Collaborative Filtering via Group-Structured Dictionary Learning

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1. Introduction

- Help users in decision making.
- Recommender systems: collaborative filtering (CF) [1]:
  - users' preferences = ratings,
  - estimation based on (i) his/her rating history, (ii) ratings of similar users.
- Novel advances in CF: dictionary learning (latent, unstructured features).
- **Our goal**:
  - structured dictionaries [2] to CF.
  - +requirements:
    - ∗ online learning: changing item-/user set/preferences; adaptation.
    - ∗ incomplete observations: missing rating values.

2. The OSDL Problem

Definition [3]:

- Group structure inducing on the hidden representation $\boldsymbol{\alpha}$ through regularization:

$$\Omega(\boldsymbol{\alpha}) = \|(\|\boldsymbol{\alpha}_G\|_2)_{G\in\mathcal{G}}\|_\eta, \quad \eta \in (0,2). \tag{1}$$

- Approximate on the observed coordinates ($\mathbf{x}_O$) using dictionary $\mathbf{D}$:

$$\frac{1}{2}\|\mathbf{x}_O - \mathbf{D}_O\boldsymbol{\alpha}\|_2^2. \tag{2}$$

- Loss for a fixed observation ($\kappa > 0$):

$$l(\mathbf{x}_O, \mathbf{D}_O) = \min_{\boldsymbol{\alpha}\in\mathcal{A}} \left[\frac{1}{2}\|\mathbf{x}_O - \mathbf{D}_O\boldsymbol{\alpha}\|_2^2 + \kappa\Omega(\boldsymbol{\alpha})\right]. \tag{3}$$

- Goal: minimize online the average loss of the dictionary ($\rho = 0$)

$$\min_{\mathbf{D}\in\mathcal{D}} f_t(\mathbf{D}) := \frac{1}{t}\sum_{i=1}^{t} l(\mathbf{x}_{O_i}, \mathbf{D}_{O_i}). \tag{4}$$

Inclusion of forgetting ($\rho \geq 0$) is possible/motivated:

$$\min_{\mathbf{D}\in\mathcal{D}} f_t(\mathbf{D}) := \frac{1}{\sum_{j=1}^{t}(j/t)^\rho}\sum_{i=1}^{t}\left(\frac{i}{t}\right)^\rho l(\mathbf{x}_{O_i}, \mathbf{D}_{O_i}). \tag{5}$$

Special cases for $\mathcal{G}$:

'Traditional' sparse dictionary	$\mathcal{G} = \{\{1\},\{2\},\ldots,\{d_\alpha\}\}$.
Group Lasso	$\mathcal{G}$ = partition.
Hierarchical dictionary	$\mathcal{G}$ = descendants of the nodes.
Grid adopted dictionary	$\mathcal{G}$ = nearest neighbors of the nodes.

Online optimization of dictionary $\mathbf{D}$ through alternations[1]:

1. Representation update ($\boldsymbol{\alpha}_t$): variational property of $\|\cdot\|_\eta$.
2. Dictionary update ($\mathbf{D}_t$):
   - update statistics of the cost $\hat{f}_t$: matrix recursions.
   - block-coordinate descent optimization.

3. CF Task via OSDL

- t^{th} user's known ratings = OSDL observations $\mathbf{x}_{O_t}$} $\Rightarrow \mathbf{D}$.
- Test user ($\mathbf{x}_O \in \mathbb{R}^{|O|}$):
  1. Estimate $\boldsymbol{\alpha}$: using $\mathbf{x}_O$ and $\mathbf{D}_O$ (rows of $\mathbf{D}$ restricted to O; solve (3)).
  2. Estimate ratings: $\hat{\mathbf{x}} = \mathbf{D}\boldsymbol{\alpha}$.
- Neighbor correction for further improvement:
  - assumption: similar items are rated similarly (s_{ij}).
  - OSDL estimation ($\mathbf{d}_k\boldsymbol{\alpha}_t$, $k \notin O_t$) corrected by the similarity weighted (s_{kj}) prediction errors ($\mathbf{d}_j\boldsymbol{\alpha}_t - x_{jt}$) of the observable items ($j \in O_t$).

4. Numerical Results

- Dataset: joke recommendation (Jester), 100 jokes $\times$ $73,421$ users ($4,136,360$ ratings).
- Performance measure:

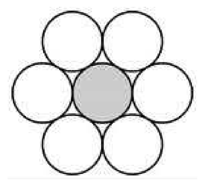
$$RMSE = \sqrt{\frac{1}{|\mathcal{S}|}\sum_{(i,t)\in\mathcal{S}}(x_{it} - \hat{x}_{it})^2}. \tag{6}$$

- Baseline: best known RMSE = 4.1123 (item neighbor), 4.1229 (unstructured dictionary).
- Applied similarities [$s_{ij} = s_{ij}(\mathbf{d}_i, \mathbf{d}_j)$, $\beta > 0$]:

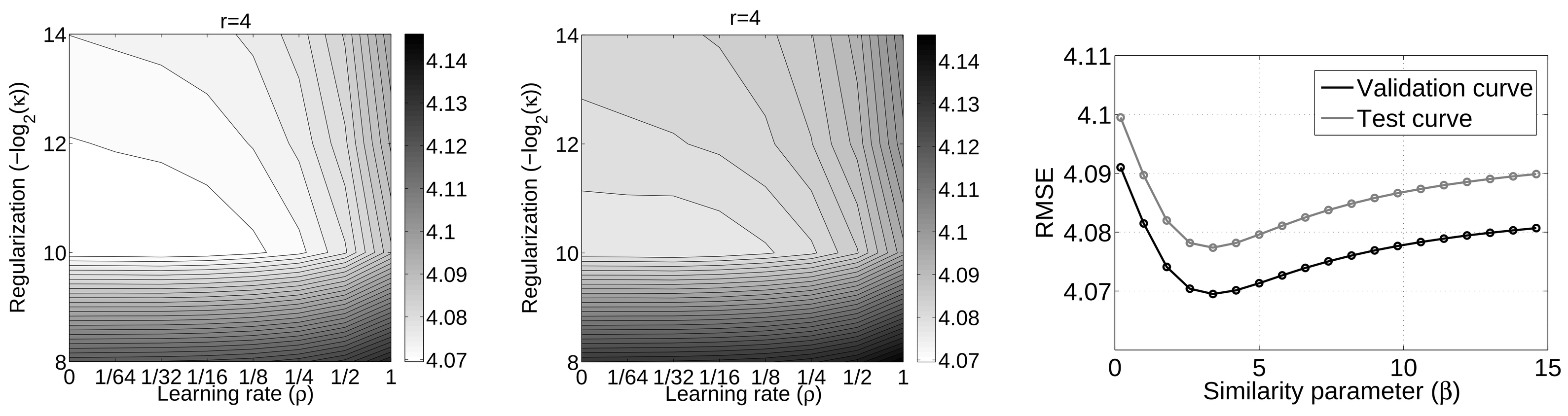
$$S_1: \quad s_{ij} = \left(\frac{\max\left(0, \mathbf{d}_i\mathbf{d}_j^T\right)}{\|\mathbf{d}_i\|_2\|\mathbf{d}_j\|_2}\right)^\beta, \text{ and } S_2: \quad s_{ij} = \left(\frac{\|\mathbf{d}_i - \mathbf{d}_j\|_2^2}{\|\mathbf{d}_i\|_2\|\mathbf{d}_j\|_2}\right)^{-\beta}. \tag{7}$$

$s_{ij} \geq 0$, close to zero (large) = very different (very similar) items.

Toroid group structure: varying neighbor size ($r \in \{0,1,\ldots,5\}$), Fig. 1, Table 2.



- validation, test surfaces: very similar.
- the same holds for similarity parameter (β) dependence.
- structured dictionaries ($r > 0$) are advantageous over unstructured ones ($r = 0$).
- best result ($r = 4$): RMSE $= 4.0774 <$ state-of-the-art (RMSE $= 4.1123$).
- robust estimation w.r.t. forgetting factor (ρ), similarity (S_i) and mini-batch size (R).



(a) (b) (c)

Figure 1: *(a)-(b): validation and test surface – forgetting factor and regularization dependence. (c): validation and test curves – similarity parameter dependence.*

Table 2: *OSDL prediction – performance summary. Group structure ($\mathcal{G}$): toroid.*

	$R=8$					$R=16$				
	$r=0$	$r=1$	$r=2$	$r=3$	$r=4$	$r=0$	$r=1$	$r=2$	$r=3$	$r=4$
S_1	4.1594	4.1326	4.1274	4.0792	**4.0774**	4.1611	4.1321	4.1255	4.0804	**4.0777**
S_2	4.1765	4.1496	4.1374	4.0815	4.0802	4.1797	4.1487	4.1367	4.0826	4.0802

Hierarchical group structure: varying hierarchy level ($l \in \{3,\ldots,6\}$).

- Results: similar to that of the toroid structure.
- Best RMSE = 4.1220 ($l = 4$, i.e., $d_\alpha = 15$).
  - much smaller d_α compared to unstructured dictionaries ($d_\alpha = 100$, RMSE$= 4.1229$).
  - competitive to the state-of-the-art (RMSE$= 4.1123$).

References

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[1]Matlab code available at `http://nipg.inf.elte.hu/szzoli`.