Indispensability Arguments and Mathematical Explanation in Science

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I, Josephine Marie Salverda, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

The aim of this thesis is to find a way to undermine the indispensability argument for mathematical platonism. In chapter 1 I provide a brief survey of the indispensability argument, arguing that the explanatory indispensability argument is stronger than earlier forms of the argument. This is because it has less controversial premises, appealing neither to confirmational holism nor to a strong naturalism but rather to inference to the best explanation, a principle of inference which both sides in the indispensability debate are taken to accept. Hence I take the explanatory indispensability argument as my target.

In chapter 2, I provide a more detailed account of the way in which inference to the best explanation, or IBE, is involved in the explanatory indispensability argument. I present two readings of the argument, rejecting the first reading and arguing that a second reading, which involves an instance of IBE, is the most plausible.

Chapter 3 considers whether there are genuine cases of mathematical explanation in science, focusing on an explanation from evolutionary biology provided by Alan Baker. I draw on the biological literature to argue that there is some reason to doubt that Baker's explanation meets the conditions for a successful application of IBE.

In chapter 4 I examine a number of restrictions on IBE recently suggested in the indispensability debate. Firstly, I argue that the indexing account suggests a reasonable restriction on IBE, but that proponents of the indexing account have not yet shown that this restriction is successful in undermining the explanatory indispensability argument. Secondly, I examine a restriction on IBE proposed by Pincock, arguing that this restriction is also unsuccessful in blocking the support of mathematical claims through IBE. Thirdly, I propose a restriction on IBE motivated by scientific practice and which, I argue, successfully undermines the explanatory indispensability argument.

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Introduction

In this thesis, I search for a way to undermine the indispensability argument for mathematical platonism, a philosophical position which holds that there are abstract mathematical objects. The main opponent of the indispensability argument is the nominalist, who claims that there are no mathematical objects. Although I aim to undermine an important argument for mathematical platonism, my thesis does not serve as an argument for nominalism, since platonism and nominalism could both be incorrect.

Since my aim is to undermine the indispensability argument, it will be advantageous first to locate a strong form of the indispensability argument, by which I mean a version of the indispensability argument with the fewest and least controversial premises, so that my arguments will have maximum effect.

In chapter 1 I provide a brief survey of the indispensability argument, which allows me to locate a form of the argument known as the explanatory indispensability argument, which does not appeal to confirmational holism or to a strong reading of naturalism. Since these premises are controversial, an argument that does without these premises is more difficult to undermine. The explanatory indispensability argument involves an implicit appeal to inference to the best explanation, a principle of inference which both sides in the indispensability debate are taken to accept. This suggests that the explanatory indispensability argument will be difficult for the nominalist to undermine. I will thus have located a version of the indispensability argument which is particularly difficult to undermine, and which I will take as my target in the rest of the thesis. Since the explanatory indispensability argument is the subject of much recent discussion in the literature, my thesis also functions as a survey of recent attempts to undermine the explanatory indispensability argument.

In chapter 2, I will clarify the way in which inference to the best explanation, or IBE, is involved in the explanatory indispensability argument. In most formulations of the explanatory indispensability argument the appeal to IBE is implicit. However, since inference to the best explanation is taken to be central in the recent indispensability debate, it will be useful to understand the role played by IBE in the explanatory indispensability argument in more detail. Therefore I will first provide a basic account of inference to the best explanation.

Then, I will examine an application of IBE in the No Miracles argument for scientific realism. Platonists sometimes appeal to scientific realism in support of the explanatory indispensability argument, because, as I will argue, accepting scientific realism involves endorsing inference to the best explanation. However, the talk of scientific realism is often misleading and I will argue that it is best dropped. Hence I will motivate a version of the explanatory indispensability argument which does not rely on scientific realism, but involves an instance of inference to the best explanation. According to this argument, the existence of a mathematical explanation in science motivates the claim that we ought rationally to believe in the existence of mathematical entities.

In chapter 3, therefore, I examine an example of mathematical explanation in science drawn from evolutionary biology. This example, provided by Alan Baker, has been widely discussed in the recent literature. I will argue that any attempts to show that Baker has not provided a genuine case of mathematical explanation in science must be made on a scientific basis, and, for this reason, I will argue that various nominalistically acceptable alternatives fail. I will appeal to the biological literature to argue that there is some evidence for a lack of consensus on Baker's explanation amongst biologists, which suggests that Baker's example needs further scientific defence.

Nevertheless, since further examples of mathematical explanation in science have been provided in science, and I wish to avoid a case-by-case study of mathematical application in science, I will seek a more general strategy of undermining the explanatory indispensability argument. In chapter 4, I examine three restrictions on IBE, proposed in order to block the instance of IBE that is involved in the explanatory indispensability argument.

First, I will consider the indexing account, which, I will argue, involves a reasonable restriction on IBE. However, I will argue that proponents of the indexing account have not yet shown that this restriction blocks the support of mathematical claims via IBE. I then examine a second restriction proposed by Pincock, which I will argue is also unsuccessful. Finally, I propose a restriction on IBE which is motivated by scientific practice, and which, I argue, is successful in blocking the instance of IBE involved in the explanatory indispensability argument. Therefore, I can claim to have succeeded in my aim to undermine the explanatory indispensability argument.

Chapter 1: Indispensability arguments for mathematical platonism

Introduction

In this chapter, I will give a basic account of indispensability arguments for mathematical platonism. I will look at the premises of such arguments and examine which of these premises are required for the argument to go through. My aim is to find a strong version of the indispensability argument, meaning a version which is difficult to undermine. In general, since less controversial premises help to make an argument stronger than controversial premises, I aim to isolate a form of indispensability argument with the fewest and least controversial premises. The idea is for each premise to be as 'weak' as possible, in the sense that the premise claims no more than it must for the argument to go through.

In section 1.1 I will present a basic form of indispensability argument for mathematical platonism in order to introduce the players in the debate. In this form, the argument has two premises: one indispensability claim and one 'naturalist' claim about belief in science. In section 1.2, I argue that the indispensability premise cannot be weakened, since dropping the indispensability requirement leaves the argument invalid. I then discuss the second, naturalist premise in section 1.3, and argue following Dieveney that on a sufficiently strong reading of this premise, a third premise of confirmational holism often taken to be implicit in the argument is superfluous, even in response to a separation objection.

Since I want to avoid strong, or controversial, premises, I then examine in section 1.4 an enhanced form of indispensability argument due to Baker that does without either confirmational holism or strong naturalism, while still answering the separation objection. This 'enhanced indispensability argument' focuses on the explanatory role of mathematics in science, and following Busch I argue that this enhanced version is essentially an appeal to inference to the best explanation (IBE).

The 'IBE version' of the indispensability argument has two premises, an indispensability premise as before and a premise which reads as an endorsement of IBE. In section 1.5 I argue that the indispensability premise cannot be weakened, and note that the 'IBE premise' seems at first sight to be granted by both sides in the debate. Since both premises will be difficult to deny, I can claim to have found a strong form of the indispensability argument, which will be my target in the rest of the thesis.

1.1: Setting up the debate

The debate surrounding indispensability arguments in the literature takes place between platonists and nominalists. I will take platonism, or mathematical realism, to be the view that there are (at least some) abstract mathematical objects. Nominalism, or anti-realism, is taken to label any view holding that there are no mathematical objects.

Platonism/realism: there are at least some abstract mathematical objects.

Nominalism/anti-realism: there are no mathematical objects.

Roughly speaking, indispensability arguments hold that quantification over mathematical objects is indispensable to our best scientific theories, and, claiming that we should rationally believe our best scientific theories to be true or approximately true, conclude that we should rationally believe in the existence of mathematical objects[1]. That is:

i. (Indispensability) Quantification over mathematical objects is indispensable to our best scientific theories.

ii. (Naturalism) We ought rationally to believe our best scientific theories to be true.

iii. (Platonist conclusion) We ought rationally to believe in the existence of mathematical objects.

Accepting this conclusion would be inconsistent with the nominalist position.

Now, there is substantial variation in the positions held on each side of the debate. On the platonist side, philosophers range from advocating 'plenitudinous platonism', according to which all possible mathematical objects exist [Balaguer 1998], and more restrictive forms of platonism, for example claiming only that enough mathematical objects exist to make most of our mathematical statements literally true [Baker 2003]. I will keep the platonist claim in its current weak form so that most platonists agree with it, allowing the discussion to have maximum applicability. Additionally, it is best for an argument to claim only as much as it needs to in order to convince or defeat the opposition.

Within nominalism, accounts vary depending on the semantic claims which are added to the metaphysical thesis that there are no mathematical objects. Note that the metaphysical claim that there are no mathematical objects entails that mathematics, on a face-value reading, is false: this is because our mathematical statements seem to refer to mathematical objects, e.g. 'there are

1 For example, Baker writes that '[the indispensability] argument claims that we ought rationally to believe in the existence of mathematical objects because we ought to believe our best available scientific theories, and quantification over mathematical objects is indispensable for science' [Baker 2005: 223].

infinitely many prime numbers'. Some nominalists provide an alternative semantics for mathematics, for example claiming that our mathematical statements have a real content as well as a literal content, where the real content is the 'conventional' content of the statement and consists of a logical truth [Yablo 2002: 230]. Others accept the conclusion that mathematics is false, and hold that mathematical objects are merely useful fictions which allow mathematics to be empirically useful without being true [e.g. Leng 2010]. For the purposes of this thesis, I will simply consider the nominalist position as resisting the platonist claim that there are abstract mathematical objects.

So far, though, I have not clarified what is meant by the term 'abstract object'. By 'abstract' I mean acausal and non-spatiotemporally located. Since I take it that most players in the debate agree that if mathematical objects were to exist, they would be abstract[2], I will often drop the 'abstract' and simply write 'mathematical objects' from now on.

Consider next the term 'object'. I do not want the claim that there are no mathematical objects to mean that there is nothing mathematical, since whatever the outcome of the debate I want to allow at least that there is a mathematical discipline, for example, and such things as mathematical textbooks. So 'object' cannot mean 'any kind of thing'. But equally I do not want the term to be too restrictive. Some philosophers use the term 'object' to contrast with 'property', for example – the textbooks may have the property of being mathematical without being mathematical objects – but it is unclear that the indispensability argument would be able to establish which, if either, of objects or properties exist. Indeed, the indispensability argument has been criticised for not characterising what mathematical objects would be like, if they exist. For example, Colyvan writes that 'the indispensability argument, on the face of it at least, does not tell us anything about either mathematical epistemology or the nature of mathematical entities' [Colyvan 2009: 5-6].

It will therefore be beyond the scope of this thesis to attempt to characterise what mathematical objects would be like, if the indispensability argument is successful in showing that they exist. In general, I will take mathematical objects simply to be those things that mathematicians and scientists refer to and quantify over when they make mathematical claims, if such objects exist[3].

2 Leng, for example, a prominent nominalist, writes: 'while it is not *inconceivable* that a plausible account of mathematical objects that, for example, viewed them to be causally efficacious or spatiotemporally located may be defensible … I take it to be safe to assume that belief in mathematical objects amounts to belief in abstracta' [Leng 2010: 19].

3 See Juhl's paper 'On the Indispensability of the Distinctively Mathematical' for discussion of the question whether 'to be mathematical is simply to be the referent of a mathematical term', and a separation of 'the question whether abstracta in general exist from whether distinctively mathematical abstracta exist' [Juhl 2012: 1].

Now, among various arguments for platonism, the indispensability argument is often taken to be one of the most important: Dieveney, for example, states that it ‘has traditionally been considered the strongest argument for realism about mathematical objects’ [Dieveney 2007: 105], and Colyvan holds that mathematical realists of his persuasion ‘think that indispensability arguments offer the *only* good reason for that realism’ [Colyvan 1998: 39]. Let us now examine the argument in more detail.

Premise i holds that 'Quantification over mathematical objects is indispensable to our best scientific theories'. However, it would be best not to make quantification the important factor, since this excludes reference to mathematical objects by singular terms, for example terms for ratios or functions[4]. Therefore, I will take *allusion* to mathematical objects to include both quantification over and reference to mathematical objects by constant terms, function terms or other singular terms, and reformulate premise i to read as follows: 'Allusion to mathematical objects is indispensable to our best scientific theories'.

I will take allusion to mathematical objects to be *indispensable* to a theory, T, iff T is scientifically better than any rival theory without allusion to mathematical objects[5]. Let us say that T_1 is *scientifically better than* T_2 iff T_1 is predictively superior to T_2, i.e. T_1 has greater accuracy and wider scope of predictions, or if T_1 and T_2 are predictively equal but T_1 is superior with respect to other scientific features such as explanatory power. I will say more about premise i in the next section.

Premise ii, which holds that we should rationally believe our best scientific theories to be true, may be taken to advocate simply a ‘healthy respect for science, taken at face value’ [Colyvan 2006: 3], or it may be formulated as part of a stronger view that ‘Science is our ultimate arbiter of truth and existence’ [Resnik 1995: 166]. The latter claim is usually made against a Quinean backdrop of naturalism as a rejection of ‘first philosophy’, following ‘the recognition that it is within science itself, and not in some prior philosophy, that reality is to be identified and described’ [Quine 1981: 21, quoted in Leng 2010: 20]. My project here is not to identify or defend a concept of naturalism as recognisably Quinean, but rather to examine which reading of the premise is advisable for the platonist. I will examine various readings of premise ii in section 1.3.

Now, note the implicit assumption in the indispensability argument that (belief in) the truth

4 Thanks to Marcus Giaquinto for this point.

5 This roughly follows Colyvan's definition, although I consider rival theories rather than modifications:
An entity is dispensable to a theory iff the following two conditions hold:

1. *There exists a modification of the theory in question resulting in a second theory with exactly the same observational consequences as the first, in which the entity in question is neither mentioned nor predicted*.
2. *The second theory must be preferable to the first.* [Colyvan 2001: 77].

of theories that quantify over mathematical objects entails (belief in) the existence of those mathematical objects. An anti-platonist who disagrees with this literal reading of mathematical statements could resist this inference. For example, a philosopher providing an alternative semantics for mathematical claims, such as Yablo, may deny that the truth of a mathematical statement implies the existence of objects quantified over in that statement and thus take the indispensability argument to have only a weaker conclusion, namely:

iii*. (Limited conclusion) We should rationally believe the mathematical statements contained in our best scientific theories to be true.

I will say more about such positions in chapter 2, but in general I will assume a literal reading of mathematical claims.

So, I take it that the indispensability argument has particular force against philosophers who accept a literal reading of our mathematical statements and simultaneously wish to resist the conclusion that there are mathematical objects. A proponent of this view must hold that our mathematical theories are literally false, so at this point the following question may arise: why try to defend such an account, which as Balaguer notes 'can seem a bit crazy' given our early acquaintance with apparent arithmetical truths like '2+2=4'? [Balaguer 2011: 2].

Note that truth is not the only factor in the intellectual value of a theory; as Leng suggests, 'it is plausible that the acknowledged successes of a given practice might be down to something other than the truth of utterances made in the context of that practice' [Leng 2010: 26]. Theories can be instrumentally useful, for example, without being true (as Newtonian theory is in science, especially the law of gravity), and it is not necessarily on the basis of truth that mathematical theories are chosen or discarded.

Charles Fisher gives the example of Invariant Theory, a mathematical theory that in 1886 was taken to be 'as necessary a part of mathematical knowledge as the differential and integral calculus' [Fisher: 146], and yet which is now 'a dead subject … the problems of Invariant Theory having become uninteresting' (though its results not changing in truth value) [ibid.: 151]. We see that a theory may be false but useful, or true but uninteresting or even useless, and hence that the intellectual value of a theory is not wholly determined by its truth value. So perhaps the claim that our mathematical theories are literally false is not as 'crazy' as it first seems.

More importantly, however, note that I will not be arguing *for* any nominalist account in this thesis, since undermining the platonist position does not entail that nominalism must be true rather than some other anti-platonist account. As formulated, platonism and nominalism can both be false but not both true [Baker 2003: 51]. Rather, my aim is to find a way to undermine an argument which as Colyvan puts it ‘is a very powerful and persuasive device for warding off nominalism’ [Colyvan 1998: 39]. If I do manage to provide strong reasons to doubt the indispensability argument, this will be a significant result for the nominalist.

The aim of this chapter is to find a strong form of indispensability argument for mathematical platonism, where I take this to mean a form with the fewest and least controversial premises. In the next few sections, I examine which premises (including apparently implicit ones) can be weakened or dispensed with entirely.

1.2: Dispensability

In this section, I will discuss the 'indispensability premise' of the indispensability argument, premise i, which claims that allusion to mathematical objects is indispensable to our best scientific theories. Is this premise as weak as possible, or could the premise be reformulated to do without the indispensability claim, simply holding that i*: 'our best scientific theories allude to mathematical objects'?

Recall that I took allusion to mathematical objects to be indispensable to a theory, T, iff T is scientifically better than any rival theory without allusion to mathematical objects. To see whether i* is sufficient, we must consider a situation, S, where i* holds and i does not. This is a situation where our best scientific theories allude to mathematical objects, but this allusion is dispensable, so that rival scientific theories which do not allude to mathematical objects are scientifically as good or better; in this case, as good.

Now, premise ii, which claims that we ought rationally to believe our best scientific theories to be true, gives us reason to believe only our *best* scientific theories to be true. In situation S, the rival theories with and without allusion to mathematical objects, respectively, are equally good. Premise ii does not tell us which of these rival theories to believe (we cannot believe both, if they are rivals). In situation S, therefore, we need not believe in the existence of mathematical objects. Therefore, premise i cannot be replaced by i*, and thus the indispensability claim is not dispensable.

Now, the indispensability premise is very hard to refute. Hartry Field attempts to show that quantification over mathematical objects is dispensable to at least one of our scientific theories by giving an account of Newtonian gravitational theory without quantification over mathematical objects [Field 1980]. His account is very technical and has been disputed on various grounds; the details are not important here, since my aim here is to find out if any premises of the indispensability argument are superfluous and how much each premise must claim in order for the conclusion to go through. The indispensability premise, it turns out, cannot be weakened on my reading of indispensability.

1.3: Naturalism and confirmational holism

In the previous section, I examined the indispensability premise that gives the indispensability argument its name. I now turn my attention to the other premise, premise ii, which claims that we ought rationally to believe our best scientific theories to be true, as well as another premise of confirmational holism, which various philosophers claim to be implicit in the indispensability argument [e.g. Maddy 1997 and Colyvan 2001, quoted in Dieveney: 109]. In this section I argue, following Dieveney, that the confirmational holism claim is in fact dispensable on a certain (strong) naturalist reading of premise ii.

I will examine a recent discussion of the indispensability argument by Mary Leng as an example. Leng frames the argument as follows, including a confirmational holism premise:

'P1 (Naturalism): We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe.

P2 (Confirmational Holism): The confirmation our theories receive extends to all their statements equally.

P3 (Indispensability): Statements whose truth would require the existence of mathematical objects are indispensable in formulating our best confirmed scientific theories.

Therefore

C (Mathematical Realism): We ought to believe that there are mathematical objects'.

[Leng 2010: 7].

The argument essentially runs as before with an extra step emphasised, holding that statements quantifying over mathematical objects are indispensable to our best confirmed scientific theories, and since we should rationally believe *all statements in* our best confirmed scientific theories to be true or approximately true, we should rationally believe in the existence of mathematical objects.

Leng argues against P2, noting from Maddy's work that there are many cases in science where scientists do not take confirmation of a theory to apply to all of the theory's statements – for example, when theories involving frictionless planes, continuous fluids or other idealised objects are confirmed, scientists do not take the idealised objects to exist, although they may be indispensable to the theory as useful fictions [Leng 2002: 399, and see Maddy 1992]. Dieveney calls this type of argument, which holds that our scientific theories can be separated into ontologically committing and non-ontologically-committing parts, a *separation objection* [Dieveney: 113].

I will not examine the details of Leng's separation objection, since my focus here is simply on finding out whether the confirmation holism premise is dispensable.

Even without confirmational holism it seems that the platonist conclusion is justified, since the platonist can proceed as follows: our best confirmed scientific statements tell us what we ought to believe, that is, we ought to believe our best confirmed scientific statements; our best confirmed scientific statements include mathematical statements whose truth entails the existence of mathematical objects; hence we ought to believe some statements whose truth entails the existence of mathematical objects; hence we ought to believe there are mathematical objects. Leng's reading of the indispensability argument can be reformulated as follows:

P1 (Naturalism): We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe.

P3* (Indispensability): Allusion to mathematical objects is indispensable to some statements which are, or follow from, statements considered best confirmed according to our ordinary scientific standards.

Therefore

C (Mathematical Realism): We ought to believe that there are mathematical objects.

This argument does not require the Confirmational Holism premise. At first glance, then, it seems that Leng has not successfully undermined the indispensability argument, since the premise she argues against is superfluous.

However, Leng can respond to this charge by pressing us on the details of our understanding of the naturalism premise. Note that we understood the claim that we 'should look to science … to discover what we ought to believe' as not distinguishing between parts of scientific theories, in order to infer that ‘we ought to believe our scientific statements’ without qualification. That is, we seem to have been assuming that ‘it is the truth of *all* of the utterances used to express our best theories … that is confirmed by our successful use of those theories’ [Leng 2010: 40]; perhaps we have smuggled confirmational holism into the naturalism premise. In this case, the revised argument will still be subject to Leng’s separation objection.

Dieveney notes, however, that 'confirmational holism is not *required* in order to respond to the separation objection' [Dieveney: 125, emphasis mine]. Rather than relying on confirmational holism, the platonist could advance a view that we should look to science as the ultimate arbiter of truth and existence, as noted in section 1.1. Leng writes that ‘We trust our best science to tell us what we ought to believe that there is, just because that is *all we have to go by*’ [Leng 2010: 2, italics mine] and claims that 'we ought not to believe in entities beyond those whose existence is confirmed according to our best scientific theories' [ibid: 13], which indicates that she accepts a strong reading of the naturalist premise. Dieveney calls this reading of naturalism *theory naturalism*: ‘We look to our best scientific theories as the ultimate arbiter of existence and truth’ [Dieveney: 127].

Replacing P1 with the alternative premise of theory naturalism provides the platonist with a response to the separation objection, because theory naturalism holds that ‘we have no more fundamental means of determining what exists than appealing to our best scientific theories. Given that our scientific theories are the ultimate arbiter of what exists, we cannot justifiably distinguish within these theories those parts whose ontological commitments we accept and those we do not accept’ [Dieveney: 114-5].

Nevertheless, we can further defend Leng's separation objection here. Leng's argument against confirmational holism involved the claim that mathematical statements could be representationally useful without being true, in the same way as claims about idealised objects like frictionless planes. Perhaps Leng could still claim that we are not ontologically committed to those parts of our theories shown to be capable of ‘usefulness without truth’, arguing that this way of distinguishing between different parts of our theories takes place within science, or at least within the scientific community, because it is the attitude of scientists that tells us we need not take these representationally useful posits to exist.

In this sense, Leng need only rely on scientific practice to motivate her separation objection, without appealing to some more fundamental arbiter than science. Taking this option would imply acceptance of some kind of *belief naturalism*: ‘We look to the beliefs of scientists as a source of our ontological commitments’ [Dieveney: 121].

However, the platonist can modify his indispensability claim to hold that statements whose truth would require the existence of mathematical objects are indispensable to the beliefs of scientists, in order to formulate their other scientific thoughts. Indeed, many scientists explicitly believe that there are mathematical objects. Hence, we are rationally required to believe in the existence of mathematical objects, if we take the attitude of scientists as an ontological guide.

It will not be possible to separate out ontologically committing beliefs from non-ontologically-committing beliefs, not because some version of confirmational holism requires us to accept all beliefs held by scientists, but because neither science nor the beliefs themselves can tell us which beliefs are ontologically privileged. Attempts at a separation objection thus falter here, with no appeal to confirmational holism required on the platonist's behalf.

Of course, adapting the naturalist premise in this manner in order to do without confirmational holism may make that premise much less plausible. For example, belief naturalism seems quite unconvincing, since there are various cases where the beliefs of scientists do not converge even where physical entities are concerned, as evidenced by conflicting opinions amongst scientists on the ontological status of atoms before Perrin’s experiments [Dieveney: 119]. Similarly, theory naturalism, which takes science to be our ultimate arbiter of existence and truth, is itself controversial.

The point reached in this section is simply that there are versions of the indispensability argument that do not rely on confirmational holism. In the next section, I will present a more plausible indispensability argument of this kind.

1.4: An explanatory indispensability argument

We saw in the last section that the confirmational holism premise sometimes held to be implicit in the indispensability argument is superfluous, since a premise of theory naturalism is sufficient to entail the desired platonist conclusion as well as to respond to separation objections.

However, theory naturalism is a strong reading of the original naturalist premise in section 1.1, which claimed merely that we should believe in our best scientific theories. That is, we have dispensed with the confirmational holism premise at the cost of committing to a view of science as our ultimate arbiter of truth and existence, which may not be convincing to all anti-platonists (or indeed platonists) and is easier to undermine than a weaker reading of mere ‘healthy respect’ for science. Remember that my aim was to find a form of the indispensability argument with the fewest *and least controversial* premises. Ideally, then, there would be a way of responding to the separation objection using a less contentious premise than either theory naturalism or confirmational holism.

Fortunately, such a form of the indispensability argument has recently been given by platonists in response to objections to the original argument. The idea of this revised form of indispensability argument is to focus on the theoretical contribution mathematics makes to science beyond representational usefulness, in particular its explanatory role. The revised argument runs as follows:

(1) 'We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.

(2) Mathematical objects play an indispensable explanatory role in science.

(3) Hence, we ought rationally to believe in the existence of mathematical objects'.

The Enhanced Indispensability Argument, in [Baker 2009: 613].

A separation objection cannot readily be used here to deny the conclusion of the argument, because the argument itself has separated our scientific theories into explanatory and non-explanatory parts and taken mathematical statements to fall into the former category, arguing that they must therefore be true.

Neither side in the debate has provided a comprehensive account of what it means for objects to play an explanatory role; as we will see in chapter 4, both sides in the debate tend to place the burden of providing such an account on their opponent. On the platonist's behalf, note that singling out the genuinely explanatory parts of a theory and showing that they can be formulated without quantifying over mathematical objects may plausibly be as difficult for the nominalist as Field’s approach. Furthermore, platonists have provided examples where it is claimed that mathematical entities do play a genuinely explanatory and indispensable role. I will examine these issues in later chapters, focusing on Baker's cicadas example from [Baker 2005, 2009] in chapter 3.

In this chapter, my focus is on finding a strong version of the indispensability argument, which is not easily undermined.

Therefore, I take it that a less controversial claim will be preferable. For example, consider the premise:

(1*) We ought rationally to believe in the existence of objects that feature indispensably in our best scientific explanations.

The notion of 'featuring in an explanation' here can be taken very broadly to mean that the explanation alludes to mathematical objects. With a literal reading of truth in the background, we can see that (1*) is implied by:

(1**) We ought rationally to believe in the truth of our best scientific explanations.

But this is just an endorsement of inference to the best explanation (IBE) in a scientific context. As Busch notes, the entire argument can be recast in this form[6] [Busch 2011a]:

(1**) We ought rationally to believe in the truth of our best scientific explanations;

(2*) Allusion to mathematical objects is indispensable to our best scientific explanations;

(3) Hence, we ought rationally to believe in the existence of mathematical objects.

I will focus on this version of the argument over Baker's, because it involves an endorsement of IBE rather than Baker's claim that we should believe in the existence of entities which play an explanatory role. As we will see in chapter 4, Baker's claim reads as a further condition on IBE and is thus more vulnerable to attack.

For now, note that Baker claims that 'the indispensability debate only gets off the ground if both sides take IBE seriously' [Baker 2005: 225], and according to Pincock, 'All sides start from the position of some form of scientific realism that accepts at least some instances of IBE' [Pincock: 211]. This suggests that the implicit endorsement of IBE in my version of the explanatory indispensability argument makes that argument quite difficult for the nominalist to undermine. For this reason, and because my version of the explanatory indispensability argument does not appeal to confirmational holism or a strong understanding of naturalism, I will take this version of the explanatory indispensability argument as my target in the rest of the thesis. In the next section, I will say a little more about its premises.

1.5: Two new premises

1.5.1: Dispensability again

It may seem as though the revised indispensability premise of the explanatory indispensability argument provided in the previous section can be weakened; perhaps it is sufficient for premise (2*) to read simply 'Mathematical objects feature in our best scientific explanations'. This is because, if there were two equally good explanations (by some measure of explanatory virtue),

6 Thanks also to Mark Kalderon for this point.

where one posited mathematical objects and one did not, then the explanation with fewer ontological commitments would be preferable. So if mathematical objects feature in our best scientific explanations, this means that an alternative explanation not positing mathematical objects must already be a worse explanation by our definition of explanatory virtue; that is, indispensability does no work [Busch 2011a: 154].

However, this is to equivocate about 'best' on two different conceptions of explanatory virtue, one including and one not including ontological considerations. Recall our understanding of indispensability from sections 1.1 and 1.2. Allusion to mathematical objects is indispensable to a theory, T, iff T is scientifically better than any rival theory without allusion to mathematical objects. Adapting this definition for explanations, if allusion to mathematical objects is dispensable to a scientific explanation, then there is a rival explanation without allusion to mathematical objects which is equally good or better on a particular reading of explanatory virtue. We took the original explanation to be our best, so this rival explanation must be equally good rather than better *on the reading of explanatory virtue selected.* But premise (1**) tells us simply to believe the best explanation, and since both rival explanations are equally good, does not tell us to select the one featuring mathematical objects, as required for the argument to go through. Thus the indispensability premise cannot be weakened, on my reading of indispensability.

1.5.2: Inference to the best explanation

As I noted in section 1.4, premise (1**) of the explanatory indispensability argument reads as an endorsement of inference to the best explanation in a scientific context. At first sight, this endorsement seems to be granted by both sides in the debate: indeed, Baker claims that there is already an 'implicit endorsement of scientific realism by both the platonist and nominalist sides in the indispensability debate. A crucial plank of the scientific realist position involves inference to the best explanation (IBE) to justify the postulation in particular cases of unobservable theoretical entities' [Baker 2005: 225].

I will say more about the way inference to the best explanation is involved in the explanatory indispensability argument, and about the way scientific realism is connected to the debate, in chapter 2. For now, note that the fact that both sides in the indispensability debate are taken to accept inference to the best explanation in a scientific context suggests that premise (1**) is relatively uncontroversial, as desired.

Conclusion

I have examined various forms of the indispensability argument and found that the explanatory indispensability argument has the least controversial premises, appealing neither to confirmational holism nor to a strong naturalism. I have argued that the indispensability premise in the explanatory indispensability argument cannot be weakened, and noted that the IBE premise is, at least at first sight, granted by both sides in the debate. In this sense, the explanatory indispensability argument can be seen as a particularly strong form of the indispensability argument, and it is also a version currently under discussion in the literature. Therefore, I will take the explanatory indispensability argument as my target in the rest of this thesis. In the next chapter, I will examine the role played by IBE in the explanatory indispensability argument in more detail.

Chapter 2: How is inference to the best explanation involved in the explanatory indispensability argument?

Introduction

In chapter 1, I formulated the explanatory indispensability argument as follows:

1) We ought rationally to believe in the truth of our best scientific explanations.
2) Allusion to mathematical objects is indispensable to our best scientific explanations.
3) Hence, we ought rationally to believe in the existence of mathematical objects.

I claimed that premise 1 should be read as an endorsement of inference to the best explanation, or IBE, in a scientific context. This appeal to IBE is taken to be central to the indispensability debate. For example, Baker claims that the idea behind the explanatory indispensability argument, 'shared by both sides in the above debate, is that … it restricts attention to cases where we can posit the existence of a given entity by inference to the best explanation' [Baker 2009: 613]. Furthermore, 'the indispensability debate only gets off the ground if both sides take IBE seriously', according to Baker [Baker 2005: 225]. In order to fully understand the indispensability debate, therefore, it will be necessary to examine in more detail the role played by IBE in the explanatory indispensability argument. That will be the focus of this chapter.

First, I will need to provide some background for inference to the best explanation. In section 2.1, I lay out the basic structure of an inference to the best explanation, formulating various conditions that must be fulfilled for such an inference to be acceptable. In section 2.2, I examine the use of IBE in a well-known argument for scientific realism, the 'No Miracles' argument. As well as providing an example of IBE in application, section 2.2 is also important because of a link commonly drawn between scientific realism, IBE and mathematical realism by platonists.

In section 2.3, I examine the endorsement of inference to the best explanation that I claim is implicit in premise 1) above. Examining the literature in order to elaborate on this claim, I will provide two readings of the explanatory indispensability argument, connecting the argument to scientific realism and directly to IBE, respectively. I will argue that the second reading of the explanatory indispensability argument is the most plausible.

After examining the explanatory indispensability argument in more detail and clarifying its connection to inference to the best explanation, I will examine the options available to the nominalist in trying to undermine the argument. These fall into four categories. I will discuss each of these briefly in section 2.4 and select the two most promising strategies to be examined in chapters 3 and 4.

2.1: Inference to the Best Explanation

Let D be the data we want to explain and let B be the background information and already established theories.

1. $E_1, \ldots, E_n$ are all of the competing potential explanations we have of D.

2. E_k is a good potential explanation of D given B, and is significantly better than all of the other explanations among $E_1, \ldots, E_n$.

3. Therefore E_k is correct.

The step from 1 and 2 to 3 is an *inference to the best explanation*: an inference from the claim that E_k is the best explanation available to the claim that E_k is true. This inference is not deductively valid, since it is at least logically possible for an excellent explanation to be false. Similarly, it is at least logically possible for a less 'good' explanation which conflicts with E_k to be true, in which case E_k could not be correct (since the potential explanations compete with one another).

Neither is inference to the best explanation clearly an inductive form of reasoning (though see e.g. [Fumerton 1980] for discussion of this matter). As an inference form, however, IBE seems to accord well with our common intuitions. For example, on seeing a picture of Winston Churchill traced in the sand, the best explanation is usually that a person was recently on the beach and drew this picture, and hence we usually believe this explanation to be true, rather than believing that the picture was created accidentally by ants [Putnam 1981].

For an inference to the best explanation to be appropriate, however, certain conditions must be fulfilled. First, the best explanation had better be quite *good*. As Lipton puts it, 'The best explanation must be good enough to merit inference' [Lipton 2004: 63]. For if none of the explanations are held to be any good, then we do not usually infer to the truth of any of them (unless it can be shown that there are no other options available – vis. Sherlock Holmes' well-known remark that 'when you have eliminated the impossible, whatever remains, however improbable, must be the truth').

Second, the best explanation must be *significantly* better than its competitors, otherwise the inference is shaky – if the best explanation is only marginally better than the runner-up, then inference to the best explanation does not justify belief in one of these explanations over the other.

Third, step 1 must involve listing all *competing* potential explanations of D, since if the explanations are all compatible, then nothing is to stop us from inferring to all of them – but in that case IBE seems to be an empty form of inference, since it has not provided us with much information about which beliefs we should form[7]. The aim, of course, is to outline a form of inference which can be epistemically informative[8]. With that aim, note that 'best' cannot simply mean 'actual', since if we already knew which explanation were actual, we would not need a guide to belief [Lipton 2004: 58]. So, the fourth condition is that the choice is taken to be among competing *potential* explanations.

Now, the criteria involved in selecting the best explanation are clearly critical to the success of any inference to the best explanation: if the criteria are in some sense arbitrary, it is unlikely that the best explanation will indeed be true. However, it is unlikely that one set of criteria will be able to cover all domains, since the evaluation of 'best' is likely to depend on principles specific to the field of inquiry. It is thus implausible that a single uniform method could be found to determine which explanation is the best in any field. There will, though, be domain-specific methods within each field.

In chapter 3, I will back up this claim and argue that it is scientists who are best placed to determine which scientific explanations are the best. Some applications of inference to the best explanation, though, involve philosophical evaluation of what makes for the best explanation; in the next section I will examine one such application, the 'No Miracles' argument for scientific realism.

2.2: The No Miracles argument for scientific realism

Take scientific realism to be the claim that our current scientific theories are (approximately) true and that our scientific terms typically refer [e.g. Putnam 1975: 73]. Some philosophers have argued that scientific realism is the best, indeed the only explanation of the ongoing predictive success of our scientific theories, and hence that scientific realism must be true. This argument, also known as the 'No Miracles' argument, contends that the truth of our scientific theories is the only explanation that makes their success non-miraculous.

For example, Putnam writes that 'The positive argument for realism is that it is the only philosophy that doesn't make the success of science a miracle. That terms in mature scientific theories typically refer …, that the theories accepted in a mature science are typically approximately true, that the same term can refer to the same thing even when it occurs in different

7 Lipton also argues that the explanations must be competing, because, if the potential explanations were not competing, an inference to just one explanation would be too restrictive [Lipton 2004: 62].

8 The aim [as in Lipton 2004] is often also to describe our actual inferential practices, which we hope are not empty.

theories – these statements are viewed by the scientific realist … as part of the only scientific explanation of the success of science' [Putnam 1975: 73].

For clarity, I will formulate this No Miracles argument in the IBE format as presented in the previous section. Let D be the data we wish to explain, namely the ongoing predictive success of our best scientific theories. Then:

1. E_1 = scientific realism, E_2, …, E_n are all of the competing potential explanations we have of D, where E_2, …, E_n are other philosophical explanations of D.

2. E_1 is a good potential explanation of D given B, and it is significantly better than all of the other explanations among E_2, …, E_n, since E_1 is the only explanation that does not make the success of science a miracle.

3. Therefore E_1, scientific realism, is correct.

Some objections to the No Miracles argument focus on its use of inference to the best explanation. For example, van Fraassen, an anti-realist, suggests the availability of a 'rival hypothesis: we are always willing to believe that the theory which best explains the evidence, is empirically adequate (that all the observable phenomena are as the theory says they are)', rather than true [van Fraassen: 20]. That is, inference to the best explanation is not the only available account of explanatory inference.

Lipton describes another problem with inference to the best explanation as the problem of *underconsideration,* also originally posed by van Fraassen. This questions whether our choice and ranking of explanations is ever likely to get us to the truth, since it is quite possible that the process by which we generate explanations does not guarantee that the true one will ever be among them [Lipton 2004: 152].

I mention the first two objections to illustrate the fact that the No Miracles argument for scientific realism is by no means uncontentious. However, I will not have space to examine the success of arguments for scientific realism in this thesis. Since IBE is taken to be accepted by both sides in the indispensability debate, I will also leave objections to inference to the best explanation to one side, and now examine a different type of objection to the No Miracles argument, which is relevant for my purposes.

A third objection sometimes made to the No Miracles argument is that the argument is circular, or question-begging. As presented above, the argument does not appear circular, but the idea here is that the No Miracles argument is sometimes used to justify the common use of inference to the best explanation in a scientific context: after all, if our current scientific theories are true, then these applications of IBE, which have taken us towards our current theories, have taken us towards the truth. This reading of the No Miracles argument is offered by Lipton, according to whom the argument 'says we ought to infer first that successful theories are true or approximately true, since this is the best explanation of their success, and then that Inference to the Best Explanation is truth-tropic, since this is the method of inference that guided us to these theories' [Lipton 2004: 191].

The circularity in this reading of the No Miracles argument is apparent when the reasoning is dissected as follows:

a) By inference to the best explanation, scientific realism is true.

b) Since scientific realism is true, our current scientific theories are approximately true.

c) Hence, the applications of inference to the best explanation that were used to construct these theories have taken us towards the truth.

d) Hence, inference to the best explanation is a reliable form of inference.

This argument is circular because the conclusion is needed in order to justify premise a).

However, argument a)-d) is not equivalent to my earlier formulation of the No Miracles argument, but rather takes several further steps to arrive at a claim about inference to the best explanation. The No Miracles argument as I presented it concludes only that scientific realism is true; this conclusion is not used to justify either of its premises, which make claims only about the strength of scientific realism as an explanation of the data. Neither does the conclusion that scientific realism is true assert anything about the rule of inference used. Of course, it is clear that the truth of scientific realism cannot be both justified by inference to the best explanation and simultaneously used to justify applications of inference to the best explanation; but as long as the No Miracles argument is not used to do the latter, it is not circular.

The objection can be recast to claim that the No Miracles argument is question-begging, because it seeks to convince those who do not accept IBE as a form of inference (for example van Fraassen) that scientific realism is true, by using an inference to the best explanation. But this is merely a psychological version of the same point[9]; granted, the No Miracles argument cannot be used to justify inference to the best explanation, or to convince any philosopher who does not yet accept inference to the best explanation. As long as it does not seek to do these things, the

9 Thanks to Marcus Giaquinto for this point.

argument is not circular[10].

Now, the arguments proposed for scientific realism tend to be some version of the No Miracles argument, and many scientific realists base their acceptance of scientific realism on some kind of No Miracles argument, which, as we have seen, involves IBE. If scientific realism is motivated in some other way, then part of the argument on the previous page can be given to show that scientific realism nevertheless involves endorsing IBE: 'If scientific realism is true, our current scientific theories are approximately true. Hence, the applications of inference to the best explanation that were used to construct these theories have taken us towards the truth. Hence, inference to the best explanation is a reliable form of inference'. This argument is not circular, and is fairly persuasive, so motivates the view that accepting scientific realism entails the acceptance of inference to the best explanation.

The point to take away from this discussion is that accepting scientific realism involves endorsing inference to the best explanation as a principle of inference. This is important in the context of the explanatory indispensability argument because of the link drawn by some philosophers from nominalists' putative acceptance of scientific realism to mathematical realism. In the next section I will examine this link in more detail.

2.3: Connecting the explanatory indispensability argument to IBE and scientific realism

At the beginning of this chapter I formulated the explanatory indispensability argument as follows:

1) We ought rationally to believe in the truth of our best scientific explanations.
2) Allusion to mathematical objects is indispensable to our best scientific explanations.
3) Hence, we ought rationally to believe in the existence of mathematical objects.

Having examined the background of inference to the best explanation in section 2.1, we can now see that the explanatory indispensability argument does not itself involve an inference to the best explanation: it is not claimed, for example, that the existence of mathematical entities provides the best explanation of the success of our scientific theories or explanations and hence that mathematical realism is correct. Neither does the argument contain an inference of the form: E is the best explanation of data D; hence E is true.

10 A question may be of course be raised about the strength of an argument that cannot convince its main opponents – but as Lipton suggests, providing a justification for the non-sceptic can also be a valuable project [Lipton 2004: 186].

Rather, as I claimed at the beginning of this chapter, premise 1 of the argument should be read as an endorsement of the acceptability of inference to the best explanation in a scientific context. This is a fairly vague claim, however, so I will now examine the details of this endorsement in more detail.

As we saw in section 2.2, accepting scientific realism involves endorsing inference to the best explanation. For this reason, I think there is sometimes a focus on scientific realism as a way of committing nominalists to IBE. For example, Busch states that he is 'interested in the relationship (or lack thereof) between arguments for scientific realism and IA' (the indispensability argument) [Busch 2011: 308]. Baker claims that there is an 'implicit endorsement of scientific realism by both the Platonist and nominalist sides in the indispensability debate' [Baker 2005: 225], while Pincock writes more cautiously that 'All sides start from the position of some form of scientific realism' [Pincock: 211].

It seems, therefore, that scientific realism is centrally involved in the indispensability debate. In the next section I will examine a reading of the explanatory indispensability argument that clarifies the way in which a commitment to scientific realism is sometimes seen as an implicit premise in the argument.

2.3.1: Version 1 of the explanatory indispensability argument

As we just saw, various philosophers claim that nominalists are committed to scientific realism. Additionally, accepting scientific realism involves accepting inference to the best explanation as a principle of inference. This fact can be used to put pressure on nominalists who supposedly accept scientific realism to admit that they thereby endorse inference to the best explanation, and hence to accept premise 1 of the explanatory indispensability argument. The full explanatory indispensability argument then runs as follows:

i. All participants in the indispensability debate, nominalists included, accept scientific realism.

ii. Accepting scientific realism involves endorsing inference to the best explanation.

iii. Hence we ought rationally to believe in the truth of our best scientific explanations.

iv. Allusion to mathematical objects is indispensable to our best scientific explanations.

v. Hence, we ought rationally to believe in the existence of mathematical objects.

In this argument, 'we' refers to the participants in the indispensability debate. I will call this argument version 1 of the explanatory indispensability argument.

Consider the two additional claims, i) and ii). Claim ii) was established in section 2.2. As we saw earlier, claim i), which holds that nominalists already accept scientific realism, is endorsed by various authors in the literature. In fact, however, claim i) is not very plausible, for two reasons.

Firstly, a direct inference can be drawn between scientific realism and mathematical realism, so accepting scientific realism would be in direct tension with the nominalist position. For, if we believe our scientific theories to be true, and 'these theories include amongst their laws assertions that imply the existence of mathematical objects', then we should believe in the existence of mathematical objects [Leng 2005b: 65][11]. This is simply the original indispensability argument from chapter 1, as Leng notes [ibid.: 75]. The assertion that nominalists accept scientific realism thus amounts to claiming that the only way for the nominalist to avoid the original indispensability argument is to show that allusion to mathematical objects is dispensable to our scientific theories, as Leng points out [Leng 2010: 10]. This charge is unfair, unless further justified.

Secondly, nominalists explicitly reject the claim that they accept scientific realism. Leng, for example, writes that she is 'defending an anti-realist view of science in rejecting the claim that we ought to believe that our best scientific theories are true or approximately true' [Leng 2010: 11-12]. So Baker's claim that nominalists implicitly endorse scientific realism goes too far; claim i) is false.

Pincock's more cautious claim that 'All sides start from the position of *some form of* scientific realism', on the other hand, is more apt [Pincock: 211, italics mine]. Leng, for example, goes on to say that the view she 'will be defending is realist in a different sense of scientific realism: it amounts to realism about (many of) the unobservable *physical* objects posited by our theories' [Leng 2010: 12]. So, it seems that the nominalist can be charged with accepting *modified realism*: a position which is realist about physical objects but anti-realist about mathematical ones, rejecting the claim that our scientific theories are true.

So, claim i), which holds that nominalists are committed to scientific realism, is false, because nominalists explicitly commit themselves only to modified realism. Hence version 1 of the explanatory indispensability argument is unsound.

11 This is with a literal reading of truth in the background, to which Leng subscribes, taking Platonism to be the 'claim that at least some of our mathematical theories that posit the existence of mathematical objects are true at face-value' [ibid.: 68]).

However, I will now argue that the platonist intended only to point out that nominalists endorse inference to the best explanation in certain cases. The talk of scientific realism, I will then argue, is simply misleading. Although version 1 of the explanatory indispensability argument is unconvincing, a second more plausible reading can be found in which the talk of scientific realism can be dropped.

If we examine Baker's claim that nominalists are committed to scientific realism more closely, we see that he continues as follows: ‘A crucial plank of the scientific realist position involves inference to the best explanation (IBE) to justify the postulation *in particular cases* of unobservable theoretical entities’ [Baker 2005: 225, italics mine]. Similarly, Pincock's claim is that 'All sides start from the position of some form of scientific realism that accepts *at least some instances* of IBE' [Pincock: 211, italics mine]. The focus is on claims about the existence of unobservable objects that are justified using inference to the best explanation.

Now, note that existence claims about unobservable objects like electrons are commonly called scientific realist claims, which may be why talk of scientific realism is so common in the indispensability literature. Since the truth of claims about unobservable objects is the main point of debate between scientific realists and their main opponents, constructive empiricists, scientific realism is sometimes characterised solely in terms of claims about unobservables. Colyvan, for example, describes scientific realism only in terms of being 'happy to go beyond what is unobservable and posit unobservable entities' [Colyvan 2006: 2], rather than as making claims about the truth of our scientific theories.

However, we have seen that Leng, for example, holds a position I called modified realism, which is in tension with scientific realism, yet agrees with the scientific realist on many claims involving unobservable objects. In the context of a debate between scientific realists and modified realists, it is misleading to label claims about unobservable entities as scientific realist claims, since both sides agree on the existence of such entities. The distinctively scientific realist claim in this debate is the assertion that our scientific theories are true. Hence, if platonists wish to draw attention instead to claims involving unobservable objects, it would improve the clarity of the debate to avoid talk of scientific realism.

In the next section I will provide a second, more plausible reading of the explanatory indispensability argument that shows the talk of scientific realism can be dropped.

2.3.2: Version 2 of the explanatory indispensability argument

In the last section we saw that nominalists are taken to accept inference to the best explanation in cases where the existence of unobservable theoretical objects is at issue. Is this indeed the case? Let us examine a case where the existence of unobservable theoretical entities is argued for using inference to the best explanation. Psillos writes:

> 'Suppose that a background theory T asserts that method M is reliable for the generation of effect X in virtue of the fact that M employs causal processes $C_1, \ldots, C_n$ which, according to T, bring about X. … Suppose, finally, that one follows M and X obtains. What else can better explain the fact that the expected (or predicted) effect X was brought about than that the theory T – which asserted the causal connections between $C_1, \ldots, C_n$ and X – has got those causal connections right, or nearly right? If this reasoning to the best explanation is cogent, then it is reasonable to accept T as approximately true' [Psillos: 79, quoted in Leng 2005b: 79].

Although this argument is couched in terms of an inference to the best explanation, the reasoning is not explicit. For clarity I will therefore formulate Psillos' argument in the IBE form I presented in section 2.1.

Take the data, D, in need of explanation to be the success of a theory T in predicting an observable effect, X. Theory T asserts the existence of various causal processes which, according to T, bring about X. Let $E_1, E_2, \ldots, E_n$ be potential explanations of D.

1. E_1 = *T has 'got it right' about the causal processes it posits,* $E_2, \ldots, E_n$ are all of the competing potential explanations we have of D.

2. E_1 is a good potential explanation of D given B, and it is significantly better than all of the other explanations among $E_2, \ldots, E_n$, since Psillos suggests that nothing else can better explain the data.

3. Therefore E_1 is correct.

As Leng notes, 'The claim that the theory has 'got it right' about the causal connections is used by Psillos to support the final, realist, claim that it is reasonable to accept T as approximately true' [Leng 2005b: 79]. So the argument above is an argument for scientific realism, on the assumption that ' 'getting it right' about unobservable causes simply amounts to having a true theory of those objects' [ibid.: 80].

Although Leng will not accept the last inference to scientific realism, she does claim that 'As a response to scepticism regarding the existence of electrons, for example, this argument has a great deal of plausibility' [Leng 2005b: 79]. So at least one prominent nominalist seems to endorse the use of inference to the best explanation in this case.

Now, platonists can argue that an analogous argument exists for the existence of mathematical objects, and that it would be 'intellectually dishonest' for nominalists to accept inference to the best explanation in one case and not in the other. But what exactly is the 'analogous argument'?

Psillos' argument involves the claim that the existence of unobservable causal processes best explains the predictive success of a theory positing those causal processes. The existence of unobservable causes can plausibly form part of a *scientific* explanation of a theory's predictive success, because of the empirical nature of such causes.

The strictly analogous argument for mathematical objects would be to claim that the existence of mathematical objects would best explain the predictive success of a theory positing those mathematical objects. But this argument seems less intuitively plausible than Psillos', because of the acausal nature of mathematical objects, as Leng notes [Leng 2005b: 80]. Can the existence of mathematical objects explain an empirical fact, if mathematical objects are causally isolated? Such an explanation would be philosophical rather than scientific, and surely quite controversial. It may be defensible, but a better 'analogous argument', I take it, would be the following.

Let D be some scientific data we wish to explain. Let E_1, E_2, …, E_n be all of the potential explanations of D. Suppose that the best of these explanations, E_1, involves indispensable allusion to mathematical objects. Then:

a) E_1, E_2, …, E_n are all of the potential explanations we have of D.

b) E_1 is a good potential explanation of D, and it is significantly better than the other explanations among E_2, …, E_n.

c) The truth of E_1 follows from a) and b) by inference to the best explanation, and IBE is an acceptable form of inference.

d) Hence we ought rationally to believe in the truth of E_1.

e) E_1 involves indispensable allusion to mathematical objects.

f) Hence we ought rationally to believe in the existence of mathematical objects.

I will call this argument version 2 of the explanatory indispensability argument. The inference from a) and b) to c) in this argument involves an instance of inference to the best explanation, and the explanation in question, E_1, is a scientific one. Hence, since nominalists accept inference to the best explanation in other cases of scientific explanation, it seems that they should accept the inference here. Additionally, the inference from c) to d) seems reasonable, and the inference from d) and e) to f) just relies on a face-value reading of mathematical claims.

So, if a scientific explanation fulfilling the conditions on E_1 can be found, version 2 of the explanatory indispensability argument seems convincing. I therefore take version 2 to be a good reading of the explanatory indispensability argument, and I will now examine the ways in which a nominalist might try to undermine it.

2.4: Laying out the options

I think there are four main options open to the nominalist in order to undermine the most plausible reading of the explanatory indispensability argument, version 2 from the previous section.

Firstly, the nominalist can deny that our best scientific explanations indispensably involve allusion to mathematical objects. Since platonists have provided putative examples of such explanations, the nominalist must either show that the allusion to mathematical objects is dispensable, or that there are better explanations of the same data not involving such allusion. This strategy will be discussed in chapter 3.

Secondly, the nominalist can deny that the truth of an explanation entails the existence of the objects to which that explanation alludes. In that case, the fact that we ought rationally to believe in the truth of an explanation involving indispensable allusion to mathematical objects does not entail that we should believe in the existence of such objects. An example of a philosopher who takes this strategy is Jody Azzouni, who provides an alternative semantics for mathematical claims [e.g. Azzouni 2004, *Deflating Existential Consequence*].

Dialectically, the second strategy is not the best option for my purposes, since nominalists such as Leng do accept a face-value reading of mathematical claims, and I prefer to look for a way of undermining the explanatory indispensability argument that is on the platonist's terms and hence maximally effective against the platonist. Therefore, further discussion of Azzouni's work will lie beyond the scope of this thesis.

A third strategy for the nominalist is to reject inference to the best explanation as an account of explanatory inference, following e.g. van Fraassen. However, we have already seen that there are cases where it seems that the nominalist accepts inference to the best explanation. Additionally, I sympathise with Baker's claim that 'the indispensability debate only gets off the ground if both sides take IBE seriously' [Baker 2005: 225]. To reject IBE completely would arguably be not really to engage with the indispensability debate.

I will therefore pursue the fourth strategy I take to be open to the nominalist in chapter 4, where I will argue that a commitment to IBE does not commit the nominalist to the existence of mathematical entities. This strategy is to search for a well-motivated restriction on IBE that blocks the support of mathematical entities through IBE. I will examine three possible restrictions in chapter 4.

Conclusion

In order to answer the question of this chapter, 'How is inference to the best explanation involved in the explanatory indispensability argument?', I first examined inference to the best explanation in terms of conditions that must be fulfilled in order for the inference rule to apply, and in application in the No Miracles argument for scientific realism.

I suggested that the use of IBE in justifying scientific realism has led various authors to accuse nominalists, who accept IBE in certain cases, of accepting scientific realism. This led to a first reading of the explanatory indispensability argument, in which the argument involves an endorsement of IBE through a putative commitment to scientific realism. Version 1 of the explanatory indispensability argument was shown to be implausible, since nominalists explicitly reject scientific realism.

I therefore provided a second reading of the explanatory indispensability argument in which the talk of scientific realism is dropped, and which draws attention to specific cases of inference to the best explanation. I take this reading, version 2 of the explanatory indispensability argument, to be the most plausible. Hence I can answer the title question of this chapter as follows: inference to the best explanation is directly involved in the explanatory indispensability argument, since the best reading of the argument involves an instance of inference to the best explanation.

In the next two chapters, I will examine two strategies that the nominalist can adopt in trying to undermine the explanatory indispensability argument.

Chapter 3: Periodical cicadas and best explanation in science

Introduction

In Chapter 2, I concluded that the best reading of the explanatory indispensability argument runs as follows. Let D be some scientific data we wish to explain. Let E_1, E_2, …, E_n be all of the potential explanations of D. Suppose that the best of these explanations, E_1, involves indispensable allusion to mathematical objects. Then:

a) E_1, E_2, …, E_n are all of the potential explanations we have of D.

b) E_1 is a good potential explanation of D, and it is significantly better than the other explanations among E_2, …, E_n.

c) The truth of E_1 follows from a) and b) by inference to the best explanation, and IBE is an acceptable form of inference.

d) Hence we ought rationally to believe in the truth of E_1.

e) E_1 involves indispensable allusion to mathematical objects.

f) Hence we ought rationally to believe in the existence of mathematical objects.

In this chapter, I will examine whether platonists have provided an explanation meeting the conditions set out above. Platonists need to provide an example of such an explanation for the explanatory indispensability argument to go through. That is, platonists must provide an explanation, E_1, which fulfils the following conditions: i) E_1 is a good potential explanation of a data set, D; ii) E_1 is the best potential explanation of that data set, D; and iii) E_1 involves indispensable allusion to mathematical objects.

In section 3.1, I will describe an example presented by Baker which is drawn from evolutionary biology, and note that at first glance it seems to meet all three conditions. In section 3.2, I will argue that any attempt by the nominalist to show otherwise must be defended on scientific grounds. In section 3.3, I will examine two attempts to show that alternative explanations are available, arguing that these fail due to lack of specific scientific evidence.

In section 3.4, I will draw on the biological literature to argue that even if Baker's explanation is the only potential explanation of its data set, there is some reason to think that Baker's explanation is not as good an explanation of this data set as it initially seemed. Therefore, I conclude that Baker needs to provide further scientific evidence for his example.

3.1: Periodical cicadas

Baker claims to provide an example of a case where our best scientific explanation of a set of data involves indispensable allusion to mathematical objects. The case he presents is taken from the field of evolutionary biology, and concerns the life cycle of three species of periodical cicadas, a type of insect commonly found in North America and part of the genus *Magicicada*. These cicadas spend most of their life-cycle in the nymphal stage, remaining in the soil, until they emerge after either 13 or 17 years, depending on the region, then mate and die, leaving the next generation of nymphs in the soil to repeat the cycle [Baker 2005: 229]. According to Baker, one question raised by biologists is: 'why are these life cycles prime?' [Baker 2009: 614]. The data to be explained are the prime life-cycle periods of *Magicicada* cicadas, and in particular, the fact that the two life-cycle periods observed in *Magicicada* cicadas are 13 and 17 years, respectively.

According to Baker, a putative explanation of some of these data runs as follows:

1. 'Having a life-cycle period that minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous. (biological law)

2. Prime periods minimize intersection (compared to non-prime periods). (number theoretic theorem)

3. Hence organisms with periodic life cycles[12] are likely to evolve periods that are prime. (‘mixed’ biological/mathematical law)

4. Cicadas in ecosystem-type E are limited by biological constraints to periods from 14 to 18 years. (ecological constraint)

5. Hence cicadas in ecosystem-type E are likely to evolve 17-year periods'.

[Baker 2009: 614].

I will refer to this explanation as Baker's explanation, or the cicadas explanation. The explanation is arrived at through two different routes. Firstly, it is held to be evolutionarily advantageous for periodical cicadas to minimise the frequency of intersection with periodically emerging predators [Baker 2005: 230].

Secondly, it is held to be evolutionarily advantageous for periodical cicadas to minimise the frequency of intersection with other similar subspecies of cicada, in order to avoid hybridization with those subspecies [ibid.: 231]. To see why, note that if cicadas from two

12 Strictly speaking, this should read 'organisms with periodic life cycles greater than a year'.

subspecies with periods of 10 and 15 years were to mate, then their offspring would be likely to have a life-cycle period of 12 or 13 years, with possible variation in life-cycle length amongst the offspring [Baker 2005: 231]. These offspring would have limited mating opportunities compared to their parents, since the offspring nymphs would emerge together with only those offspring nymphs of the same life-cycle length. Hence, the mating of cicadas from different subspecies would not be evolutionarily advantageous.

These two routes lead to the conclusion that it is in general evolutionarily advantageous for cicadas to minimise intersection with other periodically emerging species.

The claim that prime periods minimise intersection follows from a few definitions and results in number theory[13]. I will not consider these in detail, but to get a rough idea of how prime number periods minimise intersection, imagine that a brood of cicadas emerges at the same time[14] as any predators that also emerge periodically. Then, consider subsequent intersections between the two species. If the cicadas had a period of 16 years, the first offspring emergence would coincide with an emergence of predators for any predators with period 1, 2, 4, 8, or 16 years. That is, the cicadas would emerge together with as many as five different species of predator. On the other hand, if the cicadas had a period of 17 years, then the offspring emergence would coincide only with those predators of period 1 or 17 years, i.e. together with two different species of predator, at most. This result is generalised to cover future emergences, and extended to any life-cycle period, *p*, using the definitions and results from number theory.

Now, Baker's explanation for the prime life-cycle lengths of *Magicicada* cicadas is supported by the biological literature on which he draws, as we will now see. For example, Goles et al write that their 'work is based on the hypothesis that the cycle length is a prime number in order to optimally escape predators' [Goles et al: 33].

Cox and Carlton write that 'principles of number theory dictate that the frequency at which the emergence of a life cycle of composite (nonprime) length coincides with the emergences of other cycle lengths will be significantly greater than that of a life cycle of prime length' [Cox & Carlton 1988: 188], while Yoshimura writes 'Because it is a prime number, the 17-yr population is the least likely to emerge with other cycles' [Yoshimura: 115]. Both parties connect these claims to selection for prime number cycles in order to avoid hybridization. So, it seems that various biologists endorse the cicadas explanation.

13 Namely: 'two numbers, *m* and *n*, are coprime if they have no common factors other than 1 … the lowest common multiple of two numbers *m* and *n* is maximal if and only if *m* and *n* are coprime …[which] implies that the intersection frequency of two periods of length *m* and *n* is [minimized] when *m* and *n* are coprime … a number, *m,* is coprime with each number $n < 2m$, $n \neq m$ if and only if *m* is prime' [Baker 2005: 232].

14 This assumption is not often noted, but we will later see that it is important.

In a later paper, Cox and Carlton write that 'While we do not agree with some important assertions made by Yoshimura … we are encouraged by the overall similarity of these two, independently derived explanations of this intriguing biological phenomenon. If independent derivation from similar data can be taken as support for a complex hypothesis, then this general model of periodical cicada evolution is considerably strengthened by Yoshimura's contribution' [Cox and Carlton 1998: 164]. So, if their original endorsement of the explanation was tentative, Cox and Carlton later take the cicadas explanation to be quite convincing.

At first glance, then, it seems that Baker's explanation, E_1, fulfils condition i), which stipulates that 'E_1 is a good potential explanation of a data set'. Additionally, Baker does not mention any alternative explanation in the biological literature, so his explanation also seems to fulfil condition ii), which holds that 'E_1 is the best potential explanation of that data set'. I will argue in the next section that any attempt to deny that Baker's explanation meets these two conditions must be made on scientific grounds.

Now, Baker has not shown that his explanation meets condition iii), which holds that 'E_1 involves indispensable allusion to mathematical objects'. However, the burden of proof here reasonably falls on the nominalist, if she wishes to show otherwise.

Recall that my reading of indispensability from chapter 1 runs as follows: allusion to mathematical objects is indispensable to an explanation, E, if and only if E is scientifically better than any rival explanation without allusion to mathematical objects. So, if Baker's explanation is the best explanation of the data among all potential explanations, then on my reading of indispensability, allusion to mathematical objects is indispensable to the explanation. That is, if Baker's explanation fulfils condition ii), then it fulfils condition iii). If the nominalist wishes to undermine both conditions, she should focus on showing that Baker's explanation does not fulfil condition iii).

This amounts to showing that there is a rival explanation without allusion to mathematical objects that is scientifically as good as E. Since it seems that no rival explanations are presented in the biological literature, the nominalist has a lot of work to do here. Additionally, since the rival explanation provided must be scientifically as good as E, any attempt to deny that Baker's explanation meets condition iii) must also be justified on scientific grounds, as I will now argue.

3.2: Science over philosophy?

In this section, I will argue that it is science rather than philosophy that should determine whether a putative scientific explanation is a good explanation, and whether a rival explanation is scientifically as good or better. By this, I mean simply that a scientific explanation should be justified or criticised on scientific grounds. I am not claiming, for example, that it would be 'comically immodest' for philosophers to criticise science[15].

Similarly, I do not claim that scientists have the last say on explanatory worth. As Baker points out, scientists may be reluctant to change or reformulate explanations and theories proposed for extraneous reasons such as 'institutional inertia, epistemological conservativeness, and the costs of 'retooling'' [Baker 2001: 90]. Moreover, it is in principle possible for a scientific explanation proposed by a philosopher to be accepted in a leading scientific journal, as in the challenge set by Burgess and Rosen [Burgess & Rosen 1997, quoted in Baker & Colyvan 2011: 330].

Rather, all I want to claim is that any grounds given to cast doubt on the claim that a putative scientific explanation is a good scientific explanation should be scientific grounds. Philosophers must be scientifically competent in order to provide such grounds.

Now, the prominent platonists in the indispensability debate agree with all of my claims so far: for example, Baker and Colyvan write that 'We are not suggesting that philosophers should never criticise science or that philosophers should not propose revisions to current science. We do think that before such criticisms and revisions are advanced, philosophers need to be up to speed on the relevant science and have good reasons for the revisions in question' [Baker & Colyvan: 332]. It is thus important to be clear on the dialectic here.

Platonists claim to have provided an example of a good scientific explanation, to be used in support of the explanatory indispensability argument. Nominalists may object to the *use* of this example in the indispensability debate: for example, one such objection holds that the use of the cicadas explanation by platonists is question-begging, as we will see later. In this context, philosophical objections are appropriate. However, if nominalists choose to argue that a given scientific explanation is not a good one, or that an alternative explanation better explains the data, it is reasonable to request scientific grounds for that assertion.

This, I think, is in keeping with the general attitude of current nominalists. For example, as we saw in chapter 1, Leng accepts the view that 'We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe' [Leng 2010: 7]. An attempt to cast doubt on Baker's

15 A charge sometimes levelled at nominalists in a different context: Burgess, following Lewis, writes that 'given the comparative historical records of success and failure of philosophy on the one hand, and of mathematics on the other, to propose philosophical 'corrections' to mathematics is *comically immodest'* [Burgess: 30].

explanation by claiming that its statements are not best confirmed, for example, thus requires use of our ordinary scientific standards.

Now, a great deal of subject-specific knowledge may be necessary to become familiar with these ordinary scientific standards: it is no small challenge for the nominalist to get 'up to speed' on science. As I claimed in chapter 2, some of the criteria determining what makes a good explanation are likely to be quite specific to each field. For example, in section 3.4 we will see that scientists working in evolutionary biology tend to look for support for claims that a posited ecological mechanism explains some set of data by looking for a mathematical model of the mechanism, which predicts those data. So, it will be challenging for a philosopher not familiar with mathematical modelling to examine explanations in evolutionary biology, for example.

In sum, this section has tried to make clear the extent of the task facing the nominalist who tries to show that Baker's explanation does not meet conditions i)-iii). In the next section, I will examine whether nominalists meet this challenge.

3.3: Alternative explanations

I will examine two alternative explanations put forward in the philosophical literature. I will assume that Baker's explanation has already been shown to fulfil condition i), i.e. that it is a good potential explanation.

First, Baker considers a suggestion made to him by Saatsi that a ‘quasi-geometrical’ version of the cicadas explanation not involving the property of primeness may be available [Baker 2009: 616]. Imagine we lay a series of sticks of length 17 end to end, next to further series in turn of sticks of length 14, 15, 16 and 18 (the alternative possible lengths of the life-cycle within the ecological constraint). For each pair of series we keep track of how many sticks we need to lay down before the two series are of the same length, and we will see that pairs involving one series of sticks with length 17 require the most sticks. This shows that a life cycle of length 17 will minimise intersection with nearby periods and hence that the periods of cicadas in ecosystem-type E are likely to be of length 17 [ibid.].

Now, Saatsi's sticks explanation still alludes to mathematical objects, namely, the numbers 14-18, although it does not mention primeness. Baker grants the nominalist that reference to individual natural numbers can reasonably be paraphrased away using first-order logic [Baker 2009: 619], though. So the sticks explanation may be nominalistically acceptable, in that allusion to mathematical objects can be paraphrased away.

If Saatsi's explanation is scientifically as good as Baker's, then Baker's explanation, E_1, does not meet condition ii), which stipulates that E_1 must be the best potential explanation. If Saatsi's explanation is scientifically as good as Baker's, then Baker's explanation also does not meet condition iii), which holds that E_1 involves indispensable allusion to mathematical objects. This is because condition iii) fails if there is a rival explanation without allusion to mathematical objects that is scientifically as good as E_1.

Note that because Saatsi's explanation does not mention primeness, it fails to provide an answer to the question 'Why do *Magicicada* cicadas have prime life-cycle length?', instead answering only the question 'Why is the life-cycle length of *Magicicada* cicadas in ecosystem-type E 17?'. This means that the sticks explanation does not explain all of the data that Baker's explanation does. Furthermore, Baker notes that Saatsi's explanation is not generalisable, since it does not allow for predictions about likely life-cycle periods for cicadas in a different ecosystem, or for other periodic species [Baker 2009: 617]. So, Saatsi's explanation is scientifically less good than Baker's by my definition from chapter 1, where scientific superiority included predictive superiority.

Additionally, it seems quite unlikely that biologists would accept the sticks explanation over Baker's explanation, if they accept the sticks explanation at all: although I do not claim to know the full criteria for what makes a good explanation in evolutionary biology, I think it is plausible to claim that the sticks explanation does not fulfil those conditions[16]. Therefore, since the sticks explanation is scientifically less good, by my definition, and unlikely to be accepted in biological practice – hence also less good by whatever criteria biologists use here – we may now discard the sticks explanation as a viable alternative. So far, then, Baker's explanation still meets criteria ii) and iii).

Consider now a second alternative explanation, proposed by Daly and Langford. These authors suggest that perhaps 'The first question to ask is 'Why, in the case of any particular species of cicada, is their periodic life-cycle of this duration rather than any other? … The answer, supplied by evolutionary theory, will be along the following lines: given that certain relevant creatures also present in the cicada habitat have periodic life-cycle of some other duration, it is advantageous for the cicada life-cycle to be of the particular duration it is, for this minimizes the encounters between the organisms' [Daly & Langford: 656-7]. The idea here seems to be that a detailed ecological account will suffice to explain why the life-cycle period of *Magicicada* cicadas

16 N.B. I don't think Saatsi claims it would; he proposes the sticks explanation because he thinks 'the point is that the explanandum of the biological theory is only that the periods are 13 or 17, not that the period is some *n*, where *n* is prime', and he accepts that 'It's a different question, of course, what scientists write' [personal correspondence from Saatsi to Baker, Baker 2009: 616-7].

in ecosystem-type E is 17 years: for example, by noting that there are predators in ecosystem-type E with a period of 4 years, and perhaps another subspecies of cicada with a period of 16 years, and hence that a period of 17 years would be advantageous for the *Magicicada* cicadas.

Note that this ecological explanation, like the sticks explanation, does not explain why the life-cycle period of *Magicicada* cicadas is prime, and does not allow for straightforward predictions about likely life-cycle periods for cicadas in a different ecosystem, or for other periodic species [Baker 2009: 617]. Nevertheless, the ecological explanation is much more promising, because it claims to be drawn from evolutionary theory, and thus has a much higher chance of being accepted by biologists. The problem, of course, is that the explanation is entirely hypothetical. Baker and Colyvan claim that 'It seems unlikely that biologists would be at all impressed with Daly and Langford's proffered alternative' [Baker & Colyvan: 330]; but, in my opinion, Daly and Langford have not yet proffered a real alternative.

If Daly and Langford filled out the details of their alternative ecological explanation, providing biological evidence for the past predators of *Magicicada* cicadas, it is possible that this explanation might be accepted by biologists. Although the ecological explanation only explains life-cycle periods in a given ecosystem, since '*Magicicada* spp. are confined to the deciduous forests of eastern North America' [Cox & Carlson 1988: 184], the explanation could perhaps be expanded to include all ecosystems in that area.

Nevertheless, Daly and Langford have not yet provided a detailed explanation of this kind. Additionally, even if their ecological explanation were held to be a good explanation by biologists, Baker's explanation might yet be preferred, in which case Baker's explanation still meets conditions ii) and iii). So, in the absence of further argument from Daly and Langford, we may drop this alternative explanation from consideration also.

Baker's explanation, then, seems to be safe from alternatives, so far as the philosophical literature goes. Both alternative explanations considered lacked the detailed biological claims that are plausibly required in order to be considered a good explanation in evolutionary biology. In addition, both alternative explanations failed to explain the datum that the life-cycle periods of *Magicicada* cicadas are prime.

However, it could be argued that the nominalist will never succeed in providing a scientifically convincing alternative explanation that does not allude to mathematical objects, for a reason which makes the set-up of the example unfair: the explanandum itself already contains allusion to mathematical objects, namely prime numbers. Bangu claims that this property of the explanandum makes Baker's example question-begging against the nominalist, in the context of the indispensability debate. For an explanation must in general have a true explanandum; but, if we assume that the explanandum of Baker's explanation is true, in order to submit that explanation to IBE, then we have already assumed that the platonist's desired conclusion is correct [Bangu: 17].

I am not sure where the burden of proof lies here: on the platonist, to show that Baker's example is not question-begging – as Baker tries to do [Baker 2009: 619-622] – or on the nominalist, to show that allusion to mathematical objects can be removed from the data, since its presence in the data is backed up by scientific practice. I will not try to settle this issue here. Instead, I will now argue that even if Baker's explanation is the only potential explanation of the prime-numbered periods of *Magicicada* cicadas, there is some reason to think that Baker's explanation is not as good an explanation of the data as it initially seemed. I will back up this claim with reference to the biological literature.

3.4: An appeal to the biological literature

In section 3.1, I noted that Baker's explanation is endorsed in [Cox and Carlton 1988, 1998, Yoshimura 1997]. However, further examination reveals a number of worries with the explanation elsewhere in the biological literature.

For example, Goles et al note that 'a drawback, however, is that there is as yet no evidence for relevant periodic predators of cicadas' [Goles et al: 33], while Webb notes that 'there is little field data to argue [for] the existence of perfect 2 and 3 year cycling cicada predators' [Webb: 389]. Kon discusses a hypothetical periodic predator of cicadas which attacks cicadas only above ground, and claims that 'It is unlikely that such a predator exists and it is unclear that such a predator has existed' [Kon: 856]. So, in the absence of biological evidence for the existence of relevant periodic predators, it seems that cicadas may not need to avoid intersection with periodic predators, in which case the fact that prime numbers minimise intersection is not relevant in explaining why cicada periods are prime.

Now, minimising intersection with other species was also posited to be advantageous in order to avoid hybridization; but, claim Lehmann-Ziebarth et al, 'A difficulty of this explanation is that prime-period phenotypes might in fact be more likely to hybridize; if, for example, 12- and 13-year phenotypes co-occur, they will emerge together at least within 156 years, while 12- and 14-year phenotypes will never emerge together if they initially emerge 1 year apart' [Lehmann-Ziebarth et al: 3202]. Here we see the importance of the implicit assumption noted earlier that *Magicicada* cicadas initially emerge at the same time as their predators or other periodic cicadas.

So far, I have merely pointed out that a number of assumptions involved in Baker's explanation may be problematic. But a further problem is that his explanation is drawn from papers in which the models are described as 'largely verbal' by other biologists [Lehmann-Ziebarth et al: 3202, describing Cox and Carlton 1988, Yoshimura 1997]. Baker's explanation involves a bare framework of mathematics, whereas many other papers provide a much more complex simulation model.

Yoshimura, for example, who was seen to endorse Baker's explanation under the assumption that it is advantageous for cicadas to avoid hybridization, later provides a population model in order to simulate the dynamics of the cicadas case [Yoshimura et al: 290]. The conclusion drawn from the results of the simulation model is that 'the current model shows that various intermediate life cycles extinguish each other, so that only long, prime-numbered cycles are left. This outcome is achieved surprisingly fast with a relatively simple deterministic model … and it is fairly resistant at least to some ranges of initial conditions. Thus the current results support the selective advantage of prime-numbered cycles in the hybridization hypothesis' [ibid.: 293].

This suggests that biologists working in this field tend to look for support for claims that a posited ecological mechanism explains some data set by looking for a mathematical model of the mechanism that is able to predict those data, as I suggested in section 3.2. So, Baker's explanation alone may be too simplistic to warrant full support; further models of the situation are required, it seems, for biologists to fully endorse his explanation.

So far, of course, Baker need not worry. For it seems that Yoshimura et al have provided support for Baker's explanation using a simulation model, and moreover this model certainly involves allusion to mathematical objects and the property of primeness.

However, not all models support Baker's explanation, which indicates a lack of consensus in the biological literature. For example, Kon suggests that the assumptions involved in some models may be faulty, in particular two assumptions that 'the predator dynamics is independent of the cicada dynamics' and that 'periodical cicadas initially emerge when periodically oscillating predators are abundant', that is, together with their periodic predators [Kon: 856]. Kon develops a model that does without those assumptions. As we saw earlier, Kon doubts that there are any relevant periodic predators of cicadas, but he assumes there are, for the sake of argument, and 'derive[s] the conclusion that prime periodicities are not advantageous even under periodic predator pressure' [Kon: 856].

Lehmann-Ziebarth et al also develop one model which fails to indicate selection for prime-numbered cycles: on restricting the possible range of periods from 12-16 years 'to encompass 13 but no other prime number', they found that 'the 13-years phenotype does not persist' [Lehmann-Ziebarth et al: 3206]. A second model is developed which does result in selection for prime cycles, but involving assumptions that are taken to be problematic, and the authors 'do not think that our model gives a plausible explanation for prime-numbered periods, and we could not devise another explanation and model that generated prime numbers' [ibid.: 3210]. The authors conclude that 'our difficulty in deriving ecological scenarios that could lead to prime-numbered periods suggests looking for nonecological explanations' [ibid.]. In particular, they suggest that 'the explanation for prime-numbered periods, rather than just fixed periods, may reside in physiological or genetic mechanisms or constraints' [ibid.: 3200]. Here, it seems, they posit the existence of an entirely different kind of explanation for the prime periods of *Magicicada* cicadas.

Now, the difficulty Lehmann-Ziebarth et al have in deriving appropriate ecological scenarios may be due to their lack of training, compared to Yoshimura, who we just saw had more success[17]. Nevertheless, the point here is that Baker's example may require more detailed biological defence than he at first thought necessary.

In summary, there is some reason to think that Baker's explanation is not as good as it first seemed, because it involves some assumptions that are taken to be problematic by biologists, and because it seems to be in need of support through a more complex simulation model. Baker's explanation finds some support through simulation models in [Yoshimura et al], but other biologists disagree with the claim that the evolution of cicadas selects for prime periods, because their simulation models do not support this claim [Kon, Lehmann-Ziebarth et al]. So, there is some evidence for a lack of consensus on this matter in the biological literature, and hence there is some ground to doubt that Baker's explanation has been established as a good explanation, that is to doubt that Baker's explanation meets condition i).

Additionally, in order to meet condition ii), Baker's explanation needs to be established as the *best* explanation of the prime periods of *Magicicada* cicadas. But, although Baker does not mention the existence of alternative explanations in the biological literature, some biologists suggest that other explanations for prime periods may be found by considering physiological or genetic mechanisms [Lehmann-Ziebarth et al]. Supposing that such an alternative explanation is found, it may turn out that Baker's explanation is not the best explanation for the prime periods of *Magicicada* cicadas, even if it is a good explanation, in which case Baker's explanation does not meet condition ii)[18].

17 Thanks to Marcus Giaquinto for this point.

18 Although note that the two explanations might not conflict, in which case IBE does not preclude the possibility that both explanations are true.

Now, I do not claim that Baker's explanation does not meet conditions i) and ii), but only that it has not been conclusively established that Baker's explanation does meet these conditions. Although I noted that it would be hard for nominalists to find alternative explanations in the complex field of evolutionary biology, it may also be quite challenging for Baker to defend his explanation in the face of doubt in the biological literature. So, Baker's explanation may be less troubling for the nominalist than it first seems.

Nevertheless, the platonist can try to provide further examples of explanations meeting conditions i)-iii), and it would not be a very appealing strategy for the nominalist to argue against each of these examples on a case-by-case basis. Additionally, it is plausible that any rival explanations to Baker's explanation may also involve indispensable allusion to mathematical objects. Therefore, I will now move on and look at more general strategies for the nominalist in the next chapter.

Conclusion

In this chapter I have examined whether Baker has provided a convincing example of an explanation, E_1 , meeting the following conditions: i) E_1 is a good potential explanation of a data set, D; ii) E_1 is the best potential explanation of that data set, D; and iii) E_1 involves indispensable allusion to mathematical objects. I presented Baker's cicadas explanation as a contender for E_1. I noted that on my reading of indispensability from chapter 1, a denial of condition iii) amounts to the following claim: there is a rival explanation without allusion to mathematical objects that is scientifically as good as E_1. Conditions ii) and iii) are thus closely linked.

I argued that all three conditions should be justified or denied on scientific rather than philosophical grounds, since the claim that a scientific explanation is a good or bad explanation, or that a scientific explanation is a better or worse explanation than its competitors, can most reasonably be made within science. I then examined two alternative explanations from the philosophical literature and concluded that they failed due to a lack of detailed scientific evidence. It therefore seemed that Baker's explanation was a good contender for the best explanation, and hence that the platonist could draw on Baker's example in defending the explanatory indispensability argument, if Bangu's charge of question-begging could be overcome, or if the burden of proof was placed on the nominalist in response to that charge.

However, I drew on the biological literature to argue that there is some reason to doubt the claim that Baker's explanation meets condition i), and some speculative reason to doubt that it meets condition ii). Hence, Baker needs to provide further defence for his example, but this may be very difficult due to the detailed biological knowledge required. Therefore, I concluded that Baker's example may not be fatal for the nominalist. Nevertheless, since platonists may provide further contenders, I will consider more general strategies for the nominalist in chapter 4.

One last note: I have not said anything about the account of explanation at play in this debate, claiming only that science should decide what makes a putative scientific explanation a good explanation. For my purposes in this chapter, I only needed to consider whether certain explanations were endorsed by scientists; there might not be a general account of acceptability of explanation in science, given the subject-specific criteria I discussed. However, this does not mean that there is no room for a philosophical account of (mathematical) explanation in science. I will say more on this in the next chapter, in which I consider restrictions on IBE proposed in order to undermine the explanatory indispensability argument.

Chapter 4: Restrictions on inference to the best explanation

<u>Introduction</u>

In this chapter, my aim is to find a way for the nominalist to undermine the explanatory indispensability argument while allowing that our best scientific explanations may involve indispensable allusion to mathematical objects. This is because I think it is plausible that some of our best scientific explanations do involve indispensable allusion to mathematical objects. Even without this intuition, I think the nominalist's best strategy is to allow that there may be such explanations, because the alternative is to argue against each of the examples proposed by platonists in turn.

Although I argued in the last chapter that Baker's cicadas example needed further defence, Baker may well be able to provide this defence, and further examples have been provided by platonists. It would be an arduous task, requiring a great deal of scientific knowledge, to argue that none of these cases is an instance of best scientific explanation. In general, I would like to find a way to undermine the explanatory indispensability argument without examining every case of mathematical application in science.

So, suppose there is a case where our best scientific explanation, E_1, of some data set involves indispensable allusion to mathematical objects. Recall that the explanatory indispensability argument uses inference to the best explanation to argue that E_1 is true, and hence, on a face-value reading of truth, that we should believe in the mathematical entities alluded to by E_1. Since I accept a face-value reading of truth, I will focus on undermining the application of IBE in the explanatory indispensability argument. I will examine three restrictions on the use of IBE in this chapter.

In section 4.1, I will examine the indexing account proposed by Melia and defended by Daly and Langford. The indexing account places the following restriction on IBE: *Putative mathematical entities can receive support via IBE only if mathematical entities play an explanatory role in science*. I will argue that this is a reasonable restriction for the nominalist to place on IBE, because this restriction is also accepted by the platonist. However, I will argue that the nominalist has so far failed to show that our scientific theories never assign an explanatory role to mathematical entities, and hence that the restriction imposed by the indexing account does not succeed in allowing the nominalist to undermine the explanatory indispensability argument.

In section 4.2, I will examine a second restriction on IBE suggested by Pincock. I will argue that Pincock's proposed restriction on IBE is reasonable, and well-motivated by scientific practice, but that Pincock has not shown that the support of mathematical claims by IBE is ruled out by his restriction. I will also refute Pincock's claim that the explanatory indispensability argument is circular. Hence, Pincock's arguments do not successfully undermine the explanatory indispensability argument.

Nevertheless, I think that Pincock provides some important insights into the indispensability debate. In section 4.3, I will propose a restriction on IBE which is motivated by scientific practice, and, drawing on Pincock's insights, I will argue that the explanatory indispensability can be successfully undermined.

4.1: The indexing account

First, I suggest that any restriction imposed on IBE by the nominalist should be well-motivated. To illustrate what I mean by this claim, consider briefly Busch's discussion of 'the requirements that scientific realists point out as relevant for establishing existence claims on the background of explanationist arguments. … [One] central requirement is that the entities referred to in our scientific theories, and which we take to be responsible for their success, can be identified across theory change' [Busch 2011b: 309]. This suggests the following restriction on IBE: *A putative entity can receive support via IBE only if (reference to) that entity can be identified across theory change*.

While this restriction might be helpful to the nominalist[19], it is motivated, according to Busch, by the scientific realist's response to a fairly contentious objection to scientific realism, the pessimistic meta-induction [Busch 2011b: 313]. That is, the restriction is motivated by philosophical views quite tangential to the indispensability debate, and in particular quite tangential to the nominalist's view, since as I argued in chapter 2, the nominalist of this debate is not a scientific realist. Therefore, I take it that Busch's restriction is not well-motivated.

A well-motivated restriction should not be overly reliant on a particular philosophical standpoint. In section 4.1.1, I will argue that the restriction provided by the indexing account is well-motivated in that sense, since it is accepted by both sides in the indispensability debate.

19 Actually I am inclined to think it is not, since putative mathematical entities can arguably be identified across theory change: all the platonist needs to do is to find one such example, for which the natural numbers may suffice.

4.1.1: A restriction on IBE

According to the indexing view, mathematical objects, if they exist, only index or pick out physical features of the world and do not add to the explanatory value of our scientific theories. For example, suppose it is a fact that *a* is $^7/_{11}$ metres away from *b*. According to Melia, 'nobody thinks that this fact holds *in virtue of* some three place relation connecting *a*, *b* and the number $^7/_{11}$. Rather, the various numbers are used merely to index different distance relations' [Melia 2000: 473]. So, numbers merely allow us to point to and describe distance relations, without entering into those relations. Similarly, Melia writes that:

> 'it may be the case that the explanation for some physical fact *F* is that a certain path *P* has a certain length. It may be the case that the only or the simplest or most elegant way of picking out this length is to use a real number: the length is √2 as long as some standard metre. Accordingly, when we come to explain *F*, our best theory may offer as an explanation '*F* occurred because *P* is √2 metres long'. But we all recognise that, though the number √2 is cited in our explanation, it is the *length* of *P* that is responsible for *F*, not the fact that the length is picked out by a real number' [Melia 2002: 76].

The idea here seems to be that once we accept that putative mathematical objects index physical features of the world, then we should accept that these physical features are the things doing explanatory work. Hence, according to Melia, the postulation of mathematical objects does not provide a 'genuinely more attractive picture of the world', and therefore we need not believe in the existence of mathematical objects [Melia 2000: 474].

Daly and Langford, who have recently defended the indexing account, describe the indexing strategy as follows:

> '… to establish Platonism it is not enough to show that mathematics is indispensable to best science. What must be shown is that the entities quantified over in mathematics have an indispensable and *genuinely explanatory* role to play in best science. Otherwise it can be argued that mathematics is indispensable merely in *indexing* physical facts, and that the mathematics itself is not explanatory. Melia claims that if the role of mathematics is one of indexing, not explaining, there is no good reason to believe that there are mathematical entities' [Daly & Langford: 642].

This argument can be presented as follows:

i. If mathematics plays only an indexing role in science, and not an explanatory role[20], then there is no good reason to believe in the existence of mathematical entities.

ii. Mathematics plays only an indexing role in science, and not an explanatory role.

iii. Hence, there is no good reason to believe in the existence of mathematical entities.

20 As is common in the literature, I will from now on take 'explanatory role' to mean 'indispensable explanatory role'.

I will call this the indexing argument. According to Daly and Langford, premise ii of this argument may be defended by the nominalist, or attacked by the platonist, as follows:

> 'The anti-Platonist, or nominalist, needs to show that the application of mathematics to the concrete world can reasonably be taken to have only an indexing role. If he can show this, then he is not unreasonable in taking mathematics to lack an explanatory role. … The Platonist, on the other hand, needs to show that the application of mathematics to the concrete world has more than an indexing role. … If the Platonist can show this, then he is not unreasonable in taking mathematics to have an explanatory role' [Daly & Langford: 646].

I will examine whether the burden of proof suggested by Daly and Langford is fairly divided between the nominalist and platonist in the next section. For now, note from this quote that a merely indexing role is taken to be the opposite of an explanatory role: according to Daly and Langford, it is reasonable to think that mathematics has an explanatory role if and only if it has more than an indexing role.

Note also that Daly and Langford speak of mathematics playing an indexing/explanatory role, but tend to focus on the role of mathematical entities. For example, they write that 'The role of mathematical objects is not to *explain* concrete facts … but merely to *index those facts'* [Daly & Langford: 644]. Therefore, premise i of the indexing argument can be interpreted as placing one of two equivalent restrictions on IBE[21]: either 'Putative mathematical entities cannot receive support through IBE if mathematical entities play a merely indexing role in science', or 'Putative mathematical entities can receive support via IBE only if mathematical entities play an explanatory role in science'. I choose the latter formulation. It can be generalised – since the restriction is meant to apply in the case of unobservable theoretical entities as well as mathematical entities – to read: *Putative theoretical entities can receive support via IBE only if such entities play an explanatory role in science*.

Strictly speaking, the talk of putative entities playing an explanatory or indexing role is incoherent from those who do not believe that the entities in question exist, though it is common in the literature. The proposed restriction can instead be rephrased to run as follows: 'The claim that entities of a kind posited by theory T exist can receive support via IBE only if T assigns an explanatory role to entities of that kind'[22]. Similarly, the claim that mathematical entities play an indexing/explanatory role in science can reformulated to read as follows: 'our scientific theories assign an indexing/explanatory role to mathematical entities'.

Nevertheless, we will now see that Baker, one of the most prominent platonists in the

21 Premise i does not explicitly mention IBE, but since IBE is prominent in the background of this debate, I think interpreting premise i as a restriction on IBE is justified.

22 Thanks to Marcus Giaquinto for this formulation.

indispensability debate, accepts the proposed restriction on IBE in its original form: 'Putative theoretical entities can receive support via IBE only if such entities play an explanatory role in science'.

As we saw in chapter 1, the first premise of Baker's version of the explanatory indispensability argument claims that 'We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories' [Baker 2009: 613]. So, Baker condones a focus on entities that play an explanatory role in science. We now need to establish that Baker accepts that only such entities are supported by IBE.

Baker responds to Melia's claim that putative mathematical objects do not add to the explanatory power of a theory by writing that 'What needs to be checked … is that the mathematical component of the explanation is explanatory in its own right, rather than functioning as a descriptive or calculational framework for the overall explanation' [Baker 2005: 234]. Baker goes to some effort to check this in his cicadas example [ibid.: 234-6]. So, Baker seems to have some sympathy for the view that we need not believe in the non-explanatory posits of our explanations.

It might be suggested, however, that Baker adopts this attitude only for the sake of argument with the nominalist. Perhaps Baker himself would be happy to claim that all of the entities posited by our best explanations exist.

This is quite possible, but, I think, made less likely by noting that Baker has a 'hunch' that the platonist could distinguish between mathematical entities and idealised physical objects such as frictionless slopes by arguing that 'reference to idealized concrete objects may provide a descriptive framework for scientific theorizing, but that such reference is not genuinely explanatory' [Baker 2005: 237]. So, Baker seems to envisage cases where he might wish to reject the existence of idealised physical objects on the grounds that these putative objects play no explanatory role. This, I think, is a good reason to suppose that Baker accepts the proposed restriction on IBE.

Finally, Baker does not reject the indexing account by arguing that we should believe in the existence of mathematical entities that play a merely indexing role. For these reasons, I conclude that Baker accepts the proposed restriction on IBE: 'Putative theoretical entities can receive support via IBE only if such entities play an explanatory role in science'.

In the next section, I will examine whether this restriction on IBE allows the nominalist to undermine the explanatory indispensability argument.

4.1.2: (Only) an indexing role?

Recall the indexing argument from the last section:

i. If mathematics plays only an indexing role in science, and not an explanatory role, then there is no good reason to believe in the existence of mathematical entities.

ii. Mathematics plays only an indexing role in science, and not an explanatory role.

iii. Hence, there is no good reason to believe in the existence of mathematical entities.

In the last section, we saw that premise i can be interpreted as a restriction on IBE, which is accepted by the platonist: 'Putative mathematical entities can receive support via IBE only if mathematical entities play an explanatory role in science'. In this section, I will examine whether the nominalist succeeds in establishing premise ii of the indexing argument.

First, I will note that both sides agree that mathematics has an indexing role in certain cases in science, but that there are many cases where it is unclear what could be indexed, so it is by no means straightforward for the nominalist to show that mathematics plays an indexing role in all cases. I will suggest that Melia seems content to verify his indexing view on a case-by-case basis, but since I wish to find a more general strategy for the nominalist, I will consider Daly and Langford's claim that Melia's strategy can be extended to cover all cases. I will consider two ways in which this generalisation might be justified, arguing that the first fails and the second is incomplete. Therefore, I will conclude that the nominalist has not yet succeeded in establishing premise ii.

Daly and Langford claim that 'Both parties agree that mathematics has an indexing role' [Daly & Langford: 646]. It seems fair to say that both parties agree that mathematics has an indexing role in some cases of mathematical application in science. Consider the cases involving distance relations on which Melia draws: it seems reasonable to think that the role of numbers in measuring distances is to index these distances, allowing us to name and compare them. Baker and Colyvan, the most prominent platonists in this debate, agree that 'Were science to do nothing more than make descriptive claims about distances, temperatures, and the like, the indexing account would be on firm ground' [Baker & Colyvan: 325]. So, both parties agree that mathematics has an indexing role in some applications of mathematics in science.

In other cases, however, it seems much less clear that mathematics plays an indexing role, because it is unclear which physical features the mathematics could be indexing. For example, Baker and Colyvan ask what might be indexed by the negative time dimension of the R^3 model of the special relativistic space-time manifold [ibid.]. Similarly, cases involving imaginary numbers may be problematic for the nominalist, since there is no obvious physical feature indexed by imaginary numbers, as there is with distance. This is part of the reason imaginary numbers were long thought difficult to conceive of: even three centuries after terms for imaginary numbers were introduced, Cauchy preferred to lay out a complex analysis without use of the term '√–1', since he took this to be a term that 'we can abandon without regret because one does not know what this pretended sign signifies' [Kline: 155].

Proponents of the indexing view may be able to provide a convincing account of the application of negative and imaginary numbers in science in order to establish that mathematics plays only an indexing role in science, but as Baker and Colyvan note, such an account has not yet been provided [Baker & Colyvan: 325]. Therefore, let us now consider the steps that might be taken in order to provide a complete indexing account.

Melia seems content to verify the indexing view on a case-by-case basis, writing that 'there *may* be applications of mathematics that do result in a genuinely more attractive picture of the world' [Melia 2000: 474] and that 'only by a careful analysis of the uses to which mathematics is put will we be able to judge whether or not the indispensability argument supports Platonism' [Melia 2002: 76]. Note that Melia seems open to the possibility that there may be cases where mathematics plays a genuinely explanatory role in science.

However, as I stated at the beginning of this chapter, my aim is to find a way to undermine the explanatory indispensability argument without examining every case of mathematical application in science. So I will now examine Daly and Langford's bold claim that 'If [Melia's] strategy works against some cases of putative mathematical explanation, it works against all possible putative mathematical explanations' [Daly & Langford: 656]. If this is true, then the difficulty of establishing premise ii, which claims that mathematics plays only an indexing role in science, is significantly reduced.

So far the talk has been of mathematics playing an explanatory role, rather than of mathematical explanations, but I think a mathematical explanation in science[23] is taken to be a scientific explanation where mathematics plays an explanatory role. As I noted earlier, Baker checks that his cicadas example is a genuine case of mathematical explanation by checking 'that the mathematical component of the explanation is explanatory in its own right', so my interpretation seems reasonable [Baker 2005: 234].

Now, the platonist has not yet provided an account of what it means for mathematics to

23 The topic of mathematical explanation within mathematics unfortunately lies beyond the scope of this thesis.

play an explanatory role in science. Baker checks that the mathematical component of the cicadas explanation is explanatory by subjecting the cicadas explanation to some of the existing accounts of scientific explanation, including the deductive-nomological and pragmatic accounts [Baker 2005: 235]. However, it is not clear that these accounts are suitable for that purpose: if mathematical explanation in science is scientific explanation with an extra condition, then it is plausible that the two kinds of explanation may require different accounts. As Baker and Colyvan concede, 'a philosophical account of mathematical explanation is something sorely needed for both philosophy of mathematics and philosophy of science' [Baker & Colyvan: 333].

Nevertheless, the fact that an account of mathematical explanation in science is currently lacking does not help to establish Daly and Langford's claim that the indexing strategy works against all cases of putative mathematical explanation, if it works against any. Let us examine Daly and Langford's claim in context:

> 'We suggest that the nominalist's view should be that there could not be mathematical explanations. … we have tried to show the resilience of Melia's indexing strategy. If his strategy works against some cases of putative mathematical explanation, it works against all possible putative mathematical explanations. Indeed, much of the interest of his strategy lies precisely in its potentially sweeping range of application. Certainly it would be dogmatic simply to deny the possibility of mathematical explanation. But that should not be the nominalist's position. Thanks to Melia's indexing strategy, he has an argument which calls into question the possibility of mathematical explanation' [Daly & Langford: 655-6].

I suggest two ways in which this view could be supported. Firstly, the idea could be that the platonist puts forward explanations E_1, …, E_n as the best available cases of genuine mathematical explanation in science, but lacks a general account of mathematical explanation in science. If Melia's indexing strategy succeeds in showing that the role of mathematics in E_1, …, E_n is merely to index, then the platonist's most promising cases have been undermined, calling the very possibility of mathematical explanation into question.

There are a number of problems with this option. For one, platonists deny that their best cases of putative mathematical explanation have been undermined: Baker and Colyvan write that 'Daly and Langford have given us no reason to doubt that we have mathematical explanations in any of the key examples in the literature' [Baker & Colyvan: 332]. Secondly, even if E_1, …, E_n were successfully undermined, the platonist could simply have been wrong to put forward E_1, …, E_n as the best cases of mathematical explanation in science. This would not automatically call into question the very possibility of mathematical explanation.

Therefore, consider a second option. Perhaps the details of Melia's indexing strategy allow us to see from a small number of cases that mathematics could never play the role it needs to in order to be explanatory. In that case, the platonist's search for an account of mathematical explanation in science is futile.

To examine this reading, we need to look at the details of Melia's indexing strategy. As we saw earlier, Melia seems open to the possibility that there may be cases where mathematics plays a genuinely explanatory role in science, but holds that such cases have not yet been found. So, Melia's strategy does not seem to deny the possibility of mathematical explanation. Nevertheless, Melia does contrast the indexing role of mathematics with the role of unobservable physical entities, drawing on our intuitions to suggest that the two cases are very different. For example, Melia writes that:

> 'Postulating quarks genuinely makes the *world* a simpler place. Under the quark hypothesis, various objects in the particle zoo do exist *in virtue of* the existence, properties and relations of quarks. … the complex objects *owe their existence* to these fundamental objects and their modes of recombination. Accordingly, these principles do genuine physical work in simplifying our account of the world. Not so, the mathematical objects – or, at least, not so according to the standard mathematical platonist. It's wholly implausible to think that the sums that exist do so in virtue of standing in certain relations to abstract objects' [Melia 2000: 474, italics in original].

Here, it seems, we are meant to conclude that mathematical objects are nothing like quarks, and could not possibly play the same role in our scientific theories as quarks do. But this, of course, is not an acceptable intuition to draw on, because quarks are causal entities, while mathematical objects are by assumption acausal: naturally, mathematical objects could not possibly play an explanatory role, if an explanatory role is a causal role. The nominalist cannot assume that an explanatory role must be a causal role without providing further arguments for that claim, since this would beg the question against the platonist, as Baker notes [Baker 2005: 228].

Now, suppose that the fact that on some occasion there was a perfect lunar eclipse of the sun, with just the sun's corona left visible, can be explained by the fact that on that occasion the ratio of the earth-moon distance to the earth-sun distance equals the ratio of the moon's diameter to the sun's[24]. This ratio cannot be replaced with a different numerical specification in the way a distance measurement can, so this ratio might plausibly be counted as having an explanatory role, even though a ratio does not have causal powers. Thus, if a non-causal explanatory role is possible, it seems quite plausible that mathematical objects play such a role. The nominalist needs to refute this possibility if she hopes to show that mathematics can play only an indexing role.

In sum, if proponents of the indexing account choose to argue that mathematics could not possibly play an explanatory role, they need to fill out the content of that claim, indicating the nature of the explanatory role that mathematics apparently cannot play. Daly and Langford need to

24 Thanks to Marcus Giaquinto for this example.

show either that a non-causal explanatory role is impossible, or indicate how a non-causal explanatory role might be possible, but argue that mathematics does not play such a role. Without having completed this task, Daly and Langford cannot claim to have established premise ii of the indexing argument, which holds that 'Mathematics plays only an indexing role in science, and not an explanatory role'.

Before concluding this section, note that Daly and Langford might object to this burden of proof. Recall their views on this matter: 'The anti-Platonist, or nominalist, needs to show that the application of mathematics to the concrete world can reasonably be taken to have only an indexing role. … The Platonist, on the other hand, needs to show that the application of mathematics to the concrete world has more than an indexing role' [Daly & Langford: 646]. Perhaps Daly and Langford will suggest that the details of a non-causal explanatory role are up to the platonist.

This suggestion does not go through, for Daly and Langford are not being asked to show how mathematics might have an explanatory role. Rather, they are asked simply to show that the explanatory role to which their indexing role is directly opposed is not by assumption acausal. It is not up to the platonist to fill in the details of this account.

Additionally, note that Daly and Langford's division of labour seems a little unfair. Why must the platonist show that mathematics *has* more than an indexing role, whereas the nominalist need only show that mathematics *can reasonably be taken to have* only an indexing role? Baker and Colyvan point out that many of the key examples in the debate 'are examples taken directly from science and the explanations in question are genuinely scientifically acceptable explanations' [Baker & Colyvan: 332]. Perhaps the mathematics involved in those explanations can therefore *reasonably be taken to have* an explanatory role.

At the very least, if the platonist is required to provide a convincing account of mathematical explanation in science, then the nominalist is required to provide a convincing account of mathematical indexing. This suggests that the indexing account may not be able to avoid dealing with mathematical application in science on a case-by-case basis; in which case justifying premise ii, which claims that 'Mathematics plays only an indexing role in science, and not an explanatory role', will be an arduous task.

In conclusion, we have seen that the indexing account has not yet succeeded in establishing premise ii of the indexing argument, since there are cases of mathematical application in science where the indexing strategy has not yet been successfully applied, and Daly and Langford's proposed generalisation of the indexing strategy is incomplete. Therefore, there may be cases where mathematics plays a genuinely explanatory role in science.

So, although the indexing account involves a reasonable restriction on IBE – 'The claim that entities of a kind posited by theory T exist can receive support via IBE only if T assigns an explanatory role to entities of that kind' – proponents of the indexing account have not yet shown that this restriction is helpful to the nominalist in undermining instances of IBE used to support the existence of mathematical entities. Hence the explanatory indispensability argument has not yet been refuted.

In the next section, therefore, I will examine an alternative restriction on IBE proposed by Pincock.

4.2: Pincock's restriction on IBE

In this section, I will examine Pincock's arguments against the explanatory indispensability argument. Pincock writes that he is 'sympathetic to the interpretation of pure mathematics known as structuralism' [Pincock: 277]. In evaluating Pincock's arguments, it is therefore important to remember that Pincock is not a nominalist. Indeed, Pincock appears to agree with the platonist on some points, since he believes 'that it is metaphysically necessary that there are infinitely many natural numbers' [ibid.: 214]. Nevertheless, Pincock proposes a restriction on IBE which he takes to undermine the explanatory indispensability argument [ibid.: 214], and additionally Pincock argues that the explanatory indispensability argument is question-begging [ibid.: 211].

I will examine whether Pincock succeeds in undermining the explanatory indispensability argument. In section 4.2.1, I examine the background of Pincock's account and see how it ties in with my reading of the explanatory indispensability argument, noting that Pincock focuses on the truth of mathematical claims rather than the existence of mathematical entities. In section 4.2.2, I will examine Pincock's argument against the explanatory indispensability argument. I will argue that Pincock's proposed restriction on IBE is reasonable, and well-motivated by scientific practice, but that Pincock has not shown that the support of mathematical claims by IBE is ruled out by his restriction. I will also refute Pincock's claim that the explanatory indispensability argument is circular. Hence, I will argue, Pincock's arguments do not successfully undermine the explanatory indispensability argument.

4.2.1: Background

In this section, I will compare Pincock's reading of the explanatory indispensability argument to mine, since it is worthwhile to consider the background of Pincock's account before considering his proposed restriction on IBE.

Recall that my reading of the explanatory indispensability argument from section 2.3.2 runs as follows. Let D be some scientific data we wish to explain. Let E_1, E_2, …, E_n be all of the potential explanations of D. Suppose that the best of these explanations, E_1, involves indispensable allusion to mathematical objects. Then:

a) E_1, E_2, …, E_n are all of the potential explanations we have of D.

b) E_1 is a good potential explanation of D, and it is significantly better than the other explanations among E_2, …, E_n.

c) The truth of E_1 follows from a) and b) by inference to the best explanation, and IBE is an acceptable form of inference.

d) Hence we ought rationally to believe in the truth of E_1.

e) E_1 involves indispensable allusion to mathematical objects.

f) Hence we ought rationally to believe in the existence of mathematical objects.

Pincock's chosen reading of the explanatory indispensability argument, following Baker's, runs as follows:

'1_{ER}. We ought rationally to believe in the truth of any claim that plays an indispensable$_C$ explanatory role in our best scientific theories.

2_{ER}. Mathematical claims play an indispensable$_C$ explanatory role in science[25].

3_{ER}. Hence, we ought rationally to believe in the truth of some mathematical claims.'

[Pincock: 207]

According to Pincock's reading of the explanatory indispensability argument, we ought rationally to believe in the truth of claims that play an indispensable explanatory role. This suggests that Pincock may accept a version of the restriction on IBE discussed in the last section: 'Mathematical claims can receive support via IBE only if mathematical claims play an explanatory role in science'.

25 Strictly speaking, this should read 'in our best scientific theories', as should Baker's formulation.

To make it clear how this fits in with my reading of the explanatory indispensability argument, Pincock's reading can be interpreted as placing a further condition on the explanation, E_1, posited in my reading of the explanatory indispensability argument given just above.

According to my reading, the explanatory indispensability argument uses IBE to claim that a certain explanation, E_1, is true, where: i) E_1 is a good potential explanation of a data set, D; ii) E_1 is the best potential explanation of that data set, D; and iii) E_1 involves indispensable allusion to mathematical objects. As we saw in section 4.1, Baker, Daly and Langford hold that we need only believe in the existence of putative entities that play an indispensable explanatory role in E_1. Thus E_1 must also fulfil a further condition: iv) the mathematical entities posited by E_1 must play an indispensable explanatory role. According to Pincock's reading, the further condition should instead read: iv*) the mathematical *claims* in E_1 must play an indispensable explanatory role.

So, both Baker's and Pincock's versions can be interpreted as placing an extra condition on any explanation E_1 that might be put forward in support of my reading of the explanatory indispensability argument. Pincock's reading differs from Baker's in focusing on the role played by mathematical claims in our scientific theories, rather than on the role played by putative mathematical entities.

This is because, in Pincock's opinion, further argument is needed from the platonist 'for the conclusion that the belief in the truth of the relevant mathematical claims rationally supports the belief in the existence of some mathematical objects' [Pincock: 208]. Now, I have simply assumed a face-value, platonist reading of mathematical claims, according to which the truth of mathematical claims entails the existence of abstract mathematical objects; so Pincock's reading differs from mine in this respect. Perhaps Pincock is right to demand further argument from the platonist in defence of a face-value reading of mathematical claims. However, this issue does not need to be settled for my purposes, since I want to find a way to undermine the explanatory indispensability argument, even granting the platonist a face-value reading[26].

Note, though, that Pincock's focus on mathematical claims rather than mathematical entities means that Pincock's notions of indispensability$_C$ and of explanatory role must be defined for claims rather than entities. Pincock adapts Colyvan's definition of indispensability for entities. According to Pincock, 'a claim is indispensable$_C$ to a theory when all competitors that remove that claim do worse by the ordinary scientific standards of theory choice. If we focus only on a single

26 A problem might arise if Pincock's arguments relied an anti-platonist interpretation of mathematical claims, in which case I would not be able to use his arguments to undermine my version of the explanatory indispensability argument – which assumes a platonist interpretation – without further discussion of the relative merits of platonist and anti-platonist interpretations. Fortunately, we will see in the next section that they do not.

explanation and its explanatory power, then it is natural to say that a claim is indispensable$_C$ to that explanation when all competing explanations that lack that claim have a lower degree of explanatory power' [Pincock: 205].

In chapter 1 I also followed Colyvan in formulating my definition of indispensability, according to which 'allusion to mathematical objects is indispensable to a theory, T, iff T is scientifically better than any rival theory without allusion to mathematical objects'. I claimed that the factors to be considered in judging what makes a theory scientifically better than another theory include explanatory power. So the notion of indispensability in Pincock's work is essentially the notion we have been working with already.

Pincock provides a more extensive account of what it takes for mathematical claims to play an explanatory role than Baker and Colyvan provide for mathematical entities. For example, in Baker's cicadas example from chapter 2, mathematical claims play an explanatory role by 'isolating recurring features of a phenomenon', according to Pincock [Pincock: 208-9]. Other ways in which mathematical claims can play an explanatory role are by tracking causes, as in Melia's distance cases, and by 'connecting different phenomena using mathematical analogies' [ibid.]. I will not examine Pincock's account of explanatory role in detail since, according to Pincock, the explanatory indispensability argument fails even though our mathematical claims play an explanatory role. This makes Pincock's approach very appealing for my purposes, since my aim is to undermine the explanatory indispensability argument while allowing for my intuition that mathematics plays an explanatory role in science.

Therefore, let us now examine the way in which Pincock tries to undermine the explanatory indispensability argument.

<u>4.2.2: Pincock's restriction on IBE</u>

Pincock undermines the explanatory indispensability argument by i) proposing a restriction on IBE such that only claims of a certain type can be supported using IBE [Pincock: 214]; and ii) arguing that mathematical claims in science are not of the right type to be supported using IBE [ibid.]. Pincock also claims that iii) his considerations show the explanatory indispensability argument to be question-begging [ibid.: 211].

In order to examine the success of Pincock's argument, we need to answer three questions in response to his three claims: 1) Is Pincock's restriction on IBE a reasonable restriction? 2) Is it true that the support of mathematical claims by IBE is ruled out by Pincock's restriction? and

3) Does Pincock's charge of question-begging go through? I will consider each question in turn as I examine Pincock's argument.

1) Is Pincock's restriction on IBE a reasonable restriction?

Pincock's proposed restriction on IBE runs as follows:

'Sensitivity: A claim that appears in an explanation can receive support via IBE only when the explanatory contribution tells against some relevant alternative epistemic possibilities'
[Pincock: 214].

Pincock does not provide a definition of the phrase 'relevant alternative epistemic possibilities', so to understand this phrase it will be best to look at the examples Pincock takes to support his restriction. Pincock considers the examples of atoms and of electrons; I will focus on the case of electrons, 'thought of as particles with a minimal negative charge' [ibid.: 216]. According to Pincock,

> 'Prior to Thomson's and Millikan's experiments, the value of this charge and the charge to mass ratio were not relevant to explanations that involved the posited electrons. But once an explanation was developed that took advantage of these particular values, our restricted form of IBE found a convincing application and the claim that electrons exist could be accepted.'
>
> [Pincock: 216]

The idea here is that early explanations involving electrons did not specify particular values of the charge to mass ratio of an electron. Thus, these early explanations did not circumscribe the range of possible values of this ratio. Now, a 'range of possible values' does not mean that any value might have turned out to be correct, because the value of the charge to mass ratio of an electron is fixed independently of our observation; it is now known to be – 1.758820150 x 10^{11} C/kg [Mohr et al: 710]. Rather, there is a range of *epistemically* possible values, because scientists working prior to Thomson's and Millikan's experiments did not know the correct value. Early explanations involving electrons did not rule out epistemically possible alternatives for the value of the charge to mass ratio.

Thomson's and Millikan's experiments, on the other hand, allowed a specific value for the charge to mass ratio to be found, and this information could be brought to bear on explanations involving electrons [Pincock: 216]. In this way alternative epistemic possibilities for the charge to mass ratio were ruled out; and it was not until this point, according to Pincock, that the existence of electrons was widely accepted. This claim is backed up by historian of physics Helge Krugh, who writes that 'With Thomson's cathode-ray experiments of 1897 the electron became [for scientists] a material reality, an elementary particle, and the basis of a theory of matter' [Krugh: 203].

So Pincock has provided a good historical reason to think that scientists are wary of accepting claims about new entities, even though they appear in scientific explanations, unless these claims in some way circumscribe the range of possibilities for relevant properties of these entities. This supports Pincock's restriction on IBE in the case of claims about unobservable physical objects, and, since this support is derived from scientific practice, I think we can therefore say that Pincock's restriction is well-motivated[27]. In particular, it is not reliant on a particular philosophical viewpoint. Therefore, I answer question 1) by concluding that Pincock's restriction is indeed a reasonable restriction on IBE, in the case of claims concerning unobservable physical entities in science.

Of course, it does not immediately follow that Pincock's restriction is reasonable in all cases; in the case of claims concerning already established entities, the restriction might be on less firm ground. Nevertheless, in the context of the indispensability debate, I think Pincock's restriction is reasonable. For platonists have drawn an analogy between claims concerning unobservable physical objects and claims concerning mathematical entities, in order to claim that rejecting the application of IBE in one case is inconsistent with accepting it in the other. To claim that there are different conditions on IBE in the two cases would be a risky strategy for the platonist, since it would leave the putative analogy on shaky ground. So, let us grant that Pincock's restriction on IBE is reasonable in both cases, and now consider question 2).

2) Is it true that the support of mathematical claims by IBE is ruled out by Pincock's restriction?

Pincock considers Baker's cicadas example in order to motivate the view that mathematical claims in our scientific explanations do not tell against relevant alternative epistemic possibilities. First, Pincock notes that the cicadas example includes the mathematical claim 'prime periods minimise intersection', but that the weaker mathematical claim 'prime periods of less than 100 years minimise intersection' would be sufficient to show that the periods of *Magicicada* cicadas are likely to be prime [Pincock: 212]. Additionally, according to Pincock, 'reasonable rules for the use of IBE suggest that, other things being equal, the explanation that employs the weaker claim is

27 Pincock seems to take the cases from scientific practice as support for his restriction, and draws initial motivation for the restriction from consideration of mathematical cases [Pincock: 214]. However, in my opinion the restriction is dialectically stronger if it is motivated by scientific practice and then applied to the mathematical case.

superior to the explanation employing the stronger claim' [ibid.]. Therefore, an application of IBE to the cicadas case tells us only to believe in the weaker mathematical claim.

This 'problem of weaker alternatives' is not too troubling for the platonist, I think, on my weak definition of platonism according to which that position holds only that there are at least some mathematical objects. Even if all of the mathematical claims used in our scientific theories were as weak as possible, the indispensability argument would, if successful, justify belief in the existence of many mathematical objects. However, Pincock goes on to argue as follows:

> 'I want to consider the choice faced by an agent who does not yet believe in any of the usual strong axioms for the natural numbers. For that person, it is an open epistemic possibility that there are only finitely many natural numbers. Thus, the choice is between believing that there are numbers *1* through *n* for some *n* or that the natural numbers continue indefinitely. … [The belief that there are infinitely many natural numbers] is not justifiable based on the cicada explanation. The totality of the natural numbers does not manifest itself in this explanation. This explanatory contribution does not tell against any of the relevant epistemic possibilities.'
>
> [Pincock: 214]

So, in the cicadas case Pincock takes two relevant epistemic alternatives to be the possibilities that 'there are only finitely many natural numbers' and that 'there are infinitely many natural numbers'. According to Pincock, the explanatory contribution of mathematics in the cicadas explanation is not 'sensitive' enough to tell between these two possibilities [ibid.]. The claim here is not that IBE tells us to believe only in a weaker mathematical claim, as in the problem of weaker alternatives; rather, the claim is that Pincock's restricted form of IBE does not support belief in any mathematical claim in the cicadas explanation, since the explanatory contribution of mathematics in the cicadas case does not rule out either of the two epistemic possibilities considered.

A possible response from the platonist here is to suggest a third epistemic possibility, namely that 'there are no natural numbers'. If the mathematical claim in the cicadas explanation tells against this possibility – which it seems to – then Pincock's restriction seems to be met and IBE can, after all, be applied to support the truth of mathematical claims. Pincock does not comment on whether the claim that 'there are no natural numbers' is a relevant epistemic possibility, but his choice of agent suggests he would not take this possibility to be relevant, since he stipulates an agent 'who does not yet believe in any of the usual strong axioms for the natural numbers' [Pincock: 214], rather than an agent who does not yet believe in the existence of mathematical objects. This motivates a choice between finitely many and infinitely many numbers rather than a choice between finitely many and no numbers.

Therefore, we need to consider whether it is reasonable to look to the explanatory contribution of mathematics to provide guidance on the choice of strong axioms for natural numbers. It seems quite plausible to say that such guidance is unlikely to be found in the cicadas case. But it also seems quite a stringent condition on relevant epistemic possibilities to claim that these possibilities should include strong claims about the structure of the domain of natural numbers[28]. In the electrons case, Thomson's and Millikan's experiments provided a value for the charge to mass ratio of electrons, and the relevant alternative possibilities were simply taken to be alternative values for this ratio, rather than strong foundational claims about the theory of electrons.

So there seems to be an imbalance in the factors used by Pincock to decide whether mathematical claims meet his restriction on IBE, and whether claims about unobservable physical entities meet the restriction. This strategy makes Pincock's restriction harder to meet for mathematical claims than it is for claims concerning unobservable physical objects in a way that is unfair to the platonist. If mathematical claims do not meet Pincock's restriction simply because it is made more difficult for them to do so, then Pincock cannot reasonably claim to have shown that the support of mathematical claims by IBE is ruled out by his restriction. Therefore the answer to question 2), which asks 'Is it true that the support of mathematical claims by IBE is ruled out by Pincock's restriction?', is negative.

Now, Pincock might respond by pointing out that the charge to mass ratio of an electron is a crucial part of the characterisation of electrons. Similarly, axioms for the domain of natural numbers are a crucial part of the characterisation of that domain. In both cases, therefore, the relevant alternative epistemic possibilities range over characterising properties of the domains in question. So there is no imbalance in the choice of relevant alternative possibilities in the two cases.

However, this response can be undermined by noting that the charge to mass ratio is only one of the important properties of electrons. Other important properties include the fact that electrons have negative charge, and spin ½, for example, and a complete picture of electrons is built up over time by considering all of these properties. Settling on axioms for the domain of natural numbers on the basis of just the cicadas explanation, on the other hand, is a much more demanding criterion. So there is an imbalance in the two cases after all.

In that case, Pincock can concede that we should not expect the explanatory contribution of mathematics in the cicadas case to help us settle on strong axioms for the domain of natural

28 Pincock may be motivated towards this view because he leans towards structuralism about mathematics. This is why it seems dialectically better to motivate Pincock's restriction by attention to the case of unobservable physical objects rather than mathematical entities; the opposite approach leaves Pincock open to the charge that his restriction is motivated to some extent by structuralist sympathies rather than scientific practice.

numbers. Nevertheless, Pincock can argue that in the electrons case a collection of explanations allows scientists to rule out alternative possibilities for each of the crucial properties of electrons, and in this way to approach a full account of electrons. But the same process is unlikely to occur for mathematical claims, because the applications of mathematics in science are unlikely ever to provide reasons to accept or deny strong axioms for mathematics.

The reason for this may be that scientists focus only on the part of a mathematical theory that is useful to them, exploiting only the part they need in order to present their chosen explanation. Therefore the explanatory contribution of mathematics in science is unlikely to provide a fuller picture of the structure in question. As Pincock puts it, 'the mathematical domains in question are highly structured, and there is unlikely to be an explanation exploiting this specific structure in a way that is sensitive enough to tell in favor of the truth of claims that are substantial enough to single out this structure' [Pincock: 216].

This response seems promising. However, it seems to rely on an alternative restriction on IBE, along the following lines:

Claims from theory T that play an explanatory role in science can receive support via IBE only if the explanatory contribution of these claims allows us to build up a fuller picture of theory T.

This restriction needs further justification from Pincock, if he chooses to follow this route; I will not discuss this restriction any further, since I will examine a restriction I take to be more promising in section 4.3.

At this point, let us take stock. I have argued that the support of mathematical claims by IBE is not ruled out by Pincock's restriction, because Pincock's choice of relevant alternatives in the mathematical case is more stringent than in the case concerning unobservable physical entities, which is unfair to the platonist. I have considered two possible responses from Pincock.

Firstly, Pincock might claim that the choice of relevant alternative epistemic possibilities is comparable in the two cases, because in both cases the relevant alternatives range over characterising properties of the domains in question. However, in the electrons case the specification of one or two relevant properties is taken to be sufficient, whereas a specification of strong axioms in the mathematical case is much more demanding. So this response is not convincing.

Secondly, Pincock's point might be that the explanatory contribution of claims about electrons allows us to build up a fuller picture of electrons, possibly across several explanations. On the other hand, the explanatory contribution of mathematical claims is unlikely to lead to a fuller picture of mathematics, since a large portion of the picture of mathematics is not brought to bear in our scientific explanations. This response relies on an alternative condition on IBE, which Pincock may accept, but which needs to be justified. In the absence of further argument from Pincock on this point, I conclude that the answer to question 2) 'Is it true that the support of mathematical claims by IBE is ruled out by Pincock's restriction?' is still negative.

Let us now examine Pincock's suggestion that the explanatory indispensability argument is question-begging.

3) Does Pincock's charge of question-begging go through?

So far, we have seen arguments claiming that the explanatory contribution of mathematics does not help us to decide on strong axioms for natural numbers, for example, and plausibly does not help us to expand the theory of mathematics in general. Hence, according to Pincock, there is good reason to think that 'scientists first justify the relevant mathematics by mathematical means and then use this mathematics to explain scientific phenomena' [Pincock: 216]. This seems a very plausible claim.

But if this claim is true, then the explanatory indispensability argument is question-begging, according to Pincock. Recall that Pincock's reading of the explanatory indispensability argument runs as follows:

'1_{ER}. We ought rationally to believe in the truth of any claim that plays an indispensable$_C$ explanatory role in our best scientific theories.

2_{ER}. Mathematical claims play an indispensable$_C$ explanatory role in science.

3_{ER}. Hence, we ought rationally to believe in the truth of some mathematical claims.'

[Pincock: 207]

According to Pincock, since mathematics is first justified by mathematical means and then used to explain scientific phenomena, there is a 'prior nonempirical source of justification for the mathematical claims that make explanatory contributions' [ibid.: 211]. Pincock continues:

> 'On this view, explanatory contributions can only provide additional boosts in justification for a belief that was already substantially justified. But if prior justification is required for explanatory contributions, then the explanatory indispensability argument is completely undermined. Mathematics is, indeed, indispensable to science, but only because its claims receive substantial support independently of its application in science. So accepting 2_{ER} presupposes that we have accepted 3_{ER}. That is, the argument begs the question at issue

between those who believe in the truth of these mathematical claims and those who do not' [Pincock: 211].

Now, premise 2_{ER} holds that some mathematical claims, say C_1, …, C_n, play an indispensable explanatory role in science. If mathematical claims must be justified by mathematical means prior to being applied in science, then premise 2_{ER} presupposes that claims C_1, …, C_n have already been justified by mathematical means. 3_{ER} claims that 'we ought rationally to believe in the truth of some mathematical claims'. Does accepting 2_{ER} indeed presuppose the acceptance of 3_{ER}, as Pincock claims? Not necessarily; this is only the case if accepting that some claims C_1, …, C_n have been justified by mathematical means entails belief in the truth of C_1, …, C_n. I think that Pincock's argument needs to make this explicit, as follows:

(a) The explanatory indispensability argument assumes that mathematical claims can contribute to scientific explanations.

(b) Mathematical claims can only contribute to scientific explanations if the mathematical claims are independently supported by purely mathematical means.

(c) If mathematical claims are supported by purely mathematical means, we ought rationally to believe in the truth of these claims.

(d) Hence the explanatory indispensability argument assumes that we ought rationally to believe in the truth of some mathematical claims.

(e) Hence the explanatory indispensability argument is circular.

Claim c) in this argument is a point of debate between .platonists and nominalists, with platonists being inclined to accept the claim[29], while nominalists reject it. If the indispensability argument relies on claim c), then it is not only question-begging but also redundant, since, according to claim c), mathematics itself can tell us that mathematical claims are true. In this case, on a face-value reading of mathematical claims, we ought rationally to believe in mathematical objects, without needing to consider cases of mathematical application in science.

However, I think the platonist can run the explanatory indispensability argument without assuming that c) holds, and thus without begging the question against the nominalist. For the platonist can reason as follows:

1. We ought rationally to believe in the truth of claims that play an indispensable explanatory

29 Although as we saw in chapter 1, some mathematical realists do not accept that pure mathematical practice tells us we ought rationally to believe in the truth of our mathematical claims, since according to realists like Colyvan, 'indispensability arguments offer the *only* good reason for that realism' [Colyvan 1998: 39].

role in science.

2. In order for mathematical claims to play an explanatory role in science, these claims must be supported by mathematical means.

3. For the sake of argument, suppose that support by mathematical means involves simply a deduction from mathematical axioms, for example.

4. Then mathematical claims can play an explanatory role in science, so long as these claims can be deduced from the mathematical axioms.

5. Our mathematical claims play an indispensable explanatory role in science.

6. Hence, we ought rationally to believe in the truth of mathematical claims.

This argument maintains a face-value reading of our mathematical claims – if a mathematical claim is true, the objects posited by that claim exist – but the argument reserves judgement on whether the truth of our mathematical claims can be confirmed by mathematical means. Hence the fact that mathematical claims need to be confirmed by mathematical means before being applied in science does not entail that we ought rationally to believe in the truth of some mathematical claims. So this argument does not beg the question against the nominalist. Unfortunately for the nominalist, then, and in answer to question 3), Pincock's claim that the explanatory indispensability argument is question-begging does not go through.

In this section, I have argued that Pincock has not succeeded in undermining the explanatory indispensability argument, since his restriction on IBE, although well-motivated, was not successful in blocking the support of mathematical claims through IBE. Additionally, Pincock fails to show that the explanatory indispensability argument is question-begging. Nevertheless, Pincock has provided an important insight into the indispensability debate, namely his focus on the independent confirmation of theoretical entities. In the next section, I will propose a restriction on IBE which shares this focus.

4.3: A new restriction on IBE

In this section, I argue that a promising restriction on IBE can be found through attention to scientific practice in the case of unobservable physical entities. Although I argued in the last section that Pincock's proposed restriction on IBE is not successful in undermining the explanatory indispensability argument, I think Pincock is right to look at conditions on IBE that fit scientific practice in the case of unobservable physical entities, as I will now argue.

The indispensability argument sets out to convince nominalists that our best science tells us to believe in mathematical entities. This is because many nominalists claim that only science can justify belief in mathematical entities. Leng, for example, writes that 'we ought not to believe in entities beyond those whose existence is confirmed according to our best scientific theories' [Leng 2010: 13]. Some mathematical realists, such as Colyvan, also reject further philosophical arguments for mathematical realism; as we saw earlier, Colyvan claims that 'indispensability arguments offer the *only* good reason for that realism' [Colyvan 1998: 39].

So, the focus in the indispensability debate is on science, and scientific practice, as an epistemic tool in forming our beliefs. In particular, inference to the best explanation is proposed as an epistemically informative rule of inference in science. Any restriction on inference to the best explanation should therefore be motivated by scientific rather than philosophical practice.

As I claimed earlier, it is dialectically better to motivate a restriction on IBE by considering the case of unobservable physical entities, and then arguing that mathematical claims do not fulfil this restriction, rather than thinking of a restriction on IBE that is not met by mathematical claims, and then showing that claims about unobservable physical entities do fulfil this restriction. This is because the latter strategy is more likely to be based on a particular philosophical viewpoint, which is to be avoided.

Therefore, the search for a restriction on IBE should examine the conditions that scientists impose on their best explanations involving unobservable physical entities before believing these explanations to be true. I will now propose a restriction based on consideration of the putative Higgs boson particle.

The Higgs boson is posited as part of the Standard Model in particle physics, which 'describes the fundamental particles from which we, and every visible thing in the universe, are made, and the forces acting between them' [CERN press release]. The existence of the Higgs boson would help to explain how some fundamental particles have mass, something currently missing from the Standard Model. According to a physicist working at CERN, the European Organization for Nuclear Research, 'The Higgs particle is the last missing ingredient of the theory of strong, weak and electromagnetic interactions, called the Standard Model' [Antoniadis: 967].

Scientists are very keen to locate this missing piece of the Standard Model, as we can gather from the billions spent on the Large Hadron Collider which was built to collect data on high-energy particles such as the Higgs boson. It is claimed that 'The Higgs boson is the one piece of the Standard Model which has not yet found experimental confirmation. The search for the Higgs boson has become one of the highest priorities in today's experimental particle physics' [De Roeck & Polesello: 1078].

The search for this particle takes place through collecting experimental data; this summer scientists at CERN announced the discovery of a new particle in the mass region near 126 GeV, which is 'consistent with the Higgs boson' [CERN press release]. Since further properties of the new particle have not yet been established, it is not yet clear whether this new particle is a Higgs boson as predicted by the Standard Model, or another kind of boson: 'Positive identification of the new particle's characteristics will take considerable time and data' [ibid.].

Now, we have seen that the existence of the Higgs boson would provide the missing link in a model that is, predictively, extremely successful. Additionally, the Higgs boson would provide a good explanation of how some fundamental particles have mass, according to scientists, and this is held to be very important; for example, in the CERN bulletin, the question 'Why have we tried so hard to find the Higgs particle?' is answered as follows: 'Because it could be the answer to the question: how does Nature decide whether or not to assign mass to particles?' [CERN bulletin].

Nevertheless, scientists seek to confirm the existence of this new entity by collecting experimental evidence in order to support the existence of the entity. However good an explanation the Higgs boson can provide, scientists do not take this explanation to be true until experimental confirmation is provided of the existence of this new entity. This suggests the following restriction on IBE:

1. *A claim positing theoretical entities can be supported via IBE only if there is independent experimental confirmation of the existence of such entities*.

For example, the claim that Higgs boson particles explain how some fundamental particles have mass can receive support via IBE only once there is experimental evidence that confirms the existence of such particles. This restriction seems to place a reasonable condition on IBE in any case where a new entity is proposed in science. Inference to the best explanation is thought of as a valuable epistemic tool in science, but to claim that inference to the best explanation can function without experimental confirmation in the case of new theoretical entities goes against scientific practice.

Now, if restriction 1 is applied in the case of mathematical entities, then IBE cannot be used to support the existence of mathematical entities, since there is not commonly thought to be experimental confirmation of the existence of such entities. (If there were such confirmation, the indispensability argument would be redundant). However, the platonist may argue that it is unreasonable to think that scientists ever look for experimental confirmation of the existence of mathematical entities. So, in order for the restriction to be applicable to the case of mathematical entities, the word 'experimental' should be dropped from the condition. The amended restriction then reads as follows:

1*. *A claim positing theoretical entities can be supported via IBE only if there is independent confirmation of the existence of such entities*.

Now, if there is no independent confirmation of the existence of mathematical entities, then IBE is blocked from supporting claims that posit mathematical entities by restriction 1*. In that case the explanatory indispensability argument is undermined.

On the other hand, if the platonist claims that there is independent confirmation of the existence of mathematical entities, the platonist can plausibly apply restriction 1* and conclude that IBE can be used to support claims that posit mathematical entities. The problem for the platonist here lies in motivating the claim that there is independent confirmation of the existence of mathematical entities. Such confirmation must be independent, but the platonist cannot rely on another of his arguments for platonism, in the context of the indispensability debate. The confirmation cannot come from mathematics, because the nominalist disagrees that confirmation within mathematics entails the truth of our mathematical claims, let alone the existence of our mathematical entities. It is difficult to see where the platonist can locate this confirmation without begging the question against the nominalist.

Instead, the platonist may try to reject restriction 1*. The restriction cannot be rejected outright, because it is motivated by scientific practice. However, the platonist might argue that an alternative restriction holds in the case of mathematical entities, as follows:

2. *A claim positing mathematical entities can be supported via IBE only if there is independent confirmation of this claim.*

This restriction can be applied by the platonist in support of the explanatory indispensability argument, since the platonist can argue that confirmation of our mathematical claims comes from mathematics. This does not beg the question against the nominalist, as we saw earlier in responding to Pincock's charge of question-begging, so long as it is not assumed that the mathematical confirmation of a claim entails the truth of that claim.

However, there are two reasons against the platonist adopting restriction 2 to deal with the case of mathematical entities.

Firstly, as I argued earlier, it is a risky strategy for the platonist to claim that there are different conditions on IBE in the two cases of mathematical entities and unobservable physical entities. For platonists have drawn an analogy between claims concerning unobservable physical objects and claims concerning mathematical entities, in order to claim that rejecting the application of IBE in one case is inconsistent with accepting it in the other. But if the platonist motivates different restrictions in the two cases, then the nominalist can accept one of the restricted forms of IBE and reject the other, without inconsistency.

Secondly, the very same claim may posit both mathematical entities and unobservable physical entities. As Leng points out, 'our theory of unobservable physical objects also includes assertions about their relation to mathematical objects' [Leng 2005b: 80]. That is, given some claim, C, about electrons, say, it may not be possible to formulate C without alluding to mathematical objects. In that case C cannot simply be split up into parts which posit mathematical entities and parts which posit electrons. So the platonist cannot apply a different restriction on IBE to each case.

For these reasons, I claim that the platonist cannot appeal to restriction 2 in order to salvage the explanatory indispensability argument.

I will consider one final response from the platonist. The platonist might object that the aim of the explanatory indispensability argument is merely to establish that we ought rationally to believe in the existence of mathematical entities, not to establish that such entities indeed exist. Hence, in the case of unobservable physical entities, IBE need only motivate belief in those entities also.

For example, in the Higgs boson case, IBE need only motivate the claim that belief in the Higgs boson is rational, and need not establish that claims about the Higgs boson are in fact true. If IBE is an inference to rational belief, then restriction 1* looks less reasonable, because it is plausible that many scientists were quite optimistic that the Higgs boson would one day be found – because of its important explanatory contribution – prior to experimental confirmation at CERN.

However, this strategy allows that the conclusion of an inference to the best explanation is not that our best explanation is true, but that our best explanation is rationally believable. In other words, an explanation need not be true to be good. I think this opens the door for nominalists to argue in a similar way, for example, that our best explanation is nominalistically adequate rather than true, since our explanations need not be true to be good [e.g. Leng 2005a and 2005b]. So this response from the platonist is unwise.

In conclusion, we have seen that Pincock was right to worry about independent confirmation, since a successful restriction on IBE holds that 'A claim positing theoretical entities can be supported via IBE only if there is independent confirmation of the existence of such entities'. This restriction is motivated by scientific practice, which means it is well-motivated in the sense I proposed in section 4.1. It is difficult for the platonist to show that the restriction can be met in the case of mathematical entities without begging the question against the nominalist. Additionally, the platonist cannot appeal to an alternative restriction on IBE in the case of mathematical entities without threatening the analogy he has drawn between mathematical entities and unobservable physical entities. Therefore, I can claim to have successfully undermined the explanatory indispensability argument by blocking the instance of IBE involved in that argument.

Conclusion

In this chapter, I have examined a number of restrictions on IBE recently suggested in the indispensability debate. I argued that the indexing account presented by Melia and defended by Daly and Langford suggests a reasonable restriction on IBE, but that proponents of the indexing account have not yet shown that this restriction is helpful to the nominalist in undermining instances of IBE used to support the existence of mathematical entities. Hence the explanatory indispensability argument has not yet been refuted by the indexing account

I then examined a restriction on IBE proposed by Pincock, arguing that this restriction is not successful in blocking the support of mathematical claims through IBE. Additionally, I argued that Pincock's claim that the explanatory indispensability argument is question-begging does not go through.

Finally, I proposed a restriction on IBE motivated by consideration of the case of the Higgs boson particle posited by the Standard Model in particle physics. I argued that this restriction is well-motivated and blocks the support of claims that posit mathematical entities by IBE. Therefore I can claim to have provided a successful way to undermine the explanatory indispensability argument.

At the beginning of this chapter, I wrote that I would like to find a way to undermine the explanatory indispensability argument without examining every case of mathematical application in science. I can claim to have done this by providing a restriction on IBE that blocks the support of mathematical posits via IBE. However, this does not mean the study of cases of mathematical application in science is unimportant. Both sides in the indispensability debate agree that an account of mathematical explanation in science is needed, and it is plausible that such an account may need to proceed on a case-by-case basis, given my argument in chapter 3 that some of the criteria determining what makes a good explanation are likely to be quite domain-specific.

My arguments in this chapter have provided reason to think that the study of cases of unobservable entities is also important in the context of the indispensability debate, since the study of scientific practice in these cases can help us to find restrictions on IBE that are well-motivated, and which can then be applied in either the nominalist's or the platonist's favour, depending on the content of the restriction. This may provide an interesting avenue for future research.

Conclusion

My aim in this thesis has been to find a way to undermine the explanatory indispensability argument, as well as examining some recent developments in the indispensability debate.

In chapter 1, I started out by examining the original indispensability argument, which claims that we ought rationally to believe in the existence of mathematical objects because quantification over mathematical objects is indispensable to our best scientific theories, and we ought rationally to believe our best scientific theories to be true. My aim was to find a strong version of this argument, meaning an argument that is difficult to undermine since it has few and uncontroversial premises.

I argued that a version of the indispensability argument can be given which does not rely on confirmational holism, and then that the strong reading of naturalism which was necessary to get this result is also dispensable. I thus arrived at the explanatory indispensability argument, which I argued was a particularly strong version of the argument in the sense I have just given. Hence I chose this argument as my target for the rest of the thesis. Since this version of the indispensability argument has been widely discussed in the recent literature, this choice allowed me to survey a number of recent attempts to undermine the explanatory indispensability argument in chapter 4.

My first reading of the explanatory indispensability argument involved an implicit endorsement of inference to the best explanation. Since inference to the best explanation is taken to be central to the recent indispensability debate, my focus in chapter 2 was on clarifying the role played by IBE in the explanatory indispensability argument. I provided a basic account of inference to the best explanation and examined the way in which IBE is involved in arguments for scientific realism. This was useful because it allowed me to establish that the talk of scientific realism in the indispensability debate is usually misleading and had better be dropped.

I then provided a reading of the explanatory indispensability argument which involves an instance of inference to the best explanation, and which seems fairly convincing if a suitable example of mathematical explanation can be found in science. In chapter 3, I examined an example provided by Baker, drawn from evolutionary biology and concerning the prime life-cycle periods of *Magicicada* cicadas. I appealed to the biological literature to suggest that Baker's explanation needs further scientific defence.

However, since I think it is plausible that there are genuine examples of mathematical explanation in science, I aimed to find a way of undermining the indispensability argument that did not rely on rejecting examples provided by the platonist on a case-by-case basis. Therefore, in chapter 4 I examined a number of restrictions on IBE recently proposed in order to block the instance of IBE that is involved in the explanatory indispensability argument.

I examined the indexing account defended by Daly and Langford, arguing that the restriction suggested by this account is reasonable, since it is accepted by both sides in the indispensability debate. However, I concluded that the indexing restriction ultimately fails to undermine the explanatory indispensability argument, because the indexing account is incomplete.

Next, I examined a restriction on IBE proposed by Pincock, which I argued is also a reasonable restriction, since it fits with scientific practice in the case of unobservable physical entities. However, I argued this restriction also fails to undermine the explanatory indispensability argument, because it is unsuccessful in blocking the support of mathematical claims through IBE.

Finally, I proposed a restriction on IBE motivated by scientific practice in the case of the Higgs boson particle posited by the Standard Model in particle physics. I argued that it is difficult for the platonist to show that the restriction can be met in the case of mathematical entities without begging the question against the nominalist. Furthermore, I argued, the platonist cannot appeal to an alternative restriction on IBE in the case of mathematical entities without threatening the analogy he has drawn between mathematical entities and unobservable physical entities. Therefore, I can claim to have successfully undermined the explanatory indispensability argument by blocking the instance of IBE involved in that argument.

I can thus claim to have undermined the explanatory indispensability argument without needing to examine every case of mathematical application in science. However, I suggested that a thorough examination of this kind is still important, because an account of mathematical explanation in science may need to proceed on a case-by-case basis. Additionally, I have argued that the study of scientific practice in the case of unobservable physical entities is also important and may provide an interesting new approach to the indispensability debate.

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