely without a more radical redesign of SEFDM systems since even optimal decoding begins to suffer large increases in bit error rate relative to OFDM.

Section 1.1 describes other research and the background. The structure of the paper is as follows. Section 2 provides a brief introduction to SEFDM. Section 3 derives the main theorem necessary for our receiver and decoder design. Section 4 describes the receiver and decoder design. Section 5 shows the designs perform well in simulation and section 6 gives conclusions and further work.

1.1 Spectrally Efficient FDM approaches

The idea of non orthogonal and spectrally efficient systems occurs in the 1975 work of Mazo et al [1]. More recently, the idea of multi-carrier spectrally efficient systems was introduced in [2] and termed SEFDM.

Similar systems use the names high compaction multi-carrier modulation (HC-MCM) [3, 4] or overlapped FDM (OvFDM) [5]. Related systems are fast OFDM (FOFDM) [6] and the M-ary amplitude shift keying OFDM [7], both proposing reducing the spectrum to the half of an equivalent OFDM but subject to the limitation that the information symbols are only one-dimensional (e.g. BPSK or ASK). In addition, offset QAM proposed in [8] succeeded in eliminating guard bands and hence supported higher spectral efficiency.

Recently the concept of non-orthogonal carriers has found its way into the very high bit rate optical communications field. The applicability of Fast OFDM concept of [6] has been demonstrated in [9] in a system termed Optical Fast OFDM that provides attractive error performance for one dimensional modulation schemes. Furthermore, [10] proposed the so called optical Dense OFDM (DOFDM) which can accommodate higher order modulations. Simulation and experimental tests confirmed almost the same error performance as conventional OFDM. By orthogonally polarizing the sub-carriers it is possible to enhance immunity to the chromatic dispersion for both conventional OFDM and DOFDM. A related system termed non-orthogonal FDM (NOFDM) proposed restoration of orthogonality from the view point of the input symbols by employing orthogonal pulse-shaping [11], where details of appropriate pulse shapes and power and bit loading provided in [12, 13] and [14].

There are two problems with SEFDM systems: efficiently generating such a signal (the transmitter problem) and efficiently detecting and decoding such a signal. For the transmitter problem, a known method (first proposed by the authors) is to use the inverse fractional Fourier transform [15]. The HC-MCM system shortens the symbol transmission time and hence transmits by using OFDM techniques with zero-padded input and truncated output [3]. Recently several techniques to generate SEFDM signals using the Inverse Discrete Fourier Transform (IDFT) have been proposed by the authors [16, 17, 18] and have been implemented in hardware [19].

Obtaining optimal solutions for the decoding problem is non polynomial (NP) hard. Various techniques are suggested: some of the better known solutions to the decoding problem are zero forcing (ZF) [20, 21], minimum mean squared error (MMSE) [22], the sphere decoder [23, 15] and semi-definite relaxation (SDR) [24, 25]. Maximum likelihood methods have extremely high complexity and cannot be used in practice for anything other than the smallest systems. Methods such as SDR, MMSE and ZF have lower complexity but introduce a significant error penalty, particularly when noise levels are high or the number of carriers large [21]. They are therefore unlikely to prove useful in systems with many carriers or practical noise levels.

By contrast, the sphere decoder (SD) is a method of dynamic programming that can handle the NP hardness of overlapped optimisation problems achieving the optimal solution — SD techniques are investigated by Kanaras et al in [15, 26]. Much promising research has taken place on the use of SD for SEFDM. Furthermore, [27] developed a new sub-optimal SD based detector that uses semi-definite programming to reduce the complexity of the SD, whereas [28] and [29] proposed the use of a fixed complexity sphere decoders (FSD) and then a combination of FSD and the truncated singular value decomposition (SVD) to solve the problem of the variable complexity of the SD whilst still providing attractive error performance. SD suffers from two basic drawbacks which have only been partially overcome. It requires the inversion of ill-conditioned matrices (regularisation helps this problem at the expense of introducing noise) and its complexity is not fixed but is, in general, worse than polynomial [30, 31]. The execution time of SD can worsen considerably with many carriers, in high noise or with low α. Consequently, a practical implementation could be possible only under very specific conditions, for relatively small signal dimensions ($N \leq 32$) and in high signal to noise ratio (SNR) regimes. Therefore, the need remains for a detector technique which can recover signals well and in a short fixed time.

In SEFDM, the channel equalisation problem needs consideration. Work has been done on the problem of accounting for channel effects in SEFDM systems and [32] shows that joint detection and equalisation are possible.

An open question remains to what extent it is even theoretically possible to recover signals. For sampled SEFDM systems the Bit Error Rate (BER) as a function of energy per bit to noise power spectral density ratio (E_b/N_0) is not known although Mazo and Landau famously made pioneering work in this area for single carrier systems [33]. A later result by Rusek et al [34, 35] demonstrates that for $\alpha > 0.802$ and 4-QAM for optimal detection the BER should be exactly that of OFDM (although technical differences in the system mean that this result may not precisely carry over to SEFDM systems as considered in this work). It also demonstrates that this is the “best possible” value of α in this setting and for lower α this the performance will diverge (see [34] for full details). However, it would be expected the BER for a “good enough” decoding system to be “close” to the BER for OFDM for $\alpha \in (5/6, 1)$ and diverge for smaller α (especially when $\alpha < 0.802$).

2 The Spectrally Efficient Frequency Division Multiplexing scheme

If spectral efficiency is defined as the bitrate transmitted divided by the amount of spectrum used (bits/s/Hz) then it can be seen that multiplying the symbol period T by a factor $\alpha < 1$ but keeping the frequency separation F the same will increase the spectral efficiency (by increasing the bitrate) by a factor of approximately $1/\alpha$ for a large number of carriers. Here then we take the spectral efficiency of the new system as being $1/\alpha$ and hence $\alpha = 5/6$ means spectral efficiency of 120% or a 20% gain. The result is the same (and the system mathematically identical) if T is kept constant and F reduced.

Assume that the system has N carrier frequencies each separated by a frequency separation F. Let S_i (with $i \in \{0, 1, \dots, N-1\}$) be the symbol (a complex number chosen from an “alphabet”) on carrier i for time $[0, t)$. Now, ignoring the frequency offset of the initial carrier for simplicity, the transmitted signal ($B(t)$ for broadcast signal) in the period $[0, T]$ is given by $B(t) = \sum_{k=0}^{N-1} S_k \exp[2\pi ikt/T]$. For OFDM, the interference between frequencies is zero when the signal is integrated over the symbol period. The discrete version of this can be considered instead where $B(t)$ is sampled at M discrete times in the set $\{0, T/M, 2T/M, \dots, (M-1)T/M\}$. This new series is U_m (with $m \in \{0, 1, \dots, M-1\}$) where $U_m = B(Tm/M)$ and (1) becomes $U_m = \sum_{k=0}^{N-1} S_k \exp[2\pi ik(mT/M)T] = \sum_{k=0}^{N-1} S_k \exp[2\pi ikm/M]$. It is this discrete version which is traditionally used in OFDM transmitters as it can be easily generated using FFT techniques and then the continuous signal approximated from this.

Now, consider the SEFDM system where $TF = \alpha < 1$. Further we assume that α is rational $\alpha = b/c$ with $b, c \in \mathbb{N}$ (the set of natural numbers). The equivalent equation for the transmitted signal is given by

$$B(t) = \sum_{k=0}^{N-1} S_k \exp[2\pi iktb/cT], \tag{1}$$

where $B(t)$ is the broadcast signal at time $t \in [0, T)$. The discretely sampled version where $U_m = B(Tm/M)$

becomes

$$U_m = \sum_{k=0}^{N-1} S_k \exp[2\pi i k m b / cM]. \tag{2}$$

Because of the b/c factor FFT techniques cannot be used in a straightforward manner to generate the transmitted wave. However, section 3.1 shows one way this can be done and section 4 shows one workable design for a transmitter and decoder based on the insight that the SEFDM system with rational α consists of interleaved OFDM systems.

A working SEFDM system will generate and receive a continuous waveform. (If the transmission is digitally generated as in this case the continuous wave form would be from a smoothed version of the digital samples). A computer simulation is by its nature discrete. It can be shown that for the continuous wave form, the interchannel interference impacting the mth channel from the nth channel in an SEFDM system (for $n \neq m$) is given by:

$$I(n,m) = S_n \left(\mathrm{sinc}[(n-m)\alpha]/\pi\right) \exp[\pi i (n-m)\alpha],$$

where $\mathrm{sinc}(x)$ is the normalised sinc function $\sin(\pi x)/x$. For the discrete simulation the interference term is

$$I'(n,m) = \frac{S_n \mathrm{sinc}[(n-m)\alpha]}{\mathrm{sinc}[(n-m)\alpha/M]} \exp[\pi i (n-m)\alpha(M-1)/M].$$

This can be thought of as the original $I(n,m)$ corrupted by a rotation factor $(M-1)/M$ (the origin of this is the non-centred sampling times) and a magnification factor $(n-m)\alpha/M \sin[(n-m)\alpha/M]$. Both tend to 1 as $M \to \infty$ (as would be expected). In short, the discrete simulation will exaggerate (sometimes greatly) the interfering effects of the SEFDM carriers.

3 Mathematics of SEFDM systems

A core insight of this paper is the viewing of SEFDM as interleaved OFDM systems. This is illustrated in Fig. 1. Here the large vertical double arrows represent an OFDM system with symbol period T and frequency separation F. (Remember that an OFDM system has $TF = 1$ and an SEFDM system has $TF = \alpha < 1$.) The smaller single arrows represent an SEFDM system with the same symbol period T and a frequency

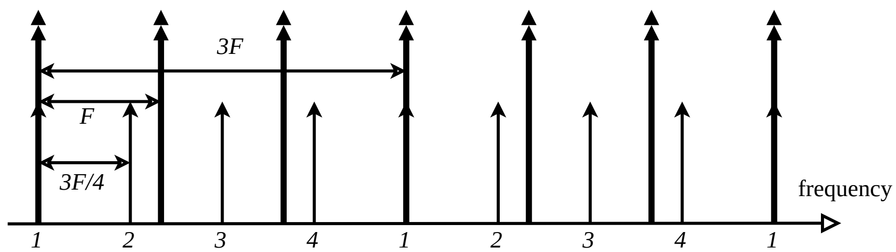

Figure 1: A representative diagram of SEFDM with $\alpha = 3/4$ compared to an OFDM system.

separation $\frac{3}{4}F$ ($\alpha = 3/4$).

It can be seen that those SEFDM frequencies labelled 1 (below the x-axis) always align exactly with OFDM frequencies (separated by $3F$). In other words, those SEFDM frequencies are an OFDM system which happens only to send symbols on every third carrier. The frequencies labelled 2 also form an OFDM system offset in frequency from the first by $\frac{3}{4}F$. In general, if α is some rational $\alpha = b/c$ with $b < c \in \mathbb{N}$ (where $\mathbb{N}$ is the set of natural numbers) this can be viewed as c interleaved OFDM systems each sending symbols on every bth carrier and offset from each other by F. This insight will be used both in transmitter and decoder design, a formal proof follows.

3.1 Proof of the equivalence of SEFDM and interleaved OFDM

For notational convenience, assume here and throughout this paper that an $N \times M$ matrix $\mathbf{C} = [c_{nm}]$ has its indices running from zero (not one as is more usual). That is $n \in \{0, 1, \ldots, N-1\}$ and $m \in \{0, 1, \ldots, M-1\}$. Equation (2) can now be written as $\mathbf{U} = \mathbf{SC}$, where $\mathbf{U} = [U_0, U_1, \ldots, U_{M-1}]$, $\mathbf{S} = [S_0, S_1, \ldots, S_{N-1}]$ and $\mathbf{C} = [c_{nm}]$ is the $N \times M$ carrier matrix given by $c_{nm} = \exp[2\pi i n m b/cM]$. The equation $\mathbf{U} = \mathbf{SC}$ does generate the sequence to be transmitted but the multiplication by an $N \times M$ matrix could be computationally intensive for large N. Assume N is some multiple of c (this is not a necessary assumption but makes the notation easier) so $N = cd$ with $d \in \mathbb{N}$.

Theorem 1. *Consider an SEFDM system with N carrier frequencies sampled $M \geq N$ times in the symbol period and with rational $\alpha = b/c$ as described in equation (2). The system can be decomposed into the sum of c separate OFDM systems each with $b\lceil N/c \rceil$ carrier frequencies and a frequency offset applied to each of* *these (where $\lceil\cdot\rceil$ is the ceiling function). Of these frequencies, a maximum of $\lceil N/c\rceil$ actually carry symbols.*

Proof. Assume without loss of generality that N is a multiple of c. For a system where N is not a multiple of c the same proof applies on the expanded system with $N' > N$ carriers such that N' is a multiple of c and no symbols are broadcast on the final carriers ($S_N = S_{N+1} = \cdots = S_{N'-1} = 0$).

Let $\mathbf{D}$ be the $Nb/c \times M$ matrix for an OFDM system with Nb/c carriers and M samples given by $\mathbf{D} = [d_{nm}]$ and $d_{nm} = \exp[2\pi inm/M]$. Let $\mathbf{R}(k)$ be the $M \times M$ rotation matrix: $\mathbf{R}(k) = \mathrm{diag}[r(k)_m]$ where $r(k)_m = \exp[2\pi imkb/cM]$ and diag is the matrix with all elements zero apart from the diagonal which has its kth element as $r(k)_m$.

The SEFDM transmission can be considered as the sum of c interleaved OFDM systems. Let $\mathbf{U}'(k)$ be the signal generated by the kth such system. Let $\mathbf{S}'(k)$ be the symbols in $\mathbf{S}$ that are transmitted on the kth system. That is $\mathbf{S}'(0) = (S_0, S_c, S_{2c}, \ldots)$ and $\mathbf{S}'(1) = (S_1, S_{c+1}, S_{2c+1}, \ldots)$ and so on. Formally, define the c symbol vectors (each of length Nb/c) $\mathbf{S}'(k)$, for $k = 1, 2, \ldots, c-1$

$$S'(k)_n = \begin{cases} S_{nc/b+k} & n \mod b = 0, \\ 0 & \text{otherwise}, \end{cases}$$

where $n \in (0, 1, \ldots, Nb/c)$. Note that each of the original symbols S_n appears in exactly one of the new symbol vectors $\mathbf{S}'(k)$. This also means that the reverse map can be constructed $S_n = S'(n \mod c)_{b(n-k)/c}$.

Consider the matrix equation $\mathbf{U}' = \sum_{k=0}^{c-1} \mathbf{S}'(k)\mathbf{D}\mathbf{R}(k)$. This is the sum of the k new symbol vectors transformed by an OFDM system and rotated. It remains to show that $\mathbf{U}' = \mathbf{U}$. Define each element of this sum as $\mathbf{U}'(k) = \mathbf{S}'(k)\mathbf{D}\mathbf{R}(k)$ and therefore $\mathbf{U}' = \sum_{k=0}^{c-1} \mathbf{U}'(k)$. For any k the mth element of $\mathbf{U}'(k)$ (referred to here as $U'(k)_m$) is given by

$$U'(k)_m = \exp[2\pi imkb/cM] \sum_{n=0}^{Nb/c-1} S'(k)_n \exp[2\pi inm/M]. \tag{3}$$

Since $S'(k)_n = 0$ if $n \mod b \neq 0$ then the sum index n can be transformed using $l = n/b$ to give $U'(k)_m = \exp[2\pi imkb/cM] \sum_{l=0}^{N/c-1} S'(k)_{lb} \exp[2\pi imlb/M]$. Since $S'(k)_{lb} = S_{lc+k}$ for all $l \in \{0, 1, \ldots, N/c-1\}$ then $U'(k)_m = \exp[2\pi imkb/cM] \sum_{l=0}^{N/c-1} S_{lc+k} \exp[2\pi imlb/M]$. The sum must be transformed again using $p =$

$lc+k$ and hence $l=(p-k)/c$ to $U'(k)_m = \exp[2\pi imkb/cM]\sum_{p=k}^{Nc-c+k} S_p\delta(p \mod c)\exp[2\pi im(pb-kb)/cM]$, where $\delta(n)$ is the delta function which is equal to 1 if $n=0$ and 0 otherwise. A final sum transformation gives $U'(k)_m = \sum_{n=0}^{N-1} S_n\delta(n+k \mod c)\exp[2\pi inmb/cM]$. Clearly then the final proof arises

$$\mathbf{U}' = \sum_{k=0}^{c-1}\sum_{n=0}^{N-1} S_n\delta(n+k \mod c)\exp[2\pi inmb/cM] = \sum_{n=0}^{N-1} S_n \exp[2\pi inmb/cM] = \mathbf{U},$$

where the removal of the sum over k at the second equality sign occurs because $n+k \mod c = 0$ is always true for exactly one value of k for any given value of n. $\square$

4 Transmitter and receiver design

The transmitter and receiver designs outlined in this section have several advantages over those in the SEFDM literature. The receiver and transmitter designs also have much in common which would help with the cost of building them.

4.1 Transmitter design

The generation of SEFDM signals using the Inverse Discrete Fourier Transform (IDFT) has been proposed by the authors in [16] and this has led to a recent hardware implementation [19]. As the SEFDM signal can be described as a sum of overlapped independent rotated OFDM signals, it can be shown that the SEFDM transmitters can be built using OFDM generation techniques. An OFDM signal is efficiently generated using the Inverse Discrete Fourier Transform (IDFT) [36].

From Theorem 1 it can be shown that adding c rotated OFDM systems can create the same signal as an SEFDM system. This can be utilised to build an SEFDM transmitter as illustrated in Fig. 2. The transmitter starts by reordering the input symbols and insert zeros at appropriate locations to generate the c symbol vectors. The symbols reorder block generates the $\mathbf{S}'(k)$ vectors. These vectors are then fed into the c IDFT modules. The outputs of the IDFTs are then rotated using the rotation matrices $\mathbf{R}(k)$ and combined to generate the time sampled sequence $\mathbf{U}$, which can be fed into a digital to analogue converter (D/A) to finally generate the continuous time signal $B(t)$.

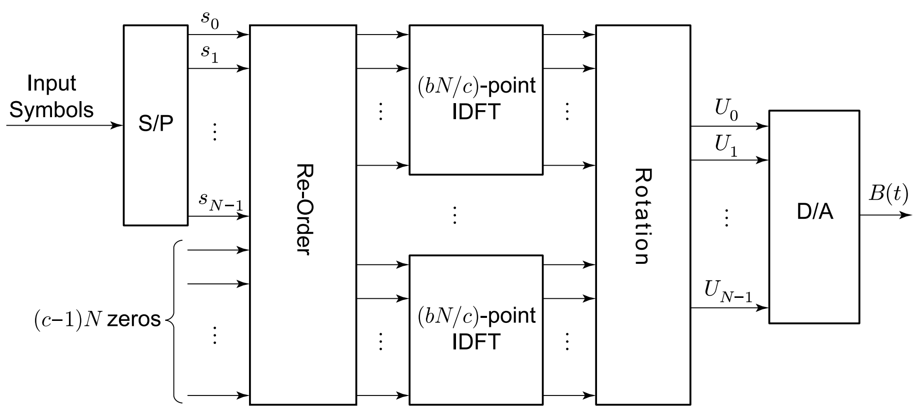

Figure 2: The IDFT implementation for SEFDM transmitter.

Generation of SEFDM with the IDFT offers many advantages. The IDFT based system is ready for digital implementation providing all the digital over analogue advantages. The structure of the SEFDM system is based on similar building blocks to the widely available OFDM system which will facilitate a smooth changeover. In addition, as will be shown later the receiver and transmitter have the same structure which can enable dual operation of the same equipment and consequently reduce the design and implementation costs.

4.2 Receiver/decoder design

Once the SEFDM signals for one symbol period have been received they must then be decoded to return the original symbol. The receiver therefore attempts to recover the original symbols by decoding the interleaved OFDM systems individually by subtracting the estimated interference from the other OFDM systems (for this reason we term this the "stripe" decoder). Note that the design here is heuristic, no proof of convergence is given (and one may not be possible). The justification for the design is that it is intuitive and it works in software tests. The receiver/decoder is shown diagrammatically as a data flow diagram in Fig. 3.

Begin with a received signal (box A in diagram) and an initial estimate that all symbols are $0 + 0i$ (box B in diagram) and iteratively improve the estimates by isolating the signal arising from each of sub OFDM systems (see Fig. 1). After several iterations the estimates converge to the correct input symbol and are eventually rounded to the closest symbol in the symbol alphabet in use.

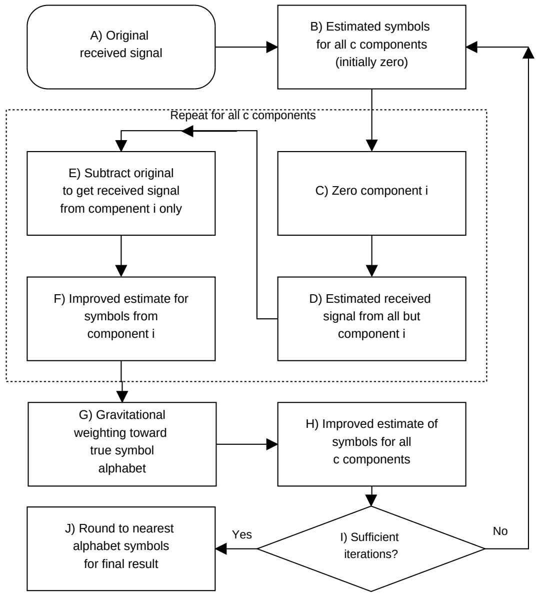

Figure 3: Receiver/decoder dataflow diagram.

Consider, again, the SEFDM system with N carriers and $\alpha = b/c$. Let $\mathbf{U}$ (as in section 3.1) be the received signal (for now assume it is not corrupted by noise). If the system is OFDM, decoding is simple. The received frequencies are orthogonal and a simple IDFT recovers the symbols on each carrier. Now, it follows that if the symbols for $c-1$ of the interleaved OFDM systems were known then the symbols on the remaining OFDM system could be obtained. This is achieved by firstly subtracting that portion of $\mathbf{U}$ which arises from the $c-1$ OFDM systems with known symbols and secondly, performing the inverse DFT. Using the notation of section 3.1, if $\mathbf{U}(k)$ is the signal arising from the kth interleaved OFDM system then $\mathbf{U} - \sum_{k=1}^{c-1} \mathbf{U}(k)$ is the signal arising from the zeroth OFDM system $\mathbf{U}(0)$. An IDFT of $\mathbf{U}(0)$ recovers the symbols. A similar process would be required if $\mathbf{U}(0), \mathbf{U}(2), \mathbf{U}(3), \ldots, \mathbf{U}(c-1)$ were known and $\mathbf{U}(1)$ were to be recovered. In that case a frequency shift $\mathbf{R}(1)$ (again as in section 3.1) would need to be applied before the inverse DFT. It should be noted that even if $\mathbf{U}$ is corrupted by AWGN, the above process can be used to produce a maximum likelihood estimate of the original broadcast symbols by rounding it to the nearest letter in the "symbol alphabet" being used.

Given estimates of the correct symbols for the c interleaved OFDM systems then improved estimates can be produced (inner dotted box in diagram). The estimates are produced by, for each OFDM system in turn: first subtract the signal from the $c-1$ other OFDM systems (box C, D and E in diagram) and secondly perform an inverse DFT with frequency shift to get an improved estimate for that OFDM subsystem (box F in diagram). To improve performance a "gravitational" model is added to pull estimates towards symbols in the symbol alphabet (box G in diagram). This is repeated for J iterations (box I in diagram). Note that estimates are "soft estimates" (complex numbers which are not necessarily members of the symbol alphabet) until the final stage of processing the estimates are mapped to the nearest member of the symbol alphabet (box J in diagram).

1. Set $\widehat{\mathbf{S}}$ the estimated symbols to $0+0i$ (box B).

2. Let $j := 1$ (j counts iterations – there are J iterations).

3. Let $\widehat{\mathbf{S}(0)}$, $\widehat{\mathbf{S}(1)}$, ... be the estimates for the symbols of the c interleaved OFDM systems. The $\widehat{\mathbf{S}(k)}$ together make $\widehat{\mathbf{S}}$ as in section 2.

   (a) For each of the c systems in turn, remove that part of the signal generated by all symbols in $\widehat{\mathbf{S}}$ apart from $\widehat{\mathbf{S}(k)}$ (box C and D). Use this to estimate a new $\widehat{\mathbf{S}(k)}$ and hence a new $\widehat{\mathbf{S}}$ (box E and F).

   (b) For each of the N symbols calculate a "gravitationally weighted" version of $\widehat{\mathbf{S}}$, $G(\widehat{\mathbf{S}})$ (box G).

4. $\widehat{\mathbf{S}} := \widehat{\mathbf{S}}(J-j)/J + (j/J)G(\widehat{\mathbf{S}})$ (box H).

5. If $j < J$ then $j := j+1$ and go to step 3 (box I).

6. Finally, $\widehat{\mathbf{S}}$ is "sliced" to the nearest alphabet symbol for each estimated symbol $\widehat{S_i}$ (box J).

The two central steps of the algorithm (a) and (b) above require slightly more explanation. If $\mathbf{r}$ is the received signal corrupted by noise then, to estimate the kth OFDM system first calculate $C(k) = \mathbf{r} - \sum_{j=0, j\neq k}^{c-1} \widehat{\mathbf{U}(j)}$ where $\widehat{\mathbf{U}(j)}$ is the estimated signal transmitted by just the estimated symbols in the jth OFDM system $\widehat{\mathbf{S}(j)}$ (box D). $C(k)$ can then be shifted in frequency by multiplication $\mathbf{R}(k)$ to produce an estimate of the signal (plus noise) arising solely from the kth OFDM system (box E). This can be decoded in the usual OFDM manner (using IDFT) as if it were an OFDM system transmitting on every bth carrier. This produces a new estimate for the symbols on the kth OFDM system $\widehat{\mathbf{U}(k)}$.

These estimate symbols are truncated to ensure that no symbol has a real or imaginary part outside the range of the signalling alphabet (for example, if the system is 4-QAM estimates are rounded so all real and imaginary parts are in the range $[-1, 1]$. The new estimate for $\widehat{\mathbf{U}(j)}$ can immediately be used to update $\widehat{\mathbf{S}}$ (box F). This takes place for each of the c OFDM systems in turn (dotted large box) to produce a new $\widehat{\mathbf{S}}$.

Note that while this part of the decoder design seems complicated, in fact, the decoder can be implemented using the transmitter. To calculate $C(k)$ from $\mathbf{r}$ the received signal and $\widehat{\mathbf{U}(j)}$ (the estimated symbols on all carriers $j \neq k$) simply feed the estimated symbol set $\widehat{\mathbf{S}}$ with symbols $k, k+c, k+2c, \ldots$ set to zero to the transmitter. This produces an estimate of the signal which would be transmitted by all but the kth OFDM system (corrupted by noise). This signal can be decoded using IDFT as in standard OFDM to produce an improved estimate for $\widehat{\mathbf{S}(k)}$ the symbols of the kth OFDM system.

The "gravitationally weighted" $G(\widehat{\mathbf{S}})$ (box G) is calculated by examining each estimated symbol in turn $\widehat{S_0}$, $\widehat{S_1}$ and so on and producing the weighted sum of each of the symbols in the alphabet weighted by the inverse of the distance to them (as a gravity law). If A is the symbol alphabet (say, $1+0i$ $-1+0i$ for BPSK) then

$$G(\widehat{S}_i) = K \sum_{a \in A} a/d(a, S_i)^2,$$

where $d(a, b)$ is the Euclidean distance between the two points in complex space and K is the normalising constant $1/\sum_{a \in A} a/d(a, S_i)^2$. If for any $a \in A$, $d(a, S_i) = 0$ then $G(S_i) = a$, that is, if the estimated symbol happens to be exactly on a point in the alphabet (to machine precision) then that point is returned. Many similar weighting schemes could be tried but this one appears sufficiently effective and quick to calculate.

The complexity of the decoder system is tied to the complexity of the transmitter. (The "gravitational" part is of negligible complexity). To subtract an estimated signal a signal is generated by the transmitter. The final complexity of the decoder then is a fixed linear multiple of the transmitter complexity – this multiple being a product of c (the number of interleaved OFDM systems) and J, the number of iterations in step (a) above (experiment found 20 to be a reasonable value).

4.3 Comments on Implementation

Numerical results modelling the work reported here demonstrate attractive error performance (as will be seen in the next section) at a much reduced complexity when compared to optimal iterative detection algorithms such as SD. Ultimately the aim is to realize the proposed designs in hardware. Examining the structure of the proposed system reveals the support of an efficient implementation path. The transmitter design relies on general purpose IDFT operations which can be efficiently evaluated with the Inverse Fast Fourier Transform (IFFT). We have recently reported the implementation of such transmitter on a reconfigurable field programmable gate array (FPGA) architecture [37] and demonstrated its operation, showing its ability to perform real time tuning of α. Furthermore, design studies as very large scale integrated circuit (VLSI) structures have also been reported in [38].

Examining the structure of the stripe decoder, shows that the main components are standard DFT modules which can in turn be realized with the FFT algorithm. Implementations of DFT based demodulators for SEFDM system have also been reported in [39]. A main difference in the design is that multiple DFT blocks are needed for the stripe decoder while a single longer DFT is implemented in [39]. However, the DFT blocks arrangements in the stripe decoder may follow the same pattern as those of the transmitter design.

5 Simulation results

Numerical simulation was carried out to determine the performance of the transmitter and decoder system (as mentioned in the introduction the transmitter has been tested in hardware this work is ongoing for the receiver/decoder). A sampled SEFDM system is characterised by N (the number of carriers in one symbol period), the symbol alphabet (what allowable complex symbols are considered), M (the number of samples obtained for decoding in one symbol period), $\alpha = b/c$ (the compression ratio) and E_b/N_0 the energy per bit to noise power spectral density ratio. As previously remarked, the spectral efficiency of an SEFDM system compared with OFDM is $1/\alpha$. So, for $\alpha = 5/6$ the spectral efficiency increases by 20% and for $\alpha = 4/5$ by 25%.

5.1 Simulation description

Code to simulate the system was written in python. The code implements transmission of SEFDM using the FFT method described in section 4. Test signals are generated from random bits. Additive White Gaussian Noise (AWGN) with a given E_b/N_0 is then added. Channel effects such as fading and frequency and phase offsets as well as system aspects such as channel estimation are not considered in this work. The assumption of a simplified AWGN channel serves to illustrate the basic concepts of the work and its practicability. More sophisticated channel models are the subject of ongoing work. It has been shown that joint detection and equalisation using sphere decoder can provide attractive BER performance [32]. The authors believe that a joint detection and equalisation technique based on the proposed detection algorithm from this paper could be used to alleviate the problem.

The simulations described here are all performed with the assumption that data is arriving as fast as the system can broadcast it (that is, the system is at maximum load and there is always a symbol on every channel in every period). The results would not be altered if this load fell (a blank symbol could be broadcast). The choice of symbols is random. As the relationship of symbol patterns to BER is unknown, completely random choices of symbols is the best way to obtain the actual BER a working system would have.

Three decoding schemes are implemented. The first is the "stripe" decoding technique from this paper (section 4.2). The maximum likelihood (ML) method explicitly tests every possible combination of alphabet symbols on each carrier and measures the difference between the time series generated and the received signal. While this is in some sense "perfect" as a decoding scheme it is computationally intractable for large N (assuming for simplicity that $M = N$). The number of tests requires increases as $O(A^N)$ where A is the number of symbols in the alphabet and N the number of carriers and assuming each test can be performed with FFTs in parallel then each is of order $O(N \log N)$. By contrast the stripe decoder complexity is $O(N \log N)$ (although this must be multiplied by the constant J the number of iterations performed). The "sphere decoder" method attempts to more intelligently assess only the "nearly correct" symbol sequences. That is it uses a dynamic programming technique over only a subset of the possible symbol space. However, because it relies on numerical matrix inversion, it suffers problems with large numbers of channels or low values of E_b/N_0 as the number of symbol combinations investigated becomes large. The three methods are referred to in the results as ML, stripe and sphere. The ML should represent a "best possible" result and the sphere decoder should also be optimal except in cases where the algorithm fails to find a solution – in practice the result coincides almost exactly with the optimal solution where that is known.

To get statistically representative results, a high number of iterations must be performed with each iteration representing one symbol period. 95% confidence intervals have been calculated for all experiments which measure bit error rate on the assumption that each decoded bit is an independent trial (in fact, for say a 128 carrier 4-QAM system the error rates on groups of 256 bits composing one symbol period will be loosely correlated but between simulated symbol periods the bits are independent trials). For most of the graphs plotted the 95% confidence intervals are too close together to show up and are omitted.

For space reasons runtime efficiency results are not shown here (and results of runtime for computer simulation are not expected to translate directly to better performance when implemented on hardware). The runtime results for transmitter and decoder were completely in line with the expected theoretical results – the time taken to produce a signal for one symbol period using the transmitter code was $O(M \log Mc)$ where c is the number of OFDM systems to be added and M the number of samples per symbol period. To get accurate experimental estimates for BER it was necessary to generate and decode tens of thousands of symbol periods (since the BER was extremely low). In our software simulations the "stripe" decoder could transmit and decode 128 symbols per period in an acceptable runtime whereas the ML decoder could perform no more than 4 and the sphere decoder no more than 8. In short, the transmitter design and receiver/decoder designs were, as predicted, a fixed, small multiple of the runtime of an FFT routine.

5.2 Decoder results, prediction accuracy

It should be emphasised throughout this section that the OFDM result (the theoretical BER line) represents the best possible result obtainable in the case of an orthogonal system. The maximum likelihood (ML) result represents (within the bounds of statistical errors) a best possible result for the simulation parameters used (N, M, α and E_b/N_0). The sphere decoding result will also be "near" optimal for the simulation parameters used.

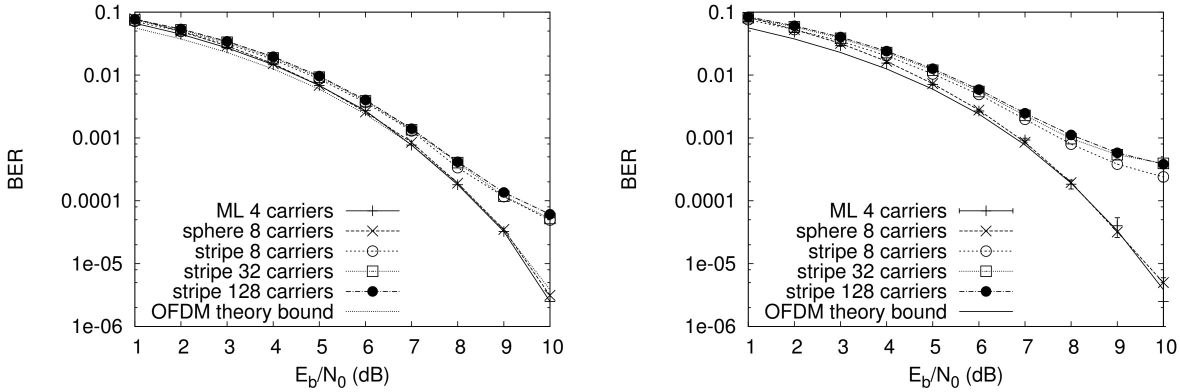

Figure 4: Bit error rate for $\alpha = 5/6$ (left) and for $\alpha = 4/5$ (right) using 4-QAM.

Fig. 4 (left) shows results for the stripe decoder using 4-QAM for $\alpha = 5/6$. The sphere decoder and the ML decoder results are a good match with the theoretical best possible except for low E_b/N_0 (where they do not quite meet the optimal bound as we might expect). However, it is worth reiterating that the theory applies to an idealised situation with complete knowledge of the whole time signal in analysis but the simulation (and a real working system) only considers samples. The stripe decoder certainly shows worse performance than the perfect theoretical performance. However, it is comparable to the sphere decoder in low E_b/N_0 and for $E_b/N_0 > 5.0$dB the worsening of performance is equivalent to approximately an extra 1 dB of E_b/N_0 (the BER for OFDM at 9.0dB is the same as is the BER for SEFDM at 8.0dB). Note that it is this horizontal separation which is relevant since the design question is "how much more power (or less noise) would be necessary to regain the lost performance?" This is certainly very good performance. In low E_b/N_0 environments the stripe decoder would certainly be as good in terms of BER as OFDM. In high E_b/N_0 environments (above 8dB) the stripe decoder is able to achieve an acceptable bit error rate for wireless systems where BER below 0.0001 are entirely reasonable although an OFDM system would have lower errors. In this case an SEFDM could either fall back to OFDM or transmit at a higher power to reduce the E_b/N_0 until the BER was acceptable.

Fig. 4 (right) shows results for the stripe decoder using 4-QAM for $\alpha = 4/5$ – this is just below the limit of $\alpha = 0.802$ which is considered the "best possible" for idealised recovery of the signal. The stripe decoder

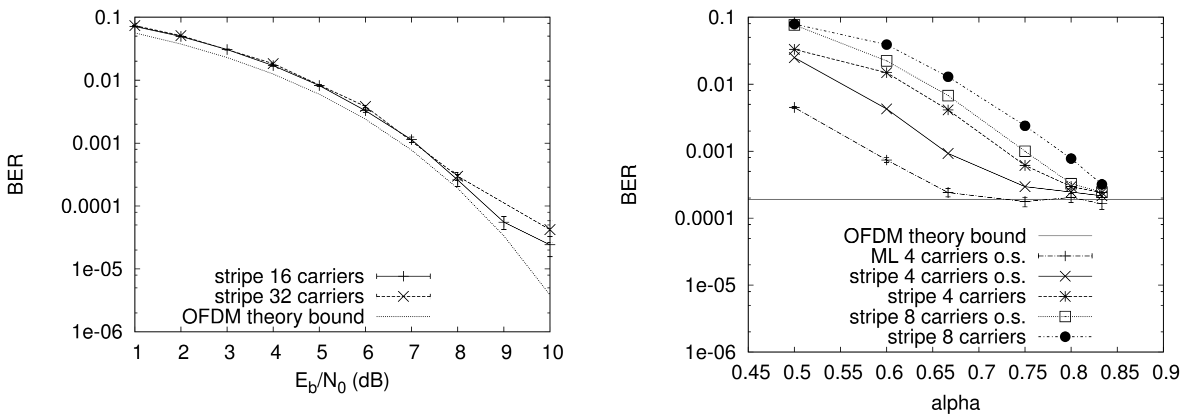

Figure 5: Bit error rate for 4-QAM with oversampling, $\alpha = 5/6$ fixed, E_b/N_0 varying (left) and $E_b/N_0 = 8.0$ dB fixed α varying (right).

again shows degraded performance. However, it is again comparable to the sphere decoder in low E_b/N_0 and for 5.0dB $< E_b/N_0 <$ 9.0dB the worsening of performance is equivalent to an extra 1 or 1.5 dB of noise (that is the BER for SEFDM at 9.0 dB is the same as the BER for OFDM at 7.5 dB). This is an acceptable power penalty/price to pay given the advantage of bandwidth saving. In low E_b/N_0 environments the stripe decoder would certainly be as good in terms of BER and much preferable in terms of spectral efficiency. However, it seems that the performance has worsened by going below the theoretical $\alpha = 0.802$ limit even slightly.

Fig. 5 shows the improvements which oversampling can bring. Recall from section 2 that the simulation in fact over estimates interference when the number of samples is “low”. More samples will produce interference levels closer to the real life (continuous) system.

Fig. 5(left) shows the results for 4-QAM with oversampling with $\alpha = 5/6$. For 16 and 32 carriers an oversampling rate such that $M = 16N$ is tried — 16 samples per carrier in every symbol period. This should be compared with Fig. 4 (left) which is the result for $N = M$ — one sample per carrier in every symbol period. For 16 and 32 carriers the BER has almost no worsening from the theory except in the highest signal to noise ratio where the degradation still remains modest. Overall, then the performance of the algorithm is extremely satisfactory with oversampling. Oversampling results for $\alpha = 4/5$ are less successful however.

Fig. 5(right) shows the results with $E_b/N_0 = 8.0$ dB and α varied. The stripe detector is tried with

4 and 8 carrier systems and heavy over sampling — in this experiment $M = 128$ when $N = 4$ or $N = 8$. In this case the improvement is marked with a significant improvement in BER using oversampling. Since the stripe detector can perfectly happily function with large channel numbers it can also work well with smaller numbers of channels and oversampling. The oversampling also improves the performance of the ML estimator, making it stay closer to the theoretical OFDM limit for smaller values of α. This figure, however, shows an important theoretical limit to what can be gained by SEFDM type systems even with optimal detection and an extremely small number of channels. When $\alpha < 2/3$ the BER begins to increase markedly. Therefore, even with perfect detection it could never be expected that spectral efficiency gains of more than 50% ($1/\alpha - 1$ with $\alpha = 2/3$) can be achieved even for the four carrier system. For more carriers the limit of $\alpha \simeq 0.802$ seems likely to apply. These oversampling results confirm the intuition from section 2 that the discrete simulation exaggerates the interference effect and more samples will bring the interference (and hence BER) down.

Finally, Fig. 6 shows results using the stripe decoder on BPSK for $\alpha = 1/2$ (with $\alpha = 1/2$ the system is that of FOFDM [6]). The results are nearly "perfect" with little deviation from the theory line for OFDM which represents the best possible BER for an OFDM system with that E_b/N_0. The decoding using the "stripe" method is ideal for this scenario. More than 128 carriers cannot be tested quickly enough to get sufficiently accurate error prediction for the lower E_b/N_0 values. However, there seems no reason to believe that the bit error rate increases with the number of carriers in this scenario. It can be seen that the results are near "perfect" for BPSK with $\alpha = 1/2$. However, this is not as useful as it might appear since BPSK with $\alpha = 1/2$ only carries the same amount of data as 4-QAM OFDM.

In summary then, the results in this section show that the transmitter and decoder perform well for 4-QAM with $\alpha = 5/6$ but these good results fall off to be a less acceptable performance for $\alpha = 4/5$ in tune with the expectation from the theoretical results of Rusek et al [34, 35] suggesting a lowest possible value of $\alpha = 0.802$ before interference cannot be compensated for. The system performed better with heavy over sampling as suggested by section 2 and performed extremely well (indeed the results were indistinguishable from optimal) for BPSK with $\alpha = 1/2$. Although no channel model was used in this simulation, complementary work in [32] shows that SEFDM detection and equalisation can give good BER performance in dispersive

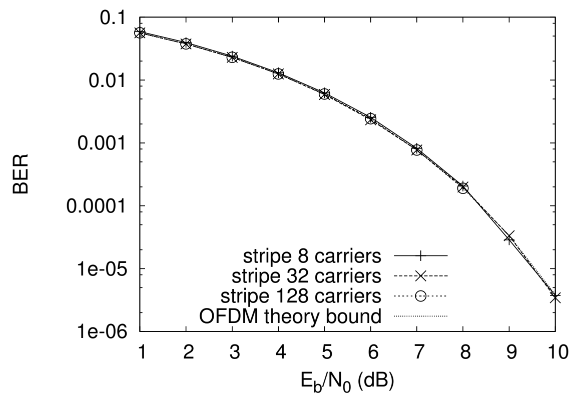

Figure 6: Bit error rate using BPSK, fixed $\alpha = 1/2$ varying E_b/N_0

channel environments when the receiver employs a regularised sphere detection mechanism.

It remains to be seen whether the system would be practical for modern systems with a very large number of carriers (512 and beyond). Detailed investigation of the properties of SEFDM in [40] have shown that the condition number of the matrix representing the carriers increases with the number of carriers and this increases the complexity of the problem for any detection method which involves matrix inversion. However, with the detection method proposed here, the error rate is not expected to be seriously compromised and we are encouraged by the results shown in Fig. 4 where there is only a slight degradation of the error when the number of carriers is increased from 16 to 128. Current software limitations for testing with larger number are being addressed by implementing the transmitter and receiver in hardware and this is underway. The building block for the system is the DFT as with ODFM and, hence, the speed of execution is not expected to be an issue in a hardware implementation.

6 Conclusions

This paper describes the design of a simple system for transmitting, receiving and decoding Spectrally Efficient FDM (SEFDM) signals. These signals can simply be generated by a transmitter mechanism very similar to that of standard OFDM with little increase in complexity. The decoder is more difficult to implement but the increase in complexity with the number of channels remains that of standard OFDM

$O(M \log M)$ (where M is the number of samples).

Detailed modelling and simulations show that the decoder described in this paper can give an increase in spectral efficiency of 20% ($\alpha = 5/6$) with little noise penalty and even 25% ($\alpha = 4/5$) in some circumstances (with a noise penalty close to 2dB for a BER of 10^{-4}). Oversampling can be used to compensate for almost all of the noise penalty for $\alpha = 5/6$. With oversampling this system is "almost perfect" producing the expected gain in spectral efficiency, relative to OFDM, with minimal error degradation.

Naturally, work remains to be done in this area. The decoder proposed here is a simple heuristic chosen because it gets a "good enough" solution in a very short time. It seems likely that similar heuristics could close much of the small gap between the solution here and the "optimal" solution. The simulations here do not account for channel fading, however, modelling using techniques similar to those of [32] to perform channel equalisation in SEFDM are currently underway.

In conclusion, the system proposed and modelled here could produce gains in spectral efficiency compared with an equivalent OFDM system. The system is only slightly more complex to implement than the OFDM system and functions in environments with similar noise levels, particularly when oversampling is used.

References

[1] J. E. Mazo, "Faster than Nyquist sampling," *Bell Systems Technical Journal*, vol. 54, no. 8, pp. 1451–1462, 1975.

[2] M. Rodrigues and I. Darwazeh, "A spectrally efficient frequency division multiplexing based communication system," in *8th International OFDM-Workshop*, Hamburg, Germany, Sep. 2003, pp. 70–74.

[3] M. Hamamura and S. Tachikawa, "Bandwidth efficiency improvement for multi-carrier systems," in *15th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 2004*, vol. 1, Sept. 2004, pp. 48–52.

[4] Y. Hou and M. Hamamura, "Bandwidth efficiency of PC-OFDM systems with high compaction multi-carrier modulation," in *Proc. International Conference on Wireless Communications, Networking and Mobile Computing*, vol. 1, Sep. 23–26, 2005, pp. 197–200.

[5] W. Jian, Y. Xun, Z. Xi-lin, and D. Li, “The Prefix Design and Performance Analysis of DFT-based Overlapped Frequency Division Multiplexing (OvFDM-DFT) System,” in *3rd Workshop on Signal Design and Its Applications in Communications,*, 23-27 Sept. 2007, pp. 361–364.

[6] M. Rodrigues and I. Darwazeh, “Fast OFDM: A proposal for doubling the data rate of OFDM schemes,” in *Proceedings of the International Conference on Telecommunications*, Beijing, China, Jun. 2002, pp. 484–487.

[7] F. Xiong, “M-ary amplitude shift keying OFDM system,” *IEEE Transactions on Communications*, vol. 51, no. 10, pp. 1638–1642, Oct. 2003.

[8] B. Hirosaki, “An Orthogonally Multiplexed QAM System Using the Discrete Fourier Transform,” *IEEE Transactions on Communications*, vol. 29, no. 7, pp. 982–989, 1981.

[9] J. Zhao and A. D. Ellis, “A novel optical fast OFDM with reduced channel spacing equal to half of the symbol rate per carrier,” in *Proc. Conf Optical Fiber Communication (OFC), collocated National Fiber Optic Engineers Conf. (OFC/NFOEC)*, 2010, pp. 1–3.

[10] S. Yamamoto, K. Yonenaga, A. Sahara, F. Inuzuka, and A. Takada, “Achievement of Subchannel Frequency Spacing Less Than symbol Rate and Improvement of Dispersion Tolerance in Optical OFDM Transmission,” *IEEE Journal of Lightwave Technology*, vol. 28, no. 1, pp. 157–163, 2010.

[11] W. Kozek and A. Molisch, “Nonorthogonal pulseshapes for multicarrier communications in doubly dispersive channels,” *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1579–1589, 1998.

[12] T. Strohmer and S. Beaver, “Optimal OFDM system design through optimal sphere coverings,” *2001 IEEE International Conference on Acoustics, Speech, and Signal Processing, 2001. Proceedings. (ICASSP ’01)*, vol. 4, pp. 2373–2376 vol.4, 2001.

[13] ——, “Optimal OFDM design for time-frequency dispersive channels,” *IEEE Transactions on Communications*, vol. 51, no. 7, pp. 1111–1122, July 2003.

[14] A. Kliks, H. Bogucka, I. Stupia, and V. Lottici, "A Pragmatic Bit and Power Allocation Algorithm for NOFDM Signalling," in *Proc. IEEE Wireless Communications and Networking Conf. WCNC 2009*, 2009, pp. 1–6.

[15] I. Kanaras, A. Chorti, M. Rodrigues, and I. Darwazeh, "Spectrally Efficient FDM Signals: Bandwidth Gain at the Expense of Receiver Complexity," *IEEE International Conference on Communications. ICC 2009. Proceedings*, June 2009.

[16] S. I. A. Ahmed and I. Darwazeh, "IDFT Based Transmitters for Spectrally Efficient FDM System," in *London Communication Symposium*, Sep 2009.

[17] S. Isam and I. Darwazeh, "Simple DSP-IDFT Techniques for Generating Spectrally Efficient FDM Signals," in *7th International Symposium on Communication Systems, Networks and Digital Signal Processing*, July 2010.

[18] ——, "On the digital design of non-orthogonal spectrally efficient frequency division multiplexed (FDM) signals," in *ISBC'10*, Jul 2010.

[19] M. R. Perrett and I. Darwazeh, "Flexible hardware architecture of SEFDM transmitters with real-time non-orthogonal adjustment," *Proc. of Int. Conference on Telecommunications*, 2011.

[20] V. Pammer, Y. Delignon, W. Sawaya, and D. Boulinguez, "A low complexity suboptimal MIMO receiver: The combined ZF-MLD algorithm," *14th IEEE Proceedings on Personal, Indoor and Mobile Radio Communications, 2003. PIMRC 2003*, vol. 3, pp. 2271–2275 vol.3, 7-10 Sept. 2003.

[21] I. Kanaras, A. Chorti, M. Rodrigues, and I. Darwazeh, "A combined MMSE-ML detection for a spectrally efficient non orthogonal FDM signal," *5th International Conference on Broadband Communications, Networks and Systems, 2008. BROADNETS 2008.*, pp. 421–425, Sept. 2008.

[22] Y.-P. Lin and S.-M. Phoong, "MMSE OFDM and prefixed single carrier systems: BER analysis," in *International Conference on Acoustics, Speech, and Signal Processing, 2003, ICASSP '03, IEEE*, Hong Kong, Apr. 2003, pp. 229–232.

[23] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1639–1642, Jul 1999.

[24] W.-K. Ma, P.-C. Ching, and Z. Ding, "Semidefinite relaxation based multiuser detection for M-ary PSK multiuser systems," *IEEE Transactions on Signal Processing*, vol. 52, no. 10, pp. 2862–2872, Oct. 2004.

[25] I. Kanaras, A. Chorti, M. Rodrigues, and I. Darwazeh, "An Investigation of Semidefinite Programming Detection for a non orthogonal FDM system," *20th Personal, Indoor and Mobile Radio Communications Symposium 2009, IEEE PIMRC'09, Japan, Tokyo*, September 2009.

[26] ——, "A New Quasi-Optimal Detection Algorithm for a Non Orthogonal Spectrally Efficient FDM," in *9th International Symposium on Communications and Information Technologies 2009, IEEE ISCIT 2009, Incheon, Korea*, September 2009.

[27] I. Kanaras, A. Chorti, M. R. D. Rodrigues, and I. Darwazeh, "A Fast Constrained Sphere Decoder for Ill Conditioned Communication Systems," *IEEE Communications Letters*, vol. 14, no. 11, pp. 999–1001, 2010.

[28] S. Isam and I. Darwazeh, "Design and Performance Assessment of Fixed Complexity Spectrally Efficient FDM Receivers," in *IEEE 73rd Vehicular Technology Conference (IEEE VTC'11)*, 2011.

[29] S. Isam, I. Kanaras, and I. Darwazeh, "A Truncated SVD Approach for Fixed Complexity Spectrally Efficient FDM Receivers," in *IEEE Wireless Communications & Networking Conference (IEEE WCNC'11)*, 2011.

[30] B. Hassibi and H. Vikalo, "On the sphere-decoding algorithm I. Expected complexity," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 2806–2818, Aug. 2005.

[31] J. Jalden and B. Ottersten, "On the complexity of sphere decoding in digital communications," *IEEE Transactions on Signal Processing*, vol. 53, no. 4, pp. 1474–1484, April 2005.

[32] A. Chorti, I. Kanaras, M. R. D. Rodrigues, and I. Darwazeh, "Joint Channel Equalization and Detection of Spectrally Efficient FDM Signals," in *21th Personal, Indoor and Mobile Radio Communications Symposium 2010, IEEE PIMRC'10,*, Sep 2010.

[33] J. Mazo and H. Landau, "On the minimum distance problem for faster-than-Nyquist signaling," *IEEE Transactions on Information Theory*, vol. 34, no. 6, pp. 1420–1427, Nov 1988.

[34] F. Rusek and J. Anderson, "The two dimensional Mazo limit," in *Proc. Int. Symposium on Information Theory*, vol. 1, 2005, pp. 970–974.

[35] ——, "Multistream Faster than Nyquist Signaling," *IEEE Transactions on Communications*, vol. 57, no. 5, pp. 1329–1340, May 2009.

[36] S. Weinstein and P. Ebert, "Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform," *IEEE Transactions on Communications,*, vol. 19, no. 5, pp. 628–634, Oct 1971.

[37] M. R. Perrett and I. Darwazeh, "Flexible Hardware Architecture of SEFDM Transmitters with Real-Time Non-Orthogonal Adjustment," in *IEEE 18th International Conference on Telecommunications (IEEE ICT'11)*, 2011.

[38] P. N. Whatmough, M. Perret, S. Isam, and I. Darwazeh, "VLSI Architecture for a Reconfigurable Spectrally Efficient FDM Baseband Transmitter," in *IEEE International Symposium on Circuits and Systems (IEEE ISCAS'11)*, 2011.

[39] R. C. Grammenos, S. Isam, and I. Darwazeh, "FPGA Design of a Truncated SVD Based Receiver for the Detection of SEFDM Signals," in *22nd Personal, Indoor and Mobile Radio Communications Symposium 2011, IEEE PIMRC'11,*, 2011.

[40] S. Isam, "Spectrally efficient FDM communication signals and transceivers: Design, mathematical modeling and system optimization," Ph.D. dissertation, University College London, 2011.