<u>**Supplemental material for**</u>

Sensitivity of laboratory based implementations of edge illumination X-ray phase-contrast imaging

P.C. Diemoz, C.K. Hagen, M. Endrizzi, and A. Olivo

Department of Medical Physics and Bioengineering, University College London, WC1E 6BT
London, United Kingdom

In this "supplemental material" section, we discuss the calculation of the angular sensitivity in polychromatic edge illumination (EI), in the general case when diffraction effects cannot be neglected.

It has been shown in previous publications that, with sufficiently large projected source dimensions, diffraction peaks are washed out and the beam distribution at the detector plane only depends on the geometrical parameters of the setup[1]. Therefore, the beam shape is independent from energy, $\rho_{ref}\left(x;E\right)=\rho_{ref}\left(x\right)$, and, as a result, so is the illumination curve, i.e. $C\left(x_e;E\right)=C\left(x_e\right)$. In the main article, we used this property in the derivation of the equation describing the angular sensitivity of a laboratory-based EI setup:

$$\sigma\left(\Delta\theta_{x,eff}\right)\simeq\frac{\sqrt{C\left(x_{e,+}\right)}}{z_{od}\sqrt{2T_{eff}I_0}\left[\rho_{ref}\left(x_{e,+}\right)-\rho_{ref}\left(x_{e,+}+d\right)\right]} \quad (1)$$

where $\Delta\theta_{x,eff}$ and T_{eff} are the effective refraction angle and sample transmission, z_{od} the object-to-detector distance, $x_{e,+}$ the position of the lower edge of the detector aperture and d its width. We now consider the case where both the beam shape and the illumination function depend on energy. To account for this, Eq. (1) needs to be rewritten in a more general form (cf. Eqs. (3) and (4) in main article):

$$\sigma\left(\Delta\theta_{x,eff}\right) \simeq \frac{\sqrt{C_{eff}\left(x_{e,+};\Delta\theta_{x,eff}=0\right)}}{\sqrt{2T_{eff}I_0}\left[\partial C_{eff}/\partial\Delta\theta_{x,eff}\right]\left(x_{e,+};\Delta\theta_{x,eff}=0\right)} \quad (2)$$

where, in the case of negligible beam attenuation from the sample, the effective illumination curve C_{eff} is related to its monochromatic counterparts by the following expression (cf. Eqs. (1) and (2) in main article):

$$C_{eff}\left(x_e - z_{od}\Delta\theta_{x,eff}\right) = \int_{E_{\min}}^{E_{\max}} dEf\left(E\right)C\left(x_e - z_{od}\Delta\theta_x\left(E\right);E\right) \quad (3)$$

The effective illumination function is therefore obtained as a weighted average of the monochromatic components, where the corresponding weighting factor $f\left(E\right)$ includes the effects of the beam spectrum and detector energy response. Note that, if the illumination curve is independent of the energy, Eq. (2) reduces to Eq. (1), since in this case $\left[\partial C_{eff}/\partial\Delta\theta_{x,eff}\right]\left(x_e;\Delta\theta_{x,eff}=0\right) = z_{od}\left(\rho_{ref}\left(x_e\right)-\rho_{ref}\left(x_e+d\right)\right)$. The first derivative of the illumination curve, instead, can be calculated by differentiating Eq. (3) with respect to $\Delta\theta_{x,eff}$ and remembering[2] that $\Delta\theta_x\left(E\right) = \Delta\theta_{x,eff}E_{eff}^2/E^2$, where E_{eff} is the effective energy for the refraction signal:

$$\frac{\partial\left[C_{eff}\left(x_e - z_{od}\Delta\theta_{x,eff}\right)\right]}{\partial\left[\Delta\theta_{x,eff}\right]} = \int_{E_{\min}}^{E_{\max}} dEf\left(E\right)\frac{E_{eff}^2}{E^2}\frac{\partial\left[C\left(x_e - z_{od}\Delta\theta_x\left(E\right);E\right)\right]}{\partial\left[\Delta\theta_x\left(E\right)\right]} \quad (4)$$

In order to obtain the derivative of the effective illumination curve, therefore, the derivatives of the monochromatic illumination curves need to be averaged using a different weighting factor, equal to $E_{eff}^2 / E^2 f(E)$. This result is a direct consequence of the energy dependency of the sample-induced refraction angles, and also bears an additional, more general consequence concerning the EI signal in the polychromatic case. In fact, since the signal is to first approximation proportional to the derivative of the illumination curve, this shows that lower energies give a larger contribution to the total signal. It is important to note that this property also holds for other X-ray phase-contrast imaging techniques, notably free-space propagation and grating interferometry[3].

The obtained formulas lead to the following general expression for the sensitivity in polychromatic EI:

$$\sigma\left(\Delta\theta_{x,eff}\right) \simeq \frac{\sqrt{\int_{E_{\min}}^{E_{\max}} dE f(E) C\left(x_{e,+};E\right)}}{\sqrt{2T_{eff} I_0} \int_{E_{\min}}^{E_{\max}} dE f(E) \frac{E_{eff}^2}{E^2} \frac{\partial\left[C\left(x_{e,+} - z_{od}\Delta\theta_x(E);E\right)\right]}{\partial\left[\Delta\theta_x(E)\right]}} \tag{5}$$

We used Eq. (5) to estimate the variation of the sensitivity with the source dimensions in the polychromatic case, by considering the parameters of our experimental setup. The polychromatic spectrum provided by the Rigaku 007HF, with a Mo target and operated at 35kV/25mA, and the energy response of the ANRAD ‘SMAM’ amorphous selenium flat panel were taken into account to provide the weight function $f(E)$. The source-to-object and object-to-detector distances are 1.6 m and 0.4 m, the pre-sample mask has apertures of 12 µm and period 66.8 µm, the detector mask has apertures of 20 µm. A 50% misalignment between the two masks is considered. The sensitivity curve in the polychromatic case is reported in Fig. 1, together with the curves obtained at 21 keV (refraction effective energy for the system) and calculated with the geometrical optics approximation.

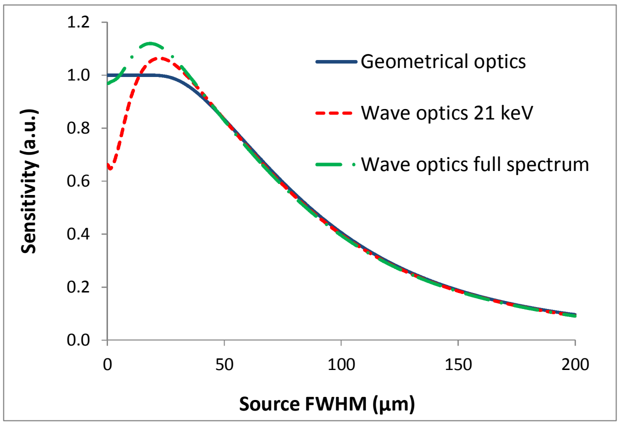

FIG. 1. Variation of the sensitivity as a function of the source dimensions: wave optics calculation with polychromatic spectrum (green dashed-dotted line), wave optics calculation at 21 keV (red dashed line) and geometrical optics approximation (blue continuous line) (see details in text).

As expected, the three curves converge when the projected source becomes sufficiently large, as in this case the beam distribution does not depend on energy and geometrical optics provides a sufficiently accurate approximation. At smaller source sizes, the profiles are different because diffraction effects, which affect the shape of the beam, are highly energy dependent (in particular, the position of the peak is shifted at the various energies).

[1]P. R. T. Munro, K. Ignatyev, R. D. Speller and A. Olivo, Opt. Express **18**, 4103 (2010).

[2]M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light* (Cambridge University Press, Cambridge, 1999).

[3]P. C. Diemoz, A. Bravin, M. Langer and P. Coan, Opt. Express **20**, 27670 (2012).