

Intelligent Urban Positioning using Multi-Constellation GNSS with 3D Mapping and NLOS Signal Detection

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BIOGRAPHY

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ABSTRACT

Reliable metres-level positioning in dense urban areas is difficult to achieve cost-effectively using a single method. The way forward is to combine multiple positioning techniques. This paper introduces the concept of intelligent urban positioning (IUP), which combines

- Multi-constellation GNSS;
 - Multiple techniques for detecting non-line-of-sight (NLOS) signal propagation; and
 - Multiple techniques using three-dimensional mapping.
- IUP may also be extended to incorporate other position-fixing and dead-reckoning sensors.

The paper begins by explaining the limitations of conventional GNSS positioning in dense urban environments. It then introduces the potential ingredients of intelligent urban positioning, a mixture of new and established techniques, and discusses how they might be combined. 3D mapping may be used for conventional map matching, height aiding, NLOS signal detection, reflection prediction and shadow matching, a new method for determining position by comparing measured and predicted satellite visibility. NLOS reception may also be detected using consistency checking, C/N_0 measurement, dual-polarization antenna technology, an antenna array and a sky-pointing camera.

The results of a preliminary demonstration of the IUP concept using GPS and GLONASS data collected in London are then presented. In this test, conventional GNSS positioning, aided by consistency-based LOS detection is combined with shadow matching. In the example presented, a horizontal position error of less than 2m was obtained, compared to about 25m for conventional GNSS positioning. This clearly demonstrates the potential of the IUP approach. Note, however, that further research is needed to improve the reliability.

1. INTRODUCTION

There are many applications that could benefit from improved urban positioning. These include location-based services (LBS), intelligent transport systems (ITS), augmented reality, vehicle lane control, advanced rail signalling and navigation for the blind. High sensitivity receivers and multiple satellite constellations have vastly improved GNSS signal availability in dense urban areas. However, accuracy remains a problem because buildings block and reflect many of the signals.

Reliable metres-level positioning in dense urban areas is difficult to achieve cost-effectively using a single method. The way forward is to combine multiple positioning techniques. Intelligent urban positioning (IUP) aims to achieve this level of performance by combining three key ingredients:

- Multi-constellation GNSS;
- Detection of non-line-of-sight (NLOS) signal propagation; and
- Three-dimensional mapping.

Making use of the signals from all visible GNSS satellites significantly increases the amount of information available to compute a position solution from. It also provides the flexibility to select which signals to use and which to discard. NLOS signals are received only via reflected surfaces and can contribute large ranging errors. If these signals can be identified and excluded, the accuracy of conventional GNSS positioning may be substantially improved. Therefore, multi-constellation GNSS and effective NLOS detection are both critical components of any initiative to improve GNSS positioning accuracy in challenging urban environments.

The combination of positioning technology, such as GNSS, with conventional mapping is sometimes known as intelligent positioning [1]. Intelligent *urban* positioning uses 3D mapping, which also provides information on the position, size and shape of the surrounding buildings. This can be used to predict which signals are blocked and reflected and where this occurs. It thus forms the third key component of accurate urban positioning.

Section 2 summarises the urban positioning problem, explaining how blockage and reflection of signals by buildings degrades the accuracy of conventional GNSS positioning. Sections 3 to 7 then describe the portfolio of new and existing techniques that may be deployed as part of an IUP system. Section 3 outlines the capabilities of conventional map-aided GNSS positioning. Section 4 reviews existing NLOS detection techniques and then Section 5 describes how NLOS detection may be aided using a 3D city model. Section 6 discusses advanced NLOS multipath mitigation using a 3D city model. Finally, Section 7 describes shadow matching, a new positioning technique, developed at UCL, that pattern matches the observed GNSS signals with those predicted from the 3D model. This is followed by a discussion in

Section 8 of how the different components of IUP may be combined.

Section 9 presents the results of a preliminary demonstration of the IUP concept using GPS and GLONASS data collected in London are presented. In this test, conventional GNSS positioning, aided by consistency-based LOS detection is combined with shadow matching.

This paper focuses on single-epoch positioning using GNSS and mapping. However the IUP concept is readily extendable to continuous positioning and can incorporate additional position-fixing and dead-reckoning sensors [2]. Sections 10 and 11 discuss some of the options. Finally, Section 12 summarises the conclusions and discusses future work.

2. THE URBAN POSITIONING PROBLEM

The urban environment presents two major challenges to GNSS signal reception. Firstly, the buildings and other obstacles, such as buses, block the direct line-of-sight (LOS) to many of the satellites, effectively reducing the number in view. Consequently, a multi-constellation receiver is essential in order to reliably obtain sufficient direct-LOS signals to compute a position solution. At many urban locations, a full global deployment of all four GNSS constellations will be required for a high position-solution availability [3].

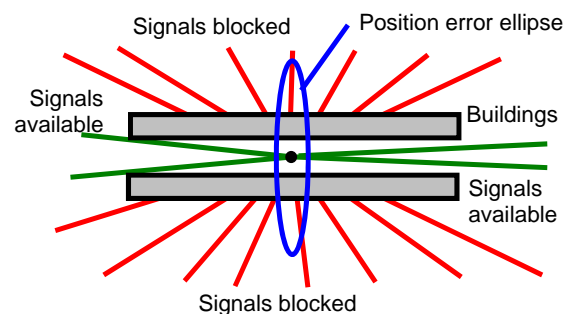
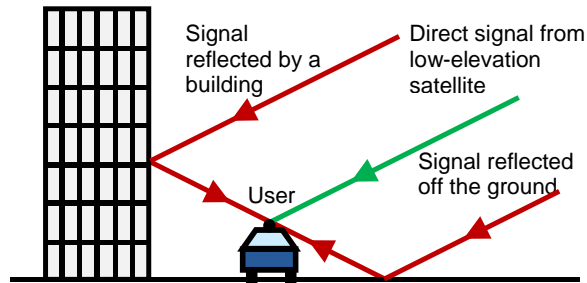


Figure 1: Signal geometry in an urban canyon

Furthermore, because most signals from across the street are blocked by buildings, leaving the along-street signals, the position solution geometry is poor. The result is that the dilution of precision across the street is much larger than along the street, leading to a much lower accuracy in the cross-street direction. Figure 1 illustrates this. By way of example, if satellites are observed at (azimuth, elevation) $(-170^\circ, 75^\circ)$, $(-10^\circ, 30^\circ)$, $(10^\circ, 75^\circ)$, and $(-170^\circ, 30^\circ)$, the north dilution of precision (DOP) is 1.4, but the east DOP is 8.2 [2]. This is a relatively extreme example. However, GNSS availability modelling using a 3D model of London urban canyons has shown that, for pedestrian users of GPS and GLONASS, the across-street DOP will be more than 5 for 22% of the time when a position solution is available while the along-street DOP exceeds 5 for 12% of the time [3].

The second problem is that urban environments contain many flat surfaces that reflect the GNSS signals. Modern glass and metal buildings are particularly strong reflectors, while water enhances the reflectivity of most surfaces. Reception of these reflected signals results in significant positioning errors due to NLOS reception and multipath interference. These are often grouped together as “multipath”. However, they are actually separate phenomena that produce very different ranging errors as Figure 2 illustrates.

Multipath interference



Non-line-of-sight reception

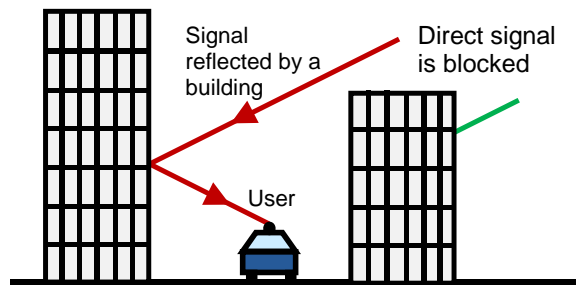


Figure 2: Multipath interference and NLOS reception

NLOS reception occurs where the direct line-of-sight signal is blocked and the signal is received only via reflections. This results in a pseudo-range measurement error equal to the path delay, which is the difference between length of the path taken by the reflected signal and the (blocked) direct path between satellite and receiver. This error is always positive and, although typically tens of metres, is potentially unlimited. Signals received via distant tall buildings can exhibit errors of more than a kilometre. The corresponding carrier-based ranging error is within half a wavelength of the pseudo-range error (noting that phase changes occur on reflection). The strength of NLOS signals varies greatly. They can be very weak, but can also be nearly as strong as the directly received signals. As high-sensitivity receivers can acquire much weaker signals their use can significantly increase the number of NLOS signals received.

Multipath interference occurs where the signal is received through multiple paths between the satellite and user

antenna. Both direct-line-of-sight and NLOS signals may be subject to multipath interference. In the latter case, the signal is received via multiple reflected paths but not directly.

Where multipath interference to directly received signals occurs, the reflected signals distort the code correlation peak within the receiver such that the code phase of the direct LOS signal cannot be accurately determined by equalising the power in the early and late correlation channels. The resulting code tracking error depends on the receiver design as well as the direct and reflected signal strengths, path delay and phase difference, and can be up to half a code chip [2][4]. Code tracking errors are largest where the path delay is about half a code chip (150m for GPS C/A code). Carrier-phase tracking errors are limited to a quarter of a wavelength (assuming the direct LOS signal is stronger than the reflections) and are largest where the path delay is short.

The pseudo-range errors due to multipath interference can be reduced significantly through careful user antenna and receiver design [2], though this does increase the cost, size and power consumption of the user equipment. For dynamic applications, such as navigation, advantage may be taken of the high spatial variation in multipath errors by implementing carrier smoothing to average out most of the code multipath error. Carrier smoothing may be implemented on a signal-by-signal basis using a Hatch filter inputting carrier-phase or Doppler-shift measurements [5]. It is also a standard feature of an extended Kalman filter (EKF)-based navigation solution as the EKF inputs carrier-phase or Doppler-shift measurements as well as the pseudo-ranges [2]. All carrier-smoothing methods are straightforward to implement on any GNSS user equipment without significantly increasing the cost, size or power consumption.

None of the multipath mitigation techniques described above have any significant effect on the errors caused by NLOS signal reception. This is why it is important to treat multipath and NLOS as separate phenomena. Techniques which *do* detect and mitigate NLOS signal reception are reviewed in Sections 4 to 6 of this paper.

Where a signal is partially blocked by an obstacle, diffraction can occur, bending the path of the signal and attenuating it. The attenuation increases with the diffraction angle with useable GNSS signals receivable at deflections of up to 5° [6] [3]. Diffracted signals are also delayed, but typically only by decimeters. They are thus useful for nonprecision positioning and navigation applications. A diffracted signal is normally received instead of the direct signal, but may occasionally be received in addition.

3. CONVENTIONAL MAP-AIDED GNSS POSITIONING

The positioning process may be initialised using a conventional single-epoch least-squares position solution [2] using all available signals, noting that this will often be of relatively poor quality. A chi-square test statistic based on the residuals of the least-squares estimation [7] may be used to estimate uncertainty bounds.

This approximate position solution will occasionally be corrupted by a distantly-reflected NLOS signal with a large path delay. However, as this will be inconsistent with the other signals, the chi-square test statistic will be large. Therefore, if the test statistic exceeds a certain threshold, the signal with the largest residual should be eliminated and a new position solution computed using the remaining signals.

Once the direct-LOS and NLOS signals have been identified (see Sections 4 to 6), a more accurate conventional position solution may be computed from the direct-LOS signals only, if a sufficient number are found. Otherwise the direct-LOS signals may be given a higher weighting than the NLOS signals in the position solution.

The combination of the initial approximate position solution and uncertainty bounds defines a search area for the rest of the intelligent urban positioning process. This also defines the region of the conventional and 3D mapping database to be used. Conventional map matching may then be used to reduce the size of the search area. For road-vehicle applications, the position may be constrained to the roads and parking areas within the search area. For pedestrian applications, the position should be constrained to the outdoor parts of the search area, noting that indoor positioning is outside the scope of this paper. Figure 3 illustrates this.

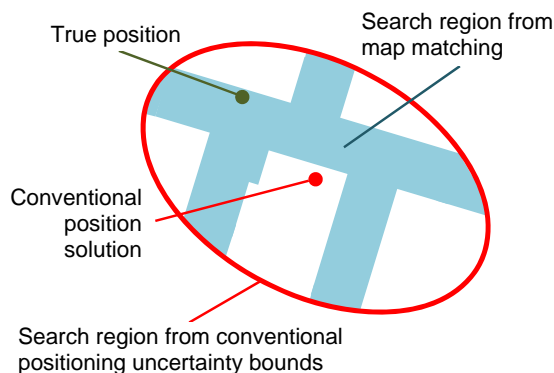


Figure 3: Positioning search area constrained using map matching

Many conventional maps and all 3D maps provide the terrain height. Land vehicle or pedestrian GNSS user equipment may be assumed to be a fixed height above the terrain. Therefore, the approximate GNSS position solution may be used to obtain a height solution from the

mapping data or a separate terrain height database. This may be used as an extra ranging measurement within a GNSS positioning algorithm, a technique known as height aiding [1]. This is particularly useful in cases where there are insufficient direct-LOS signals to determine a position solution without using NLOS signals.

Typically, the height-aiding measurement is treated as a virtual transmitter at the centre of the Earth, the range to which is equal to the (local) Earth radius plus the height. If the terrain within the search area is not flat, the height will vary with the horizontal position. If the terrain height variation is regular, this may be compensated for by adjusting the range and the virtual transmitter position to produce a curved or slanted surface of position.

The height information may also be used to score candidate position solutions computed using different combinations of signals. For each candidate, the horizontal position solution is used to extract a height from the database, which is compared with the height from the candidate position solution. The closer the two heights are, the higher the score awarded to that candidate.

Note that datum and/or geodetic-orthometric conversions may be required to use map height alongside GNSS.

4. NLOS DETECTION METHODS

A number of different methods for distinguishing NLOS from direct-LOS signals have been proposed. This section briefly summarises each approach and discusses its pros and cons. Hybrid approaches are also considered. The dual polarization, sky-pointing camera, and antenna array methods, which require additional hardware, are described in Section 4.1. The elevation, signal-to-noise, and consistency-checking methods, which do not require extra hardware, are then described in Section 4.2. Section 5 then discusses how a 3D model could be used for NLOS detection.

4.1 Methods Requiring Additional Hardware

Direct line-of-sight GNSS signals have right-handed circular polarization (RHCP), whereas most reflected signals have left-handed circular polarization (LHCP). Polarization thus provides a way of distinguishing NLOS from direct-LOS signals.

A dual-polarization antenna is a single antenna whose internal elements are combined in two different ways to produce RHCP-sensitive and LHCP-sensitive outputs. A pair of antennas, one sensitive to each polarization could also be used. NLOS signals may be identified simply by correlating the RHCP and LHCP antenna signals separately within the receiver and determining a separate signal-to-noise ratio (SNR) or carrier-power-to-noise-density ratio (C/N_0) for each polarization. If the LHCP SNR or C/N_0 is the larger of the two, the signal is assumed to be NLOS; otherwise, it is assumed to be direct-LOS

[8]. Figure 4 illustrates this. UCL has successfully demonstrated this method in a London urban canyon [9]. With suitable calibration of the antenna system, the dual-polarization method could also be used to detect severe multipath interference.

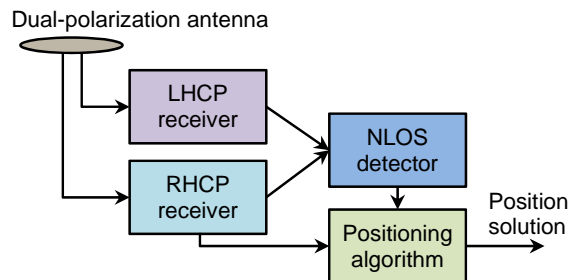


Figure 4: Dual-polarization NLOS detection method

The dual-polarization method will detect most NLOS signals, but not all. A NLOS signal that is reflected twice (or four times) between transmitter and receiver or is reflected with an angle of incidence greater than Brewster's angle will not be LHCP and so will not be detected using polarization. However, this limitation may be mitigated by using the dual-polarization technique as part of a hybrid NLOS detection scheme. By eliminating most of the NLOS signals, the dual-polarization technique makes it much easier for other methods, such as consistency checking and 3D city model techniques, to detect the remaining signals.

A sky-pointing camera with a panoramic lens or an array of cameras can be used to generate an image of the entire field of view above the receiver's masking angle. If the orientation of the camera is known, the blocked and unblocked lines of sight may be determined from the image. By comparing these with the direct lines of sight of the GNSS signals, the NLOS and direct-LOS signals may be distinguished [10][11]. This technique is suited to vehicle applications, such as mobile mapping, where there is space for the camera(s) and the orientation may be determined. However, the need for an accurate attitude and heading solution makes it more difficult to implement on hand-held devices.

A GNSS antenna array may be used to measure the angle of arrival (AOA) of the signals, essentially inverting the well-known interferometric attitude determination [12]. Where the orientation of the antenna is known, NLOS and direct-LOS signals may be distinguished simply by comparing the measuring lines of sight those determined from the satellite ephemeris data. Otherwise, AOA measurements differenced across satellites must be compared with the predictions. If they match, both signals may be assumed to be direct LOS; otherwise, either or both could be NLOS.

The main limitation of all of these techniques is the cost, size, weight, and power consumption of the additional hardware they require, noting that the dual-polarization and antenna-array methods also require additional front-

ends and correlation channels in the GNSS user equipment. Thus, although they are certainly suitable for many professional GNSS applications, they are unlikely to be practical for hand-held mobile devices.

4.2 Methods not Requiring Additional Hardware

Multipath interference and NLOS reception may be mitigated simply by selecting the highest elevation signals on the basis that the higher the elevation angle, the less likely the signal is to be blocked or reflected by a building. However, high-elevation signals can still be NLOS, particularly where a tall building is nearby. Conversely, low elevation signals can be direct-LOS as not all directions are obstructed by buildings in urban areas. Consequently, selecting the highest elevation signals will often result in some of the NLOS signals being accepted and will usually result in many of the direct-LOS signals being rejected. Selecting only high elevation signals also adversely affects the geometry of the solution. Thus, this method can only ever be partially effective.

Signal-to-noise ratio or C/N_0 can also be used as an indicator of NLOS reception. Signals are attenuated on reflection and most antennas are more sensitive to the RHCP direct signals than the mostly LHCP NLOS signals. However, although reflected signals are *on average* weaker than direct signals, it cannot be simply assumed that the strongest signals are direct LOS and the weakest ones are NLOS. Direct signals may be attenuated due to foliage, while diffracted signals, which are useful for nonprecision positioning are also typically weaker. Signals reflected from glass, metal, and wet surfaces can be almost as strong as direct signals. Furthermore, most antennas are less sensitive to polarization from low-elevation signals (assuming a level antenna). Mobile phone antennas are linearly polarized, so their gain is the same for LHCP and RHCP signals, but varies with direction.

A low C/N_0 or SNR is thus an indicator that a signal is more likely to be NLOS, but not that it is definitely NLOS. Selecting the strongest signals for use in the position solution will therefore eliminate most, but not necessarily all NLOS signals and may also eliminate some diffracted and direct-LOS signals.

A hybrid approach makes an initial signal selection based on elevation and SNR. It then compares the resulting height solution with the height obtained from a map database to verify the signal selection [13].

Consistency checking operates on the principle that NLOS measurements produce a less consistent navigation solution than direct-LOS measurements. Furthermore, multipath-contaminated direct-LOS measurements produce a less consistent navigation solution than multipath-free direct-LOS measurements. Therefore, if position solutions are computed using combinations of

signals from different satellites, those obtained using only the multipath-free direct-LOS signals should be in greater agreement than those that include multipath-contaminated and NLOS measurements. The same principle is used for fault detection in receiver autonomous integrity monitoring (RAIM).

UCL has implemented a basic recursive-elimination consistency-checking algorithm [14]. This “top down” method computes a conventional single-epoch least-squares position solution and then a chi-square test statistic based on the residuals [7]. If the test statistic exceeds the detection threshold, the solution is deemed inconsistent and the signal with the largest measurement residual is eliminated. The process continues iteratively until either the test statistic falls within the detection threshold or the number of measurements remaining is the minimum necessary to compute a navigation solution.

Testing using GPS and GLONASS signals has shown that consistency checking reduces position errors under moderate multipath conditions, but that performance is unreliable in dense urban environments with large numbers of reflected signals. This is partly because multiple signals reflected off the same surface will be consistent with each other with the result that consistency checking can sometimes reject direct-LOS signals and retain NLOS signals. Performance was significantly improved by combining the consistency checking and C/N_0 -based signal selection approaches by weighting measurements in the least-squares position solutions according to their C/N_0 [14].

Severely-multipath-contaminated direct-LOS are rejected alongside the NLOS signals using consistency checking, while short-path-delay NLOS signals can be retained. This is good for optimising the accuracy of conventional GNSS positioning. However for shadow matching (Section 7), it is useful to be able to distinguish NLOS and direct-LOS signals.

Consistency checking may also be performed using a “bottom up” approach whereby multiple position solutions are computed using different combinations of measurements and then scored according to their consistency with the remaining measurements. In UCL’s approach, solutions are computed using the minimum number of satellite signals, which is four plus the number of interconstellation timing biases estimated. Each solution is used to predict the pseudo-ranges not used to form that solution. The solution is then scored based on the difference between each of the measured and predicted pseudo-ranges [15]. Unlike the chi-square test statistic, the scoring scheme does not have to assume a Gaussian error distribution. This provides the flexibility to select a distribution and scoring scheme that accounts for the fact that NLOS reception always produces positive ranging errors.

Having identified the best scoring minimum combination of signals, those remaining signals that are consistent in terms of measured and predicted pseudo-ranges are then added to this selection form the final signal selection. In order to find the optimum signal selection, it is not necessary to compute position solutions from every possible minimum combination of signals. By using RANdom SAMple Consensus (RANSAC), a subset of signal selections may be generated which is sufficiently large to contain one or more subsets of the final signal selection. This is because the final signal selection may be found by adding consistent signals to a subset of those signals. The results are summarised in [15].

Adding map-indicated height as an additional measurement can improve the robustness of consistency checking. This is because the height measurement will be more consistent with the multipath-free direct-LOS measurements, which should make the NLOS and multipath-contaminated measurements easier to detect. Test results are presented in [15].

5. NLOS DETECTION USING A 3D CITY MODEL

Where the user position is known, it is straightforward to compare the direct-LOS signal paths with a 3D city model to determine which signals are blocked. The NLOS signals are then excluded from the position solution [16]. However, the position will often only be known to within a few tens or hundreds of meters. This will be the case if it has been determined using NLOS-contaminated GNSS pseudo-ranges, phone signals or Wi-Fi. In this case, it is necessary to consider signal blockage at multiple locations, which requires two problems to be solved:

- 1) Calculating the GNSS signal shadowing by the buildings at multiple locations in real time.
- 2) Determining which signals are NLOS when the exact user position is unknown.

The first problem has been solved by UCL using the building boundary method. The azimuth and elevation of the building boundary is pre-computed over a grid of candidate positions using ray tracing [3]. Then, to determine whether a signal is blocked or not, the elevation of the signal is simply compared with that of the building boundary at the corresponding azimuth, enabling large numbers of candidate signal paths to be tested in real time. By adjusting the boundary by a few degrees, diffraction can also be predicted. Figure 5 shows a building boundary.

Without any data compression, about 300 bytes are required to store a building boundary with a 1° resolution. By exploiting the similarities both between neighboring azimuths in the same building boundary and between building boundaries at neighboring grid points, substantial data compression should be achievable; possibly up to a factor of ten [17].

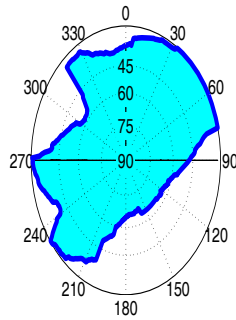


Figure 5: Example building boundary (The shaded area indicates where direct-LOS visibility is predicted).

Assuming a 2×2 meter grid spacing and streets 20 m wide, a standard 4 GB flash drive could store building boundary data for 2,500–25,000 km of road network. For comparison, the Greater London metropolitan area contains about 15,000 km of road. However, building boundary data is only needed for streets where conventional GNSS positioning is poor, maybe 10% of the total. Therefore, it should be practical to preload a mobile device with data for several cities, which could be kept up-to-date via the internet.

An alternative model is to download the building boundary data from a network server as required. A conventional GNSS position solution or Wi-Fi fix should be able to localize position to within 1000 grid points, requiring 30–300 kB of building boundary data to be downloaded in order to perform shadow matching. This takes less than two seconds using a 3G mobile phone connection with an average data rate. The final option is to perform the position calculation in a remote server that stores all of the building boundary data.

Where the exact user position is unknown, the satellite visibility at each candidate position on a search grid may be determined using the building boundary method. Signals may then be placed in three categories:

- Direct LOS available at all candidate positions;
- Direct LOS blocked at all candidate positions;
- Direct LOS available at some positions and blocked at others.

Where there are sufficient direct LOS signals available at all candidate positions, these may be selected for computation of the conventional GNSS position solution. Otherwise, there are three approaches. The first is to make use of shadow matching (described in Section 7) to reduce the number of candidate positions considered. With a smaller set of positions, there should be more direct LOS signals that are predicted to be available at all of those positions. However, this approach is based on the assumption that shadow matching is reliable, but it cannot be used to provide NLOS information for shadow matching itself, thus limiting its reliability because

shadow matching is vulnerable to measured NLOS signals being mistaken for direct.

The second approach is to combine the 3D model with another NLOS detection method, such as consistency checking. In this case, the only signal combinations considered by the consistency checking algorithm would be those which both

- 1) Include all of the signals predicted by the city model to be direct LOS at all candidate positions and
- 2) Exclude all of the signals predicted by the city model to be NLOS or unavailable at all candidate positions.

The final approach is to jointly determine the position solution and distinguish the NLOS and LOS signals using the 3D city model. This is done by searching for a position and signal selection that are mutually consistent. This can work because which signals are direct LOS and which are NLOS varies with position and this variation can be predicted using the city model. Two approaches are outlined below: searching by signal combination and searching by position.

5.1 Searching by Signal Combination

The principle behind searching by signal combination is that the correct combination of signals will lead to the correct position solution and that, at the correct position, these signals will be predicted by the city model to be direct-LOS or diffracted, noting that diffraction leads to relatively small ranging errors.

To search by signal combination, a number of combinations of signals are first proposed. It is not necessary to consider all possible combinations. If the city model is initially used to predict signal availability at all candidate positions then the combinations considered can be limited to those which both

- 1) Include all of signals predicted to be either direct LOS or diffracted at all candidate positions and
- 2) Exclude all of the signals predicted not to be either direct LOS or diffracted at any of the candidate positions.

Furthermore, a RANSAC approach may be adopted, whereby a limited set of signal combinations are initially chosen and tested. Those with the best scores are then perturbed by adding or subtracting signals and tested again. Provided the initial combinations are well chosen, this will lead to the optimal combination without having to test every possibility.

For each proposed signal combination, the following steps are implemented:

- 1) Compute a position solution using that combination of signals.
- 2) Use the 3D city model to predict which signals are available at that location using the building boundary method; this may include those signals predicted to be diffracted.

- 3) Score the proposed combination of signals according to how many are also predicted to be direct-LOS or diffracted from the city model.

Often, there will be multiple high-scoring combinations of signals. For example, subsets of the optimal combination will typically score well. These should be compared with each other to determine the overall signal selection.

A potential problem with this method is that other error sources, such as ionosphere and troposphere propagation delays, multipath interference and poor signal geometry can lead to significant errors in the position solutions with the result that the city model signal availabilities are predicted in the wrong places, resulting in both low scores for correct combinations of signals and high scores for incorrect combinations.

One way of mitigating this is to consider the uncertainty bounds of each computed position solution and use the 3D model to predict signal availability at multiple points within those bounds. The score is then based on the average of the matches at each point. However, if the uncertainty bounds are too big, many different signal combinations will produce similar scores with the result that this NLOS detection method would be of limited use.

It is therefore best to minimise the other sources of positioning error. Ionosphere propagation errors may be reduced using an SBAS-style total electron count (TEC) grid or a dual-frequency receiver, while multipath errors may be reduced through careful receiver design; however, these all increase costs and power consumption.

A full investigation is needed to optimise the design of this 3D city-model-based NLOS detection technique and establish its performance. However, even if this method were only found to be partially effective on its own, it could still be useful as part of a hybrid NLOS detection scheme.

A hybrid scheme for distinguishing NLOS from direct-LOS signals would score combinations of signals based on the following:

- The consistency with the 3D city model predictions of satellite visibility at the position solution.
- The consistency of the pseudo-range measurements with each other and with those not included in the signal combination.
- The measured C/N_0 of the constituent signals.
- The consistency of the height solution with that obtain from a map or database at the same horizontal position.

5.2 Searching by Candidate Position

The other way of jointly determining the position solution and distinguish the NLOS and LOS signals using the 3D city model and the pseudo-range measurements is to search by position. The principle behind this approach is

that if the signals predicted to be visible using the 3D city model at a candidate position are correct, the position solution produced using only those signals will be consistent with that candidate.

To search by candidate position, all candidate position solutions within a grid bounded the uncertainty bounds of the initial position solution (see Section 3) are considered. Map matching may optionally be used to eliminate some of these candidates.

For each candidate position, the following steps are implemented:

- 1) Use the 3D model to predict which signals have direct LOS visibility and which are diffracted.
- 2) Compute a position solution using only those signals, giving lower weighting to the diffracted signals.
- 3) Score the candidate position based on its consistency with the position solution.

One possible scoring scheme is to compute the relative likelihood, Λ , of each candidate based on the position difference and the uncertainty of the position solution. Thus,

$$\Lambda = \exp\left(-[\hat{\mathbf{r}} - \mathbf{r}_c]^T \mathbf{P}^{-1} [\hat{\mathbf{r}} - \mathbf{r}_c]\right),$$

where $\hat{\mathbf{r}}$ is the position solution, \mathbf{r}_c is the candidate position and \mathbf{P} is the error covariance matrix of the position solution, based on the signal geometry and the pseudo-range standard deviation due to error sources other than NLOS reception. Note that the smaller \mathbf{P} is, the more the likelihood will vary between candidate positions.

The positions compared may be either 2D or 3D. In the 3D case, the height of the candidate position is obtained from the 3D mapping or a terrain height database. Thus, the 3D likelihood score implicitly consistency of the position solution with the terrain height map.

This technique may be used to determine position directly as well as for signal selection. The candidate position with the highest likelihood may be taken as the position solution. A better position may be obtained from a weighted average of the highest likelihood positions or by fitting a bi-variate Gaussian distribution to the likelihood surface of the candidate position grid (assuming a 2D search). Note that if this technique is applied in an open environment, where the predicted visibility is the same at each candidate position, taking the peak of the bi-variate Gaussian distribution fitted to the likelihood surface will give the same position solution as a conventional least-squares positioning algorithm.

This method works on the assumption that NLOS reception produces larger position errors than the other error sources as this is the reason for wishing to exclude NLOS pseudo-range measurements from the position solution in the first place. Therefore performance is optimised by minimising the ionosphere propagation and

multipath errors as discussed in Section 5.1 for the searching by signal combination method.

At some candidate positions, it will not be possible to compute a position solution using the predicted signals because one or more of them has not been received. There are a number of possible explanations for this. The candidate position may be wrong; the city model may be out-of-date or simply wrong; or there may be a temporary obstruction, such as a bus or lorry. The way forward is thus to compute a position solution using those predicted signals that are available. The score may also be reduced, particularly where more than one signal is unavailable.

As with the other method, a full investigation is needed to optimise the design and establish its performance. Similarly, it could also form part of a hybrid scheme by also scoring the candidate positions on the consistency of the pseudo-range measurements with each other and the C/N_0 measurements of the selected signals.

6. ADVANCED NLOS AND MULTIPATH MITIGATION USING A 3D CITY MODEL

Where the user position is known, ray tracing using a 3D city model may be used to determine the path delay of the NLOS signals, enabling the pseudo-ranges to be corrected [18][19]. Where the user position is not known to the accuracy required, but is known sufficiently well to reliably identify the reflecting surfaces, the path delay may be modelled as a function of the user position, enabling NLOS signals to be used for position determination [20].

Where the position uncertainty is sufficiently large for there to be multiple candidate reflecting surfaces for each NLOS signal, NLOS correction becomes a much more complex problem

Ray tracing can also be used to identify the path delays of reflected signals that cause multipath interference to the direct signals. The maximum pseudo-range errors resulting from that multipath interference can then be determined from the path delay and a model of the receiver's response, enabling the relevant signals to be excluded from the position solution or downweighted as appropriate.

Correction of multipath errors using a city model is theoretically possible. However, this requires the relative amplitude and phase of the reflected signals as well as the path delay. This requires the model to incorporate surface reflectivity and phase shift data for it to be of sufficient resolution to determine the path delay to centimetric accuracy.

Note that tracing the path of a reflected signal is a much more complex problem than determining whether or not a direct signal is blocked. The building boundary method cannot be used to reduce the real-time processing load.

UCL is therefore prioritising simpler applications of the 3D city models in positioning. However, other research groups are investigating these more advanced approaches.

7. SHADOW MATCHING

Shadow matching is a new positioning technique using GNSS and a 3D city model and is intended for use alongside conventional GNSS positioning. The basic principle is that positioning information may be inferred from which signals are receivable and which are not [17][21][22][23]. Figure 6 illustrates the concept. If a satellite is visible in some parts of the street but not others, the user can be localised to one part of the street or the other depending on whether the signal is received. Repeating the process with another satellite, the position solution is refined further. This approach enables satellite signals that are not receivable to contribute to the position solution!

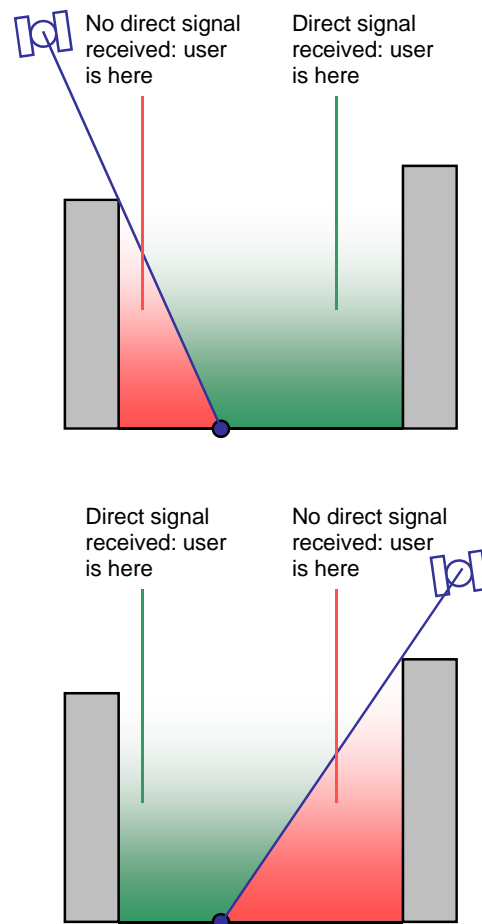


Figure 6: Principle of Shadow Matching.

In UCL's current implementation of shadow matching, the building boundary method (see Section 5) is used to predict satellite visibility over a grid of candidate positions. The grid extends to the uncertainty bounds of the initial position solution. Each candidate position is then scored according to how well the measured and

predicted satellite visibility matches. This is an example of the pattern-matching positioning method, as opposed to the ranging method used in conventional GNSS positioning [24].

A shadow-matching position solution is currently obtained simply by taking the average of the positions of the highest-scoring candidate positions in the search grid. Further research will be conducted to identify a more optimal positioning algorithm.

Experimental testing of a basic algorithm in London using a survey-grade multi-constellation GNSS receiver has demonstrated that shadow matching can reliably determine which side of the street the user is on under conditions when conventional stand-alone GNSS positioning cannot [22][17]. Thus, shadow matching provides an enhancement to conventional positioning in cases where the cross-street accuracy is poor due to the signal geometry.

In this basic algorithm, signals were assumed to be either received via a direct-LOS path or not received at all. However, in practice, NLOS and diffracted signals will also be received. If these are treated as direct-LOS performance will be degraded. The initial shadow matching testing did indeed produce signals that were observed but not predicted by the model at the correct location. In most cases, the shadow matching algorithm gave the correct solution in spite of this. However, to get the best performance out of shadow matching, direct-LOS, NLOS, and diffracted signals should be treated differently.

The NLOS detection techniques described in Sections 4 and 5 are not completely reliable, particularly those that do not require additional hardware. For conventional positioning, signals that may or may not be NLOS should generally be excluded if sufficient direct-LOS signals are available. By contrast, shadow matching makes use of all GNSS signals, including those that are not received at all. However, as a pattern-matching technique, shadow-matching is inherently probabilistic. Consequently, a signal may be treated as having a certain probability of being NLOS and a certain probability of being direct-LOS.

Current shadow-matching research is focusing on incorporating diffraction prediction and C/N_0 measurements into the matching scheme [23].

8. THE INTELLIGENT URBAN POSITION SOLUTION

Sections 3 to 7 of this paper have discussed the ingredients of an intelligent urban positioning system. Realising IUP, requires them to be combined together. There are many different ways of doing this and considerable research will be required to determine which is best. Here, two possibilities are considered: a sequential method and a parallel grid-based search.

A possible sequential approach comprises the following steps:

- 1) Use all signals to compute an approximate position using least-squares.
- 2) Use this position to obtain the height solution from a database.
- 3) Set up a search grid within the 3σ per axis uncertainty bounds of the position solution from step 1.
- 4) Use the 3D city model to identify any signals that are either direct-LOS or diffracted at all points within the search grid from step 3 and any signals that are not direct-LOS or diffracted at any of the points.
- 5) Perform NLOS detection using a mixture of pseudo-range consistency checking, database height consistency checking and C/N_0 measurements, together with the results of step 4.
- 6) Compute a least-squares position solution using only those signals considered very likely to be direct-LOS or diffracted.
- 7) Set up a search grid within the 3σ per axis uncertainty bounds of the position solution from step 6, scoring each point according to a bivariate Gaussian distribution based on the position solution from step 6 and its covariance.
- 8) Use conventional map matching to eliminate points within the search grid that are inside buildings.
- 9) Perform shadow matching at each candidate position within the search grid from step 7, using the NLOS information from step 5 and multiplying the existing score by the shadow-matching score.
- 10) Set the position solution to the weighted mean of the highest scoring candidate positions.

A possible parallel grid-based approach comprises the following steps:

- 1) Use all signals to compute an approximate position using least-squares.
- 2) Set up a search grid within the 3σ per axis uncertainty bounds of the position solution from step 1.
- 3) Use conventional map matching to eliminate points within the search grid that are inside buildings.
- 4) At each point in the search grid, compute a least-squares position solution using only those signals predicted to be direct-LOS or diffracted at the point and score the 3D position as described in Section 5.2 (making use of the terrain height database).
- 5) At each grid point, score the consistency of the pseudo-range measurements used in step 4, possibly using a chi-square test.
- 6) Perform shadow matching at each grid point, using the C/N_0 measurements as an indication of the probability of each received signal being NLOS.
- 7) Combine the scores from steps 4, 5 and 6 to produce an overall score for each grid point.
- 8) Set the position solution to the weighted mean of the highest scoring candidate positions.

Note that steps 4, 5, and 6 each use different information to produce their scores.

9. EXPERIMENTAL DEMONSTRATION

The IUP system demonstrated here is a simplified version of the sequential system proposed in Section 8. It comprises five steps:

- 1) Use RANSAC-based consistency checking [15] to identify NLOS and severely multipath contaminated signals.
- 2) Compute a conventional least-squares GNSS position solution, excluding the signals identified as NLOS or multipath contaminated in step 1.
- 3) Setup a 20m-radius search grid centred at the position solution from step 2 above.
- 4) Perform shadow matching at each grid point as described in [23], producing a position solution that is the average of the highest-scoring grid points.
- 5) Form an IUP position solution by taking the cross-street position from shadow matching (step 4) and the along-street solution from conventional GNSS positioning (step 2).

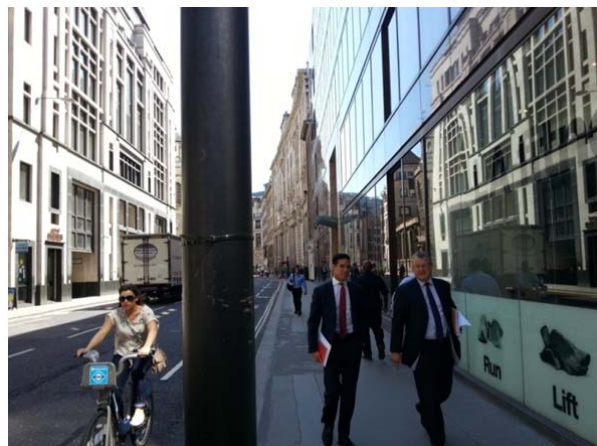


Figure 7: Street views from the test site.

Data was collected using a Leica Viva GS15 survey-grade multi-constellation GNSS receiver across several sites in central London as described in [23]. Site G004 in Leadenhall Street was selected for demonstrating IUP. Figure 7 shows the view from the site in each direction. Ten minutes of data was collected at a rate of 1 Hz. The

3D city model, described further in [23], was obtained from Z Mapping. The truth was obtained by selecting points identifiable on the city model and measuring the test positions from them using a tape measure.

Steps 1 to 4 of the IUP process were run for the whole data set and an epoch selected that clearly demonstrates how each stage of the process contributes to the overall position solution. The final position solution (step 5) was determined manually for this epoch. The intention is to demonstrate the potential of the intelligent urban positioning concept rather than to analyse the performance as the algorithms are still at an early stage of development.

Figure 8 shows two conventional least-squares GNSS position solutions, one using all of the signals received and the other excluding the NLOS and severely multipath contaminated signals identified using RANSAC-based consistency checking. In this case, the position error is almost halved.



Figure 8: Conventional GNSS position solutions before and after application of RANSAC-based consistency checking

Figure 9 shows the (post RANSAC) conventional GNSS position solution, the shadow-matching solution, and the IUP solution obtained by taking cross-street position from shadow matching and the along-street position from the conventional solution. As the figure shows, the IUP solution is substantially more accurate than any of the other solutions. Table 1 gives the position errors of each solution. For this particular epoch, IUP reduced the horizontal position error from about 25m to less than 2m. However, this is a single result and should not be taken as representative of IUP performance in general.

Considering the data set as a whole, shadow matching provided a more accurate cross-street position solution than conventional positioning at most epochs, but not all, while conventional positioning provided a more accurate

along-street position on average, but again, not on every epoch. Similarly, RANSAC-based consistency checking improved the conventional GNSS position solution at some epochs but not others.



Figure 9: Conventional (post RANSAC), shadow matching and IUP position solutions

Positioning Method	Positioning Error (m)		
	North	East	Horizontal
Conventional	25.6	1.8	25.6
Conventional with RANSAC	14.6	1.9	14.7
Shadow matching	1.6	-7.7	7.8
Intelligent urban positioning	1.0	-0.7	1.2

Table 1: Position errors obtained using each method

In summary, the experimental results show that the intelligent urban positioning approach has great potential to provide accurate positioning in dense urban areas, but that further research is needed to achieve this reliably. Given that only a simple version of IUP has been demonstrated, there is a lot of scope for improvement. In particular, the conventional GNSS and shadow-matching position solutions were combined in a somewhat ad-hoc manner and a more rigorous approach is needed to improve reliability.

10. DYNAMIC POSITIONING

For dynamic positioning applications, such as navigation, a continuous position solution is required. Consequently, information from previous epochs will normally be available to assist the computation of the current position solution. This may be used in two ways:

- To initialise the intelligent urban positioning process for the current epoch, defining the search area, and
- Scoring the current candidate positions according to their consistency with the previous position solution on the basis that the distance that can be travelled between epochs is limited.

Prior to using the previous position solution, it must be propagated forward to the current time. Where velocity is estimated, this may be used to predict forward the position, while the position uncertainty must be increased to account for unpredictable user motion between epochs. The process is essentially the same as the system propagation phase of a conventional GNSS extended Kalman filter (EKF) [2].

The position solution predicted forward from the previous epoch will usually be more accurate than the conventional single-epoch least-squares position solution using all available signals discussed in Section 3. It will also usually be more precise, enabling a smaller search area to be defined for the IUP process. With a smaller search area, it is easier to distinguish direct-LOS and NLOS signals using a 3D city model as there are fewer possibilities to consider. Similarly shadow matching is less likely to produce an ambiguous position fix when the search area is smaller.

Defining a search area from the previous position solution implicitly ensures that the new and old position solutions will be consistent to a certain extent. However, within the search area, the position probability distribution as predicted from the previous epoch will vary. The position probability distribution derived by IUP from the current set of GNSS measurements will also vary. The optimal position solution is that obtained by combining the two probability distributions. This may be thought of as reweighting the probability distribution of the new position solution based on its consistency with the old. Equally, it may be treated as a reweighting of the old distribution to make it consistent with the new. Either way, the result is the same. This process is analogous to the measurement update phase of a conventional EKF.

Both of the IUP algorithms proposed in Section 8 may easily be adapted to incorporate the position predicted forward from the previous epoch. Both incorporate a grid of candidate positions that are scored according to various criteria. Therefore, these scores may be multiplied by the probability at each grid point of the position solution predicted forward from the previous epoch. The new scores thus combine current and previous information.

A multi-epoch IUP algorithm must also output positioning information for prediction forward to the next epoch. Expressing this as a mean and covariance enables EKF-based prediction algorithms to be used. However, this can sometimes lead to a poor solution. In the most challenging environments, IUP will often produce ambiguous position solutions, whereby points in several different parts of the search area receive high scores. The highest scoring candidate position will not always be the correct position, while a weighted mean may produce a position in a low-scoring region of the search grid.

Where the user is moving around, the correct position solution will be consistent across epochs whereas the

incorrect position solutions will not be. Therefore, when the position solution is ambiguous, it is best to carry all of the potential candidates forward to the next epoch. One option is to extract a series of positions, each with an associated covariance and weighting from the scoring grid and then propagate each forward to the next epoch using EKF-based prediction. This is analogous to the iterative Gaussian mixture approximation of the posterior (IGMAP) approach to terrain-referenced navigation [25].

A second option is simply to carry the grid of position candidate scores over from one epoch to the next. This preserves all of the information. To propagate the grid forward to the next epoch, predicted user motion may be applied by moving scores from one grid point to another, interpolating as necessary. Unpredicted user motion may be represented by expressing each score as the weighted average of its own score and that of the neighbouring points, effectively blurring the distribution of scores.

11. MULTI-SENSOR POSITIONING

Intelligent urban positioning may easily be extended to incorporate other positioning technologies. Dead-reckoning systems, such as inertial navigation, odometry and pedestrian dead reckoning, measure motion [2]. Therefore they may be used to propagate the IUP position solution from one epoch to the next, limiting the growth in position uncertainty between epochs. As with conventional GNSS integration, the dead-reckoning sensors enable GNSS information to be averaged over a longer time period and provide bridging of the position solution through GNSS outages.

More basically, a single ultra-low-cost accelerometer may be used to detect whether or not a pedestrian has moved between epochs. The additional uncertainty applied to the IUP position solution between epochs can then be adjusted accordingly.

Outdoor positioning using short-range signals of opportunity (SOOP), such as Wi-Fi, is accurate to a few tens of meters. This may well provide a better initialisation of the IUP process, defining a smaller search area than a conventional GNSS position solution contaminated by NLOS errors. Received signal strength pattern matching using FM radio broadcasts [26] and/or mobile phone signals [27] could also be used for initialising IUP.

12. CONCLUSIONS AND FUTURE WORK

In order to achieve more accurate and reliable positioning in dense urban areas, the concept of intelligent urban positioning (IUP) has been introduced. This combines multi-constellation GNSS with multiple techniques for detecting non-line-of-sight (NLOS) signal propagation and multiple techniques using three-dimensional mapping. Several different techniques for detecting NLOS reception have been reviewed and compared. For best performance

without employing additional hardware, multiple NLOS detection techniques should be deployed in parallel. Similarly, 3D mapping may be used in multiple ways, including shadow matching, NLOS detection, height aiding and conventional map matching. These have also been reviewed in the paper.

The results of a preliminary demonstration of the IUP concept using GPS and GLONASS data collected in London have been presented. In this test, conventional GNSS positioning, aided by consistency-based LOS detection was combined with shadow matching. In the example presented, a horizontal position error of less than 2m was obtained, compared to about 25m for conventional GNSS positioning. This clearly demonstrates the potential of the IUP approach. However, further research is needed to improve the reliability.

Much of IUP is still at the conceptual stage, while those aspects that have been tested are still relatively immature. It is likely that some of the ideas presented here will eventually be discarded while new ideas will emerge. Determination of the eventual combination of hardware, mapping and algorithms, together with the tuning of those algorithms will therefore require considerable further research.

Many different factors will require further investigation. These include the building topology and reflectivity; the effect of human-body and vehicle shadowing; the quality of the receiver, antenna and 3D mapping; the available processing power and memory; the number of GNSS signals available. Furthermore, different versions of IUP may well evolve to meet the needs of different applications.

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