Wisdom Mathematics

Nicholas Maxwell
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For over thirty years I have argued that all branches of science and scholarship would have both their intellectual and humanitarian value enhanced if pursued in accordance with the edicts of wisdom-inquiry rather than knowledge-inquiry.

What, then, about mathematics? How would the intellectual and human value of mathematics benefit from being pursued within the framework of wisdom-inquiry? Is it not wildly implausible to suppose that this august field of mathematics could somehow benefit from a dose of wisdom-inquiry? Would it become more rigorous? Or more useful? Would wisdom-inquiry help mathematicians prove theorems, or deploy their mathematical results in wiser ways?

Is not mathematics, in any case, almost paradigmatic of knowledge-inquiry? It is here, after all, in pure mathematics, that we have proven knowledge, secure knowledge, something we do not have in any other field (except logic, itself perhaps a branch of mathematics). It almost looks as if mathematics is a counter-example to, a refutation of, my general thesis.

A second look, however, might incur some doubts. It may begin to seem highly implausible that pure mathematics can be regarded as a branch of knowledge at all.

Problems of Platonism

Suppose we ask: What is pure mathematics knowledge *about*? One answer is Platonism: mathematics embodies knowledge of the abstract entities it purports to be about: number, spaces (of various kinds), groups, fields, functional relationships - ultimately, perhaps, sets and relationships on sets. Immediately, something very odd arises. No one has ever seen any of these abstract mathematical entities. We have no real evidence for the existence of these entities whatsoever. How is it possible for there to be absolutely secure, proven knowledge of entities which we have no grounds whatsoever to hold exist? Does not Platonism demand that mathematics (and in what follows I mean pure mathematics) is held to be wildly speculative and conjectural - far more so than the wilder flights of theoretical physics?

Plato might have replied that mathematicians do directly "see" these mathematical entities with the mind's eye – proofs helping the mind's eye to see more clearly. But this kind of intellectual intuitionism is hardly very plausible even when put forward in the context of the mathematics of Plato's day – elements of Euclidean geometry (to speak somewhat anachronistically). It is wildly implausible when put forward in the context of modern mathematics, with its extremely abstract entities that resist all attempts at visualization, and with notions of proof that seem to have little to do with aiding mental visualization. Mathematicians may develop mental images associated with the mathematics they work in, and these images may have a certain heuristic value, but mathematical results can hardly be said to be about these images, proofs acquiring their certainty from the fact that mathematicians "see" these entities with the mind's eye. Either mathematics is about mental images per se, or it is about independently existing non-mental entities of which we form mental images. If the former, mathematics has to be put alongside descriptions of other imaginings, dreams and daydreams, as having the same character and epistemological status, a branch of phenomenology or psychology. If the latter, imagining such entities can provide no grounds whatsoever for holding that these entities really do exist: mathematics, interpreted to be about such entities (for whose existence there is no evidence whatsoever) would be irredeemably speculative. Platonism must plump for one or other option, but neither does justice to the actual nature of mathematics. Platonism is, it seems, untenable.

Alternatives to Plato

Much of 20th century philosophy of mathematics has been concerned to find an alternative to Platonism, one which rescues the idea that mathematics is a branch of secure, proven knowledge from the collapse of Platonism. All these attempts, in my view, fail.

The best known, perhaps, is the logicism of Frege, Russell and Whitehead. This holds that mathematics is an elaboration of logic. Logicism is generally held to fail for technical reasons. In deriving mathematics from logic, Russell and Whitehead were obliged to introduce postulates that could hardly be judged to be a part of logic. There is in my view a very much more serious objection to logicism that is never mentioned in the literature. Logicism, if successful, would reveal mathematics to be utterly intellectually disreputable. For, to put it bluntly, it would reveal that mathematics amounts to nothing more than increasingly intricate, obfuscating ways of asserting “p or not p” (which one may take to be a simple, paradigmatic truth of classical logic). What could be more intellectually disreputable? An elementary principle of intellectual integrity is that one says what one has to say in as simple, transparent a way as possible. All of mathematics would violate this principle horribly, if logicism were correct.

A modified version of this view, which might be attributed to Cantor or, perhaps with more justice, to the composite French mathematician Bourbaki, holds that mathematics is just elaborations of set theory. This is more plausible. A great deal of mathematics is formulated, at a fundamental level, in the language of set theory. I will criticize this view, briefly, later on.

Another attempt to rescue mathematics as knowledge from the downfall of Platonism is intuitionism. This can be attributed to L. E. J. Brouwer. According to intuitionism, mathematics is to be interpreted as being about, and embodying knowledge of, our mental constructs. Intuitionism is of some interests to mathematicians because it rejects “p or not p” of classical logic, and regards reductio ad absurdum proofs as invalid. It is of technical interest to see how much of classical mathematics can be derived from the impoverished means of intuitionism. As a view about the nature of mathematics, however, intuitionism seems straightforwardly untenable, for the reasons given above.

Another, almost desperate, attempt to construe mathematics as knowledge after the downfall of Platonism goes by the name of formalism. According to formalism, mathematics consists of nothing more than uninterpreted symbols, as written down on the page, manipulated by means of specified rules. Formalism hardly succeeds in doing justice to the profound significance and value of mathematics. Nor does it, in the end, succeed in representing mathematics as knowledge. Formalism is often attributed to David Hilbert but, in my view, this is a mistake. Hilbert held that it was useful to regard axiomatic systems as uninterpreted systems of symbols manipulated by precise rules – in order to prove meta-theorems about such systems, such as those having to do with consistency and completeness. But this does not mean Hilbert held formalism to give the correct account of mathematics. When told a mathematician had given up mathematics to write novels, Hilbert remarked "Ah, he did not have enough imagination to be a mathematician" – hardly the comment of a formalist.

Attempts to construe mathematics as a branch of knowledge have not, it seems, met with great success. The paradigmatic case of knowledge looks, on closer inspection, rather less clear cut than one might suppose.

Wisdom-Inquiry Mathematics

Reject knowledge-inquiry and accept wisdom-inquiry instead, and we are no longer obliged to construe mathematics as a branch of knowledge. What, then, is it? I suggest that we should see mathematics as the enterprise of developing and unifying problem-solving methods, the enterprise of exploring and delineating problematic possibilities. Mathematics is not about anything actual; it is about (problematic) possibilities. Given a piece of axiomatized mathematics - Euclidean geometry say - what matters is not whether anything, X, actually exists - such as physical space - which is such that the axioms and theorems of Euclidean geometry, when interpreted to be about X, are true of X. What matters, rather, is that *if* anything, X say, exists which is such that when the axioms of Euclidean geometry are interpreted as being about X they are true of X, *then* the theorems of Euclidean geometry are true of X as well. That is what matters to the mathematician. Not that any such X exists, but if such an X exists, the theorems will be true of X (granted that the axioms are).

Pure mathematics does not embody knowledge of anything. Rather, it is a treasure trove of inter-related problem-solving methods, highly significant and useful for a variety of reasons and purposes, a systematic survey of significant problematic possibilities. Mathematics is meaningful but indifferent, at a formal level, as to whether anything actually exists which makes it true. This view, incidentally, does justice to Hilbert's remark about imagination. One needs imagination in order to see the possibilities that a piece of mathematics would be about were these possibilities to exist in actuality.

The Problem of Mathematical Significance

My view is that 20th century philosophy of mathematics has been preoccupied with the wrong problems. There is a fundamental problem that has been ignored, namely: How do we distinguish between significant and insignificant mathematics? One could imagine endlessly many branches of mathematics existing corresponding, for example, to various board games like drafts and chess. One would have theorems stating: given such and such a position, the shortest number of moves required for mate by white is six. This kind of mathematics is insignificant, and is to be contrasted with what G. H. Hardy would call "real" mathematics: number theory, analysis, geometry, algebra, topology, and so on. What is the basis for this distinction? Given the modern proliferation of specialized kinds of mathematics which many mathematicians regard as "trivial" or insignificant, and the danger of mathematics being swamped by this sort of thing, this problem of significant mathematics is of practical importance for mathematics itself, as well as being important for our understanding of the nature of mathematics.

One could think that Platonism attempts to solve the problem. Significant mathematics is that part of mathematics which is about real, Platonic, existing mathematical entities, while insignificant mathematics is insignificant because it is not about anything. (Roger Penrose holds a version of this view: see his *The Road to Reality*.) But this attempted solution does not work. We have no reason whatsoever for holding that those and only those entities corresponding to significant mathematics actually exist. Besides, significance is a matter of degree, and may well be multi-faceted, whereas the distinction existence/non-existence is sharp, absolute, and uni-faceted.

In order to solve the problem we need to bring in values, and relate mathematics to values. My criticism of knowledge-inquiry is that it suppresses highly problematic, influential assumptions concerning metaphysics, values and politics. This is true of physics, and natural science more generally. And it is true of mathematics.

If we view mathematics from a knowledge-inquiry perspective, rigour seems to require that anything as irrational, or non-rational, as values must be excluded from mathematics. Allowing values to influence what goes on in mathematics could, it seems, only subvert mathematical rigour.

But viewed from a wisdom-inquiry perspective, it is all the other way round. We need to bring values into mathematics in order to make sense of, and improve, our judgements about what is mathematically significant and insignificant. If we exclude consideration of values from mathematics, we deprive ourselves of any rationale for making the distinction. It will become a mere matter of subjective taste - more or less the situation today.

So what is the solution to the problem of mathematical significance? It is vital to remember the links between mathematics and life. Mathematics begins with the discovery that a problem (or set of problems) in one area of life or activity is similar in certain respects to a problem (or set of problems) in another, possibly apparently very different area - so that solutions to problems in one field can be used to solve problems in the other field. An early example of this is the discovery that problems connected with counting sheep are similar to counting people, stones, or twigs. Another early, but mathematically much more profound, example is the discovery that problems connected with counting are, in some respects, similar to problems connected with measuring lengths, areas and volumes. This led to the discovery of irrational numbers. There is also Fermat's and Descartes' discovery that geometrical problems and algebraic problems can be interconnected (via Cartesian coordinates). Much of the power of mathematics resides from this feature, that a problem that may be insuperably difficult to solve in one field becomes, when translated into an equivalent problem in another field, much easier to solve - even solvable by means of standard methods. The problem-solving power of mathematics is enormously enhanced as a result of its multi-faceted interconnectedness. I am inclined to conjecture that one of the important functions of set theory may be, in providing something like a common language for mathematics, to facilitate this interconnectedness. (Mathematics should not be characterized as elaborations of set theory, but nevertheless set theory provides a common language for much of mathematics, which is of great value because it facilitates the vital inter-connectedness of mathematics.)

This, to my mind, is of the essence of mathematics. It is, as I have said, about the development and unification (or inter-relation) of problem-solving methods, the seeing of problematic possibilities related to actual problems we tackle in life.

Very, very crudely, then, we can say this. A new piece of mathematics will be significant to the extent that it satisfies two requirements: (a) it links up to the interconnected body of existing mathematics, ideally in such a way that some problems difficult to solve in other branches become much easier to solve when translated into the new piece of mathematics; (b) it has fruitful applications for (other) worthwhile human endeavours. A new piece of mathematics might well be judged to be significant even though it met only one of these two requirements. If it meets both, all the better. If it meets neither, its champions will have to struggle to convince their fellow mathematicians that what they are doing is significant mathematics.

Linking mathematics up to the problems it is designed to solve - whether practical or from some other branch of mathematics - is important, both for teaching, and in order to help clarify the nature of mathematics, and what matters and what does not within mathematics.

Conclusion

The transition from knowledge-inquiry to wisdom-inquiry does, then, have fruitful implications for mathematics. Viewed from the perspective of knowledge-inquiry, mathematics confronts us with two fundamental problems. (1) How can mathematics be held to be a branch of knowledge, in view of the difficulties that view engenders? What could mathematics be knowledge *about*? (2) How do we distinguish significant from insignificant mathematics? This is a fundamental philosophical problem concerning the nature of mathematics. But it is also a practical problem concerning mathematics itself. In the absence of the solution to the problem, there is the danger that genuinely significant mathematics will be lost among the unchecked growth of a mass of insignificant mathematics. This second problem cannot, it would seem, be solved granted knowledge-inquiry. For, in order to solve the problem, mathematics needs to be related to values, but this is, it seems, prohibited by knowledge-inquiry because it could only lead to the subversion of mathematical rigour.

Both problems are solved, however, when mathematics is viewed from the perspective of wisdom-inquiry. (1) Mathematics is not a branch of knowledge. It is a body of systematized, unified and inter-connected problem-solving methods, a body of problematic possibilities. (2) A piece of mathematics is significant if (a) it links up to the interconnected body of existing mathematics, ideally in such a way that some problems difficult to solve in other branches become much easier to solve when translated into the piece of mathematics in question; (b) it has fruitful applications for (other) worthwhile human endeavours.

If ever the revolution from knowledge to wisdom occurs, I would hope wisdom mathematics would flourish, the nature of mathematics would become much more transparent, more pupils and students would come to appreciate the fascination of mathematics, and it would be easier to discern what is genuinely significant in mathematics (something that baffled even Einstein). As a result of clarifying what should count as significant, the pursuit of wisdom mathematics might even lead to the development of significant new mathematics.