Spectrum of quantum black holes and quasinormal modes

Jonathan Oppenheim
Racah Institute of Theoretical Physics, Hebrew University of Jerusalem, Givat Ram, Jerusalem 91904, Israel
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The spectrum of multiple level transitions of a quantum black hole is considered and the linewidths calculated. Initial evidence is found for these higher order transitions in the spectrum of quasinormal modes for Schwarzschild and Kerr black holes, further bolstering the idea that there exists a correspondence principle between quantum transitions and classical "ringing modes." Several puzzles are noted, including a fine-tuning problem between the linewidth and level degeneracy. A more general explanation is provided for why setting the Immirzi parameter of loop quantum gravity from the black hole spectrum necessarily gives the correct value for the black hole entropy.

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Although there is a lack of experimental data on which to base attempts to construct a quantum theory of gravity, it is commonly regarded that the theory must give a correct accounting for black hole entropy. The fact that black hole entropy is a quarter of the black hole area then plays the role of an experimental data point on which to test any theory. String theory gives the correct prediction for extremal black holes [1], and loop quantum gravity gives the entropy up to a proportionality constant [2,3] known as the Immirzi parameter. In general, the area scaling of entropy is rather generic for gravitating systems [4].

A less ambitious program involves attempting to quantize a black hole. As early as 1974, Bekenstein [5,6] made the case that the area A of a quantum black hole is quantized WITH equal spacing between levels,

$$A=a_0 n, \quad n=1,2,3\ \ldots, \tag{1}$$

in units where $G=\hbar=k_B=1$ and a_0 a constant [7]. This has been the standard starting point for the quantum black hole [8,9], as it is based on general arguments rather than on a particular model. Bekenstein argued that since classically the black hole's area is an adiabatic invariant, it should be quantized (following an insight of Ehrenfest [10]). Furthermore, for nonextremal black holes, he argued that the minimum change in area is independent of the black hole mass, charge, and angular momentum, which naturally leads to Eq. (1).

Even in his original paper (also [6]), Bekenstein noted that Bohr's correspondence principle implies that transitions in energy levels of a quantum black hole correspond to the black hole's quasinormal "ringing modes" (QNMs) [11]. For large n, one expects a quantum black hole to correspond to a classical black hole just as a quantized oscillator in the large mass limit should give the correct normal modes of a classical oscillator. Since the mass M of a black hole is given by $\sqrt{A/16\pi}$, the energy $\omega_0=\Delta M$ emitted when the black hole looses one area quantum is given by

$$\omega_0=\frac{a_0}{32\pi M}. \tag{2}$$

It was noted by Bekenstein and Mukhanov [8,12] that the constant a_0 should be 4 times the logarithm of a natural number if one is to interpret the quantum levels of the black hole as giving rise to the Bekenstein-Hawking entropy S:

$$S=\frac{1}{4}A. \tag{3}$$

We can use the fact that that the entropy of a black hole is $\ln g_n$ where g_n is the number of states at level n. Defining the ground state degeneracy as $k=g_1$, we can use Eqs. (1) and (3) to fix $a_0=4\ln k$ with k a natural number:

$$\omega_0=\frac{\ln k}{8\pi M}. \tag{4}$$

Hod [13] then noticed that the QNM spectrum for a Schwarzschild black hole had a frequency whose real part numerically approached Eq. (4) with $k=3$ in the limit of infinite imaginary frequency. Motl [14] later confirmed this analytically. In light of this, Dreyer [15] proposed changing the gauge group of loop quantum gravity from SU(2) to SO(3). He then advocated using the spacing of quasinormal modes to fix the undetermined Immirzi parameter. He argued that the value that fixes the energy spacing also yields the correct value for the black hole entropy, thus claiming black hole entropy a prediction of the theory. This has generated a great level of excitement in the field, and since then, a large number of studies have been conducted both to extend our understanding of quasinormal modes [16] and to further understand the quantum black hole in this context [17].

To learn about quantum black holes by studying the QNM structure of classical black holes is certainly a speculative undertaking. Nonetheless, given the highly intriguing numerical coincidences which are emerging and the lack of real experimental data on which to base a quantum theory of gravity, there is merit in taking the preceding arguments seriously and seeing how far they can be pushed. Certainly a study of the phonon modes of a solid would give one insight into their quantization. Whether the QNM spectrum of the black hole can be treated in the same way as Bohr treated experimental data from the hydrogen atom remains to be seen. In the remainder of the paper, I will essentially assume that such a correspondence holds.

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Thus far, researchers have only looked for transitions involving one area quantum and focused on the QNM spectrum in the limit of large damping. No convincing explanation exists as to why this should be the significant regime, although some suggestions have been made (e.g., [18]). If one believes that the quasinormal modes arise from the underlying quantum structure, then one expects that this correspondence principle should apply to all modes. There does not appear to be any reason to single out the highly damped modes as arising from the quantum structure and the rest of the modes as arising from some other structure. One is therefore forced to look for an explanation for the less damped mode as well.

Furthermore, there has been no attempt to link the imaginary part of the spectrum with the quantum black hole. Motl [14] has noted that the spacing of the imaginary part corresponds to the expected poles in the thermal Green's function. It is unclear why this correspondence also only appears at large damping or how it arises from the quantum structure of the black hole.

If one takes Bohr's correspondence principle seriously, it is natural that the linewidth of the quantum black hole would be associated with the imaginary part of the QNMs (since one expects classical damping or dispersion to correspond to the line broadening of the quantum transition [19]). I will thus first reexamine the quantum black hole and calculate the line broadening for multiple level transitions. One other observation is that one expects not only transitions in which the black hole jumps one level, but also higher level transitions in which the black hole jumps δ levels. Then, taking the correspondence between QNMs and quantum black holes seriously one expects to see QNMs with a real frequency of $\delta\omega_0$. Indeed, I then find that the QNM spectrum contains some evidence for multiple level transitions in addition to the single level transition so far observed. I will present data from both Schwarzschild and Kerr, which, although not as clean as the data in the asymptotic regime, show initial evidence for these multiple level transitions. With regard to the imaginary part of the QNM spectrum, the expected scaling is observed for the multiple level transitions, but several puzzles remain.

After presenting the data, I return to some theoretical aspects of the quantum black hole and note a fine-tuning problem which exists in the physics governing the line broadening of the spectrum. I then discuss a puzzle, particularly if $k=3$, concerning suppression of Hawking radiation. Finally, I note that there is a general explanation (in terms of the Bekenstein model) for why fixing the Immirzi parameter from the quantum black hole spectrum necessarily gives the correct result for the black hole entropy.

Let us consider a spontaneous emission process in which a black hole with n area quanta decays δ levels. If the probability per unit time of a spontaneous decay between two levels n and $n-\delta'$ is $p_{n,\delta'}$, then under the assumption that these transitions give rise to the thermal character of black hole radiation, one expects

$$\frac{p_{n,\delta'}}{p_{n,\delta}}=e^{\beta(\delta-\delta')\omega_0}\left(\frac{\delta'}{\delta}\right)^2, \tag{5}$$

where β is the inverse Hawking temperature,

$$\beta=8\pi M, \tag{6}$$

and the factor $(\delta'/\delta)^2$ comes from the phase space [e.g., $(\delta\omega_0)^2$] of the emitted radiation. One also gets such a probability distribution if one assumes that the degeneracy g_n is what dominates the transition. We can then use Eq. (4) to write

$$p_{n,\delta'}=k^{(1-\delta')}\delta'^2 p_{n,1}. \tag{7}$$

Then the total probability Γ_n per unit time for the decay of the nth level is

$$\Gamma_{n\delta}=\sum_{\delta'=1}^{n} p_{n,\delta'} \\ =\frac{p_{n,1}k^2(1+k)}{(k-1)^3}+O(n^2k^{-n}), \tag{8}$$

where we henceforth drop terms which are exponentially suppressed for large n. Using the methods of Weisskopf and Wigner [19,20], the linewidth $\gamma_{n\delta}$ of the transition from n to $n-\delta$ is given by

$$\gamma_{n\delta}=\Gamma_n+\Gamma_{n-\delta} \\ =\frac{k^2(1+k)}{(k-1)^3}(p_{n,1}+p_{n-\delta,1}), \tag{9}$$

while the difference between two linewidths of a black hole with fixed n is

$$\gamma_{n\delta}-\gamma_{n,\delta'}=\frac{k^2(1+k)}{(k-1)^3}(p_{n-\delta,1}-p_{n-\delta',1}). \tag{10}$$

From the Stefan-Boltzmann law and the Hawking temperature, one can see that the classical black hole evaporates its mass at a rate proportional to $1/M^2$ (i.e., $1/n$) [21]. Bekenstein and Mukhanov [12] have calculated the probability distribution of the quantum black hole to make various transitions. They use the classical result to fix the decay rate of single level transitions (which dominate). Were we to follow this reasoning, we could set the total luminosity to the classical result

$$\omega_0\sum_{\delta} p_{n\delta}\delta\propto 1/n, \tag{11}$$

where a constant of proportionality accounts for the noncontinuous nature of the spectrum and would depend on the particular particle being emitted (with different values of the proportionally constant being advocated by different authors [12,22,23]).

Using Eq. (7), this gives

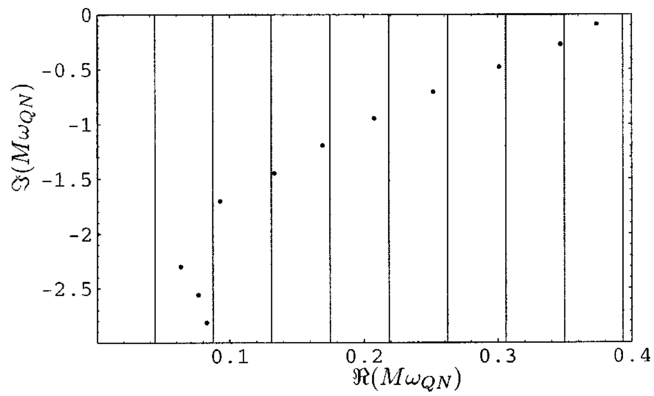

FIG. 1. Im($M\omega_{QN}$) vs Re($M\omega_{QN}$) for the Schwarzschild, $l=2$ quasinormal modes [24]. The vertical lines are the theoretically predicted values $M\delta\omega_0$.

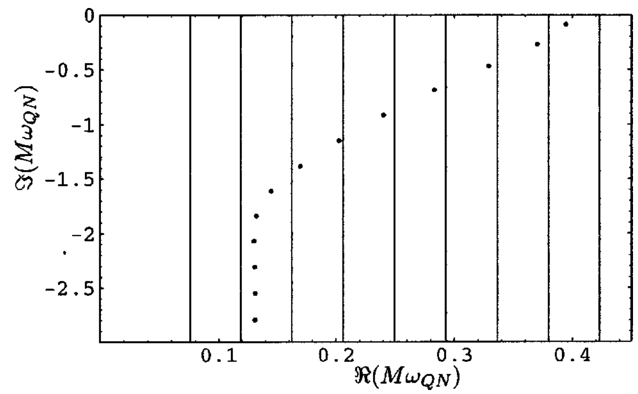

FIG. 2. Im($M\omega_{QN}$) vs Re($M\omega_{QN}$) for Kerr $J/M=0.15$, $l=m=2$ [25].

$$p_{n,1}=\frac{(k-1)^4}{k^2(1+4k+k^2)}\frac{b}{\sqrt{n\ln k}}, \tag{12}$$

with b being the constant of proportionality. This can now be substituted into Eq. (9) to give the line breadth of black hole transitions.

Now that we have the line breadth and energy spacing for multiple level transitions, let us turn to whether they are reflected in the QNM spectrum.

Figure 1 shows the gravitational perturbation with lowest angular momentum $l=2$. In addition to the level at infinite damping ($\delta=1$), one sees evidence for multiple transitions which occur close to the predicted values $\delta\omega_0$ (for $\delta=2,\ldots,9$). The QNMs which lie close to these theoretical predictions are the $n=1,\ldots,8$ QNM's which have increasingly larger imaginary part. The agreement with the theoretically predicted result is within 5%. Strongest disagreement occurs at the highest energy transition. The reason that the spectrum ends here is explained by the fact that one does not have $\delta\omega_0>l/3\sqrt{3}M$ (the peak of the black hole potential) since this is when the energy of the mode is larger than the peak of the black hole potential. At this energy, the mode must become either purely outgoing or purely ingoing (while QNMs are defined to be outgoing at infinity and "ingoing" at the horizon—i.e., falling into the black hole at the horizon).

The $n=9$ QNM is the "algebraically special" mode at $\text{Re}(\omega)=0$. Whether the latter mode is in fact a QNM is a matter of some debate [11]. Following this mode, the spectrum $n=10,11,12,\ldots$ gradually asymptotes to the $\delta=1$ line.

Since the imaginary part of the QNM should correspond to the linewidth of the quantum black hole, one expects the imaginary part of the QNM spectrum to be given by Eq. (9). It is perhaps encouraging that the higher order transitions do have an imaginary part which is proportional to $1/\sqrt{n}$ as Eq. (12) predicts. However, the spacing of Im(ω_{QN}) between different modes does not correspond to Eq. (10). The entire QNM spectrum scales like $1/M$ (i.e., $1/\sqrt{n}$). From Eqs. (9) and (10) we see that if $\gamma_{n\delta}$ scales like $1/M$, then the difference in linewidth between two successive transitions of different δ should essentially be the derivative of $\gamma_{n\delta}$ and one would expect Im(ω_{QN}) to have a term which scales like $-\delta/M^2$. This is not observed, although the slope of Im(ω_{QN}) does go in the expected direction in that higher order transitions are sharper. There are sets of modes which occur at roughly equal Im(ω_{QN}) as in Eq. (9), but these are at higher l.

The behavior of the QNMs which would correspond to multiple level transitions are in stark contrast to the QNM meant to correspond to the transition at ω_0. The latter is infinitely broad (occurring at infinite imaginary frequency) and surrounded by a huge degeneracy of other modes. While one can find many possible explanations for the splitting of the energy levels or to explain why particular transitions should be broad or narrow, I know of no general arguments which could consistently and convincingly explain the different behavior (witnessed in the QNM spectrum) between the first level transition and the multiple level ones.

While the $\delta=1$ QNM corresponds exactly to ω_0, the data for $\delta>1$ are not exact. This could be for a number of reasons. One expects the energy levels of the black hole to be shifted because of their coupling to fields. Additionally, it is not at all clear the extent to which QNMs are probing the structure of the black hole horizon. The considerations here are at best an approximation to the actual quantum structure of the black hole.

Preliminary analysis of perturbations of higher l show mixed results. The QNM data for Kerr, initially calculated by Leaver [24], also are less clear, having a very rich structure. Data from Ref. [25] are plotting in Figs. 2, 3, and 4. The theoretical prediction [5,6]

$$\delta\omega_0=T_H\ln k\,\delta+4\pi J/(MA)m \tag{13}$$

(with T_H the Hawking temperature and J the black hole's angular momentum) is also shown. Thus far, researchers have focused their attention on modes in the asymptotic regime, thus concluding that the Kerr spectrum does not show evidence for black hole quanta [since only the $4\pi J/(MA)m$ term is found in this limit] [18,25]. However, by taking into consideration the nonasymptotic part of the QNM spectrum, I would argue that the behavior of the quasinormal modes of

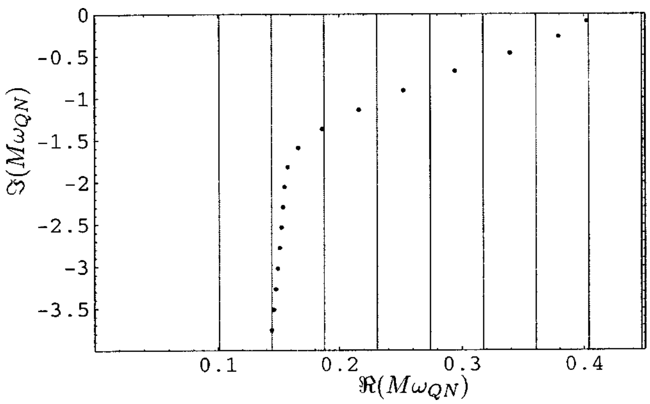

FIG. 3. Im($M\omega_{QN}$) vs Re($M\omega_{QN}$) for Kerr $J/M=0.2$, $l=m=2$ [25].

Kerr suggests the existence of transitions where the black hole area changes by some number of quanta.

One does observe fairly equal spacing as theoretically predicted, although (for example) the levels of $J/M=0.2$ occur at the half-tones. The behavior of these plots is fairly typical, with a set of modes at gradually slopping line broadening (imaginary part), followed by a sudden (and remarkable) change at the $\delta=1$ level. The spectrum then gradually asymptotes from the $\delta=1$ level to the $\delta=0$ level.

Let us now turn to two theoretical aspects of the quantum black hole which do not depend on the QNM spectrum but on which the QNM spectrum might shed light. First, there is an interesting fine-tuning problem with expression (12) which is worth noting. As explained in the discussion preceding this equation, if one wants $p_{n,1}$ to agree with the classical result, then we require that it scale like $1/\sqrt{n}$. On the other hand, if we assume that we can apply Fermi's golden rule to the black hole (this only requires that the decay be governed by some transition Hamiltonian), then we seem to get different behavior. Namely,

$$p_{n,1}=(2\pi)^2\omega_0^2 g_n T_{n,n-1}^2$$

$$=\frac{4\pi k^n \ln k}{n} T_{n,n-1}^2, \qquad (14)$$

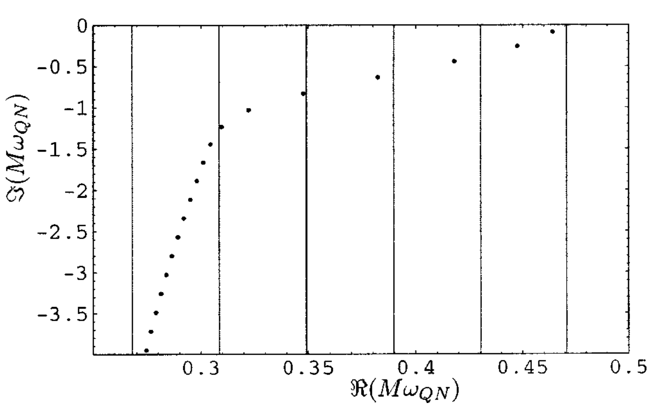

FIG. 4. Im($M\omega_{QN}$) vs Re($M\omega_{QN}$) for Kerr $J/M=0.5$, $l=m=2$ [25].

where $T_{n,n-1}$ is the transition matrix from the nth black hole state to the $(n-1)$th state. Since presumably the strength $T_{n,n-1}$ of the transition matrix and the degeneracy g_n of the levels are independent of each other, it is rather surprising that they should conspire in precisely the exact way to give Eq. (12). One possibility is that something like Eq. (14) is correct and that some other processes are involved in determining the classical emission rate. This would then explain the fact that the real part of the QNM spectrum approaches ω_0 only in the limit of infinite damping, since here the absorption time is exponentially fast in n. Such a mechanism, however, would not explain why the higher level transitions do have a lifetime of $1/\sqrt{n}$ according to the QNM spectrum. There is a natural model to circumvent this problem: assume the that decay is dominated by transitions with little change in the degrees of freedom associated with each quanta. If we label the k degrees of freedom of each area quanta by s_i, then it is rather natural to regard a transition as the disappearance of a single quantum where none of the other quanta change s_i, i.e., the other area quanta remain as passive observers of the transition. This leads to an effective degeneracy of the transition of n rather than k^n, since this process can occur by any of the quanta being annihilated. This would not affect the entropy, since there are still k^n possible states. It is arguably also more simple than the Bekenstein-Mukhanov transition, since only one quantum is involved in each decay, rather than a large collective process which involves the entire n quanta. $T_{n(n-1)}^2$ would then just have to behave like $1/M$, which rather naturally occurs in simple harmonic-oscillator-type transitions. However, this model has the disadvantage that it is harder to explain the thermal character of the emitted radiation.

Another interesting puzzle worth pointing out puts into question the thermal character of the radiation of the quantum black hole. If $k=3$ (as is popularly supposed), then the thermal emission of the classical black hole will be substantially suppressed. This is because the smallest possible emission (corresponding to ω_0) occurs at an energy almost identical to $1/\beta$. The Hawking radiation of this quanta is therefore suppressed by an amount $1/e$. Higher level transitions, such as those we have discussed, will be exponentially suppressed. Most of the Hawking emission will therefore occur at a single frequency. This also occurs for $k=2$ although to a lesser extent. The fact that these higher level transitions are so weak, lying outside the peak of the thermal spectrum, might play a role in explaining the difference between these levels, which are sharp, and the huge degeneracy of broad levels which occur at large imaginary part. This is in addition to the well-known issue that the Hawking spectrum is continuous while the Bekenstein model gives a discrete spectrum.

Finally, we address Dreyer's proposal to change the gauge group of loop quantum gravity (LQG) from SU(2) to SO(3) in light of the QNM spectrum. The proposal is to fix the Immirzi parameter γ using ω_0. This is viewed as giving an independent way to fix γ (instead of using the black hole entropy), and therefore the fact that it also gives the correct value for the black hole entropy is viewed as a prediction of the theory. We now give an explanation for this which relies on rather general arguments. I suggest that the ambiguity of γ still remains, but the theory does become more testable.

First, we note that Dreyer's arguments do not depend on the details of LQG, except insofar as LQG is believed to be consistent with the Bekenstein model. Namely, (i) LQG gives Eq. (1) with $a_0 = 8\pi l_P^2 \gamma \sqrt{j_{min}(j_{min}+1)}$ and j_{min} the minimum allowed spin of the spin-network edges which puncture the surface of the horizon (although initial conclusions were that the area spectrum was not evenly spaced [26]). (ii) In LQG each area quantum contributes a set amount of entropy $\ln k$ [with $k=(2j_{min}+1)$]. (iii) Dreyer assumes that black hole emission is given by the disappearance of one of these punctures (i.e., a decrease in n). These are precisely the same conditions that gave rise to Eq. (2).

The Bekenstein model has two undetermined parameters a_0 and k which one can fix by setting a_0 to match the black hole entropy—i.e., $a_0 = 4\log k$—and then perhaps fixing k from the QNM spectrum. Likewise, in LQG, one can first set γ to give $a_0 = 4\log k$ (as was previously done, although for a fixed k) and then set $k=3$ to match the QNM data. Here, one sees that LQG has two undetermined parameters which must be set to the data. This way of setting the parameters is physically equivalent to Dreyer's method; just the order is reversed.

However, what makes Dreyer's result very interesting is that it does provide a potential test for LQG—namely, the extent to which k can arise naturally. It might have been that one could not have three degrees of freedom per puncture; thus, one hurdle has already been cleared. Although a number of hurdles remain, strong arguments in favor of $k=3$ would provide a boost to LQG. Presumably, j_{min} is more tightly constrained than γ, making the prospect for constraining the theory in this regard brighter.

Furthermore, the fact that the QNM spectrum seems to fit with Bekenstein's prediction supports the equal area spacing model and, indeed, any quantum theory of gravity which gives rise to the same spectrum [such as LQG with assumption (iii)]. The fact that one finds some evidence for multiple level transitions further bolsters this contention, although the evidence is not unambiguous. Certainly one should retain a degree of healthy skepticism about the project of making predictions using the QNM spectrum. A number of puzzles still remain, and regardless of the QNM spectrum, we have seen that there are many open questions concerning the quantization of black holes which can perhaps serve as a guide in constructing a quantum theory of gravity.

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[1] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).

[2] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998).

[3] C. Rovelli, Phys. Rev. Lett. **77**, 3288 (1996).

[4] J. Oppenheim, Phys. Rev. D **65**, 024020 (2002).

[5] J. Bekenstein, Lett. Nuovo Cimento Soc. Ital. Fis. **11**, 467 (1974).

[6] For a review see J. Bekenstein, in *Proceedings of the 8th Marcel Grossmann Meeting on General Relativity*, edited by T. Piran (World Scientific, Singapore, 1998), p. 92.

[7] n should not be confused with the order of the quasinormal modes.

[8] V. Mukhanov, JETP Lett. **44**, 63 (1986); in *Complexity, Entropy and the Physics of Information*, edited by W. Zurek (Addison-Wesley, New York, 1990), Vol. 3.

[9] See, for example, I. Kogan, JETP Lett. **44**, 267 (1986); hep-th/9412232; C. Lousto, Phys. Rev. D **51**, 1733 (1995); M. Maggiore, Nucl. Phys. **B429**, 115 (1994); H. Kastrup, Phys. Lett. B **385**, 75 (1996); J. Makela, *ibid.* **390**, 115 (1997); J. Bekenstein and G. Gour, Phys. Rev. D **66**, 024005 (2002); A. Barvinksy, S. Das, and G. Kunstatter, Phys. Lett. B **517**, 415 (2001); J. Louko and J. Makela, Phys. Rev. D **54**, 4982 (1996); G. Gilad and A.J.M. Medved, Class. Quantum Grav. **20**, 1661 (2003); T. Padmanabhan and A. Patel, hep-th/0305165; T. Padmanabhan, Class. Quantum Grav. **4**, L107 (1987).

[10] P. Ehrenfest, work mentioned in M. Born, *Atomic Physics* (Blacki, London, 1969).

[11] For a review see K. Kokkotas and B. Schmidt, Living Rev. Relativ. **2**, 2 (1999) or H.P. Nollert, Class. Quantum Grav. **16**, R159 (1999).

[12] J. Bekenstein and V. Mukhanov, Phys. Lett. B **360**, 7 (1995).

[13] S. Hod, Phys. Rev. Lett. **81**, 4293 (1998).

[14] L. Motl, gr-qc/0212096.

[15] O. Dreyer, Phys. Rev. Lett. **90**, 081301 (2003).

[16] See, for example, recent work such as V. Cardoso, R. Konoplya, and J. Lemos, Phys. Rev. D **68**, 044024 (2003); V. Cardoso and J. Lemos, *ibid.* **67**, 084020 (2003); A. Neitzke, hep-th/0304080; K. Glampedakis and N. Andersson, Class. Quantum Grav. **20**, 3441 (2003); C. Molina, Phys. Rev. D **68**, 064007 (2003); E. Berti and K. Kokkotas, *ibid.* **67**, 064020 (2003); R. Konoplya, *ibid.* **68**, 024018 (2003); S. Musiri and G. Siopsis, Phys. Lett. B **563**, 102 (2003); A. Maassen van den Brink, Phys. Rev. D **68**, 047501 (2003); P.T. Leung, A. Maassen van den Brink, K.W. Mak, and K. Young, gr-qc/0307024; N. Andersson and C.J. Howls, gr-qc/0307020.

[17] See, for example, J. Baez, Nature (London) **421**, 703 (2003); G. Kunstatter, Phys. Rev. Lett. **90**, 161301 (2003); D. Birmingham, S. Carlip, and Y. Chen, hep-th/0305113; E. Berti and K. Kokkotas, Phys. Rev. D **68**, 044027 (2003); A. Polychronakos, *ibid.* **69**, 044010 (2004); A. Corichi, Phys. Rev. D **67**, 087502

(2003); A. Berezin, Nucl. Phys. **B66**, 409 (2003); R. Kaul and S. Rama, Phys. Rev. D **68**, 024001 (2003); A. Alekseev, A. Polychronakos, and M. Smedbck, Phys. Lett. B **574**, 296 (2003).

[18] S. Hod, gr-qc/0307060.

[19] For a review of the theory of the natural line breadth, see, for example, W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, 1939).

[20] V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1930).

[21] D. Page, Phys. Rev. D **13**, 198 (1976).

[22] J. Makela, Phys. Rev. D **54**, 4982 (1996).

[23] S. Hod, Phys. Lett. A **299**, 144 (2002).

[24] E. Leaver, Proc. R. Soc. London **A402**, 285 (1985).

[25] E. Berti, V. Cardoso, K. Kokkotas, and H. Onozawa, Phys. Rev. D **68**, 124018 (2003).

[26] C. Rovelli and L. Smolin, Nucl. Phys. **B442**, 593 (1995); A. Ashtekar and J. Lewandowski, Class. Quantum Grav. **14**, A55 (1997); L. Lenher, S. Frittelli, and C. Rovelli, *ibid.* **13**, 2921 (1996); R. Loll, Phys. Rev. Lett. **75**, 3048 (1995); Nucl. Phys. **B460**, 143 (1996).